Cournot Competition in the Loan Market: Microfoundations and Limitations

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Abstract

When firms choose their capacity and then compete à la Bertrand, the market equilibrium can correspond to the Cournot outcome (Kreps & Scheinkman, 1983). In the banking sector, a bank's lending capacity is constrained by its capital structure due to regulatory capital requirements. This paper establishes the conditions under which the Bertrand-Cournot equivalence extends to banks. I treat capital as an imperfect capacity commitment, allowing banks to distribute dividends and raise additional capital at a short-term premium during the competition stage. I show under which conditions the Cournot outcome is the unique equilibrium of the game. Such micro-foundations for Cournot competition in the loan market open new perspectives to the modeling of an elaborate, yet tractable, banking sector in macroeconomic models.

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1 Introduction

In recent years the macro-banking literature has received increased attention. An important challenge for researchers in this field is to model an elaborate banking sector in order to capture the special role of banks in the economy while keeping the framework tractable enough to be embedded in a macroeconomic model. Researchers wish to incorporate key ingredients such as risk, limited liability, regulation, and asymmetric information. When several of these features are present, perfect competition is useful to maintain the model analytically tractable. However, the banking sector is very concentrated and banks have substantial market power (Degryse & Ongena, 2008; Freixas & Rochet, 2008). Consequently, assuming perfect competition may result in outcomes or predictions that overlook important mechanisms driven by market power.

Several papers incorporate micro-founded financial frictions in a macroeconomic context, but often lenders do not have the key characteristics of banks and are simple risk-neutral lenders (Bernanke & Gertler, 1986, 1990; Brunnermeier & Sannikov, 2014). Another set of papers includes many of the relevant bank characteristics, but run into tractability issues. Thakor (1996) and Begenau (2020) include capital requirements, but bypass the challenges associated with limited liability by proxying deposit insurance with a reduced-form subsidy from the government to banks. Christiano *et al.* (2010) embed a banking sector in a DSGE framework, where banks face credit and liquidity risk but compete perfectly, whereas Abadi *et al.* (2023) account for banks' market power, but they need to rely on a reduced-form cost function for banks, which is meant to capture agency costs and regulations.

This paper proposes a partial solution of the overarching challenge of microfounding Cournot competition in the loan market under well-defined conditions, which also delineate the limitations. The Cournot approach effectively account for imperfect competition in a meaningful manner with an inherent tractability that gives researchers flexible modeling choices.

There are other approaches to imperfect competition that offer a degree of tractability. In the literature, the most commonly used forms of competition à la Salop and à la Dixit-Stiglitz [1]. In

[1] For competition à la Salop, examples include but are not limited to: Dell'Ariccia (2001); Chiappori *et al.*

a Salop model, borrowers are uniformly distributed on a circle and banks decide their location. Borrowers incur transportation costs to reach a bank. This type of competition can be narrowly interpreted as purely spatial, emphasizing that physical distance is an important factor in the lending market (Nguyen, 2019; Degryse & Ongena, 2005; Petersen & Rajan, 2002) or, in a broader sense, the unit circle can be seen as the space of products where banks offer loans with different features to gain market power. The standard interpretation of Dixit-Stiglitz also relies on product differentiation, but it implies that, *ceteris paribus*, borrowers are better off by having multiple loans with different banks rather than having one large loan with one bank. Ulate (2021) provides a plausible micro-foundation: the CES demand can be generated by a two step decision process in which first borrowers choose a bank through a taste shock and then decide on the loan quantity. However, these models are microfounded only under the assumption of horizontal differentiation (e.g. bank branding or location) where no product is objectively better than another. Vertical differentiation (e.g. differences in contract terms such as maturity, collateral, or monitoring) does not microfound the competitive structure in these models. Vertical quality differences affect borrower choice, but they do not justify the taste-shock logic. For instance, if a bank offers better terms due to superior screening or relationship lending, borrowers will strictly prefer it in financial terms. This paper provides microfoundations for Cournot competition by modeling agents as maximizing the risk-adjusted net present value of cashflows, offering a valid alternative to taste-shock approaches. These foundations highlight both the advantages and the limitations of Cournot, which, while tractable, should be adopted only when consistent with the environment under study.

Cournot competition also offers analytical advantages that are particularly useful in dynamic macroeconomic models. Li (2024) points out that monopolistic competition models à la Dixit–Stiglitz do not generate time-varying loan markups over the business cycle without additional assumptions. In the DSGE literature, such variation is typically introduced through mechanisms like exogenous shocks to the elasticity of substitution (Gerali *et al.* , 2010), changes in

(1995); Andrés & Arce (2012); Andrés *et al.* (2013); for competition à la Dixit-Stiglitz: Gerali *et al.* (2010); Ulate (2021); Wang *et al.* (2022); Abadi *et al.* (2023).

banks' marginal cost of lending (Hafstead & Smith, 2012), deep habits in financial intermediation (Airaudo & Olivero, 2019; Aliaga-Díaz & Olivero, 2010), or nominal rigidities à la Calvo or Rotemberg (Hülsewig *et al.* , 2009). By contrast, Li (2024) models the banking sector as a Cournot oligopoly, where markups vary endogenously with the number of banks and the elasticity of loan demand. This framework captures how imperfect competition can endogenously shape macroeconomic fluctuations, without the need to introduce additional frictions.

The starting point of the micro-foundations of Cournot competition is Kreps & Scheinkman (1983) (hereinafter KS) who show that in a two-stage game in which firms first choose capacity and then compete à la Bertrand, the unique subgame perfect equilibrium is the Cournot outcome. Given this setting, the banking sector seems a natural application for two reasons: (1) given a fixed amount of regulatory capital, capital requirements constrain a bank's lending capacity, therefore, bank capital choices can be interpreted as capacity choices (Schliephake & Kirstein, 2013); (2) banks typically do not raise capital and issue loans simultaneously, but have medium-term capital targets (Couaillier, 2021). However, capital does not always represent a rigid constraint. Maggi (1996) develops a model of capacity–price competition in which firms can expand capacity, though the analysis is limited to differentiated goods under linear demand. This perspective is particularly suited for banks, since bank capital appears to be a less rigid constraint than physical capital. Schliephake & Kirstein (2013) prove that Maggi's framework can be extended to banks that issue risk-free loans, and show that if the cost of raising capital in the second stage is sufficiently high, the Cournot outcome is the unique subgame-perfect equilibrium.

The contribution of this paper is to outline the conditions under which this result continues to hold in a more general setup, given the following characteristics of the banking sector. First, banks are protected by limited liability and deposits are insured by the government. Second, banks are subject to capital requirements and can increase or decrease their capital in the competition stage. In line with Schliephake & Kirstein (2013), I find that the cost of recapitalizing must be sufficiently high in order to sustain the Cournot equilibrium. The intuition is that capacity constraints must be relevant, otherwise the competition stage becomes a standard Bertrand game. Third, loans are not only risky, but their risk is endogenous. In particular I allow the probability of default to depend on the interest rate charged by the bank. This feature is important because it allows for loan demand functions derived from bank–borrower interactions that incorporate asymmetric information. The interest rate can affect the composition of the pool of borrowers or borrowers' incentives. Depending on the friction taken into account, a higher loan rate can lead to a safer (De Meza & Webb, 1987; Bernanke & Gertler, 1990) or a riskier portfolio (Stiglitz & Weiss, 1981; Martinez-Miera & Repullo, 2010) or the effect may be ambiguous (House, 2006).

The impact of the loan rate on the probability of default requires particular attention, as it threatens the Bertrand-Cournot equivalence. In KS, the Bertrand–Cournot equivalence holds because, once firms are at full capacity, the demand-stealing mechanism of Bertrand competition vanishes: price cuts reduce revenues without expanding output. In stage 1, firms anticipate that they will operate at full capacity and therefore, the strategic choice of capacity is equivalent to the strategic choice of quantity. This equivalence cannot be straightforwardly applied to environments characterized by asymmetric information: a bank, even when operating at full capacity, may want to charge a lower rate in order to improve its distribution of defaults. Whether the Bertrand–Cournot equivalence holds depends on the nature and magnitude of the underlying friction. Two conditions are required. First, the expected average residual cashflow—i.e. cashflows net of deposits—must be increasing in the bank's loan rate. This implies that in standard moral hazard setups (Boyd & De Nicolò, 2005; Martinez-Miera & Repullo, 2010), borrowers' incentives should not be overly sensitive to small changes in the loan rate. Second, a bank's default rate depends only on its own interest rate, not on that of its competitor. This condition rules out setups such as adverse selection with screening, where a bank always has the incentive to undercut its competitor to be the cheapest lender and attract a better pool of borrowers (Broecker, 1990; Marquez, 2002).

Cournot offers desirable properties of tractability and modeling flexibility. This paper contributes by establishing its microfoundations and delineating its limitations, thereby offering a tool that may open new perspectives in the macro-banking literature.

2 Model Setup

2.1 The environment

The model builds on Kreps & Scheinkman (1983) (KS) and Martinez-Miera & Repullo (2010). All agents are risk neutral and the gross risk-free rate is normalized to one. Consider the following two-bank two-stage game. In stage 1, each bank $i \in \{1,2\}$ raises capital $k_i \in \mathbb{R}_+$, and in stage 2 banks compete à la Bertrand in the loan market. Capital regulation requires banks to fund a fraction $\gamma \in (0,1)$ of their loans, l_i with capital. Loans can also be financed through deposits, d_i , which are supplied elastically. In the second stage, banks are allowed to adjust their capital: they can either reduce capital by distributing dividends at a unit cost δ or raise more capital at a short-term premium κ. These costs can be broadly interpreted as capital adjustments costs, which capture both purely transactional costs and deadweight losses (e.g. limited investor base to raise more capital in the short-term). Their role in the model is to prevent banks from frictionlessly adjusting their capital, thereby making stage 1 irrelevant.

In contrast with Schliephake & Kirstein (2013), loans are risky. Let r_i be the interest rate charged by bank i. The fraction of loans that default is governed by the random variable x, which is distributed according to the cumulative distribution function $F(x|r_i)$ which has support $[0,1]$. I assume that if a loan defaults its recovery rate is zero. Over its support, $F(x|r_i)$ is twice continuously differentiable in x and r_i, and it is strictly increasing in x. How r_i affects the distribution of defaults depends on the underlying friction. This feature will allow the model to nest some asymmetric information setups, provided that the conditions presented later in this section are respected.

The solution concept is subgame perfect Nash equilibrium, hence I solve the game by backward induction. In stage 2, banks take capital raised in stage 1 as given and every pair (k_1, k_2) represent a different subgame which I denote by $\mathcal{H}(k_1, k_2)$. In every subgame, banks compete à la Bertrand subject to capital requirements. Borrowers select banks according to a Bertrand allocation rule. All borrowers first apply to the bank offering the lowest rate. If the cheapest bank has not sufficient lending capacity to satisfy the entire demand at $r_i < r_j$, it has two options: (i) expand its capital so as to increase lending capacity; or (ii) refrain from expanding, in which case total demand is rationed. Rationing follows the efficient rule: borrowers who are more willing to pay, are served first. The residual demand is then served by the rival bank at its quoted rate. Formally, denote by $L(r)$ the loan demand as a function of the loan rate and assume that it is twice continuously differentiable and strictly decreasing where it is positive. In the subgame $\mathcal{H}(k_1, k_2)$, given the loan rates posted $\mathbf{r} = (r_1, r_2)$ and the additional capital raised $\mathbf{e} = (e_1, e_2)$ by each bank, the demand served by bank i is given by:

$$l_i(\mathrm{r}, \mathbf{e}) = \begin{cases} \min\left(\frac{k_i+e_i}{\gamma}, L(r_i)\right) & \text{if } r_i < r_j \\ \min\left(\frac{k_i+e_i}{\gamma}, \max\left(\frac{L(r_i)}{2}, L(r_i) - \frac{k_j+e_j}{\gamma}\right)\right) & \text{if } r_i = r_j \\ \min\left(\frac{k_i+e_i}{\gamma}, \max\left(0, L(r_i) - \frac{k_j+e_j}{\gamma}\right)\right) & \text{if } r_i > r_j \end{cases}$$

In every instance, a bank cannot extend loans beyond its capacity $(k_i + e_i)/\gamma$ determined by the capital requirement. If bank i sets the lowest rate, it serves the entire market up to its capacity. If banks set the same rate, they equally split the demand; however if one bank does not have sufficient capacity to serve half of the market, the other can serve the residual demand. Lastly, if bank i names the highest rate, it serves the residual demand (if any). Note that efficient rationing is not an inconsequential assumption; KS result does not hold under other types of rationing without further assumptions [2]. To ease readability let $l_i(\mathrm{r}, \mathbf{e}) = l_i$.

If cashflows generated by the loans are not sufficient to pay back deposits the bank defaults and deposit are repaid though the government insurance. Therefore, from the bank's perspective, deposits are the cheapest source of funding[3]. I assume that the unit cost of paying dividends δ is sufficiently small to ensure that the bank always prefers to pay dividends and raise deposits when

[2] See Lepore (2009).

[3] Provided that the bank's probability of default is strictly positive. If the bank's probability of default is zero, then the bank is indifferent between deposits and capital and the model reduces to a KS game where production costs are equal to capacity costs.

the capital requirement constraint is slack [4]. Given this, after stage 2 decisions have been made, the capital requirement constraint binds.

Define $\eta(l_i, k_i)$ as the capital adjustment function:

$$\eta(l_i, k_i) = \begin{cases} (1-\delta)(k_i - \gamma l_i) & \text{if } k_i \geq \gamma l_i \\ -(1+\kappa)(\gamma l_i - k_i) & \text{if } k_i < \gamma l_i \end{cases}$$

If capital raised in stage 1 is greater than what the regulation requires, then the bank pays back a dividend, whereas if the bank needs to raise more capital, it must pay the short term premium[5]. Consequently, deposits are always equal to $d_i = (1-\gamma)l_i$. Given the loan rate posted and the additional capital raised by each bank, the stage 2 payoff of bank i is given by:

$$\left(\int_0^{\tilde{x}} \left((1-x)(1+r_i) - (1-\gamma)\right) dF(x|r_i)\right) l_i + \eta(k_i, l_i)$$
$$\text{where } \tilde{x} = \frac{r_i + \gamma}{1 + r_i}$$

$\tilde{x}$ is the maximum fraction of defaults that allows the bank to repay deposits. Define $m(r) = \int_0^{\tilde{x}} \left((1-x)(1+r_i) - (1-\gamma)\right) dF(x|r_i)$ as the expected average residual cashflow, i.e. the average cashflow left to shareholders after deposits are repaid. Note that, thanks to the deposit guarantee, the bank pays deposits at the risk-free rate only in the states of the world in which it survives.

2.2 Payoffs and Equilibrium Concept

In addition to pure strategies, I also allow for mixed strategies, where players randomize over their available actions. In stage 2, bank i will choose a distribution over rates $G_i(r_i)$ and how much extra capital e_i to raise in order to maximize its expected payoff:

[4] Given the evidence of bank payout behavior (Acharya *et al.* , 2022; Belloni *et al.* , 2024) this assumption does not seem particularly restrictive. The assumption is important for tractability (for further discussion see Section 5). For the exact condition see the Appendix.

[5] Note that by construction $k_i < \gamma l_i$ only if $e_i > 0$.

$$\max_{G_i\in\mathcal{S}_r,\ e_i\in\mathbb{R}_+}\left\{M(G_i,G_j,e_i,e_j)=\int_{\underline{r}_j}^{\bar{r}_j}\int_{\underline{r}_i}^{\bar{r}_i}\left[m(r_i)l_i+\eta(k_i,l_i)\right]dG_i(r_i)dG_j(r_j)\right\}$$

where $\underline{r}_i$ and $\bar{r}_i$ are respectively the infimum and the supremum of the support of G_i, and $\mathcal{S}_r$ is the space of distributions over rates.

In stage 1, each bank chooses capital according to some distribution $\mu_i(k)$, with support $[\underline{k}_i,\bar{k}_i]\subseteq\mathbb{R}_+$, anticipating the equilibrium strategies of each subgame. Denote by $\mathcal{S}_k$ the strategy space of stage 1. Bank i aims to maximize its expected profits:

$$\max_{\mu_i(k_i)\in\mathcal{S}_k}\left\{\pi(\mu_i,\mu_j)=\int_{\underline{k}_i}^{\bar{k}_i}\int_{\underline{k}_j}^{\bar{k}_j}\left(M_i^*(k_i,k_j)-k_i\right)d\mu_j(k_j)d\mu_i(k_i)\right\}$$

where μ_j is the opponent's strategy and $M^*(k_i,k_j)$ is the expected equilibrium payoff of $\mathcal{H}(k_i,k_j)$.

Definition. The tuple $(\mu_1^*,\mu_2^*,G_1^*(r_1|k_1,k_2),G_2^*(r_2|k_1,k_2),e_1^*(k_1,k_2),e_2^*(k_1,k_2))$ is a subgame perfect Nash equilibrium (SPNE) if

- For all $(k_1,k_2)\in\mathbb{R}_+^2$, $(G_1^*(r_1|k_1,k_2),G_2^*(r_2|k_1,k_2),e_1^*(k_1,k_2),e_2^*(k_1,k_2))$ are the equilibrium strategies of the subgame $\mathcal{H}(k_1,k_2)$, i.e. for all $i=1,2$

$$M(G_i^*,G_j^*,e_i^*,e_j^*)\geq M(G_i,G_j^*,e_i,e_j^*)\qquad\forall(G_i,e_i)\in\mathcal{S}_r\times\mathbb{R}_+$$

- For all $i=1,2$:

$$\pi(\mu_i^*,\mu_j^*)\geq\pi(\mu_i,\mu_j^*)\qquad\forall\mu_i\in\mathcal{S}_k$$

2.3 Key Conditions

I now set out the sufficient conditions which allow me to prove that the Cournot outcome, defined in the next section, is the unique equilibrium of this game.

Condition 1. (monotonicity) $m(r)$ is strictly increasing in r where it is positive.

Condition 1 states that the average residual cashflow is increasing in own rate. While for firms with linear costs it is trivial that an increase in price leads to an increase in the average margin, for banks there are other channels at work:

$$\frac{\partial m(r)}{\partial r} = \underbrace{\frac{\partial \tilde{x}}{\partial r}(1+r)F(\tilde{x}|r)}_{(+)\text{ Buffer}} + \int_0^{\tilde{x}} \underbrace{F(x|r)}_{(+)\text{ Margin}} + \underbrace{(1+r)\frac{\partial F}{\partial r}}_{(\pm)\text{ Distribution shifting}} dx \gtreqless 0$$

In line with Schliephake (2016), there are three effects. First, the buffer effect: an increase in r drives the increase in the threshold $\tilde{x}$ and allows the bank to survive in more states of the world. Second, the margin effect: an increase in the rate makes the bank earn more on non-defaulting loans. Lastly, an increase in r affects the distribution of the default rate. The direction of this distribution shifting effect is ambiguous and depends on the friction that $F(\cdot|r)$ captures. The condition on monotonicity implies that if the distribution shifting effect is negative, its magnitude cannot be too large. A negative distribution shifting effect implies that when the bank charges a higher rate the portfolio becomes riskier, e.g. an entrepreneur protected by limited liability chooses a riskier project when facing a higher loan rate (Boyd & De Nicolò, 2005; Martinez-Miera & Repullo, 2010; Schliephake, 2016). In this context, Condition 1 impose that a small change in the loan rate should not lead to a dramatically different probability of default.

Condition 2. (independence) The distribution function of default is independent from the opponent's rate $F(x|r_i, r_j) = F(x|r_i)$.

As I will discuss in detail in Section 4, this condition is generally not satisfied in adverse selection settings, where relative prices affect the quality of the pool served.

2.4 The Cournot benchmark

Before proceeding to the characterization of the equilibrium, I define the one-stage Cournot benchmark. In this one-stage game, banks choose capital and loan quantities simultaneously, while facing the same fundamentals as in the two-stage game—namely, the same regulation, the same demand,

and the same risk environment. Capital requirements are binding because of the government guarantee, therefore $k_i = \gamma l_i$ and $d_i = (1-\gamma) l_i$ for $i \in \{1, 2\}$. Denote the inverse loan demand by $r(L) \equiv L^{-1}(r)$, which is the interest rate on loans as a function of total loans supplied $L = l_1 + l_2$. Taking l_j as given, bank i solves the following problem:

$$\max_{l_i \geq 0} \left(\int_0^{\tilde{x}} \left((1-x)\left(1 + r(L)\right) - (1-\gamma) \right) dF(x|r(L)) - \gamma \right) l_i$$

Define $Z(L) = \int_0^{\tilde{x}} \left((1-x)\left(1 + r(L)\right) - (1-\gamma) \right) dF(x|r(L))$. Assume that $-Z''(L)L/Z'(L) < 1$. This assumption ensures that the Cournot equilibrium is unique and therefore constitutes a valid benchmark.

Lemma 1. *Let*

$$b(l_j) = \arg\max_{l_i \geq 0} (Z(L) - \gamma) l_i$$

The best response function $b(\cdot)$ has a unique fixed point $b(l^C) = l^C$. Therefore (l^C, l^C) and $r(2l^C)$ are respectively the equilibrium quantities and the equilibrium rate of the Cournot game.

Proof. See Appendix. □

The independence condition implies that $Z(L) = m(r(L))$. In absence of this condition, the equality is not obvious. When certain frictions are introduced, it is not possible to map the two-stage game to this one-stage setup. For example, if banks can screen borrowers using an informative but imperfect signal, the relative pricing between competitors becomes relevant: the bank offering the more attractive rate will screen applicants before the other and therefore will face a better pool of borrowers. The one-stage Cournot game, which implies a single prevailing market rate, is not able to capture this kind of sorting effect across lenders. The monotonicity condition implies that $Z'(L) < 0$ where it is positive.

Auxiliary Cournot game

Before turning to the two-stage game, I define an auxiliary Cournot game, whose best response functions are useful to divide the space of the subgames into relevant regions. This auxiliary game is a stage 2 Cournot game in which the bank has already raised capital and faces the opportunity cost of paying dividends. Let:

$$\hat{b}(l_j) = \arg\max_{l_i \geq 0} (Z(L) - \gamma(1-\delta))l_i$$

It is straightforward to prove that $\hat{b}(\cdot)$ has the same properties of $b(\cdot)$ and that $\hat{b}(l_j) \leq b(l_j)$, with strict inequality when $b(l_j)$ is positive.

Now I proceed to the two stage game. In the next sections I show by backward induction that the Cournot outcome is the unique subgame perfect Nash equilibrium.

3 The baseline model

For now, to streamline the core of the analysis, I assume $\kappa = +\infty$ as in KS, therefore banks will not be able to raise capital in the short term. This assumption is relaxed in Section 5. I also assume $\delta > 0$ to rule out multiplicity of equilibria. In Section 5 I allow $\delta = 0$ and provide an alternative set of assumptions to maintain uniqueness. Following the solution concept of SPNE defined in the previous section, I proceed by backward induction.

3.1 Second stage: Bertrand competition with capital requirements

In this section, I characterize the equilibrium of every subgame $\mathcal{H}(k_1, k_2)$. Define $\rho_\gamma(\theta)$ to be the rate such that the expected average residual cashflow $m(\rho_\gamma(\theta)) = \gamma\theta$, and $\Lambda_\gamma(\theta) = L(\rho_\gamma(\theta))$ the corresponding loan demand. In every subgame $\mathcal{H}(k_1, k_2)$ such that $\min_i \frac{k_i}{\gamma} \geq \Lambda_\gamma(1-\delta)$, the capacity constraints are irrelevant and the game is a standard Bertrand competition in which the average residual cashflows equals the opportunity cost of capital in equilibrium. In KS, this is equivalent to have the price equal to the marginal cost with only two differences: first, banks care about the average residual cashflow not about the price; second, because banks can pay dividends, the cost of capacity is not completely sunk in the second stage. Optimal strategies (G_1^*, G_2^*) are $r_1 = r_2 = \rho_\gamma(1-\delta)$ with probability 1 and banks' equilibrium payoffs are equal to $M_i(G_i^*, G_j^*) = (1-\delta)k_i$. For the rest of the paper consider only subgames in which $\min_i \frac{k_i}{\gamma} < \Lambda_\gamma(1-\delta)$.

Lemma 2. *In every subgame equilibrium it must be that* $\underline{r}_i \geq r\left(\frac{k_1+k_2}{\gamma}\right) \equiv r^{FC}$ *for all* $i \in \{1,2\}$.

Proof. Given any G_j, if a bank names a rate $r \leq r^{FC}$, then it is operating at full capacity with probability one. Given that $m(r)$ is an increasing function of r, any $r < r^{FC}$ is strictly dominated by r^{FC}. In other words, when a bank reaches maximum capacity has no incentive to undercut the opponent as it would decrease the interest rate without improving the quantity. Therefore any rate $r < r^{FC}$cannot be part of an equilibrium strategy and $\underline{r}_i \geq r^{FC}$.
Denote by $\alpha_i(r) \equiv \Pr(r_i = r)$ the probability mass that the distribution G_i puts on r. □

Lemma 3. *In equilibrium, if* $\bar{r}_1 = \bar{r}_2 = \bar{r}$ *and* $\alpha_i(\bar{r}) > 0$ *for* $i \in \{1,2\}$, *then*

$$\underline{r}_i = \bar{r}_i = r^{FC} \quad \textit{and} \quad \frac{k_i}{\gamma} \leq \hat{b}\left(\frac{k_j}{\gamma}\right) \forall i \in \{1,2\}$$

Proof. See Appendix. □

Lemma 3 states that if there exists an equilibrium in which banks have the same supremum, this supremum must be smaller or equal to the full capacity rate. The intuition is the following: if $\bar{r} > r^{FC}$, then the bank with (weakly) more capital has capacity to expand lending. Hence, the strategy $\bar{r}$ would be dominated by $\bar{r} - \epsilon$, with ϵ arbitrarily small, which keeps the expected average residual cashflow constant and increases the quantity. The second part of the lemma, $\frac{k_i}{\gamma} \leq \hat{b}\left(\frac{k_j}{\gamma}\right)$, ensures that each bank has no incentive to charge a rate that is higher than $\bar{r}$ and be the monopolist of the residual demand.

Lemma 4. *In equilibrium, if* $\bar{r}_i > \bar{r}_j$ *or* $\bar{r}_i = \bar{r}_j$ *and* $\alpha_j(\bar{r}_j) = 0$, *then:*

(a) $\bar{r}_i = r\left(\hat{b}\left(\frac{k_j}{\gamma}\right) + \frac{k_j}{\gamma}\right)$ *and the equilibrium payoff of bank* i *is equal to*

$$M_i(G_i^*, G_j^*) = \underbrace{(m(\bar{r}_i) - \gamma(1-\delta))\,\hat{b}\left(\frac{k_j}{\gamma}\right)}_{=P(k_j)} + (1-\delta)k_i$$

(b) $\frac{k_i}{\gamma} > \hat{b}\left(\frac{k_j}{\gamma}\right)$

(c) $\underline{r}_i = \underline{r}_j$ *and* $\alpha_i(\underline{r}_i) = 0$ *for all* $i \in \{1,2\}$

(d) $k_i \geq k_j$

(e) the equilibrium payoff of bank j *is uniquely determined by* (k_1, k_2) *and*

$$P(k_j)\frac{k_j}{k_i} + (1-\delta)k_j \leq M_j(G_j^*, G_i^*) \leq P(k_j) + (1-\delta)k_i$$

Proof. See Appendix □

This lemma states the following: the bank that is competing less aggressively ($\bar{r}_i > \bar{r}_j$) must be the bank that has more capacity. The intuition is the following. To make the low-capacity bank indifferent across rates in the support of its equilibrium strategy, it must have a relatively lower probability of being undercut.

Under condition 1 and 2, Dasgupta & Maskin (1986) guarantee the existence of an equilibrium, therefore every $\mathcal{H}(k_1, k_2)$ has an equilibrium that must respect Lemmas 2-4. We can divide the subgames space into three relevant regions (see Figure 1 for reference) .

- Region 1 $\left\{\mathcal{H}(k_1, k_2) : \min_i \frac{k_i}{\gamma} > \Lambda_\gamma(1-\delta)\right\}$: in this region banks are so much capitalized that capacity constraints do not matter. The subgame equilibrium is $r_1 = r_2 = \rho_\gamma(1-\delta)$ with probability one and the equilibrium payoffs are $M_i(G_i^*, G_j^*) = (1-\delta)k_i$ for $i \in \{1,2\}$.
- Region 2 $\left\{\mathcal{H}(k_1, k_2) : \frac{k_i}{\gamma} \leq \hat{b}\left(\frac{k_j}{\gamma}\right) \forall i \in \{1,2\}\right\}$: in this region banks operate at full capacity. The subgame equilibrium is $r_1 = r_2 = r^{FC}$ with probability one and the equilibrium payoffs are $M_i(G_i^*, G_j^*) = m(r^{FC})\frac{k_i}{\gamma}$ for $i \in \{1,2\}$.

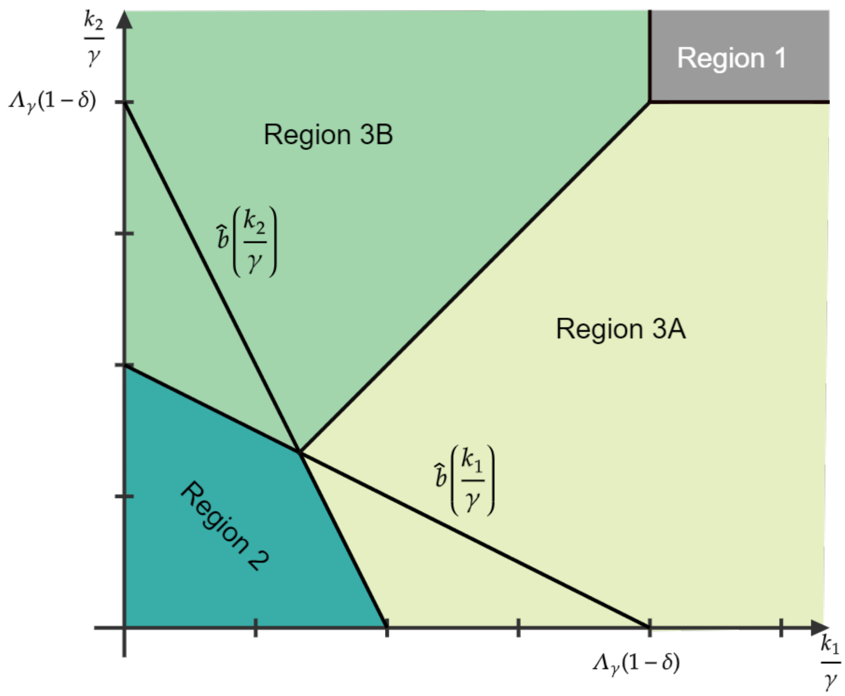

Figure 1: Equilibrium regions of the baseline model. The axes represent the banks' capacities, with each point corresponding to a specific subgame. The lines $\hat{b}(\cdot)$ are the best response functions of the auxiliary Cournot game, and $\Lambda_\gamma(1-\delta)$is the threshold such that, if both banks' capacities lie above it, capacity constraints become irrelevant. In Region 1 the equilibrium is Bertrand with marginal cost $(1-\delta)$. Region 2 is the full capacity equilibrium region. Region 3A and 3B are the mixed strategy equilibrium regions.

- Region 3A $\left\{\mathcal{H}(k_1,k_2) : k_1 \geq k_2 \text{ and } \frac{k_1}{\gamma} > \hat{b}\left(\frac{k_2}{\gamma}\right)\right\}$: in this region there is a mixed strategy equilibrium which has the characteristics described by Lemma 4. The equilibrium payoffs are $M_1(G_1^*, G_2^*) = P(k_2)+(1-\delta)k_1$ and $P(k_2)\frac{k_2}{k_1}+(1-\delta)k_2 \leq M_2(G_2^*, G_1^*) \leq P(k_2)+(1-\delta)k_1$
- Region 3B: symmetric to Region 3A

Note that stage 2 payoffs are continuous functions of k_1 and k_2.

3.2 First stage: capital choice

Recall, from Stage 1 perspective, bank i maximizes its overall profits:

$$\max_{\mu_i(k_i)\in\mathcal{S}_k}\left\{\pi(\mu_i,\mu_j) = \int_{\underline{k}_i}^{\bar{k}_i}\int_{\underline{k}_j}^{\bar{k}_j}\left(M_i^*(k_i,k_j) - k_i\right)d\mu_j(k_j)d\mu_i(k_i)\right\}$$

where $M_i^*(k_i,k_j)$ is the expected equilibrium payoff of $\mathcal{H}(k_1,k_2)$.

Proposition 1. *Under conditions 1 and 2, the Cournot outcome, $k_1 = k_2 = \gamma l^C$ and $r_1 = r_2 = r^C$, is the unique subgame perfect equilibrium of the two-stage game.*

Proof. See Appendix for a formal proof. Below I provide a sketch of the proof. □

For the sketch of the proof consider just pure strategies.

- Region 1: bank i profits are given by $\pi_i(k_i,k_j) = (1-\delta)k_i - k_i = -\delta k_i$. In this region the bank has raises too much capital. As paying dividends is costly, from a stage 1 perspective, the bank is better off by raising less capital. Hence any (k_1,k_2) that belong to region 1 cannot be a SPNE.
- Region 2: bank i profits are given by $\pi_i(k_i,k_j) = m(r^{FC})\frac{k_i}{\gamma} - k_i = \left(Z\left(\frac{k_i+k_j}{\gamma}\right) - \gamma\right)\frac{k_i}{\gamma}$. In this region banks operate at full capacity and charge the full capacity rate, hence the stage 1 strategic choice of capacity is equivalent to the strategic choice of quantity. The only possible subgame perfect equilibrium in this region the Cournot equilibrium $(k_1^*, k_2^*) = (\gamma l^C, \gamma l^C)$[6].

[6]Note that the Cournot equilibrium belongs to this region because $\hat{b}(l) \leq b(l)$ for all l.

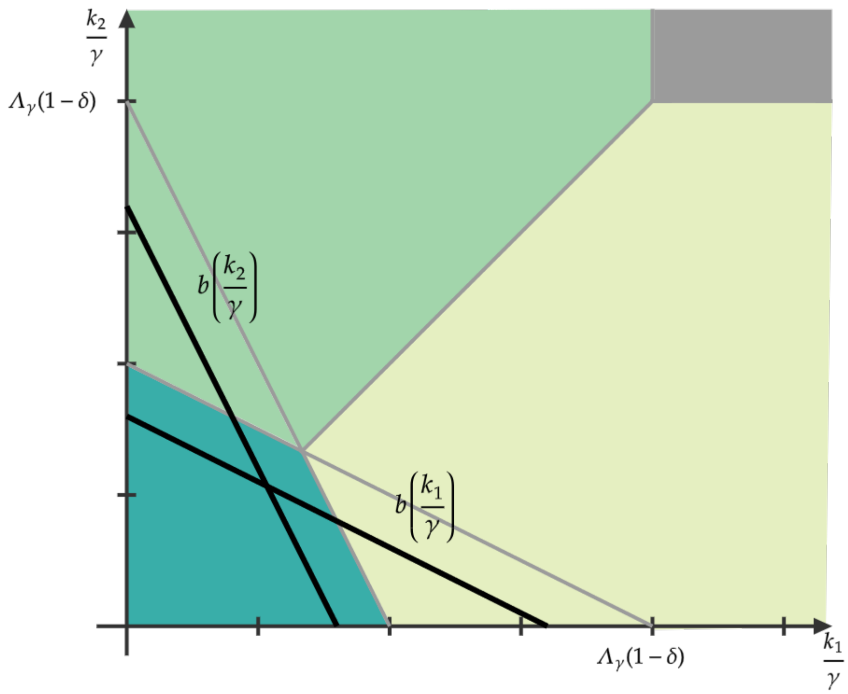

Figure 2: Full game equilibrium. The axes represent the banks' capacities, with each point corresponding to a specific subgame. The black lines $b(\cdot)$ are the best response functions of the equivalent one-stage Cournot game. The intersection of these lines is the SPNE of the game. The grey lines are the best response functions of the auxiliary Cournot game.

- Region 3A: bank 1 profits are equal to $\pi_1(k_1, k_2) = P(k_2) - \delta k_1$. Bank 1's profits are decreasing in k_1, hence bank 1 is better off by raising less capital. Note that this is true also at the border of the region when $k_1 = k_2$. The intuition is that in this region bank 1 has raised too much capital and has to pay dividends in expectation. A positive cost for paying dividends rules out the possibility of having an equilibrium in this region and ensures the uniqueness of the SPNE.

- Region 3B: symmetric to 3A.

Therefore the only subgame perfect equilibrium is $(k_1^*, k_2^*) = (\gamma l^C, \gamma l^C)$, in stage 2 $r_1 = r_2 = r^{FC} = r(2l^C)$ with probability one.

4 Modeling the banking sector: When can we assume Cournot competition?

The key conditions that must hold in order to assume Cournot competition are that (i) the average expected residual cashflow must be increasing in own rate; (ii) a bank's own default rate must not depend on the interest rate charged by the opponent. Clearly, Cournot competition can be assumed when risk is exogenous $\left(\frac{\partial F}{\partial r}=0\right)$ (e.g. Villa, 2023; Corbae & D'Erasmo, 2021; Bahaj & Malherbe, 2020). In this section I show that moral hazard can be embedded in this model, whereas adverse selection setups are often not consistent with the proposed microfoundations.

4.1 Moral Hazard

Consider a modified version of Boyd & De Nicolò (2005). Due to limited liability, when entrepreneurs face a higher loan rate, they choose a riskier project or exert less effort, hence the probability of default is increasing in loan rate. Entrepreneurs choose among projects that require one unit of investment and have the following return function:

$$X = \begin{cases} 1+\alpha(p) & \text{with prob. } (1-p) \\ 0 & \text{with prob. } p \end{cases}$$

Therefore entrepreneurs optimally pick a project by choosing the probability of default p. Assume $\alpha(p)$ to be continuous, increasing and strictly concave. Each entrepreneur t has her outside option $\bar{u}_t$ and solves the following problem

$$u(r) = \max_{p\in[0,1]} (1-p)(\alpha(p)-r)$$
$$\text{such that } u(r) \geq \bar{u}_t$$

In order to have an interior solution I further assume that $\alpha(0) - \alpha'(0) < r < \alpha(1)$. The first order condition is given by:

$$r - \alpha(p^*) + (1 - p^*)\alpha'(p^*) = 0$$

By the implicit function theorem

$$\frac{dp^*}{dr} = \frac{1}{2\alpha'(p^*) - (1 - p^*)\alpha''(p^*)} > 0$$

When charged a higher rate, entrepreneurs choose projects with higher probability of default. For simplicity assume that all loans are perfectly correlated. The expected average residual cashflow is given by:

$$m(r) = (1 - p^*(r))(r + \gamma))$$

The monotonicity condition demands $m(r)$ increasing, i.e.

$$m'(r) = -\frac{dp^*}{dr}(r + \gamma) + (1 - p^*(r)) > 0$$
$$\Leftrightarrow \frac{dp^*}{dr} < \frac{1 - p^*(r)}{r + \gamma}$$

This inequality implies that the probability of default must not be too sensitive to a marginal increase in the loan rate. For instance, if $p(r) = a + br$, then:

$$m'(r) = -b(r + \gamma) + 1 - a - br > 0$$
$$\Leftrightarrow b < \frac{1 - a}{2r + \gamma} \leq \frac{1 - a}{\gamma}$$

In conclusion, provided that the marginal residual cashflow is increasing in the loan rate, Cournot competition can be justified in frameworks that entail moral hazard (Martinez-Miera & Repullo, 2010; Schliephake, 2016; Gasparini, 2023; Corbae & Levine, 2025).

4.2 Heterogeneous borrowers, adverse or favorable selection and screening

Heterogeneity of borrowers' types requires a thorough discussion of the rationing rule. Recall the assumption on efficient rationing: when demand exceeds capacity, borrowers that have a higher reservation rate are served. Therefore, it is essential to ensure that willingness to pay is not perfectly correlated with the borrower's type. Otherwise the rationing rule would contradict the fact that borrowers' type is private information. Take a simplified version of De Meza & Webb (1987). A borrower of type t has a project that requires a unitary investment returns $(1+a)$ with probability p_t and 0 otherwise. Also assume in that all borrowers have the same outside option $\bar{u}$ constant. For a given loan rate r, a borrower of type t accepts the loan if

$$\begin{aligned} p_t(a-r) &\geq \bar{u} \\ &\Rightarrow r \leq a - \frac{\bar{u}}{p_t} \end{aligned}$$

The reservation rate perfectly predict the type, hence this would be incompatible with the efficient rationing rule. Now assume that the outside option is stochastic and depends on the type $\bar{u}_t$. For simplicity, also assume that there are two types $t \in \{H, L\}$, $\Pr(t = H) = \lambda$ and $p_H > p_L$. Type t is willing to accept the loan rate r if

$$r \leq a - \frac{\bar{u}_t}{p_t} \equiv \xi_t$$

Let $\xi_t \sim G(\xi|t)$ and assume that $G(\xi|t)$ has a monotone hazard rate, i.e.

$$\frac{g(\xi|t)}{1 - G(\xi|t)} \qquad \text{is monotonic in } t$$

where $g(\xi|t) = G'(\xi|t)$ is the probability density function. If it is monotonically decreasing, H-types are on average more willing to pay and adverse selection is modeled à la De Meza & Webb

(1987), if it is increasing L-types are more willing to pay as in Stiglitz & Weiss (1981). In this case, the willingness to pay does not fully reveal the type but induces a distribution over the types. Therefore there is no inconsistency between the rationing rule and borrowers' private information. However, independently on how we model adverse selection, whenever the correlation between type and the reservation rate is different from zero, the independence condition is violated. Take the case of DeMeza and Webb: a bank has always the incentive to be the cheapest one as it selects a better pool of borrowers in expectations.

To make the independence condition hold, very specific assumptions are needed. For example, assume that $\bar{u}_t = p_t \bar{u}$, where $\bar{u} \sim G$ and it is independent of the type, in this case a borrower of type t accepts the loan if

$$r \leq a - \frac{p_t \bar{u}}{p_t} = a - \bar{u}.$$

Thanks to this assumption, the reservation rate is independent of borrower type. However, it also implies that the rate charged by the bank does not affect the composition of the borrower pool, effectively eliminating any adverse or favorable selection mechanism. Moreover, if we add a screening technology, the independence condition is violated again. In particular, if banks receive uncorrelated signals, there is always an incentive to undercut the opponent. The reason is that the most expensive bank draws from a worse distribution as it includes borrowers that have been rejected by the cheaper bank[7]. One way to avoid the breakdown of the equilibrium is to assume that banks see the same signal (e.g. open banking). Therefore, to include heterogeneous borrowers in the model, one must make strong assumptions: borrower's willingness to pay must be independent of its type and any signal about the borrowers' type must be public (or perfectly correlated across banks).

[7]For a formal proof see Broecker, 1990; Marquez, 2002

5 Capital adjustment costs

In this section I first relax the assumption about the short-term capital premium $\kappa = +\infty$ and set out the condition under which the Cournot equilibrium is still the unique SPNE of the two-stage game. Second, I discuss the assumptions on dividends payment and provide an alternative setup to allow $\delta = 0$.

5.1 Raising more capital

Consider the same game of the baseline model, but allow $\kappa < \infty$. In this game, when the capital requirement is binding and borrowers are rationed, banks can decide to raise more capital and serve the demand it is facing. In the IO literature, Boccard and Wauthy (2000; 2004) extend KS by allowing firms to build extra capacity in the competition stage at a premium cost. They show that if the Cournot price is lower than the short-term premium, the Cournot outcome is still the unique SPNE of the two-stage game. Whereas if the short term premium is lower than the Cournot price, the subgame becomes a standard Bertrand competition with marginal cost equal to the short term premium. They seem to implicitly assume some parametric restrictions for the short-term premium, which however does not change the core of the reasoning. In the model notation:

Condition 3. $\rho_\gamma(1+\kappa) > \max\left\{r(\hat{b}(0)), r^C\right\}$.

Where $r(\hat{b}(0))$ represents the monopolist rate in the auxiliary Cournot setting. Note that also in this case the existence of an equilibrium in every subgame is guaranteed by Theorem 5 of Dasgupta & Maskin (1986).

Lemma 5. *In subgames $\mathcal{H}(k_1, k_2)$ such that $\frac{k_1+k_2}{\gamma} < \Lambda_\gamma(1+\kappa)$, the unique subgame equilibrium is $r_1 = r_2 = \rho_\gamma(1+\kappa)$ with probability one.*

Proof. See Appendix for a formal proof. The intuition is that $\frac{k_1+k_2}{\gamma} < \Lambda_\gamma(1+\kappa)$ implies $r^{FC} \geq \rho_\gamma(1+\kappa)$, hence at the full capacity rate banks find it optimal undercut the opponent and ex-

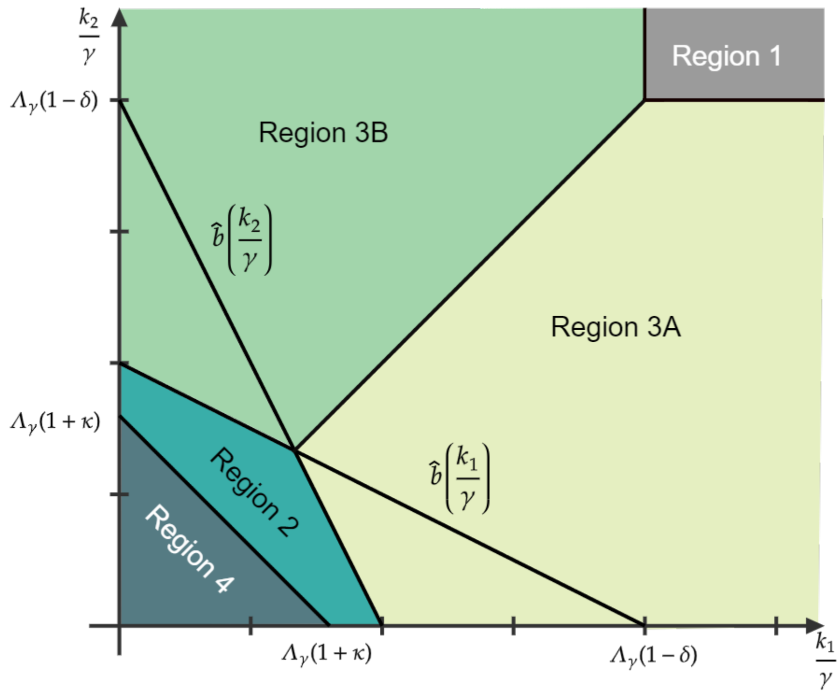

Figure 3: Equilibrium regions with capital increase in the second stage. The axes represent the banks' capacities, with each point corresponding to a specific subgame. The curves $\hat{b}(\cdot)$ are the best response functions of the auxiliary Cournot game, and $\Lambda_\gamma(1-\delta)$is the threshold such that, if both banks' capacities lie above it, capacity constraints become irrelevant. In Region 1 the equilibrium is Bertrand with marginal cost $(1-\delta)$. Region 2 is the full capacity equilibrium region. Region 3A and 3B are the mixed strategy equilibrium regions. In Region 4 the equilibrium is Bertrand with marginal cost $(1+\kappa)$. This region is delimited by $\Lambda_\gamma(1+\kappa)$: if the sum of capacities is below this threshold, the sub-game falls into Region 4.

pand their capacity. The typical demand-stealing mechanism of Bertrand competition is restored. Therefore, banks undercut each other until they make zero profits, i.e. $r_1 = r_2 = \rho_\gamma(1+\kappa)$. $\square$

Lemma 6. *In subgames $\mathcal{H}(k_1, k_2)$ such that $\frac{k_1+k_2}{\gamma} \geq \Lambda_\gamma(1+\kappa)$, lemmas 2 to 4 hold.*

Proof. See Appendix. $\square$

The intuition is that when $\frac{k_1+k_2}{\gamma} \geq \Lambda_\gamma(1+\kappa)$, which implies $r^{FC} \leq \rho_\gamma(1+\kappa)$, the possibility to raise more capital does not create any profitable deviation in the subgames equilibria found in the baseline game. The condition $\rho_\gamma(1+\kappa) > r(\hat{b}(0))$ is necessary to show that Lemma 4 is robust to short-term capital expansions. Specifically, it ensures that the equilibrium payoffs are solely determined by (k_1, k_2) and are the same of the baseline game.

Now we can divide the subgames into four relevant regions. Regions 1 and 3A/B are the same of the baseline game with the same payoffs. Region 2 has the same equilibrium strategies and payoffs of the baseline but it is now delineated by $\left\{\mathcal{H}(k_1,k_2) : \frac{k_1+k_2}{\gamma} \geq \Lambda_\gamma(1+\kappa) \text{ and } \frac{k_i}{\gamma} \leq \hat{b}\left(\frac{k_j}{\gamma}\right) \forall i = 1,2\right\}$. Finally in Region 4, defined as $\left\{\mathcal{H}(k_1,k_2) : \frac{k_1+k_2}{\gamma} < \Lambda_\gamma(1+\kappa)\right\}$, the equilibrium strategies are $r_1^* = r_2^* = \rho_\gamma(1+\kappa)$ and $e_i^* = \gamma\frac{\Lambda_\gamma(1+\kappa)}{2} - k_i$; the equilibrium payoffs $(1+\kappa)k_i$ for $i = 1,2$. Also in this case, payoffs are continuous in (k_1,k_2).

Proposition 2. *Under conditions 1-3, the Cournot outcome is the only SPNE of the two stage game.*

Proof. For a formal proof see Appendix. Below I provide a sketch of the proof. □

The only additional step with respect to the baseline model is to prove that there cannot be any SPNE in Region 4. In in this region, $\pi_i(k_i,k_j) = \kappa k_i$, hence both banks have the incentive to increase capital. The intuition is that banks anticipate that they are going to expand capacity in the second stage. As raising capital in the short term is more costly, hence they are better-off by raising more capital in stage 1.

5.2 Paying dividends

In the baseline model I assume that the cost of paying dividends δ must be positive. A positive cost is necessary to rule out multiple equilibria. Alternatively, it is possible to assume that in the first stage bank capital requires a premium $r_K > 0$ and banks can pay dividends at no cost. In this way is costly for the bank to raise excessive capital in stage 1. I also assume that the cost of paying dividends must be small enough so that the bank never wants to have capital in excess of the requirement. If δ is too high or banks just cannot pay out dividends ($\delta \to \infty$) the game needs simplifying assumptions in order to become tractable. The reason is that the amount of bank capital not only determines the lending capacity of the bank, but also the marginal cost of issuing loans. If $l_i \leq k_i$, the marginal cost of issuing loans is zero as the bank does not need to raise deposits. If $l_i > k_i$, the bank needs to raise deposits to finance its loans. Because of the deposit

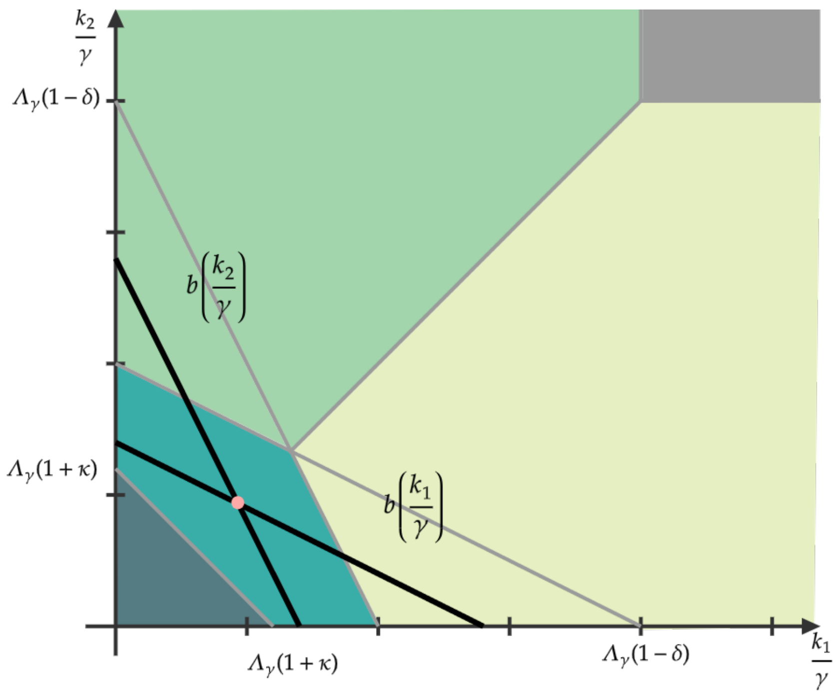

Figure 4: Full game equilibrium with capital increase in the second stage. The axes represent the banks' capacities, with each point corresponding to a specific subgame. The black lines $b(\cdot)$ are the best response functions of the equivalent one-stage Cournot game. The intersection of these lines is the SPNE of the game. The grey lines are the best response functions of the auxiliary Cournot game.

δ	$\to \infty$	with no risk, bank loans as imperfect substitutes and linear demand Schliephake & Kirstein (2013)
	> 0 but small*	this paper
	$= 0$	multiplicity of equilibria (without additional assumptions)
κ	$\to \infty$	baseline game (for firms: Kreps & Scheinkman (1983))
	$< \infty$ but large**	this paper (for firms: Boccard and Wauthy (2000; 2004))
	$= 0$	standard Bertrand competition

Table 1: This table summarizes how different values for the parameters of the model map to the literature.* small enough so that banks are always willing to pay back dividends when the capital requirement constraint is slack.**sufficiently large to make Condition 3 hold.

guarantee, the marginal cost depends on leverage: the more leveraged is a bank, the cheaper the deposits[8]. Whereas when banks can pay out dividends, banks' leverage will just depend on the capital requirement and the marginal cost of issuing loans is constant and independent of the initial capital raised. Schliephake & Kirstein (2013) show that the Cournot outcome is SPNE in a tractable model in which banks are not allowed to pay dividends, issue risk-free differentiated loans, and loan demand is linear.

6 Conclusion

Cournot competition in the banking sector can be microfounded through a two-stage game in which banks first choose capital and then compete à la Bertrand, subject to capital requirements. Three key conditions must be satisfied: (i) the average marginal residual cashflow must be increasing in the loan rate; (ii) in the two-stage game, the distribution of default rates must depend only on the bank's own rate and not on its competitor's; and (iii) the short-term premium must be sufficiently high. When these conditions hold, banks behave as if they were competing in quantities, leading to a Cournot outcome in equilibrium. Cournot provides tractability and modeling flexibility. This paper establishes the key conditions under which Cournot competition can be microfounded in

[8]Recall: from the bank's perspective, the cost of deposits is the risk-free rate times the probability of survival of the bank

the banking sector. At the same time, these conditions delineate its limitations, thereby clarifying the scope of its applicability.

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Appendix

Proposition 1: when $b(l_j)$ it is positive it must satisfy the following first order condition:

$$Z'(b(l_j)+l_j)b(l_j)+Z(b(l_j)+l_j)-\gamma=0$$

Given the equation above, the best response function has the following properties:

[a] $b(l_j)$ is strictly decreasing : by the implicit function theorem

$$\frac{db(l_j)}{dl_j}=-\frac{Z'(b(l_j)+l_j)+Z''(b(l_j)+l_j)b(l_j)}{2Z'(b(l_j)+l_j)+Z''(b(l_j)+l_j)b(l_j)}<0$$

[b] $b'(l_j)>-1$: increase l_j by ϵ and decrease $b(l_j)$ by the same amount. The FOC is equal to:

$$\begin{aligned}&Z'(b(l_j)+l_j)(b(l_j)-\epsilon)+Z(b(l_j)+l_j)-\gamma\\&=\underbrace{Z'(b(l_j)+l_j)b(l_j)+Z(b(l_j)+l_j)-\gamma}_{=0}-Z'(b(l_j)+l_j)\epsilon\\&=-Z'(b(l_j)+l_j)\epsilon>0\end{aligned}$$

Hence it must be that $b'(l_j) < -1$.

[c] If $l_j > b(l_j)$, then $b(b(l_j)) < l_j$: set $l_j = b(l_j)$ and $b(l_j) = l_j$ and evaluate the FOC:

$$\begin{aligned} & Z'(b(l_j) + l_j)l_j + Z(b(l_j) + l_j) - \gamma \\ = \; & Z'(b(l_j) + l_j)(l_j + b(l_j) - b(l_j)) + Z(b(l_j) + l_j) - \gamma \\ = \; & Z'(b(l_j) + l_j)(l_j - b(l_j)) < 0 \end{aligned}$$

as $l_j > b(l_j)$ by hypothesis. This implies that the best response to $b(l_j)$ is smaller than l_j, i.e. $b(b(l_j)) < l_j$.

The last property ensures that $b(l_j)$ is a contraction and therefore has a unique fixed point.

Capital requirement binding in Stage 2. Given any triple (k_i, l_i, r_i) if $k_i > \gamma l_i$, bank i prefers to pay dividends and raise more deposits to make the capital requirement binding

$$\int_0^{\tilde{x}} \left((1-x)(1+r_i) - (1-\gamma)\right) dF(x|r_i) l_i + (1-\delta)(k_i - \gamma l_i) \geq \int_0^{\check{x}} \left((1-x)(1+r_i) l_i - (l_i - k_i)^+\right) dF(x|r_i)$$

where $\check{x} = \frac{(1+r_i)l_i - (l_i - k_i)^+}{(1+r_i)l_i}$ and $(y)^+ = \max\{0, y\}$. It is always possible to have a positive but arbitrarily close to zero δ that make the inequality above true. Re-arranging:

$$(1-\delta)(k_i - \gamma l_i) \geq (1+r_i) l_i \int_{\tilde{x}}^{\check{x}} F(x|r_i) dx$$

The RHS is *strictly* smaller than $(1+r_i)l_i(\check{x} - \tilde{x}) = (k_i - \gamma l_i)$, hence there exists a $\bar{\delta} > 0$, such that

$$(1-\bar{\delta})(k_i - \gamma l_i) = (1+r_i) l_i \int_{\tilde{x}}^{\check{x}} F(x|r_i) dx$$

Hence for any $\delta < \bar{\delta}$ the inequality holds.

Lemma 3: WLOG let $k_1 \geq k_2$. By hypothesis $\bar{r}_1 = \bar{r}_2 = \bar{r}$. Now suppose $\bar{r} > r^{FC}$. Bank 1 would have a profitable deviation to name a rate that is lower but arbitrarily close to $\bar{r}$

$$\lim_{\epsilon \downarrow 0} M_1(\bar{r}-\epsilon, G_j) - M_1(\bar{r}, G_j) = \alpha_2(\bar{r})(m(\bar{r})-\gamma(1-\delta)) \left[\min\left(\frac{k_1}{\gamma}, L(\bar{r})\right) - \max\left(\frac{L(\bar{r})}{2}, L(\bar{r}) - \frac{k_j}{\gamma}\right)\right] > 0$$

Hence it must be that $\bar{r} \leq r^{FC}$. Now I prove the second part of the lemma. By lemma 1, it must be that $\underline{r}_i = \bar{r}_i = r^{FC}$ for all i. Then if bank i names a rate $r > r^{FC}$, its payoff must be equal to

$$(m(r) - \gamma(1-\delta))\left(L(r) - \frac{k_j}{\gamma}\right) + (1-\delta)k_i$$

Let $l_i = L(r) - \frac{k_j}{\gamma}$, then it is equivalent to maximize $\left(Z\left(l_i + \frac{k_j}{\gamma}\right) - \gamma(1-\delta)\right) l_i$. By definition it is maximized at $l_i = \hat{b}\left(\frac{k_j}{\gamma}\right)$, hence it must be that $\frac{k_i}{\gamma} \leq \hat{b}\left(\frac{k_j}{\gamma}\right)$, otherwise bank i would have a profitable deviation.

Lemma 4: WLOG assume $\bar{r}_1 > \bar{r}_2$. Before proceeding I must prove that $k_2 \geq \gamma\Lambda_\gamma(1-\delta)$ is incompatible with the hypotheses of the lemma. By hypothesis, $\min_i \frac{k_i}{\gamma} < \Lambda_\gamma(1-\delta)$, hence if $k_2 > \Lambda_\gamma(1-\delta)$, then $k_1 < \Lambda_\gamma(1-\delta)$. By naming $r \in \left(\rho_\gamma(1-\delta), r\left(\frac{k_1}{\gamma}\right)\right)$ bank 2 gets a payoff that is strictly higher than $(1-\delta)k_2$. Hence in equilibrium it must be that $\bar{r}_2 > \rho_\gamma(1-\delta)$. However if $\bar{r}_1 > \bar{r}_2$, it implies that when bank 1 names $\bar{r}_1$, the residual demand is always equal to zero and $M_1(\bar{r}_1, G_2) = (1-\delta)k_1$. However this cannot be part of an equilibrium as bank 1 has the profitable deviation to name any rate $r \in (\rho_\gamma(1-\delta), \bar{r}_2)$.

For (a) and (b): consider the function

$$\phi(r) = (m(r) - \gamma(1-\delta)) \max\left(0, L(r) - \frac{k_j}{\gamma}\right)$$

By naming any rate $r \geq \bar{r}_1$, bank 1 gets $M_1(r, G_2) = \phi(r) + (1-\delta)k_1$, hence it must be that $\phi(r)$ is maximized at $\bar{r}_1$. In order to maximize $\phi(r)$, bank 1 should choose r such that $\frac{k_2}{\gamma} \leq L(r) \leq \frac{k_1+k_2}{\gamma}$. For any level of r there is a loan quantity, namely $l(r) = L(r) - \frac{k_2}{\gamma}$, such that

$\phi(r) = \left(Z\left(l(r) + \frac{k_2}{\gamma}\right) - \gamma(1-\delta)\right) l(r)$. Picking r to maximize $\phi(r)$ is equivalent to maximize:

$$\max_{l \in \left[0, \frac{k_1}{\gamma}\right]} \left(Z\left(l + \frac{k_2}{\gamma}\right) - \gamma(1-\delta)\right) l$$

This is maximized at $\min\left(\frac{k_1}{\gamma}, \hat{b}\left(\frac{k_2}{\gamma}\right)\right)$, if the capital requirement binds we are in the case of Lemma 3, which is incompatible with the hypothesis of this lemma, hence it must be that $\frac{k_1}{\gamma} > \hat{b}\left(\frac{k_2}{\gamma}\right)$ and $\bar{r}_1 = r\left(\hat{b}\left(\frac{k_2}{\gamma}\right) + \frac{k_2}{\gamma}\right)$.

(c) Suppose that $\underline{r}_i < \underline{r}_j$. By naming $\underline{r}_i$ bank i gets $M_i(\underline{r}_i, G_j) = (m(\underline{r}_i) - \gamma(1-\delta)) \min\left(\frac{k_i}{\gamma}, L(\underline{r}_i)\right) + (1-\delta)k_i$. Clearly if $L(\underline{r}_i) > \frac{k_i}{\gamma}$, then the payoff is strictly increasing in $r$and bank i would have the profitable deviation to name $\underline{r}_i + \epsilon$; if $L(\underline{r}_i) < \frac{k_i}{\gamma}$, it must be that $\underline{r}_i = r(\hat{b}(0))$ otherwise bank i would have a profitable deviation. However $\underline{r}_i \leq \bar{r}_1 = r\left(\hat{b}\left(\frac{k_2}{\gamma}\right) + \frac{k_2}{\gamma}\right) < r(\hat{b}(0))$, therefore it cannot be an equilibrium. It must be that $\underline{r}_1 = \underline{r}_2 = \underline{r}$. Note that $\underline{r} > r^{FC}$, otherwise for bank 1 would be profitable to deviate and name $r\left(\hat{b}\left(\frac{k_2}{\gamma}\right) + \frac{k_2}{\gamma}\right)$. Now I prove that $\alpha_i(\underline{r}) = 0$ for all $i \in \{1, 2\}$. Let i denote the bank that has (weakly) more capital[9] and bank j the bank that has (weakly) less capital. Suppose bank j names $\underline{r}$ with positive probability. Then bank i prefers to name a rate that is smaller but arbitrarily close to $\underline{r}$

$$\lim_{\epsilon \downarrow 0} M_i(\underline{r} - \epsilon, G_j) - M_i(\underline{r}, G_j) = \alpha_j(\underline{r})(m(\underline{r}) - \gamma(1-\delta)) \underbrace{\left(\min\left(\frac{k_i}{\gamma}, L(\underline{r})\right) - \max\left(\frac{L(\underline{r})}{2}, L(\underline{r}) - \frac{k_j}{\gamma}\right)\right)}_{>0}$$

Therefore it must be that $\alpha_j(\underline{r}) = 0$. Bank j names $\underline{r}$ with zero probability, however $\underline{r}$ is the infimum of the support, hence it must be that bank j names a rate that is arbitrarily close and above $\underline{r}$ but not exactly $\underline{r}$

$$M_j(\underline{r}, G_i) - \lim_{\epsilon \downarrow 0} M_j(\underline{r} + \epsilon, G_i) = \alpha_i(\underline{r})(m(\underline{r}) - \gamma(1-\delta)) \underbrace{\left(\min\left(\frac{k_j}{\gamma}, \frac{L(\underline{r})}{2}\right) - \max\left(0, L(\underline{r}) - \frac{k_i}{\gamma}\right)\right)}_{>0}$$

Hence it must be that $\alpha_i(\underline{r}) = 0$.

[9] I still have to prove that this is bank 1.

(d) $\underline{r} \leq \bar{r}_1 = r\left(\hat{b}\left(\frac{k_2}{\gamma}\right) + \frac{k_2}{\gamma}\right)$, implies $L(\underline{r}) \geq \hat{b}\left(\frac{k_2}{\gamma}\right) + \frac{k_2}{\gamma} > \frac{k_2}{\gamma}$. Hence the equilibrium payoff of bank 2 must be equal to $m(\underline{r})\frac{k_2}{\gamma}$. Now suppose $k_2 > k_1$, it must be that the equilibrium payoff of bank 1 is equal to $m(\underline{r})\frac{k_1}{\gamma}$. By part (a) we also know that the equilibrium payoff of bank 1 is equal to $P(k_2) + (1-\delta)k_1$, which implies that $m(\underline{r}) = P(k_2)\frac{\gamma}{k_1} + \gamma(1-\delta)$. The payoff of bank 2 can be re-written as $P(k_2)\frac{k_2}{k_1} + (1-\delta)k_2$. If bank 2 names $r = r\left(\hat{b}\left(\frac{k_1}{\gamma}\right) + \frac{k_1}{\gamma}\right) > \bar{r}_1$, it gets $P(k_1) + (1-\delta)k_2$. Therefore if $P(k_1) + (1-\delta)k_2 > P(k_2)\frac{k_2}{k_1} + (1-\delta)k_2$, which can be re-written as $k_1P(k_1) > k_2P(k_2)$, bank 2 has a profitable deviation and $k_2 > k_1$ contradicts the hypotheses of the lemma. Define the function $\psi(k) = kP(k) = k\left(Z\left(\hat{b}\left(\frac{k}{\gamma}\right) + \frac{k}{\gamma}\right) - \gamma(1-\delta)\right)\hat{b}\left(\frac{k}{\gamma}\right)$ and compute the derivative

$$\psi'(k) = \left(Z\left(\hat{b}\left(\frac{k}{\gamma}\right) + \frac{k}{\gamma}\right) - \gamma(1-\delta)\right)\left(\hat{b}\left(\frac{k}{\gamma}\right) - \frac{k}{\gamma}\right)$$

Hence:

$$\psi(k_2) - \psi(k_1) = \int_{k_1}^{k_2} \left(Z\left(\hat{b}\left(\frac{k}{\gamma}\right) + \frac{k}{\gamma}\right) - \gamma(1-\delta)\right)\left(\hat{b}\left(\frac{k}{\gamma}\right) - \frac{k}{\gamma}\right) dk$$

Bank 2 has a profitable deviation if the expression above is negative. As $\hat{b}(\cdot)$ is decreasing, this integral is more likely to be positive when k_2 is as small as possible. From (b) we know that $k_2 > \gamma\hat{b}^{-1}\left(\frac{k_1}{\gamma}\right)$, hence:

$$\psi(k_2) - \psi(k_1) < \psi\left(\gamma\hat{b}^{-1}\left(\frac{k_1}{\gamma}\right)\right) - \psi(k_1)$$

$$= k_1\left(\left(Z\left(\hat{b}^{-1}\left(\frac{k_1}{\gamma}\right) + \frac{k_1}{\gamma}\right) - \gamma(1-\delta)\right)\hat{b}^{-1}\left(\frac{k_1}{\gamma}\right) - \left(Z\left(\hat{b}\left(\frac{k_1}{\gamma}\right) + \frac{k_1}{\gamma}\right) - \gamma(1-\delta)\right)\hat{b}\left(\frac{k_1}{\gamma}\right)\right) \leq 0$$

The term above is negative because by definition $\left(Z\left(l + \frac{k_1}{\gamma}\right) - \gamma(1-\delta)\right)l$ is maximized at $\hat{b}\left(\frac{k_1}{\gamma}\right)$. Therefore it must be that $k_1 \geq k_2$.

Finally (e): from part (a), (c) and (d) we know that

$$m(\underline{r})\frac{k_2}{\gamma} \leq (m(\underline{r}) - \gamma(1-\delta))\min\left(\frac{k_1}{\gamma}, L(\underline{r})\right) + (1-\delta)k_1 = P(k_2) + (1-\delta)k_1$$

Hence in equilibrium bank 2 can get at most $P(k_2)+(1-\delta)k_1$. We also know that $P(k_2)+(1-\delta)k_1 =$

$(m(\underline{r}) - \gamma(1-\delta)) \min\left(\frac{k_1}{\gamma}, L(\underline{r})\right) + (1-\delta)k_1 \leq m(\underline{r})\frac{k_1}{\gamma}$, which implies $m(\underline{r}) \geq P(k_2)\frac{\gamma}{k_1} + \gamma$. Hence $m(\underline{r})\frac{k_2}{\gamma} \geq P(k_2)\frac{k_2}{k_1} + (1-\delta)k_2$.

Proposition 1: WLOG let $\bar{k}_1 \geq \bar{k}_2$.

- (Step 1) In equilibrium it must be that $\bar{k}_1/\gamma \geq b(\underline{k}_2/\gamma)$. Suppose not: $\bar{k}_1/\gamma < b(\underline{k}_2/\gamma)$, which implies that $b(\bar{k}_1/\gamma) > b(b(\underline{k}_2/\gamma))$. As $\underline{k}_2 \leq \bar{k}_1$, it must be that $\underline{k}_2/\gamma < b(\underline{k}_2/\gamma)$. Then it must be that $b(b(\underline{k}_2/\gamma)) > \underline{k}_2/\gamma$[10]. By transitivity, $\underline{k}_2/\gamma < b(\bar{k}_1/\gamma)$. Therefore when bank 2 raises $\underline{k}_2$ is for sure in Region 2:

$$\pi(\underline{k}_2, \mu_1) = \int_{\underline{k}_1}^{\bar{k}_1} \left(Z\left(\frac{k_1 + \underline{k}_2}{\gamma}\right) - \gamma\right)\frac{\underline{k}_2}{\gamma} d\mu_1(k_1)$$

  The profits are strictly increasing in $\underline{k}_2$ as $\underline{k}_2 < b(k_1/\gamma)$ for all $k_1 \in [\underline{k}_1, \bar{k}_1]$, hence bank 2 can profitably deviate and name $\underline{k}_2 + \epsilon$. Therefore it must be that $\bar{k}_1/\gamma \geq b(\underline{k}_2/\gamma)$.

- (Step 2) $\bar{k}_1/\gamma \leq b(\bar{k}_2/\gamma)$. Suppose not: $\bar{k}_1/\gamma > b(\bar{k}_2/\gamma)$. Then when bank 1 raises $\bar{k}_1$ is either in region 2 or in region 3A:

$$\pi(\bar{k}_1, \mu_2) = \int_{\underline{k}_2}^{\xi(\bar{k}_1)} \left(Z\left(\frac{\bar{k}_1 + k_2}{\gamma}\right) - \gamma\right)\frac{\bar{k}_1}{\gamma} d\mu_2(k_2) + \int_{\xi(\bar{k}_1)}^{\bar{k}_2} (P(k_2) - \delta k_1) d\mu(k_2)$$

  where $\xi(k) = \frac{1}{\gamma}\hat{b}^{-1}\left(\frac{k}{\gamma}\right)$. The profits are strictly decreasing in $\bar{k}_1$, in particular the first term is decreasing because $\bar{k}_1/\gamma \geq b(\underline{k}_2/\gamma) \geq b(k_2/\gamma)$ for all k_2 in the support. Therefore bank 1 would have the profitable deviation to raise $\bar{k}_1 - \epsilon$. Hence it must be that $\bar{k}_1/\gamma \leq b(\bar{k}_2/\gamma)$.

- (Step 3) The previous step imply that $\bar{k}_1/\gamma = b(\bar{k}_2/\gamma) = b(\underline{k}_2/\gamma)$, which implies that bank 2's equilibrium strategy is a pure strategy k_2; bank 1 best response to the pure strategy k_2 is $b\left(\frac{k_2}{\gamma}\right)$. In turn, bank 2 must best respond to that hence $\frac{k_2}{\gamma} = b\left(\frac{k_1}{\gamma}\right) = b\left(b\left(\frac{k_2}{\gamma}\right)\right)$. The unique solution is $k_1 = k_2 = \gamma l^C$, and in the second stage banks name $r^{FC} = r(2l^C) = r^C$ with probability one.

[10]See proof of Proposition 1 for the properties of $b(\cdot)$

Lemma 5: Recall that demand is rationed according to the efficient rule as in the baseline game. In particular, when banks name the same rate, they can raise more capital only if they cannot serve the entire demand collectively, i.e. $r_1 = r_2 = r$ and $L(r) > \frac{k_1+k_2}{\gamma}$.

(Step 1) $\underline{r}_i \geq \rho_\gamma(1+\kappa)$ for all $i \in \{1,2\}$. Suppose not and let $\underline{r}_i < \rho_\gamma(1+\kappa)$ for some i. By hypothesis $\rho_\gamma(1+\kappa) < r^{FC}$, hence when bank i names $\underline{r}_i$it gets:

$$M_i(\underline{r}_i, G_j) = m(\underline{r}_i)\frac{k_i}{\gamma}$$

Bank i is operating at full capacity and does not find it profitable to raise more capital as $\underline{r}_i < \rho_\gamma(1+\kappa)$. However $m(\cdot)$ is an increasing function, hence bank i would be better off by naming $\underline{r}_i + \epsilon$. Hence, this cannot be an equilibrium and it must be that $\underline{r}_i \geq \rho_\gamma(1+\kappa)$ for all $i \in \{1,2\}$.

(Step 2) $\bar{r}_i \leq \rho_\gamma(1+\kappa)$ for all $i \in \{1,2\}$. Suppose not and let $\bar{r}_i > \rho_\gamma(1+\kappa)$. WLOG divide the proof into two cases:

- $\bar{r}_i > \bar{r}_j$ or $\bar{r}_i = \bar{r}_j$ and $\alpha_j(\bar{r}_j) = 0$. By naming $\bar{r}_i$ bank i gets $M_i(\bar{r}_i, G_j) = (1-\delta)k_i$. As $\underline{r}_j \geq \rho_\gamma(1+\kappa)$, bank j will always find it profitable to raise more capital and supply the entire market, therefore bank i has no residual demand to serve. Bank i is better off by naming $\rho_\gamma(1+\kappa)$ and getting $(1+\kappa)k_i$.

- $\bar{r}_i = \bar{r}_j = \bar{r}$ and $\alpha_i(\bar{r}) > 0$ for all $i = 1, 2$. If $\rho_\gamma(1+\kappa) < \bar{r} < r^{FC}$:

$$M_i(\bar{r}, G_j) = \alpha_j(\bar{r})\left[m(\bar{r})\frac{L(\bar{r})}{2} + \eta\left(\frac{L(\bar{r})}{2}, k_i\right)\right] + (1-\alpha_j(\bar{r}))(1-\delta)k_i$$

  If bank i instead names $\bar{r} - \epsilon$:

$$\lim_{\epsilon \downarrow 0} M_i(\bar{r} - \epsilon) = \alpha_j(\bar{r})\left[m(\bar{r})L(\bar{r}) + \eta(L(\bar{r}), k_i)\right] + (1-\alpha_j(\bar{r}))(1-\delta)k_i$$

Hence:

$$\lim_{\epsilon \downarrow 0} M_i(\bar{r} - \epsilon) - M_i(\bar{r}, G_j) =$$
$$\alpha_j(\bar{r}) \left[m(\bar{r}) \frac{L(\bar{r})}{2} + \eta(L(\bar{r}), k_i) - \eta\left(\frac{L(\bar{r})}{2}, k_i\right)\right] > 0$$

Finally if $\bar{r} \geq r^{FC}$ both banks have incentives to undercut as in every standard Bertrand game.

Therefore it must be that $\bar{r}_i \leq \rho_\gamma(1+\kappa)$ for all $i \in \{1, 2\}$.

(Step 3) $\rho_\gamma(1+\kappa) \leq \underline{r}_i \leq \bar{r}_i \leq \rho_\gamma(1+\kappa)$, then that $r_i = \rho_\gamma(1+\kappa)$ with probability 1 for all $i \in \{1, 2\}$ is the only possible equilibrium (existence is guaranteed by Dasgupta & Maskin (1986), however is immediate to show that given the opponent's strategy there are no profitable deviations).

Lemma 6: Start with Lemma 2. The proof is the same of the baseline model as we are working under the hypothesis that $\frac{k_1+k_2}{\gamma} \geq \Lambda_\gamma(1+\kappa)$, which implies $r^{FC} \leq \rho_\gamma(1+\kappa)$. Lemma 3 follows exactly.

the possibility of expanding does not alter the first part of the proof. Hence, if $\bar{r}_1 = \bar{r}_2 = \bar{r}$ and $\alpha_i(\bar{r}) > 0$ for all $i \in \{1, 2\}$ it must be that $\bar{r} = \underline{r} = r^{FC}$. Banks do not have incentives to undercut the opponent, so we must check that there are no incentives to charge a higher rate. Given the opponent strategy $\alpha_j(r^{FC}) = 1$, bank i maximises its payoff:

$$\max_r G_j(\rho_\gamma(1+\kappa)) \left((m(r) - \gamma(1-\delta)) \min\left(\frac{k_i}{\gamma}, L(r) - \frac{k_j}{\gamma}\right)\right) + (1-\delta)k_i$$

the payoff is multiplied by $G_j(\rho_\gamma(1+\kappa))$ because if the opponent charges a rate higher than $\rho_\gamma(1+\kappa)$ it will serve the entire market. However $\alpha_j(r^{FC}) = 1$ and $\frac{k_1+k_2}{\gamma} \geq \Lambda_\gamma(1+\kappa)$, imply $G_j(\rho_\gamma(1+\kappa)) = 1$. The rest of the proof follows.

Finally. Lemma 4. WLOG of generality let $\bar{r}_1 > \bar{r}_2$. Before proceeding I must prove that $k_2 \geq \gamma\Lambda(\gamma(1-\delta))$ is incompatible with the hypotheses of the lemma. By hypothesis, $\min_i \frac{k_i}{\gamma} < \Lambda_\gamma(1-\delta)$, hence if $k_2 > \Lambda_\gamma(1-\delta)$, then $k_1 < \Lambda_\gamma(1-\delta)$. By naming $r \in \left(\rho_\gamma(1-\delta), \min\left(r\left(\frac{k_1}{\gamma}\right), \rho_\gamma(1+\kappa)\right)\right)$

bank 2 gets a payoff that is strictly higher than $(1-\delta)k_2$. Hence in equilibrium it must be that $\bar{r}_2 > \rho_\gamma(1-\delta)$. However if $\bar{r}_1 > \bar{r}_2$, it implies that when bank 1 names $\bar{r}_1$, the residual demand is always equal to zero and $M_1(\bar{r}_1, G_2) = (1-\delta)k_1$. However this cannot be part of an equilibrium as bank 1 has the profitable deviation to name any rate $r \in (\rho_\gamma(1-\delta), \bar{r}_2)$. By naming any rate $r \geq \bar{r}_1$, bank 1 gets $M_1(r, G_2) = G_2(\rho_\gamma(1+\kappa))\phi(r) + (1-\delta)k_1$, hence it must be that $\phi(r)$ is maximized at $\bar{r}_1$. The optimization problem is equivalent to the one of the baseline model, hence $\bar{r}_1 = r\left(\hat{b}\left(\frac{k_2}{\gamma}\right) + \frac{k_2}{\gamma}\right)$. By Condition 3, $r(\hat{b}(0)) < \rho_\gamma(1+\kappa)$, hence $r\left(\hat{b}\left(\frac{k_2}{\gamma}\right) + \frac{k_2}{\gamma}\right) < \rho_\gamma(1+\kappa)$ and $\bar{r}_2 \leq \bar{r}_1 < \rho_\gamma(1+\kappa)$. This implies that $G_2(\rho_\gamma(1+\kappa)) = 1$ and the rest of the proof follows.

Proposition 2. WLOG let $\bar{k}_1 \geq \bar{k}_2$. The proof follows the steps of Proposition 1, but we need to add a preliminary step:

- (Step 1) $\underline{k}_1/\gamma \geq \Lambda_\gamma(1+\kappa) - \bar{k}_2/\gamma$ for all $i \in \{1,2\}$. Suppose not and $\underline{k}_1/\gamma < \Lambda_\gamma(1+\kappa) - \bar{k}_2/\gamma$. When bank i raises $\underline{k}_1$, it is for sure in Region 4 and gets profits equal to

$$\pi_1(\underline{k}_1, \mu_j) = (1+\kappa)\underline{k}_1 - \underline{k}_1 = \kappa\underline{k}_1$$

  this is clearly increasing in $\underline{k}_1$, hence bank i would have the profitable deviation to name $\underline{k}_1 + \epsilon$. This inequality implies that $\bar{k}_1/\gamma \geq \underline{k}_1/\gamma \geq \Lambda_\gamma(1+\kappa) - \underline{k}_2/\gamma$.

- (Step 2) In equilibrium it must be that $\bar{k}_1/\gamma \geq b(\underline{k}_2/\gamma)$. Suppose not: $\bar{k}_1/\gamma < b(\underline{k}_2/\gamma)$, which implies $\underline{k}_2/\gamma < b(\bar{k}_1/\gamma)$. Therefore when bank 2 raises $\underline{k}_2$ is either in Region 4 or in Region 2:

$$\pi(\underline{k}_2, \mu_1) = \int_{\underline{k}_1}^{\beta(\underline{k}_2,\kappa)} \kappa\underline{k}_2 d\mu_1(k_1) + \int_{\beta(\underline{k}_2,\kappa)}^{\bar{k}_1} \left(Z\left(\frac{k_1+\underline{k}_2}{\gamma}\right) - \gamma\right)\frac{\underline{k}_2}{\gamma} d\mu_1(k_1)$$

  where $\beta(\underline{k}_2, \kappa) = \gamma\Lambda_\gamma(1+\kappa) - \underline{k}_2$. Profits are strictly increasing in $\underline{k}_2$ for all $k_1 \in [\underline{k}_1, \bar{k}_1]$, hence bank 2 can profitably deviate and name $\underline{k}_2 + \epsilon$. Therefore, putting together step 0 and step 1 it must be that $\bar{k}_1/\gamma \geq \max\{b(\underline{k}_2/\gamma), \Lambda_\gamma(1+\kappa) - \underline{k}_2/\gamma\}$.

- (Step 3) $\bar{k}_1/\gamma \leq b(\bar{k}_2/\gamma)$. Suppose not: $\bar{k}_1/\gamma > b(\bar{k}_2/\gamma)$ and $\bar{k}_1/\gamma \geq \Lambda_\gamma(1+\kappa) - \underline{k}_2/\gamma$. Then when bank 1 raises $\bar{k}_1$ is either in region 2 or in region 3A:

$$\pi(\bar{k}_1, \mu_2) = \int_{\underline{k}_2}^{\xi(\bar{k}_1)} \left(Z\left(\frac{\bar{k}_1 + k_2}{\gamma}\right) - \gamma \right) \frac{\bar{k}_1}{\gamma} d\mu_2(k_2) + \int_{\xi(\bar{k}_1)}^{\bar{k}_2} (P(k_2) - \delta k_1) d\mu(k_2)$$

where $\xi(k) = \frac{1}{\gamma}\hat{b}^{-1}\left(\frac{k}{\gamma}\right)$. The profits are strictly decreasing in $\bar{k}_1$, in particular the first term is decreasing because $\bar{k}_1/\gamma \geq b(\underline{k}_2/\gamma) \geq b(k_2/\gamma)$ for all k_2 in the support. Therefore bank 1 would have the profitable deviation to raise $\bar{k}_1 - \epsilon$.

- (Step 4) Putting together the previous steps it must be that $\bar{k}_1/\gamma \leq b(\bar{k}_2/\gamma)$ and $\bar{k}_1 \geq \max\{b(\underline{k}_2/\gamma), \Lambda_\gamma(1+\kappa) - \underline{k}_2/\gamma\}$, which implies that bank 2 is playing a pure strategy k_2. Bank 1, must best respond to the pure strategy k_2, hence Bank 1 will solve:

$$\max_{k_1 \geq 0} \pi_1(k_1, k_2)$$

where

$$\pi_1(k_1, k_2) = \begin{cases} \kappa k_1 & \text{if } \frac{k_1}{\gamma} \leq \Lambda_\gamma(1+\kappa) - \frac{k_2}{\gamma} \\ \left(Z\left(\frac{k_1+k_2}{\gamma}\right) - \gamma \right) \frac{k_1}{\gamma} & \text{if } \Lambda_\gamma(1+\kappa) < \frac{k_1}{\gamma} \leq \hat{b}\left(\frac{k_2}{\gamma}\right) \\ P(k_2) - \delta k_1 & \text{if } \frac{k_1}{\gamma} > \hat{b}\left(\frac{k_2}{\gamma}\right) \end{cases}$$

Hence $\frac{k_1^*}{\gamma} = \max\left(b\left(\frac{k_2^*}{\gamma}\right), \Lambda_\gamma(1+\kappa) - \frac{k_2^*}{\gamma}\right)$. At the same time bank 2 will have to best respond to that and similarly $\frac{k_2^*}{\gamma} = \max\left(b\left(\frac{k_1^*}{\gamma}\right), \Lambda_\gamma(1+\kappa) - \frac{k_1^*}{\gamma}\right)$. As $r^C < \rho_\gamma(1+\kappa)$, $b\left(\frac{k_i^*}{\gamma}\right) > \Lambda_\gamma(1+\kappa) - \frac{k_i^*}{\gamma}$, which imply $\frac{k_i^*}{\gamma} = b\left(b\left(\frac{k_i^*}{\gamma}\right)\right) = l^C$.