The Stock Market's Two Truths: Subjective Beliefs and Objective Reality

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Abstract

Investor beliefs determine stock prices, yet surveys provide incomplete measures of these expectations. We develop a return-based framework that infers the representative investor's *subjective* beliefs from the dynamics of *objective* cash-flow and discount-rate news components of market returns, with minimal reliance on surveys. We find investors substantially underreact to fundamental news, initially incorporating only 30% of an objective cash-flow shock. Our framework also identifies which features of a subjective belief model are necessary to fit the data. A belief model that focuses on underreaction but assumes constant subjective expected returns will fit poorly; volatile, acyclical subjective expected returns are necessary.

JEL classifications: D84, G10, G12, G14, G17, G40

Key words: Subjective beliefs, underreaction, recessions, return decomposition

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1 Introduction

Investor beliefs about cash flows and discount rates determine stock returns [Campbell and Shiller (1988a, 1988b) and Campbell (1991)]. To study these beliefs, researchers often invoke the full-information rational expectations (FIRE) framework, under which subjective beliefs match objective reality. This framework facilitates the econometric estimation of beliefs, but growing evidence of deviations between rational forecasts and survey measures casts doubt on the FIRE assumption [e.g., Vissing-Jorgensen (2003), Amromin and Sharpe (2014), and Greenwood and Shleifer (2014)]. Alternatively, researchers can study beliefs directly by turning to these surveys. They provide clues, but issues with past survey designs introduce barriers when describing the investor expectations that determine stock returns [Adam and Nagel (2023)]. Measures of investor cash-flow beliefs are not readily available, and surveys show that subjective expectations vary considerably across investor types, which leaves doubt about which investor group best proxies for the representative investor. These challenges motivate our approach; we infer investor beliefs directly from stock returns with minimal survey support.

In this paper, we propose a theoretical framework for characterizing the representative investor's *subjective* beliefs through the dynamics of *objective* expectations. Campbell's (1991) seminal approach demonstrates that unexpected stock returns can be decomposed into changes in expectations about cash flows and discount rates. This decomposition is valid for objective and subjective expectation operators, so realized stock market returns must simultaneously reconcile with two truths: (i) the representative investor's subjective discount-rate and cash-flow news components and (ii) the objective discount-rate and cash-flow news counterparts. For example, an overly optimistic reaction to an objective cash-flow shock leads the representative investor to set a higher price, which manifests as lower objective discount rates. Our framework builds on this insight that objective expected returns are influenced by subjective beliefs, and we formalize how the relation between objective discount-rate news and objective cash-flow news reflects the degree and direction of investor misreaction. Our estimates imply that the representative investor substantially underreacts to fundamental news, taking up to four years to fully incorporate information. This underreaction is particularly evident around recessions, as investors exhibit a delayed reaction to deteriorating fundamentals and fail to anticipate the recession. Prices surge on pre-recession cash-flow optimism, only to crash as the recession unfolds.

In our theoretical framework, we demonstrate how the empirical properties of objective cash-flow and discount-rate news estimates are informative about subjective beliefs. We consider three cases. Each allows for general dynamics of subjective cash-flow beliefs but has a different assumption on subjective expected returns. Case 1 imposes constant subjective expected returns, which is a common simplifying assumption in subjective belief models [e.g., Nagel and Xu (2022) and Bordalo, Gennaioli, La Porta, and Shleifer (2024)]. Case 2 allows for volatile, but acyclical, variation in subjective expected returns, consistent with Nagel and Xu's (2023) broad-based survey evidence on subjective expected returns. Case 3 is general and places no restrictions on the belief dynamics of either cash flows and expected returns. Improving our collective understanding of the role of beliefs in price formation is crucial, and these cases allow us to demonstrate which features of a belief model are needed to match the empirical evidence.

The simple Case 1 demonstrates that any misreaction to objective cash-flow news by the representative investor, such as underreaction or overreaction, will increase the volatility of objective discount-rate news because fluctuations in mispricing generate additional variation in objective expected returns. Additionally, the correlation between objective discount-rate news and objective cash-flow news distinguishes between underreaction versus overreaction. A positive correlation corresponds to underreaction to fundamental cash-flow shocks. For example, a muted price reaction to a positive objective cash-flow shock increases objective expectations of future returns, so cash-flow and discount-rate news comove positively. Conversely, overreaction is associated with a negative correlation. In addition to these effects, Case 2 introduces volatility in objective discount-rate news through acyclical variation in subjective expected returns. The unrestricted Case 3 allows for cyclicality in subjective expected returns, though we show empirically that Case 3 essentially collapses to Case 2.

We investigate the implications of our framework by decomposing returns into objective components using Campbell's (1991) vector autoregression (VAR) approach. We use model forecast combinations as state variables in the VAR following Anarkulova, Cederburg, Da, and Zhou (2026), who demonstrate that their method produces stable estimates that cleanly separate cash-flow and discount-rate expectations. Over the 1927 to 2021 sample period, objective discount-rate news and cash-flow news have monthly standard deviations of 5.90% and 2.84%, respectively, and a correlation of 0.47. The high discount-rate news volatility is consistent with substantial deviations between subjective and objective expectations. The positive correlation indicates underreaction to objective cash-flow news by the representative investor. Previous studies have used survey-based expectations to test the FIRE hypothesis [e.g., Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2020)]. Our unique approach allows us to provide corroborating evidence on aggregate stock market underreaction and extend inferences to the representative investor by studying prices rather than surveys.

Market behavior around recessions also supports underreaction in subjective beliefs. We examine the 15 NBER-dated recessions in our sample. In the two years preceding economic peaks, the stock market experiences positive cumulative excess returns averaging 16%, despite deteriorating objective cash-flow news that anticipates the recession (cumulative effect of -11% on realized returns). This pre-recession price run-up without support from fundamentals causes a substantial decline in objective expectations of future returns (cumulative effect of 20% on realized returns). Once recessions begin, the market averages a statistically significant decline of -8% in the first six months, consistent with Gómez-Cram (2022) and Kroencke (2022). Objective cash-flow news remains relatively flat, so the price decline implies an increase in objective expected returns (cumulative effect of -12%). These systematic patterns indicate that investors react sluggishly to the negative objective cash-flow news that precedes recessions.

We also directly compare objective and subjective beliefs. Given the sparsity of survey evidence on investors' cash-flow expectations [e.g., Adam and Nagel (2023)], analyst forecasts are the available proxy for the representative investor's subjective beliefs. We decompose returns into subjective news components following Chen, Da, and Zhao (2013). This subjective decomposition in conjunction with the objective decomposition allows us to study the quantitative impact of economic restrictions that we impose in our framework, which is informative about the necessary elements for a subjective belief model.

To study the temporal relation between subjective and objective cash-flow news, we regress quarterly subjective cash-flow news on contemporaneous objective cash-flow news as well as up to 16 quarterly lags of objective news. The contemporaneous relation is weak with a regression coefficient of 0.26, but updates to subjective beliefs positively relate to past objective news and the sum of coefficients in the model with 16 lags is 1.09. Investors initially underreact to fundamental news and take up to four years to fully incorporate the shock.

The return decompositions further imply a linear relation between the objective and subjective news components, and our theoretical framework allows us to examine how a specific model-imposed constraint (e.g., constant subjective expected returns) will affect a model's ability to fit this relation. We first regress subjective cash-flow news on its objective counterpart and estimate that the representative investor underreacts to objective cash-flow news by 70% (t-statistic of 6.03). We also regress subjective discount-rate news on objective cash-flow news and find results that support Nagel and Xu's (2023) conclusion that subjective expected returns are largely acyclical. The point estimate of -0.15 indicates some countercyclicality, but the coefficient is statistically insignificant (t-statistic of -0.96) and economically small. These regressions estimate all of the necessary components to study the three cases in our framework.

We quantify the fit of the three cases by comparing the case-implied objective discount-rate news with the actual estimate and computing the mean squared error (MSE). Case 1, which focuses on cash-flow beliefs and constrains the subjective expected excess return to be constant, poorly fits the data. This case produces an MSE of 112.84, which exceeds the 66.93 unconditional variance of objective discount-rate news. Allowing for time-varying, but acyclical, subjective expected returns in Case 2 significantly improves the fit, reducing the MSE to just 0.71. Because Case 2 captures virtually all of the variation, Case 3 effectively collapses into it. We contribute to the burgeoning literature working to develop subjective belief models by demonstrating that a successful model will require underreaction to fundamental cash-flow shocks as well as acyclical variation in subjective discount rates.

Finally, we study objective and subjective beliefs around recessions. Whereas our prior analysis shows deteriorating objective cash-flow news accompanied by decreasing objective discount rates before recessions, subjective beliefs reveal the opposite pattern. The subjective decomposition implies that the price run-ups are driven by positive subjective cash-flow news with increasing subjective discount rates. The decidedly optimistic subjective viewpoint helps explain why investors continue to hold stocks even as objective expected returns turn negative. Only later do investors act on building evidence of poor fundamentals and overpriced stocks.

Our paper contributes to a large and growing asset pricing literature on subjective beliefs and deviations from FIRE. The FIRE hypothesis assumes investors form expectations that match objective probability distributions [e.g., Muth (1961), Lucas (1978), and Cochrane (2011)]. In order to generate volatile discount-rate news, FIRE models typically rely on volatile, countercyclical risk premiums [e.g., Campbell and Cochrane (1999), Bansal and Yaron (2004), and Gabaix (2012)]. Survey evidence, however, suggests that investors' beliefs deviate from rational expectations [e.g., Vissing-Jorgensen (2003), Amromin and Sharpe (2014), and Greenwood and Shleifer (2014)]. Recent work examines subjective cash-flow expectations [e.g., Chen, Da, and Zhao (2013) and Delao and Myers (2021)] or return expectations across investor types such as retail investors [e.g., Greenwood and Shleifer (2014) and Giglio, Maggiori, Stroebel, and Utkus (2021)] or institutional investors [e.g., Andonov and Rauh (2022); Dahlquist and Ibert (2024, 2026); Couts, Gonçalves, and Loudis (2024); and Couts, Gonçalves, Liu, and Loudis (2024)].[1] A growing theoretical literature proposes

[1]Other papers study subjective beliefs for other asset classes [e.g., Piazzesi, Salomao, and Schneider (2015) and Buraschi, Piatti, and Whelan (2022) for bonds; Piazzesi and Schneider (2009) for real estate; and Nagel and Xu (2023) and Couts, Gonçalves, and Loudis (2024) for multiple asset classes]. See Brunnermeier, Farhi, Koijen, Krishnamurthy, Ludvigson, Lustig, Nagel, and Piazzesi (2021) and Adam and Nagel (2023)

different mechanisms reconciling deviations of subjective beliefs from objective reality. For example, investors with fading memory [e.g., Nagel and Xu (2022)], with parameter uncertainty [e.g., Collin-Dufresne, Johannes, and Lochstoer (2016)], or with behavioral biases [e.g., Barberis, Greenwood, Jin, and Shleifer (2015); Jin and Sui (2022); and Bordalo, Gennaioli, La Porta, and Shleifer (2024)] experience systematic belief distortions.

Existing surveys provide sparse evidence on the representative investor's cash-flow expectations [Adam and Nagel (2023)] and long-horizon beliefs [Sias, Starks, and Turtle (2024)]. The need for more and better surveys has been recognized in the literature [e.g., Manski (2004)]. Our framework allows us to directly examine returns with minimal reliance on survey data, such that it considers all relevant horizons and studies the representative investor who sets prices.[2] We show how subjective belief distortions relative to objective reality manifest through the dynamics of objective news components and demonstrate which features of a subjective belief model are needed to match empirical evidence.

Prior literature finds consistent support for underreaction at the consensus level of macroeconomic forecasts [e.g., Coibion and Gorodnichenko (2012, 2015) and Bordalo, Gennaioli, Ma, and Shleifer (2020)].[3] Financial markets show mixed evidence of overreaction and underreaction. Investors and managers appear to extrapolate from past returns in forming stock market return expectations [e.g., Amromin and Sharpe (2014) and Greenwood and Shleifer (2014)]. At the individual stock level, there is return-based evidence of either overreaction [e.g., long-term reversal of De Bondt and Thaler (1985)] or underreaction [e.g., post-earnings announcement drift of Ball and Brown (1968) and Bernard and Thomas (1989, 1990) or

for excellent reviews.

[2]Recent studies by Chen, Hansen, and Hansen (2020) and Ghosh and Roussellet (2023) attempt to identify subjective investor beliefs using asset prices and survey data. Both studies prespecify the stochastic discount factor that captures investor preferences in order to isolate subjective beliefs. Cassella, Chen, Gulen, and Liu (2025) describe subjective beliefs conditional on a prespecified model of extrapolative beliefs. In contrast, our approach allows us to remain ex ante agnostic about how subjective beliefs are formed. Bretscher, Malkhozov, Tamoni, and Yang (2025) also observe that the return decomposition applies in subjective and objective expectations and that wedges between subjective and objective beliefs manifest in objective returns. However, they focus on quantifying the subjective-objective belief bias and reconciling findings with FIRE in the time series and cross section. Alternatively, Wu (2023) derives subjective expectations of market returns based on trades of spread option strategies.

[3]Whereas consensus-level macroeconomic forecasts show underreaction, individual forecasters typically overreact [e.g., Bordalo, Gennaioli, Ma, and Shleifer (2020)].

momentum of Jegadeesh and Titman (1993)]. There is further evidence of underreaction in analysts' short-term earnings forecasts [e.g., Abarbanell and Bernard (1992) and Bouchaud, Krüger, Landier, and Thesmar (2019)] and overreaction in long-term profitability and growth expectations by managers [e.g., Gennaioli, Ma, and Shleifer (2016) and Barrero (2022)] and analysts [e.g., Bordalo, Gennaioli, La Porta, and Shleifer (2019, 2024)]. Our study contributes to the literature by demonstrating underreaction in aggregate cash-flow expectations at the stock market level. While we remain agnostic on belief formation mechanisms, our findings for subjective cash-flow beliefs are broadly consistent with models of information frictions such as sticky information [e.g., Mankiw and Reis (2002)] or models with noisy information and limited attention [e.g., Sims (2003), Woodford (2003), and Gabaix (2014)].[4]

The remainder of the paper is organized as follows. Section 2 develops our theoretical framework for understanding subjective expectations through the lens of the objective return decomposition and introduces our stock return decomposition methods. Section 3 describes our data sources and provides descriptive statistics. Section 4 analyzes the implications of objective news components for subjective beliefs and directly compares objective and subjective news components. Section 5 concludes.

2 Return decompositions and methods

Section 2.1 develops stock market return decompositions and explores the implied connections between objective and subjective news components. Section 2.2 describes our approaches for estimating objective and subjective news components.

[4]The underreaction measured using our return-based approach could also partially reflect the representative investor's weak transmission of subjective beliefs into portfolio actions [e.g., Giglio, Maggiori, Stroebel, and Utkus (2021) and Charles, Frydman, and Kilic (2024)].

2.1 Objective and subjective return decompositions

Campbell and Shiller (1988a, 1988b) develop a framework relating prices to future cash flows and returns. After loglinearization, the log price-dividend ratio follows

$$p_t - d_t = \frac{k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j (\Delta d_{t+1+j} - r_{f,t+1+j} - r^e_{t+1+j}), \tag{1}$$

where Δd_{t+1} is log dividend growth, $r_{f,t+1}$ is the log risk-free rate, r^e_{t+1} is the log excess return, E_t is the objective expectation operator, and ρ and k are loglinearization constants. Equation (1) shows that prices are high when expected future dividend growth (in excess of the risk-free rate) is high and/or when future expected excess returns are low. Under subjective expectations,

$$p_t - d_t = \frac{k}{1-\rho} + E^*_t \sum_{j=0}^{\infty} \rho^j (\Delta d_{t+1+j} - r_{f,t+1+j} - r^e_{t+1+j}), \tag{2}$$

where E^*_t is the subjective expectation operator.[5] With FIRE, the representative investor's subjective expectations equal objective expectations, making equations (1) and (2) equivalent.

Without FIRE, investor views may depart from objective reality. The representative investor forms beliefs about future cash flows and discount rates, and she sets a price that reflects her subjective beliefs [i.e., equation (2)]. This price level and the objective reality of future cash flows determine the objective expected excess returns [i.e., equation (1)].[6] Intuitively, objective expectations of future returns are low (high) when the price is too high (low) given the true cash-flow fundamentals.

Price fluctuations reflect changes in the representative investor's beliefs. We capture

[5] We set the loglinearization constant $\rho = 1/(1 + \exp(\overline{d-p}))$ to 0.9972, which is calculated from the average log monthly dividend-price ratio $(\overline{d-p})$ in our sample. The decompositions assume the terminal condition $\lim_{j\to\infty} \rho^j (p_{t+j} - d_{t+j}) = 0$. We assume that subjective expectations are coherent following Cui, Delao, and Myers (2025) and satisfy the law of iterated expectations.

[6] For simplicity, this intuition abstracts from the potential feedback mechanism in which asset prices can influence real investment decisions. See Bond, Edmans, and Goldstein (2012) and Goldstein and Yang (2017) for excellent reviews of the real effects of financial markets.

these effects within Campbell's (1991) return decomposition framework. Unexpected excess stock returns can be decomposed into cash-flow news and discount-rate news using either expectation operator. In objective expectations,

$$\begin{aligned} r^e_{t+1} - E_t r^e_{t+1} &= (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j (\Delta d_{t+1+j} - r_{f,t+1+j}) - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^e_{t+1+j} \\ &= N_{CF,t+1} - N_{DR,t+1}. \end{aligned} \tag{3}$$

The news components $N_{CF,t+1}$ and $N_{DR,t+1}$ reflect changes in objective expectations about dividend growth and excess returns, respectively.[7] In subjective expectations,

$$\begin{aligned} r^e_{t+1} - E^*_t r^e_{t+1} &= (E^*_{t+1} - E^*_t) \sum_{j=0}^{\infty} \rho^j (\Delta d_{t+1+j} - r_{f,t+1+j}) - (E^*_{t+1} - E^*_t) \sum_{j=1}^{\infty} \rho^j r^e_{t+1+j} \\ &= N^*_{CF,t+1} - N^*_{DR,t+1}. \end{aligned} \tag{4}$$

Positive subjective cash-flow news ($N^*_{CF,t+1} > 0$) represents an upward revision in the representative investor's expectations of future cash flows. She drives the price higher, such that the return is positive. In contrast, positive subjective discount-rate news ($N^*_{DR,t+1} > 0$) reflects an increase in discount rates, which results in a price decline and a negative return.

We subtract equation (4) from equation (3) to get

$$N_{DR,t+1} = -(N^*_{CF,t+1} - N_{CF,t+1}) + N^*_{DR,t+1} + (E_t r^e_{t+1} - E^*_t r^e_{t+1}). \tag{5}$$

Collectively, equations (1) to (5) illustrate that objective expected returns reflect subjective beliefs, both in levels and in changes over time. In levels, equations (1) and (2) show that objective expected returns are low when prices, as determined by subjective beliefs, are high relative to objective fundamentals. In changes, equation (5) demonstrates that news

[7] It is standard in the literature to estimate the discount-rate news component by modeling excess returns [e.g., Campbell and Vuolteenaho (2004) and Campbell, Giglio, and Polk (2013)], such that the risk-free rate enters the definition of the cash-flow news component. In the Internet Appendix, we show robustness to this modeling choice.

about objective expected returns has three sources. First, when she becomes more optimistic about cash flows relative to objective reality ($N^*_{CF,t+1} > N_{CF,t+1}$), she drives up the price relative to fundamentals, thereby decreasing objective expected returns. Second, when the representative investor raises her subjective discount rates ($N^*_{DR,t+1} > 0$), she drives down the price and increases objective expected returns. Third, there is a (typically) small effect of the initial gap between objective and subjective expected returns ($E_t r^e_{t+1} - E^*_t r^e_{t+1}$).

We develop a theoretical framework to demonstrate how the properties of objective discount-rate and cash-flow news reflect the behavior of subjective beliefs. We present three cases. Each case allows for general, unrestricted dynamics of cash-flow beliefs, but has a different assumption about the dynamics of subjective discount rates. Case 1 has the most restrictive assumption, with subjective expected returns that are constant. This case encompasses several models of beliefs, including Nagel and Xu (2022) and Bordalo, Gennaioli, La Porta, and Shleifer (2024). Case 2 allows for time-varying, but acyclical, subjective expected returns. Nagel and Xu (2023) provide systematic evidence using a wide variety of survey evidence that subjective expected returns are volatile but acyclical, supporting the assumption of Case 2. Case 3 is the general case in which subjective belief dynamics are unrestricted for both cash flows and discount rates.

Case 1. *General subjective cash-flow belief dynamics with constant subjective expected excess returns.*

This case illustrates how subjective cash-flow beliefs affect objective expected returns, with no variation in subjective expected excess returns. Subjective cash-flow news can be expressed as $N^*_{CF,t+1} = (1+\theta)N_{CF,t+1} + \varepsilon_{t+1}$, where θ measures the degree of overreaction [$\theta > 0$] or underreaction [$\theta \in (-1,0)$] to objective cash-flow news and ε_{t+1} is orthogonal to $N_{CF,t+1}$. Given the assumption about subjective discount rates, $N^*_{DR,t+1} = 0$ and $E^*_t r^e_{t+1} = E^* r^e_{t+1}$. Equation (5) simplifies to

$$N_{DR,t+1} = -\theta N_{CF,t+1} - \varepsilon_{t+1} + (E_t r^e_{t+1} - E^* r^e_{t+1}). \tag{6}$$

When $\theta > 0$, the representative investor overreacts to cash-flow news. A positive fundamental shock ($N_{CF,t+1} > 0$) increases her subjective optimism relative to reality, and the resulting price overshoot causes a decrease in objective expected returns ($N_{DR,t+1} < 0$). Conversely, when $\theta \in (-1, 0)$, the representative investor underreacts. Given the same positive fundamental shock, underreaction in subjective beliefs causes a muted price increase relative to the change in the fundamental value, which increases objective expected returns ($N_{DR,t+1} > 0$). Thus, overreaction induces a negative relation between $N_{DR,t+1}$ and $N_{CF,t+1}$, whereas underreaction creates a positive one.

The top two rows of Panel A of Table I provide illustrative examples of the overreaction and underreaction mechanisms (with $\varepsilon_{t+1} = 0$).[8] Consider a positive 10% objective cash-flow shock. A FIRE investor would increase the price by 10%. An overreacting representative investor with $\theta = 0.5$ perceives a 15% cash-flow shock and, with constant subjective expected returns, increases the price by 15%. The −5% objective discount-rate news reconciles the cash-flow and discount-rate effects in equation (3) and implies a decrease in objective expected returns. In contrast, an underreacting investor with $\theta = -0.5$ increases the price by only 5%, with the 5% objective discount-rate news reflecting an increase in objective expected returns.

In addition to overreaction or underreaction to fundamentals, subjective belief revisions that are unrelated to the fundamental shock may be too optimistic ($\varepsilon_{t+1} > 0$) or pessimistic ($\varepsilon_{t+1} < 0$) in a given period.[9] The bottom four rows of Panel A show that the price effects of such optimism (pessimism) drive down (up) objective expected returns. Because this variation is orthogonal to objective cash-flow news, higher volatility of ε_{t+1} will lead to greater attenuation of the correlation between objective discount-rate news and cash-flow news.

From equation (6), the variance of $N_{DR,t+1}$ that is inherited from representative investor

[8]For simplicity, the examples in Table I assume $E_t r^e_{t+1} = E^*_t r^e_{t+1} = 0$ and no change to the FIRE risk premium.

[9]This optimism or pessimism (i.e., ε_{t+1}) is orthogonal to the contemporaneous objective cash-flow news (i.e., $N_{CF,t+1}$), but may be related to past news (e.g., $N_{CF,t}$ or $N_{CF,t-1}$) if the investor initially overreacts or underreacts.

misreactions is approximately $\theta^2 Var(N_{CF,t+1}) + Var(\varepsilon_{t+1})$. Objective discount-rate news is more volatile the larger is the degree of overreaction or underreaction. If we were to focus on pure overreaction or underreaction (i.e., $Var(\varepsilon_{t+1})$ near zero) and rule out extreme overreaction (i.e., $\theta \in (-1, 1)$), then objective discount-rate and cash-flow news would be very highly correlated (positively or negatively) and discount-rate news variance would likely be lower than objective cash-flow news variance because $Var(N_{DR,t+1}) \approx \theta^2 Var(N_{CF,t+1}) < Var(N_{CF,t+1})$. Cash-flow expectation errors that are unrelated to fundamentals (i.e., ε_{t+1}) will increase the volatility of $N_{DR,t+1}$ and attenuate the correlation between $N_{DR,t+1}$ and $N_{CF,t+1}$.

Case 2. *General subjective cash-flow belief dynamics with acyclical subjective expected excess returns.*

This case extends Case 1 to incorporate acyclical subjective expected excess returns, consistent with the empirical evidence of Nagel and Xu (2023). A general expression for subjective discount-rate news is $N^*_{DR,t+1} = \lambda N_{CF,t+1} + \eta_{t+1}$, with η_{t+1} orthogonal to $N_{CF,t+1}$. This case assumes that $\lambda = 0$, such that variation in $N^*_{DR,t+1}$ is acyclical with respect to fundamental cash-flow news. Equation (5) simplifies to

$$N_{DR,t+1} = -\theta N_{CF,t+1} - \varepsilon_{t+1} + \eta_{t+1} + (E_t r^e_{t+1} - E^*_t r^e_{t+1}). \tag{7}$$

Equation (7) implies that acyclical variation in subjective discount-rate news is directly transferred to objective discount-rate news. This variation in η_{t+1} attenuates the relation between $N_{DR,t+1}$ and $N_{CF,t+1}$, since η_{t+1} is orthogonal to objective cash-flow news. Objective discount-rate news variance also picks up a $Var(\eta_{t+1})$ term, such that acyclical subjective discount-rate variation increases $Var(N_{DR,t+1})$. Panel B of Table I numerically illustrates how η_{t+1} induces additional volatility in $N_{DR,t+1}$ and weakens the relation between $N_{DR,t+1}$ and $N_{CF,t+1}$.

Case 3. *General dynamics for cash-flow and discount-rate beliefs.*

This general case places no restrictions on the dynamics of subjective beliefs. In addition to the three effects above relating to θ, ε_{t+1}, and η_{t+1}, this case allows for non-zero λ in $N^*_{DR,t+1} = \lambda N_{CF,t+1} + \eta_{t+1}$. As such, subjective discount-rate news could be procyclical ($\lambda > 0$), countercyclical ($\lambda < 0$), or acyclical ($\lambda = 0$). The general expression for objective discount-rate news is given by

$$N_{DR,t+1} = (\lambda - \theta)N_{CF,t+1} - \varepsilon_{t+1} + \eta_{t+1} + (E_t r^e_{t+1} - E^*_t r^e_{t+1}). \tag{8}$$

Because there are no restrictions on cash-flow or discount-rate beliefs in this case, equations (5) and (8) are equivalent.

If $\lambda > 0$ [e.g., Giglio, Maggiori, Stroebel, and Utkus's (2021) evidence on retail investors], subjective discount rates increase with positive cash-flow news, dampening the price response. If $\lambda < 0$ [e.g., Dahlquist and Ibert's (2024) evidence on institutional investors], subjective discount rates decrease with positive cash-flow news, amplifying the price response. If $\lambda = 0$ [e.g., Nagel and Xu's (2023) broad-based evidence across multiple surveys], Case 3 collapses to Case 2.

2.2 Measuring objective and subjective news components

Our framework in Section 2.1 demonstrates that the objective cash-flow news and discount-rate news components are informative about the representative investor's subjective beliefs. The framework also has implications for the interrelations between objective and subjective news components. To study these implications, we estimate objective and subjective return decompositions.

Our objective return decompositions follow the method of Anarkulova, Cederburg, Da, and Zhou (2026), who demonstrate that their approach generates robust estimates that cleanly separate cash-flow and discount-rate news. We begin with a large set of return predictors from the literature. We then separate them into three groups by predictor persistence, such that we have low-, medium-, and high-persistence groups. Within each persistence group, we average the return forecasts to calculate model forecast combinations. We then standardize the combination return forecasts and use them as state variables in the VAR:

$$\begin{bmatrix} r^e_{t+1} \\ \tilde{r}^e_{1,t+2} \\ \tilde{r}^e_{2,t+2} \\ \tilde{r}^e_{3,t+2} \end{bmatrix} = a + A \begin{bmatrix} r^e_{t} \\ \tilde{r}^e_{1,t+1} \\ \tilde{r}^e_{2,t+1} \\ \tilde{r}^e_{3,t+1} \end{bmatrix} + \epsilon_{t+1}, \tag{9}$$

where $\tilde{r}^e_{l,t+1}$ is the low-persistence model forecast combination predictor of r^e_{t+1}, $\tilde{r}^e_{2,t+1}$ is the medium-persistence forecast, and $\tilde{r}^e_{3,t+1}$ is the high-persistence forecast.

Following Campbell (1991), the discount-rate news component implied by the VAR is

$$N_{DR,t+1} = e1'\rho A(I - \rho A)^{-1}\epsilon_{t+1}, \tag{10}$$

where $e1$ is a $(k+1) \times 1$ vector with the first element equal to one and the remaining k elements equal to zero and I is a $(k+1) \times (k+1)$ identity matrix. The cash-flow news component is

$$N_{CF,t+1} = (e1' + e1'\rho A(I - \rho A)^{-1})\epsilon_{t+1}. \tag{11}$$

We adopt the common empirical practice of estimating cash-flow news as the residual [e.g., Campbell (1991) and Campbell and Vuolteenaho (2004)]. Chen (2010) and Engsted, Pedersen, and Tanggaard (2012) show that the residual approach is equivalent to directly estimating the cash-flow news component if the log dividend-price ratio is included as a standalone state variable in the VAR. In the Internet Appendix, we demonstrate robustness to this modeling choice.

Our subjective return decomposition follows Chen, Da, and Zhao (2013), as we use analyst forecasts to measure subjective expectations of future cash flows. Each firm's implied cost of capital (ICC) is estimated by combining earnings forecasts with current stock prices [see, e.g., Pástor, Sinha, and Swaminathan (2008)].[10] Cash-flow news captures price changes attributable to revisions in expected future dividends holding ICC constant, while discount-rate news reflects price changes due to revisions in required returns holding expected dividends constant.[11] To construct market-level measures, we aggregate firm-level components using market capitalization weights, ensuring our decomposition captures broad market expectations. Capital gain equals cash-flow news plus discount-rate news in this framework, so we multiply the subjective discount-rate news by negative one for ease of comparison with our objective return decomposition. Finally, cash-flow news and discount-rate news sum to the realized capital gain in Chen, Da, and Zhao (2013) instead of the unexpected excess return as in Campbell (1991), so we demean the estimated subjective news components to estimate unconditionally mean-zero expectation shocks for comparability to the objective news components.

3 Data and descriptive statistics

The sample for our objective return decomposition is a balanced panel of monthly stock market excess returns and 25 predictor variables spanning January 1927 to December 2021. The market excess returns are from Kenneth French's website.[12] We use log excess returns multiplied by 100 to express them in percentage form. Table II lists the predictors, and details on variable construction are in the Internet Appendix. The predictors use data from Amit Goyal's website, Kenneth French's website, CRSP, and Bloomberg.[13]

The subjective return decomposition uses data from the Thomson Reuters I/B/E/S Estimates Database from 1985 to 2021. I/B/E/S Summary Statistics contains consensus analyst forecasts for earnings per share (EPS) across multiple forecast horizons. Firm-level accounting data are from Compustat. For each firm in our sample, we collect quarterly data (March,

[10] This calculation assumes steady-state growth rates in the terminal period, consistent with standard valuation models.

[11] Detailed procedures for sample selection and ICC calculation are given in Chen, Da, and Zhao (2013).

[12] We thank Kenneth French for making these data available (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

[13] We thank Amit Goyal for making these data available (https://sites.google.com/view/agoyal145).

June, September, and December) on earnings forecasts for different time horizons: current fiscal year ($FY1$), next two fiscal years ($FY2$ and $FY3$), and long-term growth (LTG). To ensure data quality, we require firms to have: (i) non-missing forecasts for $FY1$, $FY2$, and LTG; (ii) coverage by at least three analysts; and (iii) a minimum of 16 months of continuous coverage. We obtain share prices and shares outstanding from I/B/E/S, with all earnings forecasts and prices adjusted for stock splits.

4 Results

To study how subjective beliefs move stock prices using return decompositions, we use two complementary approaches. First, we examine the time-series properties of objective cash-flow and discount-rate news, which are informative about the representative investor's subjective expectations following the theoretical framework in Section 2.1. Section 4.1 reports VAR estimates, and Section 4.2 draws inferences about the representative investor's subjective beliefs based on the dynamics of objective cash-flow and discount-rate news in the full sample and around recessions. Second, we use the subjective return components estimated from analyst forecasts to directly examine subjective beliefs and relate them to objective beliefs. Section 4.3 considers the joint behavior of objective and subjective return components in the historical time series and around recessions.

4.1 VAR estimates

Table III shows the A matrix of VAR coefficients, the R^2 from each regression, and the covariance matrix for the VAR residuals (Σ). Ordinary least squares (OLS) t-statistics for the VAR coefficients are in parentheses. The estimates for the excess market return predictability regression in the VAR show that the low- and medium-persistence model forecast combinations are significant predictors (t-statistics of 2.28 and 5.89, respectively), and the R^2 of 4.93% is high for a monthly predictive regression.

The VAR coefficient estimates also show that our approach produces predictors with substantial differences in persistence. The estimated half-lives are about one month for the low-persistence predictor, 13 months for the medium-persistence predictor, and 84 months for the high-persistence predictor. These persistence-based state variables allow the VAR to capture rich horizon-based patterns in expected returns [Anarkulova, Cederburg, Da, and Zhou (2026)]. For example, the covariance matrix estimate implies short-term continuation (due to a positive covariance between the shocks to r^e_{t+1} and $\tilde{r}^e_{1,t+2}$) and mean reversion in the medium and long terms (due to negative covariances between shocks to r^e_{t+1} and both of $\tilde{r}^e_{2,t+2}$ and $\tilde{r}^e_{3,t+2}$).

4.2 Evidence on subjective beliefs from objective news

In this section, we make inferences about the representative investor's subjective expectations from the properties of objective cash-flow and discount-rate news. Section 4.2.1 studies the full time series of the news components. Section 4.2.2 concentrates on the periods around recessions.

4.2.1 Full sample inferences

Table IV shows time-series statistics for objective discount-rate news and cash-flow news. We implement a stationary block bootstrap following Politis and Romano (1994) for inferences.[14] Two important empirical patterns emerge. The monthly standard deviation of N_{DR} is 5.90%, which is twice as high as the N_{CF} standard deviation of 2.84%. About 97% of bootstrap draws show this pattern of higher discount-rate news volatility. The correlation between N_{DR} and N_{CF} is positive and equal to 0.47 (90% bootstrap confidence interval of 0.25 to 0.88), which is near estimates from Vuolteenaho (2002), Campbell, Giglio, and Polk

[14]The stationary block bootstraps in the paper use average block lengths of 120 months and 16 quarters for the monthly and quarterly analyses, respectively, based on the block length choice approach of Politis and White (2004) and Patton, Politis, and White (2009). Inferences are robust to alternative block lengths or an independent and identically distributed (IID) bootstrap. Our bootstrap resamples the contemporaneous and lagged return and predictor data to maintain the proper structure for the VAR, and the model forecast combinations are reformed in each of the 100,000 bootstrap draws based on the bootstrap sample before estimating the VAR and the dependent quantities. We reject bootstrap draws in which the VAR is not stationary.

(2013), and Gonçalves (2021). These empirical patterns in objective return components provide valuable insights into the representative investor's subjective beliefs.

The larger volatility of discount-rate news relative to cash-flow news is consistent with Campbell's (1991) seminal finding that stock return variance is largely driven by fluctuating discount rates. Whereas this volatility pattern is often accommodated within FIRE frameworks, the positive correlation between discount-rate news and cash-flow news contrasts with the predictions of many existing FIRE models. Campbell and Cochrane (1999) and Gabaix (2012), for example, predict that discount-rate news and cash-flow news should be reinforcing. Poor cash-flow news in these models is accompanied, on average, by an increase in risk premiums that further decreases the price. This reinforcing behavior manifests as a model-implied negative correlation between discount-rate and cash-flow news, which is inconsistent with our positive estimate of 0.47.

Our framework suggests alternative mechanisms to explain these two empirical patterns. Under Case 1, underreaction ($\theta < 0$) generates a positive correlation between N_{DR} and N_{CF} as prices adjust sluggishly. The positive estimated correlation of 0.47 is consistent with underreaction. Underreaction alone cannot explain the facts in Table IV, however, because isolating this mechanism would imply that the variance N_{CF} should exceed that of N_{DR} and that the correlation between the objective news components is nearly perfect. The pure underreaction mechanism, therefore, provides an incomplete description of beliefs.

Case 1 also allows for noisy subjective cash-flow expectations, which can generate substantial additional variation in objective discount-rate news. Whereas FIRE models attribute the high volatility of objective discount-rate news to time-varying risk premiums, belief models that comply with Case 1 [e.g., Nagel and Xu (2022) and Bordalo, Gennaioli, La Porta, and Shleifer (2024)] can generate this empirical fact even with constant subjective risk premiums. Noise in subjective cash-flow expectations will also break the near-perfect positive correlation between objective cash-flow and discount-rate news caused by underreaction.

Cases 2 and 3 relax the strong assumption of constant subjective expected returns. In Case 2, acyclical variation in subjective discount rates generates additional objective discount-rate news volatility and attenuates the positive correlation induced by underreaction. Under the general Case 3, the full-sample moments of objective discount-rate and cash-flow news do not allow us to separately identify θ (i.e., the parameter for over/underreaction) and λ (i.e., the parameter for pro/countercyclical subjective discount rates). Nagel and Xu (2023) provide systematic empirical evidence that subjective discount rates are largely acyclical, suggesting $\lambda \approx 0$. Our subsequent analyses with direct measures of subjective beliefs also support this conclusion. With acyclical subjective expected returns, Case 3 collapses to Case 2. Thus, the moments of N_{DR} and N_{CF} in Table IV point to underreaction.[15]

4.2.2 Recession period inferences

Given the importance of understanding market downturns, we study dynamics around recessions. Recession onsets are coincident with many of the large stock market declines in our sample. Transitions from boom to bust are also likely to correspond with the largest revisions in objective cash-flow expectations.

Figure 1 plots the cumulative market returns, expected return contributions, discount-rate news contributions, and cash-flow news contributions in the two years leading up to a recession and the first two years of recession and recovery. The figure averages across the 15 NBER-dated recessions in our sample. Panel A tracks the average cumulative excess market return over the 24 months preceding a recession with month zero representing the month of the economic peak, and the red dotted lines provide a 90% bootstrap confidence interval.[16] Panel B plots the cumulative excess market return during the first 24 months of the recession and recovery period, with month one representing the first month of the recession. Panels C and D show the cumulative effects of objective expected excess returns. Panels E and F provide the cumulative contributions of objective discount-rate news (i.e., $-N_{DR}$, such that the plotted discount-rate effects contribute positively to returns), and Panels G and H are analogous for objective cash-flow effects (i.e., N_{CF}). The cumulative effects for the pre-

[15] The positive correlation between N_{DR} and N_{CF} also indicates underreaction even if subjective expected returns are countercyclical [e.g., Dahlquist and Ibert (2024) and Boons, Ottonello, and Valkanov (2026)]. In this case, $\lambda < 0$ so equation (8) of Case 3 shows that the positive correlation implies $\theta < 0$ and $|\theta| > |\lambda|$.

[16] Details about the bootstrap procedure are available in the Internet Appendix.

recession (recession/recovery) period in Panels C, E, and G (Panels D, F, and H) sum to the cumulative excess returns in Panel A (Panel B).

Figure 1 demonstrates dynamics around recessions. Panel A shows that pre-recession periods are marked, on average, by stock market increases that reach nearly 20% before leveling off around six months before the recession onset. The average cumulative two-year excess return is 16%. Expected excess returns (Panel C) initially contribute positively to realized returns, but the slope flattens and turns negative as the peak approaches. The negative slope implies that the expected excess return is negative on average in the months before a recession. Our estimates suggest a very large, positive cumulative effect from discount-rate news (Panel E), as declining future expected returns contribute about 20% to the pre-recession price run-up. Finally, Panel G shows that the declining discount rates are accompanied by poor cash-flow news, with an average cumulative effect of about -11%. The partially offsetting effects of objective discount-rate news ($-N_{DR}$ as plotted) and cash-flow news (N_{CF}) are consistent with the positive correlation between N_{DR} and N_{CF} reported in Table IV. On the whole, the pre-recession periods have rising prices in the face of poor objective cash-flow news, such that objective expected returns fall precipitously.

The recession/recovery periods in Figure 1 initially have large, statistically significant market declines (Panel B), as recently emphasized by Gómez-Cram (2022). The cumulative excess return reaches an average low of -8% in month six of the recession. Panel D implies that a slight negative expected return contributes somewhat to this decline, but Panel F shows that the poor average excess return realization is accompanied by a large -12% discount-rate contribution from unexpected increases in objective expected returns. Once the recession starts, objective cash-flow news (Panel H) is roughly flat throughout the recession and recovery.

In summary, our recession analysis reveals systematic patterns that seem inconsistent with FIRE. It is unsurprising that objective cash-flow news is negative on average leading up to the onset of a recession. What is surprising is that, as objective cash-flow growth prospects worsen, the stock market enjoys substantial gains. The pre-recession advances in the stock market drive objective expected returns into negative territory on average, a pattern that contrasts with predictions of positive risk premiums from popular FIRE frameworks (e.g., standard parameterizations of Campbell and Cochrane (1999) and Bansal and Yaron (2004)). The patterns are, however, consistent with a representative investor who initially underreacts to fundamental news and has optimistic cash-flow beliefs. Rather than properly perceiving the worsening growth prospects leading up to the recession, investors drive prices ever higher, which causes objective expected returns to decline. Investors only sluggishly react to the objectively poor cash-flow news as the signals become more overt with the onset of a recession. Stocks suffer as the recession begins, as anticipated by the negative objective discount-rate news in the pre-recession period.

4.3 Direct comparison of objective and subjective beliefs

We now turn to direct measures of subjective beliefs and compare them to objective beliefs. We use analyst forecasts to proxy for the representative investor's subjective expectations of future cash flows. Analyst data are available beginning in 1985. Section 4.3.1 considers the full sample, and Section 4.3.2 concentrates on periods around recessions.

4.3.1 Full sample inferences

Panel A of Table V provides time-series properties for the objective and subjective return components, where the subjective news components are quarterly estimates following Chen, Da, and Zhao (2013). Discount-rate news robustly accounts for more return variance than does cash-flow news under both objective and subjective expectations. Whereas the objective effects of discount-rate news and cash-flow news have a 0.31 correlation over the shorter sample period (90% bootstrap confidence interval of 0.07 to 0.48), the subjective effects are even more strongly positively correlated at 0.69 (confidence interval of 0.48 to 0.80). Regardless of whether we examine objective or subjective expectations, the positive correlation is in line with Campbell, Giglio, and Polk's (2013) conclusion that market downturns reflect declining cash-flow expectations or increasing discount rates, but usually not both.

Panel B of Table V shows a large disconnect between objective and subjective beliefs. The correlation between the discount-rate effects from the two models is moderate at 0.46. More striking is the correlation between cash-flow news measures, which is just 0.21 (90% bootstrap confidence interval of -0.02 to 0.35). This low contemporaneous correlation could indicate either noise in subjective cash-flow expectations or a lag in the subjective response to objective cash-flow news if investors initially underreact.

Figure 2 investigates underreaction by examining the temporal relation between subjective and objective cash-flow news. We regress quarterly subjective cash-flow news on the contemporaneous objective cash-flow news along with zero to 16 lags of objective news. Figure 2 plots the sum of regression coefficients as a function of the number of included lags and the 90% confidence interval based on OLS standard errors.

Figure 2 demonstrates underreaction in subjective beliefs. The relation between contemporaneous subjective and objective cash-flow news is relatively weak, with a regression coefficient of 0.26, consistent with the low correlation in Table V. Updates to subjective beliefs are, however, related to past objective news. When we include four lags of objective cash-flow news, the sum of coefficients is 0.72, indicating that the representative investor is slowly learning about fundamental news. The sum is 1.09 when we include 16 lags, which indicates that investor beliefs have fully responded to objective cash-flow news after four years have passed.

The estimates of objective and subjective return components, along with our theoretical framework in Section 2.1, allow us to explore which features of a model of subjective beliefs are necessary to match the empirical evidence. Specifically, we can compare the estimated objective discount-rate news with the objective discount-rate news implied under the three cases studied in Section 2.1 and consider the impact on model fit of the economic restrictions imposed by each case. For example, Case 1 allows for general dynamics of subjective cash-flow expectations but constrains subjective expected returns to be constant, so this case is informative about how well a model with constant subjective discount rates can fit the data.

Table VI presents parameter estimates and summarizes model fit. The sample is quarterly data over the June 1985 to December 2021 period. We begin with the regressions

$$N^*_{CF,t+1} = a + (1+\theta)N_{CF,t+1} + \varepsilon_{t+1} \tag{12}$$

and

$$N^*_{DR,t+1} = b + \lambda N_{CF,t+1} + \eta_{t+1}. \tag{13}$$

Recall that θ reflects the degree of overreaction [$\theta > 0$] or underreaction [$\theta \in (-1, 0)$] to objective cash-flow news. The λ parameter shows procyclical ($\lambda > 0$), countercyclical ($\lambda < 0$), or acyclical ($\lambda = 0$) variation in subjective discount-rate news.

Panel A of Table VI reports regression results. The estimate of θ is -0.70 (t-statistic of -6.03). This negative estimate of θ indicates underreaction to fundamental cash-flow news, in line with our previous conclusions. The point estimate of λ is -0.15, but this estimate is statistically insignificant with a t-statistic of -0.96. The regression R^2 is only 0.63%. This evidence is consistent with Nagel and Xu (2023), who generally find countercyclical point estimates but weak statistical significance when relating subjective expected returns to economic cycles, leading to their conclusion that subjective discount rates are volatile but largely acyclical.[17]

Panel B of Table VI shows a measure of model fit for each case from Section 2.1. We first calculate, for each case, the fitted value of the objective discount-rate news implied by the right-hand side of the case-specific equation. The regressions above provide estimates of θ, λ, ε_{t+1}, and η_{t+1}. We include each one for the model specifications in which it appears. Each specification also includes the $E_t r^e_{t+1} - E^*_t r^e_{t+1}$ term in equation (5) and the gap between return and capital gain, which is needed in the regression because the subjective decomposition is based on capital gains whereas our objective decomposition is based on returns. We then calculate the mean squared error for each case, where the error is the difference between the objective discount-rate news estimate and the case-implied fitted value of objective discount-

[17] In the Internet Appendix, we demonstrate that these conclusions are robust to excluding crisis periods, focusing exclusively on crisis periods, or allowing for asymmetries with respect to positive versus negative objective cash-flow news periods.

rate news.

The first column of Panel B of Table VI investigates Case 1, which constrains subjective expected returns to be constant. Modeling objective discount-rate news only as a function of $-\theta N_{CF,t+1}$, $-\varepsilon_{t+1}$, the expected-return gap, and the return-capital gain gap results in a mean squared error of 112.84 (confidence interval of 58.16 to 183.71). This model fit is exceedingly poor given that the variance of quarterly objective discount-rate news over this period is 66.93. In untabulated results, an unrestricted OLS regression of objective discount-rate news on Case 1 determinants produces a coefficient of -0.09 for ε_{t+1}, but Case 1 restricts it to -1.00 which results in a very high MSE.[18] These results show that modeling underreaction and noise in subjective cash-flow beliefs, in isolation, cannot explain the substantial variation in objective discount rates. Models that assume constant subjective expected excess returns [e.g., Nagel and Xu (2022) and Bordalo, Gennaioli, La Porta, and Shleifer (2024)] counterfactually imply an MSE of 0.00 for Case 1.[19] Such models will be limited in their ability to fit the empirical properties of subjective and objective return components.

In untabulated results, we also find that a model that assumes constant subjective cash-flow beliefs with unrestricted dynamics on subjective discount rates also provides a poor fit, with an MSE of 80.42 (confidence interval of 51.84 to 111.88). Thus, modeling either cash-flow beliefs or discount-rate beliefs in isolation will not be empirically successful.

The Case 2 results in the next column of Panel B introduce time-varying subjective expected returns, but constrain subjective discount-rate news to be acyclical. The MSE is just 0.71 (confidence interval of 0.01 to 6.42), which is a reduction of nearly 99% relative to the objective discount-rate news variance of 66.93. Case 2 achieves outstanding model fit for objective discount-rate news. Case 3 has an MSE of 0.00, which is by construction given that it allows for general subjective belief dynamics so equations (5) and (8) are equivalent.

[18]The small OLS coefficient for ε_{t+1} reflects an omitted variable bias, since ε_{t+1} and η_{t+1} are positively correlated and enter equation (8) of the general Case 3 with coefficients of -1.00 and 1.00, respectively.

[19]We formally map the model of Bordalo, Gennaioli, La Porta, and Shleifer (2024) into Case 1 and derive the model implications for objective and subjective cash-flow and discount-rate news in the Internet Appendix.

Relatively little is gained, however, when the acyclical restriction of Case 2 is relaxed in Case 3. The λ parameter estimate is therefore shown to be statistically insignificant in Panel A and economically insignificant in Panel B.

The results in Table VI indicate that a successful model of subjective beliefs must incorporate variation in subjective expectations of both cash flows and discount rates. The model fit is quite poor when either subjective cash-flow beliefs or subjective expected returns are constrained to be constant. The results indicate that including volatile, acyclical subjective discount-rate news greatly improves model fit. A successful theory of subjective beliefs must, therefore, incorporate rich features of both cash-flow and discount-rate expectations in order to match the empirical properties of the data.

4.3.2 Recession period inferences

The results in Section 4.3.1 demonstrate that the representative investor substantially underreacts to objective cash-flow news and takes up to four years to fully incorporate fundamental news. This initial underreaction manifests around recessions. Figure 3 is analogous to Figure 1, but plots the cumulative discount-rate news contribution and cash-flow news contribution from the subjective return decomposition. The sample period in Figure 3 covers only four recessions, but we verified that the important patterns in objective expectations about discount-rate news and cash-flow news from Figure 1 carry over to this sample.

Figure 3 reveals a fundamental disconnect between objective reality and subjective beliefs. Whereas the objective decomposition shows deteriorating cash-flow news accompanied by decreasing discount rates before recessions (Figure 1), subjective expectations have the opposite pattern. Increases in market valuations are accompanied by positive, rather than negative, subjective cash-flow news. In untabulated results, we estimate that the representative investor's subjective cash-flow expectations are overly optimistic, with an average pre-recession cumulative ε_{t+1} above 5%.[20] Subjective discount-rate news has a slight negative contribution, consistent with investors extrapolating recent positive returns [e.g., Amromin

[20]This optimism could reflect noise or delayed reaction to positive news from the boom period.

and Sharpe (2014) and Greenwood and Shleifer (2014)]. This reversal in the perceived drivers of market performance suggests investors misinterpret and underreact to negative objective economic signals preceding downturns. The observed wedge between objective and subjective cash-flow expectations leading up to recessions explains why investors were readily holding stocks when objective expected returns were negative. In the first year of the recession/recovery period, we estimate that an average cumulative subjective cash-flow news effect of -15% contributes to market crashes. Thus, investors initially underreact to negative objective cash-flow signals as the recession approaches, and their overly optimistic expectations only adjust as the recession unfolds.

5 Conclusion

We develop a return-based approach to infer the representative investor's subjective beliefs from stock market returns with minimal reliance on survey evidence. Our theoretical framework demonstrates how the dynamics of objective discount-rate and cash-flow news are informative about subjective beliefs. We also quantitatively assess the effects of economic restrictions imposed by subjective belief models. Therefore, our framework provides guidance for the necessary features of belief models.

The representative investor systematically underreacts to fundamental shocks, initially underreacting by 70% and taking up to four years to fully incorporate objective cash-flow news. Volatile, but acyclical, subjective discount rates are also necessary to match the dynamics of subjective and objective return components. The effects of deviations between subjective beliefs and objective reality are particularly pronounced in the periods leading up to recessions, and overly optimistic subjective cash-flow expectations help explain why investors happily invest in stocks despite negative objective expected returns.

Our findings identify and quantify the dynamics of representative investor beliefs, but we remain agnostic on the specific mechanisms that produce these patterns. The underreaction to cash-flow news and noisy subjective cash-flow beliefs that we document are consistent with several existing frameworks. Models of sticky information with agents who infrequently update [Mankiw and Reis (2002)] and models with agents who allocate limited attention across signals [Sims (2003), Woodford (2003), Gabaix (2014)] can produce both gradual absorption of fundamental news and noisy subjective cash-flow beliefs. Our results show, however, that volatile but acyclical subjective discount rates are also necessary to match the data. Pure information-friction models do not typically address this feature. A complete theory of the representative investor's beliefs must jointly account for underreaction to cash-flow news, noise in subjective cash-flow beliefs, and volatile acyclical subjective discount rates to clear our diagnostic framework.

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Table I

Illustrative examples for the relation between subjective beliefs and objective reality.

The table provides illustrative examples for Cases 1 and 2 of the relation between subjective beliefs and objective reality, introduced in Section 2.1. The parameters govern subjective cash-flow overreaction ($\theta > 0$) or underreaction ($\theta < 0$), noise in subjective cash-flow news that is too optimistic ($\epsilon_{t+1} > 0$) or pessimistic ($\epsilon_{t+1} < 0$), and variation in subjective discount-rate news orthogonal to fundamentals that is positive ($\eta_{t+1} > 0$) or negative ($\eta_{t+1} < 0$). The examples consider the effects of a positive 10% objective cash-flow shock.

			FIRE	Subjective					Objective				
θ	ε_{t+1}	η_{t+1}	$r^e_{t+1} - E_t r^e_{t+1}$	$r^e_{t+1} - E^*_t r^e_{t+1}$	=	$N^*_{CF,t+1}$	−	$N^*_{DR,t+1}$	$r^e_{t+1} - E_t r^e_{t+1}$	=	$N_{CF,t+1}$	−	$N_{DR,t+1}$
Panel A: Case 1 parameter scenarios													
0.5	0%	—	10%	15%	=	15%	−	0%	15%	=	10%	−	−5%
−0.5	0%	—	10%	5%	=	5%	−	0%	5%	=	10%	−	5%
0.5	3%	—	10%	18%	=	18%	−	0%	18%	=	10%	−	−8%
0.5	−3%	—	10%	12%	=	12%	−	0%	12%	=	10%	−	−2%
−0.5	3%	—	10%	8%	=	8%	−	0%	8%	=	10%	−	2%
−0.5	−3%	—	10%	2%	=	2%	−	0%	2%	=	10%	−	8%
Panel B: Case 2 parameter scenarios													
0.5	0%	3%	10%	12%	=	15%	−	3%	12%	=	10%	−	−2%
0.5	0%	-3%	10%	18%	=	15%	−	−3%	18%	=	10%	−	−8%
−0.5	0%	3%	10%	2%	=	5%	−	3%	2%	=	10%	−	8%
−0.5	0%	-3%	10%	8%	=	5%	−	−3%	8%	=	10%	−	2%

Table II
Predictor variables.

The table lists the 25 market return predictor variables in our sample and the study we follow to construct each predictor. The predictors are sorted into subgroups based on their autoregression coefficients, and the table shows the low-persistence group (estimated half-lives of less than three months) in Panel A, the medium-persistence group (estimated half-lives greater than or equal to three months but less than 48 months) in Panel B, and the high-persistence group (estimated half-lives greater than or equal to 48 months) in Panel C.

Variable	Study
Panel A: Low persistence	
Stock variance	Goyal and Welch (2008)
Long-term return	Goyal and Welch (2008)
Default return	Goyal and Welch (2008)
Inflation	Goyal and Welch (2008)
Relative bill rate	Campbell (1991)
Tail risk	Kelly and Jiang (2014)
Average skewness	Jondeau, Zhang, and Zhu (2019)
Panel B: Medium persistence	
Net stock issues	Goyal and Welch (2008)
Term spread	Goyal and Welch (2008)
Default spread	Goyal and Welch (2008)
Small-stock value spread	Campbell and Vuolteenaho (2004)
Payout yield	Boudoukh, Michaely, Richardson, and Roberts (2007)
Average correlation	Pollet and Wilson (2010)
PLS aggregated book-to-market ratio	Kelly and Pruitt (2013)
Amihud illiquidity measure	Chen, Eaton, and Paye (2018)
Zero return frequency	Chen, Eaton, and Paye (2018)
Panel C: High persistence	
Dividend-price ratio	Goyal and Welch (2008)
Dividend yield	Goyal and Welch (2008)
Earnings-price ratio	Goyal and Welch (2008)
Book-to-market ratio	Goyal and Welch (2008)
Dividend-earnings ratio	Goyal and Welch (2008)
Treasury bill yield	Goyal and Welch (2008)
Long-term yield	Goyal and Welch (2008)
Smoothed earnings-price ratio	Campbell and Vuolteenaho (2004)
Nearness to Dow historical high	Li and Yu (2012)

Table III
Vector autoregression for baseline model.

The table summarizes results from a vector autoregression with the model forecast combinations with $k = 3$ predictor subgroups based on persistence. The predictors are sorted into subgroups based on their autoregression coefficients, and we create model forecast combinations for low-persistence ($\tilde{r}^e_{1,t+1}$, half-lives less than three months), medium-persistence ($\tilde{r}^e_{2,t+1}$, three months to four years), and high-persistence ($\tilde{r}^e_{3,t+1}$, more than four years) predictors. The state variables in the VAR are the log excess market return and the fitted log excess market return forecasts from the low-, medium-, and high-persistence subgroups using the mean combination approach. The fitted expected returns are standardized. We report the VAR coefficient matrix A, its OLS t-statistics (in parentheses), the R^2 (in percent) from each regression, and the covariance matrix Σ. The sample period is January 1927 to December 2021. An * (**) [***] on a VAR coefficient estimate indicates statistical significance at the 10% (5%) [1%] level in a two-tailed test.

	A					Σ			
	r^e_t	$\tilde{r}^e_{1,t+1}$	$\tilde{r}^e_{2,t+1}$	$\tilde{r}^e_{3,t+1}$	R^2	r^e_{t+1}	$\tilde{r}^e_{1,t+2}$	$\tilde{r}^e_{2,t+2}$	$\tilde{r}^e_{3,t+2}$
r^e_{t+1}	0.04	0.38**	0.99***	0.08	4.93	27.27	1.86	−0.24	−0.55
	(1.36)	(2.28)	(5.89)	(0.49)					
$\tilde{r}^e_{1,t+2}$	−0.03***	0.45***	0.07**	−0.02	17.63	1.86	0.82	−0.02	−0.03
	(−5.54)	(15.36)	(2.53)	(−0.73)					
$\tilde{r}^e_{2,t+2}$	−0.01***	−0.01	0.97***	0.02***	93.76	−0.24	−0.02	0.06	0.01
	(−9.16)	(−0.97)	(119.79)	(2.69)					
$\tilde{r}^e_{3,t+2}$	−0.00	−0.00	−0.02***	1.00***	98.57	−0.55	−0.03	0.01	0.01
	(−1.32)	(−0.81)	(−4.45)	(260.79)					

Table IV
Objective discount-rate and cash-flow news variability.

The table reports the standard deviations of objective discount-rate news and objective cash-flow news and the correlation between them. The standard deviations are reported in percent per month. The table also reports the bootstrap probability that the standard deviation of objective cash-flow news exceeds the standard deviation of discount-rate news and the bootstrap 90% confidence interval for the correlation. The sample period is January 1927 to December 2021.

Statistic	Estimate
$\sigma(N_{DR})$	5.90
$\sigma(N_{CF})$	2.84
$\Pr[\sigma(N_{CF}) > \sigma(N_{DR})]$	0.03
$\rho(N_{DR}, N_{CF})$	0.47
90% CI	$[0.25, 0.88]$

Table V
Objective and subjective discount-rate and cash-flow news variability.

The table reports the variability statistics for both objective and subjective decompositions. Panel A reports the standard deviations of discount-rate news and cash-flow news and the correlation between them for each decomposition. The standard deviations are reported in percentage per month. Panel B reports the correlations between subjective and objective news components. The table also reports the bootstrap probability that the standard deviations of objective cash-flow news exceeds the corresponding standard deviations of discount-rate news and the bootstrap 90% confidence interval for the indicated correlation. The subjective series are quarterly, so the statistics are based on quarterly observations and the quarterly standard deviations are reported in percentage per month by multiplying them by $1/\sqrt{3}$. The sample period is June 1985 to December 2021.

Panel A: News component moments		
Statistic	Objective	Subjective
σ(DR news)	4.72	6.15
σ(CF news)	3.33	4.75
Pr[σ(CF news) > σ(DR news)]	0.01	0.00
ρ(DR news, CF news)	0.31	0.69
90% CI	$[0.07, 0.48]$	$[0.48, 0.80]$
Panel B: Objective-subjective correlations		
Statistic	DR news	CF news
ρ	0.46	0.21
90% CI	$[0.36, 0.56]$	$[-0.02, 0.35]$

Table VI
Model fit for Cases 1 to 3.

The table reports results from implementing the three cases in Section 2.1. Panel A shows regression results to estimate the θ parameter calculated from the $(1+\theta)$ coefficient in a regression of subjective cash-flow news on objective cash-flow news and the λ parameter in a regression of subjective discount-rate news on objective cash-flow news. We report OLS t-statistics (in parentheses) and the R^2 (in percent) from each regression. An * (**) [***] indicates statistical significance at the 10% (5%) [1%] level in a two-tailed test. Panel B reports the model fit, as measured by the mean squared error of model-implied objective discount-rate news relative to the estimated objective discount-rate news. The panel also shows the 90% bootstrap confidence interval for each mean squared error. The sample period is June 1985 to December 2021.

Panel A: Parameter estimates from regressions on $N_{CF,t+1}$			
	$N^*_{CF,t+1}$	$N^*_{DR,t+1}$	
θ	-0.70^{***}		
	(-6.03)		
λ		-0.15	
		(-0.96)	
R^2	20.03	0.63	
Panel B: Model fit for each case			
	Case 1	Case 2	Case 3
Mean squared error	112.84	0.71	0.00
90% CI	[58.16,183.71]	[0.01,6.42]	[0.00,0.00]

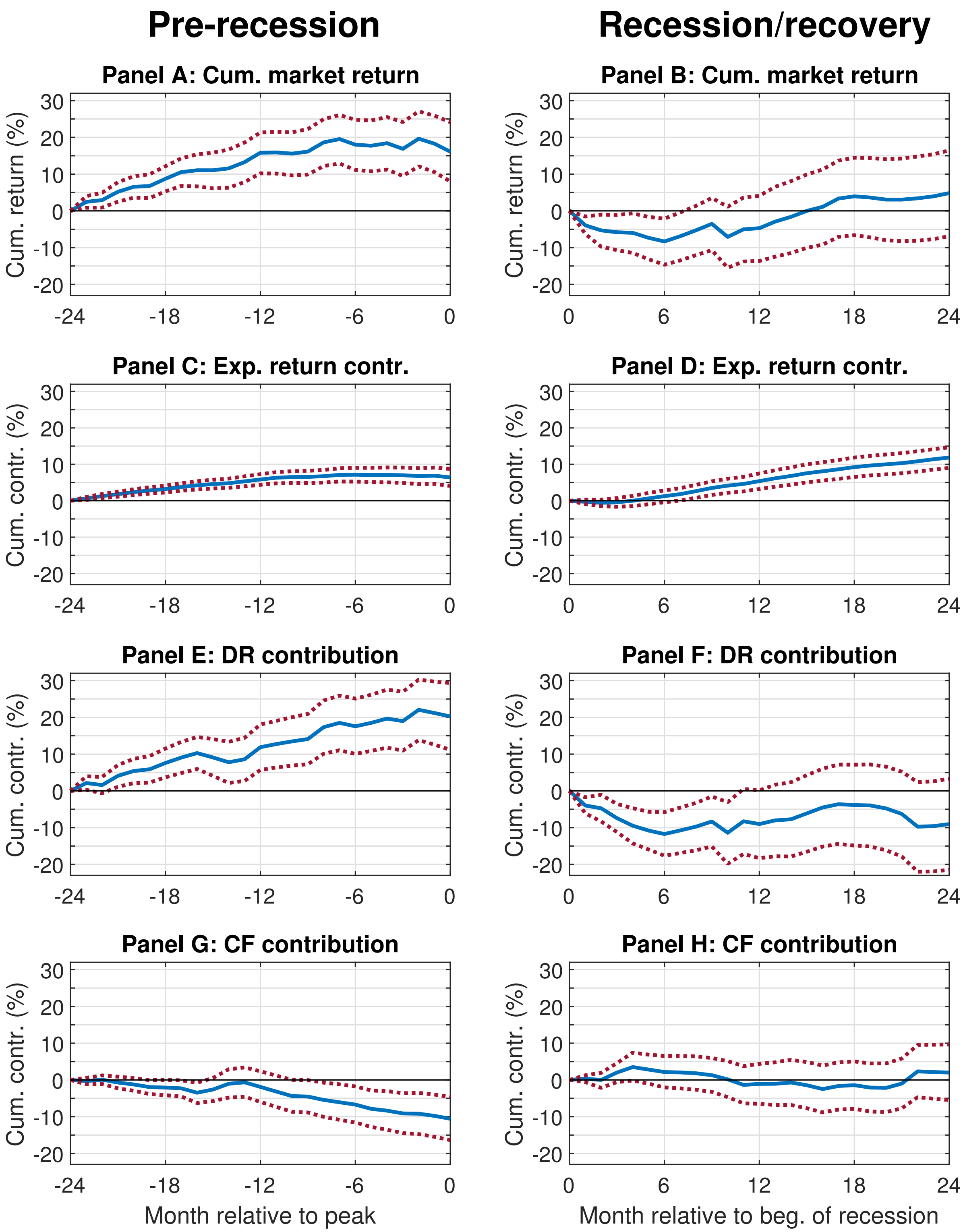

Figure 1. Baseline VAR objective news about returns, discount rates, and cash flows around recessions. The figure shows the cumulative market returns (Panels A and B), expected returns (Panels C and D), negative of discount-rate news (Panels E and F), and cash-flow news (Panels G and H) in the periods leading up to economic peaks (Panels A, C, E, and G) and following NBER recession starts (Panels B, D, F, and H). The 90% confidence intervals from a bootstrap simulation are shown as red dotted lines. The sample period is January 1927 to December 2021.

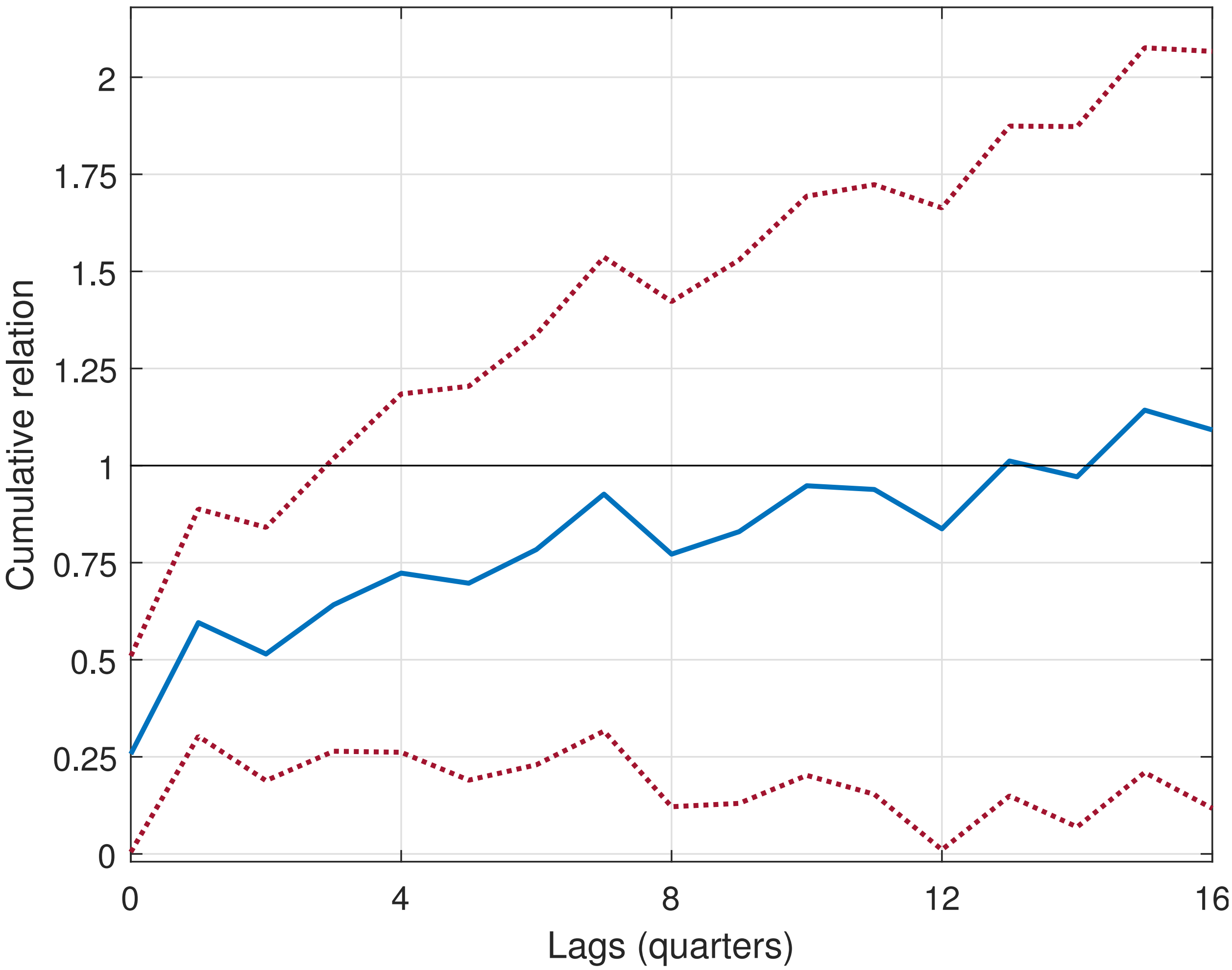

Figure 2. Subjective belief underreaction to objective cash-flow news. The figure plots the sum of regression coefficients from regressing quarterly subjective cash-flow news on the contemporaneous objective cash-flow news and zero to 16 lags of objective cash-flow news. The 90% confidence interval using OLS standard errors is shown as red dotted lines. The sample period is June 1985 to December 2021.

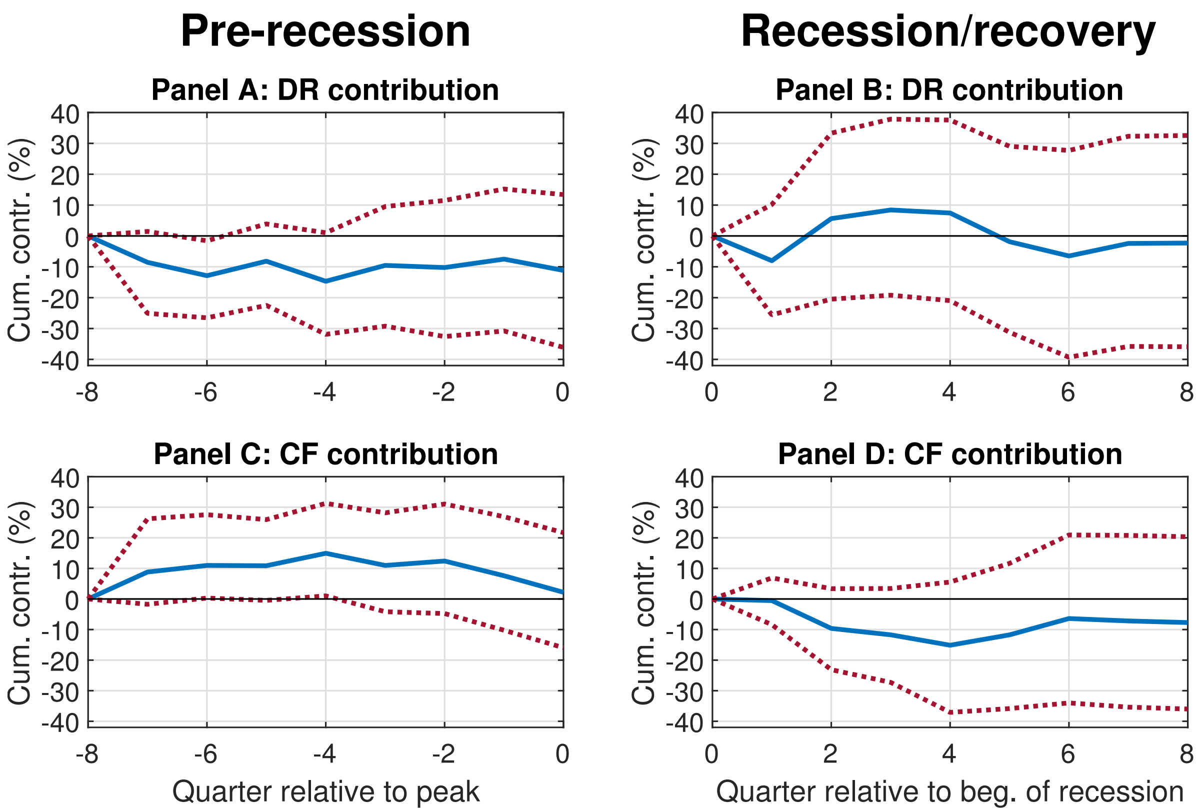

Figure 3. Subjective news about capital gains, discount rates, and cash flows around recessions. The figure shows the cumulative discount-rate news (Panels A and B) and cash-flow news (Panels C and D) in the periods leading up to economic peaks (Panels A and C) and following NBER recession starts (Panels B and D). The 90% confidence intervals from a bootstrap simulation are shown as red dotted lines. The sample period is June 1985 to December 2021.

Internet Appendix

"The Stock Market's Two Truths: Subjective Beliefs and Objective Reality"

A Variable descriptions

This appendix describes the return predictor variables used in the vector autoregressions. We require predictors to have historical data by January 1927. We estimate VARs using monthly data, so we also require that we can calculate each variable on a monthly basis.

We consider a total of 25 predictors. The first 14 predictors are from Goyal and Welch (2008).

1. **Dividend-price ratio**: The log of the ratio of dividends paid on the S&P 500 over the past 12 months and the index level of the S&P 500.

2. **Dividend yield**: The log of the ratio of dividends paid on the S&P 500 over the past 12 months and the 12-month lag of the index level of the S&P 500.

3. **Earnings-price ratio**: The log of the ratio of earnings on the S&P 500 over the past 12 months and the index level of the S&P 500.

4. **Book-to-market ratio**: The ratio of book value and market value of the Dow Jones Industrial Average.

5. **Dividend-earnings ratio**: The log of the ratio of dividends on the S&P 500 over the past 12 months and earnings on the S&P 500 over the past 12 months.

6. **Stock variance**: The sum of squared daily returns on the S&P 500 over the past month.

7. **Net stock issues**: The ratio of the sum of net issues by NYSE stocks over the past 12 months and the total market capitalization of NYSE stocks.

8. **Treasury bill yield**: The yield on three-month Treasury bills.

9. **Long-term yield**: The yield on long-term Treasury bonds.

10. **Long-term return**: The return on long-term Treasury bonds over the past month.

11. **Term spread**: The difference between the yield on long-term Treasury bonds and the yield on three-month Treasury bills.

12. **Default spread**: The difference between the yield on BAA-rated corporate bonds and the yield on AAA-rated corporate bonds.

13. **Default return**: The difference between the return on long-term corporate bonds and the return on long-term Treasury bonds over the past month.

14. **Inflation**: The one-month percentage change in the Consumer Price Index (All Urban Consumers) lagged by one month to account for reporting delays.

The next three predictors are variables used in VARs by John Campbell that are not analogous to any of the variables listed above.

15. **Relative bill rate**: Following Campbell (1991), we compute the difference between the yield on one-month Treasury bills and the average yield on one-month Treasury bills over the prior 12 months. Data for one-month Treasury bill yields are from Kenneth French's website.[21]

16. **Smoothed earnings-price ratio**: Following Campbell and Vuolteenaho (2004), the smoothed earnings-price ratio is the log of the ratio of the average annual earnings on the S&P 500 over the past ten years and the index level of the S&P 500. We thank Amit Goyal for providing these data.

17. **Small-stock value spread**: Following Campbell and Vuolteenaho (2004), the small-stock value spread is the log of the ratio of the book-to-market ratio of the small value portfolio and the book-to-market ratio of the small growth portfolio, where the small value and small growth portfolios are from a two-by-three sort on size and book-to-market. The June book-to-market ratio for each portfolio is the value-weighted average of the book value from fiscal year $t-1$ divided by the market value from December of year $t-1$. The log difference in the book-to-market ratios in subsequent months is adjusted by adding the log of the cumulative value-weighted return on the small growth portfolio since the past June and subtracting the log of the cumulative value-weighted return on the small value portfolio since the past June. All data are from Kenneth French's website.

The remaining eight predictors are from recent studies published in the *Journal of Finance*, *Journal of Financial Economics*, or *Review of Financial Studies*.

18. **Net payout yield**: Following Boudoukh, Michaely, Richardson, and Roberts (2007), we compute the net payout yield as the log of 0.1 plus the ratio of the aggregate net payout by nonfinancial firms over the past 12 months and the aggregate market capitalization of nonfinancial firms. We use data from CRSP for NYSE-, AMEX-, and Nasdaq-listed firms with share codes of 10 and 11, and we exclude firms with Standard Industrial Classification (SIC) codes between 6000 and 6999. The aggregate dividend for each month is calculated from firm-level dividends that are imputed from the returns and ex-dividend returns on nonfinancial firms. The aggregate net issuance for each month is calculated from firm-level net issuance, which is the product of the split-adjusted price per share and the monthly change in the split-adjusted number of shares

[21] We thank Kenneth French for making these data available (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

outstanding, in which the split-adjusted price per share is the month-beginning and month-ending average. Aggregate net payout is aggregate dividends minus aggregate net issuances. We calculate $PY_t = \log(0.1 + NP_t)$, where NP_t is the ratio of the aggregate net payout by nonfinancial firms over the past 12 months and the aggregate market capitalization of nonfinancial firms.

19. **Average correlation**: Following Pollet and Wilson (2010), we calculate the value-weighted average cross-sectional correlation of daily returns for the 500 largest stocks by market capitalization and the end of each month. We use daily returns from CRSP for NYSE-, AMEX-, and Nasdaq-listed firms with share codes of 10 and 11. For each month t, we compute the standard deviation for each stock i, $\hat{\sigma}_{i,t}$, and the covariance between each pair of stocks i and j, $\hat{\sigma}_{ij,t}$, over the period from month $t-2$ to month t using non-missing daily returns. We require that each firm exists for all three months by ensuring it has a price observation at the end of month $t-3$. We then compute the correlation between each pair of stocks i and j as

$$\hat{\rho} = \frac{\hat{\sigma}_{ij,t}}{\hat{\sigma}_{i,t}\hat{\sigma}_{j,t}}. \tag{A1}$$

    Finally, we calculate the average correlation measure as

$$CORR_t = \sum_{i=1}^{500} \sum_{j \neq i} w_{i,t} w_{j,t} \hat{\rho}_{ij,t}, \tag{A2}$$

    where $w_{i,t}$ and $w_{j,t}$ are the value weights of stocks i and j at the end of month t in a portfolio of the largest 500 stocks at the end of month t.

20. **Nearness to Dow historical high**: Following Li and Yu (2012), we calculate the ratio of the Dow Jones Industrial Index at the end of each month t and the historical high of daily closing index values of the Dow Jones Industrial Index. Data are from Bloomberg.

21. **Partial least squares aggregated book-to-market ratio**: Following Kelly and Pruitt (2013), we aggregate information from the book-to-market ratios of portfolios sorted on size and book-to-market using a partial least squares (PLS) approach. We use data on returns and book-to-market ratios of 100 portfolios sorted on size and book-to-market and data on returns of the CRSP value-weighted stock market index from Kenneth French's website. The June book-to-market ratio for each portfolio is the value-weighted average of the book value from fiscal year $t-1$ divided by the market value from June of year t. The log book-to-market ratios in subsequent months for each portfolio are adjusted by subtracting the log of the cumulative value-weighted return on the portfolio since the past June. In the first step of the PLS approach, we estimate the time-series regression

$$\log(BM_{i,t}) = \phi_{i,0} + \phi_{i,1} \log R_{t+1} + e_{i,t}, \tag{A3}$$

    for each portfolio i, where $BM_{i,t}$ is the book-to-market of portfolio i in month t and

R_{t+1} is the stock market return in month $t+1$. In the second step, we estimate the cross-sectional regression

$$\log(BM_{i,t}) = c_t + ABM_t\hat{\phi}_{i,1} + w_{i,t}, \tag{A4}$$

for each month t. The ABM_t measure is the estimated latent factor in the cross-sectional regressions.[22] We estimate the model using data from January 1927 to January 2022 to obtain a time series of the predictor that spans January 1927 to December 2021.

22. **Tail risk**: Following Kelly and Jiang (2014), we estimate tail risk using Hill's (1975) power law estimator. We use daily returns from CRSP for NYSE-, AMEX-, and Nasdaq-listed firms with share codes of 10 and 11. The tail risk measure in month t is calculated as

$$TAIL_t = \frac{1}{K_t}\sum_{k=1}^{K_t}\log\left(\frac{R_{k,t}}{u_t}\right), \tag{A5}$$

where $R_{k,t}$ is the kth daily return in the pooled cross section in month t that falls below the extreme value threshold (u_t) and K_t is the total number of returns below the threshold in month t. The threshold parameter is the fifth percentile of the pooled cross-sectional distribution of daily returns in month t. We standardize the measure to have a mean of zero and a standard deviation of one.

23. **Amihud illiquidity measure**: Following Chen, Eaton, and Paye (2018), we measure the log of the equal-weighted average of the Amihud (2002) illiquidity measure with adjustments for breaks and market volatility. We use daily returns from CRSP for NYSE-listed stocks with share codes of 10, 11, and 12. For each firm i in each month t, the Amihud illiquidity measure is calculated as

$$AMI_{i,t} = \left(\frac{1}{D_{i,t}}\sum_{d=1}^{D_{i,t}}\frac{|R_{i,d,t}|}{DVOL_{i,d,t}}\right)CPI_t, \tag{A6}$$

where $D_{i,t}$ is the number of trading days with positive volume for stock i in month t, $|R_{i,d,t}|$ is the absolute value of the return on stock i on day d of month t, $DVOL_{i,d,t}$ is the dollar trading volume in millions for stock i on day d, and CPI_t is the Consumer Price Index (CPI). CPI data are from the Federal Reserve Bank of St. Louis.[23] We use the real-time break adjustment procedure from Chen, Eaton, and Paye (2018) to adjust the time series for the effects of changes in tick sizes in 1997 and 2001. Finally, we subtract the market volatility component as the cross-sectional average of the log of estimated firm-level volatility, $vol_t = \overline{\log(\hat{\sigma}_{i,t})}$.

24. **Zero return measure**: Following Chen, Eaton, and Paye (2018), we measure the log of the equal-weighted average of the proportion of zero-return days with an adjustment

[22] We thank Seth Pruitt for making code available (https://sethpruitt.net/research/).

[23] See https://fred.stlouisfed.org/series/CPIAUCNS.

for breaks. We use daily returns from CRSP for NYSE-listed stocks with share codes of 10, 11, and 12. For each firm i in each month t, we calculate the proportion of trading days with a return of zero. We use the real-time break adjustment procedure from Chen, Eaton, and Paye (2018) to adjust the time series for the effects of changes in tick sizes in 1997 and 2001.

25. **Average skewness**: Following Jondeau, Zhang, and Zhu (2019), we measure the value-weighted average of the skewness of excess daily returns within each month. We use daily returns from CRSP for NYSE-, AMEX-, and Nasdaq-listed firms with share codes of 10 and 11. We exclude stocks with an Amihud (2002) illiquidity measure in the top 0.1% or with prices below $1. For a given month, we use all stocks that have at least ten valid return observations.

B Additional discussion and results

This appendix contains additional discussion and empirical results referenced in the paper.

B.1 Bootstrap approach

Figure 1 (Figure 3) shows 90% bootstrap confidence intervals for cumulative returns (capital gains) and the cumulative effects from return (capital gain) components. We generally follow Kroencke (2022) to construct these bootstrap confidence intervals. We describe our approach for returns and return components in Figure 1, and the approach for Figure 3 is analogous with capital gain components. Figure 1 is an event-time analysis over the two years preceding and the two years following an economic peak. We bootstrap 100,000 samples of returns and return components that each cover this four-year window. Each bootstrap draw samples from the existing recession data by randomly selecting sets of the return and the matched return components from the same month in the data, such that the relations between returns and the return components are preserved. The random sets are drawn from the data for the 15 recession periods while maintaining event time. For example, the bootstrap draws for the month that precedes the economic peak by six months are matched returns and return components from the corresponding 15 sets of datapoints that occurred six months prior to the economic peaks in the data. Once we draw a full sequence of returns for the four-year period, we cumulate the returns and return components within each bootstrap draw. The plotted 90% bootstrap confidence intervals are the 5% and 95% quantiles of the bootstrap distributions of the cumulative sums for each event month.

B.2 Definition of cash-flow news

Following Campbell and Vuolteenaho (2004), Campbell, Giglio, and Polk (2013), and many others, we model excess returns to calculate the discount-rate news term in equation (3). We make this modeling choice because the return predictors proposed by the literature are typically shown to forecast excess returns. A natural consequence of this choice to model excess returns is that the cash-flow news term in equation (3) includes the risk-free rate. An alternative is to separately model the risk-free rate using the following equation

$$
\begin{aligned}
r^e_{t+1} - E_t r^e_{t+1} =& (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{f,t+1+j} \\
& - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^e_{t+1+j} \\
=& N_{CF,t+1} - N_{RF,t+1} - N_{RP,t+1},
\end{aligned} \tag{B7}
$$

where $N_{RF,t+1}$ is risk-free rate news and $N_{RP,t+1}$ is risk-premium news. We can then calculate discount-rate news as $N_{DR,t+1} = N_{RF,t+1} + N_{RP,t+1}$. We estimate the following VAR,

$$
\begin{bmatrix} r^e_{t+1} \\ r_{f,t+1} \\ \tilde{r}^e_{1,t+2} \\ \tilde{r}^e_{2,t+2} \\ \tilde{r}^e_{3,t+2} \end{bmatrix} = a + A \begin{bmatrix} r^e_{t} \\ r_{f,t} \\ \tilde{r}^e_{1,t+1} \\ \tilde{r}^e_{2,t+1} \\ \tilde{r}^e_{3,t+1} \end{bmatrix} + \nu_{t+1}, \tag{B8}
$$

and calculate the news terms,

$$
N_{RP,t+1} = e1' \rho A (I - \rho A)^{-1} \nu_{t+1}, \tag{B9}
$$

$$
N_{RF,t+1} = e2' \rho A (I - \rho A)^{-1} \nu_{t+1}, \tag{B10}
$$

$$
N_{CF,t+1} = (e1' + e1' \rho A (I - \rho A)^{-1} + e2' \rho A (I - \rho A)^{-1}) \nu_{t+1}. \tag{B11}
$$

The Treasury bill yield ($r_{f,t}$) is not included as a state variable in the model forecast combinations, but is included directly in the VAR as a state variable. Tables B.I and B.II and Figure B.1 show estimates and results that demonstrate that our conclusions hold using this alternative modeling and estimation approach.

B.3 Residual method versus direct approach

Following Campbell (1991) and many others, our base method estimates the cash-flow and discount-rate news components from equation (3) by estimating discount-rate news as in equations (9) to (10) and calculating cash-flow news as the residual as in equation (11). As noted by Chen (2010) and Engsted, Pedersen, and Tanggaard (2012), using this residual approach to calculate cash-flow news is equivalent to estimating cash-flow news directly if the VAR includes the log dividend-price ratio as a standalone state variable. We examine sensitivity to this alternative approach by estimating cash-flow and discount-rate news using this VAR specification,

$$\begin{bmatrix} r^e_{t+1} \\ d_{t+1} - p_{t+1} \\ \tilde{r}^e_{1,t+2} \\ \tilde{r}^e_{2,t+2} \\ \tilde{r}^e_{3,t+2} \end{bmatrix} = a + A \begin{bmatrix} r^e_t \\ d_t - p_t \\ \tilde{r}^e_{1,t+1} \\ \tilde{r}^e_{2,t+1} \\ \tilde{r}^e_{3,t+1} \end{bmatrix} + \nu_{t+1}. \tag{B12}$$

In this approach, the dividend-price ratio is not included as a state variable in the model forecast combinations, but is included directly in the VAR as a state variable. Tables B.III and B.IV and Figure B.2 show estimates and results that demonstrate that our conclusions hold using this alternative modeling and estimation approach.

B.4 Alternative specifications for estimating θ and λ

In Section 4.3.1, we estimate θ by regressing subjective cash-flow news on objective cash-flow news and λ by regressing subjective discount-rate news on objective cash-flow news. In this section, we explore the robustness of the estimates. Panel A of Table B.V reports the base results for convenience. Panel B examines the sensitivity to crisis periods. We specifically exclude quarters that occurred during the global financial crisis (GFC), the COVID period, or NBER recessions. The estimates of θ and λ are insensitive to excluding these periods. We also run a specification only within the six quarters that are spanned by the GFC and COVID periods. The θ estimate is very close to the base case but is insignificant due to the small sample size. The λ estimate is larger in magnitude and the point estimate indicates countercyclical subjective expected returns, but the coefficient estimate is statistically insignificant so we do not draw strong conclusions. Panel C investigates whether investors have asymmetric underreaction/overreaction or countercyclical/procyclical patterns in relation to objective cash-flow news. The θ regression is suggestive that underreaction to good news is stronger than underreaction to bad news, but underreaction is pervasive and there is no statistical difference between the coefficient estimates in the good and bad periods. The λ regression indicates acyclicality with respect to positive cash-flow shocks and countercyclicality with respect to negative cash-flow shocks, but the coefficient estimates are statistically insignificantly different from zero and the difference is also statistically insignificant. Overall, the results in Table B.V show that our primary specification is robust to these alternative regression designs.

B.5 Implications of Bordalo, Gennaioli, La Porta, and Shleifer (2024)

In Section 4.3.1, we empirically study Case 1. For comparison with the empirical results, we introduce the model of Bordalo, Gennaioli, La Porta, and Shleifer (2024) and map it into Case 1 by deriving $N_{CF,t+1}$, $N_{DR,t+1}$, $N^*_{CF,t+1}$, $N^*_{DR,t+1}$, θ, ε_{t+1}, $E_t(r^e_{t+1})$, and $E^*_t(r^e_{t+1})$ and showing that the components are related as in Case 1.

Bordalo, Gennaioli, La Porta, and Shleifer (2024) specify dividend growth for the market as

$$g_{t+1} = \mu g_t + \tau_{t+1} + \omega_t, \tag{B13}$$

where $\tau_{t+1} \sim N(0, \sigma_\tau^2)$ is tangible news, $\omega_t \sim N(0, \sigma_\omega^2)$ is intangible news, and τ_{t+1} and ω_t are uncorrelated. Objective beliefs for future cash-flow growth at a horizon $(1+j)$ therefore follows

$$E_t(g_{t+1+j}) = \mu^j(\mu g_t + \omega_t). \tag{B14}$$

Bordalo, Gennaioli, La Porta, and Shleifer (2024) incorporate biases in subjective expectations as

$$E_t^*(g_{t+1+j}) = E_t(g_{t+1+j}) + \mu^j v_t, \tag{B15}$$

with

$$v_t = \phi v_{t-1} + w_t \tag{B16}$$

and

$$w_t = \tilde{\theta}(\mu \tau_t + \omega_t). \tag{B17}$$

With a parameter $\tilde{\theta} > 0$ ($\tilde{\theta} < 0$), investors overreact (underreact) to cash-flow news. Given this design, the conditional objective expected return is

$$E_t[r_{t+1}] = r + \left(\frac{1-\rho\phi}{1-\rho\mu}\right) E_t[g_{t+1} - E_t^*(g_{t+1})], \tag{B18}$$

where r is the constant subjective expected return and ρ is a loglinearization constant, and returns are given by

$$r_{t+1} = E_t[r_{t+1}] + \left(\frac{1+\rho\mu\tilde{\theta}}{1-\rho\mu}\right)\tau_{t+1} + \left(\frac{\rho(1+\tilde{\theta})}{1-\rho\mu}\right)\omega_{t+1}. \tag{B19}$$

B.5.1 Subjective return decomposition

The subjective risk premium is constant in the model. Subjective cash-flow news is, therefore, equal to the realized return minus the subjective expected return,

$$N_{CF,t+1}^* = \left(\frac{1+\rho\mu\tilde{\theta}}{1-\rho\mu}\right)\tau_{t+1} + \left(\frac{\rho(1+\tilde{\theta})}{1-\rho\mu}\right)\omega_{t+1} - \left(\frac{1-\rho\phi}{1-\rho\mu}\right) v_t, \tag{B20}$$

and the subjective discount-rate news is zero in each period,

$$N_{DR,t+1}^* = 0. \tag{B21}$$

B.5.2 Objective return decomposition

The risk-free rate is constant, so objective cash-flow growth reflects expectations about dividend growth as implied by the forms of equations (B13) and (B14). Objective cash-flow news in the model is

$$
\begin{aligned}
N_{CF,t+1} =& (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j g_{t+1+j} \\
=& g_{t+1} - \mu g_t - \omega_t \\
& + \sum_{j=1}^{\infty} \rho^j \mu^j (g_{t+1} - \mu g_t + \mu^{-1} \omega_{t+1} - \omega_t) \\
=& \tau_{t+1} + \sum_{j=1}^{\infty} \rho^j \mu^j (\tau_{t+1} + \mu^{-1} \omega_{t+1}) \\
=& \frac{1}{1-\rho\mu} \tau_{t+1} + \frac{\rho}{1-\rho\mu} \omega_{t+1}. \qquad \text{(B22)}
\end{aligned}
$$

Given the forms of equations (B19) and (B22), objective discount-rate news is

$$
N_{DR,t+1} = \frac{-\rho\mu\tilde{\theta}}{1-\rho\mu} \tau_{t+1} + \frac{-\rho\tilde{\theta}}{1-\rho\mu} \omega_{t+1}. \qquad \text{(B23)}
$$

Thus, when $\tilde{\theta} > 0$, objective discount-rate news has a reinforcing effect that accompanies objective cash-flow news.

B.5.3 Case 1 analysis

We demonstrate how Bordalo, Gennaioli, La Porta, and Shleifer's (2024) model maps into Case 1. Define

$$
\alpha = \frac{\sigma_\tau^2}{\sigma_\tau^2 + \rho^2 \sigma_\omega^2}. \qquad \text{(B24)}
$$

From equations (B18),

$$
E_t[r_{t+1}] - E^*[r_{t+1}] = -\left(\frac{1-\rho\phi}{1-\rho\mu} \right) v_t. \qquad \text{(B25)}
$$

Consider the following proposed relation between θ in Case 1 and $\tilde{\theta}$ from Bordalo, Gennaioli, La Porta, and Shleifer (2024):

$$
\theta = \tilde{\theta}(1 - (1-\rho\mu)\alpha). \qquad \text{(B26)}
$$

In Case 1, ε_{t+1} will be orthogonal to $N_{CF,t+1}$ if and only if equation (B26) holds. Given this relation,

$$\begin{aligned}
\varepsilon_{t+1} =& N^*_{CF,t+1} - (1+\theta)N_{CF,t+1} && \text{(B27)}\\
=& \left(\frac{1+\rho\mu\tilde{\theta}}{1-\rho\mu}\right)\tau_{t+1} + \left(\frac{\rho(1+\tilde{\theta})}{1-\rho\mu}\right)\omega_{t+1} - \left(\frac{1-\rho\phi}{1-\rho\mu}\right)v_t \\
& - (1+\tilde{\theta}(1-(1-\rho\mu)\alpha))\left(\frac{1}{1-\rho\mu}\tau_{t+1} + \frac{\rho}{1-\rho\mu}\omega_{t+1}\right) && \text{(B28)}\\
=& -\tilde{\theta}(1-\alpha)\tau_{t+1} + \tilde{\theta}\rho\alpha\omega_{t+1} - \left(\frac{1-\rho\phi}{1-\rho\mu}\right)v_t. && \text{(B29)}
\end{aligned}$$

Given this specification for ε_{t+1}, it is orthogonal to $N_{CF,t+1}$,

$$\begin{aligned}
Cov(N_{CF,t+1}, \varepsilon_{t+1}) =& \frac{1}{1-\rho\mu}(-\tilde{\theta}(1-\alpha))\sigma_\tau^2 + \frac{\rho}{1-\rho\mu}\tilde{\theta}\rho\alpha\sigma_\omega^2 && \text{(B30)}\\
=& -\frac{\tilde{\theta}}{1-\rho\mu}\frac{\rho^2\sigma_\tau^2\sigma_\omega^2}{\sigma_\tau^2+\rho^2\sigma_\omega^2} + \frac{\tilde{\theta}}{1-\rho\mu}\frac{\rho^2\sigma_\tau^2\sigma_\omega^2}{\sigma_\tau^2+\rho^2\sigma_\omega^2} && \text{(B31)}\\
=& 0, && \text{(B32)}
\end{aligned}$$

which demonstrates that the relation in equation (B26) holds. Finally, we have

$$\begin{aligned}
-\theta N_{CF,t+1} - \varepsilon_{t+1} + E_t[r_{t+1}] - E^*[r_{t+1}] =& -\frac{\tilde{\theta}(1-(1-\rho\mu)\alpha)}{1-\rho\mu}\tau_{t+1} \\
& - \frac{\tilde{\theta}(1-(1-\rho\mu)\alpha)\rho}{1-\rho\mu}\omega_{t+1} + \tilde{\theta}(1-\alpha)\tau_{t+1} \\
& - \tilde{\theta}\rho\alpha\omega_{t+1} + \left(\frac{1-\rho\phi}{1-\rho\mu}\right)v_t - \left(\frac{1-\rho\phi}{1-\rho\mu}\right)v_t && \text{(B33)}\\
=& -\frac{\tilde{\theta}(1-(1-\rho\mu)\alpha-(1-\alpha)(1-\rho\mu))}{1-\rho\mu}\tau_{t+1} \\
& - \frac{\tilde{\theta}\rho((1-(1-\rho\mu)\alpha)+\alpha(1-\rho\mu))}{1-\rho\mu}\omega_{t+1} && \text{(B34)}\\
=& -\frac{\rho\mu\tilde{\theta}}{1-\rho\mu}\tau_{t+1} - \frac{\tilde{\theta}\rho}{1-\rho\mu}\omega_{t+1} && \text{(B35)}\\
=& N_{DR,t+1}, && \text{(B36)}
\end{aligned}$$

which confirms that the Bordalo, Gennaioli, La Porta, and Shleifer (2024) model conforms to Case 1.

Table B.I
Vector autoregression for model with risk-free rate.

The table summarizes results from a vector autoregression with the model forecast combinations with $k = 3$ predictor subgroups based on persistence. The predictors are sorted into subgroups based on their autoregression coefficients, and we create model forecast combinations for low-persistence ($\tilde{r}^e_{1,t+1}$, half-lives less than three months), medium-persistence ($\tilde{r}^e_{2,t+1}$, three months to four years), and high-persistence ($\tilde{r}^e_{3,t+1}$, more than four years) predictors. The state variables in the VAR are the log excess market return, the log risk-free rate, and the fitted log excess market return forecasts from the low-, medium-, and high-persistence subgroups using the mean combination approach. The fitted expected returns are standardized. We report the VAR coefficient matrix A, its OLS t-statistics (in parentheses), the R^2 (in percent) from each regression, and the covariance matrix Σ. The sample period is January 1927 to December 2021. An * (**) [***] on a VAR coefficient estimate indicates statistical significance at the 10% (5%) [1%] level in a two-tailed test.

	A					R^2	Σ				
	r^e_t	$r_{f,t}$	$\tilde{r}^e_{1,t+1}$	$\tilde{r}^e_{2,t+1}$	$\tilde{r}^e_{3,t+1}$		r^e_{t+1}	$r_{f,t+1}$	$\tilde{r}^e_{1,t+2}$	$\tilde{r}^e_{2,t+2}$	$\tilde{r}^e_{3,t+2}$
r^e_{t+1}	0.04	−0.33	0.38**	0.99***	0.05	4.95	27.27	−0.01	1.86	−0.24	−0.58
	(1.34)	(−0.53)	(2.25)	(5.87)	(0.31)						
$r_{f,t+1}$	−0.00	0.97***	−0.00**	−0.00	−0.00	95.36	−0.01	0.00	−0.01	−0.00	0.00
	(−0.34)	(149.87)	(−2.12)	(−0.86)	(−0.07)						
$\tilde{r}^e_{1,t+2}$	−0.03***	0.05	0.45***	0.08***	−0.02	17.65	1.86	−0.01	0.82	−0.02	−0.04
	(−5.54)	(0.44)	(15.34)	(2.61)	(−0.85)						
$\tilde{r}^e_{2,t+2}$	−0.01***	−0.01	−0.01	0.97***	0.02***	93.76	−0.24	−0.00	−0.02	0.06	0.00
	(−9.14)	(−0.31)	(−0.95)	(119.56)	(2.62)						
$\tilde{r}^e_{3,t+2}$	−0.00	−0.02	−0.01**	−0.02***	1.00***	98.52	−0.58	0.00	−0.04	0.00	0.01
	(−0.56)	(−1.04)	(−2.14)	(−4.72)	(257.50)						

Table B.II

Objective discount-rate and cash-flow news variability for model with risk-free rate.

The table reports the standard deviations of objective discount-rate news and objective cash-flow news and the correlation between them from the VAR model that measures discount-rate news as the sum of risk-free rate news and risk premium news. The standard deviations are reported in percent per month. The table also reports the bootstrap probability that the standard deviation of objective cash-flow news exceeds the standard deviation of discount-rate news and the bootstrap 90% confidence interval for the correlation. The sample period is January 1927 to December 2021.

Statistic	Estimate
$\sigma(N_{DR})$	5.36
$\sigma(N_{CF})$	2.39
$\Pr[\sigma(N_{CF}) > \sigma(N_{DR})]$	0.04
$\rho(N_{DR}, N_{CF})$	0.28
90% CI	$[-0.02, 0.85]$

Table B.III
Vector autoregression for model with dividend-price ratio.

The table summarizes results from a vector autoregression with the model forecast combinations with $k = 3$ predictor subgroups based on persistence. The predictors are sorted into subgroups based on their autoregression coefficients, and we create model forecast combinations for low-persistence ($\tilde{r}^e_{1,t+1}$, half-lives less than three months), medium-persistence ($\tilde{r}^e_{2,t+1}$, three months to four years), and high-persistence ($\tilde{r}^e_{3,t+1}$, more than four years) predictors. The state variables in the VAR are the log excess market return, the log dividend-price ratio, and the fitted log excess market return forecasts from the low-, medium-, and high-persistence subgroups using the mean combination approach. The fitted expected returns are standardized. We report the VAR coefficient matrix A, its OLS t-statistics (in parentheses), the R^2 (in percent) from each regression, and the covariance matrix Σ. The sample period is January 1927 to December 2021. An * (**) [***] on a VAR coefficient estimate indicates statistical significance at the 10% (5%) [1%] level in a two-tailed test.

	A						Σ				
	r^e_t	$d_t - p_t$	$\tilde{r}^e_{1,t+1}$	$\tilde{r}^e_{2,t+1}$	$\tilde{r}^e_{3,t+1}$	R^2	r^e_{t+1}	$d_{t+1} - p_{t+1}$	$\tilde{r}^e_{1,t+2}$	$\tilde{r}^e_{2,t+2}$	$\tilde{r}^e_{3,t+2}$
r^e_{t+1}	0.03	−1.43**	0.44**	0.93***	0.72**	5.29	27.17	−0.27	1.88	−0.24	−0.54
	(1.03)	(−2.01)	(2.58)	(5.39)	(2.06)						
$d_{t+1} - p_{t+1}$	−0.00	1.00***	−0.01***	−0.01***	−0.00	98.69	−0.27	0.00	−0.02	0.00	0.01
	(−0.41)	(135.90)	(−3.16)	(−4.70)	(−0.27)						
$\tilde{r}^e_{1,t+2}$	−0.03***	0.37***	0.43***	0.09***	−0.19***	18.30	1.88	−0.02	0.82	−0.01	−0.03
	(−5.04)	(3.01)	(14.74)	(3.08)	(−3.09)						
$\tilde{r}^e_{2,t+2}$	−0.01***	−0.04	−0.01	0.97***	0.04**	93.77	−0.24	0.00	−0.01	0.06	0.01
	(−9.27)	(−1.20)	(−0.74)	(117.32)	(2.39)						
$\tilde{r}^e_{3,t+2}$	−0.00	0.01	−0.00	−0.02***	0.99***	98.55	−0.54	0.01	−0.03	0.01	0.01
	(−1.28)	(0.77)	(−0.58)	(−4.09)	(123.03)						

Table B.IV
Objective discount-rate and cash-flow news variability for model with dividend-price ratio.

The table reports the standard deviations of objective discount-rate news and objective cash-flow news and the correlation between them from the VAR model that includes the log dividend-price ratio. The standard deviations are reported in percent per month. The table also reports the bootstrap probability that the standard deviation of objective cash-flow news exceeds the standard deviation of discount-rate news and the bootstrap 90% confidence interval for the correlation. The sample period is January 1927 to December 2021.

Statistic	Estimate
$\sigma(N_{DR})$	7.10
$\sigma(N_{CF})$	5.08
$\Pr[\sigma(N_{CF}) > \sigma(N_{DR})]$	0.06
$\rho(N_{DR}, N_{CF})$	0.68
90% CI	$[0.39, 0.91]$

Table B.V
Parameter estimates with alternative specifications.

The table reports regression results to estimate the θ parameter calculated from the $(1+\theta)$ coefficient in a regression of subjective cash-flow news on objective cash-flow news and the λ parameter in a regression of subjective discount-rate news on objective cash-flow news. Panel A repeats the base specification from the paper for convenience. Panel B excludes various crisis period observations from the sample: the global financial crisis (GFC) period, the COVID period, and NBER recession periods. The panel also reports a specification (Crisis only) that includes only observations from the global financial crisis and COVID periods. Panel C examines the potential for asymmetry in the response of subjective beliefs to objective cash-flow news by regressing subjective news on the positive and negative components of objective cash-flow news. The panel also reports the difference in coefficients across positive and negative cash-flow news. We report OLS t-statistics (in parentheses) and the R^2 (in percent) from each regression. An * (**) [***] indicates statistical significance at the 10% (5%) [1%] level in a two-tailed test. The sample period is June 1985 to December 2021.

Panel A: Base specification		
	θ	λ
Full sample	-0.70^{***}	-0.15
	(-6.03)	(-0.96)
Panel B: Sensitivity to crisis periods		
	θ	λ
Excl. GFC	-0.85^{***}	-0.19
	(-6.48)	(-1.06)
Excl. COVID	-0.69^{***}	-0.11
	(-6.21)	(-0.80)
Excl. both	-0.84^{***}	-0.14
	(-6.73)	(-0.89)
Excl. NBER	-0.84^{***}	-0.08
	(-6.67)	(-0.56)
Crisis only	-0.86	-0.82
	(-1.23)	(-0.65)
Panel C: Asymmetry		
	θ	λ
$N_{CF} \geq 0$	-0.85^{***}	0.02
	(-4.99)	(0.09)
$N_{CF} < 0$	-0.49^{**}	-0.37
	(-2.38)	(-1.36)
Difference	-0.37	0.39
	(-1.23)	(1.00)

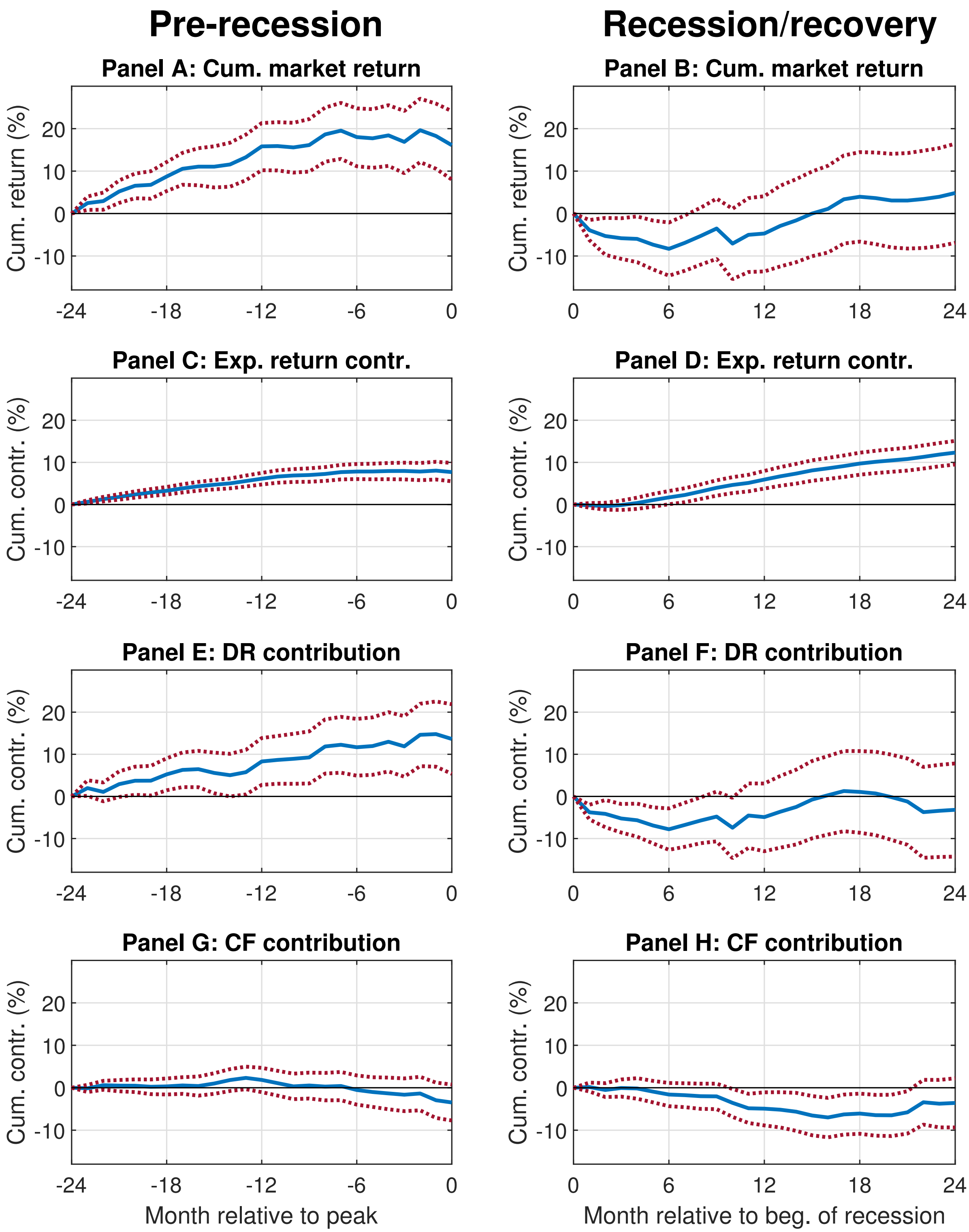

Figure B.1. Objective news about returns, discount rates, and cash flows around recessions from VAR with risk-free rate. The figure shows the cumulative market returns (Panels A and B), expected returns (Panels C and D), negative of discount-rate news (Panels E and F), and cash-flow news (Panels G and H) in the periods leading up to economic peaks (Panels A, C, E, and G) and following NBER recession starts (Panels B, D, F, and H). The 90% confidence intervals from a bootstrap simulation are shown as red dotted lines. The sample period is January 1927 to December 2021.

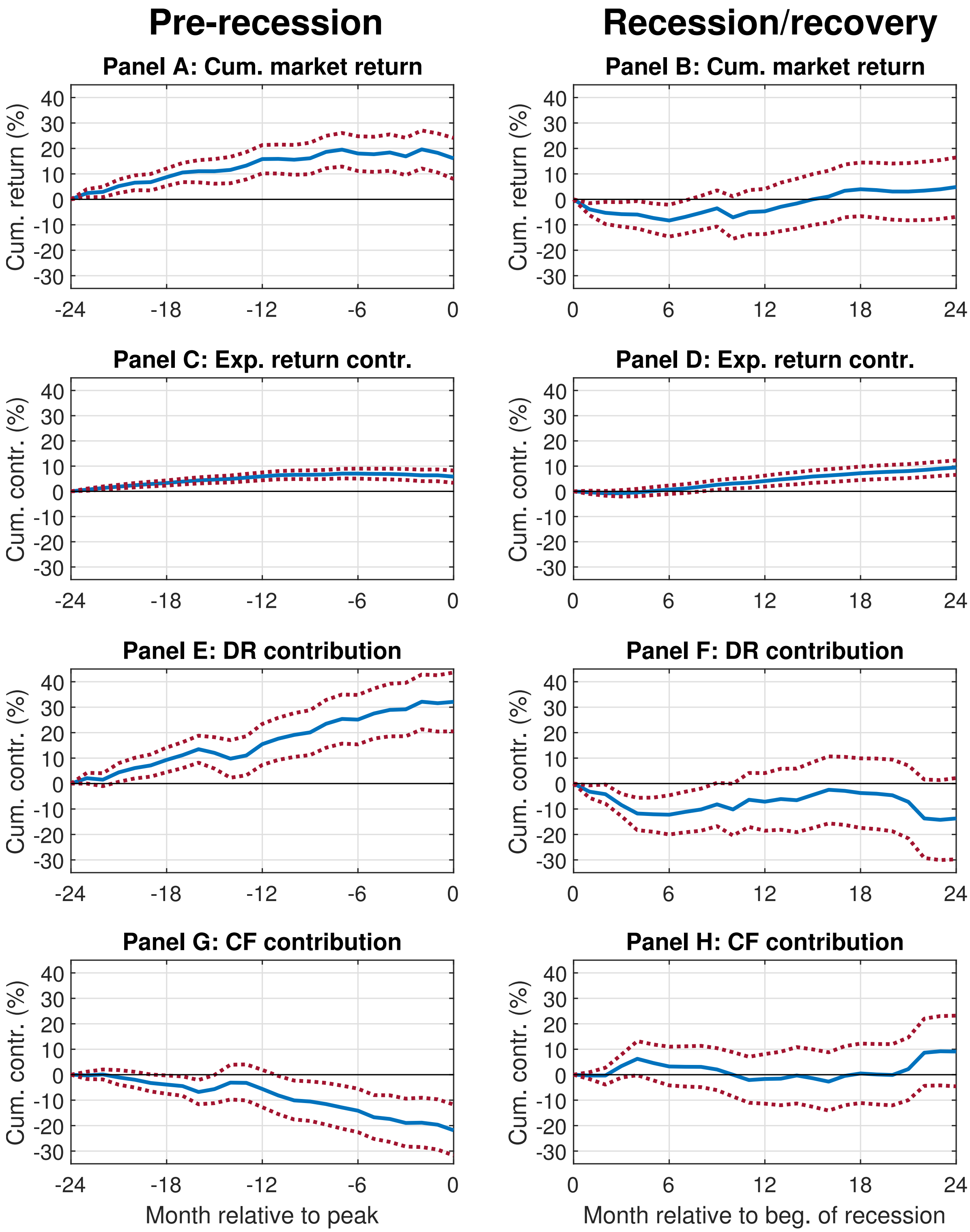

Figure B.2. Objective news about returns, discount rates, and cash flows around recessions from VAR with dividend-price ratio. The figure shows the cumulative market returns (Panels A and B), expected returns (Panels C and D), negative of discount-rate news (Panels E and F), and cash-flow news (Panels G and H) in the periods leading up to economic peaks (Panels A, C, E, and G) and following NBER recession starts (Panels B, D, F, and H). The 90% confidence intervals from a bootstrap simulation are shown as red dotted lines. The sample period is January 1927 to December 2021.