# **Economic Uncertainty and Investor Attention**

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#### Abstract

This paper develops a multi-firm equilibrium model of information acquisition based on differences in firms' characteristics. It is shown that higher economic uncertainty attracts investor attention to firm-level earnings announcements. Increased investor attention magnifies the earnings response coefficients of all announcing firms. However, reactions to increased attention differ by firm characteristics (e.g., firms with higher systematic risk attract more investor attention, and their prices react more to earnings announcements). More importantly, heightened investor attention caused by high economic uncertainty implies a steeper CAPM relation and higher betas for announcing firms. Empirical tests using firm-level attention measures yield support to the model's predictions.

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## 1 Introduction

We explore the impact of economic uncertainty and investor attention on asset prices in a multi-firm equilibrium model of information acquisition. The motivation for this inquiry starts with a large body of theoretical and empirical research that studies the tradeoffs imposed by the limited attention theory (Sims, 2003; Hirshleifer and Teoh, 2003; Peng, 2005). Limited attention models can explain a wide array of phenomena, such as the home bias puzzle (Van Nieuwerburgh and Veldkamp, 2009), investment and attention allocation behavior (Van Nieuwerburgh and Veldkamp, 2010), the attention allocation of mutual fund managers (Kacperczyk, Van Nieuwerburgh, and Veldkamp, 2016), or the comovement of asset returns (Peng and Xiong, 2006; Veldkamp, 2006). Nevertheless, the question remains whether these attention choices are priced in equilibrium. In other words, do firms' prices react to investors' attention decisions; and if so, does the Capital Asset Pricing Model reflect these reactions? This question is important given that the CAPM is a paradigm of modern finance and that attention has a first-order effect on financial markets (Da, Engelberg, and Gao, 2011; Andrei and Hasler, 2015).

To answer the question, we develop an equilibrium model of information acquisition. Our model is a multi-firm variant of Grossman and Stiglitz (1980), in which firms make earnings announcements and investors tailor their attention to any combination of firms' announcements. We focus on earnings announcements because they are salient information releases by firms that convey firm-specific and, potentially, macroeconomic/systematic information. We allow for investors' attention decisions to depend on the economic uncertainty investors face. This facilitates predictions about how aggregate uncertainty affects information acquisition, investor demand for shares, and the intertwined CAPM pricing of both corporate announcements and macroeconomic risk.

The first prediction of our model is that investors' attention to earnings announcements increases in uncertainty. In turn, increased investor attention magnifies stock price reactions to the earnings announcements, hereafter referred to as earnings response coefficients (ERCs). Furthermore, the effect varies predictably with firm-specific factors: ERCs increase incrementally more for firms that have (i) a stronger exposure to systematic risk; (ii) more informative earnings announcements; (iii) a more volatile idiosyncratic component in their earnings; and (iv) more noise trading. The intuition behind all these four cases is that the benefit of collecting information outweighs its cost for these firms, which attracts more investor attention to their announcements.

One central goal of our multi-asset framework is to study how investor attention impacts the CAPM. The model predicts that when a firm announces earnings, its beta increases proportionally to the fraction of investors who pay attention to its announcement. Furthermore, the model predicts a higher risk premium and thus a steeper CAPM relation on days of heightened investor attention caused by high uncertainty. While the increase in the market risk premium due to higher uncertainty is an obvious equilibrium outcome in asset pricing models, the higher risk premium caused by heightened investor attention is a novel result. In our model, investors earn a risk premium by paying attention because they are rewarded for resolving uncertainty (Robichek and Myers, 1966; Epstein and Turnbull, 1980).

In extensions of the model, we show that the impact of uncertainty on attention is more substantial for investors with lower information processing costs (e.g., institutional investors). We also show that the relation between uncertainty and attention is preserved in a dynamic version of the model. Thus, although in our main model we obtain results using the standard setting from Grossman and Stiglitz (1980), the static information choice imposed by that setting is not critical for our findings.

We test the model's predictions using the VIX as a time-varying measure of economic uncertainty and SEC EDGAR downloads to proxy for investor attention.<sup>1</sup> The results generally support our predictions. First, we find that investors pay more attention to earnings announcements on days with higher VIX. Second, we find that ERCs are larger for firms that announce on days with higher VIX. We attribute this effect primarily to the increase in investor attention. Third, we show that our ERC results are concentrated in firms with high CAPM beta (whose announcements are more likely to convey systematic information), firms with higher institutional ownership (whose cost of information acquisition is likely lower), firms with higher idiosyncratic volatility, and firms with more noise trading (captured by trading volume). Finally, we find strong empirical support for a steeper CAPM relation on days with heightened investor attention. Our findings indicate that investor attention is responsible for increased market betas on earnings announcement days.

Our study extends previous theories of attention in two ways, and offers rational explanations for several empirical findings in the literature that were attributed to behavioral factors. First, in our setting, firm-level announcements provide valuable information about the prospects of the announcing firms and the entire economy. In contrast, extant theories ignore such *information spillovers* and limit investors' attention to either systematic or idiosyncratic news (e.g., Peng and Xiong, 2006; Kacperczyk et al., 2016). In our model, information spillovers lead to a positive relation between uncertainty and attention and to an impact of attention on firms' market betas on announcement days. Information spillovers also lead to weaker ERCs when more firms are announcing: with a greater number of announce-

<sup>&</sup>lt;sup>1</sup>As an alternative attention proxy, we confirm our results using Google stock ticker searches attributable to investors (deHaan, Lawrence, and Litjens, 2021).

ments, prices reveal more market-wide information for free, weakening attention incentives. This spillover effect contrasts with the explanation advanced in Hirshleifer, Lim, and Teoh (2009) that ERCs are weaker because multiple announcements compete for investors' limited attention (a cognitive constraint effect). Finally, related theories study information spillovers in similar contexts (Patton and Verardo, 2012; Savor and Wilson, 2016), but are silent about the interaction between information spillovers and investor attention and the impact of attention on ERCs and the CAPM equilibrium.

Second, the aggregate amount of attention in our economy fluctuates with incentives tied to economic uncertainty, whereas previous models bind attention to a fixed capacity constraint (Sims, 2003; Peng and Xiong, 2006; Kacperczyk et al., 2016). Our setup relaxes this constraint and instead assumes that investors face disclosure processing costs (Blankespoor, deHaan, and Marinovic, 2020). As a result, the aggregate amount of attention increases on days with higher uncertainty, which explains the steepening of the securities market line. Conversely, attention decreases on days with lower uncertainty or less informative announcements. This latter finding offers an alternative and rational explanation for investors' inattention to Friday announcements (DellaVigna and Pollet, 2009; Louis and Sun, 2010; Michaely, Rubin, and Vedrashko, 2016b). That is, Friday announcers may have different firm characteristics than non-Friday announcers, a prediction consistent with the empirical findings of Michaely, Rubin, and Vedrashko (2016a).

In a related empirical paper, Hirshleifer and Sheng (2022) also challenge the idea of fixed attention capacity constraints. They provide evidence that investors can potentially devote more or less attention to *both* macro and micro news (see also Eberbach, Uhrig-Homburg, and Yu, 2021). While our empirical findings are consistent with those in Hirshleifer and Sheng (2022), different from that study, we build a theory to explain these findings. In addition, we derive and analyze the cross-sectional implications of investors' rational responses to heightened uncertainty using EDGAR (Google) searches.<sup>2</sup>

Our study adds to the rapidly growing literature that documents a robust beta-return relation on various occasions: on macroeconomic announcement days; when investor attention is strong; in months after the U.S. midterm elections; on leading earnings announcement days; or overnight (Savor and Wilson, 2014; Ben-Rephael, Carlin, Da, and Israelsen, 2021; Chan and Marsh, 2021a,b). We contribute to this literature by showing theoretically that heightened investor attention leads to a steeper beta-return relation and increases firms' market betas on the days of their announcements.

<sup>&</sup>lt;sup>2</sup>Several recent studies use EDGAR data to explore different issues in corporate finance and asset pricing (e.g., Loughran and McDonald, 2011; DeHaan, Shevlin, and Thornock, 2015; Lee, Ma, and Wang, 2015; Drake, Roulstone, and Thornock, 2015; Bauguess, Cooney, and Hanley, 2018; Chen, Cohen, Gurun, Lou, and Malloy, 2020; Chen, Kelly, and Wu, 2020; Gao and Huang, 2020).

Overall, our paper shows that economic uncertainty is an essential driver of investors' attention to firm-level information and that investors' rational attention behavior has critical asset pricing implications. First, attention is the primary channel through which stock prices react to earnings announcements. Second, heightened attention leads to higher market betas for the announcing firms and a steeper securities market line. Investor attention, thus, might be an overlooked factor in explaining the cross-section of asset returns.

The rest of the paper proceeds as follows. Section 2 describes our model and its main predictions, and Section 3 examines extensions and other implications. Section 4 tests our model's predictions for investor attention and market pricing around earnings announcements and CAPM pricing on days with high versus low investor attention. Finally, Section 5 provides concluding remarks.

## 2 Model

Consider an economy populated by a continuum of investors, indexed by  $i \in [0, 1]$ . The economy has three dates  $t \in \{0, 1, 2\}$ . At t = 0, each investor makes an information acquisition decision that we will describe below. At t = 1, investors trade competitively in financial markets. At t = 2, financial assets' payoffs are realized, and investors derive utility from consuming their terminal wealth. Investors trade a riskless asset and N risky assets indexed by  $n \in \{1, ..., N\}$ . The riskless asset is in infinitely elastic supply and pays a gross interest rate of 1 per period. Each risky asset ("firm") has an equilibrium price  $P_n$  at t = 1 and pays a risky dividend at t = 2:

$$D_n = b_n f + e_n, \text{ for } n \in \{1, ..., N\}.$$
 (1)

The payoff  $D_n$  has a systematic component f and a firm-specific component  $e_n$ . The parameters  $b_n$ , which are heterogeneous across firms and known by investors, dictate the exposures of firms' payoffs to the systematic component. Without loss of generality, we assume that the average of  $b_n$  across firms is 1.

We denote by **D** the  $N \times 1$  vector of asset payoffs, by **P** the  $N \times 1$  the vector of asset prices, and by  $\mathbf{R}^e \equiv \mathbf{D} - \mathbf{P}$  the vector of dollar excess return of the risky assets.<sup>3</sup> We fix the total number of shares for all assets to **M** (hereafter the *market portfolio*), an equallyweighted vector whose elements are all equal to 1/N. The future market return is then  $R^e_{\mathbf{M}} \equiv \mathbf{M}'\mathbf{R}^e$ . The assumption of an equally-weighted market portfolio **M** does not affect

<sup>&</sup>lt;sup>3</sup>Throughout the paper, we will adopt the following notation: we use letters in plain font to indicate univariate variables and bold letters to indicate vectors and matrices; we use subscripts to indicate individual assets and superscripts to indicate individual investors. Appendix A.1 provides further details.

our results. However, it is useful for interpreting the results in terms that are empirically measurable (as discussed in detail below).

At t = 0, all investors have a common information set  $\mathcal{F}_0$  that consists of the prior distributions of f and  $e_n$ :

$$f \sim \mathcal{N}(0, U^2) \tag{2}$$

$$e_n \sim \mathcal{N}(0, \sigma_{en}^2), \quad \text{for } n \in \{1, \dots, N\}.$$

$$(3)$$

We allow for variances  $\sigma_{en}^2$  to vary in the cross-section of firms. Firm-specific components  $e_n$  are independent across firms, and f and  $e_n$  are independent,  $\forall n \in \{1, ..., N\}$ .

We refer to U as uncertainty for the rest of the paper. It represents investors' expected forecasting error conditional on information available at time 0,  $U^2 \equiv \text{Var}[f|\mathcal{F}_0]$ . As we will show below, in our model U is closely related to investors' pre-announcement uncertainty about the future return on the market, which helps us confront the theory with the data.

Defining U as uncertainty is the simplest way to derive theoretical predictions. Alternatively, we could be more specific about the information set  $\mathcal{F}_0$ , without any impact on the results. Assuming, for instance, that *before* time 0 investors hold the prior  $f \sim \mathcal{N}(0, \sigma_f^2)$ , and that at time 0 they observe public information about f under the form of a signal G = f + gwith  $g \sim \mathcal{N}(0, \sigma_g^2)$ , Bayesian updating implies

$$U^2 = \operatorname{Var}[f|\mathcal{F}_0] = \frac{\sigma_f^2 \sigma_g^2}{\sigma_f^2 + \sigma_g^2}.$$
(4)

A higher variance  $\sigma_f^2$  of the fundamental or a higher variance  $\sigma_g^2$  of the noise in public information increases investors' uncertainty at time 0. Thus, our results come through whether U measures uncertainty in macro fundamentals or captures noise in the available public information at time 0. We, therefore, keep our model agnostic about what determines U.<sup>4</sup>

A total of  $A \leq N$  firms issue earnings announcements at t = 1. We denote the set of announcing firms by  $\mathcal{A} \equiv \{1, ..., A\}$ . As in Teoh and Wong (1993), earnings announcements convey information about firms' future dividends:

$$E_a = D_a + \varepsilon_a, \quad \text{for } a \in \mathcal{A},\tag{5}$$

where the earnings noise shocks  $\varepsilon_a$  are independently distributed,  $\varepsilon_a \sim \mathcal{N}(0, \sigma_{\varepsilon a}^2)$ , and drawn independently from f and  $e_n$ ,  $\forall a \in \mathcal{A}$  and  $\forall n \in \{1, ..., N\}$ .

At t = 0, each investor i chooses whether or not to be attentive to the earnings announce-

 $<sup>^{4}</sup>$ We discuss the introduction of an additional layer of information acquisition in Section 3.2 using a dynamic version of the model. We show that including this feature does not qualitatively change our results.

ments. Investor *i* can pay attention to announcements made by the firms in any of the  $2^A$  possible subsets of  $\mathcal{A}$ . (The set of all subsets of  $\mathcal{A}$  represents the *power set* of  $\mathcal{A}$ , or  $\mathscr{P}(\mathcal{A})$ , and includes the empty set  $\emptyset$  and  $\mathcal{A}$  itself.) Thus, there are potentially  $2^A$  investor types, indexed by  $k \in \mathscr{P}(\mathcal{A})$ . For instance, investors who choose to stay uninformed are of type  $k = \emptyset$ ; investors who pay attention to all earnings announcements are of type  $k = \mathcal{A}$ . We use the dummy variable  $I_a^k$ , with  $a \in \mathcal{A}$  and  $k \in \mathscr{P}(\mathcal{A})$ , to indicate type k investor's decision to pay attention to  $E_a$ : if  $a \in k$ , then  $I_a^k = 1$ ; otherwise,  $I_a^k = 0$ .

Each investor starts with zero initial wealth and maximizes expected utility at time 0,

$$\max_{k \in \mathscr{P}(\mathcal{A})} \mathbb{E}_0 \left[ \max_{\mathbf{q}^k} \mathbb{E}_1^k \left[ -e^{-\gamma \left( W^k - c|k| \right)} \right] \right],\tag{6}$$

where  $\mathbf{q}^k$  is the optimal portfolio of a type k investor and |k| denotes the cardinality of the set k, or  $|k| = \sum_{a \in \mathcal{A}} I_a^k$ .

At time 0, investor *i* decides her type *k*, knowing that at time 1 she will choose an optimal portfolio based on the information set pertaining to the type *k*. The first optimization is a combinatorial discrete choice problem.<sup>5</sup> The second optimization is a standard Markowitz (1952) portfolio choice problem, where  $\gamma$  is the risk aversion coefficient,  $W^k = (\mathbf{q}^k)'\mathbf{R}^e$  is investor's final wealth at t = 2 (which depends on her type *k*), and *c* is the monetary cost of paying attention to one earnings announcement—e.g., an information-processing cost, or time and opportunity cost. The attention cost *c* is strictly positive and is the same across investors and firms. (We derive additional predictions in a model with heterogeneous costs across investors—e.g., retail versus institutional investors—in Section 3.)

At t = 1, investors build optimal portfolios:

$$\mathbf{q}^{k} = \frac{1}{\gamma} \operatorname{Var}_{1}^{k}[\mathbf{D}]^{-1}(\mathbb{E}_{1}^{k}[\mathbf{D}] - \mathbf{P}), \quad \text{for } k \in \mathscr{P}(\mathcal{A}),$$
(7)

where the superscripts k in  $\mathbb{E}_1^k[\cdot]$  and  $\operatorname{Var}_1^k[\cdot]$  read "under the information set of a type k investor." That is,  $\operatorname{Var}_1^k[\mathbf{D}]$  is the  $N \times N$  covariance matrix of assets' payoffs, conditioned on the type k investor's information set.

We assume that an unmodeled group of agents trades for non-informational reasons or liquidity needs. This is a common assumption in noisy rational expectations models, which ensures that equilibrium prices do not fully reveal investors' information. Consistent with much of the prior literature, we often interpret liquidity trading as noise (Grossman and Stiglitz, 1980; He and Wang, 1995). Liquidity traders have inelastic demands of  $\mathbf{x}$  shares,

<sup>&</sup>lt;sup>5</sup>Examples of combinatorial discrete (binary) choice problems in economics include plant location problems, country selection by multinational firms, and selection of which goods to produce. Hu and Shi (2019) and Arkolakis, Eckert, and Shi (2021) made recent theoretical advances in this field.

where each element of **x** is normally and independently distributed,  $x_n \sim \mathcal{N}(0, \sigma_{xn}^2)$ .

Denoting by  $\lambda^k$  the fraction of type k investors, the prices of risky assets are determined in equilibrium by the market-clearing condition:

$$\sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k \mathbf{q}^k + \mathbf{x} = \mathbf{M}.$$
(8)

Before turning to the equilibrium analysis, we define the fraction of investors who observe the announcement  $E_a$  as

$$\Lambda_a \equiv \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k I_a^k.$$
(9)

Importantly, in our model the attention capacity of investors is not constrained, in the sense that an equilibrium in which  $\Lambda_a = 1 \ \forall a \in \mathcal{A}$  is possible, as we will describe below.

## 2.1 Equilibrium search for information

As is customary in noisy rational expectations models, prices take the linear form

$$\mathbf{P} = \boldsymbol{\alpha}\mathbf{E} + \boldsymbol{\xi}\mathbf{x} - \boldsymbol{\zeta}\mathbf{M},\tag{10}$$

where  $\mathbf{E} \equiv [E_1, E_2, \cdots, E_A]'$ ,  $\boldsymbol{\alpha}$  is a  $N \times A$  matrix, and  $\boldsymbol{\xi}$  and  $\boldsymbol{\zeta}$  are  $N \times N$  matrices.

Solving for the equilibrium price coefficients is not necessary to determine the equilibrium demand for information. Instead, it is sufficient to make the following conjecture (equivalent to Lemma 3.2 in Admati, 1985), which we will verify in Proposition 3.

Conjecture 1.

$$\widehat{\mathbf{P}} \equiv \boldsymbol{\xi}^{-1}(\mathbf{P} + \boldsymbol{\zeta}\mathbf{M}) = \sum_{a=1}^{A} \frac{\Lambda_a}{\gamma \sigma_{\varepsilon a}^2} \boldsymbol{\iota}_a E_a + \mathbf{x}, \qquad (11)$$

where  $\widehat{\mathbf{P}} \equiv [\widehat{P}_1, \widehat{P}_2, \cdots, \widehat{P}_N]'$  and  $\boldsymbol{\iota}_a$  is a standard basis vector of dimension N with all components equal to 0, except the a-th, which is 1.

This conjecture transforms the equilibrium prices into simple signals about  $E_a$ ,  $a \in \mathcal{A}$ . In equilibrium, all investors except the fully informed (of type  $k = \mathcal{A}$ ) use prices to learn. Accordingly, the information sets of investors at time 1 are

$$\begin{cases} \mathcal{F}^{k} = \{ E_{a} \mid a \in k \} \cup \widehat{\mathbf{P}} & \text{if } k \in \mathscr{P}(\mathcal{A}) \setminus \mathcal{A}, \\ \mathcal{F}^{k} = \{ E_{a} \mid a \in \mathcal{A} \} & \text{if } k = \mathcal{A}. \end{cases}$$
(12)

Before characterizing the information acquisition decision for each investor type, we define the following *learning coefficients*:

$$\ell_a^k = I_a^k + (1 - I_a^k)\ell_a, \quad \text{where } \ell_a \equiv \frac{\Lambda_a^2}{\Lambda_a^2 + \gamma^2 \sigma_{xa}^2 \sigma_{\varepsilon a}^2}.$$
(13)

If a type k investor observes the earnings announcement  $E_a$ , then  $I_a^k = 1$  and the learning coefficient  $\ell_a^k$  reaches its maximum value, 1. Without observing  $E_a$ ,  $I_a^k = 0$  and the investor relies on prices to learn, which yields  $\ell_a^k = \ell_a < 1$ . Prices are informative about  $E_a$  to the extent that *someone* pays attention to the signal  $E_a$ , that is, if  $\Lambda_a > 0$ . In this case,  $\ell_a$  increases with the fraction of informed investors (investors learn more from prices when a higher fraction of them pay attention to  $E_a$ ) and decreases with the amount of noise in supply  $\sigma_{xa}$  and the amount of noise in the earnings announcement  $\sigma_{\varepsilon a}$  (investors learn less from prices when there is more noise in supply or when earnings announcements are noisier).

Investors' demand for information ultimately depends on the reduction in uncertainty achieved by observing new information. Because in our setup the vector of final payoffs **D** is a multidimensional normally distributed random variable, the reduction in uncertainty from observing new information is conveniently measured using the notion of entropy: under the information set of any investor type  $k \in \mathscr{P}(\mathcal{A})$ , the vector **D** has entropy

$$H^{k}[\mathbf{D}] = \frac{N}{2} \ln(2\pi + 1) - \frac{1}{2} \ln(\det(\operatorname{Var}_{1}^{k}[\mathbf{D}]^{-1})).$$
(14)

From this definition, it follows that the uncertainty perceived by the investor decreases with the determinant of the posterior precision matrix of  $\mathbf{D}$  (i.e., the inverse of the posterior covariance matrix  $\operatorname{Var}^{k}[\mathbf{D}]$ , hereafter  $\boldsymbol{\tau}^{k}$ ).

Defining  $\operatorname{Var}[\mathbf{D}] \equiv \sigma_f^2 \mathbf{b} \mathbf{b}' + \operatorname{Var}[\mathbf{e}]$ , where  $\mathbf{e}$  is the vector of idiosyncratic components  $e_n$  in firms' payoffs given in (1), we can state the following proposition.

**Proposition 1.** The posterior precision matrix for each investor type  $k \in \mathscr{P}(\mathcal{A})$  is

$$\boldsymbol{\tau}^{k} \equiv \operatorname{Var}_{1}^{k}[\mathbf{D}]^{-1} = \operatorname{Var}[\mathbf{D}]^{-1} + \sum_{a=1}^{A} \frac{\ell_{a}^{k}}{\sigma_{\varepsilon a}^{2}} \boldsymbol{\iota}_{a} \boldsymbol{\iota}_{a}',$$
(15)

and its determinant is given by

$$\det(\boldsymbol{\tau}^k) = \det(\operatorname{Var}[\mathbf{D}]^{-1}) \left( \prod_{a=1}^A \frac{\ell_a^k \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\sigma_{\varepsilon a}^2} \right) \left( 1 + U^2 \sum_{a=1}^A \frac{\ell_a^k b_a^2}{\ell_a^k \sigma_{ea}^2 + \sigma_{\varepsilon a}^2} \right).$$
(16)

Proposition 1 shows how the heterogeneity in the learning coefficients  $\ell_a^k$  across investors of different types  $k \in \mathscr{P}(\mathcal{A})$  drives the heterogeneity in the determinants  $\det(\boldsymbol{\tau}^k)$ . Because a higher determinant means less uncertainty (Eq. 14), the determinants  $\det(\boldsymbol{\tau}^k)$  provide a clear ranking of the informational distances between the  $2^A$  investor types. For instance the most informed investors (of type  $\mathcal{A}$ ) have the highest  $\det(\boldsymbol{\tau}^k)$  because  $\ell_a^{\mathcal{A}} = 1$ ,  $\forall a \in \mathcal{A}$ , whereas the least informed investors (of type  $\emptyset$ ) have the lowest  $\det(\boldsymbol{\tau}^k)$ .

The ranking in det( $\tau^k$ ) dictated by Proposition 1 allows for a simple characterization of the information market equilibrium. Consider a type k investor who decides whether to migrate to any alternative type in  $\mathscr{P}(\mathcal{A}) \setminus k$ . The key quantity that regulates the investor's decision is the *benefit-cost ratio*, which we define as

$$B_{\emptyset}^{k} \equiv \frac{\det(\boldsymbol{\tau}^{k})}{\det(\boldsymbol{\tau}^{\emptyset})} e^{-2\gamma c|k|}.$$
(17)

The ratio  $\det(\boldsymbol{\tau}^k)/\det(\boldsymbol{\tau}^{\emptyset})$  in  $B_{\emptyset}^k$  measures the gain in precision obtained from observing the earnings announcements made by all the firms in the set k, whereas  $e^{-2\gamma c|k|}$  measures the cost of paying attention to these announcements. With this benefit-cost ratio in hand, we can formulate the following result.

**Proposition 2.** A type k investor changes type from k to  $k' \in \mathscr{P}(\mathcal{A}) \setminus k$  if and only if

$$\frac{B_{\emptyset}^{k'}}{B_{\emptyset}^{k}} > 1 \quad \iff \quad \frac{1}{2\gamma} \ln \frac{\det(\boldsymbol{\tau}^{k'})}{\det(\boldsymbol{\tau}^{k})} > c(|k'| - |k|).$$
(18)

Assume, without loss of generality, that |k'| - |k| > 0. On the left-hand side of (18),  $\frac{1}{2\gamma} \ln \frac{\det(\boldsymbol{\tau}^{k'})}{\det(\boldsymbol{\tau}^{k})}$  measures the benefit of migrating from k to k' as a reduction in entropy divided by investor's risk aversion,  $(H^{k}[\mathbf{D}] - H^{k'}[\mathbf{D}])/\gamma$ ; the right-hand side measures the attention cost. The type k investor changes type if and only if the benefit from the reduction in entropy achieved by becoming of type k' outweighs its cost. Risk aversion lowers the benefit of information: because more risk-averse investors trade less aggressively, they benefit less from paying attention to firm disclosures.

The ratio  $\det(\boldsymbol{\tau}^{k'})/\det(\boldsymbol{\tau}^{k})$  in (18) is greatly simplified by means of Proposition 1: all the heterogeneity pertaining to non-announcing firms enters only in  $\det(\operatorname{Var}[\mathbf{D}]^{-1})$  and thus vanishes in the ratio. To gain further insight into this ratio, let us focus on a simplified version where investors in aggregate pay attention to one firm only (i.e., there is only one announcing firm, a). In this case, a type  $\emptyset$  investor changes type to  $\{a\}$  if and only if

$$\frac{1}{2\gamma} \ln \frac{1 + \frac{\operatorname{Var}[D_a]}{\sigma_{\varepsilon_a}^2}}{1 + \frac{\operatorname{Var}[D_a]}{\sigma_{\varepsilon_a}^2} \frac{\Lambda_a^2}{\Lambda_a^2 + \gamma^2 \sigma_{xa}^2 \sigma_{\varepsilon_a}^2}} > c.$$
(19)

On the left-hand side the benefit of information increases with  $\operatorname{Var}[D_a]/\sigma_{\varepsilon a}^2$ , which measures the quality of information provided by the earnings announcement; decreases with the fraction of informed investors  $\Lambda_a$ , in which case prices are more informative and the signal  $E_a$  becomes less valuable; increases with the amount of noise in supply  $\sigma_{xa}$ , in which case prices are less informative and the signal  $E_a$  becomes more valuable; and decreases with the risk aversion. (See also Grossman and Stiglitz, 1980, for similar tradeoffs.)

The same tradeoffs are at play when multiple firms are announcing, with the significant difference that heterogeneity in firms characteristics  $(b_a, \sigma_{\varepsilon a}, \sigma_{ea}, \text{and } \sigma_{xa})$  yields heterogeneous information choices across firms. We will analyze this heterogeneity in Section 2.4, where we discuss the model's theoretical predictions and continue to focus here on the information market equilibrium, which we characterize in the following theorem.

**Theorem 1.** There exist two positive values  $c_{min} < c_{max}$ , strictly increasing in U, such that:

- (A) If  $c \in [c_{max}, \infty)$ , then the cost of information is prohibitive and no investor finds it optimal to pay attention to the earnings announcements:  $\lambda^{\emptyset} = 1$ .
- (B) If  $c \in (c_{\min}, c_{\max})$ , then there exists a set  $\{\lambda^k \mid k \in \mathscr{P}(\mathcal{A})\}$  such that, in equilibrium:  $\sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k = 1; \ \lambda^{\emptyset} < 1; \ \lambda^{\mathcal{A}} < 1; \ and \ the \ benefit-cost \ ratios \ \{B_{\emptyset}^k \mid k \in \mathscr{P}(\mathcal{A})\}\ are$ determined such that for any pair  $\{k, k'\} \in \mathscr{P}(\mathcal{A})$ :
  - (i) If  $\{\lambda^k > 0\} \land \{\lambda^{k'} > 0\}$ , then  $B_{\emptyset}^{k'}/B_{\emptyset}^k = 1$ .
  - (ii) If  $\{\lambda^k = 0\} \land \{\lambda^{k'} > 0\}$ , then  $B_{\emptyset}^{k'}/B_{\emptyset}^k \ge 1$ .

Conditions (i) and (ii) are both necessary and sufficient for the stability of the information market equilibrium when  $c \in (c_{\min}, c_{\max})$ .

(C) If  $c \in [0, c_{min}]$ , then the cost of information is small enough such that all investors pay attention to all the earnings announcements:  $\lambda^{\mathcal{A}} = 1$ .

Cases (A) and (C) are trivial equilibria in which the information cost is too high or too low. In these cases, investors unanimously choose to remain uninformed or to pay attention to all earnings announcements. Case (B), which will be the focus of our analysis in Section 2.4, defines a set of conditions such that, in equilibrium, no investor can unilaterally improve their utility by changing their type.<sup>6</sup> We explain in Section 2.4 how investors arrive at this self-sustaining equilibrium, and describe an iterative algorithm that converges to equilibrium from any initial conditions  $\{\lambda_0^k > 0 \mid k \in \mathscr{P}(\mathcal{A})\}$ .

## 2.2 Equilibrium prices and earnings response coefficients

We now aggregate investors' demands in order to solve for equilibrium prices. Define first the weighted average precision matrix for the population of informed investors as

$$\boldsymbol{\tau} \equiv \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k \boldsymbol{\tau}^k.$$
<sup>(20)</sup>

**Lemma 1.** The weighted average precision is given by

$$\boldsymbol{\tau} = \operatorname{Var}[\mathbf{D}]^{-1} + \begin{bmatrix} \operatorname{diag}[\pi_a(\Lambda_a) \mid a \in \mathcal{A}] & \mathbf{0}_{A \times (N-A)} \\ \mathbf{0}_{(N-A) \times A} & \mathbf{0}_{(N-A) \times (N-A)} \end{bmatrix},$$
(21)

where each coefficient  $\pi_a(\Lambda_a)$  is a strictly increasing function of  $\Lambda_a$ ,

$$\pi_a(\Lambda_a) = \frac{\Lambda_a^2 + \Lambda_a \gamma^2 \sigma_{xa}^2 \sigma_{\varepsilon a}^2}{\Lambda_a^2 \sigma_{\varepsilon a}^2 + \gamma^2 \sigma_{xa}^2 \sigma_{\varepsilon a}^4}, \quad a \in \mathcal{A},$$
(22)

and diag $[y_j \mid j \in z]$  is a diagonal matrix with  $\{y_j \mid j \in z\}$  on its diagonal.<sup>7</sup>

Each function  $\pi_a(\Lambda_a)$  determines the aggregate precision gains from observing  $E_a$ . A key property of these functions, which will prove useful shortly, is that they depend on the economic uncertainty U only *indirectly* through  $\Lambda_a$ .

**Proposition 3.** The equilibrium prices in this economy satisfy

$$\boldsymbol{\tau} \mathbf{P} = \sum_{a=1}^{A} \pi_a(\Lambda_a) \boldsymbol{\iota}_a E_a + \gamma \begin{bmatrix} \operatorname{diag} \left[ \frac{\pi_a(\Lambda_a) \sigma_{\varepsilon a}^2}{\Lambda_a} \mid a \in \mathcal{A} \right] & \mathbf{0}_{A \times (N-A)} \\ \mathbf{0}_{(N-A) \times A} & \mathbf{I}_{N-A} \end{bmatrix} \mathbf{x} - \gamma \mathbf{M}, \quad (23)$$

where  $\mathbf{I}_z$  is the identity matrix of dimension z.

<sup>6</sup>Conditions (i) and (ii) can be grouped by means of a Kronecker product. Consider the column vector  $\mathbf{B} = \{B_{\emptyset}^{k} \mid k \in \mathscr{P}(\mathcal{A})\}$  and let  $\mathbf{B}^{-1}$  be its element-wise inverse. The Kronecker product  $\mathbf{B}^{-1} \otimes \mathbf{B}'$ , whose rows correspond to  $\lambda^{k}$  and columns to  $\lambda^{k'}$ , groups all the necessary elements, e.g., if  $\{\lambda^{k} > 0\} \land \{\lambda^{k'} > 0\}$ , then the element (k, k') of  $\mathbf{B}^{-1} \otimes \mathbf{B}'$  should equal 1.

<sup>&</sup>lt;sup>7</sup>The off-diagonal elements in the second term of Eq. (21) are all zero, potentially suggesting that an earnings announcement  $E_a$  is only informative about  $D_a$ . This may seem unexpected, given that all final payoffs share a common systematic component. However, the precision matrix does not have the usual element-wise interpretation of the covariance matrix (e.g., the diagonal terms of the precision matrix are *not* asset-specific precisions). Inverting the precision matrix  $\tau$  would restore the common interpretation.

The earnings response coefficients (ERCs) measure the reactions of the equilibrium prices to the earnings announcements. In a simpler model with a sole announcer the ERC is the coefficient of  $E_a$  in the equilibrium price. In our model with N firms and A announcers, ERCs form the principal diagonal of the  $N \times A$  matrix  $\boldsymbol{\alpha}$  in the price conjecture (10). That is, ERCs measure the price reactions of the announcing firms to their own announcements. Denoting by  $\mathbf{D}_{\mathcal{A}}$  the final payoffs of all announcing firms, we derive the following corollary.

**Corollary 3.1.** The earnings response coefficients are given by the diagonal of the  $A \times A$  matrix  $\alpha_A$ , which solves:

$$\boldsymbol{\alpha}_{\mathcal{A}} = \mathbf{I}_{A} - (\mathbf{I}_{A} + \operatorname{Var}[\mathbf{D}_{\mathcal{A}}] \operatorname{diag}[\pi_{a}(\Lambda_{a}) \mid a \in \mathcal{A}])^{-1}.$$
(24)

The  $A \times A$  matrix  $\boldsymbol{\alpha}_{\mathcal{A}}$  is zero if  $\Lambda_a = 0 \ \forall a \in \mathcal{A}$ . An important separation result helps us interpret  $\boldsymbol{\alpha}_{\mathcal{A}}$ : as shown in Lemma 1, the coefficients  $\pi_a(\Lambda_a)$  do not directly depend on U. Therefore, in the following analysis, we can separately assess the effects of an increase in economic uncertainty on ERCs and, in particular, the additional effect that arises from changes in investor attention.

## 2.3 Illustration

To illustrate how investors' search for information converges to a stable equilibrium, it is helpful to write the individual optimization problem (6) in a more straightforward form. Appendix A.7 shows that at time 0, each investor makes the following choice:

$$\max_{k \in \mathscr{P}(\mathcal{A})} \ln B^k_{\emptyset},\tag{25}$$

where the benefit-cost ratios  $B_{\emptyset}^k$  have been defined in (17).

A key property of the function  $f(k) = \ln B_{\emptyset}^k$  is submodularity—the difference in the incremental value of f(k) that one element *a* makes when added to the type *k* decreases as the size of *k* increases. Submodularity can be interpreted as a property of *diminishing returns*. It implies that an individual investor's incentive to become more informed (e.g., to increase her type from *k* to  $k \cup \{a\}$ ) decreases with her current level of attention. Furthermore, we show in Appendix A.7 that a migration of a positive mass of investors from any type *k* to a different type *k'* decreases the relative attractiveness of type *k'* with respect to type *k*, i.e., decreases the fraction  $B_{\emptyset}^{k'}/B_{\emptyset}^k$ . This implies that an individual investor's incentive to choose *k'* over *k* decreases if in aggregate more investors choose *k'* over *k*. Hence we recover the Grossman and Stiglitz (1980) result that individual action and the aggregate of (others) individual actions are strategic substitutes. Hu and Shi (2019) and Arkolakis et al. (2021) derive an evolutionary learning algorithm that reaches the equilibrium of a submodular game from any initial point. Starting from a set of initial values  $\{\lambda_0^k > 0 \mid k \in \mathscr{P}(\mathcal{A})\}$  such that  $\sum_k \lambda_0^k = 1$ , the algorithm allows some small fraction of the population of investors of a given type k to revise their strategy as the best response to the current total population strategy. This process is iterated over all types until it converges to a self-sustaining equilibrium in which no investor changes strategy, as in Theorem 1. We relegate the details of this algorithm to Appendix A.7 and focus here on a numerical example, which we illustrate in Figure 1.

#### (Insert Figure 1 about here)

This numerical example considers an economy with three announcers. The announcing firms differ through their exposure to systematic risk,  $b_1 > b_2 > b_3$ , while other firm-level parameters are homogeneous across firms. The parameters that we chose are provided in the caption of the figure. Note that this example is only illustrative—in Section 4, we propose a realistic calibration with a larger number of announcers.

The dashed and solid lines in the figure depict the values  $c_{min}$  and  $c_{max}$ , respectively. The plot confirms the results of Theorem 1: (i)  $c_{min} < c_{max}$  and (ii)  $c_{min}$  and  $c_{max}$  increase with the amount of uncertainty U. When  $c \leq c_{min}$ , all investors are attentive to all earnings announcements,  $\lambda^{\mathcal{A}} = 1$ ; when  $c \geq c_{max}$ , no investor pays attention to earnings announcements,  $\lambda^{\emptyset} = 1$ ; when  $c \in (c_{min}, c_{max})$ , the two dotted lines that split the middle zone show that investors always find the announcement of firm 1 most valuable—they pay attention to  $E_1$  in cases (B1), (B2), and (B3)—whereas the announcement of firm 3 least valuable—they pay attention to  $E_3$  only in case (B3). Since  $b_1 > b_2 > b_3$ ,  $E_1$  is the most informative announcement about the systematic factor f, and investors turn their attention first to firm 1. Thus, in this equilibrium investors behave as if they queue announcements based on their exposure to systematic risk. Frederickson and Zolotoy (2016) document a similar queuing result: investors devote more immediate attention to announcing firms that are comparatively more visible (i.e., larger firms, firms with more media coverage, higher advertising expense, or higher analyst coverage). In the case discussed here, attention queueing is based on firms' exposures to the systematic factor f. Indeed, as we show in the next section, firms' exposures to the systematic factor yield a clear ranking of investor attention across firms.

### 2.4 Implications for attention and earnings response coefficients

Building on the previous illustration, we derive several testable implications of the model. The first result that emerges from Theorem 1 and Figure 1 is the effect of an increase in uncertainty on the information market equilibrium. Suppose uncertainty is low enough that all investors are inattentive—this corresponds to case (A), depicted with the hashed area in the plot. Then, after an increase in uncertainty the equilibrium moves to the right, anywhere from case (B) to case (C): a positive fraction of investors become attentive first to  $E_1$ , and if the increase in uncertainty is sufficiently substantial, to  $E_2$  and ultimately to  $E_3$ . The main implication is that an increase in uncertainty triggers investor attention to firm-level information. Moreover, investors direct their attention to an increasing number of firms as uncertainty increases.

The previous implication refers to the number of firms: more announcing firms become the focus of investor attention as uncertainty increases. We now turn to the effect of uncertainty on the number of investors who pay attention to the earnings announcements. The fractions  $\Lambda_a$  of investors who observe each earnings announcement, defined in (9), are not apparent from Figure 1, which only shows when these fractions are positive or zero. To analyze how these fractions vary with uncertainty, assume for simplicity that no investor in the economy observes the announcement of firm a, or  $\Lambda_a = 0$ . Note that similar intuition holds without the assumption but with a more complicated expression. Then, for a type k investor the benefit of paying attention to  $E_a$  follows from (17):

$$\frac{\det(\boldsymbol{\tau}^{k\cup\{a\}})}{\det(\boldsymbol{\tau}^k)} = 1 + \frac{1}{\sigma_{\varepsilon a}^2} \left( \sigma_{ea}^2 + \frac{b_a^2}{\frac{1}{U^2} + \sum_{\alpha=1, \ \alpha \neq a}^A \frac{b_\alpha^2 \ell_\alpha^k}{\ell_\alpha^k \sigma_{ej}^2 + \sigma_{\varepsilon \alpha}^2}} \right).$$
(26)

The first implication of (26) is that the benefit of paying attention to  $E_a$  strictly increases with uncertainty (this holds for all investor types and all announcing firms). Moreover, the benefit of attention is higher for firms with a stronger exposure  $b_a$  to the systematic component, a higher volatility  $\sigma_{ea}$  of their idiosyncratic component, and less noise  $\sigma_{\varepsilon a}$  in their announcement. Eq. (26) also implies that the benefit of attention decreases with the amount of attention that investors pay to *other* earnings announcements, as reflected in the summation term: if a large number of firms announce at the same time (i.e., A is high), and large fractions of investors are attentive (i.e.,  $\Lambda_{\alpha}$  is large,  $\forall \alpha \neq a$ ), then prices are highly informative about f and paying attention to  $E_a$  becomes less valuable. This implication is similar to the *investor distraction hypothesis* (Hirshleifer et al., 2009): when multiple announcements compete for investor attention, prices underreact to the new information. In our model, this result arises not because investors are distracted by the simultaneous announcements but because information spillovers increase aggregate price informativeness, diminishing the benefit of attention.

A critical implication of (26) emerges once we fix  $b_a = 0$ , which results in a constant benefit of paying attention to  $E_a$ . In this case, an increase in uncertainty does not lead to an increase in attention to firm-level information because no information spillover occurs from firm *a* to the rest of the economy. This implication, coupled with evidence from recent empirical work (Hirshleifer and Sheng, 2022; Ben-Rephael et al., 2021; Chan and Marsh, 2021b) and our empirical analysis in Section 4, highlights the importance of information spillovers in theories of firm-level information acquisition.

Panel (a) of Figure 2 illustrates the impact of an increase in uncertainty in our calibrated economy with three announcers. The three lines depict the fractions of the population of investors attentive to each earnings announcement. This example assumes that  $b_1 > b_2 > b_3$ . Confirming Eq. (26), the fractions  $\Lambda_1$ ,  $\Lambda_2$ , and  $\Lambda_3$  increase with U. We note that for low levels of economic uncertainty the fractions  $\Lambda_a$  are all zero for  $a \in \{1, 2, 3\}$ , which corresponds to case (A) of Theorem 1. As uncertainty increases the economy moves successively to all the subcases of (B) and ultimately to case (C).

#### (Insert Figure 2 about here)

The increase in investor attention caused by an increase in uncertainty has additional implications for the response of prices to firm-level information. To gain more intuition, we write the ERC in an economy with a sole announcer (a particular case of Corollary 3.1):

$$ERC_a = 1 - \frac{1}{1 + (U^2 b_a^2 + \sigma_{ea}^2) \pi_a(\Lambda_a)}.$$
(27)

The ERC increases with uncertainty directly through an increase in the variance of the firm's payoff  $\operatorname{Var}[D_a] = U^2 b_a^2 + \sigma_{ea}^2$  and indirectly through an increase in investors' attention to the earnings announcement. Firms with a stronger exposure  $b_a$  to the systematic component, or a higher volatility  $\sigma_{ea}$  of their idiosyncratic component, observe a larger increase in their ERC as uncertainty and investor attention increase. Panel (b) of Figure 2 revisits our economy with three announcers. It confirms that ERCs increase with uncertainty and that firms with stronger exposure to the systematic components have higher ERCs.

Eq. (27) implies that ERCs are driven both by the exogenous increase in uncertainty and the endogenous increase in investor attention and that the two effects compound each other. We disentangle these two effects in Figure 3. The gray bars depict the impact on ERCs of an increase in U. The hashed bars include the additional impact of the increase in investor attention, confirming the direct and indirect effects from (27). Note that in this example the ERC of the third announcer increases from zero to a positive value only through the indirect effect of an increase in attention.

(Insert Figure 3 about here)

We now turn to other dimensions of heterogeneity across firms and summarize the results in Figure 4. Panels (a) and (d) analyze the effect of the volatility of the idiosyncratic component,  $\sigma_{e1} > \sigma_{e2} > \sigma_{e3}$  (while all other parameters are constant across firms). Eqs. (26) and (27) imply that firms with higher  $\sigma_{ea}$  should observe stronger investor attention and ERCs to their announcements because the informativeness of an earnings announcement,  $Var[D_a]/\sigma_{\varepsilon a}^2$ , is higher for firms with higher  $\sigma_{ea}$ . Thus, investors focus on those firms first after an increase in uncertainty. Panels (a) and (d) confirm these effects for the fractions of informed investors and ERCs.

#### (Insert Figure 4 about here)

Assuming that firms differ through the noise in their signals,  $\sigma_{\varepsilon 1} < \sigma_{\varepsilon 2} < \sigma_{\varepsilon 3}$ , implies that the signal of firm 1 is more valuable for investors for the same reason as above:  $E_1$  is more informative about f than  $E_2$ , which itself is more informative than  $E_3$ . Panels (b) and (e) of Figure 4 illustrate this. Finally, we also analyze the case of different noisiness of supply. Panels (c) and (f) consider an economy in which  $\sigma_{x1} > \sigma_{x2} > \sigma_{x3}$  and show that after an increase in U, investors turn their attention more to firm 1, causing an increase in ERCs. The intuition stems from price informativeness: the equilibrium prices of firms with more substantial noise in supply reveal less information to investors, which increases the ex-ante increase to acquire information from earnings (as in Grossman and Stiglitz, 1980). This intuition explains the greater attention and stronger ERCs for firms with higher  $\sigma_{xa}$ .

To summarize, the testable implications of our model concerning the impact of uncertainty on investor attention and on ERCs are: (i) when uncertainty increases, investors focus on earnings announcements of a larger number of firms, and more investors pay attention to each announcing firm; (ii) investors' incentives to pay attention to earnings announcements decrease with the number of firms that announce their earnings simultaneously; (iii) when uncertainty (investor attention) increases, ERCs strengthen for all announcing firms; and (iv) increases in ERCs caused by higher uncertainty (investor attention) are incrementally stronger for firms with higher  $b_a$ , higher  $\sigma_{ea}$ , lower  $\sigma_{\varepsilon a}$ , and higher  $\sigma_{xa}$ .

### 2.5 Implications for the CAPM

We now turn to the implications of investor attention for the CAPM. The derivation of a model-implied CAPM on earnings announcement days requires endogenous prices at time 0. Thus, maintaining the same model assumptions as in the previous analysis, we assume that at time 0 agents trade in the market and observe additional information. (The type of this information—public or private—is inconsequential for the results derived here.) As such, time 0 and time 1 represent the close of two consecutive trading days, with earnings being announced on the second day. Denoting equilibrium prices at times 0 and 1 by  $\mathbf{P}_0$  and  $\mathbf{P}_1$ , asset returns on the announcement day are  $\mathbf{R}_1^e \equiv \mathbf{P}_1 - \mathbf{P}_0$ .

At time 0, all agents observe a publicly available signal about the aggregate market payoff,

$$G = \mathbf{M}'\mathbf{D} + g$$
, where  $g \sim \mathcal{N}(0, \sigma_g^2)$ , (28)

where the noise in the public signal g is independent of all the random variables previously defined. In an economy with a large number of firms (i.e., when  $N \to \infty$ ), one can interpret G as a signal about the systematic component f.

As in the baseline model, noise traders at time 0 have inelastic demands of  $\mathbf{x}_0$  shares, with  $x_{0,n} \sim \mathcal{N}(0, \sigma_{xn}^2)$ , and we denote noise trading at time 1 by  $\mathbf{x}_1$ , which is defined as before. Thus, the total supply of assets available for trading to informed investors is  $\mathbf{M} - \mathbf{x}_0$  at time 1 and  $\mathbf{M} - \mathbf{x}_0 - \mathbf{x}_1$  at time 1. This follows He and Wang (1995) and Brennan and Cao (1997).

To summarize, in this slightly modified setup, investors trade before and after earnings announcements, making their information acquisition decision at any time between 0 and 1. Then the following proposition describes investor asset demands and the risky asset prices at each market session in this model.

**Proposition 4.** There exists a partially revealing rational expectations equilibrium in the two trading session economy in which

(i) Individual asset demands for a type-k investor are given by:

$$\mathbf{q}_0 = \frac{1}{\gamma} \boldsymbol{\tau}_0(\mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0), \qquad (29)$$

$$\mathbf{q}_1^k = \frac{1}{\gamma} \boldsymbol{\tau}_1^k (\mathbb{E}_1^k [\mathbf{D}] - \mathbf{P}_1), \tag{30}$$

where  $\boldsymbol{\tau}_0 \equiv \operatorname{Var}[\mathbf{D}|\mathcal{F}_0]^{-1}$  and  $\boldsymbol{\tau}_1^k \equiv \operatorname{Var}[\mathbf{D}|\mathcal{F}_1^k]$ ,  $\mathcal{F}_0 = \{G\}$ , and  $\mathcal{F}_1^k = \{G\} \cup \mathcal{F}^k$ , with  $\mathcal{F}^k$  defined in (12).

(ii) The vectors of risky asset prices are given by

$$\mathbf{P}_{0} = \frac{1}{\sigma_{q}^{2}} \boldsymbol{\tau}_{0}^{-1} \mathbf{M} G - \gamma \boldsymbol{\tau}_{0}^{-1} (\mathbf{M} - \mathbf{x}_{0}), \qquad (31)$$

$$\mathbf{P}_{1} = \boldsymbol{\tau}_{1}^{-1} \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^{k} \boldsymbol{\tau}_{1}^{k} \mathbb{E}_{1}^{k} [\mathbf{D}] - \gamma \boldsymbol{\tau}_{1}^{-1} (\mathbf{M} - \mathbf{x}_{0} - \mathbf{x}_{1}),$$
(32)

where  $\boldsymbol{\tau}_1 \equiv \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k \boldsymbol{\tau}_1^k$ .

The proof is provided in Appendix A.8 and follows He and Wang (1995) and Brennan

and Cao (1997), adapted to our Grossman and Stiglitz (1980) setup. Proposition 4 leads to a CAPM relation, which we describe in the following corollary.

**Corollary 4.1.** (CAPM) Define the market excess return as  $\mathbf{R}^{e}_{\mathbf{M}} = \mathbf{M}'\mathbf{R}^{e}$ . The following CAPM relation holds on earnings announcement day:

$$\mathbb{E}[\mathbf{R}^e] = \boldsymbol{\beta} \mathbb{E}[\mathbf{R}^e_{\mathbf{M}}], \quad \text{with } \boldsymbol{\beta} = \frac{(\boldsymbol{\tau}_0^{-1} - \boldsymbol{\tau}_1^{-1})\mathbf{M}}{U_0^2 - \mathbf{M}' \boldsymbol{\tau}_1^{-1}\mathbf{M}},$$
(33)

where the market risk premium is given by

$$\mathbb{E}[\mathbf{R}_{\mathbf{M}}^{e}] = \gamma U_{0}^{2} - \gamma \mathbf{M}' \boldsymbol{\tau}_{1}^{-1} \mathbf{M}, \qquad (34)$$

and  $U_0^2 \equiv \mathbf{M}' \boldsymbol{\tau}_0^{-1} \mathbf{M}$  represents the market-wide uncertainty (variance) that investors face before making information decisions and before the earnings announcements.

Eq. (34) shows that the market risk premium is made up of two terms. The first term increases with  $U_0^2$ , which is a direct measure of the uncertainty investors face before earnings are announced. This ex-ante uncertainty increases with both U and with  $\sigma_g$ , as can be intuitively understood by considering an economy with a large number of firms:

$$\lim_{N \to \infty} U_0^2 = \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2}.$$
(35)

Investors' attention to earnings announcements governs the second term in (34). Without attention (if  $\Lambda_a = 0 \ \forall a \in \mathcal{A}$ ),  $\tau_1 = \tau_0$  and the market risk premium is zero—that is, buying the market portfolio at time 0 and selling it at time 1 involves on average no risk. However, when investors are attentive,  $\mathbf{M}' \tau_1^{-1} \mathbf{M}$  decreases with investor attention  $(\partial \mathbf{M}' \tau_1^{-1} \mathbf{M} / \partial \Lambda_a < 0 \ \forall a \in \mathcal{A}$ ; see Appendix A.8) and yields a positive risk premium. Investors earn a risk premium by paying attention because they are rewarded for resolving uncertainty (Robichek and Myers, 1966; Epstein and Turnbull, 1980). Eqs. (33)-(34), thus, lead to the following prediction: a higher level of ex-ante uncertainty  $U_0$  and heightened investor attention to firmlevel news contribute to a higher risk premium and a steeper market beta-return relation.

Corollary 4.1 yields additional predictions about the market betas of announcing firms. These predictions emerge most transparently in a large economy in which, as  $N \to \infty$ , firms' market betas converge to (the predictions do not hinge on taking this limit, but the intuition is easier to convey in a large economy; see Appendix A.8):

$$\lim_{N \to \infty} \boldsymbol{\beta} = \mathbf{b} + h \begin{bmatrix} \frac{\pi_1(\Lambda_1)\sigma_{e_1}^2}{1 + \pi_1(\Lambda_1)\sigma_{e_2}^2} b_1 \\ \frac{\pi_2(\Lambda_2)\sigma_{e_2}^2}{1 + \pi_2(\Lambda_2)\sigma_{e_2}^2} b_2 \\ \vdots \\ \frac{\pi_A(\Lambda_A)\sigma_{e_A}^2}{1 + \pi_A(\Lambda_A)\sigma_{e_A}^2} b_A \\ \mathbf{0}_{N-A} \end{bmatrix},$$
(36)

where h > 0 and the scalars  $\pi_a(\Lambda_a)$ ,  $a \in \mathcal{A}$  are defined in Lemma 1 and are increasing in  $\Lambda_a$ .

Eq. (36) has two predictions. First, betas are stronger for announcing firms. Consider two firms, one announcer and one non-announcer, with the same exposure to the systematic factor  $b_a = b_n > 0$ . The last term in (36) shows that the beta of the announcing firm increases on its announcement date (Patton and Verardo, 2012; Chan and Marsh, 2021b). Second, and more specific to our information acquisition setting, investor attention is the channel through which the announcing firm's beta increases. Without attention,  $\pi_a(0) = 0$ , and the betas of the two firms remain the same. On the other hand, when attention is positive the announcing firm's beta increases proportionally to the fraction of investors who pay attention to its announcement.

## **3** Additional implications and extensions

### **3.1** Heterogeneous attention costs

Our analysis so far has focused on an economy in which firms are heterogeneous, but investors are ex-ante identical. In reality, different investors may have different information acquisition costs. For instance, institutional owners presumably have lower information acquisition costs than retail investors. When choosing whether to pay attention to firm-level information, an institutional investor's alternative is generally to pay attention to a different financial signal or other job-related tasks (e.g., human resources, calling investors). In addition, institutional investors subscribe to services that lower the direct costs of information acquisition. In contrast, retail investors pay attention to a primary job, family matter, hobby, or the back of their eyelids, which may carry higher opportunity costs.

To study the implications of heterogeneous information costs, we extend our model to two groups of investors, with information costs  $c_l < c_h$ . (These low-cost  $(c_l)$  and high-cost  $(c_h)$  investors can be thought of in different ways, such as institutions vs. individuals, local vs. non-local investors, or industry-focused vs. generalist investors.) The additional layer of heterogeneity requires re-writing the equilibrium conditions of Theorem 1 separately for each investor group. Importantly,  $c_l < c_h$  implies that

$$B_{l,k}^{k \cup \{a\}} > B_{h,k}^{k \cup \{a\}}, \quad \forall k \in \mathscr{P}(\mathcal{A}) \text{ and } a \notin k,$$
(37)

where  $B_{j,k}^{k\cup\{a\}} = \exp(-2\gamma c_j|k|) \det(\boldsymbol{\tau}^{k\cup\{a\}}) / \det(\boldsymbol{\tau}^k)$  for  $j \in \{l, h\}$ . In words, paying attention to one extra announcement has a larger net benefit for a low-cost investor than for a highcost investor. The condition (37), labeled "monotonicity in types" by Hu and Shi (2019), guarantees the existence of an equilibrium and ensures that the solution method described in Appendix A.7 reaches the equilibrium.

Figure 5 plots the attention of low-cost (left) and high-cost (right) investors as functions of uncertainty. We use the same calibration with  $b_1 > b_2 > b_3$  as in Figure 1, split the population of investors into 50% low-cost and 50% high-cost (other splits lead to similar results), and fix  $c_l = 0.045$  and  $c_h = 0.055$ . The two panels show that for any level of uncertainty, larger fractions of low-cost investors pay attention to the earnings announcements. The steeper lines in the left-hand side plot suggest that low-cost investors respond faster to the increase in uncertainty than high-cost investors, confirming the intuition from (37) that low-cost investors benefit comparatively more from increasing their attention.

Assuming different attention costs has further implications for ERCs. As shown in (27), ERCs increase with the amount of attention in the economy, which implies that the *investor* base of firms has an impact on ERCs: ERCs for firms with high ownership by low-cost investors should show a more robust response to an increase in uncertainty, through the stronger increase in attention. We test this theoretical implication in Section 4.

## 3.2 Dynamic model

We have derived our main results under the simplifying assumption of a one-period economy. This section shows that the same comparative statics results hold in a dynamic setup with time variation in uncertainty. The dynamic setup consists of an overlapping-generations economy in which a new generation of investors is born every period. We refer to the generation of investors born at time t as generation t. Each generation is present in the economy for three dates and makes information acquisition and trading decisions sequentially, as in the static model. Focusing on generation t - 1, each investor  $i \in [0, 1]$  makes an information acquisition choice between t-1 and t, trades to take positions in securities at t, and consumes final wealth at t + 1. As such, generation t - 1 investors liquidate their holdings at time t + 1 by selling them at market prices to generation t investors. Figure 6 shows the timeline.

#### (Insert Figure 6 about here)

Our primary purpose is to understand how the dynamic feature of the economy impacts the results obtained in the previous section. For this purpose, it is sufficient to assume that investors trade a single risky asset and a riskless asset. (Assuming multiple risky assets would considerably complicate the analysis without additional insights.) The riskless asset is in infinitely elastic supply and pays a gross interest rate of  $R_f > 1$  per period. The risky asset pays a risky dividend per period,

$$D_{t+1} = bf_{t+1} + e_{t+1}, (38)$$

which, as in (1), has two components: a systematic component,  $f_{t+1} \sim \mathcal{N}(0, U_t^2)$ , and a firm-specific component,  $e_t \sim \mathcal{N}(0, \sigma_e^2)$ .

The key difference with the static model is that we allow for economic uncertainty,  $U_t$ , to be time-varying. More precisely, we assume that  $U_t$  takes one of  $S \ge 2$  possible values,  $u_s, s \in \{1, ..., S\}$ , and we denote the probability of the event  $U_t = u_s$  by  $p_s$ . Furthermore,  $U_t$  is observable to generation t - 1 investors, who make an information acquisition choice between t - 1 and t and trade in the market at t. One could assume, for instance, that  $U_t$  is revealed at time  $t - \epsilon$ , where  $\epsilon$  is very small (e.g., a fraction of a second). This assumption preserves the sequence of the information acquisition and trading decisions, as in Grossman and Stiglitz (1980).

At time t, the firm issues an earnings announcement,

$$E_t = D_{t+1} + \varepsilon_t, \tag{39}$$

with  $\varepsilon_t \sim \mathcal{N}(0, \sigma_{\varepsilon}^2)$ . We denote the investors who pay attention to  $E_t$  as I investors, and those who decide to remain uninformed as  $\emptyset$  investors. The indicator variable  $I^k$  takes the value 1 if k = I and 0 if  $k = \emptyset$ . The cost of paying attention to  $E_t$  is c > 0.

Each investor  $i \in [0, 1]$  of generation t - 1 starts with zero initial wealth and maximizes expected utility:

$$\max_{k \in \{I,\emptyset\}} \mathbb{E}_{t-1} \left[ \max_{q_t^k} \mathbb{E}_t^k \left[ -e^{-\gamma \left( W_{t+1}^k - cI^k \right)} \right] \right], \tag{40}$$

where  $W_{t+1}^k \equiv q_t^k (D_{t+1} + P_{t+1} - R_f P_t) \equiv q_t^k R_{t+1}^e$  is type k investor's terminal wealth.

The risky asset demand of liquidity (noise) traders equals  $x_t$ , with  $x_t$  being independently and identically distributed,  $x_t \sim \mathcal{N}(0, \sigma_x^2)$ . We conjecture the following linear structure for the price, which is the dynamic equivalent of (10) from the static version of the model:

$$P_t = \alpha_t E_t + \xi_t x_t. \tag{41}$$

The equilibrium in this dynamic model follows the same steps as in the static model.<sup>8</sup> One difficulty is that time variation in uncertainty creates a non-linearity. With time variation in U, the distribution of the future price  $P_{t+1}$  becomes non-Gaussian, and thus the equilibrium can only be solved using an approximation. The method commonly used in the literature (Vayanos and Weill, 2008; Gârleanu, 2009) preserves risk aversion towards diffusion risks while inducing risk neutrality towards future changes in U, restoring linearity. We refer the reader to Appendix A.9 for details and proceed here to discuss the main results.

**Proposition 5.** (a) Investor i is attentive to the earnings announcement if and only if

$$\frac{\operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]}{\operatorname{Var}_{t}^{I}[R_{t+1}^{e}]} > e^{2\gamma c}.$$
(42)

(b) The benefit of information,  $\operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]/\operatorname{Var}_{t}^{I}[R_{t+1}^{e}]$ , increases in  $\operatorname{Var}_{t}[D_{t+1}] = b^{2}U_{t}^{2} + \sigma_{e}^{2}$ .

We recover the same result as in the static model: the benefit of paying attention to  $E_t$  increases with economic uncertainty. Moreover, the benefit of attention is higher when b is higher and when the volatility  $\sigma_e$  of the idiosyncratic component is higher. Thus, we should observe stronger investor attention when economic uncertainty is high in the dynamic model, as in the static model.

The next proposition shows that in the dynamic model the ERC is a weighted average of price responses from different investors, with weights  $w_t$  on I investors and  $1 - w_t$  on  $\emptyset$ investors (see Appendix A.9 for an expression of  $w_t$  in  $\Lambda_t$ ), as in Hirshleifer and Teoh (2003).

**Proposition 6.** The earnings response coefficient in this economy is given by

$$\operatorname{ERC}_{t} = \frac{w_{t}}{R_{f}} \frac{\operatorname{Var}_{t}[D_{t+1}]}{\operatorname{Var}_{t}[D_{t+1}] + \sigma_{\varepsilon}^{2}} + \frac{1 - w_{t}}{R_{f}} \frac{\operatorname{Var}_{t}[D_{t+1}]}{\operatorname{Var}_{t}[D_{t+1}] + \sigma_{\varepsilon}^{2}/\ell_{t}},\tag{43}$$

where  $w_t \in [0, 1]$ ,  $\ell_t \in [0, 1)$ . Both  $w_t$  and  $\ell_t$  are increasing with the fraction  $\Lambda_t$  of investors who pay attention to  $E_t$ . Thus, the earnings response coefficient increases in  $\Lambda_t$ .

<sup>&</sup>lt;sup>8</sup>Dynamic models of trading of this type have multiple equilibria, i.e., a model with N risky assets has  $2^N$  equilibria (e.g. Banerjee, 2011; Andrei, 2018). Thus, this model has two equilibria: a low-volatility equilibrium and a high-volatility equilibrium. The results that we present here hold in both equilibria.

In (43),  $\ell_t$  is the dynamic counterpart of the learning coefficient defined in (13) for the static model. Two effects take place when uncertainty increases. The first effect is an increase in both terms of (43) through  $\operatorname{Var}_t[D_{t+1}]$ . The second effect follows from Proposition 5. The increase in economic uncertainty increases investor attention, and therefore both  $w_t$  and  $\ell_t$  increase, further strengthening the ERC. We thus recover the intuition from the static model: the ERC increases with economic uncertainty, both directly through an increase in the variance of the firm's payoff  $\operatorname{Var}_t[D_{t+1}]$  and indirectly through an increase in investor attention. The two effects are stronger for firms with a higher b or idiosyncratic volatility  $\sigma_e$ .

Our focus on investor attention to earnings announcements is motivated by both the existing literature on investor attention and the notion that earnings announcements convey valuable information about the macroeconomy (e.g., Patton and Verardo, 2012; Savor and Wilson, 2016). However, our model also accounts for the possibility that investors may scale up their information acquisition ahead of the earnings announcements. In particular, Proposition 5 shows that regardless of prior information acquisition decisions, the benefit of paying attention to the earnings announcement increases with uncertainty at time t. Moreover, Proposition 6 shows that greater investor attention increases  $w_t$  and  $\ell_t$ , strengthening the ERC. Although investors' search for information beforehand may dampen the effect of an increase in uncertainty on the conditional variance  $\operatorname{Var}_t[D_{t+1}]$ , the second effect characterized in Proposition 6 still guarantees that heightened investor attention increases ERCs.<sup>9</sup>

## 4 Empirical analyses

In this section, we conduct empirical tests of our theoretical predictions regarding the effect of aggregate uncertainty on investors' information acquisition, on ERCs, and on the CAPM. In our first set of tests, we examine the relation between uncertainty and investor attention around the announcement of quarterly earnings. That is, the unit of measurement in our analyses is the quarterly earnings announcement.

## 4.1 Variable definitions and summary statistics

We use the VIX, an option-based measure of expected S&P 500 volatility, to measure timevarying market-wide uncertainty. The VIX proxies for forward-looking stock market uncer-

<sup>&</sup>lt;sup>9</sup>Benamar, Foucault, and Vega (2021) show that heightened attention in the face of greater uncertainty does not fully neutralize the effect of uncertainty (see their Proposition 1). Put differently, a higher uncertainty at time t-1 results in a higher  $\operatorname{Var}_t[D_{t+1}]$ , despite investors' heightened attention at t-1. Their argument follows from the first-order condition in a standard information acquisition problem with convex attention costs. Since the marginal cost of attention increases with attention, the effect of the increase in uncertainty on  $\operatorname{Var}_t[D_{t+1}]$  is only partially offset by investors' heightened attention.

tainty, risk, or volatility and its direct counterpart in our model is  $U^{10}$ . To mitigate the potential for reverse causality, we use the closing *VIX* from the trading day prior to the earnings announcement.

To capture investor search for information, we exploit the SEC's EDGAR download logs. EDGAR is a publicly available central repository for companies' SEC filings. The SEC makes the records of EDGAR search activity public, where a search is defined as accessing a given filing. We use the natural log of the company-day total volume of completed EDGAR searches, ESV, as a search-driven proxy for investor attention. Completed EDGAR searches are those that result in successful delivery of the requested document (code=200), which is not an index page (idx=0). We also use the natural log of the number of downloads of a company's filings from unique IP addresses, ESVU, to capture the extensive margin of investor search based on the number of investors accessing the firm's filings. The EDGAR search records are available from February 14, 2003 to June 30, 2017.<sup>11</sup> Note that a change in ESV(U) is equivalent to a change in  $\log \Lambda_a$  in our model.<sup>12</sup>

A secondary measure of investor search is the Investor Search Volume Index (ISVI) based on investor searches for stock tickers via Google, as calculated and generously provided by deHaan et al. (2021). We view ISVI as a secondary measure as it is available only from 2010 to 2018 and for a smaller sample of firms, and is a 0 - 100 index rather than a more easily interpretable raw count of searches.

As in prior studies (e.g., Livnat and Mendenhall, 2006; Hirshleifer et al., 2009; DellaVigna and Pollet, 2009), we use standardized earnings surprise (*SUE*) deciles based on calendarquarter sorts in our analyses of market reactions to earnings announcements. Our inferences remain similar when we use raw *SUEs* instead of *SUE* deciles. We measure earnings surprises as  $SUE_{i,t} = (X_{i,t} - \mathbb{E}[X_{i,t}])/P_{i,t}$ , where *i* denotes firm, *t* denotes quarter,  $X_{i,t}$  are IBES reported actual earnings,  $\mathbb{E}[X_{i,t}]$  are expected earnings, taken as the latest median forecast from the IBES summary file (following Dai, 2020), and  $P_{i,t}$  is the share price at the end of quarter *t*. Daily excess returns are calculated as CRSP-reported daily returns adjusted for size

<sup>&</sup>lt;sup>10</sup>Formally, denoting by **b** the vector of firms' exposures to f and by  $\Sigma_e$  the covariance matrix of firmspecific shocks, then investors' uncertainty at t = 0 about the future market return,  $\operatorname{Var}[\mathbf{M'R}^e|\mathcal{F}_0]$ , equals  $U^2\mathbf{M'bb'M} + \mathbf{M'\Sigma}_e\mathbf{M}$ . Our assumptions of an equally weighted market portfolio and an average of 1 for firms' exposures to the systematic factor imply  $\mathbf{M'b} = 1$ . Moreover, the matrix of idiosyncratic shocks  $\Sigma_e$  is diagonal, and its diagonal has a finite mean, thus  $\lim_{N\to\infty} \mathbf{M'\Sigma}_e\mathbf{M} = 0$  and  $\operatorname{Var}[\mathbf{M'R}^e|\mathcal{F}_0] = U^2$ .

<sup>&</sup>lt;sup>11</sup>EDGAR downloads may come from humans or from automated programs or robots (e.g., Ryans, 2017). We use all downloads for three reasons: 1) automated downloads may be used by services that provide information to investor clients; 2) automated downloads may be programmed to access EDGAR files conditional on other inputs to the program capturing, for instance, macroeconomic conditions; and 3) our use of year fixed effects in regressions controls for a secular trend of increasing robot downloads over time.

<sup>&</sup>lt;sup>12</sup>In the model,  $\Lambda_a$  can be approximated with  $Q_a/Q$ , where Q is a large number that measures the total population of investors and  $Q_a$  measures the number of investors who observe  $E_a$ . Hence,  $\Delta \log \Lambda_a = \Delta \log Q_a$ , and thus a change in  $\log \Lambda_a$  is equivalent to a change in ESV(U).

decile.<sup>13</sup> Earnings announcement returns, *EARET*, are calculated as the two-day compounded excess returns from the day of the earnings announcement through the day after.

In our analyses of market reactions to earnings announcements, we use the following variables as controls, following prior literature (e.g., Hirshleifer et al., 2009): compound excess returns from ten to one days before the earnings announcement, *PreRet*; the market value of equity on the day of the earnings announcement, *Size*; the ratio of book value of equity to the market value of equity at the end of the quarter for which earnings are announced, *Book-to-Market*; earnings persistence based on estimated quarter-to-quarter autocorrelation in reported earnings, *EPersistence*; institutional ownership as a fraction of total shares outstanding at the end of the quarter for which the earnings are announced, *IO*; earnings volatility, *EVOL*; the reporting lag measured as the number of days from quarter end to the earnings announcement, *ERepLag*; analyst following defined as the number of analysts making quarterly earnings forecasts according to the IBES summary file, *#Estimates*; average monthly share turnover over the preceding 12 months, *TURN*; an indicator variable for negative earnings, *Loss*; the number of other firms announcing earnings on the same day, *#Announcements*; year indicators; and day-of-week indicators.<sup>14</sup>

Our subsample analyses use partitions based on proxies for the underlying constructs. Although the exposures of firms' payoffs to the systematic factor f (the parameters  $b_n$ ) are not perfectly observed in the data, they can be proxied by firms' CAPM betas. More precisely, in our model firms with larger exposures to f necessarily have higher market betas (we provide this link in Eq. (36)). We use forecast dispersion (*DISP*) and idiosyncratic volatility (*IDVOL*), defined in detail in Appendix B, as proxies for total earnings variance (Var $[E_a]$ in our model) and firm-specific payoff variance ( $\sigma_{ea}^2$ ).<sup>15</sup> The volatility of noise trade ( $\sigma_{xa}^2$ ) is reflected in share turnover (*TURN*), though we caution that turnover also captures other constructs, such as information asymmetry and disagreement. Finally, we split the sample on institutional ownership (*IO*) to capture variation in the cost to investors of acquiring information (c), as these costs are likely to be lower for institutional than retail owners. We provide detailed variable definitions in Appendix B.

Our sample begins in 1995, as earnings announcement dates tended to be identified unreliably prior to 1995 (DellaVigna and Pollet, 2009; Hirshleifer et al., 2009). We further limit our sample to firms for which we can calculate analyst forecast-based earnings surprises,

<sup>&</sup>lt;sup>13</sup>Our main results on earnings announcement window returns are robust to defining excess daily returns as firm-specific returns adjusted for either equal-weighted or value-weighted market returns.

<sup>&</sup>lt;sup>14</sup>To mitigate the influence of outliers among skewed/fat-tailed controls, we winsorize *Size*, *EPersistence*, and *EVOL* at the first and 99th percentiles.

<sup>&</sup>lt;sup>15</sup>Note that Forecast Dispersion could be driven by variation and unpredictability in either earnings fundamentals (Var[ $D_a$ ] =  $b_a^2 \sigma_f^2 + \sigma_{ea}^2$ ) or earnings noise ( $\sigma_{\varepsilon a}$ ). As can be seen in a comparison of panels (d) and (e) of Figure 4,  $\sigma_{ea}^2$  and  $\sigma_{\varepsilon a}^2$  have opposing effects on the relation between uncertainty and ERCs.

firms with a stock price greater than \$5, and firms with average monthly share turnover in the past year no lower than 1. The latter restrictions drop the smallest and least actively traded firms from the sample. Finally, we restrict the sample to observations for which data for all variables used in the respective analyses are available. This results in a sample of 224,675 firm-quarter observations for the analyses that do not require data on investor attention measures and 119,341 (62,757) for the analyses that require data availability on EDGAR (Google) searches. Table 1 provides the descriptive statistics for the variables used in our analyses.<sup>16</sup>

#### (Insert Table 1 about here)

Table 2 provides correlations. All correlations in bold are significant at the one percent level. VIX is negatively correlated with EDGAR search volume measures and *ISVI*, but these are raw correlations that do not correct for other factors, such as time factors affecting both VIX and search volume (e.g., higher VIX and lower search in some years). VIX is not generally significantly related to earnings announcement returns or earnings surprises, suggesting that prior-day economic uncertainty is not directly linked to firm-level earnings surprises.

(Insert Table 2 about here)

## 4.2 Attention and earnings response coefficients

As we elaborate on in Section 2, our first hypotheses relate to the effects of economic uncertainty on investor attention to firm-level information, which we test for using investor searches and market reactions around earnings announcements.

Our first set of tests examines whether aggregate uncertainty affects firm-level search activity in and of itself. For these tests, we exploit the SEC EDGAR records of access to company-specific filings around quarterly earnings announcements as well as investors' Google searches captured by *ISVI*. We estimate the following regression equation:

$$SEARCH_{it} = c_0 + c_1 \times VIX_t + c_2 \times ESV_{it-1} + c_3 \times SUE_{it} + c_4 \times abs (SUE_{it}) + \gamma \cdot X_{it} + u_{it},$$
(44)

where SEARCH is either the log of daily EDGAR search volume (ESV), the log of daily EDGAR search volume from unique IP addresses (ESVU), or ISVI. We also include the

<sup>&</sup>lt;sup>16</sup>The number of observations for some variables in Table 1 is greater than 224,675, in part because we require some lagged variables to be non-missing in the regression tests but not in Table 1.

lagged dependent variable (ESV, ESVU, or ISVI on the previous earnings announcement), the standardized SUE decile, and the absolute standardized SUE decile to control for differences in average search volume across firms and in response to earnings news. Table 3 presents the results from the estimation of (44). In Table 3 and the remaining tests, we standardize all variables to a mean of zero and unit variance for ease of interpretation.

The results in Table 3 provide strong evidence for more active searching for firm-level information on days with higher VIX, as the coefficients of interest on VIX are positive and statistically significant for all three dependent variables. The coefficients of interest can be interpreted as the approximate percent change in search volume or unique searchers for a standard deviation change in the VIX. A one standard deviation change in VIX is associated with a 3.0 (3.4) percent increase in the number of EDGAR searches (from unique IP addresses) for the announcer's filings on the earnings announcement date, and a 1.8 percent increase in ISVI relative to its standard deviation (recall that ISVI is an index rather than a logged count variable as for ESV(U)).<sup>17</sup> Lagged dependent variables are significantly associated with announcement day searches, as are the signed and absolute earnings surprise deciles (except for absolute SUE in the ESV specification). Coefficients on #Announcements are negative, though only statistically significant in the ESV and ISVI specifications, providing support for the effect of multiple announcements shown in Hirshleifer et al. (2009). In the remainder, we focus on EDGAR search volume measures (ESV and ESVU), as these are available for a longer time span covering roughly twice the number of earnings announcements as ISVI.

#### (Insert Table 3 about here)

Our next set of tests exploits the model's predictions regarding price reactions to firmlevel information. We examine how economic uncertainty interacts with firm-level news in the price formation process. We focus on the association between size decile-adjusted stock returns in the two-day earnings announcement window and the earnings surprise, the *VIX*, the interaction between the *VIX* and the earnings surprise, and a set of controls. We interact each of these controls with our earnings surprise variable to mitigate concerns that a correlated omitted interaction drives the coefficient on our interaction of interest. Standard errors are clustered at the earnings announcement date level.

To test the hypotheses developed in Section 2, we estimate the following regressions at

<sup>&</sup>lt;sup>17</sup>In unreported analysis replacing VIX with VIX centile indicators in the specifications presented in Table 3 (i.e.,  $SEARCH_{it} = c_0 + \sum_{j=1}^{100} c_{1j} \times VIX$  Centile<sub> $ij</sub> + \cdots$ ), we find that the relation between VIX and attention measures is convex, consistent with the convexity in Figure 2(a) when attention to announcements begins increasing from around  $U \in [0.19, 0.32]$ . We focus on the linear empirical effect identified by estimating Eq. (44) for ease of interpretation.</sub>

the firm-quarter level:

$$EARET_{it} = c_0 + c_1 \times SUE_{it} + c_2 \times VIX_t + c_3 \times SUE_{it} * VIX_t + \gamma \cdot X_{it} + u_{it}, \text{ and}$$
$$EARET_{it} = c_0 + c_1 \times SUE_{it} + c_2 \times ESVU_t + c_3 \times SUE_{it} * ESVU_t + \gamma \cdot X_{it} + u_{it}, \quad (45)$$

where the dependent variable  $EARET_{it}$  represents the announcement-window return and  $X_{it}$  represents a set of controls.

Column (1) of Table 4 reports our estimates of the first equation in (45).<sup>18</sup> The coefficient on *SUE* decile is positive and significantly different from zero (0.204, p < 0.01), consistent with positive market responses to earnings surprises. Our coefficient of interest, the interaction between *VIX* and *SUE*, is also positive and significantly different from zero (0.015, p < 0.01). We infer from this that market responses to firm-level information are higher on days with greater uncertainty. Specifically, a one standard deviation change in *VIX* yields an ERC that is approximately seven percent higher than the average response to earnings surprises (7% = 0.015/0.204).

Columns (2) and (3) of Table 4 explore the mediating role of attention. In column (2), we replace VIX with ESVU. The sample shrinks considerably because EDGAR search data is available for a shorter window (2003-2017 relative to the earnings announcement sample from 1995 to 2020). Even with the smaller sample, the coefficient on ESVU\*SUE is positive and significant (0.028, p < 0.01), consistent with earnings announcements that attract greater investor attention receiving stronger market reactions in the announcement window. In column (c), we include both VIX and ESVU as well as their interactions with SUE. The coefficients of interest are both positive, although the ESVU\*SUE interaction (0.027, p < 0.01) is significant while the VIX\*SUE interaction (0.010, p > 0.10) becomes insignificant at traditional cutoffs. Overall, the coefficient pattern is consistent with the indirect effect of VIX on market responses, operating through investor attention allocation as reflected in EDGAR search activity, in line with the prediction of our model illustrated in Eq. (27) and Figure 3.<sup>19</sup>

<sup>&</sup>lt;sup>18</sup>We use SUE decile in our analyses. Empirically, raw SUE dispersion within the bottom decile is higher on high-VIX days. To ensure that this dispersion is not driving higher ERCs, we re-ran our Table 4 analyses on a subsample that excludes the bottom SUE decile and obtained similar results (untabulated, available from the authors).

<sup>&</sup>lt;sup>19</sup>Our results are consistent with Drake et al. (2015), who also find a positive association between EDGAR search volume and ERCs. Additionally, they present evidence that EDGAR search volume around the earnings announcement is associated with less post-earnings announcement drift (*PEAD*). While we do not find a similar effect on average, we find in untabulated analysis a moderate negative association between EDGAR search volume and near-term *PEAD* for firms with above-median CAPM beta. This is consistent with our theoretical prediction of stronger effects for firms with greater systematic risk and in line with the higher ERC for high-beta firms documented in our Table 6.

(Insert Table 4 about here)

We estimate (45) in several subsamples to provide additional support for the theoretical predictions derived above. As shown in Figures 4 and 5, the effect of macroeconomic uncertainty on ERCs is not generally monotonic in the splitting variables: the plots show monotonic relations for  $b_n$ ,  $\sigma_{\varepsilon n}$ , and  $\sigma_{xn}$ , but not for and  $\sigma_{en}$  and  $c_n$ .

Table 5 presents estimates from these cross-sectional splits, where the variable of interest is the  $VIX^*SUE$  interaction. In these tests, we split our sample from the annual median values of: CAPM beta, forecast dispersion (*DISP*), idiosyncratic volatility (*IDVOL*), trailing share turnover (*TURN*), and institutional ownership (*IO*).

In the CAPM beta split subsamples, the coefficient of interest has a positive sign but is not statistically significant for low-beta firms. In contrast, the coefficient for high-beta firms is positive and significantly different from both zero (p < 0.01) and the corresponding lowbeta coefficient (p < 0.10). This is consistent with our result in Figure 2(b), that the effect of economic uncertainty on ERCs is greater for firms with larger exposures to systematic risk.

#### (Insert Table 5 about here)

In the subsamples split on forecast dispersion and idiosyncratic volatility, the coefficients of interest are all positive and significantly different from zero (0.012-0.022, p < 0.05). However, they are not significantly different from each other.

For the splits using share turnover to capture the expected magnitude of noise trade,  $\sigma_{xa}$ , the effects of economic uncertainty on ERCs are concentrated in subsamples with abovemedian *TURN*. The coefficient on *VIX\*SUE* in the high-*TURN* sample is positive and significantly different from both zero (0.022, p < 0.01) and the coefficient in the low-*TURN* sample (0.04, p < 0.10 for the test of difference in coefficients). This plausibly captures the predicted positive effect shown in Figure 4, panel (f), where the effect of economic uncertainty on ERCs is greater when the volatility of noise trade is larger. Similar to noise trade in our model, high turnover can make it difficult to infer fundamental information from price, making attention to earnings incrementally more valuable during periods of high uncertainty.

Our last sample splits are based on institutional ownership (IO). It is plausible to assume that retail investors face greater opportunity costs than institutional investors when choosing whether to pay attention to firm-level information. Indeed, recent empirical evidence supports the view that retail investors are more susceptible to distractions than institutional investors (Israeli, Kasznik, and Sridharan, 2021; Da, Hua, Hung, and Peng, 2022). Consistent with this interpretation and our predictions illustrated in Figure 5, we find that the effect of economic uncertainty on ERCs is concentrated in the high-IO subsample (0.024, p < 0.01), while the estimated effect for the low-*IO* subsample is insignificantly different from zero (0.007, p > 0.10). The difference in coefficients is large in percentage terms (0.024/0.007 = 343%) and significantly different from zero at the 10% level, consistent with higher information acquisition costs reducing the effects of economic uncertainty on ERCs.

Table 6 re-estimates the regressions from Table 5 with ESVU replacing VIX, to provide evidence that the effects are attributable to attention rather than the VIX itself and other co-varying constructs, in line with Figure 3 from our theoretical analysis. The pattern is generally similar, albeit weaker, plausibly due to the smaller sample size. Interestingly, the results for the forecast dispersion and idiosyncratic volatility splits are stronger than those in Table 5, as the effect of ESVU on ERCs is concentrated in the high forecast dispersion (0.036, p < 0.01) and idiosyncratic volatility (0.034, p < 0.01) subsamples. These coefficients are also significantly different from those in the corresponding below-median subsamples (p < 0.10 for both), consistent with heightened uncertainty (Var $[D_a]$ ) leading to stronger relations between attention and ERCs.

(Insert Table 6 about here)

### 4.3 Calibration around earnings announcements

Can our model generate quantitatively similar attention responses to changes in economic uncertainty? To answer this question, we calibrate our model based on historical data. First, we match historical data on VIX: in our sample period from 1995 to 2020, the VIX averaged 20, with a daily standard deviation of 8.5. Then, we define  $U \equiv VIX/100$  (VIX values are quoted in percentage points) and standardize it, i.e.,  $\hat{U} \equiv (U-0.2)/0.085$ . In our illustration, we will allow U to take values between 0.1 and 0.4, since during our sample period the 10th and 90th percentile of VIX were 11.6 and 28.7, respectively.

In our sample the average number of firms per quarter is 2,264, and the average number of announcements per trading day is 53, with a standard deviation of 67.<sup>20</sup> To compare, Frederickson and Zolotoy (2016) report an average of 41 announcements per trading day with a standard deviation of 61, and Ferracuti and Lind (2021) report an average of 63 and a standard deviation of 83. Hirshleifer and Sheng (2022) report a higher average, 118, and a standard deviation of 79. These studies do not separately report the number of unique firms per year or quarter. Accordingly, we set the total number of firms in the economy as N = 3,000 and assume that between 10 and 100 firms announce their earnings on a given trading day. The remaining calibration parameters are:  $\gamma = 10$ ;  $\sigma_e = \sigma_{\varepsilon} = 0.4$  for all firms;

<sup>&</sup>lt;sup>20</sup>Note that this differs from the mean #Announcements at the firm-announcement level reported in Table 1, as a trading day with N announcements would be counted once in an average across trading days N times in an average across firm-announcements.

the market portfolio **M** is a vector whose values are all equal to 1/3000; the volatility of noise in supply is  $\sigma_x = 1/(3000 \times 4)$  for all firms (which ensures that the probability of having negative supplies is negligible); all the betas of the announcing firms are 1; and the cost of information is c = 0.03.<sup>21</sup>

Panel (a) of Figure 7 plots the response of  $\log \Lambda_a$  to a change in  $\hat{U}$ , or  $\partial \log \Lambda_a / \partial \hat{U}$ , in two cases: when 10 firms are announcing earnings (solid line) and when 100 firms are announcing (dashed line). On the horizontal axis we let U vary from 0.1 to 0.4, while the vertical axis measures the sensitivity of  $\log \Lambda_a$  to changes in  $\hat{U}$ , consistent with the coefficient  $c_1$  in (44). The plot shows that our calibrated model can match the numbers in Table 3. Furthermore, the model also correctly implies a lower coefficient when the number of announcers is higher (in which case price informativeness is higher), in line with the negative coefficients for #Announcements obtained in Table 3.

#### (Insert Figure 7 about here)

Panel (b) plots the model-implied ERCs as functions of U when 10 firms are announcing earnings (solid line) and when 100 firms are announcing (dashed line). Our model generates plausible magnitudes for ERCs, comparable with coefficients on *SUE* Decile in Table 4. The plot also shows that ERCs increase with U but are smaller when more firms announce earnings, consistent with panel (a) showing that attention is a substitute for price informativeness. (See also Chen et al., 2020, who document a similar substitution effect between the acquisition of private information and the supply of public information.)

## 4.4 CAPM tests

We now turn to the predictions of our model for the CAPM. Corollary 4.1 shows that the market risk premium is increasing in both ex-ante uncertainty and investor attention, which implies a steeper securities market line (SML). Eq. (36) further implies that firms' betas increase on earnings announcement days, but only if investors pay attention to announcements.

To test these predictions, we estimate firm and portfolio betas using classical Fama and MacBeth (1973) two-step regressions. In our firm-level estimation, we estimate betas separately for high-attention days and earnings dates. In particular, for each firm i, we define four indicator variables:  $\mathbf{1}_{\text{EA}}^{i}$  equals one on days when the firm i announces earnings;  $\mathbf{1}_{\text{HighAtt}}^{i}$  equals one on days when investor attention to firm i is high (i.e., time-detrended ESV(U))

<sup>&</sup>lt;sup>21</sup>To the best of our knowledge, the only attempt in the literature to estimate the parameters of Hellwig's (1980) noisy rational expectations model is Cho and Krishnan (2000). In line with the estimation in their Table 2, our calibration assumes that noise in supply is considerably smaller than noise in private information ( $\sigma_x \ll \sigma_{\varepsilon}$ ), and also a reasonable value of ten for the coefficient of risk aversion.

of firm *i* is above the sample median);  $\mathbf{1}_{\text{EA}}^{\text{high},i}$  equals one if both  $\mathbf{1}_{\text{EA}}^{i}$  and  $\mathbf{1}_{\text{HighAtt}}^{i}$  are one; and  $\mathbf{1}_{\text{EA}}^{\text{low},i}$  equals one if  $\mathbf{1}_{\text{EA}}^{i}$  is one and  $\mathbf{1}_{\text{HighAtt}}^{i}$  is zero. We then estimate three time-series regressions for each firm:

$$r_{i,t}^{e} = \alpha_{\text{Other}}^{i} + \alpha_{\Delta EA}^{i} \mathbf{1}_{\text{EA}}^{i} + \beta_{\text{Other}}^{i} r_{M,t}^{e} + \beta_{\Delta EA}^{i} (\mathbf{1}_{\text{EA}}^{i} \times r_{M,t}^{e}) + \varepsilon_{i,t}$$

$$(46)$$

$$r_{i,t}^{e} = \alpha_{\text{Other}}^{i} + \alpha_{\Delta A}^{i} \mathbf{1}_{\text{HighAtt}}^{i} + \beta_{\text{Other}}^{i} r_{M,t}^{e} + \beta_{\Delta A}^{i} (\mathbf{1}_{\text{HighAtt}}^{i} \times r_{M,t}^{e}) + \varepsilon_{i,t}$$
(47)

$$r_{i,t}^{e} = \alpha_{\text{Other}}^{i} + \alpha_{\Delta EA}^{low,i} \mathbf{1}_{\text{EA}}^{\text{low},i} + \alpha_{\Delta EA}^{high,i} \mathbf{1}_{\text{EA}}^{\text{high},i} + \beta_{\text{Other}}^{i} r_{M,t}^{e} + \beta_{\Delta EA}^{low,i} (\mathbf{1}_{\text{EA}}^{\text{low},i} \times r_{M,t}^{e}) + \beta_{\Delta EA}^{high,i} (\mathbf{1}_{\text{EA}}^{\text{high},i} \times r_{M,t}^{e}) + \varepsilon_{i,t},$$

$$(48)$$

where  $r_{M,t}^e$  is the excess return on the market and  $r_{i,t}^e$  is the excess return for firm  $i^{22}$ 

The first regression tests whether firm betas increase on earnings announcement days. That is,  $\beta_{\Delta EA}^i$  in (46) measures the change in the firm *i*'s beta on announcement days. The second regression investigates whether firm betas vary with investors' attention:  $\beta_{\Delta A}^i$  in (47) measures the change in the firm *i*'s beta on days when investors' attention to the firm's information is above its sample median. Finally, the third regression is a direct test of (36):  $\beta_{\Delta EA}^{high,i}$  in (48) measures the change in the firm *i*'s beta on earnings announcement days when investors' attention to the firm's information is above its sample median.

We estimate (46)-(48) for each firm, then compute averages betas across firms, together with their standard errors. Table 7 presents estimates. Column (1) confirms Patton and Verardo (2012)'s finding that firm betas increase on earnings announcement days. On average, betas increase by 0.081 (p < 0.05) on announcement days. Columns (2) and (3) show that, on average, firm betas increase with investor attention when attention is measured using ESV and ESVU. On days when the detrended ESV(U) is above its median, betas increase by 0.043, p < 0.01, (0.020, p < 0.01).

#### (Insert Table 7 about here)

In columns (4) and (5), we further split the earnings announcement days into high- and low-attention days, as in (48). Betas increase on earnings announcement days *only* when investors' attention is high. In both columns the average  $\beta_{\Delta EA}^{high,i}$  is positive and statistically significant, whereas  $\beta_{\Delta EA}^{low,i}$  is negative and marginally significant in one case. The results are consistent across the two attention measures: using ESV(U) the average  $\beta_{\Delta EA}^{high,i}$  is 0.103, p < 0.01 (0.077, p < 0.05). Overall, the evidence confirms our model's prediction that betas of announcing firms increase only when investors pay attention to announcements.

 $<sup>^{22} \</sup>rm Firm-level excess returns are available from CRSP. In addition, daily excess returns on 10 value-weighted beta-sorted portfolios are available from global-q.org/testingportfolios.html and on 25 size/BM portfolios and the market from https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Appendix B.1 describes the dataset and discusses the robustness of this section's results.$ 

Next, we explore whether the effects we document at the firm-level in Table 7 extend to portfolio-level analyses. We estimate the following regression, in which the intercepts and portfolio betas are allowed to vary conditionally on the type of day:

$$r_{j,t}^{e} = \alpha_{\text{Other}}^{j} + \alpha_{\Delta A}^{j} \mathbf{1}_{\text{HighAtt}} + \beta_{\text{Other}}^{j} r_{M,t}^{e} + \beta_{\Delta A}^{j} (\mathbf{1}_{\text{HighAtt}} \times r_{M,t}^{e}) + \varepsilon_{j,t},$$
(49)

where  $\mathbf{1}_{\text{HighAtt}}$  is a dummy variable for high attention days (days with the detrended *aggregate* ESV(U) above its median);  $r_{j,t}^{e}$  is the portfolio excess return;  $\beta_{\text{Other}}^{j}$  is the beta on other days; and  $\beta_{\Delta A}^{j}$  measures the change in the portfolio's beta on high ESV(U) days. Tables 8 and 9 present results for 10 beta-sorted and 25 size/BM portfolios. We focus on the coefficients  $\beta_{\Delta A}$ , which our theory predicts to be positive.

(Insert Table 8 about here)

(Insert Table 9 about here)

Table 8 supports our model's prediction, with nine out of ten portfolio betas being significantly higher on high-attention days. The increase in betas is consistent across the two attention measures (*ESV* in panel A and *ESVU* in panel B) and ranges from 0.067 (p < 0.01) to 0.131 (p < 0.01) in panel A and from 0.023 (p < 0.1) to 0.122 (p < 0.01) in panel B. Table 9 further confirms these results with the 25 size/BM portfolios.<sup>23</sup> The majority of portfolio betas (21 out of 25, both in panel A and panel B) increase on high-attention days, with their increase ranging from 0.031 (p < 0.1) to 0.401 (p < 0.01) in panel A and from 0.016 (p < 0.05) to 0.414 (p < 0.01) in panel B.

Finally, we estimate day-specific CAPMs. According to Corollary 4.1, both a high level of ex-ante uncertainty and heightened investor attention increase the market risk premium and thus imply a steeper SML. Table 10 shows the regression estimates for day-specific CAPMs, with results for 10 beta-sorted portfolios in panel A and 25 size/BM portfolios in panel B (estimates are in basis points per day). Our benchmark is the all-days CAPM relation, shown in column (1). Then, columns (2) and (3) classify trading days into subsamples with  $VIX_{t-1}$  above its median and in its top quartile, respectively. In panel A the SML becomes steeper when ex-ante uncertainty is higher, an effect that strengthens with the level of uncertainty. Next, columns (4) and (5) classify trading days into subsamples with detrended *ESV* above its median and in its top quartile, and columns (6) and (7) do the same for *ESVU*. All columns confirm our hypothesis that more attention steepens the SML. Finally, columns (8) and (9)

 $<sup>^{23}</sup>$ In unreported analysis, we find that the increase in betas is exclusively driven by attention and not by an increase in ex-ante uncertainty. When considering high-*VIX* days in the regression equation (49), we obtain that portfolio betas increase only when these high-*VIX* days are simultaneously high-attention days.

document the combined effect of high ex-ante uncertainty and heightened attention on days when the  $VIX_{t-1}$  and ESV(U) are both above their medians. Both columns support the prediction of Corollary 4.1. The slope coefficients are noticeably stronger when high ex-ante uncertainty precedes heightened attention. Overall, the magnitudes of the slope increases due to higher VIX or/and ESV(U) are economically significant, ranging from 4.76 (p < 0.01) to 11.78 (p < 0.01) basis points per day.

#### (Insert Table 10 about here)

Panel B yields similar inferences, albeit with weaker statistical significance. This is not too surprising given the well-documented results that CAPM performs poorly in these portfolios (Fama and French, 1993, 1996, 2004; Cochrane, 2009). However, even in this case, columns (4)-(7) show evidence that the SML steepens on days with high aggregate attention. This effect strengthens with the level of attention. The slope of the SML on high-attention days is positive and statistically significant in most cases, ranging from 4.31 (p < 0.05) to 9.99 (p < 0.01) basis points per day. Comparing these results with the ones in columns (2)-(3), we notice that attention has a stronger effect on the slope of the SML than uncertainty.

Figure 8 provides a graphical representation of Table 10. It plots average daily excess returns in basis points against betas estimated from time-series regressions. The top (bottom) panels present results for 10 beta-sorted portfolios (25 size/BM-sorted portfolios). All plots are day-specific. The left panels plot the CAPM relation estimated on all days versus days when  $VIX_{t-1}$  is in its top quartile; the center and right panels plot the CAPM relation on all days versus days when the detrended aggregate ESV(U) measures are in their top quartiles.

### (Insert Figure 8 about here)

The top panels confirm that the SML, when estimated on 10 beta-sorted portfolios, is steeper on high-uncertainty and high-attention days. For the 25 size/BM portfolios (the bottom panels) the estimated all-days SML has a negative slope, which becomes positive on high-uncertainty and high-attention days. Finally, on high-attention days, portfolio betas are noticeably higher (see Tables 8 and 9). This agrees with Eq. (36) and its premise that investors learn about the economy from firm-specific information.

To summarize, according to our theory, two effects occur when investors are more attentive to firm-level news. First, heightened attention increases firm betas; Tables 7 to 9 support this prediction. Second, the increase in attention resolves uncertainty, which steepens the SML; Table 10 and Figure 8 support this second prediction. Our paper provides a unified theoretical explanation for the relation between attention and the CAPM. It shows that investors' attention to firm-level news is the channel through which betas increase on announcement days and the CAPM relation steepens.

More generally, because in our model heightened attention resolves uncertainty, we should observe a steeper CAPM relation on days with strong uncertainty resolution. A quick test of this statement is readily available: broadly defining days with high values of  $\log(VIX_{t-1}/VIX_t)$ as days with strong uncertainty resolution, testing the CAPM on days when this proxy is high yields robust CAPM relations in any portfolio sorts and at the individual stock level. Admittedly,  $\log(VIX_{t-1}/VIX_t)$  is a coarse proxy for uncertainty resolution. Nevertheless, these results suggest that models in which uncertainty and attention fluctuations generate time variation in the resolution of uncertainty (e.g., Andrei and Hasler, 2015, 2019; Benamar et al., 2021) might be particularly suitable for studying the cross section of asset returns.

## 5 Conclusion

This paper examines the relationship between economic uncertainty and investor attention to firm-level earnings announcements. In a multi-firm equilibrium model, we show that heightened economic uncertainty causes investors to allocate more attention to firm-level information. Investors pay incrementally more attention to the earnings announcements of high-beta firms, firms with more informative earnings announcements, higher idiosyncratic volatility of earnings, less informative prices, and lower information acquisition costs.

The central premise of our model is that investors learn valuable information about the economy from earnings announcements. Consequently, investors' learning intensifies when market-wide uncertainty is high. This implies a steeper beta-return relation on days of heightened investor attention. Moreover, our model predicts that betas of announcing firms increase with investors' attention to earnings announcements.

The data support these predictions. Using two proxies for investor attention to firmlevel information (SEC EDGAR search traffic and Google stock ticker searches), we find that investors pay more attention to firm-level earnings announcements on days with high economic uncertainty. Our analysis further reveals that prices respond to earnings news more strongly when there is more significant economic uncertainty. These results are concentrated in firms with high CAPM beta, higher institutional ownership, idiosyncratic volatility, and prior share turnover. We view these as consistent with our theoretical predictions related to cross-sectional variation in the benefit-to-cost ratio of information. Finally, we find strong empirical support for higher betas on high-attention days and a steeper CAPM relation on days of heightened investor attention to firm-level information.

In conclusion, these results suggest that economic uncertainty is an essential driver of

investor attention to firm-level information. They highlight the critical role of information spillovers in information acquisition models. They also suggest that more reliable market risk pricing occurs not only when uncertainty is high but when investors respond to high uncertainty by intensifying their learning—in other words, when information gets processed and resolves uncertainty. Thus, models in which uncertainty and attention fluctuations generate time variation in the resolution of uncertainty might be particularly suitable for studying the cross-section of asset returns.

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# A Appendix

#### A.1 Proof of Proposition 1

Notation used thorough the Appendix:

- We denote **I** as the identity matrix, **1** as a vector of ones, and **0** as a vector/matrix of zeros. These vectors and matrices are always assumed to have the conformable dimension, which we do not specify below in order to avoid overly cumbersome notation.
- The set of announcing firms is  $\mathcal{A} = \{1, 2, ..., A\}$ . Within this set, firms are indexed by a.
- The set of investor types is the power set of  $\mathcal{A}$ ,  $\mathscr{P}(\mathcal{A})$ , of dimension  $2^{\mathcal{A}}$ . Within this set, investor types are indexed by k.
- $\overline{k}$  denotes the complement of an investor type  $k \subseteq \mathcal{A}$ , that is,  $\overline{k} = \mathcal{A} \setminus k$ .
- |k| denotes the cardinality of the set k.
- $\iota_a$  is a standard basis vector of dimension N with all components equal to 0, except the *a*-th, which is 1.  $\iota_k$  ( $\iota_{\bar{k}}$ ) represents the matrix with all the column vectors { $\iota_a \mid a \in k$ } ({ $\iota_a \mid a \in \bar{k}$ }).  $\iota$  represents the matrix with all the column vectors { $\iota_a \mid a \in \mathcal{A}$ }.
- $h_a \equiv \frac{\Lambda_a}{\gamma \sigma_{\varepsilon a}^2}$ , for  $a \in \mathcal{A}$ .  $\mathbf{h}_k$  and  $\mathbf{h}_{\bar{k}}$  denote the column vectors  $\{h_a \mid a \in k\}$  and  $\{h_a \mid a \in \bar{k}\}$ .
- diag $[y_j | j \in z]$  denotes a diagonal matrix whose diagonal is  $\{y_j | j \in z\}$ .  $\delta \mathbf{h}_k$  ( $\delta \mathbf{h}_{\bar{k}}$ ) is a diagonal matrix whose diagonal is  $\mathbf{h}_k$  ( $\mathbf{h}_{\bar{k}}$ ), e.g.,  $\delta \mathbf{h}_k = \text{diag}[\{h_a | a \in k\}]$ .
- $\varepsilon_k$  and  $\varepsilon_{\bar{k}}$  denote the column vectors  $\{\varepsilon_a \mid a \in k\}$  and  $\{\varepsilon_a \mid a \in \bar{k}\}$ , and  $\varepsilon = \begin{bmatrix} \varepsilon_k \\ \varepsilon_{\bar{k}} \end{bmatrix}$ . Similarly for  $\mathbf{x}_k, \mathbf{x}_{\bar{k}}$ , and  $\mathbf{x}$ .
- $\Sigma_{\varepsilon k}$  denotes the covariance matrix of the vector  $\varepsilon_k$  (a diagonal matrix whose elements are  $\{\sigma_{\varepsilon a}^2 \mid a \in k\}$ ).  $\Sigma_{\varepsilon \bar{k}}$  denotes the covariance matrix of the vector  $\varepsilon_{\bar{k}}$ .  $\Sigma_{x\bar{k}}$  denotes the covariance matrix of the vector  $\mathbf{x}_{\bar{k}}$ .

#### Learning for type k investors

Type k investors observe the earnings announcements  $\{E_a \mid a \in k\}$ , and learn from prices. Conjecture 1 implies that the only prices useful for learning are  $\{\hat{P}_a \mid a \in \bar{k}\}$ . (If an investor observes  $E_a$  then the price signal  $\hat{P}_a$  is a noisy version of  $E_a$  and is redundant for learning.)

Group the information set of type k investors into two vectors,  $\mathbf{E}_k$  of dimension |k| and  $\mathbf{P}_{\bar{k}}$  of dimension  $|\bar{k}|$ . Then we can write

$$\begin{bmatrix} \mathbf{D} \\ \mathbf{E}_k \\ \widehat{\mathbf{P}}_{\bar{k}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \boldsymbol{\iota}'_k \\ \boldsymbol{\delta}\mathbf{h}_{\bar{k}}\boldsymbol{\iota}'_{\bar{k}} \end{bmatrix} \mathbf{D} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\delta}\mathbf{h}_{\bar{k}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}_k \\ \boldsymbol{\varepsilon}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{\iota}'_{\bar{k}} \end{bmatrix} \mathbf{x},$$
(A.1)

and thus

$$\begin{bmatrix} \mathbf{D} \\ \mathbf{E}_{k} \\ \widehat{\mathbf{P}}_{\bar{k}} \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \operatorname{Var}[\mathbf{D}] & \operatorname{Var}[\mathbf{D}] & \left[ \boldsymbol{\iota}_{k} & \boldsymbol{\iota}_{\bar{k}} \delta \mathbf{h}_{\bar{k}} \right] \\ \begin{bmatrix} \boldsymbol{\iota}_{k} \\ \delta \mathbf{h}_{\bar{k}} \boldsymbol{\iota}_{\bar{k}}' \end{bmatrix} \operatorname{Var}[\mathbf{D}] & \begin{bmatrix} \boldsymbol{\iota}_{k} \\ \delta \mathbf{h}_{\bar{k}} \boldsymbol{\iota}_{\bar{k}}' \end{bmatrix} \operatorname{Var}[\mathbf{D}] \begin{bmatrix} \boldsymbol{\iota}_{k} & \boldsymbol{\iota}_{\bar{k}} \delta \mathbf{h}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} \mathbf{\Sigma}_{\varepsilon k} & \mathbf{0} \\ \mathbf{0} & \delta \mathbf{h}_{\bar{k}}^{2} \mathbf{\Sigma}_{\varepsilon \bar{k}} + \mathbf{\Sigma}_{x \bar{k}} \end{bmatrix} \right) \right).$$
(A.2)

We will apply the Projection Theorem, which we write here for convenience.

**Projection Theorem.** Consider the n-dimensional normal random variable

$$\begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{s} \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} \boldsymbol{\mu}_{\boldsymbol{\theta}} \\ \boldsymbol{\mu}_{\mathbf{s}} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{\boldsymbol{\theta},\boldsymbol{\theta}} & \boldsymbol{\Sigma}_{\boldsymbol{\theta},\mathbf{s}} \\ \boldsymbol{\Sigma}_{\mathbf{s},\boldsymbol{\theta}} & \boldsymbol{\Sigma}_{\mathbf{s},\mathbf{s}} \end{bmatrix} \right).$$
(A.3)

Provided  $\Sigma_{\mathbf{s},\mathbf{s}}$  is non-singular, the conditional density of  $\theta$  given  $\mathbf{s}$  is normal with conditional mean and conditional variance-covariance matrix:

$$\mathbb{E}[\boldsymbol{\theta}|\mathbf{s}] = \boldsymbol{\mu}_{\boldsymbol{\theta}} + \boldsymbol{\Sigma}_{\boldsymbol{\theta},\mathbf{s}} \boldsymbol{\Sigma}_{\mathbf{s},\mathbf{s}}^{-1} \left(\mathbf{s} - \boldsymbol{\mu}_{\mathbf{s}}\right)$$
(A.4)

$$\operatorname{Var}[\boldsymbol{\theta}|\mathbf{s}] = \boldsymbol{\Sigma}_{\boldsymbol{\theta},\boldsymbol{\theta}} - \boldsymbol{\Sigma}_{\boldsymbol{\theta},\mathbf{s}} \boldsymbol{\Sigma}_{\mathbf{s},\mathbf{s}}^{-1} \boldsymbol{\Sigma}_{\mathbf{s},\boldsymbol{\theta}}.$$
 (A.5)

Applied to (A.2), the Projection Theorem together with the Woodbury Matrix Identity imply:

$$\operatorname{Var}^{k}[\mathbf{D}] = \left(\operatorname{Var}[\mathbf{D}]^{-1} + \begin{bmatrix} \boldsymbol{\iota}_{k} & \boldsymbol{\iota}_{\bar{k}} \delta \mathbf{h}_{\bar{k}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_{\varepsilon k}^{-1} & \mathbf{0} \\ \mathbf{0} & (\delta \mathbf{h}_{\bar{k}}^{2} \boldsymbol{\Sigma}_{\varepsilon \bar{k}} + \boldsymbol{\Sigma}_{x \bar{k}})^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{\iota}_{k}' \\ \delta \mathbf{h}_{\bar{k}} \boldsymbol{\iota}_{\bar{k}}' \end{bmatrix} \right)^{-1}$$
(A.6)

$$= \left( \operatorname{Var}[\mathbf{D}]^{-1} + \begin{bmatrix} \boldsymbol{\iota}_k & \boldsymbol{\iota}_{\bar{k}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_{\varepsilon k}^{-1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\delta} \mathbf{h}_{\bar{k}}^2 (\boldsymbol{\delta} \mathbf{h}_{\bar{k}}^2 \boldsymbol{\Sigma}_{\varepsilon \bar{k}} + \boldsymbol{\Sigma}_{x \bar{k}})^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{\iota}_k' \\ \boldsymbol{\iota}_{\bar{k}}' \end{bmatrix} \right)^{-1}$$
(A.7)

$$= \left( \operatorname{Var}[\mathbf{D}]^{-1} + \boldsymbol{\iota} \operatorname{diag} \left[ \frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \mid a \in \mathcal{A} \right] \boldsymbol{\iota}' \right)^{-1},$$
(A.8)

with  $\ell_a^k$  defined in (13). We have thus obtained  $\tau^k \equiv \operatorname{Var}^k[\mathbf{D}]^{-1}$  as in Proposition 1. This simple form for  $\tau^k$  allows us to compute its determinant using the Matrix Determinant Lemma:

$$det(\mathbf{A} + \mathbf{U}\mathbf{W}\mathbf{V}') = det(\mathbf{W}^{-1} + \mathbf{V}'\mathbf{A}^{-1}\mathbf{U}) det(\mathbf{W}) det(\mathbf{A}),$$
(A.9)

where  $\mathbf{A} = \operatorname{Var}[\mathbf{D}]^{-1}$ ,  $\mathbf{U} = \boldsymbol{\iota}$ ,  $\mathbf{W} = \operatorname{diag}\left[\frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \mid a \in \mathcal{A}\right]$ , and  $\mathbf{V}' = \boldsymbol{\iota}'$ . The Matrix Determinant Lemma implies

$$\det(\boldsymbol{\tau}^{k}) = \det\left(\operatorname{Var}[\mathbf{D}]^{-1}\right) \left(\prod_{a=1}^{A} \frac{\ell_{a}^{k}}{\sigma_{\varepsilon a}^{2}}\right) \det\left(\operatorname{diag}\left[\frac{\sigma_{\varepsilon a}^{2}}{\ell_{a}^{k}} \mid a \in \mathcal{A}\right] + \boldsymbol{\iota}'\operatorname{Var}[\mathbf{D}]\boldsymbol{\iota}\right)$$
(A.10)

$$= \det\left(\operatorname{Var}[\mathbf{D}]^{-1}\right)\left(\prod_{a=1}^{A} \frac{\ell_{a}^{k}}{\sigma_{\varepsilon a}^{2}}\right) \det\left(\operatorname{diag}\left[\frac{\sigma_{\varepsilon a}^{2}}{\ell_{a}^{k}} + \sigma_{ea}^{2} \mid a \in \mathcal{A}\right] + U^{2}\mathbf{b}_{\mathcal{A}}\mathbf{b}_{\mathcal{A}}'\right), \quad (A.11)$$

where  $\mathbf{b}_{\mathcal{A}}$  is the vector of announcer firms' exposure to the systematic component f.

Further apply the Matrix Determinant Lemma to the last term:

$$\det(\boldsymbol{\tau}^{k}) = \det\left(\operatorname{Var}[\mathbf{D}]^{-1}\right) \left(\prod_{a=1}^{A} \frac{\ell_{a}^{k}}{\sigma_{\varepsilon a}^{2}}\right) \left(\prod_{a=1}^{A} \left(\frac{\sigma_{\varepsilon a}^{2}}{\ell_{a}^{k}} + \sigma_{ea}^{2}\right)\right) \left(1 + \mathbf{b}_{\mathcal{A}}^{\prime} \operatorname{diag}\left[\frac{\ell_{a}^{k}}{\ell_{a}^{k}} \sigma_{ea}^{2} + \sigma_{\varepsilon a}^{2}\right] | a \in \mathcal{A}\right] U^{2} \mathbf{b}_{\mathcal{A}}\right)$$
(A.12)

$$= \det\left(\operatorname{Var}[\mathbf{D}]^{-1}\right) \left(\prod_{a=1}^{A} \frac{\ell_a^k \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\sigma_{\varepsilon a}^2}\right) \left(1 + U^2 \sum_{a=1}^{A} \frac{\ell_a^k b_a^2}{\ell_a^k \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}\right),\tag{A.13}$$

which completes the proof of Proposition 1.

#### A.2 Proof of Proposition 2

The expected utility of a type  $\emptyset$  investor (uninformed) at time 1 is:

$$\mathcal{U}_{1}^{\emptyset} = \max_{\mathbf{q}^{k}} \mathbb{E}_{1}^{\emptyset} \left[ -e^{-\gamma \left( W^{\emptyset} - c \sum_{a=1}^{A} I_{a}^{\emptyset} \right)} \right] = \max_{\mathbf{q}^{k}} \mathbb{E}_{1}^{\emptyset} \left[ -e^{-\gamma \left( \mathbf{q}^{\emptyset} \right)' \mathbf{R}^{e}} \right].$$
(A.14)

Further replacing the optimal portfolio choice from Eq. (7) yields

$$\mathcal{U}_{1}^{\emptyset} = -\mathbb{E}_{1}^{\emptyset} \left[ e^{-\mathbb{E}_{1}^{\emptyset}[\mathbf{R}^{e}]' \operatorname{Var}_{1}^{\emptyset}[\mathbf{R}^{e}]^{-1} \mathbf{R}^{e}} \right]$$
(A.15)

$$= -e^{-\frac{1}{2}\mathbb{E}_{1}^{\emptyset}[\mathbf{R}^{e}]' \operatorname{Var}_{1}^{\emptyset}[\mathbf{R}^{e}]^{-1}\mathbb{E}_{1}^{\emptyset}[\mathbf{R}^{e}]}.$$
 (A.16)

Assume that a type  $\emptyset$  investor considers acquiring information and becoming of type  $k \in \mathscr{P}(\mathcal{A})$ , where |k| > 0. At time 1, from the perspective of the type  $\emptyset$  investor,  $\mathbb{E}_1^k[\mathbf{R}^e]$  is a random vector. Denote this random vector by  $\mathbf{z} + \mathbf{m}$ , with mean  $\mathbf{m}$  and variance  $\Sigma$  (i.e.,  $\mathbf{z}$  has mean  $\mathbf{0}$  and variance  $\Sigma$ ). By the law of iterated expectations,

$$\mathbf{m} \equiv \mathbb{E}_{1}^{\emptyset}[\mathbb{E}_{1}^{k}[\mathbf{R}^{e}]] = \mathbb{E}_{1}^{\emptyset}[\mathbf{R}^{e}], \qquad (A.17)$$

and by the law of total variance,

$$\boldsymbol{\Sigma} \equiv \operatorname{Var}_{1}^{\emptyset}[\mathbb{E}_{1}^{k}[\mathbf{R}^{e}]] = \operatorname{Var}_{1}^{\emptyset}[\mathbf{R}^{e}] - \operatorname{Var}_{1}^{k}[\mathbf{R}^{e}].$$
(A.18)

Therefore, for the type  $\emptyset$  investor,  $-\frac{1}{2}\mathbb{E}_1^k[\mathbf{R}^e]'\operatorname{Var}_1^k[\mathbf{R}^e]^{-1}\mathbb{E}_1^k[\mathbf{R}^e]$  (that is, the random exponent in (A.16), written for type k) is a random scalar that can be written as (define  $\Sigma^{\emptyset} \equiv \operatorname{Var}_1^{\emptyset}[\mathbf{R}^e]$  to simplify notation):

$$-\frac{1}{2}\mathbb{E}_{1}^{k}[\mathbf{R}^{e}]^{\prime}\operatorname{Var}_{1}^{k}[\mathbf{R}^{e}]^{-1}\mathbb{E}_{1}^{k}[\mathbf{R}^{e}] = -\frac{1}{2}(\mathbf{z}+\mathbf{m})^{\prime}(\boldsymbol{\Sigma}^{\emptyset}-\boldsymbol{\Sigma})^{-1}(\mathbf{z}+\mathbf{m})$$
(A.19)

$$= \mathbf{z}' \underbrace{\left(-\frac{1}{2}(\boldsymbol{\Sigma}^{\emptyset} - \boldsymbol{\Sigma})^{-1}\right)}_{\mathbf{F}} \mathbf{z} + \underbrace{\left(-\mathbf{m}'(\boldsymbol{\Sigma}^{\emptyset} - \boldsymbol{\Sigma})^{-1}\right)}_{\mathbf{G}'} \mathbf{z} + \underbrace{\mathbf{m}'\left(-\frac{1}{2}(\boldsymbol{\Sigma}^{\emptyset} - \boldsymbol{\Sigma})^{-1}\right)}_{\mathbf{H}} \mathbf{m}.$$
 (A.20)

Our aim is to compute  $\mathbb{E}_1^{\emptyset}[\mathcal{U}_1^k]$ , i.e., the type  $\emptyset$  agent's expectation of what her expected utility will be if she changes type to k. We will apply the following Lemma (Veldkamp, 2011, p. 102):

**Lemma A2.** Consider a random vector  $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$ . Then,

$$\mathbb{E}\left[e^{\mathbf{z'Fz}+\mathbf{G'z}+\mathbf{H}}\right] = \det(\mathbf{I}-2\mathbf{\Sigma}\mathbf{F})^{-\frac{1}{2}}e^{\frac{1}{2}\mathbf{G'}(\mathbf{I}-2\mathbf{\Sigma}\mathbf{F})^{-1}\mathbf{\Sigma}\mathbf{G}+\mathbf{H}}.$$
(A.21)

Compute first

$$\mathbf{I} - 2\boldsymbol{\Sigma}\mathbf{F} = \mathbf{I} - 2\boldsymbol{\Sigma}\left(-\frac{1}{2}(\boldsymbol{\Sigma}^{\emptyset} - \boldsymbol{\Sigma})^{-1}\right)$$
(A.22)

$$= \Sigma^{\emptyset} (\Sigma^{\emptyset} - \Sigma)^{-1}, \qquad (A.23)$$

which, using (A.18), leads to the determinant in Lemma A2:

$$\det(\mathbf{I} - 2\mathbf{\Sigma}\mathbf{F}) = \frac{\det(\mathbf{\Sigma}^{\emptyset})}{\det(\operatorname{Var}_{1}^{k}[\mathbf{R}^{e}])} = \frac{\det(\boldsymbol{\tau}^{k})}{\det(\boldsymbol{\tau}^{\emptyset})}.$$
(A.24)

The exponent in Lemma A2 is:

$$\frac{1}{2}\mathbf{G}'(\mathbf{I} - 2\boldsymbol{\Sigma}\mathbf{F})^{-1}\boldsymbol{\Sigma}\mathbf{G} + \mathbf{H}$$
(A.25)

$$=\frac{1}{2}\left(-\mathbf{m}'(\boldsymbol{\Sigma}^{\emptyset}-\boldsymbol{\Sigma})^{-1}\right)(\boldsymbol{\Sigma}^{\emptyset}-\boldsymbol{\Sigma})(\boldsymbol{\Sigma}^{\emptyset})^{-1}\boldsymbol{\Sigma}\left(-\mathbf{m}'(\boldsymbol{\Sigma}^{\emptyset}-\boldsymbol{\Sigma})^{-1}\right)'-\mathbf{m}'\frac{1}{2}(\boldsymbol{\Sigma}^{\emptyset}-\boldsymbol{\Sigma})^{-1}\mathbf{m} \quad (A.26)$$

$$= \frac{1}{2}\mathbf{m}'(\mathbf{\Sigma}^{\emptyset})^{-1}\mathbf{\Sigma}(\mathbf{\Sigma}^{\emptyset} - \mathbf{\Sigma})^{-1}\mathbf{m} - \frac{1}{2}\mathbf{m}'(\mathbf{\Sigma}^{\emptyset} - \mathbf{\Sigma})^{-1}\mathbf{m}$$
(A.27)

$$= \frac{1}{2}\mathbf{m}'\left((\mathbf{\Sigma}^{\emptyset})^{-1}\mathbf{\Sigma} - \mathbf{I}\right)(\mathbf{\Sigma}^{\emptyset} - \mathbf{\Sigma})^{-1}\mathbf{m}$$
(A.28)
$$(\mathbf{I}_{\mathbf{I}}, \mathbf{I}_{\mathbf{I}}, \mathbf{I}, \mathbf{I}, \mathbf{I}, \mathbf{I}, \mathbf{I}, \mathbf{I}, \mathbf{I}, \mathbf{I}, \mathbf{I$$

$$= -\frac{1}{2}\mathbf{m}'(\mathbf{\Sigma}^{\emptyset})^{-1}\mathbf{m}.$$
 (A.29)

We can then use Lemma A2 to write

$$\mathbb{E}_{1}^{\emptyset}[\mathcal{U}_{1}^{k}] = -e^{\gamma c|k|} \mathbb{E}_{1}^{\emptyset} \left[ e^{-\frac{1}{2} \mathbb{E}_{1}^{k}[\mathbf{R}^{e}]' \operatorname{Var}_{1}^{k}[\mathbf{D}]^{-1} \mathbb{E}_{1}^{k}[\mathbf{R}^{e}]} \right]$$
(A.30)

$$= -e^{\gamma c|k|} \sqrt{\frac{\det(\boldsymbol{\tau}^{\emptyset})}{\det(\boldsymbol{\tau}^{k})}} e^{-\frac{1}{2} \mathbb{E}_{1}^{\emptyset}[\mathbf{R}^{e}]' \operatorname{Var}_{1}^{\emptyset}[\mathbf{R}^{e}]^{-1} \mathbb{E}_{1}^{\emptyset}[\mathbf{R}^{e}]}$$
(A.31)

$$= \mathcal{U}_{1}^{\emptyset} e^{\gamma c|k|} \sqrt{\frac{\det(\boldsymbol{\tau}^{\emptyset})}{\det(\boldsymbol{\tau}^{k})}}.$$
(A.32)

At time t = 0, the type  $\emptyset$  investor compares  $\mathbb{E}_0[\mathcal{U}_1^{\emptyset}]$  with  $\mathbb{E}_0[\mathcal{U}_1^k]$  and acquires the additional signals if and only if

$$\mathbb{E}_0[\mathcal{U}_1^{\emptyset}] < \mathbb{E}_0[\mathcal{U}_1^k] = \mathbb{E}_0[\mathbb{E}_1^{\emptyset}[\mathcal{U}_1^k]], \qquad (A.33)$$

which, after replacement of (A.32), yields  $e^{\gamma c|k|} \sqrt{\det(\boldsymbol{\tau}^{\emptyset})/\det(\boldsymbol{\tau}^k)} < 1$  (the division by  $\mathbb{E}_0[\mathcal{U}_1^{\emptyset}] < 0$  flips the inequality sign). Thus, an investor of type  $\emptyset$  changes type to k if and only if

$$B_{\emptyset}^{k} \equiv \frac{\det(\boldsymbol{\tau}^{k})}{\det(\boldsymbol{\tau}^{\emptyset})} e^{-2\gamma c|k|} > 1.$$
(A.34)

Consider now two investor types k and k' as in Proposition 2. The empty set  $\emptyset$  is the only common subset of both k and k', for all  $k, k' \in \mathscr{P}(\mathcal{A})$ . Thus, the uninformed investor is a common reference point for type k and type k' investors, and therefore the investor with the lowest benefitcost ratio among  $\{B_{\emptyset}^k, B_{\emptyset}^{k'}\}$  will always choose to migrate to the other type. In other words, a type k investor changes type from k to  $k' \in \mathscr{P}(\mathcal{A}) \setminus k$  if and only if

$$\frac{B_{\emptyset}^{k'}}{B_{\emptyset}^{k}} > 1 \quad \iff \quad \frac{1}{2\gamma} \ln \frac{\det(\tau^{k'})}{\det(\tau^{k})} > c(|k'| - |k|). \tag{A.35}$$

This holds regardless of the sign of |k'| - |k|.

## A.3 Proof of Theorem 1

An important property of the benefit-cost ratios  $B_{\emptyset}^k$ , for  $k \in \mathscr{P}(\mathcal{A}) \setminus \emptyset$ , is that they can be decomposed into the product of consecutive *one-step* benefit-cost ratios. Formally, let k(i) be the  $i^{\text{th}}$ element of k and  $\kappa(i)$  the subset of k that contains all its elements up to and including k(i). Using

the convention  $\kappa(0) = \emptyset$  and defining  $B_{\kappa(i-1)}^{\kappa(i-1)\cup\{k(i)\}} \equiv \frac{\det(\boldsymbol{\tau}^{\kappa(i-1)\cup\{k(i)\}})}{\det(\boldsymbol{\tau}^{\kappa(i-1)})}e^{-2\gamma c}$ , we can write

$$B_{\emptyset}^{k} = \prod_{i=1}^{|k|} B_{\kappa(i-1)}^{\kappa(i-1)\cup\{k(i)\}}.$$
(A.36)

We first establish the following Lemma.

**Lemma A3.** Consider an announcer  $a \in A$  and any type  $k \subseteq A \setminus \{a\}$ . Then

$$\underset{k}{\arg\min}B_{k}^{k\cup\{a\}} = \mathcal{A}\setminus\{a\}$$
(A.37)

$$\underset{k}{\arg\max}B_{k}^{k\cup\{a\}} = \emptyset \tag{A.38}$$

Lemma A3 states that the type k for which the one-step benefit-cost ratio  $B_k^{k\cup\{a\}}$  attains its minimum is the highest cardinality type that excludes a, that is,  $\mathcal{A} \setminus \{a\}$ ; and the type k for which  $B_k^{k\cup\{a\}}$  attains its maximum is the empty set  $\emptyset$ . In other words, attention has diminishing returns: the lowest benefit from observing  $E_a$  belongs to the investor who already observes all the other earnings announcements; and the highest benefit belongs to the uninformed investor. The proof of Lemma A3 follows from writing explicitly  $B_k^{k\cup\{a\}}$  by means of Proposition 1,

$$B_k^{k \cup \{a\}} = \frac{\det(\boldsymbol{\tau}^{k \cup \{a\}})}{\det(\boldsymbol{\tau}^k)} e^{-2\gamma c}$$
(A.39)

$$= \left(\frac{\sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2} + \frac{b_a^2}{\frac{1}{U^2} + \sum_{\alpha=1}^A \frac{\ell_\alpha^k b_\alpha^2}{\ell_\alpha^k \sigma_{e\alpha}^2 + \sigma_{\varepsilon \alpha}^2}} \frac{(1-\ell_a)\sigma_{\varepsilon a}^2}{(\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2)^2}\right) e^{-2\gamma c}, \tag{A.40}$$

which is indeed minimized when  $\ell_{\alpha}^{k} = 1$ ,  $\forall \alpha \in \mathcal{A} \setminus \{a\}$ , and maximized when  $\ell_{\alpha}^{k} < 1$ ,  $\forall \alpha \in \mathcal{A} \setminus \{a\}$ . In the former case, k must be  $\mathcal{A} \setminus \{a\}$ ; in the latter, k must be  $\emptyset$ . (NB: Lemma A3 is a direct consequence of the fact that the function  $\ln(B_{\emptyset}^{k})$  is linearly related to the entropy defined in (14):  $\ln(B_{\emptyset}^{k}) = 2(H^{\emptyset}[\mathbf{D}] - H^{k}[\mathbf{D}] - \gamma c|k|)$ . By the submodularity property of the entropy,  $\ln(B_{\emptyset}^{k})$  is submodular and therefore  $B_{k}^{k \cup \{a\}}$  has diminishing returns. See also Appendix A.7.)

Lemma A3, together with the multiplicative property (A.36), will allow us obtain the bounds  $c_{min}$  and  $c_{max}$ . We will first derive the lower bound  $c_{min}$ . When the information cost is below  $c_{min}$ , all investors are informed, i.e.,  $\lambda^{\mathcal{A}} = 1$ . In order for this to be a stable equilibrium, the following conditions must hold simultaneously:

$$B^{\mathcal{A}}_{\mathcal{A}\setminus\{a\}} \ge 1 \quad \forall a \in \mathcal{A},\tag{A.41}$$

meaning that no investor of type  $\mathcal{A}$  finds it optimal to renounce being attentive to any signal  $E_a$ . If these conditions hold simultaneously, then one can easily show using the multiplicative property (A.36) and Lemma A3 that

$$B_k^{\mathcal{A}} \ge 1$$
, for any type  $k \subset \mathcal{A}$ , (A.42)

meaning that no investor of type  $\mathcal{A}$  finds it optimal to be of any other possible type. (This can be shown by writing  $B_k^{\mathcal{A}}$  as a product as in (A.36) and using Lemma A3 for each individual term of the product; it is a direct consequence of the property of diminishing returns to attention.)

Conditions (A.41) further imply  $\min_a B^{\mathcal{A}}_{\mathcal{A} \setminus \{a\}} \geq 1$ , which will pin down  $c_{min}$ . Using the fact that  $\lambda^{\mathcal{A}} = 1$ , the definition of  $\ell_a$  in Eq. (13) yields upper limits for all the learning coefficients  $\ell_a$ ,

$$\bar{\ell}_a = \frac{1}{1 + \gamma^2 \sigma_{xa}^2 \sigma_{\varepsilon a}^2} \quad \forall a \in \mathcal{A},$$
(A.43)

and thus  $c_{min}$  solves

$$e^{2\gamma c_{min}} = \min_{a} \left( \frac{\sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\bar{\ell}_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2} + \frac{b_a^2}{\frac{1}{U^2} + \sum_{\alpha=1}^A \frac{\bar{\ell}_\alpha b_\alpha^2}{\bar{\ell}_\alpha \sigma_{e\alpha}^2 + \sigma_{\varepsilon a}^2}} \frac{(1 - \bar{\ell}_a) \sigma_{\varepsilon a}^2}{(\bar{\ell}_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2)^2} \right).$$
(A.44)

Since the right hand side equals  $\min_a(\det(\boldsymbol{\tau}^{\mathcal{A}})/\det(\boldsymbol{\tau}^{\mathcal{A}\setminus\{a\}}))$  and thus is always larger than one, equation (A.44) has a unique, strictly positive solution  $c_{min}$ . It can be easily checked that  $c_{min}$  is strictly increasing in U.

Consider now an equilibrium in which no investor is informed, or  $\lambda^{\emptyset} = 1$ . In order for this to be a stable equilibrium, the following conditions must hold simultaneously:

$$B^a_{\emptyset} \le 1 \quad \forall a \in \mathcal{A}. \tag{A.45}$$

If these conditions hold, then a consequence of the property of diminishing returns to attention is that  $B_{\emptyset}^k \leq 1$  holds for any type  $k \subseteq \mathcal{A}$ . (This can be shown by writing  $B_{\emptyset}^k$  as a product as in (A.36) and using Lemma A3 for each individual term of the product.)

Conditions (A.45) further imply  $\max_{a} B^{a}_{\emptyset} \leq 1$ , and  $\lambda^{\emptyset} = 1$  leads to  $\ell_{a} = 0 \ \forall a \in \mathcal{A}$ . Thus,  $c_{max}$  solves

$$e^{2\gamma c_{max}} = \max_{a} \left( 1 + \frac{b_a^2 U^2 + \sigma_{ea}^2}{\sigma_{\varepsilon a}^2} \right).$$
(A.46)

This equation has a unique, strictly positive solution  $c_{max}$ , which is strictly increasing in U. Furthermore, since  $B^a_{\emptyset} > B^A_{\mathcal{A} \setminus \{a\}} \quad \forall a \in \mathcal{A}$  (by Lemma A3), it is clear that  $\max_a B^a_{\emptyset} > \min_a B^A_{\mathcal{A} \setminus \{a\}}$  and therefore  $c_{max} > c_{min}$ . This completes the proofs of cases (C) and (A) of Theorem 1.

In case (B) of Theorem 1, the information cost is  $c \in (c_{min}, c_{max})$ . Clearly, when  $c \in (c_{min}, c_{max})$ both conditions (A.41) and (A.45) are violated and thus the equilibrium cannot be  $\lambda^{\emptyset} = 1$  or  $\lambda^{\mathcal{A}} = 1$ . Thus, in equilibrium there exists a set  $\{\lambda^k \mid k \in \mathscr{P}(\mathcal{A})\}$  such that:  $\sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k = 1$ ;  $\lambda^{\emptyset} < 1$ ; and  $\lambda^{\mathcal{A}} < 1$ . Consider now all the pairs of types  $\{k, k'\} \in \mathscr{P}(\mathcal{A})$ . For each pair, there are four cases:

- (i)  $\{\lambda^k > 0\} \land \{\lambda^{k'} > 0\}$ : this can be a stable equilibrium (meaning that no investor has an incentive to migrate from type k to type k' or vice versa) only if  $B_{\emptyset}^{k'}/B_{\emptyset}^{k} = 1$ .
- (ii)  $\{\lambda^k = 0\} \land \{\lambda^{k'} > 0\}$ : this can be a stable equilibrium (meaning that no investor of type k' has an incentive to migrate to type k) only if  $B_{\emptyset}^{k'}/B_{\emptyset}^k \ge 1$ .
- (iii)  $\{\lambda^k > 0\} \land \{\lambda^{k'} = 0\}$ : this is the reversal of the previous case and requires  $B_{\emptyset}^{k'}/B_{\emptyset}^k \leq 1$ .
- (iv)  $\{\lambda^k = 0\} \land \{\lambda^{k'} = 0\}$ : in this case there is no condition on  $B^{k'}_{\emptyset}/B^k_{\emptyset}$  since there are no investors of types k and k'.

Conditions (i)-(iv) are both necessary and sufficient for the stability of the information market equilibrium. See Appendix A.7 for an algorithm that converges to the equilibrium for any set of positive initial values  $\{\lambda_0^k > 0 \mid k \in \mathscr{P}(\mathcal{A})\}$  such that  $\sum_k \lambda_0^k = 1$ .

## A.4 Proof of Lemma 1

Lemma 1 results directly after writing  $\boldsymbol{\tau}^k$  for each investor type under this form:

$$\boldsymbol{\tau}^{k} = \operatorname{Var}[\mathbf{D}]^{-1} + \boldsymbol{\iota} \operatorname{diag}\left[\frac{\ell_{a}^{k}}{\sigma_{\varepsilon a}^{2}} \mid a \in \mathcal{A}\right] \boldsymbol{\iota}', \tag{A.47}$$

where  $\iota$  is a  $N \times A$  matrix whose columns are the standard basis vectors  $\iota_a$  for all the announcing firms (vectors having all components equal to 0, except the *a*-th, which is 1).

The weighted average precision is then

$$\boldsymbol{\tau} = \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^{k} \boldsymbol{\tau}^{k} = \operatorname{Var}[\mathbf{D}]^{-1} + \boldsymbol{\iota} \operatorname{diag} \left[ \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^{k} \frac{\ell_{a}^{k}}{\sigma_{\varepsilon a}^{2}} \mid a \in \mathcal{A} \right] \boldsymbol{\iota}',$$
(A.48)

with  $\ell_a^k$  defined in (13). Furthermore,

$$\sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k \frac{\ell_a^k}{\sigma_{\varepsilon a}^2} = \frac{(1 - \Lambda_a)\ell_a}{\sigma_{\varepsilon a}^2} + \frac{\Lambda_a}{\sigma_{\varepsilon a}^2} = \frac{\Lambda_a^2 + \Lambda_a \gamma^2 \sigma_{xa}^2 \sigma_{\varepsilon a}^2}{\Lambda_a^2 \sigma_{\varepsilon a}^2 + \gamma^2 \sigma_{xa}^2 \sigma_{\varepsilon a}^4} = \pi_a(\Lambda_a),$$
(A.49)

which yields (21).

# A.5 Proof of Proposition 3

We will use the market clearing condition to solve for the undetermined price coefficients:

$$\sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k \frac{\operatorname{Var}^k[\mathbf{D}]^{-1}}{\gamma} \mathbb{E}^k[\mathbf{D}] - \frac{\tau}{\gamma} \mathbf{P} + \mathbf{x} = \mathbf{M}.$$
 (A.50)

Using the Projection Theorem and  $h_a \equiv \frac{\Lambda_a}{\gamma \sigma_{\varepsilon a}^2}$  we can compute

$$\operatorname{Var}^{k}[\mathbf{D}]^{-1} \mathbb{E}^{k}[\mathbf{D}] = \left(\operatorname{Var}[\mathbf{D}]^{-1} + \begin{bmatrix} \boldsymbol{\iota}_{k} & \boldsymbol{\iota}_{\bar{k}} \delta \mathbf{h}_{\bar{k}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_{\varepsilon k}^{-1} & \mathbf{0} \\ \mathbf{0} & (\delta \mathbf{h}_{\bar{k}}^{2} \boldsymbol{\Sigma}_{\varepsilon \bar{k}} + \boldsymbol{\Sigma}_{x \bar{k}})^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{\iota}_{k}' \\ \delta \mathbf{h}_{\bar{k}} \boldsymbol{\iota}_{\bar{k}}' \end{bmatrix} \right) \times \\ \times \operatorname{Var}[\mathbf{D}] \begin{bmatrix} \boldsymbol{\iota}_{k} & \boldsymbol{\iota}_{\bar{k}} \delta \mathbf{h}_{\bar{k}} \end{bmatrix} \left( \begin{bmatrix} \boldsymbol{\iota}_{k}' \\ \delta \mathbf{h}_{\bar{k}} \boldsymbol{\iota}_{\bar{k}}' \end{bmatrix} \operatorname{Var}[\mathbf{D}] \begin{bmatrix} \boldsymbol{\iota}_{k} & \boldsymbol{\iota}_{\bar{k}} \delta \mathbf{h}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Sigma}_{\varepsilon k} & \mathbf{0} \\ \mathbf{0} & \delta \mathbf{h}_{\bar{k}}^{2} \boldsymbol{\Sigma}_{\varepsilon \bar{k}} + \boldsymbol{\Sigma}_{x \bar{k}} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{E}_{k} \\ \mathbf{\widehat{P}}_{\bar{k}} \end{bmatrix},$$
(A.51)

which simplifies to

$$\operatorname{Var}^{k}[\mathbf{D}]^{-1} \mathbb{E}^{k}[\mathbf{D}] = \begin{bmatrix} \boldsymbol{\iota}_{k} & \boldsymbol{\iota}_{\bar{k}} \delta \mathbf{h}_{\bar{k}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_{\varepsilon k}^{-1} & \mathbf{0} \\ \mathbf{0} & (\delta \mathbf{h}_{\bar{k}}^{2} \boldsymbol{\Sigma}_{\varepsilon \bar{k}} + \boldsymbol{\Sigma}_{x \bar{k}})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{E}_{k} \\ \widehat{\mathbf{P}}_{\bar{k}} \end{bmatrix}$$
(A.52)

$$= \begin{bmatrix} \boldsymbol{\iota}_{k} & \boldsymbol{\iota}_{\bar{k}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_{\varepsilon k}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \operatorname{diag} \begin{bmatrix} \frac{\gamma \Lambda_{a}}{\Lambda_{a}^{2} + \gamma^{2} \sigma_{\varepsilon a}^{2} \sigma_{xa}^{2}} \mid a \in \bar{k} \end{bmatrix} \begin{bmatrix} \mathbf{E}_{k} \\ \widehat{\mathbf{P}}_{\bar{k}} \end{bmatrix}.$$
(A.53)

According to Conjecture 1,

$$\widehat{\mathbf{P}}_{\bar{k}} = \operatorname{diag}\left[\frac{\Lambda_a}{\gamma \sigma_{\varepsilon a}^2} \mid a \in \bar{k}\right] \mathbf{E}_{\bar{k}} + \mathbf{x}_{\bar{k}},\tag{A.54}$$

which, after replacement into (A.53), yields:

$$\operatorname{Var}^{k}[\mathbf{D}]^{-1} \mathbb{E}^{k}[\mathbf{D}] = \boldsymbol{\iota}_{k} \operatorname{diag} \left[ \frac{1}{\sigma_{\varepsilon a}^{2}} \mid a \in k \right] \mathbf{E}_{k} + \boldsymbol{\iota}_{\bar{k}} \operatorname{diag} \left[ \frac{\Lambda_{a}^{2}}{\Lambda_{a}^{2} \sigma_{\varepsilon a}^{2} + \gamma^{2} \sigma_{\varepsilon a}^{4} \sigma_{xa}^{2}} \mid a \in \bar{k} \right] \mathbf{E}_{\bar{k}} + \boldsymbol{\iota}_{\bar{k}} \operatorname{diag} \left[ \frac{\gamma \Lambda_{a}}{\Lambda_{a}^{2} + \gamma^{2} \sigma_{\varepsilon a}^{2} \sigma_{xa}^{2}} \mid a \in \bar{k} \right] \mathbf{x}_{\bar{k}}.$$
(A.55)

We now go back to (A.50), which we write as

$$\boldsymbol{\tau} \mathbf{P} = \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k \operatorname{Var}^k[\mathbf{D}]^{-1} \mathbb{E}^k[\mathbf{D}] + \gamma \mathbf{x} - \gamma \mathbf{M},$$
(A.56)

which, after replacement of (A.55) becomes

$$\boldsymbol{\tau} \mathbf{P} = \begin{bmatrix} \operatorname{diag} \left[ \pi_a(\Lambda_a) \mid a \in \mathcal{A} \right] \\ \mathbf{0} \end{bmatrix} \mathbf{E} + \gamma \begin{bmatrix} \operatorname{diag} \left[ \frac{\pi_a(\Lambda_a)\sigma_{\varepsilon_a}^2}{\Lambda_a} \mid a \in \mathcal{A} \right] & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N-\mathcal{A}} \end{bmatrix} \mathbf{x} - \gamma \mathbf{M}, \quad (A.57)$$

where **E** is the column vector of earnings announcements and the functions  $\pi_a(\Lambda_a)$ ,  $a \in \mathcal{A}$  are defined in Lemma 1. We can now verify Conjecture 1:

$$\widehat{\mathbf{P}} = \frac{1}{\gamma} \begin{bmatrix} \operatorname{diag} \begin{bmatrix} \underline{\Lambda}_{a} \\ \pi_{a}(\Lambda_{a})\sigma_{\varepsilon_{a}}^{2} & | \ a \in \mathcal{A} \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N-\mathcal{A}} \end{bmatrix} \begin{bmatrix} \operatorname{diag} \begin{bmatrix} \pi_{a}(\Lambda_{a}) & | \ a \in \mathcal{A} \end{bmatrix} \\ \mathbf{0} & \mathbf{I}_{N-\mathcal{A}} \end{bmatrix} \mathbf{E} + \mathbf{x}$$
(A.58)

$$= \begin{bmatrix} \operatorname{diag} \left[ \frac{\Lambda_a}{\gamma \sigma_{\varepsilon_a}^2} \mid a \in \mathcal{A} \right] \\ \mathbf{0} \end{bmatrix} \mathbf{E} + \mathbf{x}, \tag{A.59}$$

which completes the proof of Proposition 3.

## A.6 Proof of Corollary 3.1

Define first  $\mathbf{\Pi} \equiv \text{diag} [\pi_a(\Lambda_a) \mid a \in \mathcal{A}]$ . From (A.57), the matrix of response coefficients to **E** for all firms in the economy,  $\boldsymbol{\alpha}$ , is given by

$$\boldsymbol{\alpha} = \boldsymbol{\tau}^{-1} \boldsymbol{\iota} \boldsymbol{\Pi} = (\operatorname{Var}[\mathbf{D}]^{-1} + \boldsymbol{\iota} \boldsymbol{\Pi} \boldsymbol{\iota}')^{-1} \boldsymbol{\iota} \boldsymbol{\Pi}, \qquad (A.60)$$

where  $\iota$  represents the matrix with all the column vectors  $\{\iota_a \mid a \in \mathcal{A}\}$ . Multiplying with  $\Pi \iota'$  and applying the Woodbury matrix identity yields:

$$\Pi \iota' \alpha = \Pi - (\Pi^{-1} + \iota' \operatorname{Var}[\mathbf{D}]\iota)^{-1}.$$
(A.61)

We recognize that  $\iota' \alpha = \alpha_A$  and  $\iota' \operatorname{Var}[\mathbf{D}]\iota = \operatorname{Var}[\mathbf{D}_A]$ , where  $\mathbf{D}_A$  is the  $A \times 1$  vector of payoffs for the announcing firms. Thus, after multiplication with  $\mathbf{\Pi}^{-1}$ , we obtain Eq. (24):

$$\boldsymbol{\alpha}_{\mathcal{A}} = \mathbf{I} - (\mathbf{I} + \operatorname{Var}[\mathbf{D}_{\mathcal{A}}]\mathbf{\Pi})^{-1}.$$
 (A.62)

The earnings response coefficients of the announcing firms are given by the diagonal elements of the matrix  $\boldsymbol{\alpha}_{\mathcal{A}}$ . We also note that Eq. (24) can alternatively be written  $\boldsymbol{\alpha}_{\mathcal{A}}^{-1} = \mathbf{I} + \mathbf{\Pi}^{-1} \operatorname{Var}[\mathbf{D}_{\mathcal{A}}]^{-1}$ , by means of the Woodbury matrix identity.

#### A.7 Equilibrium solution algorithm

We will first show that the maximization problem (6) is equivalent with the simplified form (25):

$$\max_{k \in \mathscr{P}(\mathcal{A})} \mathbb{E}_0 \left[ \max_{\mathbf{q}^k} \mathbb{E}_1^k \left[ -e^{-\gamma \left( W^k - c|k| \right)} \right] \right] = \max_{k \in \mathscr{P}(\mathcal{A})} e^{\gamma c|k|} \mathbb{E}_0 \left[ \max_{\mathbf{q}^k} \mathbb{E}_1^k \left[ -e^{-\gamma \left( q^k \right)' \mathbf{R}^e} \right] \right]$$
(A.63)

$$= \max_{k \in \mathscr{P}(\mathcal{A})} e^{\gamma c|k|} \mathbb{E}_0 \left[ -e^{-\frac{1}{2} \mathbb{E}_1^k [\mathbf{R}^e]' \operatorname{Var}_1^k [\mathbf{R}^e]^{-1} \mathbb{E}_1^k [\mathbf{R}^e]} \right], \quad (A.64)$$

which, after using (A.32) and the law of iterated expectations, yields

$$\max_{k \in \mathscr{P}(\mathcal{A})} e^{\gamma c|k|} \mathbb{E}_0 \left[ \sqrt{\frac{\det(\boldsymbol{\tau}^{\emptyset})}{\det(\boldsymbol{\tau}^k)}} \mathcal{U}_1^{\emptyset} \right] = \max_{k \in \mathscr{P}(\mathcal{A})} e^{\gamma c|k|} \sqrt{\frac{\det(\boldsymbol{\tau}^{\emptyset})}{\det(\boldsymbol{\tau}^k)}} \mathbb{E}_0 \left[ \mathcal{U}_1^{\emptyset} \right].$$
(A.65)

We notice that  $\mathbb{E}_0\left[\mathcal{U}_1^{\emptyset}\right]$  is a constant that does not depend on the individual choice of the investor. Dividing by this (negative) constant yields

$$\max_{k \in \mathscr{P}(\mathcal{A})} \frac{1}{2} \ln(\det(\boldsymbol{\tau}^k)) - \frac{1}{2} \ln(\det(\boldsymbol{\tau}^{\emptyset})) - \gamma c|k| = \max_{k \in \mathscr{P}(\mathcal{A})} \frac{1}{2} \ln B_{\emptyset}^k,$$
(A.66)

and therefore the optimization problem at time 0 for each investor in this economy is (25).

To prove that the function  $\ln B^k_{\emptyset}$  is submodular, consider two types  $k, k' \in \mathscr{P}(\mathcal{A})$  with  $k \subseteq k'$ and  $a \in \mathcal{A} \setminus k'$ , then use (A.39)-(A.40) to compute

$$\ln B_{\emptyset}^{k \cup \{a\}} - \ln B_{\emptyset}^{k} = \ln B_{k}^{k \cup \{a\}} \tag{A.67}$$

$$= \ln\left(\frac{\sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2} + \frac{b_a^2}{\frac{1}{U^2} + \sum_{\alpha=1}^A \frac{\ell_\alpha^k b_\alpha^2}{\ell_\alpha^k \sigma_{e\alpha}^2 + \sigma_{\varepsilon a}^2}} \frac{(1-\ell_a)\sigma_{\varepsilon a}^2}{(\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2)^2}\right) - 2\gamma c.$$
(A.68)

The same difference is lower when written for k' instead of k, due to the term  $\sum_{\alpha=1}^{A} \frac{\ell_{\alpha}^{k} b_{\alpha}^{2}}{\ell_{\alpha}^{k} \sigma_{e\alpha}^{2} + \sigma_{\varepsilon\alpha}^{2}}$  in the denominator (this term is larger when written for k' because  $k \subseteq k'$ ). Therefore,

$$\ln B^{k\cup\{a\}}_{\emptyset} - \ln B^k_{\emptyset} \ge \ln B^{k'\cup\{a\}}_{\emptyset} - \ln B^{k'}_{\emptyset}, \tag{A.69}$$

and thus the function  $\ln B_{\emptyset}^k$  is indeed submodular. We further prove the following Lemma.

**Lemma A4.** For any two types  $k, k' \in \mathscr{P}(\mathcal{A})$  and  $\lambda^k > 0$ , a migration of a positive mass of investors  $z < \lambda^k$  from k to k' decreases  $B_{\emptyset}^{k'}/B_{\emptyset}^k$ .

*Proof.* Consider a type  $k \in \mathscr{P}(\mathcal{A})$  and its complement  $\bar{k} = \mathcal{A} \setminus k$ . Using Proposition 1, write

$$\det(\boldsymbol{\tau}^{k}) = \det(\operatorname{Var}[\mathbf{D}]^{-1}) \left( \prod_{a \in k} \frac{\sigma_{ea}^{2} + \sigma_{\varepsilon a}^{2}}{\sigma_{\varepsilon a}^{2}} \right) \left( \prod_{a \in \bar{k}} \frac{\ell_{a} \sigma_{ea}^{2} + \sigma_{\varepsilon a}^{2}}{\sigma_{\varepsilon a}^{2}} \right) \times \left( 1 + U^{2} \sum_{a \in k} \frac{b_{a}^{2}}{\sigma_{ea}^{2} + \sigma_{\varepsilon a}^{2}} + U^{2} \sum_{a \in \bar{k}} \frac{\ell_{a} b_{a}^{2}}{\ell_{a} \sigma_{ea}^{2} + \sigma_{\varepsilon a}^{2}} \right).$$
(A.70)

A migration from  $k \to k'$  increases the terms  $\prod_{a \in \bar{k}} \frac{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\sigma_{\varepsilon a}^2}$  and  $\sum_{a \in \bar{k}} \frac{\ell_a b_a^2}{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}$ , while all

the other terms of the decomposition (A.70) remain constant. Thus,  $\det(\boldsymbol{\tau}^k)$  increases. One can similarly show that  $\det(\boldsymbol{\tau}^{k'})$  decreases, and therefore  $B_{\emptyset}^{k'}/B_{\emptyset}^k$  decreases.

The submodularity property of the function  $\ln B_{\emptyset}^k$ , coupled with the monotonicity of  $B_{\emptyset}^{k'}/B_{\emptyset}^k$ implied by Lemma A4, justify the use of an iterative algorithm that converges towards a stable equilibrium. The algorithm is adapted from Hu and Shi (2019) and Arkolakis et al. (2021) and consists of the following steps:

- 1. Start from any set of positive initial values  $\{\lambda_0^k > 0 \mid k \in \mathscr{P}(\mathcal{A})\}$  such that  $\sum_k \lambda_0^k = 1$ . Compute the benefit-cost ratios  $\{B_{\emptyset}^k \mid k \in \mathscr{P}(\mathcal{A})\}$ .
- 2. For any two types  $k, k' \in \mathscr{P}(\mathcal{A})$ , compute  $B_{\emptyset}^{k'}/B_{\emptyset}^k$ :
  - (a) if  $B_{\emptyset}^{k'}/B_{\emptyset}^{k} = 1$ , no further changes in  $\lambda^{k}$  and  $\lambda^{k'}$  are needed at this step.
  - (b) if  $B_{\emptyset}^{k'}/B_{\emptyset}^{k} > 1$ , then allow a small fraction of the population of type k investors to migrate to type k', which will decrease  $B_{\emptyset}^{k'}/B_{\emptyset}^{k}$  (Lemma A4). In the illustration below, the dot A depicts the initial values  $\{\lambda^{k}, \lambda^{k'}\}$ , located on a line with slope  $\lambda^{k'}/\lambda^{k}$ . The algorithm multiplies the slope of the line by m > 1 and finds two new values  $\lambda_{new}^{k}$  and  $\lambda_{new}^{k'}$  such that  $\lambda_{new}^{k} + \lambda_{new}^{k'} = \lambda^{k} + \lambda^{k'}$  and  $\lambda_{new}^{k} < \lambda^{k}$ , thus reaching the dot B:

After the multiplication, the new values for  $\lambda^k$  and  $\lambda^{k'}$  are given by

$$\lambda_{new}^{k} = \lambda^{k} \frac{\lambda^{k} + \lambda^{k'}}{\lambda^{k} + m\lambda^{k'}} \quad \text{and} \quad \lambda_{new}^{k'} = \lambda^{k'} \frac{\lambda^{k} + \lambda^{k'}}{\lambda^{k} / m + \lambda^{k'}}.$$
 (A.71)

To ensure stability of the solution, m is set to increase with  $(B_{\emptyset}^{k'}/B_{\emptyset}^{k}-1)$ . Finally, compute the benefit-cost ratios  $\{B_{\emptyset}^{k} \mid k \in \mathscr{P}(\mathcal{A})\}$  using the new values  $\{\lambda_{new}^{k}, \lambda_{new}^{k'}\}$ .

- (c) if  $B_{\emptyset}^{k'}/B_{\emptyset}^{k} < 1$ , apply a similar procedure as in the previous step, moving from A to C.
- 3. Iterate step 2 until the algorithm has converged to the desired accuracy and the conditions of Theorem 1 are satisfied. Convergence is guaranteed by Lemma A4.

#### A.8 (CAPM) Proofs of Proposition 4 and Corollary 4.1

Investors' learning and uncertainty at time 0 Given information at time 0, investors form beliefs about  $\mathbf{D}$ ,  $\mathbb{E}_0[\mathbf{D}]$  and  $\operatorname{Var}_0[\mathbf{D}]$ . The prior variance of  $\mathbf{D}$  is  $\operatorname{Var}[\mathbf{D}] = U^2 \mathbf{b} \mathbf{b}' + \operatorname{Var}[\mathbf{e}]$ .

Based on investors' information set  $\mathcal{F}_0 = \{G\}$ , we apply the Projection Theorem (page 42), with:

$$\Sigma_{\theta\theta} = \operatorname{Var}[\mathbf{D}] = U^2 \mathbf{b} \mathbf{b}' + \operatorname{Var}[\mathbf{e}]$$
(A.72)

$$\Sigma_{\theta s} = \operatorname{Cov}[\mathbf{D}, G] = \operatorname{Var}[\mathbf{D}]\mathbf{M} = U^2 \mathbf{b} + \operatorname{Var}[\mathbf{e}]\mathbf{M}$$
(A.73)

$$\Sigma_{s\theta} = \operatorname{Cov}[G, \mathbf{D}] = \mathbf{M}' \operatorname{Var}[\mathbf{D}] = U^2 \mathbf{b}' + \mathbf{M}' \operatorname{Var}[\mathbf{e}]$$
(A.74)

$$\Sigma_{ss} = \operatorname{Var}[G] = \mathbf{M}' \operatorname{Var}[\mathbf{D}]\mathbf{M} + \sigma_g^2 = U^2 + \mathbf{M}' \operatorname{Var}[\mathbf{e}]\mathbf{M} + \sigma_g^2, \qquad (A.75)$$

where **b** is the  $N \times 1$  vector of firms' exposures to the systematic component f,  $\mathbf{M'b} = 1$  by assumption, and **e** is the vector of idiosyncratic components in firms' payoffs. Then

$$\operatorname{Var}_{0}[\mathbf{D}] = U^{2}\mathbf{b}\mathbf{b}' + \operatorname{Var}[\mathbf{e}] - (U^{2}\mathbf{b} + \operatorname{Var}[\mathbf{e}]\mathbf{M}) \frac{1}{U^{2} + \mathbf{M}' \operatorname{Var}[\mathbf{e}]\mathbf{M} + \sigma_{g}^{2}} (U^{2}\mathbf{b}' + \mathbf{M}' \operatorname{Var}[\mathbf{e}]), \quad (A.76)$$

or, using the Woodbury Matrix Identity:

$$\boldsymbol{\tau}_0 \equiv \operatorname{Var}_0[\mathbf{D}]^{-1} = \operatorname{Var}[\mathbf{D}]^{-1} + \frac{1}{\sigma_g^2} \mathbf{M} \mathbf{M}'.$$
(A.77)

Investors' posterior expectation at time 0 is:

$$\mathbb{E}_0[\mathbf{D}] = \operatorname{Var}[\mathbf{D}]\mathbf{M}(\sigma_g^2 + \mathbf{M}' \operatorname{Var}[\mathbf{D}]\mathbf{M})^{-1}G = \frac{1}{\sigma_g^2} \boldsymbol{\tau}_0^{-1}\mathbf{M}G.$$
(A.78)

where the second equality results from multiplying the first equality with  $\tau_0^{-1}\tau_0$  and simplifying.

The market-wide uncertainty at time 0 is defined as  $U_0^2 \equiv \operatorname{Var}_0[\mathbf{M'D}]$ :

$$U_0^2 = \mathbf{M}' \operatorname{Var}_0[\mathbf{D}] \mathbf{M} = \sigma_g^4 \frac{1}{\sigma_g^2} \mathbf{M}' \left( \operatorname{Var}[\mathbf{D}]^{-1} + \mathbf{M} \frac{1}{\sigma_g^2} \mathbf{M}' \right)^{-1} \mathbf{M} \frac{1}{\sigma_g^2}$$
(A.79)

$$=\sigma_g^4 \left[\frac{1}{\sigma_g^2} - \left(\sigma_g^2 + \mathbf{M}' \operatorname{Var}[\mathbf{D}]\mathbf{M}\right)^{-1}\right] = \frac{\sigma_g^2 \mathbf{M}' \operatorname{Var}[\mathbf{D}]\mathbf{M}}{\sigma_g^2 + \mathbf{M}' \operatorname{Var}[\mathbf{D}]\mathbf{M}} = \frac{1}{\frac{1}{\mathbf{M}' \operatorname{Var}[\mathbf{D}]\mathbf{M}} + \frac{1}{\sigma_g^2}}.$$
 (A.80)

Since  $\mathbf{M}' \operatorname{Var}[\mathbf{D}]\mathbf{M} = U^2 + \mathbf{M}' \operatorname{Var}[\mathbf{e}]\mathbf{M}$ ,  $U_0$  increases if U increases or if  $\sigma_g$  increases. Furthermore,  $\lim_{N\to\infty} \mathbf{M}' \operatorname{Var}[\mathbf{D}]\mathbf{M} = U^2$  and we recover Eq. (35) in the text.

**Equilibrium** To solve for the equilibrium prices, conjecture the following linear forms:

$$\mathbf{P}_0 = \mathbf{\Gamma}_0 G + \boldsymbol{\xi}_{00} \mathbf{x}_0 - \boldsymbol{\zeta}_0 \mathbf{M} \tag{A.81}$$

$$\mathbf{P}_1 = \mathbf{\Gamma}_1 G + \boldsymbol{\xi}_{01} \mathbf{x}_0 - \boldsymbol{\zeta}_1 \mathbf{M} + \boldsymbol{\alpha}_1 \mathbf{E} + \boldsymbol{\xi}_1 \mathbf{x}_1$$
(A.82)

Noise traders hold  $\mathbf{x}_0$  at time 0 and  $\mathbf{x}_0 + \mathbf{x}_1$  at time 1.

**Time 1** At time 1, investors' learning follows Appendix A.1, with the addition that all investors observe G from the previous period and the conjecture (11) must change to take this into account (Note: investors' information at time 0 is public, and thus  $\mathbf{x}_0$  is observed):

$$\widehat{\mathbf{P}}_{1} \equiv \boldsymbol{\xi}_{1}^{-1}(\mathbf{P}_{1} - \boldsymbol{\Gamma}_{1}G - \boldsymbol{\xi}_{01}\mathbf{x}_{0} + \boldsymbol{\zeta}_{1}\mathbf{M}) = \sum_{a=1}^{A} \frac{\Lambda_{a}}{\gamma \sigma_{\varepsilon a}^{2}} \boldsymbol{\iota}_{a} E_{a} + \mathbf{x}_{1}.$$
(A.83)

The posterior variance  $\operatorname{Var}_{1}^{k}[\mathbf{D}]$  is now

$$\boldsymbol{\tau}_{1}^{k} \equiv \operatorname{Var}_{1}^{k} [\mathbf{D}]^{-1} = \boldsymbol{\tau}_{0} + \sum_{a=1}^{A} \frac{\ell_{a}^{k}}{\sigma_{\varepsilon a}^{2}} \boldsymbol{\iota}_{a} \boldsymbol{\iota}_{a}', \qquad (A.84)$$

where  $\tau_0$  is defined in (A.77). The weighted average precision at time 1 is then

$$\boldsymbol{\tau}_{1} = \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^{k} \boldsymbol{\tau}_{1}^{k} = \boldsymbol{\tau}_{0} + \begin{bmatrix} \operatorname{diag}[\pi_{a}(\Lambda_{a}) \mid a \in \mathcal{A}] & \mathbf{0}_{A \times (N-A)} \\ \mathbf{0}_{(N-A) \times A} & \mathbf{0}_{(N-A) \times (N-A)} \end{bmatrix},$$
(A.85)

where  $\pi_a(\Lambda_a)$  is defined as in (22). We can then write a modified version of (A.55):

$$\boldsymbol{\tau}_{1}^{k} \mathbb{E}_{1}^{k} [\mathbf{D}] = \boldsymbol{\iota}_{k} \operatorname{diag} \left[ \frac{1}{\sigma_{\varepsilon a}^{2}} \mid a \in k \right] \mathbf{E}_{k} + \boldsymbol{\iota}_{\bar{k}} \operatorname{diag} \left[ \frac{\Lambda_{a}^{2}}{\Lambda_{a}^{2} \sigma_{\varepsilon a}^{2} + \gamma^{2} \sigma_{\varepsilon a}^{4} \sigma_{xa}^{2}} \mid a \in \bar{k} \right] \mathbf{E}_{\bar{k}} + \boldsymbol{\iota}_{\bar{k}} \operatorname{diag} \left[ \frac{\gamma \Lambda_{a}}{\Lambda_{a}^{2} + \gamma^{2} \sigma_{\varepsilon a}^{2} \sigma_{xa}^{2}} \mid a \in \bar{k} \right] \mathbf{x}_{\bar{k}} + \frac{1}{\sigma_{g}^{2}} \mathbf{M}G,$$
(A.86)

which leads to a new market clearing condition (the counterpart of (8) in the baseline setup):

$$\sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k \mathbf{q}_1^k + \mathbf{x}_0 + \mathbf{x}_1 = \mathbf{M}, \quad \text{where } \mathbf{q}_1^k = \frac{1}{\gamma} \boldsymbol{\tau}_1^k (\mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_1).$$
(A.87)

Thus, prices at time 1 solve a modified version of (A.56) in the baseline setup, and one can check that they verify the new conjecture (A.83). We thus obtain (32) in Proposition 4:

$$\boldsymbol{\tau}_1 \mathbf{P}_1 = \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^k \boldsymbol{\tau}_1^k \mathbb{E}_1^k [\mathbf{D}] + \gamma \mathbf{x}_0 + \gamma \mathbf{x}_1 - \gamma \mathbf{M}.$$
(A.88)

**Time 0** Consider an investor who at time 0 knows that she will be of type k at time 1. We prove here that knowing her future type does not change her portfolio choice at time 0,

$$\mathbf{q}_0 = \frac{1}{\gamma} \boldsymbol{\tau}_0(\mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0), \qquad (A.89)$$

which is Eq. (29) in Proposition 4. The proof of this statement follows Brennan and Cao (1997), adapted to our Grossman and Stiglitz (1980) setup with information acquisition.

The final wealth of a type-k investor at time 2 is (taking into account the cost of information):

$$W^{k} = (\mathbf{q}_{0}^{k})'(\mathbf{P}_{1} - \mathbf{P}_{0}) - c|k| + (\mathbf{q}_{1}^{k})'(\mathbf{D} - \mathbf{P}_{0}),$$
(A.90)

and he expected utility at time 1 for this investor is then given by

$$\mathcal{U}_{1}^{k} = -\exp\left[-\gamma(\mathbf{q}_{0}^{k})'(\mathbf{P}_{1} - \mathbf{P}_{0}) + \gamma c|k| - \frac{1}{2}(\mathbb{E}_{1}^{k}[\mathbf{D}] - \mathbf{P}_{1})'\boldsymbol{\tau}_{1}^{k}(\mathbb{E}_{1}^{k}[\mathbf{D}] - \mathbf{P}_{1})\right].$$
 (A.91)

Defining

$$\mathbf{a}^k \equiv \mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_0 \tag{A.92}$$

$$\mathbf{c}^k \equiv \mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_1,\tag{A.93}$$

we can write the expected utility at time 1 as

$$\mathcal{U}_{1}^{k} = -\exp\left[-\gamma(\mathbf{q}_{0}^{k})'(\mathbf{a}^{k}-\mathbf{c}^{k})+\gamma c|k|-\frac{1}{2}(\mathbf{c}^{k})'\boldsymbol{\tau}_{1}^{k}\mathbf{c}^{k}\right].$$
(A.94)

To compute the expected utility at time 0,  $\mathbb{E}_0[\mathcal{U}_1^k]$ , we need the joint distribution at time 0 of  $\mathbf{a}^k$  and  $\mathbf{c}^k$  (both of them are random for a time 0 investor). The law of iterated expectations implies:

$$\mathbb{E}_0[\mathbf{a}^k] = \mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0. \tag{A.95}$$

Using that  $\mathbb{E}_0[\mathbf{x}_1] = \mathbf{0}$ , Eq. (A.88) implies

$$\boldsymbol{\tau}_{1} \mathbb{E}_{0}[\mathbf{P}_{1}] = \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^{k} \boldsymbol{\tau}_{1}^{k} \mathbb{E}_{0}[\mathbf{D}] + \gamma \mathbf{x}_{0} - \gamma \mathbf{M}$$
(A.96)

$$= \boldsymbol{\tau}_1 \mathbb{E}_0[\mathbf{D}] - \gamma(\mathbf{M} - \mathbf{x}_0), \qquad (A.97)$$

and thus

$$\mathbb{E}_0[\mathbf{P}_1] = \mathbb{E}_0[\mathbf{D}] - \gamma \boldsymbol{\tau}_1^{-1}(\mathbf{M} - \mathbf{x}_0), \qquad (A.98)$$

which leads to

$$\mathbb{E}_0[\mathbf{c}^k] = \mathbb{E}_0[\mathbf{D}] - \mathbb{E}_0[\mathbf{D}] + \gamma \boldsymbol{\tau}_1^{-1}(\mathbf{M} - \mathbf{x}_0) = \gamma \boldsymbol{\tau}_1^{-1}(\mathbf{M} - \mathbf{x}_0).$$
(A.99)

We now compute variances and covariances of  $\mathbf{a}^k$  and  $\mathbf{c}^k$ :

$$\operatorname{Var}_{0}[\mathbf{a}^{k}] = \operatorname{Var}_{0}[\mathbb{E}_{1}^{k}[\mathbf{D}]] = \operatorname{Var}_{0}[\mathbf{D}] - \operatorname{Var}_{1}^{k}[\mathbf{D}] = \boldsymbol{\tau}_{0}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}, \quad (A.100)$$

and, defining  $\mathbf{\Omega} \equiv \operatorname{Var}_0[\mathbf{D} - \mathbf{P}_1],$ 

$$\operatorname{Var}_{0}[\mathbf{c}^{k}] = \operatorname{Var}_{0}\left[\mathbb{E}_{1}^{k}[\mathbf{D} - \mathbf{P}_{1}]\right] = \mathbf{\Omega} - (\boldsymbol{\tau}_{1}^{k})^{-1}.$$
(A.101)

Finally, the covariance  $\operatorname{Cov}_0[\mathbf{a}^k, \mathbf{c}^k]$  is

$$\operatorname{Cov}_{0}[\mathbf{a}^{k}, \mathbf{c}^{k}] = \operatorname{Cov}_{0}[\mathbb{E}_{1}^{k}[\mathbf{D}], \mathbb{E}_{1}^{k}[\mathbf{D}] - \mathbf{P}_{1}]$$
(A.102)

$$= \operatorname{Var}_{0}[\mathbb{E}_{1}^{k}[\mathbf{D}]] - \operatorname{Cov}_{0}[\mathbb{E}_{1}^{k}[\mathbf{D}], \mathbf{P}_{1}]$$
(A.103)

$$= \boldsymbol{\tau}_0^{-1} - (\boldsymbol{\tau}_1^k)^{-1} - \operatorname{Cov}_0[\mathbb{E}_1^k[\mathbf{D}], \mathbf{P}_1].$$
(A.104)

To solve for  $\operatorname{Cov}_0[\mathbb{E}_1^k[\mathbf{D}], \mathbf{P}_1]$ , consider the most informed type, denoted by  $\tilde{k}$ . Then

$$\operatorname{Cov}_{0}[\mathbb{E}_{1}^{\tilde{k}}[\mathbf{D}], \mathbf{P}_{1}] = \underbrace{\operatorname{Cov}_{1}^{k}[\mathbb{E}_{1}^{\tilde{k}}[\mathbf{D}], \mathbf{P}_{1}]}_{=0} + \operatorname{Cov}_{0}[\mathbb{E}_{1}^{k}[\mathbf{D}], \mathbf{P}_{1}], \qquad (A.105)$$

and thus all the covariances  $\operatorname{Cov}_0[\mathbb{E}_1^k[\mathbf{D}], \mathbf{P}_1]$  take the same value,  $\operatorname{Cov}_0[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbf{P}_1]$ .

Then, using (A.88):

$$\operatorname{Cov}_{0}[\mathbb{E}_{1}^{\tilde{k}}[\mathbf{D}],\mathbf{P}_{1}] = \operatorname{Cov}_{0}\left[\mathbb{E}_{1}^{\tilde{k}}[\mathbf{D}],\boldsymbol{\tau}_{1}^{-1}\left(\sum_{k\in\mathscr{P}(\mathcal{A})}\lambda^{k}\boldsymbol{\tau}_{1}^{k}\mathbb{E}_{1}^{k}[\mathbf{D}] + \gamma\mathbf{x}_{0} + \gamma\mathbf{x}_{1} - \gamma\mathbf{M}\right)\right],\qquad(A.106)$$

and since  $\tilde{k}$  is the most informed investor, she does not learn from prices and therefore  $\mathbb{E}_1^{\tilde{k}}[\mathbf{D}]$  does not depend on  $\mathbf{x}_1$ . Thus, we can further write the covariance above as

$$\operatorname{Cov}_{0}[\mathbb{E}_{1}^{\tilde{k}}[\mathbf{D}], \mathbf{P}_{1}] = \boldsymbol{\tau}_{1}^{-1} \sum_{k \in \mathscr{P}(\mathcal{A})} \lambda^{k} \boldsymbol{\tau}_{1}^{k} \operatorname{Cov}_{0}[\mathbb{E}_{1}^{\tilde{k}}[\mathbf{D}], \mathbb{E}_{1}^{k}[\mathbf{D}]]$$
(A.107)

$$=\boldsymbol{\tau}_{1}^{-1}\sum_{k\in\mathscr{P}(\mathcal{A})}\lambda^{k}\boldsymbol{\tau}_{1}^{k}\left(\underbrace{\operatorname{Cov}_{1}^{k}[\mathbb{E}_{1}^{\tilde{k}}[\mathbf{D}],\mathbb{E}_{1}^{k}[\mathbf{D}]]}_{0}+\operatorname{Cov}_{0}[\mathbb{E}_{1}^{k}[\mathbf{D}],\mathbb{E}_{1}^{k}[\mathbf{D}]]\right)$$
(A.108)

$$=\boldsymbol{\tau}_{1}^{-1}\sum_{k\in\mathscr{P}(\mathcal{A})}\lambda^{k}\boldsymbol{\tau}_{1}^{k}\left(\boldsymbol{\tau}_{0}^{-1}-(\boldsymbol{\tau}_{1}^{k})^{-1}\right)$$
(A.109)

$$= \boldsymbol{\tau}_0^{-1} - \boldsymbol{\tau}_1^{-1},$$
 (A.110)

and thus, going back to (A.104), we obtain

$$\operatorname{Cov}_{0}[\mathbf{a}^{k},\mathbf{c}^{k}] = \boldsymbol{\tau}_{0}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1} - (\boldsymbol{\tau}_{0}^{-1} - \boldsymbol{\tau}_{1}^{-1}) = \boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}.$$
 (A.111)

Eqs. (A.95), (A.99), (A.100), (A.101), and (A.111) imply the joint distribution of  $\mathbf{a}^k$  and  $\mathbf{c}^k$ :

$$\begin{bmatrix} \mathbf{a}^{k} \\ \mathbf{c}^{k} \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} \mathbb{E}_{0}[\mathbf{D}] - \mathbf{P}_{0} \\ \gamma \boldsymbol{\tau}_{1}^{-1}(\mathbf{M} - \mathbf{x}_{0}) \end{bmatrix}, \begin{bmatrix} \boldsymbol{\tau}_{0}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1} & \boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1} \\ \boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1} & \boldsymbol{\Omega} - (\boldsymbol{\tau}_{1}^{k})^{-1} \end{bmatrix} \right).$$
(A.112)

We are now ready to compute

$$\mathbb{E}_{0}[\mathcal{U}_{1}^{k}] = \mathbb{E}_{0}\left[-\exp\left(-\gamma(\mathbf{q}_{0}^{k})'(\mathbf{a}^{k}-\mathbf{c}^{k})+\gamma c|k|-\frac{1}{2}(\mathbf{c}^{k})'\boldsymbol{\tau}_{1}^{k}\mathbf{c}^{k}\right)\right]$$
(A.113)

using Lemma A2. To simplify notation, denote by  $\mathbb{E}_0[\mathbf{a}^k] = \mathbf{m}_a$  and  $\mathbb{E}_0[\mathbf{c}^k] = \mathbf{m}_c$  (these do not depend on k) and  $\mathbf{z}$  the demeaned vector,  $\begin{bmatrix} \mathbf{a}^k \\ \mathbf{c}^k \end{bmatrix} = \begin{bmatrix} \mathbf{z}_a \\ \mathbf{z}_c \end{bmatrix} + \begin{bmatrix} \mathbf{m}_a \\ \mathbf{m}_c \end{bmatrix}$ . The exponent above is

$$\mathbf{z}'\underbrace{\begin{bmatrix}0 & 0\\0 & -\boldsymbol{\tau}_1^k/2\end{bmatrix}}_{\mathbf{F}}\mathbf{z} + \underbrace{\begin{bmatrix}-\gamma(\mathbf{q}_0^k)' & \gamma(\mathbf{q}_0^k)' - \mathbf{m}_c'\boldsymbol{\tau}_1^k\end{bmatrix}}_{\mathbf{G}'}\mathbf{z} + \underbrace{\gamma c|k| + \gamma(\mathbf{q}_0^k)'(\mathbf{m}_c - \mathbf{m}_a) - \frac{1}{2}\mathbf{m}_c'\boldsymbol{\tau}_1^k\mathbf{m}_c}_{\mathbf{H}}.$$
 (A.114)

Let  $\Sigma$  be the covariance matrix in (A.112). Then,

$$\mathbf{I} - 2\mathbf{\Sigma}\mathbf{F} = \mathbf{I} - 2\begin{bmatrix} 0 & -(\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1})\frac{\boldsymbol{\tau}_1^k}{2} \\ 0 & -(\boldsymbol{\Omega} - (\boldsymbol{\tau}_1^k)^{-1})\frac{\boldsymbol{\tau}_1^k}{2} \end{bmatrix}$$
(A.115)

$$= \mathbf{I} - \begin{bmatrix} 0 & -\boldsymbol{\tau}_1^{-1}\boldsymbol{\tau}_1^k + \mathbf{I} \\ 0 & -\boldsymbol{\Omega}\boldsymbol{\tau}_1^k + \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \boldsymbol{\tau}_1^{-1}\boldsymbol{\tau}_1^k - \mathbf{I} \\ 0 & \boldsymbol{\Omega}\boldsymbol{\tau}_1^k \end{bmatrix}.$$
 (A.116)

Use block inversion to obtain  $(\mathbf{I} - 2\Sigma \mathbf{F})^{-1}$ . The diagonal blocks are both invertible, and thus

$$(\mathbf{I} - 2\Sigma \mathbf{F})^{-1} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & (\Omega \boldsymbol{\tau}_1^k)^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{I} & -(\boldsymbol{\tau}_1^{-1} \boldsymbol{\tau}_1^k - \mathbf{I})(\Omega \boldsymbol{\tau}_1^k)^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$
(A.117)

$$= \begin{bmatrix} \mathbf{I} & [(\boldsymbol{\tau}_1^k)^{-1} - \boldsymbol{\tau}_1^{-1}] \boldsymbol{\Omega}^{-1} \\ \mathbf{0} & (\boldsymbol{\tau}_1^k)^{-1} \boldsymbol{\Omega}^{-1} \end{bmatrix},$$
(A.118)

and thus  $(\mathbf{I} - 2\boldsymbol{\Sigma}\mathbf{F})^{-1}\boldsymbol{\Sigma}$  equals

$$\begin{bmatrix} \boldsymbol{\tau}_{0}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1} - [\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}] \boldsymbol{\Omega}^{-1} [\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}] & [\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}] \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_{1}^{k})^{-1} \\ (\boldsymbol{\tau}_{1}^{k})^{-1} \boldsymbol{\Omega}^{-1} [\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}] & (\boldsymbol{\tau}_{1}^{k})^{-1} [\mathbf{I} - \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_{1}^{k})^{-1}] \end{bmatrix}.$$
(A.119)

In Lemma A2, the term  $\frac{1}{2}\mathbf{G}'\left(\mathbf{I}-2\boldsymbol{\Sigma}\mathbf{F}\right)^{-1}\boldsymbol{\Sigma}\mathbf{G}$  equals

$$\frac{1}{2} \begin{bmatrix} -\gamma(\mathbf{q}_{0}^{k})' & \gamma(\mathbf{q}_{0}^{k})' - \mathbf{m}_{c}'\boldsymbol{\tau}_{1}^{k} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{\tau}_{0}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1} - [\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}]\boldsymbol{\Omega}^{-1}[\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}] & [\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}]\boldsymbol{\Omega}^{-1}(\boldsymbol{\tau}_{1}^{k})^{-1} \\ & (\boldsymbol{\tau}_{1}^{k})^{-1}\boldsymbol{\Omega}^{-1}[\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}] & (\boldsymbol{\tau}_{1}^{k})^{-1}[\mathbf{I} - \boldsymbol{\Omega}^{-1}(\boldsymbol{\tau}_{1}^{k})^{-1}] \end{bmatrix} \quad (A.120) \times \begin{bmatrix} -\gamma \mathbf{q}_{0}^{k} \\ \gamma \mathbf{q}_{0}^{k} - \boldsymbol{\tau}_{1}^{k} \mathbf{m}_{c} \end{bmatrix},$$

and thus it has the following form

$$\frac{1}{2}\mathbf{G}'(\mathbf{I} - 2\Sigma\mathbf{F})^{-1}\Sigma\mathbf{G} + \mathbf{H} = \frac{1}{2}\begin{bmatrix}g_1' & g_2'\end{bmatrix}\begin{bmatrix}a & b\\b' & d\end{bmatrix}\begin{bmatrix}g_1\\g_2\end{bmatrix} = \frac{1}{2}g_1'ag_1 + g_1'bg_2 + \frac{1}{2}g_2'dg_2 + \mathbf{H}, \quad (A.121)$$

with,

$$\frac{1}{2}g_1'ag_1 \equiv \frac{1}{2}\gamma^2(\mathbf{q}_0^k)' \left(\boldsymbol{\tau}_0^{-1} - (\boldsymbol{\tau}_1^k)^{-1} - [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}]\boldsymbol{\Omega}^{-1}[\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}]\right) \mathbf{q}_0^k \tag{A.122}$$

$$g_1' b g_2 \equiv -\gamma (\mathbf{q}_0^k)' [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1} [\gamma \mathbf{q}_0^k - \boldsymbol{\tau}_1^k \mathbf{m}_c]$$
(A.123)

$$\frac{1}{2}g_2'dg_2 \equiv \frac{1}{2}[\gamma(\mathbf{q}_0^k)' - \mathbf{m}_c'\boldsymbol{\tau}_1^k](\boldsymbol{\tau}_1^k)^{-1}[\mathbf{I} - \boldsymbol{\Omega}^{-1}(\boldsymbol{\tau}_1^k)^{-1}][\gamma\mathbf{q}_0^k - \boldsymbol{\tau}_1^k\mathbf{m}_c].$$
(A.124)

Taking the first order condition with respect to  $\mathbf{q}_0^k$  yields (using matrix differentiation rules:  $\partial x' A x / \partial x = (A + A') x$  and  $\partial x' A / \partial x = A$ ):

$$\frac{\partial \frac{1}{2} g_1' a g_1}{\partial \mathbf{q}_0^k} = \gamma^2 \left( \boldsymbol{\tau}_0^{-1} - (\boldsymbol{\tau}_1^k)^{-1} - [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \right) \mathbf{q}_0^k$$
(A.125)

$$\frac{\partial g_1' b g_2}{\partial \mathbf{q}_0^k} = -2\gamma^2 [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \mathbf{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1} \mathbf{q}_0^k + \gamma [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \mathbf{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1} \boldsymbol{\tau}_1^k \mathbf{m}_c$$
(A.126)

$$= -2\gamma^{2} [\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}] \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_{1}^{k})^{-1} \mathbf{q}_{0}^{k} + \gamma [\boldsymbol{\tau}_{1}^{-1} - (\boldsymbol{\tau}_{1}^{k})^{-1}] \boldsymbol{\Omega}^{-1} \mathbf{m}_{c}$$
(A.127)

$$\frac{\partial \frac{1}{2} g_2' dg_2}{\partial \mathbf{q}_0^k} = \gamma^2 (\boldsymbol{\tau}_1^k)^{-1} [\mathbf{I} - \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1}] \mathbf{q}_0^k - \frac{1}{2} \gamma (\boldsymbol{\tau}_1^k)^{-1} [\mathbf{I} - \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\tau}_1^k \mathbf{m}_c - \frac{1}{2} \gamma [\mathbf{I} - (\boldsymbol{\tau}_1^k)^{-1} \boldsymbol{\Omega}^{-1}] \mathbf{m}_c$$
(A.128)

$$=\gamma^{2}(\boldsymbol{\tau}_{1}^{k})^{-1}[\mathbf{I}-\boldsymbol{\Omega}^{-1}(\boldsymbol{\tau}_{1}^{k})^{-1}]\mathbf{q}_{0}^{k}-\gamma[\mathbf{I}-(\boldsymbol{\tau}_{1}^{k})^{-1}\boldsymbol{\Omega}^{-1}]\mathbf{m}_{c}$$
(A.129)

$$\frac{\partial \mathbf{H}}{\partial \mathbf{q}_0^k} = \gamma(\mathbf{m}_c - \mathbf{m}_a). \tag{A.130}$$

All the terms with  $\mathbf{q}_0^k$  sum up to (there are 3 terms; add first term with first half of second term; add third term with second half of second term; take total;  $\boldsymbol{\tau}_1^{-1} \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1}$  is symmetric):

$$\gamma^{2} (\boldsymbol{\tau}_{0}^{-1} - \boldsymbol{\tau}_{1}^{-1} \boldsymbol{\Omega}^{-1} \boldsymbol{\tau}_{1}^{-1}) \mathbf{q}_{0}^{k}, \qquad (A.131)$$

whereas all the terms without  $\mathbf{q}_0^k$  sum up to (there are 3 terms):

$$\gamma \boldsymbol{\tau}_1^{-1} \boldsymbol{\Omega}^{-1} \mathbf{m}_c - \gamma \mathbf{m}_a, \tag{A.132}$$

and thus the first order condition with respect to  $\mathbf{q}_0^k$  is

$$(\boldsymbol{\tau}_{0}^{-1} - \boldsymbol{\tau}_{1}^{-1}\boldsymbol{\Omega}^{-1}\boldsymbol{\tau}_{1}^{-1})\mathbf{q}_{0}^{k} = \frac{1}{\gamma}(\mathbf{m}_{a} - \boldsymbol{\tau}_{1}^{-1}\boldsymbol{\Omega}^{-1}\mathbf{m}_{c}).$$
(A.133)

From (A.95) and (A.99), we know that  $\mathbf{m}_a = \mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0$  and  $\mathbf{m}_c = \gamma \boldsymbol{\tau}_1^{-1}(\mathbf{M} - \mathbf{x}_0)$ . Since none of them depend on k, it follows that  $\mathbf{q}_0^k$  is independent on k and thus

$$\boldsymbol{\tau}_{0}^{-1}\mathbf{q}_{0} - \boldsymbol{\tau}_{1}^{-1}\boldsymbol{\Omega}^{-1}\boldsymbol{\tau}_{1}^{-1}\mathbf{q}_{0} = \frac{1}{\gamma}(\mathbb{E}_{0}[\mathbf{D}] - \mathbf{P}_{0}) - \frac{1}{\gamma}\boldsymbol{\tau}_{1}^{-1}\boldsymbol{\Omega}^{-1}\gamma\boldsymbol{\tau}_{1}^{-1}(\mathbf{M} - \mathbf{x}_{0}).$$
(A.134)

The last terms on each side cancel out by market clearing. We therefore obtain (A.94):

$$\mathbf{q}_0 = \frac{1}{\gamma} \boldsymbol{\tau}_0(\mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0). \tag{A.135}$$

Using (A.78) and market clearing yields (31) and completes the proof of Proposition 4.

Proof of Corollary 4.1 (CAPM) Eqs. (A.135) and (A.99) imply  $\mathbb{E}_0[\mathbf{D}-\mathbf{P}_0] = \gamma \boldsymbol{\tau}_0^{-1}(\mathbf{M}-\mathbf{x}_0)$ and  $\mathbb{E}_0[\mathbf{D}-\mathbf{P}_1] = \gamma \boldsymbol{\tau}_1^{-1}(\mathbf{M}-\mathbf{x}_0)$ . Thus,

$$\mathbb{E}_0[\mathbf{P}_1 - \mathbf{P}_0] = \gamma(\boldsymbol{\tau}_0^{-1} - \boldsymbol{\tau}_1^{-1})(\mathbf{M} - \mathbf{x}_0).$$
(A.136)

Taking unconditional expectation and defining  $\mathbf{R}^e \equiv \mathbf{P}_1 - \mathbf{P}_0$  yields

$$\mathbb{E}[\mathbf{R}^e] = \gamma(\boldsymbol{\tau}_0^{-1} - \boldsymbol{\tau}_1^{-1})\mathbf{M}, \qquad (A.137)$$

which, written for the market portfolio is

$$\mathbb{E}[\mathbf{R}_{\mathbf{M}}^{e}] = \gamma(\mathbf{M}'\boldsymbol{\tau}_{0}^{-1}\mathbf{M} - \mathbf{M}'\boldsymbol{\tau}_{1}^{-1}\mathbf{M}) = \gamma(U_{0}^{2} - \mathbf{M}'\boldsymbol{\tau}_{1}^{-1}\mathbf{M}), \qquad (A.138)$$

where  $U_0^2$  is the market-wide uncertainty at time 0, defined in (A.80). The second term in brackets,  $\mathbf{M}' \boldsymbol{\tau}_1^{-1} \mathbf{M}$ , decreases with  $\Lambda_a$ ,  $\forall a$ . To see this, we know from (A.85) that

$$\boldsymbol{\tau}_{1} = \boldsymbol{\tau}_{0} + \begin{bmatrix} \operatorname{diag}[\pi_{a}(\Lambda_{a}) \mid a \in \mathcal{A}] & \mathbf{0}_{A \times (N-A)} \\ \mathbf{0}_{(N-A) \times A} & \mathbf{0}_{(N-A) \times (N-A)} \end{bmatrix},$$
(A.139)

and that  $\pi_a(\Lambda_a)$  increases in  $\Lambda_a$  (Lemma 1). Thus

$$\frac{\partial \mathbf{M}' \boldsymbol{\tau}_1^{-1} \mathbf{M}}{\partial \pi_a(\Lambda_a)} = \mathbf{M}' \frac{\partial \boldsymbol{\tau}_1^{-1}}{\partial \pi_a(\Lambda_a)} \mathbf{M}$$
(A.140)

$$= -\mathbf{M}' \boldsymbol{\tau}_1^{-1} \frac{\partial \boldsymbol{\tau}_1}{\partial \pi_a(\Lambda_a)} \boldsymbol{\tau}_1^{-1} \mathbf{M}$$
(A.141)

$$= -\mathbf{M}'\boldsymbol{\tau}_1^{-1}\boldsymbol{\iota}_a\boldsymbol{\iota}_a'\boldsymbol{\tau}_1^{-1}\mathbf{M} < 0, \qquad (A.142)$$

where we have used that the derivative of the inverse of a matrix K is  $-K^{-1}K^dK^{-1}$  (*d* meaning derivative: start with  $\mathbf{I}^d = (KK^{-1})^d = K^dK^{-1} + K(K^{-1})^d$  and solve for  $(K^{-1})^d$ ). Thus,  $\mathbf{M'}\boldsymbol{\tau}_1^{-1}\mathbf{M}$ 

decreases when investors are more attentive and  $\Lambda_a$  increases.

From (A.137)-(A.138), we obtain the CAPM in Corollary 4.1:

$$\mathbb{E}[\mathbf{R}^e] = \frac{(\boldsymbol{\tau}_0^{-1} - \boldsymbol{\tau}_1^{-1})\mathbf{M}}{U_0^2 - \mathbf{M}'\boldsymbol{\tau}_1^{-1}\mathbf{M}} \mathbb{E}[\mathbf{R}^e_{\mathbf{M}}]. \qquad \Box$$
(A.143)

Firms' betas: proof of Eq. (36) To understand how market betas relate to firms' exposures  $\mathbf{b}$  to the systematic factor, and how betas are governed by investor attention, we start by analyzing the numerator in (A.143), or  $(\boldsymbol{\tau}_0^{-1} - \boldsymbol{\tau}_1^{-1})\mathbf{M}$ . From (A.85),  $\boldsymbol{\tau}_1 = \boldsymbol{\tau}_0 + \boldsymbol{\iota} \boldsymbol{\Pi} \boldsymbol{\iota}'$ , where  $\boldsymbol{\Pi}$  is a  $A \times A$ diagonal matrix with the scalars  $\pi_a(\Lambda_a)$  on its diagonal,  $\mathbf{\Pi} = \text{diag}[\pi_a(\Lambda_a) \mid a \in \mathcal{A}]$ . This yields

$$\boldsymbol{\tau}_{1}^{-1} = \boldsymbol{\tau}_{0}^{-1} - \boldsymbol{\tau}_{0}^{-1} \boldsymbol{\iota} (\boldsymbol{\Pi}^{-1} + \boldsymbol{\iota}' \boldsymbol{\tau}_{0}^{-1} \boldsymbol{\iota})^{-1} \boldsymbol{\iota}' \boldsymbol{\tau}_{0}^{-1},$$
(A.144)

and thus

$$(\boldsymbol{\tau}_{0}^{-1} - \boldsymbol{\tau}_{1}^{-1})\mathbf{M} = \boldsymbol{\tau}_{0}^{-1}\boldsymbol{\iota}(\mathbf{\Pi}^{-1} + \boldsymbol{\iota}'\boldsymbol{\tau}_{0}^{-1}\boldsymbol{\iota})^{-1}\boldsymbol{\iota}'\boldsymbol{\tau}_{0}^{-1}\mathbf{M}.$$
 (A.145)

The term  $\boldsymbol{\tau}_0^{-1}\boldsymbol{\iota}$  represents the first A columns of  $\boldsymbol{\tau}_0^{-1} = \operatorname{Var}_0[\mathbf{D}]$ . Using (A.76), removing all terms that vanish when  $N \to \infty$ , and denoting by  $\mathbf{b}_A = [b_1 \quad b_2 \quad \cdots \quad b_A]'$ ,

$$\boldsymbol{\tau}_0^{-1}\boldsymbol{\iota} = \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b} \mathbf{b}'_A + \operatorname{Var}[\mathbf{e}]\boldsymbol{\iota}.$$
 (A.146)

This implies (using  $\mathbf{b'M} = 1$  and further removing vanishing terms):

$$\boldsymbol{\iota}' \boldsymbol{\tau}_0^{-1} \mathbf{M} = \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \tag{A.147}$$

$$\mathbf{\Pi}^{-1} + \boldsymbol{\iota}' \boldsymbol{\tau}_0^{-1} \boldsymbol{\iota} = \mathbf{\Pi}^{-1} + \operatorname{Var}[\mathbf{e}_A] + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \mathbf{b}'_A, \qquad (A.148)$$

where  $\mathbf{e}_A = [e_1 \ e_2 \ \cdots \ e_A]'$ . Using (A.146)-(A.148), the term  $(\boldsymbol{\tau}_0^{-1} - \boldsymbol{\tau}_1^{-1})\mathbf{M}$  is then

$$\left(\frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b} \mathbf{b}_A' + \operatorname{Var}[\mathbf{e}] \boldsymbol{\iota}\right) \left( \mathbf{\Pi}^{-1} + \operatorname{Var}[\mathbf{e}_A] + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \mathbf{b}_A' \right)^{-1} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A$$
(A.149)

$$= \mathbf{b} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A' \left( \mathbf{\Pi}^{-1} + \operatorname{Var}[\mathbf{e}_A] + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \mathbf{b}_A' \right)^{-1} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A$$
(A.150)

$$+ \begin{bmatrix} \operatorname{Var}[\mathbf{e}_{A}] \\ \mathbf{0} \end{bmatrix} \left( \mathbf{\Pi}^{-1} + \operatorname{Var}[\mathbf{e}_{A}] + \frac{U^{2}\sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2}} \mathbf{b}_{A} \mathbf{b}_{A}' \right)^{-1} \frac{U^{2}\sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2}} \mathbf{b}_{A}$$
(A.151)

$$= \mathbf{b} \underbrace{\left[ \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} - \left( \frac{U^2 + \sigma_g^2}{U^2 \sigma_g^2} + \mathbf{b}_A' (\mathbf{\Pi}^{-1} + \operatorname{Var}[\mathbf{e}_A])^{-1} \mathbf{b}_A \right)^{-1} \right]}_{\text{a strictly positive scalar,} \equiv \omega_1}$$
(A.152)

trictly positive scalar, 
$$\equiv \omega_1$$

$$+ \begin{bmatrix} \operatorname{Var}[\mathbf{e}_{A}] \\ \mathbf{0} \end{bmatrix} \left( \mathbf{\Pi}^{-1} + \operatorname{Var}[\mathbf{e}_{A}] + \frac{U^{2} \sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2}} \mathbf{b}_{A} \mathbf{b}_{A}' \right)^{-1} \frac{U^{2} \sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2}} \mathbf{b}_{A}$$
(A.153)

The scalar  $\omega_1$  is strictly positive because the diagonal matrix  $\mathbf{\Pi}^{-1} + \operatorname{Var}[\mathbf{e}_A]$  is positive definite. The last term above equals (solving only for its non-zero part):

$$\operatorname{Var}[\mathbf{e}_{A}] \left[ \left( \mathbf{\Pi}^{-1} \operatorname{Var}[\mathbf{e}_{A}]^{-1} + \mathbf{I} + \frac{U^{2} \sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2}} \mathbf{b}_{A} \mathbf{b}_{A}' \operatorname{Var}[\mathbf{e}_{A}]^{-1} \right) \operatorname{Var}[\mathbf{e}_{A}] \right]^{-1} \frac{U^{2} \sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2}} \mathbf{b}_{A} \qquad (A.154)$$

$$= \left(\underbrace{\mathbf{\Pi}^{-1}\operatorname{Var}[\mathbf{e}_{A}]^{-1} + \mathbf{I}}_{\operatorname{an} A \times A \operatorname{matrix}, \mathbb{A}} + \mathbf{b}_{A} \frac{U^{2}\sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2}} \mathbf{b}_{A}' \operatorname{Var}[\mathbf{e}_{A}]^{-1} \right)^{-1} \frac{U^{2}\sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2}} \mathbf{b}_{A}$$
(A.155)

$$= \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbb{A}^{-1} \mathbf{b}_A \left[ 1 - \left( \frac{1}{\frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2}} + \underbrace{\mathbf{b}'_A \operatorname{Var}[\mathbf{e}_A]^{-1} \mathbb{A}^{-1} \mathbf{b}_A}_{\operatorname{a \ strictly \ positive \ scalar, \ \omega_2}} \right)^{-1} \mathbf{b}'_A \operatorname{Var}[\mathbf{e}_A]^{-1} \mathbb{A}^{-1} \mathbf{b}_A \right]$$
(A.156)

$$= \frac{U^{2}\sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2} + \omega_{2}U^{2}\sigma_{g}^{2}} \begin{bmatrix} \frac{\frac{\pi_{1}(\Lambda_{1})\sigma_{e_{1}}^{2}}{1 + \pi_{1}(\Lambda_{1})\sigma_{e_{1}}^{2}}b_{1}}{\frac{\pi_{2}(\Lambda_{2})\sigma_{e_{2}}^{2}}{1 + \pi_{2}(\Lambda_{2})\sigma_{e_{2}}^{2}}b_{2}} \\ \vdots \\ \frac{\pi_{A}(\Lambda_{A})\sigma_{e_{A}}^{2}}{1 + \pi_{A}(\Lambda_{A})\sigma_{e_{A}}^{2}}b_{A} \end{bmatrix}.$$
(A.157)

The scalar  $\omega_2$  is strictly positive because the diagonal matrix  $\operatorname{Var}[\mathbf{e}_A]^{-1}\mathbb{A}^{-1}$  is positive definite. We can then write market betas,  $\boldsymbol{\beta} = \frac{(\boldsymbol{\tau}_0^{-1} - \boldsymbol{\tau}_1^{-1})\mathbf{M}}{\mathbf{M}'(\boldsymbol{\tau}_0^{-1} - \boldsymbol{\tau}_1^{-1})\mathbf{M}}$ , as

$$\frac{1}{\omega_{1} + \frac{U^{2}\sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2} + \omega_{2}U^{2}\sigma_{g}^{2}}\sum_{a=1}^{A}\frac{\pi_{a}(\Lambda_{a})\sigma_{ea}^{2}}{1 + \pi_{a}(\Lambda_{a})\sigma_{ea}^{2}}\frac{b_{a}}{N}} \left(\omega_{1}\mathbf{b} + \frac{U^{2}\sigma_{g}^{2}}{U^{2} + \sigma_{g}^{2} + \omega_{2}U^{2}\sigma_{g}^{2}} \begin{bmatrix} \frac{\pi_{1}(\Lambda_{1})\sigma_{e1}^{2}}{1 + \pi_{1}(\Lambda_{1})\sigma_{e1}^{2}}b_{1} \\ \frac{\pi_{2}(\Lambda_{2})\sigma_{e2}^{2}}{1 + \pi_{2}(\Lambda_{2})\sigma_{e2}^{2}}b_{2} \\ \vdots \\ \frac{\pi_{A}(\Lambda_{A})\sigma_{eA}^{2}}{1 + \pi_{A}(\Lambda_{A})\sigma_{eA}^{2}}b_{A} \\ \mathbf{0}_{N-A} \end{bmatrix} \right). \quad (A.158)$$

In a large economy  $(N \to \infty)$ , the denominator in the first term converges to  $\omega_1$  and thus we recover Eq. (36) in the text, with h > 0 defined as:

$$h \equiv \frac{1}{\omega_1} \frac{U^2 \sigma_g^2}{U^2 + \sigma_q^2 + \omega_2 U^2 \sigma_q^2}.$$
 (A.159)

The result that the announcing firms' betas increase with attention does not depend on taking the limit  $N \to \infty$ . In unreported analysis, we verify this result through simulations in a smaller economy. We find that the result always holds in our simulations, which we have performed for a wide range of parameter values.

### A.9 (Dynamic model) Proofs of Propositions 5 and 6

**Proof of Proposition 5** We start by making the following conjecture for equilibrium prices:

$$\widehat{P}_t \equiv \xi_t^{-1} P_t = \frac{\Lambda_t}{\gamma \sigma_{\varepsilon}^2} Z_t E_t + x_t, \qquad (A.160)$$

where  $\Lambda_t$  is the fraction of informed investors and  $Z_t$  will be determined in equilibrium below.

**Learning for the informed investor** For the informed investor, the only informative signal at time t is  $E_t$ . Application of the Projection Theorem yields

$$\operatorname{Var}_{t}^{I}[D_{t+1}] = \frac{\operatorname{Var}_{t}[D_{t+1}]\sigma_{\varepsilon}^{2}}{\operatorname{Var}_{t}[D_{t+1}] + \sigma_{\varepsilon}^{2}} = \left(\operatorname{Var}_{t}[D_{t+1}]^{-1} + \frac{1}{\sigma_{\varepsilon}^{2}}\right)^{-1},$$
(A.161)

and

$$\mathbb{E}_t^I[D_{t+1}] = \frac{\operatorname{Var}_t[D_{t+1}]}{\operatorname{Var}_t[D_{t+1}] + \sigma_{\varepsilon}^2} E_t = \frac{\operatorname{Var}_t^I[D_{t+1}]}{\sigma_{\varepsilon}^2} E_t.$$
(A.162)

**Learning for the uninformed investor** The uninformed investor learns from the price signal  $\hat{P}_t$ , and thus the Projection Theorem implies:

$$\operatorname{Var}_{t}^{\emptyset}[D_{t+1}] = \frac{\operatorname{Var}_{t}[D_{t+1}]\sigma_{\varepsilon}^{2}}{\frac{\Lambda_{t}^{2}Z_{t}^{2}}{\Lambda_{t}^{2}Z_{t}^{2} + \gamma^{2}\sigma_{x}^{2}\sigma_{\varepsilon}^{2}}} \operatorname{Var}_{t}[D_{t+1}] + \sigma_{\varepsilon}^{2} = \left(\operatorname{Var}_{t}[D_{t+1}]^{-1} + \frac{\Lambda_{t}^{2}Z_{t}^{2}}{\Lambda_{t}^{2}Z_{t}^{2} + \gamma^{2}\sigma_{x}^{2}\sigma_{\varepsilon}^{2}} \frac{1}{\sigma_{\varepsilon}^{2}}\right)^{-1}, \quad (A.163)$$

and

$$\mathbb{E}_t^{\emptyset}[D_{t+1}] = \operatorname{Var}_t^{\emptyset}[D_{t+1}] \frac{\gamma \Lambda_t Z_t}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_{\varepsilon}^2 \sigma_x^2} \widehat{P}_t$$
(A.164)

$$= \frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_x^2 \sigma_\varepsilon^2} \frac{\operatorname{Var}_t^{\emptyset}[D_{t+1}]}{\sigma_\varepsilon^2} E_t + \operatorname{Var}_t^{\emptyset}[D_{t+1}] \frac{\gamma \Lambda_t Z_t}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_x^2} x_t.$$
(A.165)

**Equilibrium** When forming optimal portfolios at time t, both I and  $\emptyset$  investors form expectations about  $P_{t+1} + D_{t+1}$ . Using that  $\mathbb{E}_t^k[P_{t+1}] = 0 \ \forall k \in \{I, \emptyset\}$ , informed investors' beliefs are:

$$\mathbb{E}_{t}^{I}[P_{t+1} + D_{t+1}] = \mathbb{E}_{t}^{I}[D_{t+1}]$$
(A.166)

$$\operatorname{Var}_{t}^{I}[P_{t+1} + D_{t+1}] = \operatorname{Var}_{t}^{I}[D_{t+1}] + \sum_{s=1}^{S} p_{s} \left[ \alpha_{s,t+1}^{2} (b^{2} U_{s}^{2} + \sigma_{e}^{2} + \sigma_{\varepsilon}^{2}) + \xi_{s,t+1}^{2} \sigma_{x}^{2} \right],$$
(A.167)

where  $p_s$  represents the probability of reaching the state  $U_s$ . The last term in (A.167) is the variance of the future price,  $\operatorname{Var}_t[P_{t+1}]$ , which is the same for I and  $\emptyset$  investors, and does not change over time (the information that investors have at t becomes irrelevant at t + 1; furthermore, at any time t investors face the same probability distribution over future values of  $U_s$ , and thus over the values of the price coefficients at time t + 1). Thus, we denote the last term in (A.167) by  $\operatorname{Var}[P_{t+1}]$ .

Similar reasoning leads to uninformed investors' beliefs:

$$\mathbb{E}_{t}^{\emptyset}[P_{t+1} + D_{t+1}] = \mathbb{E}_{t}^{\emptyset}[D_{t+1}]$$
(A.168)

$$\operatorname{Var}_{t}^{\emptyset}[P_{t+1} + D_{t+1}] = \operatorname{Var}_{t}^{\emptyset}[D_{t+1}] + \operatorname{Var}[P_{t+1}].$$
(A.169)

Consider now the optimization problem that all investors face:

$$\max_{k \in \{I,\emptyset\}} \mathbb{E}_{t-1} \left[ e^{\gamma c I^k} \max_{q_t^k} \mathbb{E}_t^k \left[ -e^{-\gamma q_t^k (P_{t+1} + D_{t+1} - R_f P_t)} \right] \right], \tag{A.170}$$

which leads to the following portfolio choice problem of  $k \in \{I, \emptyset\}$  investors:

$$\max_{q_t^k} \mathbb{E}_t^k \left[ -e^{-\gamma q_t^k (P_{t+1} + D_{t+1} - R_f P_t)} \right].$$
(A.171)

In the expectation above, the future price  $P_{t+1}$  is normally distributed *conditional* on the future value of  $U_s$ . One can write the expectation as

$$\mathbb{E}_{t}^{k}\left[-e^{-\gamma q_{t}^{k}(P_{t+1}+D_{t+1}-R_{f}P_{t})}\right] = \sum_{s=1}^{S} p_{s} \mathbb{E}_{t}^{k}\left[-e^{-\gamma q_{t}^{k}(P_{s,t+1}+D_{t+1}-R_{f}P_{t})}\right],$$
(A.172)

where  $P_{s,t+1}$  is the future price in the state  $U_s$ . Defining  $R_{s,t+1}^e \equiv P_{s,t+1} + D_{t+1} - R_f P_t$ , the expectation can be further written as

$$\mathbb{E}_{t}^{k}\left[-e^{-\gamma q_{t}^{k}(P_{t+1}+D_{t+1}-R_{f}P_{t})}\right] = \sum_{s=1}^{S} p_{s}\left(-e^{-\gamma q_{t}^{k}\mathbb{E}_{t}^{k}[R_{s,t+1}^{e}] + \frac{1}{2}\gamma^{2}(q_{t}^{k})^{2}\operatorname{Var}_{t}^{k}[R_{s,t+1}^{e}]}\right).$$
(A.173)

We now resort to an approximation of this function as in Vayanos and Weill (2008) and Gârleanu (2009). Economically, this approximation preserves risk aversion towards diffusion risks (i.e., risks created by normally distributed variables), but creates risk neutrality towards discrete jump risks (i.e., risks created by future changes in  $U_s$ ). The approximation is very accurate in this setting, particularly because  $\mathbb{E}_t^k[R_{s,t+1}^e] = \mathbb{E}_t^k[D_{t+1}] + \mathbb{E}_t^k[P_{s,t+1}] - R_fP_t$  does not vary across future states ( $\mathbb{E}_t^k[P_{s,t+1}] = 0 \ \forall s$ ), and thus the future distribution of prices remains symmetric, unimodal, and elliptical (only the variance  $\operatorname{Var}_t^k[R_{s,t+1}^e]$  changes across future states). First, define

$$\overline{\operatorname{Var}}_{t}^{k}[R_{s,t+1}^{e}] \equiv \gamma \operatorname{Var}_{t}^{k}[R_{s,t+1}^{e}], \qquad (A.174)$$

and replace this above to obtain a function of  $\gamma$ :

$$f(\gamma) = \sum_{s=1}^{S} p_s \left( -e^{-\gamma q_t^k \mathbb{E}_t^k [R_{s,t+1}^e] + \frac{1}{2}\gamma(q_t^k)^2 \overline{\operatorname{Var}}_t^k [R_{s,t+1}^e]} \right).$$
(A.175)

The Taylor expansion of  $f(\gamma)$  around zero is given by  $f(\gamma) = f(0) + \gamma f'(0) + \mathcal{O}(\gamma)$ , where  $\mathcal{O}(\gamma)$  represents higher-order terms that go to zero faster than  $\gamma$  as  $\gamma \to 0$ . Therefore

$$f(\gamma) \approx -1 + \sum_{s=1}^{S} p_s \left( \gamma q_t^k \mathbb{E}_t^k [R_{s,t+1}^e] - \frac{1}{2} \gamma (q_t^k)^2 \overline{\operatorname{Var}}_t^k [R_{s,t+1}^e] \right)$$
(A.176)

$$= -1 + \sum_{s=1}^{S} p_s \left( \gamma q_t^k \mathbb{E}_t^k [R_{s,t+1}^e] - \frac{1}{2} \gamma^2 (q_t^k)^2 \operatorname{Var}_t^k [R_{s,t+1}^e] \right)$$
(A.177)

$$= -1 + \gamma q_t^k \mathbb{E}_t^k [R_{t+1}^e] - \frac{1}{2} \gamma^2 (q_t^k)^2 \operatorname{Var}_t^k [R_{t+1}^e].$$
(A.178)

The first order condition with respect to  $q_t^k$  leads to the optimal portfolio of the informed and uninformed investors:

$$q_t^I = \frac{\mathbb{E}_t^I[R_{t+1}^e]}{\gamma \operatorname{Var}_t^I[R_{t+1}^e]} \quad \text{and} \quad q_t^{\emptyset} = \frac{\mathbb{E}_t^{\emptyset}[R_{t+1}^e]}{\gamma \operatorname{Var}_t^{\emptyset}[R_{t+1}^e]}.$$
 (A.179)

Market clearing requires:

$$\Lambda_t \frac{\mathbb{E}_t^I [D_{t+1} + P_{t+1}] - R_f P_t}{\gamma \operatorname{Var}_t^I [R_{t+1}^e]} + (1 - \Lambda_t) \frac{\mathbb{E}_t^{\emptyset} [D_{t+1} + P_{t+1}] - R_f P_t}{\gamma \operatorname{Var}_t^{\emptyset} [R_{t+1}^e]} = -x_t,$$
(A.180)

and since the price conjecture (41) implies  $\mathbb{E}_t^I[P_{t+1}] = \mathbb{E}_t^{\emptyset}[P_{t+1}] = 0$ , this yields

$$\frac{\Lambda_t \mathbb{E}_t^I[D_{t+1}]}{\operatorname{Var}_t^I[R_{t+1}^e]} + \frac{(1 - \Lambda_t) \mathbb{E}_t^{\emptyset}[D_{t+1}]}{\operatorname{Var}_t^{\emptyset}[R_{t+1}^e]} - \left(\frac{\Lambda_t}{\operatorname{Var}_t^I[R_{t+1}^e]} + \frac{1 - \Lambda_t}{\operatorname{Var}_t^{\emptyset}[R_{t+1}^e]}\right) R_f P_t = -\gamma x_t,$$
(A.181)

and we recognize the weighted average precision across investors, denoted hereafter by  $\tau_t$ :

$$\tau_t \equiv \frac{\Lambda_t}{\operatorname{Var}_t^I[R_{t+1}^e]} + \frac{1 - \Lambda_t}{\operatorname{Var}_t^{\emptyset}[R_{t+1}^e]}.$$
(A.182)

Eq. (A.181) further leads to

$$\tau_t R_f P_t = \frac{\Lambda_t \mathbb{E}_t^I [D_{t+1}]}{\operatorname{Var}_t^I [D_{t+1}]} \frac{\operatorname{Var}_t^I [D_{t+1}]}{\operatorname{Var}_t^I [R_{t+1}^e]} + \frac{(1 - \Lambda_t) \mathbb{E}_t^{\emptyset} [D_{t+1}]}{\operatorname{Var}_t^{\emptyset} [D_{t+1}]} \frac{\operatorname{Var}_t^{\emptyset} [D_{t+1}]}{\operatorname{Var}_t^{\emptyset} [R_{t+1}^e]} + \gamma x_t.$$
(A.183)

After replacement of (A.161)-(A.162) and (A.163)-(A.165), we obtain

$$P_{t} = \frac{\tau_{t}^{-1}}{R_{f}} \left( \frac{\Lambda_{t}}{\sigma_{\varepsilon}^{2}} \frac{\operatorname{Var}_{t}^{I}[D_{t+1}]}{\operatorname{Var}_{t}^{I}[R_{t+1}^{e}]} + \frac{1 - \Lambda_{t}}{\sigma_{\varepsilon}^{2}} \frac{\Lambda_{t}^{2} Z_{t}^{2}}{\Lambda_{t}^{2} Z_{t}^{2} + \gamma^{2} \sigma_{x}^{2} \sigma_{\varepsilon}^{2}} \frac{\operatorname{Var}_{t}^{\emptyset}[D_{t+1}]}{\operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]} \right) E_{t} + \frac{\tau_{t}^{-1}}{R_{f}} \left( \gamma + (1 - \Lambda_{t}) \frac{\gamma \Lambda_{t} Z_{t}}{\Lambda_{t}^{2} Z_{t}^{2} + \gamma^{2} \sigma_{\varepsilon}^{2} \sigma_{x}^{2}} \frac{\operatorname{Var}_{t}^{\emptyset}[D_{t+1}]}{\operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]} \right) x_{t},$$
(A.184)

which determines the coefficients in the price conjecture  $P_t = \alpha E_t + \xi x_t$ . Moreover, the conjecture (A.160), which requires that  $\frac{\alpha_t}{\xi_t} = \frac{\Lambda_t}{\gamma \sigma_{\varepsilon}^2} Z_t$ , together with (A.184) imply that  $Z_t$  must be

$$Z_{t} = \frac{\operatorname{Var}_{t}^{I}[D_{t+1}]}{\operatorname{Var}_{t}^{I}[R_{t+1}^{e}]}.$$
(A.185)

We now solve for the equilibrium  $\Lambda_t$  in the dynamic model. The approximated expected utility of uninformed investors in (A.178), after replacement of the optimal portfolio choice (A.179), is

$$\mathcal{U}_{t}^{\emptyset} = -1 + \gamma \frac{\mathbb{E}_{t}^{\emptyset}[R_{t+1}^{e}]}{\gamma \operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]} \mathbb{E}_{t}^{\emptyset}[R_{t+1}^{e}] - \frac{1}{2}\gamma^{2} \left(\frac{\mathbb{E}_{t}^{\emptyset}[R_{t+1}^{e}]}{\gamma \operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]}\right)^{2} \operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]$$
(A.186)

$$= \frac{1}{2} \frac{\mathbb{E}_{t}^{\emptyset}[R_{t+1}^{e}]^{2}}{\operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]} - 1 \approx -e^{-\frac{1}{2} \frac{\mathbb{E}_{t}^{\theta}[R_{t+1}^{e}]^{2}}{\operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]}}.$$
(A.187)

where we have used the approximation  $x - 1 \approx -e^{-x}$ . This approximation restores the expected utility in exponential form and is highly accurate when  $\mathbb{E}_t^{\emptyset}[R_{t+1}^e]^2/(2\operatorname{Var}_t^{\emptyset}[R_{t+1}^e])$  is small, which is likely to be the case:  $\mathbb{E}_t^{\emptyset}[R_{t+1}^e]^2/\operatorname{Var}_t^{\emptyset}[R_{t+1}^e]$  represents the squared Sharpe ratio of the stock from the perspective of uninformed investors. Similarly, for an informed investor,

$$\mathcal{U}_{t}^{I} \approx -e^{\gamma c} e^{-\frac{1}{2} \frac{\mathbb{E}_{t}^{I} [R_{t+1}^{e}]^{2}}{\operatorname{Var}_{t}^{I} [R_{t+1}^{e}]}}.$$
(A.188)

For an uninformed investor,  $\mathbb{E}_t^I[R_{t+1}^e]$  is a normally distributed random variable with mean  $\mathbb{E}_t^{\emptyset}[R_{t+1}^e]$  (by the law of iterated expectations) and variance  $\Sigma_t \equiv \operatorname{Var}_t^{\emptyset}[R_{t+1}^e] - \operatorname{Var}_t^I[R_{t+1}^e]$  (by the law of total variance). Taking expectation at t-1 of (A.188) as in (A.170) and applying Lemma A2 yields

$$\mathbb{E}_{t-1}\left[-e^{\gamma c}e^{-\frac{1}{2}\frac{\mathbb{E}_t^I[R_{t+1}^e]^2}{\operatorname{Var}_t^I[R_{t+1}^e]}}\right] = \mathcal{U}_t^{\emptyset}e^{\gamma c}\left(\frac{\operatorname{Var}_t^I[R_{t+1}^e]}{\operatorname{Var}_t^{\emptyset}[R_{t+1}^e]}\right)^{1/2}.$$
(A.189)

Since  $\mathcal{U}_t^{\emptyset} < 0$ , the uninformed investor is attentive to the earnings announcement if and only if

$$\frac{\operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]}{\operatorname{Var}_{t}^{I}[R_{t+1}^{e}]} > e^{2\gamma c},\tag{A.190}$$

which proves part (a) of Proposition 5. Using (A.161) and (A.163), the benefit of information is

$$\frac{\operatorname{Var}_{t}^{\emptyset}[R_{t+1}^{e}]}{\operatorname{Var}_{t}^{I}[R_{t+1}^{e}]} = \frac{\operatorname{Var}[P_{t+1}] + \frac{\operatorname{Var}_{t}[D_{t+1}]\sigma_{\varepsilon}^{2}}{\frac{\Lambda_{t}^{2}Z_{t}^{2}}{\Lambda_{t}^{2}Z_{t}^{2}+\gamma^{2}\sigma_{x}^{2}\sigma_{\varepsilon}^{2}}} \operatorname{Var}_{t}[D_{t+1}] + \sigma_{\varepsilon}^{2}}{\operatorname{Var}[P_{t+1}] + \frac{\operatorname{Var}_{t}[D_{t+1}]\sigma_{\varepsilon}^{2}}{\operatorname{Var}_{t}[D_{t+1}] + \sigma_{\varepsilon}^{2}}}.$$
(A.191)

Since  $\frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_x^2 \sigma_{\varepsilon}^2} < 1$ ,  $\frac{\operatorname{Var}_t^{\emptyset}[R_{t+1}^e]}{\operatorname{Var}_t^I[R_{t+1}^e]}$  increases in  $\operatorname{Var}_t[D_{t+1}]$ , proving part (b) of Proposition 5.  $\Box$ 

**Proof of Proposition 6** The ERC (i.e., the sensitivity  $\alpha_t$  of the price  $P_t$  to the earnings announcement  $E_t$ ) follows directly from (A.182) and (A.184):

$$\alpha_t = \frac{1}{R_f} \frac{1}{\frac{\Lambda_t}{\operatorname{Var}_t^I[R_{t+1}^e]} + \frac{1 - \Lambda_t}{\operatorname{Var}_t^\theta[R_{t+1}^e]}} \left( \frac{\Lambda_t \operatorname{Var}_t^I[D_{t+1}]}{\operatorname{Var}_t^I[R_{t+1}^e]\sigma_{\varepsilon}^2} + \frac{(1 - \Lambda_t) \operatorname{Var}_t^\theta[D_{t+1}]}{\operatorname{Var}_t^\theta[R_{t+1}^e]\sigma_{\varepsilon}^2} \frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_x^2 \sigma_{\varepsilon}^2} \right) \quad (A.192)$$

$$= \frac{1}{R_f} \left( w_t \frac{\operatorname{Var}_t[D_{t+1}]}{\operatorname{Var}_t[D_{t+1}] + \sigma_{\varepsilon}^2} + (1 - w_t) \frac{\operatorname{Var}_t[D_{t+1}]}{\operatorname{Var}_t[D_{t+1}] + \sigma_{\varepsilon}^2/\ell_t} \right),$$
(A.193)

where  $w_t$  and  $\ell_t$  are defined as:

$$w_{t} = \frac{\frac{\Lambda_{t}}{\operatorname{Var}_{t}^{I}[R_{t+1}^{e}]}}{\frac{\Lambda_{t}}{\operatorname{Var}_{t}^{I}[R_{t+1}^{e}]} + \frac{1-\Lambda_{t}}{\operatorname{Var}_{t}^{\theta}[R_{t+1}^{e}]}} \quad \text{and} \quad \ell_{t} = \frac{\Lambda_{t}^{2}Z_{t}^{2}}{\Lambda_{t}^{2}Z_{t}^{2} + \gamma^{2}\sigma_{x}^{2}\sigma_{\varepsilon}^{2}} < 1.$$
(A.194)

ERC<sub>t</sub> is a weighted average. A higher fraction of informed investors  $\Lambda_t$  increases  $w_t$ , and thus the weighted average places a higher weight on  $\frac{\operatorname{Var}_t[D_{t+1}]}{\operatorname{Var}_t[D_{t+1}]+\sigma_{\varepsilon}^2}$ . Because  $\ell_t < 1$ , this higher weight increases the weighted average. Moreover, a higher  $\Lambda_t$  increases  $\ell_t$ , which further increases  $\frac{\operatorname{Var}_t[D_{t+1}]}{\operatorname{Var}_t[D_{t+1}]+\sigma_{\varepsilon}^2/\ell_t}$  and thus the weighted average. Overall, these two effects confirm that a higher  $\Lambda_t$  increase the ERC, proving Proposition 6.

Variable	Description
VIX	Closing value of <i>VIX</i> on the trading day prior to the earnings announcement. Source: CRSP.
ESV	Log daily number of EDGAR downloads of the company's filings from SEC EDGAR. Source: SEC.
ESVU	Log daily number of EDGAR downloads of the company's filings from unique IP addresses. Source: SEC.
ISVI	Investor Search Volume Index based on investors' Google searches of stock tickers. Source: DeHaan, Lawrence, and Litjens (2021).
EARET	Compound excess return over the size decile portfolio for earnings announcement trading date and one trading day after. Source: CRSP.
SUE Decile	Earnings surprise relative to analyst consensus forecasts deflated by quarter-end share price. Source: IBES Summary File, CRSP.
PreRet	Compound excess return over the size decile portfolio for earnings announcement trading date -10 to -1. Source: CRSP.
Size	Market value of equity on the earnings announcement date in \$M. Source: CRSP.
Book-to-Market	Book to market ratio at the end of quarter for which earnings are announced. Source: Compustat.
EPersistence	Earnings persistence based on $AR(1)$ regression with at least 4, up to 16 quarterly earnings. Source: Compustat.
ΙΟ	Institutional ownership as a fraction of total shares outstanding. Source: Thomson- Reuters 13F Data, CRSP.
EVOL	Standard deviation of seasonally differenced quarterly earnings over the prior 16 (at least 4) quarters. Source: Compustat.
ERepLag	Days from quarter-end to earnings announcement. Source: Compustat.
#Estimates	Number of analysts making quarterly earnings forecasts. Source: IBES Summary File.
TURN	Average monthly share turnover for the 12 months preceding the earnings announce- ment. Source: CRSP.
Loss	Indicator for negative earnings. Source: Compustat.
#Announcements	Number of concurrent earnings announcements. Source: Compustat, IBES.
CAPM Beta	CAPM Beta estimated using the CRSP value-weighted market return index for the 250 (at least 60) trading days prior to the earnings announcement. Source: CRSP
IDVOL	Idiosyncratic volatility estimated using the CAPM model with the CRSP value- weighted market return index for the 250 (at least 60) trading days prior to the earnings announcement. Source: CRSP
DISP	Earnings forecast dispersion calculated as standard deviation of analyst forecasts deflated by mean absolute forecast. Source: IBES Summary File.

# **B** Variable definitions

# B.1 CAPM tests: data description and robustness checks

The analysis in Section 4.4 starts by merging by GVKEY and DATE the database that contains firm daily excess returns with the EDGAR search database. The individual returns sample limits are from January 2002 to December 2020. The EDGAR sample limits are from 2003-02-14 to 2017-06-30, which dictates the final limits of the merged sample. This initial merged sample has 11,097,305 observations and 4497 unique firms.

To identify high/low attention days, we build detrended time series of log search data at the individual firm level. Then, we add the value of 1 to the EDGAR search data to be able to take

the log on days with zero EDGAR search. (These days commonly occur at the beginning of the sample; as another option, we have removed the first five years of data, from 2003 to 2007, and the results are robust to this alternative.) After detrending the log EDGAR search data, we split the residuals according to their sample median, with high-attention days ( $\mathbf{1}_{\text{HighAtt}}^{i} = 1$ ) corresponding to residuals being above the median.

Before estimating the regressions (46)-(48) for Table 7, we clean up the data as follows:

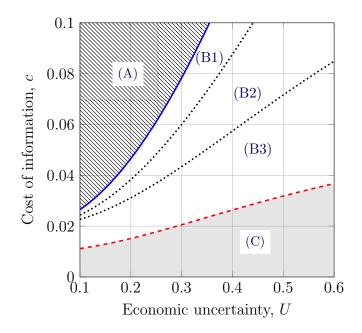
- (i) Using the Thomson Reuters I/B/E/S database, we remove announcements recorded after 4:00 PM on a given date. While one can measure investor attention (EDGAR downloads) on days when these announcements are released, investors trading on a U.S. exchange will react to the announcements only on the following trading day. This non-synchronicity prevents us from properly aligning  $\mathbf{1}_{EA}^{low,i}$  and  $\mathbf{1}_{EA}^{high,i}$ . (The results are robust and even gain statistical significance if we do not remove these announcements.)
- (ii) We remove firms that have less than 20 earnings announcements. This ensures that there are enough earnings announcement days for the regression (48), which further splits the earnings announcement days into low/high attention days. (The results are similar if we use a tighter threshold, e.g., 40.)
- (iii) We remove firms that have more than 500 zero EDGAR search values. (The results are similar if we use a tighter threshold, e.g., 250.)

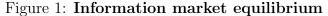
Furthermore, the results in Table 7 are robust to alternative splits of the earnings announcement days in (48): once the earnings announcement days are selected, re-compute the median of detrended EDGAR searches *within this sub-sample* (instead of using the median across all search days), then split the days based on this new median.

The analysis that yields Tables 8-10 and Figure 8 uses return data from January 1990 to December 2021, which corresponds to the sample limits for the *VIX*. The results are similar if we limit the sample to June 2017 (the upper limit of the EDGAR search data) or if we use the entire dataset for portfolio excess returns (starting from January 1967 for ten beta-sorted portfolios and from July 1926 for 25 size/BM portfolios).

Finally, the results in Table 10 are robust to several alternative specifications. The first robustness check concerns our definition of high-attention days. Rather than using the raw detrended ESV(U) measures, we regress ESV(U) on VIX and use the residuals instead, with similar results. Second, the results are stronger in panel A and remain confirmatory in panel B after removing the first five years of EDGAR data, years during which search numbers are relatively lower.

# C Figures and Tables





This figure depicts the three cases of Theorem 1, (A), (B), and (C). We further split case (B) in three sub-cases: (B1)  $\Lambda_1 > 0, \Lambda_2 = \Lambda_3 = 0$ , in which investors only pay attention to the announcement of firm 1; (B2)  $\Lambda_1 > 0, \Lambda_2 > 0, \Lambda_3 = 0$ , in which investors pay attention to the announcements of firms 1 and 2 but not 3; (B3)  $\Lambda_1 > 0, \Lambda_2 > 0, \Lambda_3 > 0$ , in which investors pay attention to the announcements of all firms. The calibration used for this illustration is:  $\gamma = 1, b_1 = 1.2, b_2 = 1, b_3 = 0.8, \sigma_{e1} = \sigma_{e2} = \sigma_{e3} = 0.2, \sigma_{\varepsilon_1} = \sigma_{\varepsilon_2} = \sigma_{\varepsilon_3} = 1$ , and  $\sigma_{x1} = \sigma_{x2} = \sigma_{x3} = 1$ .

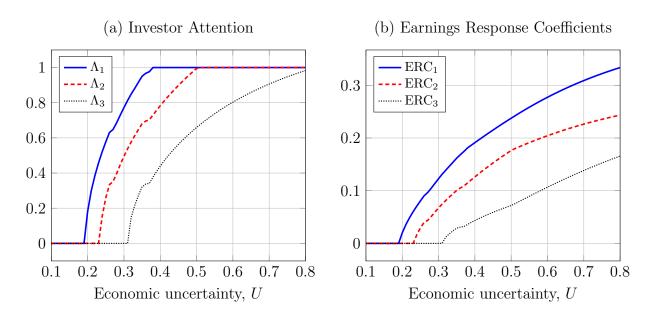
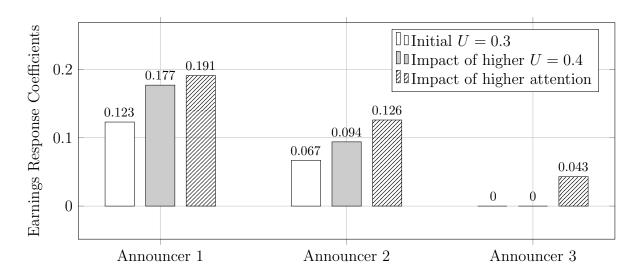


Figure 2: The impact of economic uncertainty on investor attention and ERCs Panel (a) plots the fractions of attentive investors to each one of the three earnings announcements. Panel (b) plots the earnings response coefficients. In this economy,  $b_1 > b_2 > b_3$ , c = 0.045, and the rest of the calibration is provided in Figure 1.



# Figure 3: The separate impact of an increase in uncertainty and an increase in investor attention on ERCs

This figure plots the successive changes in ERCs of the announcing firms after an increase of economic uncertainty from 0.3 to 0.4. The grey bars plot ERCs resulting exclusively from the increase in U. The hashed bars plot the final ERCs, including also the impact of the increase in investor attention. In this economy,  $b_1 > b_2 > b_3$ , c = 0.045, and the rest of the calibration is provided in Figure 1.

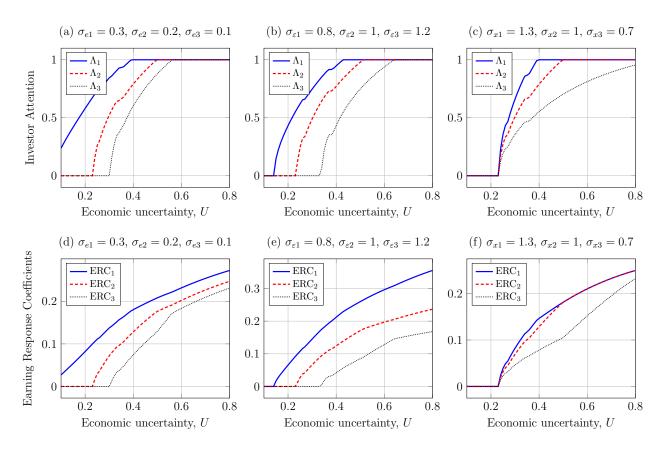


Figure 4: The impact of economic uncertainty on investor attention and ERCs This figure plots the fractions of investors attentive to each earnings announcement (above) and ERCs (below), as functions of economic uncertainty, for different  $\sigma_{ea}$ ,  $\sigma_{\varepsilon a}$ , and  $\sigma_{xa}$ . The rest of the calibration is provided in Figure 1, and c = 0.045.

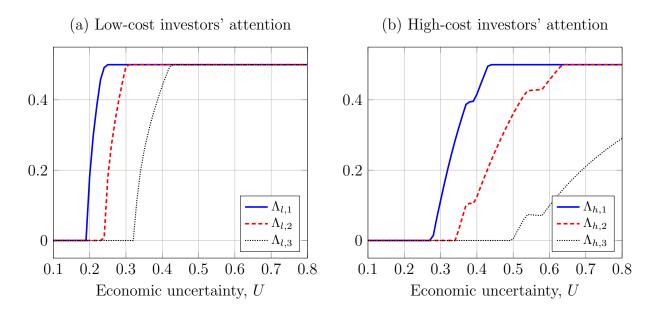


Figure 5: The impact of economic uncertainty on investor attention in an economy with heterogeneous attention costs

Each panel of the figure plots the fractions of attentive investors as functions of economic uncertainty, with low-cost investors in panel (a) and high-cost investors in panel (b). In this economy,  $b_1 > b_2 > b_3$ ,  $c_l = 0.045$ ,  $c_h = 0.055$ , the fractions of low-cost and high-cost investors are of equal size (50%), and the rest of the calibration is provided in Figure 1.

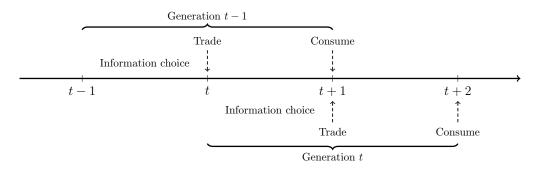


Figure 6: Overlapping generations economy

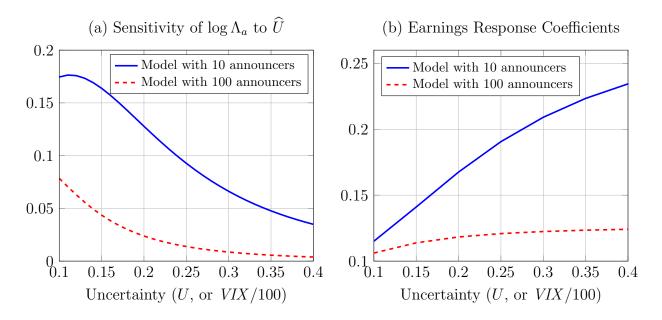


Figure 7: Response of investor attention to changes in uncertainty and ERCs in an economy with 3000 firms

Panel (a) plots the partial derivative of  $\log \Lambda_a$  with respect to standardized uncertainty  $\widehat{U}$  when  $U \in [0.1, 0.4]$ , and is thus the theoretical counterpart of the coefficient  $c_1$  in (44) and in Table 3. Panel (b) plots ERCs implied by the theoretical model when  $U \in [0.1, 0.4]$  and is thus the theoretical counterpart of the coefficient  $c_1$  in (45) and in Table 4. Both panels consider two alternatives, one with 10 announcers (solid lines), and one with 100 announcers (dashed lines). See Section 4.3 for a detailed description of the calibration.

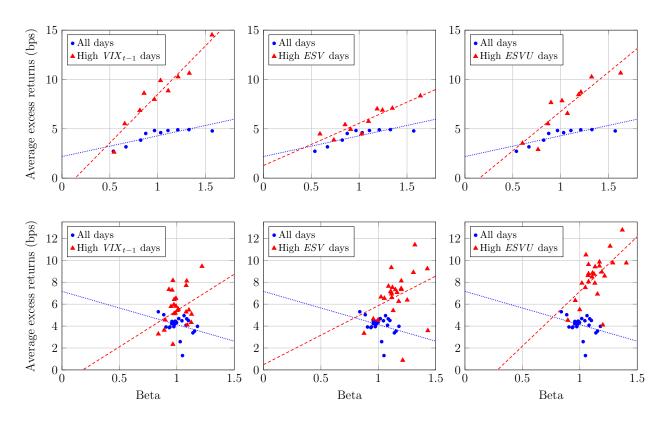


Figure 8: The impact of uncertainty and attention on the CAPM This figure plots average daily excess returns in basis points (bps) against full-sample betas for ten value-weighted beta-sorted portfolios (top panels) and 25 size/BM-sorted portfolios (bottom panels). The estimates are reported separately for: all days versus days with  $VIX_{t-1}$ in its top quartile (left panels); all days versus days with the detrended aggregated ESV in its top quartile (middle panels); and all days versus days with the detrended aggregate ESVUin its top quartile (right panels). The lines represent day-specific CAPM relations (Table 10). Daily excess returns are available from January 1990 to December 2021, and EDGAR search records are available from February 2003 to June 2017 (see Appendix B.1).

## Table 1: Descriptive statistics

This table reports descriptive statistics for the sample used in analyses of returns around earnings announcements. Detailed definitions of all variables are available in Appendix B.

Variable	Obs. Count	Mean	Std. Dev.	25%	50%	75%
VIX	234,874	19.626	8.162	13.770	18.030	23.010
ESV	$124,\!660$	4.719	1.999	3.367	4.934	6.319
ESVU	$124,\!660$	3.664	1.506	2.639	3.912	4.883
ISVI	$66,\!534$	4.419	13.315	0.000	0.000	0.000
EARET	$234,\!874$	0.001	0.080	-0.033	0.001	0.037
SUE Decile	$234,\!874$	5.536	2.705	3.000	6.000	8.000
PreRet	$234,\!851$	0.002	0.081	-0.035	-0.001	0.035
Size	$234,\!874$	4973.899	13764.427	282.266	854.312	2980.429
$Book\-to\-Market$	234,727	0.534	0.382	0.274	0.458	0.701
EPersistence	$234,\!206$	0.226	0.398	-0.040	0.180	0.500
IO	$225,\!437$	0.633	2.288	0.430	0.666	0.842
EVOL	$234,\!232$	0.822	2.115	0.116	0.272	0.654
ERepLag	$234,\!874$	30.765	13.609	22.000	28.000	37.000
#Estimates	$234,\!874$	7.799	6.573	3.000	6.000	11.000
TURN	$234,\!874$	17.446	17.605	6.935	12.826	22.120
Loss	234,874	0.194	0.396	0.000	0.000	0.000
#Announcements	234,874	150.476	92.544	73.000	137.000	221.000

Correlations	
5.	
Table	

This table presents Spearman (Pearson) correlations above (below) the diagonal for all variables used in the analyses. Detailed definitions of all variables are available in Appendix B. Bold indicates significance at the one percent level.

	Variable	1	2	ဂ	4	ю	9	2	×	6	10	11	12	13	14	15	16	17
	XIX		-0.069		-0.083	0.004	0.005	-0.006	-0.124	0.079	0.029	-0.133	-0.047	-0.065	-0.061	-0.003	0.029	0.051
2	ESV	-0.025		0.961	0.199	0.000	0.010	0.017	0.316	-0.041	-0.105	0.207	0.162	0.112	0.263	0.151	0.035	-0.091
e C	ESVU	-0.008	0.967		0.223	-0.001	0.011	0.016	0.335	-0.058	-0.101	0.214	0.165	0.102	0.293	0.172	0.044	-0.077
4	IA SI	-0.050	0.153	0.168		0.007	0.016	0.010	0.208	-0.108	-0.005	0.051	0.051	-0.023	0.179	0.117	0.014	-0.035
r0 L	EARET	-0.002	-0.001	-0.002	0.002		0.315	-0.045	0.023	0.012	0.001	0.023	-0.023	-0.023	0.007	-0.015	-0.094	0.005
9	SUE Decile	0.004	0.012	0.015	0.013	0.299		0.095	0.031	-0.009	0.004	0.035	0.030	-0.035	0.018	0.045	-0.145	0.025
2	PreRet	-0.007	0.004	0.003	0.003	-0.043	0.096		0.053	-0.011	0.009	0.005	-0.017	-0.019	0.011	0.025	-0.050	-0.004
8	Size	-0.050	0.215	0.242	0.169	0.002	0.002	0.007		-0.313	0.003	0.442	0.134	-0.143	0.746	0.274	-0.179	0.018
6	Book-to-Market	0.097	0.003	-0.005	-0.070	0.017	-0.019	-0.005	-0.142		-0.087	-0.114	0.198	0.023	-0.254	-0.217	-0.006	-0.019
10	EPersistence	0.028	-0.101	-0.097	0.000	0.001	0.004	0.010	0.010	-0.070		0.002	-0.359	-0.096	0.049	0.068	-0.052	0.015
11	OI	-0.007	0.016	0.015	0.058	0.000	0.002	-0.002	0.009	-0.008	0.000		0.192	0.094	0.465	0.508	-0.027	0.006
12	EVOL	-0.004	0.028	0.031	0.016	-0.018	0.003	-0.017	0.004	0.108	-0.113	0.004		0.162	0.084	0.194	0.195	0.011
13	ERepLag	-0.024	0.071	0.060	-0.019	-0.022	-0.049	-0.019	-0.128	0.032	-0.074	0.004	0.073		-0.120	0.109	0.199	-0.178
14	#Estimates	-0.047	0.262	0.291	0.145	0.000	0.014	-0.004	0.534	-0.195	0.059	0.043	0.006	-0.140		0.453	-0.067	0.014
15	TURN	0.046	0.102	0.121	0.084	-0.022	0.032	0.018	-0.023	-0.086	0.077	0.037	0.100	0.041	0.298		0.156	-0.003
16	Loss	0.033	0.037	0.044		-0.091	-0.149	-0.040	-0.101	0.050	-0.055	-0.004	0.128	0.185	-0.070	0.157		-0.002
17	#Announcements	0.027				0.002	0.025	0.004	-0.032	-0.019	0.012	0.004	0.008	-0.231	-0.019	0.000	-0.004	

#### Table 3: Investor attention and economic uncertainty.

This table presents results of regressions of announcement-window EDGAR and (investordriven) Google searches on prior day's closing value of *VIX* and controls (Eq. 44). All variables are standardized to be mean-zero and unit-variance. Detailed definitions of all variables are available in Appendix B. Standard errors for the coefficients are clustered by date. \*\*\*, \*\*, and \* indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

Dep. Var.	ESV	ESVU	ISVI
	(1)	(2)	(3)
VIX	0.030***	0.034***	0.018*
	(0.005)	(0.005)	(0.010)
lag(Dep. Var.)	0.455***	0.535***	0.238***
0(1)	(0.013)	(0.013)	(0.008)
SUE Decile	0.005***	0.005***	0.009**
	(0.001)	(0.001)	(0.004)
abs(SUE Decile)	0.005	0.011***	0.015*
	(0.003)	(0.003)	(0.008)
Size	0.059***	0.059***	0.086***
	(0.002)	(0.003)	(0.005)
Book-to-Market	-0.008***	-0.011***	-0.014***
	(0.001)	(0.001)	(0.004)
EPersistence	-0.016***	-0.013***	0.005
	(0.002)	(0.002)	(0.004)
IO	0.002	0.001	0.107***
	(0.001)	(0.001)	(0.034)
EVOL	0.006***	0.007***	0.009*
	(0.001)	(0.001)	(0.005)
ERepLag	0.025***	0.016***	0.010**
	(0.006)	(0.006)	(0.005)
#Estimates	0.045***	0.045***	0.034***
	(0.003)	(0.003)	(0.005)
TURN	0.027***	0.028***	0.055***
	(0.002)	(0.002)	(0.006)
Loss	-0.001	0.004***	0.002
	(0.001)	(0.001)	(0.004)
#Announcements	-0.024**	-0.010	-0.019***
	(0.011)	(0.012)	(0.005)
Date-clustered SE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Day-of-week FE	Yes	Yes	Yes
Ν	119,341	119,341	62,757
R-sq	0.803	0.817	0.122

#### Table 4: ERCs, economic uncertainty and investor attention

This table presents results of regressions of earnings announcement returns (*EARET*) on earnings surprise deciles interacted with the *VIX* (column a), with *ESVU* (column b), and with both the *VIX* and *ESVU* (column c) (Eq. 45). All variables are standardized to be mean-zero and unit-variance. Control variables include: *PreRet*, *Size*, *Book-to-Market*, *EPersistence*, *IO*, *EVOL*, *ERepLag*, *#Estimates*, *Turn*, *Loss*, *#Announcements*, year indicators, day-of-week indicators, and each of these interacted with *SUE* Decile. Detailed definitions of all variables are available in Appendix B. Standard errors for the coefficients are clustered by date. \*\*\*, \*\*, and \* indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

		Dep. Var. $= EA$	RET
	(1)	(2)	(3)
VIX*SUE Decile	0.015***		0.010
	(0.005)		(0.007)
$ESVU^*SUE$ Decile	· · · ·	0.028***	0.027***
		(0.007)	(0.007)
SUE Decile	$0.204^{***}$	0.295***	0.297***
	(0.012)	(0.018)	(0.018)
VIX	-0.007	. ,	-0.007
	(0.005)		(0.006)
ESVU	. ,	-0.018***	-0.018***
		(0.006)	(0.006)
lag(ESVU)		-0.001	-0.001
		(0.007)	(0.007)
$lag(ESVU)^*SUE$ Decile		-0.011	-0.010
		(0.007)	(0.007)
PreRet	-0.075***	-0.080***	-0.080***
	(0.004)	(0.006)	(0.006)
$PreRet^*SUE$ Decile	-0.012***	-0.012**	-0.011**
	(0.003)	(0.005)	(0.005)
Controls	Yes	Yes	Yes
$Controls^*SUE$ Decile	Yes	Yes	Yes
Date-clustered SE	Yes	Yes	Yes
Year and Day-of-week FE	Yes	Yes	Yes
N	$224,\!675$	119,332	119,332
R-Square	0.111	0.143	0.143

	Subsan	Subsamples: based	on within-ye	Dep. Var. = $EARET$ ed on within-year-of-earnings-announcement median splits on announcing firm characteristics	Dep. Var. = gs-announce	= EARET ement media	n splits on $\varepsilon$	unnouncing f	firm charact	eristics
Sample:	Low CAPM Beta	High CAPM Beta	Low Forecast Dispersion	High Forecast Dispersion	Low Idiosync. Volatility	High Idiosync. Volatility	Low TURN	$\operatorname{High}_{TURN}$	Low IO	High <i>IO</i>
<i>VIX*SUE</i> Decile	0.007	$0.022^{***}$	$0.022^{***}$	$0.017^{**}$	$0.012^{**}$	$0.014^{**}$	0.004	$0.022^{***}$	0.007	$0.024^{***}$
SUE Decile	$(0.006)$ $0.206^{***}$	(0.007) $0.221^{***}$	(0.007) $0.296^{***}$	(0.007) $0.150^{***}$	$\left \begin{array}{c} (0.005) \\ 0.180^{***} \end{array}\right $	(0.007) $0.229^{***}$	(0.006) $0.360^{***}$	(0.008) $0.216^{***}$	(0.005) $0.237^{***}$	(0.008) $0.197^{***}$
	(0.015)	(0.019)	(0.019)	(0.018)	(0.013)	(0.018)	(0.019)	(0.019)	(0.016)	(0.019)
VIA	(0.005)	(800.0)	-0.005	-0.007 (0.008)	(0.005)	-0.002 ( $0.008$ )	(900.0)	100.0-	-0.006)	-0.00 (0.006)
PreRet	-0.078***	-0.076***	$-0.082^{***}$	-0.069***	-0.072***	-0.080***	-0.071***	-0.080***	-0.083***	-0.067***
$PreRet^*SUE$ Decile	(0.004) -0.015*** (0.004)	(0.004) -0.011*** (0.004)	(0.006) -0.015*** (0.006)	(0.004) -0.012*** (0.004)	(0.004)	(0.004) -0.014*** (0.003)	(0.004) -0.018*** (0.004)	(0.004) -0.011** (0.004)	(0.003) -0.010*** (0.003)	(0.005) -0.015*** (0.005)
Controls	Yes	$\mathbf{Yes}$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls*SUE Decile	${ m Yes}_{ m Vec}$	Yes Ves	Yes Ves	${ m Yes}_{ m Ves}$	${ m Yes}_{ m Voc}$	${ m Yes}_{ m Vec}$	${ m Yes}_{ m Ves}$	Yes Vas	${ m Yes}_{ m Ves}$	Yes Ves
Year and Day-of- week FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N R-Square	$112,431 \\ 0.123$	112,128 0.108	100,029 0.12	98,367 0.11	$\begin{array}{c} 112,575 \\ 0.124 \end{array}$	$111,984 \\ 0.114$	112,503 0.14	$112,172 \\ 0.103$	112,322 0.115	112,353 $0.111$

This table presents results of regressions of earnings announcement returns (EARET) on earnings surprise deciles based on quarterly sorts (*SUE* Decile) interacted with the *VIX*. All variables are standardized to be mean-zero and unit-variance. Control Table 5: Cross-sectional analyses: Economic uncertainty and  $ERC_s$ 

variables include: PreRet, Size, Book-to-Market, EPersistence, IO, EVOL, ERepLag, #Estimates, Turn, Loss, #Announcements,

75

Table 6: Cross-sectional analyses: Investor attention and $\mathbf{ERC}_s$
This table presents results of regressions of earnings announcement returns $(EARET)$ on earnings surprise deciles based on
quarterly sorts (SUE Decile) interacted with ESVU. All variables are standardized to be mean-zero and unit-variance. Control
variables include: PreRet, Size, Book-to-Market, EPersistence, IO, EVOL, ERepLag, #Estimates, Turn, Loss, #Announcements,
year indicators, day-of-week indicators, and each of these interacted with SUE Decile. Detailed definitions of all variables are
available in Appendix B. Standard errors for the coefficients are clustered by date. ***, **, and * indicate statistical significance
at the two-sided $1\%$ , $5\%$ , and $10\%$ levels, respectively.

	Su	Subsamples: ba	sed on within-	-year-of-earni	Dep. Var. ngs-announc	= EARET sement media	n splits on a	$\label{eq:Dep.Var.} Dep. \ Var. = EARET$ based on within-year-of-earnings-announcement median splits on announcing firm characteristics	m characteri	stics
Sample:	Low CAPM Beta	High CAPM Beta	Low Forecast Dispersion	High Forecast Dispersion	Low Idiosync. Volatility	High Idiosync. Volatility	${ m Low} TURN$	$\operatorname{High}_{TURN}$	Low IO	High IO
$ESVU^*SUE$ Decile	$0.014^{*}$	$0.035^{***}$	0.009	$0.036^{***}$	0.012	$0.034^{***}$	0.013*	$0.028^{**}$	$0.024^{***}$	$0.026^{**}$
	(0.008)	(0.011)	(0.011)	(0.010)	(0.007)	(0.010)	(0.008)	(0.012)	(0.00)	(0.010)
SUE Decile	$0.284^{***}$	0.313*** (0.025)	0.305*** (0.030)	$0.261^{***}$	0.221*** (0.020)	$0.348^{***}$	$0.430^{***}$	$(0.304^{***})$	$(0.354^{***})$	$0.240^{***}$
ESVU	-0.010	$-0.025^{**}$	$-0.020^{**}$	$-0.021^{*}$	$-0.018^{***}$	-0.017	0.001	$-0.035^{***}$	$-0.016^{*}$	$-0.018^{*}$
	(0.008)	(0.010)	(0.008)	(0.011)	(0.006)	(0.011)	(0.007)	(0.012)	(0.009)	(0.010)
lag(ESVU)	-0.001	0.004	-0.010	0.011	-0.002	0.004	-0.005	0.014	0.004	-0.002
	(0.008)	(0.011)	(0.008)	(0.011)	(0.007)	(0.011)	(0.007)	(0.012)	(0.009)	(0.010)
$lag(ESVU)^*SUE$ Decile	-0.008	$-0.026^{**}$	$-0.021^{*}$	-0.022**	-0.009	-0.015	-0.001	$-0.051^{***}$	-0.003	$-0.029^{***}$
	(0.009)	(0.012)	(0.012)	(0.011)	(0.008)	(0.011)	(0.008)	(0.012)	(0.010)	(0.011)
PreRet	-0.084***	-0.078***	-0.098***	-0.067***	-0.082***	-0.082***	-0.075***	-0.085***	-0.082***	-0.079***
	(0.007)	(0.008)	(0.008)	(0.008)	(0.007)	(0.007)	(0.007)	(0.008)	(0.007)	(0.008)
$PreRet^*SUE$ Decile	$-0.016^{**}$	-0.009	-0.011	-0.011	$-0.017^{**}$	$-0.010^{*}$	$-0.016^{***}$	-0.009	$-0.011^{**}$	-0.011
	(0.007)	(0.007)	(0.010)	(0.007)	(0.007)	(0.006)	(0.006)	(0.007)	(0.006)	(0.009)
Controls	Yes	Yes	Yes	Yes	$\mathbf{Y}_{\mathbf{es}}$	Yes	Yes	Yes	Yes	$\mathbf{Y}_{\mathbf{es}}$
$Controls^*SUE Decile$	$Y_{es}$	Yes	Yes	Yes	$\mathbf{Yes}$	Yes	$\mathbf{Yes}$	$\mathbf{Y}_{\mathbf{es}}$	$\mathbf{Yes}$	Yes
Date-clustered SE	$\mathbf{Y}_{\mathbf{es}}$	Yes	Yes	Yes	$\mathbf{Yes}$	Yes	$\mathbf{Yes}$	$\mathbf{Yes}$	$\mathbf{Yes}$	Yes
Year and Day-of-week FE	$\mathbf{Y}_{\mathbf{es}}$	$\mathbf{Yes}$	$\mathbf{Y}_{\mathbf{es}}$	Yes	$\mathbf{Yes}$	$\mathbf{Y}_{\mathbf{es}}$	$\mathbf{Yes}$	$\mathbf{Yes}$	Yes	Yes
N	59131	60089	54710	53347	60313	58907	59827	59505	59158	60174
R-Square	0.148	0.144	0.158	0.138	0.156	0.145	0.171	0.134	0.148	0.142

Table 7: Firm betas and investor attention on earnings announcement days This table reports averages of beta estimates from three regressions:

$$r_{i,t}^{e} = \alpha_{\text{Other}}^{i} + \alpha_{\Delta EA}^{i} \mathbf{1}_{\text{EA}}^{i} + \beta_{\text{Other}}^{i} r_{M,t}^{e} + \beta_{\Delta EA}^{i} (\mathbf{1}_{\text{EA}}^{i} \times r_{M,t}^{e}) + \varepsilon_{i,t}$$
(i)

$$r_{i,t}^{e} = \alpha_{\text{Other}}^{i} + \alpha_{\Delta A}^{i} \mathbf{1}_{\text{HighAtt}}^{i} + \beta_{\text{Other}}^{i} r_{M,t}^{e} + \beta_{\Delta A}^{i} (\mathbf{1}_{\text{HighAtt}}^{i} \times r_{M,t}^{e}) + \varepsilon_{i,t}$$
(ii)

$$r_{i,t}^{e} = \alpha_{\text{Other}}^{i} + \alpha_{\Delta EA}^{iow,i} \mathbf{1}_{\text{EA}}^{iow,i} + \alpha_{\Delta EA}^{ing,i} \mathbf{1}_{\text{EA}}^{\text{ing},i} + \beta_{\text{Other}}^{i} r_{M,t}^{e} + \beta_{\Delta EA}^{low,i} (\mathbf{1}_{\text{EA}}^{\text{how},i} \times r_{M,t}^{e}) + \beta_{\Delta EA}^{high,i} (\mathbf{1}_{\text{EA}}^{\text{high},i} \times r_{M,t}^{e}) + \varepsilon_{i,t},$$
(iii)

where  $\mathbf{1}_{\text{EA}}^{i}$  equals one on days when the firm *i* announces earnings;  $\mathbf{1}_{\text{HighAtt}}^{i}$  equals one on days when investor attention to firm *i* is high (i.e., time-detrended ESV(U) of firm *i* is above the sample median);  $\mathbf{1}_{\text{EA}}^{\text{high},i}$  equals one if both  $\mathbf{1}_{\text{EA}}^{i}$  and  $\mathbf{1}_{\text{HighAtt}}^{i}$  are one; and  $\mathbf{1}_{\text{EA}}^{\text{low},i}$  equals one if  $\mathbf{1}_{\text{EA}}^{i}$  is one and  $\mathbf{1}_{\text{HighAtt}}^{i}$  is zero;  $r_{M,t}^{e}$  is the excess return on the market; and  $r_{i,t}^{e}$  is firm *i*'s excess return. The table reports average values for the  $\beta$  coefficients across firms and their standard errors (in parenthesis). \*\*\*, \*\*, and \* indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)
Attention measure:		ESV	ESVU	ESV	ESVU
Average $\beta_{Other}$	$1.028^{***}$ (0.012)	$1.018^{***}$ (0.012)	$1.028^{***}$ (0.012)	$1.039^{***}$ (0.012)	$1.039^{***}$ (0.012)
Average $\beta_{\Delta EA}$	$0.081^{**}$ (0.034)	· · ·	<b>`</b>	· · · ·	
Average $\beta_{\Delta A}$	<b>`</b>	$0.043^{***}$ (0.005)	$0.020^{***}$ (0.005)		
Average $\beta_{\Delta EA}^{low}$		~ /		$-1.920^{*}$ (1.049)	-1.551 (1.063)
Average $\beta_{\Delta EA}^{high}$				$(0.103^{***})$ (0.039)	$(0.077^{**})$ (0.038)
Average Adj. R <sup>2</sup> Firm Count	$0.252 \\ 1,368$	$0.253 \\ 1,260$	$0.253 \\ 1,260$	$0.258 \\ 1,260$	$0.257 \\ 1,260$

# Table 8: Portfolio betas and investor attention, 10 beta-sorted portfoliosThis table reports beta estimates from the regression

$$r_{j,t}^{e} = \alpha_{\text{Other}}^{j} + \alpha_{\Delta A}^{j} \mathbf{1}_{\text{HighAtt}} + \beta_{\text{Other}}^{j} r_{M,t}^{e} + \beta_{\Delta A}^{j} (\mathbf{1}_{\text{HighAtt}} \times r_{M,t}^{e}) + \varepsilon_{j,t},$$

where  $\mathbf{1}_{\text{HighAtt}}$  is an indicator variable for high attention days (days with the detrended aggregate ESV(U) above the median);  $r_{M,t}^e$  is the excess return on the market;  $r_{j,t}^e$  is the portfolio excess return;  $\beta_{\text{Other}}^j$  is the beta on other days; and  $\beta_{\Delta A}^j$  measures the change in the portfolio's beta on high attention days. The regression is estimated on 10 beta-sorted portfolios. ESV(U) results are reported in Panel A (Panel B). Standard errors are given in parentheses. \*\*\*, \*\*, and \* indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

			Panel	<b>A:</b> 10 beta	a-sorted po	ortfolios ar	nd high $ES$	SV days		
	Low	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	High
$\beta_{Other}$	$\begin{array}{c} 0.508^{***} \\ (0.006) \\ 0.093^{***} \end{array}$	$\begin{array}{c} 0.631^{***} \\ (0.006) \\ 0.120^{***} \end{array}$	$\begin{array}{c} 0.789^{***} \\ (0.006) \\ 0.107^{***} \end{array}$	$\begin{array}{c} 0.841^{***} \\ (0.006) \\ 0.107^{***} \end{array}$	$\begin{array}{c} 0.930^{***} \\ (0.006) \\ 0.116^{***} \end{array}$	$\begin{array}{c} 0.990^{***} \\ (0.005) \\ 0.131^{***} \end{array}$	$\begin{array}{c} 1.067^{***} \\ (0.006) \\ 0.127^{***} \end{array}$	$\begin{array}{c} 1.184^{***} \\ (0.007) \\ 0.083^{***} \end{array}$	$\begin{array}{c} 1.306^{***} \\ (0.008) \\ 0.067^{***} \end{array}$	$ \begin{array}{r} 1.561^{***} \\ (0.011) \\ 0.029 \end{array} $
$\beta_{\Delta A}$	(0.093) $(0.011)$	(0.011)	(0.017)	(0.010)	(0.010)	(0.131) $(0.010)$	(0.011)	(0.085) $(0.012)$	(0.007) $(0.014)$	(0.029)
			Panel I	<b>B:</b> 10 beta	-sorted por	rtfolios and	d high $ES$	VU days		
	Low	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	High
$\beta_{Other}$	0.504***	0.626***	0.785***	0.837***	0.925***	0.989***	1.065***	1.190***	1.319***	1.574***
$\beta_{\Delta A}$	(0.007) $0.094^{***}$ (0.011)	(0.006) $0.121^{***}$ (0.010)	(0.006) $0.108^{***}$ (0.010)	(0.006) $0.109^{***}$ (0.010)	(0.006) $0.119^{***}$ (0.010)	$\begin{array}{c} (0.006) \\ 0.120^{***} \\ (0.009) \end{array}$	$(0.006) \\ 0.122^{***} \\ (0.010)$	(0.007) $0.057^{***}$ (0.012)	(0.008) $0.023^{*}$ (0.014)	(0.012) -0.009 (0.019)

Table 9: Portfolio betas and investor attention, 25 size/BM-sorted portfolios This table reports beta estimates from the regression

$$r_{j,t}^e = \alpha_{\text{Other}}^j + \alpha_{\Delta A}^j \,\mathbf{1}_{\text{HighAtt}} + \beta_{\text{Other}}^j r_{M,t}^e + \beta_{\Delta A}^j (\mathbf{1}_{\text{HighAtt}} \times r_{M,t}^e) + \varepsilon_{j,t},$$

where  $\mathbf{1}_{\text{HighAtt}}$  is an indicator variable for high attention days (days with the detrended aggregate ESV(U) above the median);  $r_{M,t}^e$  is the excess return on the market;  $r_{j,t}^e$  is the portfolio excess return;  $\beta_{\text{Other}}^j$  is the beta on other days; and  $\beta_{\Delta A}^j$  measures the change in the portfolio's beta on high attention days. The regression is estimated on 25 size/BM-sorted portfolios. ESV(U) results are reported in Panel A (Panel B). Standard errors are given in parentheses. \*\*\*, \*\*, and \* indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

			Panel A	: 25 size/I	3M-sorted	portfolios	and high <i>l</i>	ESV days		
			$\beta_{Other}$					$\beta_{\Delta A}$		
	Growth	(2)	(3)	(4)	Value	Growth	(2)	(3)	(4)	Value
Small	0.999***	0.919***	0.814***	0.780***	0.726***	0.160***	0.220***	0.290***	0.336***	0.358***
	(0.010)	(0.009)	(0.008)	(0.009)	(0.009)	(0.018)	(0.017)	(0.015)	(0.015)	(0.016)
(2)	1.130***	$0.984^{***}$	0.900***	$0.897^{***}$	0.960***	0.031*	$0.193^{***}$	0.283***	$0.317^{***}$	0.401***
	(0.009)	(0.008)	(0.008)	(0.008)	(0.009)	(0.016)	(0.014)	(0.013)	(0.014)	(0.016)
(3)	$1.175^{***}$	$0.988^{***}$	$0.921^{***}$	$0.920^{***}$	$0.985^{***}$	-0.062***	$0.091^{***}$	$0.175^{***}$	$0.204^{***}$	$0.246^{***}$
	(0.008)	(0.006)	(0.006)	(0.007)	(0.009)	(0.014)	(0.011)	(0.011)	(0.013)	(0.017)
(4)	$1.129^{***}$	$0.925^{***}$	$0.905^{***}$	$0.894^{***}$	$0.999^{***}$	-0.088***	$0.102^{***}$	$0.217^{***}$	$0.192^{***}$	$0.258^{***}$
	(0.006)	(0.005)	(0.006)	(0.007)	(0.009)	(0.011)	(0.008)	(0.010)	(0.012)	(0.016)
Large	$1.028^{***}$	$0.933^{***}$	$0.909^{***}$	$0.949^{***}$	$1.086^{***}$	-0.144***	0.007	$0.100^{***}$	$0.253^{***}$	$0.295^{***}$
	(0.004)	(0.004)	(0.005)	(0.008)	(0.011)	(0.007)	(0.007)	(0.009)	(0.014)	(0.020)

Panel B: 25 size/BM-sorted portfolios and high ESVU days

	$\beta_{Other}$					$\beta_{\Delta A}$				
	Growth	(2)	(3)	(4)	Value	Growth	(2)	(3)	(4)	Value
Small	0.998***	0.920***	0.808***	0.771***	0.707***	0.146***	0.193***	0.278***	0.328***	0.373***
	(0.011)	(0.010)	(0.008)	(0.009)	(0.009)	(0.018)	(0.016)	(0.014)	(0.015)	(0.016)
(2)	1.144***	$0.985^{***}$	0.896***	0.882***	$0.941^{***}$	-0.010	$0.169^{***}$	$0.264^{***}$	0.327***	$0.414^{***}$
	(0.009)	(0.008)	(0.008)	(0.008)	(0.009)	(0.016)	(0.013)	(0.013)	(0.014)	(0.016)
(3)	1.187***	$0.994^{***}$	$0.919^{***}$	$0.914^{***}$	0.968***	-0.092***	$0.064^{***}$	$0.162^{***}$	0.199***	0.270***
	(0.008)	(0.006)	(0.006)	(0.007)	(0.010)	(0.013)	(0.010)	(0.011)	(0.012)	(0.016)
(4)	1.138***	0.928***	0.896***	0.879***	0.958***	-0.103***	0.083***	0.220***	0.216***	0.346***
	(0.006)	(0.005)	(0.006)	(0.007)	(0.009)	(0.011)	(0.008)	(0.010)	(0.012)	(0.015)
Large	1.040***	0.930***	0.900***	0.907***	1.064***	-0.161***	0.016**	0.113***	0.348***	0.329***
	(0.004)	(0.004)	(0.005)	(0.008)	(0.012)	(0.006)	(0.007)	(0.009)	(0.013)	(0.019)

Table 10:	CAPM,	economic	uncertainty,	and	investor	attention
	- )					

This table reports the results of regressions of excess returns on 10 value-weighted beta-sorted portfolios (Panel A) and 25 size/BM-sorted portfolios (Panel B) on the excess return on the market. Estimates are in basis points per day and are computed separately for: all days; days with  $VIX_{t-1}$  above the median and in its top quartile; days with the detrended aggregate ESV above the median and in its top quartile; days with the detrended aggregate ESVU above the median and in its top quartile; and days with both  $VIX_{t-1}$  and ESV(U) above the median. Standard errors are given in parentheses. \*\*\*, \*\*, and \* indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

	Panel A: 10 beta-sorted portfolios										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)		
	All days	$\left  \begin{array}{c} V \\ > 50\% \end{array} \right $	$\begin{array}{l} IX \\ > 75\% \end{array}$	$\left  \begin{array}{c} ES \\ > 50\% \end{array} \right $	SV > 75%	ES $> 50%$	VU > 75%	$\begin{vmatrix} VIX > 50\% \\ ESV > 50\% \end{vmatrix}$	VIX > 50% $ESVU > 50%$		
Intercept $\beta$	$2.186^{***} \\ (0.546) \\ 2.102^{***} \\ (0.518)$	$\begin{array}{c} -0.577 \\ (0.606) \\ 7.117^{***} \\ (0.578) \end{array}$	$\begin{array}{c} -1.355 \\ (1.104) \\ 9.843^{***} \\ (1.050) \end{array}$	$ \begin{array}{c c} 1.216 \\ (0.764) \\ 4.761^{***} \\ (0.680) \end{array} $	$1.274 \\ (0.734) \\ 4.282^{***} \\ (0.666)$	$ \begin{array}{c} -0.333 \\ (0.785) \\ 5.096^{***} \\ (0.708) \end{array} $	$\begin{array}{c} -1.211 \\ (1.410) \\ 7.967^{***} \\ (1.289) \end{array}$	$\begin{array}{c} -2.559^{**} \\ (1.003) \\ 11.786^{***} \\ (0.883) \end{array}$	-5.528*** (1.550) 10.297*** (1.383)		
R-Square Nb. days	$0.673 \\ 8058$	$0.950 \\ 4027$	$0.917 \\ 2012$	0.860 1808	0.838 904	$0.866 \\ 1808$	$0.827 \\ 904$	0.957 848	0.874 752		

	·								
	All days	VIX		ESV		ESVU		VIX > 50%	VIX > 50%
		> 50%	> 75%	> 50%	> 75%	> 50%	> 75%	ESV > 50%	ESVU > 50%
Intercept	7.164***	8.963***	-1.200	1.525	0.475	-1.274	-2.829	6.783*	-0.127
	(1.930)	(2.460)	(4.910)	(2.188)	(3.806)	(2.347)	(3.131)	(3.395)	(3.141)
$\beta$	-3.028	-3.298	6.618	4.314**	5.389	6.052***	$9.990^{***}$	3.422	$5.409^{*}$
	(1.904)	(2.452)	(4.842)	(1.894)	(3.278)	(2.085)	(2.750)	(2.948)	(2.813)
R-Square	0.099	0.073	0.075	0.184	0.105	0.268	0.365	0.055	0.138
Nb. days	8058	4027	2012	1808	904	1808	904	848	752