How do Households Suppress the Price of Tail Risk?

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Abstract

We examine how households dampen volatility prices through their demand for Short Put Products (SPPs) – a globally popular structured product that offers high headline rates in exchange for selling a put option. Using a comprehensive dataset covering all index-linked SPP issuances worldwide since market inception, we empirically show that SPP issuance volumes rise when the volatility of the underlying asset is high, as higher volatility allows to offer higher headline rates. In turn, increased SPP issuance drives down the prices of deeply out-of-the-money put options, suppressing the corresponding volatility risk premium. To uncover the underlying mechanism and quantify the equilibrium effects of these innovative products on volatility prices, we develop a structural model in which households underweight left-tail risk, driving demand for SPPs. Risk-averse financial intermediaries optimize the headline rate offered on these products while imperfectly hedging their exposure. As volatility rises, stronger demand for SPPs – driven by higher headline rates – exerts downward pressure on option prices, particularly at strikes below 100%. Our findings reveal a novel channel for enhancing financial stability: household demand for innovative security designs lowers the cost of insuring against left-tail risk for other market participants.

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I. Introduction

Over the past 25 years, financial institutions have increasingly designed and marketed products with non-linear payoffs – embedding the sale or purchase of options – to cater to households' "non-standard" preferences. These products include retail structured products (Celerier and Vallee (2017), Vokata (2023)), structured funds (Gao, Hu, Kelly, Peng and Zhu, 2024), or structured annuities (Koijen and Yogo, 2022). Collectively, these financial products represent several trillion dollars of assets under management as of 2024. While their penetration rate has historically been high in Europe and Asia, this class of products has experienced rapid growth in the U.S. in recent years.[1] Despite their growing importance, the asset pricing implications of these products remain under-explored.

Among these instruments, Short Put Products (SPPs) – the most widely sold retail structured product globally – may particularly affect volatility pricing. Through SPPs, households implicitly sell large quantity of deeply out-of-the-money put options in exchange for attractive coupons, thereby increasing their exposure to left-tail risk. Consistent with the growing literature on demand pressure effects in option markets (beginning with Gârleanu, Pedersen and Poteshman (2009)), the hedging activities of intermediaries issuing these products may exert downward pressure on put option prices. By allowing households to bear left tail risk, the development of SPPs may thus, in equilibrium, lower the price of insurance against such risk. This phenomenon could be sufficiently pronounced to have implications for financial stability, given the large size of SPP markets relative to underlying option markets, especially at low moneyness levels. As shown in Figure 1, for the three main equity indices – Eurostoxx, Nikkei 225, and S&P 500, – SPP outstanding volumes represent approximately 25%, 15% and 5%, respectively, of the open interest for out-of-the-money put options with maturity over three months.

INSERT FIGURE 1

[1] In Europe, approximately 15% of households hold retail products with non-linear payoffs.

This study therefore aims to address the following questions: Can households suppress volatility prices through their demand for SPPs? What are the underlying mechanism and equilibrium effects? How does the magnitude of these pricing effects vary with market conditions, for instance in times of stress?

To address these questions, we follow a two-step approach. First, we leverage a comprehensive dataset that includes all SPPs issued globally since the market's inception, merged with data on option prices and open interest, to provide reduced-form evidence of a robust link between realized volatility, SPP issuance, and volatility prices. Second, we develop and estimate a structural model to flesh out the economic mechanism at play, allowing us to precisely characterize and quantify the equilibrium effects of the SPP market development through counterfactual analysis.

Our dataset covers the global issuance of SPPs over the 2005-2019 period and includes key product characteristics for each SPP, including the underlying asset, maturity, detailed payoff design, issuance volume, and issuance date.[2] The payoff formula of these products depends on the performance of an underlying asset – typically an equity index or a stock – and is determined by two parameters: the *headline rate*, a fixed coupon paid periodically until maturity, and the *barrier*, usually set between 50% and 70% of the underlying price at issuance. At maturity, the investor receives their capital back if and only if the underlying asset is above the barrier. If the final value of the underlying falls below the barrier, the investor's final payment is adjusted for the negative performance of the underlying asset. Hence, SPPs offer a fixed coupon in most scenarios, while fully exposing retail investors to left-tail risk.[3] These products are also often labeled as Yield-Enhancement Products (YEPs), although this terminology does not accurately capture their economic substance in our view.

We focus our analysis on equity index-linked SPPs, which account for 188 000 products

[2] Data source is similar to the one in Celerier and Vallee (2017), except that the dataset for this study extends beyond Europe and covers a longer period.

[3] Many of these products also embed an autocall feature, meaning that the product is redeemed early if the underlying asset is above the barrier at any coupon date. This features limits the upside for the investor, while keeping the downside largely unchanged.

and \$650 billion in cumulated issuances across North America, Europe, and Asia over the 2005-2019 period, and represent around half of total SPP volumes. The twelve most popular underlying indices, for which we collect detailed data on the volatility surface and open interest, account for 99% of the equity index-linked SPPs. These indices also represent over 70% of global market capitalization, illustrating their relevance for financial stability. Median maturity, headline rate, and barrier levels in our sample are three years, 7.5%, and 60%, respectively. We extract headline rates – a central outcome of interest– using a text analysis algorithm applied to the product payoff descriptions. Finally, outstanding volumes and the open interest for put option of corresponding strikes and maturity amount to 13 and 67 billion dollars on average in 2015, suggesting a possible significant impact of SPPs on the price of tail risk.

Our empirical analysis yields two key findings. On the SPP market front, we show that issuance volumes increase as realized volatility rises. Specifically, higher realized volatility predicts higher headline rates offered by SPPs, as the premium of the embedded put options increases with underlying the asset volatility. We estimate that the elasticity of headline rates to 90-day realized volatility is approximately 0.4. In turn, higher realized volatility– through its impact on headline rates– drives higher net issuance volumes, consistent with household demand particularly responding to this design feature.

On the volatility market front, we find that higher SPP issuance volumes predict lower implied volatility and a lower volatility premium, defined as the spread between implied and realized volatility. This relationship is strongest around the 60% moneyness level, corresponding to the most common SPP barrier, but is evident across all moneyness levels below 100%. The effect is also larger when the size of the corresponding option market is smaller, such as for the Kospy 200, or the Hang Seng China Enterprise (HSCEI) indices. Second, the suppressing effect of SPP issuance on volatility prices is particularly pronounced following significant drops in the underlying index, when early redemption of outstanding products becomes unlikely. To strengthen causal identification, we exploit a natural experiment in

South Korea, where regulators temporarily banned the issuance of HSCEI-linked SPPs in September 2015. We find that, while realized volatility declines after the ban– as it followed a market crash – implied volatility increases, and open interest in corresponding put options decreases. This result holds in a difference-in-differences specification, supporting the causal link between SPP issuance and volatility pricing.

These empirical facts support the case that SPP issuances suppress volatility prices, and particularly so in times of stress when equity markets are down or volatility is high. Household demand for SPPs would therefore make them play the somewhat unexpected role of volatility price stabilizer. These patterns are persistent at a relatively long horizon, and therefore do not correspond to the micro-structure concept of price impact. This persistence suggests that the asset pricing phenomenon we are exploring results from long run demand and supply equilibrium effects, motivating a demand-based option pricing framework that delivers a stationary equilibrium.

We therefore theoretically investigate the economic mechanism that can rationalize these empirical findings. Financial intermediaries, who issue SPPs to households and trade options and the underlying asset to hedge SPP contracts, are a natural focus of our framework. When structuring short put products, intermediaries hedge the associated market risks. The typical hedge for a SPP requires the intermediaries to short a portfolio of puts on the underlying asset and short the dividend exposure through dividend swaps. If competitive intermediaries could hedge their risks perfectly, as would be the case in a Black-Scholes-Merton economy, options would be redundant assets and hedging would have no effect on option prices. In reality, however, even intermediaries cannot hedge options perfectly because trading cannot be implemented in continuous time due to transactions costs and because the volatility of the underlying evolves stochastically over time (Figlewski, 1989). In addition, intermediaries are sensitive to risk, for instance due to capital constraints and agency costs.

We accordingly develop a structural model with these features, which we estimate to rigorously describe and quantify the impact of household demand for SPPs on volatility prices. The economy is populated by a representative household, a representative dealer, and outside investors with exogenous demand for options, in the style of Gârleanu et al. (2009), who all trade in discrete time. The investment set includes a riskless asset, an equity index, an SPP, and a set of plain-vanilla European puts on the equity index. Financial markets are driven by an exogenous state vector consisting of the prevailing interest rate and the stochastic volatility of the stock index. The dealer sets the headline rate and put prices, matches the demand for the SPP from households and puts from institutional investors, and partially hedges by trading the stock index. In line with the intermediary asset pricing literature, the dealer has CARA utility. The household can invest in the risk-free asset, the equity index, and the SPP, but cannot trade options. Each period, household demand for the SPP is affected by their time-invariant preferences, the endogenous SPP design (that is, the headline rate that it offers given a fixed maturity and barrier), and time-varying market conditions. This setting allows for substantial flexibility in modeling household preferences and beliefs.

We estimate the model by likelihood-based inference, as follows. For each level of the risk-free rate and the equity index volatility, the model provides the household demand for the SPP, the SPP headline rate set by the dealer, and the implied volatility of each put. We compare these values with monthly data on the swap rate, equity index return, SPP headline rate, and implied volatility of puts, which allows us to estimate the structural parameters of the model by maximum likelihood. We apply this method to products tied to the Eurostoxx 50, which is the index with the largest market share of SPPs.

The model predicts that household demand for SPPs and headline rates increase when volatility increases and interest rates decrease, which is consistent with the data. The demand for the SPP is null when the representative household has Epstein-Zin Utility. However, we do obtain significant household demand for SPPs when households underweight the left-tail risk, for instance when the household neglects to some extent the probability that the SPP may not pay the headline rate.

We conduct a counterfactual analysis showing that the implied volatility of a put contract on the equity index with a 60% strike is ca. 2 percentage points higher in the absence of the SPP compared to a market in which the SPP is traded. Furthermore, the equilibrium impact of SPPs increases with both the volume of issuance of SPPs, and the volatility of the equity index.

We conclude our analysis by first noting that a similar phenomenon is at play for dividend futures, and by developing a trading strategy based on our empirical findings. When the outstanding volume of SPPs goes up, the term structure of dividends flattens. We show that a long-short trading strategy in volatility markets that sort stocks by their associated SPP issuance volumes yields attractive Sharpe ratios, which further speaks to the significance and persistence of the equilibrium effects we document.

Our results suggest that the development of markets for innovative securities can affect the supply and demand equilibrium for volatility prices by incorporating non-standard household preferences into prices through intermediaries hedging strategies. Crucially, this impact is concentrated on the left-tail risk as households' tolerance for taking this exposure drives a security design that implicitly sells deeply out-of-the-money put options. As option markets prices are key inputs for financial institutions risk management models, and as certain economic agents likely have elastic demand for downside protection, the effects we document have the potential to influence risk-taking, and in turn financial stability. While the development of retail structured products has raised concerns over investor protection, our findings uncovers a potential trade-off between retail investor protection and financial stability. Banning SPPs may have a hidden cost, as illustrated by our event study on the ban on HSCEI-linked SPPs in South Korea in 2015.[4] More broadly, our results speak to the equilibrium effects of a change in the macro-structure of financial markets, in our case the introduction of a new asset class that allows a class of investor to express preferences they could not express before.

[4]There also was a more recent ban on Kospi2-linked SPPs in February 2024.

This paper makes contributions to multiple strands of the literature. Our work expands the burgeoning literature of market macrostructure covered in Haddad and Muir (2025) in a new dimension: asset prices are impacted by how financial institutions connect different asset classes or capital market segments to respond to a specific demand from a class of investor previously excluded. This role is different from the traditional intermediation, and is economically closer to what banks do when structuring securitization.

Relatedly, our findings are consistent with models that emphasize supply and demand factors, such as funding constraints of financial intermediaries or limits to arbitrage, as key drivers of asset prices. Gârleanu et al. (2009) develop a demand-based option pricing model and show that demand pressures from the put-call imbalance explain cross-sectional variations in volatility skewness across U.S. equity options. Vayanos and Vila (2009), Greenwood and Vayanos (2014) and Greenwood and Vissing-Jørgensen (2018) explain the term structure of riskless returns in a segmented supply and demand framework. Risk-averse intermediaries trade with end clients with strong preferences for specific-maturity bonds, hence driving price and return variations across different maturities. Brown, Davies and Ringgenberg (2018) study non-fundamental demand in ETF market. Our contribution is to provide a micro-foundation for demand pressures by pinning down household as a source of demand for left-tail risk and quantify its asset pricing implications.

The paper also adds to the literature on the volatility risk premium (Bakshi, Carr and Wu, 2008; Bollerslev, Tauchen and Zhou, 2009; Todorov, 2010; Han and Zhou, 2012; Cao and Han, 2013) and on spillovers between distinct but related capital markets, such as derivative markets and their underlying asset markets (Calvet, Gonzalez-Eiras and Sodini, 2004; Henderson, Pearson and Wang, 2015).

Finally, this study speaks to the general equilibrium effects of the class of financial products studied in Celerier and Vallee (2017), Vokata (2021), Calvet, Célérier, Sodini and Vallee (2023), and Vokata (2023), retail structured products, which have been shown to be effective at catering to, or mitigating, household behavioral biases.

II. Background and Data

A. SPPs: Definition and Market Development

Short Put Products (SPPs), also commonly referred to as *Yied Enhancement Products* (YEPs), are the most popular category of retail structured products. Retail structured products include any fixed-maturity investment products marketed to retail investors that offer a payoff varying automatically and non-linearly with the performance of an underlying asset.[5] Typically designed with embedded options, these products leave no room for discretionary investment decisions before maturity. While the underlying assets are primarily equity indices and individual stocks, they may also include commodities, fixed-income assets, or alternative indices.

A defining feature of SPPs is that their payoff at maturity embeds the sale of a put option, exposing retail investors to tail risk. More precisely, the capital remains protected on the downside as long as the underlying asset stays *above* a predefined *barrier*. If the underlying falls below this barrier, the investor participates in its negative performance. This downside exposure allows intermediaries to enhance the coupon payments offered at regular intervals until maturity – referred to as the *headline rate*. As a result, the payoff formula of a standard SPP is determined by two key parameters, the *headline rate* – the fixed coupon paid at regular interval until maturity, and the *barrier* – typically set between 50% and 80% of the underlying asset's price at issuance. Figure 2 illustrates the payoff diagram of a typical SPP at maturity, while Panel A of Figure 3 presents a histogram of barriers observed in our dataset.

INSERT FIGURE 2

INSERT FIGURE 3

[5]Exchange traded funds, which have payoffs that are a linear function of a given underlying financial index, are not retail structured products.

The first SPPs were introduced in Europe in the early 2000s, since the inception of the retail market for structured products. Since then, their adoption has grown steadily worldwide, with volumes surging across Asia, Europe, and the U.S. alongside the broader expansion of retail structured products. As of 2023, global outstanding volumes of retail structured products exceed $2 trillion, with Asia accounting for 50% and the U.S and Europe each representing 25%.[6] While synthetic Capital Guarantee Products (CGPs) initially dominated the market, SPPs have since emerged as the leading design in most regions, representing nearly 40% of total issuances in 2017 and around $400 billion in outstanding volumes by 2024.[7] A large share of SPPs also embed an autocall feature, meaning that the product is redeemed early if the underlying asset trades above a pre-defined level at a coupon date. This features limits the upside for the investor as the headline rate is likely to be paid only during a short period, and therefore allows to raise the headline rate the product offer. This feature keeps the downside for the investor largely unchanged.

B. Hedging SPP Issuances and the Market for Out of the Money Puts

When issuing structured products, banks typically mitigate the risk of large payouts to investors by buying or selling options that offset these liabilities. In the case of SPPs, the issuing bank sells put options that mirror the exposure of the end user, or roll similar positions with a shorter maturity. Banks often rely on a combination of initial hedging with options and dynamic vega-hedging using the underlying asset. This latter approach is necessary because the autocall feature creates a Vanna exposure, i.e. a sensitivity of the product vega to changes in the underlying price (and volatility) .

A key characteristic of SPPs is the large relative size of their market compared to the put option markets where banks hedge. To hedge these products, banks typically sell deeply out-of-the money puts with relatively long maturity. However, the open-interest on these markets is generally limited, often not exceeding a few hundred billion dollars. The largest

[6]See SRP Global Market Review: H1 2024.

[7]See Calvet et al. (2023) for more details on CGPs.

option market is in the U.S., where institutional investors such as pension funds and hedge funds actively participate and buy puts to protect their portfolios against pronounced drops. Europe follows as the second-largest market, while Asia – despite having the largest share of SPP outstanding volumes as of 2015 – has a relatively small options market.

C. Data

C.1. SPPs Volumes, Underlying Assets, and Payoff Formula

We obtain detailed information on all retailed structured products issued globally since market inception, including SPPs, from a specialized private data provider. For each product, we observe key characteristics such as the underlying asset, maturity, exact payoff structure, issuance volume, issuance and redemption date.

While existing literature on retail structured products has primarily focused on specific geographic areas, such as Europe and the U.S., this study leverages the dataset's global coverage, exploiting data from products issued in North America, Europe and Asia. This comprehensive coverage provides a rich cross-section for analysis and is crucial for our study, as a large share of these products are structured on foreign underlying assets. Consequently, observing both domestic and foreign issuances is essential to fully capture the demand pressure on put option markets.

Our dataset includes all SPPs issued globally since market inception that matured before 2020. In total, it covers 362,000 products issued across 52 countries, representing $1.3 trillion of cumulated issuance (in current dollars). To enhance the accuracy of each product's headline rate and barrier, we apply a textual analysis algorithm to the payoff description, following the approach in Celerier and Vallee (2017). This algorithm also enables us to identify key payoff features that influence headline rates.[8]

We focus our analysis on index-linked SPPs, which represent 45% of SPP total volumes, 188 000 products, and $650 billion in cumulated issuances over the 2005-2019 period. Our

[8]The code is available in the Internet Appendix.

focus on index-linked SPPs is motivated by two key factors. First, we have access to high-quality volatility surfaces for the top 12 indices, which collectively cover more than 99% of index-linked SPPs. Second, these 12 indices represent over 70% of global market capitalization, making their impact on financial stability particularly relevant. The 12 indices, ranked by decreasing SPP market share size, are Eurostoxx 50, Nikkei, S&P 500, Hang Seng China Enterprise Index, Kospi 200, FTSE 100, Russell, Hang Seng Index, CAC 40, Swiss Market Index, DAX, and MIB Index.

C.2. Implied Volatility

For the three main SPP underlying indices – Eurostoxx 50, Nikkey, and S&P500 – we obtain a monthly panel dataset of implied volatility surfaces from a trading desk of a global investment bank. This dataset spans 2002 to 2019 and provides implied volatility at the underlying asset, moneyness, and maturity levels. For each index at time t, we observe implied volatility for moneyness levels ranging from 50% to 150% in 10% increments, with maturities of 3, 12, and 24 months. This data is of higher quality than the one from commercial vendor because the trading desk continuously model the volatility surface, even in the absence of listed trades.

For all 12 indices, we obtain volatility surfaces, prices, realized volatility, and open interest from the OptionMetrics IvyDB Global Indices dataset. For each index at time t, we observe implied volatility across different strike prices. We merge these volatility surfaces with index price data to express strikes as a percentage of the underlying asset price. To ensure data reliability, we validate OptionMetrics data for strikes above 80% by comparing it against the investment bank's volatility surface.

Finally, we complement our dataset with swap rates for distributing countries, sourced from Bloomberg. Specifically, we include German swap rates for all European countries, along with swap rates for Canada, China, Hong Kong, Japan, South Korea, Singapore, Sweden, Switzerland, Taiwan, the UK, and the U.S.

C.3. Monthly Panel

To perform our empirical analysis, we construct a monthly panel of underlying indices spanning the 2005-2019 period. For each month-underlying observation, the panel includes issuance and outstanding volumes of SPPs, volatility prices at three maturity across different strikes, as well as average headline rates, realized volatility, and swap rates.

For each index, issuance volumes represent the total volume of SPPs that use the index as an underlying asset, divided by the total number of underlying assets in each product. This adjustment accounts for the fact that 88% of SPPs have more than one underlying asset, with an average of 2.5 underlying assets per product. To measure outstanding volumes, we compute the rolling sum of issuances, subtracting the rolling sum of matured products while accounting for early redemptions.

The final panel consists of 2,160 observations spanning 180 months across twelve indices.

D. Summary Statistics

Figure 4 illustrates the growth of the SPP market across the U.S., Europe, and Asia since 2005, highlighting its significant size relative to corresponding put option markets. The figure demonstrates the rapid adoption of SPPs in Europe and Asia, as well as their more recent expansion in the U.S.. On average over the period, global outstanding volumes of index-linked SPPs accounted for 16% of the open interest in index-linked put options worldwide.

INSERT FIGURE 4

Tables I provides summary statistics on all variables included in our empirical analysis. The strike of the embedded put option ranges from 50% of the initial value of the underlying (10^{th} percentile) to 70% (90^{th} percentile). The median maturity is 3 years. On average, SPPs offer a headline rate and excess ex-post return of 6.5% (median 6%) and 4.9% (median 5.1%), respectively. In addition, most SPPs earned positive excess returns, as confirmed in

Panel B of Figure 3. Panel B of Figure 3 reports the distribution of ex post excess returns across SPPs.

INSERT TABLE I

The second panel of Tables I provides summary statistics on our monthly panel of SPP volumes, volatility prices, and put option open interest. While the median outstanding volumes of SPPs amount to 3.4 billion \$, the median open interest of put options of strikes at 80% or below amounts and maturity higher than three months amounts to 12.5 billion \$, suggesting a possible large impact of SPPs on the price of tail risk.

III. Empirical Results

We establish two sets of reduced-form findings. On the SPP market side, we provide evidence that net issuance volumes increase when realized volatility is higher, as the headline rates SPPs offer are higher under these market conditions. On the option market side, we find that the price of volatility decreases when SPP volumes increase. This relationship is more pronounced when the market has experienced a significant drop recently, as the SPP autocall feature is then unlikely to be triggered. A natural experiment in South Korea leveraging a targeted SPP ban strengthens a causal interpretation of the relationship between SPP volumes and volatility prices.

A. Realized Volatility and SPP Issuances

We first document that the headline rate of SPPs significantly varies with market conditions, namely realized volatility and the level of interest rates. In Figure 5, we plot the average headline rates of SPP linked to the EuroStoxx 50 and the 90-day realized volatility for that index between 2005 and 2020. We observe a clear positive correlation between these two quantities. This relationship is intuitive as the premium of a put increases when the volatility of the underlying asset is high, and SPPs embed a short position in puts. The relationship between headline rates and interest rates has been first uncovered in Celerier and Vallee (2017), although on a shorter and narrower sample.

INSERT FIGURE 5

To more precisely measure the relationship between headline rates and market conditions, we run the following regressions at the SPP issuance level:

$$\log(HeadlineRate_i) = \alpha + \beta_v HistVol_{a,t} + \beta_r IR_t \gamma_i + \epsilon_i, \tag{1}$$

where $HeadlineRate_i$ is the headline rate of issuance i, $HistVol_t$ is the 90-day realized volatility in month t of the underlying asset a, and IR_t is the 3-year CMS swap rate in the country of issuance in month t, and γ_i are a set of fixed effects, namely month, underlying assets, product design and country fixed effects. Regression coefficients are presented in Table II. We observe that even in this saturated specification the level of realized volatility is highly predictive of the headline rate offered by a SPP, with an elasticity around 0.4. Interest rates also predict headline rate, which is consistent with banks aiming to preserve the headline rate when interests come down, as documented in Celerier and Vallee (2017). The economic magnitude of this relationship is less pronounced than the one with volatility, which is to be expected as given that volatility has a more direct effect on the designs that banks can structure.

INSERT TABLE II

We also investigate whether headline rates, and relatedly realized volatility, are predictive of SPP net issuance volumes.[9] We regress the net issuance volume of issuance for SPP linked

[9]Net issuance is the volume of SPP issuance in a month minus the volume that is redeemed in that month, early or not. Banks typically issue a new SPP when a given issuance is redeemed, to encourage investors to roll-over their capital. Similar to mutual funds, net flows better capture the demand for the asset class, and the change of notional that banks need to hedge.

to a given index on quartiles of realized volatility, calculated at the index level. Consistent with household preferences for high headline rates, we observe that SPP issuance volume is significantly higher when headline rates offered are high, and relatedly when realized volatility is high. Together, these findings highlight the endogenous nature of SPP design and volume.

INSERT TABLE IV

B. SPP Volumes and Volatility Prices

We now turn to studying whether SPP issuances predicts the level of implied volatility. In Panel A of Figure 1, we plot the average yearly issuance volume of SPP linked to a given index, scaled by the index open interest. In Panel B, we plot the volatility risk premium for the three indices, i.e. the difference between the 1 year implied volatility and the 90-day realized volatility, at the 60 to 100% moneyness, with 10% increments. We observe that the volatility risk premium is negatively correlated with the amount of SPP issuance, and that this relationship is more pronounced for moneyness around 60%.

This relationship is confirmed when looking at the broader sample of indices. We plot the average volatility premium over SPP outstanding volumes scaled by open interest in figure 6 over the 12 indices we previously study. We observe a strong negative relationship in this cross-section.

INSERT FIGURE 6

To gain identification and identify the existence of this relationship in the time-series, we now run the following specification:

$$IV^u_{K=k,t} = \alpha + \beta SPPVolumes + \beta_v HistVol_{a,t} + \beta_r IR_t + \gamma_t + \gamma^u + \varepsilon^u_t, \qquad (2)$$

where $IV^u_{K=k,t}$ is the 1-year implied volatility of underlying asset u and moneyness k, in month t, $SPPVolume_t^u$ is the SPP volume with underlying asset u in month t. γ_t and γ^u are month and underlying fixed effects. Standard errors are clustered at the month level.

Table V provides the coefficient estimates when we use the proprietary volatility surface for the three main indices, S&P 500, Eurostoxx 50, and Nikkei 225. We use absolute SPP issuance volumes in Column 1, and scale SPP issuance volumes by indices market capitalization in Columns 2 to 8. The main advantage of this proprietary dataset of volatility surface is that we can span moneyness from 50 to 100% (Columns 3 to 8).

Overall, we observe that the amount of SPP issuance is predictive of lower implied volatility. This result is robust when running regressions in levels or in logs, and when scaling volumes by indices market capitalization. When looking at the cross-section of moneyness, we observe that the relationship is more pronounced at 60%, and decreases as moneyness rises, consistent with the cross-sectional figure

INSERT TABLE V

To better visualize these relationships, we plot in Figure 7 the coefficient estimates for dummies for quintiles of issuance volumes in Panel A, and strikes, in Panel B in a regression where the dependent variable is the 1 year implied volatility (in level). These figures suggest a concave relationship between SPP volumes and implied volatility, and confirm the monotonic relationship relative to moneyness.

INSERT FIGURE 7

Finally, Figure 2 confirm that we obtain similar result when using the full sample of index-linked SPP and volatility surface data from option metrics.

INSERT TABLE VI

C. Market Downturns

Table VII shows the dampening effects of SPP outstanding volumes in periods of market downturns. We create dummies for index yearly returns below 15%, 25% and 35%. As expected given the well known negative correlation between volatility and returns, we observe that implied volatility increases during market downturns. However, this increase is significantly lower when outstanding SPP volumes are high. The economic magnitude is large, as changing quartiles of SPP outstanding volume can affect implied volatility under market stress by several percentage points. The magnitude is larger when the market drop is more pronounced. This result is consistent with SPPs playing a particularly pronounced dampening role on the price of tail risk in times of stress.

INSERT TABLE VII

D. Natural Experiment

In September 2015, following a warning from the South Korean Financial Services Commission (FSC), the four largest SPP providers in South Korea decided to halt the issuance of HSCEI-linked SPPs. The warning came after a SPP provider announced that HSCEI-linked SPPs issued in April 2015 were expected to hit the barrier and trigger capital losses due to the sharp decline in the HSCEI index. The FSC cautioned SPP providers, stating that it might consider banning SPPs linked to certain indices. In response, all major SPP providers in South Korea ceased issuing HSCEI-linked SPPs. Panel A of Figure 8 illustrates the drop in the issuance of HSCEI-linked SPPs after the warning.

INSERT FIGURE 8

Panel B of Figure 8 displays the associated impact on the volatility surface: following the regulator's warning, volatility prices increased, with the effect being particularly pronounced at lower strike prices. The figure plots the HSCEI volatility surface before and after the ban, highlighting the shift. The sharpness of the shock allows us to gain causal identification.

A natural concern relates to the endogeneity of the regulator's decision, as it was issued in response to prevailing market conditions, namely a strong drop in the index value. However, if anything, the primary endogeneity concern would bias results in the opposite direction of our findings. The FSC's warning was issued after a 35% drop in the HSCEI index and several months of high volatility, as shown in Panel A of Figure 8. The endogeneity bias would therefore predict lower implied volatility after the warning, as markets tamper off, not higher as we observe.

IV. Structural Model

Having established that SPP volumes are endogenous to the security design, and that SPP volumes are predictive of implied volatility levels, we develop a quantitative model that captures the equilibrium that jointly determines these three outcome variables. This theoretical exercise allows us to quantify these equilibrium effects by implementing counterfactual exercises, and to investigate how household preferences affect volatility prices through the development of innovative security design. In this framework, we consider market conditions, namely the level of historical volatilty and interest rate, as state variables, which also allows to condition our findings for these dimensions. Motivated by our empirical analysis, we focus on the headline rate as the main parameter of the SPP design, and consider the barrier and maturity as fixed.

A. Assets

We consider a discrete-time financial market defined at dates $t = 0, \dots, \infty$. In the empirical implementation of the model, the time index t will refer to a month. At each date t, financial market participants can trade four securities: (i) a riskless zero-coupon bond, (ii) an equity index fund, (iii) a short put product (SPP) written on the equity index, and (iv) a set of plain-vanilla European puts on the equity index. The zero-coupon bond, the SPP, and the European puts all reach maturity at date $t + T$. In the empirical application, we set T equal to 36 months, which is the median maturity of an SPP on the Eurostoxx 50.

Zero-coupon bond and equity index. Let $R_{f,t:t+T}$ denote the net arithmetic return on the zero-coupon bond between t and $t+T$, and let $R_{\mathrm{M},t:t+T}$ denote the net arithmetic return on the equity index over the same period. The zero-coupon bond and the equity index have infinitely elastic supplies, so that their returns are taken as exogenous to the model.

The exogenous state vector:

$$z_t = (z_{1,t}, z_{2,t})'. \tag{3}$$

captures time-varying conditions in the bond and equity markets. The state variable $z_{1,t}$ drives relevant interest rates and the state variable $z_{2,t}$ the volatility of the equity index return, as we now explain.

We define the state variable $z_{1,t}$ as the yield on the riskless zero-coupon bond issued at t and maturing at date $t+T$, expressed in annual units. We assume that the yield satisfies a first-order autoregression:

$$z_{1,t} - \mu_1 = \phi_1(z_{1,t-1} - \mu_1) + \sigma_1\, \varepsilon_{1,t}$$

for every t. The return on the zero-coupon bond over its life, $R_{f,t:t+T}$, is linked to the annualized yield, $z_{1,t}$, via the usual relationship: $R_{f,t:t+T} = (1 + z_{1,t})^{T/12} - 1$.

For simplicity, we assume that the spread between the T-period zero yield and the 1-period zero yield is time-invariant, and we denote by π denote this constant spread. The net arithmetic interest rate per period is therefore $R_{f,t:t+1} = (1 + z_{1,t} - \pi)^{1/12} - 1$ for every t.

We model the equity index by assuming that its log excess return follows a process with constant mean and stochastic volatility. Specifically, let $r^e_{\mathrm{M},t:t+1} = \ln(1 + R_{\mathrm{M},t:t+1}) - \ln(1 + R_{f,t:t+1})$ denote the log excess return on equity between t and $t+1$. We posit that:

$$r^e_{\mathrm{M},t:t+1} = \mu_r + e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1}, \tag{4}$$

where μ_r is a constant, $z_{2,t+1}$ is the state variable controlling stochastic volatility, and $\varepsilon_{\mathrm{M},t+1}$

is a stochastic shock. The state variable $z_{2,t}$ satisfies the first-order autoregression:

$$z_{2,t} - \mu_2 = \phi_2(z_{2,t-1} - \mu_2) + \sigma_2\, \varepsilon_{2,t} \tag{5}$$

in every period t. The dynamics of the equity index return between t and $t+T$ follows from the identity $R_{\text{M},\,t:t+T} = (1 + R_{\text{M},t:t+1}) \dots (1 + R_{\text{M},T-1:T}) - 1$.

We close the specification of bond and equity returns by assuming that the vector of stochastic shocks, $\varepsilon_{t+1} = (\varepsilon_{1,t+1}, \varepsilon_{2,t+1}, \varepsilon_{\text{M},t+1})'$, is independent through time. Furthermore, it is Gaussian with zero mean and variance-covariance matrix:

$$\Sigma = \begin{pmatrix} 1 & \rho_{1,2} & \rho_{1,\text{M}} \\ \rho_{1,2} & 1 & \rho_{2,\text{M}} \\ \rho_{1,\text{M}} & \rho_{2,\text{M}} & 1 \end{pmatrix} \tag{6}$$

in every period t.

To sum up, equations (3)-(6) fully specify the dynamics of bond and equity returns. Throughout the paper, we consider the state vector, z_t, the vector of shocks, ε_t, and the parameters μ_1, ϕ_1, σ_1, μ_r, μ_2, ϕ_2, σ_2, $\rho_{1,2}$, $\rho_{1,\text{M}}$, and $\rho_{2,\text{M}}$ as exogenous inputs of the general equilibrium model.

Short put product. The SPP is issued at t and is illiquid until it reaches maturity at date $t+T$. Its payoff at date T depends on a barrier, B, defined at the contract's origination, and is contingent on the performance of the equity index over the period, $R_{\text{M},t:t+T}$. If the return is above the barrier, the SPP pays off a coupon rate y_t. Otherwise, the SPP has the same return as the equity index. More compactly, the net payoff of the SPP per unit of capital invested is given by:

$$R_{\text{SPP},\,t:t+T} = y_t\, \mathbb{1}_{\left\{R_{\text{M},\,t:t+T} > B\right\}} + R_{\text{M},\,t:t+T}\, \mathbb{1}_{\left\{R_{\text{M},\,t:t+T} \leq B\right\}},$$

where y_t is the coupon rate per period and B is the barrier. For SPPs written on the Eurostoxx 50, the barrier B has a median value of -40%. To avoid arbitrage opportunities, we we assume that $B < 0$ and $R_{f,t:t+T} > B/2$. in every state of the world.

In the general equilibrium model defined in Section IV.C, the barrier is taken as exogenous, but the coupon rate per period is endogenously determined by supply and demand forces. The coupon yield will then be a function of the state:

$$y_t = Y(z_t), \tag{7}$$

where the mapping $Y(\,\cdot\,)$ is determined in general equilibrium.

Puts. Investors can trade a set of plain-vanilla puts written on the equity index, which are issued at t and reach maturity at date $t+T$. Their strikes are exogenous and denoted by $K_1, \ldots, K_N$. A European put with strike K_n pays off $(K_n - 1 - R_{\text{M},\,t:t+T})_+$ at date $t+T$, where x_+ denotes the maximum of x and 0 for every real number x. The prices of the European puts, $P_{\text{PUT},n,t}$, and their net returns, $R_{\text{PUT},n,t} = (K_n - 1 - R_{\text{M},\,t:t+T})_+/P_{\text{PUT},n,t} - 1$, are endogenously determined in general equilibrium.

An important property of our setup is that the SPP can be hedged partially, but not fully, by trading the zero-coupon bond, the equity index, and European puts at date t. The explanation is that the payoff of the SPP is discontinuous at $R_{\text{PUT},n,t} = B$, while the assets used for hedging purposes (bond, equity, puts) have continuous payoffs. Put slighlty differently, an SPP contract delivers the same payoff at maturity as a portfolio containing (i) a short position in 1 put option on the equity index with strike $1 + B$, (ii) B units of the zero-coupon bond, and (iii) $(y_t - B)$ units of a digital call options paying \$1 if the equity index return exceeds the barrier. We do not assume that a digital call option of maturity T is traded in our setting, a reflection of the fact that digital call options are thinly traded and difficult to hedge in practice (Gallus, 1999). Hence, the SPP is not a redundant asset in our setting.

B. *Agents*

Every period t, financial markets are populated by three types of investors: a long-lived representative household, a representative dealer, and a group of investors with an inelastic demand for puts.

Households. The representative household at t trades and consumes at dates t, $t+T$, $t+2\,T, \ldots, \infty$. It can trade the riskless asset, the equity index fund, and the SPP. Importantly, it does not have access to put markets.

The household's consumption-portfolio problem is defined as follows. The household starts period t with wealth W_t^H. It allocates C_t to consumption and invests the remaining wealth, $FW_t^H = W_t^H - C_t^H$, in financial assets through the dealer. The dealer receives a fraction ϕ of gross invested wealth, FW_t^H, as compensation for his services. The household's net invested wealth is therefore

$$X_t^H = (1-\phi)\, FW_t^H$$

at the end of period t. The household allocates a fraction $\alpha_{\mathrm{M},t}^H$ of net invested wealth to the index fund, a fraction $\alpha_{\mathrm{SPP},t}^H$ to the SPP, and the remaining fraction, $1-\alpha_{\mathrm{M},t}^H-\alpha_{\mathrm{SPP},t}^H$, to the zero-coupon bond. The financial wealth available to the household at the beginning of period $t+T$ is therefore

$$W_{t+T}^H = X_t^H\,\left(1 + R_{f,t:t+T} + \alpha_{\mathrm{M},t}^H R_{\mathrm{M},t:t+T}^e + \alpha_{\mathrm{SPP},t}^H\, R_{\mathrm{SPP},t:t+T}^e\right),$$

where $R_{\mathrm{M},t:t+T}^e = R_{\mathrm{M},t:t+T} - R_{f,t:t+T}$ and $R_{\mathrm{SPP},t:t+T}^e = R_{\mathrm{SPP},t:t+T} - R_{f,t:t+T}$ denote, respectively, the excess return on the index fund and the SPP over the period.

The household selects consumption and portfolio weights by maximizing the Epstein-Zin recursive utility:

$$V^H(W_t^H; z_t, Y) = \max_{\{C_t^H, \alpha_{\mathrm{M},t}^H, \alpha_{\mathrm{SPP},t}^H\}} \left[(1-\delta^T)\left(C_t^H\right)^{1-1/\psi} + \delta^T \left(\mu_{t+T}^H\right)^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}, \tag{8}$$

where Y is the state-contingent SPP coupon yield defined in (7) and

$$\mu_t^H = \left\{\mathbb{E}_t^{\mathbb{P}}[V^H(W_{t+T}^H)^{1-\gamma}]\right\}^{\frac{1}{1-\gamma}}$$

is the certainty equivalent of future consumption. The household is a price-taker viewing the state-contingent SPP yield and other asset prices as exogenous. We denote by

$$\alpha_{\mathrm{SPP},t}^H = \alpha_{\mathrm{SPP}}^H(z_t, Y)$$

the optimal SPP weight in period t.

Dealers. The dealer available in period t has no initial wealth but receives from the household the management fee $X_t^D = \phi F W_t^H = \phi X_t^H/(1-\phi)$, or equivalently

$$X_t^D = X_t^H/\lambda^H,$$

where $\lambda^H = (1-\phi)/\phi$. The dealer allocates a fraction $\alpha_{\mathrm{M},t}^D$ of his wealth X_t^D to the index fund, a fraction $\alpha_{\mathrm{SPP},t}^D$ to the SPP, a fraction $\alpha_{\mathrm{PUT},n,t}^D$ to each put n, and the remaining fraction to the zero-coupon bond.

The dealer's wealth in period $t+T$ is

$$W_{t+T}^D = X_t^D\left(1 + R_{t:t+T}^D\right),$$

where

$$R^D_{t:t+T} = R_{f,\,t:t+T} + \alpha^D_{\text{M},t}\, R^e_{\text{M},\,t:t+T} + \alpha^D_{\text{SPP},t}\, R^e_{\text{SPP},\,t:t+T} + \sum_{n=1}^{N} \alpha^D_{\text{PUT},n,t}\, R^e_{\text{PUT},n,\,t:t+T} \tag{9}$$

denotes the dealer's portfolio return, and $R^e_{\text{PUT},n,\,t:t+T} = R_{\text{PUT},n,\,t:t+T} - R_{f,t:t+T}$ is the excess return of each put n over the period.

The dealer has constant absolute risk aversion (CARA) utility, $U_t = -\mathbb{E}^{\mathbb{P}}_t(e^{-A_t\, W^D_{t+T}})$. The CARA specification is commonly used in intermediary asset pricing (see, e.g., He and Krishnamurthy (2018)) because it permits the dealer to absorb the losses than can be caused by imperfect hedging or mismatches between supply and demand. We assume that the CARA coefficient in period t satisfies $A_t = \gamma^D / X^D_t$, which allows us to obtain stationary portfolio weights, as we now explain.

The dealer solves the optimization problem

$$\max_{\alpha^D_t} \mathbb{E}_t \left[-e^{-\gamma^D\,(W^D_{t+T}/X^D_t)} \right],$$

where $\alpha^D_t = (\alpha^D_{\text{M},t}, \alpha^D_{\text{SPP},t}, \alpha^D_{\text{PUT},1,t}, \ldots, \alpha^D_{\text{PUT},N,t})$. Since $W^D_{t+T}/X^D_t = 1 + R^D_{t:t+T}$, the solution to this problem is independent of the wealth level X^D_t. We denote by

$$\alpha^D[z_t; Y(z_t)]$$

the optimal portfolio of the dealer.

We observe that since the dealer is short-lived, his portfolio depends on the current state of the bond and equity markets, z_t, and on the SPP coupon yield in the current state, $Y(z_t)$. By contrast, the household's optimal portfolio depends on the coupon yield in *all* states, as the household is long-lived and the mapping Y characterizes all current and future investment opportunities.

Exogenous Demand for Puts. We assume that an unmodelled group of investors has an inelastic demand for puts, as in Gârleanu et al. (2009). Specifically, this group invests

$$d_n^E \, X_t^D$$

units of each put n, where $d_n^E \in \mathbb{R}$. We take the number of puts demanded per unit of wealth, d_n^E, as exogenous. Importantly, the exogenous demand, $d_n^E \, X_t^D$, grows with the size of the economy, so that the pricing impact of the demand pressure remains sizable over time.

C. Stationary General Equilibrium

We focus on a stationary general equilibria, in which the joint distribution of asset returns conditional on the state, z_t, remains the same over time.

Definition 1 (Stationary General Equilibrium): *A stationary general equilibrium consists of a state-contingent SPP yield, $Y(z_t)$, and state-contingent put prices, $P_{\text{PUT},n}(z_t)$, such that markets clear:*

$$\begin{aligned} \alpha^D_{\text{SPP},t}[z_t; Y(z_t)] \, X_t^D + \alpha^H_{\text{SPP},t}(z_t, Y) \, X_t^H &= 0 \\ \alpha^D_{\text{PUT},n}[z_t; Y(z_t)] \, X_t^D + d_n^E \, X_t^D \, P_{\text{PUT},n}(z_t) &= 0 \end{aligned}$$

in every period t and state z_t.

We divide the equilibrium equations by the dealer's wealth and obtain:

$$\alpha^D_{\text{SPP},t}[z_t; Y(z_t)] = -\lambda^H \, \alpha^H_{\text{SPP},t}(z_t, Y) \tag{10}$$

$$\alpha^D_{\text{PUT},n}[z_t; Y(z_t)] = -d_n^E \, P_{\text{PUT},n}(z_t). \tag{11}$$

These equations show that the dealer offsets the position of the household sector in the SPP market, and the exogenous demand in the market for each put. In general equilibrium the dealer finds it optimal to offset the positions of other agents.

PROPOSITION 1: *In a stationary general equilibrium, the SPP yield, $Y(z_t)$, and the dealer's hedge, $\alpha^D_{\text{M}}[z_t, Y(z_t)]$, satisfy:*

$$\mathbb{E}\left(R^e_{\text{M},t:t+T}\, M_{t+T}\middle|\, z_t\right) = 0, \tag{12}$$

$$\mathbb{E}\left(R^e_{\text{SPP},t:t+T}\, M_{t+T}\middle|\, z_t\right) = 0, \tag{13}$$

where the excess return on the SPP satisfies

$$R^e_{\text{SPP},t+1} = Y(z_t)\, \mathbb{1}_{\left\{R_{\text{M},t:t+T}>B\right\}} + R_{\text{M},t:t+T}\, \mathbb{1}_{\left\{R_{\text{M},t:t+T}\leq B\right\}} - R_{f,t:t+T},$$

and the stochastic discount factor is given by:

$$M_{t+T} = e^{-\gamma^D\left\{\alpha^D_{\text{M}}[z_t,Y(z_t)]\, R^e_{\text{M},t:t+T} - \lambda^H\, \alpha^H_{\text{SPP}}(z_t,Y)\, R^e_{\text{SPP},t:t+T} - \sum_{n=1}^{N} d^E_n\, (K_n - 1 - R_{\text{M},t:t+T})_+\right\}}.$$

The price of each put n is given by:

$$P_{\text{PUT},n}(z_t) = \frac{\mathbb{E}_t\left[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T}\middle|\, z_t\right]}{(1 + R_{f,t:t+T})\mathbb{E}\left(M_{t+T}\middle|\, z_t\right)}, \tag{14}$$

for every t and z_t.

We henceforth denote by $\mathcal{V}_n(z_t)$ the implied volatility of put n resulting from $P_{\text{PUT},n}(z_t)$.

PROPOSITION 2: *There exists a unique stationary equilibrium.*

D. Estimation Strategy

The estimation is based on the following structural equations:

$$
\begin{aligned}
z_{1,t} &= \mu_1 + \phi_1(z_{1,t-1} - \mu_1) + \sigma_1\,\varepsilon_{1,t}, && (15)\\
z_{2,t} &= \mu_2 + \phi_2(z_{2,t-1} - \mu_2) + \sigma_2\,\varepsilon_{2,t}, && (16)\\
r^e_{\mathrm{M},t} &= \mu_r + e^{z_{2,t}/2}\varepsilon_{\mathrm{M},t} && (17)\\
y_{SPP,t} &= Y(z_t) + \sigma_{spp} u_t && (18)\\
IV_{n,t} &= \mathcal{V}_n(z_t) + \sigma_{IV}\eta_{n,t} && (19)
\end{aligned}
$$

The model is specified by the following parameters. The interest rate and equity index returns are parameterized by $\mu_1, \phi_1, \sigma_1, \mu_r, \mu_2, \phi_2$, and σ_2. In simulations, we have verified that the demand for the SPP, α exhibits limited variation with respect to the patience parameter, δ, and the elasticity of intertemporal subsitution, ψ. For this reason, we calibrate and we estimate the model parameters

$$
\theta = (\underbrace{\mu_1, \phi_1, \sigma_1, \mu_r, \mu_2, \phi_2, \sigma_2, \rho_{1,2}, \rho_{1,\mathrm{M}}, \rho_{2,\mathrm{M}}}_{\text{assets}}, \underbrace{\gamma, \lambda^H}_{\text{household}}, \underbrace{\gamma^D}_{\text{dealer}}, \underbrace{d^E}_{\text{demand pressure}}, \underbrace{\sigma_{spp}, \sigma_{IV}}_{\text{measurement}}).
$$

The vector θ has $K + 15$ components, where K is the number of puts.

The estimation of θ requires the following. For a given θ, we compute the household's demand schedule for the SPP, $\alpha^H_{\text{SPP}}[z_t; Y(z_t)]$, by solving the Bellman equation (1). We then obtain the dealer's demand for the equity index and the SPP yield spread in each state z_t by solving the set of first-order conditions (10) and (11).

We estimate the parameter vector θ by likelihood-based inference. The observation vector contains the interest rate, the equity excess return, and the endogenous prices:

$$
y_t = (z_{1,t}, r^e_{\mathrm{M},t}, X_t), \tag{20}
$$

where

$$X_t = (y_{SPP,t}, IV_{1,t}, \ldots, IV_{n,t}) \tag{21}$$

contains equilibrium quantities;

The one-step likelihood of y_{t+1} is given by

$$f(y_{t+1}|y_{1:t}) \;\; = \;\; \iint h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) f(z_{2,t}, z_{2,t+1}|y_{1:t}) dz_{2,t} dz_{2,t+1}.$$

We note that $h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) = h(y_{t+1}|z_{1,t}, z_{2,t}, z_{2,t+1})$ and therefore

$$\begin{aligned} h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) \;\; &= \;\; f(X_{t+1}|z_{1,t+1}, z_{2,t+1}) \;\; f(r^e_{M,t+1}|z_{1,t}, z_{2,t}, z_{1,t+1}, z_{2,t+1}) \\ & \quad \times \; f(z_{1,t+1}|z_{1,t}, z_{2,t}, z_{2,t+1}), \end{aligned}$$

where $f(X_{t+1}|z_{1,t+1}, z_{2,t+1})$, $f(R^e_{M,t+1}|z_{1,t+1}, z_{2,t+1}, z_{1,t}, z_{2,t})$ and $f(z_{1,t+1}|z_{2,t+1}, z_{1,t}, z_{2,t})$ have closed-form expressions provided in the Appendix.

Since the volatility state is not observed directly, we sequentially approximate its distribution by a particle filter $\{z^{(b)}_{2,t}\}^B_{b=1}$.

1. We simulate forward each particle and obtain $\{(z^{(b)}_{2,t}, \tilde{z}^{(b)}_{2,t+1})\}^B_{b=1}$, which targets the distribution of $(z_{2,t}, z_{2,t+1})$ conditional on $y_{1:t}$.

2. We approximate the one-step ahead likelihood by:

$$\hat{f}(y_{t+1}|y_{1:t}) = \frac{1}{B} \sum_{b=1}^{B} h(y_{t+1}|y_t, z^{(b)}_{2,t}, \tilde{z}^{(b)}_{2,t+1})$$

3. We assign to each particle the nonormalized importance weight:

$$p^{(b)}_{t+1} = \frac{h(y_{t+1}|y_t, z^{(b)}_{2,t}, z^{(b)}_{2,t+1})}{\sum_{b'=1}^{B} h(y_{t+1}|y_t, z^{(b)}_{2,t}, z^{(b)}_{2,t+1})}$$

4. We sample the particle $z^{(b)}_{2,t+1}$ from the candidate particles $\tilde{z}^{(1)}_{2,t+1}, \ldots, \tilde{z}^{(B)}_{2,t+1}$ with prob-

abilities $p_{t+1}^{(1)}, \ldots, p_{t+1}^{(B)}$. We repeat the same procedure to obtain $z_{2,t+1}^{(2)}, \ldots, z_{2,t+1}^{(B)}$.

E. Estimation Results

Preliminary results from the model estimation are provided in Table VIII. We consider a version of the model under which investors neglect the probability that the equity index may generate a return lower than the barrier and the SPP may therefore pay less than the headline rate. The demand for the SPP is then substantial. The unrestricted estimate in column 1 corresponds to our baseline model. The restricted estimate in column2 corresponds to model where λ_h is set to zero, i.e. where household demand for SPP cannot play a role in the stochastic discount factor. The comparison between these two estimations allows us to reject the null hypothesis that household demand for SPPs do not play a role in the stochastic discount factor.

The SPP yield spread increase with the interest rate and equity volatility, which is consistent with the empirical evidence in Figure 5.

INSERT TABLE VIII

In Figure 9, we compare the implied volatility curve produced by the model estimation with the average one observed in the data, along SPP volumes and across moneyness. The model outputs closely match their empirical counterparts, which provides a validation for the framework.

INSERT FIGURE 9

V. Conclusion

This study uncovers how the development of Short Put Products (SPPs) has significant implications for volatility pricing and financial stability. By allowing households to implicitly sell deep out-of-the-money put options, SPPs create a persistent demand for left-tail risk exposure, which in turn affects its equilibrium pricing on the option market. Higher SPP issuance volumes suppress implied volatility and the volatility risk premium, particularly at the moneyness levels corresponding to common SPP barriers. These effects are amplified in periods of market stress, when early redemption of existing products becomes unlikely, reinforcing the role of SPPs in volatility price suppression.

To explain these empirical patterns, we develop a demand based asset pricing framework that captures the interactions between households, intermediaries, and option markets. Our estimation results suggest that household preferences?such as underweighting left-tail risk?drive demand for SPPs, which in turn influences equilibrium option prices through intermediaries? hedging behavior. Counterfactual analysis confirms that, in the absence of SPPs, implied volatility would be significantly higher, highlighting the role of these instruments in shaping volatility market dynamics.

These insights have broad implications for financial market participants and policymakers. First, by altering the pricing of downside protection, SPPs can impact risk management strategies for institutional investors. Second, our results suggest a trade-off between investor protection and financial stability: while regulators may seek to limit household exposure to complex financial products, restricting SPPs may also increase the cost of insuring against tail risk, as seen in the South Korean regulatory intervention. Finally, our study underscores the broader equilibrium effects of market innovation, showing that the introduction of new financial instruments can reshape asset pricing by incorporating previously unaccounted-for investor preferences.

Overall, our research contributes to the growing literature on the macrostructure of financial markets by highlighting how financial innovation and household demand interact to influence option pricing and volatility dynamics, reinforcing the need to consider equilibrium effects when assessing whether a given financial innovation is beneficial or harmful.

REFERENCES

Bakshi, Gurdip, Peter Carr, and Liuren Wu, "Stochastic Risk Premiums, Stochastic Skewness in Currency Options, and Stochastic Discount Factors in International Economies," *Journal of Financial Economics*, January 2008, *87* (1), 132–156.

Bollerslev, Tim, George Tauchen, and Hao Zhou, "Expected Stock Returns and Variance Risk Premia," *Review of Financial Studies*, November 2009, *22* (11), 4463–4492.

Brown, David C, Shaun Davies, and Matthew Ringgenberg, "ETF arbitrage and return predictability," *available at SSRN*, 2018.

Calvet, Laurent E, Claire Célérier, Paolo Sodini, and Boris Vallee, "Can Security Design Foster Household Risk-Taking?," *Journal of Finance*, 2023, *78* (4), 1917–1966.

Calvet, Laurent E., Martin Gonzalez-Eiras, and Paolo Sodini, "Financial Innovation, Market Participation, and Asset Prices," *Journal of Financial and Quantitative Analysis*, 2004, *39* (3), 431–459.

Cao, Jie and Bing Han, "Cross Section of Option Returns and Idiosyncratic Stock Volatility," *Journal of Financial Economics*, April 2013, *108* (1), 231–249.

Celerier, Claire and Boris Vallee, "Catering to Investors Through Security Design: Headline Rate and Complexity," *Quarterly Journal of Economics*, 02 2017, *132* (3), 1469–1508.

Figlewski, Stephen, "Options Arbitrage in Imperfect Markets," *Journal of Finance*, 1989, *44* (5), 1289–1311.

Gallus, Christoph, "Exploding Hedging errors for Digital Options," *Finance and Stochastics*, 1999, *3*, 187–201.

Gao, Pengjie, Allen Hu, Peter Kelly, Cameron Peng, and Ning Zhu, "Asset Complexity and the Return Gap," *Review of Finance*, 2024, *28*, 511–550.

Gârleanu, Nicolae, Lasse Heje Pedersen, and Allen M. Poteshman, "Demand-Based Option Pricing," *Review of Financial Studies*, October 2009, *22* (10), 4259–4299.

Greenwood, Robin and Annette Vissing-Jørgensen, "The Impact of Pensions and Insurance on Global Yield Curves," *Harvard Business School Working Paper*, June 2018, *18-109.*

_ **and Dimitri Vayanos**, "Bond Supply and Excess Bond Returns," *Review of Financial Studies*, 2014, *27* (3), 663–713.

Haddad, Valentin and Tyler Muir, "Market Macrostructure: Institutions and Asset Prices," Technical Report, National Bureau of Economic Research 2025.

Han, Bing and Yi Zhou, "Variance Risk Premium and Cross-Section of Stock Returns," *unpublished paper, University of Texas at Austin*, 2012.

He, Zhighuo and Arvind Krishnamurthy, "Intermediary Asset Pricing and the Financial Crisis," *Annual Review of Financial Economics*, 2018, *10*, 173–197.

Henderson, Brian J, Neil D Pearson, and Li Wang, "New Evidence on the Financialization of Commodity Markets," *Review of Financial Studies*, 2015, *28* (5), 1285–1311.

Koijen, Ralph SJ and Motohiro Yogo, "The Fragility of Market Risk Insurance," *Journal of Finance*, 2022, *77* (2), 815–862.

Todorov, Viktor, "Variance Risk-Premium Dynamics: The Role of Jumps," *Review of*

Financial Studies, January 2010, *23* (1), 345–383.

Vayanos, Dimitri and Jean-Luc Vila, "A Preferred-Habitat Model of the Term Structure of Interest Rates," Technical Report w15487, National Bureau of Economic Research, Cambridge, MA November 2009.

Vokata, Petra, "Engineering Lemons," *Journal of Financial Economics*, 2021, *142* (2), 737–755.

_ , "Salient Attributes and Household Demand for Security Designs," *Working Paper*, 2023.

Proof of Proposition 1

The dealer selects the portfolio weight of equity, $\alpha^D_{\text{M},t}$, the weight of the SPP, $\alpha^D_{\text{SPP},t}$ and weight of each put, $\alpha^D_{\text{PUT},n,t}$, that maximizes expected utility. The corresponding first-order conditions are:

$$\begin{aligned}
\mathbb{E}_t\left[R^e_{\text{M},\,t:t+T}\, e^{-\gamma^D\left(\alpha^D_{\text{M},t}\, R^e_{\text{M},\,t:t+T}+\alpha^D_{\text{SPP},t}\, R^e_{\text{SPP},\,t:t+T}+\sum_{n=1}^N \alpha^D_{\text{PUT},n,t}\, R^e_{\text{PUT},n,\,t:t+T}\right)}\right] &= 0,\\
\mathbb{E}_t\left[R^e_{\text{SPP},t+1} e^{-\gamma^D\left(\alpha^D_{\text{M},t}\, R^e_{\text{M},t+1}+\alpha^D_{\text{SPP},t}\, R^e_{\text{SPP},t+1}+\sum_{n=1}^N \alpha^D_{\text{PUT},n,t}\, R^e_{\text{PUT},n,t+1}\right)}\right] &= 0,\\
\mathbb{E}_t\left[R^e_{\text{PUT},n,t+1} e^{-\gamma^D\left(\alpha^D_{\text{M},t}\, R^e_{\text{M},t+1}+\alpha^D_{\text{SPP},t}\, R^e_{\text{SPP},t+1}+\sum_{n=1}^N \alpha^D_{\text{PUT},n,t}\, R^e_{\text{PUT},n,t+1}\right)}\right] &= 0,
\end{aligned}$$

for every $n \in \{1,\ldots,N\}$. Let

$$M^0_{t+T} = e^{-\gamma^D\left(\alpha^D_{\text{M},t}\, R^e_{\text{M},\,t:t+T}+\alpha^D_{\text{SPP},t}\, R^e_{\text{SPP},\,t:t+T}+\sum_{n=1}^N \alpha^D_{\text{PUT},n,t}\, R^e_{\text{PUT},n,\,t:t+T}\right)}. \tag{1}$$

We infer from (4) and (5) that

$$M^0_{t+T} = e^{-\gamma^D\left[\alpha^D_{\text{M},t}\, R^e_{\text{M},\,t:t+T}-\lambda^H\alpha^H_{\text{SPP},t}\, R^e_{\text{SPP},\,t:t+T}-\sum_{n=1}^N d^E_n\,(K_n-1-R_{\text{M},t:t+T})_+ +(1+R_{f,t:t+T})\sum_{n=1}^N d^E_n P_n\right]}. \tag{2}$$

Since $(1+R_{f,t:t+T})\sum_{n=1}^N d^E_n P_n$ is known conditional on z_t, we infer that is also a stochastic discount factor.

The first-order condition for holdings of put n can be rewritten as:

$$\mathbb{E}_t\left[(K_n-1-R_{\text{M}})_+\, M_{t+T}\right] = (1+R_{f,t})P_n\mathbb{E}_t\left[(K_n-1-R_{\text{M}})_+\, M_{t+T}\right],$$

and we conclude that the proposition holds.

Equity Premium

We assume the following:

$$r^e_{\text{M},t+1} = \mu_r(z_t) + e^{z_{2,t+1}/2}\varepsilon_{\text{M},t+1} \tag{1}$$

$$\mathbb{E}_t(r^e_{\text{M},t+1}) = \lambda\, Var_t(r^e_{\text{M},t+1}) \tag{2}$$

$$\mathbb{E}_t(R_{\text{M},t:t+T} - R_{f,t:t+T}) = RP, \tag{3}$$

where RP is an exogenous constant. The values of $\mu_r(z_t)$ and λ are unique and can be computed as follows.

LEMMA 2: *The drift in each state is given by*

$$\mu_r(z_t,\lambda) = \lambda\,\mathbb{E}_t\left\{\left[e^{z_{2,t+1}/2}\varepsilon_{\text{M},t+1} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\text{M},t+1})\right]^2\right\} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\text{M},t+1}). \tag{4}$$

The parameter λ is the unique solution to the equation:

$$\rho(\lambda) = RP, \tag{5}$$

where

$$\rho(\lambda) = \mathbb{E}\left\{\prod_{i=1}^{T}(1+z_{1,t+i-1})^{\frac{1}{12}}\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i})\right] - (1+z_{1;t})^{T/12}\right\} \tag{6}$$

for every $\lambda \in \mathbb{R}$.

Proof. The system has the following properties. Since

$$\mathbb{E}_t(r^e_{\mathrm{M},t+1}) = \mu_r(z_t) + \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1}),$$

we obtain that

$$Var_t(r^e_{\mathrm{M},t+1}) = \mathbb{E}_t\left\{\left[e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1})\right]^2\right\},$$

and therefore

$$\mu_r(z_t) = \lambda\,\mathbb{E}_t\left\{\left[e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1})\right]^2\right\} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1}). \tag{7}$$

Under the assumptions of the model, we obtain:

$$\begin{aligned}1 + R_{\mathrm{M},t:t+T} &= (1+R_{f,t:t+1})\dots(1+R_{f,t+T-1:t+T})\exp\left(r^e_{M,t:t+1} + \dots + r^e_{\mathrm{M},t+T-1:t+T}\right)\\ &= \prod_{i=1}^{T}(1+R_{f,t+i-1:t+i})\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i})\right]\\ &= \prod_{i=1}^{T}(1+z_{1,t+i-1})^{\frac{1}{12}}\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i})\right].\end{aligned}$$

and

$$1 + R_{f,t:t+T} = (1+z_{1,t})^{T/12}, \tag{8}$$

Let

$$\rho(\lambda) = \mathbb{E}\left\{\prod_{i=1}^{T}(1+z_{1,t+i-1})^{\frac{1}{12}}\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i})\right] - (1+z_{1;t})^{T/12}\right\} \tag{9}$$

We obtain λ by solving □

Existence and Uniqueness of One-Step Equilibrium

The consumption-portfolio problem of the household at date t depends both on (i) the value of the SPP yield spread on contracts issued at date t and (ii) the value of the SPP yield spread in future states, which determines the continuation value of future wealth. To prove the existence and uniqueness of a stationary general equilibrium, it is useful to distinguish between current and future spreads.

We accordingly rewrite the household's decision problem as follows. Let $\mathcal{Z}$ denote the state space and let $Y_0 : \mathcal{Z} \to \mathbb{R}_+$ denote a continuous mapping providing the state-contingent SPP yield spread at dates $t+T, t+2T, \ldots, \infty$. The household selects consumption and portfolio weights at date t by maximizing the Epstein-Zin recursive utility:

$$V^H[W_t^H; y(z_t), Y_0, z_t] = \max_{\{C_t^H, \alpha_{\text{M},t}^H, \alpha_{\text{SPP},t}^H\}} \left[(1-\delta^T)\,(C_t^H)^{1-1/\psi} + \delta^T\,(\mu_{t+T}^H)^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}, \tag{1}$$

where the excess return on the SPP is given by:

$$R^e_{\text{SPP},\,t:t+T} = y(z_t)\,\mathbb{1}_{\left\{R_{\text{M},\,t:t+T} > B\right\}} + R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\left\{R_{\text{M},\,t:t+T} \leq B\right\}}, \tag{2}$$

the wealth available at the beginning of date $t+T$ by:

$$W_{t+T}^H = X_t^H\,\left(1 + R_{f,t:t+T} + \alpha_{\text{M},t}^H R^e_{\text{M},t:t+T} + \alpha_{\text{SPP},t}^H\,R^e_{\text{SPP},t:t+T}\right),$$

and the certainty equivalent of future wealth by:

$$\mu_{t+T}^H = \left(\mathbb{E}_t^{\mathbb{P}}\left\{V^H\left[W_{t+T}^H; Y_0(z_{t+T}), Y_0, z_{t+T}\right]^{1-\gamma}\right\}\right)^{\frac{1}{1-\gamma}}.$$

We denote by

$$\alpha_{\text{SPP},t}^H = \alpha_{\text{SPP}}^H[y(z_t), Y_0, z_t]$$

the household's optimal SPP weight in period t. We know that $\alpha_{\text{SPP}}^H(y_t, Y_0, z_t)$ is contained in $[0,1]$ and is a continuous function of y_t, Y_0, and z_t. We do not need to reformulate the dealer's problem since her decisions do not depend on future investments opportunities.

More generally, we establish the existence of a one-step equilibrium under the following set of sufficient conditions.

CONDITION 1 (Sufficient conditions for existence of one-step equilibrium): *The state vector z_t is first-order Markov and contained in a compact subset, $\mathcal{Z}$, of a Euclidean space. The household share of the SPP,*

$$\alpha_{\text{SPP}}^H(y_t, Y_0, z_t), \tag{3}$$

is contained in $[0,1]$, is continuous and increasing in y_t, and satisfies $\alpha_{\text{SPP}}^H(0, Y_0, z_t) = 0$ for every z_t.

The condition $\alpha_{\text{SPP}}^H(0, Y_0, z_t) = 0$ states that if the SPP yield spread is zero, the SPP is dominated by the bond and the household allocates no funds to the SPP. Let $\mathcal{C}_+$ denote the set of continuous mappings define on $\mathcal{Z}$ and taking values in $\mathbb{R}_+$. In the rest of the section, we derive the following result.

PROPOSITION 3 (Existence and uniqueness of one-step equilibrium): *Assume that Condition 2 holds. For every $Y_0 \in \mathcal{C}_+$, there exists a unique $Y_1 \in \mathcal{C}_+$ such that for every state*

$z_t \in \mathcal{Z}$, the dealer maximizes her utility and the SPP and put markets clear:

$$\alpha^D_{\text{SPP},t}[z_t, Y_1(z_t)] \;=\; -\lambda^H \, \alpha^H_{\text{SPP},t}[Y_1(z_t), Y_0, z_t], \tag{4}$$

$$\alpha^D_{\text{PUT},n}[z_t; Y_1(z_t)] \;=\; -d^E_n \, P_{\text{PUT},n}(z_t). \tag{5}$$

We denote by $\mathcal{T}$ the transformation on $\mathcal{C}_+$ that maps the future yield spread, Y_0, to the current equilibrium yield spread: $Y_1 = \mathcal{T}(Y_0)$.

The proof of the proposition proceeds as follows. Consider the stochastic discount factor

$$M_{t+T}(a^D_{\text{M}}, y_t, a^H_{\text{SPP}}) = e^{-\gamma^D \left[a^D_{\text{M}} \, R^e_{\text{M},\, t:t+T} - \lambda^H \, a^H_{\text{SPP}} \, R^e_{\text{SPP},\, t:t+T}(y_t) - \sum_{n=1}^N d^E_n \, (K_n - 1 - R_{\text{M},t:t+T})_+\right]}$$

where $R^e_{\text{SPP},\, t:t+T}(y_t) \;=\; y_t \, \mathbb{1}_{\left\{R_{\text{M},\, t:t+T} > B\right\}} + R^e_{\text{M},\, t:t+T} \, \mathbb{1}_{\left\{R_{\text{M},\, t:t+T} \le B\right\}}$ expresses the SPP's excess return as a function of the the current yield.

LEMMA 3 (Auxiliary function G): *For a given state, z_t, SPP yield spread, y_t, and household demand for the SPP, $\alpha^H_{\text{SPP},t}$, the dealer's first-order condition in z_t:*

$$\mathbb{E}_t\left[R^e_{\text{M},\, t:t+T} \, M_{t+T}(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t})\right] = 0. \tag{6}$$

has a unique solution, $\alpha^D_{\text{M},t}$, which we denote by $G(y_t, \alpha^H_{\text{SPP},t}, z_t)$. Furthermore, the dealer's equity share, $\alpha^D_{\text{M},t} = G(y_t, \alpha^H_{\text{SPP},t}, z_t)$, is strictly increasing in the SPP yield spread, y_t, and in the household's demand for the SPP, $\alpha^H_{\text{SPP},t}$.

Proof. Consider the function $f(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t}) = \mathbb{E}_t\left[R^e_{\text{M},\, t:t+T} \, M_{t+T}(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t})\right]$, or equivalently

$$f(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t}) = \mathbb{E}_t\left\{R^e_{\text{M},\, t:t+T} \, e^{-\gamma^D\left[\alpha^D_{\text{M},t} \, R^e_{\text{M},\, t:t+T} - \lambda^H \alpha^H_{\text{SPP},t} \, R^e_{\text{SPP},\, t:t+T}(y_t) - \sum_{n=1}^N d^E_n \, (K_n - 1 - R_{\text{M},t:t+T})_+\right]}\right\}.$$

We note that

$$\frac{\partial f}{\partial \alpha^D_{\text{M},t}} = -\gamma^D \, \mathbb{E}_t\left[(R^e_{\text{M},\, t:t+T})^2 \, M_{t:t+T}\right] < 0,$$

$$\frac{\partial f}{\partial y_t} = \gamma^D \, \lambda^H \, \alpha^H_{\text{SPP},t} \, \mathbb{E}_t\left(R^e_{\text{M},\, t:t+T} \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M_{t:t+T}\right),$$

$$\frac{\partial f}{\partial \alpha^H_{\text{SPP},t}} = \gamma^D \, \lambda^H \, \mathbb{E}_t\left(R^e_{\text{M},\, t:t+T} \, R^e_{\text{SPP},\, t:t+T} \, M_{t+T}\right).$$

By the Implicit Function Theorem, the partial derivatives of G are given by:

$$\frac{\partial G}{\partial y_t} = -\frac{\partial f}{\partial y_t} \times \left(\frac{\partial f}{\partial \alpha^D_{\text{M},t}}\right)^{-1} = \lambda^H \, \alpha^H_{\text{SPP},t} \, \frac{\mathbb{E}_t\left(R^e_{\text{M},\, t:t+T} \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M_{t:t+T}\right)}{\mathbb{E}_t\left[(R^e_{\text{M},\, t:t+T})^2 \, M_{t+T}\right]}, \tag{7}$$

$$\frac{\partial G}{\partial \alpha^H_{\text{SPP},t}} = -\frac{\partial f}{\partial \alpha^H_{\text{SPP},t}} \times \left(\frac{\partial f}{\partial \alpha^D_{\text{M},t}}\right)^{-1} = \lambda^H \, \frac{\mathbb{E}_t\left(R^e_{\text{M},\, t:t+T} \, R^e_{\text{SPP},\, t:t+T} \, M_{t+T}\right)}{\mathbb{E}_t\left[(R^e_{\text{M},\, t:t+T})^2 \, M_{t+T}\right]}. \tag{8}$$

Since $\mathbb{E}_t\left(R^e_{\text{M},\,t:t+T}\,M_{t+T}\right)=0$, we know that

$$\mathbb{E}_t\left(R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}M_{t+T}\right)=-\mathbb{E}_t\left[R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t+T}\leq B\}}M_{t:t+T}\right]>0. \tag{9}$$

Hence the function G strictly increases in y_t.

Similarly, since $R^e_{\text{M},\,t:t+T}\,R^e_{\text{SPP},\,t:t+T}=(R^e_{\text{M},\,t:t+T})^2\mathbb{1}_{\{R_{\text{M},\,t:t+T}\leq B\}}+y_t\,R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}$, we infer that

$$\begin{aligned}\mathbb{E}_t\left(R^e_{\text{M},\,t:t+T}\,R^e_{\text{SPP},\,t:t+T}\,M_{t+T}\right) &= \mathbb{E}_t\left[(R^e_{\text{M},\,t:t+T})^2\mathbb{1}_{\{R_{\text{M},\,t:t+T}\leq B\}}M_{t+T}\right]\\ &\quad +y_t\,\mathbb{E}_t\left[R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}M_{t+T}\right]>0.\end{aligned}$$

Hence the function G strictly increases in $\alpha^H_{\text{SPP},t}$. □

In a one-step equilibrium, the following first-order conditions hold:

$$\mathbb{E}_t\left(R^e_{\text{M},\,t:t+T}\,M_{t+T}\right) = 0, \tag{10}$$
$$\mathbb{E}_t\left(R^e_{\text{SPP},t+1}\,M_{t+T}\right) = 0. \tag{11}$$

By equation (2), the first-order condition (11) can be rewritten as:

$$y_t\,\mathbb{E}_t\left(\mathbb{1}_{\left\{R_{\text{M},\,t:t+T}>B\right\}}M_{t+T}\right)+\mathbb{E}_t\left(R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\left\{R_{\text{M},\,t:t+T}\,\leq B\right\}}\,M_{t+T}\right)=0, \tag{12}$$

or

$$y_t=-\frac{\mathbb{E}_t\left(R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\left\{R_{\text{M},\,t:t+T}\,\leq B\right\}}\,M_{t+T}\right)}{\mathbb{E}_t\left(\mathbb{1}_{\left\{R_{\text{M},\,t:t+T}>B\right\}}M_{t+T}\right)}>0. \tag{13}$$

We infer from equation (10) that

$$y_t=\frac{\mathbb{E}_t\left(R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}\,M_{t+T}\right)}{\mathbb{E}_t\left(\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}\,M_{t+T}\right)}. \tag{14}$$

Since the return on the SPP is deterministic for $R_{\text{M},\,t:t+T}>B$, this equation reduces to:

$$y_t=\frac{\mathbb{E}_t\left[R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}\,M^*_{t+T}(\alpha^D_{\text{M},t})\right]}{\mathbb{E}_t\left[\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}\,M^*_{t+T}(\alpha^D_{\text{M},t})\right]}, \tag{15}$$

where

$$M^*_{t+T}(\alpha^D_{\text{M},t})=e^{-\gamma^D\left[\alpha^D_{\text{M},t}\,R_{\text{M},t:t+T}-\sum_{n=1}^N d^E_n\,(K_n-1-R_{\text{M},t:t+T})_+\right]}. \tag{16}$$

We now study the properties of the function on the right-hand side of equation (15).

LEMMA 4 (Auxiliary function H): *The function $H(\alpha^D_{\text{M},t},z_t)$ strictly decreases in $\alpha^D_{\text{M},t}$ for every state z_t.*

Proof. We let

$$H(a, z_t) = \frac{u(a, z_t)}{v(a, z_t)} - R_{f,t,t:T}, \tag{17}$$

where

$$\begin{aligned} u(a, z_t) &= \mathbb{E}_t \left[R_{\text{M}, t:t+T} \, \mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T}(a) \right], \\ v(a, z_t) &= \mathbb{E}_t \left[\mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T}(a) \right]. \end{aligned}$$

We note that

$$\frac{\partial u}{\partial a} = -\gamma^D \, \mathbb{E}_t \left[(R_{\text{M}, t:t+T})^2 \, \mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T} \right]$$

$$\frac{\partial v}{\partial a} = -\gamma^D \, \mathbb{E}_t \left[R_{\text{M}, t:t+T} \, \mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T} \right] = -\gamma^D \, u_t(a).$$

We infer that

$$\frac{\partial H}{\partial a} = \gamma^D \, \frac{\left\{ \mathbb{E}_t \left[R_{\text{M}, t:t+T} \, \mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T} \right] \right\}^2 - \left\{ \mathbb{E}_t \left[(R_{\text{M}, t:t+T})^2 \, \mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T} \right] \right\} \left\{ \mathbb{E}_t \left[\mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T} \right] \right\}}{\left\{ \mathbb{E}_t [\mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T}] \right\}^2}.$$

By the Cauchy Schwarz inequality, the numerator is negative.[10] Hence, H is strictly decreasing in a. We also note that

$$\left| \frac{\partial H}{\partial a}(a, z_t) \right| \leq \gamma^D \, \frac{\mathbb{E}_t \left[(R_{\text{M}, t:t+T})^2 \, \mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T} \right]}{\mathbb{E}_t [\mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, M^*_{t+T}(a)]}.$$

for every a and z_t. □

We consider the function

$$\Phi(y_t, Y_0, z_t) = y_t - H \left\{ G[y_t, \alpha^H_{\text{SPP}}(y_t, Y_0, z_t), z_t], z_t \right\}.$$

Under Condition 2, the function α^H_{SPP} increases in y_t. By Lemmas 3 and 4, the function Φ therefore increases in y_t.

When $y_t = 0$, the household has zero demand for the SPP; $\alpha^H_{\text{SPP}}(0, Y_0, z_t) = 0$, as per Condition 2. Hence,

$$\Phi(0, Y_0, z_t) = -H \left[\hat{\alpha}(z_t), z_t \right],$$

where $\hat{\alpha}(z_t) = G(0, 0, z_t)$ denotes the dealer's optimal equity share in the absence of the SPP contract. The equity share, $\hat{\alpha}(z_t)$, satisfies the first-order condition:

$$\mathbb{E}_t \left\{ R^e_{\text{M}, t:t+T} \, e^{-\gamma^D \left[\alpha^D_{\text{M},t} \, R^e_{\text{M}, t:t+T} - \sum_{n=1}^N d^E_n \, (K_n - 1 - R_{\text{M}, t:t+T})_+ \right]} \right\} = 0, \tag{18}$$

[10] Let $X = R_{\text{M}, t:t+T} \, \mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, [M^*_{t+T}(a)]^{1/2}$ and $Y = \mathbb{1}_{\{R_{\text{M}, t:t+T} > B\}} \, [M^*_{t+T}(a)]^{1/2}$. By the Cauchy Schwarz inequality, $[E(XY)]^2 < E(X^2)E(Y^2)$. The inequality is strict because X and Y are not perfectly correlated.

or, more compactly, $\mathbb{E}_t\left\{R^e_{\text{M},\,t:t+T}\, M^*_{t+1}[\hat{\alpha}(z_t)]\right\} = 0$. Since

$$\mathbb{E}_t\left\{R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}\, M^*_{t+1}[\hat{\alpha}(z_t)]\right\} = -\mathbb{E}_t\left\{R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t:t+T}\leq B\}}\, M^*_{t+1}(\hat{\alpha}(z_t))\right\} > 0,$$

we infer that $H\left[\hat{\alpha}(z_t), z_t\right] > 0$ and therefore $\Phi(0, Y_0, z_t) < 0$.

We also note that

$$\frac{\partial \Phi(y_t, Y_0, z_t)}{\partial y_t} \geq 1. \tag{19}$$

Hence $\Phi(y_t, Y_0, z_t)$ is positive for y_t sufficiently large. We infer that the equation $\Phi(y_t, Y_0, z_t) = 0$ has a unique solution, so that there exists a unique one-step equilibrium for every z_t.

We further note that

$$Y_1(z_t, Y_0) \leq |\, H\left[G(0, 0, z_t), z_t\right] | \tag{20}$$

for every $z_t \in \mathcal{Z}$ and $Y_0 \in \mathcal{C}_+$.

Proof of Proposition 2

Let $\mathcal{C}$ denote the set of continuous functions defined on $\mathcal{Z}$ and taking values on the real line. Let $\|f\|_\infty = \sup_{z\in\mathcal{Z}} |f(z)|$ denote the sup norm. We know that $(\mathcal{C}, \|f\|_\infty)$ is a Banach space.

Consider

$$\bar{y} = \max_{z_t\in\mathcal{Z}} |H\left[G(0, 0, z_t), z_t\right]| < \infty. \tag{1}$$

We know that for every function $Y_0 \in \mathcal{C}$, the transform $Y_1 = \mathcal{T}\, Y_0$ takes values between 0 and $\bar{y}$, as equation (20) implies. For this reason, we henceforth focus on the subset of functions:

$$\mathcal{K} = \{f \in \mathcal{C} : 0 \leq f(z) \leq \bar{y} \text{ for every } z \in \mathcal{Z}\}.$$

The set $\mathcal{K}$ is convex and closed.

Consider the function $f(y_t, a_{\text{SPP}}, z_t) = H\left[G(y_t, a_{\text{SPP}}, z_t), z_t\right]$. By the chain rule,

$$\frac{\partial f}{\partial a_{\text{SPP}}}(y_t, a_{\text{SPP}}, z_t) = \frac{\partial H}{\partial a_M}\left[G(y_t, a_{\text{SPP}}, z_t), z_t\right]\, \frac{\partial G}{\partial a_{\text{SPP}}}(y_t, a_{\text{SPP}}, z_t). \tag{2}$$

The functions on the right-hand side are bounded and we let

$$K_1 = \sup\left\{\left|\frac{\partial f}{\partial a_{\text{SPP}}}(y_t, a_{\text{SPP}}, z_t)\right|;\ (y_t, a_{\text{SPP}}, z_t) \in [0, \bar{y}] \times [0, 1] \times \mathcal{Z}\right\}.$$

Importantly, we note that the upper bounds $\bar{y}$ and K_1 do not depend on the household demand for the SPP.[11]

[11] Furthermore, we infer from and that

$$K_1 \leq \lambda^H\, \gamma^D \sup_{(y_t, a_{\text{SPP}}, z_t)\in[0,\bar{y}]\times[0,1]\times\mathcal{Z}} \frac{\mathbb{E}_t\left\{R^e_{\text{M},\,t:t+T}\, R^e_{\text{SPP},\,t:t+T}\, M_{t+T}[G(a_{\text{SPP}}, z_t), y_t, z_t]\right\}}{\mathbb{E}_t\left\{\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}\, M_{t+T}[G(a_{\text{SPP}}, z_t), y_t, z_t]\right\}}. \tag{3}$$

CONDITION 2 (Sufficient conditions for existence of stationary equilibrium): *There exists* $k_{\text{SPP}} > 0$ *such that the household share of the SPP,* $\alpha^H_{\text{SPP}}(y_t, Y_0, z_t)$*, satisfies*

$$|\alpha^H_{\text{SPP}}(y_t, Y_0', z_t) - \alpha^H_{\text{SPP}}(y_t, Y_0, z_t)| \leq k_{\text{SPP}} \, \|Y_0' - Y_0\|_\infty$$

for every $Y_0, Y_0' \in \mathcal{K}$*,* $y_t \in [0, \bar{y}]$*, and* $z_t \in \mathcal{Z}$*. Furthermore, the constant* k_{SPP} *satisfies* $k_{\text{SPP}} < 1/K_1$.

We consider $Y_0, Y_0' \in \mathcal{K}$. Let $Y_1 = \mathcal{T} Y_0$ and $Y_1' = \mathcal{T} Y_0'$. For a given state z_t, we let $y_t = Y_1(z_t)$ and $y_t' = Y_1'(z_t')$. Since $\Phi(y_t', Y_0', z_t) = \Phi(y_t, Y_0, z_t) = 0$, we infer that

$$\Phi(y_t', Y_0', z_t) - \Phi(y_t', Y_0, z_t) = \Phi(y_t, Y_0, z_t) - \Phi(y_t', Y_0, z_t). \tag{4}$$

We infer from (19)

$$|y_t' - y_t| \leq |\Phi(y_t, Y_0, z_t) - \Phi(y_t', Y_0, z_t)|. \tag{5}$$

Sufficient Condition 2 implies

$$|\Phi(y_t', Y_0', z_t) - \Phi(y_t', Y_0, z_t)| \leq k_{\text{SPP}} \, K_1 \, \|Y_0' - Y_0\|_\infty. \tag{6}$$

We infer from (4), (5) and (6) that

$$|y_t' - y_t| \leq k_{\text{SPP}} \, K_1 \, \|Y_0' - Y_0\|_\infty.$$

Since this inequality holds for every state z_t, the transformation $\mathcal{T}$ is a contraction:

$$\|\mathcal{T} Y_0' - \mathcal{T} Y_0\|_\infty \leq k_{\text{SPP}} \, K_1 \, \|Y_0' - Y_0\|_\infty.$$

Hence, the contraction mapping $\mathcal{T}$ has a unique fixed point.

Alternative specification of the dealer

The wealth of the dealer in period t is

$$X_t^D = \phi F W_t^H = \phi X_t^H / (1 - \phi).$$

We write

$$X_t^H = \lambda^H X_t^D,$$

where $\lambda^H = (1 - \phi)/\phi$.

The dealer's wealth in period $t + T$ is

$$W_{t+T}^D = X_t^D \left(1 + R_{t:t+T}^D\right).$$

where

$$R^D_{t:t+T} = R_{f,\,t:t+T} + \alpha^D_{\text{M},t}\, R^e_{\text{M},\,t:t+T} + \alpha^D_{\text{SPP},t}\, R^e_{\text{SPP},\,t:t+T} + \sum_{n=1}^{N} \alpha^D_{\text{PUT},n,t}\, R^e_{\text{PUT},n,\,t:t+T} \tag{1}$$

denotes the dealer's return over the period.

The dealer has constant absolute risk aversion (CARA) utility:

$$\mathbb{E}^{\mathbb{P}}_0 \sum_{t=1}^{\infty} \delta^t u_t(W^D_t)$$

where $u_{t+T}(W^D_{t+T}) = -e^{-A_t\, W^D_{t+T}}/A_t$ and $A_t = \gamma^D/X^D_t$.

Let $\alpha^D_t = (\alpha^D_{\text{M},t}, \alpha^D_{\text{SPP},t}, \alpha^D_{\text{PUT},1,t}, \ldots, \alpha^D_{\text{PUT},K,t})$. Assume that to minimize attention costs etc., the dealer chooses the portfolio α^D_t and is then precluded from modifying it thereafter.

The dealer solves the optimization problem

$$\max_{\alpha^D_t} \mathbb{E}_t \left[-e^{-\gamma^D\,(W^D_{t+T}/X^D_t)} \right].$$

Since $W^D_{t+T}/X^D_t = 1 + R^D_{t:t+T}$, the solution to this problem is independent of the wealth level X^D_t. We denote by

$$\alpha^D[z_t; Y(z_t)]$$

the optimal portfolio of the dealer.

We can also assume that the dealer consumes X^D_t at date t and keeps the residual risk:

$$X^D_t \left(\alpha^D_{\text{M},t}\, R^e_{\text{M},\,t:t+T} + \alpha^D_{\text{SPP},t}\, R^e_{\text{SPP},\,t:t+T} + \sum_{n=1}^{N} \alpha^D_{\text{PUT},n,t}\, R^e_{\text{PUT},n,\,t:t+T} \right). \tag{2}$$

Likelihood

As we explain in the main text, the one-step likelihood of y_{t+1} is

$$f(y_{t+1}|y_{1:t}) \;=\; \iint h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) f(z_{2,t}, z_{2,t+1}|y_{1:t})\, dz_{2,t}\, dz_{2,t+1},$$

where

$$h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) \;=\; f(z_{1,t+1}|z_t, z_{2,t+1}) \;\; f(r^e_{M,t+1}|z_t, z_{t+1}) \;\; f(X_{t+1}|z_{t+1}). \tag{1}$$

We explain below how to compute the conditional densities on the right-hand side of (1).

First, conditional on z_t, the state vector z_{t+1} is Gaussian with mean $(\mu_1+\phi_1(z_{1,t}-\mu_1), \mu_2+\phi_2(z_{2,t}-\mu_2))'$ and variance-covariance matrix

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\,\sigma_1\,\sigma_2 \\ \rho_{12}\,\sigma_1\,\sigma_2 & \sigma_2^2 \end{pmatrix}$$

Hence

$$f(z_{1,t+1}|z_t, z_{2,t+1}) = n\left[z_{1,t+1}; \mu_1 + \phi_1(z_{1,t} - \mu_1) + \sigma_1\rho_{12}\varepsilon_{2,t+1}; \sigma_1^2(1-\rho_{12}^2)\right],$$

where

$$\varepsilon_{2,t+1} = \frac{1}{\sigma_2}[z_{2,t+1} - \mu_2 - \phi_2(z_{2,t} - \mu_2)]$$

denotes the normalized innovation on state 2.

Second, we turn to the conditional distribution of

$$r^e_{M,t+1} = \mu_r + e^{z_{2,t+1}/2}\varepsilon_{M,t+1} \tag{2}$$

conditional on density z_t and z_{t+1}). The variance-covariance matrix of the Gaussian vector $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{m,t})'$:

$$V_\varepsilon = \begin{pmatrix} 1 & \rho_{12} & \rho_{1M} \\ \rho_{12} & 1 & \rho_{2M} \\ \rho_{1M} & \rho_{2M} & 1 \end{pmatrix}$$

The Cholesky decomposition of V_ε can be written as

$$V_\varepsilon = L\,L^\top$$

where $L = (L_{ij})$ is a lower triangular matrix. We consider the random vector $u_t = L^{-1}u_t$, or equivalently

$$\varepsilon_{t+1} = L\,u_{t+1}, \tag{3}$$

The vectors are u_t is iid $\mathcal{N}(0, I)$ and satisfy:

$$\begin{aligned}
\varepsilon_{1,t+1} &= L_{1,1}\,u_{1,t+1} \\
\varepsilon_{2,t+1} &= L_{2,1}\,u_{1,t+1} + L_{2,2}\,u_{2,t+1} \\
\varepsilon_{\mathrm{M},t+1} &= L_{3,1}\,u_{1,t+1} + L_{3,2}\,u_{2,t+1} + L_{3,3}\,u_{3,t+1}.
\end{aligned}$$

If the vectors z_t and z_{t+1} are known, then $\varepsilon_{1,t+1}$, $\varepsilon_{2,t+1}$ $u_{1,t+1}$ and $u_{2,t+1}$ are also known.

We note that $r^e_{M,t+1} = \mu_r + e^{z_{2,t+1}/2}\varepsilon_{M,t+1}$ satisfies

$$r^e_{M,t+1} = \mu_r + e^{z_{2,t+1}/2}(L_{3,1}u_{1,t+1} + L_{3,2}u_{2,t+1} + L_{3,3}u_{3,t+1}) \tag{4}$$

Hence

$$f(r^e_{\mathrm{M},t+1}|z_t, z_{t+1}) = n\left[r^e_{\mathrm{M},t+1}; \mu_r + e^{z_{2,t+1}}(L_{3,1}u_{1,t+1} + L_{3,2}u_{2,t+1}); e^{z_{2,t+1}}L_{33}^2)\right], \tag{5}$$

where

$$\begin{aligned}
u_{1,t+1} &= \varepsilon_{1,t+1}/L_{1,1}, & (6)\\
u_{2,t+1} &= (\varepsilon_{2,t+1} - L_{2,1}\,u_{1,t+1})/L_{2,2}. & (7)
\end{aligned}$$

Price Elasticity

Let v_t^{SPP} denote the volume of SPP. The stochastic discount factor can be written as $M_{t+T} = e^{-m_{t+T}(z_t, v_t^{\text{SPP}})}$, where

$$m_{t+T}(z_t, v_t^{\text{SPP}}) = \gamma^D \left[\alpha_{\text{M}}^D(z_t, v_t^{\text{SPP}})\, R^e_{\text{M},t:t+T} - \lambda^H\, v_t^{\text{SPP}}\, R^e_{\text{SPP},t:t+T} - \sum_{n=1}^{N} d_n^E\, (K_n - 1 - R_{\text{M},t:t+T})_+ \right].$$

We know that

$$\begin{aligned} \mathbb{E}[R^e_{\text{M},t:t+T} M_{t+T}(z_t, v_t^{\text{SPP}}) \big|\, z_t] &= 0 \\ \mathbb{E}[R^e_{\text{SPP},t:t+T} M_{t+T}(z_t, v_t^{\text{SPP}}) \big|\, Z_t] &= 0, \end{aligned}$$

for every v_t^{SPP} and z_t.

By (14), the price of a put satisfies:

$$\begin{aligned} \ln[P_{\text{PUT},n}(z_t, v_t^{\text{SPP}})] &= \ln\{\mathbb{E}\left[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T}(z_t, v_t^{\text{SPP}}) \middle|\, z_t\right]\} \\ &\quad - \ln\left\{\mathbb{E}\left[M_{t+T}(z_t, v_t^{\text{SPP}}) \middle|\, z_t\right]\right\} - \ln(1 + R_{f,t:t+T}). \end{aligned}$$

We observe that

$$\frac{\partial \mathbb{E}\left(M_{t+T} \middle|\, z_t\right)}{\partial v_t^{\text{SPP}}} = \gamma^D\, \mathbb{E}\left(\left[\lambda^H\, R^e_{\text{SPP},t:t+T} - \frac{\partial \alpha_{\text{M}}^D(z_t, v_t^{\text{SPP}})}{\partial v_t^{\text{SPP}}}\, R^e_{\text{M},t:t+T}\right] M_{t+T} \middle|\, z_t \right) = 0.$$

Hence,

$$\begin{aligned} \frac{\partial \ln[P_{\text{PUT},n}(z_t)]}{\partial v_t^{\text{SPP}}} &= \frac{\partial \ln\left\{\mathbb{E}\left[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T} \middle|\, z_t\right]\right\}}{\partial v_t^{\text{SPP}}} \\ &= \frac{1}{\mathbb{E}\left[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T} \middle|\, z_t\right]} \frac{\partial\left\{\mathbb{E}\left[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T} \middle|\, z_t\right]\right\}}{\partial v_t^{\text{SPP}}} \\ &= \frac{1}{P_{\text{PUT},n}(1 + R_{f,t:t+T})\mathbb{E}\left(M_{t+T} \middle|\, z_t\right)} \frac{\partial\left\{\mathbb{E}\left[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T} \middle|\, z_t\right]\right\}}{\partial v_t^{\text{SPP}}} \\ &= \frac{\gamma^D}{P_{\text{PUT},n}(1 + R_{f,t:t+T})\mathbb{E}\left(M_{t+T} \middle|\, z_t\right)} \times \\ &\quad \mathbb{E}\left\{ (K_n - 1 - R_{\text{M},t:t+T})_+ \left[\lambda^H\, R^e_{\text{SPP},t:t+T} - \frac{\partial \alpha_{\text{M}}^D(z_t, v_t^{\text{SPP}})}{\partial v_t^{\text{SPP}}}\, R^e_{\text{M},t:t+T}\right] M_{t+T} \middle|\, z_t \right\}. \end{aligned}$$

By the chain rule,

$$\begin{aligned} \frac{\partial \ln P_{\text{PUT},n}}{\partial v_t^{\text{SPP}}} &= \frac{1}{P_{\text{PUT},n}} Vega_{n,t} \frac{\partial IV_{n,t}}{\partial v_t^{\text{SPP}}} \\ &= \frac{Vega_{n,t}}{P_{\text{PUT},n}} \frac{IV_{n,t}}{v_t^{\text{SPP}}} \frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} \end{aligned}$$

Hence

$$\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} = \frac{\gamma^D v_t^{\text{SPP}}}{Vega_{n,t} IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}\right| z_t)} \times \mathbb{E}\left\{ (K_n - 1 - R_{\text{M},t:t+T})_+ \left[\lambda^H R^e_{\text{SPP},t:t+T} - \frac{\partial \alpha^D_{\text{M}}(z_t, v_t^{\text{SPP}})}{\partial v_t^{\text{SPP}}} R^e_{\text{M},t:t+T} \right] M_{t+T} \middle| z_t \right\}.$$

LEMMA 5: *The sensitivity of the dealer's optimal risky share with respect to the SPP volume is given by*

$$\frac{\partial \alpha^D_{\text{M}}}{\partial v_t^{\text{SPP}}} = \lambda^H \frac{\kappa_0 \kappa_2 + \kappa_1^2}{\kappa_0(\kappa_2+\kappa_3) + \lambda^H \gamma^D v_t \kappa_1 (\kappa_3 \kappa_0 - \kappa_1^2)/\kappa_0}, \tag{1}$$

where

$$\begin{aligned}
\kappa_0 &= \mathbb{E}\left(1_{\{R_M > B\}} M_{t+T}\right) > 0 \\
\kappa_1 &= -\mathbb{E}\left(R^e_{\text{M}} 1_{\{R_M < B\}} M_{t+T}\right) > 0 \\
\kappa_2 &= \mathbb{E}\left[(R^e_{\text{M}})^2 1_{\{R_M < B\}} M_{t+T}\right] > 0 \\
\kappa_3 &= \mathbb{E}\left[(R^e_{\text{M}})^2 1_{\{R_M > B\}} M_{t+T}\right] > 0
\end{aligned}$$

Proof. The dealer's optimal risky share and the SPP yield solve:

$$f(\alpha^D_{\text{M}}, y_t; v_t^{\text{SPP}}) = E(R^e_{\text{M}} M) = 0 \tag{2}$$

$$g(\alpha^D_{\text{M}}, y_t; v_t^{\text{SPP}}) = E(R^e_{\text{SPP}} M) = 0 \tag{3}$$

By the Implicit Function Theorem,

$$\frac{\partial \alpha^D_{\text{M}}}{\partial v_t^{\text{SPP}}} = \frac{\frac{\partial f}{\partial v}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial v}}{\frac{\partial f}{\partial y}\frac{\partial g}{\partial \alpha^D_{\text{M}}} - \frac{\partial f}{\partial \alpha^D_{\text{M}}}\frac{\partial g}{\partial y}}, \tag{4}$$

We note that

$$\begin{aligned}
\frac{\partial f}{\partial \alpha^D_{\text{M}}} &= -\gamma^D(\kappa_2 + \kappa_3) \\
\frac{\partial f}{\partial y} &= \lambda^H \gamma^D v_t^{\text{SPP}} \kappa_1 \\
\frac{\partial f}{\partial v} &= \lambda^H \gamma^D (\kappa_2 + y_t \kappa_1) \\
\frac{\partial g}{\partial \alpha^D_{\text{M}}} &= -\gamma^D (\kappa_2 + y_t \kappa_1) \\
\frac{\partial g}{\partial y} &= (1 + \lambda^H \gamma^D v_t y_t)\kappa_0 \\
\frac{\partial g}{\partial v} &= \lambda^H \gamma^D (\kappa_2 + y_t^2 \kappa_0),
\end{aligned}$$

where $y_t = \kappa_1/\kappa_0$. We verify that

$$\frac{\partial f}{\partial v}\,\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\,\frac{\partial g}{\partial v} = \lambda^H\,\gamma^D\,\left(\kappa_0\kappa_2 + \kappa_1^2\right)$$

$$\frac{\partial f}{\partial y}\,\frac{\partial g}{\partial \alpha^D_{\text{M}}} - \frac{\partial f}{\partial \alpha^D_{\text{M}}}\,\frac{\partial g}{\partial y} = \gamma^D\,\left[\kappa_0(\kappa_2+\kappa_3) + \lambda^H\,\gamma^D\,v_t\kappa_1(\kappa_3\kappa_0 - \kappa_1^2)/\kappa_0\right]$$

Hence,

$$\frac{\partial \alpha^D_{\text{M}}}{\partial v_t^{\text{SPP}}} = \lambda^H\,\frac{\kappa_0\kappa_2 + \kappa_1^2}{\kappa_0(\kappa_2+\kappa_3) + \lambda^H\,\gamma^D\,v_t\kappa_1(\kappa_3\kappa_0 - \kappa_1^2)/\kappa_0}. \tag{5}$$

□

The sensitivity is thus

$$\begin{aligned}\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} &= \gamma^D\,\lambda^H\,v_t^{\text{SPP}}\times \\ &\frac{\mathbb{E}\left\{(K_n - 1 - R_{\text{M},t:t+T})_+\,\left[R^e_{\text{SPP},t:t+T} - \frac{\kappa_0\kappa_2+\kappa_1^2}{\kappa_0(\kappa_2+\kappa_3)+\lambda^H\,\gamma^D\,v_t\kappa_1(\kappa_3\kappa_0-\kappa_1^2)/\kappa_0}\,R^e_{\text{M},t:t+T}\right]M_{t+T}\right|z_t\right\}}{Vega_{n,t}IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}|\,z_t\right)}.\end{aligned}$$

If $K_n \leq 1 + B$,

$$\begin{aligned}\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} &= \lambda^H\,\gamma^D\,v_t^{\text{SPP}}\,\frac{(\kappa_0\,\kappa_3 - \kappa_1^2)(1+\lambda^H\,\gamma^D\,v_t\,\kappa_1/\kappa_0)}{\kappa_0(\kappa_2+\kappa_3) + \lambda^H\,\gamma^D\,v_t\kappa_1(\kappa_3\kappa_0-\kappa_1^2)/\kappa_0} \\ &\times\frac{\mathbb{E}\left\{(K_n - 1 - R_{\text{M},t:t+T})_+\;R^e_{\text{M},t:t+T}\,M_{t+T}\right|z_t\right\}}{Vega_{n,t}IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}|\,z_t\right)},\end{aligned}$$

Finally, if $K_n = 1 + B$,

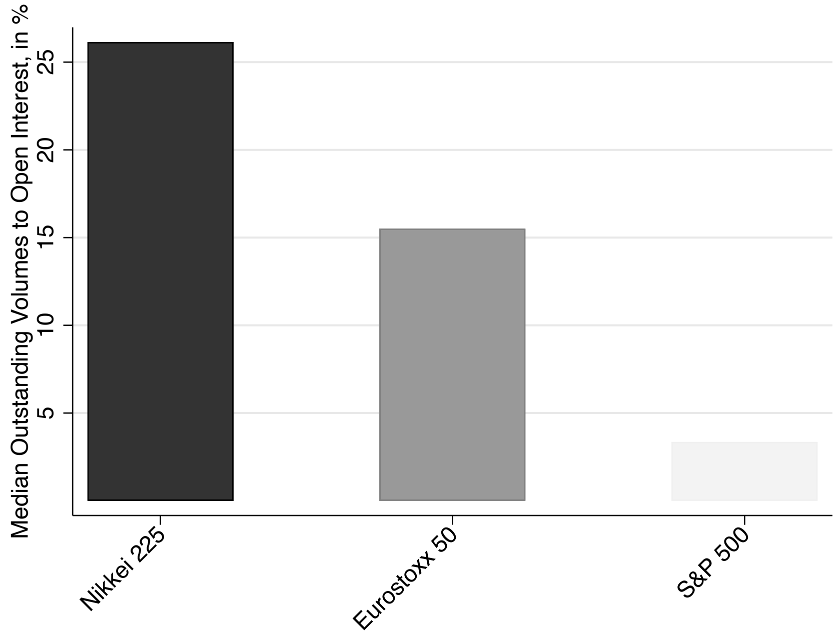
$$\begin{aligned}\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} &= -\lambda^H\,\gamma^D\,v_t^{\text{SPP}}\,\frac{(\kappa_0\,\kappa_3 - \kappa_1^2)(1+\lambda^H\,\gamma^D\,v_t\,\kappa_1/\kappa_0)}{\kappa_0(\kappa_2+\kappa_3) + \lambda^H\,\gamma^D\,v_t\kappa_1(\kappa_3\kappa_0-\kappa_1^2)/\kappa_0} \\ &\times\frac{\kappa_2 - (R_{f,t} - B)\kappa_1}{Vega_{n,t}IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}|\,z_t\right)},\end{aligned}$$

which is approximately equal to

$$\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} = -\lambda^H\,\gamma^D\,v_t^{\text{SPP}}\,\frac{\kappa_0\,\kappa_3 - \kappa_1^2}{\kappa_0(\kappa_2+\kappa_3)}\,\frac{\kappa_2 - (R_{f,t} - B)\kappa_1}{Vega_{n,t}IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}|\,z_t\right)}.$$

Appendix A. Figures

Panel A. Scaled SPP Issuance Volumes across Indices



Panel B. Volatility Premium across Indices

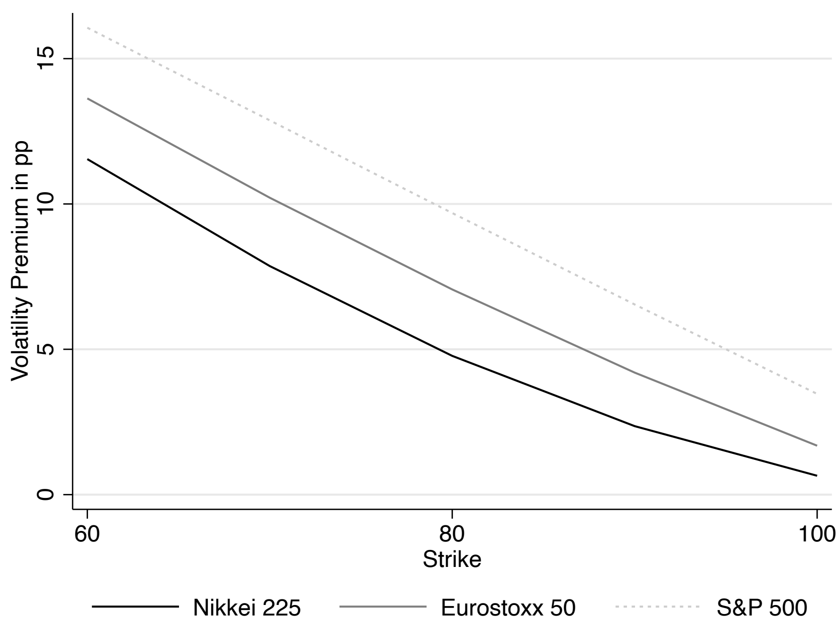


Figure 1. Average Scaled Volumes and Volatility Premium across Indices

Panel A plots the average yearly issuance volumes of SPP scaled by market capitalization across the three indices, S&P500, Nikkei225, and Eurostoxx50. Panel B plots the volatility premium, i.e. the difference between 1 year implied volatility and 90Day-realized volatility, at the 60, 70, 80, 90, and 100% strikes for the three indices, S&P 500, Nikkei 225, and Eurostoxx 50.

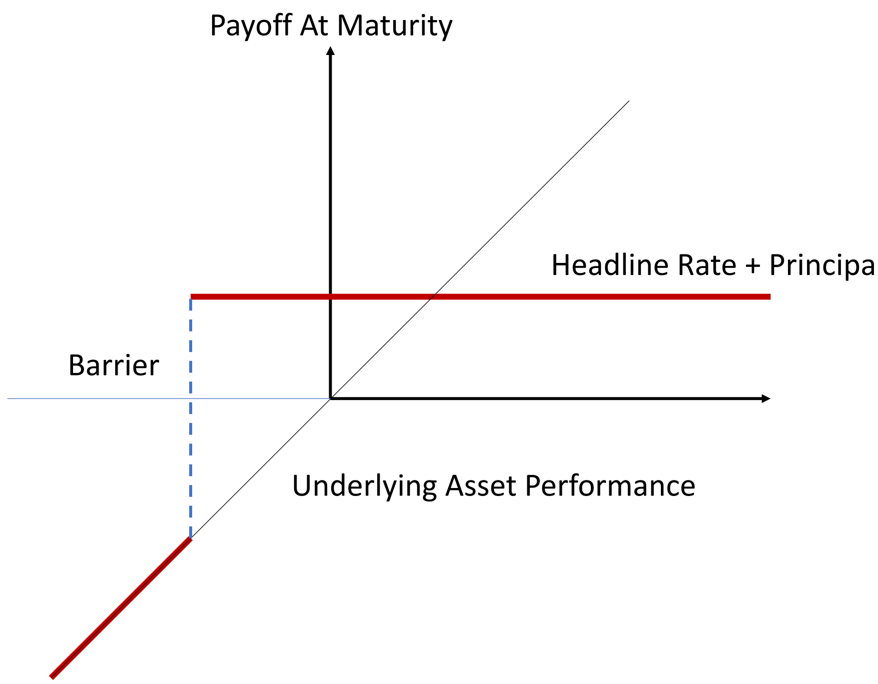


Figure 2. Design of a Simple Short Put Product

This figure shows the pay-off diagram of a typical Short Put Product.

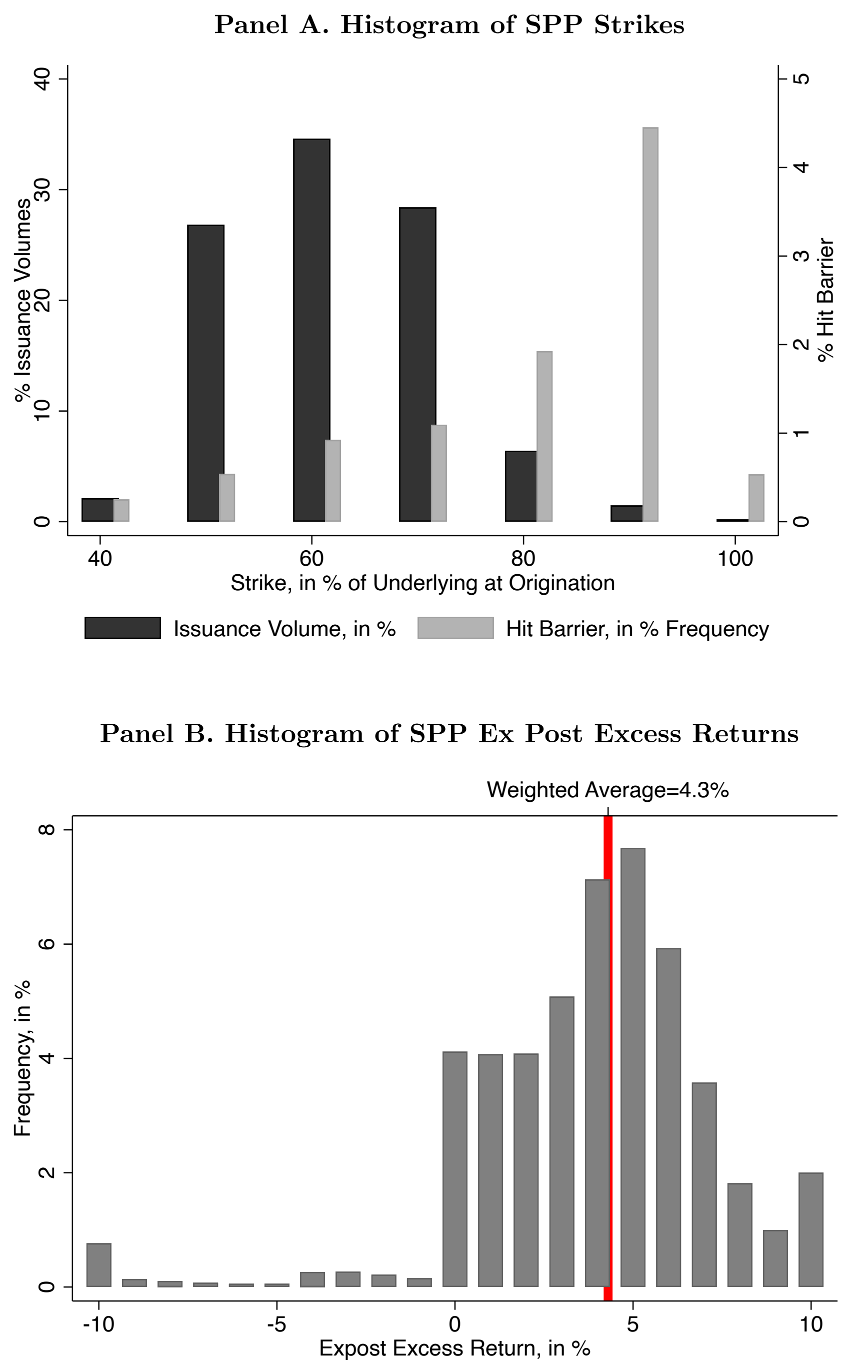


Figure 3. SPP Issuance volumes across Strikes and Ex-Post Excess Returns

Panel A presents the distribution of SPP issuance volumes by percentage strikes over the 2005-2019 period, along with the percentage of products that hit the barrier. Panel B shows the histogram of SPP ex-post excess returns over the same period. Ex-post excess returns are calculated as the difference between the SPP realized return at maturity and the corresponding 3-year swap rate. The sample consists of 100,000 SPPs issued globally since 2005, with indices as their underlying assets.

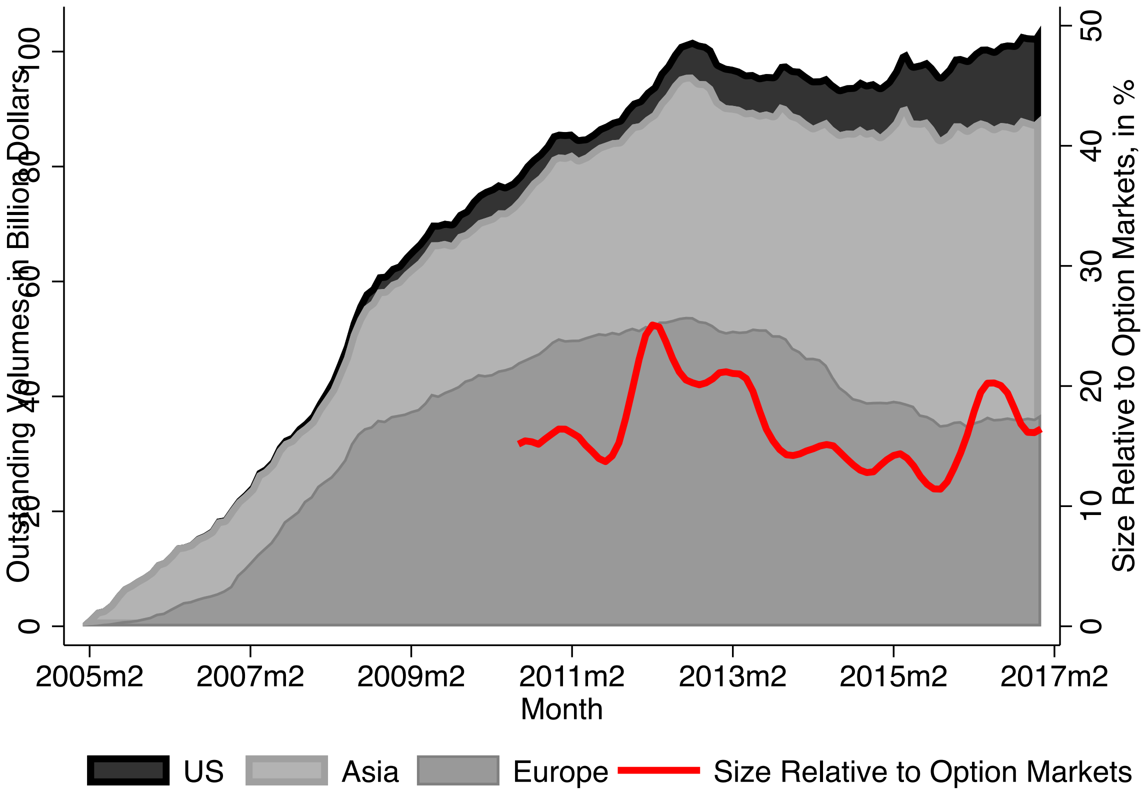


Figure 4. Outstanding Volumes of Index-linked SPPs Across Regions

The figure plots the evolution of outstanding volumes of index-linked SPPs across Europe, Asia and the U.S. in billion dollars from 2005 to 2016. The red line represents the size of these outstanding volumes relative to the open interest of all index-linked put options worldwide with strikes at or below 80% and maturities exceeding three months.

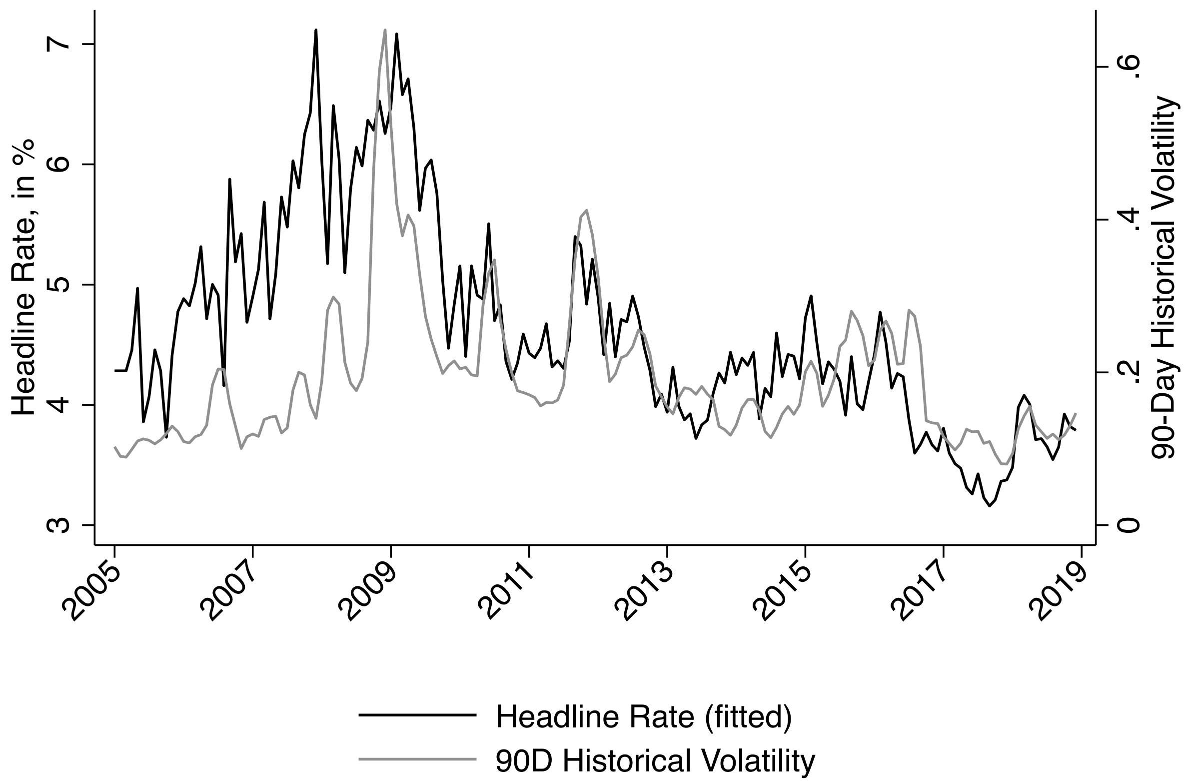


Figure 5. Eurostoxx SPP Headline Rates and realized volatility

This figure displays the fitted headline rates of SPPs issued globally from 2005 to 2019, with the Eurostoxx 50 as the underlying asset, alongside the 90-day realized volatility of the Eurostoxx 50. The sample is structured as a monthly panel, where realized volatility represents the average 90-day realized volatility for each month, and the fitted headline rate corresponds to the predicted headline rate for a simple SPP with a 3-year maturity and a 60% strike. We obtain the predicted headline rate by regressing the headline rates of 50,000 products in our sample on month, maturity, strike, the number of payoffs, the number of underlying assets (fixed effects), and dummies for key payoff features, such as worst-off, protection tracker, and knock-in.

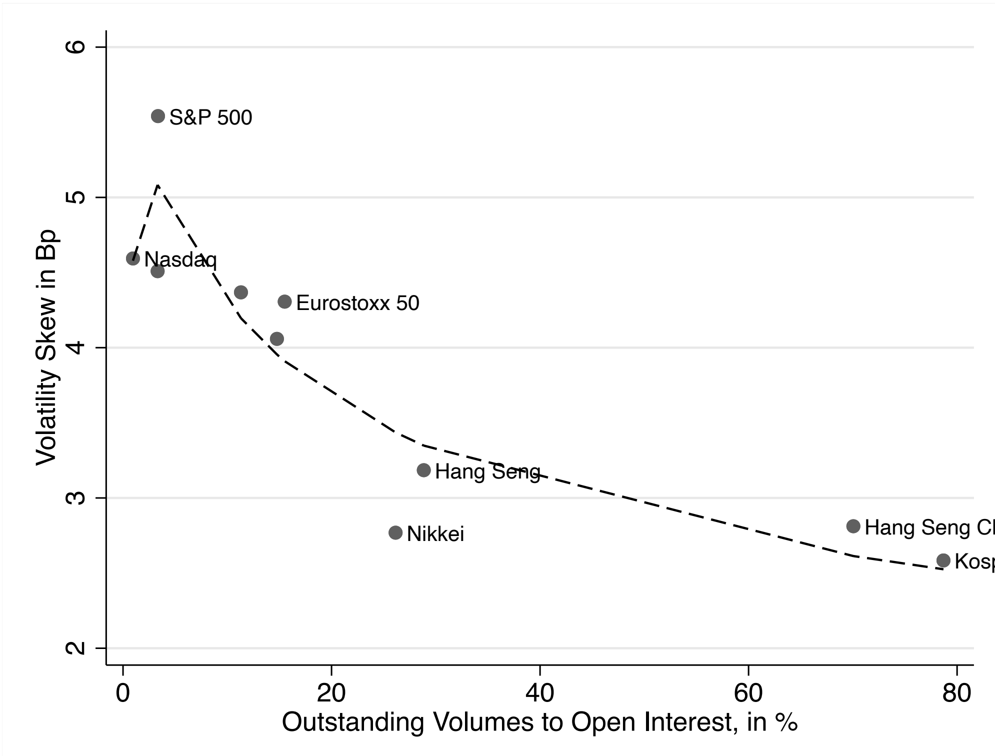
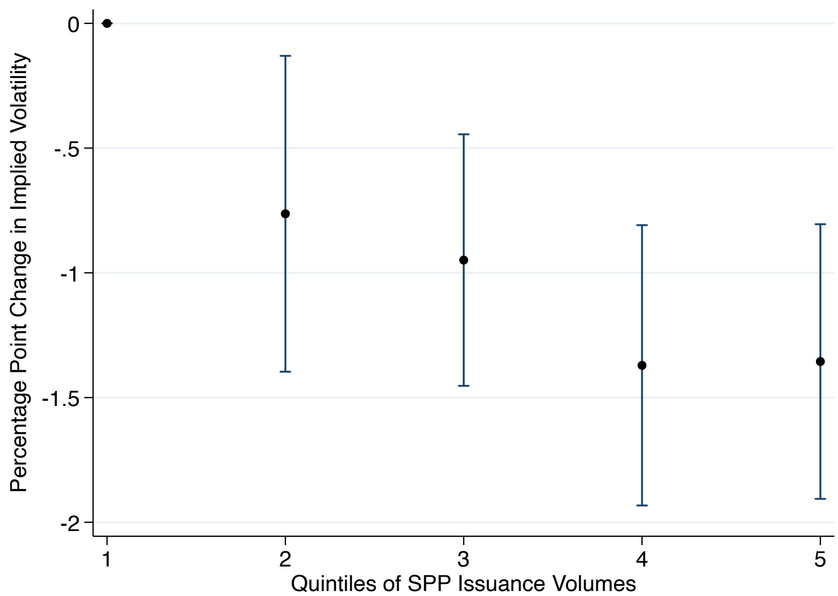


Figure 6. SPP Outstanding Volumes and Volatility Skewness

This figure displays the average volatility skewness over the 2005-2019 period versus outstanding volumes of SPP in percent of the open interest for options with a strike below 80% and a maturity above three months. The average volatility skewness is measured using monthly measure of the difference between mean implied volatility at the 80% strike and mean implied volatility at the 100% strike. Out sample focused on the 10 indices used as undrlyings for 99% of SPPs using indices as underlyings during the 2005-2019 period.

Panel A. Across SPP Issuance Volumes



Panel B. Across Strikes

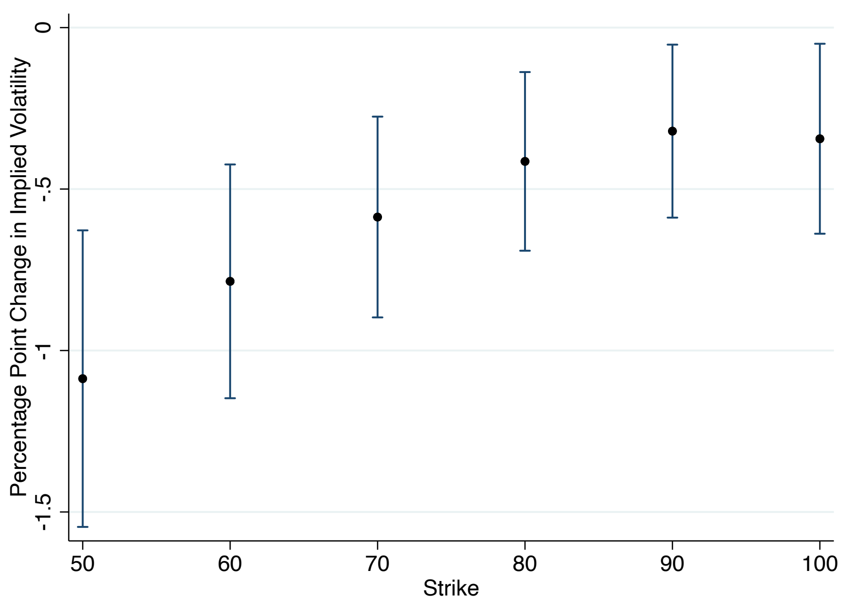
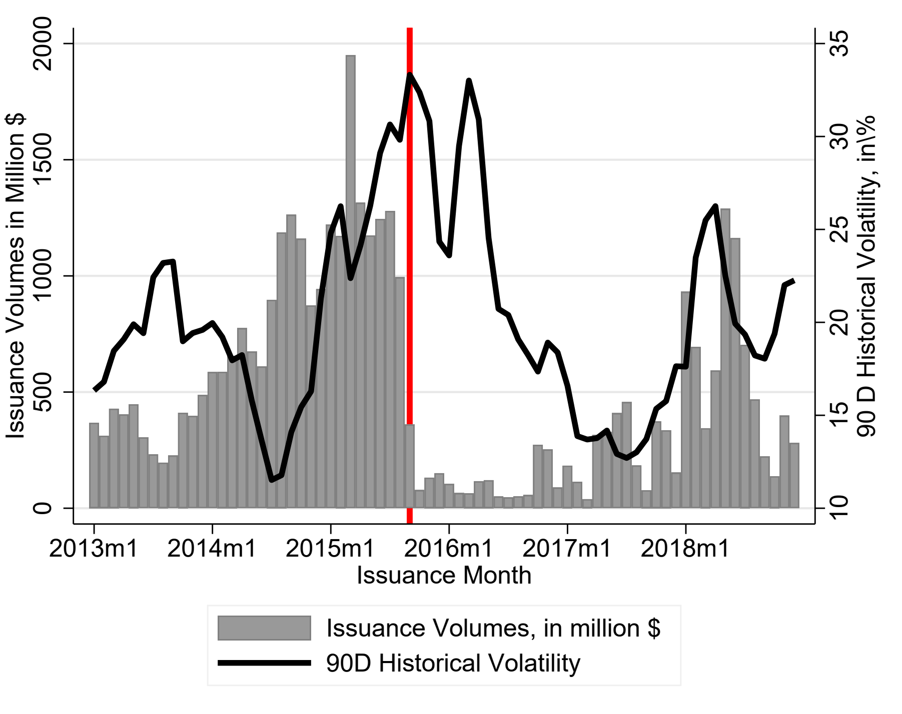


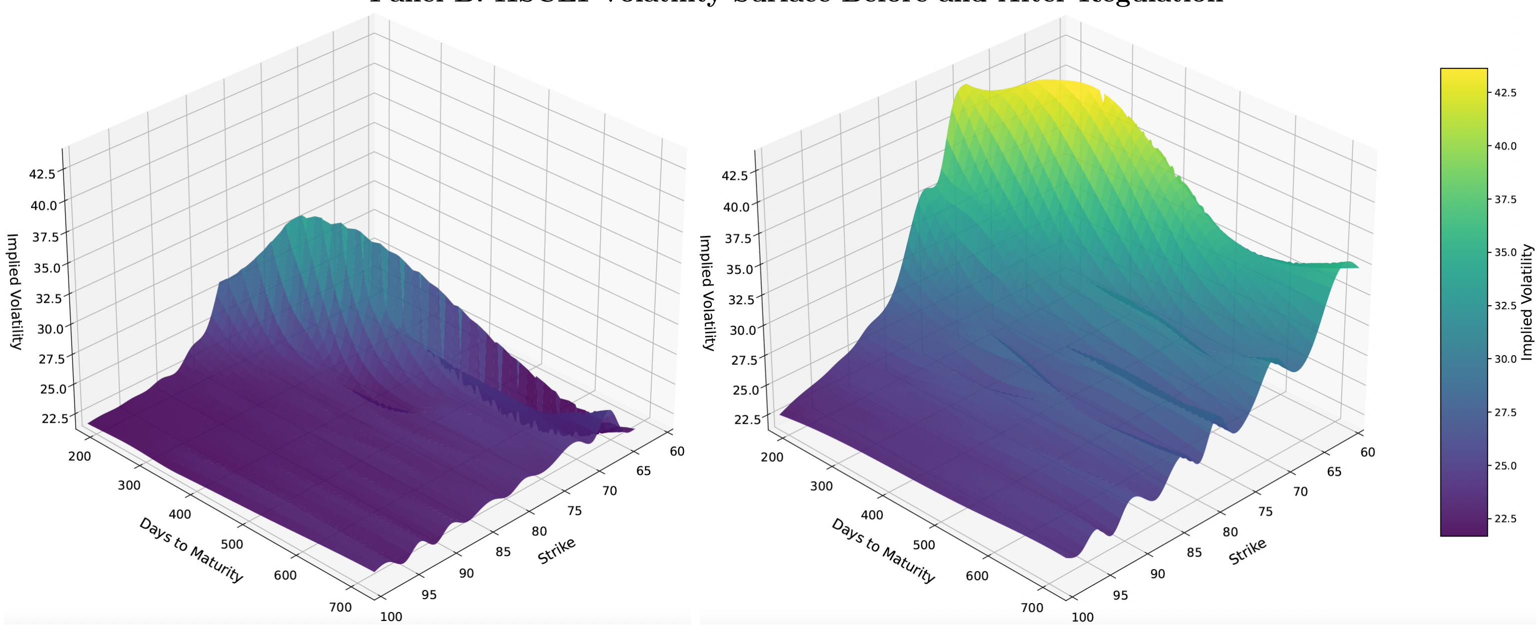
Figure 7. Impact of SPP Issuance Volumes on Implied Volatility: Coefficient Estimates

This figure plots the coefficient estimates of dummies for quintiles of issuance volumes in Panel A, and strikes, in Panel B in a regression where the dependent variable is the 1 year implied volatility. The sample is a monthly panel of the three indices, S&P500, Nikkei225, and Eurostoxx50. The regression includes month and underlying fixed effects.

Panel A. HSCEI-linked SPPs - Issuance Volumes



Panel B. HSCEI Volatility Surface Before and After Regulation

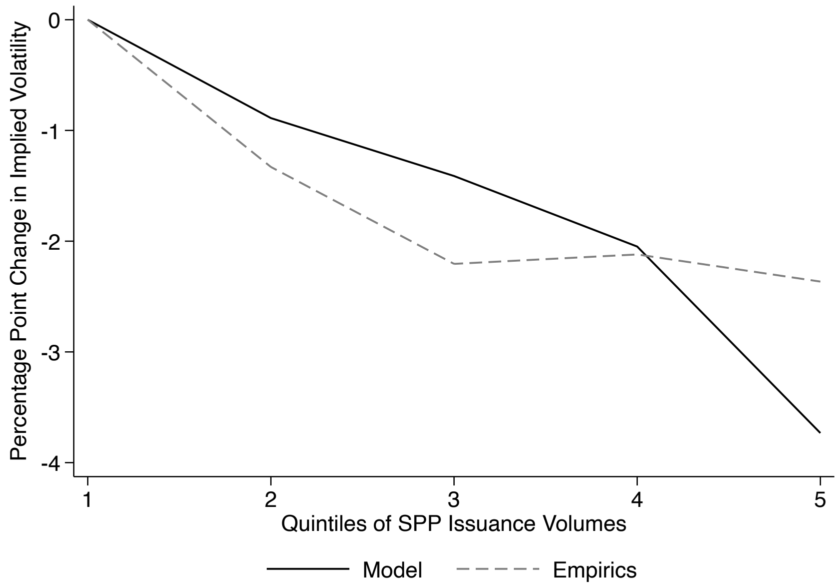


Before **After**

Figure 8. Natural Experiment: 2015 South Korean Ban on HSCEI-Linked SPPs

Panel A plots the issuance volumes of HSCEI-linked SPPs over the 2005-2019 period. The vertical line indicates the date of the informal ban of HSCEI-linked SPPs by the South Korean market authority. Panel B plots the average volatility surfaces in the year before and the year after the ban.

Panel A. Across SPP Issuance Volumes



Panel B. Across Strikes

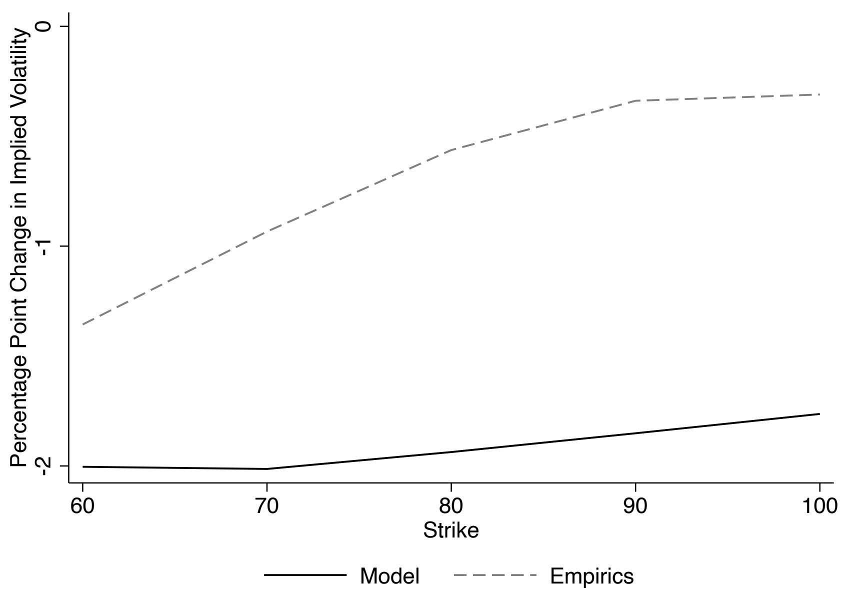


Figure 9. Impact of SPP Issuance Volumes on Implied Volatility: Model versus Empirics

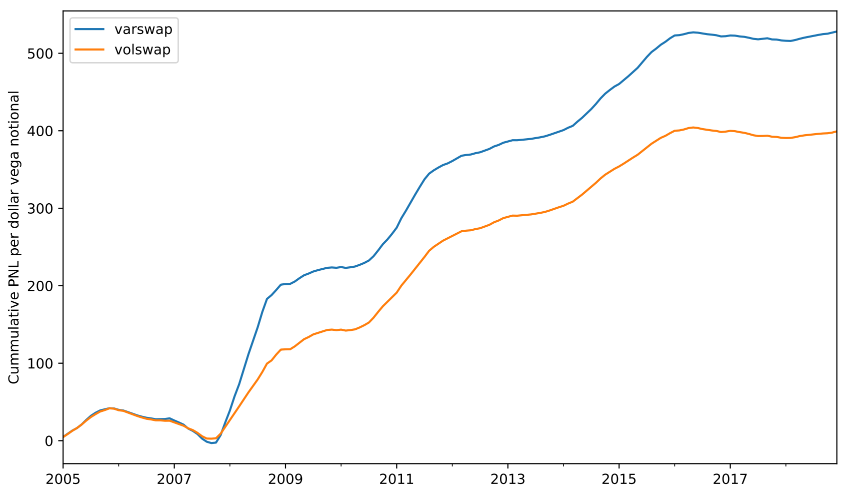


Figure 10. Cumulative P&L on long-short volatility and variance swaps portfolio

This figure plots the cumulative gains from implementing a monthly long-short trading strategy held to its 1 year maturity, based on sorting stocks on the amount of outstanding volumes of short put products.

Appendix B. Tables

Table I
Summary Statistics on Short Put Products (2005-2019)

	Mean	**S. D.**	**p10**	**p25**	**p50**	**p75**	**p90**
Product Characteristics, N =178,161							
Maturity, in years	3.0	1.1	1.5	3.0	3.0	3.0	3.1
Headline Rate							
Yearly Coupon, in %	6.5	2.4	4.0	4.8	6.0	7.8	10.0
Spread to Benchmark, in %	4.5	2.0	2.2	3.0	4.2	5.6	7.2
Strike, in % of the index initial price	58.2	8.9	50.0	50.0	60.0	65.0	70.0
Ex-Post Returns							
Yearly Return, in %	5.5	8.6	1.1	3.9	5.4	7.2	9.5
Excess to 1-year Risk Free Rate, in %	4.9	4.4	1.2	3.2	5.1	6.7	8.5
Issuance Volumes							
In million $	5.1	15.3	0.1	0.6	2.0	5.4	12.3
Underlying Index, Market Share in %							
Eurostoxx 50	29.2						
S&P 500	18.2						
Nikkei 225	17.1						
Hang Seng China	13.6						
Kospi 200	10.5						
Hang Seng	2.8						
Russel	2.7						
FTSE	2.6						
Swiss Market Index	1.6						
Cac 40	0.8						
Dax	0.7						
Nasdaq	0.2						
Monthly Panel across 12 Indices and 180 months, N =2,160							
Market Parameters, in %							
3-year Swap Rate	2.4	1.6	0.6	1.3	2.1	3.4	4.6
Realized Volatility over 90Days	18.1	7.9	10.5	12.6	16.0	21.3	29.2
Implied Volatility - Three Main Indices, in %							
60% Strike	31.7	6.2	25.1	27.8	30.6	34.8	39.2
80% Strike	25.1	5.6	19.7	21.2	23.9	27.5	31.8
100% Strike	19.9	5.4	14.4	16.1	18.9	22.2	25.8
Implied Volatility - All Indices, in %							
80% Strike	25.0	6.7	18.3	20.7	23.6	27.5	32.8
90% Strike	22.8	6.8	16.2	18.6	21.4	25.2	30.4
100% Strike	21.0	6.8	14.3	16.6	19.7	23.2	28.5
Volumes							
Issuance Volumes, In billion $	0.5	0.8	0.0	0.0	0.1	0.7	1.4
Outstanding Volumes, In billion $	9.8	13.9	0.1	0.7	3.4	12.5	32.5
Open Interest, In billion $	67.3	116.1	2.1	4.1	12.5	53.8	267.9

This table reports summary statistics on our total sample of index-linked SPPs, as well as our aggregated monthly panel of SPP volumes, volatility prices, and market parameters. SPP data are from a dataset that covers the global issuance of SPPs over the 2005-2019 period. Volatility prices for the three main indices, S&P 500, Eurstox 50, and Nikkei 225, are from a proprietary dataset of a major investment bank. Volatility prices and open interest for all indices are from OptionMetrics. Open interest is the open interest of put options with strikes at 80% and maturity over three months.

Table II
Market Conditions and Product Design

	Headline Rate (spread to benchmark) in %			in Log		
	(1)	(2)	(3)	(4)	(5)	(6)
90D realized volatility, in %	0.11***	0.05***	0.05***			
	(0.01)	(0.01)	(0.01)			
3Y Swap Rate	0.11	-0.32***	-0.72***			
	(0.10)	(0.08)	(0.11)			
90D realized volatility, Log				0.43***	0.19***	0.19***
				(0.05)	(0.04)	(0.03)
3 Year Swap Rate, Log				-0.08*	-0.26***	-0.59***
				(0.04)	(0.03)	(0.06)
Fixed Effects						
Underlying	Yes	Yes	Yes	Yes	Yes	Yes
Year	-	Yes	Yes	-	Yes	Yes
Design	-	-	Yes	-	-	Yes
Market Country	-	-	Yes	-	-	Yes
Observations	178,161	178,161	178,156	177,892	177,892	177,887
R^2	0.12	0.24	0.36	0.10	0.21	0.34

This table displays the coefficients of OLS regressions in which the dependent variable is the spread between the product headline rate and the benchmark interest rate. Headline rate is defined as the fixed yearly rate that the investor receives in the best possible scenario. The benchmark rate is the 3 year swap rate of the distribution country. The sample includes all SPP products issued over the 2005-2019 period with one or several indices as underlyings. Design fixed effects include fixed effects for maturity, strike, number of payoff features, number of scenari. Standard errors are clustered at the month and the index-year levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table III
Realized Volatility and SPP Volumes

	Net Issuances, in Billion $				**Outstanding Volumes, in Billion $**			
Estimation Samples	**Indices**		**Period**		**Indices**		**Period**	
	3 main (1)	**All (2)**	**≤2011 (3)**	**>2011 (4)**	**3 Main (5)**	**All (6)**	**≤2011 (7)**	**>2011 (8)**
Volatility 2nd Quartile	0.02	0.04	0.01	0.02	1.87*	1.32**	2.01***	0.59
	(0.08)	(0.03)	(0.02)	(0.04)	(1.09)	(0.57)	(0.35)	(0.36)
Volatility 3rd Quartile	0.25***	0.10***	0.03*	0.10***	3.58***	2.38***	2.69***	1.61***
	(0.08)	(0.04)	(0.02)	(0.04)	(1.28)	(0.84)	(0.51)	(0.45)
Volatility 4th Quartile	0.26***	0.12***	0.04**	0.19***	3.31**	2.85**	3.19***	1.82**
	(0.08)	(0.05)	(0.02)	(0.04)	(1.52)	(1.35)	(0.67)	(0.71)
3Y Swap Rate	0.20***	0.03***	0.01	0.05***	-7.56***	-0.08	0.42***	-1.12***
	(0.04)	(0.01)	(0.01)	(0.02)	(0.64)	(0.31)	(0.13)	(0.14)
Fixed Effects								
Month					Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	540	2,136	1,008	1,128	540	2,136	1,008	1,128
R^2	0.13	0.05	0.34	0.12	0.81	0.77	0.78	0.88

This table displays the coefficients from regressing monthly index implied volatility on SPP issuance volumes with corresponding underlying. Monthly issuance volumes are scaled by the market cap of corresponding indices. The model includes month and index fixed effects, as well as strike fixed effects in Column 1. The sample includes monthly implied volatility data at 1 year for the 12 leading global indices over the 2005-2019 period: S&P500, Eurostoxx 50, Nikkey, Hang Send, Han Seng China Enterprise, Swiss Market Index, CAC 40, DAX, Nasdaq, Russel, FTSE, Kosip 200. Standard errors are clustered at the month and at the index-year levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table IV
Headline Rates and SPP Volumes

	Net Issuances, in Billion $			
Estimation Samples	**Indices**		**Period**	
	3 main	**All**	**≤2011**	**>2011**
	(1)	**(2)**	**(3)**	**(4)**
Headline Rate	0.11***	0.03***	0.01***	0.04***
	(0.03)	(0.01)	(0.00)	(0.01)
3Y Swap Rate	0.06	0.02***	0.01	0.02
	(0.04)	(0.01)	(0.01)	(0.01)
Constant	-0.81***	-0.22***	-0.01	-0.32***
	(0.15)	(0.04)	(0.04)	(0.07)
Fixed Effects				
Index	Yes	Yes	Yes	Yes
Observations	534	1,761	727	1,034
R^2	0.12	0.05	0.31	0.10

This table displays the coefficients from regressing SPP net issuance volumes on the average headline rate offered by these products. The model includes index fixed effects, as well as strike fixed effects in Column 1. Standard errors are clustered at the month level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table V
The Impact of SPP Volumes on Implied Volatility: Effects on the Volatility Surface of S&P500, Eurostoxx 50, and Nikkei 225

	Dependent Variable : Implied Volatility, in %							
Strike	**All**		**50%**	**60%**	**70%**	**80%**	**90%**	**100%**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Monthly Issuance Volumes, in Billion $	-0.43***							
	(0.12)							
Scaled Issuance Volumes, in basis points		-0.23***	-0.40***	-0.31***	-0.26***	-0.20***	-0.13*	-0.08
		(0.07)	(0.11)	(0.09)	(0.08)	(0.07)	(0.07)	(0.08)
90-Day Realized Volatility, in %	0.23***	0.23***	0.19***	0.20***	0.21***	0.23***	0.26***	0.29***
	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)
3-Year Swap Rate, in %	-0.05	-0.06	-0.05	-0.11	-0.15	-0.13	-0.06	0.12
	(0.17)	(0.17)	(0.25)	(0.21)	(0.20)	(0.19)	(0.17)	(0.16)
Fixed Effects								
Strike	Yes	Yes						
Month	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3,240	3,240	540	540	540	540	540	540
R^2	0.96	0.96	0.97	0.97	0.97	0.97	0.97	0.97

This table displays the coefficients from regressing monthly index implied volatility on SPP issuance volumes with corresponding underlying. In Column 1, SPP volumes are issuance volumes, in Columns 2 to 8, SPP issuance volumes are scaled by market cap. The model includes month and index fixed effects, as well as strike fixed effects in Columns 1 and 2. The sample includes monthly implied volatility data at 1 year for the three main indices of the US, Europe and Asia: S&P500, Nikkei225, and Eurostoxx50. The sample covers the 2005-2019 period. Standard errors are clustered at the month and at the index-year levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table VI
The Impact of SPP Volumes on Implied Volatility: Effects for All Indices across Strikes, Time, and Exposure

	Dependent Variable : Implied Volatility, in %							
	Strikes				**Time Period**		**Option Market Size**	
	All	**80%**	**90%**	**100%**	**<2011**	**≥2011**	**All Strikes**	**80%**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Scaled Issuance Volumes, in BP	-0.34***	-0.39***	-0.35***	-0.30***	-0.39***	-0.13**	0.27	0.35
	(0.10)	(0.11)	(0.10)	(0.11)	(0.14)	(0.05)	(0.30)	(0.32)
Volumes × Medium Option Market							-0.54*	-0.62**
							(0.29)	(0.31)
Volumes× Small Option Market							-0.65**	-0.80***
							(0.28)	(0.31)
90-Day realized volatility, in %	0.41***	0.40***	0.41***	0.41***	0.41***	0.25***	0.41***	0.40***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.07)	(0.03)	(0.03)	(0.03)
3-Year Swap Rate	-0.21*	-0.27**	-0.20*	-0.16	-0.53***	-0.15	-0.21***	-0.27***
	(0.12)	(0.12)	(0.12)	(0.12)	(0.17)	(0.09)	(0.07)	(0.07)
Fixed Effects								
Strike	Yes				Yes	Yes	Yes	
Month	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6,398	2,128	2,135	2,135	983	1,145	6,398	2,128
R^2	0.92	0.91	0.92	0.92	0.91	0.92	0.92	0.91

This table displays the coefficients from regressing monthly index implied volatility on SPP issuance volumes with corresponding underlying. Monthly issuance volumes are scaled by the market cap of corresponding indices. The model includes month and index fixed effects, as well as strike fixed effects in Column 1. The sample includes monthly implied volatility data at 1 year for the 12 leading global indices over the 2005-2019 period: S&P500, Eurostoxx 50, Nikkei 2255, Hang Seng, Han Seng China Enterprise, Swiss Market Index, CAC 40, DAX, Nasdaq, Russel, FTSE, Kospi 200. Standard errors are clustered at the month and at the index-year levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table VII
SPP Volumes and Implied Volatility in Times of Market Stress

	Dependent Variable : Implied Volatility, in %									Realized Vol
Market Stress Indicator	**Yearly Returns**$< -15\%$			**Yearly Returns**$< -25\%$			**Yearly Returns**$< -35\%$			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**
Market Stress	9.21***	14.02***	1.24	14.60***	21.97***	3.92**	17.34***	25.86***	7.72***	1.73
	(1.45)	(3.15)	(1.06)	(1.66)	(2.84)	(1.74)	(1.47)	(3.31)	(2.40)	(1.66)
SPP Volume Quartiles	-0.58***	-0.36**	-0.30***	-0.45***	-0.29*	-0.27***	-0.39**	-0.30*	-0.31***	-0.19*
	(0.17)	(0.17)	(0.07)	(0.16)	(0.16)	(0.06)	(0.16)	(0.16)	(0.07)	(0.11)
Market Downturn × Quartiles		-1.74**	-0.61*		-2.84***	-1.74***		-3.48***	-2.56***	-0.34
		(0.68)	(0.33)		(0.63)	(0.46)		(1.05)	(0.61)	(0.34)
3-Year Swap Rate	1.76***	1.73***	-0.56***	1.70***	1.68***	-0.56***	1.97***	1.96***	-0.54***	-0.43***
	(0.15)	(0.15)	(0.09)	(0.14)	(0.14)	(0.09)	(0.14)	(0.14)	(0.08)	(0.10)
Fixed Effects										
Month	-	-	Yes	-	-	Yes	-	-	Yes	Yes
Strike	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	5,177	5,177	5,177	5,177	5,177	5,177	5,177	5,177	5,177	1,728
R^2	0.56	0.56	0.90	0.62	0.63	0.90	0.58	0.59	0.90	0.86

This table displays the coefficients from regressing monthly index implied volatility on an indicator dummy for market stress and its interaction with quartiles of SPP outstanding volumes. The model includes index and strike fixed effects, as well as month fixed effects in Columns 3, 6, 9 and 10. The sample includes monthly implied volatility data at 1 year for the 12 leading global indices over the 2005-2019 period: S&P500, Eurostoxx 50, Nikkei225, Hang Seng, Han Seng China Enterprise, Swiss Market Index, CAC 40, DAX, Nasdaq, Russel, FTSE, Kospi 200. Standard errors are clustered at the month level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table VIII
Maximum Likelihod Estimation of Structural Model

	Unrestricted Estimate (1)	Restricted Estimate (2)
Interest Rate		
Mean μ_1	0.01928	0.01924
Persistence parameter ϕ_1	0.99699	0.99676
Volatility σ_1	0.00212	0.00211
Equity Index		
Mean log volatility μ_2	-5.49322	-5.50605
Persistence parameter ϕ_2	0.93439	0.93319
Volatility of log volatility σ_2	0.33565	0.33668
Correlations		
ρ_{12}	0.10121	0.10291
ρ_{1M}	0.25972	0.28945
ρ_{2M}	-0.13300	-0.14236
Stochastic discount factor		
Sensitivity of SDF to SPP volume, λ_H	0.38810	0
Payoff of Exogenous Put Portfolio:		
$S_T = 0.0$	0.63072	0.53537
$S_T = 0.5$	-1.11587	-1.14221
$S_T = 0.6$	-0.82323	-0.82332
$S_T = 0.7$	-0.66744	-0.65700
$S_T = 0.8$	-0.88680	-0.86121
$S_T = 0.9$	-0.41482	-0.33398
Measurement errors		
Yield spread	0.01556	0.01615
Put prices	0.00629	0.00633
Test of the restriction $\lambda_H = 0$		
Log likelihood	6286.82283	6283.32066
LR statistic	7.00325	
p value	0.00814	

This table displays the ML estimates of the structural model outlined in Section IV.

Table IX
Volatility and Variance Swap Portfolios
Sorted on Outstanding Short Put Products

		2004-2018		2007-2018		2009-2018	
Portfolio		Var Swap	Vol Swap	Var Swap	Vol Swap	Var Swap	Vol Swap
Low	Mean	-14.79	-18.46	-13.19	-17.71	-47.91	-47.53
	Sd	31.36	28.98	33.78	31.21	18.60	20.03
	Sharpe	-0.47	-0.64	-0.39	-0.57	-2.58	-2.37
2	Mean	-11.43	-17.10	-6.82	-13.58	-39.93	-41.43
	Sd	31.10	28.25	33.32	30.28	18.64	19.94
	Sharpe	-0.37	-0.61	-0.20	-0.45	-2.14	-2.08
3	Mean	-4.49	-11.26	0.83	-7.36	-29.64	-32.94
	Sd	30.85	27.94	33.01	29.96	21.08	21.61
	Sharpe	-0.15	-0.40	0.03	-0.25	-1.41	-1.52
4	Mean	3.28	-5.66	8.61	-2.31	-24.21	-28.44
	Sd	34.58	29.32	37.07	31.49	21.08	21.20
	Sharpe	0.09	-0.19	0.23	-0.07	-1.15	-1.34
High	Mean	22.93	10.06	28.41	13.42	-15.24	-19.36
	Sd	42.11	33.26	45.07	35.53	23.95	22.77
	Sharpe	0.54	0.30	0.63	0.38	-0.64	-0.85
High-Low	Mean	37.72	28.52	41.60	31.12	32.67	28.17
	Sd	16.80	10.99	17.51	11.08	10.27	8.57
	Sharpe	2.25	2.59	2.38	2.81	3.18	3.29

This table presents return statistics associated with variance and volatility swap portfolios formed by sorting on the total outstanding of short put products scaled by the underlying stocks' market capitalization. At each month, stocks are sorted into five portfolios based on the total outstanding of structured retail products associated with each stock scaled by the market cap. Variance and volatility swaps with one year of maturity are formed for each stock and grouped into the sorted portfolios. The ex-post hold-to-maturity returns in units of vega notionals are reported for each portfolio group. The returns for each monthly portfolio groups are annualized (hold-to-maturity return means are multiplied by 12 and standard deviations are multiplied by square root of 12).

Table X
Sharpe Ratios for Long Short Volatility and Variance Swap Portfolios with Alternative Sorting Variables

Type	2004-2018 VarSwap	VolSwap	2007-2018 VarSwap	VolSwap	2009-2018 VarSwap	VolSwap
Sort Variable						
Outstanding_MarketCapScaled	2.25	2.59	2.38	2.81	3.18	3.29
Outstanding	1.78	1.92	1.91	2.05	3.20	3.13
NetIssuance_MarketCapScaled	1.87	2.11	1.89	2.15	2.31	2.27
NetIssuance	1.28	1.28	1.40	1.45	1.92	1.73
Maturity	-1.24	-1.28	-1.30	-1.38	-1.85	-1.72
Issuance	0.86	0.74	0.96	0.84	0.83	0.65

This table presents the long-short portfolio sharpe ratio associated with variance and volatility swap portfolios formed by sorting on variables capturing the stock and flow of short put products. At each month, stocks are sorted into five portfolios based on the sorting variable. Variance and volatility swaps with one year of maturity are formed for each stock and grouped into the sorted portfolios. The ex-post hold-to-maturity returns are reported in units of vega notionals for each portfolio group. The sharpe ratio is based on annualized returns (each porfolio's hold-to-maturity mean return is multiplied by 12, and standard deviations are multiplied by square root of 12).