Self-inflicted debt crises*

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Abstract

In a dynamic model of optimal bailouts, we show how borrower myopia affects the severity of debt crises and the pricing of credit risk. Myopic borrowers misprice the option to default with a U-shaped negative pricing error. The myopia discount changes default incentives, bailout policies, and credit spreads. Optimal bailouts punish myopia when the distortions from default mispricing outweigh future bailout costs, but reward myopia otherwise. The model explains why (i) defaults get procrastinated and debt crises protracted, (ii) credit spread dynamics are more asymmetric for lower-quality borrowers, (iii) default can be cheaper to resolve for lower-quality borrowers, and (iv) countries can benefit from populist governments.

JEL: H63, G01, G4, D86

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Debt crises are often self-inflicted by myopic borrowers that make time-inconsistent consumption-savings decisions. Sovereign myopia manifests itself either as a behavioral trait of the population (Laibson, 1997) or as result of an inefficient political system that sacrifices the future for short-term electoral benefits (Aguiar and Amador, 2011). Outright default is rare in sovereign debt crises. Sovereigns typically procrastinate default and call on bailout agencies to rescue them from the brink of default. Political risk and the threat of spillovers to other countries hinder efficient debt renegotiations. The result are protracted crises and prolonged episodes of debt relief (Reinhart and Rogoff, 2009). Sovereign credit markets in turn are 'sleepy' in normal times, that is, sovereign spreads are low and insensitive to fundamentals, until credit spreads suddenly spike once crisis hits.

The European debt crisis is a prime example. Article 125 of the Lisbon Treaty states that the European Union (EU) *shall not be liable for the commitments of [...] any Member State. A Member State shall not be liable for the commitments of [...] another Member State.* From this rule it looks clear that all countries are responsible for their own debt, and no country has to rescue another. The disclaimer has proven vacuous. The IMF, ECB, and Eurogroup granted bailouts to Ireland in 2010, Greece in 2010, 2012 and 2015, and other Eurozone countries between 2011 and 2016.

In this paper, we show how borrower myopia affects the severity and resolution of debt crises and the pricing of sovereign credit risk. Bailout agencies, such as IMF, World Bank, Paris Club, or ESM, face a policy dilemma. Generous financial transfers avoid outright default and quickly fix a debt crisis that can spill over to other countries and spark systemic risk. But they are expensive to implement and encourage moral hazard, as borrowers can repeated plead for financial aid. How myopia affects a bailout agency' policy tradeoff, borrowers' default incentives, and sovereign credit spreads ultimately determines the severity of debt crises. In particular, should bailouts be smaller or larger for more myopic borrowers? Does borrower myopia lead to accelerated or procrastinated default? How does borrower myopia affect debt and credit spread dynamics? Is rational or myopic default cheaper to resolve? Do myopic governments harm or benefit the country? Who is to blame for procrastinated default and protracted crises?

We address these questions in a dynamic equilibrium model of strategic default and optimal bailouts in which we introduce borrower myopia and compare default and debt crisis dynamics to the rational case as benchmark. The starting point for our analysis is the interplay between bailouts (that is, direct financial transfers, debt relief, or haircuts) and default incentives when the borrower is strategic about the timing of default, while the bailout agency cannot commit to a policy ex-ante. The bailout agency designs an incentive-compatible financial transfer schedule in a Markovian subgame perfect Nash equilibrium to minimize intertemporal bailout costs. The bailout schedule depends on the wealth threshold at which the borrower requests aid. We focus on the dynamic Stackelberg game in which the borrower moves first by requesting financial aid and the bailout agency responds (Mella-Barral and Perraudin, 1997; Mella-Barral, 1999).

The novel assumption in our continuous-time model with endogenous consumption, saving, borrowing, and default policies is that the borrower values disproportionately within-tenure consumption. Decisions are taken with instantaneous gratification preferences (Harris and Laibson, 2012), which is the limit of quasi-hyperbolic preferences when tenure becomes infinitesimally short. In this setup, a single parameter captures the borrower's degree of myopia, which allows to keep tractability in the model and retain rationality as a special case of the myopia parameter.

In the model, the borrower requests financial aid from the bailout agency when financial conditions deteriorate beyond an endogenous critical threshold. Negotiations take place with a bailout agency that acts in the lenders' interests. We allow for two types of default. In a soft default, the bailout agency provides debt relief through financial transfers, which guarantees the debt. Debt renegotiation frictions and the threat of a hard default keep the borrower from requesting financial aid excessively. A hard default in which, for simplicity, the borrower loses access to financial markets and enters autarky acts as threat point in the negotiation, as it yields immediate benefits for a troubled borrower at the expense of loss of access to borrowing and future bailouts. Reinhart and Rogoff (2009) document that sovereign defaults are lengthy and cluster in time. We capture this fact by assuming that the bailout agency incurs spillover costs in a hard default.

Our analysis yields several important insights. Myopia distorts the borrower's intertemporal consumption, saving, borrowing, and default decisions. While rational borrowers perfectly smooth consumption, myopic borrowers imperfectly smooth consumption by overconsuming more in good than bad times, which yields a procyclical consumption-to-GDP ratio and countercyclical expected consumption growth. Myopia creates hence a retractive force that, like a rubber band, pulls time-inconsistent borrowers back into crisis.

Bailout agencies take into account the distortions caused by time-inconsistent consumption-savings decisions when they design a bailout schedule and, as a result, they assist myopic borrowers differently than rational borrowers. The key tradeoff in determining the size of bailout is that financial transfers are less effective at staving off default for myopic than rational borrowers, due to the retractive force. In turn, future expected costs of default are higher under myopia. The bailout agency balances, at the optimum, the marginal reduction in the risk-neutral default probability with the total cost of future defaults. Bailouts thus depend on how effective today's financial transfer is at reducing the risk of future defaults. The bailout agency's strategic response to myopia alters, in turn, borrowers' incentives to default.

Optimal bailouts punish myopia through smaller financial transfers when myopia's retractive force is strong and, otherwise, reward myopia through larger financial transfers. Equilibrium transfers increase with myopia if spillovers are high, renegotiation frictions or political risk are high, risk aversion is high and growth opportunities are poor, and default costs are low. Larger bailouts incentivize earlier default, but not always. Small transfers tend to delay default, lead to frequent debt relief, prolonged crisis, but discipline overspending. In contrast, large transfers tend to resolve crisis quickly, put the borrower back on a growth trajectory, but accelerate strategic default and worsen overspending.

The model yields closed-form expressions for sovereign credit spreads and the intertemporal costs faced by the bailout agency. The latter summarize the effect of current bailouts, the frequency of future bailouts, and the likelihood of negotiation failure. A decomposition of default costs shows that myopic default can be cheaper to resolve than rational default. This occurs when myopia gets punished, but in some cases also when myopia gets rewarded. It is not even clear that myopia always harms a country. We show that rational agents can benefit from a myopic sovereign. This occurs when myopia gets rewarded and debt relief occurs frequently.

The policy distortions arise from how myopic borrowers value the option to default. Myopic borrowers overvalue the consumption claim compared to rational borrowers but, more importantly, they undervalue the option to default. The pricing error is U-shaped in the distance to default, as it is zero in default and when wealth tends to infinity and negative in between. As a result, myopic borrowers ignore during normal times the impact of default and behave as if they were richer than they actually are.

To understand the role that this myopia discount plays in debt renegotiation, consider how the borrower's default policy changes with myopia when the bailout policy is given and, alternatively, how the bailout policy changes with myopia when the borrower's default policy is given. A myopia irrelevance result holds for the borrower's default policy. Both the hard-default and the soft-default thresholds are invariant to myopia, so long as the bailout agency offers the same bailout schedule. This implies that myopic borrowers are neither more nor less inclined to default than rational borrowers. However, myopia affects default through the strategic response of the bailout agency and its impact on equilibrium transfers.

Rational agents price risky bonds and credit-linked securities, such as credit default swaps, differently when the borrower is myopic. We show that default-contingent claims have a mixed power-hypergeometric shape that nests the standard power shape of Merton (1974) and Black and Cox (1976) in the limiting rational case. As a result, credit spreads are more asymmetric than predicted by a model with rational borrower. They initially rise more slowly than rational spreads, but then spike once the crisis starts to unfold. In the cross-section, credit prices are ***ceteris paribus*** higher for more myopic countries since their growth is lower. In time-series, there are two countervailing forces: Myopic governments travel the distance to default faster, as they consume more. The distance to default is, however, larger when myopic governments receive higher transfers and default later. Standard credit metrics based on the power shape are inadequate proxies for the risk of sovereign default under myopia.

Related Literature: Our model studies the implications of time-inconsistency for sovereign default and debt relief, both cross-sectionally and over time. Leland (1994) offers an early analysis on debt renegotiation. Strategic debt service and debt renegotiation have since then been extensively studied in corporation finance (Anderson and Sundaresan, 1996; Mella-Barral and Perraudin, 1997; Mella-Barral, 1999; Fan and Sundaresan, 2000; Lambrecht, 2001). Bolton (2016) applies the corporate finance setting to sovereign debt based on the analogy between the fiat liability of sovereigns and corporate equity. We extend the debt renegotiation literature is several important ways. Most importantly, we introduce time-inconsistent borrower behavior. Grenadier and Wang (2007) study the impact of myopia on investment under equity financing. By contrast, we solve the joint intertemporal consumption, investment, borrowing, and debt relief problem.

The political economy literature underlines the inclination of governments to show preference for consumption within their time in office (Persson and Svensson, 1989; Alesina and Tabellini, 1990). Based on this premise, Aguiar and Amador (2011) map the inefficiency of the political system into a model of consumption with hyperbolic discounting (Laibson, 1997). Their model is aimed at showing how politically induced myopia affects the rate of convergence of poor countries, the tendency for expropriation of foreign assets and the effect of openness on growth. By contrast, ours focuses on the design of debt relief in the presence of default spillovers. Acharya and Rajan (2013) model a myopic government that maximizes single-period public spending. They show that myopia can induce a government to default later, since it seeks to maintain current spending capacity and disregards the effects of borrowing on future taxes and output. In our model, myopia affects the default decision through its implications for bailout policy. In particular, when default spillovers are low myopia leads to late default and when spillovers are high, myopia leads to early default.

Default in our model happens on the equilibrium path. In this manner, our paper is related to models of non-contingent sovereign debt. In the seminal contribution by Eaton and Gersovitz (1981) external debt financing can be sustained using exclusion from international debt markets as a deterrent for repudiation. Later contributions by Aguiar and Gopinath (2006) and Arellano (2008) indicate that the costs of financial exclusion are relatively small and insufficient to justify the levels of debt typically observed in the market. Higher levels of debt can be justified by richer models that control for maturity structure and output costs of default (Benjamin and Wright, 2009; Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012). The idea of trade and financial sanctions as a method for preventing debt repudiation, while popular in the theoretical literature (Bulow and Rogoff, 1989; Broner and Ventura, 2011) is still an open empirical question (Rose, 2005; Blundell and Bond, 1998; Martinez and Sandleris, 2011). Tomz and Wright (2013) provide a review of the empirical literature on sovereign debt. Our model abstracts from the detailed modeling of the costs of debt default and assumes for simplicity that default implies loss of access to the risk-free asset and a proportional loss of financial wealth.

Models of contingent sovereign debt have been the subject of study of reputation based models (Thomas and Worrall, 1988; Chari and Kehoe, 1990; Kehoe and Levine, 1993). These models lead to "self-enforcing equilibria" in which debt obligations adjust to economic shocks so that it is feasible and incentive compatible for the government to respect the contract. In a narrow interpretation of these models, default never happens on the equilibrium path. In a more flexible interpretation, the optimal contract embeds automatic debt relief (Grossman and Van Huyck, 1988). Either way, it is an undisputable empirical fact that default does take place. Reinhart and Rogoff (2009) record the history of sovereign default since the 14th century and show that most countries have defaulted at least once. They further show that overborrowing is pronounced in good times; serial default on external debt is a phenomenon common to all regions of the world; and that new countries go through repeated debt restructuring before they "graduate" to an advanced rare-default status. For suitable parameter levels, our model incorporates all of these empirical facts. Furthermore debt relief is contingent on the level of wealth at which the country defaults, but debt is raised at a fixed interest rate. Importantly, our paper differs from the existing literature in incorporating a probability of failure in debt relief negotiations. This creates important implications for policy design. In particular, because the bailout agency realizes that default can happen on the equilibrium path, default spillovers do affect debt relief even if the bailout agency holds all bargaining power.

Our paper also relates to the finance literature on sovereign credit spreads and sovereign excess returns. Duffie et al. (2003) develop a reduced-form model for pricing sovereign debt and use it to examine the pricing of Russian bonds during the late 1990s. We develop a structural model that can be used to price sovereign default. We introduce time inconsistency, in contrast to models by Uhrig-Homburg (2013) and Jeanneret (2015). Hyperbolic discounting in our setup leads to default-contingent claim prices that have a mixed power-hypergeometric shape. They are thus more nonlinear than in a Merton-type credit risk model. Standard market-based credit metrics are therefore poor proxies for sovereign default risk. They underestimate the (risk-neutral) default probability and are too insensitive to credit risk for too long. Only when default becomes imminent do standard credit metrics spike up.

The remainder of the paper is organized as follows. Section 1 sets up the model and derives the optimal investment and financing policy outside of default. Section 2 illustrates the endogenous mispricing of sovereign default arising from borrower myopia. Section 3 explores the optimal bailout policy. Section 4 conducts policy analysis. Section 5 explores the suitability of sovereign credit metrics and Section 6 concludes. All proofs are relegated to an Appendix.

1 Model

We develop a dynamic equilibrium model of self-inflicted debt crises. We allow the borrower to be myopic by seeking instantaneous gratification (Harris and Laibson, 2012), while the borrower retains an option to default on its debt burden. Default plays a dual role. It changes the incentive effects of myopia as the borrower approaches default and it serves as a bargaining chip to repeatedly obtain concessions from the lenders. We first characterize the borrower's endogenous consumption, investment, and borrowing decisions, describe the distortions caused by myopia, and show how the threat of default disciplines the borrower's incentives. We then determine the optimal bailout policy from the perspective of a bailout agency that acts on behalf of lenders.

1.1 Assumptions

Time is continuous and the horizon infinite. Financial markets for securities are complete and frictionless, except for debt renegotiation frictions. A sovereign borrower with net wealth W_t at time t decides on consumption, savings, borrowing, and default. Consumption is c_t with consumption share $\psi_t = \frac{c_t}{W_t}$. There exist riskless and risky savings technologies. The investment share θ_t equals the fraction of wealth invested in the risky savings technology that follows a geometric Brownian motion with drift rate μ and volatility σ. The risk-free rate is r and the price of risk is $\nu = \frac{\mu - r}{\sigma}$. Funding gaps are financed by short-term debt, $D_t = c_t - W_t(1-\theta_t)$, that matures at $t+dt$. Funding surpluses are invested in a riskless bond. Away from the default threshold, debt is riskless since it is short term and the path of wealth continuous. At the default threshold, debt remains riskless since it is fully backed by a bailout agency, such as IMF and ESM. Risky long-term debt can be priced in the model, but the analysis is more complicated when its issuance is endogenous.

Wealth dynamics at any time t before default are given by

$$dW_t = [W_t(r + \theta_t(\mu - r)) - c_t]dt + \sigma W_t \theta_t dw_t. \tag{1}$$

In choosing its policies, the sovereign faces self-control problems as it seeks instantaneous gratification (Harris and Laibson, 2012). Specifically, the borrower has CRRA preferences over consumption with relative risk aversion γ. Utility streams are discounted at rate $\rho > r$ until a Poisson-distributed random time τ with intensity λ at which a new self of the sovereign arrives. The current self discounts utility streams after the new self arrives using the discount factor $\delta exp(-\rho t)$ where $\delta \in (0, 1)$ and $\gamma + \delta > 1$. Instantaneous gratification assumes that future selves arrive instantaneously, that is, $\lambda \to \infty$. In this case, the parameter δ captures the degree of myopia.

A crucial feature of the model is that the borrower has the option to default on its debt and, as a result, can repeatedly extract concessions from the lenders. We allow for two default regimes, hard default and soft default, that both can occur in equilibrium. Hard default can be optimal in equilibrium and, alternatively, in a soft default it serves as threat point for debt renegotiations.

We focus on the dynamic Stackelberg game between myopic borrower and rational bailout agency in which the borrower moves first and the bailout agency responds optimally (Mella-Barral and Perraudin, 1997; Mella-Barral, 1999). The contracting space is incomplete: The bailout agency can incentivize the borrower country through repeated financial transfers according to an optimal schedule, denoted $T(W)$. Absent an international bankruptcy court, the bailout agency can neither impose austerity nor enforce a first-best debt relief policy.[1]

In this setting, we determine the Markovian subgame perfect Nash equilibria in threshold strategies, $(\underline{W}^a, \underline{W}^-, \underline{W}^+)$. When a hard default is optimal, the borrower enters autarky once wealth drops to $W \leq \underline{W}^a$. When a soft default is optimal, the borrower chooses the wealth threshold $\underline{W}^-$ when to ask for financial aid. The bailout agency responds by offering a financial transfer $T(\underline{W}^-)$ that, if negotiations are successful, improves the borrower's financial situation to $\underline{W}^+ = \underline{W}^- + T(\underline{W}^-)$. This setup is equivalent to the bailout agency posting an ex-post incentive-compatible transfer schedule $T(W)$ for any wealth level and the borrower optimally responding by picking a $\underline{W}^-$ and

[1] In an extension, we have analyzed alternative settings in which the bailout agency moves first or both parties lack commitment. The borrower moves first by declaring soft default. The agency then decides on the bailout T. The bailout policy is subgame perfect as it minimizes the cost to the guarantor given W. Optimal debt forgiveness, by contrast, requires commitment by the guarantor. The guarantor is the 'Stackelberg leader' in this setting. The bailout agency commits to a bailout T and the borrower decides when to plead for aid given T. That is, one solves for $W(T)$. When the guarantor can commit to the bailout, it can manipulate when the borrower defaults. We find our main setting to be more realistic.

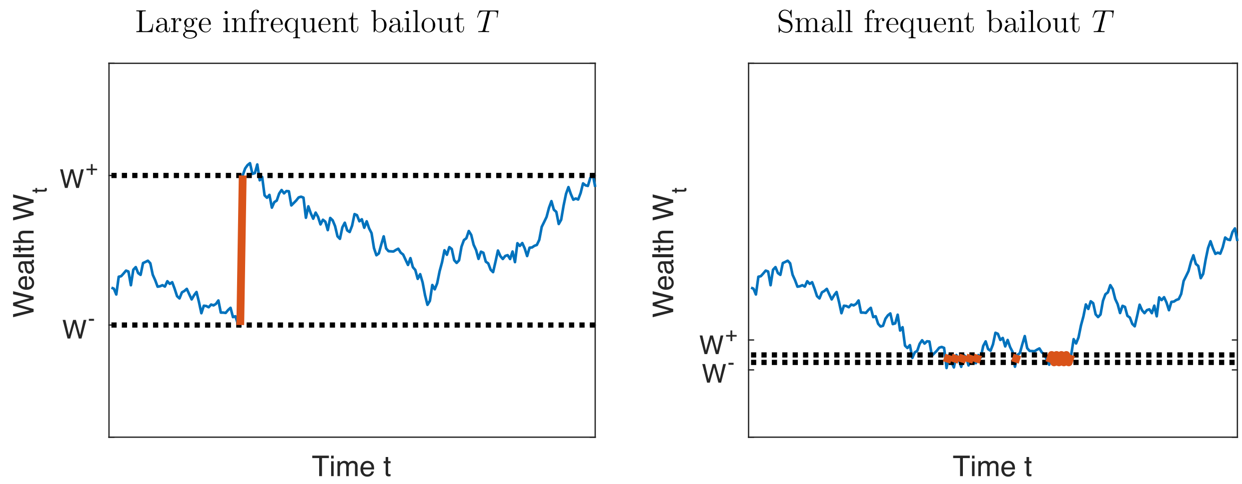

Figure 1: **Model illustration.** The figure plots wealth W_t over time, default threshold $\underline{W}^-$, and target wealth threshold $\underline{W}^+$ after a bailout. The vertical red lines indicate a bailout of size $T = \underline{W}^+ - \underline{W}^-$. The two figures assume different default and target thresholds $(\underline{W}^-, \underline{W}^+)$.

corresponding transfer $T(\underline{W}^-)$. The bailout agency acts in the interests of the lenders by designing the transfer schedule $T(W)$ to minimize the intertemporal cost $I(c(W); T)$ of rescuing the country, optimally balancing financial transfers with the frequency at which governments request financial aid.

[Figure 1 About Here.]

Figure 1 illustrates alternative bailout policies. In the left plot, infrequent large bailouts occur when $\underline{W}^-$ is high and $T(\underline{W}^-)$ is large. In the right plot, frequent small bailouts occur when $\underline{W}^-$ is low and $T(\underline{W}^-)$ is small. The latter resembles a tight-leash policy in which the lenders focus on addressing the borrower's incentive problem by keeping the borrower close to the default threshold and thereby trying to minimize policy distortions.

Debt renegotiation is not frictionless. Otherwise, the borrower would call for financial aid any time the promised aid is positive. Negotiating relief deals requires in practice the approval from several committees and parties with sometimes conflicting incentives. We incorporate debt renegotiation frictions by assuming that the negotiations between borrower country and agency fail with probability $p \in (0, 1)$. Once the country requests aid, it receives the financial transfer so long as the negotiations succeed and continues operating with access to borrowing and lending markets.

If negotiations fail, the country enters a hard default.

Default is costly to both borrower and lenders. In a hard default, the borrower suffers from austerity and the lenders from the debt guarantee and the threat of spillovers. The borrower incurs direct cost of bankruptcy, as its wealth changes from W to $W^a(W) = \omega_0 + \omega_1 W$ with parameters ω_0 and ω_1 that depend on the enforcability of debt. The borrower faces indirect costs as it stays in autarky forever and can no longer borrow or lend. The bailout agency seizes assets and picks up the net cost $W^a(W) - W$ from its debt guarantee. Reinhart and Rogoff (2009) document that during the last two centuries at least five waves of sovereign defaults occurred. The median duration of default episodes was 3–6 years, with a median duration of 6 (3) years pre (post) World War II. To capture the stylized facts that default episodes are lengthy and defaults across countries cluster in time, we assume the bailout agency incurs indirect costs in the form of expected spillover costs amounting to $\kappa \geq 0$.

1.2 Value functions and optimality conditions

The value function $M(W)$ of a myopic sovereign prior to its replacement differs from the corresponding value function $R(W)$ after the current sovereign (or "self") has been replaced. Let c^* be the consumption and θ^* the investment share that maximize $M(W)$. The Hamilton-Jacobi-Bellman (HJB) equations for $M(W)$ and, respectively, $R(W)$ at any wealth level W above the default threshold $\underline{W}^-$ are given by:

$$\rho M(W) - \lambda(\delta R(W) - M(W)) = \max_{c,\theta} \; u(c) + M'(W)[W(r + \theta(\mu - r)) - c] + \frac{\sigma^2}{2} W^2 \theta^2 M''(W), \tag{2}$$

$$\rho R(W) = u(c^*) + R'(W)[W(r + \theta^*(\mu - r)) - c^*] + \frac{\sigma^2}{2} W^2 (\theta^*)^2 R''(W). \tag{3}$$

The above formulation assumes that the sovereign is aware of its myopia and correctly anticipates that its policies will continue to be myopic, even after its current self is replaced by a new one. This assumption is commonly termed "sophisticated" myopia (Phelps and Pollak, 1968; Laibson, 1997).

Instantaneous gratification assumes that future selves arrive instantaneously ($\lambda \to \infty$) and implies $M(W) = \delta R(W)$. For this to be the case, the second term on the left-hand side of expression (2) needs to satisfy $\lim_{\lambda\to\infty} \lambda(\delta R(W) - M(W)) = -(1-\delta)\lim_{\lambda\to\infty} u(c^*)$.

In a soft default (SD), the value-matching condition requires that the borrower's value functions are probabilistically matched before and after default. The borrower optimally decides when to call on the agency's help. The optimality condition requires that marginal benefit and cost are equalized. With probability $0 \leq p < 1$ that negotiations fail and value $\Omega(W)$ after a hard default, the value-matching and smooth-pasting conditions for the soft-default threshold $\underline{W}^-$ read

$$VMC \text{ SD}: \qquad M(\underline{W}^-) = (1-p)M(\underline{W}^+) + p\Omega(\underline{W}^-), \tag{4}$$

$$SPC \text{ SD}: \qquad M'(\underline{W}^-) = (1-p)M'(\underline{W}^+)(1 + T'(\underline{W}^-)) + p\Omega'(\underline{W}^-). \tag{5}$$

The term $T'(\underline{W}^-)$ in condition (5) arises because the borrower is strategic in default about how transfer $T(\underline{W}^-)$ depends on the wealth level $\underline{W}^-$ at which the borrower defaults. The default incentive depends on the steepness T' of the bailout scheme. The bailout agency, in turn, can design the transfer to influence the borrower's default decision. A steeper contract with larger T' induces the borrower to default later, potentially reducing the cost to the agency.

A hard default (HD) is optimal if no threshold satisfying (4) and (5) exists or the bailout costs exceed the cost of outright default. If a hard default is optimal, it occurs when wealth drops to the threshold $\underline{W}^a$ that satisfies the value-matching and smooth-pasting conditions

$$VMC \text{ HD}: \qquad M(\underline{W}^a) = \Omega(\underline{W}^a), \tag{6}$$

$$SPC \text{ HD}: \qquad M'(\underline{W}^a) = \Omega'(\underline{W}^a). \tag{7}$$

1.3 Myopic consumption and investment policy

The first-order conditions for optimal consumption yield

$$u'(c^*) = M'(W) = \delta R'(W), \tag{8}$$

and, respectively, optimal investment satisfies

$$\theta^* = -\frac{R'(W)}{R''(W)W}\frac{\nu}{\sigma}. \tag{9}$$

Condition (8) illustrates the static effect of myopia. Consumption is higher when δ is lower, i.e., when the sovereign is more time-inconsistent. The limiting consumption-to-wealth ratio as $W \to \infty$ equals

$$\psi = \frac{\rho - (1-\gamma)(r + \frac{1}{\gamma}\frac{\nu^2}{2})}{\gamma - (1-\delta)}. \tag{10}$$

It rises with myopia due to the term $1-\delta$ in the denominator of (10). Absent default options, there is an isomorphism between myopia and impatience: For every value of the consumption-to wealth ratio that obtains under myopia, there exists a value of of the impatience parameter ρ that produces exactly the same consumption-to-wealth ratio assuming time-consistency. We will show below that this isomorphism ceases once default options are present.

Condition (9) shows that myopia does not directly change the nature of the investment policy. The same formula (9) applies if the country is time-consistent. Myopia affects investment only to the extent it changes the curvature of the value function.

To provide explicit solutions, we conjecture that the equilibrium consumption policy $c = c^*(W)$ is increasing in wealth and denote by $Y(c)$ the borrower's (subjective) wealth given any level of equilibrium consumption, defined as $Y(c^*(W)) = W$. One can show that Y satisfies the following linear ODE (Karatzas et al., 1986; Demchuk, 2003):

$$rY(c) = (\gamma - (1-\delta))\frac{1}{\gamma}c + \left(r - \rho - (1-\frac{1}{\gamma})\frac{\nu^2}{2}\right)\frac{1}{\gamma}cY'(c) + \frac{\nu^2}{2}(\frac{1}{\gamma}c)^2Y''(c), \tag{11}$$

subject to two boundary conditions. At the lower boundary $\underline{c}^- = c^*(\underline{W}^-) = M'(Y(\underline{c}^-))^{-\frac{1}{\gamma}}$, $Y(\underline{c}^-) = \underline{W}^-$, and $\lim_{c\to\infty} c/(Y(c)) = \psi$. The solution to (11) has the form $Y(c) = \frac{c}{\psi} + Ac^{-\gamma\underline{h}} + Bc^{-\gamma h}$, where $\underline{h}$ and h are the negative and, respectively, positive solution to the characteristic equation

$\frac{\nu^2}{2}h^2 - (r - \rho - \frac{\nu^2}{2})h - r = 0$ and independent of δ:

$$
\begin{aligned}
\underline{h} &= \tfrac{1}{\nu^2}\left(r - \rho - \tfrac{\nu^2}{2} - \sqrt{\left(r - \rho - \tfrac{\nu^2}{2}\right)^2 + 2r\nu^2}\right), \\
h &= \tfrac{1}{\nu^2}\left(r - \rho - \tfrac{\nu^2}{2} + \sqrt{\left(r - \rho - \tfrac{\nu^2}{2}\right)^2 + 2r\nu^2}\right).
\end{aligned}
\tag{12}
$$

We can discard the rapidly growing term $c^{-\gamma\underline{h}}$ in the expression for Y by setting $A = 0$. The value-matching condition yields $B = -(\frac{\underline{c}^-}{\psi} - \underline{W}^-)(\underline{c}^-)^{\gamma h} < 0$, so that wealth in terms of consumption is

$$
Y(c) = \frac{c}{\psi} - \underbrace{(\frac{\underline{c}^-}{\psi} - \underline{W}^-)(\frac{c}{\underline{c}^-})^{-\gamma h}}_{\text{Default value} > 0}. \tag{13}
$$

The expression (13) has an intuitive interpretation. The first term measures wealth without default, and the second term captures the impact of the option to default. Its impact on $Y(c)$ is negative since default is valuable and increases consumption c for any wealth level W. The last term in expression (13) is the private valuation by a myopic borrower of a state-contingent claim that pays one unit when default occurs:

$$
\text{MDef}(c) = (\frac{c}{\underline{c}^-})^{-\gamma h}. \tag{14}
$$

It is crucial that MDef(c) corresponds to the price of a default-contingent claim if the borrower is rational, but not if the borrower is myopic.

The myopic value function in terms of consumption, defined as $N(c^*(W)) = M(W)$, is given by

$$
N(c) = \frac{1}{\rho}\left[(\gamma - (1-\delta))u(c) + u'(c)\left(rY(c) + \frac{1}{\gamma}\frac{\nu^2}{2}Y'(c)c\right)\right]. \tag{15}
$$

Expression (15) highlights the impact of myopia on valuations. The instantaneous utility of consumption, $u(c)$, gets scaled by $\gamma - (1-\delta)$ instead of γ, but more importantly, subjective wealth $Y(c)$ and its derivative are distorted due to the misvaluation of default claims by a myopic borrower that we explore now.

2 (Mis)pricing of Sovereign Default

To understand the nature of the distortions arising from borrower myopia, it will be useful to explore how a myopic borrower prices the default option. As we will show, myopic borrowers undervalue default-contingent claims, which means they ignore in normal times the consequences of default. To see this, consider the shadow value of a claim on default for a myopic borrower, MDef(c) given in (14), compared to the market price of a default-contingent derivative claim, denoted Def(c).

The price of a default-contingent derivative claim can be computed in closed form (all derivations are relegated to the Appendix), as follows:

Proposition 1 *The price of a default-contingent derivative claim equals*

$$\textit{Def}(c) = (\frac{c}{\underline{c}^-})^{-\gamma i}\frac{H(c)}{H(\underline{c}^-)}, \tag{16}$$

where the coe cient i is the positive solution to the characteristic equation $\frac{\nu^2}{2}i^2 - (r - \rho - \frac{\nu^2}{2} - (1 - \delta)\psi)i - r = 0$ with solution

$$i = \frac{1}{\nu^2}\left(r - \rho - \frac{\nu^2}{2} - (1-\delta)\psi + \sqrt{\left(r - \rho - \frac{\nu^2}{2} - (1-\delta)\psi\right)^2 + 2r\nu^2}\right), \tag{17}$$

and H is the hypergeometric function $H(c) = {}_2F_1(\alpha_1, \alpha_2, \alpha_3; z(c))$ evaluated at the default-option wedge

$$z(c) = 1 - \psi Y'(c). \tag{18}$$

The coefficient i varies with δ and coincides with h in the rational case when $\delta = 1$. The function $H(c)$ in expression (16) is the main departure from the standard Merton (1974) and Black and Cox (1976) type solution. It denotes the hypergeometric function ${}_2F_1$ with coefficients $(\alpha_1, \alpha_2, \alpha_3)$ derived in the Appendix and evaluated at the default-option wedge $z(c)$. This wedge equals $z(\underline{c}^-) = 1$ at the default boundary and $\lim_{c\to\infty} z(c) = 0$ in good times.

The default claim can be used to infer how capital markets assess the borrower's default risk and to check how it differs from the borrower's distorted assessment. Comparing expression (14) to the market value of a default claim, $\text{Def}(c)$, yields the following result.

Theorem 1 *A myopic borrower undervalues the default claim compared to a rational borrower. That is, MDef*$(c) \leq$ *Def*(c) *for* $\delta < 1$. *In the two limits* $c = \underline{c}^-$ *and* $c \to \infty$, *MDef*$(c) =$ *Def*(c).

Theorem 1 highlights that the risk-neutral probability of default under the borrower's subjective beliefs is lower than the one of a rational borrower, and it is lower the more myopic is the borrower. Only in the rational case do private and market values coincide.

2.1 Shadow wealth

Myopic borrowers act as if they were richer than they actually are. This misconception originates from the misvaluation of the default option in Theorem 1. It leads to a gap between the private valuation and the market value of the consumption claim that is absent under rationality. Myopic borrowers overvalue the consumption claim compared to the rational market.

Suppose the borrower valued the default option at its market value. Wealth for a given level of consumption would be

$$\Psi(c) = \frac{c}{\psi} - (\frac{\underline{c}^-}{\psi} - \underline{W}^-)\text{Def}(c). \tag{19}$$

We can write the subjective wealth $Y(c)$, which equals the private value of consumption, in terms of the $\Psi(c)$ claim and shadow wealth $G(c) \geq 0$ as

$$\begin{aligned} \text{(Subjective) wealth } Y(c) &= \frac{c}{\psi} - (\frac{\underline{c}^-}{\psi} - \underline{W}^-)(\frac{c}{\underline{c}^-})^{-\gamma h} \\ &= \Psi(c) + \underbrace{(\frac{\underline{c}^-}{\psi} - \underline{W}^-)[\text{Def}(c) - \text{MDef}(c)]}_{\text{Shadow wealth } G(c) \geq 0}. \end{aligned} \tag{20}$$

The comparison between the expressions for $\Psi(c)$ and $Y(c)$ illustrates that a myopic borrower acts richer than (s)he actually is: $\Psi(c) \leq Y(c)$ since a myopic borrower undervalues the default claim compared to a rational borrower, $\mathrm{MDef}(c) \leq \mathrm{Def}(c)$.[2]

The consequence of the misperception is that in good times the myopic borrower tends to ignore the consequences of his actions on the likelihood of default. Only when default becomes imminent the myopic borrower cannot any longer ignore the impact on default. Close to default the myopic borrower starts to behave more like a rational borrower. Default acts as a disciplining device for a myopic borrower. The marginal shadow wealth $G'(c)$ Q 0 determines the magnitude of the consumption and investment distortions.

[**Figure 2 About Here.**]

Figure 2 illustrates the shape of $G(c)$ (left) and its derivative $G'(c)$ (right) as a function of the probability of default. The blue line assumes that the government is time-consistent ($\delta = 1$), which represents our benchmark economy. The red line assumes that the government exhibits myopia ($\delta = 0$). The shadow wealth G vanishes under rationality. It also disappears in the myopic case in the limit as $W \to \infty$ (that is, when Pr(Default)=0) and at the default threshold $\underline{W}^-$ (that is, when Pr(Default)=1).

To illustrate the nature of the misvaluation, consider an otherwise identical rational borrower with distorted beliefs who makes two mistakes. He wrongly believes the level of consumption is a fraction $1 - \frac{1-\delta}{\gamma}$ of its true value and, in addition, he assumes perfect consumption smoothing and therefore overestimates the growth rate in consumption by its time-varying component $\frac{1-\delta}{Y'(c)}$.

An alternative interpretation is the following: Consider an otherwise identical borrower with distorted beliefs whose only mistake is to misjudge the consumption delta of the default option, $\mathrm{MDef}'(c)$. So long as the borrower beliefs $\mathrm{MDef}'(c) = \mathrm{Def}'(c)$, $\Psi'(c) = Y'(c)$ and market and private

[2] Note that $\Psi(c)$ is not the market value of the consumption claim, since the borrower also misvalues the consumption stream. That is, the present value of future equilibrium consumption absent default is not equal to $\frac{c}{\psi}$. A more elaborate decomposition as (20) holds when $\Psi(c)$ is market value of equilibrium consumption until default and $p(c)$ denotes the market price of the consumption claim. We then have

$$Y(c) = \Psi(c) + \Phi(c) + G(c),$$

with

$$\begin{aligned}
\Psi(c) &= p(c) - (p(\underline{c}^-) - \underline{W}^-)\mathrm{Def}(c), && \text{(Market value of equilibrium } c^* \text{ until default)} \\
\Phi(c) &= \tfrac{c}{\psi} - p(c) - (\tfrac{\underline{c}^-}{\psi} - p(\underline{c}^-))\mathrm{Def}(c), && \text{(Mispricing of consumption until default)} \\
G(c) &= (\tfrac{\underline{c}^-}{\psi} - \underline{W}^-)[\mathrm{Def}(c) - \mathrm{MDef}(c)]. && \text{(Mispricing of default-contingent claim)}
\end{aligned}$$

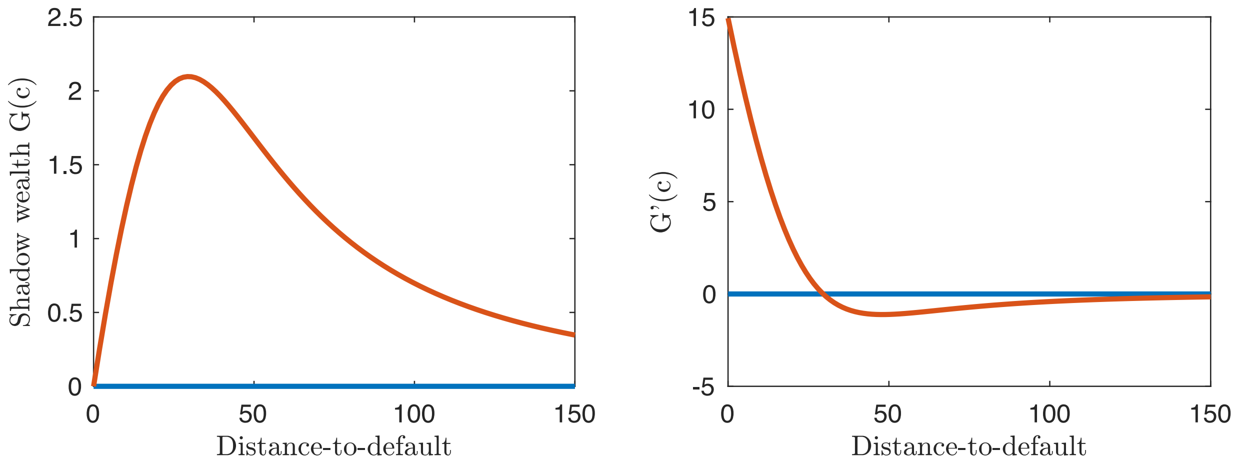

Figure 2: **Shadow wealth.** The figure plots the myopic wealth gap, $G(c)$ (left), and its marginal change with the consumption level, $G'(c)$ (right), as a function of the probability of default. The blue line assumes that the government is time-consistent ($\delta = 1$). The red line assumes the borrower is myopic with $\delta = 0$.

values of the consumption claim coincide, that is, $\Psi(c) = Y(c)$.

2.2 Imperfect consumption smoothing and misinvestment

Myopia leads to overconsumption and imperfect consumption smoothing due to the undervaluation of the default option (Theorem 1). The consequence is a reduction and countercyclical variation in expected consumption growth. Consumption growth exhibits equilibrium dynamics

$$\frac{dc_t}{c_t} = \left(r - \rho + (1 + \frac{1}{\gamma})\frac{\nu^2}{2} - \frac{1-\delta}{Y'(c_t)}\right)\frac{1}{\gamma}dt + \frac{\nu}{\gamma}dw_t, \tag{21}$$

with $Y'(\underline{c}^-) = [1 + \gamma h(1 - \frac{\psi}{\underline{c}^-/(\underline{W}^-)})]\frac{1}{\psi} > \frac{1}{\psi} = \lim_{c\to\infty} Y'(c)$. The last term in parenthesis captures the imperfect smoothing through a reduction and countercyclical variation in consumption growth as $Y'(c) = \Psi'(c) + G'(c) > 0$ declines with c.

Myopia produces a rubber band effect: Expected consumption growth decreases with the distance to default. In simple terms, myopic countries tend to overconsume more in good times rather than in bad times. By contrast, impatient countries have a constant consumption growth and their tendency to overconsume stays constant over the business cycle. This is a qualitative difference between myopia and impatience and only materializes when default options are present. To understand its nature, first consider the "election" effect, i.e the change in value of the government upon the arrival of a new self:

$$E(W) = \lim_{\lambda \to \infty} \lambda(\delta R(W) - M(W)) \tag{22}$$

We can easily show that $-\frac{1-\delta}{Y'(c)} = \frac{E'(W)}{u'(c)}$, which is the marginal rate of substitution between present utility and the cost of a new self arrival. Equation (13) shows that there is a higher expected consumption growth when the marginal value of the election effect is high. In those cases, the government is "saving" against a drop in its value by an arrival of a new self, which allows consumption to grow stronger over time.

Why is the ratio $\frac{E'(W)}{u'(c)}$ higher closer to default? To understand this effect, notice that consumption is a constant fraction ψ of the sum of wealth the value of the default option (see formula 9). Since the value of this option increases close to default, consumption is convex in wealth. As the government relies more on the value of the default option in bad times to finance its consumption, it becomes more "concerned" with the possibility of a new government arriving, which implies that the marginal rate of substitution between present utility and the election effect increases.

[**Figure 3 About Here.**]

Figure 3 documents the consumption distortions arising from government myopia. The figures plot consumption growth (left) and consumption over GDP (right) as functions of the endogenous distance to default. The yellow line assumes that the borrower is time-consistent ($\delta = 1$), which is the benchmark economy. The red line assumes that the borrower exhibits myopia ($\delta = 0$). The blue line assumes that the borrower is time-consistent ($\delta = 1$), but less patient compared to the benchmark economy. The impatience parameter ρ is calibrated so that consumption-to-wealth ratio in normal times is equal to the case of borrower myopia. The right figure shows that myopic borrowers overconsume relative to first best. The left figure plots the expected consumption growth rate. The figure shows that myopic countries smooth consumption imperfectly. Defaults acts as a disciplining device. As the country approaches default, consumption declines and the expected growth rate in consumption approaches the rational benchmark.

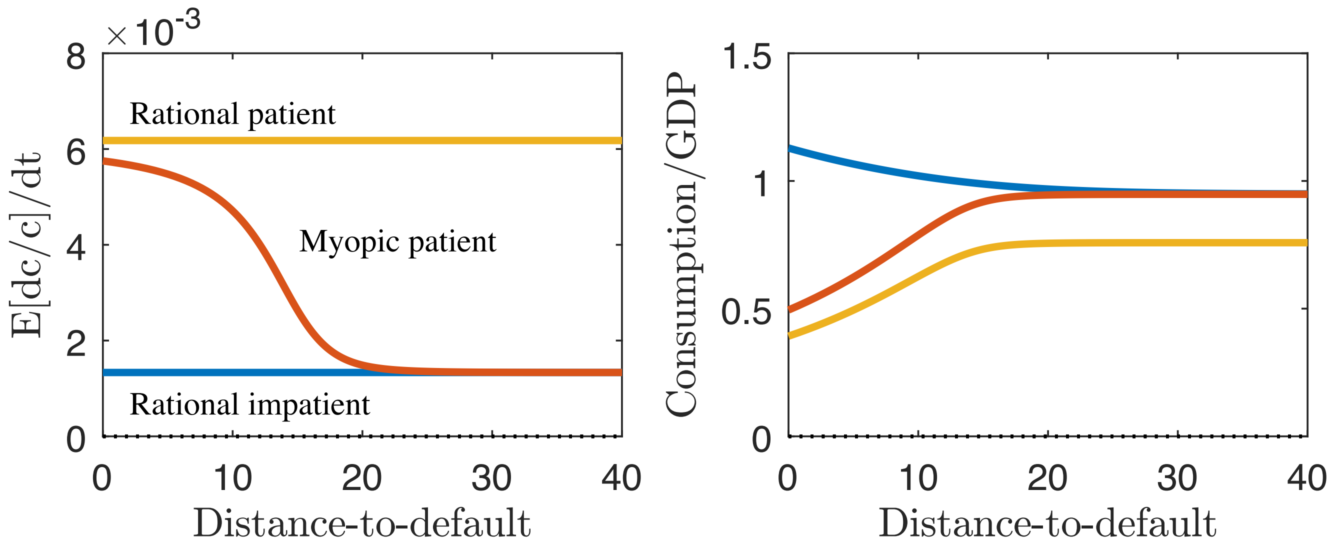

Figure 3: **Imperfect consumption smoothing.** The left figure plots expected consumption growth. The right figure plots consumption over GDP $= W(r+\theta(\mu-r))$. The red line assumes the borrower is myopic ($\delta = 0$). The yellow line corresponds to a patient rational borrower ($\delta = 1$) with same impatience parameter ρ as the myopic borrower. The blue line corresponds to an impatient rational borrower ($\delta = 1$) with higher impatience parameter ρ as the myopic borrower, calibrated so that the consumption-to-wealth ratios in normal times are equalized.

The myopic investment share equals

$$\theta^* = \frac{Y'(c^*)c^*}{Y(c^*)}\theta^\star = \left(\frac{c}{W} + \gamma h(\frac{c^-}{\underline{W}^-} - \psi)\frac{W^-}{W}\text{MDef}(c)\right)\frac{1}{\psi}\theta^\star, \tag{23}$$

where $\theta^\star = \frac{1}{\gamma}\frac{\nu}{\sigma}$ is Merton's fixed investment share. Total investment relative to consumption at time t can be decomposed as

$$\frac{\theta_t^* W_t}{c_t^*} = (\underbrace{\Psi'(c_t^*)}_{\text{Effect of default}} + \underbrace{G'(c_t^*)}_{\text{Effect of myopia (marginal shadow wealth)}}) \times \theta^\star. \tag{24}$$

The first term, Ψ', leads to risk shifting in crisis times (gambling for resurrection). The second term in expression (24) shows that myopia distorts investment further. Combined with expression (20) and the right panel of Figure 2, it becomes clear that myopia leads to undersaving at times when consumption is high and to oversaving relative to the rational benchmark at times when consumption is low, since $G'(c)$ is negative in good times (high c) and positive in bad times (low c).

2.3 Self-inflicted debt crises: Debt spirals and growth traps

A debt crisis is self-inflicted when overconsumption and underinvestment lead to a period over which the expected growth rate in wealth becomes negative, such that the country enters a death zone associated with abysmal growth in combination with rising debt.

We distinguish two types of debt crises. A debt spiral is a situation in which the threat of default does not discipline the borrower country. In normal times, expected wealth growth is positive. But when the country crosses a "point of no return," expected wealth growth turns negative. The country slips deeper into the crisis and, since default is not an effective threat, the country's finances quickly spiral out of control. A wealth trap is the reverse situation in which the threat of default does discipline the borrower country.

Wealth dynamics are given by

$$\frac{dW_t}{W_t} = \left(r - \frac{c_t}{Y(c_t)} + \frac{\nu^2}{\gamma} \frac{Y'(c_t)c_t}{Y(c_t)} \right) dt + \frac{\nu}{\gamma} \frac{Y'(c_t)c_t}{Y(c_t)} dw_t = \mu_t dt + \sigma_t dw_t, \tag{25}$$

with

$$\begin{aligned} \mu_t &= r - (1 - \tfrac{\nu^2}{\gamma}[\Psi'(c_t^*) + G'(c_t^*)])\psi_t^*, \\ \sigma_t &= \tfrac{\nu}{\gamma}[\Psi'(c_t^*) + G'(c_t^*)]\psi_t^*. \end{aligned} \tag{26}$$

Wealth exhibits countercyclical volatility and either pro- or countercyclical growth rate under myopia. Both the portfolio share θ^* and the consumption share ψ^* rise when W drops. Which of the two effects dominates the expected growth rate in wealth depends on myopia and other parameters.

The growth rate and volatility in wealth are $r - \psi + \frac{\nu^2}{\gamma}$ and, respectively, $\frac{\nu}{\gamma}$ for $W \to \infty$. The growth rate of wealth drops as the borrower approaches default if $\psi > (1 + \gamma h)\frac{\nu^2}{\gamma}$ and it reaches $r - \frac{c}{\underline{W}} + [1 + \frac{1}{\psi}(1 + \gamma h)(\frac{c}{\underline{W}} - \psi)]\frac{\nu^2}{\gamma}$. By contrast, the volatility rises up to $[1 + \frac{1}{\psi}(1 + \gamma h)(\frac{c}{\underline{W}} - \psi)]\frac{\nu}{\gamma}$ by the time of default. Proposition 2 provides sufficient conditions for the expected growth rate in wealth to be negative.

Proposition 2 *A debt spiral ($\mu(\underline{c}^-) < 0$) occurs if the borrower is su ciently myopic so that*

$$\frac{\underline{c}^-}{\underline{W}^-} > \psi + \frac{r + \frac{\nu^2}{\gamma} - \psi}{1 - \frac{\nu^2}{\gamma}\frac{1}{\psi}(1 + \gamma h)}. \tag{27}$$

There exists a threshold δ^* such that a self-in icted debt spiral occurs starting from any wealth level if $\delta < \delta^*$, i.e. the borrower is su ciently myopic.

3 Optimal Bailout Policy

This section derives the optimal incentive-compatible bailout policy that minimizes the intertemporal costs to the bailout agency given the borrower's incentive to plead for financial aid. Default can be hard or soft. In a hard default, the borrower country enters autarky. In a soft default, the country renegotiates its debt with the bailout agency. We start by characterizing the hard-default threshold $\underline{W}^a$ and the soft-default threshold $\underline{W}^-$. We then determine the equilibrium bailout scheme.

3.1 Hard and soft default

In a hard default, the country becomes autark and the bailout agency seizes assets so that wealth changes from W to $W^a(W) = \omega_0 + \omega_1 W$ with parameters ω_0, ω_1 determined by the enforcability of debt. Optimal consumption in autarky is a constant fraction of wealth, $c^a(W) = \psi^a W$, with autark consumption-wealth ratio

$$\psi^a = \frac{\rho - (1-\gamma)(\mu - \gamma\frac{\sigma^2}{2})}{\gamma - (1-\delta)}. \tag{28}$$

The borrower's value function in autarky equals

$$\Omega(W) = \frac{1}{\psi^a}\frac{c^a(W)^{1-\gamma}}{1-\gamma} = (\psi^a)^{-\gamma}\frac{W^a(W)^{1-\gamma}}{1-\gamma}. \tag{29}$$

Hard default occurs when wealth drops to the threshold $\underline{W}^a$ that satisfies the value-matching condition (6) and smooth-pasting condition (7). These conditions determine the hard-default threshold and the consumption policy. Appendix C shows the hard-default threshold depends on the ratio of the propensities to consume in autarky and good times, ψ^a/ψ. Myopia increases both propensities, yet in a symmetric fashion, so that their ratio remains independent of myopia.

Lemma 1 shows that myopia does not distort a myopic borrower's default decision in the absence of a bailout agency that provides optimal incentives. Any distortions in default decisions thus arise from the strategic interaction between borrower and bailout agency.

Lemma 1 *The hard-default threshold $\underline{W}^a$ is invariant to the myopia parameter δ, with*

$$\underline{W}^a = \frac{\left(\mu - \gamma\frac{\sigma^2}{2} - \frac{\nu^2}{2}\frac{\psi^a}{\psi}\left(\frac{1}{\gamma} + h\right) - \rho\frac{1-(\omega_1)^{\frac{1}{\gamma}-1}}{1-\gamma}\right)(\omega_1)^{-\frac{1}{\gamma}}\omega_0}{r - \frac{\nu^2}{2}h - \left(\mu - \gamma\frac{\sigma^2}{2} - \frac{\nu^2}{2}\frac{\psi^a}{\psi}\left(\frac{1}{\gamma} + h\right) - \rho\frac{1-(\omega_1)^{\frac{1}{\gamma}-1}}{1-\gamma}\right)(\omega_1)^{1-\frac{1}{\gamma}}}. \tag{30}$$

Consumption at the hard-default threshold equals $\underline{c}^a = (\omega_1)^{-\frac{1}{\gamma}}\psi^a(\omega_0 + \omega_1\underline{W}^a)$.

In a soft default, the borrower and the bailout agency negotiate over a financial transfer to keep the borrower from outright default. We model the interaction as a dynamic game in which the borrower acts as the Stackelberg leader and the bailout agency acts as the follower. The borrower pleads for financial aid at the soft-default threshold $\underline{W}^-$. The bailout agency responds by offering an incentive-compatible transfer that changes wealth to $\underline{W}^+ = \underline{W}^- + T(\underline{W}^-)$. The agency offers $T(\underline{W}^-)$ to keep the borrower afloat, which requires $M(\underline{W}^- + T(\underline{W}^-)) \geq \Omega(\underline{W}^- + T(\underline{W}^-))$ with $M(W)$ from (15) and $\Omega(W)$ from (29). The threshold $\underline{W}^-$ is a best response given the transfer policy T and the history of past transfers and default thresholds. In turn, the transfer policy is a best response to any default threshold W chosen by the borrower, given the history of past transfers and default thresholds. We analyze a stationary subgame-perfect Nash equilibrium in which $\underline{W}^-$ and $T(\underline{W}^-)$ are time invariant.

3.2 Bailout costs

To determine the optimal bailout policy, we need to value the agency's payment obligations given an incentive-compatible default threshold $\underline{W}^-$ and an equilibrium transfer schedule $T = T(\underline{W}^-)$.

The bailout agency minimizes the intertemporal costs, trading off infrequent large against frequent small transfers, or not bailing out at all. Expected bailout costs $I(c)$ are the present value of costs conditional on default:

$$I(c) = I(\underline{c}^-)\text{Def}(c). \tag{31}$$

Upon reaching the soft-default threshold, the agency's cost incorporate the current transfers, the cost of future bailouts, and the possibility of a hard default:

$$I(\underline{c}^-) = (1-p)(\underbrace{T}_{\text{Current transfer}} + \underbrace{I(\underline{c}^+)}_{\text{Future bailouts}}) + p(\underbrace{\kappa + W^a(\underline{W}^-) - \underline{W}^-}_{\text{Hard default}}). \tag{32}$$

Evaluating (31) at the post-transfer equilibrium consumption $\underline{c}^+ = c(\underline{W}^- + T)$ and substituting in (32) yields the annuity value of transfers and hard default costs. Solving forward, one obtains

$$I(c) = \frac{(1-p)T + p(\kappa + W^a(\underline{W}^-) - \underline{W}^-)}{1-(1-p)\text{Def}(\underline{c}^+)}\text{Def}(c). \tag{33}$$

Expression (33) is crucial for characterizing the optimal bailout policy.

3.3 Equilibrium bailout scheme

The optimal incentive-compatible bailout policy minimizes the intertemporal costs to the bailout agency in (31), given that the borrower follows a threshold strategy and asks for financial aid whenever wealth drops below the endogenous threshold $\underline{W}^-$ determined by (4) and (5). A hard default is optimal if no such threshold exists or the bailout costs exceed the cost of outright default.

We now characterize the optimal bailout, given the agency's and the borrower's objectives and sequence of moves. By the one-shot deviation principle (Fudenberg and Tirole, 1991), the optimal bailout is pinned down by deviations from the subgame-perfect Nash equilibrium in a single round of bailout negotiations. The bailout agency when deciding its transfer policy $T(W)$ takes into account that the borrower changes its default policy with the size of the offered bailout. The optimality condition for $T(W)$ equates the marginal benefit of the transfer to its marginal cost. As a result, the optimal bailout policy depends on borrower myopia.

Proposition 3 *The optimal bailout is determined by setting*

$$\underbrace{-\frac{d\mathit{Def}(\underline{c}^{+})}{dT}}_{\substack{\textit{Marginal decline in}\\ \textit{price of default}\\ (\searrow\textit{ under myopia})}} \times \underbrace{I(\underline{c}^{-})}_{\substack{\textit{Discounted cost}\\ \textit{of future defaults}\\ (\nearrow\textit{ under myopia})}} = \underbrace{1}_{\substack{\textit{Marginal cost}\\ \textit{of bailout}}} . \tag{34}$$

Condition (34) illustrates how the optimal bailout policy depends on myopia. The marginal benefit of the transfer is the reduction in the cost of future defaults through a reduction in the risk-neutral default probability (first term in expression (34)) times the discounted losses given default to the bailout agency (second term in expression (34)). The marginal cost of the bailout on the right-hand side of (34) is 1.

Equilibrium consumption dynamics (21) with subsistence level $\underline{c}^{-}$ and jump size $\underline{c}^{+} - \underline{c}^{-}$, and wealth dynamics (25) with soft-default threshold $\underline{W}^{-}$ and financial transfer $T(\underline{W}^{-}) = Y(\underline{c}^{+}) - \underline{W}^{-}$ are pinned down by the conditions

$$\underline{c}^{-} = (p\omega_1)^{-\frac{1}{\gamma}}\psi^{a}(\omega_0 + \omega_1\underline{W}^{-}), \tag{35}$$

$$N(\underline{c}^{-}) = (1-p)N(\underline{c}^{+}) + p\Omega(\underline{W}^{-}), \tag{36}$$

$$Y'(\underline{c}^{+}) = -\frac{\partial \mathrm{Def}(\underline{c}^{+})}{\partial \underline{c}^{+}} I(\underline{c}^{-}), \tag{37}$$

with the expressions for ψ^{a} from (28), $N(c)$ from (15), $\Omega(W)$ from (29), $Y(c)$ from (13), $\mathrm{Def}(c)$ from (16), and $I(c)$ from (33).

Figure 4 illustrates the determination of equilibrium bailouts. The impact of myopia on the optimal bailout depends on the sensitivity to wealth of the price of default and on the magnitude of bailout costs. Financial transfers are less effective under myopia than rationality at reducing the risk-neutral default probability. Therefore, the risk-neutral default probability is not only higher but

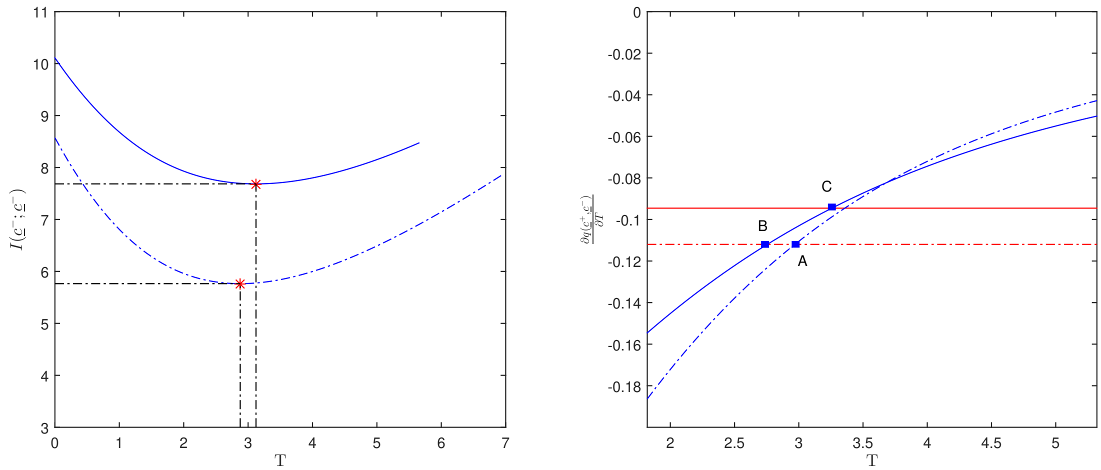

Figure 4: **Determination of equilibrium bailouts.** The figure shows the best response function for the government and the bailout agency. The dotted blue line traces the optimal soft default threshold $\underline{W}$ for different levels of transfers offered by the bailout agency. The solid red line traces the optimal transfer that the bailout agency offers, for different levels of the soft default threshold at which the government pleads for aid. The intersection of the best response curves indicates the equilibrium transfer and default threshold. The parameters used are $(\gamma, \rho, r, \omega_0, \omega_1, \delta, p, \mu, \sigma, \kappa) = (5, 0.02, 0.015, 1, 0.5, 0.5, 0.2, 0.025, 0.05, 2)$.

also less sensitive to wealth under myopia than rationality, and the cost of default is higher. These forces imply that optimal financial transfers can be higher or lower under myopia than rationality. In case financial transfers are less effective at staving off default and reducing the price of the default claim, the claim is less sensitive to wealth under myopia than rationality. As a result, optimal transfers are lower under myopia. On the other hand, the need for larger bailouts is higher when future bailouts arrive sooner. Transfers are therefore higher under myopia if default is sufficiently more costly.

[**Figure 4 About Here.**]

3.4 Tradeoff in optimal bailout

To illustrate the tradeoff inherent in the optimal bailout and its interaction with the borrower's incentive to default, we consider two counterfactuals. First, we fix the policy of the bailout agency and ask how does a borrower's default policy change with myopia. Second, we fix the default policy and ask how does the bailout policy change with myopia.

How does a borrower's default policy change with myopia, given the policy of the bailout agency? In deciding whether to default or not, the government balances the benefits of default (immediate increase in wealth) against its future costs (loss of access to the risk free asset and eligibility for future bailouts). Myopia diminishes the importance the government attaches to the future, either after a bailout or after a hard default. The following result shows that myopia leaves unaffected the default policy of the government, holding fixed the policy of the bailout agency.

Theorem 2 *(Myopia irrelevance) Fix a transfer T that is o ered by the bailout agency once the country reaches a soft-default threshold $\underline{W}^-$. Then $\underline{W}^-$ is invariant to the myopia parameter δ.*

How does the bailout policy change with myopia, given the default policy of the borrower? Holding fixed the default policy, the equilibrium bailout scheme has the following important property.

Proposition 4 *Given an equilibrium default threshold $\underline{W}^-$, an optimal bailout scheme has the property $T'(W) = -1$ for all W and independent of the myopia parameter δ.*

The result follows from the first-order condition, $\frac{\partial I(c(W);T)}{\partial T} = (1-p)(1 + I(\underline{c}^-)\frac{\partial \mathrm{Def}(c^+)}{\partial c}\frac{\partial c^+}{\partial T}) = 0$, that is independent of W. In a soft default, if the borrower receives a bailout, one-shot deviations thus have the property that $\underline{W}^+$ and $\underline{c}^+ = c(\underline{W}^+)$ are independent of when the borrower defaults. Different bailout schemes differ, as a result, in the intercept but not the slope of the bailout schedule.

4 Policy Analysis

4.1 Comparing bailouts: When does myopia get rewarded?

We now return to the question of how a bailout agency should adapt its policy depending on a country's myopia? Below we concentrate on bailout policy, leaving aside any structural policy aimed at strengthening the time-consistency of the government.

To gain intuition, consider first how a government changes its default boundary with the size of the offered bailout. The dotted blue line of Figure 4 traces the combinations of the bailout T

and the default threshold $\underline{W}^{-}$ that satisfy simultaneously the government's value matching and smooth pasting conditions. The higher the bailout, the earlier the government defaults. Intuitively, a higher bailout increases the value of the inside alternative, i.e. the value of non-default. For the value matching condition to continue to hold, the outside alternative needs to increase as well, which implies an increase in the default threshold.

Consider now the financial policy of the bailout agency. The solid red line of Figure 4 traces the optimal response of the bailout agency to a given default threshold chosen by the government. The equilibrium lies at the intersection of the best response functions of the government and the bailout agency. The result above implies that the best response function of the government does not change with myopia. Therefore, myopia can only produce equilibrium changes in the default threshold, transfers, and investment through the effect it induces in the policy of the bailout agency.

Whether myopic governments are treated more strictly or more leniently by bailout agencies ultimately depends on economic determinants that affect the growth rate of the economy, the costs of a hard default, and the risk of negotiation failure. To demonstrate the economic intuition, we show how transfers differ between myopic ($\delta = 0$) and rational governments ($\delta = 1$) as we vary the parameters of the model.

Lemma 2 *Myopia is punished (rewarded) when a myopic borrower receives a smaller (larger) debt relief and, correspondingly, defaults later (sooner) than a rational borrower. Punishment and reward depend on the following conditions:*

1. *Myopia is punished for low levels of political risk, and rewarded otherwise.*
2. *Myopia is punished when spillover costs are low, and rewarded otherwise.*
3. *Myopia is punished for low values of risk aversion, and rewarded otherwise.*
4. *Myopia is punished for low values of impatience, and rewarded otherwise.*
5. *Myopia is punished when the market price of risk is high, and rewarded otherwise.*
6. *Myopia is punished when default costs are low, and rewarded otherwise.*

[Figure 5 About Here.]

Figure 5 documents when myopia gets punished in equilibrium (red region) and when it gets rewarded (blue region). The white and grey region indicate no bailout in equilibrium. Myopia gets punished when the equilibrium transfer is smaller under myopia than under rationality, $T(\underline{W}^{-};\delta{=}0) > T(\underline{W}^{-};\delta{=}1)$. Myopia gets rewarded when the equilibrium transfer is larger under myopia than under rationality, $T(\underline{W}^{-};\delta{=}0) < T(\underline{W}^{-};\delta{=}1)$.

illustrates the parameter regions over which myopia gets punished and over which it gets rewarded. The white region indicates that myopia gets punished, that is, the equilibrium transfer is smaller under myopia than under rationality and, correspondingly, default occurs at a lower wealth threshold. The shaded region indicates that myopia gets rewarded, that is, the equilibrium transfer is larger under myopia than under rationality and, correspondingly, default occurs at a higher wealth threshold. The figures plot the parameter region over which the equilibrium transfer is larger under myopia than rationality. We start with the base case $(\gamma, \rho, r, \omega_0, \omega_1, p, \mu, \sigma, \kappa) = (5, 0.02, 0.015, 1, 0.5, 0.2, 0.025, 0.05, 10)$, and in each plot we vary two parameters. The top left figure varies spillover cost κ and renegotiation friction p. The top right figure varies impatience ρ and risk aversion γ. The bottom left figure varies volatility σ and growth rate μ. The bottom right figure varies the recovery parameters ω_0 and ω_1.

The plots in Figure 5 confirm Lemma 2. The equilibrium transfer increases under myopia relative the rational case when spillover cost κ, renegotiation frictions p, risk aversion γ, and volatility σ are higher, and when impatience ρ, growth rate μ, and the fixed recovery ω_0 are lower. The differential in equilibrium transfers is nonmonotonic in the recovery fraction ω_1. For small and large values of ω_1, myopia gets punished while it gets accommodated for intermediate values of ω_1.

4.2 Procrastinated default

Procrastinated default and protracted crisis episodes are a recurrent theme in sovereign debt crisis. Under what conditions does myopia lead to early or late default? When does myopia lead to procrastinated default?

Figure 6 documents when myopia leads to early default (red region) and when it leads to late default (blue region). The white and grey region indicate no bailout in equilibrium and, hence, a hard default occurs at W^a. Soft default occurs early under myopia when it occurs earlier than hard default ($\underline{W}^- > W^a$). Soft default occurs late under myopia when it occurs later than hard default ($\underline{W}^- < W^a$).

The figure provides a simple intuition. Larger bailouts incentivize earlier default. What is more important for policy makers, however, are the interaction between bailout packages and the incentive to default.

Figure 7 documents how the parameters affect whether myopia leads to procrastinated soft default (red region) or accelerated soft default (blue region). The white and grey region indicate no bailout in equilibrium and, hence, a hard default occurs at W^a. Default is procrastinated under myopia when it occurs later than rational default ($\underline{W}^-(\delta{=}0) > \underline{W}^-(\delta{=}1)$). Default is accelerated under myopia when it occurs earlier than rational default ($\underline{W}^-(\delta{=}0) < \underline{W}^-(\delta{=}1)$).

The figure supports the notion that default procrastination and borrower myopia are intricately related. Myopic borrowers tend to procrastinate default when myopia is punished. This occurs for low spillovers, low negotiation frictions, impatience, high growth, and low variable default cost.

[Figures 6 and 7 About Here.]

4.3 Is rational or myopic default cheaper to resolve?

Proposition 5 *The bailout costs upon default, $I(\underline{c}^-)$, monotonically fall with δ. The expected bailout costs $I(c(W))$ can fall or rise with δ.*

Expected bailout costs given W are $I(c(W)) = I(\underline{c}^-)\text{Def}(c(W))$. Bailout costs upon default are equal to $I(\underline{c}^-) = \frac{(1-p)T+p(\kappa+W^a(\underline{W}^-)-\underline{W}^-)}{1-(1-p)\text{Def}(\underline{c}^+)}$. Hence,

$$\frac{d}{d\delta}I(c(W)) = [\underbrace{\frac{\partial I(\underline{c}^-)}{\partial \delta}}_{<0} + \underbrace{I'(\underline{c}^-)\frac{\partial \underline{c}^-}{\partial \delta}}_{>0}]\text{Def} + I(\underline{c}^-)[\underbrace{\frac{\partial \text{Def}(c)}{\partial \delta}}_{>0} + \underbrace{\text{Def}'(c)\frac{\partial c(W)}{\partial \delta}}_{<0}]. \tag{38}$$

Expression (38) illustrates that competing forces determine if the expected bailout costs $I(c(W))$

fall or rise with δ. The first and last terms in the two brackets are negative, while the second and third terms are positive. Depending on the parameters, the negative or the positive force dominates.

[**Figure 8 About Here.**]

Figure 8 documents when myopia is more expensive to resolve than rational default and when it is cheaper to resolve. The red area indicates regions where myopic default is more expensive to resolve than rational default, such that $I(c(\underline{W}^-); \delta = 0) > I(c(\underline{W}^-); \delta = 1)$. The blue area indicates regions where myopic default is cheaper to resolve than rational default, such that $I(c(\underline{W}^-); \delta = 0) < I(c(\underline{W}^-); \delta = 1)$. The figure reveals an important intuition. Myopic default tends to be cheaper to resolve when myopia is punished, but sometimes also when myopia is rewarded.

4.4 Do myopic sovereigns harm or benefit a country?

Myopic sovereigns tend to harm a country through imperfect consumption smoothing. But can myopic sovereigns, in fact, benefit rational agents in the country? The size and frequency of bailouts depend on the myopia of the sovereign. The following result shows that the strategic impact on the bailout policy and the endogenous timing of soft default can dominate the consumption-savings distortions.

Proposition 6 *The value function $R(W)$ of a rational agent under a myopic sovereign rises or falls with δ. In particular, $\frac{d}{d\delta}R(\underline{W}^-; \delta = 1)$ Q 0.*

The value function of a rational agent under a myopic sovereign equals the value function $R(W)$ after the current self of the sovereign has been replaced. From Section 1.3, it is given by

$$R(W) = \frac{1}{\delta\rho}\left[(\gamma - (1-\delta))u(c^*) + u'(c^*)\left((r - \frac{\nu^2}{2}h)W + \frac{\nu^2}{2}\frac{c^*}{\psi}(\frac{1}{\gamma} + h)\right)\right]. \tag{39}$$

Its total derivative with respect to δ, $\frac{d}{d\delta}R(W) = \frac{\partial}{\partial\delta}R(W) + \frac{\partial}{\partial c^*}R(W)\frac{\partial}{\partial\delta}c(W)$, can be either positive or negative depending on the parameters. As a result, a rational agent may benefit from a myopic sovereign due to the bailout concessions and, more interestingly, the change in the endogenous timing of default.

5 Indebtedness and Credit Spread Dynamics

The dynamics of indebtedness: An explicit formula for the maximum indebtedness illustrates how borrower myopia affects debt dynamics. A myopic borrower both consumes and invests a disproportionately large fraction of wealth in crisis times. Risk aversion in wealth falls as the country approaches default, so that gambling for resurrection starts to dominate investment choices. Instead of funding investment by a reduction in consumption, the borrower uses excessive borrowing to fill the funding gap. As a result, debt issuance skyrockets in crisis times.

The funding gaps are financed by rising short-term debt $D(W)$ that amounts to

$$\begin{aligned} D(W) &= c^* - W(1-\theta^*) = c^* - Y(c^*) + Y'(c^*)c^*\frac{1}{\gamma}\frac{\nu}{\sigma} \\ &= (\psi - (1-\theta^\star))\frac{c_t}{\psi} + (1 + h\frac{\nu}{\sigma})(\frac{c_t}{\psi} - W_t) \\ &= (1 - \frac{1}{\psi}(1-\theta^\star))c^* + (1 + h\frac{\nu}{\sigma})(\frac{\underline{c}^-}{\psi} - \underline{W}^-)(\frac{c^*}{\underline{c}^-})^{-\gamma h}, \end{aligned} \quad (40)$$

where we have used that $(\frac{\underline{c}}{\psi} - \underline{W})(\frac{c_t}{\underline{c}^-})^{-\gamma h} = \frac{c_t}{\psi} - W_t$.

Expression (40) shows that myopia raises the borrower's indebtedness. Whether myopia generally raises or lowers debt issuance depends on the level of consumption and the magnitude of the rational investment share. So long as $\theta^\star < 1$, the first term in (40) shows that debt issuance is higher by more myopic borrowers (since ψ is higher). In addition, the second term in (40) rises when c^* falls as we get closer to default—raising debt issuance further.

The dynamics of the debt outstanding, as shown in the Appendix, are given by

$$\frac{dD_t}{D_t} = \left\{[(1+\gamma h)\phi\frac{c_t}{D_t} - \gamma h]E[\frac{dc_t/c_t}{dt}] - \frac{1}{2}(\frac{\nu}{\gamma})^2\gamma h(1+\gamma h)(\phi\frac{c_t}{D_t} - 1)\right\}dt + \left[(1+\gamma h)\phi\frac{c_t}{D_t} - \gamma h\right]\frac{\nu}{\gamma}dw_t, \quad (41)$$

with coefficient $\phi = \frac{\psi-(1-\theta^\star)}{\psi} < 1$.

Proposition 7 *The borrower's debt, as a multiple of consumption, peaks at*

$$1 + \frac{1}{\psi}(\frac{1}{\gamma} + h)\frac{\nu}{\sigma} - (1 + h\frac{\nu}{\sigma})\frac{\underline{W}^-}{\underline{c}^-}, \tag{42}$$

which is the level of debt, $D(\underline{W}^-)/\underline{c}^-$*, at which the borrower defaults. Maximum indebtedness increases in myopia so long as*

$$\frac{\partial \psi^{\max}}{\partial \delta} > \frac{1 + h\frac{\nu}{\sigma}}{\theta^\star + h\frac{\nu}{\sigma}} \left(\gamma - [r - \rho - (1 - \frac{1}{\gamma})\frac{\nu^2}{2}] \right). \tag{43}$$

Expression (42) shows that how myopia affects a borrower's maximum indebtedness, that is, ultimately its inclination to default, depends on the consumption-to-wealth ratio in good times, ψ, relative to the endogenously determined consumption-to-wealth ratio at default, $\psi^{\max} = \underline{c}^-/\underline{W}^-$.

The dynamics of credit spreads and credit risk metrics: The time-inconsistency of consumption and risk-taking policies under myopia renders standard market-based credit metrics unsuitable proxies for sovereign default risk. Tomz and Wright (2013) document that sovereign credit spreads are highly nonlinear and excessively sensitive to fundamentals. These features arise in our setting and are natural characteristics of sovereign credit risk dynamics.

In the standard Merton (1974) and Black and Cox (1976) settings, the price of a derivative claim has a power shape, exactly like MDef(c), such that on a log-log scale it increases linearly as the distance to default declines. The market price of a default-contingent derivative claim, Def(c), has, by contrast, a mixed power-hypergeometric shape in our setting:

$$\text{MDef}(c) = (\frac{c}{\underline{c}^-})^{-\gamma h}, \qquad \text{Def}(c) = (\frac{c}{\underline{c}^-})^{-\gamma i}\frac{H(c)}{H(\underline{c}^-)}. \tag{44}$$

The mixed power-hypergeometric function has a distinctive shape. For low default probability, it is less convex than the power function and plots concave in the distance to default on a log-log scale. Hence, the default claim is very sensitive to small changes in the likelihood of default for high distance to default. There exists a critical point, however, where the relation switches. For high default probability, the mixed power-hypergeometric function is more convex than the power function and plots convex in the distance to default on a log-log scale. This means the price of the derivative claim rises at an accelerating rate as the distance to default declines.

[**Figure 9 About Here.**]

Figure 9 illustrates the relation between default probability and the price of a default-contingent claim when the government is myopic. The left figure plots the price of a default-contingent claim, Def(c), as a function of the distance-to-default. The blue line assumes that the borrower is rational ($\delta = 1$). The red line assumes that the borrower is myopic ($\delta = 0$). The right figure plots the price of a default-contingent claim, Def(c), against the private valuation by a myopic borrower of a state-contingent claim that pays one unit when default occurs, MDef(c). The blue line assumes that the borrower is rational ($\delta = 1$). The red line assumes that the borrower is myopic ($\delta = 0$).

6 Conclusion

In the recent euro-crisis the ECB, the EFSF, ESM, and IMF were involved in the bailout of Ireland, Portugal, Spain, Greece, Cyprus, Latvia, Romania, and Hungary. The varying degree of duration, size, and success of these programs stirs policy debates regarding their appropriate design. Our paper analyzes one of the critical aspects of this debate: To what extent is bailing out myopic governments a waste of money? To what extent can countries use a myopic government as bargaining chip to extract higher financial transfers? The answer is that one 'size' does not fit all.

Our continuous-time model of optimal debt relief for borrowers with time-inconsistent preferences shows that the optimal policy for a bailout agency that lacks commitment is to get tougher the more myopic a government is when default spillovers are low; and vice versa to be more lenient towards myopic governments when default spillovers are high. Repeated rounds of financial aid and a protracted period of sovereign crisis are more likely outcomes when an indebted country can impose low spillover costs to the bailout agency in the event of default. The varying degree of leniency of bailout agencies towards myopia is a natural candidate to explain the heterogeneity in the speed at which governments seek financial aid and the response of CDS markets to negative economic shocks.

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A Derivations in Section 1

After plugging in optimal consumption c^* and investment θ^* that maximize (2),

$$\begin{aligned} \rho M(W) &= \delta u(c^*) + M'(W)[W(r+\theta^*(\mu-r)) - c^*] + \frac{\sigma^2}{2}W^2(\theta^*)^2 M''(W) \\ &= (\frac{\delta}{1-\gamma} - 1)M'(W)^{1-\frac{1}{\gamma}} + M'(W)(Wr - \frac{\nu^2}{2}\frac{M'(W)}{M''(W)}), \end{aligned} \quad \text{(A1)}$$

The ODE for $Y(c)$ obtains as follows. We have

$$\begin{aligned} &Y'(c^*(W))\tfrac{\partial c^*(W)}{\partial W} = 1, \\ &Y''(c^*(W))(\tfrac{\partial c^*(W)}{\partial W})^2 + Y'(c^*(W))\tfrac{\partial^2 c^*(W)}{(\partial W)^2} = 0, \end{aligned}$$

and, hence,

$$\tfrac{\partial c^*(W)}{\partial W} = \tfrac{1}{Y'(c^*(W))}, \qquad \tfrac{\partial^2 c^*(W)}{(\partial W)^2} = -\tfrac{Y''(c^*(W))}{Y'(c^*(W))^3}.$$

Differentiating both sides of (8) with respect to W yields

$$M''(W) = u''(c^*(W))\frac{\partial c^*(W)}{\partial W} = u''(c^*(W))\frac{1}{Y'(c^*(W))},$$

and

$$\theta^* = -\frac{u'(c^*)}{u''(c^*)}\frac{Y'(c^*)}{Y(c^*)}\frac{\nu}{\sigma}. \quad \text{(A2)}$$

Substituting in (A1),

$$\rho M(Y(c^*)) = (\frac{\delta}{1-\gamma} - 1)u'(c^*)^{1-\frac{1}{\gamma}} + u'(c^*)(Y(c^*)r - \frac{\nu^2}{2}\frac{u'(c^*)}{u''(c^*)}Y'(c^*)), \quad \text{(A3)}$$

and differentiating both sides yields

$$\begin{aligned} \rho M'(Y(c^*))Y'(c^*) &= (1-\frac{1}{\gamma})(\frac{\delta}{1-\gamma} - 1)u'(c^*)^{-\frac{1}{\gamma}}u''(c^*) + u''(c^*)(Y(c^*)r - \frac{\nu^2}{2}\frac{u'(c^*)}{u''(c^*)}Y'(c^*)) \\ &\quad + u'(c^*)(Y'(c^*)r - \frac{\nu^2}{2}\frac{u''(c^*)^2 - u'(c^*)u'''(c^*)}{u''(c^*)^2}Y'(c^*) - \frac{\nu^2}{2}\frac{u'(c^*)}{u''(c^*)}Y''(c^*)) \\ \Leftrightarrow rY(c^*) &= (\frac{1}{\gamma} - 1)(\frac{\delta}{1-\gamma} - 1)c^* - \frac{u'(c^*)}{u''(c^*)}(Y'(c^*)(r - \rho - \nu^2) + \frac{\nu^2}{2}\frac{u'(c^*)u'''(c^*)}{u''(c^*)^2}Y'(c^*) - \frac{\nu^2}{2}\frac{u'(c^*)}{u''(c^*)}Y''(c^*)) \\ &= (\frac{\delta}{1-\gamma} - 1)(\frac{1}{\gamma} - 1)c^* + [r - \rho + (\frac{1}{\gamma} - 1)\frac{\nu^2}{2}]\frac{1}{\gamma}c^*Y'(c^*) + \frac{\nu^2}{2}(\frac{1}{\gamma}c^*)^2 Y''(c^*), \end{aligned} \quad \text{(A4)}$$

where we have used

$$\begin{aligned}
u(c) &= \frac{1}{1-\gamma}c^{1-\gamma} = \frac{1}{1-\gamma}u'(c)c, \\
u'(c) &= c^{-\gamma}, \text{ so that } c = u'(c)^{-\frac{1}{\gamma}}, \\
u''(c) &= -\gamma c^{-(1+\gamma)}, \text{ so that } \frac{u'(c^*)}{u''(c^*)} = -\frac{1}{\gamma}c^*, \\
u'''(c) &= \gamma(1+\gamma)c^{-(2+\gamma)}.
\end{aligned}$$

The ODE for $M(W) = N(c^*(W))$ in (A1) yields the following ODE for $N(c)$:

$$\rho N(c) = (\frac{\delta}{1-\gamma} - 1)\left(\frac{N'(c)}{Y'(c)}\right)^{1-\frac{1}{\gamma}} + N'(c)\left(\frac{Y(c)}{Y'(c)}r - \frac{\nu^2}{2}\frac{Y'(c)N'(c)}{Y'(c)N''(c) - Y''(c)N'(c)}\right), \tag{A5}$$

since

$$\begin{aligned}
M'(W) &= N'(c^*(W))\frac{\partial c^*(W)}{\partial W}, \\
M'(W) &= \frac{N'(c^*(W))}{Y'(c^*(W))}, \\
M''(W) &= N''(c^*(W))(\frac{\partial c^*(W)}{\partial W})^2 + N'(c^*(W))\frac{\partial^2 c^*(W)}{(\partial W)^2} = \frac{Y'(c^*(W))N''(c^*(W)) - Y''(c^*(W))N'(c^*(W))}{Y'(c^*(W))^3}.
\end{aligned}$$

The value functions $N(c)$ and $M(W)$ can now be solved for. The ODE in (A5) yields

$$\begin{aligned}
N(c) &= \frac{1}{\rho}\left[(\gamma-(1-\delta))u(c) + u'(c)\left(rY(c) - \frac{\nu^2}{2}\frac{u'(c)}{u''(c)}Y'(c)\right)\right] \\
&= \frac{1}{\rho}\left[(\gamma-(1-\delta))u(c) + u'(c)\left(rY(c) + \frac{1}{\gamma}\frac{\nu^2}{2}Y'(c)c\right)\right] \\
&= \frac{1}{\rho}\left[(\gamma-(1-\delta))u(c) + u'(c)\left((r - \frac{\nu^2}{2}h)Y(c) + \frac{\nu^2}{2}\frac{c}{\psi}(\frac{1}{\gamma} + h)\right)\right],
\end{aligned}$$

since

$$\begin{aligned}
M'(W) &= N'(c^*(W))\frac{\partial c^*(W)}{\partial W} = N'(c^*)\frac{1}{Y'(c^*)} \\
M''(W) &= N''(c^*(W))(\frac{\partial c^*(W)}{\partial W})^2 + N'(c^*(W))\frac{\partial^2 c^*(W)}{(\partial W)^2} \\
&= \left[N''(c^*) - N'(c^*)\frac{Y''(c^*)}{Y'(c^*)}\right]\frac{1}{Y'(c^*)^2} = u''(c^*)\frac{1}{Y'(c^*)}
\end{aligned}$$

Hence,

$$
\begin{aligned}
M(W) &= \frac{1}{\rho}\left[(\gamma-(1-\delta))u(c^*(W)) + u'(c^*(W))(rW - \frac{\nu^2}{2}\frac{u'(c^*(W))}{u''(c^*(W))}Y'(c^*(W)))\right] \\
&= \frac{1}{\rho}\left[(\gamma-(1-\delta))u(c^*(W)) + u'(c^*(W))\left((r-\frac{\nu^2}{2}h)W + \frac{\nu^2}{2}\frac{c^*(W)}{\psi}(\frac{1}{\gamma}+h)\right)\right],
\end{aligned}
$$

where we have used

$$
\begin{aligned}
-\gamma h(Y(c)-\frac{c}{\psi}) &= \gamma h(\frac{\underline{c}^-}{\psi}-\underline{W}^-)(\frac{c}{\underline{c}^-})^{-\gamma h}, \\
(cY'(c)-\frac{c}{\psi}) &= \gamma h(\frac{\underline{c}^-}{\psi}-\underline{W}^-)(\frac{c}{\underline{c}^-})^{-\gamma h}, \\
cY'(c) &= \frac{c}{\psi}-\gamma h(Y(c)-\frac{c}{\psi}).
\end{aligned}
$$

B Derivations in Section 2

Consider how a rational borrower would value a claim on equilibrium consumption up until default, denoted by $\Psi(c)$. The no-arbitrage condition

$$
r\Psi(c) = c + \left(r-\rho-(1-\frac{1}{\gamma})\frac{\nu^2}{2}-\frac{1-\delta}{Y'(c)}\right)\frac{1}{\gamma}c\Psi'(c) + \frac{\nu^2}{2}(\frac{1}{\gamma}c)^2\Psi''(c), \tag{B6}
$$

with boundary conditions $\Psi(\underline{c}^-) = \underline{W}^-$ and $\lim_{c\to\infty}\Psi(c)/c = 1/\psi$ has two solutions. The solution that imposes $\Psi'(c) = Y'(c)$ for all c yields the subjective wealth equation (13). This can be seen from a comparison between (11) and (B6). The ODE (11) is a special case of (B6) when $\Psi'(c) = Y'(c)$. The general solution that does not impose this restrictive, myopic condition yields the market value of a consumption claim.

Wealth dynamics are given by

$$
\begin{aligned}
dW_t &= [W_t(r+\theta_t(\mu-r))-c_t]dt + \sigma W_t\theta_t dw_t \qquad &(B7)\\
&= [Y(c_t)(r-\frac{u'(c^*)}{u''(c^*)}\frac{Y'(c^*)}{Y(c^*)}\nu^2)-c_t]dt - Y(c_t)\frac{u'(c^*)}{u''(c^*)}\frac{Y'(c^*)}{Y(c^*)}\nu dw_t \qquad &(B8)\\
&= [rY(c_t)+\frac{\nu^2}{\gamma}Y'(c_t)c_t - c_t]dt + \frac{\nu}{\gamma}Y'(c_t)c_t dw_t \qquad &(B9)\\
&= [r(Y(c_t))-(1-\frac{\nu^2}{\gamma}Y'(c_t))c_t]dt + \frac{\nu}{\gamma}Y'(c_t)c_t dw_t \qquad &(B10)\\
\frac{dW_t}{W_t} &= \left(r-\frac{c_t}{Y(c_t)}+\frac{\nu^2}{\gamma}\frac{Y'(c_t)c_t}{Y(c_t)}\right)dt + \frac{\nu}{\gamma}\frac{Y'(c_t)c_t}{Y(c_t)}dw_t \qquad &(B11)
\end{aligned}
$$

since

$$\theta^* = -\frac{u'(c^*)}{u''(c^*)}\frac{Y'(c^*)}{Y(c^*)}\frac{\nu}{\sigma}.$$

and

$$\begin{aligned} u'(c) &= c^{-\gamma}, \\ u''(c) &= -\gamma c^{-(1+\gamma)}, \\ \frac{u'(c^*)}{u''(c^*)} &= -\frac{c^*}{\gamma}, \end{aligned}$$

and

$$\begin{aligned} Y(c) &= \frac{c}{\psi} - (\frac{\underline{c}^-}{\psi} - \underline{W}^-)(\frac{c}{\underline{c}^-})^{-\gamma h} \\ Y'(c)c &= \frac{c}{\psi} + \gamma h(\frac{\underline{c}^-}{\psi} - \underline{W}^-)(\frac{c}{\underline{c}^-})^{-\gamma h} \\ \frac{Y'(c)c}{Y(c)} &= \frac{\frac{c}{\psi} + \gamma h(\frac{\underline{c}^-}{\psi} - \underline{W}^-)(\frac{c}{\underline{c}^-})^{-\gamma h}}{\frac{c}{\psi} - (\frac{\underline{c}^-}{\psi} - \underline{W}^-)(\frac{c}{\underline{c}^-})^{-\gamma h}} \end{aligned}$$

Consumption dynamics are given by

$$\begin{aligned} dc_t &= dc^*(W_t) = \frac{\partial c^*(W)}{\partial W} dW_t + \frac{1}{2}\frac{\partial^2 c^*(W)}{(\partial W)^2} < dW_t > \\ &= \frac{1}{Y'(c_t)}\left[dW_t - \frac{1}{2}\frac{Y''(c_t)}{Y'(c_t)^2} < dW_t >\right] \\ &= \frac{1}{Y'(c_t)}\left[dW_t - \frac{\nu^2}{2}(\frac{1}{\gamma}c_t)^2 Y''(c_t)dt\right] \\ &= \frac{1}{Y'(c_t)}\left[\left(\left(r - \rho + (1+\frac{1}{\gamma})\frac{\nu^2}{2}\right)\frac{1}{\gamma}Y'(c_t)c_t - (1-\delta)\frac{1}{\gamma}c_t\right)dt + \frac{\nu}{\gamma}Y'(c_t)c_t dw_t\right] \qquad \text{(using the ODE for } Y(c)) \\ \frac{dc_t}{c_t} &= \left(r - \rho + (1+\frac{1}{\gamma})\frac{\nu^2}{2} - \frac{1-\delta}{Y'(c_t)}\right)\frac{1}{\gamma}dt + \frac{\nu}{\gamma}dw_t \end{aligned}$$

C Value of the government in autarky

After default, there exists no risk-free lending and savings technology. Then the value function $\Omega(W)$ satisfies

$$\rho\Omega(W) = \delta u(c^a) + \Omega'(W)(W\mu - c^a) + \frac{\sigma^2}{2}W^2\Omega''(W),$$

with $u'(c^a) = \Omega'(W)$. The ODE becomes

$$\rho\Omega(W) = (\gamma - (1-\delta))\frac{1}{1-\gamma}\Omega'(W)^{1-\frac{1}{\gamma}} + \Omega'(W)W\mu + \frac{\sigma^2}{2}W^2\Omega''(W),$$

which yields

$$\Omega(W) = (\psi^a)^{-\gamma}\frac{W^{1-\gamma}}{1-\gamma},$$

The autarky consumption-wealth ratio ψ^a is equal to

$$\psi^a = \frac{\rho - (1-\gamma)(\mu - \gamma\frac{\sigma^2}{2})}{\gamma - (1-\delta)}.$$

D Hard default policy

The smooth pasting condition (7) can be written as

$$u'(\underline{c}^{-a}) = \Omega'(\underline{W}^{-a}), \text{ or} \tag{D12}$$

$$\underline{c}^{-a} = (\omega_1)^{-\frac{1}{\gamma}}\psi^a(\omega_0 + \omega_1\underline{W}^{-a}) \tag{D13}$$

The left hand side of the value matching condition (6) rewrites as

$$M(\underline{W}^{-a}) = \frac{1}{\rho}\left[(\gamma-(1-\delta))u(\underline{c}^{-a}) + u'(\underline{c}^{-a})\left((r - \frac{\nu^2}{2}h)(\underline{W}^{-a}) + \frac{\nu^2}{2}\frac{\underline{c}^{-a}}{\psi}(\frac{1}{\gamma}+h)\right)\right] \tag{D14}$$

From (29) the right hand side of the same condition rewrites as

$$\Omega(\underline{W}^{-a}) = (\psi^a)^{-\gamma}\frac{(\omega_0 + \omega_1\underline{W}^{-a})^{1-\gamma}}{1-\gamma}$$

Substituting (D13) in (D14) and solving the value matching condition (6) for the hard default threshold we obtain

$$\underline{W}^{-a} = \frac{\left(\mu - \gamma\frac{\sigma^2}{2} - \frac{\nu^2}{2}\frac{\psi^a}{\psi}(\frac{1}{\gamma}+h) - \rho\frac{1-(\omega_1)^{\frac{1}{\gamma}-1}}{1-\gamma}\right)(\omega_1)^{-\frac{1}{\gamma}}\omega_0}{r - \frac{\nu^2}{2}h - \left(\mu - \gamma\frac{\sigma^2}{2} - \frac{\nu^2}{2}\frac{\psi^a}{\psi}(\frac{1}{\gamma}+h) - \rho\frac{1-(\omega_1)^{\frac{1}{\gamma}-1}}{1-\gamma}\right)(\omega_1)^{1-\frac{1}{\gamma}}}$$

which is independent of δ.

E Valuing default claim and cost of debt relief $I(c)$

The price $P(c)$ of any claim with payoff linked to consumption c, assuming complete markets, satisfies the no-arbitrage condition

$$rP(c) = \left(r - \rho - (1 - \frac{1}{\gamma})\frac{\nu^2}{2} - \frac{1-\delta}{Y'(c)}\right)\frac{1}{\gamma}cP'(c) + \frac{\nu^2}{2}(\frac{1}{\gamma}c)^2 P''(c). \tag{E15}$$

We conjecture that the general solution to equation (E15) is additively linear in compound functions of the form

$$P(c) = Ac^{-\gamma i}H(c), \tag{E16}$$

with constant A, coefficient i that satisfies the quadratic equation

$$\frac{\nu^2}{2}i^2 - \left(r - \rho - \frac{\nu^2}{2} - (1-\delta)\psi\right)i - r = 0, \tag{E17}$$

and $H(c)$ is a hypergeometric function.

We have

$$\begin{aligned} P'(c) &= -\gamma i A c^{-\gamma i}H(c)/c + Ac^{-\gamma i}H'(c), \\ P''(c) &= -\gamma i(-\gamma i - 1)Ac^{-\gamma i}H(c)/c^2 - 2\gamma i A c^{-\gamma i}H'(c)/c + Ac^{-\gamma i}H''(c). \end{aligned}$$

This yields

$$\begin{aligned} &rH(c) = \left(r - \rho - (1 - \tfrac{1}{\gamma})\tfrac{\nu^2}{2} - \tfrac{1-\delta}{Y'(c)}\right)\left(-iH(c) + \tfrac{1}{\gamma}H'(c)c\right) + \tfrac{\nu^2}{2}(\tfrac{1}{\gamma})^2[\gamma i(1+\gamma i)H(c) - 2\gamma i H'(c)c + H''(c)c^2], \\ &\Leftrightarrow \left[r + \left(r - \rho - (1+i)\tfrac{\nu^2}{2} - \tfrac{1-\delta}{Y'(c)}\right)i\right]H(c) = \left(r - \rho - (1 - \tfrac{1}{\gamma} + 2i)\tfrac{\nu^2}{2} - \tfrac{1-\delta}{Y'(c)}\right)\tfrac{1}{\gamma}cH'(c) + \tfrac{\nu^2}{2}(\tfrac{1}{\gamma}c)^2 H''(c). \end{aligned}$$

We make the change of variables

$$z = 1 - \psi Y'(c) = -\frac{\gamma h(\frac{\underline{c}^-}{\psi} - \underline{W}^-)(\frac{c}{\underline{c}^-})^{-\gamma h}}{\frac{c}{\psi}} < 0, \tag{E18}$$

and let

$$\begin{aligned} H(c) &= G(z), \\ H'(c) &= G'(z)z'(c), \\ H''(c) &= G''(z)z'(c)^2 + G'(z)z''(c), \end{aligned}$$

with

$$
\begin{aligned}
z(c) &= 1-\psi Y'(c) = -\psi B c^{-\gamma h}/c, \\
z'(c) &= -\psi Y''(c) = -\psi(-\gamma h-1)Bc^{-\gamma h}/c^2 = (-\gamma h-1)z/c, \\
z''(c) &= -\psi Y'''(c) = -\psi(-\gamma h-2)(-\gamma h-1)Bc^{-\gamma h}/c^3 = (-\gamma h-2)(-\gamma h-1)z/c^2,
\end{aligned}
$$

and $B=\gamma h(\frac{\underline{c}^-}{\psi}-\underline{W}^-)(\frac{1}{\underline{c}^-})^{-\gamma h}$.

We can transform the ODE for $G(z)$ into a hypergeometric differential equation, where condition (E17) helps to simplify, as follows:

$$
\begin{aligned}
&\left[r+\left(r-\rho-(1+i)\tfrac{\nu^2}{2}-\tfrac{(1-\delta)\psi}{1-z}\right)i\right]G(z)= \\
&-\left(r-\rho-(1+2i+h+\tfrac{1}{\gamma})\tfrac{\nu^2}{2}-\tfrac{(1-\delta)\psi}{1-z}\right)(h+\tfrac{1}{\gamma})G'(z)z+\tfrac{\nu^2}{2}(h+\tfrac{1}{\gamma})^2G''(z)z^2 \\
&\Leftrightarrow \left[\underbrace{\left\{r+\left(r-\rho-(1+i)\frac{\nu^2}{2}-(1-\delta)\psi\right)i\right\}}_{0}\tfrac{1}{z}-\underbrace{\left\{r+\left(r-\rho-(1+i)\frac{\nu^2}{2}\right)i\right\}}_{(1-\delta)\psi i}\right]G(z)= \\
&-\left[\left(r-\rho-(1+2i+h+\tfrac{1}{\gamma})\tfrac{\nu^2}{2}\right)(1-z)-(1-\delta)\psi\right](h+\tfrac{1}{\gamma})G'(z)+\tfrac{\nu^2}{2}(h+\tfrac{1}{\gamma})^2G''(z)z(1-z)\ \text{(by (E17))} \\
&\Leftrightarrow (1-\delta)\psi iG(z)-\left[r-\rho-(1+2i+h+\tfrac{1}{\gamma})\tfrac{\nu^2}{2}-(1-\delta)\psi-\left(r-\rho-(1+2i+h+\tfrac{1}{\gamma})\tfrac{\nu^2}{2}\right)z\right](h+\tfrac{1}{\gamma})G'(z) \\
&+\tfrac{\nu^2}{2}(h+\tfrac{1}{\gamma})^2G''(z)z(1-z)=0 \\
&\Leftrightarrow \frac{(1-\delta)\psi i}{\frac{\nu^2}{2}(h+\frac{1}{\gamma})^2}G(z)+\left[\frac{\frac{r}{i}+(i+h+\frac{1}{\gamma})\frac{\nu^2}{2}}{\frac{\nu^2}{2}(h+\frac{1}{\gamma})}+\frac{r-\rho-(1+2i+h+\frac{1}{\gamma})\frac{\nu^2}{2}}{\frac{\nu^2}{2}(h+\frac{1}{\gamma})}z\right]G'(z)+G''(z)z(1-z)=0.
\end{aligned}
$$

We obtain the hypergeometric differential equation for $G(z)$:

$$
-\alpha_1\alpha_2 G(z)+\left[\alpha_3-(\alpha_1+\alpha_2+1)z\right]G'(z)+G''(z)z(1-z)=0. \tag{E19}
$$

The differential equation for $G(z)$ is of hypergeometric form and has the solution $G(z)={}_2F_1(\alpha_1,\alpha_2;\alpha_3;z)=(1-z)^{-\alpha_1}\,{}_2F_1(\alpha_1,\alpha_3-\alpha_2;\alpha_3;\frac{z}{z-1})$. The second equality follows from the Pfaff transformation of the hypergeometric function and guarantees that $\frac{z}{z-1} \in [0,1]$. The coefficients α_1, α_2 are the roots to the characteristic equation

$$
\begin{aligned}
\alpha_3 &= \frac{\frac{r}{i} + (i + h + \frac{1}{\gamma})\frac{\nu^2}{2}}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})} \\
\alpha_1 \alpha_2 &= -\frac{(1-\delta)\psi i}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})^2} \\
\alpha_1 + \alpha_2 &= -\frac{r - \rho - (1+2i)\frac{\nu^2}{2}}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})} \\
0 &= \frac{\nu^2}{2} i^2 - \left(r - \rho - \frac{\nu^2}{2} - (1-\delta)\psi\right) i - r \\
\alpha_1 &= \frac{i - h}{h + \frac{1}{\gamma}}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_2 &= -\frac{(1-\delta)\psi i}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})^2 \alpha_1} \\
(\alpha_1)^2 + \frac{r - \rho - (1+2i)\frac{\nu^2}{2}}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})}\alpha_1 - \frac{(1-\delta)\psi i}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})^2} &= 0
\end{aligned}
$$

$$
\alpha_{1/2}^2 - \frac{\sqrt{\left(r - \rho - \frac{\nu^2}{2} - (1-\delta)\psi\right)^2 + 2r\nu^2} - (1-\delta)\psi}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})}\alpha_{1/2} - \frac{(1-\delta)\psi i}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})^2} = 0,
$$

since

$$
\begin{aligned}
\alpha_1 \alpha_2 &= -\frac{(1-\delta)\psi i}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})^2}, \\
\alpha_1 + \alpha_2 &= \frac{\sqrt{\left(r - \rho - \frac{\nu^2}{2} - (1-\delta)\psi\right)^2 + 2r\nu^2} - (1-\delta)\psi}{\frac{\nu^2}{2}(h + \frac{1}{\gamma})},
\end{aligned}
$$

and

$$
\begin{aligned}
\alpha_{1/2} &= \frac{\sqrt{\left(r-\rho-\frac{\nu^2}{2}-(1-\delta)\psi\right)^2+2r\nu^2}-(1-\delta)\psi \pm \sqrt{\left(\sqrt{\left(r-\rho-\frac{\nu^2}{2}-(1-\delta)\psi\right)^2+2r\nu^2}-(1-\delta)\psi\right)^2+2(1-\delta)\psi i\nu^2}}{2\frac{\nu^2}{2}(h+\frac{1}{\gamma})} \\
&= \frac{-(1-\delta)\psi+\sqrt{\left(r-\rho-\frac{\nu^2}{2}-(1-\delta)\psi\right)^2+2r\nu^2} \pm \sqrt{\left(r-\rho-\frac{\nu^2}{2}\right)^2+2r\nu^2}}{2\frac{\nu^2}{2}(h+\frac{1}{\gamma})} \\
&= \frac{i\nu^2-\left(r-\rho-\frac{\nu^2}{2} \mp \sqrt{\left(r-\rho-\frac{\nu^2}{2}\right)^2+2r\nu^2}\right)}{2\frac{\nu^2}{2}(h+\frac{1}{\gamma})} \\
&= \frac{i-h}{h+\frac{1}{\gamma}} \text{ or } = \frac{i-h}{h+\frac{1}{\gamma}}.
\end{aligned}
$$

The coefficients of the hypergeometric functions are

$$\begin{aligned}
\alpha_1 &= \tfrac{i-h}{h+\frac{1}{\gamma}} = \tfrac{1}{2\frac{\nu^2}{2}(h+\frac{1}{\gamma})}\left(\sqrt{\left(r-\rho-\tfrac{\nu^2}{2}-(1-\delta)\psi\right)^2+2r\nu^2}-(1-\delta)\psi-\sqrt{\left(r-\rho-\tfrac{\nu^2}{2}\right)^2+2r\nu^2}\right),\\
\alpha_2 &= \tfrac{i-h}{h+\frac{1}{\gamma}} = \tfrac{1}{2\frac{\nu^2}{2}(h+\frac{1}{\gamma})}\left(\sqrt{\left(r-\rho-\tfrac{\nu^2}{2}-(1-\delta)\psi\right)^2+2r\nu^2}-(1-\delta)\psi+\sqrt{\left(r-\rho-\tfrac{\nu^2}{2}\right)^2+2r\nu^2}\right),\\
\alpha_3 &= \tfrac{i-i}{h+\frac{1}{\gamma}}+1 = 1+\tfrac{1}{\frac{\nu^2}{2}(h+\frac{1}{\gamma})}\sqrt{\left(r-\rho-\tfrac{\nu^2}{2}-(1-\delta)\psi\right)^2+2r\nu^2},
\end{aligned} \tag{E20}$$

To obtain the constant A, we use that

$$\begin{aligned}
I(\underline{c}^-) &= A(\underline{c}^-)^{-\gamma i}H(\underline{c}^-),\\
I(\underline{c}^+) &= A(\underline{c}^+)^{-\gamma i}H(\underline{c}^+),
\end{aligned}$$

and

$$I(\underline{c}^-) = (1-p)(T(\underline{W}^-)+I(\underline{c}^+))+p(\kappa+W^a(\underline{W}^-)-\underline{W}^-).$$

Plugging in, we obtain

$$A = \frac{(1-p)T(\underline{W}^-)+p(\kappa+W^a(\underline{W}^-)-\underline{W}^-)}{(\underline{c}^-)^{-\gamma i}H(\underline{c}^-)-(1-p)(\underline{c}^+)^{-\gamma i}H(\underline{c}^+)}. \tag{E21}$$

Therefore,

$$I(\underline{c}^-) = \frac{(1-p)T(\underline{W}^-)+p(\kappa+W^a(\underline{W}^-)-\underline{W}^-)}{1-(1-p)(\frac{\underline{c}^+}{\underline{c}^-})^{-\gamma i}\frac{H(\underline{c}^+)}{H(\underline{c}^-)}}.$$

F Optimal Contract

First, any bailout offer $T>0$ that makes the government indifferent between defaulting and not, i.e. $M(W+T)=\Omega(W+T)$ is dominated by an offer $T+\varepsilon$ for some $\varepsilon>0$, so that the government does not ask for a new transfer immediately upon receiving T. Therefore the restriction $M(W+T)\geq\Omega(W+T)$ holds as a strict inequality and the associated Lagrange multiplier in the agency's cost minimization problem is zero.

Second, by the one-shot deviation principle (see Fudenberg Tirole 91), the optimum in the bailout agency's problem is pinned down by considering deviations from the Subgame Perfect Nash Equilibrium in a single round of bailout negotiations, reverting to the equilibrium afterwards.

In particular, assume that the government pleads for aid when its wealth reaches W, retaining threshold $\underline{W}$ for all future soft default events.

The bailout agency minimizes

$$I(c(W)) = (1-p)(T+I(c(W+T))+p(\kappa+W^a(W)-W)$$

Since in the future both the government and the agency will stick to the equilibrium,

$$I(c(W+T) = \text{Def}(c(W+T))I(\underline{c}^-)$$

Writng $c^+ = c(W+T)$, the first order condition of the agency reads as:

$$\frac{\partial I(c(W);T)}{\partial T} = (1-p)\left[1 + \frac{\partial \text{Def}(c^+)}{\partial c^+}\frac{\partial c^+}{\partial T} I(\underline{c}^-)\right] = 0, \text{ or}$$

$$1 + \frac{\partial \text{Def}(c^+)}{\partial c^+}\frac{1}{Y'(c^+)} I(\underline{c}^-) = 0 \tag{F22}$$

The above condition shows that the target consumption level c^+ and, hence, the target wealth level $W^+ = W+T$ are independent of W. In equilibrium, $c^+ = \underline{c}^+$ and $W^+ = \underline{W}^+$, so that the contract curve offered by the bailout agency has the property $T'(W) = -1$ for all W, and $\underline{c}^+$ and $\underline{W}^+ = Y(\underline{c}^+)$ are given implicitly by the solution to the algebraic equation (37).

From the SPC $M'(\underline{W}^-) = u'(\underline{c}^-) = p\Omega'(\underline{W}^-) = p\omega_1(\psi^a)^{-\gamma}(\omega_0 + \omega_1\underline{W}^-)^{-\gamma}$, we obtain the lower consumption threshold

$$\begin{aligned}\underline{c}^- &= (p\omega_1)^{-\frac{1}{\gamma}}\, c^a(\underline{W}^-) \\ &= (p\omega_1)^{-\frac{1}{\gamma}}\, \psi^a(\omega_0 + \omega_1\underline{W}^-),\end{aligned} \tag{F23}$$

or, equivalently, $\underline{W}^- = \frac{1}{\omega_1}((p\omega_1)^{\frac{1}{\gamma}}\frac{\underline{c}^-}{\psi^a} - \omega_0)$. Therefore whenever a positive bailout is offered by the agency, it satisfies the agency's optimality condition (37), the government's smooth pasting condition (35) and the value matching condition (36).

G Proof of theorem 2

Totally differentiate the value-matching condition (4) with respect to δ

$$\begin{aligned}&\frac{dM(\underline{W})}{d\delta} = (1-p)\frac{dM(\underline{W}+T)}{d\delta} + p\frac{d\Omega(\underline{W})}{d\delta}, \text{ or} \\ &\frac{\partial M(\underline{W})}{\partial\delta} + M'(\underline{W})\frac{d\underline{W}}{d\delta} = (1-p)\left[\frac{\partial M(\underline{W}+T)}{\partial\delta} + M'(\underline{W}+T)\frac{d\underline{W}}{d\delta}\right] + p\frac{d\Omega(\underline{W})}{d\delta}, \text{ or} \\ &\frac{1}{\rho}u(\underline{c}) + M'(\underline{W})\frac{d\underline{W}}{d\delta} = (1-p)\left[\frac{1}{\rho}u(\underline{c}^+) + M'(\underline{W}+T)\frac{d\underline{W}}{d\delta}\right] + p\left[\frac{\partial\Omega(\underline{W})}{d\delta} + \Omega'(\underline{W})\frac{d\underline{W}}{d\delta}\right]\end{aligned} \tag{G24}$$

Substituting in (G24) the smooth-pasting condition (5)

$$\frac{1}{\rho}u(\underline{c}) = (1-p)\frac{1}{\rho}u(\underline{c}^{+}) + p\frac{\partial\Omega(\underline{W})}{\partial\delta} \tag{G25}$$

From (F23) and (29)

$$(\gamma-(1-\delta))u(\underline{c}) = \gamma-(1-\delta)\psi^{\alpha}(p\omega_1)^{\frac{\gamma-1}{\gamma}}\Omega(\underline{W}) = (\gamma-(1-\delta))^{\gamma}\,\Lambda u(W^{\alpha}(\underline{W})) \tag{G26}$$

where $\Lambda = (\rho-(1-\gamma)(\mu-\gamma\frac{\sigma^2}{2}))^{1-\gamma}(p\omega_1)^{\frac{\gamma-1}{\gamma}}$.

Solving (G25) for $\underline{c}$

$$u'(\underline{c}) = ((1-\gamma)u(\underline{c}))^{-\frac{\gamma}{1-\gamma}} = (\gamma-(1-\delta))^{\gamma}\left((1-\gamma)\Lambda u(W^{\alpha}(\underline{W}))\right)^{-\frac{\gamma}{1-\gamma}} \tag{G27}$$

Substituting (G26) in (G25) and rearranging

$$\begin{aligned}
&(1-p)u(\underline{c}^{+}) = \psi^{\alpha}(p\omega_1)^{\frac{\gamma-1}{\gamma}}\Omega(\underline{W}) - p\rho\frac{\partial\Omega(\underline{W})}{d\delta} = \left[\psi^{\alpha}(p\omega_1)^{\frac{\gamma-1}{\gamma}} + p\rho\gamma\frac{1}{\psi^{\alpha}}\frac{\partial\psi^{\alpha}}{\partial\delta}\right]\Omega(\underline{W}) = \\
&\frac{1}{\gamma-(1-\delta)}\left[(\rho-(1-\gamma)(\mu-\gamma\frac{\sigma^2}{2}))(p\omega_1)^{\frac{\gamma-1}{\gamma}} - p\rho\gamma\right]\Omega(\underline{W}) \Rightarrow \\
&(1-p)u(\underline{c}^{+}) = \left(\frac{1}{\gamma-(1-\delta)}\right)^{1-\gamma} Ku(W^{\alpha}(\underline{W})) \Rightarrow \\
&(1-p)(\gamma-(1-\delta))u(\underline{c}^{+}) = (\gamma-(1-\delta))^{\gamma}\,Ku(W^{\alpha}(\underline{W}))
\end{aligned} \tag{G28}$$

where $K = \left[(\rho-(1-\gamma)(\mu-\gamma\frac{\sigma^2}{2}))(p\omega_1)^{\frac{\gamma-1}{\gamma}} - p\rho\gamma\right](\rho-(1-\gamma)(\mu-\gamma\frac{\sigma^2}{2}))^{-\gamma}$.

Solving (G28) for $\underline{c}^{+}$

$$u'(\underline{c}^{+}) = ((1-\gamma)u(\underline{c}^{+}))^{-\frac{\gamma}{1-\gamma}} = (\gamma-(1-\delta))^{\gamma}\left(\frac{1-\gamma}{1-p}Ku(W^{\alpha}(\underline{W}))\right)^{-\frac{\gamma}{1-\gamma}} \tag{G29}$$

From (F23) and (10)

$$\frac{\underline{c}}{\psi} = (p\omega_1)^{-1/\gamma}\frac{\psi^{\alpha}}{\psi}W^{\alpha}(\underline{W}) = (p\omega_1)^{-1/\gamma}\frac{\rho-(1-\gamma)(\mu-\gamma\frac{\sigma^2}{2})}{\rho-(1-\gamma)(r+\frac{1}{\gamma}\frac{\sigma^2}{2})}W^{\alpha}(\underline{W}) \tag{G30}$$

Solving (G28) for $\underline{c}^{+}$ and dividing by ψ:

$$\frac{\underline{c}^{+}}{\psi} = \frac{1}{\psi}\left(\frac{1}{\gamma-(1-\delta)}\right)\left(\frac{(1-\gamma)Ku(W^{\alpha}(\underline{W}))}{(1-p)}\right)^{1/(1-\gamma)} = \left(\rho-(1-\gamma)(r+\frac{1}{\gamma}\frac{\sigma^2}{2})\right)\left(\frac{(1-\gamma)Ku(W^{\alpha}(\underline{W}))}{(1-p)}\right)^{1/(1-\gamma)} \tag{G31}$$

Evaluating (**??**) at $W = \underline{W}$, substituting (G26) and (G27), and factoring out $(\gamma - (1-\delta))^\gamma$

$$\rho M(\underline{W}) = (\gamma - (1-\delta))^\gamma \left[\Lambda u(W^\alpha(\underline{W})) + ((1-\gamma)\Lambda u(W^\alpha(\underline{W})))^{-\frac{\gamma}{1-\gamma}} \left(\left(r - \frac{\nu^2}{2}h\right)\underline{W} + \frac{\nu^2}{2}\frac{\underline{c}}{\psi}\left(\frac{1}{\gamma} + h\right)\right)\right] \tag{G32}$$

Similarly, evaluating (**??**) at $W = \underline{W} + T$

$$\rho(1-p)M(\underline{W}+T) =$$
$$(\gamma - (1-\delta))^\gamma \left[Ku(W^\alpha(\underline{W}+T)) + (1-p)\left(\frac{1-\gamma}{1-p}Ku(W^\alpha(\underline{W}))\right)^{-\frac{\gamma}{1-\gamma}} \left(\left(r - \frac{\nu^2}{2}h\right)(\underline{W}+T) + \frac{\nu^2}{2}\frac{\underline{c}^+}{\psi}\left(\frac{1}{\gamma} + h\right)\right)\right] \tag{G33}$$

Rewrite the government's value in autarky as

$$\Omega(\underline{W}) = (\gamma - (1-\delta))^\gamma((\rho - (1-\gamma)(\mu - \gamma\frac{\sigma^2}{2})))^{-\gamma}u(W^\alpha(\underline{W})) \tag{G34}$$

Divide the value matching condition (4) by $(\gamma - (1-\delta))^\gamma$ and substitute (G26), (G27),(G28) and (G29)

$$\begin{aligned}
&\Lambda u(W^\alpha(\underline{W})) + ((1-\gamma)\Lambda u(W^\alpha(\underline{W})))^{-\frac{\gamma}{1-\gamma}} \left(\left(r - \frac{\nu^2}{2}h\right)\underline{W} + \frac{\nu^2}{2}\frac{\underline{c}}{\psi}\left(\frac{1}{\gamma} + h\right)\right) = \\
&Ku(W^\alpha(\underline{W}+T)) + (1-p)\left(\frac{1-\gamma}{1-p}Ku(W^\alpha(\underline{W}))\right)^{-\frac{\gamma}{1-\gamma}} \left(\left(r - \frac{\nu^2}{2}h\right)(\underline{W}+T) + \frac{\nu^2}{2}\frac{\underline{c}^+}{\psi}\left(\frac{1}{\gamma} + h\right)\right) + \\
&\rho((\rho - (1-\gamma)(\mu - \gamma\frac{\sigma^2}{2})))^{-\gamma}u(W^\alpha(\underline{W}))
\end{aligned} \tag{G35}$$

From (G30) and (G31) $\frac{\underline{c}}{\psi}$ and $\frac{\underline{c}^+}{\psi}$ are both invariant to δ. Therefore (G35)is an expression invariant to δ which shows that $\underline{W}$ does not change with δ given T.

H Dynamics of indebtedness

$$\begin{aligned}
dD_t &= \left[(\psi-(1-\theta^{**}))\frac{c_t}{\psi}-\gamma h(1+h\frac{\nu}{\sigma})(\frac{\underline{c}}{\psi}-\underline{W})(\frac{c_t}{\underline{c}})^{-\gamma h}\right]\frac{dc_t}{c_t}+\frac{1}{2}\gamma h(1+\gamma h)(\frac{\nu}{\gamma})^2(1+h\frac{\nu}{\sigma})(\frac{\underline{c}}{\psi}-\underline{W})(\frac{c_t}{\underline{c}})^{-\gamma h}dt \\
&= \left(\left[(\psi-(1-\theta^{**}))\frac{c_t}{\psi}-\gamma h(1+h\frac{\nu}{\sigma})(\frac{c_t}{\psi}-W_t)\right]E[\frac{dc_t}{c_t}/dt]+\frac{1}{2}\gamma h(1+\gamma h)(\frac{\nu}{\gamma})^2(1+h\frac{\nu}{\sigma})(\frac{c_t}{\psi}-W_t)\right)dt \\
&+ \left[(\psi-(1-\theta^{**}))\frac{c_t}{\psi}-\gamma h(1+h\frac{\nu}{\sigma})(\frac{c_t}{\psi}-W_t)\right]\frac{\nu}{\gamma}dw_t \\
&= \left(\left[\frac{c_t}{\phi}-\gamma h(D_t-\frac{c_t}{\phi})\right]E[\frac{dc_t}{c_t}/dt]+\frac{1}{2}\gamma h(1+\gamma h)(\frac{\nu}{\gamma})^2(D_t-\frac{c_t}{\phi})\right)dt+\left[\frac{c_t}{\phi}-\gamma h(D_t-\frac{c_t}{\phi})\right]\frac{\nu}{\gamma}dw_t \\
&= \{[(1+\gamma h)\frac{c_t}{\phi}-\gamma hD_t]E[\frac{dc_t/c_t}{dt}]+\frac{1}{2}\gamma h(1+\gamma h)(\frac{\nu}{\gamma})^2(D_t-\frac{c_t}{\phi})\}dt+[(1+\gamma h)\frac{c_t}{\phi}-\gamma hD_t]\frac{\nu}{\gamma}dw_t
\end{aligned}$$

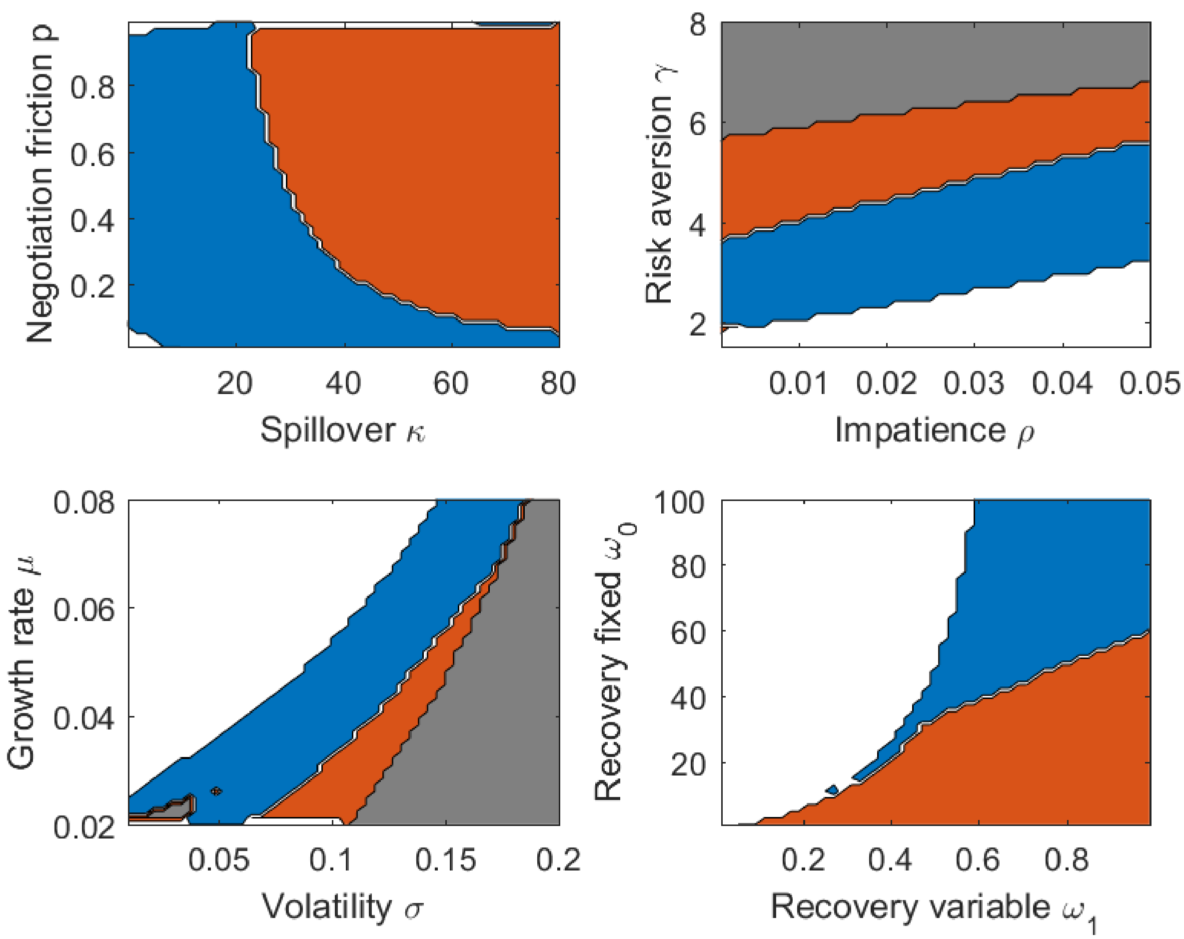

Figure 5: **Is myopia punished or rewarded?** The figure documents when myopia gets punished in equilibrium (red region) and when it gets rewarded (blue region). The white and grey region indicate no bailout in equilibrium. Myopia gets punished when the equilibrium transfer is smaller under myopia than under rationality, $T(\underline{W}^{-};\delta{=}0) > T(\underline{W}^{-};\delta{=}1)$. Myopia gets rewarded when the equilibrium transfer is larger under myopia than under rationality, $T(\underline{W}^{-};\delta{=}0) < T(\underline{W}^{-};\delta{=}1)$. We start with the base case $(\gamma, \rho, r, \omega_0, \omega_1, p, \mu, \sigma, \kappa) = (5, 0.02, 0.015, 1, 0.5, 0.2, 0.025, 0.05, 10)$, and in each plot we vary two parameters. The top left figure varies spillover cost κ and renegotiation friction p. The top right figure varies impatience ρ and risk aversion γ. The bottom left figure varies volatility σ and growth rate μ. The bottom right figure varies the recovery parameters ω_0 and ω_1.

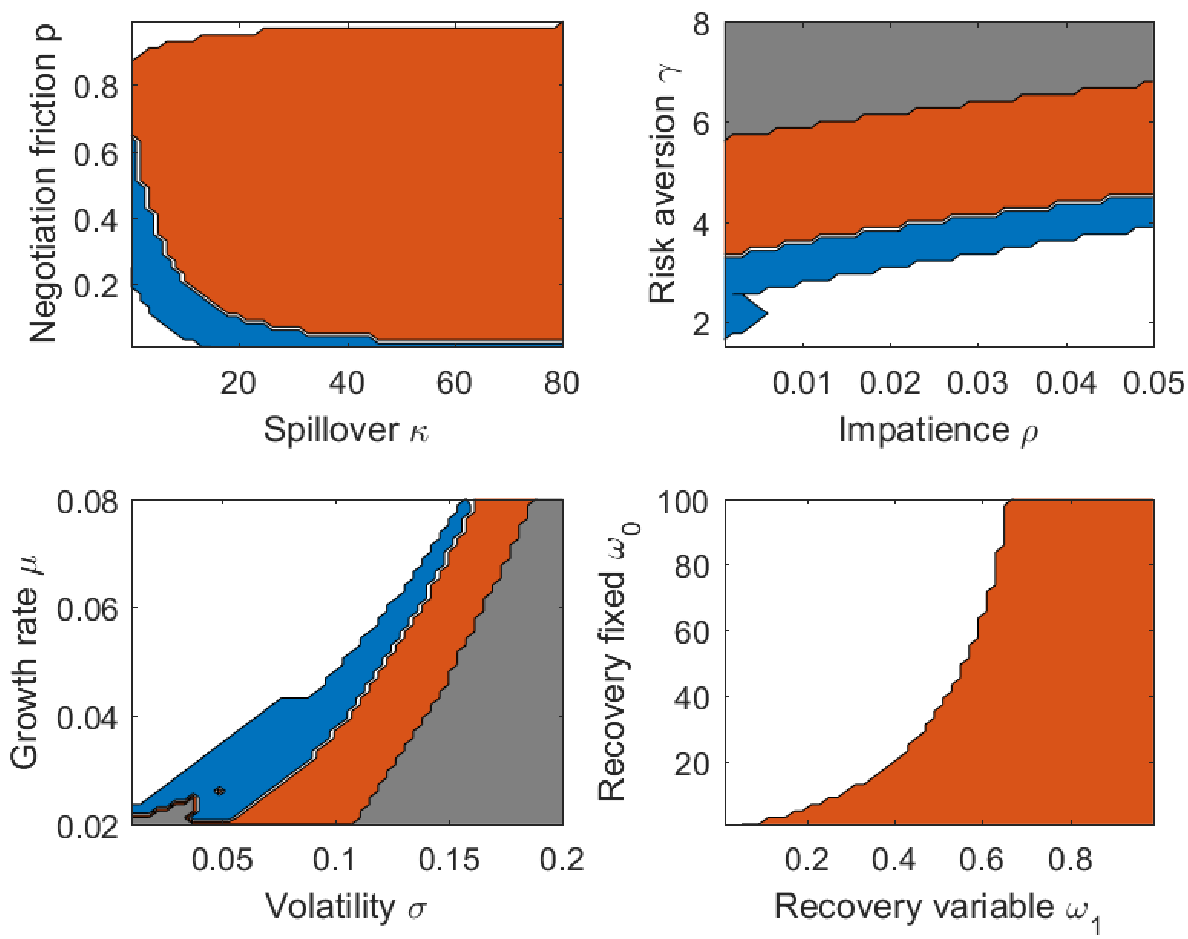

Figure 6: **Does myopia lead to early or late default?** The figure documents when myopia leads to early default (red region) and when it leads to late default (blue region). The white and grey region indicate no bailout in equilibrium and, hence, a hard default occurs at W^a. Soft default occurs early under myopia when it occurs earlier than hard default ($\underline{W}^- > W^a$). Soft default occurs late under myopia when it occurs later than hard default ($\underline{W}^- < W^a$). We start with the base case $(\gamma, \rho, r, \omega_0, \omega_1, p, \mu, \sigma, \kappa) = (5, 0.02, 0.015, 1, 0.5, 0.2, 0.025, 0.05, 10)$, and in each plot we vary two parameters. The top left figure varies spillover cost κ and renegotiation friction p. The top right figure varies impatience ρ and risk aversion γ. The bottom left figure varies volatility σ and growth rate μ. The bottom right figure varies the recovery parameters ω_0 and ω_1.

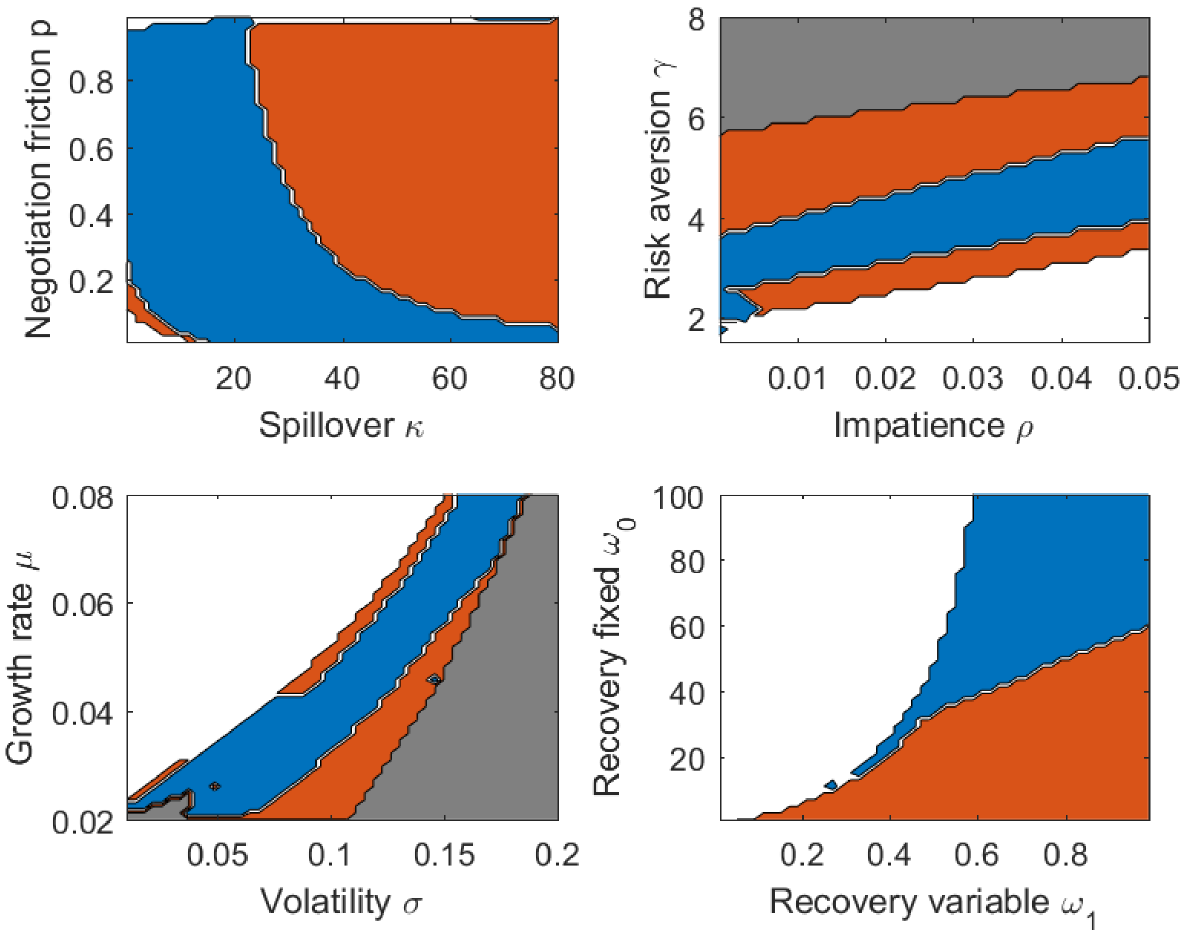

Figure 7: **When does myopia lead to procrastinated default?** The figure documents when myopia leads to procrastinated soft default (red region) and when it leads to accelerated soft default (blue region). The white and grey region indicate no bailout in equilibrium and, hence, a hard default occurs at W^a. Default is procrastinated under myopia when it occurs later than rational default ($\underline{W}^-(\delta{=}0) > \underline{W}^-(\delta{=}1)$). Default is accelerated under myopia when it occurs earlier than rational default ($\underline{W}^-(\delta{=}0) < \underline{W}^-(\delta{=}1)$). We start with the base case $(\gamma, \rho, r, \omega_0, \omega_1, p, \mu, \sigma, \kappa) = (5, 0.02, 0.015, 1, 0.5, 0.2, 0.025, 0.05, 10)$, and in each plot we vary two parameters. The top left figure varies spillover cost κ and renegotiation friction p. The top right figure varies impatience ρ and risk aversion γ. The bottom left figure varies volatility σ and growth rate μ. The bottom right figure varies the recovery parameters ω_0 and ω_1.

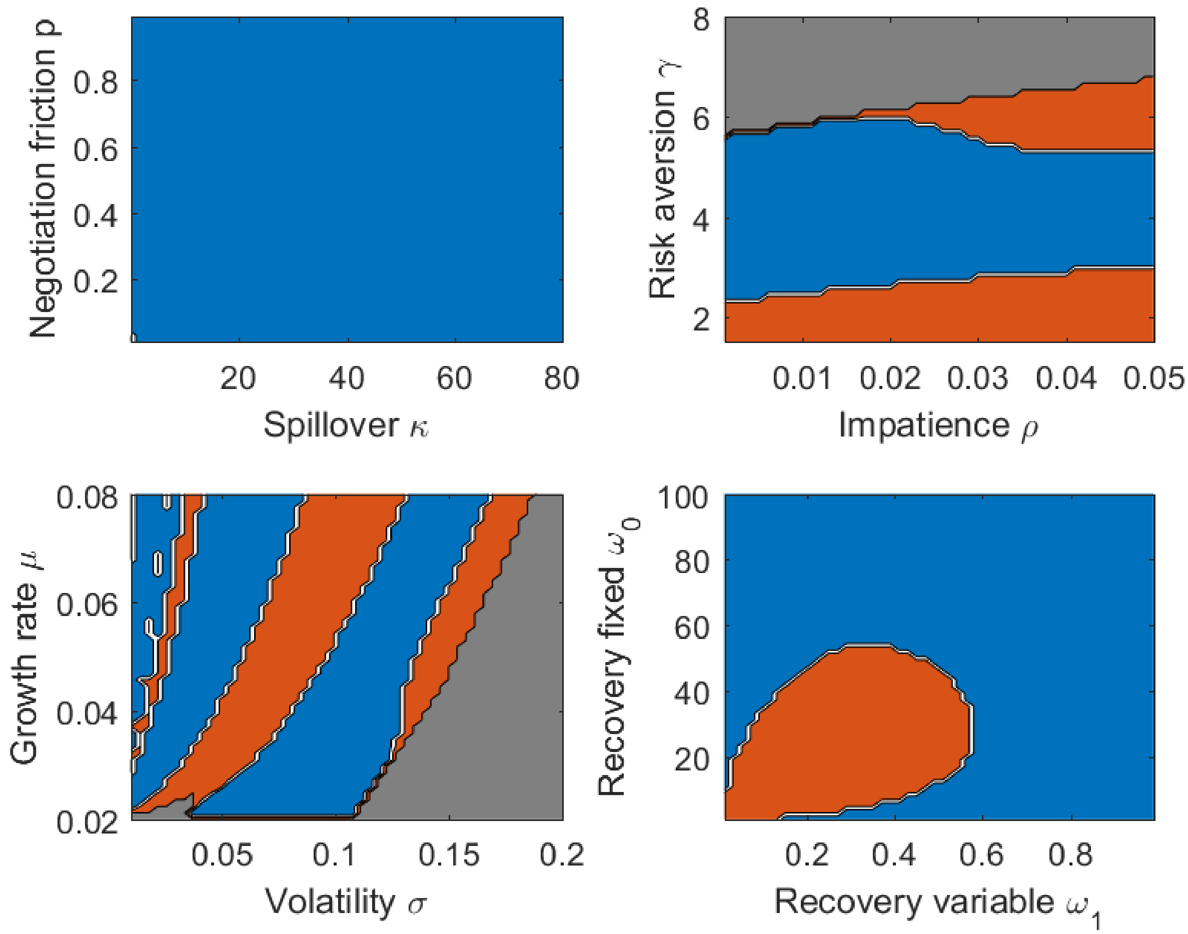

Figure 8: **Is rational or myopic default cheaper to resolve?** The figure documents when myopia is more expensive to resolve than rational default (red region) and when it is cheaper to resolve (blue region). Myopia is more expensive to resolve than rational default when $I(c(\underline{W}^{-}); \delta = 0) > I(c(\underline{W}^{-}); \delta = 1)$. Myopia is cheaper to resolve than rational default when $I(c(\underline{W}^{-}); \delta = 0) < I(c(\underline{W}^{-}); \delta = 1)$. We start with the base case $(\gamma, \rho, r, \omega_0, \omega_1, p, \mu, \sigma, \kappa) = (5, 0.02, 0.015, 1, 0.5, 0.2, 0.025, 0.05, 10)$, and in each plot we vary two parameters. The top left figure varies spillover cost κ and renegotiation friction p. The top right figure varies impatience ρ and risk aversion γ. The bottom left figure varies volatility σ and growth rate μ. The bottom right figure varies the recovery parameters ω_0 and ω_1.

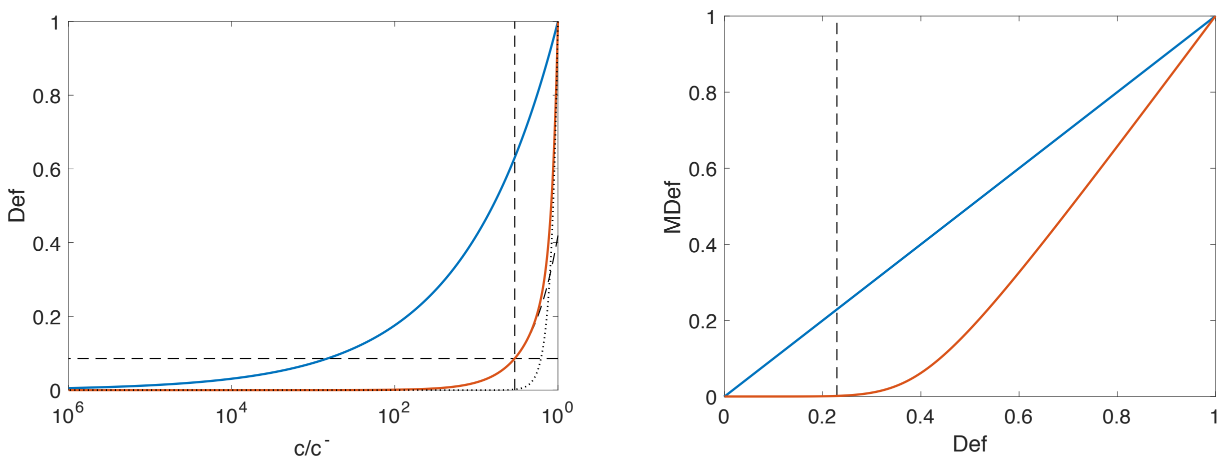

Figure 9: **Credit spread dynamics and credit risk metrics.** The left figure shows the price of a default-contingent claim, Def(c), as a function of the distance-to-default. The blue line assumes that the borrower is rational ($\delta = 1$). The red line assumes that the borrower is myopic ($\delta = 0$). The right figure plots the price of a default-contingent claim, Def(c), against the private valuation by a myopic borrower of a state-contingent claim that pays one unit when default occurs, MDef(c). The blue line assumes that the borrower is rational ($\delta = 1$). The red line assumes that the borrower is myopic ($\delta = 0$).

Internet Appendix

Figures IA.1 to IA.8 illustrate the economic intuition using partial dependence plots. For each parameter, we show how the cost of future bailout negotiations, $I(\underline{c}^-)$, the value of the default claim, $\mathrm{Def}(\underline{c}^+)$, and the sensitivity of the default claim to transfers, $-\frac{\partial \mathrm{Def}(\underline{c}^+)}{\partial T}$, vary while $\underline{c}^-, \underline{W}^-, T$ are kept constant across parameters. Plots a-c examine the direct effect of parameter changes on the determinants of the agency's first-order condition, keeping the incentives for the government to default constant. Plots d-f examine the government's side: The consumption policy at a fixed default boundary $\underline{W}^-$, the chosen soft default boundary, and the transfers received by the government.

Political risk. Negotiation failure for non-economic reasons, captured by p, reflects political risk. The value of the default claim and its sensitivity to transfers are invariant to political risk, for fixed transfers and default policy (Figures IA.1b and IA.1c). Hard default is more expensive than sequential bailouts, which means that the higher the cost of a future negotiation failure, the higher the perceived future cost $I(\underline{c}^-)$ for the agency, holding transfers fixed (Figure IA.1a). This induces the agency to offer higher transfers today, in the hope of delaying future bailouts. Consumption at the default boundary decreases in political risk as a beneficial deal with the agency becomes less likely. This in turn diminishes the value of a future default claim, the cost of future bailouts and the incentives for the agency to offer debt relief. Figure IA.1e shows that in general the direct effect of political risk on future bailout costs dominates the effect it has on consumption, and transfers increase in the likelihood of negotiation failure.

Political risk affects the soft-default boundary through three channels: First, it induces lower consumption and a lower soft default threshold (see the SPC condition (35)). Second, it induces higher transfers and a lower soft default threshold, holding consumption at the soft default boundary fixed (see the VMC condition (36)). Third, it reduces the expected gains from averting hard default $N(\underline{c}^+) - \Omega(\underline{W})$ which induces a higher soft default threshold (see the the VMC (36)). Figure IA.1f shows that the first two channels dominate when political risk is low, whereas the third, direct channel dominates for high values of political risk.

Myopia has a direct effect on the value of the default claim and the cost of future bailouts, holding consumption policy fixed (Figure IA.1a-IA.1c). Its effect manifests primarily through the increase in the consumption rate it induces (Figure IA.1d).

This is the optimal outcome of the following tradeoff. First, myopia reduces the sensitivity of the default claim on transfers (Figure IA.1c), which discourages transfers and induces a tougher stance of the agency towards myopia. Second, it increases consumption, the value of the default claim and the cost of future bailouts, which compels the agency to offer more help to myopic governments. The behavior of the government at the default boundary is more important when there is higher risk of negotiation failure, and therefore myopia ends up rewarded when there is high political risk.

Spillover costs. Spillover costs are an externality that the government imposes on the bailout agency. As such, it does not affect directly the consumption policy, the value of the default claim or its sensitivity to transfers. Instead, it changes the incentives for the bailout agency to offer help, given the policy of the government. In particular, higher spillovers increase the cost of hard default, and induce higher transfers, as the agency is interested in delaying future bailout negotiations, which induce political risk and a hard default possibility. A reinforcing mechanism operates at this stage: The bailout agency understands that the same economic logic that applies in the current round of bailout negotiations, will apply in the next round as well, and will induce higher future transfers. This further increases the cost of a future bailout negotiation and further increases the incentives to provide debt relief today.

Lemma 2 shows that myopia is punished when spillover costs are low and rewarded otherwise. On the one hand, myopia reduces the sensitivity of the default claim (Figure IA.2c) on transfers and discourages debt relief. On the other hand, myopia encourages consumption, increases the value of the default claim and shortens the arrival time of future defaults. The higher the spillover costs, the more important the second factor becomes, while the first factor stays unaffected.

Because spillover costs induce higher transfers, they also encourage earlier default (Figure IA.2f). When spillover costs are low, transfers are lower for myopic governments and they default later versus rational governments. When spillover costs are high, transfers are more generous towards myopic governments and induce earlier default.[3]

Borrower risk aversion. Risk aversion induces prudence, discourages consumption, reduces the value of the default claim and improves the sensitivity of the default claim on transfers (Figure IA.3). Because the value of the default claim reduces, the agency has less of an incentive to offer debt relief as risk aversion increases. And because the sensitivity of the default claim to transfers increases, the agency is induced to offer more relief. The former effect dominates for high values of risk aversion and the latter dominates for low values of risk aversion (Figure IA.3e). Indeed, for sufficiently low levels of risk aversion the agency may not offer any help at all. The soft default threshold reflects the U-shaped pattern of the transfers, as higher transfers induce earlier default.

Lemma 2 and Figure IA.3e show that myopia is punished for low values of risk aversion and rewarded otherwise. As previously mentioned, the incentive for the agency to punish myopia is that the myopic government pours funds into over-consumption as the bail-out agency pours funds in. This type of behavior reduces the sensitivity of the default claim to transfers. The incentive for the agency to accommodate myopia is that the over-consumption it induces increases the value of the default claim and the cost of future bailouts. Figure IA.3e shows that the former channel is dominant for low values of risk aversion and the latter dominates for high values of risk aversion.

[3]The kinks in the left part of Figure IA.2f) reflect spillover thresholds for which no bailout is offered by the agency.

Rational borrower impatience. In the model, the parameter ρ reflects the rational degree of the government's impatience. The distance of δ to unity instead, reflects the impatience induced by myopia, which in turns reflects either behavioral traits or political preference for immediate consumption. Fixing the default policy $(\underline{c}^-, \underline{W}^-)$, a higher degree of rational impatience reduces consumption growth, which increases the value of the default claim, and increases current consumption, which reduces the value of the default claim. Figure IA.4b shows that the default claim is U-shaped in rational impatience. In addition, just as time-inconsistent impatience (i.e. myopia) reduces the sensitivity of the default claim to transfers, so does rational impatience (Figure IA.4c).

For low values of rational impatience, the default claim and its sensitivity to transfers are high, which induces high transfers, but consumption is low which diminishes the default claim and reduces transfers. For high values of rational myopia, the sensitivity of the default claim is low but consumption and the value of the default claim are high. Figure (IA.4e) shows that transfers are inverse U-shaped in rational impatience. Furthermore, myopia is punished if the government is patient and rewarded otherwise. The main reason behind the latter effect is the increase in consumption induced in myopia which increases the value of the default claim and the cost of future bailouts. Given the more generous terms that patient but myopic governments receive, they tend to plead for aid earlier compared to rational ones (Figure IA.4f).

Investment opportunities. As the mean return of the risky asset of the government increases, consumption growth improves but the market price of risk $\nu = \frac{\mu - r}{\sigma}$ also increases. The risk effect dominates and the value of the default claim increases in the mean asset return (Figure IA.5b) [4]. At the same time, higher asset returns encourage consumption today (Figure IA.5d) which further increases the value of the default claim and the cost of future bailouts $I(\underline{c}^-)$. Both of these factors induce the agency to offer higher debt relief as mean asset returns become higher. However, for high levels of μ the default claim becomes less responsive to transfers. Intuitively, consumption becomes more strongly dependent on future growth prospects rather than on the current wealth. Since transfers become less effective in altering the value of the default claim, the incentives for the agency to offer debt relief decline (Figure IA.5e).

In a similar fashion, higher asset volatility decreases the market price of risk, decreases the value of the default claim and the level of consumption (Figures IA.6b and IA.6d), but increases the sensitivity of the default claim to transfers. Both an increase in mean asset returns and an increase in the volatility of returns produce an inverse U-shape pattern in transfers. Myopia increases consumption, increases the value of the default claim and reduces its sensitivity to transfers.

Figures IA.6e and IA.6f produce the key result of this section: Myopia is punished when the market price of risk is high and accommodated otherwise. Myopic governments default early when the market price of risk is low and

[4] To see this directly notice that under rationality, $Def(\underline{c}^+) = \left(\frac{\underline{c}^+}{\underline{c}^-}\right)^{-\gamma h}$ where h is decreasing in ν

default late otherwise.

Default costs and benefits. The default benefits ω_0 do not directly affect the default claim, but only implicitly through their effect on the consumption policy. The SPC condition 35 shows that consumption increases in post-hard-default wealth. Therefore ω_0 positively affects the value of the default claim, the cost of future bailouts, and the debt relief offered by the agency. However, at higher values of boundary consumption $\underline{c}^-$ the sensitivity of the default claim to transfers decreases. This is particularly so if the government is myopic. Figure IA.7e shows that in case of myopia, transfers begin to decline after a certain level of bankruptcy benefits.[5]

Overall myopic governments are treated more leniently versus rational governments if default benefits are low and more strictly if default benefits are high. In the former case myopic governments default sooner and in the latter they delay default.

The marginal cost of bankruptcy on wealth, $1 - \omega_1$, operates in a different fashion. First, it affects consumption policy in a non-linear way, since it negatively affects post-hard-default wealth but positively affects the consumption-to-wealth ratio at the default boundary (see the SPC condition 35). This produces a U-shaped boundary consumption in pre-bankruptcy wealth (see Figure IA.8 d). This non-linearity carries over to the transfer policy, as higher consumption at the default boundary increases the value of the default claim and induces, as before, higher transfers today. The difference between myopic and rational governments is that the former tend to be treated more leniently when the marginal cost of bankruptcy is high and more strictly otherwise. Hence, myopia is punished when default costs are low and accommodated when default costs are high.

[5] The kink in the right part of the plot reflects a threshold value of ω_0 beyond which no relief is offered by the bailout agency.

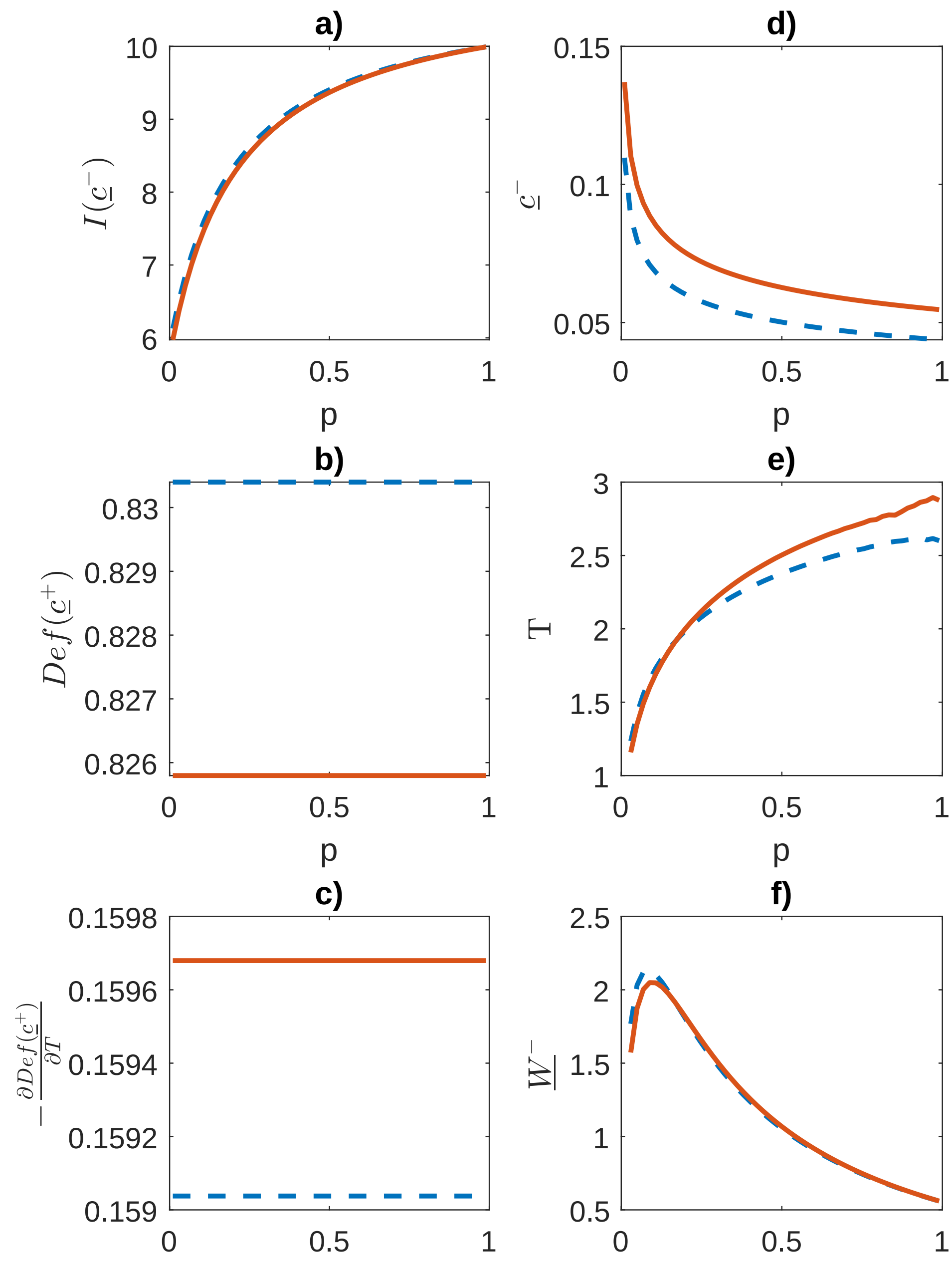

Figure IA.1: **The effect of political risk on bailouts.** This figure shows how debt relief T varies as a function of the probability that bailout negotiations fail, p. Figure a) shows the cost of a future round of bailouts $I(\underline{c}^{-})$, Figure b) the value of a default claim upon receiving a transfer $\text{Def}(\underline{c}^{+})$ and Figure c) the absolute value of the sensitivity of that claim to transfers, $-\frac{\partial \text{Def}(\underline{c}^{+}}{\partial T}$. All figures a)-c) are produced assuming $\underline{c}^{-} = 0.1, \underline{W}^{-} = 2, T = 1$. Figure d) shows the value of consumption at the default boundary assuming $\underline{W}^{-} = 2, T = 1$. Figure e) shows the equilibrium value of transfers at the government's optimal soft default boundary and Figure f) the value of that boundary. Throughout plots, the dotted solid line corresponds to the rational case $\delta = 1$ and the solid blue line corresponds to a highly myopic case $\delta = 0$.

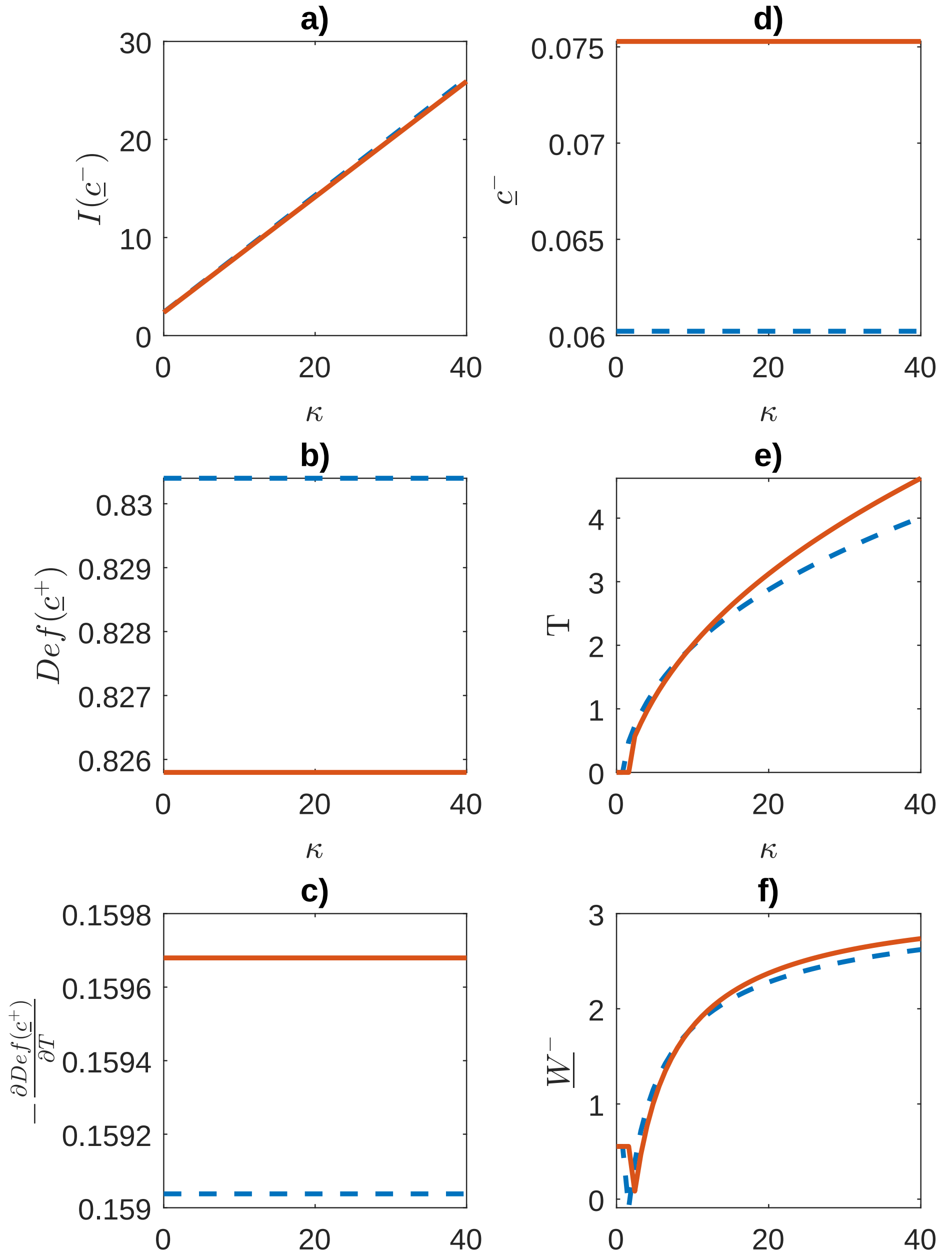

Figure IA.2: **The effect of spillovers on bailouts.** This figure shows how debt relief T varies as a function of the spillover cost κ. Figure a) shows the cost of a future round of bailouts $I(\underline{c}^-)$, Figure b) the value of a default claim upon receiving a transfer $\text{Def}(\underline{c}^+)$ and Figure c) the absolute value of the sensitivity of that claim to transfers,$-\frac{\partial \text{Def}(\underline{c}^+}{\partial T}$. All figures a)-c) are produced assuming $\underline{c}^- = 0.1, \underline{W}^- = 2, T = 1$. Figure d) shows the value of consumption at the default boundary assuming $\underline{W}^- = 2, T = 1$. Figure e) shows the equilibrium value of transfers at the government's optimal soft default boundary and Figure f) the value of that boundary. Throughout plots, the dotted solid line corresponds to the rational case $\delta = 1$ and the solid blue line corresponds to a highly myopic case $\delta = 0$.

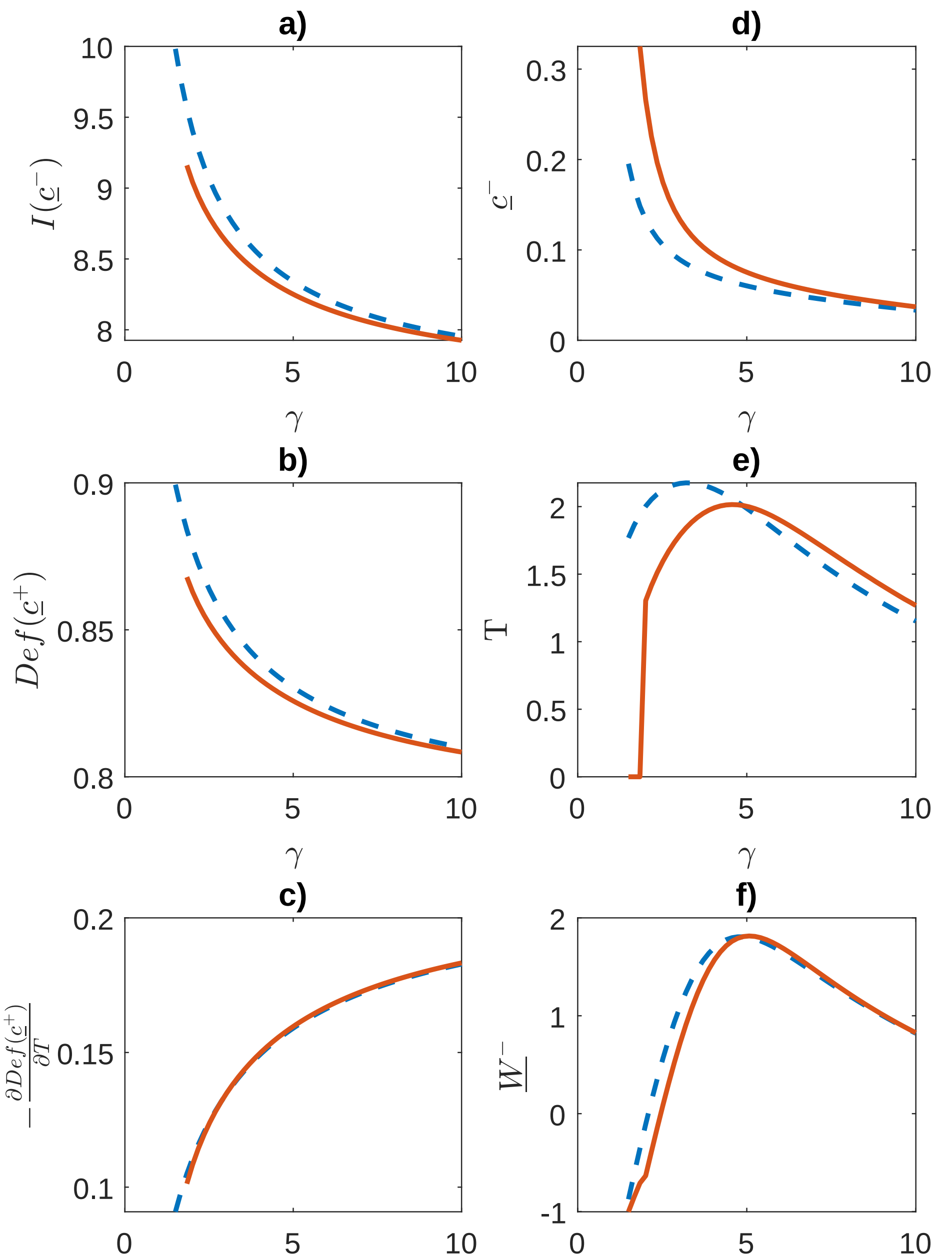

Figure IA.3: **The effect of risk aversion on bailouts.** This figure shows how debt relief T varies as a function of the borrower risk aversion γ. Figure a) shows the cost of a future round of bailouts $I(\underline{c}^-)$, Figure b) the value of a default claim upon receiving a transfer Def($\underline{c}^+$) and Figure c) the absolute value of the sensitivity of that claim to transfers,$-\frac{\partial \text{Def}(\underline{c}^+}{\partial T}$. All figures a)-c) are produced assuming $\underline{c}^- = 0.1, \underline{W}^- = 2, T = 1$. Figure d) shows the value of consumption at the default boundary assuming $\underline{W}^- = 2, T = 1$. Figure e) shows the equilibrium value of transfers at the government's optimal soft default boundary and Figure f) the value of that boundary. Throughout plots, the dotted solid line corresponds to the rational case $\delta = 1$ and the solid blue line corresponds to a highly myopic case $\delta = 0$.

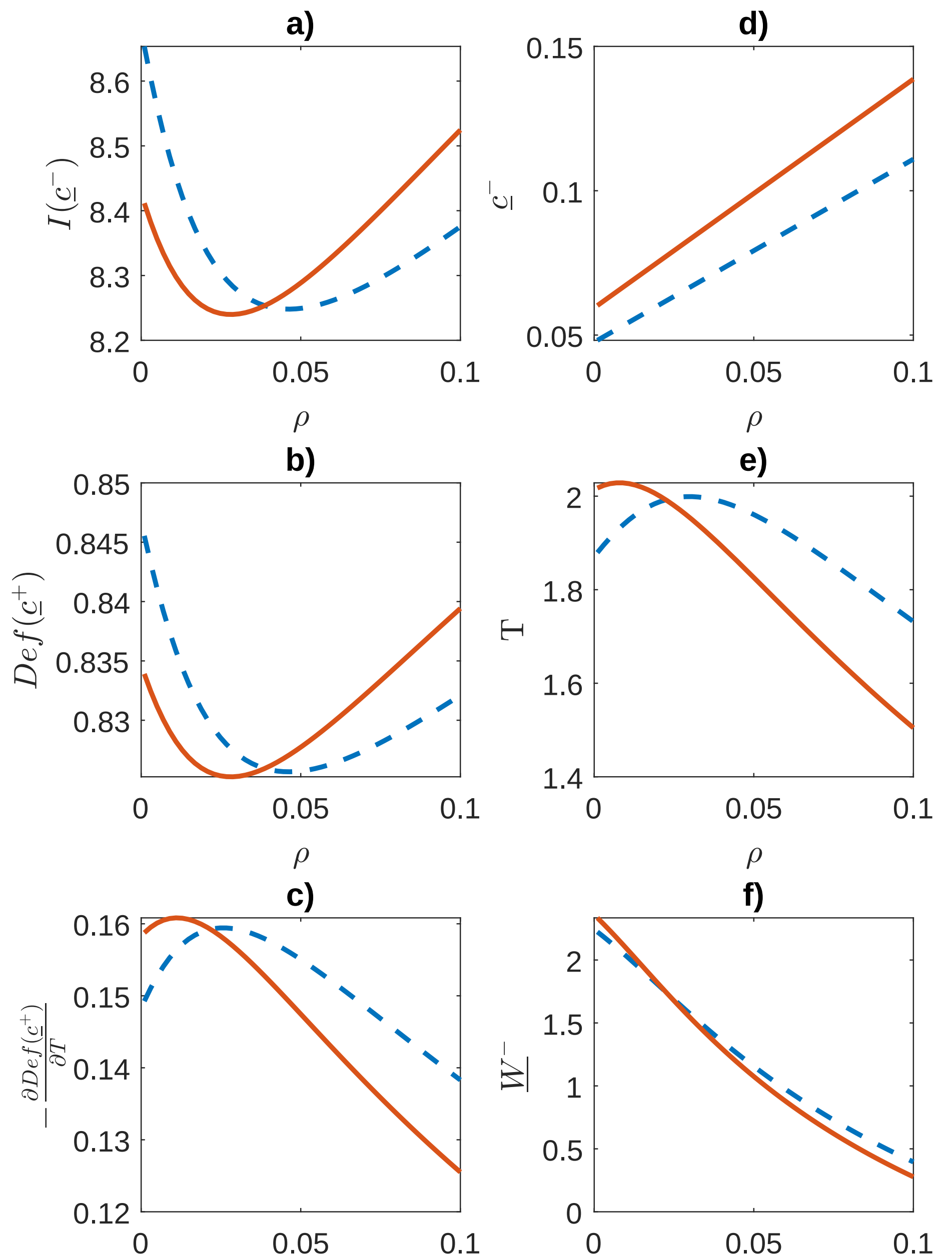

Figure IA.4: **The effect of impatience on bailouts.** This figure shows how debt relief T varies as a function of the government's imatience parameter ρ. Figure a) shows the cost of a future round of bailouts $I(\underline{c}^-)$, Figure b) the value of a default claim upon receiving a transfer Def$(\underline{c}^+)$ and Figure c) the absolute value of the sensitivity of that claim to transfers,$-\frac{\partial \text{Def}(\underline{c}^+}{\partial T}$. All figures a)-c) are produced assuming $\underline{c}^- = 0.1, \underline{W}^- = 2, T = 1$. Figure d) shows the value of consumption at the default boundary assuming $\underline{W}^- = 2, T = 1$. Figure e) shows the equilibrium value of transfers at the government's optimal soft default boundary and Figure f) the value of that boundary. Throughout plots, the dotted solid line corresponds to the rational case $\delta = 1$ and the solid blue line corresponds to a highly myopic case $\delta = 0$.

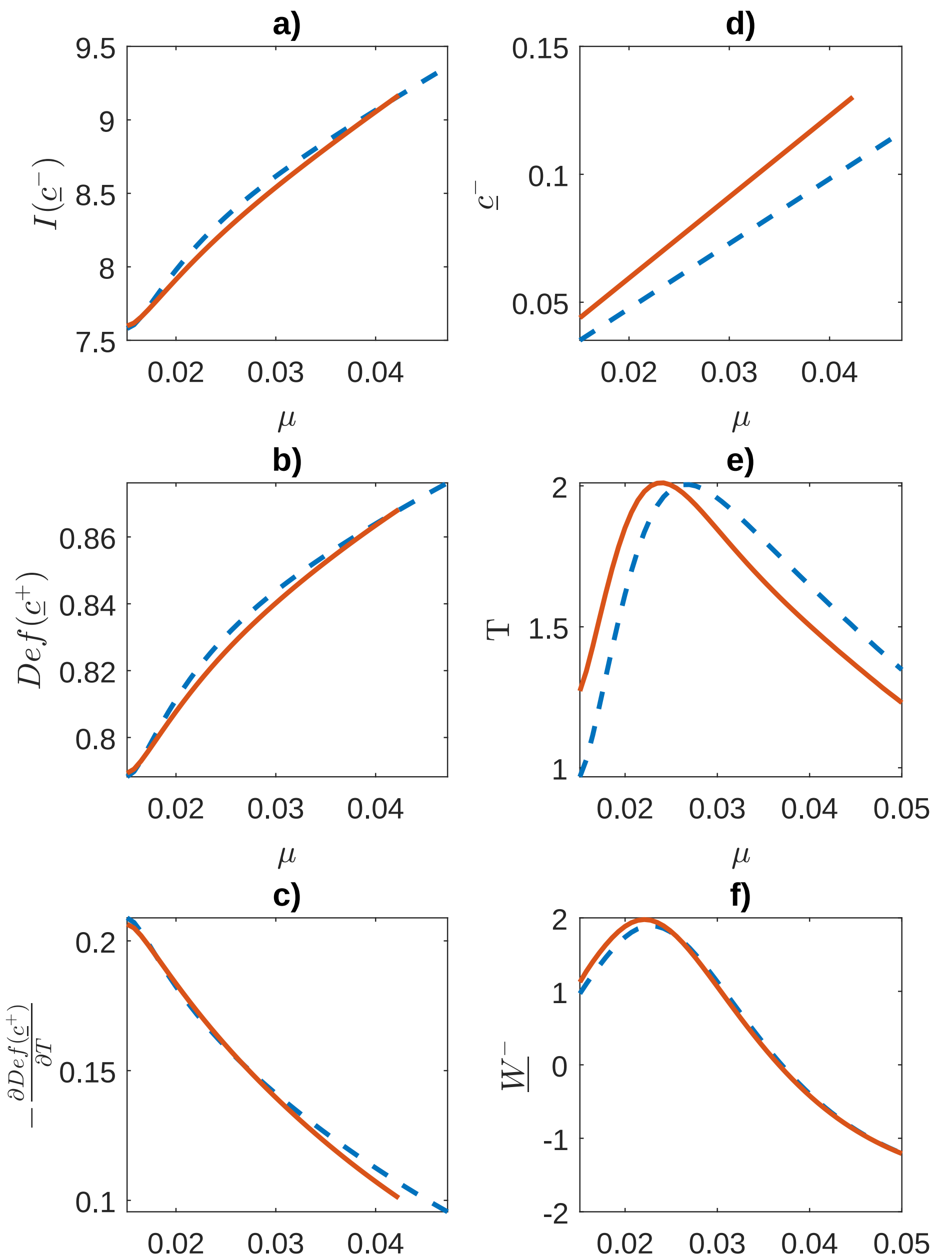

Figure IA.5: **The effect of mean asset return on bailouts.** This figure shows how debt relief T varies as a function of the mean return of the risky asset μ. Figure a) shows the cost of a future round of bailouts $I(\underline{c}^-)$, Figure b) the value of a default claim upon receiving a transfer $\text{Def}(\underline{c}^+)$ and Figure c) the absolute value of the sensitivity of that claim to transfers, $-\frac{\partial \text{Def}(\underline{c}^+}{\partial T}$. All figures a)-c) are produced assuming $\underline{c}^- = 0.1, \underline{W}^- = 2, T = 1$. Figure d) shows the value of consumption at the default boundary assuming $\underline{W}^- = 2, T = 1$. Figure e) shows the equilibrium value of transfers at the government's optimal soft default boundary and Figure f) the value of that boundary. Throughout plots, the dotted solid line corresponds to the rational case $\delta = 1$ and the solid blue line corresponds to a highly myopic case $\delta = 0$.

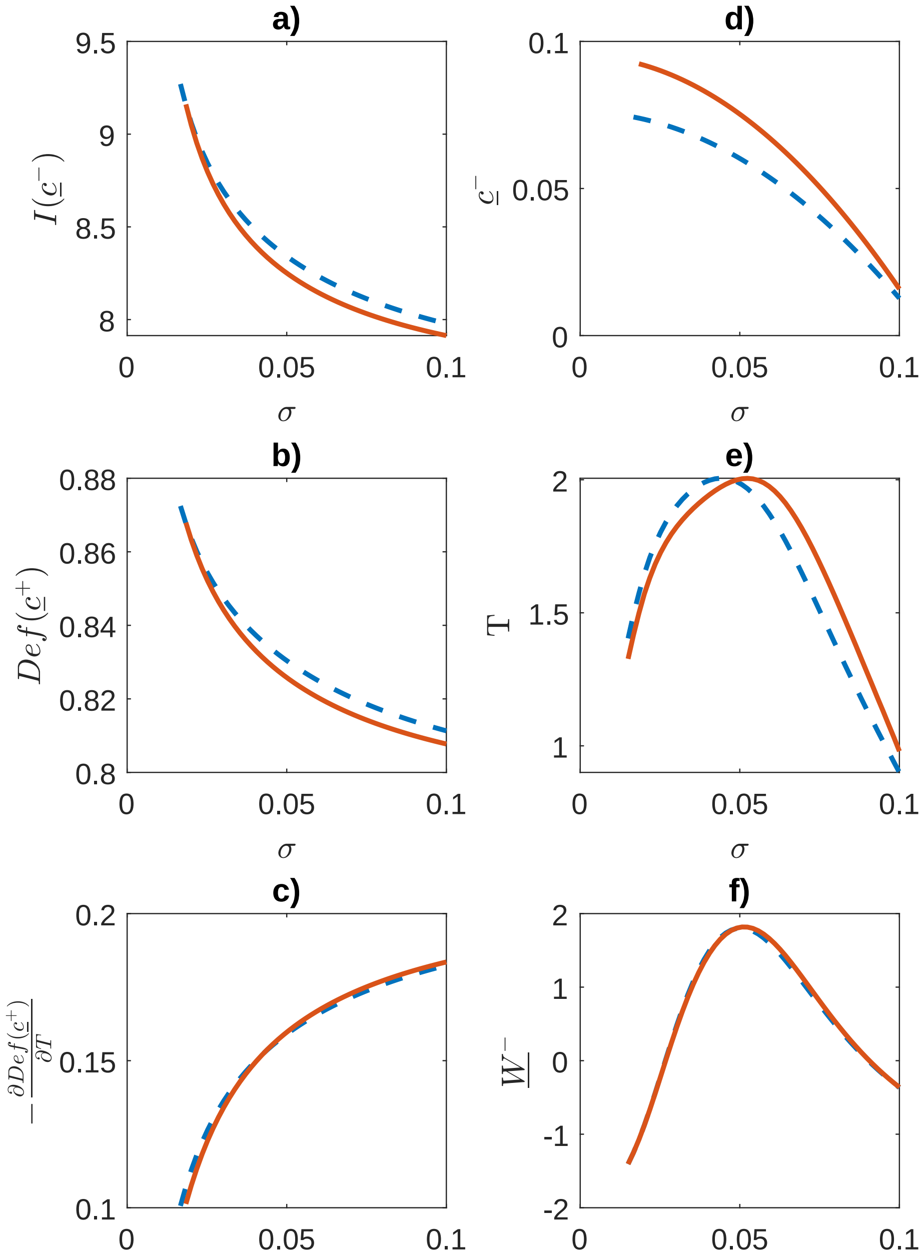

Figure IA.6: **The effect of volatility of asset returns on bailouts.** This figure shows how debt relief T varies as a function of the volatility of returns of the risky assset σ. Figure a) shows the cost of a future round of bailouts $I(\underline{c}^{-})$, Figure b) the value of a default claim upon receiving a transfer $\text{Def}(\underline{c}^{+})$ and Figure c) the absolute value of the sensitivity of that claim to transfers, $-\frac{\partial \text{Def}(\underline{c}^{+})}{\partial T}$. All figures a)-c) are produced assuming $\underline{c}^{-} = 0.1, \underline{W}^{-} = 2, T = 1$. Figure d) shows the value of consumption at the default boundary assuming $\underline{W}^{-} = 2, T = 1$. Figure e) shows the equilibrium value of transfers at the government's optimal soft default boundary and Figure f) the value of that boundary. Throughout plots, the dotted solid line corresponds to the rational case $\delta = 1$ and the solid blue line corresponds to a highly myopic case $\delta = 0$.

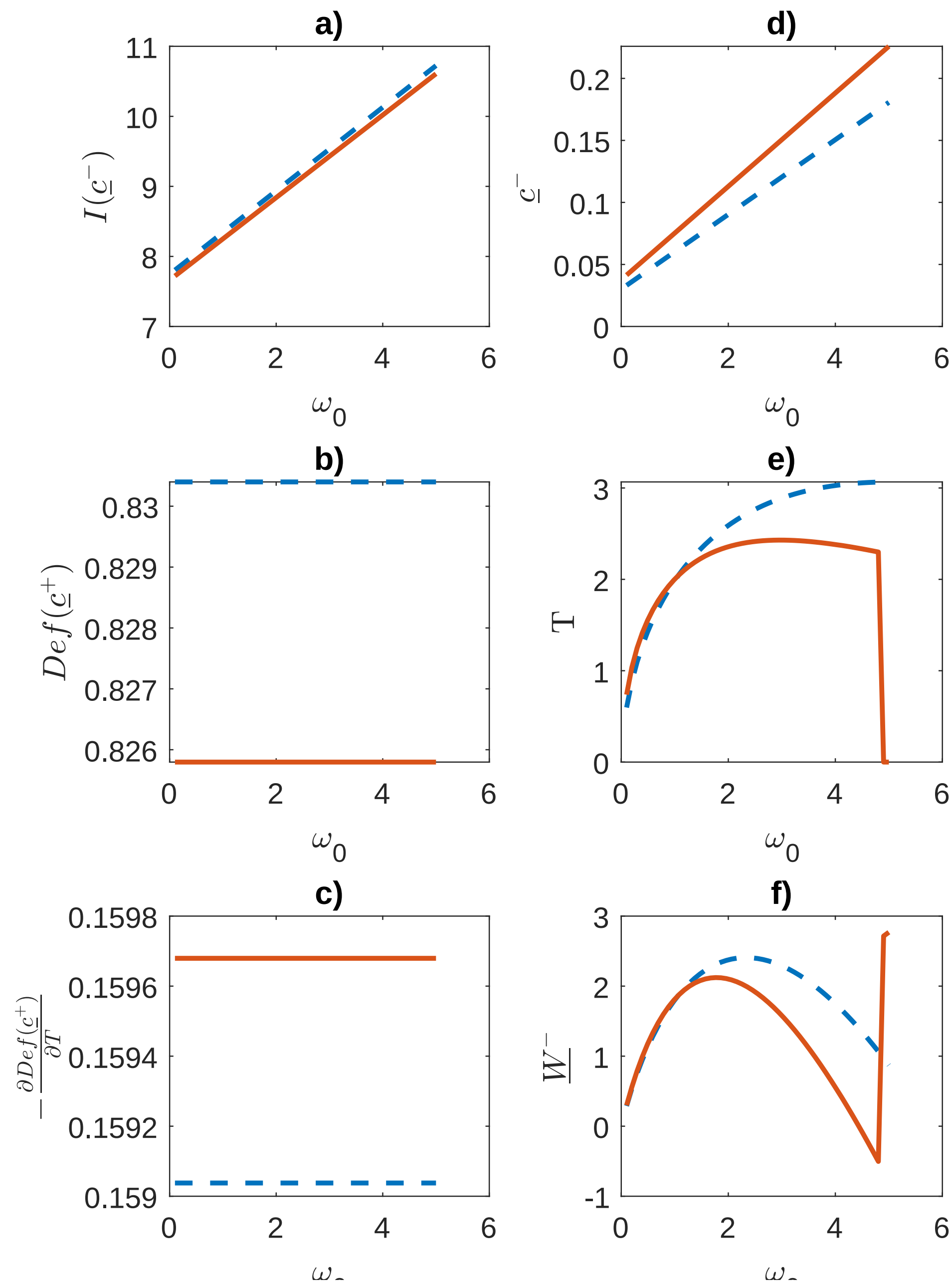

Figure IA.7: **The effect of the deadweight benefit of default on bailouts.** This figure shows how debt relief T varies as a function of the fixed benefit of default ω_0.Figure a) shows the cost of a future round of bailouts $I(\underline{c}^{-})$, Figure b) the value of a default claim upon receiving a transfer $\text{Def}(\underline{c}^{+})$ and Figure c) the absolute value of the sensitivity of that claim to transfers,$-\frac{\partial \text{Def}(\underline{c}^{+}}{\partial T}$. All figures a)-c) are produced assuming $\underline{c}^{-} = 0.1, \underline{W}^{-} = 2, T = 1$. Figure d) shows the value of consumption at the default boundary assuming $\underline{W}^{-} = 2, T = 1$. Figure e) shows the equilibrium value of transfers at the government's optimal soft default boundary and Figure f) the value of that boundary. Throughout plots, the dotted solid line corresponds to the rational case $\delta = 1$ and the solid blue line corresponds to a highly myopic case $\delta = 0$.

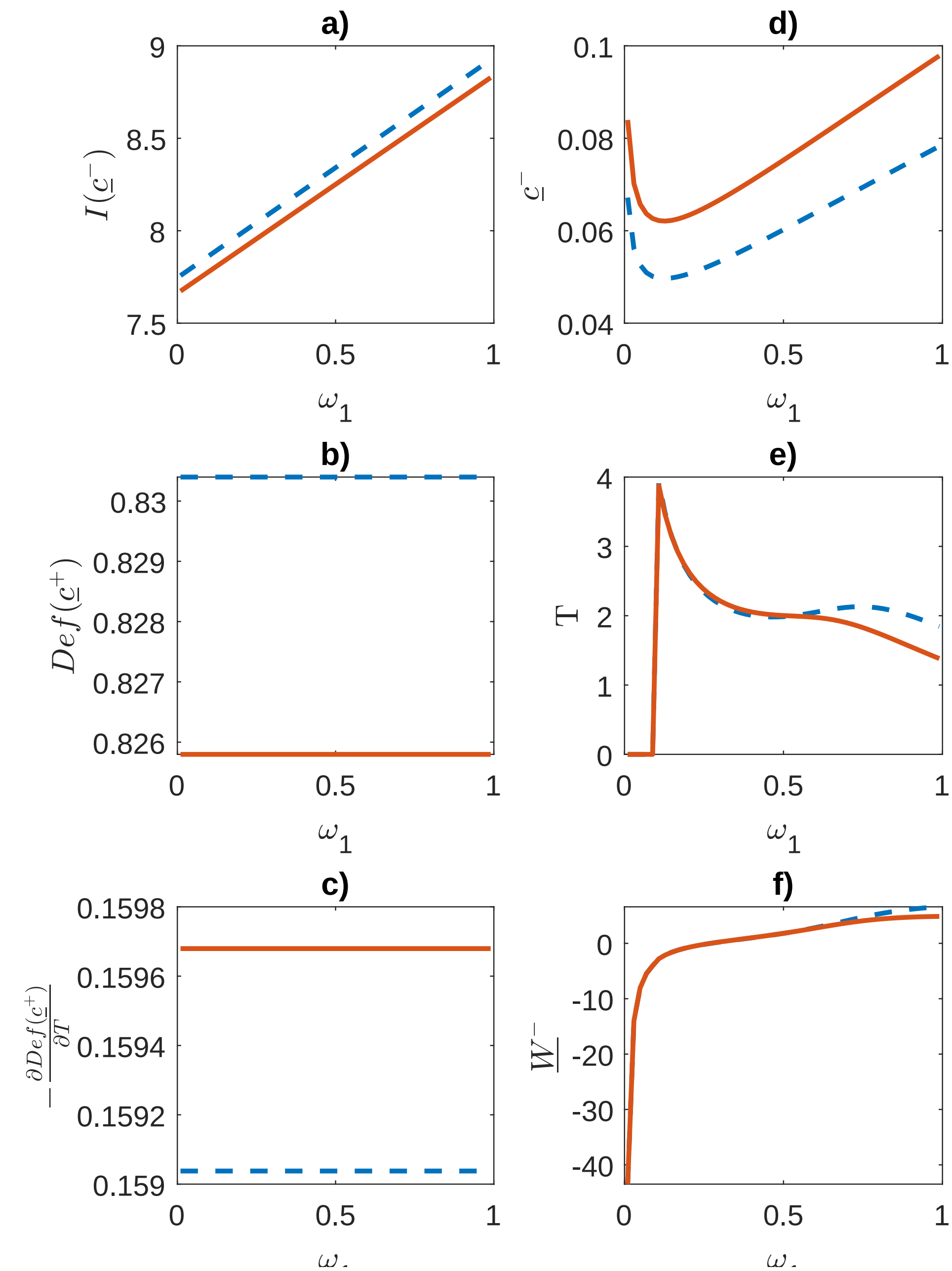

Figure IA.8: **The effect of the marginal cost of default on bailouts.** This figure shows how debt relief T varies as a function of the marginal effect of default on wealth ω_1. Figure a) shows the cost of a future round of bailouts $I(\underline{c}^-)$, Figure b) the value of a default claim upon receiving a transfer Def$(\underline{c}^+)$ and Figure c) the absolute value of the sensitivity of that claim to transfers,$-\frac{\partial \text{Def}(\underline{c}^+}{\partial T}$. All figures a)-c) are produced assuming $\underline{c}^- = 0.1, \underline{W}^- = 2, T = 1$. Figure d) shows the value of consumption at the default boundary assuming $\underline{W}^- = 2, T = 1$. Figure e) shows the equilibrium value of transfers at the government's optimal soft default boundary and Figure f) the value of that boundary. Throughout plots, the dotted solid line corresponds to the rational case $\delta = 1$ and the solid blue line corresponds to a highly myopic case $\delta = 0$.