Can Stablecoins be Stable?

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Abstract

This paper proposes a framework to analyze the stability of stablecoins – cryptocurrencies designed to peg their price to a currency. We study the problem of a monopolist platform earning seignorage revenues from issuing stablecoins and characterize equilibrium stablecoin issuance-redemption and pegging dynamics, allowing for various degrees of commitment over the system's key policy decisions. Because of two-way feedback between the value of the stablecoin and its ability to peg the currency, uncollateralized (pure algorithmic) platforms always admit zero price equilibrium. However, with full commitment, an equilibrium in which the platform maintains the peg also exists. This equilibrium is stable locally but vulnerable to large demand shocks. Without a commitment technology on supply adjustments, a stable solution may still exist if the platform commits to paying an interest rate on stablecoins contingent on its implicit leverage. Collateral and decentralizing stablecoin issuance help stabilize the peg.

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1 Introduction

A stablecoin is a cryptocurrency with embedded smart contracts designed to maintain a stable value vis-à-vis an official currency. It aims to avoid a fundamental drawback of conventional cryptocurrency: being too volatile to be used as a means of payment or store of value. Stablecoins therefore allegedly combine the benefits of the blockchain technology with the stability of well-established currencies and have gained in popularity in the last couple of years, with their combined market capitalization growing from $3 billion in 2019 to $181 billion in April 2022.[1] Confronted with the rapid growth of stablecoin platforms, legislators have become increasingly concerned about the financial stability risks posed by stablecoins and have introduced new regulatory initiatives to balance the perceived risks and benefits associated with this new technology.[2]

Stablecoin protocols rely on a wide variety of pegging mechanisms to fulfill their promise of price stability: algorithmic supply adjustments (e.g., Terra), over-collateralization with dynamic liquidation (e.g., Frax), and decentralization of the issuance process (e.g., DAI). To this date, however, the academic literature provides little guidance about the efficiency of these tools and their optimal design. This paper aims to fill this gap by developing a general model of stablecoins to analyze the performance of various pegging mechanisms.

We propose a framework to study the dynamic problem of a stablecoin platform that caters to a time-varying demand from investors. Investors, who value stability, enjoy liquidity benefits from owning stablecoins when the price is stable. The platform acts as a monopolistic issuer and taps in these liquidity benefits while trying to maintain a peg with respect to some unit of account. The existence of seignoriage revenues and the focus on price stability make a stablecoin platform similar to a central bank. Like a central bank who may overprint money, a stablecoin platform has a tendency to overissue stablecoins, which ultimately undermines the peg. A central bank's ability to perform its tasks thus relies to a large extent on its credibility. The main technological proposition of stablecoins in this regard is the possibility to commit to specific key policies such as issuance and redemption, interest rates and fees, and collateral liquidation rules via smart contracts.

Our objective is to characterize the stablecoin price, the value of the platform's equity

[1]https://www.statista.com/statistics/1255835/stablecoin-market-capitalization/

[2]For instance, the US Congress is working on a STABLE (Stablecoin Tethering and Bank Licensing Enforcement) Act while in the UK, the Treasury has launched the "UK regulatory approach to cryptoassets and stablecoins: Consultation and call for evidence".

shares—referred to in the crypto-space as *governance tokens*— and provide conditions under which the peg holds and under which it doesn't. An equilibrium in our model has two components. The monopolistic platform chooses a dynamic issuance-repurchase policy, an interest policy paid in stablecoins and a collateralization policy. Investors price the stablecoin competitively given the liquidity benefits they derive from owning stablecoins and the interest paid by the platform. In our model the unique state variable is the demand ratio between current stablecoin demand and supply by the platform.

We first study stablecoin protocols that can fully commit to issuance-redemption and interest rate rules. In other words, all platform policies can be programmed ex-ante through *credible* smart contracts. This analysis provides an upper bound for the value of *algorithmic stablecoin* protocols that rely on programmable adjustments of the quantity of stablecoin such as Terra, NuBits, and Basis. We show that even under full commitment, there exists an equilibrium in which stablecoins and governance tokens are worth zero. This equilibrium always arises because both stablecoin dividends, that is, liquidity benefits and interest payments depend themselves on the value of stablecoins. As is known in other contexts, the self-referential value of money implies zero value fixed point.

We also show that a second equilibrium exists in which the peg is *locally* stable. In this equilibrium, the system generates seigniorage revenues and governance tokens have positive value. The platform maintains a constant demand ratio and sets interest payments to peg the price. To maintain the peg, the system reacts to a positive demand shock by creating new stablecoins and distributing them to governance token holders as seigniorage dividends. Conversely, it reacts to a negative demand shock by buying back stablecoins and, thereby, reducing supply. In a pure algorithmic setting, the platform finances these repurchase operations by issuing additional governance tokens and diluting legacy holders.

We show, however, that even in this favorable equilibrium the platform cannot implement a strict peg. Like any financial institution, the platform is subject to limited liability as it cannot force stablecoin owners to finance any repurchase beyond governance token dilution. After a large negative demand shock, the value of future seignoriage revenues may be so low that the platform cannot finance a buyback and restore the optimal demand ratio even with full dilution. The peg is then broken as a too high stablecoin supply implies that market clears at a price below par. Although the peg is lost and governance tokens are worth zero, the stablecoin price may still be positive and fluctuates with demand as investors "hope for resurrection". At some point, stablecoin demand may recover enough so that governance token holders can recapitalize the platform to repurchase the quantity of stablecoin that is necessary to re-establish the peg.

We then investigate the stability properties of a stablecoin scheme under a weaker form of commitment. More precisely, we relax our initial assumption that all policies can fully be programmed via smart contracts. In this setting, the platform can commit to an interest rate rule but retains flexibility over its issuance-redemption policy. For a constant interest rate rule, the leverage ratchet effect of Admati, DeMarzo, Hellwig, and Pfleiderer (2018) and DeMarzo and He (2021) applies: it is never optimal for the system to reduce its leverage and the peg cannot be maintained. We find, however, that an equilibrium with local stability still exists if the interest rate payment decreases with the demand ratio. Such a rule penalizes over-issuance and forces the platform to implement repurchase. We stress that the strength of this punishment is purely endogenous as the interest is paid in stablecoins and the platform faces no direct issuance cost.

Next, we study how escrowing an external collateral asset on which smart contracts can be written—such as another crypto-currency—affects the system's ability to maintain the peg. This design is common in practice, with many stablecoins such as DAI or Frax partly relying on external crypto-currency holdings to improve their stability. When the collateralization rate falls below a certain threshold, a smart contract triggers the liquidation of the platform. Imposing a minimum collateralization rate is a double-edged sword: On the one hand, it improves the stability of the stablecoin price as guarantees a residual value for stablecoin owners when the system liquidates its assets. On the other hand, locking crytpo-assets in the platform is costly and future seigniorage revenues are lost when platform shuts down.

Last, we examine the stability of a stablecoin scheme that decentralizes the issuance and redemption of its stablecoin. This feature is present in DAI: a stablecoin that anyone with access to the Ethereum platform can mint freely. We find that this decentralization can act as an effective substitute for a commitment technology on stablecoin redemption and issuance. In this setting, investors acting as arbitrageurs prevents the price from moving away from the peg by creating more (redeeming) stablecoins in reaction to a positive (negative) demand shock. This decentralization allows the system to maintain the peg because the decisions affecting that system's leverage have been externalized to agents that—unlike governance token holders—are not hurt by a reduction of leverage.

Literature review Our paper contributes to an interdisciplinary literature on stablecoins. From the computer sciences literature, Klages-Mundt and Minca (2019, 2020) develop models featuring endogenous stablecoin price and an exogenous collateral and find deleveraging spirals and liquidation in a system with imperfectly elastic stablecoin demand. Gudgeon, Perez, Harz, Livshits, and Gervais (2020) simulate a stress-test scenario for a DeFi protocol and find that excessive outstanding debt and drying up of liquidity can lead the lending protocol to become undercollateralized. Our paper also relates to a descriptive literature on stablecoins (Arner, Auer, and Frost, 2020; Berentsen and Schär, 2019; Bullmann, Klemm, and Pinna, 2019; ECB, 2019; Eichengreen, 2019; G30, 2020). In closely-related contemporaneous work, Li and Mayer (2022) study the peg dynamics of stablecoin platforms under the assumption that stablecoins generate network externalities and the systems' reserves are subject to stochastic shocks. Our paper differs by considering various commitment technologies and demand shocks that affect the system's franchise value.

In studying the stabilization mechanisms across stablecoin types and the failure of governance incentives to recapitalize undercollateralized systems, our paper draws from the corporate finance literature which examines firm shareholders' attitudes towards leverage. Black and Scholes (1973) first documented that, in a frictionless capital structure setting of Modigliani and Miller (1963), firm shareholders do not have incentives to voluntarily buy back debt and reduce leverage as this always implies a transfer of wealth to existing creditors, and they will give up their default option. Myers (1977) attributes this resistance to the reduction in dilution of existing debt since shareholders do not internalize the benefits from lower bankruptcy costs accruing to debtholders but rather pay a higher post-recapitalization price. Admati, DeMarzo, Hellwig, and Pfleiderer (2018) generalize these findings to multiple asset classes of debt and with agency frictions and document a "leverage ratchet effect", whereby shareholders have no incentives to delever the firm and instead always find it optimal to further increase leverage by issuing new debt, even when leverage exceeds its optimal level. The results are consistent with agency cost models where debt overhang distorts incentives, for instance, through under-investment as in Myers (1977), or asset substitution as in Jensen and Meckling (1976)—where shareholders shift risk towards debtholders by engaging in riskier projects—or under the "control hypothesis" for debt creation in Jensen (1986), in the presence of free cash flow agency costs. DeMarzo and He (2021) also show delevering resistance effects, although in their model leverage mean-reverts to a target because of asset growth and debt maturity. Our paper contributes to this literature by considering cases in which the firm (stablecoin platform in our setting) can commit or decentralize the buybacks and coupon payment decisions through a smart contract algorithm. These features can be seen as an extreme form of debt-convenants as studied in Smith and Warner (1979), Bolton and Scharfstein (1990), Aghion and Bolton (1992), and Jason Donaldson and Gromb (2020) that would apply in each state of the world.

More broadly, our paper contributes to the literature applying corporate finance and asset pricing models to model digital platforms and token valuations. While not mainly focusing on stablecoins, Cong, Li, and Wang (2020a) develop a continuous-time model of token-based platform economy with network effects and endogenous token price and also document conflicts of interests between platform owners and users, resulting in an under-investment outcome. In their model as in ours, platform insiders restrict the supply of token to preserve the franchise value, a mechanism stabilizing its price. We highlight the existence of limitations to such a quantity adjustment mechanism. Cong, Li, and Wang (2020b) build a dynamic asset pricing model with network effects and inter-temporal linkages in endogenous token price and user adoption, and analyze the Markov equilibrium with platform productivity as the state variable.

2 General Environment

In this section, we describe our model of stablecoins. The premise of our analysis is that investors enjoy liquidity benefits from holding stablecoins issued by the platform, as they would do for money or bank deposits. Investors value stablecoins to the extent that their price is stable. Hence, the stablecoin platform can generate seignoriage revenues if it can maintain a peg between the stablecoin price and some target unit of account. We describe the formal building blocks of the model below.

2.1 Stablecoin Demand

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions. All agents are risk neutral with an exogenous discount rate of $r > 0$.[3] Time is continuous with $t \in [0, \infty)$.

[3] Alternatively, we can interpret the model as written under a fixed risk-neutral measure that is independent of the stablecoin platform policies.

We consider a platform that issues stablecoins. Stablecoins are a liability of the platform that trade at (endogenous) price p_t expressed in the unit of account. The outstanding stock of these stablecoins at date t is C_t. Stablecoins have value because investors enjoy direct utility from holding them: at date t, holding stablecoins generates utility flow $p_t\ell(A_t, p_tC_t)$ per unit with A_t an exogenous driver of stablecoin value. To fix ideas, we say that variable A_t represents the value of some cryptoassets, which proxies for investors' demand for alternative means of payment. The liquidity benefit from holding stablecoins can be thought as a convenience yield investors derive because stablecoins are a form of money.[4]

Assumption 1. *The convenience yield of stablecoins $\ell(A, pC)$ is (i) positive and continuously differentiable in both arguments; (ii) strictly increasing in A; (iii) bounded with $0 \leq \ell(A, pC) \leq r$; (iv) homogeneous of degree 0 and (v) equal to 0 if the stablecoin price p is not pegged to 1. Finally, (vi) the product of the convenience yield with the total value of stablecoins $\ell(A, pC)pC$ is single-peaked with* $\lim_{x\to\infty} \ell(A, x)x = 0$

Property (i) is standard. Property (ii) states formally that the value of stablecoins increases with demand driver A_t. Property (iii) and (iv) are technical assumptions ensuring respectively that the stablecoin price is well-defined and that the problem ultimately economizes on one state-variable. Property (v) states that stablecoin owners enjoy a liquidity benefit only if it is pegged to the unit of account. This assumption is meant to capture in a simple way a trust element whereby investors value the stablecoin as means of transactions to the extent that issuers can maintain a pre-announced peg.[5] The peg at 1 is chosen for convenience and because it corresponds to market practice but our results do not depend upon it; only the real value of stablecoin holdings matters. Finally, Property (vi) will ensure that the optimal amount of stablecoins is interior. An example of a class of functions that satisfy Assumption 1 is $\ell(A, C) = r \exp(-\alpha C/A)$ for $\alpha > 0$.

The cryptoasset value A_t that drives stablecoins' demand has the following law of motion

$$dA_t = \mu A_t dt + \sigma A_t dZ_t + A_{t^-}(S_t - 1)dN_t, \tag{1}$$

[4]Our reduced-form specification can be microfounded assuming that stablecoins are essential to carry some transactions. The utility flow from this liquidity service would then be a function $u(A_t, p_tc_t)$ of the real value of stablecoin holdings, p_tc_t. A representative investors' marginal utility for stablecoins (in addition to any dividend payment) is then given by $p_t\ell(A_t, p_tC_t) \equiv p_tu_c(A_t, p_tc_t)$, equating the representative investor's holdings c_t with the total stock of stablecoins C_t.

[5]We can relax this "extreme-peg" assumption by assuming that the liquidity benefit is still positive for small deviations from the peg, but decreasing in price volatility.

where dZ_t is the increment of a standard Brownian motion and dN_t is a standard Poisson process with constant intensity $\lambda > 0$ adapted to $\mathcal{F}$. The size of a downward jump, $-\ln(S)$ is exponentially distributed with parameter $\xi > 0$ and the expected jump size is $\mathbb{E}[S-1] = -1/(\xi+1)$. The Poisson process generates large negative shocks to stablecoin demand that can be thought of as speculative attacks. Overall, the expected growth rate of stablecoin demand is given by $\mu - \lambda/(\xi+1)$, which we assume is lower than the discount rate r. We use notation A_{t^-} to denote the cryptoasset's value before a jump.[6]

Finally, there exists a safe asset the platform can hold as collateral to back the issuance of stablecoins. This collateral trades in a perfectly competitive market at price p_t^k with

$$dp_t^k = \mu^k p_t^k dt, \tag{2}$$

with μ^k the price drift. Collateral can be thought as safe asset with rate of return $\mu^k \leq r$. We interpret the difference between the discount rate and the rate on collateral, $r - \mu^k$ as a convenience yield enjoyed by collateral owners. As we will see, this feature generates a cost from holding collateral for the stablecoin platform.[7]

2.2 Platform Operation

We will analyze both a centralized and a decentralized platform. For clarity, we postpone the description of a decentralized platform to Section 5. The main policy choice of a centralized stablecoin platform is its issuance/redemption policy $\{d\mathcal{G}_t\}_{t\geq 0}$. A positive (negative) value of $d\mathcal{G}_t$ means the platform issues (repurchases) stablecoins. The platform also chooses its coupon policy $\{\delta_t\}_{t\geq 0}$ paid in stablecoins, with $\delta_t > 0$, and its collateral purchase policy $\{d\mathcal{M}_t\}_{t\geq 0}$. Finally, the platform may default at stochastic time τ_D.

To describe these policies, consider an analogy between the stablecoin platform and a central bank. When it issues stablecoins ($d\mathcal{G}_t > 0$), the platform receives a payment $p_t d\mathcal{G}_t$ from investors in the unit of account. Similarly, when it credits the account of a depository institution with reserves, the central bank receives an asset in exchange. The stablecoin's

[6] A_{t^-} denotes the left limit $A_{t^-} = \lim_{h\to 0} A_{t-h}$. Also note that $A_{t^-}dt = A_t dt$ as the set $\{T_k\}_{k\geq 1}$ of jump times has zero measure of length.

[7] Our assumption of a safe collateral asset comes with some loss of generality because some stablecoin protocols are implicitly or explicitly backed by cryptoassets. In this case, the collateral price would likely be correlated with demand process A_t. It is intuitive, however, that such correlation would reduce the usefulness of collateral as a hedge against demand fluctuations. In addition, introducing correlation significantly complicates the analysis.

coupon policy whereby every stablecoin investor is credited with δ_t units of free stablecoins per unit owned is akin to an interest payment on reserves. Finally, collateral holdings of the platform correspond to a central bank's asset holdings.

Law of Motions The platform's policies imply the following law of motions for the amount of stablecoin outstanding, C_t and the value of its collateral, denoted K_t

$$dC_t = \delta_t C_t dt + d\mathcal{G}_t, \tag{3}$$

$$dK_t = \mu^k K_t dt + d\mathcal{M}_t. \tag{4}$$

Consider first law of motion (3) for stablecoins. The first term on the right-hand-side captures the contribution of the coupon policy δ_t to stablecoin issuance. It is treated separately from the active issuance component $d\mathcal{G}_t$, however, because the platform is not paid when transferring stablecoins as coupon payments. Equation (4) is the law of motion for the value of collateral. The first term on the right-hand side corresponds to the passive change in collateral value. The last term corresponds instead to the active change in value from purchases or sales of collateral by the platform expressed in the unit of account.[8]

Jump Notation There are both Brownian shocks and jumps to the value of cryptoassets in our model. The platform's policies can also feature jumps. A jump represents a discrete, instantaneous change of a variable. We denote the value of a variable X just before and after the jump by X_{t^-} and X_t, respectively. It is useful to decompose the stablecoin issuance and collateral purchase policies into their absolutely continuous and jump parts as[9]

$$d\mathcal{G}_t = G_t dt + (\mathcal{G}_t - \mathcal{G}_{t^-}),$$
$$d\mathcal{M}_t = M_t dt + (\mathcal{M}_t - \mathcal{M}_{t^-}).$$

[8]This law of motion can be alternatively written $dK_t = S_t^k dp_t^k + p_t^k dS_t^k$ with S_t^k the quantity of collateral held by the platform. The term $d\mathcal{M}_t$ in (4) corresponds to $p_t^k dS_t^k$.

[9]By Lebesgue decomposition of a right-continuous function $\boldsymbol{f} : \boldsymbol{I} \to \mathbb{R}$ of bounded variation where $\boldsymbol{I}$ is an interval, we can can represent $\boldsymbol{f}$ as a sum of three functions $\boldsymbol{f}_a + \boldsymbol{f}_c + \boldsymbol{f}_j$, which is unique up to constants, where $\boldsymbol{f}_j$ is a jump function, $\boldsymbol{f}_c$ is a singular function, and $\boldsymbol{f}_a$ is an absolutely continuous function. Going forward, we abstract from the presence of singular functions $\boldsymbol{f}_c$ (e.g., a "devil's staircase"). We refer the reader to Appendix C.3. of DeMarzo and He (2021) for a discussion.

2.3 Stablecoin Pricing and Platform's Objective

Stablecoin Pricing Formula Investors price the stablecoin competitively taking as given the platform's policies. They enjoy two dividend streams from holding stablecoins: the direct utility benefits when the price is pegged and coupon payments when the stablecoin platform pays interest, with resective value $\ell_t p_t$ and $\delta_t p_t$ per unit. Should the platform default, a standard bankruptcy procedure applies in which stablecoin owners are treated as pari-passu debt creditors. They thus receive any platform's collateral up to the parity value of stablecoins. The platform shareholders receive any residual proceeds. At date t, the competitive stablecoin price given the platform's continuation policies is thus

$$p_t = \mathbb{E}\left[\int_t^{\tau_D} e^{-r(s-t)} \left(\ell_s + \delta_s\right) p_s ds + e^{-r(\tau_D - t)} \min\left\{K_{\tau_D}/C_{\tau_D}, 1\right\} \middle| A_t, \{d\mathcal{G}_s, d\mathcal{M}_s, \delta_s\}_{s\geq t}\right] \tag{5}$$

Platform's Objective The platform's objective is to maximize its date-0 value E_0, which is the sum of the issuance benefits net of the collateral purchases.

$$E_0 = \max_{\tau_D, \{\delta_t, d\mathcal{G}_t, d\mathcal{M}_t\}_{t\geq 0}} \mathbb{E}_0\left[\int_0^{\tau_D} e^{-rt}\big(p_t d\mathcal{G}_t - d\mathcal{M}_t\big) + e^{-r\tau_D} \max\left\{0, K_{\tau_D} - C_{\tau_D}\right\}\right] \tag{6}$$

where the price p_t is given by equation (5). When it defaults, the platform enjoys the residual value of collateral, if any, after stablecoin owners have been paid. As a monopolistic issuer, the platform has price impact. Hence, it pays the post-issuance (post-repurchase) price when it issues (repurchases) stablecoins. As we will see, this feature is important when studying the platform's problem under limited commitment because it weakens the platform's incentives to buy back blocks of debt.

2.4 Discussion of the Environment

Monopolistic Stablecoin Platform We focus on the analysis of a single stablecoin platform for simplicity. In practice, several stablecoins would compete to cater to investors' demand for alternative means of payment. We refrain from modeling competition and entry of platforms for simplicity; one can interpret the platform's convenience yield as investors' residual demand for one platform's stablecoins after accounting for supply from other platforms. All that is needed is that the platform enjoys some market power. A source of market power in this context is network effects as in Cong, Li, and Wang (2020a).

Commitment Problem and Smart Contracts Our analysis will reveal that stablecoin platforms face a fundamental commitment problem. Policies that have been chosen at date 0 may not be optimal at date t from the point of view of the platform's equity holders. A key technological proposition of stablecoins is that rules and procedures can be programmed within transparent algorithms, so-called *smart contract*. In many cases, however, platforms retain flexibility over parts of the algorithm for technical maintenance, future adaptability, or to decrease the vulnerability to hacking. To reflect these considerations, we will characterize optimal policies under various degree of commitment to identify robust tools that can foster stability of stablecoin platforms.

3 Credible Smart Contracts

In this section, we analyze the problem of a stablecoin platform that can commit to all future policies. A platform with commitment can be viewed as a stablecoin protocol with credible smart contracts governing issuance, repurchase, coupon and collateral policies. The analysis under full commitment provides minimal necessary conditions for a stablecoin platform to have positive value and to be able to maintain its peg. This analysis delivers two main results. Stablecoins may have no value even under commitment. Second, when they have positive value, the platform is vulnerable to large shocks unless it is fully collateralized.

Inspired by market practice, we focus on a constant collateralization rule for the platform:

$$K_t = \varphi C_t, \tag{7}$$

that is, the platform must maintain a constant ratio $\varphi \in [0, 1]$ between the value of its collateral and the par value of stablecoins. This specification means that the platform's other policies $\{d\mathcal{G}_t, \delta_t\}$ fully determines its collateral purchase policy $d\mathcal{M}_t = \varphi dC_t$. The case $\varphi = 0$ ($\varphi = 1$) corresponds to a so-called algorithmic stablecoin (narrow bank).

For this analysis, the only constraint on the platform's policy choices at date 0 is that its equity cannot become negative at some future date t, that is, limited liability applies. To clearly highlight the role of this constraint, we first consider a benchmark with unlimited liability in Section 3.1 and then reintroduce limited liability in Section 3.2.

3.1 Unlimited Liability Benchmark

We first assume the platform's equity value may become negative. In this case, the platform never has to default so we set default time τ_D to infinity. The platform chooses a stablecoin issuance-redemption policy $\{d\mathcal{G}_t\}_{t\geq 0}$, an interest policy $\{\delta_t\}_{t\geq 0}$ and a collateralization rate φ to maximize the value of the platform at date 0 given by

$$E_0 = \max_{\varphi, \{\delta_\tau, d\mathcal{G}_\tau\}_{\tau \geq 0}} \mathbb{E}\left[\int_0^\infty e^{-rt} \left(p_t d\mathcal{G}_t - \varphi dC_t \right) \middle| A_0, C_0 = 0 \right] \tag{8}$$

$$\text{subject to} \quad (5), (3) \text{ and } (7).$$

As it owns no asset, the platform's payoff is the net present value of issuance proceeds. Equation (3) is the law of motion for stablecoins implied by the issuance policy and the initial condition $C_0 = 0$. Equation (5) is the competitive pricing function for stablecoins at any date t, given policies chosen by the platform for dates $\tau \geq t$.

Our first result is that even under full commmitment, there exists an equilibrium with zero stablecoin price and zero platform value if the platform does not hold collateral.

Proposition 1. *For an uncollateralized platform with $\varphi = 0$, there always exists a zero-price equilibrium in which $p_t = 0$, $\forall t \geq 0$.*

The zero-price equilibrium arises because there is no anchor between the stablecoin and the unit of account for an uncollateralized platform. In particular, the coupon is paid in stablecoins, not in the unit of account. To see why this implies a zero-price equilibrium exists, suppose the price is indeed 0. Then both components of the stablecoin dividend in pricing function (5) are equal to 0. Stablecoin owners enjoy no liquidity benefit because the price is not pegged to 1 and the real coupon δp is also worth 0 even if the platform promises a very large nominal coupon payment δ. Finally, without collateral, the price is not supported by an external asset. Hence, the the initial conjecture that the price is zero is valid. The platform has no value because it captures no stablecoin issuance benefits.

Proposition 1 highlights that stablecoins, like any form of fiat money, is fragile: stablecoins may be worth zero even when issuance and repurchase can be fully programmed with credible smart contracts. We show below that there also exists an equilibrium in which the stablecoin has value and the platform enjoys seigniorage revenues.

Proposition 2. *With full commitment and unlimited liability, the equilibrium with positive stablecoin price features a constant demand ratio* $\frac{A_t}{C_t} = \frac{A_t}{C^\star(A_t)} = a^*$ *for all* t *with*

$$C^\star(A) = \arg\max_C \left\{\ell(A, C)C\right\}. \tag{9}$$

The coupon policy at demand ratio a^* *is* $\delta^\star = r - l(a^\star)$ *to peg the stablecoin price to* 1 *and the platform sets the collateralization ratio* $\varphi^\star = 0$.

As we show formally in the proof, the platform value is the present value of liquidity benefits enjoyed by investors net of the collateral holding costs,

$$E_0 = \mathbb{E}\left[\int_0^\infty e^{-rt}\Big(\ell(A_t, C_t)C_t \mathbb{1}_{\{p_t=1\}} + (\mu^k - r)\varphi C_t\Big)dt \,\Big|\, A_0\right], \tag{10}$$

with $\ell(A, C)C$ the instantaneous total seignoriage revenues for the platform when the price is pegged. This is intuitive because the platform captures all gains from trade. Maximizing the platform value E_0 becomes a static optimization problem if it can maintain the peg. In this case, the optimal collateralization rate is $\varphi^\star = 0$ because holding collateral is costly. Given demand A, an interior optimum $C^\star(A)$ for stablecoin supply exists under Assumption 1. Homogeneity of the liquidity benefit, $\ell(A, C)$ further implies that $C^\star(A)$ is linear in A; we call $a^\star$ the constant *demand ratio*. The platform issues (buys back) stablecoins when demand, captured by A_t increases (decreases) to maintain this optimal demand ratio.

It remains to show that the platform can maintain the peg using its coupon policy as stablecoin owners enjoy no liquidity benefit otherwise. In equilibrium, the demand ratio a_t is constant, so we only need to specify $\delta^\star \equiv \delta(a^\star)$. It is easy to verify that the peg holds when $\delta^\star$ is given as in Proposition 2 because

$$p_t = \frac{\ell(a^\star) + \delta^\star}{r} = 1. \tag{11}$$

3.2 Limited Liability: Preliminary Analysis

The full commitment policy with unlimited liability requires the platform to conduct large stablecoin repurchases when the underlying cryptoasset value drops in order to restore an optimal demand ratio. For a large drop, however, the repurchase cost might exceed the post-repurchase platform value. In practice, the platform would then be unable to finance the repurchase by issuing new equity even if it fully dilutes current equity.

To capture these concerns, we now assume that policies must satisfy limited liability. That is, the platform's equity value must be positive in all contingencies. In other words, even credible smart contracts cannot force the platform to conduct repurchases if the platform's net continuation value is negative. The limited liability constraint, $E_t \geq 0$ can be expressed as follows, using similar steps as in Proposition 2

$$E_t = \mathbb{E}\left[\int_t^\infty e^{-r(s-t)}\Big(\ell(A_s, C_s)C_s + (\mu^k - r)\varphi C_s\Big)ds \middle| A_t, C_t = 0\right] - (p_t - \varphi)C_{t^-} \geq 0. \quad (12)$$

This expression is compatible with the expression for the date-0 value of the platform, Equation (10), because he platform starts with zero stablecoins ($C_{0^-} = 0$). In Equation (12), the term, $(p_t - \varphi)C_{t^-}$, is the net cost of repurchasing all outstanding stablecoins.[10] The net cost is $p_t - \varphi$ because buying back one stablecoin frees up collateral value φ. Limited liability constraint (12) admits the usual interpretation that the unlevered platform value given current demand A_t must exceed the net value of outstanding stablecoins (debt value). After a large enough negative shock to A_t, the platform's equity value can thus become negative if the platform were to implement the policy in Proposition 2.

We now analyze the solution under full commitment and limited liability, which is problem (8) adding constraint (12). In doing so, we focus on the following set of policies.

$$d\mathcal{G}(a_t, C_{t^-}) = \begin{cases} G(a_t, C_{t^-})dt & \text{if } a_t < \overline{a}, \\ \frac{A_t}{a_L^\star} - C_{t^-} & \text{if } a_t \geq \overline{a} \end{cases}, \qquad \delta(a_t) = \begin{cases} \underline{\delta} & \text{if } a_t < \overline{a}, \\ \delta^\star & \text{if } a_t \geq \overline{a} \end{cases}, \quad (13)$$

where policy parameters $\boldsymbol{\theta} = \{\underline{a}, a_L^\star, \underline{\delta}, \delta^\star, G\}$ are chosen by the platform at date 0. We remind that collateral holdings K_t must still satisfy collateralization rule (7). The policy set described by equation (13) includes the optimal policy under unlimited liability. In target region $[\overline{a}, \infty)$, the platform implements a constant demand ratio a_L^* but it stops targeting a_L^* when the demand ratio falls below a threshold $\overline{a}$. Intuitively, this new feature helps to ensure that limited liability can be satisfied after a large negative shock to demand. We note that in region $[0, \overline{a}]$ the issuance policy is smooth, that is, of order dt by opposition

[10] As shown by our analysis with unlimited liability, the policy is not to repurchase all stablecoins. The formulation in (12) simply breaks down the optimal policy into two steps which happen simultaneously at the same price: (i) repurchase all outstanding stablecoins C_{t-}, and (ii) issue new stablecoins to the optimal level, $C_t = C^\star(A_t)$. Both transactions would take place at the post-issuance price p_t.

to discrete jumps over $[\overline{a}, \infty)$. Because this region is visited with positive probability in equilibrium, we also need to specify the coupon policy when $a \in [0, \overline{a}]$.

Our motivation to focus on the set of policies described by Equation (13) is twofold. First, the policy set is directly inspired by the optimal policy under unlimited liability. Second, the policy is Markovian in that it only depends on the value of state variables (A_t, C_{t^-}), not on the full history of demand shocks up to period t. This last feature considerably simplifies our analysis in the presence of limited liability constraints.[11]

Given the set of policies considered, we can define functions $E(A, C)$ and $p(A, C)$ for the platform's equity and the stablecoin price, now omitting the time index. Given the homogeneity of the problem, the ultimate state variable for equity and price is $a = \frac{A}{C}$ so we define $e(a) \equiv E(A, C)/C$ and $p(a) \equiv p(A, C)$ where $e(a)$ is the platform's equity value per stablecoin oustanding.

We guess and verify that the price function satisfies $p(a) = 1$ for $a \in [0, \overline{a}]$ and $p(a) < 1$ for $a \in [0, \overline{a})$, that is, investors enjoy liquidity benefits only in the target region. We first characterize the optimal repurchase policy in the region in which the peg is lost.

Lemma 1. *An optimal policy under commitment and limited liability satisfies* $\underline{\delta} = 0$ *and*

$$g(a) \equiv \frac{G(a_t, C_t^-)}{C_t^-} = -\frac{\mu^k \varphi}{p(a) - \varphi}, \tag{14}$$

that is, the platform does not pay coupon when the peg is lost and it uses collateral proceeds to repurchase stablecoins. Under repurchase policy (14)*,* $e(a) = 0$ *for all* $a \in [0, \overline{a}]$*.*

The intuition for this result is as follows. As shown by (10), the platform's value rests on its ability to capture investors' liquidity benefits. As a result, the platform seeks to minimize the time it will spend in region $[0, \overline{a}]$ where the peg is lost. To increase $a_t = \frac{A_t}{C_t}$ when $a_t \in [0, \overline{a}]$, stablecoin issuance should be minimized in this region. This involves paying no coupon to investors, $\underline{\delta} = 0$ and using returns on collateral to buy back stablecoins. To see why the latter condition yields equation (14), observe that each stablecoin is backed by an collateral value φ that grows at rate μ^k. The downpayment for a stablecoin is $p - \varphi$

[11] The general problem is not standard because limited-liability constraints (12) are forward-looking, which means equity value E_t is not the solution to a standard Hamilton-Jacobi-Bellman (HJB) equation. Techniques developed by Marcet and Marimon (2019) do not apply to our problem; the additional complexity comes from the term $(p_t - \varphi)C_{t^-}$ on the right-hand side of (12) as a state variable C_{t^-} multiplies forward-looking variable p_t, which depends on all future policy choices. Our focus on Markovian policies ensure the equity value and the stablecoin price solve HJB equations.

because buying back a stablecoin frees up collateral value φ. Hence, the maximum rate at which the platform can repurchase stablecoins is given by (14). The fact that the platform's equity value is equal to 0 when the peg is lost is intuitive. Given the platform's objective to maximize time spent in the peg region $[\overline{a}, \infty)$, it should avoid any slack in the limited liability constraint in the region in which the peg is lost.

We are left to solve for the subset $\{\underline{a}, a_L^\star, \delta^\star\}$ of policy parameters $\boldsymbol{\theta}_0$. This second step requires characterizing the equilibrium price $p(a)$ and the equity value $e(a)$ over the state space $[0, \infty]$ for given values of these parameters.

Lemma 2. *Under a policy characterized by* (13) *and Lemma 1, the following holds*

1. *The platform's equity value is characterized by the following two equations*

$$e(a) = \begin{cases} 0 & \textit{if} \quad a \le \overline{a} \\ \left[e(a^\star) + (p(a^\star) - \varphi)\right] \frac{a}{a^\star} - (p(a^\star) - \varphi) & \textit{if} \quad a \ge \overline{a} \end{cases} \tag{15}$$

$$(r + \lambda - \mu) e(a^\star) = \mu^k \varphi + \mu(p(a^\star) - \varphi) + \lambda \mathbb{E}[e(Sa^\star)] \tag{16}$$

2. *The coupon policy to maintain the peg $p(a^\star) = 1$ in target region $[\overline{a}, \infty]$ is*

$$\delta^\star = r - l(a^\star) + \lambda \left(1 - \mathbb{E}[p(a^\star S)]\right) \tag{17}$$

3. *For $a \le \overline{a}$, the equilibrium price function satisfies the following equation*

$$(r + \lambda) p(a) = (\mu - g(a)) a p'(a) + \frac{\sigma^2}{2} a^2 p''(a) + \lambda \mathbb{E}[p(Sa)] \tag{18}$$

where $g(a)$ is given by (14).

Consider first the platform's equity value. We already showed in Lemma 1 that the equity value should be zero when the peg is lost. In the peg region $[\overline{a}, \infty)$, the platform issues or repurchase stablecoins to maintain a constant demand ratio $a^\star$. By definition, equity value is then given by

$$E(A, C) = E(A, C^\star(A)) + (p(a^\star) - \varphi)(C^\star(A) - C) \tag{19}$$

where $p(a^\star) = p(A, C^\star(A))$. Equation (15) obtains from (19) by dividing each side by the stablecoin quantity C. Equation (16) is the Hamilton-Jacobi-Bellman (HJB) equation for equity value at the constant demand ratio $a^\star$. Equity holders receive two cash flows: interest on collateral and expected issuance proceeds, respectively $\mu^k\varphi$ and $\mu(p(a^\star)-\varphi)$ per unit of collateral. Expected issuance proceeds are positive if demand A_t grows on expectation ($\mu>0$) and if the platform is less than fully collateralized ($\varphi<1$).

The second part of Lemma **??** characterizes the coupon policy necessary to maintain the peg in region $[\overline{a},\infty)$. In the absence of Poisson shocks ($\lambda=0$), equation (17) is the same as in Proposition 2 with unlimited liability. Large negative demand shocks force the platform to abandon the peg, in which case the stablecoin price drops below 1. This effect requires the platform to pay a larger coupon in order to compensate for this expected price devaluation as shown by the last term of (17).

Finally, the third part of Lemma **??** characterizes the equilibrium price dynamics in the region $[0,\overline{a}]$ where the peg is lost. Note that the optimal repurchase policy derived in Lemma 1 enters HJB equation (18) because it governs the rate at which the demand ratio a_t increases in region $[0,\overline{a}]$. As shown by equation (14), the stablecoin repurchase rate depends itself on the price.

3.3 Limited Liability: Optimal Platform Design

We may now consider the optimal platform design under limited commitment. The platform chooses remaining policy parameters $\{a^\star,\overline{a}\}$ so as to maximize its date-0 value,

$$E_0 = A_0\frac{e(a^\star)+p(a^\star)-\varphi}{a^\star},$$

where $\underline{\delta}$, G are given by Lemma 1 and $e()$, $p()$ and $\delta^\star$ are characterized by Lemma 2. Lemma 2 provides an explicit solution for $e(a^\star)$ as a function of policy parameters $\{a^\star,\overline{a}\}$. Solving for $p(a^\star)$ analytically, however, proves impossible in most cases because of the feedback loop in dynamic price equation (18) via the repurchase decision $g(a)$ given by (14). While we provide numerical solutions in other cases below, two cases of interest allow for an explicit characterization of the platform's policy choice: the uncollateralized platform ($\varphi=0$) and the fully-collateralized platform ($\varphi=1$).

Consider first an uncollateralized platform with $\varphi=0$. Equation (14) then shows that the optimal repurchase policy is $g=0$ in the smooth region $[0,\overline{a}]$. This is intuitive because the platform receives no collateral proceeds to finance stablecoin repurchases in this case.

Hence, it can only hope for positive exogenous shocks to demand to recover the peg. This feature allows us to provide an explicit solution for p over the whole state space and thus to characterize the optimal platform policy choice.

Proposition 3. *Consider an uncollateralized (purely-algorithmic) stablecoin platform with commitment. The following results apply*

1. *In the region $[0, \overline{a}]$ in which the peg is lost, the equilibrium stablecoin price is given by $p(a) = \left(\frac{a}{\overline{a}}\right)^{-\gamma}$ where $\gamma < -1$ is the unique negative root of*

$$r + \lambda = -\mu\gamma + \frac{\sigma^2}{2}(1+\gamma)\gamma + \frac{\lambda\xi}{\xi - \gamma}. \tag{20}$$

2. *The optimal policy $\boldsymbol{\theta}_0^\star$ is characterized by Lemma 1 and 2 and $\{\overline{a}, a^\star\}$ that solve*

$$\max_{\{\overline{a}, a^\star\}} E_0 = \frac{\ell(a^\star)}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}} \frac{1}{a^\star} \tag{21}$$

$$\text{subject to} \qquad \frac{\overline{a}E_0}{a^\star} \geq 1 \tag{22}$$

The optimal policy has a non-empty inaction region, that is, $\overline{a} > 0$ in which the platform's equity value is 0. The optimal interest rate policy is $\underline{\delta} = 0$ and $\delta^\star = r - \frac{\lambda\gamma}{\xi-\gamma}\left(\frac{a_L^\star}{\overline{a}}\right)^{-\xi} - \ell(a_L^\star)$. Furthermore, there exists $\Omega < 0$ such that the optimal demand $a_L^\star > a^\star$ is characterized by

$$\ell'(a_L^\star)a_L^\star = \ell(a_L^\star) + \lambda\Omega \tag{23}$$

The first key finding with limited liability is that there must be an inaction region. When demand A_t falls suddenly to a very low level, the platform would need to repurchase a large quantity of stablecoins to restore the optimal demand ratio $a_L^\star$. On the other hand, the future value of seignoriage revenues is low in this case because current demand A_t predicts future demand. Hence, the platform's future value is not sufficient to finance the repurchase, even if it fully dilutes equity. To satisfy limited liability, the platform thus stops buying back stablecoins when a_t is too small.

The existence of the inaction region under limited liability affects the optimal demand target $a_L^\star$. In particular, in the peg region, the platform issues less stablecoins per cryptoasset, that is, $a_L^\star > a^\star$. Remember that with unlimited liability, the optimal demand ratio is determined by equation (23) setting $\Omega = 0$. With limited liability, equation (23) implies the optimal demand ratio $a_L^\star$ is higher than $a^\star$ because $\ell'' < 0$ by assumption. The platform issues less stablecoins in the peg region to protect itself against large negative demand shocks that take it to the inaction region. Consistent with this interpretation, the second term is positive if and only if demand A_t is subject to negative jumps ($\lambda > 0$). Without Poisson jumps, the platform would never have to buy large blocks of debt to restore its target demand, which means limited liability would have no bite.

The optimal coupon policy has two components. In the peg region, the coupon is set again to ensure that the price $p(a_L^\star)$ is pegged to one. In the inaction region $[0, \overline{a}]$, the optimal coupon is 0. In this case, the stablecoin value is driven by the probability that the demand ratio a_t reaches the peg region $[\overline{a}, \infty)$. Paying stablecoin coupons increases the stock of stablecoins, which decreases a_t and delays the time before the peg region is reached. Hence, paying no coupon in the inaction region is optimal.

Finally, observe that the platform's equity value is zero in the inaction region as the limited liability constraint binds. Ex-post, the platform is indifferent between defaulting or staying in operation. From an ex-ante perspective, however, it is important that the platform stays open. In the inaction region, positive shocks to the cryptoasset value can improve the position of the platform until it reaches threshold $\overline{a}$. Then, the platform repurchases a block of stablecoins and regains the peg. The possibility of a resurrection implies stablecoins have a positive value in the inaction region. This feeds back into the price in the peg region and thus increases the platform's value at date 0.

Our analysis characterizes conditions on liquidity benefits for an equilibrium to exist under commitment and limited liability. We report necessary and sufficient conditions in the proof of Proposition 3 and report an intuitive condition here.

Corollary 1. *With full commitment and limited liability, an equilibrium with positive platform equity value exists only if*

$$\lim_{a \to \infty} \ell(a) \geq r - \mu + \frac{\lambda}{\xi + 1}, \tag{24}$$

This new condition must hold for a policy with limited liability to exist. Informally, it requires that the value of liquidity benefits is large enough to compensate for repurchase costs.[12] This condition implies that a stablecoin platform is feasible only if the growth

[12] Formally, as we show in the proof of Proposition 1, this condition must hold for there to be a policy

rate of stablecoin demand μ is high enough to compensate large negative shocks. To see this, remember that the liquidity benefit can be no greater than the discount rate, that is, $\ell(a) \leq r$. For condition (24), the growth rate of the demand for stablecoins must satisfy

$$\mu \geq \frac{\lambda}{\xi + 1}. \tag{25}$$

We illustrate our results under full commitment in Figure 1. In both cases, the equilibrium variable only depends on the current level of demand A_t and the outstanding stock of stablecoins C_t. Furthermore, due to homogeneity of ℓ, these variables only depend on the demand ratio a_t, that is, for any variable X, we have $X(A, C) = x(a)C$. As we showed, the peg can be maintained in all circumstances only under full commitment. The first panel shows that limited liability protects equity holders, as their equity value is always positive after large negative shocks. From an ex-ante perspective, however, the inability to conduct large repurchases lowers the initial platform value. In both cases, the initial platform value is given by the rightmost panel of Figure 1 taking $a \to \infty$.

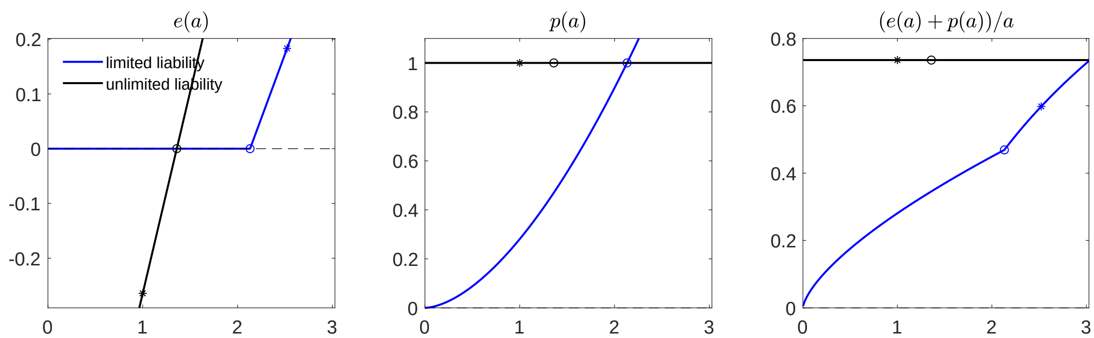

Figure 1: Full-Commitment Solution with limited liability (blue) and unlimited liability (black). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(A, C) = r\exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$.

4 Centralized Protocols

In this section, we relax our full-commitment assumption. In other words, we now consider the case in which policies cannot be fully programmed via smart contracts at date

$(\underline{a}, a_L^\star)$ such that the present value of liquidity benefit at demand ratio $a_L^\star$ exceeds the cost of buying back all the debt when at ratio $\underline{a}$

0. In particular, the platform now chooses its issuance policy sequentially without commitment. Motivated by market practice, we assume that the platform may still commit to two policies: its coupon policy $\{\delta_t\}_{t\geq 0}$ and a minimum collateralization rule $\{\underline{K}_t\}_{t\geq 0}$. The collateralization rule specifies the minimum amount of cryptoassets the platform should hold as collateral at any point in time. We can show that if the platform cannot commit to any policy, no equilibrium with non-negative stablecoin price exists.[13]

In what follows, we introduce our equilibrium concept under partial commitment in Section 4.1. We then first analyze the case of an uncollateralized platform in Section 4.2 and then consider a minimum collateralization rule in Section 4.4.

4.1 Equilibrium Concept with Partial Commitment

Under limited commitment, the issuance-repurchase policy must be sequentially optimal, that is, the continuation policy $\{d\mathcal{G}\}_{\tau\geq t}$ a date t of the sequence $\{d\mathcal{G}\}_{t\geq 0}$ must be optimal for the platform at t. With full commitment instead, the platform could choose a sequence that was ex-post suboptimal in order to increase its value at date 0.

The platform's incentives to follow a given policy depends on investors' reaction to a deviation from that policy. In our competitive model, this reaction is embedded in the stablecoin pricing function out of equilibrium. To discipline our analysis with respect to out-of-equilibrium behaviors, we focus on Markov-perfect equilibria. In a Markov equilibrium, the platform's strategies and the pricing function depend only on the value of a restricted set of state variables, rather than on the complete history of the economy.

The state variables for a Markov equilibrium are the market value of cryptoassets A, the stock of stablecoins C, and the value of cryptoassets collateral owned by the platform K. For consistency, we also specify coupon and collateralization policies as a function of these state variables. Our equilibrium concept under partial commitment is defined as follows.

Definition 1. *Given an interest rate rule $\delta(A, C)$ homogeneous of degree 0 and a minimum collateral requirement $\underline{K}(A, C)$, a Markov equilibrium is given by an equity owner value function $E(A, C, K)$, a stablecoin pricing function $p(A, C, K)$, a governance token owner's stablecoin issuance policy $d\mathcal{G}(A, C, K)$, collateral purchase policy $d\mathcal{M}(A, C, K)$, and optimal default time $\tau_D(A, C, K)$ such that the platform owners maximize the platform's equity value*

[13] See Appendix U for a proof of this claim.

at any date

$$E(A_t, C_{t^-}, K_{t^-}) = \max_{\tau_D, d\mathcal{G}, d\mathcal{M}} \mathbb{E}_t \left[\int_t^{\tau} e^{-r(s-t)} \left(p_s d\mathcal{G}_t - d\mathcal{M}_t \right) + e^{-r(\tau - t)} \max\{0, K_\tau - C_\tau\} \right] \tag{26}$$

given collateral requirement $\underline{K}(A, C)$*, coupon policy* $\delta(A, C)$*, the law of motion for stablecoins* (3)*, the law of motion for collateral* (4)*, and debt pricing function*

$$p(A_t, C_{t^-}, K_{t^-}) = \mathbb{E}_t \left[\int_t^{\tau} e^{-r(s-t)} (\ell_s + \delta_s) p_s ds + e^{-r(\tau-t)} \min\{K_\tau / C_\tau, 1\} \middle| A_t, C_{t-}, K_{t-} \right], \tag{27}$$

where the expectation in (27) *is taken under the law of motions implied by the platform's policies. The stopping time* τ *is* $\equiv \tau_D \wedge \tau_K$ *where* τ_K *is the first time the mimimum collateralization rule* $K_t \geq \underline{K}(A_t, C_{t^-})$ *is violated.*

The key equilibrium object is the stablecoin pricing function $p(A, C, K)$. Stablecoins are priced competitively given the continuation policies of the platform. Should the owners "deviate" from a policy, they would face the same price map, that is, the price may change only if state variables change as a result of a deviation, not *because* it is a deviation.

Under partial commitment, the platform chooses at date 0 its coupon policy and its minimum collateral policy for all future dates. In doing so, the platform takes into account that the issuance policy, the collateral policy and the default policy form part of a Markov equilibrium. The key difference with the commitment case is thus that the latter policies must be sequentially optimal, a requirement stronger than date-0 optimality.

4.2 Uncollateralized protocols

We first consider uncollateralized protocols with no minimum collateralization rule. While the platform may still hold collateral voluntarily, we show in Section 4.4, that it would choose not to hold any, that is, $K_t = 0$. The only state variables are thus A_t and C_t. Under our homogeneity assumptions for the liquidity benefit ℓ and the coupon payment δ, the demand ratio $a_t = \frac{A_t}{C_t}$ introduced in Section 3 is the ultimate state variable.

The first step of the analysis is to characterize an equilibrium stablecoin issuance policy $d\mathcal{G}(A, C)$ for the platform given a coupon policy δ chosen at date 0. There are two differences with the full-commitment case. First, by definition, the equilibrium issuance policy must be Markovian. Second, and most importantly, the issuance policy must be sequentially optimal. As we will show, these two features simplify the analysis, and we can provide a tight characterization of the equilibrium issuance policy. In the commitment case, instead, we had to assume a specific class of issuance policies.

Our first objective is to show that an issuance policy that forms part of a MPE must belong to the following class of stable debt policies defined below.

Definition 2 (Stable Issuance Policy). *A stablecoin issuance policy is stable if there exists a default boundary $\underline{a}$, a repurchase boundary $\overline{a} > \underline{a}$, a demand ratio target $a^\star \geq \overline{a}$, and a mapping $G(a_t, C_{t^-}) = g(a)C_{t^-}$ such that*

$$d\mathcal{G}(a_t, C_{t^-}) = \begin{cases} 0 & \text{if } \; a_t < \underline{a}, \\ G(a_t, C_{t^-})dt & \text{if } \; \underline{a} \leq a_t < \overline{a}, \\ A_t/a^\star - C_{t^-} & \text{if } \; a_t \geq \overline{a}. \end{cases}$$

The policy we assumed under full commitment forms part of this class. A stable debt policy features a target $a^\star$ for the stablecoin demand ratio. There exists a peg region $[\underline{a}, \infty)$ in which the platform implements the target ratio via issuance or repurchases. In the region $[\underline{a}, \bar{a}]$, the peg is abandoned. In this region, we say that the platform issues stablecoins *smoothly* because the quantity issued is of the order dt. We thus call this region the smooth region. In general, we refer to such an issuance policy as a smooth policy. Below demand ratio $\underline{a}$, the platform neither issues, nor repurchases and defaults.

We will show that if the coupon policy is optimally chosen at date 0, the equilibrium issuance policy that forms part of a MPE must be a stable debt policy. We proceed with a series of preliminary results. The following Lemma shows that the debt policy may not be smooth everywhere in an equilibrium with positive platform and stablecoin value. It also provides sufficient conditions for the equilibrium policy to be smooth.

Lemma 3 (Issuance Policy). *First, if the equity value in a MPE is strictly convex in C over some region, the equilibrium debt policy is smooth in that region. Second, there is no MPE with positive stablecoin price if the equilibrium issuance policy is smooth everywhere.*

For the first result, observe that for any two stablecoin levels C and C', the issuance policy form part of an equilibrium if

$$E(A, C) \geq E(A, C') + p(A, C')(C' - C) \tag{28}$$

Equation (28) simply states that by definition of $E(A, C)$, the platform should weakly prefer its equity value at C than a discrete jump at C'. If E is strictly convex in C, it can further be shown inequality (28) which means any discrete stablecoin issuance or repurchase is dominated. Hence, the issuance policy should be smooth.

The second result from Lemma 3 is a consequence of the "leverage ratchet effect". If the equilibrium issuance policy is smooth, platform owners may not capture any of the liquidity benefits from stablecoin issuance. This result is similar to DeMarzo and He (2021) who study a dynamic debt issuance problem. The leverage ratchet effect implies that the protocol value in any equilibrium with a smooth issuance policy is equal to its value without stablecoins, which is zero, because the protocol owns no asset. In our model, we can further show that the platform is unable to maintain a peg, which implies the stablecoin has no value in equilibrium if the issuance policy is smooth everywhere.

The second preliminary result is the counterpart of Lemma 3. It shows that if the equity value is linear over some segment, the equilibrium issuance policy must feature jumps.

Lemma 4. *If the equity value $e(a)$is linear over some interval $[a_L, a_U]$, the equilibrium issuance policy features a a target demand ratio $a^{jump} \in [a_L, a_U]$ such that the issuance policy for any $a \in [a_L, a_U]$ is to jump at a^{jump}.*

Lemma 3 and 4 restrict the set of issuance policies compatible with an equilibrium with strictly positive stablecoin value. They show that the issuance policy must feature jumps, but leave open the possibility that there could be several non-overlapping regions with discrete issuance or repurchase. We can show however that the equilibrium issuance policy must be part of the class of stable issuance policies.

Proposition 4 (Equilibrium Issuance Policy). *An issuance policy $d\mathcal{G}$ that forms part of a Markov equilibrium induced by an optimal coupon policy (chosen at date 0) must be a stable debt policy with default boundary $\underline{a} = 0$.*

The first standard step of the proof consists in showing that the equilibrium equity value must be weakly convex. It implies that on any segment, the equity value is either strictly convex or linear. We then show that the only possibility is that the equity value is strictly convex close to 0 and there is only one linear segment $[\bar{a}, \infty)$. This second step requires using the optimality of a coupon policy chosen at date 0. In other words, there could be coupon policies and associated MPEs in which the properties of Proposition 4 do not hold but our argument then shows that these coupon policies may not be optimal. Given these two steps, the result follows from Lemma 3 and 4.

Proposition 4 also shows that the platform never defaults, that is, $\underline{a} = 0$. This result arises because the platform is never forced to make payments in the unit of account. It chooses its issuance-repurchase policy ex-post, and can thus always choose not to repurchase debt. Second, the only transfer to stablecoin owners via the coupon policy is made in stablecoins, not in the unit of account. As the platform can issue stablecoins at no cost, it cannot gain from defaulting. Hence, it is always preferable to hope for resurrection.

Knowing that a MPE with positive stablecoin value must feature a stable debt policy, we can provide conditions on the equilibrium equity value and the stablecoin price.

Proposition 5. *Let $\delta(a)$ be an optimal coupon policy chosen at date-0. A non-zero MPE induced by this coupon policy satisfies the following properties*

1. *Equity value $e(a)$ is strictly convex in a for $a \in [0, \overline{a}]$ and linear for $a \geq \overline{a}$ with*

$$(r+\lambda)e(a) = -\delta(a)p(a) + \mu a e'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda \mathbb{E}[e(Sa)], \quad \forall a \leq \overline{a}. \tag{29}$$

2. *The stablecoin price $p(a)$ is strictly increasing (equal to 1) for $a \in [0, \overline{a}]$ ($a \geq \overline{a}$) with*

$$(r+\lambda)p(a) = \delta(a)p(a) - (g(a) + \delta(a) - \mu)ap'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)], \quad \forall a \leq \overline{a}. \tag{30}$$

3. *In the smooth region $[0, \overline{a}]$, the platform's issuance rate is equal to*

$$g(a) = \frac{\delta'(a)p(a)}{p'(a)} \tag{31}$$

and the platform's equity value is the same as if it issued no debt.

4. *The coupon policy must satisfy*

$$\delta(a) \geq (r+\lambda) + \lambda \mathbb{E}[e(Sa)] - (\ell(a^\star) + \lambda \mathbb{E}[e(Sa^\star) + p(Sa^\star)])\,\frac{a}{a^\star} \quad \textit{for all } a \geq \overline{a}. \tag{32}$$

We show in the proof of Proposition 4 that a stable debt issuance policy is only compatible with equity value and stablecoin price functions that satisfy Conditions 1 and 2. When the platform is in the peg region, it implements a constant demand ratio $a^\star$. In this region, from any debt level C the platform issues a discrete block of stablecoins $C^\star(A)-C$ at price $p(A, C^\star(A))$. Hence, equity value is given by

$$\begin{aligned} E(A,C) &= E(A,C^\star(A)) + p(A,C^\star(A))(C^\star(A)-C) \\ e(a) &= \left[e(a^\star)+p(a^\star)\right]\frac{a}{a^\star} - p(a^\star) \end{aligned} \tag{33}$$

which implies $e(a)$ is linear in a. As before, the platform sets the coupon policy $\delta(a^\star)$ to ensure $p(a^\star)=1$ as otherwise stablecoin owners would not enjoy liquidity benefit.

In the smooth region, that is, when a falls below $\overline{a}$, the platform abandons the peg. The stablecoin price is strictly positive and increasing in the smooth region although stablecoin owners do not currently enjoy liquidity benefits. As the peg may be restored following a series of positive demand shocks, however, future stablecoin owners will enjoy liquidity benefits, which supports the price today. Equations (29) and (30) are the Hamilton-Jacobi-Bellman equations in the smooth region for equity value and stablecoin price respectively

Equilibrium requirement 3 states that the platform is indifferent between issuing stablecoins and staying idle in the smooth region. A smooth debt policy can be optimal only if the return to issuance is zero. This result is similar to the leverage ratchet effect of DeMarzo and He (2021) in a similar context. Because the platform can freely and continuously issue stablecoins, it is unable to capture any issuance benefit under limited commitment because it competes against its future self, a version of the Coase (1972) problem for monopolists. Unlike in DeMarzo and He (2021), however, our equilibrium also features a peg region in which the platform does enjoy issuance benefits.

The result that the platform is indifferent about stablecoin issuance in the smooth region does not mean that the platform does not issue stablecoin in this region. Stablecoin issuance may be necessary to support the equilibrium price.[14] Equation (31) shows that the issuance policy is determined by the endogenous price function and the coupon policy. The steepest the price function (high $p'(a)$), the largest the platform's price impact and the lower issuance or repurchase is. The platform issues (repurchases) if $\delta'(a)>0$ ($\delta'(a)<0$),

[14]A good analogy is with a mixed-stategy Nash equilibrium in which a player is indifferent between options but one specific randomization should be chosen to support the equilibrium.

that is, it the ex-post equilibrium issuance policy leans against the coupon policy chosen ex-ante. In particular, if the coupon decreases with a, $\delta'(a) > 0$, so as to penalize low demand ratios, the platform repurchases debt to avoid the increase in coupon payments.

Finally, Criterion 4 is a condition on the coupon payment policy such that no deviation to a smooth debt policy is preferred to the equilibrium policy in the target region. To get some intuition about this condition, consider a demand ratio $a = \frac{A}{C} \in [\overline{a}, a^\star]$ in the peg region for which the platform is supposed to jump to $a^\star$. Instead of the equilibrium policy, suppose the platform deviates by not jumping and then revert back to the equilibrium policy after an interval dt. We show that this deviation is unprofitable if[15]

$$\underbrace{(r+\lambda)(C - C^\star(A))}_{\substack{\text{gain from} \\ \text{postponing repurchase}}} \leq \delta(a)C - \delta(a^\star)C^\star(A) \underbrace{+\lambda(\mathbb{E}[E\left(SA, C^\star(A)\right)] - \mathbb{E}[E(SA, C)])}_{\substack{\text{protection against} \\ \text{large negative shocks}}} \quad (34)$$

The left-hand-side of (34) is the gain from postponing the repurchase, equal to the effective interest rate $r + \lambda$ multiplied by the equilibrium repurchase quantity $C - C^\star$ at equilibrium price of 1. The right-hand side of (34) corresponds to the punishment from the deviation. The first term is the difference in coupon payments evaluated at the stablecoin price of 1. The second term is the benefit from implementing a larger demand ratio $a^\star$ vs. a which protects against large negative demand shocks. Condition (32) follows directly from (34).

The platform chooses the coupon policy at date 0 under the constraint $\delta(a) \geq 0$. It is thus always possible to specify a coupon policy such that (32) is satisfied because the coupon policy on $a \in [\overline{a}, \infty)\backslash\{a^\star\}$ does not impact equilibrium objects. Indeed, the optimal issuance policy is to jump to $a^\star$ when $a \in [\overline{a}, \infty)$, which implies states $a \in [\overline{a}, \infty)\backslash\{a^\star\}$ are not visited in equilibrium. Hence, given equilibrium objects, one can always set δ on $[\overline{a}, \infty)\backslash\{a^\star\}$ so as to satisfy (32). The coupon policy on $[\overline{a}, \infty)\backslash\{a^\star\}$ plays the role of an out-of-equilibrium threat to discourage deviations in the target region.

Having characterized the MPE without commitment to the issuance policy, we may now analyze the optimal coupon policy $\delta(a)$ chosen to maximize the date-0 platform value. At date 0, the platform takes as given the equilibrium played by its future selves who have full discretion over the repurchase-issuance policy. Hence, unlike in the full commitment case, the coupon policy plays a new role: it can help discipline the platform in the future

[15] The same condition applies if instead $a \geq a^\star$ but in this case the platform is supposed to issue stablecoins rather than repurchase them, which means the commitment problem does not bind.

to act in its own interest at date 0. To simplify the analysis, we assume the coupon policy is fixed in the smooth region, similar to the restriction imposed under full commitment.

Assumption 2. *The coupon policy in the smooth region is such that $\delta(a) = \underline{\delta}$ for $a \leq \underline{a}$.*

Assumption 2 simplifies the problem because we can provide analytical functional forms for the MPE equity value and price in the smooth region. Setting $\delta(a) = \underline{\delta}$ in HJB equations (29) and (30), we can guess and verify the following functional forms

$$e(a) = \begin{cases} \sum_{k=1}^{3} c_k (a/\underline{a})^{-\gamma_k} & \text{if } \ a < \overline{a}, \\ (e^\star + 1)a/a^\star - 1 & \text{if } \ a \geq \overline{a}, \end{cases}, \tag{35}$$

$$p(a) = \begin{cases} \sum_{k=1}^{3} b_k (a/\underline{a})^{-\gamma_k} & \text{if } \ a < \overline{a} \\ 1 & \text{if } \ a \geq \overline{a}, \end{cases}. \tag{36}$$

where γ_ks are roots of the characteristic equation

$$r + \lambda - \underline{\delta} = -(\mu - \underline{\delta})\gamma + \frac{\sigma^2}{2}(1+\gamma)\gamma + \frac{\lambda\xi}{\xi - \gamma}. \tag{37}$$

We may now characterize the optimal choice of the paltform at date 0. Under Assumption 2 optimization problem over the coupon policy can be thought as a choice over $\boldsymbol{\theta}_0 = \{\underline{\delta}, \delta^\star, \overline{a}, a^\star\}$. The platform chooses the optimal policy $\boldsymbol{\theta}_0^\star$ to maximize the platform's value at date 0 subject to the requirement that the stable issuance policy with parameters $\boldsymbol{\theta}_0^\star$ is part of an MPE. We show that the optimization problem can be characterized as follows.

Lemma 5 (Optimal Policy). *The optimal date-0 policy $\boldsymbol{\theta}_0^\star$ for a centralized uncollateralized platform is the solution to the following maximization problem*

$$E_0/A_0 = \max_{\{\overline{a}, a^\star, \underline{\delta}\}} \frac{\ell(a^\star)/a^\star}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}}, \tag{38}$$

$$\textit{subject to} \quad \frac{\gamma}{1+\gamma}\frac{1}{\overline{a}} = \frac{\ell(a^\star)/a^\star}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}}, \tag{39}$$

$$\overline{a} \in [0, a^\star] \tag{40}$$

with γ the only negative root of equation (37).

Observe first that $\delta^\star$ does not appear in the maximization program. As before, the only role of this parameter is to ensure the price is pegged at one in the target region, that is, $p(a^\star) = 1$. We thus substituted for the corresponding value of $\delta^\star$ to write the platform's objective function (38). The second constraint is the smooth-pasting condition for the equity value between the target and the smooth regions. This constraint reflects the requirement that the issuance policy must be optimal ex-post for the platform. In particular, the platform chooses optimally to switch from the peg to the smooth region. This feature generates condition (97). We show in the proof of Lemma 5 that this constraint implies that limited liability is satisfied for all values of a. Finally, the last constraint must be satisfied for a MPE to exist by definition of a stable debt policy.

Unlike in the full commitment case, a precise analytical characterization of the solution is difficult. The proof of Lemma 5 reports partial characterization for some cases. In the main text, we provide a numerical illustration of the dominant MPE in Figure 2. For each panel, the solution without commitment is compared to the solution where the only constraint is limited liability. We show that under limited commitment, the protocol chooses a lower target inverse supply $a^\star$ and it abandons the peg for a higher value of a. These differences are reflected in a lower value of equity and of total protocol value compared to the commitment case.

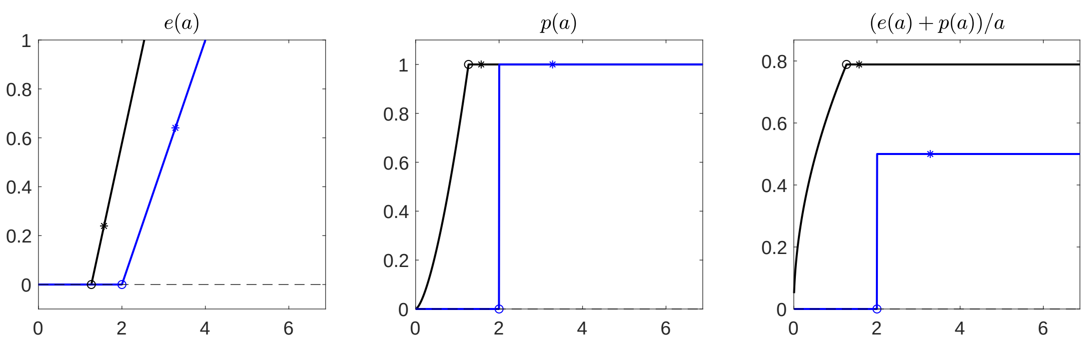

Figure 2: Solution with commitment and limited liability (black) and without commitment (blue). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(a) = r\exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$.

4.3 Centralized Platform with Collateral

[Preliminary.]

In this section, we consider a minimum collateralization rule as a tool to improve the stability of a centralized stablecoin protocol. This analysis sheds lights on crypto-collateralized protocols, for which we provide a balance sheet representation in Figure **??**. Unlike an uncollateralized protocol with no asset, a collateralized protocol holds the cryptoasset on its balance sheet to back stablecoin issuance. The difference between the market value of the cryptoasset and the par value of stablecoins is akin to overcollateralization. Overcollateralization creates a buffer in case the collateral asset suddenly looses its value.

In the following section, we solve for equilibrium issuance, collateralization, and default strategies $d\mathcal{G}$, $d\mathcal{M}$, and τ_D given commitment to the interest policy $\delta(a)$ and a minimum collateralization rule $\underline{K} = \varphi C$. The collateralization rule forces the protocol to maintain a minimum ratio between its cryptoasset holdings and the stock of stablecoins issued. The protocol shuts down automatically when he collateralization rule, given by $K_t \leq \underline{K}$, is breached, which happens at stochastic time τ_K

4.4 Stability Benefits of Collateral

In this section, we characterize the optimal collateral policy of the protocol. The collateral policy $d\mathcal{M}$ specifies the change in the value of collateral held by the protocol as a function of current state variables K_{t^-}, C_t, A_t. The law of motion for collateral is given by (4).

Our first result is that in equilibrium, the protocol's equity value is linear in the amount of collateral it holds.

Lemma 6. *In any MPE in (A, C, K), $E(A, C, K)$ is linear and increasing in K. Hence,*

$$\forall K, K' \geq \varphi C, \qquad E(A, C, K) = E(A, C, K') + K - K'. \tag{41}$$

The intuition for Lemma 6 is that the protocol can freely add or remove collateral to the platform subject to the collateralization constraint, and the protocol has no influence on the cryptoasset price. These features imply that the equilibrium equity value is linear in the value of collateral held by the platform.[16] To form intuition about (41), it is useful to remember the optimality decision for debt , given by equation (**??**) in Proposition 5. The derivative of the equity value with respect to stablecoin issuance is equal to the price of

[16]The cryptoasset price is not equal to one, but K_t is the value of collateral held by the protocol, as opposed to the amount of collateral. Hence, a change from K to say $K' > K$ costs $K' - K$ to the platform.

stablecoins. Equation (41) shows the same equation holds with respect to collateral with the only difference that the collateral price is exogenously given.

Lemma 6 motivates the following conjecture for the optimal collateral policy. For any value (A, C) of cryptoasset value and stablecoin stock, there exists a collateral target such that the protocol buys/sells collateral discretely to reach that target from any level $K \geq \varphi C$. Formally, we define the following class of policies

Definition 3 (Class of Targeted Collateral Policies). *A collateral policy is part of the class of targeted collateral policies $\mathbb{T}$ if it is characterized by a lower bound $\tilde{a}$ and a collateral target function $k^\star(a)$ such that, denoting $k \equiv \frac{K}{C}$,*

$$d\mathcal{M}(a_t, k_{t^-}, C_{t^-}) = \begin{cases} 0 & \text{if } \; a_t < \underline{a}, \\ (\varphi - k_{t^-})C_t & \text{if } \; \underline{a} \geq a_t \geq \tilde{a}. \\ (k^\star(a_t) - k_{t^-})C_t & \text{if } \; a_t > \tilde{a}. \end{cases} \tag{42}$$

Equation (41) shows that if the protocol considers a discrete collateral change from any level K_t, any adjustment is weakly optimal by the linearity of the equity value function. In particular, an adjustment to some specified target $K^\star(A, C)$ is optimal. To characterize the target $k^\star(a) = \frac{K^*(A,C)}{C}$, we provide conditions such that there is no smooth deviation - of the order dt - from the discrete adjustment policy. We obtain the following characterization

Lemma 7 (Equilibrium Collateral Policy Characterization). *A targeted policy is part of a MPE with positive stablecoin price if*

$$\lambda \left. \frac{\partial \mathbb{E}_t[E(SA, C, SK]}{\partial K} \right|_{K = K^\star(A,C)} = r + \lambda - \mu \tag{43}$$

Any equilibrium collateral policy is part of the class of targeted collateral policies.

Condition (43) captures the trade-off that pins down the optimal collateral target $K^*(A, C)$. The right-hand-side is the marginal cost of adding one unit of cryptoasset as collateral for the protocol. It is equal to the effective opportunity cost, $r + \lambda$, of the protocol minus the growth rate of the cryptoasset μ. As explained before, this difference can be interpreted as the cost of locking up the cryptoasset as collateral. The left-hand-side of (43) is the marginal benefit of adding one unit of collateral evaluated at the target $K^\star(A, C)$. Collateral protects the protocol against the risk that a negative Poisson shock to the cryptoasset triggers a breach of the minimum collateral requirement. The right-hand-side is indeed proportional to the probability of such as shock. Intuitively, if the protocol sets collateral equal to the minimum requirement φC, any negative discrete shock triggers a shutdown. Setting too high a collateral target is not optimal either as the protocol finds it optimal not to fight some very negative Poisson shocks under limited liability.

5 Decentralized Protocols

[To be completed.]

In this section, we adapt our general model to account for *decentralized* stablecoin protocols. Such protocols—with Dai as its most prominent example—allow for the decentralized creation of new stablecoins by anyone with enough collateral. To do so, individual investors have to lock some collateral asset in a smart contract generated by the protocol—a *vault*—and can issue some stablecoins against it. Once the stablecoins are sold to outside investors, the vault represents for its owner a leveraged position in the collateral asset. Moreover, the newly issued stablecoins are not tied to a particular vault and are fully indistinguishable from other stablecoins—i.e., decentralized stablecoins are perfectly fungible. Vault owners can unlock their collateral assets by repurchasing and "burning" enough stablecoins to liquidate the vault. The system's stability, therefore, relies on providing the right set of incentives for individual investors to adopt prudent risk management practices and not to over-extend the supply of stablecoins. As in the centralized case, the protocol also issues governance tokens with voting rights on the system's key parameters and claims to the system's seigniorage revenues.

5.1 Tokens Valuation

In a decentralized crypto-collateralized protocol, individual vaults indexed $i \in [0, 1]$ are created using collateral value K_t^i in exchange for a quantity C_t^i of stablecoins with price p_t. When the loan is repaid by the vault owner, the stablecoin is "burned" and removed from the supply. As for centralized crypto-collateralized protocols, a vault with the value of its collateral K_t^i below the threshold φC_t^i is liquidated. In such a case, the vault owner receives the value of the collateral after repayment of the stablecoins, if any. As there is no heterogeneity across infinitesimal vault owners, the state variables of the system is given by total stock of collateral, stablecoins, and the market capitalization of crypto-assets: $K_t = \int K_t^i di$, $C_t = \int C^i di$, and A_t.

We differentiate vault-specific variables from their aggregate counterparts by the superscript i. Thus, the value of a vault (decentralized equity) can be written as

$$V(A_t, C_{t^-}, C^i_{t^-}, K_{t^-}, K^i_{t^-}) = \max_{\tau^i, d\mathcal{G}^i, d\mathcal{M}^i} \mathbb{E}_t \bigg[\int_t^{\tau^i} e^{-r(s-t)} \big(p_s d\mathcal{G}^i_s - d\mathcal{M}^i_s \big) + e^{-r(\tau^i - t)} \max\{K^i_{\tau^i} - C^i_{\tau^i}, 0\} \bigg]$$

such that

$$dC^i_t = s_t C^i_t dt + d\mathcal{G}^i_s, \qquad \text{and} \qquad dK^i_t = \mu K^i_t dt + \sigma K^i_t dZ_t + K^i_{t^-}(S_{N_t} - 1) dN_t + d\mathcal{M}^i_t.$$

Fees accrue through time between the different investors according to predetermined parameters. Stablecoin holders receive the convenience yield ℓC as well as some interest rate paid in stablecoins by the governance system δ. Besides paying this interest to stablecoin holders, the governance system charges vault owners the stability fee—denoted s. As a result, the net nominal spread earned by the governance token is $(s - \delta)C$. Whenever a vault is liquidated, the platform must dilute the equity of the system to sustain the loss at time t given by $\int \min\{K^i_t - C^i_t, 0\} \mathbb{1}\{\tau^i = t\} di$. Thus, an equity token value is given by

$$E(A_t, C_{t^-}, K_{t^-}) = \max_{\tau, \delta, s} \mathbb{E}_t \bigg[\int_t^{\tau} e^{-r(s-t)} \bigg((s_s - \delta_s) p_s - \int \min\{K^i_s - C^i_s, 0\} \mathbb{1}\{\tau^i = s\} di \bigg) ds \bigg].$$

Finally, the price of one stablecoin per unit of debt is given by

$$p(A_t, C_{t^-}, K_{t^-}) = \mathbb{E} \bigg[\int_t^{\tau} e^{-r(s-t)} \left(\ell(A_s, C_s) + \delta_s p_s \right) ds + e^{-r(\tau - t)} \min\{K_\tau / C_\tau, 1\} \bigg].$$

5.2 Arbitrage

Given any amount of stablecoin issued by an individual vault C^i, vault owners have the option to adjust to $\widetilde{C}^i$ by issuing $\widetilde{C}^i - C^i$ at the price of $p(A, C, K)$. Therefore, the value of the vault must be at least as high as the value of the vault after the adjustment:

$$V(A, C, C^i, K, K^i) \geq V(A, C, \widetilde{C}^i, K, K^i) + p(A, C, K)(\widetilde{C}^i - C^i).$$

Contrarily to the centralized setting, the price of the stablecoin is not a function of leverage of the individual vault and the previous equation must hold with equality. The same argument holds for the collateral and we get that the value of a vault must be linear in C^i and K^i:

$$V(A, C, C^i, K, K^i) = V(A, C, \widetilde{C}^i, K, \widetilde{K}^i) + p(A, C, K)(\widetilde{C}^i - C^i) - (\widetilde{K}^i - K^i).$$

for all and $K^i \geq \varphi C^i \geq 0$ and $\widetilde{K}^i \geq \varphi \widetilde{C}^i \geq 0$. Furthermore, because the value of creating an empty vault must be 0, otherwise there are arbitrage opportunities, we get

$$V(A, C, C^i, K, K^i) = K^i - p(A, C, K)C^i. \tag{44}$$

This has non-trivial consequences. Whenever the discounted value of owning a vault deviates from that equality, arbitrage opportunities arises to either create or burn vaults and stablecoins. By adjusting the stability fee s, the maker can incentivize arbitrageurs to adjust the supply to a desired target. As atomistic agents, vault owners cannot capture any value from the option to default and do not internalize the impact of their issuance on the stablecoin price.

Another consequence of condition (44) is that the minimum collateral requirement must be at least as high as the value of stablecoin issued: $\varphi \geq 1$. Otherwise, a vault owner might issue more stablecoin than the value of the collateral and dispose of the vault. In the following lemma, we establish that if the minimum collateralization rate is not too high, it is never optimal to inject more collateral than the strict minimum to protect the vault against unintended liquidation due to large negative demand shocks.

Lemma 8. *If*

$$\varphi \leq 1 + \frac{1}{\xi + 1},$$

then $K^i(A, C, K, C^i) = \varphi C^i$.

Thus, we can simplify the notation by defining the value of a vault per unit of stablecoin issued:

$$V(A, C, C^i, K(A, C), K^i(A, C, C^i)) \equiv v(a)C^i = (\varphi - p(a))C^i. \tag{45}$$

If the stability fee $s(a)$ is too high or the price of the stablecoin $p(a)$ is too low such that

$v(a) < \varphi - p(a)$, vault owners will be able to make arbitrage profits by purchasing and burning stablecoins to get back the collateral in their vault. Equity holders then internalize that any deviation from the pair of stability fee $s(a)$ and interest payment $\delta(a)$ such that the arbitrage condition (45) is not satisfied triggers immediate changes in the supply of stablecoins. In the following proposition, we characterize the unique MPE policies for a decentralized platform.

Proposition 6 (Targeted MPE). *The unique MPE policies $s(a_t)$ and $\delta(a_t)$ are such that $d\mathcal{G}_t = \int d\mathcal{G}_t^i di$ is given by*

$$d\mathcal{G}(a_t, C_{t^-}) = \begin{cases} 0 & \textit{if} \;\; a_t < \overline{a}, \\ C^\star(A_t) - C_{t^-} & \textit{if} \;\; a_t \geq \overline{a}. \end{cases}$$

where $C^\star(A_t) \equiv A_t / a^\star$ is defined by

$$C^\star(A) = \arg\max_C \left\{ \ell(A, C)C - (r + \lambda - \mu)\varphi C + \lambda \mathbb{E}[S\varphi C - C] \right\}. \tag{46}$$

At $a^\star$, the policies are given by

$$s(a^\star) = \mu\varphi - (r + \lambda)(\varphi - 1) + \lambda \mathbb{E}[\max(0, S\varphi - 1)]$$

and

$$\delta(a^\star) = r - \ell(a^\star).$$

The value of equity is given by

$$E(A) = \frac{\ell(a^\star) - (r + \lambda - \mu)\varphi + \lambda \mathbb{E}[S\varphi - 1]}{a^\star} \frac{A}{r - \mu}.$$

The key insight is that in the presence of arbitrageurs, equity holders are able to target an optimal ratio $a^\star$ with the stability fee and the interest payment policies without incentives to deviate. Any deviation from the equilibrium policies $s(a^\star)$ and $\delta(a^\star)$ triggers an immediate adjustment of the supply of stablecoins to a suboptimal level. Thus, a decentralized platform does not require commitment to any of its policies to enforce a stable equilibrium.

6 Conclusion

In this work, we propose a general model of stablecoins and examine the merit and vulnerability of various stabilization mechanisms. Our analysis points out that stablecoin protocols share some—but not all—features with conventional financial institutions such as mutual funds, banks, and central banks. In particular, collateralization and liquidation covenants play a crucial role in the stabilization of crypto-collateralized protocols. We demonstrate that these schemes are highly dependent on the market liquidity of their collateral assets and are vulnerable to fire-sales spirals of the type observed during the 2008 financial crisis. In contrast, uncollateralized algorithmic schemes rely on irredeemable stablecoins and quantity adjustments with alleged inspiration from central banks. As for a central bank, we show that issuing irredeemable liabilities does not dispense from holding tangible assets. Otherwise, there is always a limit to how many stablecoins can be withdrawn when facing a negative demand shock, and the scheme loses its control over prices. Importantly, decentralizing the issuance and redemption of stablecoins to atomistic agents resolves the commitment issues of the platform. Overall, our work has practical implications for the design and regulation of future stablecoins. In particular, we point to collateralization, automatization, and decentralization as essential stabilization tools.

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Appendices

A Proof of Proposition 2

Integrating the first term of (8) by parts, we obtain

$$\begin{aligned}E_0 &= \max_{\{\delta_t, d\mathcal{G}_t\}_{t\geq 0}} \mathbb{E}_0 \left\{ \left[p_t C_t e^{-rt}\right]_0^\infty - \left[\int_0^\infty e^{-rt} C_t \left(dp_t - rp_t dt\right)\right]\right\} \\ &= \max_{\{d\mathcal{G}_t\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \ell(A_t, C_t) C_t dt\right],\end{aligned}$$

To obtain the second line, we guess and verify that $\lim_{t\to\infty} \mathbb{E}_0[p_t C_t e^{-rt}] = 0$ and we use the pricing equation to substitute for $\mathbb{E}_0[dp_t - rp_t]$. Finally, δ_t is only determined to the extent that it maintains the price peg. The platform's problem under full commitment is thus static and the optimal issuance rule is such that C_t maximizes $\ell(A_t, C_t)C_t$. By Property (iii) in Assumption 1, this maximizer exists, is unique, and is given by (9). The fact that $C^\star(A) = \frac{A}{a^\star}$ follows from Assumption 1. Finally, our conjecture $\lim_{t\to\infty} \mathbb{E}_0[p_t C_t e^{-rt}] = 0$ and the fact that the objective function is bounded follows from the fact that A_t grows at a rate inferior to r.

To conclude, the optimal issuance-repurchase policy $\{d\mathcal{G}_t\}_{t\geq 0}$ features a jump from 0 to $C^*(A_0)$ at date 0 and is such that $d\mathcal{G}_t + \delta_t C_t dt = dA_t$ for $t > 0$.

B Proof of ??

In this section, we first solve for the value of equity and the stablecoin price given the following policies for δ_t and $\mathcal{G}_t$:

$$d\mathcal{G}_t = \begin{cases} 0 & \text{if } \; A_t/C_{t^-} < \overline{a}, \\ C^\star(A_t) - C_{t^-} & \text{if } \; A_t/C_{t^-} \geq \overline{a}. \end{cases}$$

where $C^\star(A_t) \equiv A_t/a_L^\star$ and

$$\delta_t = \begin{cases} \underline{\delta} & \text{if } \; A_t/C_{t^-} < a_L^\star, \\ \delta^\star & \text{if } \; A_t/C_{t^-} = a_L^\star. \end{cases}$$

There is no need to specify the value of $\delta(A_t, C_{t^-})$ for $A_t/C_{t^-} \in [\overline{a}, a_L^\star) \cup (a_L^\star, \infty)$ as the platform never stays in that state. A policy $\boldsymbol{\theta}_0 = \{\overline{a}, a_L^\star, \underline{\delta}, \delta^\star\}$ chosen at date 0 is thus given by four parameters. For ease of notation, we write $a^\star$ instead of $a_L^\star$ in this proof.

In the first step of the proof, we solve for the equity value and the stablecoin price over the whole state space $a \in [0, \infty)$. We then derive the platform value at date 0 and maximize over the policy

B.1 Equity Value

As explained in the main text, the relevant state variable is $a = A/C$. Hence, we only need to characterize $e(a) \equiv E(A, C)/C$.

We conjecture then verify that the equity value per unit of stablecoin is given by

$$e(a) = \begin{cases} \sum_{k=1}^{3} c_k a^{-\gamma_k} & \text{if } \; 0 \geq a < \overline{a}, \\ (e(a^\star) + p(a^\star))a/a^\star - p(a^\star) & \text{if } \; a \geq \overline{a}, \end{cases} \tag{47}$$

where γ_ks (ordered in decreasing order) are the roots of the characteristic equation

$$r + \lambda - \underline{\delta} = -(\mu - \underline{\delta})\gamma_k + \frac{\sigma^2}{2}(1 + \gamma_k)\gamma_k + \frac{\lambda \xi}{\xi - \gamma_k}. \tag{48}$$

The roots of that polynomial are given by

$$\gamma_k = -\frac{1}{2t_1}\left(t_2 + \zeta^\nu R + \frac{\Delta_0}{\zeta^\nu R}\right)$$

where

$$\Delta_0 = t_2^2 - 3t_1 t_3, \quad \Delta_1 = 2t_2^3 - 9t_1 t_2 t_3 + 27 t_1^2 t_4,$$
$$R = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}, \quad \zeta = \frac{-1 + \sqrt{-3}}{2}, \quad \nu = \{0, 1, 2\},$$
$$t_1 = -\frac{\sigma^2}{2}, \quad t_2 = \mu - \underline{\delta} + \frac{\sigma^2}{2}(\xi - 1), \quad t_3 = -(\mu - \underline{\delta})\xi + \frac{\sigma^2}{2}\xi + r - \underline{\delta} + \lambda, \quad t_4 = -(r - \underline{\delta})\xi.$$

According to Descartes' rule of sign, this polynomial has 2 positive roots and 1 negative root if $\underline{\delta} < r$ and 1 positive root and 2 negative roots if $r < \underline{\delta} < r + \lambda$. We can show numerically that for $\{r, \sigma, \lambda, \xi, \mu, \underline{\delta}\} \in (0, 1) \times \mathbb{R}_+ \times (0, 1) \times \mathbb{R}_{>0} \times [-\infty, r + \lambda/(\xi + 1)] \times \mathbb{R}_+$, it always holds that $\gamma_2 > -1$, $\gamma_3 < -1$, and $\partial\gamma_3/\partial\underline{\delta} < 0$. (If $\lambda = 0$, it can be shown algebraically that only 1 root is strictly lower than 1 and the derivative of that root with respect to is negative.)

No-Active-Issuance Region $a \in [0, \overline{a})$ Given that the policies depends only on the state (A, C), we can solve the value of $E_t = E(A_t, C_t)$ recursively thanks to the Hamilton-Jacobi-Bellman equation (HJB). The HJB is given by

$$(r+\lambda)E(A,C) = \mu A E_A(A,C) + \underline{\delta} C E_C(A,C) + \frac{\sigma^2}{2} A^2 E_{AA}(A,C) + \lambda \mathbb{E}[E(SA,C)].$$

Given $E(A,C) = e(a)C$,

$$(r+\lambda)e(a) = \mu a e'(a) + \underline{\delta}(e(a) - e'(a)a) + \frac{\sigma^2}{2} a^2 e''(a) + \lambda \mathbb{E}[e(Sa)]. \tag{49}$$

We will use equation (49) to determine the coefficients of our functional guess in (47). We first need to compute term $\mathbb{E}[e(Sa)]$ using the conjectured $e(a)$. We have

$$\mathbb{E}[e(Sa)] = \int_0^\infty \left\{ e(e^{-s}a)\xi e^{-\xi s} \right\} ds = \int_0^\infty \left\{ \sum_{k=1}^{3} c_k e^{s\gamma_k} a^{-\gamma_k} \right\} \xi e^{-\xi s} ds = \sum_{k=1}^{3} \frac{c_k \xi a^{-\gamma_k}}{\xi - \gamma_k}.$$

We then plug in guess (47) into the HJB to obtain

$$(r+\lambda-\underline{\delta})e(a) = -(\mu - \underline{\delta}) \sum_{k=1}^{3} \gamma_k c_k a^{-\gamma_k} + \frac{\sigma^2}{2} \sum_{k=1}^{3} (1+\gamma_k)\gamma_k c_k a^{-\gamma_k} + \lambda \mathbb{E}[e(Sa)]. \tag{50}$$

Equation (48) is a necessary condition for (101) to hold, which confirms our characterization of γ is satisfied. Next, we used standard boundary conditions. The first boundary condition is that equity value should be equal to 0 when $a = 0$, that is,

$$\left. \sum_{k=1}^{3} c_k a^{-\gamma_k} \right|_{a=0} = 0.$$

This is only possible if coefficients c_1 and c_2 associated with positive roots of (48), respectively γ_1 and γ_2 are equal to 0. The second boundary condition is a value matching condition at the boundary $\overline{a}$ between the no-issuance region $[0, \overline{a}]$ and the target region

$[\overline{a}, \infty)$. Given $c_1 = c_2 = 0$, we have

$$c_3 \overline{a}^{-\gamma} = e(\overline{a}) \equiv (e(a^\star) + p(a^\star))\overline{a}/a^\star - p(a^\star),$$

where, with a slight abuse of notation, we also denote e for the equity value function in the target region. We will prove the second equality below in the analysis of the target region. Putting our results for the no-issuance region together, we have

$$\forall\, a \leq \overline{a}, \qquad e(a) = \left((e(a^\star) + p(a^\star)) \frac{\overline{a}}{a^\star} - p(a^\star) \right) \left(\frac{a}{\overline{a}} \right)^{-\gamma} \tag{51}$$

Target Region $a \in [\overline{a}, \infty)$ By definition when (A, C) is such that $a = A/C \geq \overline{a}$, the platform issues a discrete amount of debt $C^\star(A) - C$. Hence, any such (A, C), the equity value satisfies

$$E(A, C) = E(A, C^\star(A)) + p(A, C^\star(A))(C^\star(A) - C) \tag{52}$$

$$e(a) = \left[e(a^\star) + p(a^\star) \right] \frac{a}{a^\star} - p(a^\star) \tag{53}$$

where to obtain the second equation, we divided the first by C. To determine the optimal policy $\boldsymbol{\theta}_0^*$, we need to solve for the equity value at the target demand ratio $a^\star = A/C^\star(A)$. The recursive equation for the equity value is

$$\begin{aligned} E(a^\star C_{t^-}, C_{t^-}) &= (1 - rdt)\mathbb{E}\left[E(a^\star C_{t^-} + dA_t, C_{t^-} + dC_t) \right] \\ &= (1 - rdt)(1 - \lambda dt)\mathbb{E}\left[E(a^\star C_{t^-} + dA_t, C_{t^-} + dC_t) \middle| dN_t = 0 \right] \\ &+ (1 - rdt)\lambda dt \mathbb{E}\left[E(a^\star C_{t^-} + dA_t, C_{t^-} + dC_t) \middle| dN_t = 1 \right]. \end{aligned} \tag{54}$$

We first derive the expectation if no jumps occur in the interval $[t, t + dt]$ (i.e., $dN_t = 0$). Using equation (52), we have

$$\begin{aligned} &\mathbb{E}\left[E(a^\star C_{t^-} + dA_t, C_{t^-} + dC_t) \middle| dN_t = 0 \right] \\ &= \mathbb{E}\left[e(a^\star) C^\star(a^\star C_{t^-} + dA_t) + p(a^\star)(C^\star(a^\star C_{t^-} + dA_t) - C_{t^-} - dC_t) \middle| dN_t = 0 \right]. \end{aligned}$$

By Ito's Lemma

$$\mathbb{E}\left[C^\star(a^\star C_{t^-} + dA_t) \middle| dN_t = 0 \right] = C_{t^-}(1 + \mu dt).$$

We thus obtain

$$\mathbb{E}\left[E(a^\star C_{t^-} + dA_t, C_{t^-} + dC_t)\right| dN_t = 0] = e(a^\star)C_{t^-}(1 + \mu dt) + p(a^\star)C_{t^-}(\mu - \delta^\star)dt. \quad (55)$$

Now we consider the last term of (54) that corresponds to Poisson jumps to A. If the jump $dA_t/A_{t^-} = S_t - 1$ is such $a^\star S_t > \overline{a}$, then the equity holders compensate this jump and the state returns to $a^\star$. Thus, they repurchase $C_{t^-}(1 - S_t)$ units of debt and $d\mathcal{G}_t = C_{t^-}(S_t - 1)$. Similarly,

$$\mathbb{E}\left[C^\star(a^\star C_{t^-} + dA_t)\middle| dN_t = 1, S_t \geq \frac{\overline{a}}{a^\star}\right] = C_{t^-}(S_t + \mu dt).$$

Thus,

$$\begin{aligned}\mathbb{E}\left[E(a^\star C_{t^-} + dA_t, C_{t^-} + dC_t)\middle| dN_t = 1, S_t \geq \frac{\overline{a}}{a^\star}\right] &= e(a^\star)C_{t^-}(S_t + \mu dt) + p(a^\star)C_{t^-}(S_t - 1) \\ &\quad + p(a^\star)(\mu - \delta^\star)dt.\end{aligned}$$

Suppose now $S_t < \overline{a}/a^\star$, by definition of the policy, there are no instantaneous adjustments and we cannot use the identity in (52). Therefore,

$$\begin{aligned}\mathbb{E}\left[E(a^\star C_{t^-} + dA_t, C_{t^-} + dC_t)\middle| dN_t = 1, S_t < \frac{\overline{a}}{a^\star}\right] &= E(a^\star C_{t^-}S_t, C_{t^-}) + E_A(a^\star C_{t^-}S_t, C_{t^-})\mu dt \\ &\quad + \frac{\sigma^2}{2}E_{AA}(a^\star C_{t^-}S_t, C_{t^-})dt + E_C(a^\star C_{t^-}S_t, C_{t^-})\delta(a^\star S_t)dt\end{aligned}$$

Note that all terms except the first one are of the order dt. When plugged back into (54), these terms multiply λdt and are thus of order $(dt)^2$. Keeping only terms of order dt in (54), we obtain

$$\begin{aligned}e(a^\star)C_{t^-} &= p(a^\star)C_{t^-}(\mu - \delta^\star)dt + (1 - rdt - \lambda dt)e(a^\star)C_{t^-}(1 + \mu dt) \\ &\quad + \lambda dt C_{t^-}\int_0^{\ln(a^\star/\overline{a})} \left(e^{-s}(e(a^\star) + p(a^\star)) - p(a^\star)\right)\xi e^{-\xi s}ds \\ &\quad + \lambda dt C_{t^-}\int_{\ln(a^\star/\overline{a})}^{\infty} e(a^\star e^{-s})\xi e^{-\xi s}ds\end{aligned}$$

Scaling by C_t-dt, we get

$$(r+\lambda-\mu)e(a^\star) = p(a^\star)(\mu-\delta^\star) + \lambda\int_0^{\ln(a^\star/\overline{a})} \left(e^{-s}(e(a^\star)+p(a^\star)) - p(a^\star)\right)\xi e^{-\xi s}ds$$
$$+ \lambda\int_{\ln(a^\star/\overline{a})}^{\infty} e(a^\star e^{-s})\xi e^{-\xi s}ds. \tag{56}$$

Using equation (51), which gives $e(a)$ for $a \leq \overline{a}$, we can solve for the last term of (56),

$$\int_{\ln(a^\star/\overline{a})}^{\infty} e(a^\star e^{-s})\xi e^{-\xi s}ds = \int_{\ln(a^\star/\overline{a})}^{\infty}\left((e(a^\star)+p(a^\star))\frac{\overline{a}}{a^\star} - p(a^\star)\right)\left(\frac{a^\star e^{-s}}{\overline{a}}\right)^{-\gamma}\xi e^{-\xi s}ds$$
$$= \left((e(a^\star)+p(a^\star))\frac{\overline{a}}{a^\star} - p(a^\star)\right)\frac{\xi}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}.$$

Solving for all integrals, we get

$$(r+\lambda-\mu)e(a^\star) = p(a^\star)(\mu-\delta^\star) + \frac{\lambda\xi}{\xi+1}\left(1-\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}\right)(p(a^\star)+e(a^\star)) - \lambda p(a^\star)\left(1-\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}\right)$$
$$+ \lambda\left((e(a^\star)+p(a^\star))\frac{\overline{a}}{a^\star} - p(a^\star)\right)\frac{\xi}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}$$

Putting all terms in $e(a^\star)$ on the left hand side gives

$$\left(r+\frac{\lambda}{\xi+1}-\mu+\frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}\right)e(a^\star) = p(a^\star)\left(\mu-\delta^\star-\frac{\lambda}{\xi+1}-\frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}+\lambda\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}\right)$$
$$+ \lambda\left((e(a^\star)+p(a^\star))\frac{\overline{a}}{a^\star} - p(a^\star)\right)\frac{\xi}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}$$

or

$$\left(r+\frac{\lambda}{\xi+1}-\mu+\left(\frac{\lambda\xi}{\xi+1}-\frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}\right)e(a^\star)$$
$$= p(a^\star)\left(\mu-\delta^\star-\frac{\lambda}{\xi+1}-\left(\frac{\lambda\xi}{\xi+1}-\frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}-\frac{\lambda\gamma}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}\right). \tag{57}$$

Denoting

$$c \equiv r+\frac{\lambda}{\xi+1}-\mu, \quad b(\gamma) \equiv \frac{\lambda\xi}{\xi+1}-\frac{\lambda\xi}{\xi-\gamma} \tag{58}$$

we can rewrite this last equation as follows

$$e(a^\star) + p(a^\star) = \frac{r - \delta^\star - \frac{\lambda\gamma}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}}{c + b(\gamma)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}} p(a^\star) \tag{59}$$

B.2 Stablecoin Price

Similarly, we conjecture then verify that the price of a stablecoin is given by

$$p(a) = \begin{cases} \sum_{k=1}^{3} b_k a^{-\gamma_k} & \text{if } \ 0 \leq a < \overline{a}, \\ p^\star & \text{if } \ a \geq \overline{a}. \end{cases}$$

Smooth Region $a \in [0, \overline{a})$ In the smooth region, the HJB is given by

$$(r + \lambda - \underline{\delta})p(a) = (\mu - \underline{\delta})ap'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)]. \tag{60}$$

Let us compute the integral using the conjectured $p(a)$:

$$\mathbb{E}[p(Sa)] = \int_0^\infty p(e^{-s}a)\xi e^{-\xi s} ds = \int_0^\infty \sum_{k=1}^{3} b_k e^{s\gamma_k} a^{-\gamma_k} \xi e^{-\xi s} ds = \sum_{k=1}^{3} \frac{b_k \xi a^{-\gamma_k}}{\xi - \gamma_k}.$$

We can plug in the guess into the HJB and solve for the undetermined coefficients:

$$(r + \lambda - \underline{\delta})p(a) = -(\mu - \underline{\delta}) \sum_{k=1}^{3} \gamma_k b_k a^{-\gamma_k} + \frac{\sigma^2}{2} \sum_{k=1}^{3} (1 + \gamma_k)\gamma_k b_k a^{-\gamma_k} + \lambda \mathbb{E}[p(Sa)].$$

For the previous equation to hold, it must be that characteristic equation (48) holds. Imposing the boundary conditions $p(0) = 0$ and $p(\overline{a}) = p^\star$, we obtain that the coefficients with roots bigger than -1 are equal to 0 and

$$\forall\, a \leq \overline{a}, \quad p(a) = p(a^\star)\left(\frac{a}{\overline{a}}\right)^{-\gamma}, \tag{61}$$

where γ is the same parameter as above, that is, the unique negative root smaller than -1 of equation (48).

Target Region $a \in [\overline{a}, \infty)$ Using the same steps as for the equity value, we can derive the HJB for the target price at $a^\star$. We obtain

$$(r+\lambda)p(a^\star) = \ell(a^\star)\mathbf{1}_{p(a^\star)=1} + \delta^\star p(a^\star) + \lambda \overline{p}(a^\star)\int_0^{\ln(a^\star/\overline{a})} \xi e^{-\xi s} ds + \lambda \int_{\ln(a^\star/\overline{a})}^{\infty} p(a^\star e^{-s})\xi e^{-\xi s} ds. \tag{62}$$

Given the expression for p on $[0, \overline{a}]$, it clear that the price can only be positive if is is equal to 1 as otherwise the liquidity benefit term is equal to 0. Using equation (61), we can express the last term of the equation above as follows

$$\int_{\ln(a^\star/\overline{a})}^{\infty} p(a^\star e^{-s})\xi e^{-\xi s} ds = \int_{\ln(a^\star/\overline{a})}^{\infty} p(a^\star)\left(\frac{a^\star e^{-s}}{\overline{a}}\right)^{-\gamma} \xi e^{-\xi s} ds = p(a^\star)\frac{\xi}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}$$

or

$$\begin{aligned}
&\left(r + \lambda\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}\right)p(a^\star) = \ell(a^\star)\mathbf{1}_{p(a^\star)=1} + \delta^\star p(a^\star) + p(a^\star)\frac{\lambda\xi}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi} \\
\Leftrightarrow \quad &\left(r - \frac{\lambda\gamma}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}\right)p(a^\star) = \ell(a^\star)\mathbf{1}_{p(a^\star)=1} + \delta^\star p(a^\star).
\end{aligned} \tag{63}$$

Hence, the target price is pegged to 1 if

$$\delta^\star = r - \frac{\lambda\gamma}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi} - \ell(a^\star). \tag{64}$$

B.3 Optimal Policy

The maximization problem over the set of policy variables $\boldsymbol{\theta}_0 = \{\overline{a}, a^\star, \underline{\delta}, \delta^\star\}$ at time 0 is given by

$$E_0 = \max_{\boldsymbol{\theta_0}}\left\{\frac{e(a^\star)+p(a^\star)}{a^\star}A_0\right\} \tag{65}$$

$$\text{subject to} \qquad \forall\, A, C \geq 0 \qquad E(A,C) = Ce(a) \geq 0,$$
$$\overline{a} \in [0, a^\star], \quad \underline{\delta} \geq 0, \quad \delta^\star \geq 0.$$

Equations (51) and (52) show that the limited liability constraint $e(a) \geq 0$ is satisfied for all values of a if and only if it is satisfied for $a = \overline{a}$, that is, if

$$\frac{e(a^\star) + p(a^\star)}{a^\star} \geq \frac{1}{\overline{a}}. \tag{66}$$

Observe also that $\delta^\star \geq 0$ holds because $r - \ell > 0$ by assumption and the second term of (95) is positive because $\gamma < 0$. Finally, using equations (59) and (95) that ensures a price peg, we can rewrite the objective as follows

$$\frac{e(a^\star) + p(a^\star)}{a^\star} = \frac{r - \delta^\star - \frac{\lambda\gamma}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}}{c + b(\gamma)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}} p(a^\star) = \frac{\ell(a^\star)}{c + b(\gamma)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}} \frac{1}{a^\star}$$

with $c = r + \frac{\lambda}{\xi+1} - \mu$ and $b(\gamma) = \frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}$ as defined in (58). Hence, we can rewrite the optimization problem as follows:

$$\begin{aligned} E_0/A_0 = \max_{\{\overline{a}, a^\star, \underline{\delta}\}} & \left\{ \frac{\ell(a^\star)}{c + b(\gamma)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}} \frac{1}{a^\star} \right\}, \\ \text{subject to} \quad 1 \leq & \frac{\ell(a^\star)}{c + b(\gamma)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}} \frac{\overline{a}}{a^\star}, \\ & \overline{a} \in [0, a^\star], \quad \underline{\delta} \geq 0 \end{aligned} \tag{67}$$

Optimality Conditions Consider first the optimal choice of $\underline{\delta} \geq 0$. This variable enters the objective function or constraints only via the value of γ and equation (48) shows this is the only policy variable in $\boldsymbol{\theta}_0$ that determines γ. The objective function increases and constraint (67) is relaxed when γ increases because $b'(\gamma) < 0$. The implicit function theorem applied to (48) yields

$$\frac{\partial\gamma}{\partial\underline{\delta}} = \frac{1 + \gamma}{\mu - \underline{\delta} - \frac{\sigma^2}{2} - \sigma^2\gamma - \frac{\lambda\xi}{(\xi-\gamma)^2}} < 0, \tag{68}$$

where the inequality follows from $\gamma < -1$. This implies $\underline{\delta}^{opt} = 0$.

Next, consider the optimal choice of $\overline{a}$. The objective function strictly decreases with $\overline{a}$, but constraint (67) is violated for $\overline{a} = 0$. Hence, this constraint must hold as an equality. Let $\overline{a}^{opt}(a^\star)$ be the minimum value of $\overline{a} \in (0, a^\star]$, if any, that satisfies the constraint as an equality. To characterize $\overline{a}^{opt}(a^\star)$, it is convenient to denote $H(\overline{a}, a^\star)$ for the right-hand-side of constraint (67). Computing the first-order derivative of H with respect to $\overline{a}$, we obtain

$$H_{\overline{a}}(\overline{a}, a^\star) \propto c - b\xi \left(\frac{\overline{a}}{a^\star}\right)^{\xi+1},$$

which is strictly positive for $\overline{a} = 0$ (because $c > 0$ by assumption) and decreasing with $\overline{a}$. If $H_{\overline{a}}(a^\star, a^\star) > 0$ ($H_{\overline{a}}(a^\star, a^\star) < 0$), H attains its maximum at $\overline{a} = a^\star$ ($\overline{a} = a^\star \left(c/(\xi b)\right)^{\frac{1}{\xi+1}}$). A necessary and sufficient condition for $\overline{a}^{opt}(a^\star)$ to exist is that this maximum is above 1, that is,

$$\ell(a^\star) \geq \frac{c + b \min\left\{1, \frac{c}{b\xi}\right\}}{\min\left\{1, \frac{c}{b\xi}\right\}^{\frac{1}{\xi+1}}}. \tag{69}$$

A necessary condition for a solution to exist is that $\lim_{a \to \infty} \ell(a)$ is above the right-hand-side of (69) because ℓ is strictly increasing by assumption. If that latter condition is satisfied, constraint (69) can be written as $a^\star \geq a^\star_{lb}$, again because ℓ strictly increases with $a^\star$.

The platform's objective is to minimize $\overline{a}^{opt}(a^\star)$ with respect to $a^\star$ subject to $a^\star \geq a^\star_{lb}$. Two cases are possible. Suppose first $a^\star > a^\star_{lb}$. Then optimality requires $\frac{\partial \overline{a}^{opt}(a^\star)}{\partial a^\star} = 0$ where

$$\begin{aligned}\frac{\partial \overline{a}^{opt}}{\partial a^\star} = -\frac{\frac{\ell'(a^\star)}{\ell(a^\star)} H(\overline{a}^{opt}, a^\star) - \frac{\overline{a}^{opt}}{(a^\star)^2} H_x(\overline{a}^{opt}, a^\star)}{\frac{H_x(\overline{a}^{opt}, a^\star)}{a^\star}} &\propto \frac{\overline{a}^{opt}}{a^\star} - \frac{\ell'(a^\star)\overline{a}^{opt}}{\ell(a^\star)} \frac{c + b\left(\frac{\overline{a}^{opt}}{a^\star}\right)^{\xi+1}}{c - b\xi\left(\frac{\overline{a}^{opt}}{a^\star}\right)^{\xi+1}} \\ &= \frac{\overline{a}^{opt}}{a^\star}\left[1 - \frac{\ell'(a^\star)\overline{a}^{opt}}{c - b\xi\left(\frac{\overline{a}^{opt}}{a^\star}\right)^{\xi+1}}\right].\end{aligned} \tag{70}$$

with H_x the first-order-derivative of H with respect to $x = \overline{a}/a^\star$. Setting the right-hand-side of (70) to 0, one obtain an implicit characterization for $a^\star$, and is left to verify that this value satisfies $a^\star \geq a^\star_{lb}$, with $a^\star_{lb}$ the lower bound imposed by (69). The second case is when $a^\star = a^\star_{lb}$. Then we have $\overline{a}^{opt} = a^\star_{lb} \min\left\{1, \frac{c}{b\xi}\right\}$. This case arises if the right-hand-side of (70) is strictly positive for $a^\star = a^\star_{lb}$, as otherwise the platform could increase $a^\star$ to achieve its objective to decrease $\overline{a} = \overline{a}^{opt}(a^\star)$.

In the first case discussed above, the optimal value of $a^\star$ is interior, the solution is characterized by

$$\ell'(a^\star)\overline{a}^{opt} = c - b\xi\left(\frac{\overline{a}^{opt}}{a^\star}\right)^{\xi+1},$$
$$\ell(a^\star)\frac{\overline{a}^{opt}}{a^\star} = c + b\left(\frac{\overline{a}^{opt}}{a^\star}\right)^{\xi+1}.$$

We thus obtain

$$\ell'(a^\star)a^\star = c\frac{a^\star}{\overline{a}^{opt}} - b\xi\left(\frac{\overline{a}^{opt}}{a^\star}\right)^{\xi},$$
$$c\frac{a^\star}{\overline{a}^{opt}} = \ell(a^\star) - b\left(\frac{\overline{a}^{opt}}{a^\star}\right)^{\xi}.$$

Hence, we get

$$\ell'(a^\star)a^\star = \ell(a^\star) - b(\xi+1)\left(\frac{\overline{a}^{opt}}{a^\star}\right)^{\xi} = \ell(a^\star) + \lambda\xi\frac{1+\gamma^{opt}}{\xi-\gamma^{opt}}\left(\frac{\overline{a}^{opt}}{a^\star}\right)^{\xi}$$

where we used the definition of b to obtain the last equation.

C Proof of Corollary 1

Observe that the denominator of the right-hand-side of (69) is above 1. Hence, equation (69) can be satisfied only if

$$\lim_{a\to\infty}\ell(a) \geq c = r + \frac{\lambda}{\xi+1} - \mu.$$

where the equality is by definition of c in (58).

D Proof of Lemma 3

We first show that if the equity value is strictly convex in C over some interval, the issuance policy is smooth in this region. Given any debt level $\widehat{C}$, equity holders have the option to adjust the stock of stablecoins to C by issuing $C - \widehat{C}$ at the price of $p(A, C)$. Therefore, by optimality of the debt issuance policy, the equity value at $\widehat{C}$ must satisfy

$$E(A,\widehat{C}) \geq E(A,C) + p(A,C)(C-\widehat{C}), \tag{71}$$

To show that discrete repurchases are suboptimal, we prove that inequality (71) is strict if the equity value is strictly convex with respect to its second argument. Suppose to the contrary there exists $C' \neq C$ such that $E(A,C') = E(A,C) + p(A,C)(C-C')$. By strict convexity of E, we get that for all $x \in]0,1[$

$$E(A, xC + (1-x)C') < xE(A,C) + (1-x)E(A,C') = E(A,C) + (1-x)p(A,C)(C-C'). \tag{72}$$

Using then condition (71) for $\widehat{C} = xC + (1-x)C'$, we obtain

$$E(A, xC + (1-x)C') \geq E(A,C) + (1-x)p(A,C)(C-C'),$$

which is a contradiction with (72). Thus, it must be that

$$E(A,C') > E(A,C) + p(A,C)(C-C').$$

Hence, any discrete issuance with $|C-C'| > 0$ would be suboptimal for shareholders, that is, the debt policy must be smooth everywhere E is strictly convex in C

Second, we show that equity value is equal to 0 everywhere if the debt policy is smooth everywhere. With a smooth debt policy, $d\mathcal{G}_t = g_t C_t dt$ where $g_t = G_t/C_t$, the HJB for e is given by

$$(r+\lambda)e(a) = \max_{g(a)} \left\{ g(a)p(a) + \mu a e'(a) + (g(a)-\delta(a))(e(a)-e'(a)a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda \mathbb{E}[e(Sa)] \right\}. \tag{73}$$

where $e(a) = E(A/C,1)$ by homogeneity. The smooth debt policy to be optimal, if the firs-order condition with respect to g is satisfied, that is, if

$$p(a) = e'(a)a - e(a). \tag{74}$$

Thus, equation (73) simplifies to

$$(r+\lambda)e(a) = -\delta(a)p(a) + \mu a e'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda \mathbb{E}[e(Sa)]. \tag{75}$$

Equity owners' dividend is weakly negative and it is strictly negative when $\delta(a)p(a) > 0$. Hence, given limited liability, equity owners strictly prefer to default if $\delta(a)p(a) > 0$ on any part of the state space visited with positive probability. This implies $e(a) = 0$. Besides for the stablecoin to have value, it is necessary that there exists a subset of values of a visited with positive probability such that $\delta(a)p(a) > 0$. Indeed, the liquidity benefit is only captured if the price is pegged to one on such subset, but pegging the price requires $\delta(a) > 0$ on that subset. We showed, however, that the platform then strictly prefers to default so the stablecoin cannot have strictly positive value.

E Proof of Lemma 4

We first show that if the equity value e is linearly increasing in a over some segment $a \in [a_L, a_U]$, the equilibrium issuance policy cannot be smooth over this interval. We then show that for any such interval $[a_L, a_U]$, there is a single jump point.

Step 1 Non-smooth issuance The proof is by contradiction. Suppose $d\mathcal{G}_t = G(a)dt$ over $[a_L, a_U]$ with $g(a) \equiv G(a)/C$ the stablecoin issuance rate per unit of stablecoins. We showed in the proof of Lemma 3 that a smooth issuance policy is optimal over this interval if and only if

$$p(a) = -e(a) + e'(a)a. \tag{76}$$

Thus, $p'(a) = e''(a)a = 0$ where the second equality follows from the linearity of e. Hence, the price is constant over the interval $[a_L, a_U]$, that is, $p(a) = p$. Rewriting HJB equation (75) in this case, we obtain

$$(r+\lambda)e(a) = -\delta(a)p + \mu a e'(a) + \lambda \mathbb{E}[e(Sa)]. \tag{77}$$

Taking the first-order-derivative of the expression above, we respect to a, we obtain

$$e'(a) = -\delta'(a)p\mu e'(a) + \lambda \mathbb{E}[e'(Sa)S] \tag{78}$$

The HJB equation for the stablecoin price is given by

$$(r+\lambda)p(a) = \ell(a) + \delta(a)p(a) - (g(a)+\delta(a))ap'(a) + \mu a p'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)]. \tag{79}$$

which, in this case, simplifies to

$$(r+\lambda)p = \ell(a) + \delta(a)p + \lambda\mathbb{E}[p(Sa)], \tag{80}$$

because the price is constant. Combining equations (77), (78) and (78), we obtain

$$0 = (r+\lambda)(p(a)+e(a)-e'(a)a) = \ell(a) + \delta(a)p(a) + \lambda\mathbb{E}[p(Sa)] - \delta(a)p(a) + \mu a e'(a) \tag{81}$$

$$\begin{aligned} &+ \lambda\mathbb{E}[e(Sa)] + \delta'(a)ap(a) - \mu a e'(a) - \lambda\mathbb{E}[e'(Sa)Sa], \\ &= \ell(a) + \delta'(a)ap(a) + \lambda\mathbb{E}[p(Sa)+e(Sa)-e'(Sa)Sa], \\ &= \ell(a) + \delta'(a)ap(a). \end{aligned} \tag{82}$$

The last equality follows from equation (76). We proved this relationship for segments where the equilibrium issuance policy is smooth. For segments over which the issuance policy features jumps, equation (52) shows that for any a, a' in this segment, we have

$$e(a') = \big[e(a)+p(a)\big]\frac{a'}{a} - p(a) \tag{83}$$

Taking the first-order derivative with respect to a' and then setting $a = a'$ we obtain equation (76).

We now establish a contradiction. First, for relationship (83) to hold over $[a_L, a_U]$, it must be $a_L > 0$. Indeed, the left-hand-side of (83) can be no lower than 0 by limited liability. The limit when $a \to 0$ of the right-hand side is $-p$ where p is the constant price over $[a_L, a_U]$. Hence, $a_L > 0$.

We now consider two cases for price p. Suppose first $p(a) = p \neq 1$ in which case $\ell(a) = 0$ by definition of the liquidity benefit. Equations (82) and (80) then imply that $\delta'(a) = 0$ and

$$\delta(a) = \delta = (r+\lambda)p - \lambda\mathbb{E}[p(Sa)] \tag{84}$$

for $a \in [a_L, a_U]$. As $a_L > 0$, there must be an interval to the left of a_L over which $e(a)$ is strictly convex. As $p'(a) = e''(a)$ by equation (76), $p(a)$ is strictly increasing on this interval. Hence, for $a \in [a_L, a_U]$, the term $\mathbb{E}[p(Sa)]$ cannot be constant in a because the random variable Sa may take any value on $[0, a]$. This is incompatible with equation (85).

Second, suppose $p(a) = 1$. Equations (80) and (82) imply together that

$$\frac{\ell(a) - \ell'(a)a}{a} = \lambda \mathbb{E}[p'(Sa)S]. \tag{85}$$

We have

$$\begin{aligned}
\mathbb{E}[p'(Sa)S] &= \int_0^{\infty} p'(e^{-s}a)\xi e^{-s(\xi+1)} ds \\
&= \int_{\ln(a/a_L)}^{\infty} p'(e^{-s}a)\xi e^{-s(\xi+1)} ds = \kappa \left(\frac{a}{a_L}\right)^{-(\xi+1)}
\end{aligned}$$

where $\kappa \equiv \int_0^{\infty} p'(e^{-s}a_L)\xi e^{-s(\xi+1)} ds$ is a positive constant. To obtain the second line, we used the fact that p is constant over $[a_L, a_U]$. Thus, we must have

$$\ell(a) = \ell'(a)a + \lambda\kappa \left(\frac{a}{a_L}\right)^{-(\xi+1)}$$

for $a \in [a_L, a_U]$. A general solution to this equation is of the form

$$\ell(a) = \alpha a + \beta + f a^{-\xi-1}$$

with $f \geq 0$. Hence, assuming the issuance policy is smooth pins down a function form for ℓ. This leads to a contradiction because $\ell(.)$ is an exogenous function in this problem.

Step 2 Single jump point We want to show that there can only be one jump point $a_{jump} \in [a_L, a_U]$ when the equilibrium issuance policy features jump over segment $[a_L, a_U]$. Suppose there are two such jump points (the argument generalizes for more jump points) labeled a^1_{jump} and a_2^{jump}. Then, there must be one jump point, say, a^1_{jump} for which liquidity benefits $l(a)/a * A$ are larger given the single-peak property in Assumption 1. Hence, to maximize its date-0 value, the platform would strictly prefer jumping to a^1_{jump} from any point in $[a_L, a_U]$ rather than to a^2_{jump}.

We are left to show that jumping to a^1_{jump} instead of a^2_{jump} is compatible with the equilibrium issuance policy. By Lemma 3 and 4, the issuance policy features jumps on $[a_L, a_U]$ only if equity value is linear and price is constant. Hence, from any state a with jump point a^2_{jump}, we have

$$e(a) = \left[e(a^2_{jump}) + p(a^2_{jump})\right] \frac{a}{a^2_{jump}} = \left[e(a^1_{jump}) + p(a^1_{jump})\right] \frac{a}{a^1_{jump}}$$

Hence, jumping to a^1_{jump} is also an optimal equilibrium issuance policy. This equality simply reflects the fact that the platform is indifferent ex-post between all points in $[a_L, a_U]$. At date-0, however, the platform would choose jump point a^1_{jump} as the sole jump point.

F Proof of Proposition 4

In this section, we show that if a coupon policy $\delta(a)$ is optimally chosen at date 0, then a Markov equilibrium with positive stablecoin value must feature a stable issuance policy. In this proof, we write equilibrium variables as functions of the state variable a.

The result that the default boundary is $\underline{a} = 0$ is trivial. The platform does not have to make payments in the unit of account because the coupon is paid in stablecoins. Second, because the issuance policy is chosen ex-post, the platform can always choose to issue or to stay inactive if a repurchase is too costly. Hence continuing always weakly dominates defaulting.

To prove the main part of the result about stable debt policies, we will show that the equity value is strictly convex over some interval $[0, \overline{a}]$ and linear over $[\overline{a}, \infty)$. Then we use Lemma 3 and 4 to prove the result.

Step 1 The equity value $e(a)$ is weakly convex, continuously differentiable, and stablecoin price function $p(a)$ is continuous and increasing.

These properties follow from the same arguments of Lemma A.1 in DeMarzo and He (2021).

Since the equity value $e(a)$ is weakly convex, there must be a strictly ordered sequence $\{a^{(n)}\}_{n\geq 0}$ such that $a^{(0)} = 0$ and $\lim_{n\to\infty} a^{(n)} = \infty$ such that on each segment $[a^{(n)}, a^{(n+1)}]$, e is either strictly convex or linear, with different convexity on two consecutive segments.

Step 2 If e is strictly convex over some interval $[a_L, a_U]$, the price function is strictly increasing.

If e is strictly convex, Lemma Lemma 3 shows that the equilibrium issuance policy is smooth over $[a_L, a_U]$ and that equation (74) holds. Taking the first-order-derivative of this expression with respect to a, we obtain $p'(a) = e''(a)a$ and thus p is strictly increasing over $[a_L, a_U]$.

Step 3 There is at least 1 segment with e strictly convex, and one segment with e linear. Equity value cannot be linear on segment $[0, a^{(1)}]$.

If the equity value $e(a)$ is strictly convex everywhere, Lemma 3 shows that $e(a) = 0$ and $p(a) = 0$ for all a. Hence, for a Markov equilibrium with positive stablecoin value to exist, there must be a segment on which e is linear.

Suppose that e is linear on $[0, a^{(1)}]$. By Lemma 4 and **??**, there must be $a^{jump} \in [0, a^{(1)}]$ such that the issuance policy is to jump at a^{jump} from any point in $[0, a^{(1)}]$. For the jump to a^{jump} to be optimal, it must be that for any $a \in [0, a^{(1)}]$, we have

$$e(a) = \left[e(a^{jump}) + p(a^{jump})\right]\frac{a}{a^{jump}} - p(a^{jump})$$

which becomes negative as $a \to 0$. Hence, there is a contradiction. The equity value is strictly convex on $[0, a^{(1)}]$.

Step 4 The last step of the proof is to show there exists $\bar{a}$ such that the equity value is strictly convex over $[0, \bar{a}]$ and linear over $[\bar{a}, \infty)$. The characterization of the equilibrium issuance policy as a stable issuance policy then follows from Lemma 3, 4 and **??**.

Let $\delta(a)$ be an interest policy that induces a non-zero Markov equilibrium with issuance policy $d\mathcal{G}$ such that there exists a segment $[a^{(2)}, a^{(3)}]$ over which e is strictly convex—the original (interest) policy. We want to show that there exists an alternative coupon policy $\hat{\delta}(a)$ that induces a Markov equilibrium with issuance policy $d\widehat{\mathcal{G}}$ such that e has the desired properties and the date-0 platform value if strictly higher.

(a) We first construct an alternative policy and its induced equilibrium. Let $a^\star$ be the target value in the first linear region $[a^{(1)}, a^{(2)}]$ for equity in the equilibrium induced by the original policy. Construct the alternative policy and the induced equilibrium as follows. Set $\widehat{\delta}(a) = \delta(a)$ and $d\hat{\mathcal{G}}(a, C) = d\mathcal{G}(a, C)$ for all $a \leq a^\star$ and conjecture a linear equity value $\hat{e}$ and a constant price $\hat{p}$ for all $a \in [a^{(1)}, \infty)$ with $a^{(1)}$. In the equilibrium induced by the alternative policy, the issuance policy is to jump at a^* for all $a \geq a^{(1)}$.

We argue that the issuance policy $d\hat{\mathcal{G}}(a, C)$ forms part of an equilibrium induced by the alternative coupon policy. The subspace $[0, a^\star]$ is absorbing for the alternative policy, so the specification of $\hat{\delta}(a)$ for $a \geq a^\star$ is irrelevant. This is also an absorbing subspace for the equilibrium induced by the original policy because there are only downward jumps to A and the equilibrium issuance policy is such that $a \leq a^\star$ when $a \in [a^{(1)}, a^{(2)}]$. Hence, the fact that $d\mathcal{G}(a, C)$ for $a \in [0, a^{(2)}]$ is an equilibrium issuance policy induced by the original coupon policy implies that $d\widehat{\mathcal{G}}(a, C)$ for $a \in [0, a^{(2)}]$ is also an equilibrium issuance policy induced by the alternative coupon policy. The fact that $d\hat{\mathcal{G}}(a, C)$ is an equilibrium issuance policy on the rest of the state space, $a \in [a^{(2)}, \infty)$ follows from the observation that $\hat{e}$ is linear over $a \in [a^{(1)}, \infty)$ and p is constant. This implies that jumping to any point in $a \in [a^{(1)}, \infty)$ including a^* is an equilibrium issuance policy.

The argument above also implies that $\hat{e}(a) = e(a)$ and $\hat{p}(a) = p(a)$ for all $a \in [0, a^\star]$

(b) Second, we show that $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$ in the equilibrium induced by the original policy, and thus $\hat{p}(a) = 1$ for all $a \in [a^{(1)}, \infty)$ by Step 4a.

The equity value is linear over $[a^{(1)}, a^{(2)}]$ and the equilibrium issuance policy is to jump at $a^\star \in [a^{(1)}, a^{(2)}]$ when $a \in [a^{(1)}, a^{(2)}]$. Hence, the price $p(a) = p$ must be constant over $[a^{(1)}, a^{(2)}]$. Since $[0, a^\star]$ is an absorbing subspace for the equilibrium induced by the original policy, it must be that $p = 1$. If not, investors never enjoy any liquidity benefit for $a \in [0, a^\star]$ and thus $p(a) = e(a) = 0$ for all $a \in [0, a^\star]$, which is a contradiction. To see this, suppose first $p < 1$. By monotonicity of p, we have $p(a) < 1$ for all $a \in [0, a^{(2)}]$ which implies investors never enjoy the liquidity benefit. Conversely, if $p > 1$ over $[a^{(1)}, a^{(2)}]$, we have $p(a) = 1$ for a unique $a \in [0, a^{(1)})$ because p is strictly increasing over $[0, a^{(1)})$ since e is strictly convex (Step 2). Given the smooth equilibrium issuance policy on $[0, a^{(1)}]$ this state is not visited with positive probability and thus investors enjoy liquidity benefit with zero probability, which again leads to a contradiction. Hence $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$. This implies $\hat{p}(a) = 1$ for all $a \in [a^{(1)}, \infty)$ in the equilibrium induced by the alternative policy.

(c) We can now show that the platform value at date 0 is higher under the alternative policy than under the original policy. The platform's value at date 0 is given by equation (10), which we rewrite here for convenience.

$$E_0 = \mathbb{E}\left[\int_0^\infty e^{-rt}\ell(A_t, C_t)C_t \mathbf{1}_{p(A_t,C_t)=1} dt \middle| A_0, C_0 = 0\right]$$

By Step 4b, liquidity benefits are only enjoyed when $a \in [a^{(1)}, a^{(2)}]$ because $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$. Under the alternative policy $a^\star \in [a^{(1)}, a^{(2)}]$ is reached immediately at date 0 by design because the equilibrium issuance policy is to jump to $a^\star$ when no stablecoins are outstanding ($a = \infty$). In the equilibrium induced by the original policy, the optimal choice at date 0 is some $a > a^{(2)}$ by design of the original policy. Denote τ_f the first (stochastic) time the platform enters the region $[a^{(1)}, a^{(2)}]$ under the original policy. We have

$$E_0 = \mathbb{E}[E^{-r\tau_f}]\hat{E}_0 < E_0$$

because no liquidity benefit is enjoyed before the platform reaches $[a^{(1)}, a^{(2)}]$. The inequality follows from the fact that $\mathbb{E}[\tau_f] > 0$ by design of the original policy.

We showed that the original policy is strictly dominated. Hence, in an equilibrium induced by an optimal coupon policy, the issuance policy must belong to the class of stable policies.

This concludes the proof of Proposition Proposition 4.

G Proof of Proposition 5

Points 1 and 2 The properties of e and p in Points 1 and 2 follow directly from the proof of Proposition 4 in Appendix F. The HJB equation (29) for the equity value follows from equations (75) in the proof of **??**. The HJB equation for the price is given by equation

(79) in the proof of Proposition 4. For future reference, we also provide the smooth-pasting condition at the boundary $\overline{a}$ between the smooth region and the target region. We have

$$e'(\overline{a}) = \frac{e^\star + p^\star}{a^\star}, \tag{86}$$

with e the value of equity in the region $[0, \overline{a}]$. The right-hand side is the derivative of the equity value in the target region. We showed in the proof of Proposition 4 that equation (76). must hold in this region. Computing the derivative with respect to a yields the right-hand-side of (86).

Point 3 Next, we derive the equilibrium stablecoin issuance rate in the smooth region. From HJB equation (73) and the optimality of a smooth debt policy, we obtained equation (74) in the proof of Lemma **??**. Taking the first-order derivative of e in equation (73) at $g = 0$, we obtain

$$(r+\lambda)e'(a) = \mu e'(a) + \mu a e''(a) - \delta'(a)p(a) - p'(a)\delta(a) + \frac{\sigma^2}{2}a^2 e''(a) + \sigma^2 a e'''(a) + \lambda \mathbb{E}[Se'(Sa)]$$

The HJB for the stablecoin price is given by

$$(r+\lambda)p(a) = \ell(a) + \delta p(a) - (g(a) + \delta(a))ap'(a) + \mu a p'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)].$$

with $\ell(a) = 0$ because the price p is strictly below one by construction. We can then use (74) to obtain a condition on g. We have

$$\begin{aligned}
0 &= (r+\lambda)(p(a) + e(a) - e'(a)a), \\
&= \delta(a)p(a) - (g(a) + \delta(a))p'(a)a + \mu a p'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)], \\
&\quad - \delta(a)p(a) + \mu a e'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda \mathbb{E}[e(Sa)], \\
&\quad + \delta(a)p'(a)a + \delta'(a)p(a)a - \mu a^2 e''(a) - \mu a e'(a) - \frac{\sigma^2}{2}a^3 e'''(a) - \sigma^2 a^2 e''(a) - \lambda \mathbb{E}[e'(Sa)Sa], \\
&= -g(a)ap'(a) + \delta'(a)ap(a) + \mu a \underbrace{(p'(a) - e''(a))}_{=0} + \frac{\sigma^2}{2}a^2 \underbrace{(p''(a) + e''(a) - ae'''(a))}_{=0} \\
&\quad + \lambda \mathbb{E}[\underbrace{p(Sa) + e(Sa) - e'(Sa)Sa}_{=0}] \\
&= -g(a)ap'(a) + \delta'(a)ap(a)
\end{aligned}$$

The terms above can be set to 0 because equation (74) was shown to hold for all values of a in the proof of Proposition 4. This implies directly that the last term is equal to 0. Besides, taking derivatives of equation (74), we also have

$$p'(a) = e''(a)a,$$
$$p''(a) = e'''(a)a + e''(a),$$

which allow us to set other terms to 0. This proves out claim.

Point 4 . We showed in **??** that a smooth issuance is strictly optimal when the equity value is convex. Hence, we are left to show that the jump to a^* is optimal when $a \in [\overline{a}, \infty)$. By Point 1 and **??**, equity value is linear and price is constant. Hence, from any value of $a \in [\overline{a}, \infty)$ the platform is ex-post indifferent about jumping to any value in $[\overline{a}, \infty)$. In particular, jumping to $a^\star$ is weakly optimal.

We are left to characterize conditions such that for any $a \in [\overline{a}, \infty)$, deviating with a smooth issuance policy is suboptimal under condition (32). Let us derive the value $\Delta E(A_t, C_t)$ of selling a quantity of stablecoins $\Delta C_t dt$ instead of repurchasing $C_t^\star - C_t$ where $C_t^\star = A_t/a^\star$ over the time interval dt for $A_t/C_t \in [\overline{a}, \infty]$:

$$\Delta E(A_t, C_t) = \Delta C_t p(A_t, C_t)dt + (1 - rdt)\mathbb{E}_t\left[E(A_{t+dt}, C_{t+dt})\right] - E(A_t, C_t).$$

Using Ito's lemma, we get

$$\begin{aligned}\mathbb{E}_t\left[E(A_{t+dt}, C_{t+dt})\right] = E(A_t, C_t) + \mu A_t E_A(A_t, C_t)dt + (\Delta C_t + \delta(A_t, C_t))E_C(A_t, C_t)dt \\ + \frac{\sigma^2}{2}E_{AA}(A_t, C_t)dt + \lambda dt(\mathbb{E}_t\left[E(SA_t, C_t)\right] - E(A_t, C_t)).\end{aligned} \tag{87}$$

From equation (76), we get

$$E(A_t, C_t) = \frac{e(a^\star) + p(a^\star)}{a^\star}A_t - C_t p(a^\star), E_A(A_t, C_t) = \frac{e(a^\star) + p(a^\star)}{a^\star}, E_C(A_t, C_t) = -p(a^\star),$$
$E_{AA}(A_t, C_t) = E_{CC}(A_t, C_t) = E_{AC}(A_t, C_t) = 0.$

Substituting in (87), we get

$$\begin{aligned}\mathbb{E}_t\left[E(A_{t+dt},C_{t+dt})\right] &= E(A_t,C_t)+\mu(E(A_t,C_t)+p(a^\star)C_t)dt-(\Delta C_t+\delta(A_t,C_t)C_t)p(a^\star)dt\\&\quad+\lambda dt\mathbb{E}_t\left[E(SA_t,C_t)-E(A_t,C_t)\right].\end{aligned}$$

We can follow the same steps as in Appendix B to obtain

$$(r+\lambda)E(A_t,C_t^\star)=-\delta(A_t,C_t^\star)p(a^\star)C_t^\star+(p(a^\star)C_t^\star+E(A_t,C_t^\star))\mu+\lambda\mathbb{E}[E(SA_t,C_t^\star)].$$

Consequently,

$$(r+\lambda)E(A_t,C_t)=-\delta(a^\star)p(a^\star)C_t^\star+(p(a^\star)C_t^\star+E(A_t,C_t^\star))\mu+\lambda\mathbb{E}[E(SA_t,C_t^\star)]+(r+\lambda)p(a^\star)(C_t^\star-C_t).$$

Also note that $p(A_t,C_t)=p(A_t,C_t^\star)=p(a^\star)$. Hence, the net benefit of a smooth deviation is

$$\begin{aligned}\Delta E(A_t,C_t) &= -\delta(a_t)p(a^\star)C_tdt+\delta(a^\star)p(a^\star)C_t^\star dt-(r+\lambda)p(a^\star)(C_t^\star-C_t)dt\\&\quad+\lambda(\mathbb{E}[E(SA_t,C_t)]-\mathbb{E}[E(SA_t,C_t^\star)])dt.\end{aligned}$$

Thus, as $p(a^\star)=1$, a deviation is optimal if

$$-\delta(a_t)+\delta(a^\star)a_t/a^\star-(r+\lambda)p(a^\star)(a_t/a^\star-1)+\lambda(\mathbb{E}[e(Sa_t)]-\mathbb{E}[e(Sa^\star)a_t/a^\star])>0.$$

In other words, to ensure time-consistency, it must be that

$$\delta(a_t)\geq(r+\lambda)p(a^\star)+(\delta(a^\star)-(r+\lambda)p(a^\star)-\lambda\mathbb{E}[e(Sa^\star)])\,\frac{a_t}{a^\star}+\lambda\mathbb{E}[e(Sa_t)]$$

for $a_t\geq\overline{a}$. We have $p(a^\star)=1$ and by the HJB for p at $a^\star$, equation (62), we have

$$(r+\lambda)p(a^\star)=\ell(a^\star)p(a^\star)+\delta(a^\star)p(a^\star)+\lambda\mathbb{E}[p(Sa^\star)].$$

Thus we obtain the following condition to rule out a smooth deviation:

$$\forall a\in[\overline{a},\infty),\qquad \delta(a)\geq(r+\lambda)+\lambda\mathbb{E}[e(Sa)]-(\ell(a^\star)+\lambda\mathbb{E}[e(Sa^\star)+p(Sa^\star)])\,\frac{a}{a^\star}$$

which is equivalent to (32) in the main text.

H Proof of ??

Maximization Program We first show that the solutions for e and p are as derived in the proof of Proposition **??** for the commitment case. The statement is obvious in the target region in which equity is linear and the price is constant and equal to 1. For the smooth region, observe that the stablecoin issuance rate is $g(a) = \delta'(a)p(a)/p'(a)$ by Proposition 5. By Assumption 2, $\delta'(a) = 0$, which implies $g(a) = 0$. This feature implies that HJB equations (29) and (30) for the equity value and the price in the smooth region are the same as in the commitment case, respectively (49) and (60).

Hence, given a policy set $\boldsymbol{\theta}_0 = \{\underline{\delta}, \delta^*, \overline{a}, a^\star\}$, the only difference when constructing the equity value and price function is the smooth pasting condition (86) at $\overline{a}$ derived in the proof of Proposition 5. Using the functional form for $e(a)$ in the smooth region given by (51), this condition becomes

$$-\frac{\gamma}{\overline{a}}\left(\left[e(a^\star)+p(a^\star)\right]\frac{\overline{a}}{a^\star}-p(a^\star)\right)=\frac{e(a^\star)+p(a^\star)}{a^\star} \quad\Leftrightarrow\quad \frac{e(a^\star)+p(a^\star)}{a^\star}=\frac{\gamma}{1+\gamma}\frac{p(a^\star)}{\overline{a}} \tag{88}$$

We can now characterize the program of the platform at date 0. The platform maximizes its date-0 value subject to the limited liability constraint, the smooth-pasting condition (88), and the relevant conditions on parameter. We obtain

$$E_0 = \max_{\boldsymbol{\theta}_0} \frac{e(a^\star)+p(a^\star)}{a^\star}A_0 \tag{89}$$

$$\text{subject to} \quad \frac{e(a^\star)+p(a^\star)}{a^\star} = \frac{\gamma}{1+\gamma}\frac{p(a^\star)}{\overline{a}} \tag{90}$$

$$e(a) \geq 0, \quad \forall\, a \geq 0, \tag{91}$$

$$p(a^\star) = 1, \tag{92}$$

$$\overline{a} \in [0, a^\star], \qquad \underline{\delta} \geq 0, \quad \delta^\star \geq 0. \tag{93}$$

Equations (51) and (52) show that limited liability constraint (91) is satisfied if and only if

$$\left[e(a^\star)+p(a^\star)\right]\overline{a} - p(a^\star)a^\star \geq 0. \tag{94}$$

This condition is implied by equality (88) because $\gamma < -1$ which means (91) is redundant.

Next, observe that peg constraint $p(a^\star) = 1$ pins down $\delta^\star$. Equation (63) shows that to ensure $p(a^\star) = 1$, we must have

$$\delta^\star = r - \frac{\lambda\gamma}{\xi - \gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi} - \ell(a^\star). \tag{95}$$

Using then equation (57) to substitute for $e(a^\star)$, we obtain the following program

$$E_0/A_0 = \max_{\boldsymbol{\theta}}\left\{\frac{p(a^\star)}{a^\star}\frac{r - \delta^\star - \frac{\lambda\gamma}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}}\right\}. \tag{96}$$

The stablecoin price in the target region must be pegged because investors enjoy no liquidity benefit otherwise. E Furthermore, because $\gamma < -1$, the sequential optimality of $\overline{a}$ contraint implies that the limited liability constraint is always satisfied. Plugging (95) into (96) and using $p(a^\star) = 1$ we can rewrite the optimization problem as follows:

$$\begin{aligned}
E_0/A_0 = \max_{\{\overline{a}, a^\star, \underline{\delta}\}} & \frac{\ell(a^\star)/a^\star}{c + b(\gamma)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}}, \\
\text{subject to} \quad \frac{\gamma}{1+\gamma}\frac{1}{\overline{a}} = & \frac{\ell(a^\star)/a^\star}{c + b(\gamma)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}}, \\
& \overline{a} \in [0, a^\star], \\
& \gamma \text{ is the lowest negative root of (37).}
\end{aligned} \tag{97}$$

where c and $b(\gamma)$ are the reduced-form parameters given in equation (58).

Optimal Policies

Building on our analysis in Section B, constraint (97) can be written $H(\bar{a}, a^*) = \frac{\gamma}{1+\gamma} > 1$ where H is defined in (**??**). We now call $\overline{a}_{opt}(a^*, \gamma)$ the lowest value of $\overline{a}$ if any that satisfies constraint (97). Following similar steps as in Section B, this value exists only if

$$\ell(a^\star) \geq \frac{\gamma}{1+\gamma}\frac{c + b\min\left\{1, \frac{c}{b(\gamma)\xi}\right\}}{\min\left\{1, \frac{c}{b(\gamma)\xi}\right\}^{\frac{1}{\xi+1}}}. \tag{98}$$

This condition is more restrictive than condition (69) which obtains under full commitment. To see this, observe first that $\gamma/(1+\gamma) > 1$ for all $\gamma < -1$. The second term on the right-hand-side of (98) is decreasing with γ because it is increasing with b and $b'(\gamma) < 0$. Hence, it is higher than the term on the right-hand-side of (69) because γ is equal to its maximum value under full commitment.

We showed in Section B that the variable $\underline{\delta}$ only affects γ and that γ strictly decreases with $\underline{\delta}$. Hence, we can write the problem as if the platform directly chose γ.

$$\begin{aligned} &\max_{\{a^\star,\gamma\}} \quad \frac{\gamma}{1+\gamma}\frac{1}{\overline{a}_{opt}(a^\star,\gamma)}, \\ \text{subject to} \quad &\overline{a}_{opt}(a^\star,\gamma) \in [0,a^\star]. \end{aligned}$$

In what follows, we derive optimality constraints with respect to $a^\star$ and γ, assuming the constraint does not bind. The optimality condition with respect to $a^\star$ writes

$$\begin{aligned} &0 = \frac{\partial \overline{a}_{opt}(a^\star,\gamma)}{\partial a^\star} \\ \Leftrightarrow \quad &0 = \ell'(a^\star)\overline{a}_{opt} - \frac{\gamma}{1+\gamma}\left[c - \xi b(\gamma)\left(\frac{\overline{a}}{a^\star}\right)^{\xi+1}\right] \end{aligned}$$

using the implicit characterization for $\overline{a}_{opt}(a^\star,\gamma)$ and the steps for the derivation from Section B. The optimality condition with respect to γ is

$$0 = \frac{1}{(1+\gamma)^2}\frac{1}{\overline{a}_{opt}(a^\star,\gamma)} - \frac{\gamma}{1+\gamma}\frac{\frac{\partial \overline{a}_{opt}(a^\star,\gamma)}{\partial\gamma}}{(\overline{a}_{opt}(a^\star,\gamma))^2}. \tag{99}$$

Applying the Implict Function Theorem to the equation $H(\overline{a},a^\star)$, we obtain

$$\begin{aligned} \frac{\partial \overline{a}_{opt}(a^\star,\gamma)}{\partial\gamma} &= \frac{b'(\gamma)\left(\frac{\overline{a}}{a^\star}\right)^{\xi+1}\frac{H(\overline{a},a^\star)}{c+b(\gamma)\left(\frac{\overline{a}}{a^\star}\right)^{\xi+1}} + \frac{1}{(1+\gamma)^2}}{\frac{H_x}{a^\star}} \\ &= \frac{\frac{b'(\gamma)\left(\frac{\overline{a}}{a^\star}\right)^{\xi+1}}{c+b(\gamma)\left(\frac{\overline{a}}{a^\star}\right)^{\xi+1}}\frac{\gamma}{1+\gamma} + \frac{1}{(1+\gamma)^2}}{\frac{H_x}{a^\star}} \end{aligned}$$

with H_x the derivative of H with respect to $x = \frac{\overline{a}}{a^\star}$.

The first term on the right-hand-side of (99) is strictly positive. If the derivative of $\overline{a}_{opt}(a^\star,\gamma)$ is negative, it is optimal to maximize γ subject to constraint (98) as in the full commitment case. If not, there exists a counteracting force implying that the highest possible γ and thus the lowest possible $\underline{\delta}$ are not optimal.

I Proof of ??

The condition for $\overline{a}$ is given by

$$-\frac{\gamma}{\overline{a}}\left(\frac{(e(a^\star)+1)\overline{a}}{a^\star}-1\right)=\frac{e(a^\star)+1}{a^\star}.$$

Thus,

$$\frac{\overline{a}}{a^\star}=\frac{\gamma}{1+\gamma}\frac{a^\star}{e(a^\star)+1}.$$

If $\lambda=0$,

$$\frac{\overline{a}}{a^\star}=\frac{\gamma}{1+\gamma}\frac{r-\mu}{\ell(a^\star)}.$$

Since

$$\lim_{\mu\to r}\gamma=-1,$$

we can use the Hospital's rule

$$\lim_{\mu\to r}\frac{\overline{a}}{a^\star}=\lim_{\mu\to r}\frac{1}{\gamma_\mu\ell(a^\star)}$$

where

$$\gamma_\mu=\frac{1-\frac{(\mu-\underline{\delta}-\sigma^2/2)}{\sqrt{(\mu-\underline{\delta}-\sigma^2/2)^2+2\sigma^2(r-\underline{\delta})}}}{\sigma^2}.$$

As

$$\gamma=\frac{\mu-\underline{\delta}-\sigma^2/2-\sqrt{(\mu-\underline{\delta}-\sigma^2/2)^2+2\sigma^2(r-\underline{\delta})}}{\sigma^2},$$

the implicit function theorem yields

$$\lim_{\mu\to r}\gamma_\mu = \frac{1-\frac{r-\underline{\delta}-\sigma^2/2}{r-\underline{\delta}+\sigma^2/2}}{\sigma^2} = \frac{1}{r-\underline{\delta}+\sigma^2/2}.$$

Thus,

$$\lim_{\mu\to r}\frac{\overline{a}}{a^\star} = \frac{r-\underline{\delta}+\sigma^2/2}{\ell(a^\star)}.$$

For an equilibrium, we need $\overline{a} \le a^\star$, which is satisfied if and only if $\ge r - \ell(a^\star) + \sigma^2/2$.

J Solution for Centralized Equity Price with Collateral

We assume that $\varphi \le 1$ so there is never residual value of collateral for equity holders after liquidation. We conjecture then verify that the equity value per unit of stablecoin is given by

$$e(a,k) = \begin{cases} 0 & \text{if } \ k < \varphi, \\ e(a) + k - k^\star(a) & \text{if } \ k \ge \varphi, \end{cases}$$

and

$$e(a) = \begin{cases} 0 & \text{if } \ a < \underline{a}, \\ c_0 + \sum_{k=1}^3 c_k a^{-\gamma_k} & \text{if } \ \underline{a} \le a < \overline{a}, \\ e(a^\star)a/a^\star + p(a^\star)(a/a^\star - 1) + k(a) - k^\star a/a^\star & \text{if } \ a \ge \overline{a}. \end{cases}$$

where γ_ks are roots of the characteristic equation

$$r + \lambda = -\mu\gamma + \frac{\sigma^2}{2}(1+\gamma)\gamma.$$

Smooth Region $a \in [\underline{a}, \overline{a}]$ Given the optimal collateral policy $k(a)$ and the result of Lemma 6, we can define $E(A,C) + K - K(A,C) \equiv E(A,C,K)$. In the smooth region, the

HJB is given by

$$(r+\lambda)E(A_t,C_t) = p(A_t,C_t)G_t - M_t + \mu A_t E_A(A_t,C_t) + \frac{\sigma^2}{2}A_t^2 E_{AA}(A_t,C_t) \\ + (G_t+\delta_t C_t)E_C(A_t,C_t) + \lambda\mathbb{E}[E(SA_t,C_t) + SK(A_t,C_t) - K(SA_t,C_t)].$$

where $\mathbb{E}[d\mathcal{M}_t|dN_t=0] = M_t dt$ and $\mathbb{E}[d\mathcal{M}_t|dN_t=1] = \mathbb{E}[(k(Sa_t)-Sk(a_t))C_{t^-}]$. Given the definition of $e(a)C \equiv E(A,C)$, we can rewrite

$$(r+\lambda)e(a) = \max_g\left\{gp(a) - m + \mu ae'(a) + (g+\delta)(e(a)-e'(a)a) + \frac{\sigma^2}{2}a^2e''(a) + \lambda\mathbb{E}[e(Sa)+Sk(a)-k(Sa)]\right\}.$$

Thus, m_t is such that

$$dK_t = C_t dk(A_t/C_t) + k(A_t/C_t)dG_t + k(A_t/C_t)C_t\delta_t dt.$$

Since

$$dK_t = \mu K_{t^-}dt + \sigma K_{t^-}dZ_t + d\mathcal{M}_t,$$

we get

$$M_t dt = k'(A_t/C_t)(\mu A_t dt - A_t(g_t+\delta)dt) + k''(A_t/C_t)\frac{\sigma^2}{2}A_t^2/C_t dt + k(A_t/C_t)(g_t+\delta_t)C_t dt - \mu k(A_t/C_t)C_t dt$$

and

$$m_t = \mu a_t k'(a_t) + \frac{\sigma^2}{2}a_t^2k''(a_t) + (g_t+\delta)(k(a_t)-k'(a_t)a_t) - \mu k(a_t).$$

Plugging it in the HJB yields

$$(r+\lambda)e(a) = \max_g\Big\{\mu k(a) + gp(a) + (g+\delta)(e(a)-e'(a)a-k(a)+k'(a)a) + \mu a(e'(a)-k'(a)) \\ + \frac{\sigma^2}{2}a^2(e''(a)-k''(a)) + \lambda\mathbb{E}[e(Sa)+Sk(a)-k(Sa)]\Big\}.$$

The first order condition for $g(a)$ becomes

$$p(a) = -e(a) + e'(a)a + k(a) - k'(a)a. \tag{100}$$

Thus, we get

$$(r+\lambda-\delta)e(a) = (\mu-\delta)k(a) + (\mu-\delta)a(e'(a)-k'(a)) + \frac{\sigma^2}{2}a^2(e''(a)-k''(a)) + \lambda\mathbb{E}[e(Sa)+Sk(a)-k(Sa)].$$

Assume $k(a)=\varphi$. We get

$$(r+\lambda-\delta)e(a) = (\mu-\delta)\varphi + (\mu-\delta)ae'(a) + \frac{\sigma^2}{2}a^2e''(a) + \lambda\mathbb{E}[\max(0,S\varphi-1)].$$

We can plug in the guess into the HJB and solve for the undetermined coefficients:

$$\begin{aligned}(r+\lambda-\delta)e(a) = (\mu-\delta)\varphi - (\mu-\delta)\sum_{k=1}^{3}\gamma_k c_k a^{-\gamma_k} \\ + \frac{\sigma^2}{2}\sum_{k=1}^{3}(1+\gamma_k)\gamma_k c_k a^{-\gamma_k} + \lambda\mathbb{E}[\max(0,S\varphi-1)]. \end{aligned} \tag{101}$$

Let us compute the expectation $\mathbb{E}[e(Sa)]$ assuming that $\varphi \geq 1$:

$$\begin{aligned}\mathbb{E}[\max(0,S\varphi-1)] &= \int_0^{\ln(\varphi)} \left\{(\varphi e^{-s}-1)\xi e^{-\xi s}\right\} ds \\ &= \frac{\xi\varphi}{\xi+1}\left(1-\varphi^{-(\xi+1)}\right) - \left(1-\varphi^{-\xi}\right) \\ &= \frac{\xi\varphi}{\xi+1} - 1 + \frac{\varphi^{-\xi}}{\xi+1}.\end{aligned}$$

For the previous equation to hold, constant terms must be such that

$$(r+\lambda-\delta)c_0 = (\mu-\delta)\varphi + \lambda\mathbb{E}[\max(0,S\varphi-1)].$$

Additionally, terms in $c_k a^{-\gamma_k}$ must be such that

$$r+\lambda-\delta = -(\mu-\delta)\gamma_k + \frac{\sigma^2}{2}(1+\gamma_k)\gamma_k.$$

Thus, γ_ks must be the roots of that characteristic equation. The first boundary condition is given by

$$c_0 + \sum_{k=1}^{2} c_k \underline{a}^{-\gamma_k} = 0. \tag{102}$$

The second boundary condition is given by

$$c_0 + \sum_{k=1}^{2} c_k \overline{a}^{-\gamma_k} = e(\overline{a}). \tag{103}$$

Thus, the two coefficients c_ks must satisfy conditions (102) and (103).

Target Region The value of equity at $a^\star$ is equal to:

$$\begin{aligned} E(a^\star C_{t^-}, C_{t^-}) = p(a^\star)\mathbb{E}\left[d\mathcal{G}_t\right] - \mathbb{E}\left[d\mathcal{M}_t\right] \\ + (1 - r dt - \lambda dt)\mathbb{E}\left[E(a^\star C_t, C_t) \middle| dN_t = 0\right] \\ + (1 - r dt)\lambda dt \mathbb{E}\left[E(a^\star C_t, C_t) \middle| dN_t = 1\right]. \end{aligned}$$

If no jumps occur in the interval $[t, t + dt]$ (i.e., $dN_t = 0$), then the equity holders issue/repurchase debt to compensate for all Brownian shocks and reissue maturing debt so that

$$da_t/a_{t^-} = dA_t/A_{t^-} - d\mathcal{G}_t/C_{t^-} - \delta^\star dt = 0.$$

Thus,

$$\mathbb{E}\left[d\mathcal{G}_t \middle| dN_t = 0\right] = \mathbb{E}\left[C_{t^-}(dA_t/A_{t^-} - \delta^\star dt)\right] = C_{t^-}(\mu - \delta^\star)dt.$$

Furthermore, they need to issue/repurchase collateral at market value such that

$$dk_t/k_{t^-} = dK_t/K_{t^-} - d\mathcal{G}_t/C_{t^-} - \delta^\star dt = 0.$$

Thus,

$$\mathbb{E}\left[d\mathcal{M}_t \middle| dN_t = 0\right] = \mathbb{E}\left[d\mathcal{G}_t K_{t^-}/C_{t^-} + \delta^\star K_{t^-} dt - \mu K_{t^-} dt\right] = 0.$$

The continuation value in this case is equal to

$$\mathbb{E}\left[E(a^\star C_t, C_t) \right| dN_t = 0] = \mathbb{E}\left[e(a^\star) C_t \right| dN_t = 0] = e(a^\star) C_{t^-}(1 + \mu dt).$$

If there is a Poisson jump $dA_t/A_{t^-} = S_t - 1$ so that $k^\star S_t > \varphi$, then the equity holders compensate this jump so that the state returns to $a^\star$. Thus, they repurchase $C_{t^-}(1 - S_t)$ units of debt and $d\mathcal{G}_t = C_{t^-}(S_t - 1)$. In that case,

$$\mathbb{E}[E(a^\star C_t, C_t)|dN_t = 1, S_t = \tilde{S}] = \mathbb{E}[e(a^\star)C_t|dN_t = 1, S_t = \tilde{S}_t] = e(a^\star)C_{t^-}\tilde{S}.$$

Therefore, we can write

$$\mathbb{E}\left[E(A_t, C_t) \right| dN_t = 1] = e(a^\star) C_{t^-} \int_0^{\ln(k^\star/\varphi)} e^{-s} \xi e^{-\xi s} ds.$$

Also,

$$\begin{aligned}
\mathbb{E}\left[d\mathcal{G}_t \right] &= \mu C_{t^-} dt + \lambda dt \int_0^{\ln(k^\star/\varphi)} (e^{-s} - 1) C_{t^-} \xi e^{-\xi s} ds \\
&= \mu C_{t^-} dt - \lambda C_{t^-} dt \left(\frac{\xi e^{-(\xi+1)s}}{\xi + 1} - e^{-\xi s} \right) \Bigg|_0^{\ln(k^\star/\varphi)} \\
&= \mu C_{t^-} dt + \lambda C_{t^-} dt \left(\frac{\xi}{\xi + 1} \left(1 - \left(\frac{k^\star}{\varphi} \right)^{-(\xi+1)} \right) - \left(1 - \left(\frac{k^\star}{\varphi} \right)^{-\xi} \right) \right).
\end{aligned}$$

In the case of a Poisson jump, the collateral value jumps by the same amount as the crypto asset. Thus, there is no need to adjust the value of the collateral as the debt level is also adjusted for. Therefore,

$$\mathbb{E}\left[d\mathcal{M}_t \right] = 0.$$

Regrouping all terms and scaling by C_{t^-}, we get

$$
\begin{aligned}
e(a^\star) \equiv e(a^\star) = p(a^\star)(\mu - \delta(a^\star))dt + p(a^\star)\lambda dt \int_0^{\ln(k^\star/\varphi)} (e^{-s} - 1)\xi e^{-\xi s} ds \\
+ (1 - rdt - \lambda dt)e(a^\star)(1 + \mu dt) \\
+ (1 - rdt)\lambda dt \left(e(a^\star) \int_0^{\ln(k^\star/\varphi)} e^{-s}\xi e^{-\xi s} ds \right).
\end{aligned}
$$

Removing terms in $dtdt$ and scaling by dt, we have

$$
(r + \lambda - \mu)e(a^\star) = p(a^\star)(\mu - \delta(a^\star)) + \lambda \int_0^{\ln(k^\star/\varphi)} (p(a^\star)(e^{-s} - 1) + e^{-s}e(a^\star))\xi e^{-\xi s} ds.
$$

Another useful way to write that equation is

$$
(r - \mu)e(a^\star) = p(a^\star)(\mu - \delta(a^\star)) + \lambda(\mathbb{E}[e(Sa^\star)] - e(a^\star)).
$$

Solving for the integral, we get

$$
(r + \lambda - \mu)e(a^\star) = p(a^\star)(\mu - \delta(a^\star)) + \frac{\lambda\xi}{\xi + 1} \left(1 - \left(\frac{k^\star}{\varphi} \right)^{-(\xi+1)} \right) (p(a^\star) + e(a^\star)) - \lambda p(a^\star) \left(1 - \left(\frac{k^\star}{\varphi} \right)^{-\xi} \right).
$$

Putting all terms in $e(a^\star)$ on the left hand side gives

$$
\left(r + \frac{\lambda}{\xi + 1} - \mu + \frac{\lambda\xi}{\xi + 1} \left(\frac{k^\star}{\varphi} \right)^{-(\xi+1)} \right) e(a^\star) = p(a^\star) \left(\mu - \delta(a^\star) - \frac{\lambda}{\xi + 1} - \frac{\lambda\xi}{\xi + 1} \left(\frac{k^\star}{\varphi} \right)^{-(\xi+1)} + \lambda \left(\frac{k^\star}{\varphi} \right)^{-\xi} \right).
$$

K Solution for Centralized Stablecoin Price with Collateral

We conjecture then verify that the price of a stablecoin is given by

$$
p(a) = \begin{cases} 0 & \text{if } \ a < \underline{a}, \\ b_0 + \sum_{k=1}^2 b_k (a/\underline{a})^{-\gamma_k} & \text{if } \ \underline{a} \le a < \overline{a}, \\ p(a^\star) & \text{if } \ a \ge \overline{a}. \end{cases}
$$

where γ_ks are roots of the characteristic equation

$$r+\lambda=-\mu\gamma+\frac{\sigma^2}{2}(1+\gamma)\gamma.$$

Smooth Region $a\in[\underline{a},\overline{a}]$ In the smooth region, the HJB is given by

$$(r+\lambda)p(a)=\ell(a)+\delta(a)p(a)-g(a)ap'(a)+\mu ap'(a)+\frac{\sigma^2}{2}a^2p''(a)+\lambda\mathbb{E}[p(Sa)].$$

To solve for $g(a)$, we need the first order derivative of $e(a)$

$$\begin{aligned}(r+\lambda)e'(a)&=\mu k'(a)-\delta p'(a)+\mu ae''(a)+\mu e'(a)+\frac{\sigma^2}{2}a^2(e'''(a)-k'''(a))+\sigma^2a(e''(a)-k''(a))\\&\quad+\lambda\mathbb{E}[e'(Sa)+Sk'(a)-k'(Sa)S]\end{aligned}$$

together with the first order condition for $g(a)$ from equation (**??**) and its derivatives:

$$\begin{aligned}p(a)&=-e(a)+e'(a)a+k(a)-k'(a)a,\\p'(a)&=e''(a)a-k''(a)a,\\p''(a)&=e'''(a)a+e''(a)-k'''(a)a-k''(a).\end{aligned}$$

Thus, we get

$$\begin{aligned}0&=(r+\lambda)(p(a)+e(a)-e'(a)a-k(a)+k'(a)a),\\&=\ell(a)+\delta p(a)-(g(a)+\delta)ap'(a)+\mu ap'(a)+\frac{\sigma^2}{2}a^2p''(a)+\lambda\mathbb{E}[p(Sa)]\\&\quad+\mu k(a)-\delta p(a)+\mu ae'(a)+\frac{\sigma^2}{2}a^2(e''(a)-k''(a))+\lambda\mathbb{E}[e(Sa)+Sk(a)-k(Sa)]\\&\quad-\mu k'(a)a+\delta p'(a)a-\mu a^2e''(a)-\mu e'(a)a-\frac{\sigma^2}{2}a^3(e'''(a)-k'''(a))-\sigma^2a^2(e''(a)-k''(a))\\&\quad-\lambda\mathbb{E}[e'(Sa)Sa+k'(a)Sa-k'(Sa)Sa]\\&\quad-(r+\lambda)(k(a)-k'(a)a)\\&=(\mu-r-\lambda)(k(a)-k'(a)a)+\ell(a)-g(a)ap'(a)+\lambda\mathbb{E}[S](k(a)-k'(a)a).\end{aligned}$$

That is,

$$g(a)=\frac{(\mu-r-\lambda+\lambda\mathbb{E}[S])(k(a)-k'(a)a)+\ell(a)}{ap'(a)}.$$

Then, the HJB for $p(a)$ becomes

$$(r+\lambda)p(a)=(r+\lambda-\mu-\lambda\mathbb{E}[S])(k(a)-k'(a)a)+\delta p(a)-\delta p'(a)+\mu ap'(a)+\frac{\sigma^2}{2}a^2p''(a)+\lambda\mathbb{E}[p(Sa)].$$

Note that

$$\mathbb{E}[S]=\int_0^\infty e^{-s}\xi e^{-\xi s}ds=\frac{\xi}{\xi+1}.$$

Assume $k(a)=\varphi$. Thus we get

$$(r+\lambda)p(a)=(r+\lambda-\mu-\lambda\mathbb{E}[S])\varphi+\delta p(a)-\delta p'(a)+\mu ap'(a)+\frac{\sigma^2}{2}a^2p''(a)+\lambda\mathbb{E}[S\varphi].$$

We can plug in the guess into the HJB and solve for the undetermined coefficients:

$$\begin{aligned}(r+\lambda-\delta)p(a)=(r+\lambda-\mu-\lambda\mathbb{E}[S])\varphi-(\mu-\delta)\sum_{k=1}^{2}\gamma_k b_k a^{-\gamma_k}\\+\frac{\sigma^2}{2}\sum_{k=1}^{2}(1+\gamma_k)\gamma_k b_k a^{-\gamma_k}+\lambda\mathbb{E}[p(Sa)].\end{aligned}\tag{104}$$

For the previous equation to hold, the constant must be such that

$$(r+\lambda-\delta)b_0=(r+\lambda-\mu)\varphi.$$

Furthermore, terms in $b_k a^{-\gamma_k}$ must be such that

$$r+\lambda-\delta=-(\mu-\delta)\gamma_k+\frac{\sigma^2}{2}(1+\gamma_k)\gamma_k.$$

The first boundary condition is given by

$$b_0+\sum_{k=1}^{3}b_k\underline{a}^{-\gamma_k}=\varphi.\tag{105}$$

The second boundary condition is given by

$$b_0 + \sum_{k=1}^{3} b_k \overline{a}^{-\gamma_k} = p(a^\star). \tag{106}$$

Thus, the two coefficients b_ks must satisfy conditions (105) and (106).

Target Region $a \in [\overline{a}, \infty)]$ At the target demand ratio $a^\star$, the HJB for the price of a stablecoin is given by

$$\begin{aligned} p(a^\star) \equiv p(a^\star) &= \ell(a^\star)dt + \delta(a^\star)p(a^\star)dt + (1 - rdt - \lambda dt)\mathbb{E}\left[p(a^\star)\right| dN_t = 0] \\ &+ (1 - rdt)\lambda dt \mathbb{E}\left[p(a^\star C_t/C_{t\text{-}})\right| dN_t = 1]. \end{aligned}$$

If no jumps occur in the interval $[t, t + dt]$ (i.e., $dN_t = 0$), then the equity holders issue/repurchase debt to compensate for all Brownian shocks and reissue maturing debt so that

$$da_t = dA_t/A_t - d\mathcal{G}_t/C_t - \delta^\star dt = 0.$$

The continuation value in this case is equal to

$$\mathbb{E}\left[p(a^\star)\right| dN_t = 0] = p(a^\star).$$

If there is a Poisson jump $dA_t/A_{t\text{-}} = S_t - 1$, then there is immediate default the stablecoin holders get the collateral. Therefore, we can write

$$\mathbb{E}\left[p(a^\star)\right| dN_t = 1] = \int_0^\infty \varphi\xi e^{-\xi s} ds = \frac{\varphi\xi}{\xi + 1}.$$

Regrouping all terms, we get

$$p(a^\star) = \ell(a^\star)dt + \delta^\star p(a^\star)dt + (1 - rdt - \lambda dt)p(a^\star) + (1 - rdt)\lambda dt \frac{\varphi\xi}{\xi + 1}.$$

Removing terms in $dtdt$ and scaling by dt, we have

$$(r + \lambda - \delta^\star)p(a^\star) = \ell(a^\star) + \lambda \frac{\varphi\xi}{\xi + 1}.$$

L Optimal $a^\star$ for Centralized Platform with Collateral

We have to find $a^\star$ defined as

$$a^\star = \arg\max_{a^\star} \left\{ \frac{e(a^\star; a^\star) + p(a^\star; a^\star)}{a^\star} \right\} = \arg\max_{a^\star} \left\{ \frac{\ell(a^\star) + \lambda \mathbb{E}[e(Sa^\star; a^\star) + p(Sa^\star; a^\star)]}{a^\star} \right\}$$

where we write $e(a) = e(a; a^\star)$ and $p(a) = p(a; a^\star)$ to be explicit about the fact that $a^\star$ also enters as a parameter in the price functions. Note that because $\overline{a}$ is implicitly defined by

$$-\sum_{1}^{2} \gamma_k c_k \overline{a}^{-\gamma_k - 1} = \frac{e(a^\star) + p(a^\star)}{a^\star}$$

we get that

$$\frac{\partial \overline{a}}{\partial a^\star} \approx \frac{\partial}{\partial a^\star} \frac{e(a^\star) + p(a^\star)}{a^\star} = 0.$$

That is, $\overline{a}$ is already maximizing the value of $(e(a^\star) + p(a^\star))/a^\star$. Assume that $k(a)/\varphi \leq a^\star/\overline{a}$. The expectation is given by

$$\mathbb{E}[e(Sa^\star; a^\star) + p(Sa^\star; a^\star)] = \frac{\xi}{\xi + 1} \left(1 - \left(\frac{k(a^\star)}{\varphi} \right)^{-(\xi+1)} \right) (e(a^\star) + p(a^\star)) + \left(\frac{k(a^\star)}{\varphi} \right)^{-(\xi+1)} \frac{\xi k(a^\star)}{\xi + 1}.$$

The partial derivative of that expectation is given by

$$\frac{\partial \mathbb{E}[e(Sa^\star; a^\star) + p(Sa^\star; a^\star)]}{\partial a^\star} \approx \frac{\partial k(a^\star)}{\partial a^\star} \approx \frac{\partial}{\partial a^\star} \frac{e(a^\star) + p(a^\star)}{a^\star} = 0.$$

Thus, the first order condition for $a^\star$ is given by

$$\ell'(a^\star) a^\star = \ell(a^\star) + \lambda \mathbb{E}[e(Sa^\star; a^\star) + p(Sa^\star; a^\star)].$$

M Proof of Lemma 6

Given any collateral level K, governance token owners have the option to adjust collateral to K' by buying $K - K'$ at cost of $K - K'$. Therefore, the value of the governance token given K must be at least as high as the value that governance token owners would obtain by changing the collateral level to K'

$$E(A, K, C) \geq E(A, K', C) + K - K'.$$

Thus, 1 is the constant subgradient of $E(A, K, C)$ and $E(A, K, C)$ is linear and increasing in K.

N No-Deviation in Target Region with Collateral

Consider (A, C, K) such that the platform is in the target region. Let $\tilde{E}(A, C, K)$ be the value of deviating to a smooth issuanced $\Delta C dt$ before reverting back to the conjectured equilibrium policy. For simplicity we denote $K_t^\star$ for $K^\star(A_t, C_t)$ as an argument of the value function. Both along the equilibrium path and in the deviation, it is assumed that the optimal collateral policy is $K^\star(A, C) = \varphi C$. Let $p = p(a^\star)$ be the price in the target region.

The equity value in the deviation is

$$\begin{aligned}
\tilde{E}(A_t, C_t, K_t^\star) &= p\Delta C_t dt - dM_t + (1 - rdt)\mathbb{E}[E(A_{t+dt}, C_{t+dt}, K_{t+dt}^\star)] \\
&= (p - \varphi)\Delta C dt + (1 - rdt)\Bigg\{ E(A_t, C_t, K_t^\star) + \mu A E_A dt + \Delta C_t \underbrace{E_C}_{-p} dt + \underbrace{E_K}_{1} \left(\mu^k \varphi C_t dt + \varphi \Delta C dt\right) \\
&\quad + \lambda dt \mathbb{E}\Big[E(SA_t, C_t, K_t^\star) + K^\star(A_t, C_t)(1 + \rho(S - 1)) - K^\star(SA_t, C_t) - E(A_t, C_t, K_t^\star)\Big] \Bigg\}
\end{aligned}$$

In the region where equity is flat, we have

$$\begin{aligned}
E(A_t, C_t, K_t) &= E(A_t, C^\star, K_t^\star) + p(C^\star(A) - C_t) - (K^\star(A_t, C_t) - K_t) \\
&= A_t \frac{e(a^\star)}{a^\star} + p\frac{A_t}{a^\star} - pC_t - \varphi\frac{A_t}{a^\star} + K_t
\end{aligned}$$

We have

$$AE_A = C^\star(A)\left[e(a^\star) + p(a^\star)\right] - \varphi C^\star(A)$$

Substituting for AE_A, the deviation is not profitable, that is, $\tilde{E}(A_t, C_t, K_t^\star) \leq E(A_t, C_t, K_t^\star)$

if and only if

$$(r+\lambda)E(A_t, C_t, K_t^\star) \geq \mu(e(a^\star)+p(a^\star))C^\star(A)dt+(\mu^k-\mu)\varphi C_t dt+\lambda\mathbb{E}\Big[E(SA_t, C_t, K_t^\star)+\rho(S-1)\varphi C_t\Big]$$

We now replace the LHS of the inequality above. The conjectured equilibrium is to jump to $C^\star(A_t)$. Hence we have

$$\begin{aligned} E(A_t, C_t, K_t^\star) &= E(A_t, C^\star(A_t), K_t^\star) + p(C^\star(A_t) - C_t) + K^\star(A_t, C_t) - K^\star(A_t, C^\star(A_t)) \\ &= E(A_t, C^\star(A_t), K_t^\star) + (p - \varphi)(C^\star(A_t) - C_t) \end{aligned}$$

To substitute for the value of equity at the target $E(A_t, C^\star(A_t), K_t^\star)$, we use the derivations from the analysis without collateral. Adapting equation (54), we have

$$(r+\lambda)E(A_t, C^\star(A_t), K_t^\star) = \mu(e(a^\star)+p(a^\star))C^\star(A)+(\mu^k-\mu)\varphi C^\star(A_t)+\lambda\mathbb{E}[E(SA, C^\star(A_t), K_t)+\rho(S-1)\varphi C^\star(A_t)]$$

Replacing for $(r + \lambda)E(A_t, C_t, K_t^\star)$, we get

$$\begin{aligned} &\mu(e(a^\star) + p(a^\star))C^\star(A)dt + (\mu^k - \mu)\varphi C^\star(A_t)dt + \lambda\mathbb{E}\Big[E(SA, C^\star(A_t), K_t) + \rho(S - 1)\varphi C^\star(A_t\Big]) \\ &+ (r + \lambda)(p - \varphi)(C^\star(A_t) - C_t) \geq \mu(e(a^\star) + p(a^\star))C^\star(A)dt + (\mu^k - \mu)\varphi C_t dt + \lambda\mathbb{E}\Big[E(SA_t, C_t, K_t^\star) + \rho(S - 1)\varphi \end{aligned}$$

Hence, overall the no-deviation condition becomes

$$\Big[(r+\lambda)(p-\varphi)+(\mu^k-\mu)\varphi\Big](C-C^*(A)) \leq \lambda\mathbb{E}\Big[E[SA, C^\star(A), K^\star]-E[SA, C, K^\star]\Big]+\lambda\rho\varphi\mathbb{E}[1-S](C-C^\star(A))$$

We can rewrite this expression as follows

$$\Big[(r+\lambda)(p-\varphi)+(\mu^k-\mu)\varphi-\lambda\rho\varphi\mathbb{E}[1-S]\Big](C-C^*(A)) \leq \lambda\mathbb{E}\Big[E[SA, C^\star(A), K^\star]-E[SA, C, K^\star]\Big]$$

When $\varphi = 0$, we obtain the same expression as in the uncollateralized case for $\delta = 0$. To get some intuition about the condition suppose $C^\star(A) < C$ so that the equilibrum policy is to repurchase $C - C^\star(A)$ units of stablecoins. The LHS is the sum of three terms. The first one is the net cost of the repurchase proportional to $p-\varphi$. A repurchase frees up collateral. Hence, the net cost if equal to $p - \varphi$. The second term corresponds to the net benefit from owning collateral. As collateral value grows at rate μ^k but stablecoin issuance only grows at rate μ together with stablecoin demand, there is a net windfall $(\mu^k - \mu)\varphi$ from each unit of stablecoin. If $\mu^k - \mu > 0$, this force pushes against repurchasing stablecoins. The last term on the right-hand-side is negative. It says that when holding more stablecoins and thus more collateral, the effect of a Poisson shock is more severe.

O Proof of ??

Let us derive the value $\Delta E(A_t, C_t, \Delta K_t)$ of staying at a higher level of collateral $K_t + \Delta K_t$ instead of jumping directly to $K^\star(A_t, C_t)$ when the issuance policy $d\mathcal{G}_t$ is smooth:

$$\Delta E(A_t, C_t, K_t + \Delta K_t) = p_t G_t dt + (1 - rdt)\mathbb{E}_t\left[E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt})\right] - E(A_t, C_t, K_t).$$

Using Ito's lemma, we get

$$\begin{aligned}
\mathbb{E}_t\left[E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt})\right] &= E(A_t, C_t, K_t + \Delta K_t) + \mu A_t E_A(A_t, C_t, K_t + \Delta K_t)dt \\
&+ G_t E_C(A_t, C_t, K_t + \Delta K_t)dt \\
&+ \mu(K_t + \Delta K_t)E_K(A_t, C_t, K_t + \Delta K_t)dt \\
&+ \frac{\sigma^2}{2}E_{AA}(A_t, C_t, K_t + \Delta K_t)dt + \frac{\sigma^2}{2}E_{KK}(A_t, C_t, K_t + \Delta K_t)dt \\
&+ \lambda dt(\mathbb{E}_t[E(SA_t, C_t, S(K_t + \Delta K_t))] - E(A_t, C_t, K_t + \Delta K_t)).
\end{aligned}$$

Following Lemma 6, we have that

$$E(A_t, C_t, K_t + \Delta K_t) = E(A_t, C_t, K_t) + \Delta K.$$

Thus,

$$\begin{aligned}
E_A(A_t, C_t, K_t + \Delta K_t) &= E_A(A_t, C_t, K_t), \\
E_{AA}(A_t, C_t, K_t + \Delta K_t) &= E_{AA}(A_t, C_t, K_t), \\
E_C(A_t, C_t, K_t + \Delta K_t) &= E_C(A_t, C_t, K_t), \\
E_K(A_t, C_t, K_t + \Delta K_t) &= 1, \\
E_{KK}(A_t, C_t, K_t + \Delta K_t) &= 0,
\end{aligned}$$

and

$$\mathbb{E}_t[E(SA_t, C_t, S(K_t + \Delta K_t))] = \int_0^{\ln((K_t+\Delta K_t)/(\varphi C_t))} E(e^{-s}A_t, C_t, e^{-s}(K_t + \Delta K_t))\xi e^{-\xi s} ds.$$

Thus,

$$\begin{aligned}\mathbb{E}_t[E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt})] &= \mathbb{E}_t\left[E(A_{t+dt}, C_{t+dt}, C_{t+dt})\right] + \Delta K_t + \mu\Delta K_t dt \\ &+ \lambda dt(\mathbb{E}_t[E(SA_t, C_t, S(K_t + \Delta K_t))] - \mathbb{E}_t[E(SA_t, C_t, SK_t)] - \Delta K_t).\end{aligned}$$

Putting all of these together, we get

$$\begin{aligned}\Delta E(A_t, C_t, K_t + \Delta K_t) &= (1 - rdt)(\Delta K_t + \mu\Delta K_t dt) \\ &+ (1 - rdt)\lambda dt(\mathbb{E}_t[E(SA_t, C_t, S(K_t + \Delta K_t))] - \mathbb{E}_t[E(SA_t, C_t, SK_t)] - \Delta K_t)) - \Delta K_t \\ &= -(r + \lambda - \mu)dt\Delta K_t + \lambda dt(\mathbb{E}_t[E(SA_t, C_t, S\Delta K_t)] - \mathbb{E}_t[E(SA_t, C_t, SK_t)]).\end{aligned}$$

Furthermore,

$$\begin{aligned}&\lim_{\Delta K \to 0} \frac{\mathbb{E}_t[E(SA_t, C_t, S(K_t + \Delta K_t))] - \mathbb{E}_t[E(SA_t, C_t, SK_t)]}{\Delta K_t} \\ &= \frac{E(A_t\varphi C_t/K_t, C_t, \varphi C_t)\xi}{K_t}\left(\frac{K_t}{\rho C_t}\right)^{-\xi} + \frac{\xi}{\xi+1}\left(1 - \left(\frac{K_t}{\varphi C_t}\right)^{-(\xi+1)}\right) \\ &= \frac{(E(A_t\varphi C_t/K_t, C_t) + \varphi C_t - K(A_t\varphi C_t/K_t, C_t))\xi}{K_t}\left(\frac{K_t}{\rho C_t}\right)^{-\xi} + \frac{\xi}{\xi+1}\left(1 - \left(\frac{K_t}{\varphi C_t}\right)^{-(\xi+1)}\right) \\ &= \frac{(e(a_t\varphi/k_t) + \varphi - k(a_t\varphi/k_t))\xi}{k_t}\left(\frac{k_t}{\varphi}\right)^{-\xi} + \frac{\xi}{\xi+1}\left(1 - \left(\frac{k_t}{\varphi}\right)^{-(\xi+1)}\right).\end{aligned}$$

Thus, no infinitesimal deviation from the level $k(a)$ is optimal if and only if

$$\lambda\frac{(e(a\varphi/k(a)) + \varphi - k(a\varphi/k(a)))\xi}{k(a)}\left(\frac{k(a)}{\varphi}\right)^{-\xi} + \frac{\lambda\xi}{\xi+1}\left(1 - \left(\frac{k(a)}{\varphi}\right)^{-(\xi+1)}\right) = r + \lambda - \mu.$$

We can derive $k'(a)$ as

$$\begin{aligned}
k'(a) = -\frac{F_a}{F_k} &= -\frac{\frac{e'(a\varphi/k)-k'(a\varphi/k)}{k}\left(\frac{k}{\varphi}\right)^{-(\xi+1)}}{-\frac{e'(a\varphi/k)-k'(a\varphi/k)}{k}\frac{a}{k}\left(\frac{k}{\varphi}\right)^{-(\xi+1)}+\frac{1}{k}\left(\frac{k}{\varphi}\right)^{-(\xi+1)}-\frac{e(a\varphi/k)+\varphi-k(a\varphi/k)}{k^2}\left(\frac{k}{\varphi}\right)^{-\xi}} \\
&= -\frac{e'(a\varphi/k)-k'(a\varphi/k)}{-(e'(a\varphi/k)-k'(a\varphi/k))\frac{a}{k}+1-(e(a\varphi/k)+\varphi-k(a\varphi/k))/\varphi} \\
&= \frac{e'(a\varphi/k)-k'(a\varphi/k)}{(e'(a\varphi/k)-k'(a\varphi/k))\frac{a}{k}+(e(a\varphi/k)-k(a\varphi/k))/\varphi}
\end{aligned}$$

Assume that the optimal policy is at the lower bound. That is, $k(a) = \varphi$. The left-hand side (marginal benefit) becomes

$$\lambda\frac{e(a)\xi}{\varphi}.$$

Since $e(\underline{a}) = 0$, it is always optimal to be at the lower bound at $a = \underline{a}$.

Assume that the optimal policy is at the upper bound. That is, $k(a) = \varphi a/\underline{a}$. The left-hand side becomes

$$\frac{\lambda\xi}{\xi+1}\left(1-\left(\frac{a}{\underline{a}}\right)^{-(\xi+1)}\right) = \lambda\mathbb{E}[S\mathbb{1}\{S \geq \underline{a}/a\}],$$

where $\mathbb{E}[S\mathbb{1}\{S \geq \underline{a}/a\}]$ is the expected residual value of the collateral after a Poisson shock. If the probability of hitting the default boundary is close to zero, that is,

$$\left(\frac{a}{\underline{a}}\right)^{-\xi} \approx 0,$$

then it is optimal to deviate from the upper bound to a lower collateral level as $r > \mu - \frac{\lambda}{\xi+1}$. As $\lambda\mathbb{E}[S\mathbb{1}\{S \geq \underline{a}/a\}] \leq \frac{\lambda\xi}{\xi+1}$, it is never optimal to be at the upper bound.

When the issuance policy $d\mathcal{G}_t$ is not smooth, the continuation value conditional on no Poisson shock is equal to

$$\begin{aligned}
\mathbb{E}_t\left[E(A_{t+dt}, C_{t+dt}, K_{t+dt}+\Delta K_{t+dt})\middle|\, d\mathcal{N}_t = 0\right] &= \mathbb{E}_t\left[E(A_{t+dt}, C_{t+dt}, K_{t+dt})+\Delta K_{t+dt}\middle|\, d\mathcal{N}_t = 0\right] \\
&= \mathbb{E}_t\left[E(A_{t+dt}, C_{t+dt}, K_{t+dt})\middle|\, d\mathcal{N}_t = 0\right] + \Delta K_t(1+\mu dt).
\end{aligned}$$

If there is a Poisson shock, we get

$$\mathbb{E}_t \left[E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt}) \middle| \, d\mathcal{N}_t = 1 \right] = \mathbb{E}_t \left[E(SA_t, C_t, S(K_t + \Delta K_t)) \right].$$

Thus,

$$\begin{aligned}\mathbb{E}_t \left[E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt}) \right] = \mathbb{E}_t \left[E(A_{t+dt}, C_{t+dt}, K_{t+dt}) \right] + \Delta K_t (1 + \mu dt) \\ + \lambda dt (\mathbb{E}_t [E(SA_t, C_t, S(K_t + \Delta K_t))] - \mathbb{E}_t [E(SA_t, C_t, SK_t)] - \Delta K_t).\end{aligned}$$

Therefore, the condition is exactly the same as when the issuance policy $d\mathcal{G}_t$ is smooth.

P Proof of ??

The condition for $\overline{a}$ is given by

$$-\frac{\gamma}{\overline{a}} \left(\frac{(e(a^\star) + 1 - \varphi)\overline{a}}{a^\star} - (1 - \varphi) - \phi\varphi \right) = \frac{e(a^\star) + 1 - \varphi}{a^\star}.$$

Thus,

$$\frac{\overline{a}}{a^\star} = \frac{\gamma}{1 + \gamma} \frac{1 - (1 - \phi)\varphi}{e(a^\star) + 1 - \varphi}.$$

If $\lambda = 0$,

$$\frac{\overline{a}}{a^\star} = \frac{\gamma(r - \mu)}{1 + \gamma} \frac{1 - \left(1 - \frac{\mu -}{r -}\right) \varphi}{\ell(a^\star) - (r - \mu)\varphi}.$$

$$-\frac{\gamma(r - \mu)}{1 + \gamma} \frac{\frac{r - \mu}{r -}\varphi}{\ell(a^\star) - (r - \mu)\varphi}$$

Since

$$\lim_{\mu \to r} \gamma = -1,$$

we can use the Hospital's rule

$$\lim_{\mu \to r} \frac{\overline{a}}{a^\star} = \lim_{\mu \to r} \frac{1}{\gamma_\mu \ell(a^\star)}$$

and we get the same result as in Appendix I.

Q Proof of Lemma 5

Given any debt level $\widetilde{C}$, vault owners have the option to adjust debt to $\widetilde{C}'$ by buying $\widetilde{C} - \widetilde{C}'$ at cost of $p(A, C)$. Therefore, the value of the vault given $\widetilde{C}$ must be at least as high as the value that vault owners would obtain by changing the debt level to $\widetilde{C}'$

$$E(A, C, \widetilde{C}) \geq E(A, C, \widetilde{C}') + (p(A, C) - \varphi)(\widetilde{C}' - \widetilde{C}).$$

Thus, $p(A, C) - \varphi$ is the constant subgradient of $E(A, C, \widetilde{C})$ and $E(A, C, \widetilde{C})$ is linear and decreasing in $\widetilde{C}$.

R Proof of Lemma 8

S Proof of Proposition 6

In this section, we solve for the value of the vault given that $K^i(A_t, C_t, C^i_t) = \varphi C^i_t$. The value of a vault at $a^\star$ is equal to:

$$\begin{aligned} V(a^\star C_{t\text{-}}, C_{t\text{-}}, C^i_{t\text{-}}) = {} & p(a^\star)\mathbb{E}\left[d\mathcal{G}^i_t\right] - \mathbb{E}\left[d\mathcal{M}^i_t\right] \\ & + (1 - rdt - \lambda dt)\mathbb{E}\left[V(a^\star C_{t+dt}, C_{t+dt}, C^i_{t+dt})\middle| dN_t = 0\right] \\ & + (1 - rdt)\lambda dt \mathbb{E}\left[V(a^\star C_{t+dt}, C_{t+dt}, C^i_{t+dt})\middle| dN_t = 1\right]. \end{aligned}$$

If no jumps occur in the interval $[t, t + dt]$ (i.e., $dN_t = 0$), then the vault owners issue/repurchase collateral at market value such that

$$dk^i_t / k^i_{t\text{-}} = dK^i_t / K^i_{t\text{-}} - d\mathcal{G}^i_t / C^i_{t\text{-}} - s(a_t) = 0.$$

Thus,

$$\mathbb{E}\left[d\mathcal{M}_t | \, dN_t = 0\right] = d\mathcal{G}_t^i K_{t\text{-}}^i / C_{t\text{-}}^i + s(a_t) K_{t\text{-}}^i - \mu K_{t\text{-}}^i dt.$$

The continuation value in this case is equal to

$$\mathbb{E}\left[V(a^\star C_t, C_t, C_t^i) \big| \, dN_t = 0\right] = \mathbb{E}\left[v(a^\star) C_t^i \big| \, dN_t = 0\right] = v(a^\star)(C_{t\text{-}}^i + s(a^\star) C_{t\text{-}}^i dt + d\mathcal{G}_t^i).$$

If there is a Poisson jump, the vault is liquidated. In that case,

$$\mathbb{E}[V(a^\star C_t, C_t, C_t^i) | dN_t = 1] = \mathbb{E}[\max(0, S\varphi - 1) C_{t\text{-}}^i].$$

Regrouping all terms and scaling by $C_{t\text{-}}^i$, we get

$$\begin{aligned} v(a^\star) = {} & p(a^\star) d\mathcal{G}_t^i / C_{t\text{-}}^i - \varphi d\mathcal{G}_t^i / C_{t\text{-}}^i - s(a_t)\varphi dt + \mu\varphi dt \\ & + (1 - r dt - \lambda dt) v(a^\star)(1 + s(a^\star) dt + d\mathcal{G}_t^i / C_{t\text{-}}^i) + (1 - r dt)\lambda dt \mathbb{E}[\max\{0, S\varphi - 1\}]. \end{aligned}$$

Removing terms in $dtdt$ and scaling by dt, we have

$$(r + \lambda - s(a^\star)) v(a^\star) = (p(a^\star) - \varphi + v(a^\star)) d\mathcal{G}_t^i / C_{t\text{-}}^i + \varphi(\mu - s(a^\star)) + \lambda \mathbb{E}[\max(0, S\varphi - 1)].$$

Given that $v(a^\star) = \varphi - p(a^\star)$, we get

$$(r + \lambda)(\varphi - p(a^\star)) = \mu\varphi - s(a^\star) p(a^\star) + \lambda \mathbb{E}[\max\{0, S\varphi - 1\}].$$

Thus, with $p(a^\star) = 1$, we need

$$s(a^\star) = \mu\varphi - (r + \lambda)(\varphi - 1) + \lambda \mathbb{E}[\max(0, S\varphi - 1)]$$

Similarly, the value of an equity token at $a^\star$ is equal to:

$$\begin{aligned} E(a^\star C_{t\text{-}}, C_{t\text{-}}) = {} & (s(a^\star) - \delta(a^\star)) p(a^\star) C_{t\text{-}} dt \\ & + (1 - r dt - \lambda dt) \mathbb{E}\left[E(a^\star C_t, C_t) | \, dN_t = 0\right] \\ & + (1 - r dt)\lambda dt \mathbb{E}\left[\max\{0, E(a^\star C_t, C_t) + \min\{S\varphi - 1, 0\} C_t\} | \, dN_t = 1\right]. \end{aligned}$$

If no jumps occur in the interval $[t, t + dt]$ (i.e., $dN_t = 0$), then the vault owners issue/repurchase stablecoins such that

$$da_t/a_t = dA_t/A_t - d\mathcal{G}_t/C_{t^-} - s(a_t) = 0.$$

Thus,

$$\mathbb{E}\left[d\mathcal{G}_t \middle| dN_t = 0\right] = (\mu - s(a_t))C_{t^-}dt.$$

The continuation value in this case is equal to

$$\mathbb{E}\left[E(a^\star C_t, C_t) \middle| dN_t = 0\right] = \mathbb{E}\left[e(a^\star)C_t \middle| dN_t = 0\right] = e(a^\star)(1 + \mu dt)C_{t^-}.$$

If there is a Poisson jump, all vaults are liquidated before new ones are reopen and the equity token owners need to pay for the losses. In that case, equity after losses is equal to 0 when

$$s = \log(\varphi/(1 - e(a^\star)).$$

Thus, if $e(a^\star) < 1$,

$$\begin{aligned}
&\mathbb{E}\left[\max\{0, E(a^\star C_t, C_t) + \min\{S\varphi - 1, 0\}C_t\} \middle| dN_t = 1\right] \\
&= \int_0^{\log(\varphi/(1-e(a^\star))} \left(e(a^\star) + \min\{e^{-s}\varphi - 1, 0\}\right) C_{t^-}\xi e^{-\xi s} ds \\
&= \int_0^{\log(\varphi)} e(a^\star)C_{t^-}\xi e^{-\xi s} ds + \int_{\log(\varphi)}^{\log(\varphi/(1-e(a^\star))} \left(e(a^\star) + e^{-s}\varphi - 1\right) C_{t^-}\xi e^{-s\xi} ds \\
&= e(a^\star)C_{t^-}\left(1 - \varphi^{-\xi}\right) + (e(a^\star) - 1)C_{t^-}\left(\varphi^{-\xi} - \left(\frac{\varphi}{1 - e(a^\star)}\right)^{-\xi}\right) + \frac{\varphi\xi}{\xi + 1}C_{t^-}\left(\varphi^{-(\xi+1)} - \left(\frac{\varphi}{1 - e(a^\star)}\right)^{-(\xi+1)}\right)
\end{aligned}$$

Otherwise, if $e(a^\star) \geq 1$,

$$\mathbb{E}\left[\max\{0, E(a^\star C_t, C_t) + \min\{S\varphi - 1, 0\}C_t\} \middle| dN_t = 1\right] = E(a^\star C_t, C_t) + \mathbb{E}\left[\min\{S\varphi - 1, 0\}\right] C_t.$$

Assuming that $e(a^\star) \geq 1$, and regrouping all terms and scaling by $C^i_{t^-}$, we get

$$e(a^\star) = (s(a^\star) - \delta(a^\star))p(a^\star)dt + (1 - rdt - \lambda dt)e(a^\star)(1 + \mu dt) + (1 - rdt)\lambda dt \left(e(a^\star) + \mathbb{E}\left[\min\{S\varphi - 1, 0\}\right]\right).$$

Removing terms in $dtdt$ and scaling by dt, we have

$$(r-\mu)e(a^\star) = (s(a^\star)-\delta(a^\star))p(a^\star) + \lambda\mathbb{E}\left[\min\{S\varphi-1,0\}\right].$$

Since $p(a^\star)=1$ and

$$s(a^\star) = \mu\varphi - (r+\lambda)(\varphi-1) + \lambda\mathbb{E}[\max\{0, S\varphi-1\}],$$

we get

$$(r-\mu)e(a^\star) = \mu\varphi - \delta(a^\star) - (r+\lambda)(\varphi-1) + \lambda\left(\frac{\xi\varphi}{\xi+1}-1\right).$$

Furthermore, as $\delta(a^\star) = r - \ell(a^\star)$, we get

$$\begin{aligned}(r-\mu)e(a^\star) &= \ell(a^\star) - (r+\lambda-\mu)\varphi + \lambda\frac{\xi\varphi}{\xi+1}\\ &= \ell(a^\star) - \left(r+\frac{\lambda}{\xi+1}-\mu\right)\varphi.\end{aligned}$$

T No Loss of Generality for Policies without Brownian Component

In this section, we show that considering a policy function $d\mathcal{G}_t = g_tC_tdt$ instead of a more general functional form $d\mathcal{G}_t = g_tC_tdt + \kappa_tC_tdZ_t$ is without loss of generality. We proof the case for the centralized uncollateralized protocol in the smooth region but the proof can be adapted to any case. The intuition of the results is straightforward: If fighting brownian shocks with κ_t has any expected impact on the value of equity, it will also be taken into account in the smooth issuance decision g_t and cancel out. With a stochastic term in $d\mathcal{G}_t$ we can write the value of equity in the smooth region as

$$\begin{aligned}E(A_t,C_t) &= \mathbb{E}[p(A_t+dA_t, C_t+d\mathcal{G}_t)d\mathcal{G}_t]\\ &\quad + (1-rdt-\lambda dt)\mathbb{E}[E(A_t+dA_t, C_t+d\mathcal{G}_t)] + (1-rdt)\lambda dt\mathbb{E}[E(SA_t,C_t)].\end{aligned}$$

Using Ito's lemma and the fact that terms in $dtdt$ converge to 0 faster than terms in dt, we can get

$$\mathbb{E}[p(A_t + dA_t, C_t + d\mathcal{G}_t)d\mathcal{G}_t] = \mathbb{E}\left[p(A_t, C_t)g_t C_t dt + \sigma A p_A(A_t, C_t)\kappa_t C_t dt + \kappa_t^2 C_t^2 p_C(A_t, C_t)dt\right]$$

and

$$\begin{aligned}\mathbb{E}[E(A_t + dA_t, C_t + d\mathcal{G}_t)] = \mathbb{E}[&E(A_t, C_t) + \mu A E_A(A_t, C_t)dt + g_t C_t E_C(A_t, C_t)dt \\ &+ \frac{\sigma^2}{2} A_t^2 E_{AA}(A_t, C_t)dt + \frac{\kappa_t^2}{2} C_t^2 E_{CC}(A_t, C_t)dt + \sigma A_t \kappa_t C_t E_{AC}(A_t, C_t)dt]\end{aligned}$$

The first order condition for g_t is still given by

$$p(A, C) + E_C(A, C) = 0$$

while the first order condition for κ_t is given by

$$\sigma A p_A(A, C) + \kappa C p_C(A, C) + \kappa C E_{CC}(A, C) + \sigma A E_{AC}(A, C) = 0.$$

As

$$p_A(A, C) + E_{AC}(A, C) = 0$$

and

$$p_C(A, C) + E_{CC}(A, C) = 0$$

the first order condition for κ_t is satisfied if and only if the first order condition for g_t is satisfied. The HJB for $p(A, C)$ becomes

$$\begin{aligned}(r + \lambda - \delta(A, C))p(A, C) = &\mu A p_A(A, C) + (g(A, C) + \delta(A, C))C p_C(A, C) \\ &+ \frac{\sigma^2}{2} A^2 p_{AA}(A, C) + \frac{\kappa^2}{2} C^2 p_{CC}(A, C) + \sigma A \kappa C p_{AC}(A, C) + \lambda \mathbb{E}[p(SA, C)].\end{aligned}$$

Given that $p(A/C) = p(A, C)$, we get

$$(r + \lambda - \delta(a))p(a) = \ell(a) + \mu a p'(a) - (g(a) + \delta(a))ap'(a) \\ + \frac{\sigma^2}{2}a^2 p''(a) + \frac{\kappa(a)^2}{2}(p''(a)a^2 + 2p'(a)a) - \sigma\kappa(a)(p'(a)a^2 + p'(a)a) + \lambda\mathbb{E}[p(Sa)].$$

Similarly,

$$e(a) = -\delta(a)p(a) + \mu a e'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda\mathbb{E}[e(Sa)]$$

and

$$e'(a) = -\delta'(a)p(a) - \delta(a)p'(a) + \mu a e''(a) + \mu e'(a) + \frac{\sigma^2}{2}a^2 e'''(a) + \sigma^2 a e''(a) + \lambda\mathbb{E}[e'(Sa)].$$

Using the first order condition for $g(a)$ and its derivatives:

$$\begin{aligned} p(a) &= -e(a) + e'(a)a, \\ p'(a) &= e''(a)a, \\ p''(a) &= e'''(a)a + e''(a), \end{aligned}$$

we get

$$\begin{aligned} 0 &= (r + \lambda)(p(a) + e(a) - e'(a)a), \\ &= \ell(a) + \delta(a)p(a) - (g(a) + \delta(a))ap'(a) + \mu a p'(a) + \frac{\sigma^2}{2}a^2 p''(a) \\ &+ \frac{\kappa(a)^2}{2}(p''(a)a^2 + 2p'(a)a) - \sigma\kappa(p'(a)a^2 + p'(a)a) + \lambda\mathbb{E}[p(Sa)] \\ &- \delta(a)p(a) + \mu a e'(a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda\mathbb{E}[e(Sa)] \\ &+ \delta'(a)ap(a) + \delta(a)p'(a)a - \mu a^2 e''(a) - \mu a e'(a) - \frac{\sigma^2}{2}a^3 e'''(a) - \sigma^2 a^2 e''(a) - \lambda\mathbb{E}[e'(Sa)a] \\ &= \ell(a) + \delta'(a)ap(a) - g(a)ap'(a) + \kappa(a)^2/2(p''(a)a^2 + 2p'(a)a) - \sigma\kappa(a)(p'(a)a^2 + p'(a)a). \end{aligned}$$

Thus, in the smooth part of the equilibrium, it must be that

$$g(a) = \frac{\ell(a) + \delta'(a)ap(a) + \kappa(a)^2/2(p''(a)a^2 + 2p'(a)a) - \sigma\kappa(a)(p'(a)a^2 + p'(a)a)}{ap'(a)}.$$

Therefore, the HJB for $p(a)$ is given by

$$(r+\lambda)p(a) = \delta(a)p(a) - \delta'(a)ap(a) + \mu ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda\mathbb{E}[p(Sa)]$$

and none of the equilibrium price functions are affectec by $\kappa(a)$.

U No Commitment

In the main text, we assume that a centralized platform has some commitment power with respect to the coupon policy and the collateralization rule. As claimed in Section 4, we show that the platform has no value if it cannot commit at all.

Lemma 9. *Without commitment, there is no MPE with striclty positive equity value $E(A,C,K) > 0$ and stablecoin price $p(A,C,K) > 0$.*

The problem of a platform without any commitment to policies is similar to that of a firm that can choose whether or not to make coupon payments on perpetuity debt without defaulting. Once stablecoins/debt are issued, the firm strictly prefers not to make coupon payments because it already captured any benefits from issuance. As a result, the platform would always set the coupon payment to 0 ex-post, which means that stablecoin have no value ex-ante because the peg is not guaranteed. Lemma 9 thus shows that some commitment to a coupon policy is necessary; otherwise the platform and the stablecoin it issues have no value.

Proof of Lemma 9. Note that we have

$$dC_t = \delta_t C_t dt + G_t dt + (\mathcal{G}_t - \mathcal{G}_{t^-})$$

and

$$dK_t = \mu K_t dt + \sigma K_t dZ_t + M_t dt + K_{t^-}(S_t - 1)dN_t + (\mathcal{M}_t - \mathcal{M}_{t^-}).$$

If $(\mathcal{G}_t - \mathcal{G}_{t^-})$ and $(\mathcal{M}_t - \mathcal{M}_{t^-})$[17], using Ito's lemma we get

$$\begin{aligned}(r+\lambda)E(A_t, C_{t^-}, K_{t^-}) &= p(A_t, C_t, K_t)G_t - M_t + \mu A_t E_A(A_t, C_t, K_t)\\ &+ (G_t + \delta_t C_t)E_C(A_t, C_t, K_t) + (M_t + \mu K_t)E_K(A_t, C_t, K_t)\\ &+ \frac{\sigma^2}{2}A_t^2 E_{AA}(A_t, C_t, K_t) + \frac{\sigma^2}{2}K_t^2 E_{KK}(A_t, C_t, K_t) + \sigma^2 A_t K_t E_{AK}(A_t, C_t, K_t)\\ &+ \lambda \mathbb{E}[E(SA_t, C_t, SK_t)].\end{aligned}$$

Therefore, if $E_C(A, C, K)$ is strictly negative, given a strategy $\delta(A, C)$, there is always an optimal deviation to a lower interest payment $\delta(A, C) - \Delta$ where $\delta > 0$ until $\Delta(A, C) = 0$. By Proposition I of DeMarzo and He (2021), $E(A, C, K)$ is strictly decreasing in C when $p(A, C, K) > 0$.

Similarly, without commitment to $\underline{K}(A_t, C_{t^-})$, it is always optimal to put no collateral in the platform as $r < \mu - \frac{\lambda}{\xi+1}$ and $K(A, C) = 0$. (See Appendix O.)

If $\delta(A, C) = 0$, then $p(A, C, 0) < 1$ as $\ell(A, C) < r$.

□

[17]Otherwise, we get

$$E(A_t, C_{t^-}, K_{t^-}) = E(A_t, C_{t^-} + \mathcal{G}_t - \mathcal{G}_{t^-}, K_{t^-} + \mathcal{M}_t - \mathcal{M}_{t^-}) + p(A_t, C_{t^-} + \mathcal{G}_t - \mathcal{G}_{t^-}, K_{t^-} + \mathcal{M}_t - \mathcal{M}_{t^-})(\mathcal{G}_t - \mathcal{G}_{t^-}) - (\mathcal{M}_t - \mathcal{M}_{t^-}),$$

which is not impacted by δ_t.