A Battle of Wills: The Joint Impact of Sentiment and Benchmarking on Volatility and Mispricing*

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April 2025

ABSTRACT

Standard models predict a positive relationship between investor sentiment and volatility, yet the empirical evidence suggests otherwise. We reconcile this discrepancy in a model with retail sentiment and institutional benchmarking. The interaction of these features reshapes how fundamental risk translates into return volatility, creating an asymmetric relationship with sentiment. It also explains why institutions can reduce mispricing under heightened sentiment. Using exogenous variation in institutions' benchmarking intensity, we provide causal evidence on the predicted impact of institutions on volatility for different sentiment levels. We also offer evidence on the predicted effect of sentiment and institutions on mispricing.

Keywords: Sentiment, Excess Volatility, Mispricing, Benchmarking, Institutional Investors.

JEL Classification: G11, G12, G18, G41.

*We thank Antonio Gargano, Goutham Gopalakrishna (discussant), Jan Hanousek (discussant), Idan Hodor, and Fernando Zapatero for helpful comments and suggestions. The paper has also benefited from comments made at presentations at the 2024 NFA Annual Meeting, the USC Finance PhD Alumni Conference 2024, the Public Economic Theory 2023, the 2023 FMA European Conference, the EEA-ESEM 2023, the Universidad de los Andes, Chile, and the University of Chile – FEN. All errors are our own.

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Abstract

Standard models predict a positive relationship between investor sentiment and volatility, yet the empirical evidence suggests otherwise. We reconcile this discrepancy in a model with retail sentiment and institutional benchmarking. The interaction of these features reshapes how fundamental risk translates into return volatility, creating an asymmetric relationship with sentiment. It also explains why institutions can reduce mispricing under heightened sentiment. Using exogenous variation in institutions' benchmarking intensity, we provide causal evidence on the predicted impact of institutions on volatility for different sentiment levels. We also offer evidence on the predicted effect of sentiment and institutions on mispricing.

1 Introduction

A vast body of literature documents how psychological biases and cognitive limits can shape the investment behavior of individuals, introducing an element of irrationality, or "sentiment," in financial markets.[1] In the presence of frictions that limit the activity of rational market participants, this sentiment can create excess return volatility and systematic deviations of prices from fundamentals.[2]

Against this backdrop, the recent trend toward greater delegation of portfolios to institutional investors ("institutionalization") raises new questions. Economic intuition suggests that, to the extent that institutional investors are sophisticated and less prone to committing systematic mistakes ("smart money"), the greater institutionalization of markets should help correct sentiment-driven distortions. However, recent theoretical and empirical findings suggest a more nuanced description of their investment behavior and potential impact on markets. First, Basak and Pavlova (2013) show that institutions' performance concerns relative to benchmark indexes ("benchmarking concerns") can amplify the volatility of index stocks and the aggregate stock market and result in upward pressure on the stock index. Second, DeVault et al. (2019) find that at least part of the demand shocks captured by sentiment metrics are not necessarily due to irrational beliefs but reflect rational decisions of institutions in response to their investment styles. A natural question, then, is: Can we expect institutional investors to correct or, in contrast, worsen the financial distortions caused by sentiment? The interaction of financial institutions' features and investor beliefs underlying this question is at the core of a recent research agenda in Asset Pricing (Brunnermeier et al., 2021).

To further motivate our inquiry, we plot in Figure 1 the observed relationship between the U.S. stock market return volatility and Baker and Wurgler (2006)'s measure of investor sentiment for different levels of aggregate institutional stock ownership (IOR), over the period 1980-2021. If the participation of institutional investors in the stock market did not affect the relationship between sentiment and return volatility, the depicted patterns should be roughly similar across IOR levels. The evidence, however, indicates a significant variation in the levels and shapes of this relationship among IOR terciles. In particular, when institutional investors own a larger share of the stock

[1] For surveys of this literature see, e.g., Barberis and Thaler (2003); Hirshleifer (2015).

[2] See, e.g., Gromb and Vayanos (2010).

market (“High IOR”), volatility can fall with sentiment in times of overall optimism,[3] a pattern that is difficult to rationalize within conventional models of sentiment trading.[4]

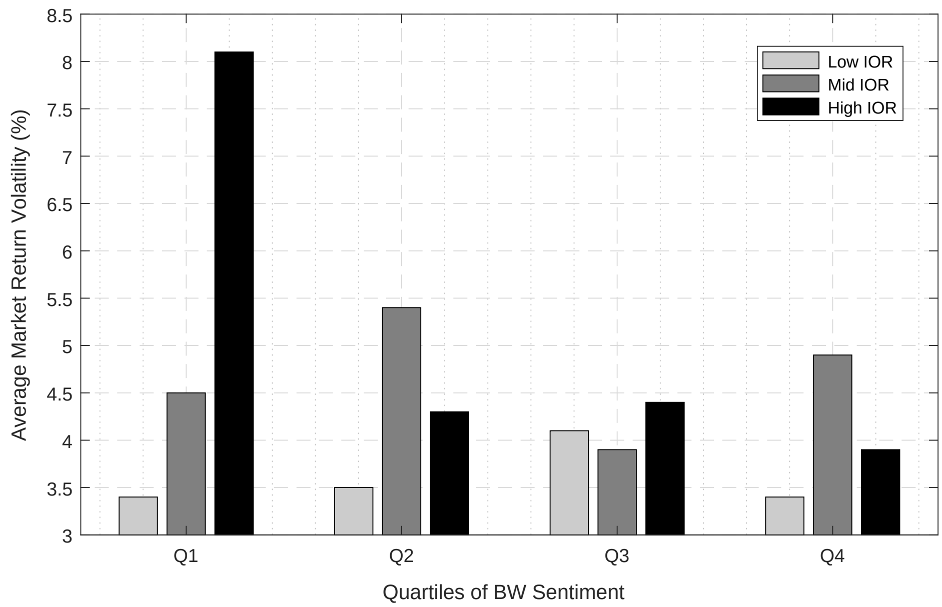


Figure 1: **Empirical relationship between market return volatility and sentiment**

This figure plots the average monthly stock market return volatility across quartiles of Baker and Wurgler (2006) (BW)'s measure of investor sentiment by terciles of institutional stock ownership (IOR). Quartiles of sentiment are created based on the overall sample from 1965/07 to 2022/06, available on Wurgler's website. Quartiles 1 and 2 correspond to strongly and moderately negative sentiment months, whereas quartiles 3 and 4 correspond to moderately and strongly positive sentiment months, respectively. This time series is normalized to have a mean value of zero. Data on quarterly institutional holdings is from Thomson/Refinitiv and covers the period from 1980 to 2021. Stock-level IOR is calculated as the ratio of shares held by 13F institutions to the number of shares outstanding. Stock-level values are then averaged across stocks each quarter using their market capitalizations as weights. Volatility is the standard deviation of the daily market returns from Ken French's website, scaled to a monthly measure and reported in percentage points.

In this paper, we examine this and other effects of investor sentiment on asset prices within a dynamic general equilibrium model that accounts for the participation of institutional investors in financial markets. In the model, risk-averse investors trade in a risky asset (a “stock”) and a riskless asset (“cash”) over a finite investment period. Investors belong to either of two classes: “retail” or “institutional.” Retail investors have standard preferences and can feature dogmatically optimistic or pessimistic beliefs about the stock's mean dividend growth rate. Thus, they can be subject to the type of positive or negative sentiment that is typically associated with retail trading in

[3]We offer regression-based evidence for this observation in Section 6.

[4]See our literature review below.

empirical studies.[5] Institutional investors have identical preferences to retail investors, except that their marginal utility of wealth is increasing in the level of a benchmark index. This assumption follows Basak and Pavlova (2013)'s reduced-form approach to capturing the fact that, as agents for their delegating investors, institutional investors are typically evaluated (and compensated) in terms of both absolute and relative performance with respect to a benchmark portfolio. Unlike retail investors, they are rational in the sense of having the correct belief about the mean dividend growth rate.

We solve for the equilibrium in this economy and explicitly characterize asset prices and portfolio allocations. Based on this characterization, we first isolate the impacts of sentiment versus benchmarking concerns on equilibrium prices. Both features introduce a wedge in the demand for the stock relative to an otherwise standard rational investor. Thus, they exert a similar upward and increasing pressure on the stock price. This similarity allows us to identify, for any given intensity of benchmarking concerns, the level of retail optimism that equalizes stock price-dividend ratios across the two reference economies where only one of the features is present.

In isolation, both features also exacerbate stock return volatility. However, they do so to different extents. The common driver of volatility exacerbation is the portfolio heterogeneity across investors that either sentiment or benchmarking concerns introduce. This portfolio heterogeneity amplifies the effect of fundamental shocks on stock returns via a relative-wealth channel. According to this channel, positive (respectively, negative) shocks to fundamentals transmit to prices not only via higher (lower) expectations of future payoffs but also via a greater (lower) demand pressure, as a result of a wealth effect, from the now relatively wealthier (poorer) traders whose portfolio is overexposed to the shock—i.e., the optimistic retail investors in one case, and the institutions in the other. The reason why the quantitative impacts of sentiment and benchmarking concerns differ is that the latter introduce a second volatility amplification channel, whereby positive (respectively, negative) shocks to prices feed back into additional positive (negative) institutional demand for the stock to hedge relative performance risk.

[5] See, among others, Kumar and Lee (2006), Barber and Odean (2008), Greenwood and Nagel (2009), Barber et al. (2009), Da et al. (2015).

We then focus the analysis on our main case of interest, namely the pricing implications of the trading between sentiment-driven retail investors and rational institutions. We aim to address two questions: (i) Is the sentiment-volatility relationship affected by the level of involvement of institutions in the market? (ii) To what extent do rational but benchmark-concerned institutions correct sentiment-induced mispricing? While the mere addition of the effects of either feature on prices described above suggests an exacerbation of the associated distortions, the equilibrium analysis reveals surprising patterns.

Concerning the first question, when the trading counterparts of the irrational investors are institutions instead of standard investors, retail optimism can actually *dampen* volatility. This result contrasts with the one prevailing in a standard economy without institutions, where sentiment (either positive or negative) unambiguously creates excess stock market volatility (DeLong et al., 1990; Dumas et al., 2009). To understand this, assume, for example, that retail investors are moderately optimistic about the stock's prospects, so they demand more (but not much more) of the stock than a standard investor. As if engaged in a "battle of wills," instead of meeting this extra demand—as rational investors would in the standard economy—institutions also demand more of the asset. They do so to the extent that, in equilibrium, both investor types end up with similar portfolios and, unlike what they would prefer, no leverage. In this equilibrium, shocks to fundamentals do not significantly alter the distribution of aggregate wealth and the relative-wealth amplification channel on volatility shuts down. The effect is such that, for moderate optimism, the stock volatility monotonically *falls* with investor sentiment.

A similar intuition explains a related result, according to which the positive relationship between the intensity of institutions' benchmarking concerns and the stock return volatility that prevails in a fully rational setting switches signs for sufficiently high levels of optimism. Moreover, the range of optimism over which the relationship between sentiment and volatility is negative widens as benchmarking concerns intensify. Thus, relative to standard investors, institutions attenuate excess volatility in the presence of retail optimism but exacerbate it under pessimism, creating an *asymmetric* sentiment-volatility pattern. The intuition behind the latter result is that, due to their benchmarking concerns, institutions are willing to buy more of the stock shares offered for sale by the pessimistic retail investors than standard investors. The resulting differences in portfolio compositions across investor types magnify the relative-wealth channel of return volatility.

An analysis of the equilibrium dynamics of the model uncovers novel patterns, as the wealth distribution across investors responds endogenously to cash-flow news, allocations, and prices. In particular, trading between rational institutions and optimistic retail investors can lead to a countercyclical pattern in return volatility, in stark contrast to the procyclical pattern prevailing when the latter investors are rational. The switch in pattern follows from a switch in sign, under certain circumstances, of the relative-wealth effect on return volatility. Specifically, when retail sentiment is so high that its impact on the stock demand is stronger than the impact of institutions' benchmarking concerns, the retail investors are relatively overinvested in the stock. Positive fundamental news then makes them relatively wealthier. The news also increases the index risk-hedging demand of institutions for the stock, leading to a sharp decline in the market price of risk. However, as their wealth increases and the market price of risk plummets, retail investors, even if highly optimistic, reduce the fraction of their wealth allocated to the stock. Because their wealth increases faster than the wealth of institutions, the lower demand of retail investors prevails in the aggregate, making the relative-wealth channel *reduce* (rather than amplify) the fundamental news sensitivity of prices. This buffering effect can push volatility levels below those that prevail under sentiment or benchmarking concerns alone. These results highlight the importance of distinguishing the degree of institutionalization of markets in empirical analyses that associate excess return volatility, as inferred from, for example, volatility-ratio tests, with irrational behavior and mispricing (e.g., Shiller, 1979, 1981; Giglio and Kelly, 2018).

Concerning the second question, whether greater institutionalization helps correct or exacerbate sentiment-induced distortions on prices depends on the relative strength of retail sentiment vis-a-vis institutions' benchmarking concerns. For low sentiment levels, even optimistic retail investors choose to sell an increasing share of their stock holdings to the institutions as the level of institutionalization increases. Because benchmarking concerns increase their risk appetite, the institutions purchase these shares at increasingly higher prices, which worsens the stock overpricing. This result verifies the conjecture of DeVault et al. (2019) that the existence of sophisticated investors could push prices further away from their fundamental value than retail sentiment. However, the opposite happens for more severe levels of optimism, when retail sentiment leads to stronger demand for the stock than institutions' benchmarking concerns. In these situations, characterized by a low (potentially negative) stock risk premium, greater institutionalization can be accompanied by aggressive stock selling by the institutions, which helps to push the stock price closer to its fundamental value. Thus, institutions help correct severe sentiment-induced overpricing that would otherwise result in financial "bubbles," understood as a stock market with a negative risk premium.

In the final part of the paper, we consolidate our findings into two novel testable implications and empirically evaluate them. The first testable implication is that the institutionalization of financial markets induces an asymmetric relationship between sentiment and volatility such that: (i) an intensification of pessimistic sentiment consistently increases volatility, whereas a similar intensification of optimistic sentiment can reduce it; and (ii) stronger benchmarking practices amplify volatility in markets dominated by pessimistic sentiment but attenuate it in markets characterized by optimistic sentiment. The second testable implication is that, in the presence of strongly optimistic investor sentiment, but not when sentiment is low or moderately optimistic, a greater institutionalization of financial markets leads to lower stock overpricing.

We provide empirical support for both implications, hence validation to our model, using data from the U.S. stock market. First, in portfolio-level analyses, we examine the volatility of stock portfolios double-sorted by institutional ownership ratios (IOR) and mispricing scores (MISP; Stambaugh et al., 2015) across different levels of market-wide sentiment (Baker and Wurgler, 2006). Consistent with part (i) of our first testable implication, the return volatility of high-IOR portfolios is, on average, 45% higher than that of low-IOR portfolios during periods of negative market-wide sentiment. Furthermore, return volatility declines monotonically as sentiment shifts from negative to positive, but this decline is more pronounced for high-IOR portfolios, consistent with the sentiment-volatility relationship becoming highly asymmetric in the presence of institutions.

Second, to assess causality, we exploit exogenous variation in the strength of institutions' benchmarking concerns using changes in the benchmarking intensity (BMI) measure of Pavlova and Sikorskaya (2023) around Russell index reconstitutions. BMI quantifies a stock's cumulative weight across all benchmarks, adjusted for the assets of mutual funds and ETFs tracking each benchmark. Pavlova and Sikorskaya (2023) argue that, conditional on stock size, liquidity, and index banding/inclusion criteria, changes in BMI from May to June, following the annual reconstitution of the Russell indices, are plausibly exogenous. To further take advantage of the cross-sectional dispersion in these changes, we follow Dong et al. (2024) in constructing a trade-based sentiment proxy that combines stock-level indicators for trading volume shocks, technical overbought/oversold conditions, and proximity to recent high prices. Cross-sectional regressions uncover a non-monotonic relationship between changes in BMI and stock return volatility, conditional on sentiment, whereby a one-standard-deviation increase in BMI changes from May to June *raises* the average change in stock return volatility by approximately 10% among strongly negative-sentiment stocks but *reduces* it by about 4% among strongly positive-sentiment stocks. In contrast, ignoring sentiment obscures these effects, as unconditionally the relationship between BMI changes and volatility becomes statistically insignificant. These results provide strong causal evidence in support of part (ii) of our first testable implication, reinforcing the model prediction that institutional benchmarking concerns interact with sentiment to shape volatility patterns in financial markets.

To contrast our second testable implication empirically, we examine how the predictive power of the MISP score in the cross-section of average stock returns is affected by the presence of institutions in periods of moderately versus strongly positive sentiment. If our model prediction holds, the documented negative relationship between MISP and subsequent abnormal returns should be weaker in the presence of institutions only when sentiment is strongly positive. Our results confirm that institutions attenuate stock overpricing only under heightened sentiment: in times of strongly positive sentiment, the next-month five-factor alpha for high-*MISP* stocks is -1.9% under low institutional presence but only -0.57% under high institutional presence, resulting in a statistically and economically significant alpha differential of 1.33% per month. In contrast, when sentiment is moderately positive, the negative relationship between MISP and alphas remains similar for stocks with high and low institutional ownership, yielding a statistically insignificant alpha differential.

Related Literature. Our paper is related to two main strands of literature. First, it relates to the literature on the implications of institutional investors' incentives on equilibrium prices. Cuoco and Kaniel (2011) find that symmetric benchmark-adjusted compensation has a significant and unambiguous positive effect on the price of benchmark assets and a negative impact on their Sharpe ratios. In contrast, asymmetric schemes have a more ambiguous effect. Using a highly tractable model, Basak and Pavlova (2013), explicitly characterize the institutions' portfolios in response to benchmarking incentives and their impact on the prices, Sharpe ratios, return volatilities, and correlations of benchmark versus non-benchmark assets. Several studies have built on this framework to rationalize observed asset pricing phenomena. Hong et al. (2014) use it to capture "status" (Keeping-Up-with-the-Joneses) concerns and explain the excessive trading of small local stocks and the trend-chasing behavior of individuals. Basak and Pavlova (2016) analyze the effect of the financialization of commodity futures markets on commodity futures prices, volatilities and correlations, and equity-commodity correlations. Buffa and Hodor (2022) study benchmark heterogeneity across asset managers to explain differences in the predictability of return comovement across cap-style and industry-sector portfolios. Hodor and Zapatero (2022) show that the interaction of institutions' short investment horizons and benchmarking concerns can rationalize a downward-sloping term structure of risk premia. Pavlova and Sikorskaya (2023) propose a theory-motivated measure (BMI) of the benchmarking-induced stock demand of asset managers and present causal evidence of the effects of benchmarking concerns on fund portfolios and stock prices. While potentially accounting for wealth effects on portfolio allocations, these studies assume that all traders are rational and, thus, are not set up to assess the impact of sentiment on prices. Other studies in this literature do allow for the existence of irrational trading to explain asset prices and volatility. Vayanos and Woolley (2013) focus on the impact of time-varying fund investors' flow, while Buffa et al. (2019) examine the impact of endogenously determined relative-performance fees in managers' compensation contracts. Breugem and Buss (2019) and Sockin and Xiaolan (2019) study the effects of benchmarking concerns on information acquisition and market efficiency. Jiang et al. (2024) find a disproportionate positive impact of flows into passive funds on the stock prices and volatilities of the largest firms, especially those already overvalued. Because these studies assume constant absolute risk aversion (CARA) preferences for rational traders and leave the investment decisions of irrational traders unmodeled (i.e., irrational trading is likened to "noise"), they cannot account for the wealth effects of either institutions or sentiment investors that are key to our analysis.

Second, our paper is related to the literature that examines the impact of sentiment on prices in general equilibrium. Several studies show that different behavioral biases such as overconfidence (Daniel et al., 2001; Scheinkman and Xiong, 2003; Dumas et al., 2009), self-attribution bias (Daniel et al., 1998), extrapolative beliefs (Hong and Stein, 1999; Barberis and Shleifer, 2003; Barberis et al., 2015, 2018), among others, can lead to sentiment-like excess trading and have a significant impact on asset returns and volatility.[6] In modeling sentiment, we focus on the type of dogmatic beliefs conducive to irrational optimism or pessimism considered by, e.g., Kogan et al. (2006). As Martin and Papadimitriou (2022) point out, this type of belief is consistent with the evidence documented by Giglio et al. (2021) and Meeuwis et al. (2021) in portfolio choice contexts. Similarly to both Kogan et al. (2006) and Martin and Papadimitriou (2022), we account for risk aversion and endogenous wealth effects on portfolio decisions and prices. Unlike these authors, we assume that the trading counterparts of the sentiment-driven investors are financial institutions rather than otherwise identical direct investors.[7] We show that due to benchmarking concerns, these institutions can either exacerbate or correct the distortions associated with sentiment depending on the relative strength of the sentiment- versus the benchmark-driven demands for the assets.

2 Model Setup

We consider a pure exchange economy with a finite horizon T populated by two classes of traders, retail and institutional investors. In principle, each investor type can exhibit irrational sentiment (optimism or pessimism), and this sentiment can differ across types. In practice, sentiment-driven trading is more commonly associated with retail investors in empirical and theoretical discussions (see references in Section 1). To accommodate the first possibility while facilitating comparison with prior literature, we set up our model to allow sentiment for both investor types in the remainder of this section and specialize it to the case of fully rational institutions in the rest of the paper.

[6]For a comprehensive survey of asset pricing models based on psychological considerations, see Barberis (2018).

[7]Similarly, Krishnamurthy and Li (2021) study the effect of sentiment on financial crises in the presence of financial intermediaries.

Financial Market. The financial market consists of a single risky security (a stock market portfolio), one share of which is available for trading. The stock only pays a dividend at the final time T. Let S and D denote the stock and dividend (cash flow) processes, respectively. For simplicity, we assume that the process D follows a geometric Brownian motion, i.e.

$$dD_t = D_t(\mu dt + \sigma dB_t),$$

where μ is the mean dividend growth rate, σ is the dividend volatility, and dB_t are the increments of the standard Wiener (cash flow "news") process under the objective probability measure $\mathbb{P}$.

In addition, a zero-coupon bond is available in zero net supply. The zero-coupon bond delivers a sure payment of one at time T. Following Kogan et al. (2006), we use the bond as the numeraire so its price equals one at all times.

Investor Preferences. Agents derive utility from terminal wealth only. Following Basak and Pavlova (2013), there are two classes of investors: retail (R) and institutional (I). Retail investors have standard logarithmic utility, i.e.

$$u_R(W_T^R) = \log W_T^R.$$

Institutional investors have otherwise identical preferences to retail investors except that their utility is scaled by the value of a benchmark index Y:

$$u_I(W_T^I) = (1 - \upsilon + \upsilon Y_T) \log W_T^I, \qquad \upsilon \in [0, 1), \tag{1}$$

where, without loss of generality, we let the benchmark index coincide with the stock market, i.e., we set $Y = S$. In the sequel, we show that a time-t measure of the strength of the institutional investor's benchmarking concern is:

$$q_t \triangleq \frac{\upsilon D_t e^{\mu(T-t)}}{1 - \upsilon + \upsilon D_t e^{\mu(T-t)}}, \qquad q_t \in [0, 1),$$

which depends positively on the benchmark weight υ in I's utility, the level of dividends D_t, and the remaining time horizon $T - t$.

Specification (1) follows Basak and Pavlova (2013)'s reduced-form approach to capturing the fact that, as agents for their delegating investors, institutions' managers are typically evaluated (and compensated) in terms of both absolute and relative performance with respect to a benchmark portfolio. Under this type of compensation arrangement, the institutional investor's marginal utility increases in the level of the benchmark. The specification can also capture relative performance concerns facing, e.g., status-conscious investors (Hong et al., 2014).

Investor Beliefs. For $k \in \{R, I\}$, investor k believes, *dogmatically*, that the mean growth rate of the dividend process D is constant and equal to μ^k. Investor k's beliefs are represented by an exponential martingale ξ^k whose evolution under $\mathbb{P}$ is given by

$$d\xi_t^k = \xi_t^k \delta^k \sigma dB_t,$$

where $\delta^k \triangleq (\mu^k - \mu)/\sigma^2$ is the level of "optimism" of investor k, and $\xi_0^k = 1$. ξ_T^k is the Radon-Nikodym derivative of $\mathbb{P}^k$, the probability measure under which the dividend mean growth rate is equal to μ^k, with respect to $\mathbb{P}$. Under $\mathbb{P}^k$, the evolution of the dividend process D is given by

$$dD_t = D_t \left((\mu + \sigma^2 \delta^k) dt + \sigma dB_t^k \right),$$

where $dB_t^k = dB_t - \sigma \delta^k dt$ is the increment of a standard Wiener process under $\mathbb{P}^k$. In the sequel, E_t^k denotes the conditional expectation at time t under investor k's beliefs.

Under $\mathbb{P}$, the dynamics of the stock price are given by

$$dS_t = S_t(\mu_{S,t} dt + \sigma_{S,t} dB_t),$$

where the stock price mean growth rate $\mu_{S,t}$ and volatility $\sigma_{S,t}$ are determined in equilibrium.

Portfolio Problem. At time 0, and without loss of generality, investor $k \in \{R, I\}$ is endowed with a fraction λ^k of the stock share (with $\lambda^I + \lambda^R = 1$) and no bond. At time $t \in [0, T]$, investor k decides the fraction $\theta_t^k \in \mathbb{R}$ of her portfolio to allocate in the stock, with the remaining fraction

$1-\theta_t^k$ allocated in the bond, to maximizes her lifetime utility of wealth

$$\begin{aligned} J_k(W_t^k) &= \max_{\theta^k} \; E_t^k[u_k(W_T^k)] \\ &= \max_{\theta^k} \; \frac{1}{\xi_t^k} E_t[\xi_T^k u_k(W_T^k)], \end{aligned}$$

subject to the budget constraint

$$dW_t^k = \theta_t^k W_t^k (\mu_{S,t}^k dt + \sigma_{S,t} dB_t^k), \tag{2}$$

where $\mu_{S,t}^k = \mu_{S,t} - \sigma_{S,t}\sigma\delta^k$, and $W_0^k = \lambda^k S_0$.

3 Equilibrium Characterization

Clearly, markets are dynamically complete. This implies the existence of a unique state price density process π with $\mathbb{P}$-dynamics:

$$d\pi_t = -\kappa_t \pi_t dB_t,$$

where κ denotes the (endogenously determined) stock market price of risk.

We define equilibrium in these markets in the usual way, as consisting of a set of portfolio allocations and asset prices such that: (i) the individual portfolio allocations of the retail and institutional investors are optimal, and (ii) the bond and stock markets clear. This definition leads to:[8]

Proposition 1. *The time-t equilibrium stock price-dividend ratio and market price of risk are given by:*

$$S_t/D_t = \overline{(S/D)}_t \frac{1}{\varpi_t^I\big(1-\gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)}, \tag{3}$$

$$\kappa_t = \bar{\kappa}\left(1 - \frac{\varpi_t^I(1-\gamma(T-t))q_t + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)\delta^R}{\varpi_t^I\big(1-\gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)}\right), \tag{4}$$

[8] As noted in Section 2, throughout the rest of the analysis, we assume that institutional investors are fully rational, meaning that $\delta^I = 0$ and $\xi_t^I = 1$ for all $t \in [0, T]$.

where $\gamma(x) \triangleq 1 - e^{-\sigma^2 x}$ $(\gamma(x) < 1, \gamma'(x) > 0)$*, and* $\overline{(S/D)}$ *and* $\bar{\kappa}$ *are the equilibrium stock price-dividend ratio and market price of risk in the standard ("STD") economy with no sentiment* ($\delta^R = 0$) *or institutional investors* ($\upsilon = 0$)*, as given by:*

$$\overline{(S/D)}_t = e^{(\mu - \sigma^2)(T-t)},$$

$$\bar{\kappa} = \sigma.$$

Both greater optimism $\delta^R > 0$ and benchmarking concerns $q > 0$ lead to higher market valuations S_t/D_t in excess of fundamental values $\overline{(S/D)}_t$, with pessimism ($\delta^R < 0$) creating the opposite effect. The higher (respectively, lower) prices translate into lower (higher) market prices of risk κ_t, reducing (increasing) the appeal of the stock in the portfolio allocation problem of investors and restoring the market equilibrium between the increased (reduced) demand and supply. Thus, the introduction of either optimistic (pessimistic) or institutional investors to an otherwise standard economy induces asset "overvaluation" ("undervaluation") from the perspective of a standard rational trading counterpart. The severity of this mispricing increases with sentiment or the intensity of benchmarking concerns.

Turning to equilibrium portfolio allocations and stock return volatility, we have the following:

Proposition 2. *The time-t portfolio weights in the stock of the retail and institutional investors are:*

$$\theta_t^R = \frac{\kappa_t}{\sigma_{S,t}} + \frac{\sigma}{\sigma_{S,t}} \delta^R, \tag{5}$$

$$\theta_t^I = \frac{\kappa_t}{\sigma_{S,t}} + \frac{\sigma}{\sigma_{S,t}} q_t, \tag{6}$$

so that the leverage $(\theta_t^R - 1)\varpi_t^R$ *of the retail investors is:*

$$(\theta_t^R - 1)\varpi_t^R = \varpi_t^I (1 - \varpi_t^I) \frac{\sigma}{\sigma_{S,t}} (\delta^R - q_t). \tag{7}$$

The equilibrium stock return volatility is:

$$\sigma_{S,t} = \bar{\sigma}_S \left(1 + \varpi_t^I \frac{\gamma(T-t)q_t(1-q_t) + (1-\varpi_t^I)\big(\gamma(\delta^R(T-t)) - \gamma(T-t)q_t\big)(\delta^R - q_t)}{\varpi_t^I\big(1-\gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)} \right) \geq \bar{\sigma}_S, \quad (8)$$

where $\bar{\sigma}_S = \sigma$ is the equilibrium stock return volatility in the STD economy with no sentiment $(\delta^R = 0)$ or institutional investors $(\upsilon = 0)$.

Equation (7) shows that the strength of retail investors' optimism relative to the institutions' benchmarking concerns, $\delta^R - q_t$, indicates whether the time-t stock allocation in R-investors' portfolio is levered $(\delta^R - q_t > 0)$ or not $(\delta^R - q_t < 0)$. In an all-rational investor economy with institutions, we have $\delta^R - q_t = -q_t < 0$, so the retail investors always lend money to the institutions. Proposition 2 shows that, because the strength q_t of their benchmarking concerns is always smaller than 1, institutional investors turn into lenders and retail investors turn into borrowers when the latter become sufficiently optimistic (i.e., $\delta^R > 1$) about the stock's prospects.

Whereas according to Eq. (8) both sentiment $(\delta^R \neq 0)$ and benchmarking concerns $(q > 0)$ create "excess volatility" with respect to the STD case (as $\sigma_{S,t} > \bar{\sigma}_S$ in both cases), the contribution of each feature is not obvious. To assess these contributions, we decompose the stock return volatility into fundamental, benchmarking, and relative-wealth components. Specifically, let us formally write:

$$\begin{aligned} \frac{dq_t}{q_t} &= \mu_{q,t}dt + \sigma_{q,t}dB_t, \\ \frac{d\varpi_t^I}{\varpi_t^I} &= \mu_{\varpi^I,t}dt + \sigma_{\varpi^I,t}dB_t. \end{aligned}$$

Further letting $\varepsilon_{S,t}^x = \frac{\partial S_t}{\partial x_t} \times \frac{x_t}{S_t}$ denote the elasticity of the stock price with respect to x at time t, we have the following:

Lemma 1. *The equilibrium stock return volatility can be decomposed as:*

$$\sigma_{S,t} = \varepsilon_{S,t}^D \sigma + \varepsilon_{S,t}^q \sigma_{q,t} + \varepsilon_{S,t}^{\varpi^I} \sigma_{\varpi^I,t}, \quad (9)$$

where $\sigma_{q,t} = (1 - q_t)\sigma$, $\sigma_{\varpi^I,t} = -(1 - \varpi_t^I)(\delta^R - q_t)\sigma$, *and*

$$\begin{aligned}
\varepsilon_{S,t}^D &= 1, \\
\varepsilon_{S,t}^q &= \frac{\varpi_t^I \gamma(T-t) q_t}{\varpi_t^I\big(1 - \gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1 - \gamma(\delta^R(T-t))\big)} > 0, && (10) \\
\varepsilon_{S,t}^{\varpi^I} &= \frac{\gamma(T-t)q_t - \gamma(\delta^R(T-t))}{\varpi_t^I\big(1 - \gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1 - \gamma(\delta^R(T-t))\big)} \varpi_t^I. && (11)
\end{aligned}$$

Thus, the excess volatility ratio, *EVR, is:*

$$EVR_t \triangleq \sigma_{S,t}/\bar{\sigma}_S - 1 = \Psi_{q,t} + \Psi_{\varpi^I,t} \geq 0, \tag{12}$$

where:

$$\begin{aligned}
\Psi_{q,t} &= \varepsilon_{S,t}^q (1 - q_t) > 0, && (13) \\
\Psi_{\varpi^I,t} &= -\varepsilon_{S,t}^{\varpi^I} (1 - \varpi_t^I)(\delta^R - q_t). && (14)
\end{aligned}$$

The first term in (9) is the direct effect of fundamental news on return volatility. It reflects the fact that positive (respectively, negative) cash flow news signals a greater (smaller) terminal dividend D_T, so the stock price S must adjust proportionally to reflect investors' updated expectations.

The second and third terms are the indirect impacts of these fundamental news on stock return volatility via the changes they induce in, respectively, the institutions' benchmarking intensity and relative-wealth share (i.e., the level of institutionalization), holding one or the other constant. Since these indirect impacts are the drivers of the excess volatility in this economy relative to the STD economy, we interpret them as the "benchmarking," Ψ_q, and "relative-wealth", Ψ_{ϖ^I}, propagation channels of fundamental shocks to excess volatility.

Benchmarking concerns create positive feedback from prices to the stock demand. In an economy with institutional investors, the higher (respectively, lower) price stemming from investors' updated expectations after positive (negative) cash flow news raises (depresses) the institution's benchmarking-related demand for the stock in order to keep up with the benchmark. Thus, the aggregate demand and the price for the stock change more than in the STD economy in response to the same cash flow news, amplifying the sensitivity of prices to dividend shocks. It is easy to see from Lemma 1 that, for a given benchmarking intensity q, the benchmarking channel is positive and increasing in the degree of optimism δ^R of the retail investors. Thus, the amplification of excess volatility induced by benchmarking concerns is always greater when trading with optimistic rather than rational retail counterparts.

The relative-wealth channel arises endogenously in equilibrium whenever $0 < \varpi_t^I < 1$, i.e., whenever no investor type absorbs the entire economy. In this case, differences in the portfolio compositions of institutional versus retail investors lead to differences in the dynamics of their relative wealth. To the extent that changes in wealth translate into changes in stock demands (as is the case with log preferences), the aggregate wealth distribution becomes a stochastic (state) variable whose volatility adds fundamental risk to the stock relative to the STD case.[9]

One can check from Eq. (11) that the relative-wealth elasticity of stock prices $\varepsilon_S^{\varpi^I}$ decreases, while the relative demand strength $\delta^R - q_t$ increases, with the degree of optimism. Moreover, each of them can be positive or negative depending on how optimistic the retail investors are. Thus, in principle, the participation of optimistic retail investors in a market where institutional investors are present could attenuate the relative wealth-induced excess volatility of stock returns. We provide conditions under which this possibility arises in Section 4.2.1.

4 Analysis of Equilibrium

4.1 Reference Economies

To further isolate the impacts of sentiment versus benchmarking concerns on financial markets equilibrium, in this section we examine two relevant reference economies: the Basak and Pavlova (2013)'s setting featuring institutional investors but no sentiment (BP), and an economy that features sentiment but no institutions (SENT). The results in this section are special cases of the

[9]One may wonder whether the condition $0 < \varpi^I < 1$ under which the relative-wealth channel arises can hold, i.e., whether both agent types can survive, in the long run $T \to \infty$ (see, e.g., Borovička 2020; Gopalakrishna et al. 2023). We address this question in Appendix B, where we show that either investor type, or both of them at the same time, can survive depending on plausible parameterizations of the model. Regardless, we note that our framework is best suited to examine the relatively short horizon (e.g., 1 to 5 years) over which institutional asset managers are typically evaluated for compensation purposes.

results in Section 3 (see Appendix A).

4.1.1 Benchmarking Concerns and No Sentiment (BP)

Basak and Pavlova (2013) introduce heterogeneity across investor types in an STD economy ($\delta^R = 0, \upsilon = 0$) by including positive benchmarking concerns ($0 < \upsilon < 1$) in the objective function (1) of the institutions.[10] The authors show that the benchmarking concerns induce an extra demand for the stock that raises the price-dividend ratio above, and depresses the stock market price of risk below, the levels prevailing in the STD economy, such that:

$$(S/D)_t^{BP} = \overline{(S/D)}_t \frac{1}{1-\gamma(T-t)\varpi_t^I q_t} \geq \overline{(S/D)}_t, \tag{15}$$

$$\kappa_t^{BP} = \bar{\kappa}\left(1 - \frac{(1-\gamma(T-t))\varpi_t^I q_t}{1-\gamma(T-t)\varpi_t^I q_t}\right) \leq \bar{\kappa}. \tag{16}$$

For both $(S/D)_t^{BP}$ and κ_t^{BP}, the differences from their equilibrium values in the STD economy $\overline{(S/D)}_t$ and $\bar{\kappa}$ increase with the "benchmarked wealth" $\varpi_t^I q_t$, which we identify with the product of the fraction of aggregate wealth in I's hands, ϖ_t^I, and the intensity of their benchmarking concerns, q_t. Basak and Pavlova (2013) refer to the upward pressure on prices and depressing effect on market price of risk resulting from benchmarking concerns as an "index effect."

The authors further show that the presence of institutions increases the stock return volatility relative to the STD economy. The effect is an increasing function of the benchmarked wealth $\varpi_t^I q_t$:

$$\sigma_{S,t}^{BP} = \bar{\sigma}_S\left(1 + \gamma(T-t)\frac{\varpi_t^I q_t(1-\varpi_t^I q_t)}{1-\gamma(T-t)\varpi_t^I q_t}\right) \geq \bar{\sigma}_S. \tag{17}$$

Using our results from Section 3, we can decompose the associated excess volatility ratio, EVR^{BP}, into its benchmarking and relative-wealth shock propagation channels as $EVR_t^{BP} = \Psi_{q,t}^{BP} + \Psi_{\varpi^I,t}^{BP}$, with:

$$\begin{aligned}
\Psi_{q,t}^{BP} &= \tfrac{\gamma(T-t)\varpi_t^I q_t}{1-\gamma(T-t)\varpi_t^I q_t}(1-q_t) > 0, \\
\Psi_{\varpi^I,t}^{BP} &= \tfrac{\gamma(T-t)\varpi_t^I(1-\varpi_t^I)q_t^2}{1-\gamma(T-t)\varpi_t^I q_t} > 0.
\end{aligned}$$

[10] In this section we only highlight the aspects of these authors' analysis that are most relevant for our purposes.

The positive sign of Ψ_q^{BP} is expected from our more general result (13), and its expression implies that the amplifying effect on return volatility of the institutions' benchmarking concerns rises with the extent of institutionalization ϖ^I of the economy, the more so, the more intense benchmarking concerns q are.

The positive sign of $\Psi_{\varpi^I}^{BP}$ implies that in the presence of institutions but no sentiment, relative-wealth effects also exacerbate the response of stock returns to fundamental shocks. Intuitively, a positive (negative) dividend shock makes institutions, which overweight the stock in their portfolios, relatively wealthier (poorer). Confronted with a higher (lower) wealth, the positive sensitivity of their demand to wealth leads institutions to demand more (less) of the stock, pushing its price even higher (lower).

4.1.2 Sentiment and No Benchmarking Concerns (SENT)

In this economy, sentiment-driven (either optimistic or pessimistic) retail investors trade in the stock and the bond alongside identical but rational investors. The specialization of our framework to this case ($\delta^R \neq 0$ and $\upsilon = 0$) resembles the setup of Kogan et al. (2006) with log preferences and is formally equivalent to a model of differences of opinion (e.g., Panageas, 2020) in which one of the two investors classes has the correct prior about the dividend growth rate μ.

Sentiment introduces a wedge between the demands for the stock of irrational and rational investors, with optimistic investors ($\delta^R > 0$) overweighting and pessimistic investors ($\delta^R < 0$) underweighting the stock in their portfolios. The following result summarizes the impact of the ensuing pressure on prices:

Lemma 2. *In the presence of sentiment and absence of institutional investors, the stock price-dividend ratio and market price of risk are*

$$(S/D)_t^{SE} = \overline{(S/D)}_t \frac{1}{1 - \varpi_t^R \gamma(\delta^R (T-t))}, \tag{18}$$

$$\kappa_t^{SE} = \bar{\kappa} \left(1 - \frac{\varpi_t^R \big(1 - \gamma(\delta^R (T-t))\big) \delta^R}{1 - \varpi_t^R \gamma(\delta^R (T-t))} \right). \tag{19}$$

In particular, the price-dividend ratio rises above (respectively, falls below) the corresponding ratio

$\overline{(S/D)}$ *in the STD economy when sentiment investors are optimistic (pessimistic), with the difference* $|(S/D)_t^{SE} - \overline{(S/D)}|$ *increasing in the intensity* $|\delta^R|$ *of sentiment. Similarly, the market price of risk under optimistic (respectively, pessimistic) sentiment falls below (rises above) its equilibrium value* $\bar{\kappa}$ *in the STD economy, with the difference* $|\kappa_t^{SE} - \bar{\kappa}|$ *increasing in sentiment intensity* $|\delta^R|$.

Importantly, sentiment also creates "excess volatility" in stock returns:

Lemma 3. *In the presence of sentiment and absence of institutional investors, the stock return volatility and excess volatility ratio are:*

$$\sigma_{S,t}^{SE} = \bar{\sigma}_S \left(1 + \frac{\varpi_t^R(1-\varpi_t^R)\gamma(\delta^R(T-t))}{1-\varpi_t^R\gamma(\delta^R(T-t))}\delta^R\right) \geq \bar{\sigma}_S, \tag{20}$$

$$EVR_t^{SE} = \Psi_{\varpi^I,t}^{SE} = \frac{\varpi_t^R(1-\varpi_t^R)\gamma(\delta^R(T-t))}{1-\varpi_t^R\gamma(\delta^R(T-t))}\delta^R \geq 0. \tag{21}$$

Thus, under heterogeneity in investor types ($0 < \varpi_t^R < 1$)*, both positive and negative sentiment exacerbate the stock return volatility relative to the STD economy* ($EVR_t^{SE} > 0$)*. Moreover,* $\sigma_{S,t}^{SE}$ *and* EVR_t^{SE} *are increasing in sentiment for fixed wealth distribution and revert to the STD equilibrium values* $\bar{\sigma}_S$ *and* 0*, respectively, when all investors are sentiment driven* ($\varpi_t^R = 1$).

When sentiment-driven and rational investors participate in the stock market, the positive impact of sentiment on volatility arises under *both* optimism and pessimism. Moreover, it increases monotonically and symmetrically with the level of (positive or negative) sentiment.[11]

The excess volatility of stock returns in the SENT economy arises solely from fluctuations in relative wealth. Similar to the institutional investor in the BP economy, a sentiment-driven *optimistic* investor—or a rational trading counterparty under *pessimistic* sentiment—holds an overexposed position in market risk relative to the other investor type. Following a positive dividend shock, the overexposed investor experiences an increase in relative wealth and demands more of the stock, further driving up its price. Conversely, a negative dividend shock reduces the investor's relative wealth, dampening demand and exerting downward pressure on the stock price.

[11] This effect is robust to more general constant relative risk aversion (CRRA) preferences, with its intensity decreasing in the coefficient of relative risk aversion.

Thus, sentiment in an all-retail investor economy *always* amplifies return volatility, just as the presence of institutional investors does in the BP setting relative to the STD case. This positive relationship between sentiment and excess volatility aligns with the predictions of earlier models on noise trading and sentiment risk (DeLong et al., 1990; Dumas et al., 2009).

4.1.3 Comparison of the BP and SENT economies

The similarities in effects on prices and return dynamics of benchmarking concerns (section 4.1.1) and sentiment (Section 4.1.2) raise the question of how the asset pricing implications of these features compare. The following result defines a specific sense in which the BP and SENT economies are comparable:[12]

Lemma 4. *At any given horizon $T - t$, there exists a unique positive level of sentiment $\check{\delta}_t^R$, with:*

$$0 < \check{\delta}_t^R = \frac{\log(1 - \gamma(T-t)q_t)}{\log(1 - \gamma(T-t))} < q_t, \tag{22}$$

that equalizes the stock price-dividend ratios across the BP and SENT economies. At this level of optimism, $\kappa_t^{BP} \geq \kappa_t^{SE}$, and $\sigma_{S,t}^{BP} \geq \sigma_{S,t}^{SE}$.

For any parameterization of the BP economy, there exists a level of optimism in the SENT economy that creates the same upward shift in stock price-dividend ratios as the "index effect" across *all* distributions of aggregate wealth. Moreover, at this level of optimism, the index effect on the market price of risk and the stock volatility is always greater than the effect of sentiment.

Fig. 2 illustrates the equilibrium initial stock price-dividend ratio, market price of risk, and return volatility, under the degree of optimism $\check{\delta}_t^R$. Equilibrium values are plotted as a function of the share of aggregate wealth $1 - \varpi_t^R$ of the institutional or the sentiment-prone (optimistic) retail investors, respectively, in the BP and SENT economies.[13] The differences in price-dividend ratios relative to the STD economy rise with the wealth share of the institutions (BP case) or of the

[12]In this exercise, we assign the distribution weights ϖ^R and $1 - \varpi^R$ in the SENT economy to, respectively, the rational and irrational investors. In this way, the irrational investor has the same weight as the institutional investor of the BP economy ($\varpi^I = 1 - \varpi^R$).

[13]For comparability, the rest of the model parameters follow the baseline parameterization of Basak and Pavlova (2013)'s single-stock economy.

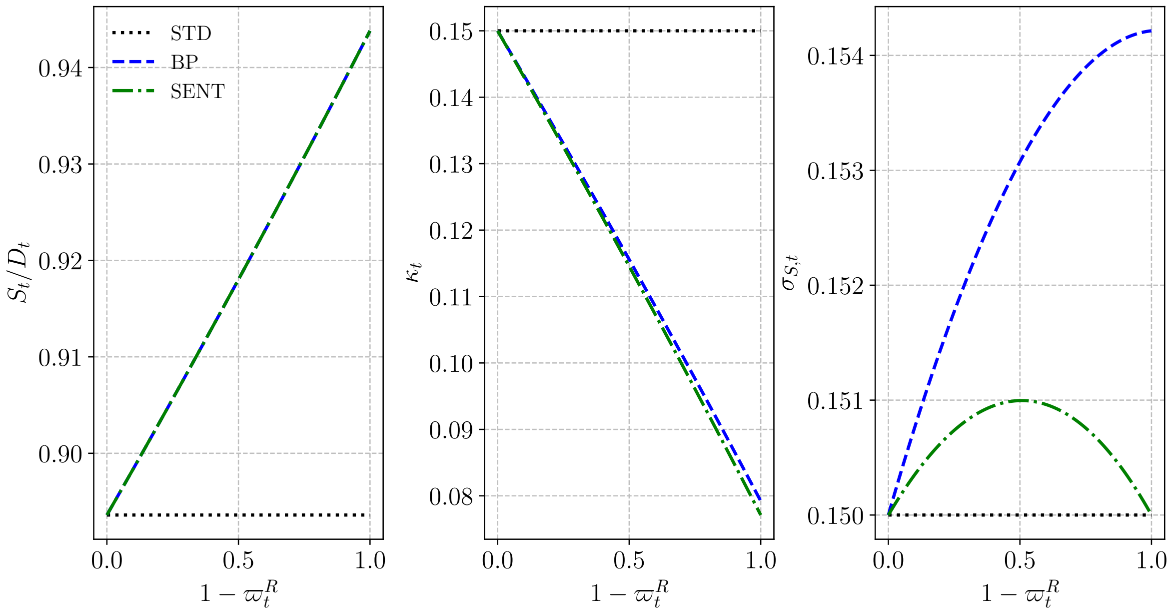


Figure 2: **Equilibrium under the reference economies**

This figure plots the equilibrium stock price-dividend ratio (leftmost panel), market price of risk (center panel), and return volatility (rightmost panel), under the STD (dotted black line), BP (dashed blue line), and SENT (dash-and-dot green line) economies. Equilibrium values are depicted as a function of the wealth shares of either I-investors (BP case) or sentiment R-investors (SENT case). Across all graphs, $\delta^R = \check{\delta}_0^R = 0.486$. The rest of the model parameters follow the parameterization in Basak and Pavlova (2013): $\mu = 0, \sigma = 0.15, t = 0, T = 5, D_0 = 1, \upsilon = 0.5$.

sentiment investors (SENT case) in each of the economies. Accordingly, equilibrium market prices of risk follow very similar decreasing patterns across the two economies, with slightly lower values in the SENT economy, in line with Lemma 4.

The right-most panel quantitatively illustrates the difference, following Lemma 4, in the amplification effects of benchmarking concerns and sentiment on stock return volatility. For low institutional or sentiment investors' shares of aggregate wealth, excess volatility increases with either share. Still, it does so more rapidly in the BP case—as expected from the presence of a benchmarking channel on excess volatility only in this case. As these shares become large enough, the pattern remains positive in the BP case but turns negative in the SENT case, where the excess volatility disappears as sentiment investors become the only investor type.[14]

[14]It can be additionally shown that whenever the wealth share of the retail rational investors ϖ_t^R is small enough (with $\varpi_t^R < 0.5$ being sufficient), the magnitude of the relative-wealth channel in the BP economy is larger, i.e.,

4.2 General Case: Interaction of Benchmarking and Sentiment

We have shown that introducing either optimistic sentiment or benchmarking concerns to an otherwise standard economy has similar boosting effects on stock prices and return volatilities, with potentially (significantly) different magnitudes in the case of volatility. In this section, we analyze the equilibrium under the general ("GE") case in which sentiment retail investors and institutional investors trade with each other.

4.2.1 Excess volatility

Comparative statics analysis of Eqs. (9)-(11) lead to the following:

Proposition 3. *In the presence of institutional ($\upsilon > 0$) and irrational retail ($\delta^R \neq 0$) investors:*

(a) *There exists a unique degree of optimism $\hat{\delta}^R(D_t, \varpi_t^I, T-t) > \check{\delta}_t^R > 0$ such that:*

$$\frac{\partial \sigma_{S,t}}{\partial \delta^R} \begin{cases} > 0, & \delta^R > \hat{\delta}^R(D_t, \varpi_t^I, T-t) \\ = 0, & \delta^R = \hat{\delta}^R(D_t, \varpi_t^I, T-t) \\ < 0, & \delta^R < \hat{\delta}^R(D_t, \varpi_t^I, T-t) \end{cases} .$$

This implies, in particular, that for $0 < \delta^R < \check{\delta}_t^R$, the effect of optimistic sentiment is to reduce *the stock return volatility across all wealth distributions ϖ_t^I relative to the BP case.*

(b) *As long as aggregate wealth is not concentrated in institutional investors' hands, the stock return volatility $\sigma_{S,t}$ monotonically* decreases *with the intensity of benchmarking concerns q_t when sentiment is sufficiently* optimistic *and monotonically increases with q_t under no or low sentiment. Otherwise, $\sigma_{S,t}$ first increases and then decreases (i.e., is hump-shaped) with q_t.*

The simple addition of the effects of benchmarking concerns (Section 4.1.1) and sentiment (Section 4.1.2) on excess volatility may suggest that in the presence of both features, the stock return

changes in relative wealth lead to greater volatility of stock returns, than in the SENT economy. More precisely,

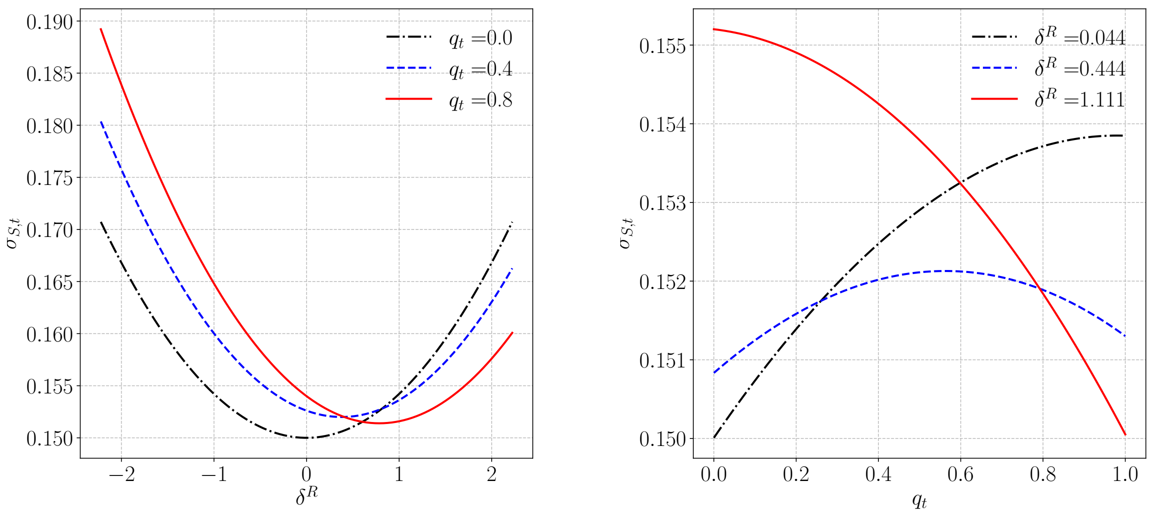
$$\Psi^{BP}_{\varpi^I,t} > \Psi^{SE}_{\varpi^I,t} \Leftrightarrow \varpi_t^R < \frac{\check{\delta}_t^R - (1-\gamma(T-t)q_t)}{1+\check{\delta}_t^R} < 0.5.$$

volatility must rise beyond the levels prevailing in the BP and SENT cases. Proposition 3 indicates that this intuition need not hold.

First, according to part (a), under moderate optimism $0 < \delta^R < \check{\delta}_t^R$ the stock excess volatility *monotonically falls* with investor sentiment across all aggregate wealth distributions. Thus, the presence of sentiment in an institutionalized economy can *dampen* the excess volatility of the stock market. This can be seen by comparing, in Panel (a) of Figure 3, the equilibrium stock return volatilities under no sentiment ($\delta^R = 0$, BP case) versus optimism ($\delta^R > 0$) when institutions face benchmarking concerns (GE case, red solid line and dashed blue lines). The negative relationship between optimism and volatility in the GE case contrasts with the pattern arising in the otherwise equivalent SENT economy ($q_t = 0$, dash-and-dot black line), whereby greater optimism always translates into *greater* excess volatility.

Second, according to part (b), the positive relationship between benchmarking intensity and volatility prevailing in a rational institutionalized (BP) economy switches signs for sufficiently high levels of optimism. This is illustrated in Panel (b) of Figure 3, which plots the stock return volatility as a function of institutions' benchmarking intensity q_t at different levels of retail sentiment. For very low (pessimistic) and null levels of sentiment, $\sigma_{S,t}$ increases monotonically with q_t, whereas for moderate levels of optimism the relationship turns nonmonotonic. Under high optimism, the stock return volatility monotonically *decreases* with q_t. The effect is such that, for high values of q_t, this volatility is smaller at high than at low levels of optimism (solid red lines versus the other two lines).

These effects can be explained in terms of the benchmarking and relative-wealth shock propagation channels. Starting with part (a) of Proposition 3, for $0 < \delta^R < \check{\delta}_t^R$ the benchmarking effect on stock demand is stronger than the sentiment effect ($q_t > \delta^R$) and creates heterogeneity in portfolios among investor types. However, as δ^R rises, the gap between the two stock demands shrinks until disappearing at $\delta^R \approx q_t$. As if engaged in a "battle of wills," at this point no investor type gets to lever up their portfolios, as they would if trading with rational retail counterparts (BP and SENT cases), and the relative-wealth channel shuts down. For even higher levels of optimism, the demand of the sentiment-driven investors prevails ($\delta^R > q_t$), the difference $\delta^R - \check{\delta}_t^R$ turns positive, and



(a) Effect of sentiment

(b) Effect of benchmarking

Figure 3: **Equilibrium volatility**

This figure plots the equilibrium stock return volatility as a function of either sentiment (Panel (a)) or institutions' benchmarking intensity (Panel (b)). In Panel (a), equilibrium relationships are illustrated for the GE cases of strong (red solid line), moderate (dashed blue line), and no (dash-and-dot green line) benchmarking concerns. In Panel (b), equilibrium relationships are illustrated for the GE cases of high (red solid line), moderate (dashed blue line) and low (dash-and-dot black line) levels of optimistic sentiment. In both Panels, we set $\varpi_t^I = 0.5$. The rest of the model parameters (other than δ^R in Panel (a), and υ in Panel (b)) are as in Fig. 2.

the ensuing difference in portfolios activates the relative-wealth channel's positive effect on return volatility.

The same changes in relative wealth as a function of the distance between δ^R and q_t explain why, according to part (b) of Proposition 3, return volatility can fall as q_t increases toward δ^R for sufficiently optimistic sentiment. When $\delta^R > 1$, however, the negative relationship between q_t and $\sigma_{S,t}$ need not turn positive (as the relationship between $\sigma_{S,t}$ and δ^R does) for sufficiently high q_t (respectively, δ^R). The reason is that the benchmark-driven demand of the institutions is limited by the stock weight in the benchmark ($q_t < 1$), so for high levels of optimism, portfolios can only become less heterogeneous as benchmarking concerns intensify.

Following this argument, one would expect that the range of optimism over which the relationship between sentiment and volatility is negative increases with the intensity q_t of institutions' benchmarking concerns. This intuition is confirmed by comparing the red solid and dashed blue lines of Figure 3(a). The figure further highlights an *asymmetry* in this relationship that is caused only by the presence of institutions: relative to rational retail investors, institutions attenuate excess volatility in the presence of optimistic sentiment but exacerbate it under widespread pessimism. To understand the latter effect, notice that institutions are willing to buy more of the stock shares sold by pessimistic retail investors than equivalently rational standard counterparts. This creates greater differences in the portfolios of R- and I-investors, making the relative-wealth channel amplify return volatility.

4.2.2 Effect of institutionalization

DeVault et al. (2019) conjecture that the existence of sophisticated investors need not help prices converge to, and might make them deviate even more from, fundamental value. We examine this conjecture within our setup by studying whether the introduction of institutions to (i.e., the institutionalization of) a market populated by optimistic retail investors exacerbates or, on the contrary, helps correct overpricing.

To this aim, we analyze how the stock price-dividend ratio S_t/D_t changes in Eq. (3) as the share ϖ_t^I of aggregate wealth in I's hands increases, for different levels of sentiment of the R investors:

Proposition 4. *In an economy populated by irrational retail and rational institutional investors, whether a higher level of institutionalization decreases, does not change, or increases the stock price-dividend ratio depends on whether the level of retail optimism δ^R exceeds, equals, or falls below the threshold $\check{\delta}_t^R$ that equalizes price-dividend ratios across the BP and SENT economies.*

Whether greater institutionalization helps correct or exacerbates sentiment-induced price distortions depends on the relative strength of retail sentiment vis-a-vis institutions' benchmarking concerns. Figure 4, which plots the equilibrium stock price-dividend ratios and market prices of risk as a function of the share of institutional investors in aggregate wealth, and Figure 5, which illustrates the associated optimal portfolios, provide the intuition for this result.

For *low levels of sentiment* ($\delta^R < \check{\delta}_t^R$), even optimistic retail investors choose to sell an increasing fraction of their stock holdings to the institutions as the level of institutionalization rises (black dotted line in the top left panel of Figure 5). Because benchmarking concerns increase

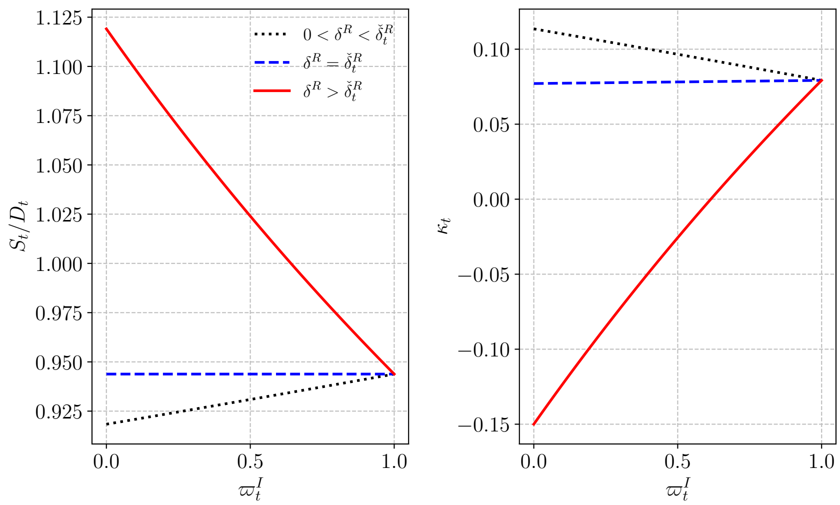


Figure 4: **Equilibrium prices in the GE case under mild to high levels of optimism**
This figure plots the equilibrium stock price-dividend ratio (left panel) and market price of risk (right panel) under the GE economy for three levels of optimism δ^R of the R-investors: mild optimism ($0 < \delta^R < \check{\delta}_t^R$, dotted black line), middle-ranged optimism ($\delta^R = \check{\delta}_t^R$, dashed blue line), and high optimism ($\delta^R > \check{\delta}_t^R$, solid red line). Equilibrium values are depicted as a function of the share of aggregate wealth of I-investors. Model parameters are as in Fig. 2.

their risk appetite, the institutions purchase these shares at increasingly higher price-dividend ratios, determining an increasing pattern of stock overpricing (black dotted line in the left panel of Figure 4). This result verifies the conjecture of DeVault et al. (2019) that the existence of sophisticated investors might push prices further away from their fundamental value than the presence of sentiment-driven retail investors.

However, the opposite holds for *high levels of optimism* ($\delta^R > \check{\delta}_t^R$, red solid lines in Figs. 4 and 5), when retail sentiment leads to stronger demand for the stock than institutions' benchmarking concerns. Such strong retail optimism can lead to severe levels of overvaluation and a negative market risk premium, akin to a financial "bubble," under low levels of institutionalization. When the risk premium is low enough, however, rational institutions, no matter how concerned about their benchmark, will find it optimal to *reduce* their portfolio allocation in the stock. As the level of institutionalization rises, aggressive selling by the institutions pushes the stock price closer to fundamental value (see the STD case in Fig. 2) and eventually reverses it to levels consistent with a positive risk premium. Notably, the threshold that separates "low" from "high" sentiment in this

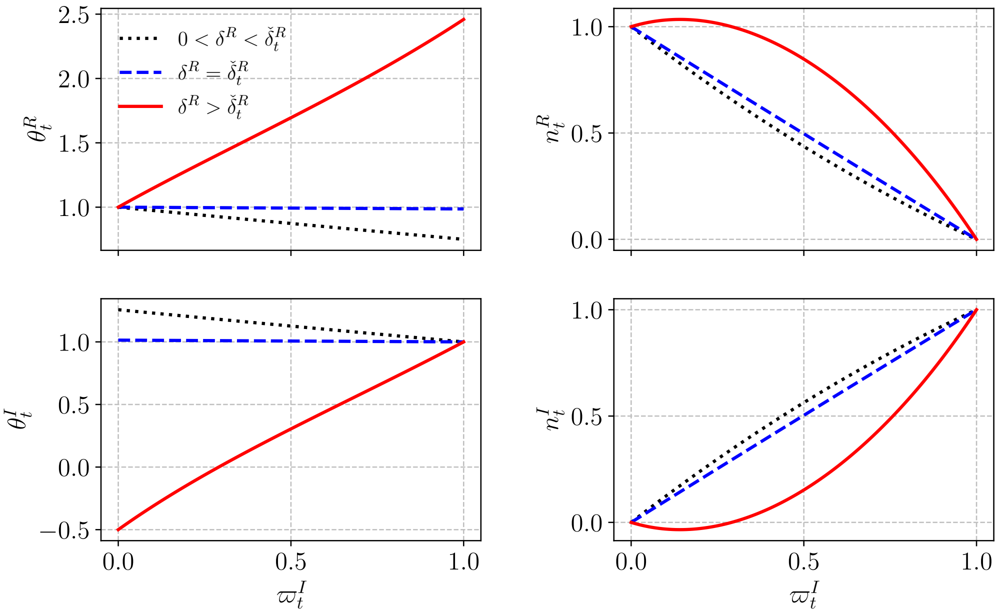


Figure 5: **Equilibrium allocations in the GE case under mild to high levels of optimism**
This figure plots the equilibrium weights (left panels) and number of shares $n_t^k = \theta_t^k W_t^k / S_t, k \in \{I, R\}$ of the stock (right panels) in the portfolios of optimistic retail (top panels) and rational institutional (bottom panels) investors under the equilibrium cases illustrated in Fig. 4: mild optimism ($0 < \delta^R < \check{\delta}_t^R$, dotted black line), middle-ranged optimism ($\delta^R = \check{\delta}_t^R$, dashed blue line), and high optimism ($\delta^R > \check{\delta}_t^R$, solid red line). Equilibrium values are depicted as a function of the share of aggregate wealth of I-investors. Model parameters are as in Fig. 2.

analysis is the same level of sentiment $\check{\delta}_t^R$, characterized in Section 4.1.3, under which the standalone impacts of sentiment and benchmarking concerns on prices equalize, and is thus invariant to the level of institutionalization (see dashed blue lines in Figs. 4 and 5).

5 Dynamic effects

Over time, the aggregate wealth distribution will vary endogenously with cash-flow news, allocations and prices. To pin down the equilibrium dynamics of all relevant variables, we fix the initial stock share endowments and solve for the corresponding time-t aggregate wealth shares to obtain the following:

Lemma 5. *Given initial wealth distribution $\varpi_0^I = \lambda$ and $\varpi_0^R = 1 - \lambda$ for, respectively, the I and R investors, the equilibrium stock price-dividend ratio, market price of risk, and return volatility* *are given by Eqs.* (3)*,* (4)*, and* (8)*, while the equilibrium portfolio allocations to the stock and the optimal borrowing are given by Eqs.* (5) *and* (7)*, for:*

$$\varpi_t^I = \frac{\lambda}{\lambda + \left(1 - \gamma(\frac{1}{2}\delta^R(\delta^R - 1)t)\right)\left(\frac{q_t}{q_0}\right)^{\delta^R}\left(\frac{1-q_t}{1-q_0}\right)^{1-\delta^R}(1-\lambda)}, \tag{23}$$

$$\varpi_t^R = 1 - \varpi_t^I. \tag{24}$$

In particular, whether positive cash flow news decrease *the institutional investors' share of aggregate wealth depends on whether the sentiment retail investors are sufficiently optimistic, as given by:*

$$\frac{\partial(\varpi_t^I)}{\partial D_t}\begin{cases} < 0, & \delta^R > q_t \\ = 0, & \delta^R = q_t \\ > 0, & \delta^R < q_t \end{cases}. \tag{25}$$

Recall, from our previous analysis, that the strength of retail optimism relative to institutional benchmarking concerns, $\delta^R - q_t$, determines the overexposure to market risk of retail investors relative to institutions. It is then intuitive to expect, as Lemma 5 states, that highly optimistic retail investors will become relatively wealthier after positive cash flow news ($dD_t > 0$) and poorer after negative cash flow news ($dD_t > 0$).

This effect can significantly reduce the equilibrium volatility of stock returns under exacerbated retail optimism. This case is illustrated by Fig. 6, where time-t equilibrium stock price-dividend raios, market prices of risk, and return volatilities, are plotted against cash flow news D_t, for $\delta^R > q_t > 0$. For comparison, the figure also plots these relationships under the BP, SENT, and STD reference cases.[15]

While the increasing pattern of stock price-dividend ratios and the decreasing pattern of market prices of risk can be anticipated from the impact of each feature (either sentiment or benchmarking concerns) when the other feature is removed (BP and SENT cases),[16] once again the pattern of volatility is more complex. In particular, the stock volatility can (i) be highly countercyclical, and

[15]The rest of the parameters are as in Fig. 2.

[16]Specifically, at all levels of cash flows D_t, stock price-dividend ratios increase, while market prices of risk fall with D_t beyond the levels prevailing in the benchmark BP and SENT economies.

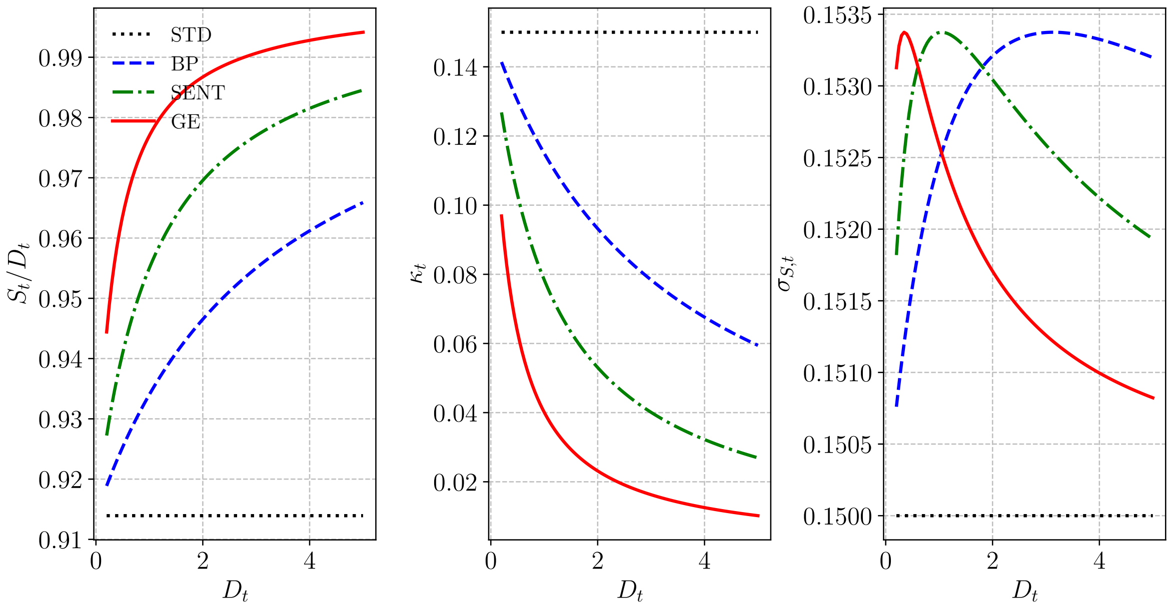


Figure 6: **Interim equilibrium: high optimism**

This figure plots the equilibrium stock price-dividend ratio (leftmost panel), market price of risk (center panel), and return volatility (rightmost panel), under the STD (dotted black line), BP (dashed blue line), SENT (dash-and-dot green line), and GE (red solid line) economies, for a relatively high level of optimism $\delta^R = 1$. Equilibrium values are depicted as a function of cash flows D_t as of $t = 1$, for a fixed initial share of aggregate wealth $\varpi_0^I = 0.5$. The rest of the model parameters are as in Fig.2.

(ii) fall below the levels prevailing in otherwise equivalent economies where either all investors are rational (BP case), or there are no institutions (SENT case). Result (i) is in line with the volatility pattern over the business cycle documented in the literature (see, e.g., Mele, 2007), and holds even in scenarios under which, in the corresponding SENT and BP cases, volatility is cyclical instead. Result (ii) highlights the importance of distinguishing the degree of institutionalization of markets in empirical analyses that associate excess return volatility, as inferred from, e.g., volatility-ratio tests, to irrational behavior and mispricing (e.g., Shiller, 1979, 1981; Giglio and Kelly, 2018).

The intuition for this result goes back to Proposition 3(b), reinforced by a feedback effect of the market risk premium on the retail stock demand. Specifically, as the stock's risk-return tradeoff worsens with D_t (center panel of Fig. 6), the optimistic retail investors reduce the fraction of their wealth invested in the stock across both GE and SENT cases (red solid and green dash-and-dot lines

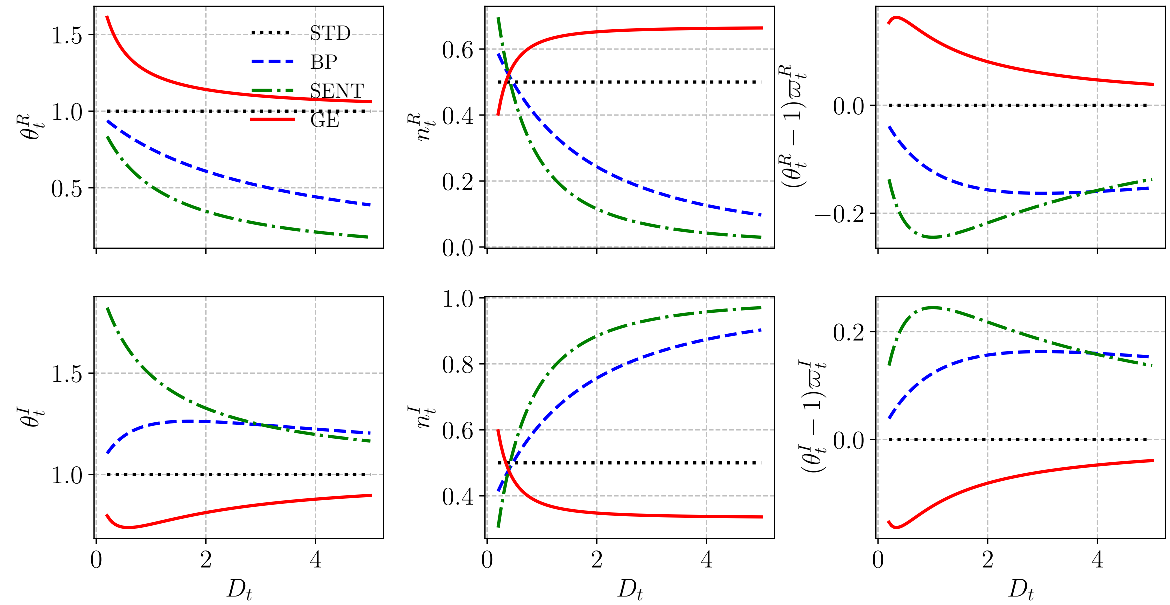


Figure 7: **Interim equilibrium portfolio allocations under high optimism**

This figure plots the equilibrium weights (leftmost panels) and number of shares $n_t^k = \theta_t^k W_t^k / S_t, k \in \{I, R\}$ of the stock (center panels) in the portfolios, and the leverage (rightmost panels), of optimistic retail (top panels) and rational institutional (bottom panels) investors. Depicted cases correspond to the STD (dotted black line), BP (dashed blue line), SENT (dash-and-dot green line), and GE (red solid line) economies. Across panels, equilibrium values are plotted against cash flows D_t as of $t = 1$, for a fixed initial share of aggregate wealth $\varpi_0^I = 0.5$. Model parameters are as in Fig. 6.

in the leftmost panels of Fig. 7).[17] In contrast to the behavior of the retail trading counterparts in the SENT case, the institutional trading counterparts in the GE case *increase*, following stronger benchmarking concerns, the stock allocation in their portfolios. When $\delta^R > q_t$, these trading patterns reduce portfolio heterogeneity across investor types much more rapidly in the GE case than in the SENT case, inducing a more aggressive attenuation effect of the relative-wealth channel on the stock return volatility. A similar comparison of the trading patterns of I and R investors in the BP case reveals that, when all agents are rational but some have benchmarking concerns, differences in portfolio allocations *widen* and the relative-wealth channel *exacerbates* the stock return volatility instead, as D_t rises. The strongly negative impact of the relative-wealth channel on volatility in the

[17]To facilitate comparison with the BP case, the portfolio of the sentiment-driven retail investors in the SENT economy is identified with the superscript "I" and plotted in the bottom panels in Fig. 7. In contrast, the superscript "R" (and the top row of panels) is reserved for their rational retail counterparts.

GE case explains both the *level* (volatility is lower in the GE case than in the SENT and BP cases) and the *slope* (volatility is countercyclical) effects. Because stronger wealth effects lead to steeper falls in volatility, both effects are exacerbated at higher levels of optimism.

6 Empirical Analysis

Our model suggests potentially large gains in explanatory power from integrating, within the same framework of analysis, both sentiment and institutions' benchmarking concerns. As illustrated by Fig. 3, the additional explanatory power could help rationalize the aggregate evidence on the changing relationship between optimism and volatility depending on the level of institutionalization of markets presented in Fig. 1. In this section, we summarize our model predictions with regard to this relationship, formulate other novel testable implications, and contrast them with the data. We end our empirical analysis with a discussion of limitations of our framework to reconcile theory and data, and with suggested directions for future research.

6.1 Testable Implications

Section 4.2.1 highlights an attenuating impact of the relative-wealth channel on stock return excess volatility. This effect implies, in particular, that in markets with a high presence of institutional investors, greater optimism need not exacerbate volatility but instead reduce it, and that whether a greater incidence of benchmarking concerns in financial markets increases or decreases volatility depends on the prevailing level of investor sentiment. These results, summarized by Proposition 3, lead to the following:

Testable Implication 1 (TI1): *The institutionalization of financial markets induces an asymmetric sentiment-volatility pattern, such that* (i) *an intensification of* pessimistic *sentiment always increases volatility while a similar intensification of* optimistic *sentiment can reduce it instead, and* (ii) *an intensification of benchmarking practices exacerbates volatility in markets with predominantly* pessimistic *sentiment but attenuates it in markets with predominantly* optimistic *sentiment.*

Second, according to Section 4.2.2, benchmarking concerns have a positive but limited influence on the demand of rational institutional investors relative to the demand of optimistic retail investors. Thus, while unlikely to help correct situations of low or moderate asset overpricing, institutions can exert a significant correcting force at more severe overpricing levels, leading to the following:

Testable Implication 2 (TI2): *A greater institutionalization of financial markets* attenuates *stock overpricing when investor sentiment is strongly optimistic but not otherwise.*

6.2 Evidence

We contrast our model implications with data on the U.S. stock market. Our baseline stock samples are sourced from Stambaugh et al. (2015) and Pavlova and Sikorskaya (2023), who provide their datasets publicly on their websites. Daily returns, prices, adjustment factors, and market capitalizations are obtained from CRSP, while market, risk-free rate, and factor returns are sourced from Ken French's database. We focus on ordinary common shares (CRSP codes 10 and 11) and exclude stocks with share prices below $5 to mitigate microstructure effects.

Testing implications TI1 and TI2 requires measures of investor sentiment and of the degree of institutionalization of markets. We use two proxies for sentiment. First, we use the *market-level sentiment* metric from Baker and Wurgler (2006), which we denote as *SentBW*. This is an aggregate monthly time-series from 1965/07 to 2022/06 and is available from Wurgler's webpage.[18] This time-series is normalized to have a mean value of zero. We create quartiles of *SentBW* over the whole sample from 1965 to 2022. Quartiles 1 and 2 correspond to strongly and moderately negative sentiment months, whereas quartiles 3 and 4 correspond to moderately and strongly positive months, respectively.

Second, following Dong et al. (2024), we create a *stock-level sentiment* measure that combines trading activity and price behavior. These authors advocate for trade-based sentiment measures because they are readily available daily, widely used by market participants, and capture investors' actual trading activity and decisions. Specifically, we combine three key indicators: trading volume shocks, reflecting sudden increases in trading activity; technical overbought/oversold conditions, indicating whether a stock is trading near recent highs or lows; and proximity to recent high

[18] https://pages.stern.nyu.edu/~jwurgler/.

prices, showing how close a stock is to its peak. These indicators are calculated over different time windows, adjusted to remove market noise, standardized annually, and then averaged to produce a single monthly sentiment score per stock, which we denote as *SentDMPZ*. More details on the construction of this measure are provided in Appendix C.

We consider two different proxies for the degree of institutionalization of markets, both available at the stock level. Our first proxy is the *institutional ownership ratio* (IOR), defined as the ratio of a stock's shares held by institutions to the number of shares outstanding. We calculate stocks' IOR based on the quarterly institutional holdings data available from 1980 to 2021 from Thomson/Refinitiv.[19]

Our second proxy is the *benchmark intensity* measure, BMI, of Pavlova and Sikorskaya (2023). BMI quantifies a stock's cumulative weight across all benchmarks, adjusted for the assets of mutual funds and ETFs tracking each benchmark, and is available for May and June each year from 1998 to 2018 from Sikorskaya's website.[20] Pavlova and Sikorskaya argue that, after controlling for stock size, liquidity, and index banding/inclusion criteria, changes in BMI around Russell reconstitutions are plausibly exogenous. Consequently, they use changes in BMI, ΔBMI, as an instrument for institutional ownership shifts to establish a causal link between ownership changes and future returns. We adopt a similar approach to examine the causal effect of ownership changes on stock volatility across different sentiment levels.

Both institutionalization proxies aim to measure investing in a stock that can be related to the benchmarking concerns of institutional investors. Empirically, we cannot differentiate, as we do in the model, the effect on this measure of the amount of capital in institutions' hands from the effect of the strength of their benchmarking concerns.[21] For this reason, in our tests we interpret either measure, depending on the context, as capturing both the degree of institutionalization and the benchmarking intensity of a stock.

[19] This data is sourced from the 13F form that investment companies and professional money managers are required to file with the SEC.

[20] https://www.sikorskaya.net/data/.

[21] In principle, two stocks with the same IOR could be exposed to different benchmarking concerns if (i) their weights in the benchmarks that institutions follow differ (as captured by BMI), or (ii) the average sensitivity of managerial pay for benchmark-adjusted performance across the institutions holding the stocks differ (not captured by BMI).

For some of our analyses, we create test portfolios by sorting and grouping stocks based on their degree of *relative mispricing* according to Stambaugh et al. (2015)'s MISP score. MISP is a combination of 11 well-known asset pricing anomalies that, according to these authors, are more likely driven by market mispricing than by rational risk premia.[22] They show that the predictability of these anomalies is higher after periods of high sentiment, and is mostly driven by the overpriced stocks (high values of MISP), as short selling is more costly than holding long positions.[23] We collect the monthly stock-level MISP from Stambaugh's webpage,[24] which covers the period from 1965 to 2016.

In summary, after combining the above datasets, our final sample when using the market-level sentiment measure *SentBW* covers the period 1980/04 to 2016/12,[25] and includes three main variables of interest with different frequencies and scopes: *SentBW* (monthly, market-level), IOR (quarterly, stock-level), and MISP (monthly, stock-level). In our cross-sectional analysis, which combines Pavlova and Sikorskaya's BMI dataset with the Russell 1000/2000 constituents obtained from the Frank Russell Company and the *SentDMPZ* dataset, the final sample spans the period from 2000 to 2018.

6.2.1 On Return Volatility

Following our testable implication TI1, we examine the empirical relationship among investor sentiment, institutions, and return volatility from two different perspectives: by focusing on the effect of market-wide sentiment at different institutionalization levels (TI1(i)), and by testing the impact of changes in a stock market's institutionalization at different sentiment levels (TI1(ii)).

[22]The individual anomalies and the studies uncovering them (in parenthesis) are: momentum (Jegadeesh and Titman, 1993), gross profitability (Novy-Marx, 2013), asset growth (Cooper et al., 2008), investment to assets (Titman et al., 2004), return on assets (Fama and French, 2006), net operating assets (Hirshleifer et al., 2004), accruals (Sloan, 1996), net stock issues (Loughran and Ritter, 1995), composite equity issues (Daniel and Titmans, 2006), failure probability (Campbell et al., 2008), and O-score (Ohlson, 1980 and Dichev, 1998).

[23]Chu et al. (2020) show that the relation between short selling costs and the predictability of these anomalies is causal, providing further support for the argument that this predictability is driven by mispricing.

[24]https://finance.wharton.upenn.edu/∼stambaug/

[25]This is 64% of the period covered by the *SentBW* dataset. Of the 441 months in our final sample, 42 months (9.5%) are strongly negative-sentiment months, that is, they fall in the lowest quartile of *SentBW*. This is the least populated quartile of *SentBW* in our final sample, which works against us finding any significant results in those months.

Effect of Market-Wide Sentiment. To test the effect of institutions on the relationship between sentiment and volatility, we form monthly rebalanced stock portfolios by assigning all the stocks in our sample to one of the 30 bins that result from sequential sorts first on their previous-quarter IOR tercile, and then on their previous-month MISP decile. Because prior literature has shown that the effects of MISP on returns are concentrated in the above-median group (Stambaugh et al., 2012; Chu et al., 2020), we exclude below-median MISP portfolios (deciles 1 to 5) from our analysis. Additionally, as we are primarily interested in contrasting high-IOR with low-IOR stocks, we exclude the mid-IOR portfolios (tercile 2).

For each of the remaining 10 portfolios, we calculate the next month's daily returns by weighting stock-level daily returns based on their gross returns in the formation month. As argued by Asparouhova et al. (2013), this weighting method places comparable weight on both large and small firms. This is especially informative for volatility and mispricing and helps mitigate statistical and microstructure biases associated with equal-weighting. The appropriately scaled standard deviation of these daily returns is used as the monthly portfolio return volatility. We test implication TI1(i) by regressing this return volatility on market-wide sentiment, institutionalization levels, and controls according to the following specification:

$$\begin{aligned} Pvol_{p,m+1} &= \alpha + \sum_{k=2}^{4} \beta_{Qk} \text{SentBW Qk}_m + \lambda \text{High IOR}_{p,m} + \\ &+ \sum_{k=2}^{4} \theta_{Qk} (\text{High IOR}_{p,m} \times \text{SentBW Qk}_m) + \nu \text{Indpro}_m + \psi \text{Recess}_m + \epsilon_{p,m} \quad (26) \end{aligned}$$

where $Pvol_{p,m+1}$ is the volatility of gross-return-weighted returns of portfolio p in month $m+1$, $SentBW$ Qk_m are dummy variables that indicate months that fall in quartile Qk (k= $1, \ldots, 4$) of the market-wide sentiment variable $SentBW_m$, and *High* $IOR_{p,m}$ is a dummy variable that equals 1 if portfolio p is in the highest tercile of institutional ownership in month m and is zero otherwise. $Indpro_m$ and $Recess_m$ are, respectively, the industrial production variable and NBER recession dummy extracted from Baker and Wurgler (2006), and aim to account for countercyclical variation in volatility.[26]

[26] In unreported analysis (available upon request), we find that without these controls our sentiment dummies for

According to TI1(i), portfolio volatility should decline as sentiment improves from strongly negative levels, particularly when institutional presence is high (*High IOR* $= 1$). This prediction implies that the interaction terms in Eq. (26) should have negative coefficients ($\theta_{Qk} < 0$ for $k \in \{2, 3, 4\}$). Moreover, the effect should be stronger at higher sentiment levels, such that $\theta_{Q4} < \theta_{Q3} < \theta_{Q2}$. Additionally, when sentiment is in the lowest quartile (i.e., strongly negative), institutions are expected to increase portfolio volatility, implying a positive coefficient λ.

Pooled OLS regression estimates of Eq. (26), reported in Table 1, are consistent with these predictions. Models (1)-(3) do not control for institutional ownership, so the reported effects reflect averages across varying levels of institutionalization. According to model (1), months with high positive sentiment are not necessarily associated with greater volatility. According to model (2), where dummy variables for negative-sentiment months (*SentBW* Q1-Q2) are omitted, volatility is, on average, 0.74% lower in moderately positive-sentiment months (*SentBW* Q3 $= 1$) and statistically significant at the 1% level. Portfolio volatility in strongly positive-sentiment months (*SentBW* Q4 $= 1$), however, remains indistinguishable from that in negative-sentiment months. These results are consistent with the nonmonotonic relationship between sentiment and volatility predicted by ours and existing models.

In model (3), where only the dummy for strongly negative-sentiment months (*SentBW* Q1) is omitted, all coefficients β_{Qk} are negative and significant, indicating that portfolio volatility declines as sentiment improves from strongly negative levels. Among these coefficients, β_{Q2} has the smallest negative effect, while β_{Q3} has the largest. Considering the average positive presence of institutions in financial markets during the period, these findings are consistent with the left-skewed U-shaped sentiment-volatility pattern predicted by TI1 (see also Fig. 3(a)).

Models (4)-(7), which explicitly account for the presence of institutions in the stock market, contain our main estimates of interest. In column (4), we replace the sentiment dummies with the high-ownership dummy (*High IOR*). The coefficient λ on *High IOR* is positive and significant, indicating that portfolios with high institutional ownership exhibit greater volatility than those with

strongly negative- and strongly positive-sentiment months are consistently significant. The business cycle variable and the recession dummy capture most of the volatility changes in the extreme sentiment months.

Table 1: **Effects of Market-Wide Sentiment on Volatility**

This table reports the results of a pooled OLS regression of next month's volatility of gross-return-weighted portfolio returns on sentiment quartile k (*SentBW* Qk), institutional ownership (IOR) highest tercile dummy (*High IOR*), and their interactions. Test assets are the 10 portfolios with MISP deciles between 6 and 10 and IOR terciles 1 and 3. Regressions include controls for industrial production (*Indpro*) and NBER recessions (*Recess*). Newey and West (1987) standard errors with 12 lags are reported in parentheses. *, **, and ***, represent significance at the 10%, 5%, and 1% levels, respectively.

Dependent Variable: Next Month Volatility of Gross-Return-Weighted Portfolio Returns							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
SentBW Q4	-0.1161	-0.4953	-1.1356**		0.0411	-0.2703	-0.6795
	(-0.47)	(-1.62)	(-2.43)		(0.16)	(-0.92)	(-1.63)
SentBW Q3		-0.7431***	-1.4017***			-0.5979***	-1.0254***
		(-3.16)	(-3.25)			(-2.88)	(-2.77)
SentBW Q2			-0.8774*				-0.5697
			(-1.88)				(-1.43)
High IOR				1.4268***	1.5152***	1.6507***	2.1131***
				(14.05)	(12.87)	(9.09)	(6.02)
High IOR × SentBW Q4					-0.3144	-0.4499*	-0.9123**
					(-1.35)	(-1.67)	(-2.26)
High IOR × SentBW Q3						-0.2903	-0.7527**
						(-1.26)	(-1.99)
High IOR × SentBW Q2							-0.6153
							(-1.52)
Indpro	0.0520***	0.0499***	0.0506***	0.0533***	0.0520***	0.0499***	0.0506***
	(8.57)	(8.30)	(8.41)	(8.39)	(8.60)	(8.34)	(8.45)
Recess	3.0184***	2.9619***	2.9030***	3.0056***	3.0184***	2.9619***	2.9030***
	(4.85)	(4.92)	(4.93)	(4.91)	(4.86)	(4.92)	(4.93)
Intercept	-0.1221	0.4067	1.0115*	-0.9623*	-0.8797*	-0.4187	-0.045
	(-0.25)	(0.82)	(1.65)	(-1.85)	(-1.77)	(-0.83)	(-0.08)

low institutional ownership. This represents an average effect across all sentiment levels, consistent with the theoretical amplification of volatility by institutional ownership, as discussed in Basak and Pavlova (2013) and reflected in our model (see, e.g., the right panel of Fig. 2).

Columns (5)-(7) allow testing the specific effect of institutional ownership on the sentiment-volatility relationship that is novel to our model. When *High IOR* equals zero, the coefficients β_{Qk} on the sentiment dummies capture the volatility-sentiment relationship for low-IOR portfolios. Column (7) shows that volatility declines in moderately positive-sentiment months for these portfolios, as indicated by the negative and significant coefficient β_{Q3}. When *High IOR* equals one, the interaction terms between *High IOR* and the sentiment dummies reveal how volatility differs between high- and low-ownership portfolios across sentiment months. According to TI1(i), the negative and asymmetric relationship between sentiment and volatility should be more pronounced for high-IOR portfolios. This implies that the coefficients θ_{Qk} for the moderately and strongly positive sentiment dummies should be negative. Consistent with this prediction, the coefficients θ_{Q3} and

θ_{Q4} in column (7) are negative and significant at the 5% level, while the coefficient θ_{Q2} on the interaction with $SentBW$ Q2 is not significant. Moreover, the coefficient λ on *High IOR*, which captures the average volatility of high-*IOR* portfolios in strongly negative-sentiment months, is positive and significant at the 1% level. Thus, as predicted, compared to low-*IOR* portfolios volatility is lower in positive-sentiment months and increases more with pessimism among high-*IOR* portfolios. Overall, the evidence presented in Table 1 largely supports the relationship between sentiment and volatility, depending on the level of institutionalization of markets, predicted by part (i) of implication TI1.

Impact of Institutions. We next leverage the BMI measure of Pavlova and Sikorskaya (2023) to examine, within a *causal* inference framework, the impact of changes in institutional benchmarking intensity on stock volatility at different sentiment levels. We identify BMI with the intensity q_t of benchmarking concerns in our model. We proxy for changes in this variable using changes in BMI, ΔBMI, from May to June of each year. We then interact these changes with the stock-level sentiment proxy *SentDMPZ* and examine their joint impact on stock volatility in June according to the following specification:

$$Mvol_{it}^{June} = \alpha + \beta \Delta BMI_{it} + \sum_{k=2}^{4} \theta_{Qk} \text{SentDMPZ Qk}_{it} + \\ + \sum_{k=2}^{4} \nu_{Qk} (\Delta BMI_{it} \times \text{SentDMPZ Qk}_{it}) + \lambda' X_{it} + \epsilon_{it}, \quad (27)$$

where $Mvol_{it}^{June}$ is the volatility of the daily returns of stock i in June of year t, scaled to a monthly measure, ΔBMI_{it} is the difference between the BMI of stock i in May of year t and its BMI in June of the same year. Pavlova and Sikorskaya (2023) argue that, conditional on the logarithm of total market value ($Log(Mcap)$), banding controls (i.e., dummies for being in the band ($InBand$), being in the Russell 2000 ($InR2000$), and their interaction in May of year $t-1$)[27] and $Float$ in May,[28] mechanical reconstitutions of the Russell index serve as a source of exogenous variation in BMI.

We include these controls in the vector X_{it}, alongside the 5-year monthly rolling beta, computed

[27]We calculate these measures using Russell constituents data obtained from the Frank Russell Company.

[28]Unlike Pavlova and Sikorskaya (2023), who use the Russell float factor, a proprietary liquidity measure affecting index weight to which we do not have access, we use the ratio of shares on float to shares outstanding from CRSP to compute $Float$.

using the CRSP total market value-weighted index (*Beta*), and the 1-year monthly rolling average bid-ask percentage spread (*BASpread*). This vector of controls also includes the daily stock volatility from January to May of year t, scaled to a monthly measure ($Mvol^{Jan-May}$), to account for the well-known persistence in stock return volatility. We perform this estimation for all stocks within 300 ranks of the Russell 1000/2000 cutoff determined, in line with the Russell reconstitution methodology, at the end of April.

According to implication TI1(ii), the relationship between ΔBMI and volatility is expected to be positive when sentiment is negative, and negative when sentiment is positive and high (see also Fig. 3(b)). Two clear predictions for the signs of the coefficients in Eq. (27) emanate from this implication: the interaction term at high sentiment levels should be negative ($\theta_{Q4} < 0$), and the coefficient on ΔBMI in the full specification (reflecting the effect of changes in benchmarking concerns when sentiment is clearly negative) should be positive ($\beta > 0$).

Cross-sectional Fama-MacBeth regression estimates of Eq. (27), reported in Table 2, offer strong causal evidence for these predictions. First, when we do not condition on sentiment (column (1)) the coefficient on ΔBMI is positive (0.019) but not statistically distinguishable from zero, consistent with a non-monotonic relationship between ΔBMI and volatility across sentiment levels. This underscores the importance of conditioning on sentiment to properly identify these effects.

Second, when we do condition on sentiment, changes in BMI have the predicted effects on stock return volatility in the cross-section depending on whether sentiment is strongly negative or strongly positive. To capture variation across sentiment levels, we introduce indicator variables for the top three quartiles of the stock-level sentiment measure, *SentDMPZ*. In column (2), we include only an indicator for the highest sentiment quartile (*SentDMPZ* Q4), implying that the coefficient β on ΔBMI reflects the average effect for the bottom three quartiles (*SentDMPZ* Q1-Q3). The estimated coefficient is 0.036, marginally significant at the 10% level. The coefficient ν_{Q4} on the interaction term ΔBMI × *SentDMPZ* Q4 is -0.076 and statistically significant at the 1% level, indicating that the ΔBMI-volatility relationship is significantly negative in the highest sentiment quartile relative to the lower three quartiles. The implied effect of ΔBMI on volatility in the highest quartile of sentiment is given by the sum of the coefficients on ΔBMI and on the interaction term ($\beta + \nu_{Q4}$),

Table 2: **Stock-Level Effects of Sentiment and Benchmarking Intensity on Volatility**

This table reports the results of Fama-MacBeth cross-sectional regressions of stock volatility in the month of June each year on changes in benchmarking intensity (ΔBMI) from May to June, following Pavlova and Sikorskaya (2023), and quartiles of stock-level sentiment in May, constructed based on the methodology described in Dong et al. (2024) and denoted as *SentDMPZ*. Following Pavlova and Sikorskaya (2023), these regressions control for the log of market capitalization ($Log(Mcap)$), the number of shares on the float as a percent of shares outstanding ($Float$), dummies for being in the band ($InBand$), being in the Russell 2000 ($InR2000$), and their interaction in May of year $t-1$. Regressions also control for the CAPM beta ($Beta$), i.e., the 5-year monthly rolling beta computed using CRSP total market value-weighted index and the 1-year monthly rolling average bid-ask percentage spread ($BASpread$), and for past volatility, computed from daily returns from January to May of each year and then scaled to a monthly value ($Mvol^{Jan-May}$). Newey and West (1987) standard errors are reported in parentheses. *, **, and ***, represent significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)
ΔBMI	0.019	0.036*	0.036*	0.096***
	(0.016)	(0.017)	(0.017)	(0.016)
SentDMPZ Q4		-0.003**	-0.003**	-0.004***
		(0.001)	(0.001)	(0.001)
SentDMPZ Q3			-0.001	-0.001*
			(0.001)	(0.001)
SentDMPZ Q2				-0.002***
				0.00
ΔBMI × SentDMPZ Q4		-0.076***	-0.076*	-0.136***
		(0.023)	(0.037)	(0.036)
ΔBMI × SentDMPZ Q3			-0.015	-0.075
			(0.051)	(0.051)
ΔBMI × SentDMPZ Q2				-0.122***
				(0.015)
$\text{Mvol}^{Jan-May}$	0.197***	0.198***	0.197***	0.199***
	(0.058)	(0.057)	(0.058)	(0.059)
InBand	-0.001	-0.001	-0.001	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)
InR2000	0.003***	0.003***	0.003***	0.003***
	(0.001)	(0.001)	(0.001)	(0.001)
InBand × InR2000	-0.002	-0.001	-0.001	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)
Log(Mcap)	-0.002	-0.002	-0.002	-0.002
	(0.001)	(0.001)	(0.001)	(0.001)
Beta	0.004***	0.004***	0.004***	0.004***
	(0.001)	(0.001)	(0.001)	(0.001)
BASpread	0.004***	0.004***	0.004***	0.004***
	(0.001)	(0.001)	(0.001)	(0.001)
Float	-0.004	-0.004	-0.004	-0.004
	(0.003)	(0.003)	(0.002)	(0.002)
Intercept	0.045*	0.045*	0.047*	0.048**
	(0.023)	(0.023)	(0.023)	(0.023)
Adj R^2	0.448***	0.450***	0.450***	0.451***
	(0.035)	(0.035)	(0.035)	(0.035)
Obs	10,531	10,531	10,531	10,531

yielding -0.04. This result is consistent with our model's prediction that ΔBMI has an inverse effect on volatility when sentiment is high. Results remain similar in column (3), when we add an indicator for the second highest sentiment quartile of *SentDMPZ*, and strengthen in column

(4), when we introduce separate indicators for sentiment quartiles Q2-Q4 and interact them with ΔBMI. In the latter specification, the coefficient on ΔBMI (0.096) captures the effect for the lowest sentiment quartile (*SentDMPZ* Q1) and is statistically significant at the 1% level, confirming the positive predicted sign for β in Eq. (27).

Results are not only statistically significant but also economically meaningful. To assess the economic magnitude of the reported causal effects, we quantify the impact of a one-standard-deviation change in ΔBMI on stock volatility. The time-series average of the cross-sectional standard deviation of ΔBMI over the 19-year sample period is 0.03167. In column (2), a one-standard-deviation increase in ΔBMI raises volatility by 0.00114 for stocks in quartiles Q1-Q3 of *SentDMPZ* ($= 0.03167 \times 0.036$). This represents 3.7% of the average absolute change in volatility from May to June, which we calculated to be 0.03077. For stocks in *SentDMPZ* Q4, a one-standard-deviation increase in ΔBMI *reduces* volatility by 0.00127 ($=-0.04 \times 0.03167$), or 4.1% of the average absolute change in volatility from May to June. Thus, a stock that moves from experiencing negative or moderate positive sentiment to high optimism (*SentDMPZ Q4*) would see its average absolute change in return volatility from May to June fall by 7.8%. In column (4), the economic magnitude of the effect is even larger: a one-standard deviation increase in ΔBMI raises volatility by 0.00304 ($= 0.03167 \times 0.096$), or 9.9% of the average absolute change from May to June. For *SentDMPZ* Q4, the effect of ΔBMI on volatility is given by the sum of the coefficient on ΔBMI and the interaction coefficient (DBMI $\times$ *SentDMPZ* Q4, -0.136), resulting in -0.04. Thus, a one-standard deviation increase in ΔBMI reduces return volatility by -0.00127 ($=-0.04 \times 0.03167$), or 4.1% of the average absolute change from May to June. This implies that a stock that moves from experiencing strong negative (*SentDMPZ Q1*) to strong positive sentiment (*SentDMPZ Q4*) would see its average absolute change in return volatility from May to June fall by 14%.

In summary, given that changes in BMI are plausibly (conditionally) exogenous around Russell index reconstitutions, we interpret our findings as evidence of an economically relevant causal link between changes in BMI from May to June and volatility in June. Consistent with part (ii) of implication TI1, this relationship is positive for low-sentiment stocks (*SentDMPZ* Q1) but reverses for high-sentiment stocks (*SentDMPZ* Q4), highlighting the role of sentiment in shaping the ΔBMI-volatility relationship.

6.2.2 On Stock Mispricing

To test implication TI2, we examine how the predictive power of the mispricing (MISP) score of Stambaugh et al. (2015) in the cross-section of average stock returns is affected by the presence of institutions in periods of moderately vs. strongly positive sentiment. If TI2 holds, stocks with similar overpricing potential—as captured by MISP—should experience lower effective overpricing—as captured by subsequent negative abnormal returns—in the presence of institutions only when sentiment is strongly positive. To assess whether this is the case, we again form 30 portfolios by sequentially sorting on IOR terciles and MISP deciles. Portfolios with high (low) MISP are potentially overpriced (underpriced). We then create two subsamples, one that includes only months with moderately positive sentiment ($SentBW$ Q3), and one that includes only months with strongly positive sentiment ($SentBW$ Q4).

Table 3, which reports average raw and risk-adjusted monthly returns across these portfolios over the period 1980/04–2016/12, confirms the attenuating effect of institutions on overpricing under heightened sentiment. High-*MISP* stocks consistently experience negative returns in the months following portfolio formation (rows (2), (4) and (6)).[29] The effect is similar across portfolios with low and high IOR during periods of *moderately positive* sentiment (difference column "(2)-(1)"), suggesting that a greater institutional presence does not mitigate overpricing at moderate levels of investor optimism. In contrast, the negative association between high values of MISP and future stock returns during periods of *strongly positive* sentiment weakens significantly for high-IOR portfolios (columns (3), (4), and (4)-(3)), indicating that institutions help reduce overpricing in such conditions. For example, within the $SentBW$ Q4 subsample, the Fama and French (2015) 5-factor (FF5) alpha for high-*MISP* stocks is -1.9% (t-stat = -8.07) under low institutional presence (*Low IOR*) but only -57 bps (t-stat = -1.94) under high institutional presence (*High IOR*). The difference in alphas, 1.33%, is statistically significant at the 1% level (t-stat = 3.96).

To facilitate a comparison of how the presence of institutions alters the empirical link between

[29] For brevity, we report only gross-return-weighted average portfolio returns. Value-weighted average returns (available from the authors upon request) yield qualitatively similar findings.

Table 3: **Effect of Institutions on Overpriced Stocks: Portfolio Sorts**

This table reports gross-return-weighted average and risk-adjusted monthly returns (using single-index and Fama and French (2015) 5-factor models, denoted as CAPM and FF5, respectively) on portfolios double-sorted by MISP deciles (rows) in the previous month and IOR terciles (columns) in the previous quarter. The first 3 columns report the results for the subsample of months with moderately positive sentiment (*SentBW* Q3=1), while the remaining columns report the results for the subsample of months with strongly positive sentiment (*SentBW* Q4=1). Out of the 30 portfolios in each subsample, we report the returns on the extreme portfolios corresponding to IOR terciles 1 and 3 and MISP deciles 1 and 10. Newey and West (1987) standard errors with 12 lags are reported in parentheses. *, **, and ***, represent significance at the 10%, 5%, and 1% levels, respectively.

			SentBW Q3 = 1			SentBW Q4 = 1		
			Low IOR	High IOR	High-Low	Low IOR	High IOR	High-Low
			(1)	(2)	(2)-(1)	(3)	(4)	(4)-(3)
Average	Low MISP	(1)	1.5313***	1.0663***	-0.4650***	2.3177***	1.9847***	-0.333
			(3.74)	(2.80)	(-2.84)	(6.51)	(6.33)	(-1.45)
	High MISP	(2)	-0.5945	-0.1889	0.4057	-1.0458**	0.3352	1.3810***
			(-1.38)	(-0.41)	(1.50)	(-2.27)	(0.87)	(4.76)
	High-Low	(2)-(1)	-2.1259***	-1.2552***	0.8706***	-3.3635***	-1.6495***	1.7140***
			(-7.60)	(-4.05)	(3.54)	(-14.51)	(-6.71)	(6.02)
CAPM	Low MISP	(3)	0.8448***	0.3075	-0.5372***	1.3041***	0.9339***	-0.3702
			(2.93)	(1.42)	(-3.18)	(3.74)	(3.40)	(-1.58)
	High MISP	(4)	-1.3274***	-1.0809***	0.2465	-2.2924***	-0.9732***	1.3192***
			(-3.87)	(-3.40)	(0.90)	(-8.26)	(-2.81)	(3.96)
	High-Low	(4)-(3)	-2.1722***	-1.3885***	0.7837***	-3.5965***	-1.9071***	1.6894***
			(-7.49)	(-4.30)	(3.09)	(-21.89)	(-6.57)	(5.90)
FF5	Low MISP	(5)	0.7377***	0.153	-0.5847***	1.0448***	0.3819**	-0.6629***
			(3.69)	(1.03)	(-2.98)	(6.62)	(2.53)	(-3.33)
	High MISP	(6)	-1.3582***	-1.1830***	0.1752	-1.8961***	-0.5698*	1.3263***
			(-5.96)	(-5.10)	(0.69)	(-8.07)	(-1.94)	(3.96)
	High-Low	(6)-(5)	-2.0959***	-1.3361***	0.7599***	-2.9410***	-0.9518***	1.9892***
			(-7.58)	(-4.33)	(2.83)	(-16.92)	(-2.62)	(5.69)

MISP and future stock returns across periods with moderately versus strongly positive sentiment, we further analyze this relationship in a regression framework. The dependent variable is the next-month return of the short leg of the MISP strategy (i.e., MISP decile 10) in excess of the risk-free rate. We restrict the sample to months with positive sentiment (*SentBW* Q3 and Q4) and stocks in the extreme terciles of IOR (terciles 1 and 3). We estimate the following model:

$$\begin{aligned} Pxret_{p,m+1} &= \alpha + \beta \text{SentBW Q4}_m + \lambda \text{High IOR}_{p,m} + \\ &+ \nu(\text{High IOR}_{p,m} \times \text{SentBW Q4}_m) + \psi' X_m + \epsilon_{p,m} \end{aligned} \tag{28}$$

where $Pxret_{p,m+1}$ is the gross-return-weighted portfolio return in excess of the risk-free rate for portfolio p in month $m+1$, $SentBW$ Q4_m is a dummy variable that indicates months that fall in quartile 4 of the market-wide sentiment variable $SentBW$, and $High\ IOR_{p,m}$ is a dummy variable that equals 1 for portfolios in the highest tercile of institutional ownership, and zero otherwise. X_m is a vector of controls that includes the five factors of Fama and French (2015), i.e., MKTRF, SMB, HML, RMW, and CMA. When both the *SentBW* Q4 and the *High IOR* dummies are set to zero, the intercept reflects the performance of the baseline portfolios of stocks with low institutional ownership during months with moderately positive sentiment.

Table 4: **Effect of Institutions on Overpricing: Regression Analysis**

This table reports the results of pooled OLS regressions of next month's gross-return-weighted portfolio excess returns on dummies for high sentiment (*SentBW* Q4) and high institutional ownership (*High IOR*). Tests assets are portfolios with overpriced stocks (i.e., MISP decile 10). Controls include the market factor in columns (4) to (6) and the Fama and French (2015) five factors in columns (7) to (9). Newey and West (1987) standard errors with 12 lags are reported in parentheses. *, **, and ***, represent significance at the 10%, 5%, and 1% levels, respectively.

	Dependent Variable: Next-Month Gross-Return-Weighted Portfolio Returns in Excess of Riskless Rate								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
SentBW Q4	-0.1483 (-0.20)		-0.6271 (-0.81)	-0.3493 (-0.90)		-0.8281* (-1.81)	-0.1631 (-0.66)		-0.6418** (-2.03)
High IOR		0.8508*** (3.13)	0.4143 (1.19)		0.8508*** (3.63)	0.4143 (1.33)		0.8508*** (3.79)	0.4143 (1.38)
High IOR × SentBW Q4			0.9575* (1.94)			0.9575** (2.31)			0.9575** (2.44)
MKTRF				1.2165*** (22.70)	1.2157*** (22.57)	1.2165*** (22.76)	1.0886*** (32.20)	1.0854*** (34.01)	1.0886*** (32.51)
SMB							0.7967*** (11.54)	0.7948*** (11.31)	0.7967*** (11.55)
HML							0.2244*** (3.24)	0.2220*** (3.20)	0.2244*** (3.25)
RMW							-0.1395 (-1.17)	-0.1429 (-1.23)	-0.1395 (-1.17)
CMA							-0.4379*** (-3.23)	-0.4435*** (-3.37)	-0.4379*** (-3.24)
Intercept	-0.751 (-1.59)	-1.2440*** (-3.29)	-0.9582** (-1.97)	-1.2360*** (-4.13)	-1.8203*** (-7.74)	-1.4432*** (-4.15)	-1.2152*** (-7.43)	-1.7078*** (-9.01)	-1.4224*** (-6.59)

Pooled OLS regression estimates of Eq. (28), presented in Table 4, confirm the results of our portfolio analysis. Because the dependent variable consists solely of short-leg returns (i.e., top decile of MISP), we expect negative intercept coefficients, indicating that the test portfolios are correctly classified as overvalued. This expectation is confirmed across specifications. Crucially, according to implication TI2, we further expect a positive sign for the coefficient ν on the interaction between *High IOR* and *SentBW* Q4, indicating that the predictive power of high MISP values in the cross-section of average returns weakens in strongly positive-sentiment months when institutional presence is high. In line with this prediction, the estimate of this coefficient ν in column (3) is positive and significant at the 10% level. Estimates of the same coefficient remain positive while statistical significance improves to the 5% level when accounting for the market excess returns in columns (4) to (6), and for the five factors of Fama and French (2015) in columns (7) to (9).

6.3 Reconciling Theory and Empirical Evidence

While our empirical results largely align with the model's predictions, some findings deviate from theoretical expectations. Below, we outline some of these discrepancies, which may indicate limitations of our framework and suggest directions for future refinement.

First, Proposition 4 predicts that institutional investors should exacerbate overpricing under moderate optimism. However, our mispricing test indicates that institutions do not systematically contribute to overpricing in this regime. Instead, we observe no significant differences in overpricing within this degree of optimism (see Table 3, columns (1) and (2)). Moreover, low-MISP stocks exhibit underpricing even under moderately positive sentiment. This result cannot be explained within our single-stock model, where positive sentiment is unambiguously associated with overpricing, and might call for a multiasset extension in which some stocks can, in principle, be underpriced even under high aggregate sentiment. Regardless, the fact that high-IOR stocks have lower subsequent risk-adjusted returns is consistent with positive price pressure by institutions.

Second, our cross-sectional regressions yield some results inconsistent with the model's predicted volatility patterns. In theory, for mid-level sentiment, volatility should initially rise with benchmarking and then decline for higher levels, implying that the coefficients on the interactions of ΔBMI with $SentDPMZ$ Q2 and Q3 should be indistinguishable from zero, as the theoretical relation is non-monotonic. However, in Table 2, the coefficient on the interaction between ΔBMI and $SentDPMZ$ Q2 is negative, significant, and larger than that for ΔBMI, implying an overall negative ΔBMI-volatility relation for moderate sentiment, for which the model predicts no effect.

Lastly, in Table 4, the lack of statistical significance for the intercept and $SentBW$ Q4 in column (1) contradicts the theoretical prediction that, in the absence of institutions, stocks with strongly positive sentiment should be more overpriced than stocks with moderately positive sentiment. However, this test shows no effects for either level of sentiment.

Overall, these deviations from theoretical predictions suggest that, while our framework captures key drivers of price dynamics, additional factors, such as heterogeneous institutional constraints, varying risk preferences, or alternative sentiment transmission mechanisms, may be at play in the data. Future work could explore extensions that incorporate these elements to better reconcile theory with empirical findings.

7 Conclusion

Despite the significant trend toward the portfolio delegation of households to institutional managers in recent years, most studies on the effect of sentiment-driven trading on asset prices assume that the other side of the trade is taken by direct investors. Similarly, despite the abundant evidence of irrational trading by individual investors, most of the literature on the role of institutions in asset pricing assumes that the trading counterparties of institutions are rational. In this paper, we account for the simultaneous presence of institutions and sentiment-driven retail trading in financial markets to find several novel equilibrium patterns.

First, the joint effect of sentiment and benchmarking concerns on stock volatility can be radically different from the addition of the effects stemming from either feature in isolation. In particular, when optimistic retail and institutional investors trade with each other, their similarly high demands for the stock can attenuate, through a relative-wealth channel, the stock return variations in response to fundamental shocks. This attenuation effect has rich implications for the level of volatility in financial markets and its dynamics over the business cycle. It can push volatility levels below those prevailing under either sentiment or benchmarking concerns. This result implies that rational institutions can have a stronger depressing effect on volatility in the presence of high sentiment than similar non-institutional peers. It also implies that in markets with a high presence of institutional investors, sentiment need not create "excess volatility." Finally, it can lead to a countercyclical return volatility pattern broadly consistent with the existing empirical evidence.

Second, for high levels of optimism institutions help correct, while for low or moderate levels they can exacerbate, the overpricing of the stock market that sentiment induces. The result highlights the often overlooked fact that the benchmarking-related demand of institutions for a benchmark stock, thus the pressure on its price, is positive but bounded. By contrast, their mean-variance driven demand for the same stock can have the opposite sign, potentially leading to an overall negative—and large—price pressure.

Empirically, we use exogenous changes in the benchmarking intensity of stocks to establish a causal link between institutional benchmarking concerns and stock return volatility across different sentiment levels. Leveraging variations in benchmark intensity around Russell 1000/2000 reconstitutions as a proxy for exogenous shifts in benchmarking concerns, we find that greater benchmarking intensity amplifies volatility in low-sentiment periods but dampens it when sentiment is high. Additionally, we show that institutions help correct overpricing in strongly positive sentiment periods but not under moderate sentiment.

Our results have several implications for the ongoing debate around the stabilizing role of institutional investors in financial markets, as well as for the empirical inference of sentiment from market-determined variables such as prices and volatility. Importantly, it implies that neither the impact of the trend toward a greater institutionalization of markets in the correction of sentiment-driven distortions nor the degree to which sentiment distorts prices and volatility in the first place is linear, but results from a complex interaction between sentiment, benchmarking, and wealth effects.

References

Asparouhova, E., H. Bessembinder, and I. Kalcheva. 2013. Noisy Prices and Inference Regarding Returns. *Journal of Finance* 68:665–714.

Baker, M., and J. Wurgler. 2006. Investor Sentiment and the Cross-Section of Stock Returns. *Journal of Finance* 61:1645–1680.

Barber, B. M., and T. Odean. 2008. All that Glitters: The Effect of Attention on the Buying Behavior of Individual and Institutional Investors. *Review of Financial Studies* 21:785–818.

Barber, B. M., T. Odean, and L. Zheng. 2009. Do Retail Trades Move Markets? *Review of Financial Studies* 22:151–186.

Barberis, N. 2018. Psychology-Based Models of Asset Prices and Trading Volume. *Handbook of Behavioral Economics* 1:79–175.

Barberis, N., R. Greenwood, L. Jin, and A. Shleifer. 2015. X-CAPM: an extrapolative capital asset pricing model. *Journal of Financial Economics* 115:1–24.

Barberis, N., R. Greenwood, L. Jin, and A. Shleifer. 2018. Extrapolation and bubbles. *Journal of Financial Economics* 129:203–227.

Barberis, N., and A. Shleifer. 2003. Style Investing. *Journal of Financial Economics* 68:161–199.

Barberis, N., and R. Thaler. 2003. A Survey of Behavioral Finance. *Handbook of the Economics of Finance* 1:1053–1128.

Basak, S., and A. Pavlova. 2013. Asset Prices and Institutional Investors. *American Economic Review* 103:1728–1758.

Basak, S., and A. Pavlova. 2016. A Model of Financialization of Commodities. *Journal of Finance* 71:1511–1556.

Borovička, J. 2020. Survival and Long-Run Dynamics with Heterogeneous Beliefs under Recursive Preferences. *Journal of Political Economy* 128:206–251.

Breugem, M., and A. Buss. 2019. Institutional Investors and Information Acquisition: Implications for Asset Prices and Informational Efficiency. *Review of Financial Studies* 32:2260–2301.

Brunnermeier, M., E. Farhi, R. S. J. Koijen, A. Krishnamurthy, S. C. Ludvigson, H. Lustig, S. Nagel, and M. Piazzesi. 2021. Review Article: Perspectives on the Future of Asset Pricing. *Review of Financial Studies* 34:2126–2160.

Buffa, A., D. Vayanos, and P. Woolley. 2019. Asset Management Contracts and Equilibrium Prices. Boston U. School of Management Research Paper No. 2492529.

Buffa, A. M., and I. Hodor. 2022. Institutional Investors, Heterogeneous Benchmarks and the Comovement of Asset Prices. Working Paper.

Campbell, J. Y., J. Hilscher, and J. Szilagyi. 2008. In Search of Distress Risk. *Journal of Finance* 63:2899–2939.

Chu, Y., D. Hirshleifer, and L. Ma. 2020. The Causal Effect of Limits to Arbitrage on Asset Pricing Anomalies. *Journal of Finance* 75:2631–2672.

Cooper, M. J., H. Gulen, and M. J. Schill. 2008. Asset Growth and the Cross-Section of Stock Returns. *Journal of Finance* 63:1609–1651.

Cuoco, D., and R. Kaniel. 2011. Equilibrium Prices in the presence of Delegated Portfolio Management. *Journal of Financial Economics* 101:264–296.

Da, Z., J. Engelberg, and P. Gao. 2015. The Sum of All FEARS Investor Sentiment and Asset Prices. *Review of Financial Studies* 28:1–32.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam. 1998. Investor Psychology and Security Market Under- and Overreactions. *Journal of Finance* 53:1839–1885.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam. 2001. Overconfidence, Arbitrage, and Equilibrium Asset Pricing. *Journal of Finance* 56:921–965.

Daniel, K., and S. Titmans. 2006. Market Reactions to Tangible and Intangible Information. *Journal of Finance* 61:1605–1643.

DeLong, J. B., A. Shleifer, L. H. Summers, and R. J. Waldman. 1990. The Survival of Noise Traders in Financial Markets. *Journal of Business* 64:1–19.

DeVault, L., R. Sias, and L. Starks. 2019. Sentiment Metrics and Investor Demand. *Journal of Finance* 74:985–1024.

Dichev, I. D. 1998. Is the Risk of Bankruptcy a Systematic Risk? *Journal of Finance* 53:1131–1147.

Dong, X., D. Mai, K. Pukthuanthong, and G. Zhou. 2024. Investor Sentiment and Asset Returns: Actions Speak Louder Than Words. Working Paper.

Dumas, B., A. Kurshev, and R. Uppal. 2009. Equilibrium Portfolio Strategies in the Presence of Sentiment Risk and Excess Volatility. *Journal of Finance* 64:579–629.

Fama, E., and K. French. 2015. A Five-Factor Asset Pricing Model. *Journal of Financial Economics* 116:1–22.

Fama, E. F., and K. R. French. 2006. Profitability, Investment and Average Returns. *Journal of Financial Economics* 82:491–518.

Garcia, D. 2013. Sentiment During Recessions. *Journal of Finance* 68:1267–1300.

Giglio, S., and B. Kelly. 2018. Excess Volatility: Beyond Discount Rates. *Quarterly Journal of Economics* 133:71–127.

Giglio, S., M. Maggiori, J. Stroebel, and S. Utkus. 2021. Five Facts about Beliefs and Portfolios. *American Economic Review* 111:1481–1522.

Gopalakrishna, G., S. J. Lee, and T. Papamichalis. 2023. Optimism, Net Worth Trap, and Asset Returns. Working Paper.

Greenwood, R., and S. Nagel. 2009. Inexperienced Investors and Bubbles. *Journal of Financial Economics* 93:239–258.

Gromb, D., and D. Vayanos. 2010. Limits of Arbitrage: The State of the Theory. *Annual Review of Financial Economics* 2:251–275.

Hirshleifer, D. 2015. Behavioral Finance. *Annual Review of Financial Economics* 7:133–159.

Hirshleifer, D., K. Hou, S. H. Teoh, and Y. Zhang. 2004. Do Investors Overvalue Firms with Bloated Balance Sheets? *Journal of Accounting and Economics* 38:297–331.

Hodor, I., and F. Zapatero. 2022. Asset Pricing Implications of Heterogeneous Investment Horizons. Working Paper.

Hong, H., W. Jiang, N. Wang, and B. Zhao. 2014. Trading for Status. *Review of Financial Studies* 27:3171–3212.

Hong, H., and J. C. Stein. 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. *Journal of Finance* 54:2143–2184.

Jegadeesh, N., and S. Titman. 1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance* 48:65–91.

Jiang, H., D. Vayanos, and L. Zheng. 2024. Passive Investing and the Rise of Mega-Firms. Working Paper.

Kogan, L., S. A. Ross, J. Wang, and M. M. Westerfield. 2006. The Price Impact and Survival of Irrational Traders. *Journal of Finance* 61:195–229.

Krishnamurthy, A., and W. Li. 2021. Dissecting Mechanisms of Financial Crises: Intermediation and Sentiment. Working Paper.

Kumar, A., and C. Lee. 2006. Retail Investor Sentiment and Return Comovements. *Journal of Finance* 61:2451–2486.

Li, J., and J. Yu. 2012. Investor Attention, Psychological Anchors, and Stock Return Predictability. *Journal of Financial Economics* 104:401–419.

Loughran, T., and J. R. Ritter. 1995. The New Issues Puzzle. *Journal of Finance* 50:23–51.

Martin, I., and D. Papadimitriou. 2022. Sentiment and Speculation in a Market with Heterogeneous Beliefs. *American Economic Review* 112:2465–2517.

Meeuwis, M., J. A. Parker, A. Schoar, and D. I. Simester. 2021. Belief Disagreement and Portfolio Choice. Forthcoming, *Journal of Finance*.

Mele, A. 2007. Asymmetric stock market volatility and the cyclical behavior of expected returns. *Journal of Financial Economics* 86:446–478.

Newey, W. K., and K. D. West. 1987. A Simple, Positive Semi-definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55:703–708.

Novy-Marx, R. 2013. The Other Side of Value: The Gross Profitability Premium. *Journal of Financial Economics* 108:1–28.

Ohlson, J. A. 1980. Financial Ratios and the Probabilistic Prediction of Bankruptcy. *Journal of Accounting Research* 18:109–131.

Panageas, S. 2020. The Implications of Heterogeneity and Inequality for Asset Pricing: A Survey. *Foundations and Trends in Finance* 12:199–275.

Pavlova, A., and T. Sikorskaya. 2023. Benchmarking Intensity. *The Review of Financial Studies* 36:859–903.

Scheinkman, J. A., and W. Xiong. 2003. Overconfidence and Speculative Bubbles. *Journal of Political Economy* 111:1183–1219.

Shiller, R. J. 1979. The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure. *Journal of Political Economy* 87:1190–1219.

Shiller, R. J. 1981. Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *American Economic Review* 71:421–436.

Sloan, R. 1996. Do Stock Prices Fully Reflect Information in Accruals and Cash Flows About Future Earnings? *Accounting Review* 71:289–315.

Sockin, M., and M. Z. Xiaolan. 2019. Delegated Learning in Asset Management. Working Paper.

Stambaugh, R. F., J. Yu, and Y. Yuan. 2012. The short of it: Investor sentiment and anomalies. *Journal of Financial Economics* 104:288–302.

Stambaugh, R. F., J. Yu, and Y. Yuan. 2015. Arbitrage Asymmetry and the Idiosyncratic Volatility Puzzle. *Journal of Finance* 70:1903–1948.

Titman, S., K.-C. Wei, and F. Xie. 2004. Capital Investments and Stock Returns. *Journal of Financial and Quantitative Analysis* 39:677–700.

Vayanos, D., and P. Woolley. 2013. An Institutional Theory of Momentum and Reversal. *Review of Financial Studies* 26:1087–1145.

Appendix

A Proofs

Proof of Proposition 1. Market completeness allows us to rewrite the investors' optimization problems as:

$$\begin{aligned} \max_{W_T^k} \quad & E_t[\xi_T^k U_k(W_T^k)] \\ \text{s.t.} \quad & E_t[\pi_T W_T^k] \leq W_0^k. \end{aligned}$$

The first order conditions are given by

$$\begin{aligned} \frac{\xi_T^R}{W_T^R} &= \psi_R \pi_T \\ \frac{\xi_T^I Y_T}{W_T^I} &= \psi_I \pi_T, \end{aligned}$$

where ψ_R and ψ_I are the Lagrange multipliers associated with the retail and institutional investors' budget constraints respectively. Using the fact that $W_T^R + W_T^I = S_T = D_T$, we find that

$$\begin{aligned} \pi_t &= E_t[\pi_T] \\ &= \frac{1}{\psi_R} E_t \left(\frac{\xi_T^R}{D_T} \right) + \frac{1}{\psi_I} E_t \left(\frac{\xi_T^I Y_T}{D_T} \right). \end{aligned}$$

It follows that

$$\begin{aligned} W_t^R &= \frac{1}{\pi_t} E_t \left(\pi_T W_T^R \right) \\ &= \frac{\xi_t^R}{\psi_R \pi_t}. \end{aligned} \tag{29}$$

and similarly

$$\begin{aligned} W_t^I &= \frac{1}{\psi_I \pi_t} E_t \left(\xi_T^I Y_T \right) \\ &= \frac{\xi_t^I}{\psi_I \pi_t} \left(1 - \upsilon + \upsilon D_t e^{\mu(T-t)} \right). \end{aligned} \tag{30}$$

Finally let ϖ_t^R and ϖ_t^I denote the shares of wealth of the retail and institutional investors, respectively, so that $\varpi_t^R + \varpi_t^I = 1$. Note that

$$\frac{\xi_t^R \psi_I}{\xi_t^I \psi_R} = \frac{\varpi_t^R}{\varpi_t^I} \left(1 - \upsilon + \upsilon D_t e^{\mu(T-t)} \right).$$

The state price density is given by

$$\pi_t = \frac{\xi_t^I}{\psi_I D_t} \left(\frac{\xi_t^R \psi_I}{\xi_t^I \psi_R} e^{-(\mu + \sigma^2(\delta^R - 1))(T-t)} + \upsilon D_t + (1-\upsilon) e^{-(\mu - \sigma^2)(T-t)} \right).$$

Writing $d\pi_t = -\pi_t \kappa_t dB_t$ and applying Ito's lemma, we find that the market price of risk κ is given by:

$$\begin{aligned}
\kappa_t &= \sigma\left(1 - \frac{\delta^R \varpi_t^R \big(1-\upsilon+\upsilon D_t e^{\mu(T-t)}\big) e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I \upsilon D_t}{\varpi_t^R \big(1-\upsilon+\upsilon D_t e^{\mu(T-t)}\big) e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I \big(\upsilon D_t + (1-\upsilon) e^{-(\mu-\sigma^2)(T-t)}\big)}\right) \\
&= \sigma\left(1 - \frac{\delta^R \varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I q_t e^{-\mu(T-t)}}{\varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I \frac{\upsilon D_t + (1-\upsilon) e^{-(\mu-\sigma^2)(T-t)}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}}\right) \\
&= \sigma\left(1 - \frac{\delta^R \varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I q_t e^{-\mu(T-t)}}{\varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I e^{-(\mu-\sigma^2)(T-t)} \big(1 - q_t(1-e^{-\sigma^2(T-t)})\big)}\right) \\
&= \sigma\left(1 - \frac{\delta^R \varpi_t^R e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I q_t e^{-\sigma^2(T-t)}}{\varpi_t^R e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I \big(1 - q_t(1-e^{-\sigma^2(T-t)})\big)}\right) \\
&= \sigma\left(1 - \frac{\delta^R(1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big) + \big(1-\gamma(T-t)\big)\varpi_t^I q_t}{(1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big) + \varpi_t^I\big(1-q_t\gamma(T-t)\big)}\right),
\end{aligned}$$

where we used to fact that $q_t = \frac{\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}$ and $\gamma(\tau) = 1 - e^{-\sigma^2\tau}$. Observe that in the absence of institutional investors, i.e., $\upsilon = 0$, we simply have

$$\kappa_t \triangleq \kappa_t^{SE} = \sigma(1-\delta^R). \tag{31}$$

The case $\delta^R = 0$ defines the BP economy of Section 4.1.1, and we have

$$\kappa_t^{BP} = \sigma\left(1 - \frac{\big(1-\gamma(T-t)\big)\varpi_t^I q_t}{1-\varpi_t^I + \varpi_t^I\big(1-q_t\gamma(T-t)\big)}\right)$$

Comparing the market price of risk given in relation (4) with its equilibrium value in an economy where there is no institutional investor, i.e., $\upsilon = 0$ given in relation (31), it is easy to verify that $\kappa_t < \kappa_t^{SE}$ whenever

$$\delta^R - q_t < q_t \frac{1-(1-q_t)\gamma(T-t)}{1-q_t\gamma(T-t)}.$$

The equilibrium stock price-dividend ratio is given by

$$\begin{aligned}
S_t/D_t &= (W_t^R + W_t^I)/D_t \\
&= \frac{1}{\varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I \frac{\upsilon D_t + (1-\upsilon)e^{-(\mu-\sigma^2)(T-t)}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}} \\
&= \frac{e^{(\mu-\sigma^2)(T-t)}}{\varpi_t^R + \varpi_t^I \frac{\upsilon D_t e^{(\mu-\sigma^2)(T-t)} + (1-\upsilon)}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}} \\
&= \overline{(S/D)}_t \frac{1}{\varpi_t^R\big(1-\gamma(\delta^R(T-t))\big) + \varpi_t^I\big(1-\gamma(T-t)q_t\big)},
\end{aligned}$$

where we used the definition of q_t, the fact that $\varpi_t^R = 1 - \varpi_t^I$ as well as $1-\upsilon = \frac{1-q_t}{q_t}\upsilon D_t e^{\mu(T-t)}$ and $\gamma(T-t) = 1 - e^{-\sigma^2(T-t)}$. □

Proof of Proposition 2. We use relations (29) and (30) to apply Ito's lemma and we identify the diffusion terms with those given by the budget constraints (2). This leads to

$$\begin{aligned}\theta_t^R \sigma_{S,t} &= \kappa_t + \sigma \delta^R \\ \theta_t^I \sigma_{S,t} &= \kappa_t + \sigma q_t.\end{aligned}$$

Then, since $\varpi_t^R \theta_t^R + \varpi_t^I \theta_t^I = 1$, we observe that leverage $(\theta_t^R - 1)\varpi_t^R$ is given by:

$$(\theta_t^R - 1)\varpi_t^R = \varpi_t^R \varpi_t^I \frac{\sigma}{\sigma_{S,t}} (\delta^R - q_t).$$

From $\varpi_t^R \theta_t^R + \varpi_t^I \theta_t^I = 1$, we also obtain that

$$\begin{aligned}\sigma_{S,t} &= \kappa_t + (1-\varpi_t^I)\sigma\delta^R + \varpi_t^I \sigma q_t \\ &= \sigma \left(1 - \frac{\delta^R (1-\varpi_t^I) e^{-\sigma^2 \delta^R (T-t)} + \big(1-\gamma(T-t)\big)\varpi_t^I q_t}{(1-\varpi_t^I)e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I \big(1 - q_t \gamma(T-t)\big)} + \varpi_t^R \delta^R + \varpi_t^I q_t\right) \\ &= \bar{\sigma}_S \left(1 + \varpi_t^I \frac{\gamma(T-t) q_t (1-q_t) + (1-\varpi_t^I)\big(1 - \gamma(T-t)q_t - \big(1 - \gamma(\delta^R(T-t))\big)\big)(\delta^R - q_t)}{(1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big) + \varpi_t^I \big(1 - \gamma(T-t)q_t\big)}\right).\end{aligned}$$

If $\upsilon = 0$, then $q_t = 0, \delta^R - q_t = \delta^R$, so we have

$$\sigma_{S,t} = \bar{\sigma}_S \left(1 + \delta^R \varpi_t^I (1-\varpi_t^I) \frac{\gamma(\delta^R(T-t))}{(1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big) + \varpi_t^I}\right)$$

It is easy to verify that in this case the volatility is increasing in the degree of optimism δ^R.

Finally, we show that $\sigma_{S,t} \geq \overline{\sigma}_S$: Given relation (8) in the paper, it is enough to show that

$$\gamma(T-t)q_t(1-q_t) + \big(\gamma(\delta^R(T-t)) - \gamma(T-t)q_t\big)(\delta^R - q_t) \geq 0$$

The first term is always positive and the second term is negative iff $\check{\delta}^R \leq \delta^R \leq q_t$, where $\check{\delta}^R$ is defined in (22). For $0 < a \leq x \leq 1$, define auxiliary function φ_a with

$$\varphi_a(x) = \gamma(T-t)x + \gamma(a(T-t))(a-x) - \gamma(T-t)ax.$$

φ_a is linear in x with $\varphi_a(a) = \gamma(T-t)a(1-a)$ and $\varphi_a(1) = (1-a)\big(\gamma(T-t) - \gamma(a(T-t))\big) > 0$ as $a \leq 1$ and function γ is increasing. We conclude that φ_a is non-negative. □

Proof of Lemma 1. Recall that

$$S_t/D_t = \overline{(S/D)}_t \frac{1}{\varpi_t^I\big(1-\gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)},$$

and $q_t = \frac{\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}$. Let us formally write

$$dq_t/q_t = \mu_{q_t} dt + \sigma_{q_t} dB_t,$$

and observe that by Ito's lemma $\sigma_{q_t} = \frac{\frac{\partial q_t}{\partial D_t}}{q_t} D_t \sigma = (1-q_t)\sigma$. Then, we have the following stock volatility decomposition

$$\sigma_{S,t} = \varepsilon_{S,t}^D \sigma + \varepsilon_{S,t}^q \sigma_{q_t} + \varepsilon_{S,t}^{\varpi^I} \sigma_{\varpi_t^I}, \tag{32}$$

where $\varepsilon^x_{S,t} = \frac{\partial S_t}{\partial x_t} \times \frac{x_t}{S_t}$ denotes the elasticity of the stock price with respect to variable x at time t, and

$$\begin{aligned}
\varepsilon^D_{S,t} &= 1, \\
\varepsilon^q_{S,t} &= \frac{\varpi^I_t \gamma(T-t) q_t}{\varpi^I_t\big(1-\gamma(T-t)q_t\big) + (1-\varpi^I_t)\big(1-\gamma(\delta^R(T-t))\big)} > 0, \\
\varepsilon^{\varpi^I}_{S,t} &= \frac{\gamma(T-t)q_t - \gamma(\delta^R(T-t))}{\varpi^I_t\big(1-\gamma(T-t)q_t\big) + (1-\varpi^I_t)\big(1-\gamma(\delta^R(T-t))\big)} \varpi^I_t.
\end{aligned}$$

Given definition (12), expressions (13) and (14) then follow in a straightforward way from decomposition (9). □

Proof of Lemma 2. This is a special case of the proof of Proposition 1 when $\upsilon = 0$. □

Proof of Lemma 3. This is a special case of the proof of Proposition 2 when $\upsilon = 0$. □

Proof of Lemma 4. Replacing variable ϖ^R_t by variable ϖ^I_t in relation (18), the price-dividend ratios in the BP and in the SENT economies are equal if and only if

$$\frac{1}{1-\varpi^I_t(1-e^{-\sigma^2\delta^R(T-t)})} = \frac{1}{1-\gamma(T-t)\varpi^I_t q_t},$$

or, equivalently, if $\gamma(T-t)q_t = 1 - e^{-\sigma^2\delta^R(T-t)}$, i.e., $\delta^R_t = \frac{\log(1-\gamma(T-t)q_t)}{\log(1-\gamma(T-t))} \triangleq \check{\delta}^R_t$.

To show that $\check{\delta}^R < q_t$, i.e., that $\log[1-\gamma(T-t)q_t] < q_t \log[1-\gamma(T-t)]$ define, for $(a,x) \in (0,1)^2$, the auxiliary function φ_a, with $\varphi_a(x) = \log[1-ax] - x\log[1-a]$. Observe that $\varphi''_a(x) = -a^2/(1-ax)^2 < 0$, so that φ_a is concave with $\varphi_a(0) = \varphi_a(1) = 0$, so φ_a must be positive on $[0,1]$. We conclude that $\check{\delta}^R < q_t$.

Next, we show that $(\kappa^{BP}_t > \kappa^{SE}_t)|_{\delta^R = \check{\delta}^R}$. When $\delta^R = \check{\delta}^R$, we have $\gamma(T-t)q_t = \gamma(\delta^R(T-t))$, and replacing ϖ^R_t by ϖ^I_t in relation (19), we find that

$$\kappa^{SE}_t = \overline{\kappa}\left(1 - \frac{\varpi^I_t(1-\gamma(T-t)q_t)\delta^R}{1-\varpi^I_t\gamma(T-t)q_t}\right).$$

Thus $\kappa^{BP}_t \geq \kappa^{SE}_t$ iff

$$(1-\gamma(T-t)q_t)\delta^R \geq (1-\gamma(T-t))q_t,$$

or equivalently

$$\frac{\log[1-\gamma(T-t)q_t]}{\log[1-\gamma(T-t)]} \geq \frac{(1-\gamma(T-t))q_t}{1-\gamma(T-t)q_t}.$$

For $a \in (0,1)$, define auxiliary function φ_a with

$$\varphi_a(x) = (1-ax)\log(1-ax) - (1-a)x\log(1-a),$$

for $x \in [0,1]$. Observe that $\varphi''_a(x) = \frac{a^2}{1-ax} > 0$. Since $\varphi_a(0) = \varphi_a(1) = 0$ and φ_a is convex, it must be the case that φ_a is negative on $[0,1]$. Since $\log[1-\gamma(T-t)] < 0$, we conclude that we always have $\kappa^{BP}_t \geq \kappa^{SE}_t|_{\delta^R = \check{\delta}^R}$.

Finally, we show that $(\sigma^{BP}_{S,t} \geq \sigma^{SE}_{S,t})|_{\delta^R = \check{\delta}^R}$. When $\delta^R = \check{\delta}^R$ we have, replacing ϖ^R_t by ϖ^I_t in relation (20),

$$\sigma^{SE}_{S,t}|_{\delta^R = \check{\delta}^R} = \overline{\sigma}\left(1 + \frac{\varpi^I_t(1-\varpi^I_t)\gamma(T-t)q_t}{1-\varpi^I_t\gamma(T-t)q_t}\frac{\log[1-\gamma(T-t)q_t]}{\log[1-\gamma(T-t)]}\right)$$

Thus $\sigma^{BP}_{S,t} \geq \sigma^{SE}_{S,t}$ iff

$$(1-\varpi^I_t)\frac{\log[1-\gamma(T-t)q_t]}{\log[1-\gamma(T-t)]} < 1 - \varpi^I_t q_t.$$

It is easy to verify that auxiliary function φ, with $\varphi(x) = -1 - \varpi x + (1-\varpi)\frac{\log[1-ax]}{\log[1-a]}$ and $(a,\varpi) \in (0,1)^2$, is increasing on $[0,1]$ and that $\varphi(1) = 0$. Thus, φ is negative on $[0,1]$. We conclude that when $\delta^R = \check{\delta}^R$, we always have $\sigma^{BP}_{S,t} \geq \sigma^{SE}_{S,t}$. □

Proof of Proposition 3. We prove part (a) first. To this end, recall that

$$\begin{aligned}
\sigma_{S,t} &= \sigma\left(1 - \frac{\delta^R \varpi_t^R e^{-\sigma^2 \delta^R (T-t)} + \varpi_t^I q_t\big(1-\gamma(T-t)\big)}{\varpi_t^R e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I\big(1-q_t\gamma(T-t)\big)} + \varpi_t^R(\delta^R - q_t) + q_t\right) \\
&= \sigma\left(1 + \varpi_t^I q_t + \varpi_t^I\Big(-x - \frac{q_t\big(1-\gamma(T-t)\big) - x\big(1-q_t\gamma(T-t)\big)}{\varpi_t^R e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I\big(1-q_t\gamma(T-t)\big)}\Big)\right) \\
&= \sigma\left(1 + \varpi_t^I q_t + \varpi_t^I\Big(-x + \frac{x-a}{\varpi_t^I + b\varpi_t^R e^{-\theta x}}\Big)\right)
\end{aligned}$$

with

$$\begin{aligned}
x &= \delta^R \\
a &= \frac{q_t\big(1-\gamma(T-t)\big)}{1-q_t\gamma(T-t)} \\
b &= \frac{1}{1-q_t\gamma(T-t)} \\
\theta &= \sigma^2(T-t).
\end{aligned}$$

Then, define

$$\varphi(x) = -x + \frac{x-a}{\varpi_t^I + b\varpi_t^R e^{-\theta x}}.$$

We have .

$$\varphi'(x) = -1 + \frac{(\varpi_t^I + b\varpi_t^R e^{-\theta x}) + \theta(x-a)b\varpi_t^R e^{-\theta x}}{(\varpi_t^I + b\varpi_t^R e^{-\theta x})^2}.$$

Set $z = \varpi_t^I + b\varpi_t^R e^{-\theta x}$ so that

$$\varphi'(x) = \frac{z-\varpi_t^I}{z^2}\Big(\frac{(1-z)z}{z-\varpi_t^I} - \log(z-\varpi_t^I) - a\theta + \log(b\varpi_t^R)\Big).$$

Finally, consider $\psi(z) = \frac{(1-z)z}{z-\varpi_t^I} - \log(z-\varpi_t^I) - a\theta + \log(b\varpi_t^R)$ with $z > \varpi_t^I$. We have

$$\psi'(z) = -1 - \frac{\varpi_t^I\varpi_t^R}{(z-\varpi_t^I)^2} - \frac{1}{z-\varpi_t^I} < 0.$$

Then, note that $\lim_{z\to(\varpi_t^I)^+}\psi(z) = \infty$ and $\lim_{z\to\infty}\psi(z) = -\infty$. It follows that there is a unique z^* such that $\psi(z^*) = 0$ and $\psi > 0$ (respectively, < 0) on (ϖ_t^I, z^*) (resp., (z^*, ∞)). Then, define

$$x^* = -\frac{1}{\theta}\log\frac{z^*-\varpi_t^I}{b\varpi_t^R}.$$

Since variable z is decreasing in variable x, we deduce that $\varphi'(x) > 0$ (resp., < 0) iff $x > x^*$ $(x < x^*)$.

Finally observe that when $\delta^R = \check{\delta}^R$, we have $x = \check{x}$ and $\check{x}$ is such that $b = e^{\theta\check{x}}$, which implies that the corresponding value of z denotes $\check{z}$ is such that $\check{z} = 1$. Then, $\psi(1) = -a\theta + \log b$. Then, set $s = 1 - e^{-\theta} \in (0,1)$

and observe that $a = \frac{q_t(1-s)}{1-q_t s}$ and $b = \frac{1}{1-q_t s}$, so that

$$\psi(1) = q_t(1-s)\log(1-s) - (1-q_t s)\log(1-q_t s).$$

Then recall that $q_t \in [0,1]$ and notice that $g(y) = y\log y$ is a convex function so that

$$g(q_t(1-s) + 1 - q_t) < q_t g(1-s) + (1-q_t)g(1).$$

As $g(1) = 0$, we obtain that $\psi(1) > 0$. Thus, we must have $\check{z} < z^*$, which implies that for all $x < \check{x}$, i.e., $\delta^R < \check{\delta}^R_t$, as δ^R increases, the stock volatility decreases for any wealth distribution. This concludes the proof of part (a).

To prove part (b), note that for the case of no sentiment (BP economy) we have:

$$\sigma_{S_t} = \overline{\sigma}\left(1 + \gamma(T-t)\frac{\varpi^I_t q_t(1-\varpi^I_t q_t)}{1-\gamma(T-t)\varpi^I_t q_t}\right).$$

Define the auxiliary function $\varphi(x)$ as:

$$\varphi(x) = \frac{x(1-ax)}{1-\gamma a x},$$

for $x \in [0,1]$ with $(a,\gamma) \in [0,1]^2$. The derivative of $\varphi(x)$ is:

$$\varphi'(x) = \frac{\gamma a^2 x^2 - 2ax + 1}{(1-\gamma a x)^2}.$$

Since $\gamma \in [0,1]$, the function $z \mapsto \gamma z^2 - 2z + 1$ is decreasing in z, with:

$$\varphi'(0) = 1 > 0 \quad \text{and} \quad \varphi'(1) = \gamma a^2 - 2a + 1.$$

The expression $\varphi'(1)$ is decreasing in a. Moreover, when $a = 0$, $\varphi'(1) = 1 > 0$, and when $a = 1$, $\varphi'(1) = \gamma - 1 < 0$. Let $0 < \overline{q}_{1t} < \overline{q}_{2t}$ be the roots of the quadratic equation:

$$\gamma(T-t)(\varpi^I_t)^2 x^2 - 2\varpi^I_t x + 1 = 0,$$

i.e.:

$$\overline{q}_{1t} = \frac{1-\sqrt{1-\gamma(T-t)}}{\gamma(T-t)\varpi^I_t}, \quad \overline{q}_{2t} = \frac{1+\sqrt{1-\gamma(T-t)}}{\gamma(T-t)\varpi^I_t}.$$

Note that $\frac{1-\sqrt{1-\gamma(T-t)}}{\gamma(T-t)} < 1$, and $\overline{q}_{1t} \leq 1$ (resp., $\overline{q}_{1t} > 1$) if $\varpi^I_t \geq \overline{\varpi}^I_t = \frac{1-\sqrt{1-\gamma(T-t)}}{\gamma(T-t)} \in [1/2, 1]$ (resp., $\varpi^I_t < \overline{\varpi}^I_t$). It is easy to check that $\overline{q}_{2t} > 1$.

Next, for $y \in (0,1]$, define the auxiliary function $g(y)$ as:

$$g(y) = \frac{1-\sqrt{1-y}}{y} = \frac{1}{1+\sqrt{1-y}}.$$

Clearly, $g(y)$ is increasing from $\lim_{y\to 0^+} g(y) = \frac{1}{2}$ up to $g(1) = 1$. Since $\gamma(T-t)$ is increasing, we conclude that as $T-t$ decreases, $\overline{\varpi}^I_t$ decreases from 1 to $\frac{1}{2}$. Summarizing the case of no sentiment, we have that: (i) When $\varpi^I_t \leq \overline{\varpi}^I_t$, σ_{S_t} is always increasing in q_t; (ii) when $\varpi^I_t \geq \overline{\varpi}^I_t$, σ_{S_t} is increasing (resp. decreasing) in q_t on $[0, \overline{q}_{1t}]$ (resp. $[\overline{q}_{1t}, 1]$).

In the presence of sentiment (GE economy), recall that

$$\sigma_{S_t} = \overline{\sigma}\left(1 + \varpi^I_t \frac{\gamma(T-t)q_t(1-q_t) + (1-\varpi^I_t)[\gamma(\delta^R(T-t)) - \gamma(T-t)q_t](\delta^R - q_t)}{\varpi^I_t[1-\gamma(T-t)q_t] + (1-\varpi^I_t)[1-\gamma(\delta^R(T-t))]}\right),$$

which can be rewritten as:

$$\sigma_{S_t} = \overline{\sigma}\left(1 + \varpi_t^I \frac{\gamma(T-t)q_t(1-\varpi_t^I q_t) - (1-\varpi_t^I)[\gamma(\delta^R(T-t)) + \gamma(T-t)\delta^R]q_t + \gamma(\delta^R(T-t))(1-\varpi_t^I)\delta^R}{\varpi_t^I[1-\gamma(T-t)q_t] + (1-\varpi_t^I)[1-\gamma(\delta^R(T-t))]}\right)$$

Define the auxiliary function φ as:

$$\varphi(x) = \frac{-a_2x^2 + a_1x + a_0}{b_0 - a_2x},$$

for $x \in [0,1]$ where:

$$\begin{aligned}
a_2 &= a\gamma \in [0,1], \\
a_1 &= \gamma - (1-a)[\gamma^R + \gamma\delta], \\
a_0 &= (1-a)\gamma^R\delta > 0, \\
b_0 &= 1 - (1-a)\gamma^R \in [0,1],
\end{aligned}$$

where we set $a = \varpi_t^I$, $\gamma = \gamma(T-t)$, $\delta = \delta^R$, and $\gamma^R = \gamma(\delta^R(T-t)$. It is easy to verify that $b_0 > a_2$. Moreover,

$$\begin{aligned}
b_0^2 - (a_2a_0 + a_1b_0) =& b_0[1-\gamma+(1-a)\gamma\delta] - a\gamma(1-b_0)\delta \\
=& b_0(1-\gamma) + \gamma\delta(b_0 - a) \\
=& b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R) > 0.
\end{aligned}$$

where $\left(a, \gamma, \gamma^R\right)$ in $(0,1)^3$ and $\delta > 0$. We have:

$$\varphi'(x) = \frac{a_2^2x^2 - 2a_2b_0x + a_2a_0 + a_1b_0}{(b_0 - a_2x)^2}.$$

The discriminant of the quadratic (numerator) is

$$\begin{aligned}
\Delta =& 4a_2^2[b_0^2 - (a_2a_0 + a_1b_0)] \\
=& 4a_2^2[b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R)] > 0.
\end{aligned}$$

This implies that there are two roots:

$$\begin{aligned}
\overline{q}_{1t} &= \frac{b_0 - \sqrt{b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R)}}{a_2}, \\
\overline{q}_{2t} &= \frac{b_0 + \sqrt{b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R)}}{a_2}.
\end{aligned}$$

Since $b_0 > a_2$, we have $\overline{q}_{2t} > 1$. This leaves us with two possible cases:

Case 1: If $a_2a_0 + a_1b_0 \geq 0$, both roots $\overline{q}_{1t}$ and $\overline{q}_{2t}$ are positive. This is (always) satisfied when δ^R is small enough, including the case of $\delta^R = 0$, as in this case $a_2a_0 + a_1b_0 = \gamma > 0$. If $a = 1$, we have

$$a_2a_0 + a_1b_0 = \gamma > 0,$$

so the condition is met, while when $a = 0$, we have

$$a_2a_0 + a_1b_0 = \gamma - (\gamma^R + \gamma\delta), \tag{33}$$

which is nonnegative if and only if δ is sufficiently small.

In this case, $\varphi'(x)$ is positive on $[0, \overline{q}_{1t}]$ and negative on $[\overline{q}_{1t}, \overline{q}_{2t}]$. Then, there are two cases:

- If $\overline{q}_{1t} < 1$, then $\sigma_{S,t}$ is increasing in q_t on $[0, \overline{q}_{1t}]$ and decreasing in q_t on $[\overline{q}_{1t}, 1]$.
- If $\overline{q}_{1t} \geq 1$, $\sigma_{S,t}$ is always increasing in q_t. The condition $\overline{q}_{1t} < 1$ is

$$(b_0 - a\gamma)^2 < b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R).$$

If $a = 1$ this last condition is always satisfied. If $a = 0$, we must have

$$0 < \gamma^R - \gamma + \gamma\delta,$$

which is incompatible with condition (33). We conclude that if ϖ_t^I is close enough to 0 we have $\overline{q}_{1t} > 1$.

Case 2: $a_2a_0 + a_1b_0 < 0$. We have $\overline{q}_{1t} < 0$. Since $\overline{q}_{2t} > 1$, we conclude that φ' is negative on $[0,1]$ so $\sigma_{S,t}$ is decreasing in q_t:

$$\begin{aligned} &\varpi_t^I \gamma(T-t) \left(1-\varpi_t^I\right) \gamma\left(\delta^R(T-t)\right)\delta^R \\ &\quad + \left(\gamma(T-t) - (1-\varpi_t^I)\left(\gamma\left(\delta^R(T-t)\right) + \gamma(T-t)\delta^R\right)\right) \\ &\quad \times \left(1 - (1-\varpi_t^I)\gamma\left(\delta^R(T-t)\right)\right) \\ &< 0. \end{aligned}$$

Note that when δ^R is large, the left-hand side of the inequality is equivalent to

$$\varpi_t^I \left(\gamma(T-t) - (1-\varpi_t^I)\right),$$

which is negative (positive) if $\varpi_t^I \leq (\geq) 1 - \gamma(T-t)$.

Summing up:

1. If δ^R is small enough, σ_S is increasing in q_t as long as ϖ_t^I is sufficiently small; otherwise, it is hump-shaped in q_t.
2. If δ^R is large enough, σ_S is decreasing in q_t as long as ϖ_t^I is sufficiently small; otherwise, it is hump-shaped in q_t. □

Proof of Proposition 4. To account for the presence of rational institutions, we let $\upsilon > 0$. Replacing ϖ_t^R by $1 - \varpi_t^I$ in Eq. (3) and taking the partial derivative of S_t/D_t with respect to ϖ_t^I, we obtain the condition

$$\frac{\partial(S_t/D_t)}{\partial \varpi_t^I} < 0 \text{ iff } e^{-\sigma^2\delta^R(T-t)} < 1 - \gamma(T-t)q_t,$$

or, equivalently, iff $\delta^R > \log\left(1-\gamma(T-t)q_t\right)/\log\left(1-\gamma(T-t)\right) = \check{\delta}_t^R$. Thus, we have:

$$\frac{\partial(S_t/D_t)}{\partial \varpi_t^I} \begin{cases} > 0, & \delta^R < \check{\delta}_t^R \\ = 0, & \delta^R = \check{\delta}_t^R \\ < 0, & \delta^R > \check{\delta}_t^R \end{cases}.$$

□

Proof of Lemma 5. Recall that

$$\frac{\xi_t^R \psi_I}{\xi_t^I \psi_R} = \frac{\varpi_t^R}{\varpi_t^I}\left(1 - \upsilon + \upsilon D_t e^{\mu(T-t)}\right).$$

It follows that

$$\frac{\varpi_0^R}{\varpi_0^I} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t + \sigma\delta^R B_t} = \frac{\varpi_t^R}{\varpi_t^I} \frac{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_0 e^{\mu T}}.$$

Then, as $D_t = D_0 e^{(\mu-\frac{\sigma^2}{2})t+\sigma B_t}$, given $D_0 > 0$, we find that

$$\frac{\varpi_0^R}{\varpi_0^I} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t - (\mu-\frac{\sigma^2}{2})\delta^R t} \left(D_t/D_0\right)^{\delta^R} = \frac{\varpi_t^R}{\varpi_t^I} \frac{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_0 e^{\mu T}},$$

i.e.,

$$\frac{\varpi_0^R}{\varpi_0^I} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t - (\mu-\frac{\sigma^2}{2})\delta^R t} \left(D_t/D_0\right)^{\delta^R} = \frac{\varpi_t^R}{\varpi_t^I} \frac{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_0 e^{\mu T}},$$

or,

$$\varpi_t^I = \frac{\varpi_0^I}{(1-\varpi_0^I)\frac{1-\upsilon+\upsilon D_0 e^{\mu T}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}} \left(D_t/D_0\right)^{\delta^R} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t - (\mu-\frac{\sigma^2}{2})\delta^R t} + \varpi_0^I}.$$

Then, from the definition of q_t, one can check that

$$\begin{aligned} D_t/D_0 &= \frac{q_t}{1-q_t}\frac{1-q_0}{q_0} e^{\mu t}, \\ \frac{1-\upsilon+\upsilon D_0 e^{\mu T}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}} &= \frac{q_t}{q_0}\frac{D_0}{D_t} e^{\mu t} = \frac{1-q_t}{1-q_0}. \end{aligned}$$

It follows that

$$\begin{aligned} \varpi_t^I &= \frac{\varpi_0^I}{\varpi_0^I + (1-\varpi_0^I)\frac{1-q_t}{1-q_0}\left(\frac{q_t}{1-q_t}\frac{1-q_0}{q_0} e^{\mu t}\right)^{\delta^R} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t - (\mu-\frac{\sigma^2}{2})\delta^R t}} \\ &= \frac{\varpi_0^I}{\varpi_0^I + (1-\varpi_0^I)\left(\frac{q_t}{q_0}\right)^{\delta^R}\left(\frac{1-q_t}{1-q_0}\right)^{1-\delta^R} e^{-\frac{1}{2}\sigma^2\delta^R(\delta^R-1)t}}. \end{aligned}$$

Note that when $\delta^R = 0$ (BP), the expression is much simpler as we get

$$\varpi_t^I = \frac{\varpi_0^I}{\varpi_0^I + (1-\varpi_0^I)\left(\frac{1-q_t}{1-q_0}\right)}.$$

Finally, since $\frac{\partial q_t}{\partial D_t} > 0$, we find that ϖ_t^I is increasing (decreasing) in cash flows D iff auxiliary function φ is decreasing (increasing) where $\varphi(q) = q^{\delta^R}(1-q)^{1-\delta^R}$. φ is a smooth function and

$$\begin{aligned} \varphi'(q) &= q^{\delta^R-1}(1-q)^{-\delta^R}\Big(\delta^R(1-q) - (1-\delta^R)q\Big) \\ &= q^{\delta^R-1}(1-q)^{-\delta^R}(\delta^R - q). \end{aligned}$$

To sum up, we have:

$$\frac{\partial(\varpi_t^I)}{\partial D_t} \begin{cases} > 0, & \delta^R < q_t \\ = 0, & \delta^R = q_t \\ < 0, & \delta^R > q_t \end{cases}.$$

□

B Long-Run Survival

Recall that

$$\frac{\psi_I}{\psi_R}\xi_t = \frac{\varpi_t^R}{\varpi_t^I}\left(1-\upsilon+\upsilon D_t e^{\mu(T-t)}\right).$$

Following Kogan et al. (2006), define $\lambda = \frac{t}{T}$ in $(0,1)$. In what follows, we shall use the fact that

$$\lim_{t\to\infty}\frac{B_t}{t} = 0 \quad \mathbb{P}-\text{a.s.}$$

We have

$$\begin{aligned}
\frac{\varpi_t^I}{\varpi_t^R} &= \frac{\psi_R}{\psi_I}\frac{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}{\xi_t} \\
&= \frac{\varpi_0^I}{\varpi_0^R}\frac{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_0 e^{\mu T}}\frac{1}{\xi_t} \\
&= \frac{\varpi_0^I}{\varpi_0^R}e^{\frac{(\delta^R)^2\sigma^2}{2}t-\delta^R\sigma B_t}\left(\frac{1-\upsilon+\upsilon D_0 e^{\left(\frac{\mu}{\lambda}-\frac{\sigma^2}{2}\right)t+\sigma B_t}}{1-\upsilon+\upsilon D_0 e^{\frac{\mu}{\lambda}t}}\right) \\
&= \frac{\varpi_0^I}{\varpi_0^R}e^{\frac{(\delta^R)^2\sigma^2}{2}t+\delta^R\sigma B_t}\left(\frac{\frac{1-\upsilon}{\upsilon D_0}e^{-\frac{\mu}{\lambda}t}+e^{-\frac{\sigma^2}{2}t+\sigma B_t}}{1+\frac{1-\upsilon}{\upsilon D_0}e^{-\frac{\mu}{\lambda}t}}\right).
\end{aligned}$$

It follows that

$$\frac{\varpi_t^I}{\varpi_t^R} \underset{t\to\infty}{\sim} \begin{cases} \frac{\varpi_0^I}{\varpi_0^R}e^{\left((\delta^R)^2-1\right)\frac{\sigma^2}{2}t}, & \text{if } \mu>0, \\ \frac{\varpi_0^I}{\varpi_0^R}\frac{1-\upsilon}{1-\upsilon+\upsilon D_0}e^{(\delta^R)^2\frac{\sigma^2}{2}t}, & \text{if } \mu=0, \\ \frac{\varpi_0^I}{\varpi_0^R}e^{\frac{(\delta^R)^2\sigma^2}{2}t}, & \text{if } \mu<0. \end{cases}$$

For $\mu>0$, this implies

$$\varpi_t^R \underset{t\to\infty}{\sim} \begin{cases} \frac{\varpi_0^R}{\varpi_0^I}e^{-\left((\delta^R)^2-1\right)\frac{\sigma^2}{2}t}, & \text{if } |\delta^R|>1, \\ 1-\frac{\varpi_0^I}{\varpi_0^R}e^{\left((\delta^R)^2-1\right)\frac{\sigma^2}{2}t}, & \text{if } |\delta^R|<1, \\ \varpi_0^R, & \text{if } |\delta^R|=1, \end{cases}$$

$$\varpi_t^I \underset{t\to\infty}{\sim} \begin{cases} 1-\frac{\varpi_0^R}{\varpi_0^I}e^{-\left((\delta^R)^2-1\right)\frac{\sigma^2}{2}t}, & \text{if } |\delta^R|>1, \\ \frac{\varpi_0^I}{\varpi_0^R}e^{\left((\delta^R)^2-1\right)\frac{\sigma^2}{2}t}, & \text{if } |\delta^R|<1, \\ \varpi_0^I, & \text{if } |\delta^R|=1. \end{cases}$$

For $\mu<0$, we have

$$\begin{aligned}
\varpi_t^R &\underset{t\to\infty}{\sim} \frac{\varpi_0^R}{\varpi_0^I}e^{-\frac{(\delta^R)^2\sigma^2}{2}t}, \\
\varpi_t^I &\underset{t\to\infty}{\sim} 1-\frac{\varpi_0^R}{\varpi_0^I}e^{-\frac{(\delta^R)^2\sigma^2}{2}t}.
\end{aligned}$$

Finally, for $\mu = 0$, we have

$$\begin{aligned}
\varpi_t^R &\underset{t\to\infty}{\sim} \frac{1-\upsilon}{1-\upsilon+\upsilon D_0}\frac{\varpi_0^R}{\varpi_0^I}e^{-\frac{(\delta^R)^2\sigma^2}{2}t},\\
\varpi_t^I &\underset{t\to\infty}{\sim} 1-\frac{1-\upsilon}{1-\upsilon+\upsilon D_0}\frac{\varpi_0^R}{\varpi_0^I}e^{-\frac{(\delta^R)^2\sigma^2}{2}t}.
\end{aligned}$$

In summary, when $\mu > 0$, institutional investors (resp., retail investors) survive in the long run whenever $|\delta^R| > 1$ ($|\delta^R| < 1$). In the special case $\delta^R = 1$, both classes of investors survive. When $\mu \leq 0$, only institutional investors survive in the long run.

C Stock-Level Sentiment Measure

Dong et al. (2024) compare the return predictability of trade-based sentiment (technical indicators) and text-based sentiment (news and social media) across four asset classes: Bitcoin, stocks, Treasury bonds, and gold. While both sentiment measures are widely used and available daily, they capture different aspects of investor behavior. News and social media sentiment reflect public beliefs, while trade-based sentiment, derived from prices and volumes, reflects actual trading activity.

We adopt their methodology to construct a stock-level trade-based sentiment measure, using trading volume sentiment (TVS), Williams' %R sentiment (WRS), and nearness to high sentiment (NHS) to capture short-term investor sentiment. TVS tracks trading intensity, WRS identifies overbought/oversold conditions, and NHS measures proximity to recent highs, offering real-time insights into investor confidence. Unlike the trend-following measures also used in Dong et al. (2024) (e.g., moving average, momentum, and on-balance volume sentiment), these three measures are more immediate and responsive, making them suitable for our short-term analysis.

We proceed as follows to construct these measures. TVS is defined as the log ratio of trading volume over the past L days:

$$TVS_t^i(L) = log\left(\frac{TV_t^i}{TV_{t-L+1}^i}\right),$$

where TV_t^i represents the trading volume of stock i on day t. A higher TVS value indicates increased trading activity, reflecting stronger investor sentiment.

WRS is based on the overbought and oversold technical indicator, defined as:

$$WRS_t^i(L) = \frac{P_{max,t}^i(L) - P_t^i}{P_{max,t}^i(L) - P_{min,t}^i(L)},$$

where $P_{max,t}^i(L)$ and $P_{min,t}^i(L)$ are the highest and lowest daily prices of stock i over the window from day $t-L+1$ to day t. If $WRS_t^i(L)$ is less than 20%, stock i is considered overbought (high-sentiment), and if $WRS_t^i(L)$ is greater than 80%, it is regarded as oversold (low-sentiment). As a low value of $WRS_t^i(L)$ indicates high sentiment, it has the opposite sign compared to TVS. To maintain consistency, we use the negative of $WRS_t^i(L)$.

NHS is based on the proximity to the recent highest price, as proposed by Li and Yu (2012):

$$NHS_t^i(L) = \frac{P_t^i}{P_{max,t}^i(L)}.$$

This measure is a slight variation from $WRS_t^i(L)$ that does not depend on the minimum price.

For each of these three measures, we consider three choices for L: 5, 10, and 20, to capture short-term investor sentiment. Dong et al. (2024) also used 50- and 100-day windows, but those are more aligned with medium- to long-term trends. Shorter windows respond more quickly to shifts in trading activity, price movements, and investor sentiment, making them better suited for short-term predictability. In contrast, longer windows smooth out temporary variations, reducing sensitivity to immediate market conditions and making them less effective for detecting rapid sentiment shifts. Thus, we construct the three sentiment measures across three window lengths, yielding a total of nine daily sentiment metrics for each stock.

Next, we orthogonalize these stock-level sentiment measures. Specifically, we adopt the method in Garcia (2013) by using the residuals in the following regression as a measure of our orthogonalized sentiment:[30]

$$x_t^i = \beta L_{0-5}(r_t^i) + \gamma L_{0-5}(r_t^{2,i}) + \lambda L_{1-5}(x_t^i) + \nu Z_t + \epsilon_t^i$$

where x_t^i is a sentiment measure of stock i on day t, $L_{0-5}(r_t^i)$ are returns of stock i on days t to $t-5$, $L_{0-5}(r_t^{2,i})$ are squared returns of stock i on days t to $t-5$, $L_{1-5}(x_t^i)$ are five lags of sentiment measure x_t^i, and Z_t is a list of control variables including a constant and weekday indicators.

After obtaining these residuals for each measure, we standardize them annually over the January-to-May window for each stock. Next, we reduce the dimensionality of the sentiment measures by computing the cross-sectional average of the nine standardized variables, resulting in a single sentiment observation per stock-day.

Lastly, we compute the monthly average of these daily observations for the month of May each year and denote it as *SentDMPZ*.

[30] The specification used in Garcia (2013) includes a dummy for days that belong to an NBER recession. Since in our cross-sectional analysis our estimation windows may not contain any recessionary days, we exclude the recession dummy variable from the orthogonalization.