Asset Prices When Investors Ignore Discount Rate Dynamics

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June 2, 2022

Abstract

I propose and test a unifying hypothesis to explain both cross-sectional return anomalies and subjective return expectation errors: some investors ignore discount rate dynamics when forming return expectations. Consistent with the hypothesis: (1) stocks' expected cash flow growth and idiosyncratic volatility explain the significant cross-sectional variation of analysts' return forecast errors; (2) a measure of mispricing at the firm level strongly predicts stock returns, even among stocks in the S&P 500 universe and at long horizons; (3) a tradable mispricing factor explains the CAPM alphas of 12 leading anomalies including investment, profitability, beta, idiosyncratic volatility, and cash flow duration.

[*]The paper is based on the second chapter of my Ph.D. thesis at Columbia Univeristy. I thank Robert Hodrick, Paul Tetlock, Stijn Van Nieuwerburgh, Milan Vidojevic and seminar participants at Barclays, CUHK (Shenzhen) for their helpful suggestions and comments. First draft: 2020-09-21. Please do not circulate without permission. All errors are my own.

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1 Introduction

The asset pricing literature has made progress towards understanding the variations of stock prices (both over time and across firms), and towards people's subjective beliefs about these variations. By examining historical data objectively, the literature shows that a highly dynamic countercyclical aggregate discount rate[1] is potentially behind volatile asset prices (Cochrane, 2011), (Koijen & Van Nieuwerburgh, 2011). In the cross section of stocks, firms with certain characteristics, such as lower idiosyncratic return volatility (Ang, Hodrick, Xing, & Zhang, 2006), asset growth (Cooper, Gulen, & Schill, 2008), cash flow duration (Dechow, Sloan, & Soliman, 2004b), or higher profitability (Novy-Marx, 2013), are associated with higher average returns, which are often termed the "cross-sectional anomalies." Regarding subjective beliefs, looking into direct surveys, researchers find that investors' actual return expectations are inconsistent with the objective distributions of the data. Their expectations can be extrapolative (Greenwood & Shleifer, 2014) over time, and they expect firms with higher idiosyncratic volatility, asset growth, and cash flow duration to deliver higher future returns (Engelberg, McLean, & Pontiff, 2019).

The search for an explanation for all these findings seems challenging, as it must simultaneously explain many anomalies and, at the same time, be consistent with facts about investors' subjective beliefs. What expectation formation process would investors adopt so that they become too optimistic about, say, the returns of stocks with high expected cash flow growth, and at the same time, too pessimistic about stocks with low idiosyncratic volatility? And would this explanation guide us to empirically explain so many cross-sectional anomalies? Indeed, to the best of my knowledge, the literature has not come up with such an explanation.

This paper proposes and tests a new hypothesis that can simultaneously explain both the empirical facts about subjective return expectations and a large set of cross-sectional anomalies. The key idea is simple: some investors ignore the dynamics of discount rates—thus disregarding a key finding in the literature—when forming their return expectations. I term this explanation the "Constant Discount Rate" (CDR) hypothesis. First, I demonstrate theoretically how such a hypothesis would lead to expectation biases and cross-sectional anomalies. Second, and more importantly, I find empirically that the predictions of the hypothesis are consistent with data on subjective return expectations and that a tradable factor constructed based on the hypothesis can explain the CAPM alphas of a large set of cross-sectional anomalies.

[1]The discount rate variation can deviate significantly from that of the underlying economic fundamentals.

The motivation for the CDR hypothesis is twofold. First, it is motivated by a large literature on heuristics, which refers to how people choose solutions that are practicable or satisfactory, rather than optimal, when facing complex problems. The literature starts from the seminal work of Simon (1956) and suggests that people may trade off accuracy for efficiency and simplicity.[2] Naturally, one would expect investors to use heuristics when making investment decisions. After all, financial markets are extremely complex and dynamic and not all investors have the time or ability to solve complicated mathematical problems. Indeed, ignoring the dynamics of discount rates greatly simplifies the valuation process for investors.[3] However, applying such a heuristic also biases investors' return expectations because discount rates do vary over time. The results found in the current paper support the view that cross-sectional asset pricing anomalies are simply a result of approximation errors from investors' heuristic decision-making processes, a form of bounded rationality.

Second, the CDR hypothesis is also motivated by the evidence of how investors make investment decisions in practice. Commonly used valuation methods, such as the discounted cash flow (DCF) models, typically assume a constant discount rate.[4] If some investors follow the logic behind these models in their valuation practice (the CDR investors), as confirmed by the survey study of Mukhlynina and Nyborg (2016),[5] the CDR assumption can have asset pricing implications. Moreover, Renxuan (2020) analyzes actual investor survey data on return expectations and finds that sell-side analysts, CFOs, and pension fund managers all underestimate the impact of discount rates in driving asset prices, providing direct evidence for the CDR hypothesis.

I start by developing a framework to formalize the hypothesis and by deriving testable implications on beliefs and stock returns, before testing the predictions of the hypothesis on data. More specifically, the framework guides my analysis on exactly how the CDR assumption leads to (1) biases in return expectation, (2) differential biases across stocks, and (3) cross-sectional asset pricing anomalies in equilibrium. I detail

[2]See Gigerenzer and Gaissmaier (2011) for a review on the literature. Notice that such an "accuracy-effort trade-off" heuristic in the psychology literature is distinct from the "representativeness" and "conservatism" heuristics proposed in Tversky and Kahneman (1974), which can generate over- and underreaction in financial markets, respectively (N. Barberis, Shleifer, & Vishny, 1998). Yet, despite its prominence in psychology literature, the finance literature has largely ignored its application in financial markets, compared to how the work of Tversky and Kahneman (1974) has been applied.

[3]See Ang and Liu (2004) for a more comprehensive valuation model, which takes into account the time variation of discount rates. As they show, allowing time variation in discount rates leads to a much more complicated valuation formula.

[4]See Damodaran (2012), Koller, Goedhart, Wessels, et al. (2010) for textbook examples of these models. Most of the treatment of the discounted cash flow models assume constant discount rates.

[5]Mukhlynina and Nyborg (2016) survey valuation professionals and find that the majority of respondents use DCF models and multiples to value stocks.

these three aspects as follows.

First, I find the return expectation biases that the CDR investors incur come mostly from an overestimation of the impact cash flow news has on stock prices. This happens because they do not account for the fact that a dynamic discount rate will *offset* part of the impact cash flow news has on stock prices.[6] Instead, they focus solely on projecting the future cash flows of the stock, and they assess the stock's return potential based on its price and projected cash flows.

As a realistic example, when Tesla, Inc (TSLA) announces plans to develop a new battery business, the firm opens a new revenue stream which might lead to higher cash flow growth in the future. As a result of the positive cash flow news, TSLA's stock price should go up. However, with the new battery business, TSLA's balance sheet also becomes riskier, so the market requires a higher premium to hold it, placing downward pressure on stock price. However, investors with the CDR belief will interpret the lowered stock price as being cheap, or as having a high expected return, because they fail to take into account the interaction between dynamic discount rates and cash flows.

Next, I find that the CDR assumption leads to biases of different degrees across stocks because the biases are incurred at each payout period and stocks are long-term assets with different payout horizons. As a result, two firm-level fundamental characteristics, namely expected cash flow growth and cash flow volatility, which proxy for stocks' cash flow duration and convexity, respectively, would drive cross-sectional differences in return expectation biases under the CDR assumption. Furthermore, any firm characteristics that forecast firm future cash flow growth and/or volatility would also forecast return expectation biases of investors with CDR beliefs.

How does the CDR assumption affect equilibrium asset prices? Intuitively, investors holding CDR beliefs incur return expectation biases, which lead them to buy too much or too little of certain stocks, causing over- or undervaluation compared to the CAPM benchmark. Since the most overvalued stocks (those with high cash flow growth and uncertainty) also exhibit a high degree of comovement in their asset payoffs (Ball, Sadka, & Sadka, 2009; Herskovic, Kelly, Lustig, & Van Nieuwerburgh, 2016), rational arbitrageurs who are averse to taking systematic risk do not trade aggressively against the CDR investors.[7] As a result, mispricing persists even in equilibrium.[8]

In the second part of the paper, I test the implications of the CDR hypothesis using

[6] Empirically, these two shocks are positively correlated on the stock level.

[7] This potentially could be due to the arbitrageurs' high exposure to aggregate discount rate shocks: stocks with higher cash flow duration are more exposed to aggregate discount rate shocks. More detailed discussion on this channel can be found in Santos and Veronesi (2010) and Lettau and Wachter (2007a).

[8] The mechanism is similar to the one discussed in Kozak, Nagel, and Santosh (2018), except here the cash flow growth and uncertainty are the key factors that drive the comovement in asset fundamentals, while in their setting the characteristics are not explicitly specified.

data on investors' subjective beliefs, firm fundamentals, and asset prices, and I find supporting evidence for my hypothesis. As for subjective beliefs, the CDR investors' return expectation biases should be higher for stocks with higher expected cash flow growth or higher cash flow volatility. Using sell-side analysts' return expectations, I find that analysts' long-term growth expectations (proxying for stocks' expected cash flow growth) and idiosyncratic volatility (proxying for cash flow volatility) strongly and positively predict future analysts' return forecast errors. Furthermore, these two characteristics alone explain 34% of the cross-sectional variation of the average log forecast errors among all stocks. In addition, the biases in analysts' return forecasts are mostly positive, consistent with the CDR hypothesis.

Second, a measure of mispricing on the firm level, based on the hypothesis, strongly predicts future stock returns. Roughly speaking, this measure is the difference between the CDR-implied expected return subtracted by a version of the expected return implied by the conditional CAPM. For the CDR-implied expected return, I chose the ICC model developed by Pástor, Sinha, and Swaminathan (2008) (henceforth PSS) as a measure.[9] For the expected return implied by the conditional CAPM, I use a measure of dynamic beta times a constant.[10] Consistent with the CDR hypothesis, the measure of mispricing negatively predicts future stock returns and the economic magnitude of the predictability is large. The stocks with the highest overvaluation significantly underperform compared to those with the least overvaluation, even within the S&P 500 universe (FF-5 alpha of 6% with a t-stat of 3.53). Notably, the underperformance persists after five years.

Third, a tradable factor-mimicking portfolio based on the CDR hypothesis explains the CAPM alphas of 12 prominent cross-sectional anomalies (including 9 out of the 11 considered in Stambaugh and Yuan (2017)). This set includes the most robust and persistent anomalies found in the literature, including investment, profitability, beta, idiosyncratic volatility, and cash flow duration. These characteristics all forecast future expected cash flow growth and/or idiosyncratic volatility with signs consistent with the CDR hypothesis.

Admittedly, this paper has limitations. First, the analysis presented here assumes that CDR investors' biases translate directly to under- or overinvestment. Exploring the amount of misinvestment due to CDR-related biases is an area for future work,

[9]The main reason for using this model instead of other ICC models is that PSS is more frequently applied in the finance literature, see, for example, Chen, Da, and Zhao (2013). I do examine other models in the Internet Appendix, including that of Gebhardt, Lee, and Swaminathan (2001), and find similar results for the tests using the PSS measure.

[10]The beta used is the pre-estimated CAPM beta found in Welch (2019). The constant is the average market return subtracted by an adjustment due to market-level bias caused by the CDR assumption. The resulting measure of mispricing uses analysts' earnings forecasts, stock prices, and payout ratios.

building on the approach of Koijen and Yogo (2019). Second, the results presented here are only implications of the CDR hypothesis, instead of direct evidence of investors applying the CDR assumption on the firm level. Renxuan (2020) provides more direct evidence, but on the aggregate level. Future studies such as Giglio, Maggiori, Stroebel, and Utkus (2021) are promising in testing the CDR in a more direct manner.

Related Literature

This paper contributes to three strands of literature. First, the paper contributes to a large literature that explains the cross-sectional asset pricing anomalies by relaxing the rational expectation assumption. Prominent examples include (N. Barberis et al., 1998; Daniel, Hirshleifer, & Subrahmanyam, 1998; Hong & Stein, 1999). To my knowledge, this paper is the first to jointly explain the cross-sectional patterns of subjective return expectations and asset pricing anomalies. A growing number of works that try to jointly explain both asset prices and investors' subjective return expectations mostly focus on *aggregate* data, rather than cross-sectional, such as: Adam, Marcet, and Beutel (2017); N. Barberis, Greenwood, Jin, and Shleifer (2015); Bordalo, Gennaioli, and Shleifer (2018); Hirshleifer, Li, and Yu (2015) and Collin-Dufresne, Johannes, and Lochstoer (2016) Nagel and Xu (2019). Also, existing works that do consider cross-sectional anomalies and subjective beliefs focus on a single anomaly and subjective cash flow expectations ((Bordalo et al., 2018; Bouchaud, Krueger, Landier, & Thesmar, 2019)), whereas I cover multiple anomalies and subjective return expectations. N. C. Barberis, Jin, and Wang (2020) proposes a model that can quantitatively explain a large number of cross-sectional anomalies based on prospect theory. While the hypothesis proposed in this paper focuses on matching data on subjective return expectations and cross-sectional anomalies.

Second, this paper proposes and tests a novel expectation formation process and thus contributes to the literature that tries to understand how investors form their subjective return expectations. One thread of this literature studies investor surveys, such as Greenwood and Shleifer (2014), Adam et al. (2017) and Adam, Matveev, and Nagel (2021). Their key findings are that investors' subjective return expectations deviate from the rational expectations typically assumed in workhorse asset pricing models. As an example, the results in Greenwood and Shleifer (2014) show that return expectations from CFOs and retail investors are strongly procyclical, a result opposite that of the countercyclical "risk premium" in models such as Campbell and Cochrane (1995). The other thread of the literature uses fund flows to infer the asset pricing models investors use, for example, Berk and Van Binsbergen (2016) and Barber, Huang, and Odean (2016), although Jegadeesh and Mangipudi (2020) disputes the validity of their results. This paper contributes to the literature by using asset pricing moments to infer what kind of expected return models investors could use. The empirical results in the current paper support the hypothesis that the expected return models investors use fail to take into account the dynamic nature of future returns, which is a new finding. Therefore, the results in this paper further restrict the set of potential candidate (subjective) return expectation models.

Third, this paper also contributes to the literature on empirically shrinking the cross-section by proposing a new empirical asset pricing model. Recent examples include Fama and French (2016), Hou, Xue, and Zhang (2015), Stambaugh and Yuan (2017), and Daniel, Hirshleifer, and Sun (2019). The current paper contributes here by showing that investors' constant discount rate assumptions can be an important force in driving many asset pricing anomalies, both theoretically and empirically. Instead of extracting a small number of factors from a larger set of anomalies, as in Fama and French (2016) and Stambaugh and Yuan (2017), this paper starts from a single expectation dynamic and constructs a factor using expectations data.

The rest of the paper is organized as follows. First, I develop a theoretical framework to formalize the CDR hypothesis and to guide the empirical analysis in Section 2. This section provides an expression for return expectation bias under CDR (Section 2.3) and stocks' average CAPM alpha (Section 2.4). Next, I empirically test the hypothesis in Section 3 starting with implications on subjective beliefs (Section 3.1), followed by implications on asset prices (3.2). I conclude in Section 4.

2 The Constant Discount Rate (CDR) Hypothesis

To guide the empirical analysis, I first develop the CDR hypothesis and derive its implications for subjective beliefs and asset prices. I start with a stylized example to provide intuition on how the CDR assumption can lead to biases in return expectations. I then extend the simple example into a more realistic setting and derive an analytical expression of the biases as a function of firms' fundamental characteristics. Subsequently, I show how return expectation biases can lead to mispricing in equilibrium, and I then derive an expression of a stock's CAPM alpha as a function of the expectation bias.

2.1 The CDR Investors' Investment Process

In this framework, an investor who holds the CDR beliefs (the CDR investor) forms their expectations as follows. First, they look at a stock's multiple, such as the price-dividend or price-earnings ratio. Additionally, they project the firm's fundamentals, including expected future cash flows and the uncertainty of the cash flows. Finally, they form their return expectations by using a present value model based on the multiple and projected cash flows. Crucially, the model they use does not assume the discount rate will vary over time, which ultimately leads to return expectation biases.

This setting is supported by how investors form return expectations in practice. As shown in Mukhlynina and Nyborg (2016), practitioners mostly use price multiples and projected future cash flows to infer future returns of stocks. The logic is that given the same projected future cash flow growth, one stock should have a higher expected return if its prices are lower than another. The process can be justified by the famous Campbell-Shiller decomposition, which states that the (log) price-dividend ratios of different stocks differ because of either different expected future cash flow growth, expected future returns, or both.[11]

2.2 A Stylized Example: The Three-Period Case

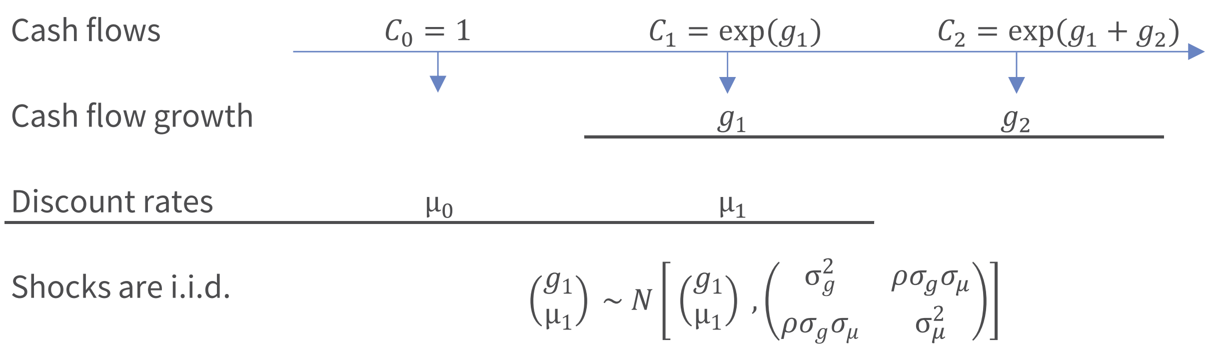
A stock is expected to pay risky dividends for two periods after paying \$1 at period 0. The (log) dividend is expected to grow at a stochastic rate of $g_t, t = 1, 2$, and the discount rate used to price the stock for the two cash flows are μ_0 and μ_1, respectively. Figure 1 illustrates this example. While the time 0 discount rate is known to investors, the discount rate at time 1, μ_1, is stochastic. This is because the risk of the stock and the market may change in period 1, and the discount rate should reflect the uncertainty. More importantly, discount rate shocks are correlated with cash flow shocks and follow a bivariate normal distribution in this example. The fair price of the stock at time 0, after the dividend payout, should be $P_0 = P_0^{(1)}\left[1 + E_0(C_1)exp\left(-E(\mu) + \frac{1}{2}\sigma_\mu^2 - \rho\sigma_\mu\sigma_g\right)\right]$, where $P_0^{(1)}$ is the present value of the period 1 cash flow.[12]

[11] Although this decomposition provides an intuitive framework to understand asset prices, it is a heuristic based on a first-order Taylor approximation that ignores the discount rate volatility. In fact, the discount rate volatility does not impact prices in the Campbell-Shiller framework.

[12] More specifically,

$$
\begin{aligned}
P_0 &= e^{-\mu_0}E_0(e^{g_1}) + E_0(e^{-\mu_0-\mu_1}C_2) \\
&= e^{-\mu_0}e^{E(g)+\frac{1}{2}\sigma_g^2} + e^{-\mu_0+2E(g)+\sigma_g^2}e^{-E(\mu)+\frac{1}{2}\sigma_\mu^2-\rho\sigma_\mu\sigma_g} \\
&=: P_0^{(1)}\left[1 + E_0(C_1)exp\left(-E(\mu) + \frac{1}{2}\sigma_\mu^2 - \rho\sigma_\mu\sigma_g\right)\right]
\end{aligned}
$$

Figure 1: A two-period example



On the other hand, the CDR investors, who ignore the discount rate volatility, would interpret the price through $P_0 = P_0^{(1)} \left[1 + E_0(C_1) exp\left(-\tilde{\mu}_1\right)\right]$, where $\tilde{\mu}_1$ is their subjective belief on how the market is discounting the stock.[13] Consequently, the CDR heuristic leads to a bias of $b = \sigma_\mu(-\frac{1}{2}\sigma_\mu + \rho\sigma_g)$.[14]

Empirically, the biases in return expectations b are positive on average. According to Vuolteenaho (2002), discount rate shocks are positively correlated with cash flow shocks at the firm level, and the magnitude of the cash flow shocks is much larger than that of the discount rate shocks. In particular, based on his estimates,[15] the CDR investors should have a bias (per year) of $b = 0.14 \times (-\frac{1}{2} \times 0.14 + 0.47 \times 0.29) = 0.0092$ for a typical stock, which means that the CDR investors would on average consider the prices to be too cheap and would consequently buy more of the stock, creating overpricing.

2.3 Biases in the Infinite-Period Case

A more realistic setting is to consider stocks paying out over infinite periods. In this case, firm-specific fundamental characteristics, namely cash flow growth and

[13] More specifically,

$$\begin{aligned} P_0 &= e^{-\mu_0} E_0(C_1) + e^{-\mu_0 - \tilde{\mu}_1} E_0(C_2) \\ &= e^{-\mu_0} E_0(C_1) + e^{-\mu_0 + 2E(g) + \sigma_g^2} e^{-\tilde{\mu}_1} \\ &=: P_0^{(1)} \left[1 + E_0(C_1) exp\left(-\tilde{\mu}_1\right)\right] \end{aligned}$$

[14] In the infinite-period, dynamic case, the unconditional bias becomes $b^i = \left\{1 - exp\left[\sigma_\mu^i(\sigma_\mu^i - \rho^i\sigma_c^i)\right]\right\} exp(g^i + \frac{1}{2}(\sigma_c^i)^2)$ for stock i. See Section 2.4 for more detail.

[15] More specifically, see Table III and Panel B of Vuolteenaho (2002).

uncertainty, drive biases in return expectations in the cross section. Appendix A derives the analytical expressions for the biases in this general setting.

Intuitively, this is because investors incur these biases each period as in the three-period case. Furthermore, stocks differ in the timing (duration) and uncertainty (convexity) of their future cash flows, which means the single-period biases are compounded to a different degree across stocks. Stocks with higher cash flow growth have higher cash flow durations, because most of their cash flows are further into the future and therefore have a longer payout horizon. As for convexity, higher cash flow volatility means a stock's price will be a more convex function of the discount rate.[16] In the bond valuation context, the same level of bias in the discount rate will translate into a larger misvaluation for bonds with a higher duration and/or a higher convexity. Since stocks' cash flow growth and uncertainty exhibit large heterogeneity in the cross section, the cross-sectional differences in misvaluations due to the CDR assumption are likely large.[17,18]

I confirm this intuition by considering a more general framework in Appendix A. The analysis shows that the biases in return expressions can be analytically linked to firm-level fundamental characteristics including expected cash flow growth and cash flow (idiosyncratic) volatility. More specifically, the unconditional bias b^i, where i denotes the stock, is given by

$$b^i = \delta^i exp(g^i + \frac{1}{2}(\sigma_c^i)^2) \tag{1}$$

and g^i and σ_c^i are expected growth and volatility of the cash flow growth, respectively. δ^i is defined as

$$\delta^i = 1 - exp\left[\sigma_\mu^i(\sigma_\mu^i - \rho^i\sigma_c^i)\right] \tag{2}$$

which also depends on the volatility of the stock's discount rate (σ_μ^i) and correlations

[16]See Dechow, Sloan, and Soliman (2004a); Gormsen and Lazarus (2019), and Weber (2018) for related discussions about the duration channel. Notice here the convexity goes in the opposite direction compared to the conventional bond convexity because for cash flow convexity, the convexity is measuring stock price's relationship with its cash flow volatility. See Pástor and Veronesi (2006) for a discussion on how cash flow uncertainty impacts stock valuations.

[17]For example, Tesla, Inc. (TSLA) has long-term cash flows that are a magnitude faster and more uncertain than those of Coca-Cola: sell-side analysts' long-term growth expectation for TSLA is at 74% as of May 2020, compared to 2.93% for Coca-Cola. Furthermore, it is reasonable to think that the cash flows of TSLA are much more uncertain than Coca-Cola.

[18]In this example, the shocks are independent and identically distributed (i.i.d.) over time. In a more realistic case when discount rate shocks are persistent, the biases are likely to be larger. Intuitively, when CDR investors ignore the discount rate dynamics, they are also ignoring the long and persistent effects that the volatility may imply.

between the discount rate and cash flows (ρ^i).

Based on previous estimates in the literature, including those of Vuolteenaho (2002), δ^i should be positive, which means CDR investors' return expectation biases are on average positive and should increase with both expected cash flow growth and cash flow volatility. I provide more discussions about the signs and the magnitude of the biases in Section 3.1.1. Moreover, I verify empirically the signs and magnitude of biases based on both analysts' return expectations and the price implied measures of the constant discount rate and confirm they are positive in Sections 3.1.1 and 3.2.1, respectively.

2.4 Biased Return Expectations and Equilibrium Asset Prices

I demonstrate in this section that when some investors hold CDR beliefs and make their investment decisions based on their own return expectations, a stock's CAPM alpha should depend on the bias in CDR investors' return expectations as well as the CDR investors' share in the economy. Intuitively, a positive bias in return expectations should lead an investor to buy more of a stock, causing overvaluation and a low CAPM alpha. Furthermore, the more CDR investors there are in the world, the more the misvaluation in equilibrium there will be.

A more formal analysis in Appendix B confirms this intuition. In this appendix, I study a multi-asset economy in which some investors with biased return expectations (CDR investors) trade with risk-averse rational investors (arbitrageurs). The setting is similar to the one studied in Kozak et al. (2018).[19] More specifically, the unconditional CAPM alpha of stock i in the model is given by

$$\alpha^i = \theta(-b^i + \beta^i b^M) \tag{3}$$

where θ is the share of CDR investors and b^i is the return expectation bias that is potentially equal to the one defined in (1).[20] The β^i are commonly defined CAPM betas and b^M is the aggregate bias CDR investors hold on the market level. So the expected return on the stock i is

$$E(R^i_{t+1}) - R_f = \alpha^i + \beta^i \left[E(R^M_{t+1}) - R_f\right] \tag{4}$$

[19]In Kozak et al. (2018), they study a model where the covariance matrix of assets' payoffs are driven by several principle components. In my model, these biases are linked to firm-level fundamental characteristics, which are the driving force behind comovement in fundamental payoffs.

[20]The model developed in Appendix B is a general one in which the bias could be due to any form of return expectation bias, not necessarily the CDR assumption. When the bias is measured empirically based on the logic of the CDR, the asset pricing test will be a test of the CDR hypothesis, as demonstrated in the following sections.

2.5 Testable Implications

The theoretical analysis yields two sets of testable implications under the CDR hypothesis for investors' subjective beliefs and asset prices.

The expression in Equations (1) and (2) leads to testable implications on investor beliefs. First, if the CDR hypothesis is true, CDR investors' unconditional return expectation biases across different firms should be largely explained by the proxies of stocks' expected cash flow growth and cash flow volatility. Second, if the term δ^i is positive, which is also empirically testable, the biases should increase with expected cash flow growth and volatility.

Equations (3) and (4) suggest that if one can measure the bias b^i on the stock level based on the CDR, the CDR can be tested by using asset prices and returns. First, the measure of the CDR-induced bias should be positive and related to both expected cash flow growth and cash flow volatility. Second, the measure for the bias or an ex ante measure of CAPM alpha according to (3) should predict the stocks' realized CAPM alpha. Finally, if the CDR hypothesis is true, the CAPM alphas of all assets should be explained by a factor-mimicking portfolio that is based on the ex ante measure of the CAPM alpha. In fact, the loading on this portfolio should be equal to 1.

I examine the extent to which these implications are true in Section 3.

3 Evaluating the Constant Discount Rate Hypothesis

In this section, I test the CDR hypothesis empirically, guided by the theoretical framework developed in the previous section. I start by testing the implications on investors' subjective beliefs based on sell-side analysts' return expectation data. Next, I develop a measure of mispricing due to the CDR assumption and use it to test implications for asset pricing.

3.1 Testing Implications on Subjective Beliefs

3.1.1 Measuring Subjective Return Expectation Biases Using Sell-Side Analysts' Price Targets

The CDR hypothesis does not make a prediction about who the CDR investors are. I test the implications of the CDR hypothesis using sell-side analysts' return expectations data. Renxuan (2020) finds that sell-side analysts do underestimate the volatility of discount rates on the aggregate. Furthermore, survey evidence from Mukhlynina and Nyborg (2016) shows that sell-side analysts do consider the commonly used discounted dividend model (DDM) as their main approach for valuation. Additionally, sell-side analysts' return expectations have comprehensive coverage on the firm level, which is unique when compared to surveys of CFOs or households, for example.

The firm-level return expectation biases are defined as 12-month realized price returns on a particular stock, minus the sell-side analysts' ex ante firm-level consensus return expectations at the end of each calendar quarter. The sell-side analysts' return expectations are defined as price targets divided by current prices subtracted by 1. Details about the data set as well as its construction are documented in Appendix E. Firm-level average return expectation biases are computed as the time-series average over the entire history of firm-level biases.

The Sign and Magnitude of Subjective Return Expectation Biases The analysis in the simple example found in Section 2.2 makes clear that the sign and magnitude of the return expectation biases are crucial for the predictions of the CDR hypothesis. Therefore, I verify the sign before testing other implications.

Figure 2 plots the empirical distribution of the average firm-level return expectation bias of sell-side analysts, together with the mean and median (bars in the middle). Subjective return expectations of sell-side analysts are on average positive at the firm level and right skewed.

The empirical results are consistent with the findings in the literature, which have previously documented the positive biases of sell-side analysts.[21] The literature has mostly attributed the positive bias to analysts' own incentives, such as their own career concerns (Hong & Kubik, 2003). The CDR hypothesis provides an alternative interpretation of such a positive bias: analysts have a higher return forecasts because they ignore discount rate dynamics, which might be an honest mistake.

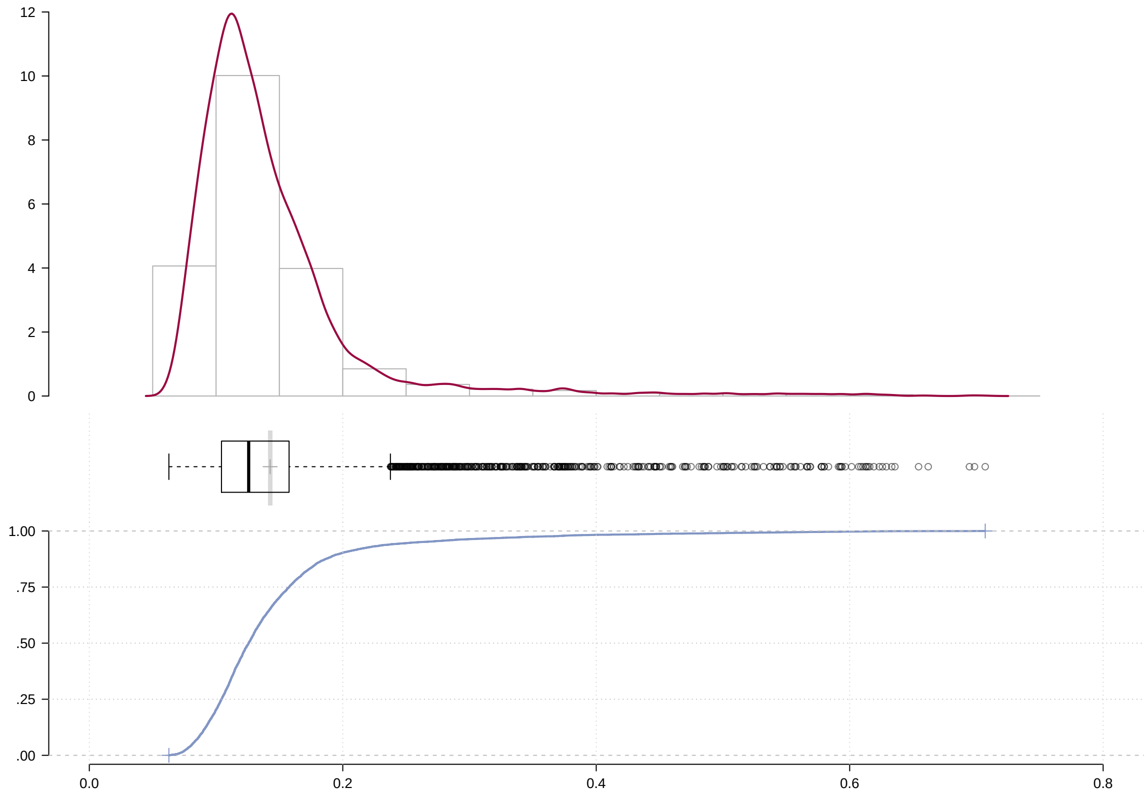
3.1.2 The Cross-Sectional Variation in Return Expectation Biases and Firm Characteristics

The CDR hypothesis predicts that the CDR investors' cross-sectional variations of unconditional return expectation biases should be driven by expected cash flow growth and cash flow volatility. Furthermore, given the positive signs of the return expectation biases, the biases should increase when the measures of these two characteristics increase.

I test the hypothesis by regressing average firm-level sell-side analysts' return forecast errors on analysts' long-term growth estimates and idiosyncratic volatility,

[21]Papers which document large positive bias of analyst price targets include Brav and Lehavy (2003) and Engelberg et al. (2019).

Figure 2: Distribution of average firm-level analyst forecast errors of 12-month ahead returns



Notes: The top and bottom panel plot the empirical probability distribution function (PDF) and the cumulative distribution function (CDF) of average sell-side analysts' return forecast errors, respectively. The dark bar in the middle represents the median while the gray bar with a cross represents the mean. The x-axis is the value of the average biases while the y-axis denotes probability in percentage points. The forecast errors are constructed based on sell-side analysts' 12-month price targets subtracted by realized average returns. More details about how the return expectations are computed are documented in Appendix E. Firm-level forecast errors are averaged over time to arrive at an average forecast error for each firm. The sample period is from 1999-Q2 to 2018-Q4.

which are proxies for the expected cash flow growth and expected cash flow volatility, respectively. Table 1 presents the regression results for both the entire stock universe with analyst return expectation coverage and the S&P 500 universe. I also contrast the results with regressions using four other firm-level characteristics known to be related to stock average returns and volatility.

The results support the CDR hypothesis. First, as Column (1) shows, both expected cash flow growth and volatility are strongly positively correlated with average return expectation errors. Analysts' biases are more positive if a stock has higher long-term growth expectation and/or higher idiosyncratic volatility. Furthermore, the two characteristics alone explain 34% of the cross-sectional variation, as indicated by the r-squared (R2) of the regression. As a benchmark, when using four other characteristics to explain average return forecast errors (Column (2)), the R2s are much lower. The results are robust across different universes ((Column (3) and (4)) and also hold for panel predictive regressions using quarterly data (see Internet Appendix).

Table 1: The cross-sectional determinants of average firm-level forecast errors of sell-side analysts

	Dependent variable:			
	average log forecast errors			
	(1)	(2)	(3)	(4)
CF growth	0.285***		0.347***	
	(0.010)		(0.031)	
I-Vol	0.520***		0.510***	
	(0.019)		(0.040)	
Investment		0.038***		0.139***
		(0.005)		(0.031)
Profitability		−0.002**		−0.001
		(0.001)		(0.001)
Beta		0.125***		0.284***
		(0.017)		(0.026)
B/M		−0.057***		0.027
		(0.013)		(0.024)
Universe	all	all	SP500	SP500
Observations	4,691	3,945	814	1,005
R^2	0.336	0.045	0.323	0.152
Adjusted R^2	0.336	0.044	0.321	0.149
Residual Std. Error	0.291 (df = 4688)	0.347 (df = 3940)	0.217 (df = 811)	0.247 (df = 1000)
F Statistic	1,186.115*** (df = 2; 4688)	46.029*** (df = 4; 3940)	193.568*** (df = 2; 811)	44.940*** (df = 4; 1000)

Note: *p<0.1; **p<0.05; ***p<0.01

Notes: "Average log forecast errors" are the log of sell-side analysts' 12-month return forecast errors defined in Section 3.1.1. Independent variables are time-series averages at the firm level based on quarterly data. "CF growth" is the average analyst long-term growth estimate; "I-Vol" is idiosyncratic return volatility measured using 60 days of daily returns and the Fama-French 3-factor model; "Investment" is the change in total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$, divided by the $t-2$ total assets at the end of each June using NYSE breakpoints; "beta" is measured using the last five years of monthly returns; "B/M" is book-to-market ratio as defined in Fama and French (2015). The sample period is from 1999-Q2 to 2018-Q4.

3.2 Testing Implications on Asset Prices and Returns

3.2.1 Measuring CDR-induced Misvaluation

I propose an intuitive measure for the bias $\widehat{b}_t^i$ that a CDR investor would incur. The measure is the difference between the implied cost of capital (ICC), $\widehat{\Pi}_t^i$, developed by Pástor et al. (2008) and the product of the dynamic beta, $\widehat{\beta}_t^i$, developed by Welch (2019) and a constant that is equal to the average market excess return ($\widehat{E(R_t^m)}$), or

$$\widehat{b}_t^i = \widehat{\Pi}_t^i - \widehat{\beta}_t^i \widehat{E(R_t^m)} \tag{5}$$

The ICC captures the essence of the return expectation of a CDR investor: it is computed using price and projected cash flows based on a present value formula that ignores the volatility of the discount rate. The second term in Equation (5) is a proxy for the "true" dynamic expected return. What a true expected return is has continued to be hotly debated in the literature. Here I take a stand similar to that of van Binsbergen and Opp (2019).[22]

Equipped with a measure of bias, I follow Equation (3) to construct a measure of misvaluation for individual stocks, $\widehat{\alpha}_t^i$, which takes the following form:

$$\widehat{\alpha}_t^i = -\widehat{b}_t^i + \widehat{\beta}_t^i \widehat{b}_t^M \tag{6}$$

$$= -\left[\widehat{\Pi}_t^i - \widehat{\beta}_t^i \widehat{E(R_t^m)}\right] + \widehat{\beta}_t^i \left[\widehat{\Pi}_t^m - \widehat{E(R_t^m)}\right] \tag{7}$$

$$= -\widehat{\Pi}_t^i + \widehat{\beta}_t^i \left[\widehat{E(R_t^m)} - \widehat{b}_t^M\right] \tag{8}$$

Equation (8) can be estimated empirically. I closely follow the procedure developed by Pástor et al. (2008) to estimate Π_t^i.[23] Appendix F details the procedure I use to estimate the ICC.[24] I estimate the dynamic β_t^i using the methodology proposed by Welch (2019).[25] I fix $\widehat{E(R_t^m)} = 0.064$, which is the average of market returns in the postwar sample, and $\widehat{b}_t^M = -2.3\%$, which is the calibration provided by Hughes, Liu,

[22] I also test the hypothesis using other proxies of true expected returns, but the results do not vary qualitatively.

[23] To determine that the results are robust, I consider alternative models developed in the literature, such as Gebhardt et al. (2001), and find similar results.

[24] One concern with using these models is that the analyst estimates are biased. However, as shown in Hou, van Dijk, and Zhang (2012) and Wang (2015), compared to the statistical models proposed in Hou et al. (2012), the analysts are not worse than statistical models when predicting future cash flows in the same universe that have analyst coverage. This is especially true for large-cap stocks for which analysts are better in accuracy.

[25] Welch (2019) shows that, empirically, his measure is superior to other estimates in some dimensions, including performing better in capturing the future realized beta.

and Liu (2009).[26]

Summary Statistics of the Misvaluation Measure I use the Institutional Broker's Estimate System (I/B/E/S) summary file for analyst earnings and price targets forecasts. I use COMPUSTAT annual data for balance sheet variables and CRSP for shares outstanding and share adjustment as well as price- and return-related variables. More detailed descriptions of the data sources are found in Appendix D.

Compared to the mostly commonly used CRSP-COMPUSTAT universe, the universe used here covers only about 40% of the number of firms and only contains larger firms. This is because analysts typically only cover larger firms and the results presented do not include microcaps.

Estimating firm-level misvaluation requires six firm-level variables, one industry-level variable, and one aggregate variable. The firm-level variables are: three analysts' consensus forecasts for a firm's earnings of the current fiscal year (FY1), the next fiscal year (FY2) and the fiscal year thereafter (FY3); one analysts' consensus long-term forecast (LTG); one payout ratio, which is based on the ratio between the firm's previous year total dividend and the firm's net income and finally, the market β. The industry-level variable is the average LTG based on 48 Fama-French industry classifications. The aggregate variable is the long-term average of gross domestic product (GDP) growth, which ranges from 7% to 6% over the 35 years in the sample. Based on these five inputs, I compute the implied cost of capital $\Pi_{i,t}$ and the entire term structure of a firm's payout ratio $PB_{i,t+s}$. There are more details on the estimation procedure documented in Appendix (F).

One important point to note is that the estimation of firm-level misvaluation α^i does not include any anomaly variables, except for β^i. However, β^i is mechanically positively related to α^i (since $\left[\widehat{E(R_t^m)} - (-\widehat{b^m})\right] > 0$ in my estimation), while the misvaluation factor is able to explain the "low-beta" anomaly. Therefore, the result of the misvaluation factor being able to explain the anomaly returns of characteristics-sorted portfolios can not be attributed to the fact that these underlying characteristics are used when constructing the misvaluation measure.

The implied cost of capital Π^i is highly persistent, with an autoregressive (AR(1)) coefficient of 0.92 based on quarterly data. I study the persistence of misvaluation α_t^i in more detail in Section 3.2.2. Table 11 presents the summary statistics for firm-level quarterly estimates of Π_t^i, together with the variables that are used to construct it. The statistics are in line with those presented in Chen et al. (2013), which also estimates the ICC based on Pástor et al. (2008).

[26]See Appendix A.3 for a more detailed discussion on the sign and magnitude of the market-level bias.

Magnitude of Misvaluation Table 2a shows the empirical distribution of the 7246 average firm-level misvaluations, or $\sum_{t+1}^{T} \widehat{\alpha}_t^i / T$, which are the empirical estimates for $E(\hat{\alpha^i}_t)$.

First, all of the misvaluations have a negative sign. This is reasonable because when ignoring the volatility of the discount rates, investors actually underestimate, on average, the discount rate and therefore overprice stocks. In fact, these results are consistent with the calibration results from Ang and Liu (2004), in which they find the average misvaluation among value-growth portfolios is about -15%.

The CDR-implied misvaluation has a cross-sectional standard deviation of more than 7% per year. Notice that the statistics presented in Table (2) potentially underestimate the magnitude of the cross-sectional dispersion of misvaluation of dynamically sorted portfolios. As shown in the second row of Table 2a, the average time-series quarterly variation on the firm-level misvaluation is 2.3%, which translates to 4.6% per year. When constructing the dynamically rebalanced portfolio annually, one would expect the spreads in ex ante misvaluation to go up substantially.[27]

To provide an intuition for the time series as well as the cross-sectional variations of misvaluations, Figure 3 plots the time-series variation of $\hat{\alpha}_t^i$ for three firms. Besides the variations, the figure also shows the persistent nature of misvaluation.

Misvaluation and Firm Characteristics Under CDR, misvaluation α^i equals the average CAPM alpha of the stocks. The CDR thus links a firm's CAPM alpha directly to its characteristics through the relationship between misvaluation and firm characteristics.

To understand what drives the variation in misvaluation, note a stock's misvaluation or α^i , under CDR, can be decomposed as follows:

$$\alpha^i = \beta^i b^m - b^i$$

Table 2b shows that the cross-sectional variation in misvaluation is mainly driven by the variation in biases b^i due to CDR. The standard errors of the bias are more than three times that of the expected return based on the conditional CAPM.

The biases can be further decomposed into two separate components as

[27] For example $5.5\% + 4.6\% * 2 = 14.7\%$ when using the inter-quartile range and time-series standard errors as an indication.

$$
\begin{aligned}
b^i &= \delta^i \times \lambda^i \\
\delta^i &= 1 - exp\left[\sigma^i_\mu(\sigma^i_\mu - \rho^i\sigma^i_c)\right] \\
\lambda^i &= exp(g^i + \frac{1}{2}(\sigma^i_c)^2)
\end{aligned}
\tag{9}
$$

As discussed before, the sign of the bias depends on δ^i, which is hard to estimate. However, λ^i involves only fundamental expectation variables that are commonly measured based on analysts' growth forecasts. I use analysts' long-term growth estimates to estimate g^i and its 36-month volatility to estimate σ^i_c. Since we observe estimates for b^i and λ^i, I back out the values of δ^i using (9).

Table 2c shows that the cross-sectional variation in biases b^i is mainly driven by λ^i, or characteristics related to expected fundamentals. Compared to δ^i, λ^i has twice the standard error (9.5% vs. 4.7%).

These empirical results help contribute to our understanding of the cross-sectional relationship between firm characteristics and CAPM alphas. Under CDR, the CAPM alpha comes entirely from misvaluation, which in turn is mostly driven by g^i and σ^i_c, via λ^i. As a result, through the CDR channel, certain characteristics can predict future returns or CAPM alphas only because these characteristics can predict future fundamental growth or fundamental volatility.

The sign of δ^i determines the relationship between fundamental characteristics and future CAPM alphas. Figure 4 shows the empirical distribution of δ^i. The figure confirms the previous conjecture that δ^i is mostly positive, as a result of the dominant cash flow news and the positive correlation between cash flow and discount rate news on the firm level. This result also leads to a prediction of the CDR hypothesis with respect to the sign when using firm characteristics to forecast future CAPM alphas: firm characteristics that positively predict future cash flow growth and/or volatility in the cross section will negatively predict the CAPM alphas.

Table 2: Empirical Distributions of Key Variables for Misvaluation (α^i)

The table presents the empirical distributions of the measure of misvaluation defined in Equation (6) and also defines its consisting variables. The data is quarterly firm-level data from 1986-01 to 2018-12. Empirical distributions are summarized based on average variable values over the entire time series for each firm. The term "ts.sd(α_t^i")" is the standard deviation of the quarterly misvaluation measure for each firm over its entire history. The term "N" is the number of firms. The notation $E(b_t^i)$ is calculated based on $\widehat{\Pi_t^i} - \widehat{\beta_t^i}0.064$ and $E(\lambda_t^i)$ is estimated based on (9) using analysts' long-term growth estimates for g^i and its 36-month volatility (using a minimum of 12 months) to estimate σ_c^i . $E(\delta_t^i)$ is calculated by dividing b_t^i by λ_t^i. The sample is winsorized at 0.5% and 0.95%. Rank correlations are Spearman rank correlations calculated using quarterly firm-level data based on the entire sample.

(a) Empirical Distribution of $E(\widehat{\alpha}_t^i)$

variable	mean	std	min	p25	median	p75	max	N
$E(\alpha_t^i)$	-0.165	0.072	-0.739	-0.181	-0.149	-0.126	-0.070	7246
ts.sd(α_t^i)	0.023	0.018	0.000	0.012	0.019	0.029	0.188	7246

(b) Empirical Distribution of $E(\widehat{\beta}_t^i\hat{\lambda})$ and $E(\hat{b}^i)$

variable	mean	std	min	p25	median	p75	max	N
$\beta^i E(R^m)$	0.063	0.023	-0.007	0.047	0.062	0.078	0.172	7246
$E(b_t^i)$	0.080	0.071	-0.059	0.040	0.064	0.098	0.633	7246

(c) Empirical Distribution of Variables in $E(\hat{b}^i)$

variable	mean	std	min	p25	median	p75	max	N
$E(\lambda_t^i)$	1.160	0.095	1.065	1.112	1.137	1.174	2.058	7246
$E(\delta_t^i)$	0.065	0.047	-0.055	0.036	0.056	0.084	0.328	7246
$E(g_t^i)$	0.143	0.071	0.063	0.104	0.126	0.158	0.707	7246
$E(\sigma_{c,t}^i)$	0.047	0.042	0.000	0.021	0.034	0.058	0.389	7246

(d) Rank Correlation Between $\widehat{\alpha}_t^i$ and Consisting Variables

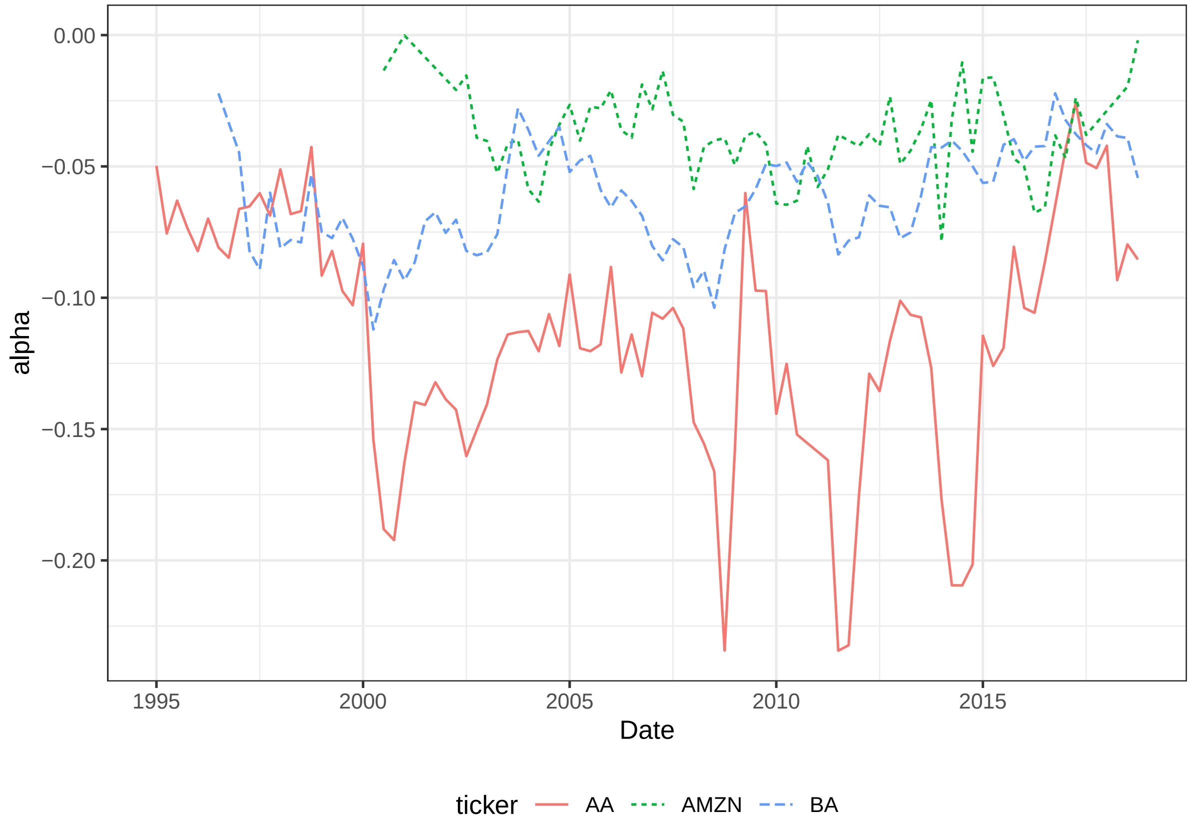
$cor(\alpha_t^i, \mu_t^i)$	$cor(\alpha_t^i, b_t^i)$	$cor(\alpha_t^i, \lambda_t^i)$
-0.319	-0.601	-0.962

(e) Rank Correlation Between b_t^i and Consisting Variables

$cov(b_t^i, \lambda_t^i)$	$cov(b_t^i, g_t^i)$	$cov(b_t^i, \sigma_{c,t}^i)$
0.747	0.776	0.034

Figure 3: Evolution of α_t^i of specific firms

The figure plots the quarterly time series of misvaluation measure $\widehat{\alpha}_t^i$ of three companies. "AA" : Alcoa Corporation; "AMZN": Amazon.com, Inc; "BA": Boeing Co.



3.2.2 Misvaluation Sorted Portfolios

I test the first asset pricing implication of CDR, namely, that the misvaluation measure $\widehat{\alpha}^i$ should positively predict a stock's CAPM alpha. Furthermore, the average spreads in the average realized CAPM alphas should be close to the magnitude suggested by the spreads in the ex ante misvaluation measures.

To test this hypothesis, I follow the convention in the asset pricing literature (for example, Fama and French (2015)) to sort stocks into quantile portfolios based on the misvaluation measure $\widehat{\alpha}^i$. I form portfolios at the end of June each year using the available information up to that point,[28] I rebalance every month based on the firms' market capitalization (value weighted). [29] Effectively, the holding period of the trading strategy is 12 months. Table 3 presents the results.

[28]For the measure, the variables used are available at least two weeks before being used to construct the ICC measures.

[29]I also present the portfolio sorts using equal weights in G.1, which shows a larger spread in CAPM alphas.

Figure 4: Probability Distribution Function (Top Panel) and Cumulative Distribution Function of $E(\delta_t^i)$ (Bottom Panel)

$E(\delta_t^i)$ is the firm-level time-series average of δ_t^i, which is calculated by dividing b_t^i by λ_t^i. The sample is winsorized at 0.5% and 0.95%.

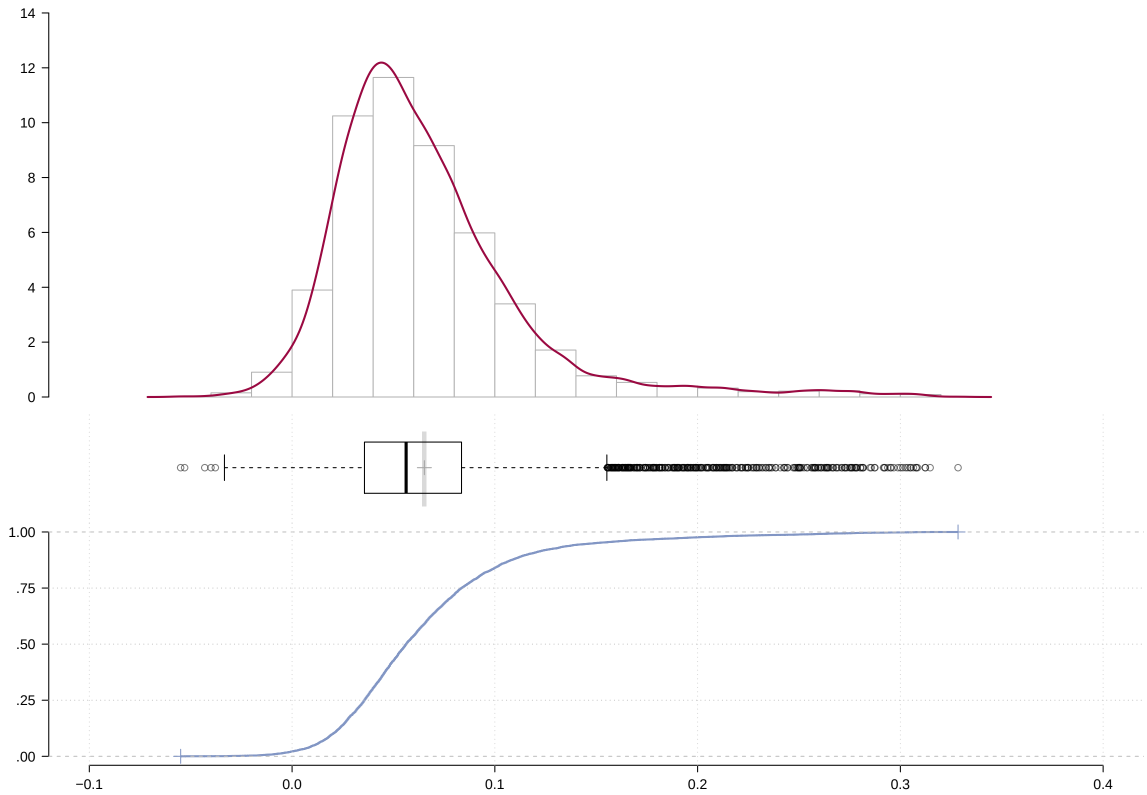


Table 3 presents the detailed results with respect to the portfolios sorted based on the misvaluation measure. These results support the hypothesis that investors do use a constant discount rate in practice and that this leads to misvaluation.

First, the results in Panel A show that stocks which are the most overvalued due to CDR have significantly lower realized CAPM alphas. The difference in realized CAPM alphas between the least overvalued ("High") and the most overvalued portfolios is 0.8% per month (9.6% per year). The spread in CAPM alphas is statistically significant with a t-stat of 5. As shown in Panel B, since the spreads in the rational expected returns (μ^i) are moderate, most overvalued portfolios end up having lower average returns than the less overvalued stocks, which amounts to 0.7% per month (8.4% per year). In fact, the return spreads also have a Fama-French 5-factor alpha of 0.69% per month (8.28% per year), with a t-statistic over 5.

Second, the magnitude of the spreads between the misvaluations of high and low portfolios are very close to those of the realized CAPM alpha spreads. This is consistent with the prediction of CDR, that the CAPM alpha should be equal to the misvaluation. As shown in Panel A, the firm-level average misvaluation within each portfolio ("Avg. $\widehat{\alpha}_t^i$") has a spread between high and low of 0.77% compared to the realized CAPM alpha of 0.80%. Furthermore, Panel C shows that what drives the misvaluation spread is consistent with what was analyzed in Section 3.2.1. The biases due to CDR, or b^i, drive the difference in $\widehat{\alpha}^i$. And λ^i appears to have a bigger role in explaining b^i than δ^i, which is positive across all portfolios. Finally, among characteristics that consist of λ^i, growth expectation seems to be more important than growth volatility (σ_c^i) in driving the spreads of misvaluation across portfolios.

Additionally, notice the realized CAPM alphas of the portfolios are negative, except for the portfolio with the highest α^i. This might seem puzzling because the value-weighted CAPM alphas should add up to zero by construction. The reason for this result is two-fold. First, stocks with higher analyst coverage have on average lower returns, as shown in Diether, Malloy, and Scherbina (2002); Hong, Lim, and Stein (2000). Stocks with a valid misvaluation measure need to have substantial analyst coverage. This dynamic universe is smaller than the CRSP universe used to construct market excess returns. More specifically, to have a valid measure of misvaluation, I require the firm to have valid analyst forecasts for short-term (1 and 2 fiscal years ahead) and long-term earnings. Second, stocks with a higher α^i are significantly larger than those with lower values, which further exacerbates the asymmetry among CAPM alphas. More details about this point are presented in Appendix D, Table 10.

The results in Table (3) also highlight two important questions to be addressed. First, as shown in Panel A, the spread in the values of misvaluation across portfolios does not seem to converge quickly after the portfolio formation. From 12 months to 60 months after the formation of the portfolio, the spread only narrows by 0.12% per month, or 1.44% per year. This means that the mispricing is highly persistent. A persistent effect means that the misvaluation has a larger economic significance since it has implications for the long-run asset returns. I explore this further in Section (3.2.2) .

Second, most overvalued portfolios tend to consist of smaller firms, as shown in Panel C. This result is intuitive as larger companies receive more media attention and therefore more analysts cover their stocks. Prices for large-cap stocks should be more efficient and the probability of being misvalued should decrease. However, it does raise the question of whether and to what extent the misvaluation is still present in the large caps. Cross-sectional phenomena that only hold in small caps carry less economic significance for asset pricing theories, especially in recent years when large caps dominate the market. I therefore further investigate this issue in Section 3.2.2.

Table 3: Pre-estimated Misvaluation ($\widehat{\alpha}_t^i$) Sorted Portfolios and Realized Average Stock Returns (1986-06 to 2019-12)

The table presents the statistics related to portfolios sorted based on the misvaluation measure created in (6). All numbers are expressed in percentages unless otherwise stated. Returns and alphas are based on monthly frequency.
Stocks are sorted into quantile portfolios based on the misvaluation measure $\widehat{\alpha}^i$ at the end of June each year, using the available information up to that point. Portfolios are rebalanced every month based on firms' market capitalization (value weighted). "Low" denotes the portfolio with lowest $\widehat{\alpha}_t^i$. "High-Low" denotes the excess returns of a portfolio that goes long on stocks with the highest $\widehat{\alpha}_t^i$ and short on those with the lowest $\widehat{\alpha}_t^i$.
Panel A presents the average misvaluation after the portfolio formation for the next 12 months and 60 months as well as the average values for firms in the portfolio throughout the firms' lives.
Panel B presents statistics related to portfolio returns. "mean ex.ret" are the monthly returns over three-month treasury rates; "SE" are standard errors which are shown in brackets. "SR" are monthly Sharpe Ratios. "FF-5 alpha" denotes Fama-French 5-factor alphas. "num_stocks" is the average number of stocks included in the portfolio over time.
Panel C presents characteristics (value weighted) associated with each of the portfolios. The terms g^i and σ_c^i denote average portfolio analysts' long-term growth expectations (LTG) and the 36-month rolling volatility of the LTG, respectively.

	Low	**2**	**3**	**4**	**High**	**High - Low**
Panel A: Ex ante Misvaluation vs. Realized Portfolio CAPM Alpha						
	Ex ante Misvaluation					
Nxt. 12m $\widehat{\alpha}_t^i$	-1.98	-1.31	-1.18	-1.05	-1.01	0.98
Nxt. 60m $\widehat{\alpha}_t^i$	-1.91	-1.28	-1.17	-1.07	-1.05	0.86
Avg. $\widehat{\alpha}_t^i$	-1.80	-1.24	-1.14	-1.05	-1.03	0.77
Realized Portfolio	Realized Portfolio CAPM Alpha					
CAPM alpha	-0.80	-0.39	-0.26	-0.09	0.01	0.80
SE CAPM alpha	(0.14)	(0.10)	(0.09)	(0.06)	(0.06)	(0.16)
Panel B: Realized Portfolio Return Statistics						
mean ex.ret	-0.03	0.27	0.33	0.48	0.66	0.70
SE ex.ret	(6.12)	(4.93)	(4.56)	(4.24)	(4.85)	(3.29)
SR	-0.01	0.05	0.07	0.11	0.14	0.21
FF-5 alpha	-0.63	-0.34	-0.41	-0.23	0.06	0.69
SE FF-5 alpha	(0.11)	(0.09)	(0.08)	(0.06)	(0.06)	(0.13)
Panel C: Portfolio Characteristics						
Mkt.Cap (Million)	15379.69	33550.85	38340.65	47129.95	88655.92	73276.23
b^i	14.32	7.29	5.51	3.85	1.41	-12.91
μ^i	7.14	6.22	6.26	6.29	7.59	0.45
π^i	21.46	13.51	11.77	10.14	9.00	-12.46
λ^i	125.02	114.65	112.63	110.77	109.54	-15.48
δ^i	10.92	6.30	4.85	3.45	1.27	-9.65
σ_c^i	5.71	3.27	2.73	2.44	2.90	-2.80
g^i	21.46	13.51	11.77	10.14	9.00	-12.46

The Persistence of Misvaluation I demonstrate the economic significance of misvaluation due to CDR by showing that it has a persistent impact on asset prices.

First, $\widehat{\alpha}_t^i$ is a persistent variable. The pooled panel regression based on annual data shows that $\widehat{\alpha}_t^i$ has an AR(1) coefficient of 0.948 (standard error 0.006, clustered by firm and year). This means that the misvaluation measure has a half life of more than 13 years.

Consistent with the highly persistent measure $\widehat{\alpha}_t^i$, the trading strategy constructed based on misvaluation has a low turnover. Table 4a shows that the average monthly portfolio turnover for the long and short side only amounts to 2%, or less than 24% annually. Compared to the trading strategies analyzed in Novy-Marx and Velikov (2015), the 2% turnover would place the misvaluation trading strategy in the lowest turnover category, on par with profitability and above only portfolios sorted based on size. This result means that transaction costs will unlikely render the CAPM alpha to zero.

Investors do not counteract the misvaluation effect quickly; stocks in the most overvalued (undervalued) portfolios underperform (outperform) even five years after the portfolio formation. Table 4b shows the returns of the trading strategy based on misvaluation for different holding periods. As shown in the table, the High-Low portfolio's CAPM alpha is still highly significant even for holding periods exceeding 60 months. The reduction in the return spreads are statistically significant, which amounts to 0.21% per month, from the 12-month to 60-month holding period. This magnitude is also inline with the decay of the spreads in the misvaluation measure itself, which amounts to 0.12% from 12 to 60 months.

For value-weighted portfolios, the persistence mainly comes from the continuing underperformance of stocks which are mostly overvalued due to CDR. On the other hand, for the equally weighted portfolios, both long and short sides continue to outperform and underperform after a prolonged period of time. This result means there might exist an interaction between firm size and the misvaluation measure. I investigate this in the next subsection.

Table 4: The Persistence of Misvaluation

The table demonstrates the persistence of misvaluation and its persistent effect in asset prices. The pooled panel regression based on annual data shows $\widehat{\alpha}_t^i$ has an AR(1) coefficient of 0.948 (standard errors 0.006, clustered by firm and year).
Panel (a) calculates the portfolio's annualized turnover, or monthly turnover multiplied by 12. Panel (b) calculates the CAPM alphas, value and equal weighted, of portfolios sorted based on $\widehat{\alpha}_t^i$ at the end of June, starting from 1986-06 and ending in 2018-12. The CAPM alphas are calculated by regressing the excess returns of the portfolios on market returns based on the universe of stocks that have estimated $\widehat{\alpha}_t^i$. The reason for using this universe is to take into account the negative CAPM alphas of stocks with higher analyst coverage.

(a) Portfolio Turnover: Misvaluation Sorted Portfolios

Portfolio	short-side	2	3	4	long-side	avg.long.short
ann.turnover	28.56%	36.44%	31.80%	27.43%	19.28%	23.92%

(b) Holding Period Returns of Misvaluation Sorted Portfolios

	portfolio holding periods (in month)					
	12	24	36	48	60	72
Panel A: CAPM alphas of value-weighted portfolios						
low α^i	-0.612	-0.524	-0.524	-0.593	-0.461	-0.568
[t-stat]	[-4.212]	[-3.356]	[-3.323]	[-3.663]	[-3.399]	[-3.529]
high α^i	0.147	0.092	0.082	0.096	0.071	0.071
[t-stat]	[2.775]	[1.793]	[1.782]	[2.262]	[1.603]	[1.841]
High - Low	0.760	0.616	0.606	0.689	0.531	0.638
[t-stat]	[4.646]	[3.573]	[3.543]	[3.964]	[3.56]	[3.755]
Panel B: CAPM alphas of equal-weighted portfolios						
low α^i	-0.626	-0.588	-0.642	-0.639	-0.627	-0.632
[t-stat]	[-2.877]	[-2.719]	[-2.964]	[-2.943]	[-2.931]	[-2.91]
high α^i	0.384	0.368	0.352	0.355	0.343	0.351
[t-stat]	[3.376]	[3.215]	[3.263]	[3.293]	[3.278]	[3.395]
High - Low	0.984	0.929	0.954	0.969	0.930	0.943
[t-stat]	[5.827]	[5.47]	[5.569]	[5.638]	[5.773]	[5.524]

Misvaluation In Different Size Segments of the Market I show that the misvaluation due to CDR also presents within the universe consisting of the largest companies. Since a few large companies take up the dominant share of the stock market, the finding that misvaluation presents in this part of the market means the channel of mispricing suggested by CDR is economically important.

Table 5 shows results of conducting an independent 3 by 3 double sort based on a stock's size and misvaluation. The CAPM alphas for the spread between the most and the least overvalued portfolio within the smallest companies is 1.08% per month (12.96% per year). However, even within the largest segment of the stock market, where the average market cap is more than 26 billion, the spread in CAPM alphas is still 0.63% per month (7.56% per year), with a t-statistic close to 5.

To further examine the economic significance of the CDR channel of misvaluation, I conduct the same portfolio sorting exercise within the S&P 500 universe, which contains the biggest U.S. companies and accounts for about 80% of all available U.S. market capitalization, as of September 2020. Table 6 shows that even within this universe, the spread in CAPM alphas between the most and least overvalued stocks is 0.39% per month (4.68% per year). In fact, the FF-5 alpha is higher, at 0.53% per month, thanks to the fact that the returns load strongly negatively on the small minus big (SMB) factor.

Finally, portfolio characteristics in both Tables 5 and 6 show that the spreads in misvaluations are in line with those of realized portfolio CAPM alphas, consistent with the prediction of the CDR. For example, for the S&P 500 universe, the model predicts the CAPM alpha would amount to about 5% per year, while the realized CAPM alpha is at 4.68% per year.

Table 5: Mean Excess Returns of Size and Misvaluation Sorted Portfolio (Value Weighted, 1986-06-01 to 2018-12-31)

This table shows the returns and characteristics for 3 by 3 portfolios independently sorted based on the misvaluation measure $\widehat{\alpha}_t^i$ in (6) and market capitalization from June of the previous year. All returns, alphas, and their standard errors are monthly and expressed in percentages. "1_1" denotes the portfolio with the lowest market capitalization from June in the previous year and the lowest α_t^i , respectively, while "3_1" denotes portfolios with the highest market capitalization and lowest α_t^i. Portfolios are value weighted each month. "SE" are standard errors which are shown in brackets. "mean ex.ret" denotes monthly returns over three-month treasury rates. "SR" are monthly Sharpe Ratios. "FF-5 alpha" denotes Fama-French 5-factor alphas. "num_stocks" denotes the average number of stocks included in the portfolio over time. Post portfolio formation average characteristics: "nxt.12m.alpha" is the average misvaluation measure 12 months after portfolio formation, "pi" is the implied cost of capital, "mu" is the average beta times 0.064, "LTG" denotes the analysts' long-term growth estimates, and "sd(LTG)" denotes the 36-month rolling volatility of LTG.

stats	1_1	1_2	1_3	high-low.small	2_1	2_2	2_3	high-low.mid	3_1	3_2	3_3	high-low.large
mean ex.ret	0.33	0.92	1.41	1.08	0.08	0.58	1.07	0.99	0.05	0.36	0.6	0.56
SE ex.ret	(6.86)	(6.26)	(7)	(2.48)	(6.48)	(5.54)	(5.81)	(2.09)	(5.48)	(4.35)	(4.57)	(2.68)
SR	0.05	0.15	0.2	0.43	0.01	0.1	0.18	0.48	0.01	0.08	0.13	0.21
CAPM beta	1.26	1.14	1.25	-0.01	1.28	1.1	1.17	-0.12	1.14	0.94	1.02	-0.12
SE CAPM beta	(0.05)	(0.04)	(0.05)	(0.03)	(0.04)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)	(0.01)	(0.03)
CAPM alpha	-0.44	0.22	0.64	1.08	-0.73	-0.1	0.36	1.07	-0.65	-0.22	-0.02	0.63
SE CAPM alpha	(0.21)	(0.19)	(0.22)	(0.13)	(0.16)	(0.14)	(0.14)	(0.1)	(0.11)	(0.07)	(0.04)	(0.13)
FF-5 alpha	-0.39	0.15	0.56	0.95	-0.74	-0.23	0.25	0.97	-0.39	-0.39	0	0.39
SE FF-5 alpha	(0.1)	(0.09)	(0.13)	(0.13)	(0.08)	(0.06)	(0.07)	(0.1)	(0.1)	(0.07)	(0.04)	(0.12)
num_stocks	429.1	208.07	134.39		225.19	281.74	256.84		116.49	274.87	372.7	
ME (million)	219.77	245.87	264.56		912.54	972.94	1024.18		26798.68	46560.25	79038.81	
nxt.12m.alpha	-0.19	-0.14	-0.13		-0.21	-0.14	-0.13		-0.2	-0.14	-0.12	
pi	0.17	0.12	0.1		0.18	0.12	0.1		0.17	0.12	0.09	
mu	0.06	0.06	0.07		0.07	0.06	0.07		0.07	0.06	0.07	
LTG	0.22	0.17	0.16		0.22	0.16	0.15		0.18	0.14	0.13	
sd(LTG)	0.06	0.05	0.05		0.06	0.04	0.04		0.04	0.03	0.03	

Table 6: Misvaluation (α^i) Sorted Portfolios and Realized Average Stock Returns for S&P 500 Firms (1986-06 to 2018-12)

The table presents the statistics related to portfolios sorted based on the misvaluation measure created in (6) for firms in the S&P 500 universe. All numbers are expressed in percentages unless otherwise stated. Returns and alphas are based on monthly frequency. Stocks are sorted into quantile portfolios based on the misvaluation measure $\widehat{\alpha}^i$ at the end of June each year, using the available information up to that point. Portfolios are rebalanced every month based on firms' market capitalization (value-weight). "Low" denotes the portfolio with the lowest $\widehat{\alpha}_t^i$. "High-Low" denotes excess returns of a portfolio that goes long on stocks with the highest $\widehat{\alpha}_t^i$ and short those with the lowest $\widehat{\alpha}_t^i$. "fwd_12m_alpha" denotes the average misvaluation measure 12 months after portfolio formation. "CAPM alpha" are calculated by regressing portfolio excess returns on returns to the universe of S&P 500 stocks that have the estimates of $\widehat{\alpha}_t^i$ available.

stats	Low	2	3	4	High	High - Low
mean ex.ret	0.2	0.39	0.47	0.56	0.65	0.45
SE ex.ret	(4.78)	(4.53)	(4.26)	(4.34)	(5.13)	(2.97)
SR	0.04	0.09	0.11	0.13	0.13	0.15
CAPM beta	0.99	0.96	0.91	0.94	1.11	0.11
SE CAPM beta	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
CAPM alpha	-0.29	-0.08	0.02	0.1	0.1	0.39
SE CAPM alpha	(0.1)	(0.08)	(0.07)	(0.07)	(0.08)	(0.15)
FF-5 alpha	-0.42	-0.4	-0.24	-0.2	0.11	0.53
SE FF-5 alpha	(0.1)	(0.09)	(0.08)	(0.07)	(0.09)	(0.15)
ME	43818.98	42726.38	54376.9	60586.93	110437.88	
fwd_12m_alpha	-0.17	-0.14	-0.12	-0.12	-0.12	

3.2.3 Explaining Cross-sectional Anomalies Using a Factor-Mimicking Portfolio

A factor-mimicking portfolio constructed based on the misvaluation measure should be able to explain completely the CAPM alphas of portfolios that are sorted on characteristics that predict future misvaluation. Under CDR, the only reason for these characteristics-based anomalies to generate CAPM alphas is that investors use a constant discount rate, which causes them to overvalue stocks associated with these characteristics.

Choosing Anomalies When choosing cross-sectional anomalies, I consider portfolios sorted on profitability, asset growth, market beta, (idiosyncratic) volatility, and cash flow duration. Furthermore, I also consider the anomalies that were used to construct the two mispricing factors in Stambaugh and Yuan (2017). I chose these anomalies because they have generated significant interest both in the academic literature and in practice. This strong interest could come from both their significance for academic theories (for example market beta, volatility) and their persistent, robust empirical performance.[30] I provide some more background behind choosing these anomalies below.

First, the beta (for example, Fama & French, 1992) and (idiosyncratic) volatility (for example, Ang et al., 2006; Haugen & Heins, 1975) anomalies generated much interest mainly because they speak directly to the failure of CAPM and also break from the positive risk-reward relationship commonly accepted in the financial markets. An extensive and continuing effort has been proposed to explain these low risk anomalies (for example, Black, 1992; Frazzini & Pedersen, 2014; Schneider, Wagner, & Zechner, 2020).

Furthermore, I include the anomalies based on profitability (Fama & French, 2015; Hou et al., 2015; Novy-Marx, 2013) and asset growth (Cooper et al., 2008; Fama & French, 2015; Hou et al., 2015), which predict future returns with positive and negative signs, respectively, because they are shown by recent literature to be able to summarize, to a large degree, the average returns of the cross section, as shown in Fama and French (2016) and Hou et al. (2015). Various theories have been proposed to explain these anomalies, both behaviorally and rationally (for example, Bouchaud et al. (2019) behaviorally and Hou et al. (2015) rationally).

Finally, I chose the cash flow duration factor (Dechow et al., 2004b; Gonçalves, 2019; Weber, 2018), which negatively predicts future stock returns, for two reasons. First, this factor is directly related to the future cash growth. Second, it has theoretical significance related to the term structure of equity (see for example, Croce, Lettau, & Ludvigson, 2014; Lettau & Wachter, 2007b). This is important for linking the macro-finance theories to help explain the time-series of aggregate stock returns to the cross section (for example, Binsbergen & Koijen, 2015; Santos & Veronesi, 2010).

In order to show that the CDR hypothesis is indeed an important channel through which mispricing occurs, I also consider the two mispricing factors constructed by

[30] I do not include the well-known "value" and "size" anomalies here because for the sample I consider (post 1986-06), it does not have a significant CAPM alpha.

Stambaugh and Yuan (2017). These two factors are constructed based on 11 anomalies and are shown in their paper to have strong power to explain the anomalies uncovered in the literature. I examine the explanatory power of the misvaluation factor on the two composite factors as well as the 11 anomalies underlying these two factors.

Constructing the Misvaluation Factor To explain the anomaly portfolio returns, I first construct a factor-mimicking portfolio of misvaluation. I follow a procedure similar to that employed by Fama and French (2015). First, I conduct 3 by 3 independent sorts based on market capitalization and $\widehat{\alpha}_t^i$. Within each of the size terciles, which consist of small-, medium-, and large-cap stocks, I subtract the returns of the stocks with the lowest $\widehat{\alpha}_t^i$ from the returns of the stocks with the highest $\widehat{\alpha}_t^i$ to obtain the returns of a long-short portfolio. More specifically, the constant discount rate (CDR) factor is

$$\begin{aligned} CDR_t = & \frac{1}{3}(R_t^{high,small} + R_t^{high,mid} + R_t^{high,big}) \\ & - \frac{1}{3}(R_t^{low,small} + R_t^{low,mid} + R_t^{low,big}) \end{aligned} \tag{10}$$

I show the return statistics of the factor together with its cumulative returns in Table 7 and Figure 5, respectively. The CDR factor has a volatility of 6.3% annually, with a mean realized return of 10.8%. The realized return mainly comes from the short leg, which contains the most overvalued stocks.

The cumulative return graph shows that the strong performance of the CDR factor is not concentrated in a specific period over the past 33 years, which confirms the results in the previous section that demonstrates the persistence of the misvaluation effect.

Explaining Five Prominent Anomalies The Constant Discount Rate Hypothesis predicts that the CAPM alphas of individual assets should be completely consumed by the CDR factor. To test this hypothesis, I construct long-short anomaly portfolios based on the five characteristics and then regress the return of the portfolios on the market excess return and the CDR factor as defined in (10):

$$R_t^i = \alpha^i + CDR_t\beta_{CDR}^i + (R_t^m - R^f)\beta_m^i + \epsilon_t^i \tag{11}$$

The constant discount rate hypothesis predicts that all of the alphas are jointly zero or

$$H_0^{CDR}: \quad \alpha^i = 0 \quad \forall i = 1, ..N$$

Figure 5: Cumulative Returns of the CDR Factor (In Log Scale)

Notes: Sample period is 1986-07-01 to 2018-12-31. Stocks are sorted independently into 3 by 3 terciles based on market capitalization and $\widehat{\alpha_t^i}$ at the end of each June. The portfolios are rebalanced each month based on market capitalization. the CDR factor is the "high minus low" and constructed by

$$CDR_t = \frac{1}{3}R_t^{high} - \frac{1}{3}R_t^{low}$$

where $R_t^{high} = \frac{1}{3}(R_t^{high,small} + R_t^{high,mid} + R_t^{high,big} - 3R_t^f)$ and $R_t^{low} = \frac{1}{3}(R_t^{low,small} + R_t^{low,mid} + R_t^{low,big} - 3R_t^f)$.

Table 7: Return Statistics CDR Factor

Notes: Sample period is 1986-07-01 to 2018-12-31. Stocks are sorted independently into 3 by 3 terciles based on the market capitalization the previous June and the $\widehat{\alpha_t^i}$ at the end of each June. The portfolios are rebalanced each month based on market capitalization. The CDR factor is constructed by

$$CDR_t = \frac{1}{3}R_t^{high} - \frac{1}{3}R_t^{low}$$

where $R_t^{high} = \frac{1}{3}(R_t^{high,small} + R_t^{high,mid} + R_t^{high,big} - 3R_t^f)$ and $R_t^{low} = \frac{1}{3}(R_t^{low,small} + R_t^{low,mid} + R_t^{low,big} - 3R_t^f)$.

	CDR	low $\hat{\alpha}$	high $\hat{\alpha}$
Annualized Return	0.108	-0.004	0.110
Annualized Std. Dev.	0.063	0.208	0.190
Annualized Sharpe	1.704	-0.021	0.578

I test the hypothesis using the Gibbons Ross Shanken (GRS) tests. I also examine the alphas of the single anomaly portfolios.

To eliminate the errors due to replication, I download the anomaly portfolios from official sources. More specifically, I download portfolios sorted based on beta, variance, and residual variance sorted portfolios directly from Ken French's website[31]and the cash flow duration sorted portfolios from Michael Weber's website.[32]

The GRS test results in Panel A of Table 8 show we can not reject the hypothesis: the CDR factor explains the CAPM alphas of all five anomaly portfolios. More specifically, the GRS test statistics based on the CDR factor is just above 1, which has a p-value of 0.42, compared to the GRS test statistics of 5.4 under the CAPM, which confirms these portfolios have high CAPM alphas.

Examining the tests on each of the five single anomalies, Table 8 shows that all of the standalone portfolios' CAPM alphas become statistically insignificant from zero after the inclusion of the CDR factor. Furthermore, all of the anomalies load strongly on the CDR factor, with point estimates on loadings greater or equal to 0.43.

The magnitude of the reduction is large, especially for the idiosyncratic variance and cash flow duration factors, which amount to 1.36% and 0.96% per month, respectively, based on the point estimates as shown in Panel B of Table 8. This large reduction in CAPM alphas is confirmed by the anomaly portfolio's large loadings on the CDR factor,

[31]Betas are measured using the last five years of monthly returns; variances are historical variances based on the past 60 days of daily returns; and residual variances are measured using 60 days of daily returns and the Fama-French 3-factor model.

[32]The details of the measure is described in Weber (2018).

Table 8: Anomalies Portfolio Alpha/Beta Before/After Controlling for CDR Factor

Sample period is 1986-07-01 to 2018-12-31. In Panel A, the GRS test statistics are presented, which test the null hypothesis that all α^is in Equation 11 are jointly zero under the CAPM or the model where market factor together with CDR factors are included. Panel B presents the tests for individual assets in Equation 11. Panel B1: the long-short anomaly portfolios are regressed on market excess returns over 3 month treasuries. Panel B2: long-short anomaly portfolios are regressed on (value-weighted) market excess returns and CDR factor defined in Equation (10). "beta" are measured using the last 5 years of monthly returns; "prof" are operating profitability defined in Fama and French (2015); "res.var" are measured using 60 days of daily returns and Fama-French 3-factor model; "asset.growth" are the change in total assets from the fiscal year ending in year t-2 to the fiscal year ending in t-1, divided by t-2 total assets at the end of each June using NYSE breakpoints; "cf.dur" are cash flow duration measure defined in Weber (2018), a composite measure based on sales and book values. Except for the "cf.dur", all other portfolios are downloaded from Ken French's website and are long-short (value-weighted) portfolios constructed by subtracting the portfolio with the lowest decile of beta, var, res.var, asset growth by the highest decile and subtracting the highest profitability portfolio by the lowest profitability portfolio. Decile portfolios of "cf.dur" are downloaded from Michael Weber's website; the portfolios end on 2014-06-30 and are equally weighted.

Panel A: GRS. Test Results

Model	CAPM	Mkt + CDR
GRS-stat	5.422	1.003
P-value	0.000	0.416

Panel B: Tests on Single Anomaly Portfolios

	Predicting Volatility of Growth			Predicting Future Growth	
	beta	**res.var**	**prof**	**asset.growth**	**cf.dur**
Panel B1: CAPM alpha of anomaly portfolios					
CAPM Alpha (%)	0.565	1.246	0.721	0.488	1.261
t-statistics	[2.288]	[3.777]	[3.593]	[2.909]	[4.124]
CAPM Beta	-1.046	-0.971	-0.456	-0.177	-0.432
t-statistics	[-18.8]	[-13.063]	[-10.077]	[-4.688]	[-6.471]
Panel B2: CAPM alpha of anomaly portfolios after controlling for CDR factor					
CAPM Alpha (%)	-0.129	-0.114	0.174	0.085	0.296
t-statistics	[-0.484]	[-0.337]	[0.799]	[0.466]	[0.926]
CAPM Beta	-0.983	-0.849	-0.406	-0.141	-0.345
t-statistics	[-18.013]	[-12.243]	[-9.137]	[-3.759]	[-5.393]
Loading on CER	0.748	1.467	0.590	0.434	1.036
t-statistics	[5.706]	[8.807]	[5.522]	[4.815]	[6.804]

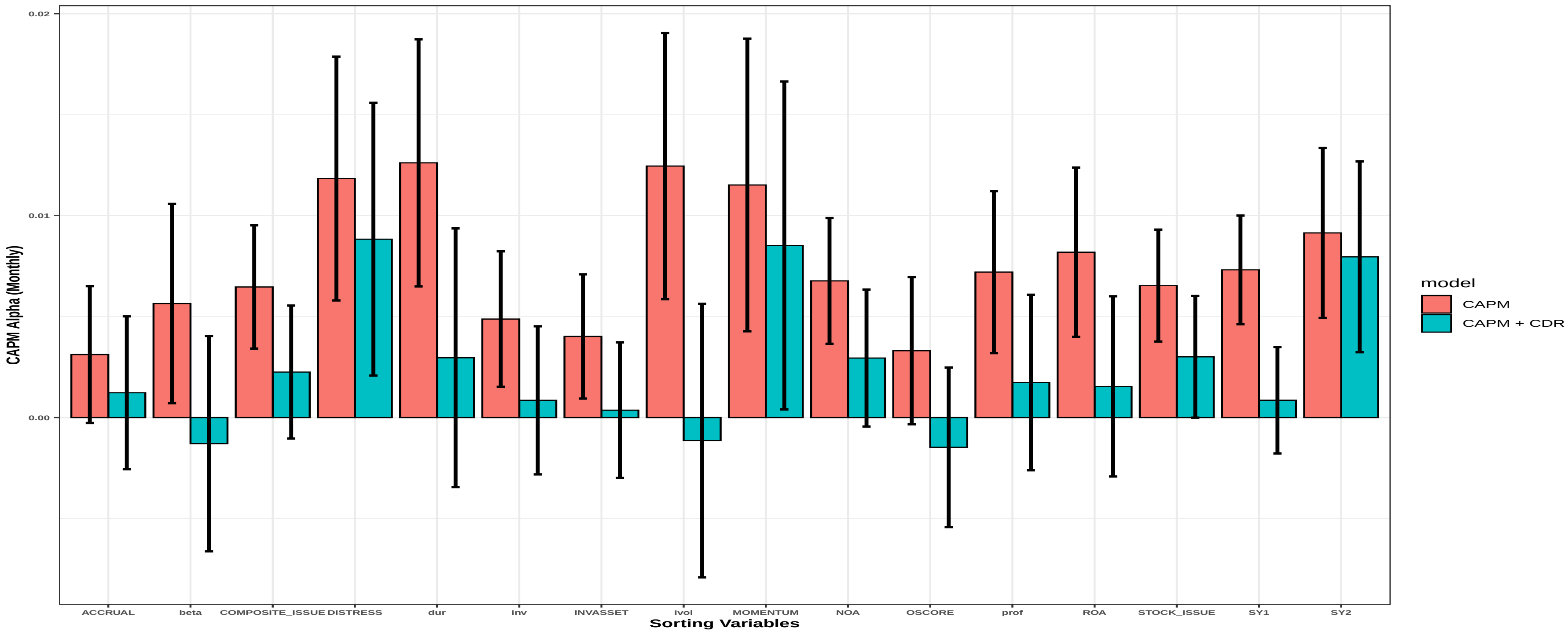
which are 1.47 and 1.04 for the residual variance and the cash flow duration factor, respectively. The strong explanatory power of the CDR factor on these two particular anomalies makes intuitive sense because residual variances closely mimic the cash flow growth volatility (σ_c^i) in the model, while the cash flow duration aims at predicting the future cash flow growth (g^i).

The Misvaluation Factor and Mispricing Factors in Stambaugh and Yuan (2017) I show that the explanatory power of the mispricing channel suggested by the CDR hypothesis goes beyond the five anomalies previously analyzed. Stambaugh and Yuan (2017) construct two mispricing factors based on 11 cross-sectional anomalies. They show that these two factors have superior performance when compared to factor models constructed by Fama and French (2015) and Hou et al. (2015) in terms of summarizing the cross section of average stock returns. I therefore examine to what extent the single misvaluation factor can explain the CAPM alphas of the misvaluation factor as well as the underlying eleven anomalies. Among these eleven anomalies, nine have not been included in the five anomalies examined earlier in Section 3.2.3.

Figure 6 shows the CAPM alphas along with the standard errors of the estimates, before and after including a CDR factor, for 14 anomalies considered (11 of them form the basis for the two mispricing factors in Stambaugh and Yuan (2017)) together with the two mispricing factors. For all the 14 standalone anomalies, as well as the two mispricing factors ("SY1" and "SY2"), the misvaluation factor constructed in this paper reduces their CAPM alphas. In fact, for all but the momentum and distress factors, the CAPM alphas become insignificant after regressing on the misvaluation factor. As a result, the CAPM alpha of the first mispricing factor (SY1) is completely explained by the misvaluation factor. The second mispricing factor (SY2) still remains unexplained by the CDR, mainly due to the momentum and distress factors. This makes sense as the misvaluation factor is a persistent, long-term factor, while the momentum and distress factor has been shown to have high turnover and to mainly generate anomalous returns in the short term.

Figure 6: CAPM Alpha of Long-Short Anomaly Portfolios Before and After Controlling for CDR Factor

The figure plots the CAPM alphas of the two mispricing factors constructed in Stambaugh and Yuan (2017) together with 11 anomalies that are used to construct the factors, together with the duration, beta, and residual variance anomalies, before and after regressing on the CDR factor.
Sample period is 1986-07-01 to 2016-12-31. "CAPM" is the intercept when regressing long-short anomaly portfolios on market excess returns over three-month treasuries. "CAPM + CDR" is the intercept when regressing the long-short anomaly portfolios on (value-weighted) market excess returns and the CDR factor defined in Equation (10). Two standard deviations above and below the estimates are indicated.
Long-short anomaly portfolio returns whose labels are in capital letters are downloaded from Robert Stambaugh's website. The "beta", "inv", "ivol", and "prof" variables are downloaded from Ken French's website and "dur" is downloaded from Michael Weber's website. "ACCRUAL" is the accrual anomaly of Sloan (1996); "beta" are measured using the last five years of monthly returns; "prof" denotes operating profitability as defined in Fama and French (2015); "ivol" is measured using 60 days of daily returns and the Fama-French 3-factor model; "inv" denotes the change in total assets (asset growth) as in Fama and French (2015) and Cooper et al. (2008); "cf.dur" denotes the cash flow duration measure defined in Weber (2018), which is a composite measure based on sales and book values. "COMPOSITE_ISSUE" is the composite equity issuance of Daniel and Titman (2006); "STOCK_ISSUE" is the equity issuance measure of Loughran and Ritter (1995); "DISTRESS" is the distress risk measures of Campbell, Hilscher, and Szilagyi (2008); "OSCORE" denotes Ohlson's O-score Ohlson (1980); "NOA" refers to the Net Operating Asset defined in Hirshleifer, Hou, Teoh, and Zhang (2004); "MOMENTUM" is the variable defined in Jegadeesh and Titman (1993); "INVASSET" is the investment to assets ratio as defined in Titman, Wei, and Xie (2013); "SY1" is the "MGMT" factor constructed in Stambaugh and Yuan (2017), which includes net stock issues, composite equity issues, accruals, net operating assets, asset growth, and investment to asset rations; and "SY2" denotes the "PERF" factor in Stambaugh and Yuan (2017), which includes distress, the O-score, momentum, profitability, and return on assets.



3.2.4 Firm Characteristics and Misvaluation

Finally, the CDR also implies that characteristics that predict future anomalous returns (CAPM alphas) should also predict future misvaluation (α_t^i) with the same sign. Since the cross-sectional variation in misvaluation is mostly driven by λ^i, these characteristics should forecast either expected future fundamental growth (g_t^i) or fundamental volatility ($\sigma_{c,t}^i$), or both, with an opposite sign as they predict future CAPM alphas.

To test the first hypothesis, I run the following predictive panel regressions with date fixed effects:

$$y_t^i = a + B' X_{t-1}^i + f_t + \epsilon_{i,t}$$

where X_{t-1}^i is a vector of characteristics including the market beta, volatility, idiosyncratic volatility, profitability, asset growth, and cash flow duration; and f_t denotes the date fixed effects. The variable y_t^i denotes either the firms' future misvaluation $\widehat{\alpha}_t^i$, the analysts' long-term growth estimates g_t^i, or the future volatility of analysts' long-term growth estimates $\sigma_{c,t}^i$.

When predicting future misvaluation, the CDR hypothesis predicts that the predictive coefficients are all negative and significant, except for profitability, which should be positive and significant. This is because, except for profitability, all the other characteristics positively predict firms' future CAPM alphas.

Panel (a) and the first column of Panel (b) of Table 9 confirm the prediction of the CDR hypothesis. All of the characteristics show significant predictive power for future misvaluation, and the coefficients have signs that correspond exactly to their CAPM alphas. Quantitatively, the R-squared is high, at more than 21%. Comparing across different characteristics, beta and residual variances show the strongest predictive powers, followed by asset growth, profitability, and cash flow duration.

I use expected growth and growth volatility as dependent variables in the regression and present the results in the Panel (b) of Table 9. These results show that the five characteristics generally can predict both the growth level and volatility, even though these two variables are moderately correlated (24% correlation in the pooled sample). One notable characteristic is profitability, which predicts future misvaluation mainly due to its ability to negatively forecast a firm's future growth volatility. For a firm's operating profitability to increase by one percent in ranking in the cross section, its future growth volatility decreases by 16% while expected growth only decreases by 3.7%. This result is intuitive: profitable firms typically have stable cash flows and are unlikely to incur high cash flow volatility in the future. A similar pattern holds for low-beta firms.

Table 9: Misvaluation and Firm Characteristics

Data are quarterly firm-level data from 1985-Q1 to 2018-Q4. Misvaluation captures the undervaluation or overvaluation following CDR, as defined in Equation (6). In Panel (a), Spearman ranked correlations are calculated. In Panel (b), results from the panel regression with date fixed effects

$$y_t^i = a + B' X_{t-1}^i + f_t + \epsilon_{i,t}$$

are presented, with standard errors clustered at the firm-quarter level. Both dependent and independent variables are transformed into cross-sectional percentiles to avoid outliers and to help with the ease of interpretation. Expected growth and growth volatility are defined as analysts' long-term growth expectations and the 36-month rolling volatility of long-term growth expectations, respectively. Both variables are downloaded from the IBES database. The measurements of "beta" are from Welch (2019) downloaded from Ivo Welch's website; "residual variances" are constructed using 60 days of daily returns (with a minimum of 20 days) and the Fama-French 3-factor model; "asset.growth" denotes the change in total assets from the fiscal year ending in year t-2 to the fiscal year ending in t-1 divided by t-2 total assets at the end of each June using New York Stock Exchange (NYSE) breakpoints; "cf.dur" denotes cash flow duration measures as defined in Gonçalves (2019) and downloaded from Andrei Gonçalves' website; "Op.Prof" are firms' operating profitabilities as defined in Fama and French (2015). Both financials and utilities sectors are excluded from the panel regressions and each firm needs to have a minimum of two years available in COMPUSTAT.

(a) Pair-wise Rank Correlation Misvaluation and Firm Characteristics

lag.asset.growth	lag.op.prof.	lag.res.var	lag.beta	lag.cf.dur
-0.155	0.116	-0.288	-0.401	-0.102

(b) Panel Regressions: Future misvaluation, expected growth, growth volatility on firm characteristics

	Dependent variable:		
	misvaluation	*expected.growth*	*growth.vol*
	(1)	(2)	(3)
lag.beta	-0.248^{***}	0.077^{***}	0.121^{***}
	(0.013)	(0.013)	(0.015)
lag.asset.growth	-0.071^{***}	0.079^{***}	0.062^{***}
	(0.007)	(0.008)	(0.009)
lag.op.prof.	0.042^{***}	-0.037^{***}	-0.160^{***}
	(0.009)	(0.010)	(0.012)
lag.res.var	-0.273^{***}	0.268^{***}	0.236^{***}
	(0.010)	(0.010)	(0.014)
lag.cf.dur	-0.046^{***}	0.040^{***}	0.051^{***}
	(0.010)	(0.010)	(0.011)
Observations	108,790	108,790	77,466
R^2	0.218	0.119	0.146
Adjusted R^2	0.216	0.117	0.145
Residual Std. Error	0.245 (df = 108615)	0.260 (df = 108615)	0.262 (df = 77312)
Note:			*p<0.1; **p<0.05; ***p<0.01

4 Conclusion

This paper proposes and tests a unifying hypothesis to explain cross-sectional asset pricing anomalies: some investors ignore the dynamics of discount rates when forming return expectations. The empirical findings in this paper show that the potential impact of the mispricing (due to the constant discount rate assumption) is economically significant. Furthermore, many prominent asset pricing anomalies can be explained by the CDR model. Additionally, data on analysts' return forecasts and firms' fundamentals are consistent with the predictions of the CDR hypothesis. The results are also consistent with the aggregate time-series estimates provided by Renxuan (2020), which show that a large set of investors underestimate the importance of the discount rate in driving the dynamic of asset prices at the market level.

The results presented in this paper also have implications for the investment community. In particular, these results provide useful suggestions to those who employ conventional discounted cash flow (DCF) models to value stocks. Namely, they could improve the accuracy of their expected returns by adjusting their estimates using the misvaluation measure developed in this paper.

The paper assumes that biased subjective return expectations would directly translate into over- and underinvestments by the CDR investors through their own portfolio optimization model. This assumption is not warranted. A natural next step is to closely examine how subjective expectations are translated into changes in investors' investment decisions, extending the methods and data considered in Koijen and Yogo (2019).

Another potential venue of research would be to connect the CDR expectation to other subjective expectation formation processes proposed in the literature, such as those proposed in Bordalo, Gennaioli, Porta, and Shleifer (2019) or Bouchaud et al. (2019), both of which focus on subjective expectations of firm fundamentals. Are these expectation formation processes consistent with each other or are they mutually exclusive?

Finally, further examination of the impact of misvaluation on the real economy is also promising. Dessaint, Olivier, Otto, and Thesmar (2021) find evidence supporting the idea that investors using CAPM distort the prices in the merger and acquisition (M&A) markets. If the channel of misvaluation suggested in this paper is valid and long lasting, those firms who receive a much higher valuation than warranted due to CDR should have a lower cost of equity capital, which could ultimately impact the firm's real activities. A logical next step is to estimate the model proposed in van Binsbergen and Opp (2019) to evaluate the loss of efficiency due to the misvaluation.

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Appendix A The Return Expectation Biases Due to the CDR Assumption

I demonstrate why assuming a constant discount rate could result in a return expectation bias, and analyze how the bias is related to firm-level characteristics.

A.1 The Setup

I start by considering a general discounted cash flow model with potentially time-varying discount rates and expected cash flows. Let V_0 be the be the value of an equity that pays c_t, $t = 1, 2, ..., \infty$ into the future. Further denote M_t as the expected return, or discount rate, known at the beginning of period t, for the cash flow to be paid on $t + 1$. For the convenience of exposition, let $\mu_t = log(M_t)$. We have

$$\begin{aligned} V_0 &= E_0 \left[\sum_{t=0}^{\infty} (\prod_{s=0}^{t} e^{-\mu_s}) c_{t+1} \right] \\ &= \sum_{t=0}^{\infty} E_0 \left[(\prod_{s=0}^{t} e^{-\mu_s}) c_{t+1} \right] \end{aligned} \tag{12}$$

If one ignores the dynamics of the discount rate and instead assumes a constant discount rate, $\pi_t = log(\Pi_t)$, the valuation becomes

$$\begin{aligned} \widetilde{V}_0 &= E_0 \left[\sum_{t=0}^{\infty} e^{-t\pi_0} c_{t+1} \right] \\ &= \sum_{t=0}^{\infty} \left[e^{-t\pi_0} E_0 \left(c_{t+1} \right) \right] \end{aligned} \tag{13}$$

Equation (13) represents the valuation formula taught in a typical undergraduate or graduate level business class: first project future cash flows to obtain $E_0 \left(c_{t+1} \right)$ and subsequently apply a discount rate, either using the weighted average cost of capital (WACC) or a CAPM model to obtain a value for π_t. This valuation formula is also a log version of the commonly used discounted cash flow model (DCF) as seen in popular valuation textbooks, such as Damodaran (2012).

To understand the implication of the constant discount rate assumption more precisely, I follow Hughes et al. (2009) to assume the dynamics of the discount rates μ_t and c_t:

$$\mu_t = r_f + \beta_t \lambda \tag{14}$$

$$\beta_t = \bar{\beta} + \sigma_\beta \epsilon_{\beta,t} \tag{15}$$

$$c_{t+1} = c_t exp\left[g + \sigma_c(\rho\epsilon_{\beta,t+1} + \sqrt{1-\rho^2}\epsilon_{c,t+1})\right] \tag{16}$$

$$\begin{pmatrix} \epsilon_{\beta,t} \\ \epsilon_{c,t+1} \end{pmatrix} \sim N\left(0, \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

The discount rate dynamic specified in (14) is different from a constant discount rate for two reasons. First, it has own volatility, which leads to uncertainty in prices with respect to the discount rate itself. More specifically, the dynamic in this particular specification is due to the conditional β_t.[33] The volatility of the risk premium is therefore

$$\sigma_\mu := \lambda\sigma_\beta.$$

This specification is consistent with a version of the conditional CAPM model.[34] If investors ignore the dynamics of β_t and instead use a static CAPM, they would use $\pi_t = \bar{\beta}\lambda$ instead. Second, the discount rate is correlated with stochastic discount cash flows, which impact the prices through ρ.[35]

The cash flow process, as specified in Equation (16), has a constant growth g and an interaction with the discount rates through ρ. Notice that the cash flow shocks are permanent growth shocks, while the discount rate shocks are temporary. This is reasonable as, in general, cash flows grow at a positive rate in the long run while the discount rate should be stationary in the long run. The specifications also mean that we can interpret a firm's discount rate volatility as mainly due to its systematic risk through β_t, while the cash flow shocks are idiosyncratic, which means that idiosyncratic

[33]The analytical difference to be presented is invariant if the risk premium λ is stochastic, as shown in Hughes et al. (2009).

[34]See for example Jagannathan and Wang (1996) for conditional CAPM.

[35]Notice that here the discount rates have a simple term structure with one shock drives the discount rates of different horizons, i.e. μ_t follows the same dynamics with respect to the horizon t. This makes the analysis less complicated. In a fully specified model as seen in Ang and Liu (2004), discount rates also have a potential term structure. Besides, the discount rate shocks are i.i.d.; Ang and Liu (2004) also consider cases where the discount rate and cash flow processes are persistent. For the sake of simplicity, the current paper uses the simple case with analytical solutions.

return volatility is driven by firms' cash flow volatility in this model.

A.2 The Biases

Under the specifications in Equation (14) through to (16), the rational "fair value" of the equity should be, according to (12):

$$V_0 = c_0 \frac{exp(g + \frac{1}{2}\sigma_c^2)}{exp(\mu_0)\left\{1 - exp\left[-(r_f + \lambda\bar{\beta} - g) - \frac{1}{2}(\rho\sigma_c - \sigma_\mu)^2 - \frac{1}{2}(1-\rho^2)\sigma_c^2\right]\right\}} \tag{17}$$

On the other hand, an investor who values the stock using a constant discount rate, or (13), would arrive at

$$\widetilde{V}_0 = c_0 \frac{exp(g + \frac{1}{2}\sigma_c^2)}{exp(\pi_0) - exp(g + \frac{1}{2}\sigma_c^2)} \tag{18}$$

To understand the impact of a dynamic discount rate in valuating a stock, consider the case where $\mu_t = \bar{\mu}$, Equation 17 becomes the familiar Gordon-Growth formula with uncertain cash flows

$$A_0 = \frac{c_0}{exp(\mu_0 - g - \frac{1}{2}\sigma_c) - 1} \tag{19}$$

which makes clear the impact of discount rates being stochastic: it adds the volatility of the discount rates, σ_μ and the interaction between discount rates and cash flow ρ into the valuation formula.

By equating Equation (17) and (18), we have the relationship between the two expected returns:

$$\begin{aligned}\Pi_0 = M_0 - exp&\left\{\mu_0 - \left[(r_f + \lambda\bar{\beta} - g) - \frac{1}{2}(\rho\sigma_c - \sigma_\mu^2) - \frac{1}{2}(1-\rho^2)\sigma_c^2\right]\right\} \\ &+ exp(g + \frac{1}{2}\sigma_c^2)\end{aligned} \tag{20}$$

The equation also implies that Π_0 will equal M_0 if $\mu_0 = \bar{\mu}$. The formula provides an analytical expression for the bias, b_t,

$$b_t^i = -M_t^i e^{-\Delta^i} + exp(g^i + \frac{1}{2}(\sigma_c^i)^2)$$

where

$$\Delta^i = \left(r_f + \lambda\bar{\beta}^i - g^i\right) - \frac{1}{2}(\rho\sigma_c^i - (\sigma_\mu^i)^2) - \frac{1}{2}(1 - (\rho^i)^2)(\sigma_\mu^i)^2.$$

I added the superscript i, which runs across different firms, to stress that the biases are related to the characteristics of different firms. The biases are dynamic because they depend on the realization of the discount rate M_t^i. Also, the bias is related to firm-level fundamental characteristics, such as expected growth and volatility of the growth. Confirming our intuition, the bias is higher for firms with higher growth rates and more uncertainty.

We are more interested in the unconditional expectation of the bias, which is given by

$$b^i = E(b_t^i) = \delta^i exp(g^i + \frac{1}{2}(\sigma_c^i)^2) \tag{21}$$

where

$$\delta^i = 1 - exp\left[\sigma_\mu^i(\sigma_\mu^i - \rho^i\sigma_c^i)\right] \tag{22}$$

A.3 A Discussion: The Sign and Magnitude of the Biases

Equation (21) relates the bias to firm-specific characteristics, and therefore will have implications for the cross section. However, is the channel suggested by the CDR important enough to have any impact on the cross section of stock returns, and is it plausible for it to explain any cross-sectional anomalies? I discuss the plausibility of the channel based on empirical findings in the literature before examining the data.

Analytically, the relationship between b^i and firm characteristics depends on the sign of δ^i. In the case that $\delta^i > 0$ (< 0), we have $b^i > 0$ (<0). Furthermore, b^i depends on fundamentals of the firm such as expected growth rates g^i and σ_c^i.

The sign of δ^i potentially differs on the market and firm level. On the market level, δ^i has been shown to be negative, leading to a negative market-level bias b^m and the magnitude of the bias has been estimated in the literature. This negative bias is mainly due to the fact that the discount rate dominates (for example, Cochrane, 2011) on the market level and that aggregate cash flows and discount rate news are weakly negatively correlated (for example, Campbell, 1990; Lochstoer & Tetlock, 2020). In fact, the negative b^m is directly supported by the empirical literature on the implied cost of capital, which assumes a constant expected return in the model. Claus and Thomas (2001) and Pástor et al. (2008) estimate that the market-level implied risk premium (Π_t^m-R^f) is around 3% or less using the constant discount rate assumption. This is significantly less than estimates of the market premium, which is typically above 5%.[36] Hughes et al. (2009) calibrates the magnitude of b^m and shows that the magnitude is at

[36] Avdis and Wachter (2017) is the latest literature estimating the market risk premium. Their estimate for the market risk premium in the U.S. is at 5.1%, lower than those in the previous literature, which typically were above 6%. However, this estimate is still more than 2% above those based on the constant discount rate assumption.

-2.3%.[37] Given the robustness of these empirical findings, I directly use $b^m = -0.023$ in my empirical tests and show that my results are not sensitive to different choices of b^m.

On the firm level, which is the focus of this paper, the sign of δ^i is likely positive, although the magnitude is unclear. This is because cash flow news likely dominates at the firm level and the shocks between the discount rate and cash flows are positive, as shown in Vuolteenaho (2002) and Cohen, Polk, and Vuolteenaho (2009), for example. Compared to the market-level evidence, no direct estimates are provided in the literature for the firm-level biases δ^i.[38] Therefore, I empirically verify the sign and magnitude of different components of b^i in Section 3.1.

Depending on the sign of δ^i on the firm level, the bias relates to a firm's fundamental characteristics, g^i and σ_c^i. In the case of a positive δ^i, firms with higher future cash flow growth and/or cash flow volatility have higher biases.

Appendix B Biased Return Expectations and Equilibrium Asset Prices: A More Formal Analysis

I study a multi-asset economy in which some investors with biased return expectations (CDR investors) trade with risk-averse rational investors (arbitrageurs). CDR investors take up $\theta \in (0,1)$ share of the economy, so arbitrageurs are left with $1-\theta$. Both of these investors live for two periods; in the first period they invest in the risky securities and have a risk-free rate r_f to maximize their terminal wealth. There are N risky assets, each of which pays a dividend of D_t^i for asset i in the next period. The number of shares outstanding of these risky assets is $x^* = (x^1, x^2, ..., x^N)'$, and risk-free assets are in unlimited supply.

Both CDR investors and arbitrageurs have the same utility function with the same risk-aversion coefficient, γ. The key difference is that the CDR investors have subjective return expectations, $\widetilde{E}(.)$, that are biased, or

$$\tilde{E}_t(R_{t+1}^i) = E_t(R_{t+1}^i) + b_t^i \tag{23}$$

[37] Their calibration is based on the parameter sets of $\sigma_c = 0.15$, $g = 0.05$, $\sigma_\mu = 0.14$ and $\rho = -0.1$, which translates into a $b^m = -0.023$.

[38] Gode and Mohanram (2003) regress the firm-level cost of capital on firm characteristics and find the estimated implied cost of capital is positively related to analysts' LTG estimates, leverage, and earnings volatility, using data from 1984 to 1998. They consider three different ICC models. Their results support a positive firm-level δ^i.

In particular, the CDR investors solve the problem

$$\max_{\omega} \sum_{i=1}^{N} \omega^i P_t^i \left[\tilde{E}_t(R_{t+1}^i) - R_f\right] - \frac{\gamma}{2}\omega' \Sigma_t \omega \tag{24}$$

where

$$R_{t+1}^i = \frac{P_{t+1}^i + D_{t+1}^i}{P_t^i}$$

While the arbitrageurs solve the problem

$$\max_{y} \sum_{i=1}^{N} y^i P_t^i \left[E_t(R_{t+1}^i) - R_f\right] - \frac{\gamma}{2} y' \Sigma_t y \tag{25}$$

Denote $\omega^* = (\omega^1, \omega^2, ..., \omega^N)'$ and $y^* = (y^1, y^2, ..., y^N)'$ the optimal demand of the CDR investors and the arbitrageurs, respectively. The market clears, and we have

$$\theta\omega^* + (1-\theta)y^* = x^* \tag{26}$$

The equilibrium asset prices and expected returns are outlined in Proposition 1.

Proposition 1. *The multi-asset economy features biased investors and arbitrageurs whose return expectations are governed by Equation* (23) *and who solve optimization problems in* (24) *and* (25)*, respectively. With market clearing conditions* (26)*, the equilibrium asset price for asset i is*

$$P_t^i = \frac{1}{1 + R_f - \theta b_t^i}\left[E_t(P_{t+1}^i + D_{t+1}^i) - \gamma e^{i'} \Sigma_t x^*\right] \tag{27}$$

where e^i is a vector of zeros with 1 on the i^{th} entry. The expected return of asset i is

$$E_t(R_{t+1}^i) - R_f = \theta(-b_t^i + \beta_t^i b_t^M) + \beta_t^i \left[E_t(R_{t+1}^M) - R_f\right] \tag{28}$$

where $b_t^M = \sum_{i=1}^N \frac{x^i P^i}{\sum_j x^j P^j} b_t^i$ is the market-level expectation bias of CDR investors, $\beta_t^i = \frac{Cov_t(R_t^i, R_t^M)}{Var_t(R_t^M)}$ (the CAPM beta in its usual definition), and $R_t^M = \sum_{i=1}^N \frac{x^i P^i}{\sum_j x^j P^j} R_t^i$ is the value-weighted market return.

Proof. See Appendix C. □

The results in Proposition 1 confirm the earlier intuition about how biases in the return expectation could cause mispricing in equilibrium. As shown in Equation (27), the more CDR investors in the economy, that is, the higher value of θ, the more serious the mispricing potentially becomes. Furthermore, when fixing the share of CDR investors, the higher the bias the CDR investors have for the return expectation of an asset, the higher its price and the lower its expected return, as shown in Equation (28). This is intuitive as the CDR investors will demand more of such an asset, leading to a lower expected returns.

Equation (28) reveals that the return expectation bias on the asset level as well as the market level together contribute to the non-zero CAPM alpha. This is intuitive as the CDR investors' irrational demand on the asset level would also lead to an equilibrium impact on the market level.

Appendix C A Proof of Proposition 1

Solving the first-order condition of (24) and (25), we have the optimal demands given by

$$\omega^* = \frac{1}{\gamma}\Sigma_t^{-1}\left[E(P_{t+1}+D_{t+1}) + B_t P_t - P_t(1+R_f)\right] \tag{29}$$

where B_t is a diagonal matrix with biases b_t^i being on the i^{th} row and i^{th} column, and

$$y^* = \frac{1}{\gamma}\Sigma_t^{-1}\left[E(P_{t+1}+D_{t+1}) - P_t(1+R_f)\right] \tag{30}$$

respectively.

Market clearing conditions imply that

$$\theta\omega^* + (1-\theta)y^* = x^*$$

or

$$\theta\frac{1}{\gamma}\Sigma_t^{-1}\left[E(P_{t+1}+D_{t+1}) + B_t P_t - P_t(1+R_f)\right] + (1-\theta)\frac{1}{\gamma}\Sigma_t^{-1}\left[E_t(P_{t+1}+D_{t+1}) - P_t(1+R_f)\right] = x^*$$
$$\theta B_t P_t + E_t(P_{t+1}+D_{t+1}) - P_t(1+R_f) = \gamma\Sigma x$$

$$\left[(1+R_f)I - \theta B_t\right] P_t = E_t(P_{t+1}+D_{t+1}) - \gamma\Sigma_t x^*$$

which leads to

$$P_t^i = \frac{1}{1+R_f-\theta b^i}\left[E_t(P_{t+1}^i + D_{t+1}^i) - \gamma e^{i'}\Sigma_t x^*\right]$$

which is Equation (27) of Proposition (1).

The expected returns follow

$$
\begin{aligned}
E_t(R^i_{t+1}) - R_f &= -\theta b^i_t + \gamma \frac{1}{P^i_t} e_i' \Sigma_t x^* \\
&= -\theta b^i_t + \gamma \frac{1}{P^i_t} e_i' Cov_t(P_{t+1} + D_{t+1}, P_{t+1} + D_{t+1}) x^* \\
&= -\theta b^i_t + \gamma Cov_t(R^i_{t+1}, P_{t+1} + D_{t+1}) x^* \\
&= -\theta b^i_t + \gamma Cov_t(R^i_{t+1}, (P_{t+1} + D_{t+1})' x^*) \\
&= -\theta b^i_t + \gamma Cov_t(R^i_{t+1}, R^M_{t+1}) P_t' x^* \qquad (31)
\end{aligned}
$$

Now define the market-cap weight for asset i as

$$
\omega^i_M = \frac{x^i P^i_t}{\sum_j x^j P^j_t}
$$

and pre-multiply Equation (31) by the weights and sum over different assets to obtain

$$
R^M_{t+1} - R_f = -\theta b^M_t + \gamma Var_t(R^M_{t+1}) P_t' x^*
$$

which gives

$$
\begin{aligned}
\gamma Var_t(R^M_{t+1}) P_t' x^* &= R^M_{t+1} - R_f + \theta b^M_t \\
P_t' x^* &= \frac{E_t(R^M_{t+1} - R_f)}{\gamma Var_t(R^M_{t+1})} \qquad (32)
\end{aligned}
$$

Substituting Equation (32) into (31), we have

$$
\begin{aligned}
E_t(R^i_{t+1}) - R_f &= -\theta b^i_t + \gamma Cov_t(R^i_{t+1}, R^M_{t+1}) \frac{E_t(R^M_{t+1} - R_f)}{\gamma Var_t(R^M_{t+1})} \\
&= \theta(-b^i_t + \beta^i_t b^M_t) + \beta^i_t \left[E_t(R^M_{t+1}) - R_f \right]
\end{aligned}
$$

the last equation is Equation (28) in Proposition (1).

Appendix D Detailed Data Descriptions

In sum, the estimation of firm-level equity requires five firm-level variables, one industry-level variable, and one aggregate variable. The firm-level variables are: three analysts' consensus forecasts for a firm's earnings of the current fiscal year (FY1), the next fiscal year (FY2), and the fiscal year thereafter (FY3); one analysts' consensus long-term forecast (LTG); and one payout ratio, which is the ratio of the firm's previous year total dividend to the firm's net income. The industry-level variable is the average LTG based on 48 Fama-French industry classifications. The aggregate variable is the long-term average of GDP growth, which goes down from 7% to 6% over the 35 years in the sample. Based on these five inputs, I compute the implied cost of capital $q_{i,t}$ and the entire term structure of a firm's payout ratio $PB_{i,t+s}$ based on (33), which is a function of the last year's payout ratio and aggregate GDP growth rate and the $q_{i,t}$.

In the IBES monthly summary history file, I use analyst earnings per share (EPS) estimates for fiscal year 1, fiscal year 2, and fiscal year 3 (fpi = 1, 2, 3) and the long-term growth estimates to take full advantage of the term structure of analyst forecasts.[39] Furthermore, I require both fiscal year one and fiscal year two consensuses to be based on no less than three available analyst estimates and at least two estimates for FY3 consensus[40] in order to be included in the sample. I only use the latest monthly consensus estimates within each calendar quarter: March, June, September, and December to obtain firm-quarter consensus estimates. In addition, the firms included in the sample need to be U.S. firms whose reporting currency is in U.S. dollars. For the base case, I consider the median estimates as the consensus estimate, but my results do not change when using the mean estimates. To obtain estimates for total dollar earnings, the EPS estimates are multiplied by shares outstanding from daily CRSP data as of the date the EPS estimates were announced. In addition, I adjust for stock splits for the shares-outstanding data. To merge the IBES database with the CRSP database, I first match them using the 8-digit historical CUSIP. Additionally, I match firms whose ticker and/or company names are the same and those who have the same 6-digit historical CUSIP. In terms of timing, I match the quarterly IBES data with the monthly CRSP-COMPUSTAT merged by calendar quarter. In all asset pricing tests, I require the analyst estimates from the IBES summary file to be announced at least one quarter before the date that the returns are observed. Since the IBES summary file's statistical period is in the middle of each month, the analyst expectation information is lagged by about three months and two weeks.

To compute the payout ratio, I collect the common dividends (DVC) and net income (IBCOM) as well as the firm's historical industry SIC code from COMPUSTAT. If a firm's net income is negative, I replace it with 6% of the asset value (AT). I winsorize the payout ratio so that it is also between zero and one. For other fundamental data and the

[39]Further horizons are available; however, the coverage is much poorer.

[40]The reason for using two FY3 estimates is that the coverage for FY3 is considerably poorer. My results are actually stronger when requiring three FY3 estimates; however, the average number of firms covered will be only 60% of the sample in the base case.

price-related variables I use the CRSP-COMPUSTAT Merged (Annual) data. I include common shares (share codes 10 and 11) in the CRSP database traded on NYSE/AMEX and NASDAQ exchanges with the beginning-of-month prices above one dollar. When forming portfolios based on fundamental variables, I follow the convention in the literature (for example Fama and French (2015)), and lag the annual fundamental information of each firm for at least six months and assume that the information on all the firms' fundamental data is observed by end of June each year. Annual and monthly stock returns, as well as market prices and gross and net of dividends are obtained from CRSP and are adjusted for stock delistings. The market capitalization (ME) of a stock is its price times the number of shares outstanding, adjusted for stock splits, using the cumulative adjustment factor provided by CRSP, which is also used to compute a firm's total expected earnings and actual earnings.

Appendix E Measuring Analyst Return Expectations Using Analyst Price Targets

Firm-level analyst return expectations are constructed using a bottom-up approach based on analyst-level return expectations per analyst issuance.

I collect a single issuance of price targets from individual analysts' 12-month[41] price targets for individual firms from the IBES unadjusted database and then match it with the closing price from CRSP on the date the price target was issued[42] to compute return expectations with price targets for individual firms. The expected returns are computed by dividing the analysts' price targets by the daily closing price on the day the estimate was issued and then subtracting one.[43] or

$$\mu^{A}_{i,f,d} = \frac{P^{A,12}_{i,f,d}}{P_{f,d}} - 1$$

where $P^{A,12}_{i,f,d}$ is the price target of analyst i for firm f, issued on day d. The superscript 12 denotes the 12-month ahead estimate. Notice this methodology ensures there is no mechanical relation between mean estimated expected returns and the level of prices. On each issuing date the analyst has the freedom to pick their own price target since they observes the prices.

[41]Other horizons are available, though the coverage is poor.

[42]In case the issuance date falls on a weekend, the last Friday prices are used. In case the issuance falls on a holiday, the previous business day closing prices are used.

[43]The same formula is used in Brav and Lehavy (2003) and Da and Schaumburg (2011).

Table 10: Returns and Alphas of the Universe with Available Estimates of Misvaluation (Analyst's Forecasts)

Sample period 1985-07 to 2018-12. Monthly value-weighted excess returns of the universe with the available firm misvaluation measure $\hat{\alpha}_t^i$, or "vw.mkt.rf.analyst", are regressed on constant (Column 1), value-weighted excess returns of the market based on the CRSP universe (Column 2), and Fama-French five-factor returns downloaded from Ken French's website (Column 3).

	Dependent variable:		
		vw.mkt.rf.analyst	
	avg.ex.ret	CAPM.alpha	FF5.alpha
	(1)	(2)	(3)
mkt.rf		1.016***	1.028***
		(0.005)	(0.006)
smb			0.001
			(0.009)
hml			0.041***
			(0.011)
cma			0.0003
			(0.016)
rmw			0.028**
			(0.011)
Constant	0.005**	−0.001***	−0.002***
	(0.002)	(0.0002)	(0.0002)
Observations	402	402	402
R^2	0.000	0.989	0.990
Adjusted R^2	0.000	0.989	0.990
Residual Std. Error	0.045 (df = 401)	0.005 (df = 400)	0.005 (df = 396)
F Statistic		35,728.420*** (df = 1; 400)	7,837.528*** (df = 5; 396)
Note:			*p<0.1; **p<0.05; ***p<0.01

Firm-level return expectations are constructed together with the stop file provided by IBES to ensure that individual estimates are not stale. IBES keeps track of the activeness of the individual estimates and provides a stop file for price targets.[44] I merge the point-in-time analyst-level expected return file with the stop file on price targets to exclude estimates that analysts and IBES have confirmed to be no longer valid. Furthermore, to avoid stale estimates, I further restrict the estimates to be no older than 90 days when entering mean consensus estimates.[45]

I construct weekly firm-level consensus expected returns by taking the mean of all active analyst-level forecasts, although using the median makes no discernible difference for the main results. I drop analyst-level estimates that are greater than five standard deviations away from the mean estimates, and I winsorize the entire analyst-level database by 1% and 99% before calculating the firm-level consensus. I take the mean of the available expected return estimates for each firm by the end of Saturday each week, or

$$\mu^A_{f,w} = \sum_i \mu^A_{i,f,w} / I_f$$

where I_f is the number of analysts for firm f at week w. For most of the application of the paper, I use firm-level return estimates based on monthly data, which is the consensus data on the last Saturday before each calendar month end.

Appendix F Estimating the Implied Cost of Capital

F.1 Methodology: The ICC Model of Pástor et al. (2008)

I follow the ICC model of Pástor et al. (2008) in estimating the implied cost of capital. Chen et al. (2013) details the way they calculate the ICC model in the cross section, and I therefore follow the procedure outlined in their appendix to estimate the ICC at the stock level.

1. Collect firm-level analyst earnings projections from the IBES monthly summary file. Include firm-level earnings projections at the end of March, June, September,

[44] According to IBES, this stop file "includes stops applied to estimates that are no longer active. This can result from several events, e.g. an estimator places a stock on a restricted list due to an underwriting relationship or the estimator no longer covers the company. Prior to June 1993, actual stop dates did not exist in the archive files used to create the Detail History. An algorithm was developed to determine the date when an estimate became invalid if, for example, a merger between companies occurred or an analyst stopped working for a firm, etc. Estimate that are not updated or confirmed for a total of 210 days, the estimate is stopped."

[45] Engelberg et al. (2019) allows the estimates to be at most 12 months old, in case the estimates are not covered by the stop file, although the choice makes little difference for the main results, as verified in that paper's appendix.

and December for the current fiscal year (the next annual reporting date), the next fiscal year, and the long-term growth forecast (LTG);

2. Estimate the firm-level Implied Cost of Capital (ICC) model. This involves assuming a firm-level long-term growth rate as well as a plowback rate (or 1− payout rate):

   (a) Assuming

$$P_t = \sum_{k=1}^{15} \frac{FE_{t+k}(1 - b_{t+k})}{(1+q_t)^k} + \frac{FE_{t+16}}{q_t(1+q_t)^{15}} = f(c^t, q_t) \tag{33}$$

   where P_t is the stock price, FE_{t+k} is the earnings forecast k years ahead, b_{t+k} is the plowback rate (1−payout), and q_t is the ICC.

   (b) Estimate FE_{t+k} :

      i. FE_{t+1} and FE_{t+2} are proxied by the current fiscal year and the next fiscal year IBES analyst summary file data. $FE_{t+3} = FE_{t+2}(1 + LTG_t)$

         A. Assuming the individual firm-level earnings growth rates to revert to industry growth forecast (LTG_t^{Ind}) by year $t + 16$:

$$\begin{aligned} g_{t+k} &= g_{t+k-1} \times exp[log(LTG_{t+3}^{ind}/LTG_{t+3})/13] \\ &\quad \forall 4 \leq k \leq 15 \\ g_{16} &= g_t^{GDP}, \\ FE_{t+k} &= FE_{t+k-1}(1 + g_{t+k}) \quad \forall 4 \leq k \leq 16 \end{aligned}$$

         where g_t^{GDP} is the GDP growth rate using an expanding rolling window since 1947.

   (c) Estimate b_{t+k} :

      i. b_{t+1} and b_{t+2} are estimated from the most recent net payout ratio for each firm. The net payout ratio is common dividends (DVC in COMPUSTAT) to net income (item IBCOM). If net income is negative, replace it with 6% of assets.[46]

[46] Notice that about 50% of the firms do not pay dividends in the last year. As a result, during the first two years, the plowback ratio is one. This does not mean that the projected earnings for the first two years have no impact on the estimation of the implied cost of capital q_t. Since FE_{t+k} are first calculated using the first two to three years of earnings projections together with the firm- and industry-level LTG, as long as any path during the first 15 years contains a non-zero payout ratio, the first three years of projections will have an impact on the estimation of the ICC.

ii. b_{t+k}, $3 \leq k \leq 16$ is assumed to be

$$b_{t+k} = b_{t+k-1} - \frac{b_{t+2} - b_t^{ss}}{15} \tag{34}$$

where $b_t^{ss} = g_t^{GDP} / q_t$

(d) The q_t is then backed out by solving Eq. (33) and (34) together numerically. When there exist multiple roots, choose the root that is closest to the risk-free rate. Exclude any stock whose price is below one dollar. Winsorize the sample at 1% and 99%. Notice that by assuming the steady-state plowback ratio, we implicitly impose the constraint that

$$q_t \geq g_t^{GDP}$$

since in the steady-state, the plowback ratio can not exceed one.

F.2 Summary Statistics of the ICC and Input Variables

Table 11: Summary Statistics

(a) Empirical distributions of variables

	statistic	Pi	b_1_2	pb_7	EP_1	EP_2	EP_3	LTG	g_ind
1	mean	0.130	0.990	0.848	0.070	0.095	0.114	0.169	0.168
2	std	0.058	0.057	0.073	0.113	0.151	0.187	0.089	0.055
3	std cs	0.056	0.055	0.072	0.111	0.149	0.184	0.086	0.047
4	std ts	0.030	0.031	0.042	0.065	0.065	0.076	0.054	0.026
5	min	0.068	0.589	0.603	-0.153	0.004	0.014	0.040	0.048
6	p25	0.094	1	0.807	0.033	0.045	0.054	0.110	0.130
7	median	0.116	1	0.848	0.053	0.066	0.076	0.150	0.159
8	p75	0.144	1	0.896	0.076	0.090	0.104	0.200	0.195
9	max	0.428	1	0.993	0.876	1.216	1.512	0.500	0.351

(b) AR(1) coefficients

	variable	Pi	b_1_2	pb_7	EP_1	EP_2	EP_3	LTG	g_ind
1	AR(1)	0.920	0.943	0.882	0.897	0.950	0.956	0.893	0.946
2	std	0.005	0.009	0.004	0.008	0.005	0.005	0.005	0.010

(c) Correlations between variables

	Pi	pb_7	b_1_2	EP_1	EP_2	EP_3	LTG	g_ind
Pi	1	-0.746	0.023	0.611	0.747	0.784	0.409	0.336
pb_7		1	0.461	-0.352	-0.417	-0.434	-0.351	-0.284
b_1_2			1	-0.006	0.006	0.010	0.079	0.102
EP_1				1	0.873	0.821	-0.136	-0.107
EP_2					1	0.970	-0.070	-0.063
EP_3						1	-0.030	-0.042
LTG							1	0.511
g_ind								1

Note: Statistics are calculated over the whole sample. Firm-level variables are winsorized at 1% and 99%. "Pi" is the implied constant discount rate (ICC); "b_1_2" is the plowback ratio from the last year; "pb_7" is the implied plowback ratio in year seven. "Ek/P", k = 1, 2, 3 are the fiscal year k earnings consensus estimates divided by the current market capitalization; "LTG" denotes long-term growth forecasts; "g_ind" denotes industry long-term growth estimates where industries are defined based on the 48 Fama-French classifications. In Panel 11a, "std" denotes the standard deviations for the variables over the entire sample, "std cs" and "std ts" are the average cross-sectional standard deviations over time and the time-series standard deviations over different firms, respectively. AR(1) coefficients are estimated by regressing the current value of the variable on its respective one-quarter lagged value based on the whole sample. Standard errors for the AR(1) coefficients are clustered by firm quarter.

Appendix G Robustness Checks

G.1 Equal-Weighted Portfolio Sorts

Table 12: Pre-estimated Misvaluation ($\widehat{\alpha}_t^i$) Sorted Portfolios and Realized Average Stock Returns (1986-06 to 2018-12, value weighted)

All returns, alphas, and their standard errors are expressed in percentages. Stocks are divided into quantile portfolios based on the misvaluation measure $\widehat{\alpha}^i$ at the end of June each year, using the available information up to that point. Portfolios are rebalanced with equal weights every month. "Low" denotes the portfolio with the lowest $\widehat{\alpha}_t^i$. "High-Low" denotes excess returns of a portfolio that goes long on stocks with the highest $\widehat{\alpha}_t^i$ and short on stocks with the lowest $\widehat{\alpha}_t^i$. "SE" are standard errors which are shown in brackets. "Mean ex.ret" are monthly returns over three-month treasury rates. "SR" denotes monthly Sharpe Ratios. "FF-5 alpha" denotes Fama-French 5-factor alphas. "num_stocks" us the average number of stocks included in the portfolio over time. "Ex Ante Misvaluation" denotes value-weighted portfolios $\widehat{\alpha}_t^i$ measured at each end of June. Their standard errors are measured using Newey-West methods based on four lags ("SE (NW-4)").

stats	Low	2	3	4	High	High - Low
Ex Ante Misvaluation	-1.8	-0.88	-0.67	-0.5	-0.3	1.5
SE (NW-4)	(0.19)	(0.32)	(0.21)	(0.18)	(0.15)	(0.12)
CAPM alpha	-0.63	-0.26	0.07	0.2	0.38	0.98
SE CAPM alpha	(0.22)	(0.19)	(0.13)	(0.11)	(0.11)	(0.17)
mean ex.ret	0.19	0.48	0.76	0.87	1.12	0.94
SE ex.ret	(7.22)	(6.28)	(5.56)	(5.23)	(5.91)	(3.31)
SR	0.03	0.08	0.14	0.17	0.19	0.28
CAPM beta	1.33	1.15	1.12	1.09	1.24	-0.08
SE CAPM beta	(0.05)	(0.04)	(0.03)	(0.02)	(0.03)	(0.04)
FF-5 alpha	-0.49	-0.29	-0.05	0.07	0.33	0.79
SE FF-5 alpha	(0.15)	(0.14)	(0.07)	(0.06)	(0.08)	(0.14)
num_stocks	456.68	453.78	456.09	456.09	453	