When Green Investors Are Green Consumers

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Preliminary (link to latest version)

May 10, 2022‡

Abstract

We bring investors with preferences for green assets to a general equilibrium setting in which they also prefer consuming green goods. Their preferences for green goods induce *consumption premia* on expected returns that counterbalance the *green premium* stemming from their preferences for green assets. Because they provide green investors with a financial hedge when green goods become expensive, brown assets command lower consumption premia on average, and green investors allocate a larger share of their wealth towards them. Empirically, the average difference in consumption premia between green and brown assets is 30 to 40 basis points per year and contributes to explaining the limited impact of green investing on polluting firms' costs of capital.

Keywords: Sustainable Finance, Asset Pricing, Portfolio Choice.
JEL codes: G11, G12.

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‡First version: April 30, 2022.

1 Introduction

The same proportion of U.S. individual investors surveyed by the Morgan Stanley Institute for Sustainable Investing (2019), namely, 33% of the them, declare that they "screen their investments according to their interests and values" and "purchase from a brand particularly because of the company's environmental or social impact." This survey suggests that the ethical motives underpinning green investors' capital allocation decisions (Riedl and Smeets, 2017; Krüger et al., 2020) are also reflected in their consumption practices.

Recent research has characterized a *green premium* induced by pro-environmental investment preferences on expected asset returns (Pastor et al., 2021b; Pedersen et al., 2021; Zerbib, 2021): this premium is higher on brown assets relative to green assets because green investors require a higher expected return to hold the assets they dislike in equilibrium. The existence of a green premium is of major importance, especially for investors willing to have an impact on corporate practices, because it can incentivize companies to mitigate their environmental footprints so as to decrease their cost of capital. However, the literature is silent on the effect of preferences for green consumption. How do pro-environmental preferences for consumption translate into investment decisions? How do they interact with pro-environmental preferences for investment?

In this paper, we address these questions by building a general equilibrium model that features a green and a neutral investor, as well as a green and a brown equity asset producing a green and a brown good, respectively. The green investor has preferences towards both investing in the green asset and consuming the green good, while the neutral investor has no preferences for tilting his investment portfolio or his consumption basket. We show that the green investor's preference for consuming the green good gives rise to *consumption premia* on expected returns. Because the brown asset has a higher payoff when the green good becomes expensive, it offers a good hedge for the green investor, and therefore commands lower consumption premia than the green asset in equilibrium. These consumption premia counterbalance the green premium that stems from the green investor's preference for the green asset. This effect arises as the green investor allocates a larger share of her wealth to the brown asset compared to the case without preferences for green consumption. Empirically as well as in the model, the impact is so large that it can offset the green premium.

There are two consumption premia. The first and most substantial one is related to the relative supply of the green good. It is associated with the willingness of the green investor to hedge against a decline in the relative supply of the green good or, equivalently, an increase in its relative price. This risk may materialize as a result of the election of a new government (e.g., the withdrawal from the Paris climate agreement, the repeal of the Clean Power Act, and the suspension of federal subsidies to the renewable energy sector following the election of Donald Trump in the U.S. in 2017), a contraction of international trade (e.g., the rise by 300% in the price of silicon, an essential component of solar panels, mostly produced by China, between August 2021 and October 2021 due to the Covid-19 crisis), the outbreak of an armed conflict (e.g., the increase in the share of coal in electricity production in Germany following the restrictions imposed on Russian gas imports since March 2022), or global energy shortages and the fear of an economic slump (e.g., the increase in coal production in China by 10% in the first two months of 2022 compared with the same period in 2021).

The mechanism of the first premium is explained as follows. When the green good becomes scarcer, the relative payoff of the brown asset increases. This effect occurs because the decrease in the relative supply of the green good is only partially compensated by the increase in its price, provided that the elasticity of substitution between goods is not too low (specifically, greater than one), as suggested by empirical evidence (Papageorgiou et al., 2017). Consequently, as a hedge against the adverse event that the green good becomes scarce or its price increases, the average investor in the economy[1] overweights the brown asset in her portfolio because it comoves positively with the price of the green good she favors. This is also the reason why the consumption premium associated with the shocks on the relative supply of the green good is larger on the green asset than on the brown asset in equilibrium. This premium is driven by a larger beta on that risk for the green asset compared to the brown asset, and a positive price of risk.

The second premium is related to the wealth share of the green investor. When her wealth increases, the resulting buying pressure increases the relative price of

[1] The average consumption preferences of the agents in the economy are tilted towards the green good because the neutral investor has equal preferences for green and brown goods.

the green good, which penalizes the average consumer in the economy who has a preference for that good. However, in that situation, the green asset provides a good hedge because its returns increase with the price of the green good. Consequently, this second consumption premium is lower for the green asset than that for the brown asset. As a result, on average, this premium works in the opposite direction to the first consumption premium in the baseline calibration. Although this effect is more pronounced when the supply of the green good is low, it is very small, of the order of a few basis points.[2] Therefore, the effect of consumption preferences on asset returns is largely dominated by the premium associated with the relative supply of the green good.

Methodologically, we build on general equilibrium models with multiple heterogeneous agents, multiple equity assets, multiple consumption goods, and general preferences, such as recently developped in Sauzet (2022a). We augment that setting by embeding preferences for specific assets, in the spirit of Pastor et al. (2021b), Pedersen et al. (2021), and Zerbib (2021). This setup allows us to (i) derive exact expressions for risk premia, portfolios, and other variables, (ii) study those variables not only on average but also in their dynamic evolution with the state of the economy, a key aspect in our analysis, and (iii) highlight the significant impact of various parameters such as the elasticity of substitution across goods, the preference for green consumption, and the preference for green investing. Each of those advances is made possible by the unique combination of general preferences, the use of continuous time, and the global solution method proposed in Sauzet (2022a).

We provide empirical evidence supporting the existence of the consumption premia on U.S. stock returns. Empirically as well as in the model, the difference in consumption premia between the greenest and the brownest companies reaches up to 40 basis points (bps) per year, which can fully offset the green premium. We also validate that the comovements of green good prices with brown asset returns are higher than those with green asset returns.

To do so, we estimate the beta-representation of the equilibrium equation for risk

[2]Formally, the sign of this relative premium depends on the comovements between the wealth share of the green investor and the relative supply of the green good. These comovements are, in turn, determined by the bias in the portfolio of the green investor towards the green asset, and therefore, by the relative strength of her preference for green investing versus green consumption. Taken together, this leads to the lower wealth share premium on the green asset in the baseline calibration. In all cases, this wealth share premium remains small regardless of its sign.

premia stemming from the model using U.S. common stocks from 2006 to 2019 at a monthly frequency. Since the factor associated with the relative supply of the green good can also be interpreted as being induced by the risk of an increase in the relative price of the green good, we construct this factor based on the prices[3] of goods from the Producer Price Indexes at the industry level and companies' carbon intensities provided by S&P–Trucost. By identifying green funds invested in U.S. equities in Bloomberg, we define the wealth share factor as the ratio of their U.S. stocks under management to the total market capitalization of the investment universe. Finally, we use the environmental rating from MSCI to build the green factor, following Pastor et al. (2021a) and because the carbon intensity is a partial and limited metric to construct a green asset allocation (as explained in Section 4). The green factor is the portfolio long the greenest companies and short the brownest ones, which reflects green investors' preferences for green assets. We control for the five Fama and French (2015) factors and the momentum factor (Carhart, 1997). As a robustness check, since we use realized returns to approximate expected returns, which move in opposite directions when investors' preferences for green assets change unexpectedly (Pastor et al., 2021a; Bolton and Kacperczyk, 2022), we also interact the predictors with a climate news variable constructed from the index in Ardia et al. (2021) to refine the estimate of the green premium by testing the model conditionally. Indeed, recent studies document the effect of climate news on asset returns, including Engle et al. (2020), Alekseev et al. (2021), and Faccini et al. (2021). In all cases, we confirm the predictions of our model regarding the risk of an increase in the relative-price of green goods: (i) the betas are lower for brown than for green stocks, and (ii) the price of risk is highly significant and negative across all specifications. Therefore, the relative price of green goods is associated with a significant consumption premium, which is 30 to 40 bps per year higher on green assets than brown assets, thereby counterbalancing the green premium.

The results of this paper have implications not only for asset pricing but also in terms of the real impact of sustainable investing. Through their preferences for green goods, green investors reduce their upward pressure on the cost of capital of polluting firms. Therefore, the consumption premia help explain the low impact of green investing on mitigating the environmental footprints of companies through the

[3]This has the benefit of being a much easier, cleaner, and higher frequency measure to come by.

cost of capital channel, as suggested by the literature (Berk and van Binsbergen, 2021; De Angelis et al., 2022). Instead, investors' preferences for green consumption make the case for a stronger focus on shareholder engagement to impact companies' practices for a dual reason: by allocating a larger share of their wealth to brown assets, green investors (i) reduce their impact on the cost of capital of brown firms and simultaneously (ii) increase their ability to actively engage with them. Several tools would be available to policymakers to counteract the effect of the consumption premia, including a price cap on green goods or the implementation of a tax on dividends from brown firms, which we briefly discuss in Section 5.

Related literature. This paper contributes to several strands of the literature in asset pricing and sustainable finance. First, to the best of our knowledge, this is the first paper that studies the effects of investors' preferences towards sustainable consumption on asset prices and investors' asset allocation. The construction of a general equilibrium model allows us to uncover these effects. From a theoretical viewpoint, Pastor et al. (2021b), Pedersen et al. (2021), and Zerbib (2021) characterize the green premium driven by investors' preferences for green assets in equilibrium on financial markets. Higher on brown assets than on green assets, this premium corresponds to the compensation required by sustainable investors for holding the assets they like least. Empirical evidence supports the existence of a green premium that is higher on the stock returns of the carbon-intensive companies (Bolton and Kacperczyk, 2021, 2022), polluting companies (Hsu et al., 2022), companies most exposed to climate change risk (Bansal et al., 2016; Barnett, 2022), and least held by green funds (Zerbib, 2021) than on the stock returns of green companies. A similar effect is documented on the cost of equity (ElGhoul et al., 2011; Chava, 2014), expected returns approximated from option-implied information (Sautner et al., 2021), bond yields (Chava, 2014; Baker et al., 2018; Zerbib, 2019; Painter, 2020; Goldsmith-Pinkham et al., 2021; Huynh and Xia, 2021; Seltzer et al., 2022), and real estate prices (Bernstein et al., 2019; Baldauf et al., 2020; Giglio et al., 2021). However, additional effects emerge in a dynamic framework. By reducing asset price informativeness, green investors' preferences may increase firms' cost of capital (Goldstein et al., 2021). In addition, green asset preference shocks increase the cost of capital of green firms (Avramov et al., 2021). Performing empirical analysis on a more recent time frame, Ardia et al. (2021) and Pastor et al. (2021a) find a higher green premium on the greenest stock returns driven by recent capital inflows, reflecting changes in investors' preferences in a transitory phase. Recent papers analyze the impact of climate risks on asset prices and allocation in general equilibrium. Barnett (2022) shows that the price of climate risk is significantly negative, notably driven by the risk of transition to a low-carbon economy. Hambel et al. (2022) highlight that investors' willingness to diversify their assets complements the attempt to mitigate economic damages from climate change in the short run, while in the longer run, a trade-off between diversification and climate action emerges. In this paper, we depart from the asset pricing literature on the impact and hedging of environmental risks, and we focus on the preferences of green investors for green consumption by constructing a two-trees, two-goods, and two-investors general equilibrium model with heterogeneous preferences for investment and consumption. We provide first theoretical and empirical evidence for the existence of significant consumption premia that counterbalance the effect of the green premium on asset returns.

Second, this paper contributes more broadly to the literature on theoretical general equilibrium asset pricing with multiple heterogeneous agents, multiple equity assets, multiple consumption goods, and general preferences, such as recently developed in Sauzet (2022a). This framework, in turn, combines models with multiple agents—they have a long and distinguished history since the seminal contributions of Dumas (1989, 1992), Wang (1996), Basak and Cuoco (1998), Chan and Kogan (2002), and more recently Brunnermeier and Pedersen (2009), Weinbaum (2009), Bhamra and Uppal (2009, 2014), Brunnermeier and Sannikov (2014), Gârleanu and Pedersen (2011), Chabakauri (2013), Gârleanu and Panageas (2015), Drechsler et al. (2018)—, with settings with multiple equity securities but one investor such as Cochrane et al. (2008), Martin (2013), and two consumption goods (Fang, 2019). In other words, the framework generalizes the contributions of Zapatero (1995), Pavlova and Rigobon (2007, 2008, 2010), Stathopoulos (2017), to non-log preferences, and a general aggregation of goods. The unique combination of general preferences, the use of continuous time, and the global solution method proposed in Sauzet (2022a), is key in allowing us to derive most of our results. To all those, we also add the preferences for specific assets: in our case, the green investor prefers the green asset, but the formulation is general and could be used in other contexts.[4]

[4]On the theoretical front, our paper is also related to contributions introducing recursive preferences in continuous-time, for example, Duffie and Epstein (1992), and contributions focusing on the

This paper also contributes to the literature on environnemtal and ecological economics. Specifically, using two goods to capture green and brown consumption is in the spirit of Guesnerie (2004), Hoel and Sterner (2007), Sterner and Persson (2008), Gollier (2010), Traeger (2011), Barro and Misra (2016), and Gollier (2019), in which the two goods are taken to represent aggregate economic capital (physical capital, labor, scientific knowledge, *etc.*) on the one hand, and various ecosystem services that are generated by natural capital on the other. While most of those contributions are based on a representative agent or social planner, we bring this intuition to a general equilibrium economy with several investors. Those investors are heterogenous in their (general) preferences for consumption and investment, and we solve for the decentralized equilibrium, which allows to meaningful discuss portfolios, in addition to risk premia, and other variables. Broadly speaking, our paper is also related to contributions in environmental macroeconomics such as, among others, Pindyck and Wang (2013), Golosov et al. (2014), Cai and Lontzek (2019), van den Bremer and van der Ploeg (2021) on the theoretical side, and Papageorgiou et al. (2017) on the empirical side.

Fourth, and importantly, this paper contributes to the literature on impact investing. Building on the seminal paper by Heinkel et al. (2001), De Angelis et al. (2022) find that the increase in the cost of capital driven by green investing has a limited impact on the practices of the most polluting companies. Through two different approaches, Oehmke and Opp (2019) and Green and Roth (2020) show the importance of investor coordination to finance the companies that need it most and increase their impact on the economy as a whole. In addition, Landier and Lovo (2020) highlight the importance of search frictions in capital markets to increase the impact of investors on corporate practices. From an impact perspective, Broccardo et al. (2020) suggest that shareholder engagement is more effective than the effect on the cost of capital of sustainable investors' asset allocation in most cases. This paper reinforces that suggestion for a different reason: green investors' preferences for green goods weaken the cost of capital channel via the consumption premia and increase the allocation of green investors towards the brownest companies, which are

existence and uniqueness of equilibria in the presence of multiple agents, and possibly multiple goods and incomplete markets, for example, Polemarchakis (1988), Geanakoplos and Polemarchakis (1986), Geanakoplos and Mas-Colell (1989), Geanakoplos (1990), Duffie et al. (1994), Berrada et al. (2007), Anderson and Raimondo (2008), Hugonnier et al. (2012), Ehling and Heyerdahl-Larsen (2015).

the preferred targets for shareholder engagement campaigns.

Outline. The paper is organized as follows. Section 2 describes the set-up of the economy and introduces the two state variables that drive economic mechanisms: the wealth share of the green investor and the relative supply of the green good. Section 3 revisits the impact of green investors on asset prices when they also have preferences for green goods. Section 4 provides empirical evidence supporting our findings. Section 5 discusses the results in light of impact investing challenges and Section 6 concludes. Proofs and additional material are provided in Appendix.

2 The Economy

This section presents the theoretical setup. We introduce a pure-exchange economy with a green and a neutral investor ($i \in \{G, N\}$), and a green and a brown tree ($j \in \{g, b\}$). The trees produce differentiated, green and brown, goods, and are traded as equity assets à la Lucas (1978). The green investor has preferences not only for investing in the green asset (Pastor et al., 2021b; Pedersen et al., 2021; Zerbib, 2021), but also for consuming the green good (Sauzet, 2022a). We show that the equilibrium can be characterized as a function of two state variables: the relative wealth of the green investor, x_t, and the relative supply of the green good, y_t. The setup is summarized in Figure B.1 in Appendix. Appendix A gathers additional results that are omitted in the main text.

Time is continuous and the horizon is infinite, $t \in [0, \infty)$. Uncertainty is represented by a probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ supporting a two-dimensional Brownian motion $\vec{Z} \equiv (Z_g, Z_b)^T \in \mathbb{R}^2$. The filtration $\mathbb{F} = (\mathcal{F}_t)_{t \in [0,\infty)}$ is the usual augmentation of the filtration generated by the Brownian motions, and $\mathcal{F} \equiv \mathcal{F}_\infty$.

2.1 Endowments, prices, assets

The two trees produce differentiated, green and brown, goods. Their output follow geometric Brownian motions

$$\frac{dY_{j,t}}{Y_{j,t}} = \mu_{Y_j} dt + \sigma_{Y_j}^T d\vec{Z}_t, \quad j \in \{g, b\}.$$

The price of the green and brown goods are $p_{g,t}$ and $p_{b,t}$, respectively. We also define the terms of trade $q_t \equiv p_{g,t}/p_{b,t}$, which is the relative price of the green good, and the real exchange rate $\mathcal{E}_t \equiv P_t^G/P_t^N$, which is the relative price of the consumption basket of the green investor. All prices are defined with respect to a numéraire taken to be a CES-basket with weight $a = 1/2$ on both goods.[5]

The green and brown trees are traded as equity assets, with returns given by

$$dR_{j,t} = \frac{dQ_{j,t}}{Q_{j,t}} + \frac{p_{j,t}Y_{j,t}}{Q_{j,t}}dt = \frac{d\left(p_{j,t}Y_{j,t}/F_{j,t}\right)}{p_{j,t}Y_{j,t}/F_{j,t}} + F_{j,t}dt \equiv \mu_{j,t}dt + \sigma_{j,t}^T d\vec{Z}_t, \quad j \in \{g, b\}, \tag{1}$$

where $Q_{j,t}$ are the equity prices, and $F_{j,t} \equiv p_{j,t}Y_{j,t}/Q_{j,t}$ are the dividend yields, for both assets. Drifts $\mu_{j,t}$, which measure conditional expected returns, and diffusion terms $\sigma_{j,t}$, which measure the loadings on the shocks and therefore the conditional volatilities, are obtained from Itô's Lemma and given in Appendix A.

The supply of each equity asset is normalized to unity, and there also exists a bond in net zero supply, which is locally riskless in units of numéraire. Its price is B_t, and the corresponding instantaneous interest rate is r_t, so that $dB_t/B_t = r_t dt$.

2.2 Preferences

Investors have recursive preferences à la Duffie and Epstein (1992) that are defined over consumption, but also over the weights on each asset in their portfolios, $\boldsymbol{w}^i$. Specifically, for the green and neutral investors, $i \in \{G, N\}$,

[5] Specifically, we normalize $\left[(1/2)p_{g,t}^{1-\theta} + (1/2)p_{b,t}^{1-\theta}\right]^{1/(1-\theta)}$ to unity.

$$V_t^i = \max_{\{C_{g,u}^i, C_{b,u}^i, w_{g,u}^i, w_{b,u}^i\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} f^i \left(C_u^i, V_u^i, \boldsymbol{w_u^i} \right) du \right], \tag{2}$$

$$f^i(C, V, \boldsymbol{w}) \equiv \left(\frac{1-\gamma}{1-1/\psi} \right) V \left[\left(\frac{C}{[(1-\gamma)V]^{1/(1-\gamma)}} \right)^{1-1/\psi} - \rho + \Phi^i(\boldsymbol{w}) \right],$$

where γ is the coefficient of relative risk aversion, ψ the elasticity of intertemporal substitution (EIS), and ρ is the discount rate.

Recursive preferences are relevant for two reasons. First, contrary to the case with *log* utility, investors are not myopic and hedging terms arise, which are important drivers of risk premia and portfolios. Second, the coefficient of relative risk aversion is not equal to the reciprocal of the EIS, $\psi \neq 1/\gamma$, which matters quantitatively to obtain risk premia that are closer to their empirical counterparts as well as for the quantitative impact of a potential tax on brown assets as discussed in Section 5. In what follows, parameters γ, ψ, ρ are taken to be identical for both investors. However, the resolution allows for any value so that exploring additional asymmetries stemming from those could be an interesting avenue for future work.

The green investor expresses her pro-environmental motives, in part, by displaying a preference towards the green asset. In this general equilibrium context, we introduce it as functions of the portfolio weights for both investors, $\Phi^i(\boldsymbol{w})$, where $\boldsymbol{w}_t^i \equiv \left(w_{g,t}^i, w_{b,t}^i \right)$, and $w_{g,t}^i$ ($w_{b,t}^i$) is the share of wealth on the green (brown) asset in the portfolio of investor $i \in \{G, N\}$. Specifically, we take

$$\Phi^i(\boldsymbol{w}^i) \equiv (1 - 1/\psi) \left(w_g^i \phi_g^i + w_b^i \phi_b^i \right) \tag{3}$$

Parameter $\phi_g^G \equiv \phi > 0$ captures the additional value that the green investor derives from holding the green asset, in the spirit of Pastor et al. (2021b) and Zerbib (2021). Without loss of generality, we assume that that the neutral investor has no preferences for the green asset ($\phi_g^N = 0$), and that neither investors have preferences for the brown asset ($\phi_b^G = \phi_b^N = 0$). In Section 3, we show that the preference of the green investor for the green asset gives rise to a green premium reducing the expected return on the green asset.[6]

[6]Because the focus of this study is the effect of preferences for green consumption, and for the

In terms of consumption, the basket of each investor is composed of the green and brown goods, which are combined according to an aggregator with constant elasticity of substitution θ, and bias in consumption α^i,

$$C_t^i = \left[{\alpha^i}^{\frac{1}{\theta}} {C_{g,t}^i}^{\frac{\theta-1}{\theta}} + (1-\alpha^i)^{\frac{1}{\theta}} {C_{b,t}^i}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \tag{4}$$

While the neutral investor has no particular preference towards any of the goods ($\alpha^N = 1/2$), the green investor also expresses her pro-environmental preferences by tilting her consumption towards the green good ($\alpha^G > 1/2$). This preference for green consumption is the key novel channel in this paper. In the theoretical characterization of Section 3, we show that it underpins large consumption premia on expected returns that can offset the effect of the green premium stemming from green asset preferences.

Allowing for a general elasticity of subsitution across goods, θ, is also important because its value determines the relative magnitude of the movement in the relative price of the goods for a given shock to relative supply. In turn, this relative magnitude governs the relative dividends on the two assets, and ultimately the tilt in portfolios and the consumption premia.[7]

From the share of wealth that investors allocate to the green and brown equity assets, $w_{g,t}^i, w_{b,t}^i$, they earn expected returns $\mu_{g,t}, \mu_{b,t}$. They allocate the remainder of their wealth $(1 - w_{g,t}^i - w_{b,t}^i)$ to the riskless bond. They use the proceeds of their investments to purchase their desired baskets of consumption $c_t^i \equiv C_t^i / W_t^i$, at price P_t^i. In other words, investors $i \in \{G, N\}$ choose their consumption and portfolios to maximize (2) subject to the following budget constraint

$$\begin{aligned} \frac{dW_t^i}{W_t^i} = & \left(r_t + w_{g,t}^i \left(\mu_{g,t} - r_t\right) + w_{b,t}^i \left(\mu_{b,t} - r_t\right) - P_t^i c_t^i\right) dt \\ & + \left(w_{g,t}^i \sigma_{g,t} + w_{b,t}^i \sigma_{b,t}\right)^T d\vec{Z}_t \end{aligned} \tag{5}$$

To complete the definition of their optimization problems, investors are subject

sake of avoiding further complexity in the model, we consider green investors' preferences for green investments constant over time.

[7]For instance, the common Cobb-Douglas case ($\theta = 1$) leads the relative price of the goods to move exactly enough to compensate relative supply so that relative dividends are unaffected. The relative payoffs of the two assets can then be perfectly correlated, and the portfolio choice between them indeterminate, at least without additional preferences for specific assets. Empirically, $\theta > 1$, which will drive the direction of the hedging terms as discussed in Section 3.

to a standard transversality condition, and W_0^i is given. Note also that $W_t^i \geqslant 0$.

The framework also allows for additional ingredients such as taxes on the dividends of each asset. This extension will be discussed in Section 5.

2.3 Equilibrium and state variables

The definition of the equilibrium is standard: (1) investors solve their optimization problems by taking aggregate stochastic processes as given, and (2) goods and equity markets clear. The detailed definition of the equilibrium is given in Appendix A.4. The bond market clears by Walras's law, which gives rise to the following useful relationship: $W_t^G + W_t^N = Q_{g,t} + Q_{b,t}$. In words, total wealth has to be held in the form of the two equity assets in aggregate.

Stationary recursive Markovian equilibrium. Most importantly, the equilibrium can be recast as a stationary recursive Markovian equilibrium in which all variables of interest are expressed as a function of a pair of state variables $X_t \equiv (x_t, y_t)'$, whose dynamics are also solely a function of X_t. x_t is the wealth share of the green investor, and y_t is the relative supply of the green good.[8] Both are defined below.

The characterization of the solution as a system of coupled algebraic and second-order partial differential equations is the focus of Section 3. For now, let us discuss the intuition behind both state variables. Note that an additional variable, which is not a state variable *per se* but is useful throughout, is $w_{g,t}^M$, the ratio of the green equity price to total wealth. It captures the weight of the green asset in the market portfolio, and it can be shown that[9]

$$w_{g,t}^M \equiv \frac{Q_{g,t}}{Q_{g,t} + Q_{b,t}} = \left(1 + \left(\frac{F_{g,t}}{F_{b,t}}\right) q_t^{-1} \left(\frac{1 - y_t}{y_t}\right)\right)^{-1} \tag{6}$$

[8] Formally, this is shown using a guess and verify approach like, for example, in Gârleanu and Panageas (2015). The variables of interest are: $\{c_{g,t}^G, c_{b,t}^G, c_{g,t}^N, c_{b,t}^N, w_{g,t}^G, w_{b,t}^G, w_{g,t}^N, w_{b,t}^G, \mu_{R_g,t}, \mu_{R_b,t}, r_t, F_{g,t}, F_{b,t}, p_{g,t}, p_{b,t}, P_t^G, P_t^N, q_t, \mathcal{E}_t\}$.

[9] Because the bond is in zero net supply $b_t^M = 0$, the weight of the brown asset in the market portfolio is $w_{b,t}^M = 1 - w_{g,t}^M$ in equilibrium.

Wealth share. The wealth share of the green investor is a measure of the average investor in the economy. It is defined as

$$x_t \equiv \frac{W_t^G}{W_t^G + W_t^N} \tag{7}$$

In this setting, the wealth share is neither constant nor solely a monotonic function of the current relative supply of the green good, y_t. It is therefore required as an additional state variable even when risk sharing is perfect (i.e., even when there are no taxes on dividends). This occurs because preferences are recursive, and due to the fundamental heterogeneity stemming from the green investor's bias towards consuming and investing green.

Relative supply. The relative supply of the green good captures the effect of current fundamentals and is defined as[10]

$$y_t \equiv \frac{Y_{g,t}}{Y_{g,t} + Y_{b,t}}. \tag{8}$$

The relative supply is a key driver of the marginal values of wealth of both investors due to their desire to consume both goods, which stems from their CES consumption baskets. This is particularly true for the green investor who has a strong willingness to consume more green goods, as discussed in Section 3. For the same reason, we show in Section 3 that the relative supply is also the main driver of the relative price of the green good, q_t, and therefore, of the relative price of the consumption basket of the green investor, $\mathcal{E}_t$. This strong monotonic relationship between y_t and $\mathcal{E}_t$ justifies using $\mathcal{E}_t$ – a much easier, cleaner, and higher frequency measure to come by – as a proxy to test our mechanisms empirically in Section 4.[11]

[10]Note that the ratio involves *quantities* of the two different goods. This poses no particular theoretical issue and is used because it simplifies the characterization of the equilibrium. This definition is a monotonic transformation of $Y_{b,t}/Y_{g,t}$: $y_t \equiv (1 + Y_{b,t}/Y_{g,t})^{-1}$, which ensures that the state variable evolves in the bounded interval $[0, 1]$. $Y_{b,t}/Y_{g,t}$ has the clear interpretation of the output of brown good produced per unit of green good. An economic intuition is that one compares the economy to the symmetric point in which relative prices are $q_t = \mathcal{E}_t = 1$.

[11]Similarly, the international finance literature has emphasized mechanisms related to the relative supply, for example, Coeurdacier (2009) and Coeurdacier and Rey (2013). Those have been tested empirically mostly using relative prices or exchange rates, and have become known as "real exchange rate hedging" mechanisms. The model could be equivalently recast using q_t as a state variable. We

As discussed in introduction, a decline in the relative supply of green goods or, equivalently, an increase in the relative price of green goods, may result from a variety of political and economic risk factors such as energy shortages, a contraction of international trade, the election of a new government, or the outbreak of an armed conflict.

Note that because $W_t^i \geqslant 0$ and $Y_{j,t} \geqslant 0$, x_t and y_t are both evolving in the bounded interval $[0, 1]$. This has the advantage that solving for unknown functions on a bounded domain is numerically more stable. Conceptually, as x_t gets closer to either of the boundaries, the economy converges (continuously) to a natural one-investor environment. As y_t gets closer to either of the boundaries, the economy converges to a one-good one-equity asset economy, but this has consequences in terms of marginal values of wealth as the investors still want to consume both goods.

Throughout, we focus on the solution to the decentralized, that is, Radner equilibrium instead of relying on the social planner's problem. The existence and uniqueness of the equilibrium should be guaranteed, for instance, following the work of Duffie and Epstein (1992), who use partial differential equation techniques to prove them in a infinite-horizon Markov diffusion setting with stochastic differential utility, or Chabakauri (2013) and Bhamra and Uppal (2014), who do so constructively for economies with heterogeneous agents and incomplete and complete markets, respectively. Both are also shown in situations with potentially dynamically complete markets[12] using a planner solution in Anderson and Raimondo (2008), and under complete markets with a full set of Arrow-Debreu securities in Hugonnier et al. (2012). As has been known since the seminal example of Hart (1975), however, the introduction of multiple goods could complicate the matter, for instance, because markets can become dynamically incomplete even if the number of assets should technically be sufficient to span risks. Those multiple-good contexts are discussed, for example, in Berrada et al. (2007) and Ehling and Heyerdahl-Larsen (2015), again for the most part through the lens of the Pareto efficient allocation obtained from a social planner. Overall, equilibrium existence and uniqueness in the context of this paper with multiple goods, a bias in consumption and investment, potential imperfect risk

focus on y_t because it makes the intuition sharper, the equations simpler, and because it is exogenous as opposed to q_t that also depends on x_t.

[12] A securities market is potentially dynamically complete if the number of securities with non-colinear payoffs is equal to one plus the number of risk factors (Brownian motions) to be spanned.

sharing (when there exists a tax on dividends), and a decentralized Radner solution, could therefore be analyzed further from a theoretical perspective. This represents an interesting avenue for further research.

2.4 Computation of the equilibrium

Section 3 characterizes all variables of interest as a function of the state variables, $X_t = (x_t, y_t)'$, and a set of unknown functions $\mathcal{G} \equiv \{J_t^G,\ J_t^N,\ F_{g,t},\ F_{b,t},\ q_t,\ w_{g,t}^G,\ w_{b,t}^G\}$.[13] Due to the stationary recursive Markovian structure of the equilibrium, those unknown functions are themselves solely functions of X_t, and are determined by a set of coupled algebraic and second-order partial differential equations.

The resolution is based on projection methods and orthogonal collocation. Specifically, each of the unknown function $g : [0,1]^2 \to \mathcal{D}^g \subseteq \mathbb{R}$ in $\mathcal{G}$ is approximated using Chebyshev polynomials and the equilibrium is solved on an grid based on the zeros of the Chebyshev polynomials. Details are provided in Sauzet (2022a).

The main appeal of this approach is that this is a global solution method, which allows us to trace out the evolution of our variables of interest as a function of the state of the economy. Combined with continuous-time, it makes it possible to cleanly express and solve for the exact subcomponents of the main variables—risk premia, portfolios, goods prices—, as well as our mechanisms of interest, in particular hedging components induced by consumption preferences. Our methodology will prove crucial, for example, when discussing the dynamic aspects of those mechanisms, and how they can be state-dependent.

Projection methods are also well-suited to contexts with multiple state variables. For settings with additional state variables that could become computationally too costly, such as those that might arise when generalizing the framework, extensions of those methods to higher-dimensional settings could prove necessary. One such method consists in naturally extending the concept of projection approaches, but to replace the Chebyshev polynomials in the approximation by neural networks, which are designed specifically to handle high-dimensional contexts. Those "projection methods

[13] J_t^G, J_t^N are introduced in Section 3 and capture (an increasing monotonic transformation of) the marginal values of wealth of each investor. In addition, as a point of notation, for any function g, g_t simply denotes $g(X_t)$, not the time-derivative of g (which is zero because the model is stationary due to the infinite horizon).

via neural networks" for continuous-time models are proposed in Sauzet (2022c).

3 Characterization of the Equilibrium

We now characterize the equilibrium theoretically. In Section 3.1, we start by discussing the marginal values of wealth of both investors, consumption, and goods prices, which are important underpinnings for other variables in the economy. Section 3.2 discusses asset prices, and we show that a preference for green consumption gives rise to consumption premia that counterbalance the green premium stemming from the preference for green investing. Section 3.3 focuses on portfolios, and describes how a preference for green consumption leads investors to allocate a larger share of their wealth to brown assets compared to when they have solely preferences for green investing. Appendix A discusses additional theoretical results such as the evolution of the state variables (Appendix A.5).

Calibration. Unless otherwise specified, parameters are set according to the calibration of Assumption 1. What matters for the preference for green consumption of the green investor is that $\alpha^G > 1/2$. Similarly, what matters for her preference for green investing is that $\phi > 0$. Their exact values mostly have a quantitative impact that is discussed below. In practice, we pick $\alpha^G = 0.85$, and $\phi_g^G = \phi = 1\%$ to broadly match the green premium and consumption premia that we obtain empirically in Section 4. The elasticity of substitution across goods, θ, is also of particular interest for the direction of portfolio biases and risk premia in equilibrium (cf. Sections 3.2 and 3.3). We follow estimations in the environmental economics literature and set it to $\theta = 2 > 1$. For instance, Papageorgiou et al. (2017) provide evidence that this parameter significantly exceeds unity, a condition that is favorable for promoting green growth.[14]

The value of other parameters mostly have a quantitative impact, as long as (i) risk aversion γ is above 1 so that there are hedging terms, and (ii) risk aversion is not equal to the reciprocal of the EIS, $\gamma \neq 1/\psi$, so that preferences are recursive. We

[14]This calibration is also consistent with the elasticity of subsitution across goods in other settings. For instance, this is the case in an international context, as discussed in Imbs and Méjean (2015) among others.

pick a relatively large risk aversion of $\gamma \in \{15, 25, 50\}$, to obtain average risk premia that are in line with the data. Indeed, as is well-known, consumption-based asset pricing models tend to generate somewhat modest risk premia. The effect is purely quantitative, however, and it impacts mostly the "market" component of risk premia, which is not our focus. Our novel consumption premia arise regardless of the exact value of γ, and remain quantitatively large. Similarly, we pick $\psi = 1.25$ to keep a relatively low average riskfree rate r_t. Consistent with the literature (e.g., Bansal and Yaron, 2004), $\psi > 1$, and investors have preference for early resolution of uncertainty ($\gamma > 1/\psi$). In what follows, parameters γ, ψ, ρ are also taken to be identical for both investors. However, the resolution allows for any value so that exploring additional asymmetries stemming from those could be an interesting avenue for future work.

Assumption 1 (Baseline calibration)**.** *Unless otherwise specified, the results in this section are obtained under the following calibration, $i \in \{G, N\}, j \in \{g, b\}$:*

- *Preference for green consumption: $\alpha^G = \alpha = 0.85$, $\alpha^N = 1/2$,*
- *Preference for green investing: $\phi_g^G = \phi = 1\%$.*
- *Elasticity of substitution between goods: $\theta^i = \theta = 2$,*
- *Numéraire basket: $a = 1/2$,*
- *Risk aversion: $\gamma^i = \gamma \in \{15, 25, 50\}$,*
- *Elasticity of intertemporal substituion: $\psi^i = \psi = 1.25$,*
- *Discount rate: $\rho^i = \rho = 1\%$,*
- *Output: $\mu_{Y_j} = \mu_Y = 2\%, \sigma_{Y_1} = (4.1\%, 0)^T, \sigma_{Y_2} = (0, 4.1\%)^T$ (no fundamental correlation).*

3.1 Marginal values of wealth, consumption, goods prices

The marginal value of wealth of the investors underly many decisions in the economy. To characterize them, note that due to the homotheticity of preferences, the value functions of the investors $i \in \{G, N\}$ can be expressed as

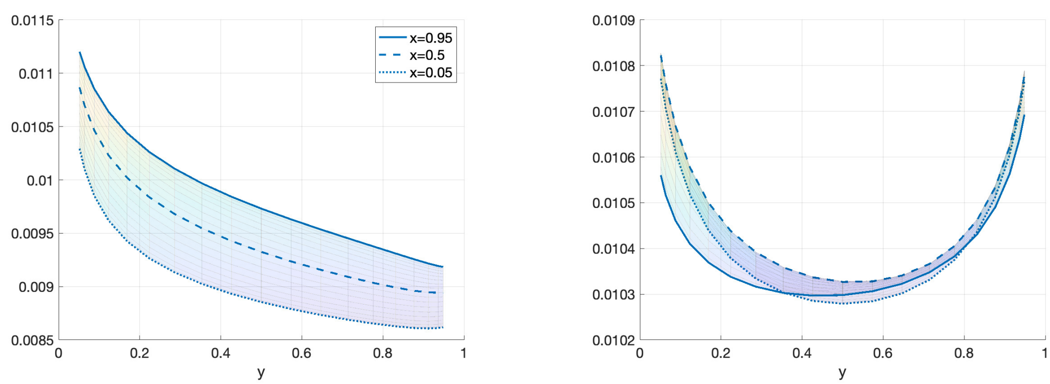
$$V^i(W_t^i, x_t, y_t) = \left(\frac{W_t^{i1-\gamma}}{1-\gamma} \right) J^i(x_t, y_t)^{\frac{1-\gamma}{1-\psi}} \tag{9}$$

Because W_t^G, W_t^N mostly have an impact in levels, the marginal values are driven primarily by functions J_t^G, J_t^N. In the remainder of the text, we therefore refer to them as (monotonic transformations of) the marginal values of wealth. Those quantities underpin the dynamics of the stochastic discount factors of both investors in the economy[15], which in turn determine portfolios, asset prices, and other economic decisions.

The evolution of J_t^i are governed by two Hamilton-Jacobi-Bellman equations, summarized in Proposition A.4 in Appendix. Figure 1 shows the result for both investors in the baseline calibration as a function of the relative supply of the green good (y_t), shown on the horizontal axis, and the wealth share of the green investor (x_t), shown as different curves.

Figure 1: Marginal values of wealth

(a) Green investor (J_t^G) (b) Neutral investor (J_t^N)



Notes: Based on the calibration of Assumption 1. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

The intuition is as follows, and will be at the core of the consumption premia and portfolio biases.

As the green good becomes relatively scarce, that is, as y_t decreases, both investors have to switch some of their consumption to the brown good. The green investor is particularly negatively affected: she prefers consuming more of the green good ($\alpha^G >$

[15] The stochastic discount factors for investors $i \in \{G, N\}$ are given by

$$\xi_t^i \equiv \xi_0^i \exp\left\{\int_0^t \left(\Theta_1 P_u^{i\,1-\psi} J_u^i + \Theta_2\right) du\right\} W_t^{i\,-\gamma} J_t^{i\,\frac{1-\gamma}{1-\psi}}$$

with $\Theta_1 \equiv -(\gamma - 1/\psi)/(1 - 1/\psi)$ and $\Theta_2 \equiv \rho(\gamma - 1)/(1 - 1/\psi)$.

1/2), but cannot due to its low relative supply, or equivalently its high relative price. Her marginal value of consumption, which is the same as her marginal value of wealth J_t^G following a standard envelope argument, therefore strongly increases. The neutral investor does not have a specific preference towards the green good ($\alpha^N = 1/2$), but still likes consuming both, due to his CES consumption basket. He is therefore also negatively impacted, and his marginal value of wealth J_t^N increases as any of the goods becomes relatively scarce ($y_t \to 0$ or $y_t \to 1$) because he would prefer a more balanced basket, that is, a more comparable relative supply or relative price of both goods. This effect for the neutral investor is, however, much more muted.

Similarly, as her share of wealth x_t increases, the preference of the green investor for green consumption puts upward pressure on the price of her preferred green good. This induces her to relunctantly tilts her consumption slightly towards the brown good, and her marginal value of wealth J_t^G increases. On the other hand, because he has no particular bias in consumption, the marginal value of wealth for the neutral investor J_t^N is little affected by x_t. In practice, the changes in the economy-wide marginal value of wealth $\widetilde{J}_t \equiv x_t J_t^G + (1 - x_t) J_t^N$ are therefore dominated by those of J_t^G.

From the Hamilton-Jacobi-Bellman equations in A.4, a first set of first-order conditions yield expressions for consumptions, summarized in Proposition A.5, which emphasize once again the underlying role of J_t^i: $c_t^i \equiv C_t^i / W_t^i = P_t^{i-\psi} J_t^i$. Details are shown in Appendix A.7 together with the corresponding figures, which are as expected. Combining with market-clearing conditions, one obtains Equation (10) for the relative price of the green good q_t, shown in Proposition 1.

Proposition 1. *The relative price of the green good, $q_t = q(X_t) \equiv p_{g,t}/p_{b,t}$, solves the following non-linear equation*

$$q_t = S_t^{1/\theta} \left(\frac{1 - y_t}{y_t} \right)^{1/\theta} \tag{10}$$

where

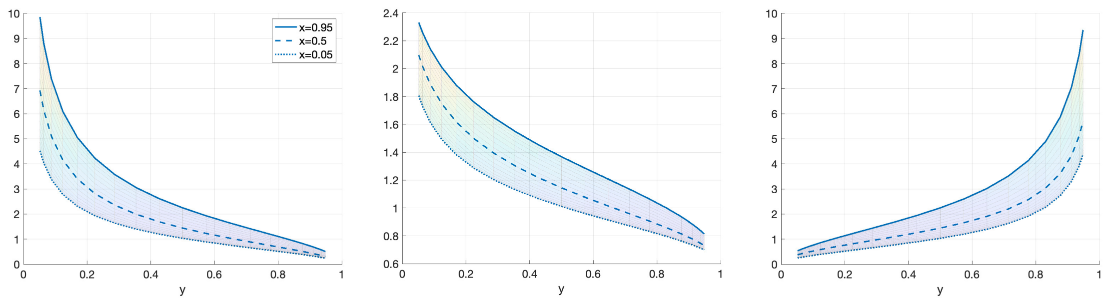
$$S_t = \frac{\alpha^G J_t^G x_t P_t^{A\theta-\psi} + \alpha^N P_t^{N\theta-\psi} J_t^N (1 - x_t)}{(1 - \alpha^G) P_t^{G\theta-\psi} J_t^G x_t + (1 - \alpha^N) P_t^{N\theta-\psi} J_t^N (1 - x_t)}$$

Prices $p_{g,t}, p_{b,t}, P_t^G, P_t^N, \mathcal{E}_t$ follow from the definition of the numéraire and Proposition A.5, and are shown in Proposition A.6.

Figure 2 shows the resulting relative price in the baseline calibration of Assumption 1. As expected, the relative price of the green good, q_t, strongly decreases as the green good becomes more abundant, that is, as y_t increases (Panel (a)). The pattern is similar for the relative price of the consumption basket of the green investor, $\mathcal{E}_t \equiv P_t^G/P_t^N$ (Panel (b)), whose evolutions are driven by q_t as shown in Proposition A.6. Because of this strong monotonic relationship, y_t, q_t, $\mathcal{E}_t$ can be used interchangeably, and we based the empirical tests of our model in Section 4 on $\mathcal{E}_t$ – a much easier, cleaner, and higher frequency measure to come by.[16] q_t and $\mathcal{E}_t$ also increase as the share of wealth of the green investor x_t increases, due to the upward pressure on the price of the green good stemming from the green investor's preference for consuming green.

Figure 2: Relative prices and dividends

(a) Green good ($q_t \equiv p_{g,t}/p_{b,t}$) (b) Green consumption basket ($\mathcal{E}_t \equiv P_t^G/P_t^N$) (c) Relative dividends ($p_{g,t}Y_{g,t}/(p_{b,t}Y_{b,t})$)



Notes: Based on the calibration of Assumption 1. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Beyond relative prices themselves, which drive relative consumption decisions, the relative dividends of the green asset are also of particular interest. They are shown in Panel (c) of Figure 2 and are obtained as

$$\frac{p_{g,t}Y_{g,t}}{p_{b,t}Y_{b,t}} = q_t\left(\frac{y_t}{1-y_t}\right) = S_t^{\frac{1}{\theta}}\left(\frac{1-y_t}{y_t}\right)^{\frac{1-\theta}{\theta}} \tag{11}$$

[16]Similarly, the international finance literature has emphasized mechanisms related to the relative supply, for example, Coeurdacier (2009) and Coeurdacier and Rey (2013). Those have been tested empirically mostly using relative prices or exchange rates, and have become known as "real exchange rate hedging" mechanisms. The model could be equivalently recast using q_t as a state variable. We focus on y_t because it makes the intuition sharper, the equations simpler, and because it is exogenous as opposed to q_t that also depends on x_t.

Let us consider a situation in which the green good becomes scarce (y_t decreases). In that case, the relative quantity of output of the green tree, $Y_{g_t}/Y_{b,t} = y_t/(1-y_t)$, decreases. As discussed above, the relative price of the green good, $p_{g,t}/p_{b,t}$, therefore, increases. However, because green and brown goods remain substitutable enough ($\theta > 1$), the effect on the relative price remains muted and the relative dividends of the green tree decrease overall. In other words, relative dividends and relative supply move in the same direction, an observation that will prove important for the direction of portfolio biases, and for risk premia. Indeed, as we discuss in Section 3.2 below, relative dividends are the main drivers of the relative returns on the two assets, while changes in dividend yields (i.e., equity prices relative to fundamentals) play a limited role.

The case in which green and brown goods are very poor substitutes (broadly $\theta < 1$)[17] would have the counterintuitive implication that the payoff of an asset would be *low* when the quantity of goods that it produces is *high*. Most importantly, it is also inconsistent with empirical estimates in the enviromental economics literature that put θ strongly above unity, a condition that is also favorable for promoting green growth (see, for instance, Papageorgiou et al., 2017).

Finally, the relative dividends of the green asset also increase as the wealth share of the green investor increases, consistent with her preference for green consumption that puts an upward pressure on the relative price of the green good. This effect is more muted in the baseline calibration, however.

3.2 Asset prices

Second moments. Let us start with second moments, which underpin part of the intuition for risk premia, and portfolios.

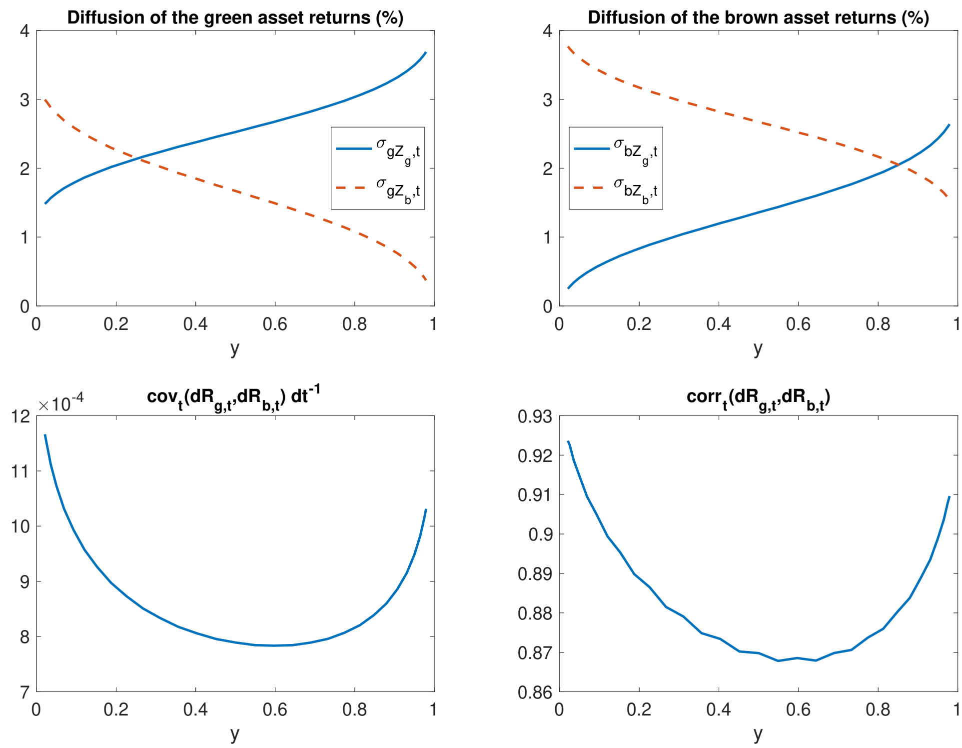
Recall that the diffusion terms for both asset returns, $j \in \{g, b\}$, are $\sigma_{j,t} \equiv \sigma_{p_j,t} + \sigma_{Y_j} - \sigma_{F_j,t}$. In practice, although dividend yields, $F_{j,t}$, are time-varying in our setting with recursive preferences and heterogeneity, the effect of their changes via $\sigma_{F_j,t}$ remains comparatively muted. The patterns in returns diffusions are, therefore, mostly driven by $\sigma_{p_j,t}, \sigma_{Y_j}$, that is, by movements in the relative dividends on both assets. In turn, the relative dividends on the green asset $p_{g,t}Y_{g,t}/(p_{b,t}Y_{b,t})$ evolve

[17] Coeurdacier (2009) shows in a CRRA context based on zero-order approximations that the exact value at which the switch occurs is a non-linear function of all parameters, although it is close to 1.

in the same direction as the relative shocks to supply, $Y_{g,t}/Y_{b,t}$. As described in Section 3.1 above, this is because goods are good enough substitutes ($\theta > 1$) so that the effect on the relative price of the goods is moderate enough to not overturn the impact of the relative supply. *In fine*, this implies that for most of the state space, the returns on the green asset tend to load more on the output shocks to the green tree ($\sigma_{gZ_g,t} > \sigma_{gZ_b,t}$), while the returns on the brown asset load more on the output shocks to the brown tree ($\sigma_{bZ_b,t} > \sigma_{bZ_g,t}$).[18] In other words, the returns on the green (brown) asset tend to increase more with positive shocks to the supply of the green (brown) tree. Figure 3, which shows the diffusion terms for the returns on both assets in the baseline calibration, confirms that this is indeed the case: on average, the loading of the green asset returns on the green output shock ($\sigma_{gZg,t}$, blue curve) is larger than their loading on the brown output shock ($\sigma_{gZb,t}$, orange curve), and vice versa for the brown asset returns.

[18]Note that if θ were to be below unity, movements in the relative prices of the goods would be so extreme that relative dividends would move invertedly with relative supply, so that the returns on the *green* asset would ultimately load more on the output shock to the *brown* tree. This implication is both counterintuitive and inconsistent with empirical estimates in the enviromental economics literature that put θ strongly above unity, a condition that is also favorable for promoting green growth (see, for instance, Papageorgiou et al., 2017) and on which we focus.

Figure 3: Second moments of returns



Notes: Based on the calibration of Assumption 1. x_t is the wealth share of the green investor. y_t is the relative supply of the green good. The figure shows a cut in which $x_t \approx 1/3$, consistent with empirical estimates.

Beyond those main patterns, note that the returns on both assets also load on both shocks, although in a more limited way. This leads the assets to be strongly correlated: on average, $\mathrm{corr}_t(dR_{g,t}, dR_{b,t}) \approx 0.9$, which is noteworthy because the outputs of the trees themselves have no fundamental correlation ($\sigma_{Y_g Z_b} = \sigma_{Y_b Z_g} = 0$). In other words, the large correlation between asset returns emerges purely endogenously. This phenomenon is driven again for a large part by movements in the relative prices of the goods, as well as by the patterns of the marginal values of wealth of both investors J_t^i (and therefore of their stochastic discount factors). Economically, this emphasizes a strong contagion taking place through asset markets: a shock on the output of a given tree has a large impact on the returns of the other tree, and can therefore impact both investors beyond its impact on goods markets. Figure B.6 in Appendix plots the (instantaneous) covariance and correlation of asset returns, as well as the volatility of both assets, $\left(\sigma_{j,t}^T \sigma_{j,t}\right)^{1/2}$, in the baseline calibration. It highlights not only that the comovements of assets are large on average, but also that they change significantly with the state of the economy. For example, the correlation of returns is highest when the green good is relatively rare (small y_t) and the green investor dominates the economy (large x_t). Indeed, the green investor is particularly unhappy in this situation because she would like to consume more of the green good but cannot. This leads her marginal value of wealth to be high, and has an impact on both assets because she needs to hold a large share of them when she holds most of the wealth in the economy. Those, in turn, impact risk premia, and the portfolio choice of investors.

Risk premia. Proposition 2 presents the expected excess returns on the green and brown assets. Proposition A.1 in Appendix generalizes those expressions to the case in which risk aversion and EIS differ across investors, and in which both investors have preferences towards both assets, $\phi_j^i \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t.

Proposition 2. *The expected risk premia on the green and brown equity assets are*

$$\begin{aligned} \mu_{g,t} - r_t &= \gamma \sigma_{g,t}^T \sigma_{\widetilde{W},t} - x_t \phi - \gamma \sigma_{g,t}^T \sigma_{\tilde{J},t} \\ \mu_{b,t} - r_t &= \gamma \sigma_{b,t}^T \sigma_{\widetilde{W},t} \qquad\quad - \gamma \sigma_{b,t}^T \sigma_{\tilde{J},t} \end{aligned} \tag{12}$$

where

$$\begin{aligned} \sigma_{\widetilde{W},t} &\equiv w_{g,t}^M \sigma_{g,t} + (1 - w_{g,t}^M) \sigma_{b,t} \\ \sigma_{\tilde{J},t} &\equiv \left(\frac{1}{\gamma}\right) \left(\frac{1-\gamma}{1-\psi}\right) \left(x_t \sigma_{J^G,t} + (1 - x_t) \sigma_{J^N,t}\right) \end{aligned}$$

and $\widetilde{W}_t$ is the total wealth, $\tilde{J}_t$ is the economy-wide marginal value of wealth, and $\sigma_{J^G,t}, \sigma_{J^N,t}$ are the geometric diffusion terms of J_t^G, J_t^N obtained as in Remark A.1 in Appendix.

The expressions for risk premia are composed of three terms.

The first term is a *wealth component* driven by the covariance of each risky asset return with the total wealth in the economy $\widetilde{W}_t$. It can be thought of as a "market" component. Intuitively, an asset that comoves a lot with total wealth provides little diversification benefits, is therefore risky, and commands a high risk premium in equilibrium. This is the usual financial diversification component that exists even when investors are myopic, and makes them want to hold some of both assets to maximize the Sharpe ratio of their portfolios.

The second term is the green premium characterized by Pastor et al. (2021b) and Zerbib (2021). Because the green investor has a preference towards investing in the green asset ($\phi > 0$), she accepts a lower expected return to hold it and the expected returns on that asset decrease. In addition, this effect scales with the wealth share of the green investor, x_t. Because we set the specific preference towards the brown asset and the specific preferences of the neutral investor to zero ($\phi_b^G = \phi_g^N = \phi_b^N = 0$), the brown asset does not display any such premium. This is without loss of generality, however, and the green premium term should be understood in a relative sense between green and brown assets.

The third term is a hedging term that constitutes our novel consumption premia, and deserves more emphasis.

From a broad perspective, this term is driven by the comovement of asset returns with the economy-wide wealth-weighted marginal value of wealth, $\widetilde{J}_t$. Intuitively, an asset whose returns are large when $\widetilde{J}_t$ is large is a good hedge because it pays when it is most valuable for the economy as a whole. Such an asset is therefore less risky, and commands a lower risk premium in equilibrium.

Importantly, note that such hedging components—and therefore our novel consumption premia—would be completely absent with log, mean-variance, or CARA preferences that have been popular in the literature, but with which investors would be myopic.

To make the intuition more precise, note that because of our Markovian setting, we can decompose the hedging term as follows[19]

$$-\gamma \sigma_{j,t}^T \sigma_{\tilde{J},t} = -\sigma_{j,t}^T \sigma_{x,t} x_t \left(\frac{1-\gamma}{1-\psi} \right) \left\{ x_t \frac{J_{x,t}^G}{J_t^G} + (1-x_t) \frac{J_{x,t}^N}{J_t^N} \right\} - \sigma_{j,t}^T \sigma_{y,t} y_t \left(\frac{1-\gamma}{1-\psi} \right) \left\{ x_t \frac{J_{y,t}^G}{J_t^G} + (1-x_t) \frac{J_{y,t}^N}{J_t^N} \right\} \tag{13}$$

In words, the novel consumption premia are composed of a wealth-hedging premium (hedging of movements in x_t), and a relative-supply-hedging premium (hedging of movements in y_t).

The quantities of risk for those two premia are driven by the (instantaneous) covariance of the asset returns with the state variables, x_t, y_t, that fully characterize the state of the economy: $\text{cov}_t \left(dR_{j,t}, dx_t \right) dt^{-1} = \sigma_{R_j,t}^T \sigma_{x,t} x_t$ and $\text{cov}_t \left(dR_{j,t}, dy_t \right) dt^{-1} = \sigma_{R_j,t}^T \sigma_{y,t} y_t$. On average, we expect the latter, $\text{cov}_t \left(dR_{j,t}, dy_t \right) dt^{-1}$, to be positive for the green asset, and negative for the brown asset. This is because, as explained above, the returns on the green asset tend to load more on shocks to the green output $dZ_{g,t}$, which also increase $Y_{g,t}$ and, therefore, increase the relative supply of the green good, $y_t \equiv Y_{g,t}/(Y_{g,t} + Y_{b,t})$. Conversely, the returns on the brown asset tend to load more on $dZ_{b,t}$, which also increase $Y_{b,t}$ and, therefore, decrease y_t. The sign of $\text{cov}_t \left(dR_{j,t}, dx_t \right) dt^{-1}$ depends on the covariance between x_t and y_t, which is endogenous and depends on investors' portfolios, which in turn depend on ϕ. It is discussed below.

The remaining pieces are the prices of those risks, which are driven by preference parameters γ, ψ, but most importantly by how the economy-wide wealth-weighted marginal value of wealth evolves with those state variables: this is captured by $J_{x,t}^i$, $J_{y,t}^i$, the derivatives of the marginal values of wealth of both investors $i \in \{G, N\}$ with respect to each state variable. The economy is composed of an investor with a preference for green consumption and an investor who is neutral, so that it has on average a tilt towards prefering the green good. In other words, because J_t^G strongly decreases with the relative supply of the green good ($J_{y,t}^G \ll 0$), the economy-wide wealth-weighted marginal value of wealth $\tilde{J}_t$ is a decreasing function of y_t on average.

[19]Again, the framework allows for potentially different γ^i, ψ^i, ρ^i for both investors. In that case, the economy-wide risk aversion is state-dependent, $\gamma_t \equiv \left(x_t/\gamma^G + (1-x_t)/\gamma^N \right)^{-1}$, and the weighting in the economy-wide marginal value of wealth $\tilde{J}_t$ also reflects differences in those parameters.

That is, situations in which y_t is low are adverse states of the world, and the price of y_t-risk is positive (recall the minus sign in Equation (13)). Therefore, an asset that comoves with the relative supply of the green good y_t is risky, because it is a poor hedge against those adverse states and hence, commands a higher risk premium in equilibrium. Conversely, the price of x_t-risk is expected to be negative on average. Indeed, as shown in Section 3.1, the evolution of $\widetilde{J}_t$ with x_t is again dominated by the marginal value of wealth of the green investor J_t^G, which tends to increase with her wealth share x_t ($J_{x,t}^G > 0$) due to the upward pressure she puts on the price of her preferred green good.

Taken together, we therefore expect the green asset, whose returns comove positively with y_t, to be riskier in terms of relative supply risk than the brown asset whose returns comove negatively with y_t. Therefore, the green asset is expected to command a higher relative-supply-hedging premium on average. The sign of the hedging of the wealth share risk is more ambiguous and is discussed below, it turns out to be negative but small in our benchmark.

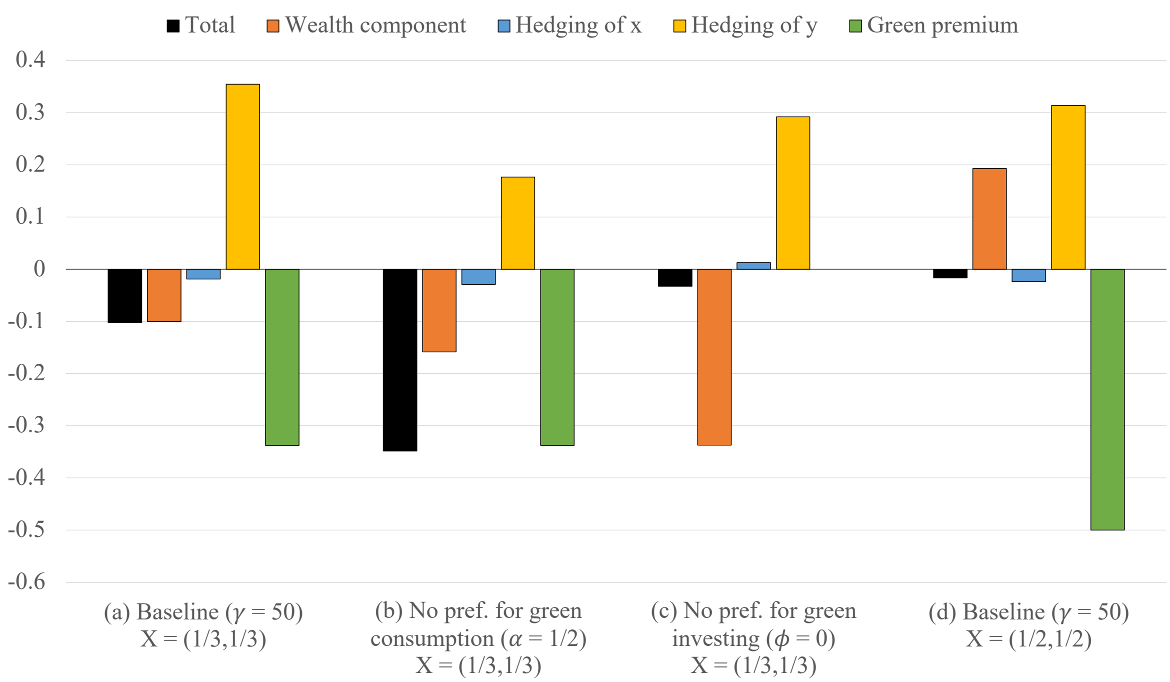
Panel (a) of Figure 4 shows the difference in expected returns between the green and the brown asset, $(\mu_{g,t} - r_t) - (\mu_{b,t} - r_t)$, and its components, in our baseline calibration of Assumption 1. To get a sense of the average premia differentials, they are shown at the point at which the green investor holds one third of the wealth ($x_t = 1/3$), and the relative supply of the green good is one third ($y_t = 1/3$), broadly consistent with empirical estimates in Morgan Stanley Institute for Sustainable Investing (2019).

The wealth component, $\gamma\sigma_{j,t}^T\sigma_{\widetilde{W},t}$, which can be understood as a market component, is important to get risk premia that are quantitatively broadly in line with the data: on average, $\mu_{j,t} - r_t \approx 4.2\%$, only slightly lower than their empirical counterparts.[20] In practice, this component depends for the most part on how dominant a given asset is in total wealth, that is, on the weights of the assets in the market portfolio ($w_{g,t}^M, w_{b,t}^M$). Those market weights are inherently related to y_t, the relative supply of both assets, as seen in Panels (a) and (b) of Figure B.10 in Appendix. In practice, the wealth components therefore drive the overall shape of the risk premia on both assets with the state of the economy, in particular with respect to the rela-

[20]Getting such values for the average risk premia is the main reason for which we pick a high calibration of risk aversion γ. Indeed, as is well-known, consumption-based asset pricing models tend to generate somewhat modest risk premia.

tive supply. They are equal for both assets broadly around the point at which their relative supply is equal, $y_t = 1/2$, although the preference for green investing leads the green asset to be slightly overvalued so that its weight in the market portfolio ($w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$) is slightly larger on average. In other words, the wealth component is slightly larger for the green asset on average, even though the difference is dominated by variations with the state of the economy. Because this term is more common, however, it is not our focus in this paper.

Figure 4: Average difference in risk premia between green and brown asset $(\mu_{g,t} - r_t) - (\mu_{b,t} - r_t)$ (%)



Notes: Based on the calibration of Assumption 1, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good. The figure shows the difference between risk premia on the green and brown asset, and their components, at $X_t \equiv (x_t, y_t)' = (1/3, 1/3)$ for Panels (a), (b), (c), and at $X_t = (1/2, 1/2)$ for Panel (d).

Of more interest are the green premium, and the novel consumption premia.

When the green investor holds about one third of total wealth, like in the data, the green premium is $-x_t\phi \approx -0.333\%$. Recall that this green premium should be interpretated in a relative way, so that on average the expected excess returns on the green asset is 33.3 basis points smaller than those on the brown asset, when we focus purely on the effect of the preference of investors for green investing. This is also visible from Panel (c) that sets $\phi = 0$, and is consistent with our empirical estimates in Section 4.

Most importantly, and this is our main result, consumption premia broadly compensate the green premium: on average (i.e., at $X_t = (1/3, 1/3)$), the expected excess returns on the green asset is 36.6 basis points larger than those on the brown asset, when we focus purely on the effect of the preference of investors for green consumption. In other words, the only reason why $\mu_{g,t} - r_t$ is larger overall at $X_t = (1/3, 1/3)$ is the mostly mechanical wealth component. This is visible in our baseline calibration of Panel (a). This can be compared to the case in which the green investor has no preference for green consumption ($\alpha = 0.5$) so that the green premium dominates,[21] and in which she has no preference for green investing ($\phi = 0$) so that consumption premia dominate. The effect can, in fact, become larger than the green premium for larger risk aversion γ, an EIS closer to $\psi = 1$, larger bias towards consumption α, and in some parts of the state space as discussed below. Consistent with our intuition above, this is driven by a positive relative-supply-hedging premium (35.5 basis points), which constitutes the vast majority of the consumption premia.

The hedging of relative wealth is negative for both assets, and in particular slightly more negative for the green asset but the effect is quantitatively small (-1.9 basis points). The intuition for this negative relative premium is as follows. In the baseline, the wealth share loads more on shocks to the output of the green tree ($dZ_{g,t}$): Figures A.3 and A.4 in Appendix indeed show that $\sigma_{xZ_g,t}x_t > \sigma_{xZ_b,t}x_t$ in magnitudes, and $\sigma_{xZ_g,t}x_t > 0$ for any X_t while the loading of x_t on shocks to the brown output $\sigma_{xZ_b,t}x_t$ flips sign, for example, as y_t increases. As shown in Proposition A.3 in Appendix, the loadings of the wealth share are themselves endogenous and follow those patterns provided that the portfolio of the green investor is biased enough towards the green asset ($w^G_{g,t} - w^M_{g,t} > w^G_{b,t} - w^M_{b,t}$). In short, in the baseline, a positive shock to the output of the green tree tends to increase the wealth share x_t.[22] Because such a shock also

[21] Note that there still exist consumption premia even without preference for green consumption ($\alpha = 1/2$). This is because investors still want to consume both goods, and therefore remain sensitive to movements in relative prices. The effect is quantitatively more muted, but continues to show the appeal of bringing green investors to this generaly equilibrium context in which they also consume, even in that case.

[22] Relatedly, because the relative supply y_t also tends to increase with positive shocks to the green output, the wealth share x_t and relative supply y_t are positively correlated in the baseline.

tends to increase the returns on the green asset more, as explained previously, this leads the covariance of the green asset returns with x_t to be larger than that of the brown asset returns. In other words, the quantity of x_t "risk" is larger for the green asset (Figure B.4). Combined with the negative price of this risk described above (Figure B.5), this leads to the negative relative wealth-share-hedging premium for the green asset in the baseline calibration. In practice, the magnitude of the bias in portfolios depends strongly on the preferences so that this conclusion depends on the exact calibration. For instance, the hedging of this risk can lead to a positive premium if preferences for green consumption (α) are strong enough, while preferences for green investing (ϕ) moderate enough, like in Panel (c) of Figure 4. In most cases however, and regardless of its sign, the magnitude of this effect remains small.[23]

Dynamics of risk premia. Interestingly, those patterns of the risk premia and their subcomponents also vary strongly with the state of the economy, an aspect that is possible to discuss only thanks to our global solution method.

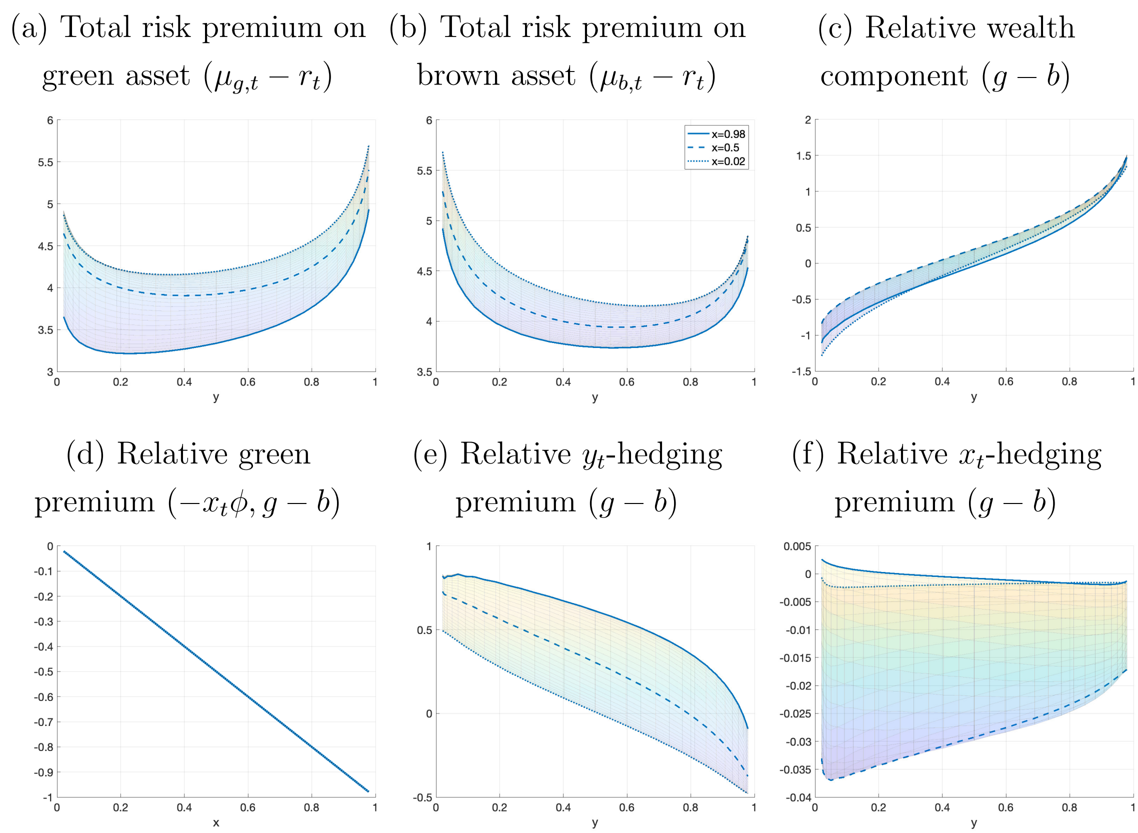
Panels (a) of Figure 5 shows that the expected risk premium on the green asset $\mu_{g,t} - r_t$ increases, in particular, as the relative supply of the underlying green tree, y_t, increases. This pattern is driven by the wealth component shown in Panel (c): as y_t becomes large, the green good starts to dominate the economy, so that the green asset also starts to dominate total wealth ($w^M_{g,t} \equiv Q_{g,t}/[Q_{g,t} + Q_{b,t}]$, the weight of the green asset in the market portfolio, increases towards 1). In such situations, the risk on the green asset is difficult to diversify away so that the green asset is risky and commands a higher risk premium.[24] Conversely, the expected returns on the brown asset increases as y_t decreases, as shown in Panel (b). Figure B.3 in Appendix also shows that states of the world in which one of the good becomes scarce (low or high y_t) are associated with a lower riskfree interest rate r_t, consistent with higher precautionary saving motives.[25]

[23]The introduction of a tax on dividends as discussed in Section 5 can reinforce the impact of x_t-hedging, because it can lead risk sharing across investors to become imperfect.

[24]In other words, because the green asset dominates total wealth as y_t becomes large, the covariance of this asset with total wealth is large because it is broadly equal to the covariance of the asset with itself. This leads the wealth component of the risk premia, which is driven by the covariance with total wealth, to be large for the green asset.

[25]Note that in some calibrations, for example, with $\gamma = 50$, r_t is negative, which is in line with real interest rates being negative empirically in the recent period even for longer maturities (e.g., Figure B.2 in Appendix shows that this is case for the 10-year market yield on inflation-indexed U.S. Treasury Securities since 2019). This has no particular impact on the equilibrium. For instance,

Figure 5: Returns



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

The expected excess returns on both assets also decrease with x_t the share of wealth held by the green investor (Panels (a) and (b)). For the green asset, this is driven mostly by the increasing impact of the preference for green investing (ϕ) as the green investor becomes larger in the economy, that is, by an increasing green premium. This can be seen in Panel (d), which plots the green premium on the green asset relative to the brown asset as a function of x_t. For the brown asset, however, this pattern is driven by the state-dependence in the hedging of relative supply risk, which becomes more strongly negative for the brown asset as the green investor—who is more worried about this risk—holds increasingly more wealth. The riskfree interest rate also increases with x_t, a fact that is consistent with the pattern of borrowing and

Figure B.3 in Appendix shows that $r_t > 0$ for $\gamma = 15$, and risk premia and portfolios in that case are similar to those with a larger γ except in terms of magnitude.

saving discussed in Section 3.3.

Panels (e) and (f) confirm that consumption premia are themselves very time-varying. The hedging of relative supply risk on the green asset relative to the brown asset is positive and large for most of the state-space, as shown in Panel (e). Again, it increases as the green investor—who is particulalry worried about this risk—becomes larger in the economy, that is, as x_t increases. This positive relative premium on the green asset also strongly increases as the relative supply of the green good y_t decreases: for example, it reaches close to 1% for large x_t and small y_t. This is consistent with the green investor being *especially* worried about relative supply risk when her preferred good becomes very scarce, and suggests that hedging terms can grow and continue to compensate the green premium even as the latter becomes larger when the green investor becomes dominant. Those increases in the positive relative y_t-premium on the green asset are driven by changes in both the quantities and price of risk as x_t increases and y_t decreases, as shown in Figures B.4 and B.5 in Appendix. Finally, as discussed above and shown in Panel (f), the relative premium on the green asset stemming from wealth share hedging is negative on average in the baseline, although more muted (and dependent on the calibration). It is largest in magnitude around $x_t = 1/2$, the point of the state space at which a switch occurs in which investor dominates the economy. Those patterns are driven by the quantities and price of x_t-risk shown in Figures B.4 and B.5.

3.3 Portfolios

We conclude this characterization by a brief discussion of the optimal portfolios of both investors. Proposition 3 shows that those are Merton (1973)-type portfolios that are composed of two pieces.[26]

The first term is common to both investors and corresponds to the myopic portfolio that would be chosen by a one-period mean-variance investor. That is, it is the usual financial diversification component driven by the risk premia on both assets, normalized by volatilities, and is partly related to the market portfolio $(w_{g,t}^M, w_{b,t}^M)$.

[26] Again, Proposition A.2 in Appendix generalizes those expressions to the case in which risk aversion and EIS differ across investors, and in which both investors have preferences towards both assets, $\phi_j^i \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t.

In this context, however, this first term also embeds the preference of the green investor for green assets (ϕ). Equation (14) shows that it is isomorphic to the expected returns on the green asset being perceived as (relatively) larger by the green investor. As expected, this term therefore makes her tilt her portfolio allocation towards the green asset in equilibrium. In other words, it is the manifestation of the green premium for portfolios.

Proposition 3. *The optimal portfolios of the green and neutral investors $j \in \{G, N\}$ are given by*

$$\begin{pmatrix} w_{g,t}^G \\ w_{b,t}^G \end{pmatrix} = \frac{1}{\gamma} \left(\Sigma_t^T \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi \\ \mu_{b,t} - r_t \end{pmatrix} + \left(\frac{1-\gamma}{1-\psi}\right) \Sigma_t^T \left(\frac{J_{x,t}^G}{J_t^G} x_t \sigma_{x,t} + \frac{J_{y,t}^G}{J_t^G} y_t \sigma_{y,t} \right) \right\}$$
$$b_t^G = 1 - w_{g,t}^G - w_{b,t}^G \tag{14}$$

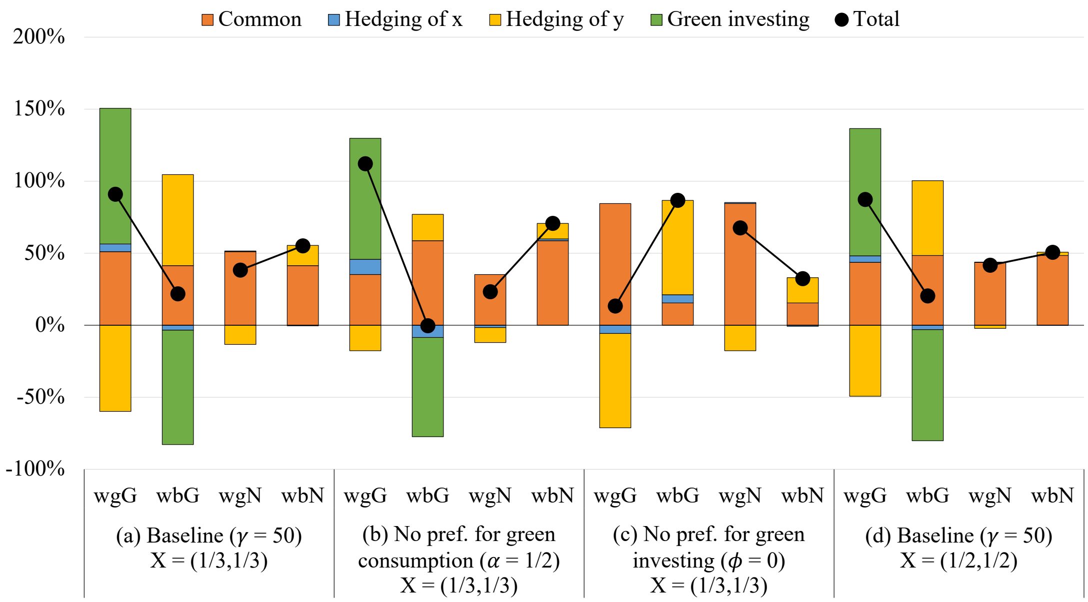
$$\begin{pmatrix} w_{g,t}^N \\ w_{b,t}^N \end{pmatrix} = \frac{1}{\gamma} \left(\Sigma_t^T \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t \\ \mu_{b,t} - r_t \end{pmatrix} + \left(\frac{1-\gamma}{1-\psi}\right) \Sigma_t^T \left(\frac{J_{x,t}^N}{J_t^N} x_t \sigma_{x,t} + \frac{J_{y,t}^N}{J_t^N} y_t \sigma_{y,t} \right) \right\}$$
$$b_t^N = 1 - w_{g,t}^N - w_{b,t}^N \tag{15}$$

where $w_{g,t}^i, w_{b,t}^i, b_t^i$ are the portfolio weights (as a share of wealth) allocated to the green equity asset, the brown equity asset, and the riskless bond, and $\Sigma_t \equiv \begin{bmatrix} \sigma_{g,t} & \sigma_{b,t} \end{bmatrix}$.

The second component are hedging terms, absent with log or myopic preferences. They are the counterpart of the *consumption risk premia* for portfolios, and capture the way investors tilt their allocation to insure against changes in the state of the economy, captured by $X_t = (x_t, y_t)'$. Investors do so by overweighting assets whose payoffs are large when they find it most valuable, that is, when their individual marginal values of wealth are high, so that hedging terms are governed by the covariance between risky returns, and individual marginal values of wealth, J_t^G, J_t^N.

Overall, the common term drives the broad pattern of the portfolios of both investors throughout the state space, corrected for the preference of the green investor for green investing (ϕ), while the hedging term captures how investors differentially deviate from this broad pattern. Hedging terms are therefore a prime variable of interest in our economy with heterogeneous investors.

Figure 6: Portfolios at $X_t = (1/3, 1/3)$, and $X_t = (1/2, 1/2)$



Notes: Based on the calibration of Assumption 1, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good. The figure shows portfolios and their components at $X_t \equiv (x_t, y_t)' = (1/3, 1/3)$ for Panels (a), (b), (c), and at $X_t = (1/2, 1/2)$ for Panel (d). $w^i_{g,t}, w^i_{b,t}$ are the weights (as % of wealth) on the green and brown asset in the portfolio of investor $i \in \{G, N\}$.

Figure 6 shows the corresponding portfolio weights on the risky assets for each investor as a percent of their wealth ($w^i_{g,t}$, $w^i_{b,t}$ for $i \in \{G, N\}$), as well as their components, for various calibrations. Figure B.7 in Appendix shows the weights in the market portfolio for comparison: $w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$, $w^M_{b,t}$.[27] Finally, we also plot the portfolio weight that each investor allocates to the riskless bond to borrow or save (b^i_t) in Figure B.8. Like for risk premia, to get a sense of average portfolios, all those variables are shown at the point at which the green investor holds one third of the wealth ($x_t = 1/3$), and the relative supply of the green good is one third ($y_t = 1/3$), broadly consistent with empirical estimates in Morgan Stanley Institute for Sustainable Investing (2019) (except for Panel (d) for which $X_t = (1/2, 1/2)$).

Panel (a) shows that in the baseline calibration, the green investor significantly

[27] Recall that the bond is in zero net supply so that $b^M_t = 0$, and $w^M_{b,t} = 1 - w^M_{g,t}$.

tilts her allocation towards the green asset: on average (i.e., at $X_t = (1/3, 1/3)$), she invests $w^G_{g,t} \approx 91\%$ of her wealth in it, as opposed to $w^G_{b,t} \approx 22\%$ in the brown asset. This is significantly more biased towards the green asset than the market portfolio, $w^M_{g,t} \approx 56\%$, $w^M_{b,t} \approx 44\%$ (Figure B.7 in Appendix). The neutral investor, because he is less sensitive to changes in relative supply and wealth share, is willing to take the other side of this trade: on average, he invests more of his wealth in the brown asset, $w^N_{b,t} = 55\%$, than in the green asset, $w^N_{g,t} = 38\%$.

As expected, the overweighting of the green asset in the portfolio of the green investor is driven by her preference towards green investing ($\phi > 0$), shown in green in Figure 6. In the baseline, this component taken separately would lead her to overweight the green asset by an *additional* 94% of her wealth, substantially beyond the 51% dictated by the common component (shown in orange) that is identical for both investors. Conversely, it would lead her to underweight the brown asset by 79% of her wealth, compared to the 41% dictated by the common component.

Most importantly, and this is our main novel result in terms of portfolios, the impact of green investing is again strongly counterbalanced once it is brought to our general equilibrium context. Indeed, the hedging term related to relative supply (shown in yellow in Figure 6), which is mostly stemming from the preferences of the green investor towards green consumption ($\alpha > 1/2$), leads her to *under*weight the green asset by about 60% of her wealth. This arises because the returns on the green asset are comparatively much smaller when the relative supply of the green good, y_t, is low (cf. Section 3.2), that is, when the green investor values it most because her marginal value of wealth J^G_t is high in those states of the world (Section 3.1).

Overall, this leads the green investor to cut the overweighting stemming from green investing by about two-third. This is also visible in Panel (c), which shows that when she has no preference for green investing ($\phi = 0$), she would end up investing much more of her wealth in the brown asset overall ($w^G_{g,t} \approx 13\%$, $w^G_{b,t} \approx 87\%$). Therefore, the green investor would pick a portfolio that is biased towards the *brown* asset in equilibrium, compared to the market portfolio. Conversely, the counterbalancing impact of hedging terms is also visible in Panel (b): without preference for green consumption ($\alpha = 1/2$), the green investor would invest an even larger share of her wealth in the green asset ($w^G_{g,t} \approx 112\%$, $w^G_{b,t} \approx 0\%$).

A few remaining comments on portfolios are in order.

First, like for risk premia, the impact of the hedging of the wealth share depends on the calibration, but remains in most cases more muted. In the baseline, it leads the green investor to increase back the weight in her portfolio on the green asset slightly as seen in Figure 6 (blue component).

Second, because the green investor is more sensitive to the risks associated with consumption preferences, especially the one related to relative supply, she is more eager to strongly tilt her portfolio according to her preferences. In practice, she is in fact willing to borrow in the riskless bond to *lever* her risky portfolio weights slightly: at $X_t = (1/3, 1/3)$, she borrows $|b_t^G| = |1 - w_{g,t}^G - w_{b,t}^G| = 13\%$ of her wealth (Figure B.8 in Appendix). The remaining investor, because he is neutral, is willing to accomodate the green investor by lending $b_t^N = 6\%$ of his wealth. Those patterns of borrowing and lending are also reflected in the riskfree rate: r_t increases as x_t increases, that is as the green investor, who is a borrower, gets a larger of total wealth. Introducing portfolio constraints, for example, such as borrowing or shorting limits, could be an interesting avenue for further research that could enrich those phenomena.

Third, because of the preference for green investing and green consumption of the "average investor" in the baseline, the equity price of the green asset $Q_{g,t}$ is slightly overvalued compared to an economy with $\phi = 0$ and $\alpha = 1/2$. In other words, the weight on the green asset in the market portfolio, which is nothing but its equity price divided by total wealth $w_{g,t}^M \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$, is slightly larger than the weight on the brown asset in the market portfolio, $w_{b,t}^M \equiv Q_{b,t}/(Q_{g,t} + Q_{b,t})$.[28] This is visible in Figure B.7 in Appendix, which plots the market portfolios in various cases, especially in Panel (d) that focuses on the symmetric point $X_t = (1/2, 1/2)$. At that point, the weight in the market portfolio would be $w_{g,t}^M = w_{b,t}^M = 50\%$ for $\alpha = 0.5$ (or more generally symmetric αs) and $\phi = 0$, as opposed to $w_{g,t}^M = 65\%, w_{b,t}^M = 35\%$ in the baseline calibration. Similarly, absent preference for green consumption and green investing, the market weights at $X_t = (1/3, 1/3)$ would be strongly tilted towards the *brown* asset unlike Panels (a), (b), (c) of Figure B.7 for which either $\phi > 0$, $\alpha^G > 1 - \alpha^N = 1/2$, or both.

Finally, portfolio weights, as well as how biased they are with respect to the market portfolio, are also strongly state-dependent. This is shown in Figures B.9 and B.10 in Appendix, which plot both as a function of the state variables $X_t = (x_t, y_t)'$ in the baseline calibration. For instance, both investors increase the share of their wealth invested in the brown asset as the relative supply of the green good y_t decreases

[28] In equilibrium, the latter is $w_{b,t}^M = 1 - w_{g,t}^M$ because the bond is in zero net supply.

(consistent with the market portfolio). They do so because of heightened relative-supply hedging motives, and despite the fact that the common component should make them want to *decrease* their portfolio weight in that asset. The green investor also increases her weight on the green asset as y_t decreases because the impact of her preference for green assets is heightened by a strongly increasing correlation across assets. Because the weight of the green asset in the market portfolio *decreases* at the same time, however, the green investor has to rely on an increasing amount of borrowing in the riskfree bond $|b_t^G|$ to tilt her risky portfolio as she desires in what she percevies as bad times (i.e., when y_t decreases). All in all, and even though those time variations in portfolios are not the focus of our empirical analysis in Section 4, they could provide interesting avenues for futher tests and research.

4 Empirical Evidence

In this section, we provide empirical evidence on the effect of the consumption premia on asset returns. Strongly supporting our theoretical results, we find that the annual consumption premia on a basket of green assets is 30 to 40 bps higher than that on a basket of brown assets on average, with a substantially significant price of risk.

4.1 Data and factor construction

To test the existence of the consumption premia, we use the beta-representation implied by the equilibrium equation for the expected returns in equilibrium (Proposition 2). Given the strong negative monotonic relationship between the relative supply of the green good, y_t, and the relative price of the green consumption basket, $\mathcal{E}_t$, discussed in Section 3.1 and shown in Panel (b) of Figure 2, we focus on testing the following beta-representation for the green and brown assets, $j \in \{g, b\}$

$$\mu_{j,t} - r_t = \alpha_{j,t} + \lambda_{M,t}\beta_{j,M,t} + \lambda_{x,t}\beta_{j,x,t} + \lambda_{\mathcal{E},t}\beta_{j,\mathcal{E},t} + \lambda_{GP,t}\beta_{j,GP,t} + \varepsilon_{j,t} \qquad (16)$$

GP_t is a factor capturing the green premium discussed below, and the quantities of risk are defined as

$$\beta_{j,x,t} \equiv \frac{\text{cov}_t\left(dR_{j,t}, dx_t\right)}{\text{var}_t(dx_t)}\ ,\ \beta_{j,\mathcal{E},t} \equiv \frac{\text{cov}_t\left(dR_{j,t}, d\mathcal{E}_t\right)}{\text{var}_t(d\mathcal{E}_t)} \tag{17}$$
$$\beta_{j,\widetilde{W},t} \equiv \frac{\text{cov}_t\left(dR_{j,t}, dR_{M,t}\right)}{\text{var}_t(dR_{M,t})}\ ,\ \beta_{j,GP,t} \equiv \frac{\text{cov}_t\left(dR_{j,t}, GP_t\right)}{\text{var}_t(GP_t)}$$

where $dR_{M,t}$ is the return on the market. We refer to $d\mathcal{E}_t$ and dx_t as the relative-price factor and wealth factor, respectively. The theoretical expressions for the prices of risk $\lambda_{M,t}, \lambda_{x,t}, \lambda_{\mathcal{E},t}$, and $\lambda_{GP,t}$ can also be derived theoretically from Equation (12) in Proposition 2.

Given the magnitude of the consumption premium related to the relative supply of the green good suggested by our theoretical resutls and the strong negative relationship between y_t and $\mathcal{E}_t$, we expect the price of risk associated with the relative-price of green goods, $\lambda_{\mathcal{E},t}$, to be significantly negative. Indeed, the average investor in the economy values the assets of which the returns are positively correlated with the prices of green goods. On the other hand, the small magnitude and the change in sign of the consumption premium driven by the wealth share of the green investor suggested by the model do not lead us to have a strong prior on the estimate of the price of risk $\lambda_{x,t}$.

We start our analysis from all the common stocks (share type codes 10 and 11) listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and National Association of Securities Dealers Automated Quotations exchange (NASDAQ; exchange codes 1, 2, and 3) in the CRSP database, and we map them to the 6-digit North American Industry Classification System (NAICS).

To construct the relative-price factor, we use the carbon intensity of companies provided by S&P–Trucost, as sustainable consumers are primarily concerned with the climate footprint of their consumption (Schanes et al., 2016), in line with the goals of the Paris Agreement. The carbon intensity of a company is defined as the amount of greenhouse gases emitted by the company across its value chain over a year, normalized by its annual revenues.[29] The firms producing the greenest (brownest) goods are, therefore, those with the lowest (highest) carbon intensity. Given our

[29] We use Trucost's default emission scope, which includes direct and first-tier indirect emissions, that is, for a given firm, the emissions related to the its activity (scope 1), induced by the generation of its purchased energy (scope 2), and those of its suppliers (upstream scope 3).

specific focus on supply shocks, we use the Producer Price Indexes computed by the U.S. Bureau of Labor Statistics as proxies for the prices of goods. Because those indexes are available by 6-digit NAICS industry only, we compute the carbon intensity of each industry as the market-value weighted carbon intensity of all firms in that industry.[30] After normalizing all prices to 100 in December 2006, we construct $\mathcal{E}_t$ as the ratio of the average production price of the 33% greenest industries to the average production price of all industries in the economy in each month

$$\mathcal{E}_t \equiv \frac{\frac{\alpha^G}{|\Omega_{33\%,t}|} \sum_{i \in \Omega_{33\%,t}} P_t^i}{\frac{\alpha^N}{|\Omega_t|} \sum_{i \in \Omega_t} P_t^i},$$

where Ω_t and $\Omega_{33\%,t}$ stand for the set of all industries and the set of the 33% greenest industries in t, respectively. $|\Omega_t|$ and $|\Omega_{33\%,t}|$ are the cardinalities of Ω_t and $\Omega_{33\%,t}$, respectively. From the perspective of the model, using this ratio corresponds to defining the good produced by the 33% greenest firms as the green good. We use a simple average as opposed to the exact theoretical expression in Proposition A.6 to avoid taking the model too literally, and having the ratio depend strongly on the elasticity of substitution θ. By the same token, we set $\alpha^G = 1$ and $\alpha^N = \frac{1}{2}$, without loss of generality, so that the ratio does not depend on specific values for the bias in consumption. The relative-price factor, $d\mathcal{E}_t$, is thus defined as the change in $\mathcal{E}$ between two consecutive months.

Following Zerbib (2021), we construct the green investor wealth share factor by first identifying 453 funds whose asset management mandates include environmental guidelines (flagged as "environmentally friendly," "climate change," and "clean energy"), of which the investment asset classes are defined as "equity," "mixed allocation," and "alternative," and with the geographical investment scope including the United States, using data from Bloomberg as of December 2019. We obtain their assets under management on a quarterly basis using FactSet, and for each quarter, we calculate the ratio of the market value of the U.S. stocks in the 453 green funds to the market value of the investment universe. We then interpolate this ratio for each month using a polynomial of degree 2, and we construct x_t by smoothing this

[30] When a price index is not available for a given N-digit NAICS industry, we use the price index for the N-1-digit industry that includes it.

interpolation.[31] The variable x_t approximates the increase in wealth dynamics of U.S. investors with pro-environmental preferences. Therefore, the wealth factor, dx_t, is defined as the change in x_t between two consecutive months.

We construct the green factor using the environmental rating provided by MSCI. We do not use carbon intensity data because building an asset allocation by minimizing its carbon intensity leads to skewing the portfolio towards a few low-emitting sectors (e.g., banking, insurance, business services, entertainment, healthcare, telecommunications), which poses a dual problem for green investors: (i) some key sectors for the ecological transition, but with higher emissions, are left out of the allocation (e.g., utilities, electrical equipment, construction materials), and (ii) the portfolio loses much in sectoral diversification. This is why other environmental metrics are often used by green investors in combination with the carbon intensity, such as the green share (Mirova, 2021) or the portfolio alignment to a temperature trajectory (Raynaud et al., 2020). In addition, MSCI is the world's largest provider of ESG ratings (Eccles and Stroehle, 2020) and covers more firms than the other ESG raters (Berg et al., 2022). We construct the green factor as a green minus brown value-weighted portfolio that is long the tercile of the greenest firms and short the tercile of the brownest firms (In et al., 2019; Pastor et al., 2021a), excluding firms without ratings.[32] Given the recent rise of green investing (Zerbib, 2021) and the availability of environmental ratings, we begin our analysis in March 2006.

Finally, we proxy for the market component by using excess returns on the market, i.e. the standard Mkt-RF factor, from Fama and French (2015). In the estimations, as is usual, we also control for the small-minus-big (SMB), high-minus-low (HML), conservative-minus-agressive (CMA), robust-minus-weak (CMA) factors (Fama and French, 2015), and the momentum (MOM) factor (Carhart, 1997). We obtain all those factors from Kenneth French's website.[33]

[31]We filter out from the series the seasonal component, which we calculate in the following standard way. First, we extract the trend of the series using a convolution filter. Then, we remove the trend from the series, and we calculate the seasonal component as the average of the detrended series for each period.

[32]Because we do not yet have historical MSCI ratings and since the order relationship between firms' ratings changes little over time, we use the latest rating for each firm to approximate its rating over the whole period under consideration. In a subsequent version of the paper, we will use the rating history to improve the estimate of the green premium.

[33]The URL is: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

All in all, we work with a scope of 1836 stocks and estimate the specification of Equation (16) using a two-pass (Fama and MacBeth, 1973) regression from March 2006 to December 2019. In the second pass, we run cross-sectional regressions of the time-series average of each asset returns on a constant and the betas, which are winsorized at the 1% level.

Table 1 provides the descriptive statistics of the variables used. Specifically, the estimated betas of the relative-price factor are consistent with the predictions of the model. Indeed, the average (winsorized) relative-price beta of the 33% greenest companies is lower than that of the 33% brownest companies by 1.04 on average, reflecting the fact that the comovements of green good prices with brown asset returns are higher than those with green asset returns.

Table 1: Summary statistics (%, monthly)

Variable	$\mu_{j,t} - r_t$	$Price\_Hedge_t$	$Wealth\_Hedge_t$	GMB_t	dC_t	$Mkt - RF_t$	HML_t	SMB_t	RMW_t	CMA_t	MOM_t
Mean	0.343	-0.019	0.001	0.142	0.051	0.755	-0.147	0.109	0.253	0.001	0.056
Std	10.969	0.192	0.001	2.418	0.158	4.062	2.574	2.406	1.604	1.420	4.507
Min	-32.920	-0.753	-0.002	-9.356	-0.249	-17.230	-11.110	-4.920	-3.880	-3.230	-34.300
25th	-5.589	-0.096	0.000	-1.227	-0.058	-1.270	-1.750	-1.710	-0.660	-1.010	-1.550
Median	0.171	0.000	0.001	0.171	0.033	1.060	-0.310	0.180	0.340	-0.010	0.300
75th	5.841	0.039	0.001	1.592	0.134	3.130	1.120	1.610	1.250	0.900	2.610
Max	38.755	0.659	0.003	9.569	0.730	11.350	8.210	7.040	4.940	3.700	12.750
Count	215217	215217	215217	215217	215217	215217	215217	215217	215217	215217	215217

4.2 Estimation

The results of the estimation strongly support the existence of the consumption premia in the cross-section of stock returns. Consistent with the characterization of the model, the effect of preferences for green goods on stock returns is driven by the relative-price factor. Table 2 summarizes the estimates for several specifications, including controling for different sets of factors. The remaining details of the estimation are shown in Table C.1 in Appendix.

The price of risk associated with the relative-price of green goods is negative and highly significant in all estimated specifications (Panel A, specifications [1] to [8]): it is -3 bps per month with t-stats ranging from -2.6 to -4.7. Moreover, the difference in the relative-price premium between the 33% greenest and the 33% brownest firms is between 26 and 40 bps per year depending on the specification considered (Panel B).

The estimates are robust to the use of a median beta by group rather than an average beta. On the other hand, although the wealth consumption premium is significant and worth almost zero when we control for the five Fama French factors, we do not find convincing evidence of a significant wealth premium across all specifications, consistent with mixed and more muted results in the model.

The price of risk associated with the GMB factor is not significant and the average green premium spread between the 33% greenest and the 33% brownest firms ranges from -16 to 38 bps per year. The low significance and the change in sign of the price of risk is likely to be explained by the shift in investors' preferences for green assets over the period considered (Pastor et al., 2021b). Indeed, recent years have witnessed a massive influx of capital into green assets, thereby increasing their returns compared to those of similar brown assets. Ardia et al. (2021) and Pastor et al. (2021a) document this effect, which we also confirm by repeating the estimation from November 2012 (cf. Table C.3 in Appendix): the price of GMB risk is positive and significant, and the difference in premium between the terciles of the green and brown companies ranges from 1.23% to 1.73% per year.

To control for the effect of changes in preferences driven by environmental news, we construct the factor dC_t following Pastor et al. (2021a), which captures the change in climate concerns from the Media Climate Change Concerns index of Ardia et al. (2021). We use dC_t as an instrument to test the model conditionally in the spirit of Cochrane (2005). Specifically, we interact the predictors in t with dC_{t-1} and repeat the estimations of specifications (4) through (8) to estimate the green premium after controlling for changes in climate concerns. The results are reported in columns (9) through (13). In most cases, the price of risk of the green premium slightly decreases and its significance slightly increases. Importantly for our study, the relative-price premium remains highly significant (with t-stats from -1.9 to -3.6) and its differential between the greenest and brownest companies ranges from 17 and 38 bps per year.

Overall, the results of our estimations strongly support the model predictions. Through the relative-price factor, the consumption premia have offset the negative green premium since March 2006 and added to the positive green premium since November 2012. As such, the consumption premia, related to pro-environmental preferences for green goods, helps explain the limited effect of green investing on the cost of capital of brown firms as discussed in Section 5.

Table 2: Empirical estimation of consumption premia

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Panel A: Risk Prices (monthly, %)													
$Relative\_Price\_Hedge_t$	-0.030	-0.030	-0.031	-0.028	-0.029	-0.030	-0.027	-0.020	-0.021	-0.018	-0.014	-0.022	-0.024
	(-3.775)	(-3.756)	(-4.213)	(-3.771)	(-4.609)	(-4.711)	(-3.774)	(-2.626)	(-3.474)	(-2.211)	(-1.921)	(-2.909)	(-3.641)
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		(0.59)	(0.47)	(0.831)	(1.366)	(1.058)	(2.056)	(-0.056)	(0.298)	(1.641)	(2.24)	(0.38)	(1.528)
GMB_t			-0.029	-0.023	0.031	0.048	0.034	0.073	-0.063	-0.023	-0.007	0.035	0.034
			(-0.451)	(-0.362)	(0.476)	(0.796)	(0.493)	(1.1)	(-1.029)	(-0.367)	(-0.118)	(0.538)	(0.644)
Panel B: Premium differential between green and brown assets (annual, %)													
$Relative\_Price\_Hedge_t$	0.150	0.165	0.358	0.385	0.365	0.331	0.404	0.265	0.383	0.280	0.167	0.374	0.308
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GMB_t			-0.193	-0.159	0.224	0.339	0.156	0.384	-0.447	-0.175	-0.046	0.174	0.195
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM	CAPM	FF3	FF3MOM	FF5	FF5MOM
Conditioning var.									ΔC_{t-1}	ΔC_{t-1}	ΔC_{t-1}	ΔC_{t-1}	ΔC_{t-1}

Notes: Variables are defined in Section 4.1 (t-stats in brackets). Full sample: Mar. 2006-Dec. 2019. Returns and betas winsorized (1% level).

5 Implications for Impact Investing

Impact investing covers several investment strategies that aim at encouraging companies to change their practices. By inducing a green premium that increases the cost of capital of polluting companies, investors' preferences for green assets are supposed to incentivize companies to mitigate their environmental footprints. Yet, empirical evidence suggests that the real impact is small. De Angelis et al. (2022) find that by internalizing the climate externalities of the companies in which they invest, green investors drive companies to reduce their carbon footprint at a very low rate, in the range of 1% to 3% per year. Oehmke and Opp (2019), Landier and Lovo (2020), and Green and Roth (2020) emphasize the fact that green investors internalizing only the environmental footprints of the companies in which they invest do not maximize their global impact. The impact is larger when they internalize the environmental footprints of all firms in the economy, irrespective of whether they invest in them (Oehmke and Opp, 2019; Green and Roth, 2020), and by prioritizing firms where the inefficiencies induced by the externalities are particularly acute and the capital search frictions are strong (Landier and Lovo, 2020).

Our findings have a dual implication from the perspective of impact investing. First, by showing that the green premium is counterbalanced by green investors' preferences for green consumption, we contribute to explaining why the impact of green investors on the cost of capital and practices of polluting firms is limited. Second, the overweighting of polluting companies in green investors' portfolios is an opportunity to leverage their shareholder position so as to increase their engagement with these companies (e.g., private or public communications, votes in general assemblies) and push them to become greener. This conclusion reinforces the one by Broccardo et al. (2020) who suggest that shareholder engagement is often more effective than green investment without accounting for consumption preferences.

Be it to accelerate the ecological transition in general, or specifically to mitigate the effect of the consumption premia on firms' cost of capital, policymakers have different options, such as capping green good prices or introducing a dividend tax. For example, we can show (see Sauzet, 2022a, for details) that when investors pay a tax τ on dividends from brown firms, the expected returns are rewritten as follows:

$$\begin{aligned} \mu_{g,t} - r_t &= \gamma \sigma_{j,t}^T \sigma_{\widetilde{W},t} - \gamma \sigma_{g,t}^T \sigma_{\tilde{J},t} - x_t \phi^G \\ \mu_{b,t} - r_t &= \gamma \sigma_{j,t}^T \sigma_{\widetilde{W},t} - \gamma \sigma_{b,t}^T \sigma_{\tilde{J},t} + \tau F_{b,t} \end{aligned} \tag{18}$$

The tax on dividends counterbalances the consumption premia on the assets of brown firms through $\tau F_{b,t}$ and hence, increases their cost of capital. From a quantitative viewpoint, dividend taxation has a substantial impact on expected returns if the dividend yields are sufficiently high, or equivalently, if asset prices at a given dividend level are sufficiently low, that is, when firms' cost of capital is high.[34] The introduction of a dividend tax is, therefore, all the more effective as the brownest companies are subject to transition risks (environmental regulations, increase in the price of carbon, change in consumer preferences, technological and reputational risks, etc.), which increase their cost of capital relative to green companies.

6 Conclusion

In this paper, we show how investors' preferences for green consumption moderate the effect of the green premium associated with their preferences for green assets on expected asset returns. In addition to being relevant for asset pricing and capital allocation, the main effect documented in this paper has implications for investors willing to contribute to the ecological transition: the increase in the cost of capital of brown firms is dampened as long as the relative prices of green goods are subject to risks of upward shocks. The allocation of a larger share of green investors' capital to brown firms could, therefore, be a welcome opportunity to reinforce their engagement with the most polluting firms.

The construction of general equilibrium models in sustainable finance, as we propose in this paper, opens up multiple avenues for future research, including the study of the effects of stochastic preferences for green investments and demand for green consumption on firms' cost of capital and investors' wealth allocation.[35] It would

[34]Technically, this occurs for instance when the elasticity of intertemporal substitution is not too large.

[35]We are exploring this avenue in ongoing research.

also be valuable to analyze alternative forms of investments and account for shareholder engagement with a view to maximizing investor impact in a general equilibrium model. Another promising avenue is to a include environment-related financial risks (van den Bremer and van der Ploeg, 2021; Hambel et al., 2022; Barnett, 2022) and production into a general equilibrium model that features green consumption and investment preferences.

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Appendix

A Additional theoretical results

A.1 Drift and diffusion terms for any variable

Remark A.1. *By Itô's Lemma, the geometric drift and diffusion term for any function $g_t = g(X_t)$ are given by:*

$$\frac{dg_t}{g_t} = \frac{dg(X_t)}{g(X_t)} \equiv \mu_{g,t}dt + \sigma_{g,t}^T d\vec{Z}_t \tag{A.1}$$

where:

$$\mu_{g,t} = \frac{g_{x,t}}{g_t}x_t\mu_{x,t} + \frac{g_{y,t}}{g_t}y_t\mu_{y,t} + \frac{1}{2}\frac{g_{xx,t}}{g_t}x_t^2\sigma_{x,t}^T\sigma_{x,t} + \frac{1}{2}\frac{g_{yy,t}}{g_t}y_t^2\sigma_{y,t}^T\sigma_{y,t} + \frac{g_{xy,t}}{g_t}x_t y_t\sigma_{x,t}^T\sigma_{y,t} \tag{A.2}$$

$$\sigma_{g,t} = \frac{g_{x,t}}{g_t}x_t\sigma_{x,t} + \frac{g_{y,t}}{g_t}y_t\sigma_{y,t} \tag{A.3}$$

This result is used repeatedly throughout the paper.

As a point of notation, recall that for any function g, g_t simply denotes $g(X_t)$, not the time-derivative of g (which is zero because the model is stationary due to infinite horizon). $g_{x,t}, g_{y,t}, g_{xx,t}, g_{yy,t}, g_{xy,t}$ denote the partial derivatives of $g(X_t)$.

A.2 Returns, and risk premia

The (geometric) drifts and diffusion terms for asset returns are obtained from Itô's Lemma and are as follows, for $j \in \{g, b\}$

$$\begin{aligned} dR_{j,t} &= \mu_{j,t}dt + \sigma_{j,t}^T d\vec{Z}_t \\ &\equiv \left(F_{j,t} + \mu_{p_j,t} + \mu_{Y_j} + \sigma_{p_j,t}^T\sigma_{Y_j} - \mu_{F_j,t} + \sigma_{F_j,t}^T\sigma_{F_j,t} - \left(\sigma_{p_j,t} + \sigma_{Y_j}\right)^T \sigma_{F_j,t}\right) dt \\ &+ \left(\sigma_{p_j,t} + \sigma_{Y_j} - \sigma_{F_j,t}\right)^T d\vec{Z}_t \end{aligned} \tag{A.4}$$

where $\mu_{p_j,t}$, $\mu_{F_j,t}$, $\sigma_{p_j,t}$, $\sigma_{F_j,t}$ are obtained using Remark A.1 above.

Proposition A.1 generalizes Proposition 2 to the case in which investors have different risk aversions, $\gamma^G \neq \gamma^N$, different elasticity of intertemporal substitutions, $\psi^G \neq \psi^N$, and in which both investors have preferences towards both assets, $\phi_j^i \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t. This poses no particular problem for the resolution, as our method allows for any value of the parameters. Exploring additional asymmetries stemming from those could be an interesting avenue for future work.

Proposition A.1. *The expected risk premia on the green and brown equity assets are*

$$\mu_{g,t} - r_t = \gamma_t \sigma_{g,t}^T \sigma_{\widetilde{W},t} - \gamma_t \sigma_{g,t}^T \sigma_{\tilde{J},t} - \gamma_t \left(x_t \frac{\phi_g^G}{\gamma^G} + (1 - x_t) \frac{\phi_g^N}{\gamma^N} \right) \tag{A.5}$$
$$\mu_{b,t} - r_t = \gamma_t \sigma_{b,t}^T \sigma_{\widetilde{W},t} - \gamma_t \sigma_{b,t}^T \sigma_{\tilde{J},t} - \gamma_t \left(x_t \frac{\phi_b^G}{\gamma^G} + (1 - x_t) \frac{\phi_b^N}{\gamma^N} \right)$$

where $\widetilde{W}_t$ is the total wealth, $\widetilde{J}_t$ is the economy-wide marginal value of wealth, γ_t is the wealth-weighted risk aversion, $\sigma_{J^G,t}, \sigma_{J^N,t}$ are the geometric diffusion terms of J_t^G, J_t^N obtained as in Remark A.1 above, and

$$\sigma_{\widetilde{W},t} \equiv w_{g,t}^M \sigma_{g,t} + (1 - w_{g,t}^M) \sigma_{b,t}$$
$$\sigma_{\tilde{J},t} \equiv x_t \left(\frac{1}{\gamma^G} \right) \left(\frac{1 - \gamma^G}{1 - \psi^G} \right) \sigma_{J^G,t} + (1 - x_t) \left(\frac{1}{\gamma^N} \right) \left(\frac{1 - \gamma^N}{1 - \psi^N} \right) \sigma_{J^N,t}$$
$$\gamma_t \equiv \left(\frac{x_t}{\gamma^G} + \frac{1 - x_t}{\gamma^N} \right)^{-1}$$

A.3 Portfolios

Proposition A.2 generalizes Proposition 3 to the case in which investors have different risk aversions, $\gamma^G \neq \gamma^N$, different elasticity of intertemporal substitutions, $\psi^G \neq \psi^N$, and in which both investors have preferences towards both assets, $\phi^i_j \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t. This poses no particular problem for the resolution, as our method allows for any value of the parameters. Exploring additional asymmetries stemming from those could be an interesting avenue for future work.

Proposition A.2. *The optimal portfolios of the green and neutral investors $j \in \{G, N\}$ are given by*

$$\begin{pmatrix} w^G_{g,t} \\ w^G_{b,t} \end{pmatrix} = \frac{1}{\gamma^G} \left(\Sigma_t^T \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi^G_g \\ \mu_{b,t} - r_t + \phi^G_b \end{pmatrix} + \left(\frac{1-\gamma^G}{1-\psi^G}\right) \Sigma_t^T \left(\frac{J^G_{x,t}}{J^G_t} x_t \sigma_{x,t} + \frac{J^G_{y,t}}{J^G_t} y_t \sigma_{y,t} \right) \right\}$$
$$b^G_t = 1 - w^G_{g,t} - w^G_{b,t} \tag{A.6}$$

$$\begin{pmatrix} w^N_{g,t} \\ w^N_{b,t} \end{pmatrix} = \frac{1}{\gamma^N} \left(\Sigma_t^T \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi^N_g \\ \mu_{b,t} - r_t + \phi^N_b \end{pmatrix} + \left(\frac{1-\gamma^N}{1-\psi^N}\right) \Sigma_t^T \left(\frac{J^N_{x,t}}{J^N_t} x_t \sigma_{x,t} + \frac{J^N_{y,t}}{J^N_t} y_t \sigma_{y,t} \right) \right\}$$
$$b^N_t = 1 - w^N_{g,t} - w^N_{b,t} \tag{A.7}$$

where $w^i_{g,t}, w^i_{b,t}, b^i_t$ are the portfolio weights (as a share of wealth) allocated to the green equity asset, the brown equity asset, and the riskless bond, and $\Sigma_t \equiv \begin{bmatrix} \sigma_{g,t} & \sigma_{b,t} \end{bmatrix}$.

A.4 Equilibrium

The definition of the equilibrium is standard.

Definition 1. *A competitive equilibrium is a set of aggregate stochastic processes adapted to the filtration generated by* $\vec{Z}$*: the price of the equity asset* $(Q_{g,t}, Q_{b,t})$*, and the interest rate* (r_t)*, together with a set of individual stochastic processes for each investor: consumption of each good* $(C^G_{g,t}, C^G_{b,t}, C^N_{g,t}, C^N_{b,t})$*, wealth* (W^G_t, W^N_t)*, and portfolio shares* $(w^G_{g,t}, w^N_{b,t}, w^G_{g,t}, w^N_{b,t})$*, such that, given the output of the two endowment trees* $(Y_{g,t}, Y_{b,t})$*:*

1. *Given the aggregate stochastic processes, individual choices solve the investor optimization problem given in Section 2.*
2. *Markets clear.*
   (a) *Good markets:*

$$
\begin{aligned}
C^G_{g,t} + C^N_{g,t} &= Y_{g,t} \\
C^G_{b,t} + C^N_{b,t} &= Y_{b,t}
\end{aligned}
\tag{A.8}
$$

   (b) *Equity markets:*

$$
\begin{aligned}
w^G_{g,t} W^G_t + w^N_{g,t} W^N_t &= Q_{g,t} \\
w^G_{b,t} W^G_t + w^N_{b,t} W^N_t &= Q_{b,t}
\end{aligned}
\tag{A.9}
$$

Most importantly, as shown in Section 2.3 of the main text, the equilibrium can be recast as a stationary recursive Markovian equilibrium in which all variables of interest are expressed as a function of a pair of state variables $X_t \equiv (x_t, y_t)'$, whose dynamics are also solely a function of X_t. x_t is the wealth share of the green investor, and y_t is the relative supply of the green good.

A.5 Evolutions of the state variables

Due to the Markovian nature of the equilibrium, the laws of motion of the state variables underlie the dynamics of the economy. They are summarized in Proposition A.3.

Proposition A.3. *The laws of motion for the wealth share of the green investor x_t, and the relative supply of the green good y_t are*

$$\frac{dx_t}{x_t} \equiv \mu_{x,t} dt + \sigma_{x,t}^T d\vec{Z}_t \tag{A.10}$$
$$\frac{dy_t}{y_t} \equiv \mu_{y,t} dt + \sigma_{y,t}^T d\vec{Z}_t$$

where

$$\begin{aligned}
\mu_{x,t} &= \left(w_{g,t}^G - w_{g,t}^M\right)\left(\mu_{g,t} - r_t\right) + \left(w_{b,t}^G - w_{b,t}^M\right)\left(\mu_{b,t} - r_t\right) \\
&\quad + \left(F_{g,t} w_{g,t}^M + w_{b,t}^M F_{b,t}\right) - P_t^G c_t^G \\
&\quad - \left((w_{g,t}^G - w_{g,t}^M)\sigma_{g,t} + (w_{b,t}^G - w_{b,t}^M)\sigma_{b,t}\right)^T \left(w_{g,t}^M \sigma_{g,t} + w_{b,t}^M \sigma_{b,t}\right) \\
\sigma_{x,t} &= \left(\left(w_{g,t}^G - w_{g,t}^M\right)\sigma_{g,t} + \left(w_{b,t}^M - w_{b,t}^M\right)\sigma_{b,t}\right) \\
\mu_{y,t} &= (1 - y_t)\left(\mu_{Y_g} - \mu_{Y_b}\right) - (1 - y_t)\left(\sigma_{Y_g} - \sigma_{Y_b}\right)^T \left(y_t \sigma_{Y_g} + (1 - y_t)\sigma_{Y_b}\right) \\
\sigma_{y,t} &= (1 - y_t)\left(\sigma_{Y_g} - \sigma_{Y_b}\right)
\end{aligned}$$

and $w_{g,t}^M \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t}), w_{b,t}^M \equiv Q_{b,t}/(Q_{g,t} + Q_{b,t})$ are the weights of the green and brown equity assets in the market portfolio, with $w_{g,t}^M$ defined in Equation (6) and $w_{b,t}^M = 1 - w_{g,t}^M$ in equilibrium because the bond is zero net supply.

Figure A.1 show the drift and diffusion terms for y_t, the relative supply of the green good. They do not depend on the wealth share of the green investor x_t or on parameters beyond $\mu_{Y_g}, \mu_{Y_b}, \sigma_{Y_g}, \sigma_{Y_b}$, because y_t is purely determined by the outputs of the green and brown trees.

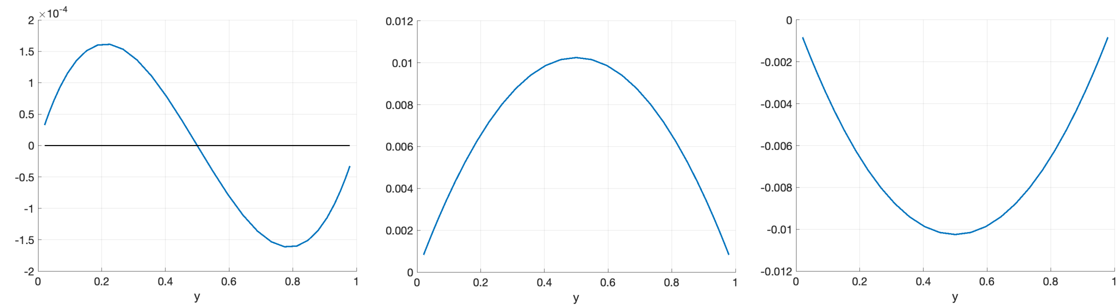
Figures A.2, A.3, A.4 show the drift and diffusion terms for x_t, the wealth share of the green investor, for various calibrations. As mentioned in the main text, the diffusion terms for x_t, and therefore the covariance between state variables, are inherently dependent on the portfolio bias of the green investor, which in turn depends strongly on her preference for green consumption (α) and green investing (ϕ).

Figure A.1: Drift and diffusion terms for the relative supply of the green good y_t

(a) Drift of y_t: $\mu_{y,t} y_t$

(b) Loading of y_t on shock to green output: $\sigma_{yZ_g,t} y_t$

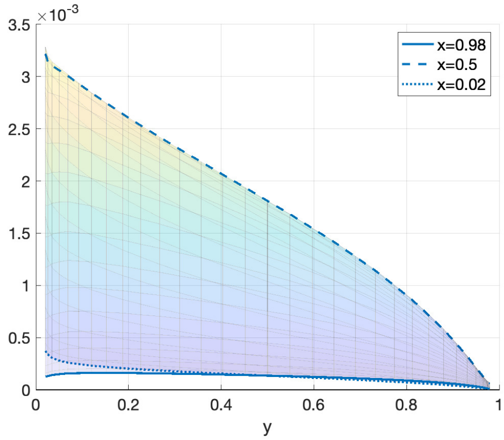
(c) Loading of y_t on shock to brown output: $\sigma_{yZ_b,t} y_t$



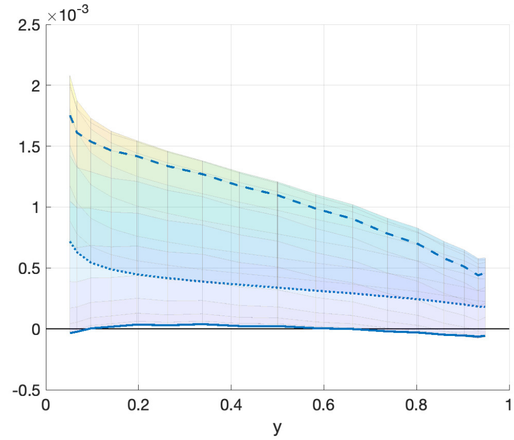
Notes: Based on the calibration of Assumption 1. y_t is the relative supply of the green good, which is exogenous so that its drift and diffusion terms do not depend on the wealth share of the green investor x_t.

Figure A.2: Drift for the wealth share of the green investor x_t: $\mu_{x,t}x_t$

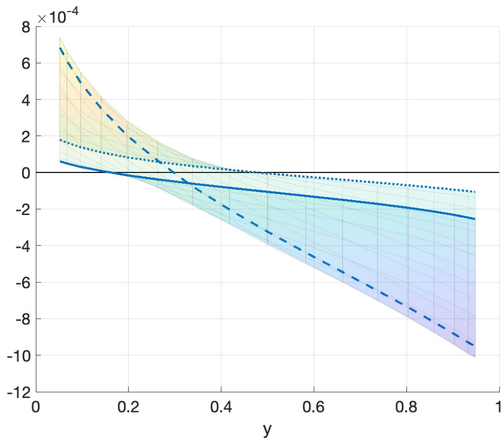
(a) Baseline calibration ($\alpha = 0.85, \phi = 1\%$)



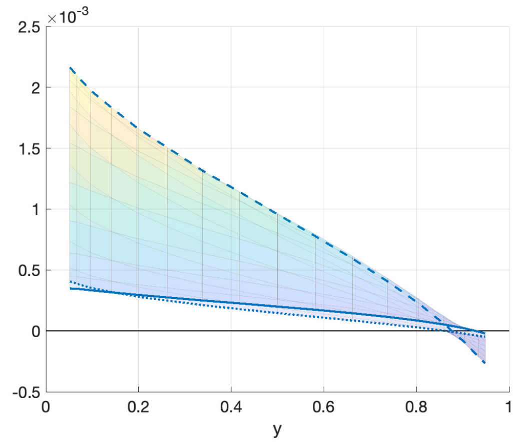
(b) No preference for green consumption ($\alpha = 1/2, \phi = 1\%$)



(c) No preference for green investing ($\alpha = 0.85, \phi = 0$)



(d) Limited preference for green investing ($\alpha = 0.85, \phi = 0.5\%$)

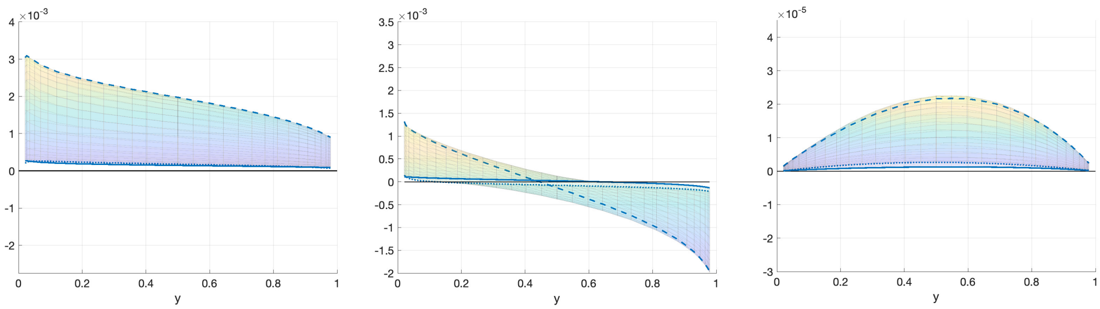


Notes: Based on the calibration of Assumption 1, with $\gamma = 50$, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure A.3: Diffusion terms for the wealth share of the green investor x_t

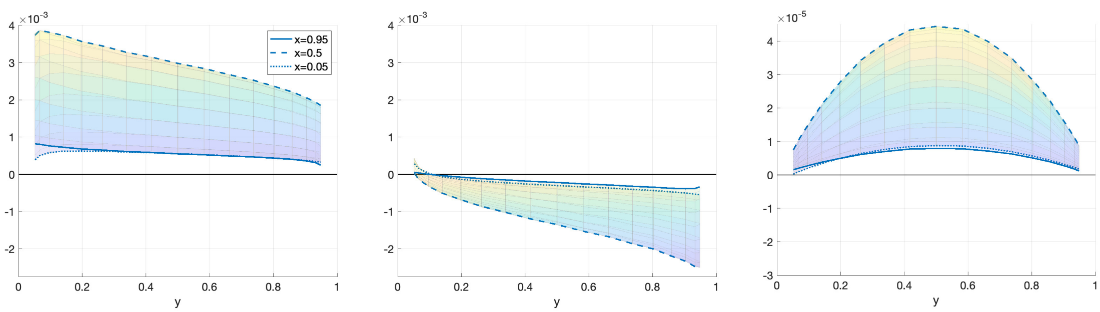
(a) Baseline calibration ($\alpha = 0.85, \phi = 1\%$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$



(b) No preference for green consumption ($\alpha = 1/2, \phi = 1\%$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$

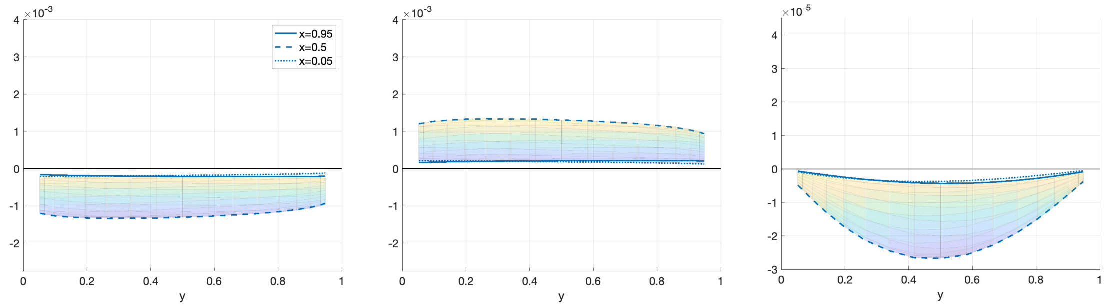


Notes: Based on the calibration of Assumption 1, with $\gamma = 50$, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure A.4: Diffusion terms for the wealth share of the green investor x_t

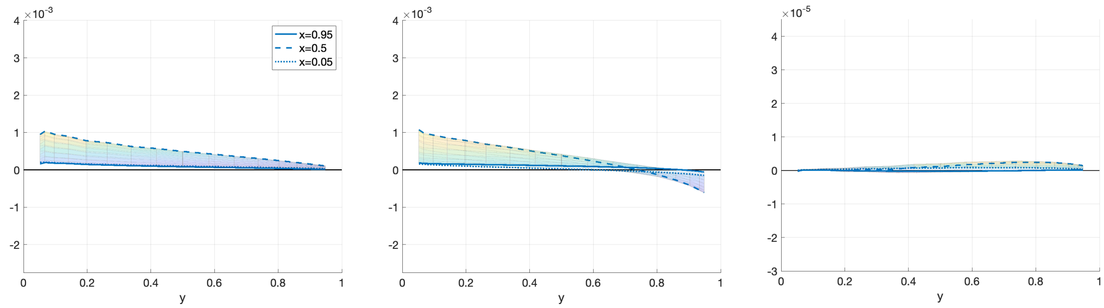
(c) No preference for green investing ($\alpha = 0.85, \phi = 0$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$



(d) Limited preference for green investing ($\alpha = 0.85, \phi = 0.5\%$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

A.6 Marginal values of wealth and Hamilton-Jacobi-Bellman equations

Proposition A.4. J_t^G, J_t^N *satisfy the Hamilton-Jacobi-Bellman equations for* $i \in \{G, N\}$

$$0 = \left(\frac{1}{\psi - 1}\right) P_t^{i1-\psi} J_t^i - \left(\frac{1}{1 - 1/\psi}\right) \rho + r_t + \frac{\gamma}{2} \left(w_{g,t}^i \sigma_{g,t} + w_{b,t}^i \sigma_{b,t}\right)^T \left(w_{g,t}^i \sigma_{g,t} + w_{b,t}^i \sigma_{b,t}\right)$$
$$+ \left(\frac{1}{1 - \psi}\right) \mu_{J^i,t} + \frac{1}{2} \left(\frac{1}{1 - \psi}\right) \left(\frac{\psi - \gamma}{1 - \psi}\right) \sigma_{J^i,t}^T \sigma_{J^i,t} \tag{A.11}$$

where $\mu_{J^i,t}, \sigma_{J^i,t}$ *are the geometric drift and diffusion terms of* J_t^i *obtained as in Remark A.1:*

$$\frac{dJ_t^i}{J_t^i} \equiv \mu_{J^i,t} dt + \sigma_{J^i,t}^T d\vec{Z}_t \tag{A.12}$$

A.7 Consumptions, goods prices

Proposition A.5. *The consumption of each investor* $i \in \{G, N\}$ *is given by*

$$c_t^i \equiv \frac{C_t^i}{W_t^i} = P_t^{i-\psi} J_t^i \tag{A.13}$$

$$c_{g,t}^i = \alpha^i \left(\frac{p_{g,t}}{P_t^i}\right)^{-\theta} c_t^i \tag{A.14}$$

$$c_{b,t}^i = (1 - \alpha^i) \left(\frac{p_{b,t}}{P_t^i}\right)^{-\theta} c_t^i \tag{A.15}$$

$$P_t^i = \left[\alpha^i p_{g,t}^{1-\theta} + (1 - \alpha^i) p_{b,t}^{1-\theta}\right]^{1/(1-\theta)} \tag{A.16}$$

Proposition A.6. *The relative price of the green good,* $q_t = q(X_t) \equiv p_{g,t}/p_{b,t}$, *solves the following non-linear equation*

$$q_t = S_t^{1/\theta} \left(\frac{1 - y_t}{y_t}\right)^{1/\theta} \tag{A.17}$$

where

$$S_t = \frac{\alpha^G J_t^G x_t P_t^{A\theta-\psi} + \alpha^N P_t^{N\theta-\psi} J_t^N (1 - x_t)}{(1 - \alpha^G) P_t^{G\theta-\psi} J_t^G x_t + (1 - \alpha^N) P_t^{N\theta-\psi} J_t^N (1 - x_t)}$$

Using the defintion of the numéraire, with $a = 1/2$, prices follow

$$p_{g,t} = \left(a + (1-a)q_t^{\theta-1}\right)^{1/(\theta-1)} \tag{A.18}$$

$$p_{b,t} = p_{g,t}q_t^{-1} = \left(aq_t^{1-\theta} + (1-a)\right)^{1/(\theta-1)} \tag{A.19}$$

$$P_t^i = \left[\alpha^i p_{g,t}^{1-\theta} + (1-\alpha^i)p_{b,t}^{1-\theta}\right]^{1/(1-\theta)} \tag{A.20}$$

$$\mathcal{E}_t = P_t^G / P_t^N \tag{A.21}$$

B Additional figures

B.1 Economic set-up

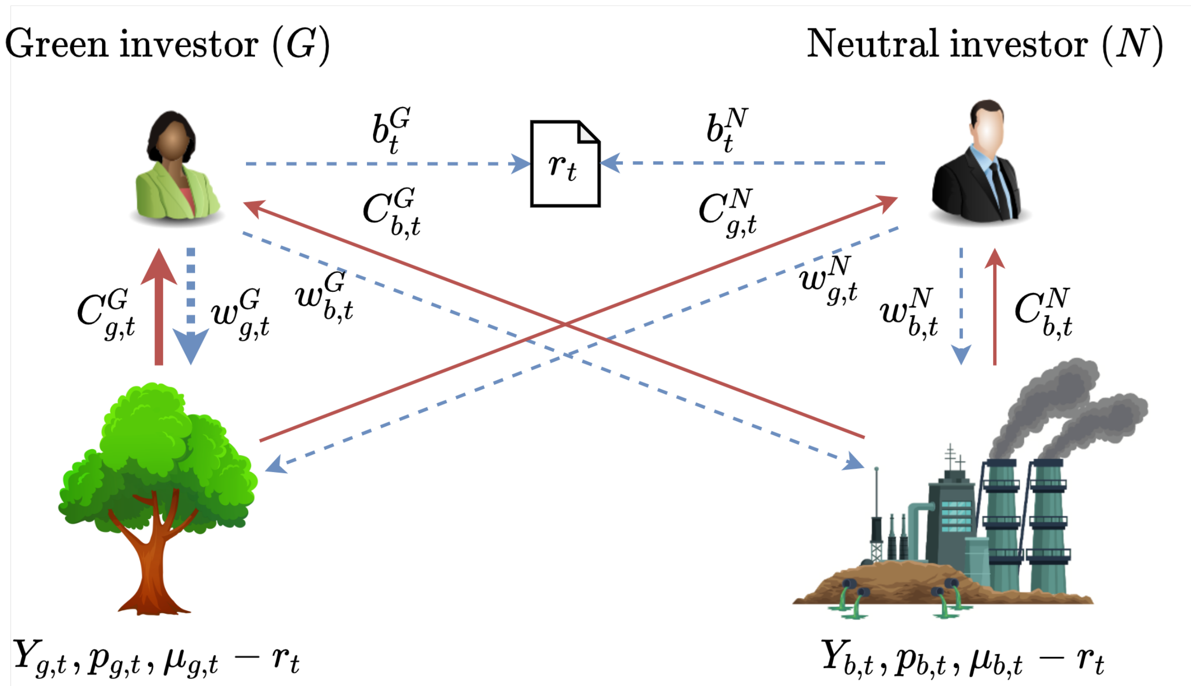
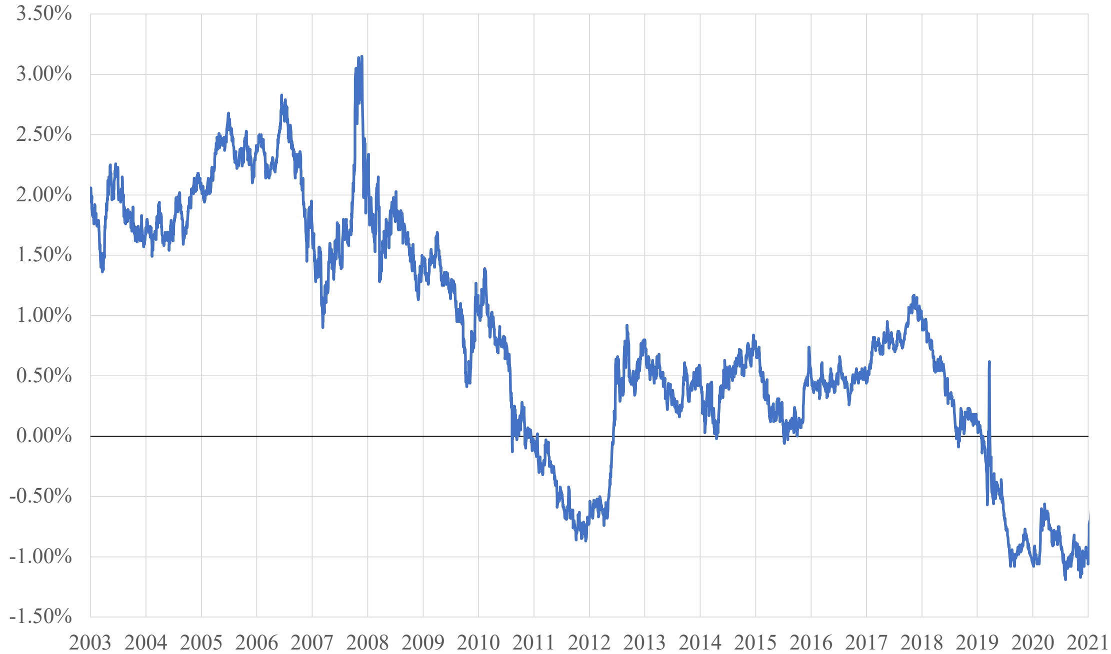


Figure B.1: The Economy

Source: Vecteezy.com. Back to main text: Section 2.

B.2 Riskfree interest rate

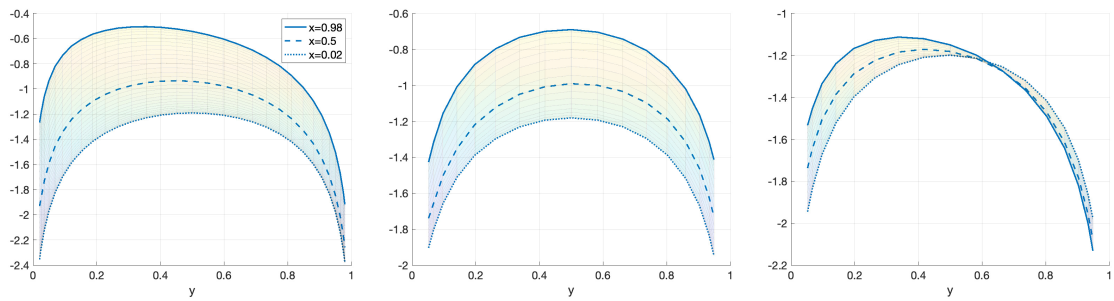
Figure B.2: Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Inflation-Indexed



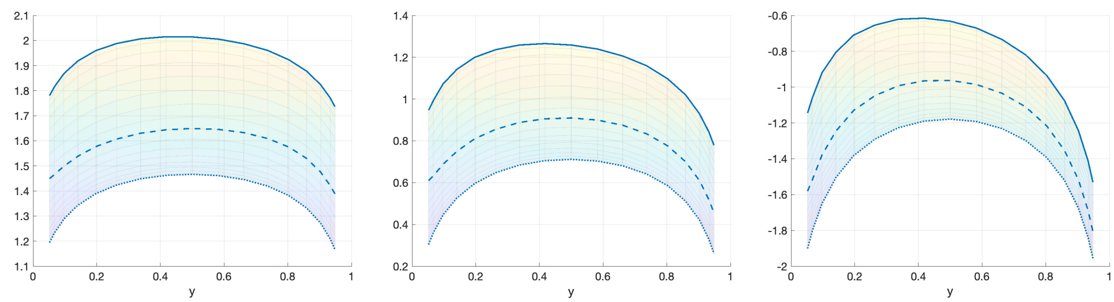
Source: Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis.

Figure B.3: Riskfree interest rate (r_t) for various calibrations

(a) Baseline ($\gamma = 50, \alpha = 0.85, \phi = 1\%$) | (b) No pref. green cons. ($\gamma = 50, \alpha = 1/2, \phi = 1\%$) | (c) No pref. green inv. ($\gamma = 50, \alpha = 0.85, \phi = 0\%$)



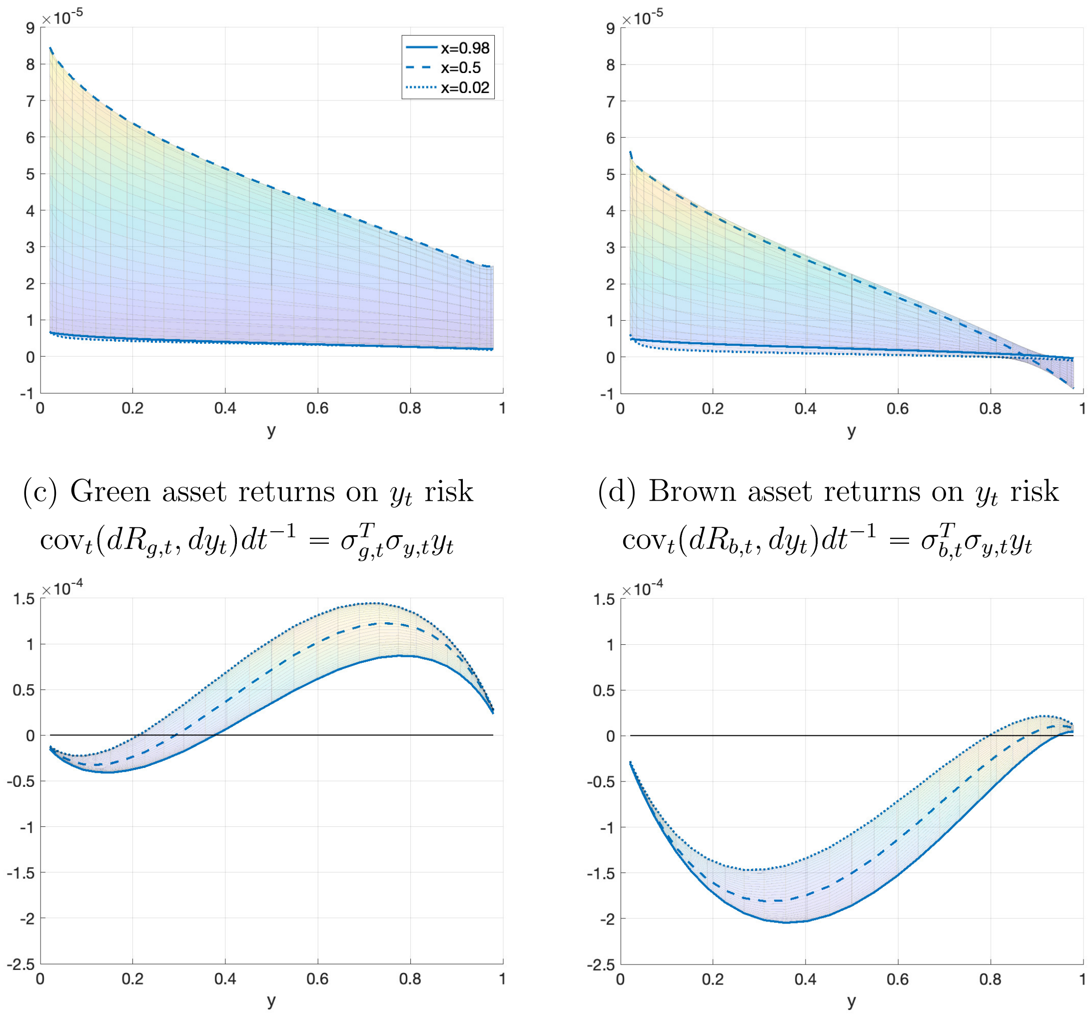
(d) $\gamma = 15, \alpha = 0.75, \phi = 1\%$ | (e) $\gamma = 25, \alpha = 0.75, \phi = 1\%$ | (f) $\gamma = 50, \alpha = 0.75, \phi = 1\%$



Notes: Based on the calibration of Assumption 1, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

B.3 Quantities and prices of risk

Figure B.4: Quantities of risk



(a) Green asset returns on x_t risk $\text{cov}_t(dR_{g,t}, dx_t)dt^{-1} = \sigma_{g,t}^T \sigma_{x,t} x_t$

(b) Brown asset returns on x_t risk $\text{cov}_t(dR_{b,t}, dx_t)dt^{-1} = \sigma_{b,t}^T \sigma_{x,t} x_t$

(c) Green asset returns on y_t risk $\text{cov}_t(dR_{g,t}, dy_t)dt^{-1} = \sigma_{g,t}^T \sigma_{y,t} y_t$

(d) Brown asset returns on y_t risk $\text{cov}_t(dR_{b,t}, dy_t)dt^{-1} = \sigma_{b,t}^T \sigma_{y,t} y_t$

Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

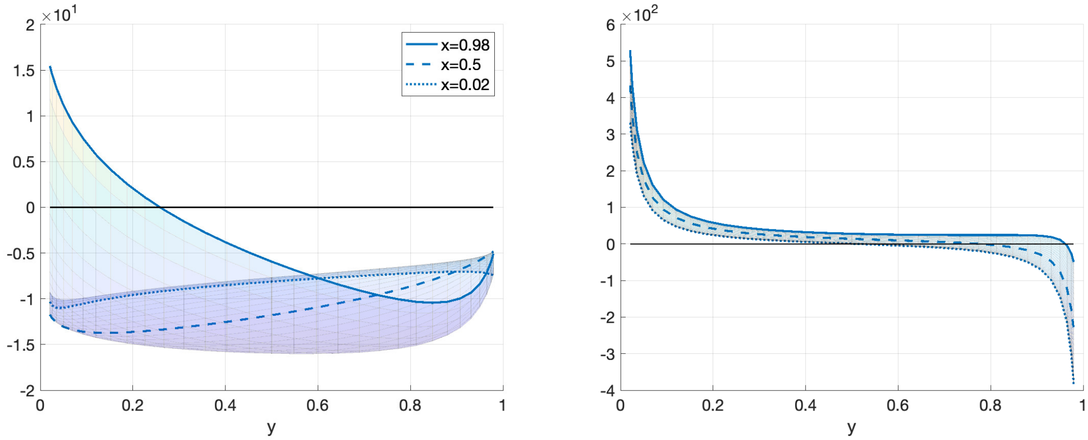
Figure B.5: Quantities of risk

(a) Price of x_t risk

$$-\left(\frac{1-\gamma}{1-\psi}\right)\left\{x_t \frac{J^G_{x,t}}{J^G_t} + (1-x_t)\frac{J^N_{x,t}}{J^N_t}\right\}$$

(b) Price of y_t risk

$$-\left(\frac{1-\gamma}{1-\psi}\right)\left\{x_t \frac{J^G_{y,t}}{J^G_t} + (1-x_t)\frac{J^N_{y,t}}{J^N_t}\right\}$$

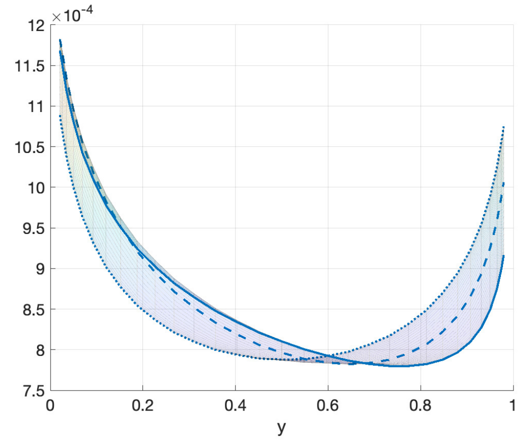


Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

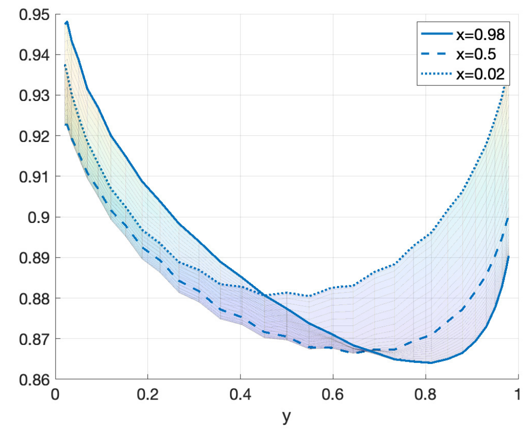
B.4 Second moment of returns

Figure B.6: (Instantaneous) Second moment of returns

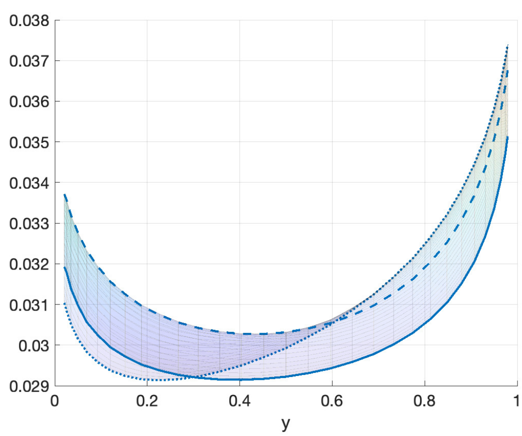
(a) Covariance of returns
$\text{cov}_t(dR_{g,t}, dR_{b,t})dt^{-1} = \sigma_{g,t}^T \sigma_{b,t}$



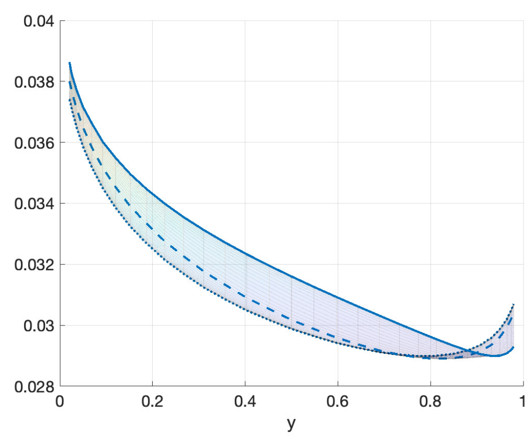
(b) Correlation of returns
$\text{corr}_t(dR_{b,t}, dR_{b,t})dt^{-1}$



(c) Volatility of green asset returns
$\text{vol}_t(dR_{g,t}) = \left(\sigma_{g,t}^T \sigma_{g,t}\right)^{1/2}$



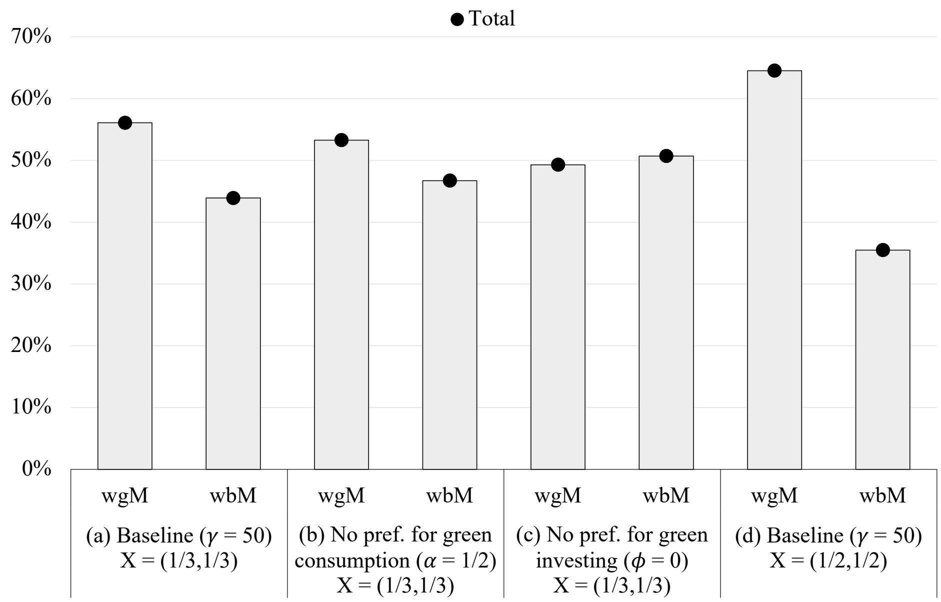
(d) Volatility of brown asset returns
$\text{vol}_t(dR_{b,t}) = \left(\sigma_{b,t}^T \sigma_{b,t}\right)^{1/2}$



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

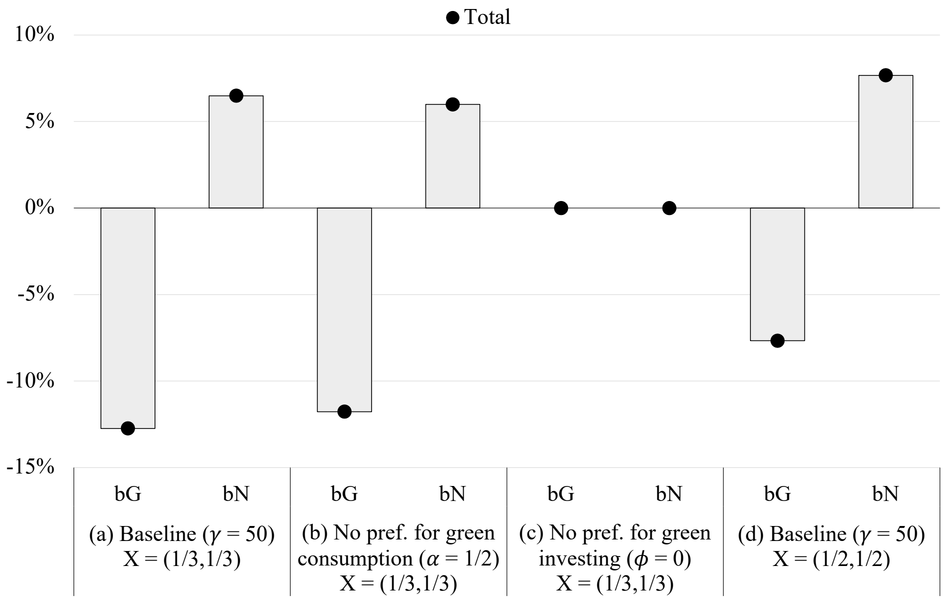
B.5 Portfolios

Figure B.7: Market portfolio at $X_t = (1/3, 1/3)$, and $X_t = (1/2, 1/2)$



Notes: Based on the calibration of Assumption 1, except for the specified parameters. The figure shows the market portfolio at $X_t \equiv (x_t, y_t)' = (1/3, 1/3)$ for Panels (a), (b), (c), and at $X_t = (1/2, 1/2)$ for Panel (d). $w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t}), w^M_{b,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$ are the weights (as % of wealth) on the green and brown asset in the market portfolio. In equilibrium, $w^M_{b,t} = 1 - w^M_{g,t}$ because the bond is in zero net supply.

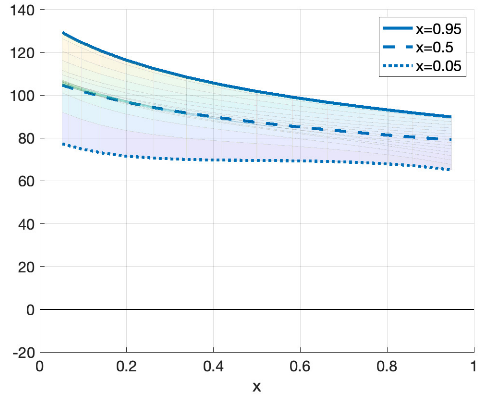
Figure B.8: Borrowing and saving in the riskless bond at $X_t = (1/3, 1/3)$, and $X_t = (1/2, 1/2)$



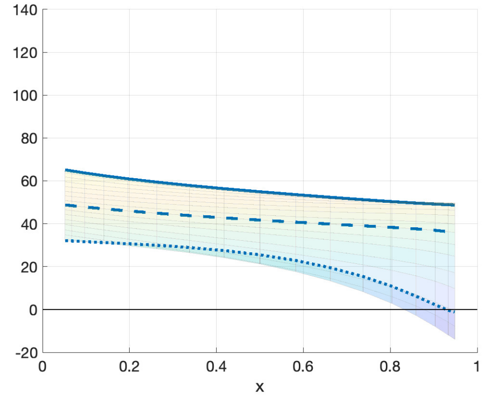
Notes: Based on the calibration of Assumption 1, except for the specified parameters. The figure shows $b^i_t = 1 - w^i_{g,t} - w^i_{b,t}$, the weight (as % of wealth) allocated to the riskfree bond by each investor, $i \in \{G, N\}$. $b^i_t > 0$ corresponds to saving in the bond, $b^i_t < 0$ corresponds to borrowing.

Figure B.9: Portfolios of both investors, $i \in \{G, N\}$ (% of wealth)

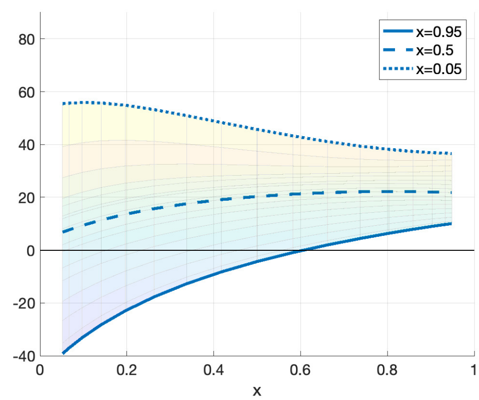
(a) Green asset in green portfolio ($w^G_{g,t}$)



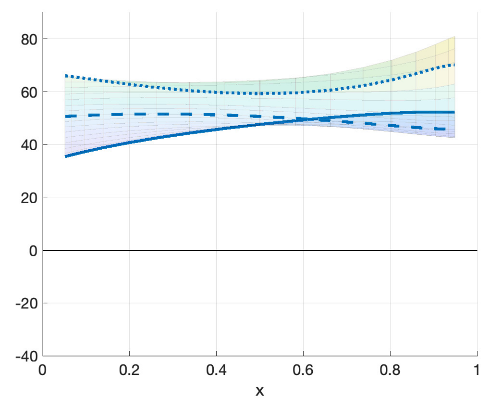
(b) Green asset in neutral portfolio ($w^N_{g,t}$)



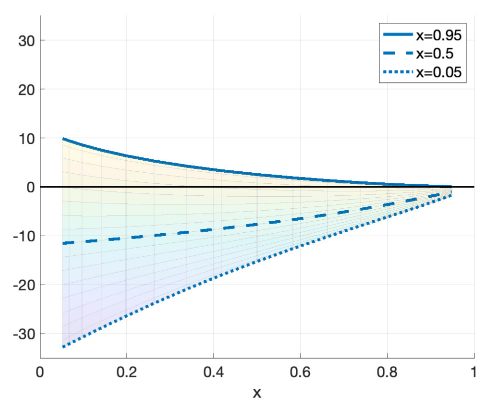
(c) Brown asset in green portfolio ($w^G_{b,t}$)



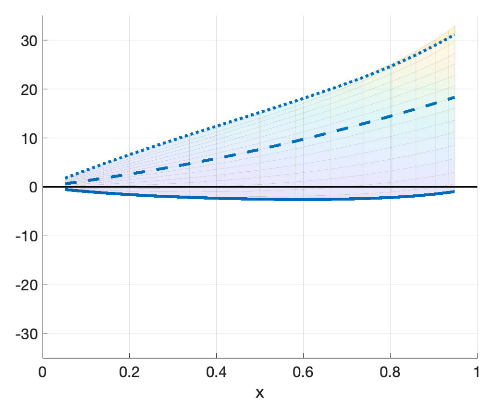
(d) Brown asset in neutral portfolio ($w^N_{b,t}$)



(e) Riskfree bond in green portfolio ($b^G_t = 1 - w^G_{g,t} - w^G_{b,t}$)



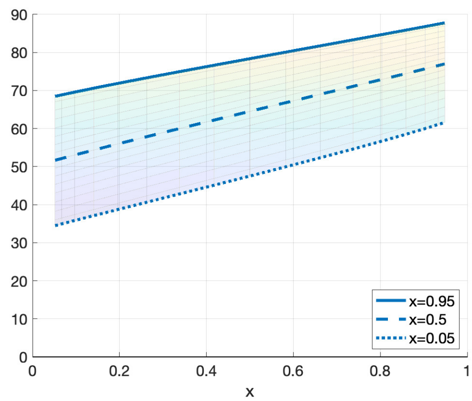
(f) Riskfree bond in neutral portfolio ($b^N_t = 1 - w^N_{g,t} - w^N_{b,t}$)



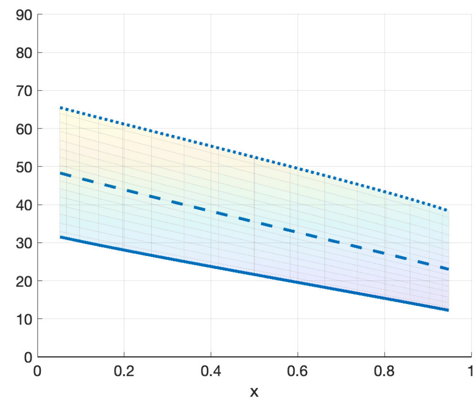
Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure B.10: Portfolios of both investors, $i \in \{G, N\}$, vs. market portfolio (%)

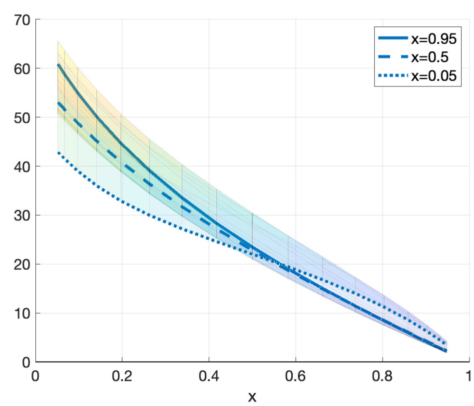
(a) Green asset in market portfolio ($w^M_{g,t}$)



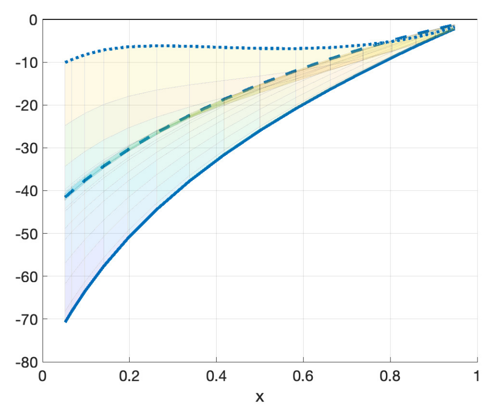
(b) Brown asset in market portfolio ($w^M_{g,t}$)



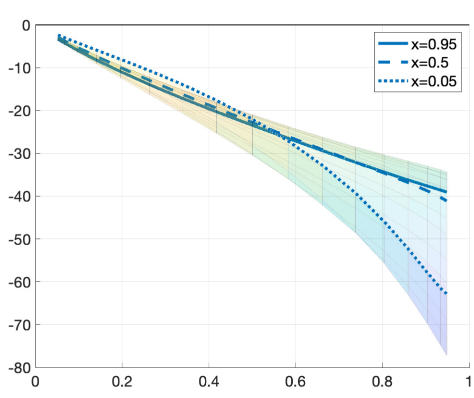
(c) Bias on green asset in green vs. market portfolio ($w^G_{g,t} - w^M_{g,t}$)



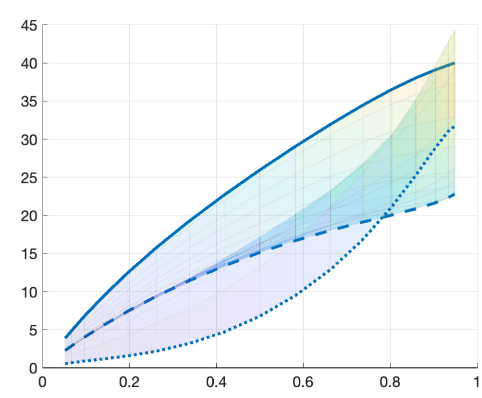
(d) Bias on brown asset in green vs. market portfolio ($w^G_{b,t} - w^M_{b,t}$)



(e) Bias on green asset in neutral vs. market portfolio ($w^N_{g,t} - w^M_{g,t}$)



(f) Bias on brown asset in neutral vs. market portfolio ($w^N_{b,t} - w^M_{b,t}$)



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

C Additional empirical results

Table C.1: Empirical estimation of consumption premia (details, full sample: March 2006-December 2019, returns and betas winsorized at the 1% level)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Panel A: Risk Prices (monthly, %)													
$Relative\_Price\_Hedge_t$	-0.030	-0.030	-0.031	-0.028	-0.029	-0.030	-0.027	-0.020	-0.021	-0.018	-0.014	-0.022	-0.024
	(-3.775)	(-3.756)	(-4.213)	(-3.771)	(-4.609)	(-4.711)	(-3.774)	(-2.626)	(-3.474)	(-2.211)	(-1.921)	(-2.909)	(-3.641)
$Price\_Hedge_t \times dC_{t-1}$									0.000	0.000	0.001	0.000	0.000
									(0.296)	(0.204)	(0.8)	(-0.136)	(-0.102)
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		(0.59)	(0.47)	(0.831)	(1.366)	(1.058)	(2.056)	(-0.056)	(0.298)	(1.641)	(2.24)	(0.38)	(1.528)
$Wealth\_Hedge_t \times dC_{t-1}$									0.000	0.000	0.000	0.000	0.000
									(-0.328)	(0.55)	(1.44)	(0.586)	(0.647)
GMB_t			-0.029	-0.023	0.031	0.048	0.034	0.073	-0.063	-0.023	-0.007	0.035	0.034
			(-0.451)	(-0.362)	(0.476)	(0.796)	(0.493)	(1.1)	(-1.029)	(-0.367)	(-0.118)	(0.538)	(0.644)
$GMB_t \times dC_{t-1}$									-0.006	-0.007	-0.010	0.001	-0.005
									(-0.602)	(-0.712)	(-1.084)	(0.084)	(-0.477)
dC_{t-1}									0.004	0.011	0.015	0.010	0.009
									(0.752)	(1.714)	(2.173)	(1.296)	(1.114)
Panel B: Premium differential between green and brown assets (annual, %)													
$Relative\_Price\_Hedge_t$	0.150	0.165	0.358	0.385	0.365	0.331	0.404	0.265	0.383	0.280	0.167	0.374	0.308
$Price\_Hedge_t \times dC_{t-1}$									0.006	0.004	0.011	-0.002	-0.001
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$Wealth\_Hedge_t \times dC_{t-1}$									0.000	0.000	0.000	0.000	0.000
GMB_t			-0.193	-0.159	0.224	0.339	0.156	0.384	-0.447	-0.175	-0.046	0.174	0.195
$GMB_t \times dC_{t-1}$									-0.023	0.012	0.020	-0.004	0.012
dC_{t-1}									0.061	0.158	0.174	0.149	0.128
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM	CAPM	FF3	FF3MOM	FF5	FF5MOM
Conditioning var.									dC_{t-1}	dC_{t-1}	dC_{t-1}	dC_{t-1}	dC_{t-1}

Notes: Variables are defined in Section 4.1 and t-stats are in brackets. Full sample: March 2006-December 2019.

Table C.2: Empirical estimation of consumption premia (details, full sample: March 2006-December 2019, non-winsorized returns, betas winsorized at the 1% level)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Panel A: Risk Prices (monthly, %)													
$Relative\_Price\_Hedge_t$	-0.026	-0.026	-0.031	-0.029	-0.021	-0.023	-0.017	-0.007	-0.018	-0.018	-0.013	-0.011	-0.020
	(-2.464)	(-2.458)	(-3.288)	(-3.01)	(-2.317)	(-2.587)	(-2.238)	(-0.851)	(-2.547)	(-2.26)	(-1.809)	(-1.677)	(-2.922)
$Relative\_Price\_Hedge_t \times dC_{t-1}$									0.001	0.001	0.001	0.002	0.001
									(1.209)	(0.732)	(0.903)	(1.349)	(0.472)
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		(0.447)	(-0.036)	(0.188)	(1.23)	(1.107)	(2.159)	(-0.258)	(0.778)	(2.058)	(2.286)	(-0.226)	(1.632)
$Wealth\_Hedge_t \times dC_{t-1}$									0.000	0.000	0.000	0.000	0.000
									(0.017)	(1.235)	(1.782)	(0.399)	(0.727)
GMB_t			-0.087	-0.095	-0.042	-0.068	-0.027	-0.031	-0.115	-0.083	-0.066	-0.009	-0.041
			(-0.824)	(-0.933)	(-0.429)	(-0.707)	(-0.28)	(-0.35)	(-1.462)	(-1.07)	(-0.98)	(-0.114)	(-0.602)
$GMB_t \times dC_{t-1}$									-0.024	-0.008	-0.014	-0.010	-0.022
									(-1.631)	(-0.563)	(-1.146)	(-0.933)	(-1.999)
dC_{t-1}									0.006	0.020	0.023	0.012	0.010
									(0.917)	(2.508)	(2.838)	(1.428)	(1.232)
Panel B: Premium differential between green and brown assets (annual, %)													
$Relative\_Price\_Hedge_t$	0.179	0.182	0.388	0.438	0.321	0.277	0.287	0.095	0.334	0.294	0.137	0.203	0.218
$Relative\_Price\_Hedge_t \times dC_{t-1}$									0.026	0.013	0.011	0.017	0.003
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$Wealth\_Hedge_t \times dC_{t-1}$									0.000	0.000	0.000	0.000	0.000
GMB_t			-0.590	-0.677	-0.313	-0.492	-0.126	-0.161	-0.845	-0.653	-0.490	-0.043	-0.227
$GMB_t \times dC_{t-1}$									-0.082	0.010	0.033	0.050	0.087
dC_{t-1}									0.095	0.331	0.333	0.208	0.163
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM	CAPM	FF3	FF3MOM	FF5	FF5MOM
Conditioning var.									dC_{t-1}	dC_{t-1}	dC_{t-1}	dC_{t-1}	dC_{t-1}

Notes: Variables are defined in Section 4.1 and t-stats are in brackets. Full sample: March 2006-December 2019.

Table C.3: Empirical estimation of consumption premia (details, Pastor et al. (2021a) sample: November 2012-December 2019, returns and β winsorized at the 1% level)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Panel A: Risk Prices (monthly, %)													
$Relative\_Price\_Hedge_t$	-0.018	-0.018	-0.016	-0.015	-0.016	-0.016	-0.018	-0.009	-0.015	-0.008	-0.007	-0.007	-0.005
	(-2.49)	(-2.391)	(-2.576)	(-2.321)	(-2.92)	(-2.798)	(-3.354)	(-1.529)	(-2.905)	(-1.21)	(-1.171)	(-1.295)	(-0.982)
$Relative\_Price\_Hedge_t \times dC_{t-1}$									0.001	0.001	0.001	0.001	0.002
									(0.863)	(1.153)	(1.363)	(0.905)	(2.039)
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		(0.455)	(0.337)	(0.004)	(1.835)	(1.513)	(2.357)	(-0.23)	(-0.645)	(1.161)	(1.732)	(0.387)	(1.422)
$Wealth\_Hedge_t \times dC_{t-1}$									0.000	0.000	0.000	0.000	0.000
									(-0.202)	(0.811)	(1.186)	(0.188)	(0.764)
GMB_t			0.080	0.174	0.188	0.176	0.189	0.208	0.104	0.095	0.097	0.072	0.083
			(0.565)	(1.978)	(3.279)	(3.077)	(3.387)	(3.639)	(1.53)	(1.908)	(1.993)	(1.538)	(1.794)
$GMB_t \times dC_{t-1}$									0.002	-0.003	-0.006	0.010	0.005
									(0.156)	(-0.302)	(-0.841)	(1.262)	(0.558)
dC_{t-1}									0.005	0.011	0.011	0.006	0.007
									(1.143)	(2.184)	(2.064)	(1.115)	(1.217)
Panel B: Premium differential between green and brown assets (annual, %)													
$Relative\_Price\_Hedge_t$	0.063	0.061	0.198	0.234	0.164	0.158	0.230	0.109	0.234	0.092	0.044	0.084	0.039
$Relative\_Price\_Hedge_t \times dC_{t-1}$									-0.003	-0.011	-0.016	-0.006	-0.022
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$Wealth\_Hedge_t \times dC_{t-1}$									0.000	0.000	0.000	0.000	0.000
GMB_t			0.774	1.735	1.728	1.575	1.232	1.486	1.037	0.932	0.861	0.485	0.619
$GMB_t \times dC_{t-1}$									0.012	-0.012	-0.030	0.060	0.034
dC_{t-1}									-0.043	-0.062	-0.055	-0.014	-0.001
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM	CAPM	FF3	FF3MOM	FF5	FF5MOM
Conditioning var.									dC_{t-1}	dC_{t-1}	dC_{t-1}	dC_{t-1}	dC_{t-1}

Notes: Variables are defined in Section 4.1 and t-stats are in brackets. Pastor et al. (2021a) sample: November 2012-December 2019.

Table C.4: Empirical estimation of consumption premia (details, Pastor et al. (2021a) sample: November 2012-December 2019, non-winsorized returns, betas winsorized at the 1% level)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Panel A: Risk Prices (monthly, %)													
$Relative\_Price\_Hedge_t$	-0.021	-0.020	-0.024	-0.023	-0.017	-0.017	-0.015	-0.005	-0.014	-0.014	-0.009	-0.004	-0.008
	(-2.364)	(-2.282)	(-2.976)	(-2.739)	(-2.22)	(-2.197)	(-2.406)	(-0.723)	(-2.455)	(-2.033)	(-1.578)	(-0.737)	(-1.186)
$Relative\_Price\_Hedge_t \times dC_{t-1}$									0.001	0.001	0.001	0.002	0.002
									(1.078)	(0.572)	(1.173)	(1.739)	(1.653)
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		(0.436)	(-0.225)	(-0.293)	(1.584)	(1.429)	(2.26)	(-0.491)	(0.49)	(1.784)	(2.064)	(-0.148)	(1.871)
$Wealth\_Hedge_t \times dC_{t-1}$									0.000	0.000	0.000	0.000	0.000
									(0.349)	(0.91)	(1.587)	(-0.189)	(0.726)
GMB_t			0.026	0.088	0.078	0.062	0.080	0.093	0.024	0.000	0.019	-0.015	-0.021
			(0.21)	(0.913)	(0.943)	(0.725)	(0.971)	(1.176)	(0.302)	(-0.005)	(0.281)	(-0.226)	(-0.321)
$GMB_t \times dC_{t-1}$									-0.018	-0.005	-0.010	-0.003	-0.016
									(-1.26)	(-0.463)	(-0.945)	(-0.365)	(-1.61)
dC_{t-1}									0.006	0.014	0.017	0.006	0.007
									(1.181)	(2.313)	(2.605)	(1.049)	(1.141)
Panel B: Premium differential between green and brown assets (annual, %)													
$Relative\_Price\_Hedge_t$	0.101	0.092	0.308	0.390	0.201	0.190	0.193	0.064	0.228	0.163	0.061	0.050	0.053
$Relative\_Price\_Hedge_t \times dC_{t-1}$									-0.005	-0.005	-0.015	-0.014	-0.024
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$Wealth\_Hedge_t \times dC_{t-1}$									0.000	0.000	0.000	0.000	0.000
GMB_t			0.264	0.929	0.761	0.575	0.555	0.676	0.257	-0.004	0.181	-0.109	-0.159
$GMB_t \times dC_{t-1}$									-0.137	-0.030	-0.055	-0.018	-0.109
dC_{t-1}									-0.049	-0.071	-0.087	-0.013	-0.010
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM	CAPM	FF3	FF3MOM	FF5	FF5MOM
Conditioning var.									dC_{t-1}	dC_{t-1}	dC_{t-1}	dC_{t-1}	dC_{t-1}

Notes: Variables are defined in Section 4.1 and t-stats are in brackets. Pastor et al. (2021a) sample: November 2012-December 2019.