Revealed Preference for Green Stocks: An Asset Demand Approach

Aaron Mora
Darla Moore School of Business
University of South Carolina

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Abstract

This paper combines a portfolio construction problem with demand estimation techniques to estimate US institutional investors' demand for green stocks. Our innovative model allows for heterogeneous investor portfolios due to varying emphasis on non-financial characteristics as well as different returns beliefs. A novel estimation approach employs a mixed logit demand specification providing realistic substitution patterns across assets. Results show that US institutional investors on average favor green stocks, a preference that varies with time and investor size. A counterfactual policy banning pension funds from green investing, results in capital gains for brown stocks and losses for green stocks.

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Contact: aaron.mora@moore.sc.edu

1 Introduction

The last decade has seen a steady rise in sustainable investments, with global funds invested in sustainable funds reaching USD 2.7 trillion in the first quarter of 2023 (Morningstar (2023*a*)). This has been accompanied by growing interest in sustainable investment from asset managers, with 85% of them already implementing or planning to implement sustainable investing (Morgan Stanley (2022)), and a growing supply of sustainable funds available to investors (see for example Morningstar (2023*b*)). Equity markets will play a fundamental role in the transition to an environmentally sustainable economy by providing incentives for listed firms to adopt cleaner technologies and practices. Understanding the investor demand for green stocks and its consequences for equity prices is a key part of understanding the incentives of listed firms to align their business strategies with an environmentally sustainable economy.

This paper presents a framework to study the demand for green stocks of US institutional investors. The key idea in this framework is to combine a traditional portfolio construction problem with demand estimation techniques to elicit the investors' revealed preferences over different stocks that are consistent with their portfolio holdings.

A major empirical challenge to study the demand for green stocks is that investors vary greatly in the portfolios they construct. One largely studied reason to explain the differences in portfolio holdings is that investors construct different portfolios because they have different beliefs over future asset returns. Another less studied reason for portfolio heterogeneity is that, even if investors have common beliefs, they can assign varying importance to the characteristics of the portfolios they construct. That is, they can exhibit taste heterogeneity over portfolio characteristics. For example, investors can tilt their portfolios toward environmentally sustainable, or green assets, for motives unrelated to future returns.

The demand for assets framework presented in this paper can accommodate both belief and taste heterogeneity, as motives for differences in portfolio holdings. Belief heterogeneity over future returns is codified via investor-specific conditional expectations, while taste heterogeneity allows investors to care about portfolio characteristics beyond those directly related to an expected return-versus-risk trade off. We apply this framework to estimate the demand for stocks of US institutional investors as explained by environmental scores and return-related stock characteristics. Moreover, using the estimated demand, we conduct a counterfactual exercise to study the effects on equity prices and aggregate holdings of a ban on green investing for pension funds, a policy discussed in the US Senate (Morgan (2023)).

This project innovates on the recent influential work by Koijen and Yogo (2019) (KY2019) that shows that an asset demand approach combined with a market clearing condition implies a valid asset pricing model. KY2019 rekindled a classic literature on modeling asset demand and advocates for applying industrial organization (IO) tools to asset pricing models. However, the microfoundations for asset demand presented in KY2019 are based exclusively on belief heterogeneity over future returns, not allowing for taste heterogeneity to influence investors' demand, and the substitution patterns between assets are restricted by the demand specification they used.

The methodological contributions of this paper are twofold. First, the microfoundations of the asset demand framework presented in this project also allow for taste heterogeneity in the portfolio construction problem in addition to belief heterogeneity. Allowing for taste heterogeneity in portfolio characteristics opens the door to studying investor behaviors that do not fit within into the traditional expected returns-versus-risk paradigm. Examples of such behaviors include: (i) investment strategies based on Environmental, Social, and corporate Governance (ESG) performance metrics of the companies, which take into account stock characteristics unrelated to returns. (ii) "Sin" stock dis-investing, where investors reduce their investments or completely avoid stocks that belong to so-called *sin* industries, like alcohol, tobacco, gambling, adult entertainment, or weapon manufacturing, based on ethical considerations alone. (iii) The deletion of a stock from a stock index can mechanically change the demand for the stock if, for example, there are hedge funds or mutual funds designed to track the index, even if the fundamentals of the company remain unchanged.

The second contribution of this paper is to present and estimate a mixed-logit demand specification for the demand for stocks.[1] In this specification, heterogeneity is captured by investor-specific coefficients that are modeled as functions of investor demographics (see Berry et al. (1995) for the seminal application of this framework, Berry

[1] The mixed logit demand is also referred to as random coefficients demand in the IO literature.

and Haile (2021), Gandhi and Nevo (2021), and Conlon and Gortmaker (2020) for modern practices on demand estimation for differentiated products). KY2019 derive a logit demand system for assets where price elasticities are proportional to portfolio shares. To see why this is a restrictive feature, imagine that the stocks for two companies have the same portfolio weights (market shares in this context), but these companies belong to different industries, for example technology and energy. These companies are likely to have very different fundamentals; however, under a logit demand system, portfolio holdings of another technology stock would react identically to a price change from both stocks. The richer investor-level heterogeneity captured by mixed logit demand delivers flexible substitution patterns between assets, improving on the restrictive price elasticities of a logit demand specification.

We apply the mixed logit demand specification to study the demand for stocks of US institutional investors. We use data on their holdings of US stocks along with data on stock characteristics. To construct stock characteristics, we combine price and accounting data with information on the environmental performance of listed companies in the form of environmental scores (E-scores) from the MSCI rating agency. This application quantifies the extent to which institutional investors value the green metrics of the stocks they select for their portfolio while also controlling for returns-related characteristics. While it is not possible to distinguish whether each characteristic is included in the demand due to belief or taste heterogeneity using only holdings data, survey evidence supports the interpretation of environmental aspects as a taste characteristic.[2]

The estimates rely on quarterly data of the stock holdings of US institutional investors and the corresponding stock characteristics from 2001-Q1 to 2019-Q4, with estimation being conducted in two-year windows. The first finding is that the revealed taste for green stocks fluctuates over time. Throughout the estimation sample, we find a positive taste for green stocks as measured by a positive semi-elasticity for E-scores that is increasing in the second half of the sample. Moreover, in the period after the Great Recession (2007-Q4 to 2009-Q2) there is an increase in the range of values for the coefficient on E-scores, showing an increase in the heterogeneity in the sensitivity to green characteristics across

[2] Giglio, Maggiori, Stroebel, Tan, Utkus and Xu (2023) examine a survey of retail investors on the motives for Enviromental Social and Governance (ESG) investing and find that generally investors expect ESG investments to underperform the market and that only 7% of investors in ESG assets were motivated by return expectations.

investors. This suggests that after periods of economic downturn, some investors may care relatively more about the return-related characteristics of the stocks and relatively less about the environmental-friendliness of the companies underlying the stocks.

We also find that the coefficient corresponding to E-scores for a particular investor is a function of the investor's assets under management. In the last ten years of the sample, institutions with more assets under management have on average a higher taste for green stocks. As a benchmark, we repeated the estimation exercise under a logit demand specification where all investors share the same sensitivity to the green metrics. Such estimates exhibit much less time-series variation.

In a counterfactual exercise, we use the estimated demand system to study the effects of a ban on green investing for pension funds on equity prices and aggregate holdings. On March 1st 2023, the US Senate passed a bill to prevent pension fund managers from basing investment decisions on factors like climate change (Morgan (2023)). President Biden vetoed the bill days later (Thomas (2023)), but various US State Legislatures have approved similar initiatives.[3] In the counterfactual exercise this policy is implemented by making the demand for stocks of pension funds perfectly inelastic to the environmental performance of the stocks, that is, investors are forced to consider only return-related characteristics when constructing their portfolios.

Using the data and estimates for 2019-Q1, we find that stock with low E-scores, commonly referred as *brown stocks*, will benefit the most with higher counterfactual prices. A portfolio containing the bottom quintile of green stocks is estimated to have an associated average price increase of 1.1% under the counterfactual. In contrast, the top quintile portfolio has an average price decrease of 1.6%. Results for the counterfactual exercise using a logit demand specification exhibit much smaller price changes due to the restrictive substitution patterns of logit demand.

Related literature. As mentioned above, this project is most closely related to Koijen and Yogo (2019). In that paper, the authors also propose a demand system approach to asset pricing and estimate a model that jointly explains asset prices and quantities. This

[3] The US House of Representatives later tried to override President Biden's veto but failed to secure the necessary votes for that measure (Foran and Wilson (2023)).

project extends the framework from KY2019 in three ways. First, our model can accommodate taste heterogeneity allowing investors to consider stock characteristics beyond those related to returns when forming their portfolios. Second, a demand system with a mixed logit demand specification provides flexible substitution patterns between assets, improving on the restrictive elasticities of the logit demand model used in KY2019. Third, KY2019 define the market as pools of investors, while this paper employs a more natural market definition: the US stock market in a given quarter. Estimation at the market level facilitates dealing with the endogeneity of prices and allows us to consider a wider range of classical instrumental variables based on demand shifters suggested by the IO literature.

More broadly, this paper contributes to the literature that models asset demand from investors. Classic works include Brainard and Tobin (1968), Rosen (1974), and Lucas (1978). This literature has recently received renewed attention with the use of new stock holdings data and strategies to tackle endogeneity problems. Recent examples include KY2019 and Koijen and Yogo (2020). Koijen and Yogo (2020) builds on the tools presented in KY2019 to study a demand system for financial assets that includes currencies, bonds, and stocks across several countries. Jiang, Richmond and Zhang (2020) use a demand approach to portfolio construction to study global imbalances in net foreign assets across countries. Another example of a demand system approach is in Han, Roussanov and Ruan (2021), where the authors use KY2019's demand approach to quantify the impact of underperforming mutual funds on the overpricing of high-beta stocks. This project contributes to these recent papers by studying more than return characteristics as determinants of investors' demand curves, the use of a mixed logit demand specification, and the use of modern instrumental variables suggested by the IO literature.

We contribute to a recent but growing literature that studies how the environmental performance of stocks matters for equity holdings and prices. Theoretical approaches include Pástor, Stambaugh and Taylor (2021), where the authors develop a model where there are non-pecuniary benefits from investing in green assets. In their model the non-pecuniary benefits boost demand for green assets, which in turn pushes up their prices and leads to lower expected returns. We contribute by showing how to combine heterogeneous non-pecuniary benefits into the micro-foundations of the asset demand framework in KY2019.

This paper also contributes to the literature that employs asset demand frameworks to empirically examine how ESG characteristics matter for investment decisions. Baker, Egan and Sarkar (2022) use a revealed preference argument to interpret the fees for ESG funds as evidence that investors are willing, on average, to pay 20 basis points more per annum to invest in a fund with an ESG mandate as compared to an otherwise identical mutual fund without an ESG mandate. Pastor, Stambaugh and Taylor (2023) employ an asset demand approach to study the degree investors tilt their portfolios between green and brown stocks; they find that on aggregate institutional investors have become increasingly green, exhibiting a positive green tilt, while non-institutional investors have become browner, exhibiting a negative green tilt. These findings are consistent with our empirical result of a positive and increasing preference for green stocks in recent years by institutional investors.

Finally, this paper expands the studies that conduct counterfactual exercises related to green investment strategies. Koijen, Richmond and Yogo (2023) use a demand system with stock characteristics related to environmental performance to study the impact of climate-related induced shifts on equity prices. They study a counterfactual exercise where there is an increased sensitivity for green stocks and find that it implies capital gains for passive investment institutions and capital losses for active investment institutions. Zhang et al. (2024) find that counterfactual scenarios lead to significant different stocks returns according to their ESG characteristics; they consider how stock returns would change if there is increased demand for ESG characteristics by investors, if there are shifts in assets under management from institutions with low demand for ESG characteristics to those with high demand, and if there are changes in the ESG characteristics of the stocks themselves. We contribute to this body of work by studying the effects of a ban on green investing on pension funds while employing the methodological contributions mentioned above.

The rest of this paper is organized as follows: section 2 presents the investor portfolio problem and how a demand for assets with a tractable logit functional form can be obtained from its solution. Section 3 presents how mixed logit demand can be estimated in the context of demand for stocks. Section 4 presents the empirical application that estimates the demand for green stocks from the institutional investors in the US. Section

5 shows the counterfactual exercise that studies the effects of a ban on green investing for pension funds. Finally, section 6 concludes and discusses future avenues of work.

2 Asset Demand with Taste Heterogeneity

In this section we show how a demand for assets with an empirically tractable logit functional form can be obtained from the solution of an traditional portfolio problem. In this set up we allow for investor belief and taste heterogeneity; we discuss how it relates to traditional portfolio problems in the asset pricing literature as well as recent key contributions that model asset demand. The exposition is divided in four subsections. The first one presents the investor portfolio problem and the second one its solution. Importantly, the third subsection presents the assumptions needed to obtain the empirically tractable demand for assets; and the fourth subsection presents the market clearing condition that pin downs equilibrium prices.

2.1 Investor Portfolio Problem

In this subsection we present the portfolio construction problem that investors solve. The key assumption is that investors differ in their beliefs about future returns and assign varying importance to non-financial characteristics of the portfolios they construct.

To facilitate exposition for the remainder of the paper we consider the assets available to investors to be stocks, but the framework in this section applies to asset classes other than stocks, and of course to combinations of assets from different classes. Let $t = 1, ..., T$ denote the stock market in given period. In our application the market definition corresponds to the US stock market at a quarter t. In each of these markets there are I_t investors indexed by $i = 1, ..., I_t$, and each investor i has to allocate A_{it} dollars of assets under management (AUM) in market t among J_t available stocks and an outside option. Stocks are treated as differentiated investment products that are demanded by investors. Let $j = 1, ..., J_t$ index one of the J_t available stocks and $j = 0$ denote the outside option.[4] In our context the outside option denotes the possibility that investors allocate a fraction of their AUM into none of the stocks in J_t.

[4] When there is no possibility of confusion, we use J_t to denote the set of inside goods and the cardinality of the set itself, that is $J_t = \{1, 2, ..., |J_t|\}$.

Let R_{t+1} denote a J_t-vector of gross returns between t and $t+1$ for the stocks available in period t; similarly R^0_{t+1}, a scalar, denotes the gross return of the outside asset. For each stock j the gross return between t and $t+1$ is computed as $R_{t+1,j} = \frac{V_{t+1,j}}{P_{t,j}}$, where $P_{t,j}$ is the price per share of stock j in t and $V_{t+1,j}$ is the payoff per share of j in $t+1$.[5]

Each investor solves a two-period problem between the current period (t) and the next period ($t+1$) where they construct a portfolio by choosing portfolio weights w_{it} (a J_t-vector), such that

$$\max_{w_{it}} \quad E_{it}[\log(A_{i,t+1})] + a_i' C_t' w_{it} \tag{1}$$

$$\text{s.t.} \quad A_{i,t+1} = A_{it}\left[R^0_{t+1} + w_{it}'[R_{t+1} - R^0_{t+1}\mathbf{1}]\right] \tag{2}$$

$$w_{it} \geq 0; \quad \mathbf{1}' w_{it} < 1 \tag{3}$$

In this problem investors value portfolios according to the characteristics they offer. The first portfolio characteristic that investors value is tomorrow's terminal wealth $A_{i,t+1}$ associated with portfolio w_{it}.[6] The second term in the objective function in (1) refers to other portfolio characteristics valued by investors and summarized by the portfolio profile based on stock characteristics contained in C_t. The part of utility that comes from tomorrow's dollar value for the portfolio enters through a log utility, while the current value for investor i of other portfolio characteristics enters with linear weights a_i, an investor-specific K_C-vector. The values of a_i capture investor preferences over the characteristics included in C_t. The log specification in the utility for tomorrow's portfolio wealth follows KY2019 and a long tradition that dates back to Samuelson (1969).[7] If the entries of a_i are set to zero, taste heterogeneity is irrelevant and we are in a context where only pecuniary factors matter for portfolio construction.

The matrix C_t denotes a $J_T \times K_C$ matrix where row j contains a K_C-vector c_{jt} of characteristics for stock j that are relevant for the profile of portfolio w_{it} beyond future wealth.

[5]In many contexts $V_{t+1,j}$ is divided as sum of the price per stock of stock j in $t+1$, $P_{t+1,j}$ plus the dividends per stock in $t+1$, $D_{t+1,j}$; however, in our setting investors only care about the overall future payoff $V_{t+1,j}$, and not whether is was generated by capital gains or dividends.

[6]This is equivalent to a set up where investors derive utility from next's period consumption while assuming they consume all their wealth in the next period; that is $C_{i,t+1} = A_{i,t+1}$ where $C_{i,t+1}$ stands for the consumption of investor i in period $t+1$.

[7]In a multi period setup, assuming Log utility collapses the portfolio problem into a two-period problem, as in our setup.

$E_{it}[\cdot]$ denotes the conditional expectation for investor i at time t, that is $E_{it}[\cdot] \equiv E[\cdot|\mathcal{I}_{it}]$, where $\mathcal{I}_{it}$ denotes the information set of investor i at time t. In this model it is assumed that investors do not learn from the actions of others investors; an assumption commonly referred in the literature as *investors agree to disagree*. The first constraint in (2), where $\mathbf{1}$ denotes a J_t-vector of ones, denotes the evolution of the portfolio's wealth by choosing portfolio w_{it}; while the constraints in (3) impose that all wealth is invested in either stocks or the outside option, as well as short-sale restrictions.[8]

Including stock characteristics into the value of selecting a portfolio w_{it} may be relevant to empirically capture investment decisions that do not entirely fit into an expected return-versus-risk investment paradigm. Examples of this type of investment behavior include: (i) green stock investing where investor value the enviromental performance of the stocks they include in their portfolios, (ii) more generally in investment strategies based on environmental, social, and corporate governance (ESG) metrics of the companies that not only take into account returns and wealth accumulation when selecting stocks to invest. (iii) "Sin" stock dis-investing, where investor avoid including stocks that belong to a so-called "sin" industry like tobacco, alcohol, gambling, adult entertainment or guns; despite the returns these stocks may offer. (iv) Another example is the addition or deletion of a stock into an stock index (e.g. S&P 500, Russell 1000 or The Dow Jones Industrial Average). The change in the index composition can mechanically induce demand for the stock in question, for example by mutual funds or hedge fund strategies designed to track the index, this despite that the stock fundamentals may remain unchanged. This modeling choice generalizes the setup in KY2019, where next's period portfolio wealth, $A_{i,t+1}$, is the only relevant characteristic to construct a portfolio.

Notice that the problem in (1) accommodate two sources of heterogeneity. First, it captures belief heterogeneity over future returns, which is codified via differential information on the expectations operator, $E_{it}[\cdot]$. Under homogenous beliefs it would be the case that $E_{it}[\cdot] = E_t[\cdot]$ for all i. And second, it captures taste heterogeneity over stock characteristics via the weights a_i, if all investors value portfolio characteristics equally then $a_i = a$ for all i. Together, the objective function in (1) accommodates these two channels for heterogeneity.

[8]For a paper that relaxes short sale constraints see Tian (2022).

2.2 Optimal Portfolio Weights

This subsection presents the solution to the portfolio construction problem. The approximate solution for positive portfolio weights has a traditional form that depends on the first two moments of expected returns but also takes into account investor preferences over non-financial characteristics.

In order to provide the solution to the investor's problem we introduce some notation. Denote by r^x_{t+1} the vector of excess log returns, $r^x_{t+1} = \log(R_{t+1}) - \log(R^0_{t+1})1$. Moreover, let $\tilde{\Sigma}_{it}$, a $J_t \times J_t$ matrix, denote the variance-covariance matrix

$$\tilde{\Sigma}_{it} = E_{it}\left[\left(r^x_{t+1} - E_{it}[r^x_{t+1}]\right)\left(r^x_{t+1} - E_{it}[r^x_{t+1}]\right)'\right],$$

and let $\tilde{\mu}_{it}$, a J_t-vector of conditional expectations adjusted by variance:

$$\tilde{\mu}_{it} = E_{it}[r^x_{t+1}] + \frac{\tilde{\sigma}^2_{it}}{2},$$

where $\tilde{\sigma}^2_{it}$ is a vector of the diagonal elements of $\tilde{\Sigma}_{it}$. Furthermore, without loss of generality, partition the asset space between the J^1_t assets with positive weights on the investor's problem, that is those assets for which short sale constraints are not binding so we can rewrite $\tilde{\Sigma}_{it}$ and $\tilde{\mu}_{it}$ as

$$\tilde{\mu}_{it} = \begin{pmatrix} \mu_{it} \\ \mu^{(2)}_{it} \end{pmatrix}, \qquad \tilde{\Sigma}_{it} = \begin{pmatrix} \Sigma_{it} & \Sigma^{(1,2)}_{it} \\ \Sigma^{(2,1)}_{it} & \Sigma^{(2,2)}_{it} \end{pmatrix}$$

where μ_{it} is a J^1_t-vector and Σ_{it} a $J^1_t \times J^1_t$ matrix, both corresponding to the assets with positive weights. The following proposition parallels Lemma 1 in the KY2019 framework in characterizing the solution for the optimal portfolio but accounts for the extra term that allows for taste heterogeneity.

Proposition 1. *The solution to the investor problem in (1)-(3), w^*_{it}, is characterized by the Euler equation*

$$E_{it}\left[\left(\frac{A_{it+1}}{A_{it}}\right)^{-1} R_{t+1}\right] = \mathbf{1} - \left(I - \mathbf{1}w^{*\prime}_{it}\right)\left(\Lambda_{it} - \lambda_{it}\mathbf{1} + C_t a_i\right) \tag{4}$$

where Λ_{it} and λ_{it} are Lagrange multipliers on (2) and (3) respectively. Moreover, the positive *optimal portfolio weights can be approximated (over a short period horizon) as*

$$w_{it}^{*} \approx \Sigma_{it}^{-1}\left(\mu_{it} - \lambda_{it}\mathbf{1} + C_t a_i\right). \tag{5}$$

The proof for Proposition 1 is shown in Appendix A. The Euler equation in (4) generalizes the set up in KY2019, since the case with no taste heterogeneity, $a_i = 0$ for all i, results in the Euler equation presented in KY2019's model.[9] Furthermore, as in KY2019, if investors do not face short-sale constraints ($\Lambda_{it} = 0$ and $\lambda_{it} = 0$) and have homogeneous beliefs ($E_{it}[\cdot] = E_t[\cdot]$ for all i), then the Euler equation becomes

$$E_t\left[\left(\frac{A_{it+1}}{A_{it}}\right)^{-1} R_{t+1}\right] = \mathbf{1},$$

which is a classic moment condition commonly tested in the literature on consumption-based asset pricing. The message in the second part of proposition 1, is that investor i's demand for stocks, given by portfolio weights w_{it}^{*}, is determined by three components: uncertainty around next period returns (Σ_{it}), expected returns (via μ_{it}) and taste sensitivity ($C_t a_i$). All else equal, if investor i has more uncertainty around next period returns for some assets then the portfolio weights on those assets will be relative smaller. Similarly, keeping Σ_{it} fixed, stock holdings are increasing on expected returns. More generally, in our setup stock holdings vary according to the additional value stocks contribute the investors' utility derived from portfolio characteristics. If a stock j contribute positively to a portfolio characteristics k valued positively by investor i, that is $(a_{i,k}c_{jt,k}) \geq 0$, then increasing such characteristic for stock j will imply an increase the in the optimal holdings of j for investor i.

2.3 An empirically tractable demand for stocks

Despite the fact that equation (5) provides a clear intuition of the determinants of the optimal positive portfolio weights, it is not very tractable empirically.[10] This subsection presents the set of assumptions that are needed to derive a form of equation (5) that uses characteristics of the stocks in a logit form to provide an empirical tractable function for the portfolio weights.

[9] Lemma 1 in Koijen and Yogo (2019), page 1481.

[10] It requires obtaining the first two investor-specific conditional moments over excess log returns; which is composed of a large cross section of stock returns.

To compute the conditional moments that determine portfolio weights in equation (5), namely Σ_{it} and μ_{it}; we need to be explicit about how next period's excess log returns are modeled and how information varies across investors. Recall that by definition $R_{t+1,j} = V_{t+1,j}/P_{t,j}$, where $V_{t+1,j}$ stands for next's period payoff for stocks j and $P_{t,j}$ is the equilibrium price per share. Similarly, $R_{t+1,0} = V^0_{t+1}/P_0$; and without loss of generality we normalize the price of the outside good to one, $P_0 = 1$. If we take logs we obtain that the vector of excess log returns is given by

$$\begin{aligned} r^x_{t+1} &= \log(V_{t+1}) - \log(V^0_{t+1})\mathbf{1} - \log(P_t) \\ &= v_{t+1} - v^0_{t+1}\mathbf{1} - p_t \\ &:= v^x_{t+1} - p_t \end{aligned}$$

Given that the excess payoff in $t+1$ is unknown at period t, we assume investors have a prior in period t about its value such that V_{t+1} and V^0_{t+1} follow a lognormal distribution:

Assumption 1. *Distribution of next period's payoff*
The J_t-vector of next period's payoff, V_{t+1}, follows the lognormal distribution

$$V_{t+1} \sim lognormal(\mu_{vt}, \Sigma_{vt}),$$

where μ_{vt} is a J_t-vector and Σ_{vt} a $J_t \times J_t$ matrix. Moreover, the outside option next period's payoff also follows a lognormal distribution

$$V^0_{t+1} \sim lognormal(\mu^0_{vt}, \Sigma^0_{vt}),$$

where μ^0_{vt} and Σ^0_{vt} are scalars. The values of $\mu_{vt}, \mu^0_{vt}, \Sigma_{vt}, \Sigma^0_{vt}$ are common knowledge to investors at time t.

Since next period's payoffs are bounded from below by zero, the log normality assumptions is an appropriate modeling choice and it is a traditional assumption in asset pricing. Assumption 1 implies that the excess log returns can be written as

$$r^x_{t+1} = \mu_{vt} - \mu^0_{vt}\mathbf{1} - p_t + e_v \tag{6}$$

where

$$e_v \sim N(0, \Sigma_{xt})$$
$$\Sigma_{xt} = E_t[(r^x_{t+1} - \mu_{xt})(r^x_{t+1} - \mu_{xt})']$$
$$\mu_{xt} = \mu_{vt} - \mu^0_{vt}\mathbf{1} - p_t,$$

and the (j,k)-entry of Σ_x is given by:

$$\Sigma_{xt,jk} = \Sigma_{vt,jk} + \Sigma^0_{vt} - cov(V_{t+1,j}, V^0_{t+1}) - cov(V_{t+1,k}, V^0_{t+1}).$$

Notice that the value of excess log returns depends on the vector of prices, p_t, which need to be pinned down in equilibrium. We also assume that equilibrium prices are observed by all investors. Then conditional on public information, excess log returns can be viewed as having a distribution inherited from the distribution assumed for next period excess payoffs.[11] In this case $r^x_{t+1} \sim N(\mu_{xt}, \Sigma_{xt})$, with both μ_{xt} and Σ_{xt} endogenously determined in the model as they depend on p_t, which is determined in equilibrium. The next assumption states a factorization for the matrix Σ_{xt}.

Assumption 2. *Factorization for the matrix Σ_{xt}*
The matrix Σ_{xt} admits the representation

$$\Sigma_{xt} = \Gamma_{xt}\Gamma'_{xt} + \sigma^2_e I. \tag{7}$$

where Γ_{xt} is a J_t-vector of factor loadings and σ^2_e is common variance across stocks.

This assumption is consistent with a factor structure for the vector of log excess returns where r^x_{t+1} admits the following single factor representation:

$$r^x_{t+1} = \mu_{xt} + \Gamma_{xt}F_{t+1} + e_{t+1},$$

the single factor F_{t+1} is distributed as $N(0,1)$ and $e_{t+1} \sim N(0, \sigma^2_e I)$, with e_{t+1} independent of F_{t+1}.[12]

[11] Unfortunately, the setup in KY2019 is not explicit about to what extent returns are endogenous or exogenous to their setup, we believe that being explicit about this and modeling how information is different across investors contribute to the clarity of the microfoundations of asset demand.

[12] Alternatively, the factorization in Assumption 2 can be obtained by assuming a factor structure on the variance matrix of future payoffs Σ_{vt}. In this case notice that $\sigma^2_e = \Sigma^0_{vt}$.

In order to compute moments that depend on the information set of each investor, we model how information varies across investors. We adapt a traditional setup from the asymmetric information literature in asset pricing (see for example Grossman (1976)).

Assumption 3. *Information Technology*
Each investor i receives a signal about next period's excess log returns, r^x_{t+1}, denoted by s_{it}, a J_t-vector given by

$$s_{it} = r^x_{t+1} + \varepsilon_{it} \tag{8}$$

where $\varepsilon_{it} \sim N(0, \sigma^2_{\varepsilon,i} I)$ and it is independent of r^x_{t+1}. The scalar $\sigma^2_{\varepsilon,i}$ is privately known by investor i.

In this set up, each investor knows s_{it} and $\sigma^2_{\varepsilon,i}$ but does not know the value of ε_{it} that produced s_{it}. This means that investors cannot back out the value of r^x_{t+1} directly from their private information and have to update their expectations over r^x_{t+1} by conditioning on the information set $\mathcal{I}_{it} = \{s_{it}, \sigma^2_{\varepsilon,i}, \mu_{xt}, \Sigma_{xt}\}$. Assumption 3 states that information is not only different across investors because they receive different signals s_{it}, but also in the precision of these signals around the true realization of r^x_{t+1}. Investors with a relative low $\sigma^2_{\varepsilon,i}$ will observe a realization of s_{it} in a smaller neighborhood of r^x_{t+1}, making s_{it} a more precise signal of r^x_{t+1}. This heterogeneity in signal precision is motivated by the possibility that some investors have better information or better capacity to process information, that allows them to produce more precise forecasts about future returns.

Next notice that $s_{it}|(r^x_{t+1}, \sigma^2_{\varepsilon,i}) \sim N(r^x_{t+1}, \sigma^2_{\varepsilon,i} I)$ and $r^x_{t+1} \sim N(\mu_{xt}, \Sigma_{xt})$ so using Bayes theorem we can compute the distribution of $r^x_{t+1}|\mathcal{I}_{it}$. The posterior distribution is given by

$$r^x_{t+1}|\mathcal{I}_{it} \sim N(\mu_{r|s_i}, \Sigma_{r|s_i}) \tag{9}$$

$$\text{with} \qquad \Sigma_{r|s_i} = \left[(\sigma^2_{\varepsilon,i})^{-1} I + \Sigma^{-1}_{xt}\right]^{-1} \tag{10}$$

$$\mu_{r|s_i} = \Sigma_{r|s_i}\left[(\sigma^2_{\varepsilon,i})^{-1} s_{it} + \Sigma^{-1}_{xt}\mu_{xt}\right]. \tag{11}$$

Details on the derivation of the posterior moments are shown in appendix A. The next proposition shows that under assumptions 1 and 2 we can obtain a convenient decomposition of the investor-specific conditional matrix Σ_{it}.

Proposition 2. *Under assumptions 1 to 3, the investor-specific conditional matrix Σ_{it} can be written as $\Sigma_{it} = \Gamma_{it}\Gamma'_{it} + \iota_{it} I$ where the J_t-vector Γ_{it} and the scalar ι_{it} are both investor-specific.*

Proof of Proposition 2 is shown in appendix A. Notably, this result is an assumption in KY2019's model.[13] In our set up we are able to obtain the investor-specific decomposition of Σ_{it} by relaying on weaker assumptions. The next assumption relates the first two moment of the vector of excess log returns with individual stock characteristics.

Assumption 4. *Return-related Stock Characteristics*
Each entry of the J_t-vectors μ_{xt} and Γ_{xt} can be expressed as a polynomial of order M over a K_x-vector of return-related stock characteristics x_{jt}, including price p_{jt}; that is

$$\mu_{xt,j} = y_{jt}'\Phi_\mu + \phi_\mu \tag{12}$$

$$\Gamma_{xt,j} = y_{jt}'\Phi_\Gamma + \phi_\Gamma \tag{13}$$

where Φ_μ and Φ_Γ are matrices of coefficients, ϕ_Γ and ϕ_μ scalars and y_{jt} is K_y-vector with dimension $K_y = \sum_{m=1}^{M} K_x^m$ and

$$y_{jt} = \begin{pmatrix} x_{jt} \\ x_{jt} \otimes x_{jt} \\ x_{jt} \otimes x_{jt} \otimes x_{jt} \\ \vdots \end{pmatrix},$$

and $\otimes$ stands for the Kronecker product.

The motivation for the previous assumption is twofold. First, modeling the mean and covariance matrices of asset returns as functions of assets characteristics is more empirically tractable than estimating the mean and covariance matrices of asset returns directly from past observations (see for example Brandt et al. (2009)).[14] Second, KY2019 show the empirical the relevance of characteristics-based demand (their appendix B), by showing that expected returns and factor loadings are well captured by a few asset characteristics, and that this approach better estimates the mean-variance portfolio compared to a benchmark that uses sample estimates of the first two moments of returns.

Notice that with Assumption 4 we now deal with two sets of asset characteristics. The first set is represented in the vector x_{jt} of stock characteristics, including prices p_{jt}, that

[13] The first part of Assumption 1 in Koijen and Yogo (2019), p. 1483.

[14] Since the mean and covariance matrices of asset returns are hard to estimate and very likely time-varying, this literature moved from estimation using historical samples of returns to using functions that map assets characteristics to their returns, and hence the first two moments of returns. One rationale for this approach is that asset characteristics are more stably related to expected returns than company names in a large cross section.

are relevant for the conditional expected returns (μ_x) and factor loadings (Γ_x), we called them *return-related characteristics*. The second set are contained in c_{jt}, that denotes the stock characteristics that are relevant to produce a portfolio profile valued by investors, we called them *taste characteristics*. Empirically, this suggest to include observable stock characteristics known to be relevant for the cross section of return into x_{jt} and those relevant for investment decisions but not directly related to stock returns into c_{jt}.

The following proposition uses all previous assumptions and results to characterize optimal portfolio weights as polynomials on asset characteristics with investor specific coefficients, and then obtain the empirically tractable logit form that relates portfolio holdings with stock characteristics.

Proposition 3. *Under Assumptions 1 to 4:*

(i) The j-th entry of the vectors Γ_{it} and μ_{it} can be written as a polynomial function on x_{jt} with investor-specific coefficients.

(ii) The optimal portfolio weights for each asset j with positive weight, $w_{it,j}$ can be written as polynomial function on asset characteristics (x_{jt}, c_{jt}) with investor specific coefficients:

$$w_{it,j} = \tilde{y}'_{jt}\Phi_{w,i} + \phi_{w,i}, \tag{14}$$

where $\Phi_{w,i}$ is vector of coefficients, $\phi_{w,i}$ is a scalar, and $\tilde{y}_{jt}$ is a $K_{\tilde{y}}$-vector with $K_{\tilde{y}} = \sum_{m=1}^{2M}(K_X + K_c)^m$, and

$$\tilde{y}_{jt} = \begin{pmatrix} \tilde{x}_{jt} \\ \tilde{x}_{jt} \otimes \tilde{x}_{jt} \\ \tilde{x}_{jt} \otimes \tilde{x}_{jt} \otimes \tilde{x}_{jt} \\ \vdots \end{pmatrix},$$

and $\tilde{x}_{jt} = (x'_{jt} \quad c'_{jt})'$ a $(K_X + K_C)$-vector.

(iii) Moreover if the polynomial order M goes to infinity, $M \to \infty$, then a restriction of parameters implies that the optimal portfolio weights for investor i can be written as:

$$w_{it,j} = \frac{\exp\left(x'_{jt}b_{it} + c'_{jt}\gamma_{it} + \xi_{jt}\right)}{1 + \sum_{j=1}^{J_t}\exp\left(x'_{jt}b_{it} + c'_{jt}\gamma_{it} + \xi_{jt}\right)}, \tag{15}$$

and the portfolio weight for the outside option is given by

$$w_{it,0} = \frac{1}{1 + \sum_{j=1}^{J_t} \exp\left(x'_{jt}b_{it} + c'_{jt}\gamma_{it} + \xi_{jt}\right)}, \tag{16}$$

where b_{it} and γ_{it} are investor-specific coefficients on observed returns-related characteristics x_{jt}, that include price p_{jt}, and taste characteristics c_{jt}. The term ξ_{jt} represents an index of unobserved (by the econometrician) return-related characteristics.

It is possible that there are asset characteristics unobserved by the econometrician but relevant for investor's portfolios. Without loss of generality we can assume those unobserved characteristics are summarized in an index ξ_{jt} and that this index is a characteristic included in x_{jt} as in part (iii) of proposition 3. In the previous proposition the coefficient on the unobserved characteristic ξ_{jt} is normalized to 1 and the portfolio value of the outside option is normalized to 1. Part (i) of this proposition is an assumption in KY2019's framework, as in proposition 2, we are able to obtain this result by relaying on weaker assumptions. Proof of proposition 3 is presented in Appendix A.[15]

This proposition tell us that the optimal weight on asset j for investor i in market t is directly explained by the characteristics of asset j, investor-specific coefficients (b_{it}, γ_{it}), and the value that investor i assigs to j relative to all the other assets available in J_t.

2.4 Market Clearing Condition

The portfolio weights in equations (15) and (16) represent the asset demand curves for investors taking stock characteristics, including price, as given. In this subsection we pair the demand system with a supply side to obtain a market clearing conditions that pin downs equilibrium prices. The key assumption is that the number of shares outstanding is fixed in the short run. The empirical application in this paper works with stock market data at a quarterly frequency, so we consider this assumption reasonable.

Let S_{jt} denote the number of shares outstanding for stock j in market t; in the short run this is assumed to be a fixed number which can be interpreted as an inelastic supply of the stock. If we multiply S_{jt} by P_{jt}, the price per share of j in t, we obtain the market

[15] Once part (i) is proved, the proof of parts (ii) and (iii) proceed similarly as in proposition 1 of KY2019.

equity for stock j, denote by ME_{jt}.

Since each stock of j should be held by an investor in the market, then the following market clearing should hold:

$$ME_{jt} = \sum_{i=1}^{I} A_{it} w_{it,j}. \tag{17}$$

This market clearing condition says that all the money invested by investors in stock j, should equal the market equity of the stock. This condition is a re-statement in dollar value, instead of quantities, that prices in equilibrium are such that in the aggregate, supply equals demand:

$$S_{jt} = \frac{\sum_{i=1}^{I} A_{it} w_{it,j}}{P_{jt}}.$$

Notice that the right hand side of (17) depends on prices, since p_{jt} enters demand via de the return-related characteristic x_{jt}. As noted in KY2019, the market clearing condition implies a fixed point equation in p_{jt}. Let p_t be the J_t-vector of log prices for the stocks in market t and define $f : \mathbb{R}^{J_t} \to \mathbb{R}^{J_t}$ as:

$$f(p_t) = \log\left(\sum_{i=1}^{I} A_{it} w_{it,j}(p_t)\right) - log(S_{jt}), \tag{18}$$

so using equation 18 we can solve for the equilibrium prices, by looking for p_t^* such that $p_t^* = f(p_t^*)$. KY2019 state the conditions for an unique fixed point to exist and present an algorithm for the determining such fixed point.

3 Demand Specification and Estimation

3.1 Demand Specification

Using data on portfolio holdings, $\{w_{it,j}\}$ and stock characteristics $\{x_{jt}, c_{jt}\}$ the goal is to estimate the asset demand coefficients defined in (15). This project uses a mixed logit demand specification (Berry et al. (1995)), also known as a random coefficients demand specification. In the mixed logit demand the investor-specific coefficients in b_{it} and γ_{it}

follow the structure:

$$b_{it} = b_0 + \Pi_b d_{it} + \Sigma_b^{1/2} v_{b,it} \tag{19}$$

$$\gamma_{it} = \gamma_0 + \Pi_\gamma d_{it} + \Sigma_\gamma^{1/2} v_{\gamma,it}, \tag{20}$$

where $b_{0,k}$ is a component of $b_{it,k}$, common to all investors; d_{it} denotes an L-vector of observable investor demographics that are relevant to characterize $b_{it,k}$, along with the corresponding coefficients in Π_b, a $K_X \times L$ matrix. The term $v_{b,it}$ is a K_X-vector of investor-specific taste shocks that are scaled by common variance covariance coefficients, Σ_b. The $v_{b,it}$ can also be interpreted as unobservable (by the econometrician) investor demographics relevant for b_{it}. Analogous interpretations apply to γ_0, Π_γ, Σ_γ and $v_{\gamma,it}$.

If we restrict to zero the matrices $\Pi_b, \Pi_\gamma, \Sigma_b$ and Σ_γ, we have that $b_{it} = b_0$ and $\gamma_{it} = \gamma_0$ for all i. This is the highly studied logit demand. In this specification all investors have the same coefficients and is the one used during estimation in KY2019.[16] Logit demand is highly tractable but delivers restrictive substitution patterns. This is because in Logit demand price elasticities are determined by market shares. Leading authors in the demand estimation literature call this "*a bug not a feature*" (Berry and Haile (2021), pg. 19). The limitations imply that for small portfolio weights, own-price elasticities are approximately proportional to the coefficient corresponding to price. Moreover, two stocks with similar portfolio weights would react identically to the price change of any other stock.[17] Imagine two stocks that have the same portfolio weights, but the companies belong to different sectors, for example technology and energy. Is easy to imagine these stocks would respond to different fundamentals, yet under a logit demand system, portfolio holdings of another technology stock would react identically to a price change from both stocks.

The restrictive substitution patterns of logit demand are a manifestation of the Independence of Irrelevant Alternatives (IIA) assumption (see, e.g. Arrow (1951), Ray (1973),

[16]In KY2019, a logit demand is estimated investor by investor, so they obtain a set of estimated coefficients by investor. However, at the investor level the substitution patters between stocks are those of logit demand.

[17]Berry and Haile (2021) develop their argument further: "These restrictions do not come from economics but from assumptions chosen for simplicity or analytical convenience. Models must, of course, abstract from reality, and finite samples require appropriate parsimony. But good modeling and approximation methods should aim to avoid strong a priori restrictions on the very quantities of interest unless those restrictions can be defended as natural economic assumptions." (pg. 19).

McFadden (1974)), that states that the relative likelihood of choosing between two options will not change on whether a third alternative is present. Because of IIA, logit demand will fail to capture substitution patterns between close substitutes, a feature that has been widely studied in the industrial organization literature. Extensions of logit demand like the nested logit demand (Cardell (1997)) and the mixed logit demand relax the IIA assumption. In nested logit the product choice is sequential, first individuals choose a product nest, and then they choose a product within the nest. Models of this type can be easily accommodated in a mixed logit demand by including the nest-defining characteristics as one of the characteristics in the demand specification.

Price Elasticities. Let $Q_{it,j} = \frac{A_{it} w_{it,j}}{P_{t,j}}$ denote the number of shares of stock j held by investor i in market t. The elasticity of stock j holdings when the price of stock k changes, denoted by $\eta_{it,jk}$ is given by:

$$\begin{aligned}\eta_{it,jk} &= \frac{\partial Q_{it,j}}{\partial P_{t,k}} \frac{P_{t,k}}{Q_{it,j}} = \frac{\partial \log(Q_{it,j})}{\partial \log(P_{t,k})} \\ &= \frac{\partial \log(w_{it,j})}{\partial \log(P_{t,j})} - 1\{j = k\} := e_{it,jk} - 1\{j = k\}.\end{aligned}$$

The term $e_{it,jk}$ is also an elasticity but with respect to portfolio weights and hence depends on the demand specification. As mentioned above log prices, p_{jt} is one of the stock characteristics included in x_{jt}. Without loss of generality, let $k = 1$ denote index for the coefficient corresponding to log prices, $b_{it,1}$. Following this notation, the term $e_{it,jk}$ under logit demand ($b_{it,1} = b_{0,1}$ for all i) is given by:

$$e^{Logit}_{it,jk} = \frac{\partial \log(w_{it,j})}{\partial \log(P_{t,k})} = b_{0,1}(1\{j = k\} - w_{it,k}).$$

The corresponding term for mixed logit demand (ML), $e^{ML}_{it,jk}$, is given by

$$e^{ML}_{it,jk} = \frac{\partial \log(w_{it,j})}{\partial \log(P_{t,k})} = \left(b_{0,1} + \sum_{\ell=1}^{L} \pi_{1,\ell} d_{\ell it} + \sigma_{t,1} v_{i,1} \right) (1\{j = k\} - w_{it,k}).$$

The term $e^{ML}_{it,jk}$ represents the price elasticity given the idiosyncratic preferences of investors as capture by the price coefficient $b_{it,1}$. However, this expression is not feasible to compute given that the taste shocks, $v_{i,1}$, are not observed. To compute the price elasticity conditional on observable data is necessary to integrate over the distribution of the taste shocks. If F_v denotes the distribution of v_{it} we can numerically integrate out its role on portfolio holdings using F_v and computing $e^{ML}_{it,jk}$ using $E_{F_V}\left[w_{it,j}\right]$:

$$
\begin{aligned}
e^{ML}_{it,jk} &= \frac{\partial \log\left(E_{F_V}[w_{it,j}]\right)}{\partial \log(P_{t,k})} = \frac{\partial E_{F_V}[w_{it,j}]}{\partial P_{t,k}} \frac{P_{t,k}}{E_{F_V}[w_{it,j}]} \\
&= \frac{1}{E_{F_V}[w_{it,j}]} \int \left[\left(b_{0,1} + \sum_{\ell=1}^{L} \pi_{1,\ell} d_{\ell it} + \sigma_{t,1} v_{i,1}\right) \tilde{w}_{it,j}(v_i)(1\{j=k\} - \tilde{w}_{it,k}(v_i))\right] dF(v_i).
\end{aligned}
$$

Comparing the terms $e^{Logit}_{it,jk}$ and $e^{ML}_{it,jk}$ shows, as mentioned at the beginning of this section, that a logit demand specification is limited in the substitutions patterns it can accommodate when stock prices change. Mixed logit demand, on the other hand, by employing a richer structure on the parameters, can deliver more flexible substitution patterns.

3.2 Estimation

The relevant equations for estimation are the mapping between asset characteristics and asset holdings, namely equation (15), paired with the random coefficients specifications that relate investor-specific coefficients with their demographics in (19) and (20). For convenience we present again these equations:

$$
\begin{aligned}
w_{it,j} &= \frac{\exp\left(x'_{jt} b_{it} + c'_{jt}\gamma_{it} + \xi_{jt}\right)}{1 + \sum_{j=1}^{J_t} \exp\left(x'_{jt} b_{it} + c'_{jt}\gamma_{it} + \xi_{jt}\right)} \\
b_{it} &= b_0 + \Pi_b d_{it} + \Sigma_b^{1/2} v_{b,it} \\
\gamma_{it} &= \gamma_0 + \Pi_\gamma d_{it} + \Sigma_\gamma^{1/2} v_{\gamma,it}.
\end{aligned}
$$

The data is composed of asset holdings, investor demographics and assets under management $\{w_{it}, d_{it}, A_{it}\}_{i=1,\dots,I_t}$, as well as asset characteristics $\{x_{jt}, c_{jt}\}_{j=1\dots,J_t}$. The parameters to estimate are the components that form the coefficients corresponding to return and taste characteristics; namely $\{b_0, \Pi_{b,}, \Sigma_b\}$ for the return characteristics x_{jt} and $\{\gamma_0, \Pi_\gamma, \Sigma_\gamma\}$ for the taste characteristics c_{jt}.

For exposition convenience of the estimation steps, let's rewrite observed asset characteristics into the $K = (K_X + K_C)$ vector $X_{jt} = (x'_{jt}, c'_{jt})'$, and accordingly define the coefficients vector $\beta_{it} = (b'_{it}, \gamma'_{it})'$ such that

$$\beta_{it} = \beta_0 + \Pi d_{it} + \Sigma^{1/2} v_{it} \tag{21}$$

where $\beta_0 = (b'_0, \gamma'_0)'$; Π is a $K \times L$ matrix with the demographics coefficients (Π_b, Π_γ); $v_{it} = (v_{b,it}, v_{\gamma,it})'$ is a K-vector, and $\Sigma^{1/2}$ is a $K \times K$ matrix composed of $(\Sigma_b^{1/2}, \Sigma_\gamma^{1/2})$. With this notation the parameters to estimate can be denoted as $\theta := (\beta_0, \Pi, \Sigma)$. In the IO literature on demand estimation $\theta_1 = \beta_0$ is commonly referred as "linear parameters" and $\theta_2 := (\Pi, \Sigma)$ as "non-linear parameters", due to the way these parameter enter the estimation procedure.

With this notation we can write the exponents in the expression for $w_{it,j}$ as

$$\begin{aligned} x'_{jt} b_{it} + c'_{jt}\gamma_{it} + \xi_{jt} &= X'_{jt}\left[\beta_0 + \Pi d_{it} + \Sigma v_{it}\right] + \xi_{jt} \\ &= \underbrace{X'_{jt}\beta_0 + \xi_{jt}}_{:=\delta_{jt}} + \underbrace{X'_{jt}\left[\Pi d_{it} + \Sigma v_{it}\right]}_{:=h_{ijt}(\theta_2, v_{it})} \\ &= \delta_{jt} + h_{ijt}(\theta_2, v_{it}). \end{aligned}$$

The term δ_{jt} is referred as the "mean utility" for option j in market t, as it is a common component for all investors; while the term $h_{ijt}(\theta_2, v_{it})$ captures investor-specific heterogeneity. With this notation we can write $w_{it,j}$ as

$$w_{it,j} = \frac{\exp\left(\delta_{jt} + h_{ijt}(\theta_2, v_{it})\right)}{1 + \sum_{j=1}^{J_t} \exp\left(\delta_{jt} + h_{ijt}(\theta_2, v_{it})\right)}. \tag{22}$$

The next step is to obtain aggregate market shares for each stock. In this project we perform estimation at the market level. This is motivated because market-level estimation facilitates dealing with the endogeneity of prices, as discussed below, so we can use instrumental variables for prices based on demand shifters that have been suggested in the IO literature. This choice is also consistent with the market definition presented in the microfoundations, namely we consider the US stock market in a given quarter.

The key idea is to construct aggregate market shares from the market clearing condition (17). Let ME_{0t} denote the aggregate investment in the outside option: $ME_{0t} = \sum_{i=1}^{I_t} A_{it} w_{i0t}$; and denote by ME_t denote the aggregate value of the market in t: $ME_t =$

$\sum_{j=0}^{J_t} ME_{jt}$. By the market clearing condition it has to be the case that the aggregate value of the market is equal to the aggregate assets under management across investors, so we have that $ME_t = \sum_{i=1}^{I_t} A_{it} := A_t$. If we divide the market clearing condition (17) by ME_t (or A_t equivalently) we obtain that

$$ME_{jt} = \sum_{i=1}^{I_t} A_{it} w_{it,j}$$
$$\Rightarrow s_{jt} := \left(\frac{ME_{jt}}{ME_t}\right) = \sum_{i=1}^{I_t} \left(\frac{A_{it}}{At}\right) w_{it,j}. \tag{23}$$

This equation tell us that in the aggregate we will compare the observed stock market shares of a "market value portfolio" (s_{jt}) with the model implied shares of a "wealth-adjusted aggregate portfolio" $\left(\sum_{i=1}^{I_t} \left(\frac{A_{it}}{At}\right) w_{it,j}\right)$. To compute the right hand side of (23), the model-implied shares, one challenge is that the investor-specific taste shocks v_{it} are not observed by the econometrician. To deal with this problem it is common practice to assume a prior distribution on these latent variables. If F_v denotes the distribution of v_{it} we can numerically integrate out its role on portfolio holdings using F_v. The model-implied shares, denoted by $\tilde{s}_{jt}$, are

$$\tilde{s}_{jt} := \sum_{i=1}^{I_t} \left(\frac{A_{it}}{A_t}\right) \int \frac{\exp\left(\delta_{jt} + h_{ijt}(\theta_2, d_{it}, v_{it})\right)}{1 + \sum_{j=1}^{J_t} \exp\left(\delta_{jt} + h_{ijt}(\theta_2, d_{it}, v_{it})\right)} dF_v(v_{it}). \tag{24}$$

It is standard practice in the IO literature to assume F_v to be a multivariate normal with zero mean and variance-covariance matrix S_v, that is $v_{it} \sim N(0, S_v)$; then the integral in (24) can be numerically approximated for example by monte carlo simulation or using Gauss-Hermite quadrature procedures. The $\tilde{s}_{jt}(\cdot)$ functions in equation (24) define a demand system that can be written as:

$$\tilde{s}(\delta_t, \theta_2; d_t, X_t, J_t) = \left(\tilde{s}_1(\delta_t, \theta_2; d_t, X_t, J_t), ..., \tilde{s}_{J_t}(\delta_t, \theta_2; d_t, X_t, J_t)\right). \tag{25}$$

The aggregation across investors in (24) with wealth-based weights (A_{it}/A_t) makes this demand system different from the standard mixed logit demand system (for example the canonical BLP demand system of Berry et al. (1995)). Nevertheless, we prove in the following proposition that this system is *invertible* in the sense that given $(\theta_2; d_t, X_t)$ there is an unique vector δ such that for all j:

$$\tilde{s}_{jt}(\delta, \theta_2; d_t, X_t, J_t) = s_{jt}.$$

Proposition 4. *Demand Inversion*
The demand system in (25) is invertible such that given $(\theta_2; d_t, X_t, J_t)$ and nonzero market shares s_{jt} with $\sum_{j=1} s_{jt} < 1$, there exists an unique vector δ such that $\tilde{s}_{jt}(\delta, \theta_2; d_t, X_t, J_t) = s_{jt}$, for all j.

The proof is included in Appendix A and it follows by verifying the conditions of the Berry's inversion theorem (Berry (1994)). Such vector δ can be found as the fixed point of the following contraction mapping. Fix $(\theta_2; d_t, X_t, J_t)$ and let $f : \mathbb{R}^J \to \mathbb{R}^J$ be given by

$$f(\delta) = \delta + \log(s_t) - \log(\tilde{s}_t(\delta, \theta_2; d_t, X_t, J_t)), \tag{26}$$

notice that if δ^* is such that $f(\delta^*) = \delta^*$ then $\log(s_t) = \log(\tilde{s}_t(\delta^*, \theta_2; X_t, J_t))$, that is, δ^* is such that model-implied shares are equal to the observed shares given a set of values for $(\theta_2; d_t, X_t, J_t)$.

The fact that the demand system is invertible allow us to operationalize an estimation strategy where we interpret the term ξ_{jt} as an structural error term. This strategy for estimation is predicated on the assumption that unobserved stock characteristics should be conditionally mean zero with respect to a vector z_{jt} of observable stock characteristics,

$$E[\xi_{jt}|z_{jt}] = 0, \tag{27}$$

this condition implies that unobservable stock characteristics in z_{jt} should not be correlated with unobserved characteristics ξ_{jt}. In the context of asset demand, as KY2019 mention, this moment condition is motivated by the literature of asset pricing in endowment economics (Lucas (1978)) that assumes that shares outstanding and asset characteristics other than price are exogenous.

The fact that the demand system (23) is invertible, allow us to exploit moment (27) for estimation. Given θ_2 we can "invert the demand" to find $\hat{\delta}_t(\theta_2)$ such that model-implied shares $\tilde{s}_t(\hat{\delta}_t, \theta_2)$ match the observed shares, s_t. With $\hat{\delta}_t(\theta_2)$ we can construct $\xi_{jt}(\theta)$ given by $\xi_{jt}(\theta) = \hat{\delta}_{jt}(\theta_2) - X'_{jt}\theta_1$ and then we can select $\theta = (\theta_1, \theta_2)$ that minimizes a GMM objective function based on (27). Formally, in the GMM strategy for estimation of the demand system in (25) we look for $\hat{\theta}_{GMM}$ in market t that solves:

$$\min_{\theta} \quad g\left(\xi_t(\theta)\right)' W_t g\left(\xi_t(\theta)\right) \tag{28}$$

$$\text{s.t.} \quad g\left(\xi_t(\theta)\right) = \frac{1}{J_t}\sum_{j=1}^{J_t} z_{jt}\xi(\theta)_{jt} \tag{29}$$

$$\xi(\theta)_{jt} = \delta_{jt}(\theta_{2t}) - X_{jt}'\theta_1 \tag{30}$$

$$\log(s_{jt}) = \log\left(\tilde{s}_{jt}(\delta_t, \theta_2; d_t, X_t, J_t)\right) \tag{31}$$

$$\tilde{s}_{jt}(\delta_t, \theta_2; d_t, X_t, J_t) = \sum_{i=1}^{I_t}\left(\frac{A_{it}}{At}\right)\int \frac{\exp\left(\delta_{jt} + h_{ijt}(\theta_2, d_{it}, v_{it})\right)}{1+\sum_{j=1}^{J_t}\exp\left(\delta_{jt} + h_{ijt}(\theta_2, d_{it}, v_{it})\right)} dF_v(v_{it}) \tag{32}$$

where W_t is a $K_Z \times K_Z$ weight matrix. Using the first order conditions of the GMM problem is possible to show that the parameter search can be simplified to be just over the non-linear parameters θ_2 (see Berry et al. (1995) and Nevo (2000) for useful derivations). To see why, it suffices to notice that the first order conditions with respect to θ_1 requires

$$0 = \frac{\partial}{\partial\theta_1}\left(g\left(\xi(\theta_t)\right)' W_t g\left(\xi(\theta_t)\right)\right) \Leftrightarrow 0 = -\frac{2}{J_t^2}[\delta_t(\theta_2) - X_t\beta_{0t}]' Z_t W_t Z_t' X_t \tag{33}$$

$$\Leftrightarrow \theta_1 = [X_t' Z_t W_t Z_t' X_t]^{-1}[X_t Z_t W_t Z_t']\delta_t(\theta_2),$$

so given a value of θ_2, there is a corresponding value for θ_1 according to the GMM objective function. If the estimation is carried out using data from multiple market periods while considering the parameters θ as common across such periods, the GMM strategy would compute (30) to (32) each period t, and then stack the moment analogs (29) for each market before computing the objective function in (28). Algorithm 1 sketches the steps necessary to implement the GMM estimation of the mixed logit demand.

Price Endogeneity and Instrumental Variables. Since the unobserved stock characteristics ξ_{jt} are part of investors demand they will also be a determinant of equilibrium prices. This means that prices and functions of prices will be correlated with ξ_{jt} and cannot be included in z_{jt}, the vector of exogenous asset characteristics for estimation. To solve for the endogeneity of prices with respect to ξ_{jt} we need instrumental variables (IVs) correlated with prices but exogenous with respect to ξ_{jt}.

We consider instrumental variables in the style of Gandhi and Houde (2019). For

Algorithm 1: Mixed Logit Demand Estimation

Input: Stock characteristics $\{X_{jt} = (x'_{jt}, c'_{jt})'\}$, aggregate stock holdings $\{s_{jt}\}$ and instrumental variables $\{Z_{jt}\}$ for $j = 1, ..., J_t$; assets under managament and demographics $\{A_{it}, d_{it}\}$ for $i = 1, ..., I_t$ for a given market t.

Output: A set of estimated parameters $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ with $\hat{\theta}_1 = \hat{\beta}_0$ and $\hat{\theta}_2 = (\hat{\Pi}, \hat{\Sigma})$.

Initialize: Pick initial values for $\theta_1^{(0)}$ and $\theta_2^{(0)} = (\Pi^{(0)}, \Sigma^{(0)})$.

During step $r \geq 1$ of the optimization routine, do:

i. Compute an initial value for $\delta_t^{(r,0)}$. If $r = 1$ the initial value can be $\delta_t^{(r,0)} = X'_{jt}\theta_1^{(0)}$.

ii. Given the current value for $\theta_2^{(r)}$, *invert* the demand system using the contraction mapping in (26). To do this, iterate until convergence an update for δ_t where the h-th update is given by:

$$\delta_t^{(h)}\left(\theta_2^{(r)}\right) = \delta_t^{(h-1)} + \log(s_t) - \log\left(\tilde{s}_t\left(\delta_t^{(h-1)}, \theta_2^{(r)}; X_t, J_t\right)\right)$$

Use $\delta_t^{(r,0)}$ for the first update. The resulting vector will be a function of $\theta_2^{(r)}$ and aggregate stock holdings s_t, denoted by $\delta_t^{(r)}$.

iii. Update the value of the linear parameters using (33)

$$\theta_1^{(r)} = [X'_t Z_t W_t Z'_t X_t]^{-1}[X_t Z_t W_t Z'_t]\delta_t^{(r)}.$$

iv. Use $\delta_t^{(r)}$ and $\theta_1^{(r)}$ to compute the GMM error term:

$$\xi_t^{(r)} = \delta_t^{(r)} - X'_t\theta_1^{(r)},$$

and the GMM moment function, notice that g is a function of the parameters in step r:

$$g\left(\theta^{(r)}\right) = \frac{1}{J_t} Z'_t \xi_t^{(r)}.$$

v. Evaluate the GMM objective function at $\theta^{(r)}$. If the objective function has converged report $\hat{\theta}_{GMM} = \theta^{(r)}$. If no convergence has been achieved update $\theta_2^{(r)}$ according to the optimization algorithm used (e.g. a Newton-Rapson update). Label this update as $\theta_2^{(r+1)}$.

vi. Repeat steps i. to v. until convergence of the GMM objective function.

each stock j denote with $J_t(j)$ the set of stocks that belong to j's industry. Next, for each exogenous dimension k in X_{jt}, we compute a metric of j's isolation along dimension k with respect to other stocks in $J_t(j)$:

$$X_{jt,k}^{GH} = \sum_{\tilde{j} \in J_t(j)} (X_{jt,k} - X_{\tilde{j}t,k})^2. \tag{34}$$

Then the vector of instrumental variables z_{jt} used for estimation will be composed of the exogenous characteristics in X_{jt} plus the Gandhi-Houde IVs (GH-IVs) constructed from such exogenous characteristics. If the dimensions considered to be exogenous with respect ξ_{jt} are in fact exogenous, then the GH-IVs would be uncorrelated with ξ_{jt} by construction, since they rely on the values of an exogenous characteristic for j and the corresponding values for other stocks in j's industry.

The case for relevance is more interesting. In our setup stocks are considered as differentiated investment products, and therefore they compete on the characteristics they offer to investors. Those stocks with more attractive characteristics to investors will have a relatively higher demand, all else equal. Then, metrics of j's isolation with respect to other stocks in $J_t(j)$ along an exogenous characteristic would capture stock j's ability to compete on such characteristic against alternative stocks in j's industry. If the alternatives of stock j in its industry offer more (less) of a characteristic positively value by investors relative to j, then there will be more (less) demand for alternatives of stock j, less (more) for stock j itself and that will decrease (increase) the price of stock j. Hence, metrics of stock j's isolation with respect to other stocks in $J_t(j)$ will be correlated with the price of stock j. In short, GH-IVs serve as relevant instruments for the price of a stock j because they proxy for demand shifts that occur when investors compare stock j with alternative stocks along an exogenous characteristic k.

Logit demand estimation. When the non-linear parameters θ_2 are restricted to zero, $\theta_2 = (\Pi, \Sigma) = (0, 0)$, we are in the case of logit demand. In this case the demand system given by $\tilde{s}_{jt}$ in (24) becomes:

$$\begin{aligned}\tilde{s}_{jt} &= \sum_{i=1}^{I_t}\left(\frac{A_{it}}{A_t}\right)\int \frac{\exp\left(\delta_{jt}+h_{ijt}(0,d_{it},v_{it})\right)}{1+\sum_{j=1}^{J_t}\exp\left(\delta_{jt}+h_{ijt}(0,d_{it},v_{it})\right)}dF_v(v_{it}).\\ &= \frac{\exp\left(\delta_{jt}\right)}{1+\sum_{j=1}^{J_t}\exp\left(\delta_{jt}\right)}\sum_{i=1}^{I_t}\left(\frac{A_{it}}{A_t}\right)\\ &= \frac{\exp\left(\delta_{jt}\right)}{1+\sum_{j=1}^{J_t}\exp\left(\delta_{jt}\right)},\end{aligned}$$

in the second line we used the fact that when $\theta_2 = 0$ then $h_{ijt} \equiv 0$, and in the third line we use the fact that wealth weights should sum up to one. In the case of logit demand we can perform the demand inversion analytically, since $\tilde{s}_{0t} = 1/(1+\sum_{j=1}^{J_t}\exp\left(\delta_{jt}\right))$ then:

$$\log\left(\tilde{s}_{jt}/\tilde{s}_{0t}\right) = \delta_{jt} = X'_{jt}\beta_0 + \xi_{jt},$$

the first equation tell us that if we set the value of δ_{jt} to $\log\left(s_{jt}/s_{0t}\right)$ then observed shares will match the (logit) model-implied shares. The second equation which is the definition of δ_{jt} tell us how to construct ξ_{jt} to use it in a GMM estimation strategy base on moment (27). Specifically, logit estimates will be obtained by linear IV-GMM based on (27) and using the Gandhi-Houde IVs described above.

4 Demand for Green Stocks

In this section we start by presenting the detailed mixed logit specification we'll use for estimation. Then we present the data sources and finally we present the demand estimation results.

Following the notation of equation (15), we estimate a demand specification given by

$$w_{it,j} = \frac{\exp\left(x'_{jt}b_0 + c_{jt}\gamma_{it} + \xi_{jt}\right)}{1+\sum_{j=1}^{J_t}\exp\left(x'_{jt}b_0 + c_{jt}\gamma_{it} + \xi_{jt}\right)}, \tag{35}$$

where the vector of return-related characteristics is given by

$$x_{jt} = (1,\ \text{mktBeta}_{jt},\ \text{lat}_{jt}, \text{lbme}_{jt},\ \text{profitability}_{jt},\ \text{investment}_{jt}),$$

and the six elements of x_{jt} correspond to: (i) an intercept, (ii) market beta (mktBeta_{jt}), (iii) log total assets (lat_{jt}), (iv) log book-to-market equity (lbme_{jt}), (v) profitability of the stock ($\text{profitability}_{jt}$), measured as operating profits to book equity, and (vi) a metric of investment for the company underlying the stock (investment_{jt}), measured as annual log growth of total assets. These return characteristics are motivated by the Fama-French five-factor model (Fama and French (2015)) that offer sensible dimensions to characterize the cross section of returns.[18]

The vector of taste characteristics c_{jt} is composed of the environmental scores of company j in t, $c_{jt} = (\text{Escore}_{jt})$. Moreover, the coefficients corresponding to the return-related characteristics are treated as homogeneous across investors.[19] The coefficient for environmental scores is modeled as heterogeneous across investors following the structure:

$$\gamma_{it} = \gamma_0 + \kappa d_{it} + \sigma v_{it}, \tag{36}$$

where the parameters $(\gamma_0, \kappa, \sigma)$ are common across investors but each investor has a different sensitivities to the environmenal scores of the stocks because they differ in their observed demographics d_{it}, as well as in their unobserved demographics, or taste shocks, v_{it}. In this demand specification we use the investor's assets under management as observed demographics, $d_{it} = \log(\text{AUM})_{it}$; and we assume unobserved demographics follow an standard normal distribution independent and identically distributed across investors. This distributional assumption is common practice in the demand estimation literature and facilitates the numerical approximation of the integral in the definition of model shares, $\tilde{s}_{jt}$ in (24). During estimation we approximate such integral using a Gauss-Hermite quadrature approximation of order 20.[20]

[18]There is a growing literature in the asset pricing questioning whether the Fama-French five-factor characteristics are sufficient to explain the cross section of returns (e.g. Han et al. (2021)). However, considering alternative returns characteristics other than those in the Fama-French five-factor is left for future research.

[19]Allowing for heterogeneity on return-related characteristics is left for future research.

[20]This guarantees the integral is exact for polynomial functions of degree up to 49.

4.1 Data

There are two main component of the data for our application: stock characteristics and portfolio holdings. Data on portfolio holdings comes from the Thomson Reuters Institutional Holdings Database that contains data on institutional investors that file the Form 13F from the Securities and Exchange Commission (SEC). Investment institutions that manage more than $100 million are required to disclose stock holdings in the Form 13F. These institutions can be banks, insurance companies, mutual, hedge, and pension funds, as well as other 13F institutions like foundations, nonfinancial corporations, and endowments.

Price and stock characteristics data for this project comes from the Compustat and Center for Research in Security Prices (CRSP) datasets which we combine to obtain fundamentals for publicly traded companies in the US stock market. Data for stock prices, dividends, returns and shares outstanding can be obtained from the CRSP Monthly Stock Database. Accounting data from the Compustat North America Fundamentals Annual and Quaterly Databases are combined with CRSP data to construct asset characteristics.

We focus on data for common stocks (with share codes 10, 11, 12, 18) that trade in the New York Stock Exchange, the American Stock Exchange and Nasdaq (exchange code 1, 2, 3 respectively); those stocks with missing data on returns or prices are filter out. Data from the CRSP database are merge with Compustat database records most recent of at least 6 months, and no more of 18 months prior to trading date. This is to guarantee that accounting data were public on the trading date.[21]

Data on environmental performance of listed companies comes from the MSCI rating agency.[22] MSCI is a pioneer rating agency in the construction of scores that evaluate the Environmental, Social, and Governance (ESG) performance of the firms they rate.[23]

[21]This follows the merging of databases as in KY2019.

[22]There is a growing number of data providers of ESG scores and a growing literature studying to what extend scores from different vendors are consistent in their evaluations (e.g. Billio et al. (2021)). Using data from other vendors would not alter the methodology here presented, but comparing how results would change when using different ESG scores is left for future research.

[23]Moreover, as mentioned in Pástor et al. (2022), MSCI has been voted "Best firm for SRI research" in the The Extel and SRI Connect Independent Research in Responsible Investment (IRRI) Survey each year from 2015 to 2019 (see `https://www.msci.com/zh/esg-ratings`).

This dataset contains firm-level annual ESG scores from 1991 to 2019.[24] We decide to work with MSCI ratings because of their long available sample, its broad coverage of publicly listed firms and the possibility to observed granular indicators that precede the construction of overall scores. In this dataset we observe series of individual performance indicators used to construct scores for each of the three pillars: E, S and G, as well as the combined ESG final score. For most vendors of ESG scores, the final score is the result of translating raw data into a numerical score using proprietary algorithms. Such methodology may change over time, complicating comparability of the scores over time. We opt to use the MSCI granular data to construct scores directly from the raw observed indicators to ensure comparability over time and ensure rated firms have the same treatment cross-sectionally.[25]

We focus on MSCI variables from the "Environmental pillar score", where firms are evaluated in several indicators that capture either positive or negative environmental performance. Positive indicators include appropriate waste management, product carbon footprint, and energy efficiency, while negative indicators include regulatory compliance, toxic emissions and waste, water stress, among others. See Appendix B for a full list of indicators.[26] Using these indicator variables on environmental performance we construct an E-score that compares firm cross-sectionally each year according to their environmental performance.

Figure 1 presents the evolution of the number of stocks and institutional investors from 2000-Q1 to 2020-Q4. Panel (b) shows that 13F institutions have grown in importance over the sample period. Towards the end of the sample institutional investor collectively managed around 70% of the US stock market from around 53% at the beginning of the sample. See Appendix B for the distribution of institutional investors according to their type over the sample period.

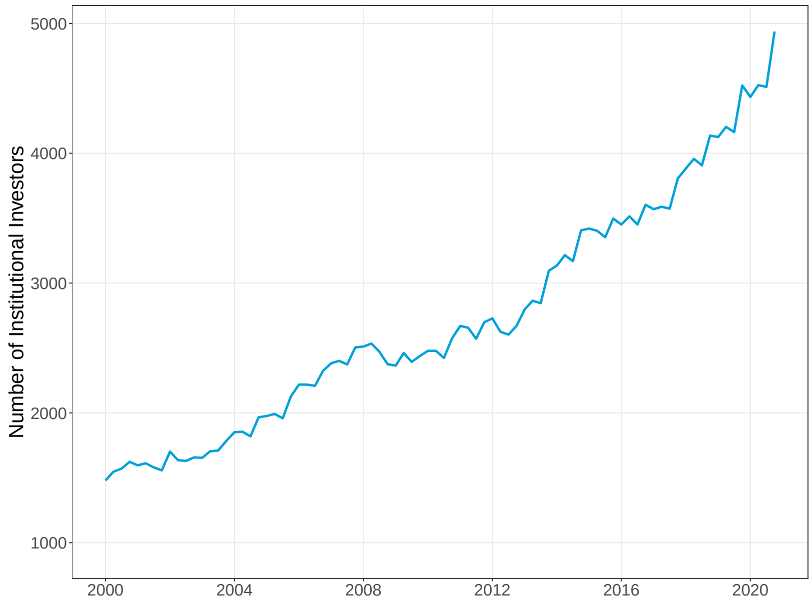
Moreover in each market, we construct a residual investor labeled as the *household sector*. The stock holdings of the household sector are defined as difference between

[24]Prior to 2010 these series were know as KLD scores. Following MSCI's acquisition of RiskMetrics in 2010, KLD scores were retooled into the MSCI KDL STATS series.

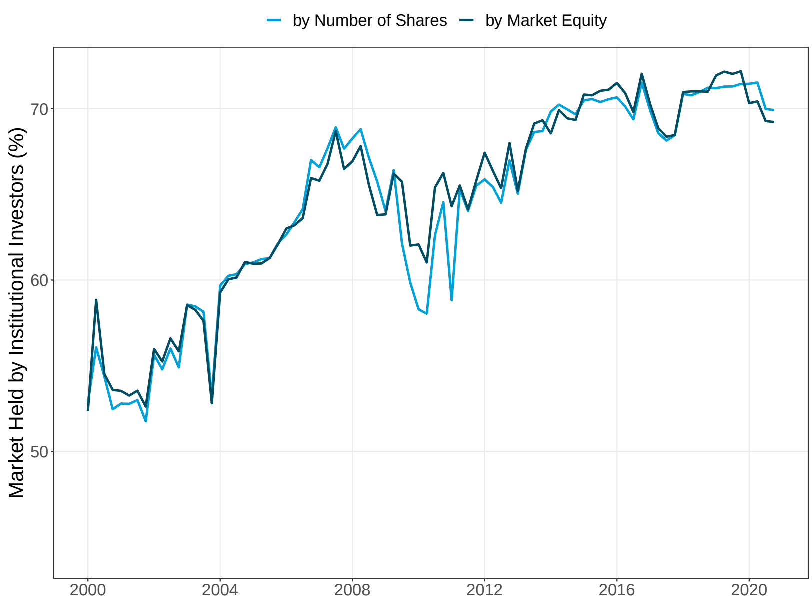
[25]In particular, the MSCI ESG KLD STATS series experienced several changes in coverage, indicators, and methodology several times after the year 2000. These changes make E, S and G pillar scores, as well as the aggregate ESG score hard to interpret consistently across vintages.

[26]The threshold for satisfying an indicator are determined by MSCI and are not disclosed with the data.

Figure 1: Institutional Stock Holdings



(a) Number of institutional investors



(b) Market held by institutional investors

Notes: Panel (a) shows the evolution on the number of institutional investors on a quarterly frequency from 2000-Q1 to 2020-Q4. Panel (b) shows the how much of the U.S. stock market is held by institutional investors from 2000-Q1 to 2020-Q1. We compute market shares according to number of shares and by market equity. By number of shares corresponds to the percentage of shares outstanding that is held by institutional investors across all stocks. Market shares according to market equity corresponds to the dollar value of stocks held by institutional investor as a percentage of the total market equity across all stocks.

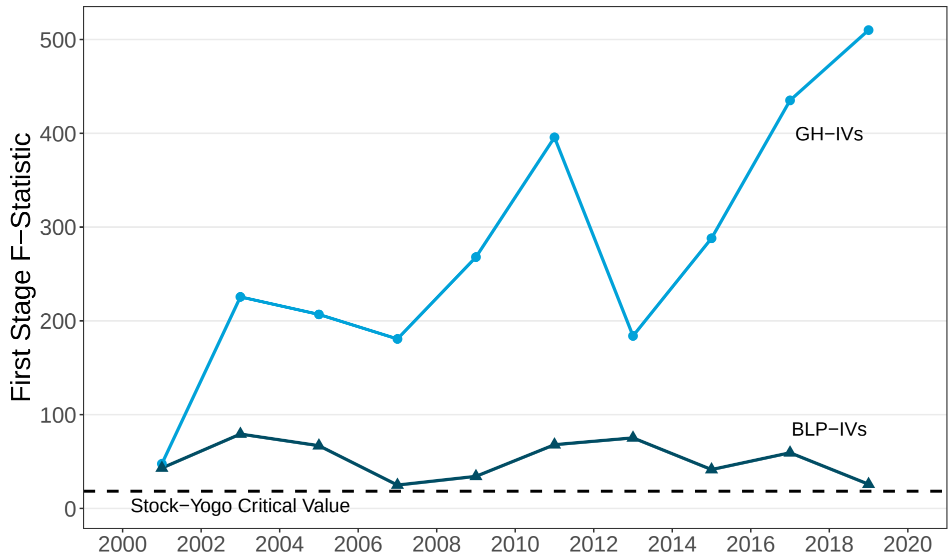
shares outstanding and the sum of shares held by 13F institutions. The introduction of the household sector is necessary for the market clearing to hold. The outside option, $j = 0$, will be set to include be stocks that are foreign (code 12), real estate investment trusts (code 18) or have missing characteristics or returns.

Data Construction. The construction of the return-related characteristics in the vector x_{jt} follows KY2019. It includes five characteristics: market beta, log total assets, log book-to-market equity, a metric of firm's profitability and a metric of firm's investment. Market beta for each stock j is compute on a 60-month rolling window where we the market return and the risk-free rate from the 3-month Treasury bill was obtained from Kenneth French's website.[27] Log book-to-market equity is computed as difference between log book equity and log market equity. We compute market equity by summing the log price per share at the end of the quarter with the log of shares outstanding expressed in millions. To compute the profitability metric we follow Fama and French (2015) and compute the ratio of operating profits to book equity. Operating profits are computed as total revenue (*revt*) minus the sum of cost of goods sold (*cogs*), selling, general and admin expenses (*xsga*), and interest and related expense-total (*xint*), or $profit = (revt - cogs - xsga - xint)$. The investment metric is the 1-year log growth of total assets. Table B1 in Appendix B shows summary statistics for the return characteristics in x_{jt} grouped by sample's year. To reduce the impact of outliers on some variables, profitability, market beta and investment are windsorized at the 2.5th and 97.5th percentile. This windsorizing is done in a quarterly fashion.

The environmental scores are constructed following the treatment of Engle et al. (2020) and Hong and Kostovetsky (2012). For each firm in the MSCI dataset we count the number of positive indicators and subtract from it the number of negative indicators, we can this difference raw E-scores. Then, after merging the raw score into the dataset of stock holdings and asset characteristics for each quarter, we rank the raw E-scores cross-sectionally and standardize to range in the interval between -1/2 and 1/2; this are the E-scores used for estimation. In this standardization the median raw score is mapped to zero, 1/2 corresponds to the stock with the highest environmental performance, that is, the *greenest* stock, and -1/2 corresponds with the lowest environmental performance, the *brownest* stock. The data on each firm on the MSCI dataset is updated at least once

[27] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Figure 2: First Stage F-stat of the instrumental variables



Notes: First stage F-statistic on instrumental variables for log Book to Market Equity. We present the F-statistic of regressing log book to market equity on the Gandhi-Houde IVs, as well as BLP-type IVs. Regressions are run over 2-year estimation windows ranging from 2000-Q1 to 2019-Q4. The dashed line corresponds to the Stock and Yogo (2005) critical value (18.37) for 1 endogenous regressor, 5 instrumental variables and 0.05 bias of two stage least squares relative to OLS.

a year, but not all firm scores get update at the same in a given year. To ensure E-scores are public on the trading date, we merge stock holdings and return characteristics in period t with the E-scores from the calendar year prior to t.[28]

For estimation, the return-related characteristics other that do not depend on price directly are assumed to exogenous (with respect to ξ_{jt}), that is the market beta, log total assets, profitability and investment. The E-score is also considered as exogenous. On these five characteristics we construct the Gandhi-Houde IVs described above, according to (34). Figure 2 shows the results of a first stage F-test for the null of weak instruments across the sample period. The figure also shows the F-statistics when using "BLP"-type instruments, another common choice of instrumental variables in the IO literature.[29] In all of the estimation windows the F-statistic of the GH-IVs is above the appropriate critical value to reject the null of weak instruments at 5 percent significance level. Moreover,

[28]For example, in 2018-Q1 and 2018-Q4 we use E-scores from 2017, whereas in 2019-Q1 we use E-scores from 2018.

[29]The BLP instruments for the price of stock j are constructed as the sum of the exogenous characteristics of other stocks in j's industry.

relatively to BLP-IVs, the null of weak IVs is rejected more easily using GH-IVs.

4.2 Estimates

Recall that in estimation the goal is to used data on stock characteristics $\{X_{jt} = (x'_{jt}, c'_{jt})'\}$, aggregate stock holdings $\{s_{jt}\}$, instrumental variables $\{Z_{jt}\}$, and assets under managament and demographics $\{A_{it}, d_{it}\}$ to estimate the parameters in θ, composed of linear parameters $\theta_1 = (b_0, \gamma_0)$ and non-linear parameters $\theta_2 = (\kappa, \sigma)$.

We use twenty years of quarterly data of stocks holdings and characteristics from 2000-Q1 to 2019-Q4. We perform estimation in two-year windows, so for every estimation window we obtain a set of parameter estimates $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ that use data from eight quarters.[30] Moreover during estimation we stardardize the variables log total assets, profitability, investment and E-scores to have mean zero and standard deviation one at each quarterly cross section of stocks. This way if we multiply the estimated coefficients by 100, we can be interpret them as approximately the semi-elasticity of a one-standard-deviation change in the corresponding stock characteristic.

Figure 3 shows the effective coefficient on E-scores, $\hat{\gamma}_{it} = \hat{\gamma}_0 + \hat{\kappa}\text{log(AUM)}_{it} + \hat{\sigma} v_{it}$, over 2-year estimation windows, and according to estimation based on logit demand or random coefficients demand (RC). The plot uses the mean value, in each window, of log assets under management and shows the 95% confidence interval, of how the coefficient on E-scores varies across investors based on the assumption that unobserved demographics, v_{it}, are normally distributed.[31]

From Figure 3 we can see that the sensitivity to E-scores varies over time but it is consistently positive throughout the estimation sample. This is true for both logit and mixed logit estimation. After the Great Recession (2007-Q4 to 2009-Q2) period there is an increase in the range of values for the coefficient on E-scores, due to larger esti-

[30] The first estimation window uses data from 2000-Q1 to a 2001-Q4, and the last estimation window uses data from 2018-Q1 to 2019-Q4.

[31] Appendix C includes additional figures where we show the effective coefficient on E-scores with alternative choices to the mean of log AUM in each estimation window. Figure C1 shows the effective coefficient also using the 25th and 75th percentile of log AUM in each estimation window. Figure C2 shows the effective coefficient on E-scores using the mean of log AUM in each quarter from 2000-Q1 to 2019-Q4. Both figures exhibit essentially the same features as Figure 3.

mated values for $\hat{\sigma}$. This suggests an increase in the heterogeneity in the sensitivity to green characteristics across investors after this period. One possible explanation is that after periods of economic downturn, some investors may be more interested in stocks with higher returns and relatively less interested in the environmental-friendliness of the companies underlying the stocks.

The range of values for the effective coefficient on E-scores after the Great Recession suggests that for some investors the sensitivity is consistent with a preference for brown stocks. For such investors, if there is a determinant of returns not captured by the return-related characteristics of the Fama-French five factor model which is higher for brown stocks, an appetite for returns could explain the preference for brown stocks in this period.[32]

From 2005 until the end of the sample, the mixed logit estimates on the coefficient for E-scores is increasing. For example, in the estimation window 2018-2019, the estimated semi-elasticity on the holdings of a stock after a 1 standard deviation increase in its E-score would result in a 73% increase in its holdings, compared with a 48% increase in the 2008-2009 estimation window.[33]

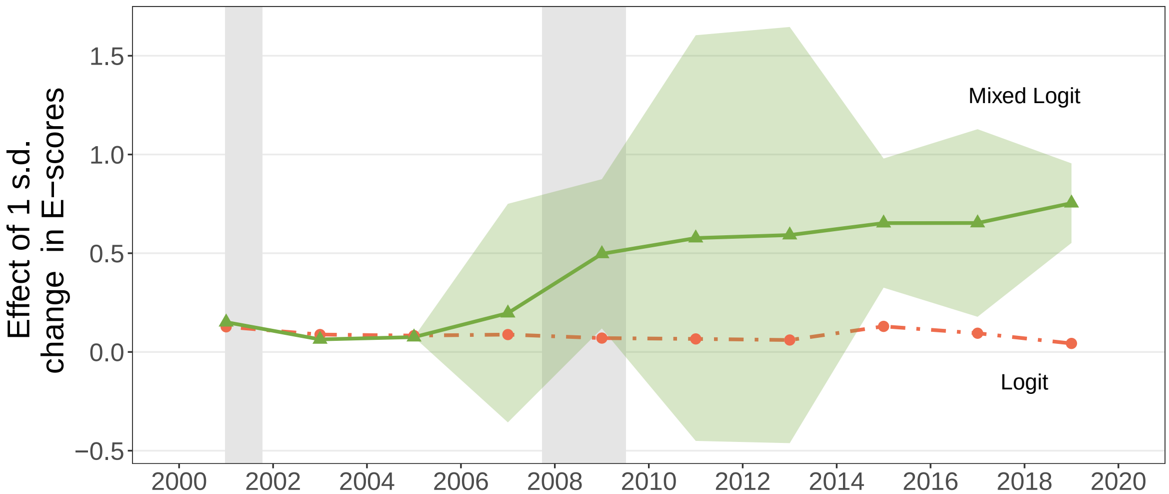
Figure 3 also shows the coefficient corresponding to E-scores if we perform estimation under a logit demand specification. In the logit case, there is no heterogeneity in the coefficient and all investors share the same sensitivity to the score, $\hat{\gamma}_{it} \equiv \hat{\gamma}_0^{logit}$ for all i. It is clear from the figure that the logit estimates exhibit much less variation and they don't increase in the second part of the sample.

Another takeaway from estimation is that the sensitivity to E-scores depends on the

[32] As presented in Pástor et al. (2021), brown stocks can have positive CAPM alphas and higher expected returns than green stocks because they are more exposed to climate risk. Similarly, Bolton and Kacperczyk (2021) find evidence for a carbon premium, in which companies with higher carbon emissions earned higher returns; they also provide evidence that such carbon premium cannot be explained entirely by traditional risk factors. Moreover, In a related study for mutual funds, Das et al. (2018) found that in the three year period after the Great Recession socially responsible mutual funds exhibited a negative and significant alpha with respect to the Fama-French five factor model. Moreover, they found that funds with lower ESG scores outperformed the fund with high ESG scores during this period.

[33] For example, the semi-elasticity of 73% implies that if a stock represents 1% of a portfolio, the increase of 1 standard deviation in the E-score would increase the holdings of that stock until it represents 1.73% of the portfolio.

Figure 3: Estimated coefficients for E-scores



Notes: This plot shows the effective coefficient corresponding to E-scores: $\hat{\gamma}_{it} = \hat{\gamma}_0 + \hat{\kappa}\log(\text{AUM})_{it} + \hat{\sigma}v_{it}$. The estimates $(\hat{\gamma}_0, \hat{\kappa}, \hat{\sigma})$ are obtained over 2-year estimation windows ranging from 2000-Q1 to 2019-Q4, and according to estimation based on logit demand or mixed logit demand. The case of logit demand corresponds to $\hat{\gamma}_{it} = \hat{\gamma}_0^{logit}$. Multiplying this effect by 100 approximates the semi-elasticity of portfolio weights with respect to a one-standard-deviation change in E-scores. The plot uses the mean value, in each window, of log assets under management, and shows the 95% confidence interval of how the coefficient on E-scores varies across investors based on the assumption that unobserved demographics, v_{it}, are normally distributed. Recession periods of the US economy are shown as shaded gray regions.

investor's assets under management. An alternative version of Figure 3 is presented in Appendix C. It shows the effective coefficient on E-scores plotted not only at the mean value but also at the 25th and 75th percentile of log assets under management in each estimation window. The main trends and features of the E-scores discussed above do not change by plotting the coefficient under various values of log assets under management; of course in a given estimation window the effective coefficient γ_{it} varies with the log assets under management of investor i according to the coefficient $\hat{\kappa}$. The average across estimation windows in the period 2010-2019 for $\hat{\kappa}$ is positive, so larger investors will are more sensitive to E-scores and will have higher demand for green stocks keeping other coefficients and stock characteristics fixed. This is consistent with Koijen et al. (2023) that find that large investors have a higher demand for stocks with higher environmental scores.[34] This is also consistent with the finding in Pastor et al. (2023) that larger

[34] The value of the average across estimation windows is 0.303. The magnitude of this average is not directly comparable with the one reported in Koijen et al. (2023) due to the type of data and demand specification they use.

Figure 4: Estimated coefficient for return-related characteristics

Notes: This plot shows the estimated coefficients corresponding to the return-related characteristics over 2-year estimation windows ranging from 2000-Q1 to 2019-Q4. Shaded regions represent 95% confidence intervals. Recession periods of the US economy are shown as vertical shaded gray regions.

Table 1: Example Estimated Price Elasticities

		Elasticities (%)	
Price Change	Portfolio Weight (%)	Logit	Mixed Logit
Apple	2.83	-0.5686	-0.5809
Alphabet	1.02	-0.0045	-0.0023
Exxon Mobile	1.02	-0.0045	-0.0013

Notes: Estimated elasticities of the aggregate holdings Apple with respect to the price change of selected stock prices. Data and estimates for 2019-Q2 under logit and mixed logit estimation.

investors tend to tilt their portfolio towards green stocks relative to smaller investors.

In estimation we also consider return-related characteristics in the demand for stocks. Figure 4 shows the coefficients corresponding to the return characteristics. For most periods, these characteristics have corresponding coefficients with the same sign. Characteristics like market beta have negative coefficients, which is consistent with the interpretation that market beta captures a basic dimension of risk and that risk is disliked by investors. An estimated negative coefficient for book-to-market equity suggests a preference for growth stocks. In periods of low interest rates, like following the Great Recession, growth stocks may be preferred by investors to value stocks and have larger equity valuations. The sensitivity to profitability and investment peak in periods where we also observe low sensitivity to market beta. This could correspond to a change in investor preferences valuing forward-looking aspects of the firms, such as profitability and investment, relatively more, and valuing backward-looking aspects of the firms, such as market beta, relatively less. Notably the 2008-2009 estimation window that includes the Great Recession period is where we observe the largest confidence intervals around most of the estimates. General uncertainty about stock market performance could be reflected in the relatively large standard errors for that period.

We also report an example of the estimated price elasticities. Table 1 shows the estimated price elasticties of the holdings of Apple according to the market value portfolio with respect to the price change of selected stocks. This example was chosen because in 2019-Q2, the market value portfolio has similar weights for Alphabet and Exxon Mobile, approximately 1.02%. As discussed in section 3, under logit demand, the cross price elasticities are proportional to portfolio weights; hence under a logit demand system, portfolio holdings of another technology stock would react identically to a price change from both stocks. This is precisely what the estimates for logit show; a 1% increase in the price of either Alphabet or Exxon Mobile lead to the same reduction in the holdings of Apple stock (-0.57%), despite Apple and Alphabet belong to the same industry while Apple and Exxon Mobile belong to different industries. On the other hand, the mixed logit estimates are flexible enough to show a larger degree of complementarity between Apple and Alphabet, than between Apple and Exxon Mobile.

5 Ban of Green Investing for Pension Funds

In this section we use the estimated demand for green stocks to study the effects of a ban of green investing for pension funds on aggregate holdings and equity prices. This counterfactual policy exercise is motivated by policy initiatives discussed in the US Senate at the beginning of 2023. On March 1st 2023, the US Senate passed a bill to prevent pension fund managers from basing investment decisions on factors like climate change (Morgan (2023)). The bill was eventually vetoed by President Biden 19 days later (Thomas (2023)) but many similar initiates have been approved in various US States legislatures.

To implement a ban on green investing for pension funds, we first identify the institutional investors that are pension funds and counterfactually make their demand for stocks perfectly inelastic to E-scores. To identify pension funds we use the classification of institutional investors from KY2019. This classification groups institutional investors into 6 categories: banks, insurance companies, mutual funds, pension funds, investment advisors (including hedge funds) and other institutions like foundations, nonfinancial corporations, and endowments. Once the pension funds have been identified, we define the following counterfactual demand curves for a stock j:

$$\tilde{w}_{it,j} = \begin{cases} \frac{\exp\left(\hat{\delta}_{jt} - \hat{\gamma}_0 c_{jt}\right)}{1+\sum_{j=1}^{J_t} \exp\left(\hat{\delta}_{jt} - \hat{\gamma}_0 c_{jt}\right)} & \text{if i is a pension fund} \\ \frac{\exp\left(\hat{\delta}_{jt} + h_{ijt}(\hat{\theta}_2, d_i, v_i)\right)}{1+\sum_{j=1}^{J_t} \exp\left(\hat{\delta}_{jt} + h_{ijt}(\hat{\theta}_2, d_i, v_i)\right)} & \text{otherwise.} \end{cases}$$

In these counterfactual demand curves pension funds are inelastic to E-scores, since in their counterfactual demand the non linear parameters are set to zero and the component corresponding to E-scores in $\hat{\delta}_{jt}$ is offset to zero. In this case pension funds adjust their demand curves as if they no longer value the portfolio profile from the environmental performance of the stocks they select. As a consequence of the demand change, prices observed in the data would not clear the market and we need to find counterfactual prices that clear the market. We rely on a market clearing condition (17) to find the new counterfactual prices. Recall that the market clearing conditions states that

$$ME_{jt} = \sum_{i=1}^{I} A_{it} w_{it,j},$$

and this condition can be expressed as a fixed point in the log vector of prices. Using the counterfactual demand curves, $\tilde{w}_{it}$, we solve for a vector of log prices p^c such that

$$p^c = f(p^c) := \log\left(\sum_{i=1}^{I} A_{it}\tilde{w}_{it}(p^c)\right) - log(S_t), \tag{37}$$

KY2019 show that a sufficient (but no necessary) condition for this fixed point to exist is that the coefficient accompanying prices has an absolute value less than one. We run the counterfactual exercise with data from the 2019-Q1 period; in this period the coefficient corresponding to prices, which is the coefficient on log book-to-market equity, is -0.443 and therefore it satisfies the sufficient condition for a fixed point to exist.

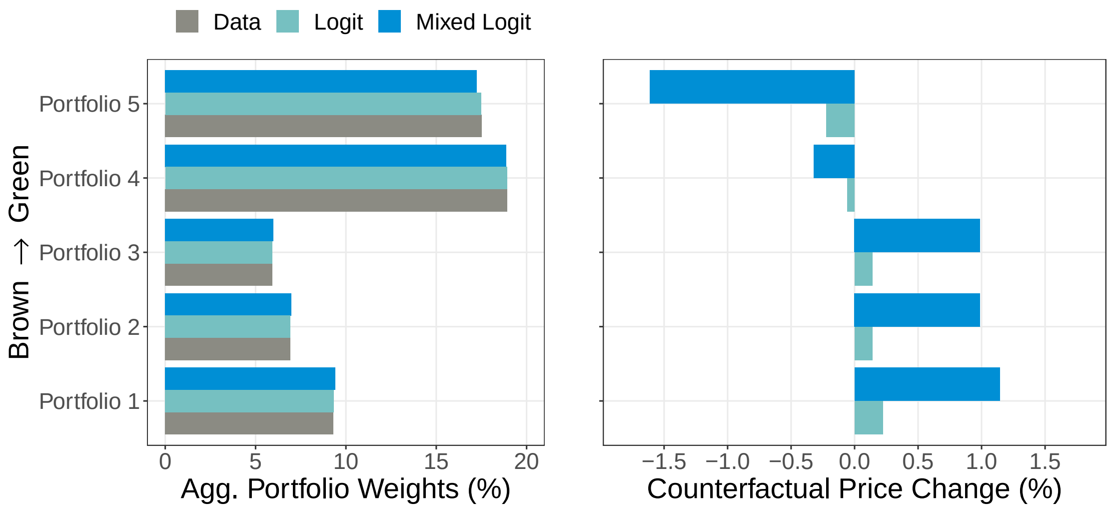
In computing counterfactual prices that satisfy the market clearing condition we assume the following. First, the number of shares outstanding of each stock is assumed to be fixed, so we have an inelastic supply of the stocks and price changes are determined by demand shifts. Second, the assets under management for each investor A_i is also assumed to remain constant during the counterfactual. That is each investor still decides how to allocate A_{it} dollars into the available stock given that prices change to counterfactual prices and in the case of pension funds they are now inelastic to the E-scores. Third, is it assumed that the coefficients associated to return-related characteristics are fixed as well as the coefficient on E-scores for institutional investors other than pension funds. Fourth, the estimated unobserved stock characteristics, $\hat{\xi}_{jt}$ is also assumed be remain constant during the counterfactual. It can be argued that for the last two assumptions a Lucas (1976) critique applies to the extent that coefficients on return characteristics and unobserved stock characteristics change with the policy. This critique apply to most counterfactual exercises in the asset demand literature and exploring ways to circumvent the critique is left for future research.

Figure 5 shows the results of the counterfactual exercise using data and estimates for 2019-Q1. The portfolios in the figure were constructed by sorting stocks by their E-score, and grouping them according to quintiles. Portfolio 1 contains the 20% of stocks with lowest E-scores, the most brown stocks, while Portfolio 5 contains the 20% of stocks with highest E-scores, the most green stocks. The left panel shows aggregate portfolio weights for each portfolio, that is the sum of the market shares for the stocks contained in each portfolio. We show the aggregate portfolio weights in three cases: first, those observed in the data. Second, under the counterfactual policy using the logit demand specification and estimates; and finally, under the counterfactual policy using the mixed logit demand specification and estimates. Results show that the aggregate holdings in the data are very similar to the counterfactual holdings. This is specially true for the logit demand case, however, compared to the counterfactual holdings under the mixed logit demand we see larger differences. Portfolio 5, which is composed of the stocks with the highest E-scores, shows a reduction in aggregate holdings under the policy, whereas Portfolios 1, 2, and 3 increase their aggregate holdings. This means that the relative importance of the stocks in Portfolio 5 diminished while the relative importance of the stocks of portfolios 1, 2 and 3 increased. This suggests that with the policy the aggregate investment share on green stocks was reduced in favor of brown stocks.

The right panel of Figure 5 shows the value-weighted average price change for the stocks in each portfolio. These price changes compare the prices observed in the data in 2019-Q1 with the counterfactual prices using both the mixed logit demand specification as well as the logit demand specification. Results show that the changes under a logit demand are much smaller than in the mixed logit demand case, this is due to the restrictive elasticities of logit demand where, as mentioned before, own-price elasticities are proportional to market shares. The results for mixed logit demand show that Portolio 5 experienced the most negative change, with an average counterfactual price change of -1.6%, while Portfolio 1 exhibited the biggest positive change with an average counterfactual change of 1.1%.[35] These result have to keep a consistency with the changes in aggregate shares, the price of green stocks will decrease under the new counterfactual prices that clear the market, because there is less demand for green stocks, but the

[35] The corresponding changes for logit demand are -0.2% for Portfolio 5 and 0.2% for Portfolio 1.

Figure 5: Counterfactual holdings and price changes of E-score-based portfolios



Notes: This figure shows the effect of a ban of green investing for pension funds on aggregate holdings and equity prices in a counterfactual exercise using data and estimates for 2019-Q1. The portfolios in the figure were constructed by sorting stocks by their E-score, and grouping them according to quintiles. Portfolio 1 contains the 20% of stocks with lowest E-scores, while Portfolio 5 contains the 20% of stocks with highest E-scores. The left panel shows the sum of the market shares for the stocks contained in each portfolio. It shows the aggregate portfolio weights observed in the data, under the counterfactual policy using estimates from a mixed logit demand specification and using estimates of a logit demand specification. The right panel shows the value-weighted average price change for each portfolio comparing the prices observed in the data with the counterfactual prices under mixed logit demand and under logit demand.

decrease will happen up to a point where the reduction in price no longer encourages more demand of the green stocks and the market clears. From the right panel of Figure 5, as with aggregate shares, the prices of green stocks decrease the most while the price of brown stocks increased with the policy.

The magnitude of the price changes in Figure 5 are commensurate to price changes observed in the data. For example, in the quarter following the data use for the counterfactual, the value-weighted price change of the stocks in Portfolio 1 between 2019-Q1 and 2019-Q2 was 1.1%; a table version of Figure 5 showing the price change over this period for all portfolios is included in Appendix C. Similarly, as studied in Rudebusch et al. (2023), policy announcements that substantially affect green and brown stocks can trigger price changes on stock indices of brown and green stocks ranging from -2% to 11% in a manner of days, as in the case of the Inflation Reduction Act approved in 2022.

6 Conclusions

This paper combines a traditional portfolio construction problem with demand estimation techniques to estimate the demand for green stocks of US institutional investors. In the framework presented, both belief and taste heterogeneity play a role. In addition to investor heterogeneity through differential beliefs about future returns, our framework allows for investors to care about the characteristics of the portfolio they are forming beyond those characteristics related directly to an expected return-versus-risk trade off. We use this framework to measure the preference for green stocks while considering return-related stock characteristics.

For estimation this paper uses a mixed logit demand specification in contrast with the logit demand specification more commonly used in the recent asset demand literature. In a logit demand specification, price elasticities are proportional to portfolio shares which restricts the substitution patterns between assets. In a mixed logit demand specification, investor heterogeneity is captured by investor-specific coefficients that are modeled as functions of investor demographics. This richer investor-level heterogeneity delivers more flexible substitution patterns between assets, and it is the modern workhorse model of demand estimation in the IO literature. By doing estimation at the market-level we can not only implement the mixed logit demand specification, but it facilitates dealing with the endogeniety of prices. Specifically, this allows us to consider instrumental variables for prices based on demand shifters that have been studied in the IO literature, such as BLP-type instruments or Gandhi-Houde price instruments, which have not been used in the asset demand literature.

The empirical exercise uses quarterly data on the stock holdings of institutional investors in the US. We pair this holdings data with return-related characteristics inspired by the Fama-French five factor model and data on the environmental performance of the listed companies in the form of E-scores. We find that the revealed taste for green stocks fluctuates over time. For both logit and mixed logit demand estimation, we find a positive taste for green stocks throughout the estimation sample. However, for mixed logit estimation the semi-elasticity for E-scores increases in the second part of the sample. Moreover, in the period after the Great Recession (2007-Q4 to 2009-Q2) there is an increase in the range of values for the coefficient on E-scores, showing an increase in the heterogeneity in the sensitivity to green characteristics across investors.

In a counterfactual exercise, we use the estimated demand system for stocks to study the effects on equity prices and aggregate holdings of a ban on green investing for pension funds. Using the data and estimates for 2019-Q1, we find that brown stocks will benefit the most in terms of counterfactual pricing. A portfolio with the bottom quintile of green stocks is estimated to have an associated average price change of 1.1% under the counterfactual, while the top quintile portfolio has an average price change of -1.6%.

Future work. Three avenues of work are left for future research. The first one deals with nontraditional stock characteristics. There is a great amount of textual information about listed companies that can be informative to investors. It is possible to extract "topics of risk" by applying a topic model to the text of regulatory risk fillings from listed companies (see, e.g. Lopez-Lira (2023)) and using the corresponding topic loadings as a stock characteristic in the demand curves of investors. This would help enrich the demand model with nontraditional but sensible characteristics related to risk. Related work includes Lopez-Lira and Roussanov (2023) that explores whether traditional common factors are enough to explain the cross section of returns.

A second avenue of future research lies at the intersection of asset pricing and corporate finance. As discussed in Brunnermeier et al. (2021) asset demand curves are flexible enough to include firm characteristics such as leverage, innovation, investment, and payout policies as relevant features signaling growth expectations and risks associated with future cash flows. Adding a model of firm corporate policies would complement asset demand systems with models of corporate decision making.

Third, dynamic considerations are at the very frontier of the asset demand estimation literature. Time-conditional statements in asset pricing are paramount and explicitly modeling the time dimension in the demand for stocks would be an important contribution to the literature. Over time portfolio optimization in one period is directly related to the stocks held in the previous period, calling for *inventory* considerations when modeling asset demand curves over time. Modeling asset demand dynamically also requires understanding how asset characteristics evolve over time, how investor funds flow in and out of the stock market, and how investor beliefs update over time. All these issues make dynamic asset demand challenging yet exciting for future research.

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Appendices

A Proofs and Mathematical Derivations

A.1 Proof of Proposition 1

Proof. The following proof follows the main steps of Lemma 1 proof's in KY2019 with the adaptation for the more general objective function. The function inside the conditional expectation in (1) takes the form

$$F_i(A_{i,t+1}, C_{it}, w_{it}) = \log(A_{i,t+1}) + a_i'C_t'w_{it}$$

we can replace the first term with

$$\log(A_{i,t+1}) = \log(A_{it}) + \log\left(\frac{A_{i,t+1}}{A_{it}}\right) = \log(A_{it}) + \log\left(R^0_{t+1} + w_{it}'(R_{t+1} - R^0_{t+1}\mathbf{1})\right)$$

by using the budget constraint (2). Then the Lagrangian for the problem is given by

$$\begin{aligned} L(w_{it}, \Lambda_{it}, \lambda_{it}) = \log(A_{it}) + E_{it}\left[\log\left(R^0_{t+1} + w_{it}'(R_{t+1} - R^0_{t+1}\mathbf{1})\right)\right] \\ + a_i'C_t'w_{it} + \Lambda_{it}'w_{it} + \lambda_{it}(1 - \mathbf{1}'w_{it}), \end{aligned}$$

the first order condition with respect to w_{it} given by

$$\begin{aligned} &E_{it}\left[\left(R^0_{t+1} + w_{it}(R_{t+1} - R^0_{t+1}\mathbf{1})\right)^{-1}\left(R_{t+1} - R^0_{t+1}\mathbf{1}\right)'\right] + \Lambda_{it}' - \lambda_{it}\mathbf{1}' + a_i'C_t' = 0 \\ &\Rightarrow E_{it}\left[\left(\frac{A_{i,t+1}}{A_{it}}\right)^{-1}\left(R_{t+1} - R^0_{t+1}\mathbf{1}\right)\right] = -\left(\Lambda_{it} - \lambda_{it}\mathbf{1} + C_t a_i\right). \qquad \text{(A1)} \end{aligned}$$

Multiplying this equation by $-\mathbf{1}w_{it}'$ yields:

$$\begin{aligned} -E_{it}\left[\left(\frac{A_{i,t+1}}{A_{it}}\right)^{-1}\mathbf{1}w_{it}'\left(R_{t+1} - R^0_{t+1}\mathbf{1}\right)\right] = \mathbf{1}w_{it}'\left(\Lambda_{it} - \lambda_{it}\mathbf{1} + C_t a_i\right) \\ \Rightarrow -E_{it}\left[\left(\frac{A_{i,t+1}}{A_{it}}\right)^{-1}\left(\frac{A_{i,t+1}}{A_{it}} - R^0_{t+1}\right)\mathbf{1}\right] = \mathbf{1}w_{it}'\left(\Lambda_{it} - \lambda_{it}\mathbf{1} + C_t a_i\right) \end{aligned}$$

using the intertemporal budget constraint. Summing the last expression with (A1) results in Euler equation in Proposition 1:

$$E_{it}\left[\left(\frac{A_{it+1}}{A_{it}}\right)^{-1}R_{t+1}\right]=\mathbf{1}-(I-\mathbf{1}w'_{it})\left(\Lambda_{it}-\lambda_{it}\mathbf{1}+C_t a_i\right).$$

Next, using the intertemporal budget constraint we can write the objective function of the investor problem as:

$$\log(A_{it})+E_{it}\left[\log\left(R^0_{t+1}+w'_{it}(R_{t+1}-R^0_{t+1}\mathbf{1})\right)\right]+a'_iC'_tw_{it}.$$

Let R^p_{t+1} denote the gross return of the portfolio with weights w_{it}, then

$$R^p_{t+1}=R^0_{t+1}+w'_{it}(R_{t+1}-R^0_{t+1}\mathbf{1}),$$

and the log excess return of the portfolio weith respect to the outside option gross return is given by

$$r^p_{t+1}-r^0_{t+1}=\log\left(\frac{R^p_{t+1}}{R^0_{t+1}}\right)=\log\left(1+w'_{it}\left(\exp(r_{t+1}-r^0_{t+1}\mathbf{1})-\mathbf{1}\right)\right).$$

Now consider the function $g(x):\mathbb{R}^J\to\mathbb{R}$ given by $g(x)=\log(1+w'(\exp(x)-\mathbf{1}))$, where the $\exp(\cdot)$ applies entry-by-entry, and w is a J-vector constant, then a second order Taylor approximation of g around $x_0=0\in\mathbb{R}^J$ is given by

$$g(x)\approx w'\left(x+\frac{1}{2}x\odot x\right)-\frac{1}{2}w'\left(xx'\right)w,$$

where $\odot$ stands for entry-by-entry multiplication. Applying this approximation to the expression for $r^p_{t+1}-r^0_{t+1}$ yields

$$\begin{aligned}r^p_{t+1}-r^0_{t+1}\approx w'_{it}&\left((r_{t+1}-r^0_{t+1})+\frac{1}{2}(r_{t+1}-r^0_{t+1})\odot(r_{t+1}-r^0_{t+1})\right)\\&-\frac{1}{2}w'_{it}(r_{t+1}-r^0_{t+1})(r_{t+1}-r^0_{t+1})'w_{it}.\end{aligned}$$

Next we apply the expectations operator $E_{it}[\cdot]$ and the second term in the objective function can be approximated by

$$
\begin{aligned}
E_{it}\left[\log\left(R^0_{t+1} + w'_{it}(R_{t+1} - R^0_{t+1}\mathbf{1})\right)\right] &\approx r^0_{t+1} + w'_{it}\left(E_{it}[r_{t+1} - r^0_{t+1}] + \frac{\tilde{\sigma}^2_{it}}{2}\right) - \frac{w'_{it}\tilde{\Sigma}_{it}w_{it}}{2} \\
&= r^0_{t+1} + w'_{it}\tilde{\mu}_{it} - \frac{w'_{it}\tilde{\Sigma}_{it}w_{it}}{2},
\end{aligned}
$$

this approximation replaces $E_{it}[(r_{t+1} - r^0_{t+1})(r_{t+1} - r^0_{t+1})']$ with $\tilde{\Sigma}_{it}$ and $E_{it}[(r_{t+1} - r^0_{t+1}) \odot (r_{t+1} - r^0_{t+1})]$ with $\tilde{\sigma}^2_{it}$, and follows from the one presented in Campbell and Viceira (2002) (Eq. 2.23). With this approximation the first order condition for becomes

$$
\begin{aligned}
&\tilde{\mu}_{it} - \tilde{\Sigma}_{it}w_{it} + \Lambda_{it} - \lambda_{it}\mathbf{1} + C_t a_i = 0 \\
&\quad \Rightarrow w_{it} = \tilde{\Sigma}^{-1}_{it}\left(\tilde{\mu}_{it} + \Lambda_{it} - \lambda_{it}\mathbf{1} + C_t a_i\right),
\end{aligned}
$$

Partition the asset space between those with non-binding short sale constraint and those binding we write $\Lambda'_{it} = [0' \quad \Lambda^{(2)'}_{it}]$ and $(C_t a_{it}) = [(C_t a_i)'_1 \quad (C_t a_i)'_2]'$, then using the partitions for $\tilde{\Sigma}_{it}$ and $\tilde{\mu}_{it}$ we have that

$$
w_{it} = \begin{pmatrix} w^{(1)}_{it} \\ 0 \end{pmatrix} = \begin{pmatrix} \Sigma_{it} & \Sigma^{(1,2)}_{it} \\ \Sigma^{(2,1)}_{it} & \Sigma^{(2,2)}_{it} \end{pmatrix}^{-1} \left(\begin{pmatrix} \mu_{it} \\ \mu^{(2)}_{it} \end{pmatrix} + \begin{pmatrix} 0 \\ \Lambda^{(2)}_{it} \end{pmatrix} - \lambda_{it}\mathbf{1} + \begin{pmatrix} (C_t a_i)_1 \\ (C_t a_i)_2 \end{pmatrix}\right).
$$

The inverse of $\tilde{\Sigma}_{it}$ is given by

$$
\tilde{\Sigma}^{-1}_{it} = \begin{pmatrix} \left(\Sigma_{it} - \Sigma^{(1,2)}_{it}\Sigma^{(2,2)-1}_{it}\Sigma^{(2,1)}_{it}\right)^{-1} & -\Sigma^{-1}_{it}\Sigma^{(1,2)}_{it}\left(\Sigma^{(2,2)}_{it} - \Sigma^{(1,2)}_{it}\Sigma^{-1}_{it}\Sigma^{(2,1)}_{it}\right)^{-1} \\ -\Sigma^{(2,2)-1}_{it}\Sigma^{(2,1)}_{it}\left(\Sigma_{it} - \Sigma^{(1,2)}_{it}\Sigma^{(2,2)-1}_{it}\Sigma^{(2,1)}_{it}\right)^{-1} & \left(\Sigma^{(2,2)}_{it} - \Sigma^{(1,2)}_{it}\Sigma^{-1}_{it}\Sigma^{(2,1)}_{it}\right)^{-1} \end{pmatrix},
$$

then w_{it} becomes

$$
\begin{pmatrix} w^{(1)}_{it} \\ 0 \end{pmatrix} = \begin{pmatrix} \left(\Sigma_{it} - \Sigma^{(1,2)}_{it}\Sigma^{(2,2)-1}_{it}\Sigma^{(2,1)}_{it}\right)^{-1}(\mu_{it} - \lambda_{it}1 + (C_t a_i)_1) - \Sigma^{-1}_{it}\Sigma^{(1,2)}_{it}\left(\Sigma^{(2,2)}_{it} - \Sigma^{(1,2)}_{it}\Sigma^{-1}_{it}\Sigma^{(2,1)}_{it}\right)^{-1}\left(\mu^{(2)}_{it} + \Lambda^{(2)}_{it} - \lambda_{it}1 + (C_t a_i)_2\right) \\ -\Sigma^{(2,2)-1}_{it}\Sigma^{(2,1)}_{it}\left(\Sigma_{it} - \Sigma^{(1,2)}_{it}\Sigma^{(2,2)-1}_{it}\Sigma^{(2,1)}_{it}\right)^{-1}(\mu_{it} - \lambda_{it}1 + (C_t a_i)_1) + \left(\Sigma^{(2,2)}_{it} - \Sigma^{(1,2)}_{it}\Sigma^{-1}_{it}\Sigma^{(2,1)}_{it}\right)^{-1}\left(\mu^{(2)}_{it} + \Lambda^{(2)}_{it} - \lambda_{it}1 + (C_t a_i)_2\right) \end{pmatrix}.
$$

We can multiply the second block by $\Sigma^{-1}_{it}\Sigma^{(1,2)}_{it}$ and sum both blocks to obtain

$$
\begin{aligned}
w^{(1)}_{it} &= \left(I - \Sigma^{-1}_{it}\Sigma^{(1,2)}_{it}\Sigma^{(2,2)-1}_{it}\Sigma^{(2,1)}_{it}\right)\left(\Sigma_{it} - \Sigma^{(1,2)}_{it}\Sigma^{(2,2)-1}_{it}\Sigma^{(2,1)}_{it}\right)^{-1}(\mu_{it} - \lambda_{it}\mathbf{1} + (C_t a_i)_1) \\
&= \Sigma^{-1}_{it}(\mu_{it} - \lambda_{it}\mathbf{1} + (C_t a_i)_1).
\end{aligned}
$$

So following the notation, for the optimal positive weights on the investor's problem can be approximated by

$$w_{it} \approx \Sigma_{it}^{-1} \left(\mu_{it} - \lambda_{it} \mathbf{1} + (C_t a_i)_1 \right).$$

To pin down the value of λ_{it}, notice that when constraint (3) is binding then

$$\mathbf{1}' w_{it} = \mathbf{1}' \Sigma_{it}^{-1} \left(\mu_{it} - \lambda_{it} \mathbf{1} + C_t a_i \right) = 1,$$

then

$$\lambda_{it} = \frac{\max\{\mathbf{1}' \Sigma_{it}^{-1} \left(\mu_{it} + C_t a_i \right) - 1, 0\}}{\mathbf{1}' \Sigma_{it}^{-1} \mathbf{1}}.$$

□

A.2 Derivation of investor-specific posterior moments

Proof. We have that

$$s_{it} | (r_{t+1}^x, \sigma_{\varepsilon,i}^2) \sim N(r_{t+1}^x, \sigma_{\varepsilon,i}^2 I)$$
$$r_{t+1}^x \sim N(\mu_{xt}, \Sigma_{xt}).$$

The pdfs of these distributions are given by:

$$p(s_{it} | r_{t+1}^x, \sigma_{\varepsilon,i}^2) = (2\pi)^{-J_t/2} det(\sigma_{\varepsilon,i}^2 I)^{-1/2} \exp\left[-\frac{1}{2} (s_{it} - r_{t+1}^x)' (\sigma_{\varepsilon,i}^2 I)^{-1} (s_{it} - r_{t+1}^x) \right]$$
$$p(r_{t+1}^x) = (2\pi)^{-J_t/2} det(\Sigma_{xt})^{-1/2} \exp\left[-\frac{1}{2} (r_{t+1}^x - \mu_{xt})' \Sigma_{xt}^{-1} (r_{t+1}^x - \mu_{xt}) \right].$$

By Bayes theorem $p(r^x_{t+1}|s_{it}, \sigma^2_{\varepsilon,i}) \propto p(s_{it}|r^x_{t+1}, \sigma^2_{\varepsilon,i})p(r^x_{t+1})$ and

$$\begin{aligned}
\log(p(r^x_{t+1}|s_{it}, \sigma^2_{\varepsilon,i})) &= -\frac{1}{2}(s_{it} - r^x_{t+1})'(\sigma^2_{\varepsilon,i}I)^{-1}(s_{it} - r^x_{t+1}) \\
&\quad -\frac{1}{2}(r^x_{t+1} - \mu_{xt})'\Sigma^{-1}_{xt}(r^x_{t+1} - \mu_{xt}) + cons \\
&= -\frac{1}{2}(r^x_{t+1})'(\sigma^2_{\varepsilon,i}I)^{-1}(r^x_{t+1}) + (r^x_{t+1})'(\sigma^2_{\varepsilon,i}I)^{-1}(s_{it}) \\
&\quad -\frac{1}{2}(r^x_{t+1})'\Sigma^{-1}_{xt}(r^x_{t+1}) - \frac{1}{2}(r^x_{t+1})'\Sigma^{-1}_{xt}(\mu_{xt}) + cons \\
&= -\frac{1}{2}(r^x_{t+1})'[(\sigma^2_{\varepsilon,i}I)^{-1} + \Sigma^{-1}_{xt}](r^x_{t+1}) + (r^x_{t+1})'[(\sigma^2_{\varepsilon,i}I)^{-1}s_{it} + \Sigma^{-1}_{xt}\mu_{xt}] + cons \\
&= -\frac{1}{2}\left(r^x_{t+1} - [(\sigma^2_{\varepsilon,i}I)^{-1} + \Sigma^{-1}_{xt}]^{-1}[(\sigma^2_{\varepsilon,i}I)^{-1}s_{it} + \Sigma^{-1}_{xt}\mu_{xt}]\right)' \\
&\quad \cdot [(\sigma^2_{\varepsilon,i}I)^{-1} + \Sigma^{-1}_{xt}]^{-1}\left(r^x_{t+1} - [(\sigma^2_{\varepsilon,i}I)^{-1} + \Sigma^{-1}_{xt}]^{-1}[(\sigma^2_{\varepsilon,i}I)^{-1}s_{it} + \Sigma^{-1}_{xt}\mu_{xt}]\right) + cons.
\end{aligned}$$

This is the pdf of a multivariate normal distribution with variance $\Sigma_{r|s_i}$ and mean $\mu_{r|s_i}$ given by

$$\begin{aligned}
\Sigma_{r|s_i} &= [(\sigma^2_{\varepsilon,i})^{-1}I + \Sigma^{-1}_{xt}]^{-1} \\
\mu_{r|s_i} &= \Sigma_{r|s_i}[(\sigma^2_{\varepsilon,i})^{-1}s_{it} + \Sigma^{-1}_{xt}\mu_{xt}].
\end{aligned}$$

□

A.3 Proof of Proposition 2

Proof. Recall that from the derivation of posterior moments in (10) we have that

$$\Sigma_{it} = \Sigma_{r|s_i} = ((\sigma^2_{\varepsilon,i})^{-1}I + \Sigma^{-1}_{xt})^{-1},$$

and from assumption 2 the term Σ_{xt} is given by $\Sigma_{xt} = (\Gamma_{xt}\Gamma'_{xt} + \sigma^2_e I)$. We can use the Woodbury matrix identity to obtain the inverse of Σ_{xt}:

$$\begin{aligned}
\Sigma^{-1}_{xt} &= [\Gamma_{xt}\Gamma'_{xt} + \sigma^2_e I]^{-1} \\
&= \frac{1}{\sigma^2_e}\left(I - \frac{\Gamma_{xt}\Gamma'_{xt}}{\sigma^2_e + \Gamma'_{xt}\Gamma_{xt}}\right),
\end{aligned}$$

and substituting into the expression for Σ_{it} yields

$$\begin{aligned}\Sigma_{it}^{-1} &= \left(\frac{1}{\sigma_{\varepsilon,i}^2} + \frac{1}{\sigma_e^2}\right) I - \frac{1}{\sigma_e^2(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})}\Gamma_{xt}\Gamma_{xt}' \\ &= \left(\frac{\sigma_e^2 + \sigma_{\varepsilon,i}^2}{\sigma_{\varepsilon,i}^2\sigma_e^2}\right) I - \frac{1}{\sigma_e^2(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})}\Gamma_{xt}\Gamma_{xt}' \\ &= \frac{1}{\sigma_e^2(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})}\left[\frac{(\sigma_e^2 + \sigma_{\varepsilon,i}^2)(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})}{\sigma_{\varepsilon,i}^2} I - \Gamma_{xt}\Gamma_{xt}'\right],\end{aligned}$$

now let's define $\delta_{it} = (\sigma_{\varepsilon,i}^2)^{-1}(\sigma_e^2 + \sigma_{\varepsilon,i}^2)(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})$ then

$$\Sigma_{it}^{-1} = \frac{1}{\sigma_e^2(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})}[\delta_i I - \Gamma_{xt}\Gamma_{xt}']$$

and using the Woodbury matrix identity again we have:

$$\begin{aligned}\Sigma_{it} &= \sigma_e^2(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})[\delta_{it} I - \Gamma_{xt}\Gamma_{xt}']^{-1} \\ &= \sigma_e^2(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})\frac{1}{\delta_{it}}\left[I + \frac{\Gamma_{xt}\Gamma_{xt}'}{\delta_{it} - \Gamma_{xt}'\Gamma_{xt}}\right] \\ &= \frac{\sigma_{\varepsilon,i}^2\sigma_e^2}{\sigma_e^2 + \sigma_{\varepsilon,i}^2} I + \frac{\sigma_e^2(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})}{\delta_{it}(\delta_{it} - \Gamma_{xt}'\Gamma_{xt})}\Gamma_{xt}\Gamma_{xt}' \\ &:= \iota_{it} I + \Gamma_{it}\Gamma_{it}'\end{aligned}$$

with

$$\begin{aligned}\iota_{it} &:= \frac{\sigma_{\varepsilon,i}^2\sigma_e^2}{\sigma_e^2 + \sigma_{\varepsilon,i}^2} \\ \Gamma_{it} &:= \left[\frac{\sigma_e^2(\sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})}{\delta_{it}(\delta_{it} - \Gamma_{xt}'\Gamma_{xt})}\right]^{1/2}\Gamma_{xt} \\ &= \left[\frac{\sigma_{\varepsilon,i}^4}{(\sigma_{\varepsilon,i}^2 + \sigma_e^2)(\sigma_{\varepsilon,i}^2 + \sigma_e^2 + \Gamma_{xt}'\Gamma_{xt})}\right]^{1/2}\Gamma_{xt}\end{aligned}$$

□

A.4 Proof of Proposition 3

Proof. **Part (i)**

We start by showing that Γ_{it} can be written as a polynomial function on x_{jt} with investor-specific coefficients. From the proof of proposition 2 we have that the j-th entry of Γ_{it} is given by:

$$\Gamma_{it,j} = k_{it}\Gamma_{xt,j} := y'_{jt}\Phi_{\Gamma,i} + \phi_{\Gamma,i},$$

where the second equality uses assumption (4), and

$$\begin{aligned}
\Phi_{\Gamma,i} &= k_{it}\Phi_{\Gamma} \\
\phi_{\Gamma,i} &= k_{it}\phi_{\Gamma} \\
k_{it} &= \sigma^2_{\varepsilon,i}(\sigma^2_{\varepsilon,i} + \sigma^2_e)^{-1/2}(\sigma^2_{\varepsilon,i} + \sigma^2_e + \Gamma'_{xt}\Gamma_{xt})^{-1/2}.
\end{aligned}$$

The next step is to show the same can be shown for $\mu_{it,j}$. Recall that μ_{it} is the term entering the approximation of optimal weights in equation (5) and it is given by

$$\mu_{it} = E_{it}[r^x_{t+1}] + \frac{\sigma^2_{it}}{2} = \mu_{r|s_i} + \frac{1}{2}diag(\Sigma_{it});$$

and the term $\mu_{r|s_i}$ is given by

$$\mu_{r|s_i} = \Sigma_{r|s_i}\left[(\sigma^2_{\varepsilon,i})^{-1}s_{it} + \Sigma^{-1}_{xt}\mu_{xt}\right].$$

Next we show that we can write $\mu_{r|s_i}$ as a function of μ_{xt}, Γ_{xt} and Γ_{it}. Notice that using assumption 2 and the Woodbury matrix identity, the term $\Sigma^{-1}_{xt}\mu_{xt}$ can be written as

$$\begin{aligned}
\Sigma^{-1}_{xt}\mu_{xt} &= [\sigma^2_e I + \Gamma_{xt}\Gamma'_{xt}]^{-1}\mu_{xt} \\
&= \frac{1}{\sigma^2_e}\left[I - \frac{\Gamma_{xt}\Gamma'_{xt}}{\sigma^2_e + \Gamma'_{xt}\Gamma_{xt}}\right]\mu_{xt},
\end{aligned}$$

also recall that by proposition 2, $\Sigma_{r|s_i} = \Sigma_{it} = \Gamma_{it}\Gamma'_{it} + \iota_{it}I$, so if we substitute these terms into the expression for $\mu_{r|s_i}$ we have:

$$\begin{aligned}
\mu_{r|s_i} &= \Sigma_{r|s_i}\left[(\sigma^2_{\varepsilon,i})^{-1}s_{it} + \Sigma^{-1}_{xt}\mu_{xt}\right] \\
&= [\iota_{it} I + \Gamma_{it}\Gamma'_{it}]\left[\left(\frac{1}{\sigma^2_{\varepsilon,1}}\right)s_{it} + \frac{1}{\sigma^2_e}\left[I - \frac{\Gamma_{xt}\Gamma'_{xt}}{\sigma^2_e + \Gamma'_{xt}\Gamma_{xt}}\right]\mu_{xt}\right] \\
&= \kappa_{1i}s_{it} + \kappa_{2i}\mu_{xt} + \kappa_{3i}\Gamma_{xt} + \kappa_{4i}\Gamma_{it}
\end{aligned}$$

with scalars

$$\begin{aligned}
\kappa_{1i} &= \left(\frac{\iota_{it}}{\sigma^2_{\varepsilon,i}}\right) \\
\kappa_{2i} &= \left(\frac{\iota_{it}}{\sigma^2_e}\right) \\
\kappa_{3i} &= -\left(\frac{\iota_{it}}{\sigma^2_e}\right)\left(\frac{\Gamma'_{xt}\mu_{xt}}{\sigma^2_e + \Gamma'_{xt}\Gamma_{xt}}\right) \\
\kappa_{4i} &= \left[\left(\frac{\Gamma'_{it}s_{it}}{\sigma^2_{\varepsilon,i}}\right) + \left(\frac{\Gamma'_{it}\mu_{xt}}{\sigma^2_e}\right) - \frac{(\Gamma'_{xt}\mu_{xt})(\Gamma'_{it}\Gamma_{xt})}{\sigma^2_e(\sigma^2_e + \Gamma'_{xt}\Gamma_{xt})}\right],
\end{aligned}$$

with the previous we have that for asset j

$$\begin{aligned}
\mu_{r|s_i,j} &= \kappa_{1i}s_{it,j} + \kappa_{2i}\mu_{xt,j} + \kappa_{3i}\Gamma_{xt,j} + \kappa_{4i}\Gamma_{it,j} \\
&= \kappa_{1i}s_{it,j} + \kappa_{2i}(y'_{jt}\Phi_\mu + \phi_\mu) + \kappa_{3i}(y'_{jt}\Phi_\Gamma + \phi_\Gamma) + \kappa_{4i}(y'_{jt}\Phi_{\Gamma,i} + \phi_{\Gamma,i}) \\
&= y'_{jt}\left[\kappa_{2i}\Phi_\mu + \kappa_{3i}\Phi_\Gamma + \kappa_{4i}\Phi_{\Gamma,i}\right] + \left[\kappa_{1i}\phi_\mu + \kappa_{2i}\phi_\Gamma + \kappa_{3i}\phi_{\Gamma,i}\right].
\end{aligned}$$

Recall that $\sigma^2_{it} = diag(\Sigma_{it})$, so $\sigma^2_{it,j} = \Gamma^2_{it,j} + \iota_{it}$. Since $\Gamma_{it,j}$ is a polynomial of degree M is clear that $\sigma^2_{it,j}$ is a polynomial of degree $2M$ on x_{jt}. To accommodate this we can define $\bar{y}_{jt}$ so it includes the degree combinations of $y_{jt} \otimes y_{jt}$, that is define

$$\bar{y}_{jt} = \begin{pmatrix} x_{jt} \\ x_{jt} \otimes x_{jt} \\ x_{jt} \otimes x_{jt} \otimes x_{jt} \\ \vdots \end{pmatrix}$$

with $\bar{y}_{jt}$ having dimension $K_{\bar{y}} = \sum_{m=1}^{2M} K^m_x$. Then we define $\Phi_{\bar{y}}$ so $(y'_{jt}\Phi_{\Gamma,i})^2 = \bar{y}'_{jt}\Phi_{\bar{y}}$.

With this notation we can write

$$\begin{aligned}\sigma^2_{it,j} &= \Gamma^2_{it,j} + \iota_{it} = (y'_{jt}\Phi_{\Gamma,i} + \phi_{\Gamma,i})^2 + \iota_{it} \\ &= (y'_{jt}\Phi_{\Gamma,i})^2 + y'_{jt}[2\phi_{\Gamma,i}\Phi_{\Gamma,i}] + [\phi^2_{\Gamma,i} + \iota_{it}] \\ &= \bar{y}'_{jt}\Phi_{\bar{y}} + y'_{jt}[2\phi_{\Gamma,i}\Phi_{\Gamma,i}] + [\phi^2_{\Gamma,i} + \iota_{it}].\end{aligned}$$

Notice that the term $\mu_{r|s_i,j}$ can be written as a polynomial on $\bar{y}_{jt}$; just pad with zeros the coefficients corresponding to powers of the elements in x_{jt} present $\bar{y}_{jt}$ but not in y_{jt}. Then it is possible to write

$$\begin{aligned}\mu_{it,j} &= \mu_{r|s_i,j} + \frac{1}{2}\sigma^2_{it,j} \\ &:= \bar{y}_{jt}\Phi_{\mu,i} + \phi_{\mu,i},\end{aligned}$$

where the $\Phi_{\mu,i}$ exact configuration of padded zeros depends on the dimensions of x_{jt} and the degree M. Finally, $\phi_{\mu,i}$ is given by

$$\phi_{\mu,i} = \kappa_{1i}s_{it,j} + \kappa_{2i}\phi_\mu + \kappa_{3i}\phi_\Gamma + \kappa_{3i}\phi_{\Gamma,i} + \frac{1}{2}\phi^2_{\Gamma,i} + \frac{1}{2}\iota_{it}.$$

Part (ii). Recall that the positive optimal portfolio weights are given by

$$w_{it} \approx \Sigma^{-1}_{it}\left(\mu_{it} - \lambda_{it}\mathbf{1} + C_t a_i\right),$$

using the expression for Σ_{it} we have

$$\begin{aligned}w_{it} &\approx [\iota_{it} + \Gamma_{it}\Gamma'_{it}]^{-1}(\mu_{it} - \lambda_{it}\mathbf{1} + C_t a_i) \\ &= \frac{1}{\iota_{it}}\left[I - \frac{\Gamma_{it}\Gamma'_{it}}{\iota_{it} + \Gamma'_{it}\Gamma_{it}}\right](\mu_{it} - \lambda_{it}\mathbf{1} + C_t a_i) \\ &= \left(\frac{1}{\iota_{it}}\right)\mu_{it} - \left(\frac{\lambda_{it}}{\iota_{it}}\right)\mathbf{1} + C_t\left(\frac{a_i}{\iota_{it}}\right) + \kappa_{it}\Gamma_{it}\end{aligned}$$

with

$$\kappa_{it} = -\frac{\Gamma'_{it}(\mu_{it} - \lambda_{it}\mathbf{1} + C_t a_i)}{\iota_{it} + \Gamma'_{it}\Gamma_{it}}$$

Now define $\tilde{x}_{jt} = [x'_{jt} \quad c'_{jt}]'$ a $K_{\tilde{x}} = (K_x + K_c)$-vector and define the $K_{\tilde{y}}$ vector $\tilde{y}_{jt}$ with

$K_{\tilde{y}} = \sum_{m=1}^{2M}(K_x + K_c)^m$ and

$$\tilde{y}_{jt} = \begin{pmatrix} \tilde{x}_{jt} \\ \tilde{x}_{jt} \otimes \tilde{x}_{jt} \\ \tilde{x}_{jt} \otimes \tilde{x}_{jt} \otimes \tilde{x}_{jt} \\ \vdots \end{pmatrix}.$$

We proceed by showing that each term in $w_{it,j}$ can be written as polynomial in $\tilde{y}_{jt}$. First

$$c'_{jt}\left(\frac{a_i}{\iota_{it}}\right) = \tilde{y}'_{jt}\begin{pmatrix} 0 \\ \left(\frac{1}{\iota_{it}}\right) a_i \\ 0 \\ \vdots \end{pmatrix} := \tilde{y}'_{jt}\Phi_{C,i},$$

next

$$\begin{aligned} \left(\frac{1}{\iota_{it}}\right)\mu_{it,j} &= \left(\frac{1}{\iota_{it}}\right)\left(\bar{y}'_{jt}\Phi_{\mu,i} + \phi_{\mu,i}\right) \\ &= \bar{y}'_{jt}\left(\frac{1}{\iota_{it}}\Phi_{\mu,i}\right) + \frac{\phi_{\mu,i}}{\iota_{it}} \\ &:= \tilde{y}_{jt}\tilde{\Phi}_{\mu,i} + \frac{\phi_{\mu,i}}{\iota_{it}} \end{aligned}$$

where $\tilde{\Phi}_{\mu,i}$ has zeros whenever a term with c_{jt} appears in $\tilde{y}_{jt}$. Finally,

$$\begin{aligned} \kappa_{it}\Gamma_{it,j} &= \kappa_{it}(y'_{jt}\Phi_{\Gamma,i} + \phi_{\Gamma,i}) \\ &= y'_{jt}(\kappa_{it}\Phi_{\Gamma,i}) + \kappa_{it}\phi_{\Gamma,i} \\ &= \tilde{y}'_{jt}\tilde{\Phi}_{\Gamma,i} + \kappa_{it}\phi_{\Gamma,i}, \end{aligned}$$

where once again $\tilde{\Phi}_{\Gamma,i}$ has zeros whenever there is a term in $\tilde{y}_{jt}$ with c_{jt} or has a power of x_{jt} not present in y_{jt}. Collecting terms we have that

$$w_{it,j} \approx \tilde{y}'_{jt}\Phi_{w,it} + \phi_{w,it}$$

with

$$\Phi_{w,it} = \tilde{\Phi}_{\mu,i} + \Phi_{C,i} + \tilde{\Phi}_{\Gamma,i}$$
$$\phi_{w,it} = \frac{\phi_{\mu,i}}{\iota_{it}} + \kappa_{it}\phi_{\Gamma,i} - \frac{\lambda_{it}}{\iota_{it}}.$$

Part (iii). Restricting the parameters so $\phi_{w,it} = w_{it0}$ and $\Phi_{w,it}/w_{it0} = [\beta_{it} \quad 1/2vec(\beta_{it}\beta'_{it}) \quad \cdots]'$ then

$$\frac{w_{ij,t}}{w_{it,0}} \approx 1 + \tilde{y}'_{jt}\frac{\Phi_{w,it}}{w_{it0}} = 1 + \tilde{x}'_{jt}\beta_{it} + \frac{1}{2}vect(\tilde{x}_{jt}\tilde{x}'_{jt})'vec(\beta_{jt}\beta'_{jt}) + \cdots$$
$$= \sum_{m=1}^{M}\frac{(\tilde{x}'_{jt}\beta_{it})^m}{m!} \longrightarrow \exp\left[\tilde{x}'_{jt}\beta_{it}\right], \quad \text{as } m \to \infty.$$

Writing $\beta'_{it} = [1 \quad b'_{it} \quad \gamma'_{it}]$ and assuming the first characteristics in x_{jt} is unobserved and denote by ξ_{jt} then we have that

$$\frac{w_{it,j}}{w_{it,0}} \approx \exp\left(\xi_{jt} + x'_{jt}b_{it} + c'_{jt}\gamma_{it}\right)$$

Finally because $w_{it,0} + \sum_{j=1}^{J_t} w_{it,j} = 1$, then $1 + \sum_{j=1}^{J_t} w_{it,j}/w_{it,0} = 1/w_{it,0}$ and

$$w_{it,j} \approx \frac{\exp\left(x'_{jt}b_{it} + c'_{jt}\gamma_{it} + \xi_{jt}\right)}{1 + \sum_{k=1}^{J_t}\exp\left(x'_{kt}b_{it} + c'_{kt}\gamma_{it} + \xi_{kt}\right)},$$

and the weight for the outside option is given by

$$w_{it,0} \approx \frac{1}{1 + \sum_{k=1}^{J_t}\exp\left(x'_{kt}b_{it} + c'_{kt}\gamma_{it} + \xi_{kt}\right)}.$$

□

A.5 Proof of Proposition 4

We start by stating Berry's Inversion theorem for demand systems. See Berry (1994) for a full proof. Then the proof consists in verying the conditions of the theorem for the demand system in (25).

Berry's Inversion Theorem Consider the metric space $(\mathbb{R}^K, d)$ with $d(x,y) = ||x-y||$ and $||\cdot||$ denoting the sup-norm. Let $f : \mathbb{R}^K \to \mathbb{R}^K$ satisfy:

i. $\forall x \in \mathbb{R}^K$, $f(x)$ is continuously differentiable such that for any j and k:

$$\frac{\partial f_j(x)}{\partial x_k} \geq 0$$
$$\sum_{k=1}^{K} \frac{\partial f_j(x)}{\partial x_k} < 1$$

ii. $\min_j \inf_x f(x) := \underline{x} > -\infty$

iii. There is a value $\bar{x}$ with the property that if for any j $x_j \geq \bar{x}$ then for some k (not necessarily equal j) $f_k(x) < x_k$.

Then there is a unique fixed point $x_0 \in \mathbb{R}^K$ to f. Moreover, let $\mathcal{X} := [\underline{x}, \bar{x}]^K$ and define the truncated function $\hat{f}_j(x) = \min\{f_j(x), \bar{x}\}$. Then $\hat{f}(x)$ is a contraction of modulus less than one on $\mathcal{X}$.

Proof. Verying the conditions of Berry's Inversion Theorem

Denote by θ the parameters to estimate in market t; s_t the vector of observed aggregate share, and $\tilde{s}_t(\delta_t, \theta_2; d_t, X_t, J_t)$ the vector of model-implied shares. Here the operator $f : \mathbb{R}^{J_t} \to \mathbb{R}^{J_t}$ for which we look a fixed point is given by:

$$f(\delta) = \delta + \log s_t - \log \tilde{s}_t(\delta, \theta_2; d_t, X_t, J_t)$$

On this operator we check the conditions for Berry's inversion. On the following we drop the t index and denote the model implied market share for asset j as $\tilde{s}_j := \tilde{s}_j(\delta, \theta_2; d_t, X_t, J_t)$.

- **Checking i.** We start by verifying that the first derivatives are non-negative, we have that:

$$\frac{\partial f_j(\delta)}{\partial \delta_k} = \begin{cases} 1 - \frac{1}{\tilde{s}_j}\frac{\partial \tilde{s}_j}{\partial \delta_j} & \text{if } k = j \\ -\frac{1}{\tilde{s}_j}\frac{\partial \tilde{s}_j}{\partial \delta_k} & \text{if } k \neq j \end{cases}$$

We know that $\tilde{s}_j \geq 0$ and using the definition for the model-implied shares we have that

$$\frac{\partial \tilde{s}_j}{\partial \delta_k} = \frac{\partial}{\partial \delta_k}\left[\sum_{i \in I}\left(\frac{A_i}{A}\right)\int w_{ij}(v_i) dF_v(v_i)\right] = \sum_{i \in I}\left(\frac{A_i}{A}\right)\int \frac{\partial w_{ij}(v_i)}{\partial \delta_k} dF_v(v_i)$$

where $\dfrac{\partial w_{ij}(v_i)}{\partial \delta_k} = \begin{cases} w_{ij}(v_i)(1 - w_{ij}(v_i)) & \text{if } k = j \\ -w_{ik}(v_i) w_{ij}(v_i) & \text{if } k \neq j \end{cases}$

From the previous we see that if $k \neq j$ then $\frac{\partial w_{ij}}{\partial \delta_k} \leq 0$ and since $(A_i/A) > 0$ then $\frac{\partial \tilde{s}_j}{\partial \delta_k} \leq 0$. For $k = j$ notice that $\frac{\partial w_{ij}}{\partial \delta_k} \leq w_{ij}$ and then $\frac{\partial \tilde{s}_j}{\partial \delta_k} \leq \tilde{s}_j$ so $\frac{\partial \tilde{s}_j}{\partial \delta_k}$. The next step is to verify that the sum of partial derivatives is less than 1. Notice that:

$$\begin{aligned}
\sum_{k=1}^{J_t} \frac{\partial f_j(\delta)}{\partial \delta_k} &= \frac{\partial f_j(\delta)}{\partial \delta_j} + \sum_{k \neq j} \frac{\partial f_j(\delta)}{\partial \delta_k} \\
&= 1 - \frac{1}{\tilde{s}_j}\frac{\partial \tilde{s}_j}{\partial \delta_j} + \sum_{k \neq j}\left(-\frac{1}{\tilde{s}_j}\frac{\partial \tilde{s}_j}{\partial \delta_k}\right) \\
&= 1 - \frac{1}{\tilde{s}_j}\left[\sum_{k=1}^{J_t}\frac{\partial \tilde{s}_j}{\partial \delta_k}\right] = 1 - \frac{1}{\tilde{s}_j}\left[\sum_{k=1}^{J_t}\sum_{i \in I}\left(\frac{A_i}{A}\right)\int \frac{\partial w_{ij}(v_i)}{\partial \delta_k} dF_v(v_i)\right] \\
&= 1 - \frac{1}{\tilde{s}_j}\left[\sum_{i \in I}\left(\frac{A_i}{A}\right)\sum_{k=1}^{J_t}\int \frac{\partial w_{ij}(v_i)}{\partial \delta_k} dF_v(v_i)\right]
\end{aligned}$$

Given the definition of $\tilde{s}_j$ it is sufficient to show that $\sum_{k=1}^{J_t}\int \frac{\partial w_{ij}(v_i)}{\partial \delta_k} dF_v(v_i) < \int w_{ij}(v_i) dF_v(v_i)$. For this we notice that

$$\begin{aligned}
\sum_{k=1}^{J_t}\frac{\partial w_{ij}(v_i)}{\partial \delta_k} &= \frac{\partial w_{ij}(v_i)}{\partial \delta_j} + \sum_{k \neq j}\frac{\partial w_{ij}(v_i)}{\partial \delta_k} = w_{ij}(v_i)(1 - w_{ij}(v_i)) - \sum_{k \neq j} w_{ij}(v_i) w_{ik}(v_i) \\
&= w_{ij}(v_i)\left[1 - \sum_{k \neq j} w_{ik}(v_i)\right] \leq w_{ij}(v_i)
\end{aligned}$$

- **Checking ii.** There first step is to rewrite the model-implied shares as

$$\tilde{s}_j = \sum_{i \in I} \left(\frac{A_i}{A} \right) \int w_{ij}(v_i) dF_v(v_i) = \exp(\delta_j) \sum_{i \in I} \left(\frac{A_i}{A} \right) D_{ij}(\delta)$$

$$\text{with } D_{ij}(\delta) = \int \frac{\exp(h_{ij}(v_i))}{1 + \sum_{j=1}^{J} \exp(\delta_j + h_{ij}(v_i))} dF_v(v_i)$$

This implies that $\ln(\tilde{s}_j) = \delta_j + \ln\left(\sum_{i \in I}(A_i/A)D_{ij}(\delta)\right)$ and that

$$f(\delta)_j = \ln(s_j) - \ln\left(\sum_{i \in I}(A_i/A)D_{ij}(\delta)\right)$$

Now notice that when $\delta_m \to -\infty$ for $m \neq j$ then the term $D_{ij}(\delta)$ tends to $\int \exp(h_{ij}(v_i))dF_v(v_i)$ so a lower bound for $f(\delta)_j$ is

$$\underline{\delta}_j > \ln(s_j) - \ln\left[\sum_{i \in I}(A_i/A) \int \exp(h_{ij}(v_i))dF_v(v_i)\right]$$

So condition ii. is satisfied with $\underline{\delta} := \min_j \underline{\delta}_j$.

- **Checking iii.** For this part set $\delta_k = -\infty$ for $k \neq j$ and defined $\bar{\delta}_j$ as the value of δ_j such that $\tilde{s}_0(\delta, \theta_2) = s_0$, that is the value of δ_j that along with $\delta_k = -\infty$ would match the observed shares for the outside good. Moreover let $\bar{\delta} > max_j \bar{\delta}_j$.

  If δ is such that $\exists j$ with $\delta_j > \bar{\delta}$ then $\tilde{s}_0(\delta) < s_0$ and hence $\sum_{k=1}^{I_t} \tilde{s}(\delta)_k > \sum_{k=1}^{J_t} s_k$ which means that there is a least one element k such that $\tilde{s}(\delta)_k > s_k$. For such k we have that $f(\delta)_k < \delta_k$ as required in part iii.

□

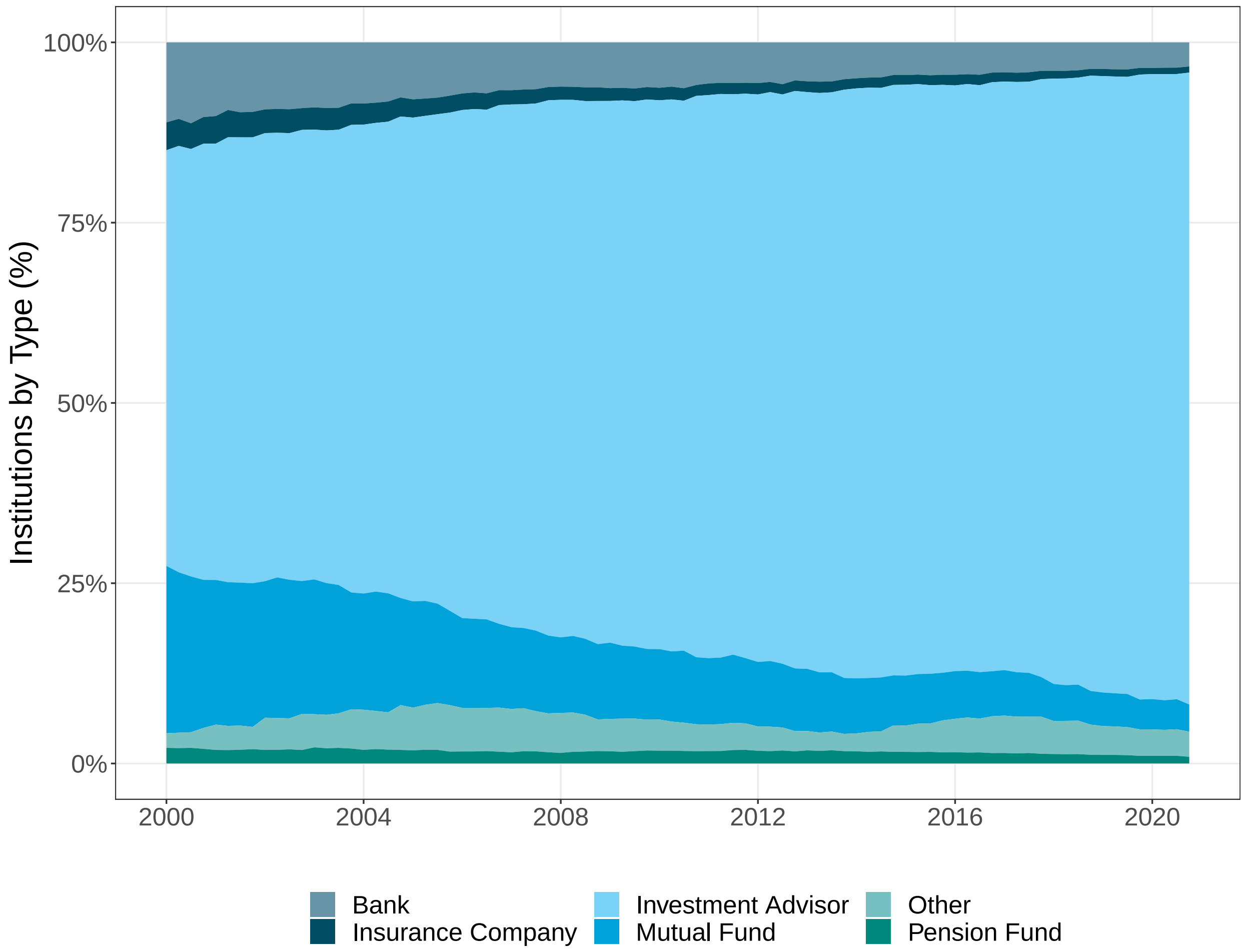
B Data Appendix

Table B1: Summary Statistics for Stock Characteristics

Variable	N. Stocks	Mean	Median	Std. Dev.	Min	Pc. 25th	Pc. 75th	Max
2000-2004								
Market Beta	499	0.684	0.639	0.554	-0.238	0.302	0.960	3.974
log Market Equity	499	8.009	7.985	1.900	0.598	6.897	9.175	13.256
log Total Assets	499	8.404	8.572	1.858	1.516	7.348	9.751	13.455
log Book-to-Market Equity	499	-0.408	-0.360	0.923	-5.521	-0.968	0.206	3.653
Profitability	499	0.211	0.192	0.227	-1.752	0.104	0.319	0.785
Investment	499	0.073	0.051	0.193	-0.811	-0.016	0.132	1.305
E-score	499	0.059	0.000	0.238	-0.500	-0.125	0.333	0.500
2005-2009								
Market Beta	696	1.208	1.099	0.713	-0.163	0.683	1.587	3.974
log Market Equity	696	8.038	7.981	1.804	0.923	6.884	9.299	13.149
log Total Assets	696	8.298	8.261	1.736	2.513	7.159	9.537	14.673
log Book-to-Market Equity	696	-0.482	-0.482	0.916	-5.659	-1.073	0.086	4.716
Profitability	696	0.223	0.211	0.251	-2.381	0.121	0.333	0.941
Investment	696	0.072	0.055	0.200	-0.741	-0.015	0.139	0.917
E-score	696	0.048	0.000	0.173	-0.500	0.000	0.200	0.500
2010-2014								
Market Beta	1122	1.294	1.223	0.646	0.009	0.812	1.678	3.182
log Market Equity	1122	8.460	8.558	1.801	1.169	7.345	9.657	13.374
log Total Assets	1122	8.665	8.650	1.869	1.534	7.539	9.871	14.697
log Book-to-Market Equity	1122	-0.599	-0.593	0.936	-7.699	-1.150	-0.002	4.218
Profitability	1122	0.227	0.209	0.241	-2.098	0.120	0.319	0.878
Investment	1122	0.062	0.046	0.151	-0.691	-0.007	0.114	0.816
E-score	1122	0.020	0.000	0.168	-0.500	0.000	0.125	0.500
2015-2019								
Market Beta	955	1.150	1.139	0.569	-0.396	0.779	1.487	3.184
log Market Equity	955	8.910	9.064	1.850	1.054	7.632	10.203	14.068
log Total Assets	955	9.003	8.949	1.903	1.534	7.803	10.263	14.780
log Book-to-Market Equity	955	-0.842	-0.794	1.061	-9.991	-1.441	-0.160	4.943
Profitability	955	0.261	0.223	0.292	-2.880	0.127	0.359	1.034
Investment	955	0.053	0.033	0.173	-0.633	-0.021	0.095	0.911
E-score	955	0.047	0.000	0.141	-0.500	0.000	0.125	0.500

Notes: Summary statistics for the stock's characteristics used during estimation. Statistics are compute over pooled quarterly observations of the variables every five years. Summary Statistics present mean, median, standard deviation, minimum, 25th percentile, 75th percentile and maximum.

Figure B1: Distribution of Institutional Investors by type



Notes: Evolution of institutional investors by type from 2000q1 to 2020q4. The classification of institutional investors follows the six categories as in Koijen and Yogo (2019).

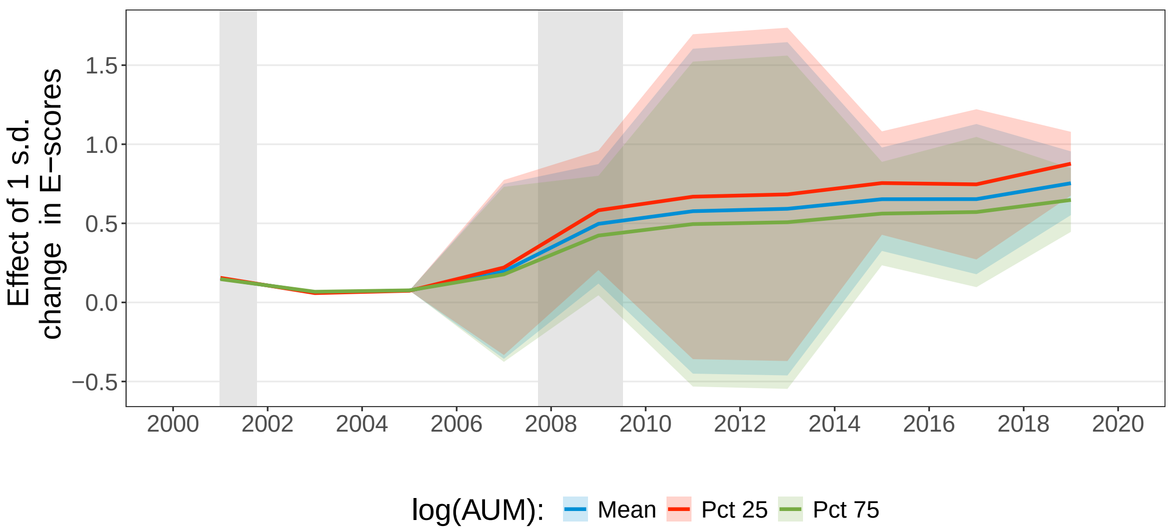
Table B2: Environmental Indicators from MSCI

Positive Indicators	Negative Indicators
Environmental Opportunities	Hazardous Waste
Waste Management	Regulatory Compliance
Packaging Materials and Waste	Ozone Depleting Chemicals
Climate Change	Toxic Spills and Releases
Environmental Management Systems	Agriculture Chemicals
Water Stress	Climate Change
Biodiversity and Land Use	Impact of Products and Services
Raw Material Sourcing	Biodiversity and Land Use
Natural Resource Use	Operational Waste
Environmental Opportunities - Green Buildings	Supply Chain Management
Environmental Opportunities in Renewable Energy	Water Management
Waste Management - Electronic Waste	Other Concerns
Climate Change - Product Carbon Footprint	
Climate Change - Insuring Climage Change Risk	
Other Strengths	

Notes: List of environmental performance indicators in the MSCI dataset. Each indicator is a dummy variable. The threshold for satisfying an indicator are determined by MSCI and are not disclosed with the data.

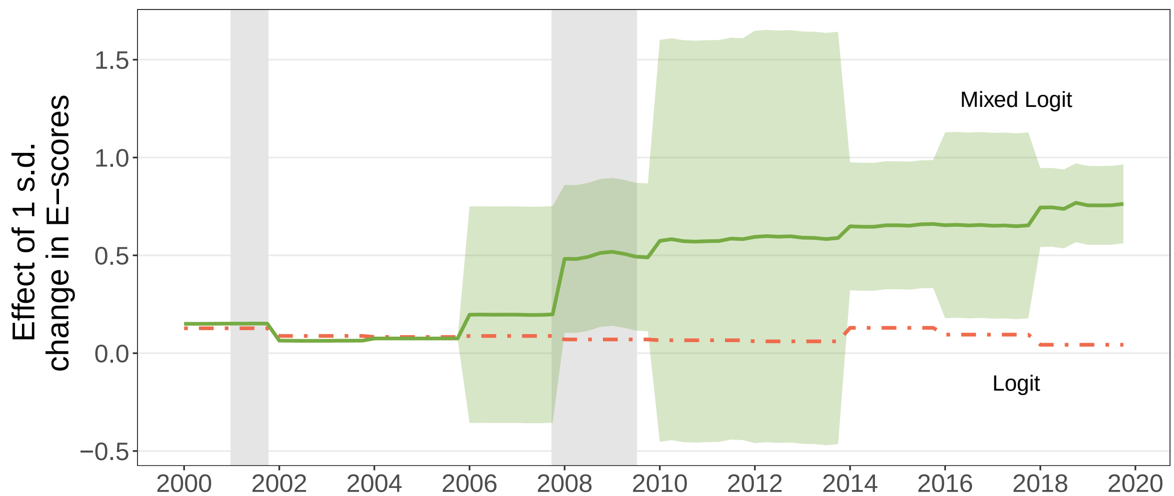
C Complementary Results

Figure C1: Estimated coefficient for E-scores at different values of AUM



Notes: This plot shows the effective coefficient corresponding to E-scores: $\hat{\gamma}_{it} = \hat{\gamma}_0 + \hat{\kappa}\log(\text{AUM})_{it} + \hat{\sigma}v_{it}$. The estimates $(\hat{\gamma}_0, \hat{\kappa}, \hat{\sigma})$ are obtained over 2-year estimation windows ranging from 2000-Q1 to 2019-Q4. The plot shows the estimated effective coefficient using the mean, 25th and 75th percentile of log assets under management in each estimation window. Shaded areas correspond to the 95% confidence intervals of how the coefficient on E-scores varies across investors based on the assumption that unobserved demographics, v_{it}, are normally distributed. Recession periods of the US economy are shown as shaded gray regions.

Figure C2: Estimated coefficient for E-scores at mean quarterly AUM



Notes: This plot shows the effective coefficient corresponding to E-scores: $\hat{\gamma}_{it} = \hat{\gamma}_0 + \hat{\kappa}\log(\text{AUM})_{it} + \hat{\sigma} v_{it}$. The estimates $(\hat{\gamma}_0, \hat{\kappa}, \hat{\sigma})$ are obtained over 2-year estimation windows ranging from 2000-Q1 to 2019-Q4 and according to estimation based on logit demand or mixed logit demand. The case of logit demand corresponds to $\hat{\gamma}_{it} = \hat{\gamma}_0^{logit}$. The plot shows the estimated effective coefficient using the mean of log assets under management in each quarter from 2000-Q1 to 2019-Q4. Shaded areas correspond to the 95% confidence intervals of how the coefficient on E-scores varies across investors based on the assumption that unobserved demographics, v_{it}, are normally distributed. Recession periods of the US economy are shown as shaded gray regions.

Table C1: Counterfactual holdings and price changes from a ban of green investing for pension funds

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5
Observed Portfolio Statistics in 2019-Q1					
N. Stocks	104	104	104	103	103
ME (USD Bill.)	2847	2115	1814	5789	5356
Agg. Port. Share (%)	9.310	6.917	5.932	18.930	17.515
Counterfactual Agg. Port Shares (%)					
Logit	9.331	6.926	5.940	18.919	17.476
Mixed Logit	9.417	6.985	5.991	18.869	17.233
Counterfactual Price change (%)					
Logit	0.222	0.138	0.138	-0.059	-0.220
Mixed Logit	1.142	0.988	0.988	-0.319	-1.611
Observed Price Change (%)					
2019 Q1 -2019-Q2	-1.046	3.496	2.606	2.950	4.734

Notes: This table shows the effect of a ban of green investing for pension funds on aggregate holdings and equity prices in a counterfactual exercise using data and estimates for 2019-Q1. The portfolios were constructed by sorting stocks by their E-score, and grouping them according to quintiles. Portfolio 1 contains the 20% of stocks with lowest E-scores, while Portfolio 5 contains the 20% of stocks with highest E-scores. The first three rows in the table show observed portfolio statistics in 2019-Q1, the quarter where the portfolios where constructed. The following 4 rows show the counterfactual changes in aggregate portfolio holdings and value-weighted prices changes according to a logit demand specification and to a mixed logit demand specification. Finally, the last row in the table shows observed value-weighted average price change between 2019-Q1 and 2019-Q2 for each portfolio.