Fiscal Imbalances and Asset Returns: Cross-Sector Fluctuations under the Aggregate Budget Constraint

Junxiong Gao, Alberto Plazzi, Rossen Valkanov, and Yan Xu*

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JEL: E62, G12, G17
Keywords: Equity predictability, public debt, fiscal imbalances, cross-sector predictability, output distortion

*Gao: Shanghai Advanced Institute of Finance, Shanghai Jiao Tong University, jxgao@saif.sjtu.edu.cn. Plazzi: Institute of Finance, Università della Svizzera italiana and Swiss Finance Institute, alberto.plazzi@usi.ch. Valkanov: Rady School of Management, University of California San Diego, rvalkanov@ucsd.edu. Xu: Hong Kong University, yanxuj@hku.hk.

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1 Introduction

Over the past three decades, present-value relations have become the empirical workhorse for extracting time-varying expectations of economic fundamentals from fluctuations in valuation ratios. A canonical example is the log-linearized present-value relation in Campbell and Shiller (1988b), a cornerstone of the equity return predictability literature,[1] which links fluctuations in the dividend–price ratio (hereafter, dp) to changing expectations of future equity returns and dividend growth. Similar present-value decompositions have also been applied to the public and foreign sectors of the U.S. economy. In the public sector, a high net surplus (taxes minus government spending) relative to government debt (nsb) predicts higher future primary surpluses and, to a lesser extent, lower future Treasury returns (Berndt et al., 2012; Campbell et al., 2023). In the foreign sector, current account imbalances, measured by net imports relative to the net value of foreign assets (nma), lead to higher future foreign portfolio returns (Gourinchas and Rey, 2007b).[2]

A common thread across this literature is that present-value relations are analyzed one sector at a time, with each sector treated in isolation. Consequently, fluctuations in a sector-specific valuation ratio can be linked to that sector's own expected fundamentals, but not to time variation in fundamentals elsewhere in the economy. Hence, this sector-specific approach cannot address cross-sector questions such as: Do fluctuations in nsb, which reflect changing fiscal conditions, lead to changes in expected equity returns or dividend growth? Or conversely, do movements in dp, a state variable capturing conditions in the equity market, lead fluctuations in expected Treasury returns or future net surpluses? More broadly, to what extent and why are shocks originating in one sector transmitted to others? These are central questions for understanding how fundamental economic shocks propagate across sectors in the economy.

In this paper, we make three main contributions. First, we develop a general framework that links the present-value relations of the private, public, and foreign sectors of the U.S. economy, allowing us to examine whether fluctuations in one sector lead time variations in expected returns or fundamentals in other sectors. The central identity linking the three sectors is the aggregate budget constraint (ABC) of the U.S. economy, an accounting relation that we use to integrate the present-value conditions of the three sectors into a single intertemporal budget constraint. The ABC implies that the aggregate consumption-to-wealth ratio is a weighted average of three sector-specific valuation ratios: dp, nsb, and nma. Similarly, the ABC yields an expression for the return to total wealth as a weighted average of expected returns across the three sectors. Hence, fluctuations in any component of the consumption-to-wealth ratio can generate variation in expected returns or fundamental growth in its own sector or in either of the other two sectors.

Embedding the three present-value relations within the ABC makes clear that an imbalance

[1] A partial list includes Campbell (1993), Vuolteenaho (2002), Campbell and Vuolteenaho (2004), Larrain and Yogo (2008), Pástor and Stambaugh (2012), Menzly et al. (2004), Lettau and Ludvigson (2005), Lettau and Van Nieuwerburgh (2008), and Cochrane (2011).

[2] Present-value relations have also been applied to other asset classes. For corporate bonds, Nozawa (2017) shows that cash-flow variation is primarily driven by time-varying expected returns rather than default losses. In commercial real estate, Plazzi et al. (2010) demonstrate that rent-price ratios forecast future returns, not future rent growth.

in one sector must be absorbed either within that sector or elsewhere in the economy. While this general framework opens the possibility of cross-sector predictability, it does not by itself guarantee that such predictability will be present in the data. Two polar cases illustrate this point. If the expected returns on equities, Treasuries, and foreign assets are all driven by a single common shock, then each sectoral present-value relation collapses to the aggregate consumption-wealth relation. In this case, the other two sectoral constraints become essentially redundant, leaving no scope for cross-sector predictability. At the other extreme, if each sector's expected returns are driven by orthogonal shocks, then valuation ratios contain no cross-sector information. While those are admittedly unrealistic scenarios, they illustrate the point that only when fluctuations in valuation ratios propagate through the ABC, affecting consumption and discount rates, do cross-sector linkages emerge. As the consumption-wealth ratio aggregates all these forces, it cannot by itself disentangle their sources. A cross-sector approach that controls for each sector's valuation ratios is therefore essential for testing whether cross-sector linkages exist and for quantifying their magnitude.

Second, we examine cross-sector predictability in U.S. data from 1952 to 2021. As a starting point, we first replicate the established within-sector results. In the private sector, the dividend–price ratio (dp) positively forecasts equity returns but contains little information about future dividend growth (Campbell and Shiller, 1988b; Cochrane, 2011). In the public sector, by contrast, most of the variation in the net-surplus–to-debt ratio (nsb) reflects expected growth in future primary surpluses, as shown by Campbell et al. (2023). Finally, in the external sector, the net foreign-asset valuation ratio (nma) forecasts foreign portfolio returns but not foreign-sector cash-flow growth, consistent with Gourinchas and Rey (2007b). We then turn to the novel empirical results investingating whether the valuation ratios in one sector lead expected returns and cash-flow growth rates in the other sectors.

Our main empirical result is that fiscal imbalances captured by nsb lead time variation in future equity returns at quarterly and annual horizons. In unrestricted regressions, the marginal impact of nsb on future returns is negative and statistically significant, implying that a higher surplus relative to outstanding debt today predicts lower equity returns in the future. Including nsb has an economically meaningful effect on the regression R^2, and does not weaken but actually reinforces the role of the dividend-to-price ratio. This finding is robust to augmenting the set of regressors with the log tax-to-GDP ratio (see Campbell et al., 2023), the Cochrane and Piazzesi (2005) forward-rates factor, and the inflation rate, which is included in the benchmark model of Berndt et al. (2012). Importantly, the predictive power of nsb for equity returns is the only cross-sector effect that is both robust and statistically significant.

The present-value restrictions in the three sectors and the ABC impose economic restrictions on the coefficients that will yield more efficient estimates of the cross-sector predictability. To expoit these restrictions, we cast cross-sector estimation in a constrained GMM framework. This approach treats the three individual sectors constraints and the ABC as linear restrictions on the slope coefficients. Imposing the constraints generally strengthens the evidence of both own- and cross-sector predictability. In particular, the net surplus-to-debt ratio remains a robust predictor of equity returns, with a negative coefficient that is significant at the 1% level across all specifications. This finding is consistent with the intuition in Cochrane (2008) that cross-equation restrictions improve inference in predictive regressions. Our framework also provides a simple test statistic based on the difference between the unconstrained OLS and constrained GMM objective functions. We cannot reject the null that the sectoral constraints and the ABC hold in the data.

Third, we examine why the net surplus-to-debt ratio forecasts expected equity returns within an asset-pricing framework. The key feature of the model is that fiscal policy shocks influence consumption growth by generating output distortions through future taxation or by stimulating activity through government spending, building on the frameworks of Barro (1979) and Jiang et al. (2024a). Under the aggregate budget constraint, a fiscal shock alters cash flows through two channels. First, taxation reallocates resources from the private to the public sector (abstracting from the foreign sector). Second, fiscal policy affects consumption dynamics and, through them, the pricing of risk across all sectors. It is this second channel that drives our main result: forward-looking fiscal adjustments that lower the surplus-to-debt ratio raise the equity risk premium and therefore predict higher future equity returns.

This mechanism also clarifies why cross-sector predictors are essential for identification. Fiscal shocks alter the common consumption-based risk component, so variation in the dividend-to-price ratio reflects not only within-sector dynamics but also fiscal spillovers transmitted through consumption. Consequently, predictive regressions that rely solely on within-sector valuation ratios confound equity-specific risk-premium variation with fiscal influences. Incorporating the surplus-to-debt ratio—anchored in the government's intertemporal budget constraint—helps isolate the fiscal component embedded in expected equity returns. In other words, the dividend-to-price ratio alone is insufficient to capture all channels of time-varying risk premia in equity returns. Consistent with the model, the surplus-to-debt ratio negatively predicts future equity returns even after controlling for the dividend-to-price ratio.

Our model builds on the government's optimal fiscal policy problem. The economy produces an exogenous endowment of raw output that accrues to the government. The government chooses tax and spending paths to maximize the representative household's lifetime utility, subject to the intertemporal public budget constraint and the value of outstanding debt. Taxation finances current spending and debt service but reduces the final output available to households by introducing distortions, while spending stimulates output and demand at the cost of higher debt. Households take these fiscal choices as given, hold equity and government debt, and price all claims using a consumption-based discount factor. Fiscal policy thus affects asset values through both cash-flow and discount-rate channels. When output distortions are absent, Ricardian Equivalence holds: consumption–wealth dynamics depend solely on the initial debt valuation, fiscal policy becomes neutral, and cross-sector predictability disappears.

Because government policy and asset prices are jointly determined in equilibrium, the model offers a tractable way to trace how fiscal expectations transmit through the economy. We study a marginal, anticipated deviation from the steady-state fiscal path—a small forward-looking fiscal shock—that captures how expectations of future tax or spending changes influence current asset prices through discount-rate effects. Such expectations need not correspond to observed policy events; valuation ratios often embed fiscal beliefs even in the absence of explicit actions, consistent with the empirical difficulty of identifying fiscal shocks. Anticipated fiscal tightening raises the public debt valuation, reduces the surplus-to-debt ratio and increases expected discount rates for the private sector, giving rise to the cross-sector predictability documented in the data.

The model's tractability further allows us to link the estimated slope coefficients from predictive regressions directly to structural parameters of output distortions. Our calibration indicates that taxation reduces output by roughly 6%, while government spending raises it by about 2%. The negative predictability of equity returns thus stems from taxation's distortionary effect dominating the stimulative impact of spending, raising risk premia for the private sector.

Several further aspects of our empirical approach are worth emphasizing. First, for the private and public sectors, we adopt the log-linearization proposed by Campbell et al. (2023), which allows for negative surpluses and therefore accommodates negative values in the numerator of the valuation ratio. We extend this framework to the foreign sector, accounting for the possibility that both cash flows and asset values can turn negative. This provides us with a unique and coherent modelling of the present value identities across the three sectors.

Second, from a statistical perspective the net surplus-to-debt ratio behaves as a stationary variable, unlike the dividend-to-price ratio for which standard tests fail to reject a unit root. Nevertheless, our results remain robust to econometric concerns arising in forecasting regressions with highly persistent predictors. Following Amihud and Hurvich (2004), we compute bias-adjusted coefficients for multivariate regressions that correct for the correlation between innovations in the dependent variable and the regressors. This adjustment substantially reduces the predictive power of the dividend-to-price ratio for equity returns, but leaves the role of the surplus-to-debt ratio largely unchanged. Bootstrapped standard errors and Bonferroni confidence intervals that account for predictor persistence likewise confirm the significance of nsb. Consistent with evidence on the dividend-to-price and debt-to-GDP ratios (Lettau and Van Nieuwerburgh, 2008; Jiang et al., 2024c), we cannot reject the null of structural breaks in the unconditional mean of nsb (in 2007) and nma (in 1968). Accounting for these breaks does not, however, alter our conclusions. Finally, simulations show that, under a benchmark where dp fully explains expected returns and dividend growth is unpredictable, the estimated nsb coefficient lies far outside the simulated range, making its predictive power unlikely to be spurious.

Third, a key advantage of our GMM formulation is that it both improves efficiency, lowering standard errors when the constraints hold, and mitigates bias when the OLS estimates are distorted by random deviations from the constraints. As confirmed by a Monte Carlo simulation calibrated to our data, imposing the constraints can substantially reduce variance and bias relative to separate OLS regressions, making the constrained GMM framework an effective tool to sharpen inference on cross-asset predictability. Our work thus contributes to methodological advances that sharpen inference in predictive regressions through structural or statistical restrictions (e.g., Timmermann, 1993; Ang et al., 2007; Campbell and Thompson, 2008; Koijen and Van Nieuwerburgh, 2011).

Our approach of decomposing the economy into three sectors and imposing the aggregate budget constraint connects different strands of research on dynamic value adjustment. For equity, the seminal contribution of Campbell and Shiller (1988b), extended by Campbell (1991) to separate cash-flow and discount-rate shocks, underpins much of the work on return and dividend-growth predictability. This body of work highlights the limitations of the dividend-to-price ratio as a stand-alone predictor. We show that fiscal policy, captured by the net surplus-to-debt ratio, contains additional information beyond dp, and rationalize this finding within a theoretical framework.

Lettau and Ludvigson (2001) show that the aggregate consumption-to-wealth ratio contains information about future equity returns. Their analysis focuses on the private sector and distinguishes between labor and non-labor income in order to construct a state variable capturing expected return variation. We build on the same consumption-wealth dynamics but take a different perspective: exploiting the national accounts identity, we decompose consumption and wealth across the private, public, and foreign sectors, and ask whether returns and cash-flow growth in one sector can be predicted by valuation ratios from others. This cross-sector decomposition, complementary to the labor- versus non-labor split in Lettau and Ludvigson (2001), enables us to investigate cross-sector predictability and uncover new interactions between sectoral budget constraints and asset pricing.[3]

Recent work exploits the log debt-to-GDP identity to predict equity and Treasury returns (e.g., Cochrane, 2022; Liu, 2023; Jiang et al., 2024c). Relative to this literature, we point out that, via the aggregate budget constraint, the consumption-to-wealth ratio can be expressed as a weighted average of three sector-specific valuation ratios. From this perspective, the surplus-to-debt ratio emerges as the relevant state variable for capturing fiscal imbalances that propagate across sectors through consumption growth and discount rates. Its central role reflects the fact that government debt represents a claim on future fiscal surpluses. Changes in nsb therefore capture how the market value of public debt adjusts in response to anticipated surplus dynamics, making it a natural object for identifying the fiscal component of risk premia.

A further line of research uses the government budget constraint to study fiscal policy adjustments and their effects on the real economy and inflation (Giannitsarou and Scott (2006), Berndt et al. (2012), and Berndt and Yeltekin (2015)). Because the net surplus (taxes minus spending) can be negative, these papers assume a stable cointegration of taxes and spending, and measure fiscal imbalances as linear combinations of tax-to-debt and spending-to-debt ratios. In contrast, we build on Campbell et al. (2023), who directly linearize the surplus-to-debt ratio using the cointegrated relation between tax–to-debt and spending-to-debt ratios. This linearization allows us to capture the high volatility of net surpluses and elevated government spending at the end of the sample.

Our model builds on the optimal fiscal policy literature in which governments manage aggregate risk through tax smoothing (Barro, 1979; Lucas and Stokey, 1983; Bohn, 1990; Jiang et al., 2024a).

[3]Conceptually, our framework could be extended by decomposing each sector's assets into human capital and investable components, in the spirit of Lettau and Ludvigson (2001). Since labor income is not directly observable, this would require constructing sector-specific analogues of "cay."

A central question in that literature is the measurement of spending and tax multipliers using either DSGE models or narrative methods based on historical data (Blanchard and Perotti, 2002; Romer and Romer, 2010; Christiano et al., 2011; Auerbach and Gorodnichenko, 2012; Ilzetzki et al., 2013; Ramey and Zubairy, 2018). By explicitly incorporating equity pricing and exploiting cross-sector predictability, our approach implies the magnitude of tax distortions and government spending directly from asset-pricing dynamics, providing a novel and complementary alternative method to the existing approaches.

Our framework allows for risky government debt, consistent with the view that the path of net surpluses provides information about the degree of protection afforded to taxpayers relative to bondholders (Jiang et al., 2026). The presence of a debt return risk premium is closely connected to the literature on public debt sustainability (Blanchard, 1985; Hamilton and Flavin, 1986; Bohn, 1998; Hall and Sargent, 2011; Blanchard, 2019; Mehrotra and Sergeyev, 2021; Ball and Mankiw, 2023; Mian et al., 2025; Elenev et al., 2025; Liu et al., 2025). Rather than focusing on expected returns on public debt, the analysis emphasizes the link between surplus dynamics and expected equity returns. This focus on the aggregate budget constraint and the cross-sector transmission of fiscal shocks complements the macro-finance literature studying how taxes and policy uncertainty affect firm-level investment and, ultimately, long-run growth (Pástor and Stambaugh, 2012; Croce et al., 2012a; Croce et al., 2012b; Croce et al., 2019; Corhay et al., 2023).

Our work is also related to Jiang et al. (2024b) who document deviations from the no-arbitrage relation between the market value of U.S. government debt and its fundamental value implied by future surpluses. This valuation puzzle relates to how the government can finance public debt at a lower cost than what its surpluses suggest (Krishnamurthy and Vissing-Jorgensen, 2012; Gorton and Ordonez, 2014; Du et al., 2018; He et al., 2019; Krishnamurthy and Lustig, 2019; Gorton and Ordonez, 2022; Atkeson et al., 2025; Chen et al., 2025). While these studies highlight the safe-asset supply and convenience-yield channels in government debt markets, our analysis deliberately focuses on the fundamental channel linking fiscal policy, surplus expectations, and asset prices. Rather than attempting to reconcile the debt valuation puzzle itself, we use the surplus-to-debt ratio to study how fiscal distortions transmit across sectors and shape expected returns.

In international finance, the "valuation channel" captures how capital gains and losses affect the external balance sheet. Lane and Milesi-Ferretti (2001) pioneered this line of inquiry, and Gourinchas and Rey (2007b) formalized the approach using a foreign-sector budget constraint to test dynamic value adjustment. Subsequent work documents the rising importance of valuation effects across countries (Gourinchas, 2008; Gourinchas and Rey, 2014). We extend this research by developing a decomposition for foreign markets that accommodates the possibility of negative flows (net exports) and negative positions (net foreign assets), allowing us to apply a unified framework across sectors and examine whether foreign imbalances spill over into domestic asset markets.

2 Present-Value Decompositions and Sector Constraints

Our starting point is the aggregate accounting identity of GDP:

$$Y_t = C_t + I_t + G_t + X_t - M_t \tag{1}$$

where Y_t denotes GDP, C_t denotes aggregate consumption, I_t is aggregate investment, G_t is government spending, X_t is exports, and M_t is imports. We can rewrite this identity as:

$$C_t = D_t + NS_t + NM_t\,, \tag{2}$$

where $D_t = Y_t - I_t - T_t$ is aggregate after-tax dividend to the private sector, $NS_t = T_t - G_t$ is the net government tax surplus computed as taxes T_t minus spending, and $NM_t = M_t - X_t$ is net imports. Conceptually, aggregate consumption is driven by flows from the private, the public, and the external sectors. The definition of dividend equal to output minus investment and taxes has been previously used in the literature to capture the net payout to the firm owners (see, e.g., Larrain and Yogo, 2008). Most asset pricing studies assume that aggregate consumption equals dividends, or $C_t = D_t$ (Lucas, 1978; Campbell, 1996). This implicitly side-steps the role of fiscal policy and net imports on asset returns, which is the focus of our paper.

We can formulate an intertemporal budget constraint for each sector. For the private sector, this is:

$$A_{t+1} + D_{t+1} = (1 + R^A_{t+1})A_t\,, \tag{3}$$

where A_t is the value of the private sector's wealth in the economy, and R^A_{t+1} its return.

The public sector's budget constraint is:

$$B_{t+1} + NS_{t+1} = (1 + R^B_{t+1})B_t\,, \tag{4}$$

where B_t is the total value of government debt outstanding, and R^B_{t+1} its return.

Finally, for the external sector, the budget constraint is:

$$F_{t+1} + NM_{t+1} = (1 + R^X_{t+1})F_t\,, \tag{5}$$

where F_t is the net value of foreign assets, i.e., assets minus liabilities, and R^X_{t+1} is the return on the foreign assets.

These budget constraints have previously been studied in isolation in the literature – see, among others, Campbell and Shiller (1988a) for the equity sector; Berndt et al. (2012) for the public sector; and Gourinchas and Rey (2007b) for the external sector.

We can define the total domestic wealth of the economy W_t as

$$W_t = A_t + B_t + F_t\,, \tag{6}$$

and using total outflows, C_t, we can write the aggregate budget constraint as:

$$W_{t+1} + C_{t+1} = (1 + R^W_{t+1})W_t\,. \tag{7}$$

The gross return to the wealth portfolio, $1 + R^W_{t+1}$, equals the weighted average of the returns to each sector, or

$$1 + R^W_{t+1} = (1 + R^A_{t+1})\frac{A_t}{W_t} + (1 + R^B_{t+1})\frac{B_t}{W_t} + (1 + R^X_{t+1})\frac{F_t}{W_t}\,. \tag{8}$$

2.1 Budget Constraints (BC) and Within-Sector Predictability

We obtain an expression relating the valuation ratio in each market to future asset returns and cash-flow growth by log-linearizing the intertemporal budget constraint and iterating forward the resulting expression. This approach has been pioneered in the equity context by Campbell and Shiller (1988b), which obtain an approximated relation with respect to the log dividend-to-price ratio. Their dynamic accounting identity forms the basis for the analysis of equity predictability that investigates the relative importance of time-variation in discount rates and expected dividend growth. The same approach cannot, however, be straightforwardly applied to the other two sectors, as cash flows to the public sector (i.e., net surplus) and to the foreign sector (i.e., exports minus imports) can turn negative, thereby invalidating the logarithmic transformation.

To overcome this issue, we adopt the approach in Campbell et al. (2023) for the public sector, and further extend it to the other two sectors. We provide a compact description of the approach to highlight the main results of interest for our empirical analysis, and then present the general framework with full derivations in Appendix A.

Campbell et al. (2023) linearize the surplus-debt ratio by exploiting the cointegrated relation between the log tax-debt and the log spending-debt ratio. Their method separately handles inflows (spending) and outflows (taxes) of the public sector and is based on an expansion of the log of one plus the surplus-to-debt ratio. Formally, the government's valuation ratio (net surplus-to-debt), denoted nsb, is approximated as:

$$nsb_{t+1} = \log\left(1 + \frac{T_{t+1} - G_{t+1}}{B_{t+1}}\right) \approx k + (1 - \rho^B)\left(\frac{1}{1-\beta}\tau b_{t+1} - \frac{\beta}{1-\beta} gb_{t+1}\right), \tag{9}$$

where τb and gb represent the log tax-to-debt and spending-to-debt ratios, respectively. The coefficient ρ^B is related to the steady-state level of nsb, while β is chosen to minimize the approximation error.

Compared to the standard Campbell and Shiller (1988b) approximation, this framework accommodates net government surpluses that turn negative, as in our sample period. Moreover, US net surplus has been historically volatile over extended periods, which proves to be a challenge for some of the alternative log-linearization approaches (Larrain and Yogo, 2008; Berndt et al., 2012). Furthermore, this approach allows a direct interpretation of nsb as a measure of fiscal space: high nsb periods are those when the government is in a strong fiscal position, while low nsb reflects periods when the government is in a weak fiscal position.

Using this method, an approximation of the net government surplus growth Δns is

$$\Delta ns_{t+1} = \frac{1}{1-\beta}\Delta\tau_{t+1} - \frac{\beta}{1-\beta}\Delta s_{t+1}, \tag{10}$$

which is also a linear combination of tax and spending growth rate $\Delta\tau$, Δs. This allows a separate identification of the impact from tax and government spending, especially related to the recent non-stationary path of fiscal imbalance, as pointed out by Campbell et al. (2023).

To maintain consistency throughout our expressions, we apply the same framework to the private sector, which implies a log-linearization for the dividend-to-asset ratio $da_t = \log\left(1 + \frac{D_t}{A_t}\right)$.

The foreign sector necessitates further modifications to the framework, as not only net imports are negative in some periods, but also net foreign assets can be close to zero or negative when net foreign assets and liabilities are close to each other. These features present a challenge for the typical log-linearization for the external sector Gourinchas and Rey, 2007a. To address this issue, we separately approximate the dynamics of foreign assets and liabilities and regard net foreign asset values as a portfolio of the two accounts. This enables us to obtain a consistent log-linearization for the valuation ratio in the foreign sector, $nma = \log\left(1 + \frac{M_t - X_t}{F_t}\right)$, which captures external sector imbalances in a similar way as nsb captures fiscal imbalances in the public sector.

The linearization framework implies the following budget constraints (BCs) relating the valuation ratio to return and cash-flow growth:

$$da_t = (1-\rho^A)r^A_{t+1} - (1-\rho^A)\Delta d_{t+1} + \rho^A da_{t+1} \tag{11}$$

$$nsb_t = (1-\rho^B)r^B_{t+1} - (1-\rho^B)\Delta ns_{t+1} + \rho^B nsb_{t+1} \tag{12}$$

$$nma_t = (1-\rho^X)r^X_{t+1} - (1-\rho^X)\Delta nm_{t+1} + \rho^X nma_{t+1}. \tag{13}$$

These expressions closely resemble those from the standard Campbell and Shiller (1988b) expansion, although they are obtained using a modified approximation that accommodates negative and volatile cash flows. These intertemporal relations have been the foundation of separate strands of literature that analyze the predictability of equity returns, Treasury returns, and foreign asset returns. The conditioning variables used are either the within-sector conditioning variables or macroeconomic quantities.

2.2 The Aggregate Budget Constraint (ABC) and Cross-Sector Predictability

We can apply the log-linearization to the aggregate wealth portfolio, defined in Eq.(7). This yields the following expression for the consumption-to-wealth ratio, cw_t:

$$cw_t = \left(1-\rho^W\right)\sum_{j=0}^{\infty}\left(\rho^W\right)^j\left(r^W_{t+j+1} - \Delta c_{t+j+1}\right) \tag{14}$$

As we show in the appendix, the consumption-to-wealth ratio is a weighted average of the state variables of the individual sectors, or

$$cw_t = \alpha_0 + w_A da_t + w_B nsb_t + w_X nma_t, \tag{15}$$

where w_A, w_B, w_X are the steady-state share of total wealth.

Eq. (15) represents an aggregate budget constraint (ABC) that ties the predictability of each sector together by enforcing the weighted average of the three valuation ratios in Eqs. (11)–(13). We provide detailed derivations in the appendix and show that

$$\begin{aligned} cw_t = w_A da_t + w_B nsb_t + w_X nma_t = \text{constant} &+ w_A \left(1-\rho^A\right) \sum_{j=0}^{\infty} \left(\rho^A\right)^j \left(r^A_{t+j+1} - \Delta d_{t+j+1}\right) \\ &+ w_B \left(1-\rho^B\right) \sum_{j=0}^{\infty} \left(\rho^B\right)^j \left(r^B_{t+j+1} - \Delta ns_{t+j+1}\right) \\ &+ w_X \left(1-\rho^X\right) \sum_{j=0}^{\infty} \left(\rho^X\right)^j \left(r^X_{t+j+1} - \Delta nm_{t+j+1}\right). \end{aligned} \tag{16}$$

In other words, if there is an imbalance in one sector, captured by a change in the state variable in that sector, this change will affect the consumption-to-wealth ratio and could generate a correspondence to future asset returns and cash flow growth rates in all the sectors.

If there is a single common factor driving the valuation dynamics across all three sectors, then each sector-specific budget constraint – Eqs. (11) through (13) – is simply a re-statement of the aggregate consumption-wealth dynamic in Eq. (15). In this case, return predictability observed in the literature for each individual sector would merely reflect a shared systematic risk factor. However, if risks originate in one specific sector and propagate to others, the consumption-wealth identity aggregates heterogeneous sources of variation, and cross-sector predictability becomes empirically necessary to capture the full transmission mechanism.

To illustrate, consider a fiscal shock such as an unanticipated increase in taxes. This alters the net government surplus and hence the valuation ratio nsb_t. Through the aggregate budget constraint in Eq. (15), this shock shifts the consumption-to-wealth ratio and, consequently, affects equilibrium valuation across all sectors. A rise in expected taxes can depress private consumption, which in turn affects firm cash flows, dividend expectations, and the equity premium. Similarly, shifts in fiscal policy may influence the trade balance through changes in domestic demand, thereby affecting the valuation of foreign assets. In other words, fluctuations in fiscal policy not only impact future Treasury returns and surplus growth but may also be transmitted across asset markets.

This transmission is particularly relevant in the context of Ricardian Equivalence (Barro, 1979), which posits that current fiscal deficits imply future tax liabilities. A lower surplus today signals future tax increases, which investors may perceive as raising the cost of capital for firms and thus demand higher risk premiums. These fiscal-induced risks may also bear on the pricing of foreign assets, especially when fiscal imbalances lead to changes in exchange rate expectations or capital flows. Given the rising prominence of debt sustainability debates in the U.S., identifying how fiscal policy affects other sectors' asset returns becomes a first-order question for macro-finance.

The extent to which state variables in one sector forecast returns in other sectors – above and beyond their own – is the key empirical question we address. If cross-sector transmission exists, then we should observe cross-predictability: variation in a sector-specific valuation ratio (such as nsb_t) should forecast returns outside its own sector, even after controlling for all sectoral state variables. Identifying this cross-sector effect requires a joint empirical framework that imposes both the aggregate constraint in Eq. (15) and the within-sector constraints in Eqs. (11)–(13). Specifically, the coefficients predicting consumption growth must align with the weighted average of the sector-specific cash-flow predictors, while the return and cash-flow predictors must satisfy each sector's intertemporal budget constraint.

Our empirical strategy is designed to disentangle these cross-sector interactions in the presence of heterogeneous risks. We adopt a system approach that jointly estimates return and cash-flow predictability equations across sectors, subject to the full set of intertemporal constraints. This enables us to trace how shocks propagate across the system and to attribute observed predictability to sector-specific or shared risk sources. Ultimately, our goal is to develop a more comprehensive and accurate framework for asset pricing that takes into account the complex interdependencies between different sectors of the economy.

Our approach differs from that of Lettau and Ludvigson (2001) in several respects. Lettau and Ludvigson (2001) use the deviation of log consumption from its long-run co-integrated relationship with labor income and wealth to predict equity returns. Their analysis focuses on the private sector and distinguishes between labor and non-labor income. Their primary goal was not to study cross-sector predictability, but rather to construct a state variable capturing expected equity return variation while controlling for labor income effects. In contrast, we build on the same consumption-wealth dynamics to investigate interactions across multiple sectors and examine predictability across them. This allows us to gain insights into the drivers of asset pricing and the interplay between sector-specific returns and cash flow growth rates.

3 Empirical Analysis

We apply the framework outlined above to US data. Section 3.1 describes the data source and presents summary statistics. Section 3.2 discusses our estimation strategy. Sections 3.3, 3.4, and 3.5 contain our main empirical results. In Section 3.6, we present additional econometric analyses and diagnostics.

3.1 Data Description and Stationarity Tests

We use quarterly data for the U.S. economy from 1952 to 2021. All return and cash-flow series are expressed in real terms by subtracting the change in CPI index.

For the private sector, we use returns and dividend growth from public equity. Specifically, we use the cum- and without-dividend return series for the CRSP index for the NYSE, NASDAQ, and

AMEX exchanges. We construct annual dividends as the trailing sum of current and past three-quarter dividends. Dividends are reinvested at the market rate, but our results are not sensitive to the reinvestment rate. Quarterly dividend growth series is obtained as the first difference in log annual dividends. To facilitate the comparison with prior studies on equity predictability, we use throughout the notation r^E in place of r^A and dp in place of da, with the understanding that dp is constructed as $\log(1 + D/P)$ for consistency with our linearization framework.

The public sector data primarily comes from the NIPA Table, which is available on the FRED or BEA websites. Following Berndt et al. (2012), we compute tax and government spending from terms in NIPA Table 3.2. Tax equals total receipts (line 37). Government spending equals current expenditures (line 41) plus gross government investment (line 42) plus capital transfer payments (line 43) minus consumption of fixed capital (line 45) minus debt interest payments (line 29). For the value of public debt B, we use the marketable only value of debt that is compiled by Hall and Sargent (2011), Hall et al. (2018).

For the external sector, we rely on the methodology of Gourinchas and Rey (2007b) to construct measures on the gross positions, flows, and returns. As Gourinchas et al. (2010) updated certain data sources and computations of these variables, we closely follow them in obtaining the variables from numerous databases including Flow of Funds Accounts, Board of Governors of the Federal Reserve System, and Bureau of Economic Advisors. Our calculation includes data from 13 countries: Australia, Canada, Denmark, France, Germany, Ireland, Italy, Japan, Mexico, Netherlands, Spain, Sweden, and the United Kingdom.

Table 1 presents descriptive statistics for our raw variables. Panel A shows that average annual real equity returns are the highest, around 7%, and exhibit the greatest volatility, with an annualized standard deviation of roughly 17%. Treasury real returns are lower, with both mean and volatility roughly one-fourth of equity, while returns on foreign assets and liabilities lie in between. In the leftmost columns we observe that the return series display rather modest correlations with each other at the quarterly frequency.

Panel B summarizes real cash flow growth rates for all components: dividends (Δd), taxes (Δt), government expenditures (Δg), total imports (Δm), and total exports (Δx). Average growth rates range between 2% and 4%, with dividends being the most volatile at 16%. Cross-sector correlations are generally low, except for a relatively high correlation of 0.61 between the two foreign-sector flows.

Panel C reports statistics for the valuation ratios that serve as key predictors in our analysis: the dividend-to-price ratio dp (denoted da in Section 2), the net surplus-to-public-debt ratio nsb, and the net foreign asset ratio nma. We also include the consumption-wealth ratio cw, which enters the ABC constraint in Eq. (15). On average, dp is about 0.03, while nsb is slightly negative at -0.01, reflecting an overall deficit. All ratios are highly persistent at a quarterly frequency, with univariate roots ranging from 0.891 for nsb to 0.987 for dp. Correlations across the three sectors are modest, never exceeding 0.11.

Given the importance of predictor stationarity for inference, the last two rows of the left-most columns report sample statistics for two tests: the Augmented Dickey-Fuller (ADF) and the

Table 1: **Summary Statistics of Returns, Cashflows, and Ratios**: This table presents summary statistics for real returns, cash-flow growth rates, and financial ratios. Panel A reports returns, where r^E is the equity return, r^D is the debt return, r^{XA} is the return on foreign assets, and r^{XL} is the return on foreign liabilities. Panel B presents growth rates in cash flows, including dividends (Δd), taxes (Δt), government expenditure (Δg), total imports (Δm), and total exports (Δx). Panel C shows valuation ratios: dp (dividend-to-price ratio), nsb (surplus-to-public debt ratio), nma (net import to net foreign asset ratio), and cw (consumption-to-wealth ratio). For all variables, we report the mean, standard deviation (Std), first-order autoregressive root (AR1), and test statistics from the Augmented Dickey-Fuller (ADF) and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests. Panel D reports the autoregressive matrix Φ from a VAR(1) estimated on the three valuation ratios. The right-hand side of each panel reports correlations among the corresponding variables, except in Panel D where correlations refer to the residuals from the VAR(1) model. Data are quarterly, 1952-2021.

	Mean, Std, AR1					Correlations		
Panel A: Returns								
	r^E	r^D	r^{XA}	r^{XL}		r^D	r^X	
Mean	0.071	0.018	0.048	0.042	r^E	0.037	-0.117	
Std	0.169	0.039	0.098	0.096	r^D		-0.009	
AR1	0.041	0.070	0.129	0.081				
Panel B: Cash-flows								
	Δd	Δt	Δg	Δm	Δx	Δns	Δnm	
Mean	0.021	0.027	0.029	0.044	0.040	Δd -0.091	-0.175	
Std	0.160	0.074	0.110	0.079	0.080	Δns	0.040	
AR1	-0.008	-0.183	-0.250	0.004	-0.080			
Panel C: Valuation Ratios								
	dp	nsb	nma	cw		nsb	nma	cw
Mean	0.031	-0.013	0.062	0.150	dp	0.015	0.110	0.656
Std	0.006	0.027	0.069	0.007	nsb		0.092	0.313
AR1	0.987	0.891	0.936	1.004	nma			0.261
ADF	-1.416	-2.669	-4.448	0.234				
KPSS	2.352	0.679	0.217	1.330				
Panel D: Valuation Ratios, VAR(1)								
	dp	nsb	nma			nsb	nma	
dp	0.986	-0.012	-0.085		dp	-0.123	-0.037	
nsb	0.003	0.889	-0.033		nsb		0.074	
nma	0.001	0.006	0.938					

Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests. The ADF null hypothesis is that the series has a unit root, while KPSS tests for stationarity. For the dp ratio, both tests clearly indicate that the series' persistence over the sample most closely resembles that of a nonstationary process. In contrast, nma largely rejects the ADF null and does not reject KPSS, indicating stationarity. The net surplus-to-debt ratio also behaves well, with an ADF statistic of -2.669 (exceeding the 10% critical value of -2.56) and a KPSS statistic of 0.679 (below the 5% critical value of 0.463). Alternative stationarity tests, omitted here for brevity, further confirm that concerns about nonstationarity primarily pertain to dp. We discuss the implications for our predictive framework in Section 3.6.2.

We further look into the cross-predictability of the three valuation ratios, $x_t = [dp_t \, nsb_t \, nma_t]'$, by estimating a first-order vector autoregressive model, $x_t = \Theta + \Phi x_{t-1} + v_t$. In the leftmost block of Panel D, we report the estimated coefficient matrix $\widehat{\Phi}$. The diagonal roots are very close to the univariate AR(1) estimates, with off-diagonal elements that are small in magnitude (and not statistically significant at conventional levels). The rightmost block reports the correlations of the VAR(1) residuals, which are also modest, lying roughly in the $[-0.15, 0.10]$ range.

We plot the three ratios in Figure 1, sampled at the end of each year. The dividend-to-price ratio dp remained above its mean until the mid-1990s, after which it experienced a sharp decline toward 0.01, followed by a rebound to around 0.02, and another decline by the end of the sample. The net surplus-to-debt ratio nsb is mostly positive until the late 2000s, when it turns negative, reaching values below -0.10 during the Global Financial Crisis and around -0.20 in the latter part of the sample as the U.S. federal government consistently ran budget deficits. The net foreign asset ratio nma fluctuates over a wider range, with negative values at both the beginning and end of the sample period, alternating with peaks as high as 0.30. In Section 3.6.2, we return to the time-series behavior of the ratios by testing for the presence of structural shifts in their means.

The plot also reveals temporary co-movements among the three sectors. For example, the dividend-to-price ratio declined during the 1990s, while both the net surplus and net foreign asset ratios increased simultaneously. However, these correlations are not persistent, and there are periods where one sector's valuation ratio falls without a corresponding movement in the other two. The time-varying mechanism through which the three sectors balance against each other largely cancels out over the full sample, resulting in the weak unconditional correlations reported above. From this perspective, our framework – which studies the joint dynamics of the three sectors while enforcing the aggregate budget constraint – provides a solid foundation. While our primary focus in this paper is asset return predictability, further investigation into the conditional correlations and potential regime-switching dynamics of the three sectors would be an interesting avenue for future research.

3.2 Estimation Procedure

We begin by replicating the own sector's predictability of future returns and cash flow growth using our data and the framework outlined in Section 2. Specifically, we estimate OLS regressions using only a sector's valuation ratio as the predictor. Next, we examine cross-sector effects by estimating OLS multivariate regressions where the set of explanatory variables is expanded to include the other sectors' valuation ratios. We further augment the regressors with standard controls for equity and bond risk premia. Finally, we conduct a constrained estimation, where we impose both the sector budget constraints (BC) and the aggregate budget constraint (ABC) that ensure consistency within and across equations. Since our study focuses on the benefits of cross-sector predictability, we discuss the key features of this constrained estimation here and provide all derivations, along with a more detailed discussion, in Appendix B.

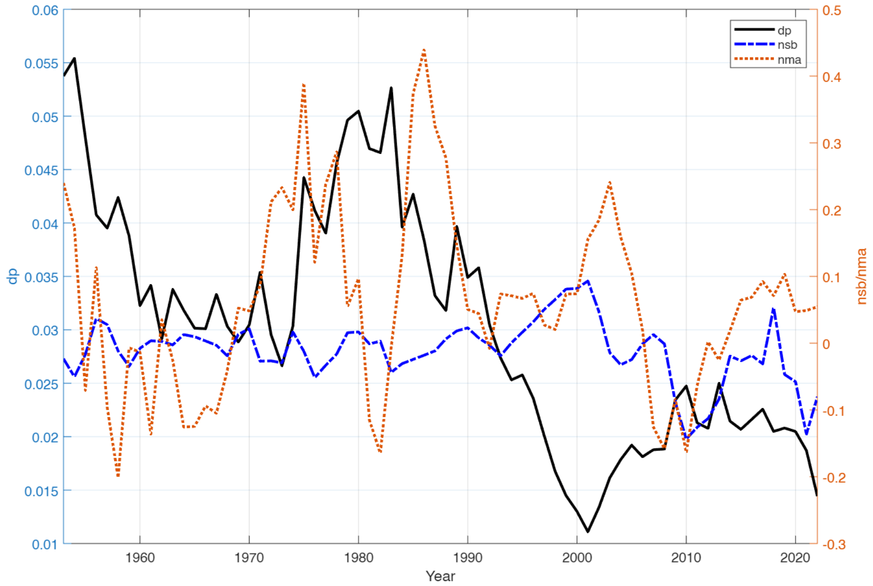

FIGURE 1: **Valuation ratios.** This figure plots the end-of-year time series of the dividend-to-price ratio (dp, left Y-axis), the net surplus-to-public-debt ratio (nsb, right Y-axis), and the net import-to-net-foreign-asset ratio (nma, right Y-axis) over the sample period 1952–2021.

The present-value identities naturally lead to predictive regressions that relate a given sector's return, cash-flow growth, and valuation ratio to the lagged valuation ratios of all sectors, plus controls. Let $\boldsymbol{b_r}$, $\boldsymbol{b_{cf}}$, and $\boldsymbol{\phi}$ be the vectors of slope coefficients in the three regressions, respectively. The budget constraints in Eq.(11)–(13) imply that these coefficients are related by:

$$\text{BC:} \qquad \boldsymbol{C} = \text{diag}(1-\rho^E, 1-\rho^B, 1-\rho^X)(\boldsymbol{b_r} - \boldsymbol{b_{cf}}) + \text{diag}(\rho^E, \rho^B, \rho^X)\boldsymbol{\phi}, \tag{17}$$

where $\boldsymbol{C} = [\boldsymbol{I}(N),\ \mathbf{0}_{N\times(M-N)}]$, with N representing the number of sectors (i.e., 3 in our case) and $M-N$ the number of controls. Additionally, the ABC linking all sectors implies that the coefficients are related by:

$$\text{ABC:} \qquad \boldsymbol{b_{cw}} = \boldsymbol{w}'\boldsymbol{\phi}, \tag{18}$$

where $\boldsymbol{b_{cw}}$ is the vector of slope coefficients in the regression of the future consumption-to-wealth ratio cw_{t+1} on the same set of predictors, and $\boldsymbol{w}$ is the vector collecting the weights in Eq.(15).

We cast the BC and ABC in a constrained GMM framework, where the objective function that consists of the standard OLS moment conditions stacked across all sectors is minimized subject to the relations in Eq.(17) and Eq. (18). Since the constraints can be formulated as a linear combination of the slope coefficients – i.e., they can be written as $\boldsymbol{R}'\widehat{\boldsymbol{\beta}}_{\text{cgmm}} - \boldsymbol{c} = 0$ for suitable R and c – the optimal constrained GMM estimator can be derived in closed form and expressed as the sum of the unconstrained OLS estimator plus a term that depends on the constraint; see Appendix Eq.(B14). This formulation clarifies that the constrained GMM estimator effectively balances the unconstrained OLS estimates with the imposed constraints. It also provides a straightforward method to test whether the constraints are rejected by the data, by comparing the objective functions of the OLS and constrained GMM estimators via a chi-square test. In Appendix B, we offer an in-depth explanation of the constrained GMM estimation procedure, which relies on a standard iterative-GMM approach, and of the construction of the optimal weight matrix.

Why should imposing the BC and ABC in a joint, constrained GMM framework improve our inference on cross-asset predictability compared to conducting separate OLS estimations? We argue that the benefits can be summarized in two key aspects. First, under ideal conditions, where the constraints hold over a sufficiently long sample period, the constrained estimator is more efficient, resulting in lower standard errors. Second, in cases where the constraints are misspecified, leading to random deviations in the data and consequently biased OLS estimates, enforcing the constraints can effectively mitigate this bias. In Appendix B4, we confirm and quantify these benefits of variance and bias reduction by comparing the OLS estimator to the GMM estimators that impose either the BC or the BC+ABC in a Monte Carlo simulation calibrated on our data. In this exercise, we assume that the covariance matrix of the moment conditions is known. Since the estimation of the weight matrix may reduce the benefits of the constrained GMM approach, the actual benefits of imposing the BC and ABC in our context remain an empirical question, which our analysis will reveal below.

3.3 Univariate Estimates

Table 2 reports OLS estimates for the predictive regressions of returns and cash-flow growth in the private, public, and foreign sectors based solely on each sector's valuation ratio. The table reports the slope coefficients and R^2 at the one-quarter ($h = 1$) and one-year ($h = 4$) horizons, with t-statistics based on Newey and West (1987) standard errors shown in parentheses.

The first two columns indicate that the dividend-to-price ratio, dp, is a significant predictor of equity returns, r^E. The positive coefficient aligns with the Campbell and Shiller (1988a) decomposition, and its statistical significance increases somewhat at the longer one-year horizon, reaching a t-statistic of 1.86 and an R^2 of 0.04. In contrast, dp does not meaningfully predict dividend growth at either forecasting horizon: the coefficient has the incorrect positive sign at the one-quarter horizon and turns negative for one-year returns. However, neither coefficient is statistically significant. These findings are consistent with, among others, Cochrane (2008) and Lettau and Van Nieuwerburgh (2008), confirming that the extent of predictability in the private sector remains unchanged when using $da = \log(1 + D/P)$ as the regressor, as we do in our analysis.

For the public sector, the predictability results are more intriguing. The log net surplus-to-debt ratio, nsb, is a strong and significant predictor of both government debt returns and cash-flow (i.e., net surplus or deficit) growth. Compared to equity, cash-flow predictability in the public sector is stronger, as the R^2s in the Δns regression are much larger at 0.05 (for $h = 1$) and 0.09 (for $h = 4$) compared to the corresponding return regression figures. This result aligns with evidence from Berndt et al. (2012), which shows that the bulk of fiscal shocks are absorbed by the surplus channel and to a much lesser extent through the debt valuation channel. The one-year estimate also aligns with that reported by Campbell et al. (2023) from an augmented VAR setting.

Finally, turning to the foreign sector, we find that the log net import-to-foreign asset ratio, nma, significantly forecasts foreign investment returns. The slope coefficient is 0.04 (t-statistic of 2.72) at the one-quarter horizon and increases to 1.31 (t-statistic of 2.96) for one-year returns, with an economically large R^2 of 0.11. Cash-flow predictability, in contrast, remains weak, with the coefficients on nma having the expected negative sign but being imprecisely estimated. These results are consistent with those of Gourinchas and Rey (2007b), who document that external imbalances predict net foreign portfolio returns at short to medium horizons.

TABLE 2: **Univariate predictive regressions**: The table presents the OLS estimates from regressing future h-period log returns and log cash-flow growth rates of the private sector (equity), public sector, and foreign sector on their current respective valuation ratios: the log dividend-to-price ratio dp, the log net surplus-to-debt ratio nsb, and the log net import-to-foreign asset ratio nma, as defined in section 2.1. From the quarterly log returns r_t series, we construct future h-period log returns as $\sum_{k=1}^{h} \rho^{k-1} r_{t+k}$. Future h-period log cash-flow growth is constructed analogously. In parentheses underneath the estimates, we report t-statistics based on Newey and West (1987) standard errors with 8 lags. Boldface coefficients are significant at the 10% confidence level. The coefficients for Δns are divided by 100. Data are quarterly, 1952-2021.

	1-quarter horizon ($h = 1$)						4-quarter horizon ($h = 4$)					
	Private		Public		Foreign		Private		Public		Foreign	
	r^E	Δd	r^D	Δns	r^X	Δnm	r^E	Δd	r^D	Δns	r^X	Δnm
dp	**0.801**	0.347					**2.986**	-0.031				
	(1.800)	(1.044)					(1.859)	(-0.027)				
nsb			**0.040**	**-1.096**					**0.185**	**-2.391**		
			(2.550)	(-2.559)					(3.401)	(-3.226)		
nma					**0.343**	-0.094					**1.312**	-0.432
					(2.715)	(-0.690)					(2.958)	(-0.989)
R^2	0.011	0.002	0.008	0.054	0.037	0.003	0.039	0.000	0.047	0.091	0.114	0.013

3.4 Multivariate OLS Estimates

Table 3 presents the estimates from multivariate regressions that augment the unrestricted model with the valuation ratios of the other two sectors. These results provide prima facie evidence of cross-sector predictability and inform us about how shocks in one sector propagate to others.

In Panel A of the table, the returns and cash flows of a given sector are regressed on all three lagged valuation ratios. Comparing these estimates with those in Panel A of Table 2, several conclusions emerge. The net surplus-to-debt ratio negatively predicts equity returns at both short and medium horizons, with coefficients of -0.21 and -0.76, both statistically significant at the 1% level or better. Including nsb has an economically substantial impact on the regression R^2 at both horizons, increasing it to 0.03 and 0.09, respectively. This result suggests that a higher than average surplus relative to outstanding debt today predicts lower equity returns in the future. Interestingly, the significance of nsb does not diminish the role of the dividend-to-price ratio dp, whose t-statistics are somewhat higher than in the univariate setting. Moreover, nsb negatively predicts dividend growth, with a coefficient of −0.122, though this result is marginally insignificant. Taken together, these findings suggest that *nsb* captures a component of equity expected returns that positively correlates with expected dividend growth and dilutes the signal in the dividend-to-price ratio. At the one-quarter horizon, the predictability of the surplus-to-debt ratio for equity returns is the only significant cross-sector effect, while at the annual horizon, the ratio also forecasts a decrease in risk premia in the foreign sector.

In Panel B of the table, we report analogous estimates after adding three control variables to all regressions: the log tax-to-GDP ratio (ty), which Campbell et al. (2023) argue ensures that the system accounts for the stationary relationship between tax and output; the Cochrane and Piazzesi (2005) tent-shaped factor for bond risk premia (CP), which is re-estimated using Fama-Bliss data over our sample period; and the inflation rate ($infl$), which is included in the benchmark model of Berndt et al. (2012). Overall, the controls enter with significant loadings across all equations. In particular, ty negatively predicts foreign asset returns at both horizons and positively forecasts annual returns to the public sector, consistent with evidence from Campbell et al. (2023). The CP factor captures an important component of expected returns to government debt, confirmed by the positive and highly significant coefficients at both the one-quarter and one-year horizons, where its inclusion raises the R^2 to 0.30, as also found by Cochrane and Piazzesi (2005). Finally, inflation negatively forecasts short-horizon cash-flow growth in the foreign sector and annual equity returns. Since our cash-flow and return series are expressed in real terms, these findings are not mechanically driven by persistence in inflation.

Despite the addition of these controls, the negative relationship between the current net surplus-to-debt ratio and future equity returns remains statistically and economically intact. The net surplus-to-debt ratio also contains relevant information on bond risk premia r^D on top of the yield factor CP. Furthermore, the coefficients and t-statistics for the dividend-to-price ratio increase further in this augmented setting, suggesting that the controls help mitigate endogeneity concerns arising from correlated movements in expected returns and dividend growth. However, in contrast to Panel A, *nsb* no longer predicts foreign asset returns, indicating that this effect is not robust to controlling for the information contained in other variables.

3.5 Multivariate Estimates Under the BC and ABC

The analysis in the previous section overlooks the restrictions imposed by budget constraints. Therefore, we carry a constrained estimation that ensures consistency across sectors by first applying the budget constraint (BC) for each sector, as outlined in Eq.(11)–(13), and then additionally enforcing the cross-sector constraint (ABC) in Eq.(15).

In Panel A of Table 4, we present GMM estimates for the return and cash-flow regressions of the three sectors that adhere to the BC. Compared to Table 3, we observe that enforcing this constraint impacts both the coefficients and, more importantly, their statistical significance, which generally increases. This is particularly evident for the dividend-to-price ratio, whose t-statistics in the equity return regression rise to 2.40 at the one-quarter horizon and 2.80 at the annual horizon,

TABLE 3: **Multivariate predictive regressions**: The table presents the OLS estimates from regressing future h-period log returns and log cash-flow growth rates of the private sector (equity), public sector, and foreign sector on all current valuation ratios. Variable definitions follow Table 2. In Panel A the regressors are the valuation ratios only, while Panel B adds the following controls: the log tax-to-GDP ratio (ty), the Cochrane and Piazzesi (2005) factor (CP), and the inflation rate ($infl$). In parentheses underneath the estimates, we report t-statistics based on Newey and West (1987) standard errors with 8 lags. Boldface coefficients are significant at the 10% confidence level. The coefficients for Δns are divided by 100. Data are quarterly, 1952–2021.

Panel A: Multivariate regressions, valuation ratios only

	1-quarter horizon (h = 1)						4-quarter horizon (h = 4)					
	Private		Public		Foreign		Private		Public		Foreign	
	r^E	Δd	r^D	Δns	r^X	Δnm	r^E	Δd	r^D	Δns	r^X	Δnm
dp	**0.822**	0.312	0.037	-0.119	0.228	1.027	**2.868**	-0.255	0.192	3.193	0.439	3.654
	(1.883)	(0.963)	(0.234)	(-0.094)	(0.181)	(0.571)	(1.948)	(-0.245)	(0.345)	(0.874)	(0.092)	(0.641)
nsb	**-0.209**	-0.122	**0.040**	**-1.111**	-0.407	-0.169	**-0.758**	-0.380	**0.192**	**-2.421**	**-2.031**	-0.406
	(-2.396)	(-1.619)	(2.478)	(-2.679)	(-1.432)	(-0.532)	(-2.524)	(-1.442)	(3.283)	(-3.246)	(-2.249)	(-0.362)
nma	-0.009	0.030	0.000	0.062	**0.355**	-0.098	-0.018	0.113	-0.020	0.239	**1.376**	-0.451
	(-0.233)	(0.996)	(-0.005)	(0.716)	(2.927)	(-0.745)	(-0.143)	(1.274)	(-0.405)	(0.842)	(3.285)	(-1.065)
R^2	0.029	0.011	0.013	0.055	0.045	0.006	0.090	0.030	0.053	0.108	0.149	0.021

Panel B: Multivariate regressions, valuation ratios plus controls

	Private		Public		Foreign		Private		Public		Foreign	
	r^E	Δd	r^D	Δns	r^X	Δnm	r^E	Δd	r^D	Δns	r^X	Δnm
dp	**1.015**	0.270	0.023	-0.652	-0.915	2.095	**3.799**	-0.427	0.338	3.349	-0.654	4.864
	(2.206)	(0.686)	(0.163)	(-0.482)	(-0.763)	(1.099)	(2.653)	(-0.372)	(0.809)	(0.902)	(-0.140)	(0.921)
nsb	**-0.203**	-0.063	**0.037**	**-1.089**	0.064	-0.284	**-0.678**	-0.145	**0.112**	**-1.916**	-0.023	-0.536
	(-2.496)	(-0.948)	(1.920)	(-1.874)	(0.277)	(-0.758)	(-2.500)	(-0.616)	(1.997)	(-1.940)	(-0.029)	(-0.375)
nma	-0.001	0.042	-0.006	0.046	**0.393**	-0.095	-0.002	0.130	-0.049	0.262	**1.570**	-0.574
	(-0.035)	(1.312)	(-0.450)	(0.481)	(3.371)	(-0.743)	(-0.016)	(1.341)	(-1.408)	(0.911)	(4.256)	(-1.446)
ty	0.009	-0.089	0.004	-0.081	**-0.786**	0.265	0.038	-0.305	**0.125**	-0.573	**-2.564**	0.371
	(0.094)	(-1.000)	(0.176)	(-0.163)	(-2.734)	(0.757)	(0.144)	(-1.324)	(1.756)	(-0.769)	(-2.984)	(0.332)
CP	-0.019	-0.287	**0.310**	0.076	-0.059	1.073	1.076	0.344	**1.534**	2.478	0.228	**9.117**
	(-0.054)	(-1.033)	(2.752)	(0.079)	(-0.050)	(1.045)	(0.954)	(0.462)	(4.564)	(0.896)	(0.067)	(3.005)
$infl$	-0.997	-0.338	0.096	2.391	1.179	**-4.141**	**-4.350**	-0.640	-0.062	-3.499	-7.195	-3.854
	(-1.322)	(-0.766)	(0.757)	(1.375)	(0.718)	(-2.102)	(-3.045)	(-0.592)	(-0.174)	(-1.269)	(-1.603)	(-0.903)
R^2	0.039	0.017	0.058	0.061	0.068	0.028	0.147	0.045	0.300	0.128	0.219	0.081

as well as for inflation. Notably, the enhanced significance of these variables does not come at the expense of nsb, which continues to robustly negatively predict equity returns, with t-statistics that are larger in absolute value. Its role in predicting government debt returns and, in particular, growth in deficit or surplus is also strengthened. Among the valuation ratios of the sectors, the net surplus-to-debt ratio stands out as the only significant cross-sector predictor, indicating that the impact of fiscal shocks on future private-sector returns is consistent with the budget constraint.

In the last row of the panel, we report the p-value for the chi-square test of the budget constraint (BC) across all sectors. At both horizons, these p-values are well beyond standard critical values, so we cannot reject the null hypothesis that the BCs hold in the data.

In Panel B of the table, we impose both the sectors' budget constraints and the aggregate budget constraint (ABC). Enforcing the constraints again leads to a general increase in the statistical significance of the coefficients at both horizons. For example, the coefficient of nsb in predicting Δns is now -2.119 (up from -2.076) and is much more precisely estimated, with a t-statistic

TABLE 4: **Multivariate predictive regressions with budget constraints**: The table presents the constrained GMM estimates from regressing future h-period log returns and log cash-flow growth rates of the private sector (equity), public sector, and foreign sector on all current valuation ratios. Variables definition follows from Table 2. In Panel A, we impose the budget constraints of Eq.(11)–(13) within each sector, while in Panel B we impose the aggregate budget constraint of Eq.(15) across the three sectors. In parentheses underneath the estimates, we report t-statistics based on Newey and West (1987) standard errors with 8 lags for the 1-quarter horizon and 12 lags for the 4-quarter horizon. Boldface coefficients are significant at the 10% confidence level. The coefficients for Δns are divided by 100. Data are quarterly, 1952–2021.

	1-quarter horizon (h = 1)						4-quarter horizon (h = 4)					
	Panel A: Imposing BC											
	Private		Public		Foreign		Private		Public		Foreign	
	r^E	Δd	r^D	Δns	r^X	Δnm	r^E	Δd	r^D	Δns	r^X	Δnm
dp	**0.973**	0.229	0.010	-0.798	-0.393	2.567	**3.730**	-0.420	0.340	3.303	-0.650	5.600
	(2.396)	(0.644)	(0.071)	(-0.940)	(-0.388)	(1.513)	(2.800)	(-0.420)	(0.810)	(1.460)	(-0.140)	(1.300)
nsb	**-0.224**	-0.059	**0.036**	**-1.229**	0.084	-0.222	**-0.670**	-0.130	**0.110**	**-2.076**	-0.030	-0.580
	(-2.975)	(-0.988)	(2.065)	(-3.630)	(0.399)	(-0.671)	(-2.550)	(-0.650)	(2.040)	(-3.630)	(-0.040)	(-0.480)
nma	0.009	**0.053**	-0.002	0.070	**0.420**	-0.055	0.000	0.130	-0.050	**0.291**	**1.580**	-0.550
	(0.281)	(1.791)	(-0.170)	(1.176)	(3.692)	(-0.463)	(-0.010)	(1.460)	(-1.490)	(1.890)	(4.510)	(-1.530)
ty	0.018	-0.123	0.004	-0.001	**-0.721**	0.228	0.040	-0.310	**0.120**	-0.485	**-2.580**	0.400
	(0.199)	(-1.639)	(0.161)	(-0.005)	(-2.896)	(0.710)	(0.150)	(-1.570)	(1.720)	(-1.090)	(-3.150)	(0.390)
CP	-0.010	-0.257	**0.264**	-0.138	-0.922	0.953	1.050	0.390	**1.530**	2.296	0.220	**8.820**
	(-0.030)	(-1.023)	(2.523)	(-0.209)	(-0.918)	(1.023)	(0.960)	(0.610)	(4.650)	(1.340)	(0.070)	(3.330)
$infl$	-1.065	-0.260	0.084	**3.589**	-0.453	**-4.652**	**-4.330**	-0.610	-0.060	**-3.304**	-7.020	-4.380
	(-1.440)	(-0.632)	(0.671)	(3.269)	(-0.293)	(-2.578)	(-3.390)	(-0.690)	(-0.170)	(-1.960)	(-1.550)	(-1.230)
	p-value constraint:		0.699				p-value constraint:		0.479			
	Panel B: Imposing BC+ABC											
	Private		Public		Foreign		Private		Public		Foreign	
	r^E	Δd	r^D	Δns	r^X	Δnm	r^E	Δd	r^D	Δns	r^X	Δnm
dp	**1.012**	0.269	0.017	-0.841	-0.936	2.283	**3.790**	-0.490	0.330	2.995	-0.760	5.170
	(2.629)	(0.761)	(0.121)	(-1.026)	(-0.964)	(1.345)	(2.960)	(-0.490)	(0.780)	(1.490)	(-0.170)	(1.210)
nsb	**-0.205**	-0.057	**0.034**	**-1.185**	0.056	-0.284	**-0.680**	-0.120	**0.110**	**-2.119**	-0.020	-0.630
	(-2.735)	(-0.969)	(1.950)	(-3.615)	(0.267)	(-0.887)	(-2.670)	(-0.610)	(2.050)	(-4.340)	(-0.020)	(-0.540)
nma	-0.001	0.041	-0.005	0.049	**0.391**	-0.085	0.010	0.120	-0.050	**0.257**	**1.570**	**-0.590**
	(-0.036)	(1.423)	(-0.429)	(0.853)	(3.592)	(-0.734)	(0.050)	(1.470)	(-1.520)	(1.910)	(4.570)	(-1.700)
ty	0.012	-0.093	0.006	-0.029	**-0.800**	0.264	0.030	**-0.320**	**0.120**	-0.383	**-2.560**	0.450
	(0.128)	(-1.273)	(0.256)	(-0.094)	(-3.221)	(0.831)	(0.100)	(-1.690)	(1.710)	(-0.910)	(-3.170)	(0.440)
CP	-0.019	-0.269	**0.302**	0.080	0.038	0.966	0.970	0.430	**1.530**	2.479	0.200	**8.630**
	(-0.060)	(-1.079)	(2.972)	(0.135)	(0.040)	(1.031)	(1.000)	(0.700)	(4.640)	(1.600)	(0.060)	(3.360)
$infl$	-1.001	-0.329	0.096	**2.769**	1.274	**-4.312**	**-4.290**	-0.530	-0.060	**-3.480**	-7.000	-4.440
	(-1.355)	(-0.809)	(0.786)	(2.777)	(0.874)	(-2.386)	(-3.720)	(-0.600)	(-0.160)	(-2.540)	(-1.610)	(-1.300)
	p-value constraint:		0.362				p-value constraint:		0.834			

of −4.340 (up from −3.630). Its relationship to future equity returns is slightly weaker at the one-quarter horizon but stronger at the one-year horizon. Interestingly, we observe that the net import-to-foreign asset ratio (nma) now significantly predicts annual cash-flow growth rates in the foreign sector with the expected negative sign. This result expands on the evidence presented in Gourinchas and Rey (2007b) and confirms that enforcing consistency across sectors improves inference. Overall, the other conclusions we draw from this setting are quantitatively similar to those from Panel A, and the p-values again cannot reject the null hypothesis.

3.6 Additional Econometric Analyses and Diagnostics

We conduct several analyses to verify that our findings are robust to econometric issues that typically arise in forecasting regressions of asset returns with persistent regressors. We focus our tests on the multivariate OLS specification in Panel A of Table 3 at the non-overlapping 1-quarter horizon, which actually delivers the most conservative figures (in terms of statistical significance) for the role of nsb in equity predictability. Consequently, we concentrate the discussion around this evidence.

3.6.1 Small-sample bias adjustment

In standard predictive regressions, it is common for regressors to be highly persistent and with innovations that correlate with those in returns. In the equity literature, the small-sample bias in OLS estimators under these conditions was first studied by Stambaugh (1999). He proposed a bias correction for the slope coefficient that depends on the bias in the OLS estimate of the autoregressive root of the predictor and negatively on the covariance between innovations. This result has recently been extended to multi-period forecast horizons by Boudoukh et al. (2022). In the case of univariate regressions of returns on the dividend–price ratio, the sign of the bias is found to be positive: positive shocks to expected returns drive up dp and down returns, which implies that we tend to reject the null of no predictability more often.

We examine the impact of small-sample bias in our multivariate system. Since Stambaugh (1999)'s bias correction applies only to univariate regressions, we rely on the method proposed by Amihud and Hurvich (2004). Their approach consists of augmenting the predictive regression with the residuals from the autoregressive process of the predictors, which are obtained after correcting for the bias in the estimator of the autoregressive root. In our context, the method assumes that the 3-dimensional vector of predictors $x_t = [dp_t \, nsb_t \, nma_t]'$ evolves according to a stationary Gaussian vector autoregressive VAR(1) model, $x_t = \Theta + \Phi x_{t-1} + v_t$. As a first step, we examine the off-diagonal elements of the matrix $\widehat{\Phi}$, which determine the exact procedure to follow. For our predictors, we cannot reject the null hypothesis that the off-diagonal elements are jointly zero: the corresponding Wald statistic takes a value of 10.73, with a p-value of 9.72%. Therefore, we proceed with the multivariate method for a diagonal Φ, as described in Section IV.B of Amihud and Hurvich (2004).

In detail, we first correct for bias the OLS estimator ϕ_i of the slope in the AR(1) process for each ratio, and adjust the intercept estimate accordingly. For nsb, the correction increases ϕ from 0.891 (Table 1) to 0.904, still quite far from a unit root. Next, we construct the corresponding corrected residuals, and finally estimate a single multivariate regression of future returns on the three current ratios and the future corrected residuals of all ratios, including an intercept. The resulting slope coefficients of the ratios are thus adjusted for small-sample bias by controlling for the (corrected) error proxies. Standard errors for the slopes are constructed as in Amihud and Hurvich (2004), taking into account the variability in the corrected AR(1) root, and then multiplied by the ratio of Newey–West to conventional standard errors to account for residual autocorrelation, following Amihud et al. (2008).

Panel A of Table 5 reports the bias-adjusted estimates. In the equity return regression, correcting for *dp* has a substantial impact, reducing its coefficient to 0.451 (nearly half the raw estimate) and its t-statistic to a mere 1.030. The adjustment has a comparably smaller effect for *nsb*, whose coefficient remains negative at -0.162 and significant at the 5% confidence level, with a t-statistic of -1.96. For the net foreign asset ratio *nma*, the table confirms the absence of significance. Overall, the small-sample bias correction leaves *nsb* as the only robust predictor of future equity returns. Overall, across all other equations, the effect of the adjustment appears quantitatively less pronounced.

3.6.2 Nonstationarities

The high persistence of valuation ratios raises concerns that the distribution of the conventional t-test for return predictability may deviate in small samples from its asymptotic approximation. We conduct three set of analyses to assess the robustness of our findings to nonstationarity concerns.

First, we use a bootstrap procedure to approximate the small-sample distribution of the OLS slope estimator. Specifically, we generate 10,000 simulated samples of the same size as our data. The simulated series are constructed using the coefficient estimates from Table 3 and the first-order autoregressive processes of the predictors, with residuals resampled jointly with replacement. Panel B of Table 5 reports the OLS estimates with bootstrapped t-statistics shown underneath in square brackets, where standard errors are computed as the standard deviation of the OLS estimates across the simulated samples. Compared to Table 3, the empirical standard errors are noticeably larger for *dp* in predicting equity returns (the t-statistic declines from 1.883 to 1.665) and for *nsb* in predicting public bond returns (from 2.478 to 1.919). In all other cases, the effect is relatively modest. Importantly, we continue to find that *nsb* negatively predicts equity returns, with a t-ratio of -2.291, implying a p-value of 2.2%.

Second, we follow the approach of Campbell and Yogo (2006). The Dickey-Fuller GLS test for the autoregressive coefficient of *nsb* yields a 95% confidence interval of [0.831, 0.947] and a t-statistic of –3.91, indicating that the series is sufficiently distant from a unit root. Accordingly, we place greater confidence in the standard t-test. We also use the DF-GLS statistic to construct a 90% Bonferroni confidence interval for the predictive coefficient. Since this framework applies to univariate models, we begin by regressing future one-quarter equity returns on the net surplus-to-debt ratio alone. The estimated coefficient is -0.21, with a t-statistic of -2.21, both very close to the multivariate estimates in Table 3. The resulting 90% Bonferroni confidence interval, $[-0.370, -0.047]$, confirms that our findings remain statistically intact.[4]

Third, we address the possibility that valuation ratios experience structural shifts in their mean, which may induce persistent deviations from fundamentals and affect full-sample predictive regressions. Lettau and Van Nieuwerburgh (2008) document a break in the mean of the dividend-to-price ratio in 1991. They propose adjusting the series by demeaning it separately before and after

[4] We obtain a very similar confidence interval, $[-0.367, -0.046]$, when approximating the multivariate system by replacing equity returns and the net surplus-to-debt ratio with the residuals from projecting them on *nma* and *dp*.

Table 5: **Multivariate predictive regressions, econometric extensions**: The table presents the OLS estimates from regressing future 1-quarter log returns and log cash-flow growth rates of the private sector (equity), public sector, and foreign sector on all current valuation ratios. Variables definition follows from Table 2. In Panel A, we account for the bias in predictive regressions using the multivariate procedure in Amihud and Hurvich (2004) and Amihud et al. (2008) for a diagonal Φ, as detailed in Section 3.6.1. In Panel B, t-statistics (in square brackets) are computed from the bootstrap procedure described in Section 3.6.2. In Panel C, we break-adjust dp in 1991Q4, nsb in 2006Q4, and nma in 1968Q1, as detailed in Section 3.6.2, and compute t-statistics based on Newey and West (1987) standard errors with 8 lags. Boldface coefficients are significant at the 10% confidence level. The coefficients for Δns are divided by 100. Data are quarterly, 1952–2021.

Panel A: Adjusting for small-sample bias

	Private		Public		Foreign	
	r^E	Δd	r^D	Δns	r^X	Δnm
dp	0.451	0.422	0.043	0.002	0.167	0.977
	(1.030)	(1.314)	(0.273)	(1.352)	(0.133)	(0.726)
nsb	**-0.162**	**-0.159**	**0.041**	**-0.963**	-0.436	0.005
	(-1.959)	(-2.152)	(2.466)	(-2.315)	(-1.540)	(0.022)
nma	0.016	0.017	0.000	-0.000	**0.341**	-0.026
	(0.413)	(0.578)	(0.028)	(-0.656)	(2.768)	(-0.196)
R^2	0.014	0.015	0.013	0.041	0.043	0.002

Panel B: Bootstrapped standard errors

	Private		Public		Foreign	
	r^E	Δd	r^B	Δns	r^X	Δnm
dp	**0.822**	0.312	0.037	-0.119	0.228	1.027
	[1.665]	[0.691]	[0.305]	[-0.082]	[0.158]	[0.677]
nsb	**-0.209**	-0.122	**0.040**	**-1.111**	-0.407	-0.169
	[-2.291]	[-1.386]	[1.919]	[-4.110]	[-1.582]	[-0.614]
nma	-0.009	0.030	0.000	0.062	**0.355**	-0.098
	[-0.227]	[0.761]	[-0.007]	[0.518]	[3.089]	[-0.791]
R^2	0.029	0.011	0.013	0.055	0.045	0.006

Panel C: Break-adjusting valuation ratios

	Private		Public		Foreign	
	r^E	Δd	r^D	Δns	r^X	Δnm
dp	**2.313**	**1.473**	0.096	**-3.446**	-2.360	0.981
	(3.071)	(2.376)	(0.359)	(-1.767)	(-1.283)	(0.311)
nsb	**-0.224**	-0.072	0.029	**-1.989**	**-0.930**	-0.517
	(-2.042)	(-0.795)	(0.981)	(-2.902)	(-2.580)	(-1.277)
nma	-0.003	0.033	-0.000	-0.014	**0.379**	-0.100
	(-0.055)	(1.018)	(-0.000)	(-0.139)	(2.918)	(-0.619)
R^2	0.053	0.021	0.004	0.103	0.069	0.010

1991, which restores its ability to consistently forecast equity returns. For public debt valuation, Jiang et al. (2024c) identify a break in the mean of the government debt-to-output ratio in 2007, following the sharp run-up in debt during the financial crisis. It is therefore natural to ask whether similar breaks occur in our series and how removing such low-frequency components affects our results.

We test for structural breakpoints in the net surplus-to-debt ratio (nsb). The null of no break is strongly rejected in 2007, with a Chow F-test p-value below 0.1%. This is consistent with the visual evidence in Figure 1. The mean of nsb up to 2006Q4 is slightly positive at 0.003, while the mean in the subsequent period falls to –0.079, reflecting the large deficits accumulated by the government during that period. We also examine the net foreign asset ratio (nma) and detect a structural break in 1968Q1: the pre-break mean of –0.023 rises to 0.087 thereafter.

Panel C of Table 5 reports the multivariate estimates when adjusting all ratios at their respective break dates. Working with the transitory component of the dividend-to-price ratio substantially enhances its predictive power: the coefficient of 2.31 and t-statistic of 3.07 are much larger than in Table 3. By contrast, for the net surplus-to-debt ratio, the break adjustment has only a modest effect, with the estimate remaining negative at –0.22 and statistically significant with a t-statistic of –2.04. The transitory component of the net foreign asset ratio again shows no predictive content.

As a more comprehensive check, we re-estimate our predictive regressions while break-adjusting either dp or nsb, allowing the break to occur at any date between 1968 and 2008. Appendix Figure B.1 plots the resulting coefficient on nsb and its 90% confidence interval from the multiple regression of future one-quarter equity returns on the three valuation ratios. In the left panel, the dp series is adjusted for a break occurring at the date on the X-axis; in the right panel, nsb is adjusted analogously. Across all break dates, the estimated coefficients remain uniformly negative and statistically significant in nearly all cases, confirming that the predictive power of nsb is not materially altered by removing low-frequency components from the valuation ratios.

3.6.3 Simulation exercise

In the spirit of Cochrane (2008), we use simulation to quantify how likely it is that nsb predicts equity returns in samples of the same length as our data when expected returns are fully captured by the dividend-to-price ratio. Specifically, we generate 10,000 samples of the private-sector system, each with the same time span as our dataset, consisting of equity returns, dividend growth, and the dividend-to-price ratio, augmented with nsb. Both dp and nsb are modeled as first-order autoregressive processes. Equity returns are generated under the assumption that dp is the sole predictor, with its coefficient implied by the budget constraint and the empirical value of ρ^E. For simplicity, the predictive coefficient on Δd is set to zero, consistent with its lack of significance in our empirical estimation. Innovations are drawn jointly from a multivariate normal distribution calibrated to the covariance matrix of the estimation residuals. This design ensures that dp fully accounts for variation in expected returns, leaving nsb with no predictive power by construction.

We perform four sets of simulations to assess robustness of our conclusions to the persistence of valuation ratios and to address potential small-sample bias. In Scenario A, the autoregressive roots of dp and nsb are set to their sample estimates. In Scenario B, ϕ_{nsb} is set to 0.904, its bias-adjusted value. In Scenario C, we allow for a unit root in dp while keeping ϕ_{nsb} at its sample estimate. Finally, in Scenario D, we consider the most conservative combination in which dp follows a unit root and ϕ_{nsb} equals 0.904. When changing ϕ_{dp}, we adjust the predictive coefficient on equity returns implied by the budget constraint accordingly.

We find that the negative predictability of equity returns is unlikely to arise under the null that only the within-sector budget constraint matters and dp is the sole predictor (see Online Appendix Figure B.2). Increasing the persistence of either dp or nsb does not change this conclusion. Correcting for small-sample bias in the AR(1) estimation of nsb further reduces the likelihood of negative predictability. Overall, these results suggest that nsb captures information beyond the budget-constraint channel.

4 Theoretical Framework and Implications

Our results underscore the central role of (sectoral and aggregate) budget constraints in asset-return dynamics: a higher surplus-to-debt ratio predicts a lower equity risk premium. This pattern reflects joint movements in fiscal policy, government spending, and private-sector profits that are hard to infer from public-sector data alone. Bringing in private- and external-sector information, together with the sector constraints and the ABC, sharpens identification of how fiscal adjustments transmit across the economy. Empirically, the strongest cross-sector effect is that the net surplus-to-debt ratio (nsb_t) negatively predicts future equity returns. To interpret this evidence, we develop a framework that links fiscal policy to private profitability through accounting identities and output distortions from taxes and spending. We present only the elements needed for the empirical connection and defer derivations to Appendix C.

4.1 Output Distortion and Cash Flows

To match our empirical focus on fiscal spillovers to the equity risk premium, we consider a closed economy and abstract from the external sector and investment. The two-sector accounting identity is

$$C_t = Y_t - G_t = (Y_t - T_t) + (T_t - G_t) = D_t + NS_t,$$

where $D_t \equiv Y_t - T_t$ denotes after-tax profits and $NS_t \equiv T_t - G_t$ is the net surplus.

Following Barro (1979) and related work (e.g., Jiang et al. (2024a)), we model taxes as distortional and government spending as output-stimulating. Let the economy produce raw output Y_t^r each period, with fiscal policy affecting finalized output via a deadweight-loss wedge:

$$Y_t = Y_t^r \left(1 - \theta(TY_t^r, GY_t^r)\right). \tag{19}$$

The wedge depends on tax and spending shares of raw output, $TY_t^r = \frac{T_t}{Y_t^r}$, $GY_t^r = \frac{G_t}{Y_t^r}$. To separate tax and spending channels in pricing outcomes, assume

$$\theta(TY^r, GY^r) = \theta_\tau(TY^r) - \theta_g(GY^r). \tag{20}$$

Consistent with Barro (1979), both θ_τ and θ_g are smooth, convex, and increasing, avoiding corner solutions. For tractability, we adopt quadratic forms:

$$\theta_\tau(TY^r) = \frac{1}{2}c^\tau\left(\frac{T}{Y^r}\right)^2, \quad \theta_g(GY^r) = \frac{1}{2}c^g\left(\frac{G}{Y^r}\right)^2, \quad \theta(TY^r, GY^r) = \frac{c^\tau}{2}(TY^r)^2 - \frac{c^g}{2}(GY^r)^2, \tag{21}$$

with $c^\tau, c^g > 0$, so taxes reduce output while spending raises it.

Households consume all output allocated by the government (goods-market clearing, as in a Lucas-tree economy):

$$C_t = Y_t - G_t = Y_t^r\left(1 - \theta(TY_t^r, GY_t^r)\right) - G_t = Y_t^r - Y_t^r\theta(TY_t^r, GY_t^r) - Y_t^r GY_t^r. \tag{22}$$

In asset markets, households hold public debt (a claim on net surplus) and private equity (a claim on dividends). Dividends are post-tax output:

$$D_t = Y_t - T_t = Y_t^r\left(1 - \theta(TY_t^r, GY_t^r)\right) - T_t = Y_t^r - Y_t^r\theta(TY_t^r, GY_t^r) - Y_t^r TY_t^r. \tag{23}$$

Eqs. (22)–(23) show that TY^r and GY^r govern bond and equity cash flows and aggregate consumption.

To isolate the role of distortions, note that if $\theta \equiv 0$ (so $Y_t = Y_t^r$), then Eq. (23) reduces to

$$D_t = Y_t - T_t = Y_t^r\left(1 - TY_t^r\right). \tag{24}$$

In that case, taxes affect dividends only mechanically through the accounting identity. With distortions, taxes (and spending) additionally shift both D_t and C_t through Eq. (21), amplifying the cash-flow effects. Below we use a consumption-based pricing-kernel approach and our GMM estimates to quantify these distortions in U.S. data.

4.2 Fiscal Policy Changes, Asset Valuation, and Cross-Sector Predictability

Motivated by the amplification channel above, we use a comparative-static exercise (as in Barro (1979)) to reconcile the magnitude of equity-return predictability from fiscal imbalances observed in the data.

The first ingredient is a steady-state equilibrium pricing benchmark linking (TY^r, GY^r) to asset values. The government takes the dynamics of raw output Y_t^r as given and chooses tax and spending shares (TY_t^r, GY_t^r) to maximize the present value of household consumption, subject to the public-sector budget constraint (as in Eq. (4)). Fiscal policy determines the allocation of output across sectors; households take consumption in Eq. (22) as given and price public debt and private equity using the CRRA consumption-based kernel. Appendix derives the government's optimal policy and the implied risk premia for equity and Treasuries.

In steady state, asset valuation prices the exogenous growth in Y_t^r, generating systematic comovement across sectors. This standard channel aligns with our GMM framework, which imposes the ABC in Eq. (18) to control for economy-wide variation in risk premia.

The second ingredient is an anticipated deviation from the steady-state fiscal path:

$$\widetilde{TY}^r_{t+1} = TY^r_{t+1} e^{u^\tau_{t+1}}, \quad \widetilde{GY}^r_{t+1} = GY^r_{t+1} e^{u^g_{t+1}}, \quad u^g_{t+1} = \alpha u^\tau_{t+1}. \tag{25}$$

The shocks (u^τ, u^g) capture changes in the expected future fiscal path (e.g., shifts in the budget constraint, political uncertainty, or related forces). We treat them as small deviations from steady state that shift equilibrium pricing; α governs the co-movement of tax and spending shocks.

Anticipated fiscal adjustments revise expectations of cash-flow growth and expected returns through output distortions and consumption-based pricing, generating cross-sector predictability via current price adjustments. In particular,

$$nsb_t \to \widetilde{nsb}_t, \qquad \mathbb{E}_t r^E_{t+1} \to \mathbb{E}_t \widetilde{r}^E_{t+1},$$

and the corresponding expected-return shift can be summarized by

$$\mathbb{E}_t \widetilde{r}^E_{t+1} - \mathbb{E}_t r^E_{t+1} = b(r^E_{t+1}, nsb_t)\, \widetilde{nsb}_t.$$

Empirically, the regressions are run on the post-shock variables $(\widetilde{r}^E_{t+1}, \widetilde{nsb}_t)$; the predictive coefficient $b(r^E_{t+1}, nsb_t)$ maps anticipated fiscal adjustments into risk-premium changes, as captured by our GMM setup.

A key implication is that $b(r^E_{t+1}, nsb_t)$ depends on how fiscal policy affects output, governed by (c^τ, c^g). To match the persistence of fiscal shifts and business-cycle durations, assume investors expect the shocks in Eq. (25) to be i.i.d. and to persist for K periods, with cumulative shock $u_{t+1\ldots t+K}$.[5] Details are in Appendix. The resulting link between (c^τ, c^g) and the predictive slope is summarized in the following proposition.

Proposition 1. *If a fiscal shock persists for K periods, denoted as $u_{t+1\ldots t+K}$, and shocks at each period are i.i.d., then:*

1. *The single-period predictive coefficient $b(nsb_t, r^E_{t+1})$ is determined by the output distortion parameters c^τ and c^g:*

$$b(nsb_t, r^E_{t+1}) = \frac{1-\beta}{(1-\rho^K)(1-\alpha\beta)} \left(-\frac{T}{D} - c^\tau \frac{T}{D} + \alpha c^g \frac{G}{D} \right), \tag{26}$$

   where $\frac{T}{D}$, $\frac{G}{D}$, $\frac{T}{Y}$, and $\frac{G}{Y}$ are steady-state constants, and $\beta = \frac{G}{T} < 1$ is the stable spending-to-tax ratio used in constructing nsb_t.

2. *The time-series response of nsb to fiscal shocks is persistent:*

$$\mathbb{E}_t \left(\widetilde{nsb}_{t+1} - nsb_{t+1} \right) = \frac{1-\rho^{K-1}}{1-\rho^K} \left(\widetilde{nsb}_t - nsb_t \right). \tag{27}$$

[5] Alternatively, one can assume an autoregressive fiscal-shock process. Relative to the i.i.d. case, the cumulative pricing impact differs only by a scalar determined by the autoregressive coefficient.

3. *The cumulative expected return over the next K periods is predicted by nsb_t with the coefficient:*

$$b\left(nsb_t, \sum_{k=1}^{K} \rho^{k-1} r^E_{t+k}\right) = \frac{1-\rho^K}{1-\rho} b(nsb_t, r^E_{t+1}). \tag{28}$$

Eq. (26) implies that higher tax distortion (c^τ) makes $b(nsb_t, r^E_{t+1})$ more negative, while stronger spending stimulus (c^g) makes it more positive. Eq. (27) shows that persistent fiscal shocks generate persistent movements in nsb, and Eq. (28) implies stronger long-horizon predictability the longer the expected fiscal episode.

We calibrate K using the significance pattern of nsb_t in predicting cumulative returns $\sum_{k=1}^{K} \rho^{k-1} r^E_{t+k}$. In Figure 2, we plot $b\big(nsb_t, \sum_{k=1}^{K} \rho^{k-1} r^E_{t+k}\big)$ as K increases and set $K = 20$, where the long-horizon coefficient begins to level off. This suggests investors expect fiscal shifts to persist for about five years.

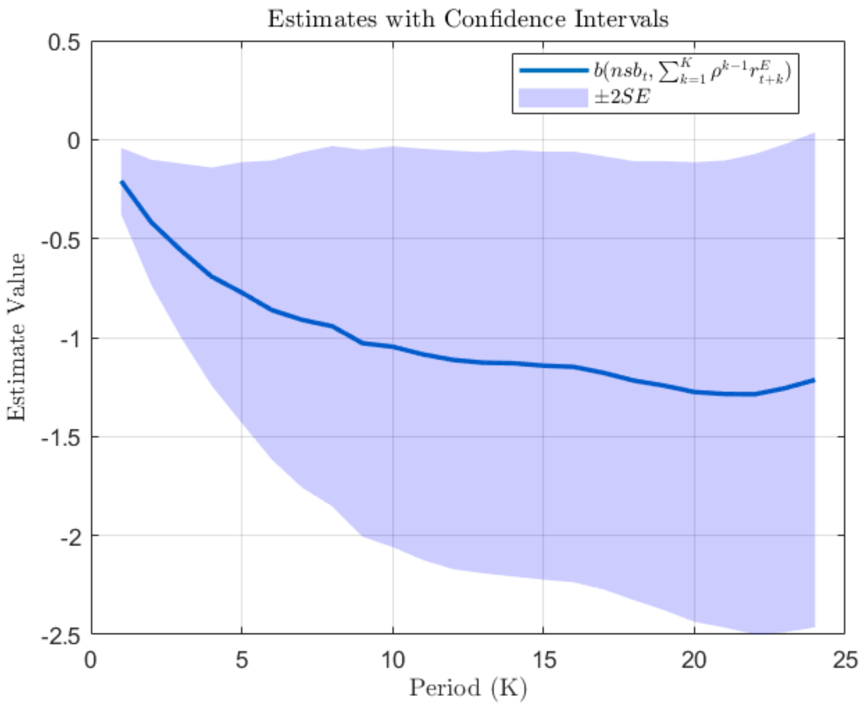

FIGURE 2: **Coefficient of long-horizon regression.** The figure plots the slope from regressing cumulative future equity returns, $\sum_{k=1}^{K} (\rho^E)^{k-1} r^E_{t+k}$, on nsb_t as K varies. Shaded areas show ± 2 standard-deviation bands.

Output distortion is necessary to match the magnitude of $b(nsb_t, r^E_{t+1})$ estimated from the data. Without distortion (i.e., $c^\tau = c^g = 0$ and $\alpha = 0$),

$$b(nsb_t, r^E_{t+1}) = -\frac{1-\beta}{1-\rho^K} \frac{T}{D}.$$

In that case, fiscal shocks affect dividends only through the ABC channel. Using our steady-state calibration,

$$\beta \approx \rho \approx 0.999, \qquad \frac{T}{D} \approx 0.2 \quad \Rightarrow \quad b(nsb_t, r^E_{t+1}) \approx -\frac{1}{K} \times 0.2 \approx -0.01,$$

whereas our empirical estimate $b(nsb_t, r^E_{t+1}) = -0.205$ implies a substantial role for output distortion beyond the ABC mechanism.

This corner case also echoes Ricardian Equivalence. If fiscal policy does not distort output ($c^\tau = c^g = 0$, $\alpha = 0$), the government would optimally forgo spending in Eq. (22) (since it does not raise output here), be indifferent among tax paths that satisfy the budget constraint, and consumption–wealth dynamics would depend only on the exogenous evolution of raw output. Eliminating fiscal policy's impact on output therefore restores Ricardian Equivalence: the government can "tax now" or "tax later" without changing equilibrium outcomes.

Overall, our model accounts for the strongly negative $b(nsb_t, r^E_{t+1})$ through fiscal policy's output effects governed by (c^τ, c^g). Since $b(nsb_t, r^E_{t+1})$ is significantly negative in the data, we can rule out the no-distortion corner solution and study the explicit mapping between $b(nsb_t, r^E_{t+1})$ and (c^τ, c^g).

The comparative-static approach is general and accommodates a wide class of steady-state benchmarks while isolating the fiscal channel through output distortion. This matches our empirical strategy: using state variables $\boldsymbol{x_t}$ (including the within-sector predictor dp) to capture the pre-shock equity risk premium,

$$r^E_{t+1} = \boldsymbol{b^E_r}\, \boldsymbol{x_t}, \quad \Rightarrow \quad \mathbb{E}_t \widetilde{r}^E_{t+1} = \boldsymbol{b^E_r}\, \boldsymbol{x_t} + b(r^E_{t+1}, nsb_t)\, \widetilde{nsb}_t.$$

As in the empirical tests, controlling for benchmark predictors isolates the incremental role of nsb; in the model, the steady-state benchmark isolates the fiscal channel. This alignment allows us to interpret $b(nsb_t, r^E_{t+1})$ structurally and infer (c^τ, c^g) from financial-market data.

We detail how to back out c^τ and c^g from $b(nsb_t, r^E_{t+1})$ in the next subsection. Finally, while we focus on a two-sector setting, the framework can accommodate the three-sector ABC in Eq. (18) by introducing an exogenous import shock u^X_{t+1}:

$$\frac{NM_{t+1}}{Y^r_{t+1}} = \frac{NM}{Y^r} e^{u^X_{t+1}}.$$

Consumption under the ABC becomes

$$C_t = Y^r_t \left(1 - \theta(TY^r_t, GY^r_t)\right) - G_t + NM_t.$$

Import shocks can then affect nma_t and sectoral risk premia. Since our empirical evidence finds no predictive power of nma_t for returns in other sectors, we omit this extension.

4.3 Distortion Implied from Return Predictability

The significantly negative estimate of $b(nsb_t, r^E_{t+1})$ implies output distortion and a violation of Ricardian Equivalence. This raises a central question: how strongly does fiscal policy affect finalized output? Our parametric specification,

$$\theta(TY^r, GY^r) = \frac{c^\tau}{2}(TY^r)^2 - \frac{c^g}{2}(GY^r)^2,$$

links $b(nsb_t, r^E_{t+1})$ directly to (c^τ, c^g), allowing financial markets' forward-looking information to discipline fiscal distortion magnitudes.

We use the GMM estimate $b(nsb_t, r^E_{t+1}) = -0.205$ from Panel B of Table 4 to calibrate (c^τ, c^g) via Eq. (26). This slope alone does not identify both parameters because raw output Y^r_t is unobservable (only finalized GDP Y_t is observed). We therefore write

$$TY^r = \frac{T}{Y}\frac{Y}{Y^r}, \quad GY^r = \frac{G}{Y}\frac{Y}{Y^r}, \tag{29}$$

and use Eq. (21) together with additional steady-state restrictions from the equilibrium model to jointly solve for c^τ, c^g, and Y/Y^r. Details are in Appendix; we summarize the solution below.

Proposition 2. *Given the predictive coefficient $b(nsb_t, r^E_{t+1})$, the output distortion parameters are solved as follows:*

- *The net distortion in output is*

$$\frac{Y}{Y^r} = \frac{-Br + \sqrt{Br^2 + 2Ar}}{Ar}, \tag{30}$$

where Ar and Br are functions of $b(nsb_t, r^E_{t+1})$, the tax–spending comovement coefficient α, and steady-state values.

- *The distortion coefficients in Eq. (21) are*

$$c^\tau = -\frac{1}{1+\alpha} - b(nsb_t, r^E_{t+1})\frac{(1-\rho^K)(1-\alpha\beta)}{(1-\beta)(1+\alpha)}\frac{D}{T} + \frac{\alpha}{1+\alpha}\left(\frac{T}{Y}\frac{Y}{Y^r}\right)^{-1}, \tag{31}$$

$$c^g = \left[\frac{1}{1+\alpha}\left(\frac{T}{Y}\frac{Y}{Y^r}\right)^{-1} + \frac{1}{1+\alpha} + b(nsb_t, r^E_{t+1})\frac{(1-\rho^K)(1-\alpha\beta)}{(1-\beta)(1+\alpha)}\frac{D}{T}\right]/\beta. \tag{32}$$

Here, the steady-state values $\frac{T}{D}$, $\frac{G}{D}$, $\frac{T}{Y}$, and $\frac{G}{Y}$ are constants, and $\beta = \frac{G}{T} < 1$ is the stable spending-to-tax ratio used in Eq. (9).

We implement this solution using $b(nsb_t, r^E_{t+1}) = -0.205$. Along with $K = 20$ from Figure 2, we calibrate α from the joint dynamics of spending and taxation by regressing $gb = \log(G/B)$ on $tb = \log(T/B)$, yielding $\alpha = 0.827$. Other steady-state ratios (e.g., T/Y and D/T) are set to long-run historical averages, while ρ and β are inferred from the public-sector identity Eq. (9).

This procedure maps an empirically observed $b(nsb_t, r^E_{t+1})$ into an implied pair (c^τ, c^g), making $b(nsb_t, r^E_{t+1})$ an indicator of how fiscal-policy expectations affect output and asset prices. We further interpret this mapping below.

4.3.1 Dividend Distortion

From Eq. (23) and the fiscal-shock specification Eq. (25), dividend growth responds as

$$\Delta \tilde{d}_{t+1} - \Delta d_{t+1} = \left(-\frac{T}{D} - c^{\tau}\frac{T}{D}\right) u^{\tau}_{t+1} + \left(c^{g}\frac{G}{D}\right) u^{g}_{t+1}. \tag{33}$$

Given $b(nsb_t, r^{E}_{t+1})$, equations Eqs. (31)–(32) pin down (c^{τ}, c^{g}) and thus the loadings of dividend-growth changes on $(u^{\tau}_{t+1}, u^{g}_{t+1})$, i.e., the implied dividend distortion from taxes and spending.

In Figure 3, we plot the tax and spending contributions in Eq. (33), normalizing $u^{\tau} = 1$ and setting $u^{g} = \alpha$ as in Eq. (25). When the predictive coefficient is zero, the two effects offset. By contrast, our negative GMM benchmark (dashed line) indicates that tax distortions dominate spending stimulus, implying a sharp drag on dividend growth.

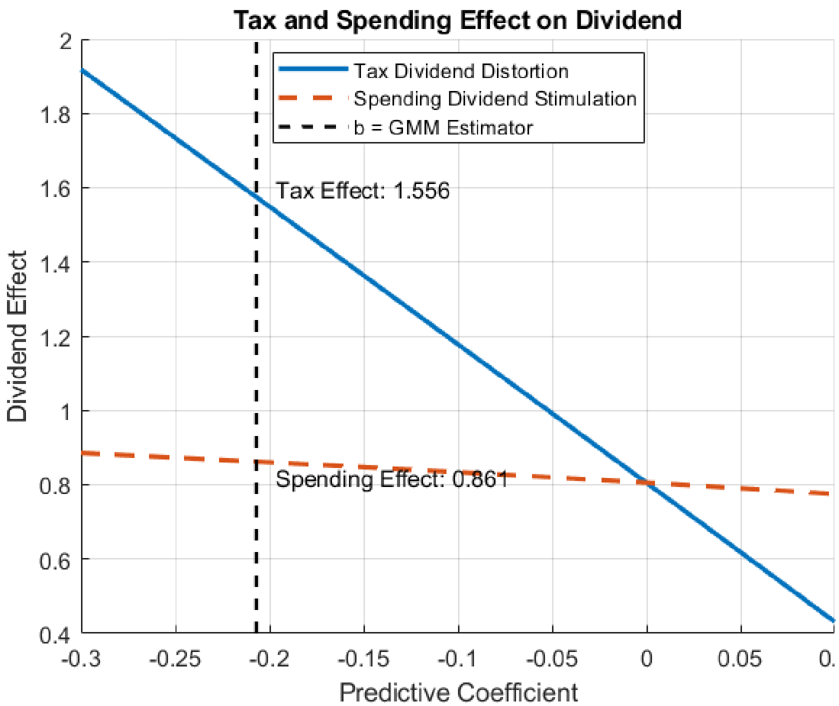

FIGURE 3: **Dividend distortion and predictive coefficient.** The figure shows how the tax and spending effects on dividend growth in Eq. (33) vary with $b(nsb_t, r^{E}_{t+1})$ in Eq. (26). The tax effect is plotted with the opposite sign for comparability. The y-axis reports the change in dividend growth for $u^{\tau} = 1$ and $u^{g} = \alpha$. Calibration uses $K = 20$ and $\alpha = 0.827$; the benchmark $b(nsb_t, r^{E}_{t+1}) = -0.205$ (Panel B, Table 4) is marked by the dashed line.

4.3.2 Output Distortion

The same mapping yields the implied tax distortion and spending stimulus in output:

$$\theta_{\tau}(TY^{r}) = \frac{1}{2}c^{\tau}\left(\frac{T}{Y}\frac{Y}{Y^{r}}\right)^{2}, \quad \theta_{g}(GY^{r}) = \frac{1}{2}c^{g}\left(\frac{G}{Y}\frac{Y}{Y^{r}}\right)^{2}.$$

Under our baseline calibration ($K = 20$, $\alpha = 0.827$),

$$\theta_\tau(TY^r) = 0.063, \quad \theta_g(GY^r) = 0.024 \quad \Rightarrow \quad \theta(TY^r, GY^r) = 0.039, \quad \frac{Y}{Y^r} = 0.961.$$

Thus taxation reduces output by about 6.3%, partially offset by a spending-induced gain of about 2.4%; the negative $b(nsb_t, r^E_{t+1})$ reflects tax distortions dominating spending stimulus.

In Figure 4, we plot $\theta_\tau(TY^r)$ and $\theta_g(GY^r)$ across values of $b(nsb_t, r^E_{t+1})$. Consistent with Eq. (26), a more negative slope corresponds to stronger tax distortion and weaker spending stimulus (and vice versa).

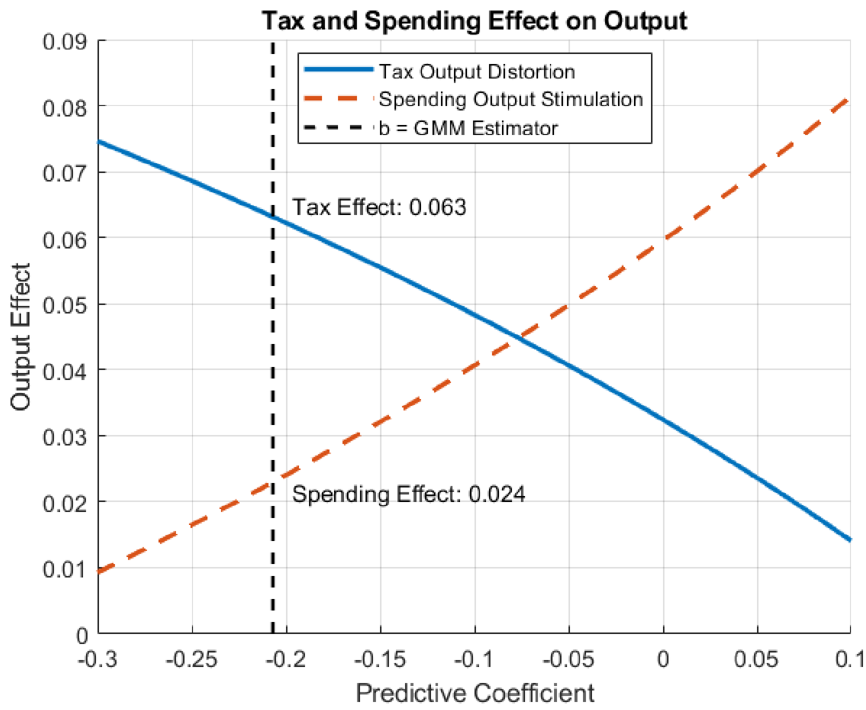

FIGURE 4: **Output distortion and predictive coefficient.** The figure plots the implied $\theta_\tau(TY^r)$ and $\theta_g(GY^r)$ from Eq. (21) as functions of $b(nsb_t, r^E_{t+1})$ in Eq. (26). The tax effect is plotted with the opposite sign for comparability. Calibration uses $K = 20$ and $\alpha = 0.827$; the benchmark $b(nsb_t, r^E_{t+1}) = -0.205$ (Panel B, Table 4) is marked by the dashed line.

4.3.3 Remarks

The coefficients (c^τ, c^g) are closely related to the literature on tax and spending multipliers. Analogous to Eq. (33), output growth responds to Eq. (25) as

$$\Delta\tilde{y}_{t+1} - \Delta y_{t+1} = \left(-c^\tau \frac{T}{Y}\right) u^\tau_{t+1} + \left(c^g \frac{G}{Y}\right) u^g_{t+1}. \tag{34}$$

Since

$$\Delta\tilde{y}_{t+1} - \Delta y_{t+1} \approx \left(\tilde{Y}_{t+1} - Y_{t+1}\right) / Y_t,$$

the implied responses in Eq. (34) translate into fiscal multipliers—how one dollar of taxes/spending changes finalized output. We defer additional discussion of multiplier measurement to Appendix, as it parallels a broad literature beyond this paper and is a natural topic for follow-up work.

Finally, the framework can accommodate channels beyond output distortion through which fiscal policy may affect equity markets (e.g., default risk or inflation dynamics, allowing fiscal decisions in real terms). While we leave such extensions to future research, our empirical analysis partially addresses them by using real returns and growth rates and controlling for debt sustainability via the tax-to-GDP ratio in estimation.

5 Conclusion

In this study, we investigate the joint dynamics and predictability of asset returns for the equity, treasury, and foreign asset investment sectors, as each represents a distinct asset class with different dynamics and potential investors. We find that the predictability of each sector's own returns remains largely intact even after accounting for state variables in other sectors.

To better understand the joint dynamics of the three sectors, we impose the aggregate budget constraint and estimate all predictive regressions jointly. The results of this estimation highlight the importance of considering the aggregate cyclical movement of the economy as a whole when examining the joint dynamics of asset returns. Based on our multivariate analysis, we find that the net surplus-to-debt ratio negatively predicts the risk premium in the equity and foreign asset investment sectors, as a lower government surplus could signal a contractionary fiscal policy in the future and higher risk premiums for investing in equity or foreign assets. However, this mechanism may be difficult to observe in the short term based solely on public sector data. Our results suggest that considering data from all three sectors and imposing aggregate budget constraints can help to better identify how this fiscal policy adjustment channel propagates throughout the economy.

Overall, our study contributes to the literature on asset return predictability by examining the joint dynamics of asset returns across different markets, considering their respective valuation ratios and cash flows, and incorporating budget constraints to better understand their interactions. Further research could explore the conditional correlations and regime-switching of the joint dynamics of the three sectors, as well as the implications of our findings for portfolio diversification and asset allocation.

Meanwhile, we propose a theoretical framework to explain our finding that fiscal surplus negatively affects the equity risk premium. This framework enables us to leverage asset market data as a forward-looking estimator of the impact of tax distortions and government spending on output. Additionally, our approach is flexible enough to incorporate alternative channels beyond output distortion through which fiscal policy can influence equity markets, such as inflation risk and public debt default risk. We leave these extensions for future research.

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Appendix A Derivation of the linearized identities

Our approach builds on Campbell et al. (2023) (CGM henceforth), who linearize the surplus-debt ratio by exploiting the cointegrated relation between the log tax-debt and the log spending-debt ratio. Their method separately handles inflows (spending) and outflows (taxes) of the public sector and can be easily reconciled with the single outflow method developed by Campbell and Shiller (1988a) (CS henceforth) for equity. To standardize the linear identities across all sectors we consider, we employ the same approach for equity and foreign asset accounts. In what follows, we present a unified linearization framework that can be consistently applied across all sectors.

A1 A Linear Approximation Method

We denote a sector's asset value by V, its asset return by R, and its net outflow by S. All budget constraints we consider in the paper share a general form:

$$V_t(1 + R_{t+1}) = V_{t+1} + S_{t+1}, \tag{A1}$$

where the net outflow consists of two components:

$$S_t = S_t^a - S_t^b.$$

Here, S^a represents the outflow of account value (e.g., dividends paid to shareholders, tax received to pay public debt), while S^b represents the inflow (e.g., fiscal spending that increases the public debt).

Dividing both sides by V_t gives:

$$1 + R_{t+1} = \frac{V_{t+1} + S_{t+1}}{V_t} = \frac{V_{t+1}}{V_t}\frac{V_{t+1} + S_{t+1}}{V_{t+1}}. \tag{A2}$$

Taking the logarithm on both sides:

$$r_{t+1} = \Delta v_{t+1} + \log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right). \tag{A3}$$

The standard CS log-linearization proceeds by replacing the last term with its first-order Taylor expansion around the average log outflow-to-value ratio:

$$sv_{t+1} = \log\left(\frac{S_{t+1}}{V_{t+1}}\right).$$

However, when S turns negative, this variable is undefined. To overcome this issue, CGM propose an alternative log-linearization based on:

$$\tilde{sv}_{t+1} = \log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) = \log\left(1 + \frac{S^a_{t+1} - S^b_{t+1}}{V_{t+1}}\right),$$

and handle S^a_{t+1}/V_{t+1} and S^b_{t+1}/V_{t+1} separately to derive a bivariate approximation. Let us denote the log inflow- and outflow-to-value ratios as:

$$av_{t+1} = \log\frac{S^a_{t+1}}{V_{t+1}}, \quad bv_{t+1} = \log\frac{S^b_{t+1}}{V_{t+1}},$$

and define steady-state values as:

$$a = \overline{e^{av_{t+1}}}, \quad b = \overline{e^{bv_{t+1}}}.$$

The steady-state values of av and bv are then $\log a$ and $\log b$, respectively. Applying a first-order approximation based on a Taylor expansion, we have:

$$\tilde{sv}_{t+1} = \log\left(1 + \frac{S^a_{t+1} - S^b_{t+1}}{V_{t+1}}\right) = \log\left(1 + e^{av_{t+1}} - e^{bv_{t+1}}\right) = k + \frac{1}{1+a-b}\left(a \times av_{t+1} - b \times bv_{t+1}\right),$$

where

$$k = \log\left(1 + a - b\right) + \frac{b \times \log b - a \times \log a}{1 + a - b}.$$

We reorganize the terms to interpret the bivariate log-linearization as:

$$\tilde{sv}_{t+1} = k + (1-\rho)\left(\frac{1}{1-\beta}av_{t+1} - \frac{\beta}{1-\beta}bv_{t+1}\right), \tag{A4}$$

with $\rho = \frac{1}{1+a-b}$, $\beta = \frac{b}{a}$. Here, $a = \overline{e^{av_{t+1}}}$ and $b = \overline{e^{bv_{t+1}}}$ represent the steady-state values of inflows and outflows, respectively. Thus, $a - b$ is the steady-state value of net outflows, and

$$\rho = \frac{1}{1+a-b}$$

should be close to 1. Similarly,

$$\beta = \frac{b}{a}$$

represents a stable relationship or co-integration coefficient between inflows and outflows (e.g., spending and taxes).

With this approximation, the log net cash flow growth is defined as:

$$\Delta s_{t+1} = \frac{1}{1-\beta}\Delta a_{t+1} - \frac{\beta}{1-\beta}\Delta b_{t+1}, \tag{A5}$$

where $\Delta a_{t+1} = \log \frac{S^a_{t+1}}{S^a_t}$ and $\Delta b_{t+1} = \log \frac{S^b_{t+1}}{S^b_t}$ are the growth rates of inflows and outflows, respectively. Accordingly, we insert the definition of ratios and cash flows into equation Eq. (A3) and derive a budget constraint (B.C.) as:

$$\tilde{sv}_t = (1-\rho)r_{t+1} - (1-\rho)\Delta s_{t+1} + \rho\tilde{sv}_{t+1}. \tag{A6}$$

Note that we can easily reconcile this expression with the classical log-linearization method in Campbell and Shiller (1988a) when the net outflow is strictly positive. CS derived:

$$\tilde{sv}_{t+1} = \log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) = k + (1-\rho)sv_{t+1}.$$

Additionally, the classical budget constraint (B.C.) is given by:

$$r_{t+1} = k + \Delta s_{t+1} + sv_t - \rho sv_{t+1}. \tag{A7}$$

Using the first relation to substitute into the B.C., we obtain:

$$r_{t+1} = k + \Delta s_{t+1} + \frac{\tilde{sv}_t - k}{1-\rho} - \rho\frac{\tilde{sv}_{t+1} - k}{1-\rho}.$$

Reorganizing terms leads us back to the bivariate approximation derived above:

$$(1-\rho)r_{t+1} = (1-\rho)\Delta s_{t+1} + \tilde{sv}_t - \rho\tilde{sv}_{t+1}.$$

From this perspective, using

$$\tilde{sv}_{t+1} = \log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right)$$

as the predictor in Eq. (A6) is equivalent to using

$$sv_{t+1} = \log\left(\frac{S_{t+1}}{V_{t+1}}\right),$$

together with Eq. (A7), when the net outflow is strictly positive. Therefore, we apply this framework consistently across all sectors to integrate our analysis.

A2 Sector budget constraints (BCs)

We apply the approximation method described above to the government's intertemporal budget constraint for public debt and extend it to equity and foreign asset accounts.

For public debt valuation, the net government surplus $NS = T - G$ represents the cash outflow from debt. Accordingly, the government's valuation ratio (net surplus-to-debt), denoted nsb, is defined as:

$$nsb_{t+1} = \log\left(1 + \frac{T_{t+1} - G_{t+1}}{B_{t+1}}\right), \tag{A8}$$

where B represents the market value of debt. The log-linearization in the previous section implies

$$nsb_{t+1} = k + (1 - \rho^B)\left(\frac{1}{1-\beta}\tau b_{t+1} - \frac{\beta}{1-\beta} gb_{t+1}\right), \tag{A9}$$

where τb and gb represent the log tax-to-debt and spending-to-debt ratios, respectively.

We can estimate β using least squares, given a value of ρ^B that is consistent with a steady-state level for nsb. Following Campbell et al. (2023), we set $\rho^B = 0.999$, which implies an estimated $\beta = 0.997$, closely aligning with their results. The corresponding log cash flow growth is defined as:

$$\Delta ns_{t+1} = \frac{1}{1-\beta}\Delta\tau_{t+1} - \frac{\beta}{1-\beta}\Delta g_{t+1}. \tag{A10}$$

The weights on tax revenues and government spending in this expression mimic those for the surplus-to-debt ratio of Eq.(A9). Moreover, a value of β close to one implies that the two weights are close to each other. The resulting budget constraint for the public sector is:

$$nsb_t = (1 - \rho^B) r^B_{t+1} - (1 - \rho^B)\Delta ns_{t+1} + \rho^B nsb_{t+1}. \tag{A11}$$

We can directly apply this framework to the equity sector. Its valuation ratio

$$da_t = \log\left(1 + \frac{D_t}{A_t}\right)$$

corresponds to the log of one plus the dividend-to-price ratio for public equity. The budget constraint for the equity sector thus becomes:

$$da_t = (1 - \rho^A) r^A_{t+1} - (1 - \rho^A)\Delta d_{t+1} + \rho^A da_{t+1}. \tag{A12}$$

The linear identity for public debt in Eq. (C12) has the same format as that in Eq. (A12) for private assets. This consistency enables us to combine the intertemporal budget constraints of each sector into an aggregate budget constraint. The linearization framework in the previous section requires a positive account value V_t and a well-defined log return, such that $1 + R_{t+1} = \frac{V_{t+1}+S_{t+1}}{V_t} > 0$. However, this assumption does not hold for the external sector, whose return is given by:

$$1 + R^X_{t+1} = \frac{F_{t+1} + NM_{t+1}}{F_t}.$$

The net foreign asset value F represents the difference between foreign assets A^X and foreign liabilities L^X. In our sample, there are periods when the net foreign asset value $F_t = A_t^X - L_t^X$ is close to zero and net imports are negative. This implies gross returns that are extremely negative, making it impossible to define a log return.

To address this issue, we separately approximate the dynamics of foreign assets and liabilities:

$$\begin{aligned} A_t^X(1 + R_{t+1}^{Ax}) &= A_{t+1}^X + M_{t+1} \\ L_t^X(1 + R_{t+1}^{Lx}) &= L_{t+1}^X + X_{t+1}. \end{aligned}$$

Both asset and liability dynamics can be log-linearized as two separate accounts, following Eq. (A6). First, the dynamics of the foreign asset account are captured by:

$$ma_t = (1 - \rho^{Ax})r_{t+1}^{Ax} - (1 - \rho^{Ax})\Delta m_{t+1} + \rho^{Ax} ma_{t+1}, \tag{A13}$$

where ma is the log import-to-asset ratio, r^{Ax} is the log return of the foreign asset account, and Δm is the log growth rate of imports. Similarly, the dynamics of the foreign liability account are captured by:

$$xl_t = (1 - \rho^{Lx})r_{t+1}^{Lx} - (1 - \rho^{Lx})\Delta x_{t+1} + \rho^{Lx} xl_{t+1}, \tag{A14}$$

where xl is the log export-to-liability ratio, r^{lx} is the log return of the foreign liability account, and Δx is the log growth rate of exports.

The net foreign asset value $F = A^X - L^X$ can be viewed as a portfolio of the two accounts, with its dynamics captured by a linear combination of Eq. (A13) and Eq. (A14). To derive the intertemporal budget constraint for the external sector as a portfolio of two accounts, we need an aggregation method for budget constraints across multiple positions. We illustrate the general aggregation method in the next section and present here the outcome of applying this technique to the foreign asset account. Specifically, we derive the following relationship for foreign asset valuation:

$$nma_t = (1 - \rho^X)r_{t+1}^X - (1 - \rho^X)\Delta nm_{t+1} + \rho^X nma_{t+1}. \tag{A15}$$

The log return, cash-flow growth rate, and valuation ratio are weighted averages of the associated variables in the asset and liability accounts, as shown in Eq. (A13) and Eq. (A14). Conceptually, the valuation ratio nma is a proxy for external sector imbalances, i.e., net import expenses divided by net asset value. Our approach to handling the foreign asset account shares similarities with Gourinchas and Rey (2007a), with differences arising from our need to derive a unified framework for all three sectors. Consistent with Gourinchas and Rey (2007a), we observe an increasing trend in U.S. net imports, alongside a corresponding rise in foreign liabilities. To ensure our results are not unduly influenced by this trend, we apply the de-trending filter proposed by Hamilton (2018) to the nma series and construct a stationary predictor.

Based on the budget constraints derived for the three sectors, we solve these equations forward and impose the standard transversality condition, which yields the long-horizon linear identities for each sector. Specifically, for the equity sector, the long-horizon equation is:

$$da_t = \left(1 - \rho^A\right) \sum_{j=0}^{\infty} \left(\rho^A\right)^j \left(r_{t+j+1}^A - \Delta d_{t+j+1}\right). \tag{A16}$$

For the public debt sector, we obtain:

$$nsb_t = \left(1 - \rho^B\right) \sum_{j=0}^{\infty} \left(\rho^B\right)^j \left(r_{t+j+1}^B - \Delta ns_{t+j+1}\right). \tag{A17}$$

Finally, for the external sector, the long-horizon identity is given by:

$$nma_t = \left(1 - \rho^X\right) \sum_{j=0}^{\infty} \left(\rho^X\right)^j \left(r_{t+j+1}^X - \Delta nm_{t+j+1}\right). \tag{A18}$$

A3 Aggregate Budget Constraint

Aggregate wealth and consumption are related by the expression

$$W_t(1 + R^W_{t+1}) = W_{t+1} + C_{t+1}, \tag{A19}$$

where W_t is the total wealth at time t, R^W_{t+1} represents the return on wealth from t to $t+1$, and C_{t+1} is consumption in period $t+1$. In analogy to the individual sectors, the linearized identity for wealth implies that

$$cw_t = (1 - \rho^W)r^W_{t+1} - (1 - \rho^W)\Delta c_{t+1} + \rho^W cw_{t+1}, \tag{A20}$$

where $cw_t = \log\left(1 + \frac{C_t}{W_t}\right)$ is the log consumption-to-wealth ratio, r^W_{t+1} is the log return on wealth, and $\Delta c_{t+1} = \log\left(\frac{C_{t+1}}{C_t}\right)$ is the growth rate of consumption. Similarly, the long-horizon identity for the aggregate economy is:

$$cw_t = \left(1 - \rho^W\right)\sum_{j=0}^{\infty}\left(\rho^W\right)^j\left(r^W_{t+j+1} - \Delta c_{t+j+1}\right), \tag{A21}$$

where $\rho^W = \frac{1}{1+\overline{CW}}$ reflects the persistence of the consumption-wealth dynamic.

The linkage between our three-sector budget constraints and the consumption-wealth dynamic comes from two fundamental identities. First, consumption equals to the sum of all cash flows:

$$C_t = D_t + NS_t + NM_t. \tag{A22}$$

Similarly, the aggregate wealth held by domestic investors equals the sum of all asset values:

$$W_t = A_t + B_t + F_t. \tag{A23}$$

We can relate the valuation ratios of the three sectors to the consumption-wealth ratio as follows:

$$\frac{C_t}{W_t} = \frac{A_t}{W_t} \times \frac{D_t}{A_t} + \frac{B_t}{W_t} \times \frac{NS_t}{B_t} + \frac{F_t}{W_t} \times \frac{NM_t}{F_t}, \tag{A24}$$

which shows that consumption relative to wealth is a weighted combination of the cashflow-to-asset ratios of each sector. Similarly, the aggregate gross return on wealth is a weighted average of the sectos's gross returns:

$$1 + R^W_{t+1} = (1 + R^A_{t+1})(A_t/W_t) + (1 + R^B_{t+1})(B_t/W_t) + (1 + R^X_{t+1})(F_t/W_t). \tag{A25}$$

To integrate our framework for the three sectors budget constraints with the consumption-wealth dynamic, we apply the log-linearization to Eq. (A24), which yields an aggregate budget constraint (A.B.C.) of the form:

$$cw_t = \alpha_{cw} + w_A da_t + w_B nsb_t + w_X nma_t, \tag{A26}$$

where the weights w_A, w_B, and w_X represent each sector's proportion of total wealth at the steady state. We calibrate these weights in the same way we calibrate ρ for each sector. The weights are:

$$w_A = \frac{e^{\overline{aw}}}{e^{\overline{aw}} + e^{\overline{bw}} + e^{\overline{fw}}} \qquad w_B = \frac{e^{\overline{bw}}}{e^{\overline{aw}} + e^{\overline{bw}} + e^{\overline{fw}}} \qquad w_X = \frac{e^{\overline{fw}}}{e^{\overline{aw}} + e^{\overline{bw}} + e^{\overline{fw}}}$$

where $\overline{aw}$ is the average value of the log A/W ratio, and similarly for $\overline{bw}$ and $\overline{fw}$. Using the same logic, we can also express the aggregate return on wealth as:

$$r^W_{t+1} = \alpha_{rw} + w_A r^A_{t+1} + w_B r^B_{t+1} + w_X r^X_{t+1}. \tag{A27}$$

Equations (A26) and (A27) highlight the key implication of our analysis: the valuation ratio in each sector captures cyclical information about consumption-wealth dynamics and thus predicts the return on total wealth, including the returns for each individual sector.

In addition, we further demonstrate a portfolio aggregation result used in our approximation framework, showing that the aggregate consumption-wealth dynamic in Eq. (A20) is a weighted average of the multi-sector dynamics. This general result aligns with our A.B.C. and is also used to derive Eq. (A15) as a linear combination of Eq. (A13) and Eq. (A14).

Consider an aggregate portfolio composed of sectors $i = 1, \ldots, n$, where each sector's return, cash flow, and stock value are denoted by R^i, S^i, and V^i, respectively. Each sector has a budget constraint, which can be approximated by:

$$\tilde{sv}_t^i = (1-\rho^i) r_{t+1}^i - (1-\rho^i)\Delta s_{t+1}^i + \rho^i \tilde{sv}_{t+1}^i.$$

The aggregate consumption-wealth dynamic can be expressed as:

$$cw_t = (1-\rho^W) r_{t+1}^W - (1-\rho^W)\Delta c_{t+1} + \rho^W cw_{t+1}.$$

To enforce the A.B.C., we show that it is a weighted average of the sectoral B.C.s. Specifically, a weight φ_i aligns all variables on the right-hand side, such that:

$$\begin{aligned} \rho^W cw_{t+1} &= \sum \varphi^i \rho^i \tilde{sv}_{t+1}^i, && \text{(A28)} \\ (1-\rho^W)\Delta c_{t+1} &= \sum \varphi^i (1-\rho^i)\Delta s_{t+1}^i. && \text{(A29)} \end{aligned}$$

Additionally, we have:

$$1 + \frac{C_{t+1}}{W_{t+1}} = \sum \frac{V_{t+1}^i}{W_{t+1}} \left(1 + \frac{S_{t+1}^i}{V_{t+1}^i}\right). \tag{A30}$$

Dividing both sides by the steady-state value gives:

$$\frac{1 + CW_{t+1}}{1 + CW} = \sum_i^n \frac{VW^i(1+SV^i)}{\sum VW^i(1+SV^i)} \left(\frac{1 + SV_{t+1}^i}{1 + SV^i}\right). \tag{A31}$$

Here, note that:

$$\rho^i = \frac{1}{1 + SV^i},$$

and

$$\rho^W = \frac{1}{1 + CW} = \frac{1}{\sum VW^i(1+SV^i)}.$$

Using the fact that $1 + CW$ can be approximated by $\log(1+CW) + 1$, and similarly for SV_t^i, we derive that:

$$\rho^W cw_{t+1} = \sum \varphi^i \rho^i \tilde{sv}_{t+1}^i,$$

where:

$$\varphi_i = \frac{VW^i(1+SV^i)}{\sum VW^i(1+SV^i)} = \rho^W \frac{VW^i}{\rho^i}.$$

Equivalently, we can express:

$$cw_{t+1} \approx \sum \frac{VW^i}{\sum VW^i} \tilde{sv}_{t+1}^i.$$

Intuitively, the weight used to combine each sector's $\tilde{sv}^i$ into cw_t is the proportion of total wealth accounted for by each sector, VW^i. To smooth this approximation, one can use $\exp\{vw^i\}$ to compute a steady-state value of the logged ratio.

Similarly, we have:

$$\varphi_i(1-\rho^i) = VW^i \frac{1-\rho^i}{\rho^i} = VW^i SV^i,$$

and,

$$1-\rho^W \approx CW.$$

The cash flow relation is then approximated by:

$$\Delta c_{t+1} \approx \sum \frac{\varphi_i(1-\rho^i)}{1-\rho^W} \Delta s^i_{t+1} \approx \sum \frac{S^i}{C} \Delta s^i_{t+1},$$

which is the difference version of:

$$c_{t+1} = \sum \frac{S^i}{C} s^i_{t+1}.$$

Intuitively, the weight used to combine each sector's Δs^i into Δc_{t+1} is the proportion of total cash flow, S^i/C, that each sector contributes. Again, to smooth this approximation, one can use $\exp\left\{sc^i\right\}$ to compute a steady-state value of the logged ratio.

Appendix B Estimation Method

To facilitate comparison with prior studies on equity predictability, we use the notation r^E in place of r^A and dp in place of da, with the understanding that dp is constructed as $\log(1+D/P)$ for consistency with our linearization framework throughout this section.

The linear identities for the three sectors (B.C.s) in Eq. (A12), Eq. (C12), and Eq. (A15), along with the A.B.C. in Eq. (A26), regulate the joint dynamics of these sectors and motivate a multi-equation regression system. This system uses the valuation ratios $[dp_t; nsb_t; nma_t]$ to predict returns and cash flows across all sectors. Furthermore, the linear identities imply a relationship between the predictive coefficients in these equations.

We first introduce how the B.C.s and A.B.C. indicate constraints in a multi-equation system and then present the general technique for estimating this system.

B1 A Predictive Regression System

We use the equity sector as an example to illustrate the multi-equation predictive regression system. The dynamics in Eq. (A12) motivate three predictive regressions:

$$r^E_{t+1} = \alpha_r + b_r dp_t + \varepsilon_{r,t+1}, \tag{B1}$$

$$\Delta d_{t+1} = \alpha_d + b_d dp_t + \varepsilon_{d,t+1}, \tag{B2}$$

$$dp_{t+1} = \alpha_\phi + b_\phi dp_t + \varepsilon_{dp,t+1}. \tag{B3}$$

With the equity sector's budget constraint:

$$dp_t = (1-\rho^E) r^E_{t+1} - (1-\rho^E)\Delta d_{t+1} + \rho^E dp_{t+1},$$

the predictive coefficients must satisfy the following linear relationship:

$$0 = (1-\rho^E)\alpha_r - (1-\rho^E)\alpha_d + \rho^E \alpha_\phi,$$
$$1 = (1-\rho^E) b_r - (1-\rho^E) b_d + \rho^E b_\phi.$$

Building on this intuition, we extend the framework to multiple sectors and present the system in matrix format. We denote vectors and matrices in boldface for clarity. Let the number of sectors be N, which in our setting is $N = 3$. Define:

$$\underset{3\times 1}{\boldsymbol{ratio_t}} = [dp_t; nsb_t; nma_t],$$

and similarly for returns and cash flow growth rates:

$$\boldsymbol{r_t} = [r_t^E; r_t^B; r_t^X],$$
$$\boldsymbol{\Delta cf_t} = [\Delta d_t; \Delta ns_t; \Delta nm_t].$$

We allow for extra control variables in the predictor set:

$$\boldsymbol{x_t} = [\boldsymbol{ratio_t}; \boldsymbol{control_t}],$$

so that the number of predictors $M > N$. The intercept of regression can be estimated by adding a column of all-one vector in $\boldsymbol{x_t}$. We propose the following class of predictive regressions:

$$\boldsymbol{r_{t+1}} = \boldsymbol{b_r x_t} + \boldsymbol{\epsilon^r_{t+1}}, \tag{B4}$$
$$\boldsymbol{\Delta cf_{t+1}} = \boldsymbol{b_{cf} x_t} + \boldsymbol{\epsilon^{cf}_{t+1}}, \tag{B5}$$
$$\boldsymbol{ratio_{t+1}} = \boldsymbol{\phi x_t} + \boldsymbol{\epsilon^{ratio}_{t+1}}. \tag{B6}$$

In addition, the A.B.C. in Eq. (A26) motivates a predictive relationship for the aggregate consumption-wealth dynamic:

$$cw_{t+1} = \boldsymbol{b_{cw} x_t} + \boldsymbol{\epsilon^{cw}_{t+1}}. \tag{B7}$$

The sector B.C.s imply a relationship between the coefficients:

$$\boldsymbol{C} = \mathrm{diag}(1-\rho^E, 1-\rho^B, 1-\rho^X)(\boldsymbol{b_r} - \boldsymbol{b_{cf}}) + \mathrm{diag}(\rho^E, \rho^B, \rho^X)\boldsymbol{\phi}, \tag{B8}$$

where the operator diag**()** denotes a diagonal matrix with its elements equal to the numbers in parentheses. Additionally, $\boldsymbol{C}$ is a constant matrix representing how the three valuation ratios are spanned by all predictors $\boldsymbol{x_t}$, such that:

$$\boldsymbol{ratio_t} = \boldsymbol{C}\ [\boldsymbol{ratio_t}; \boldsymbol{control_t}]. \tag{B9}$$

This relation indicates that:

$$\boldsymbol{C} = [\boldsymbol{I}(N),\ \boldsymbol{0}_{N\times(M-N)}],$$

where $\boldsymbol{I}(N)$ is an $N \times N$ identity matrix, and $\boldsymbol{0}_{N\times(M-N)}$ is an $N \times (M-N)$ zero matrix.

Furthermore, the A.B.C. implies the following relationship:

$$\boldsymbol{b_{cw}} = \boldsymbol{w'\phi}, \tag{B10}$$

where $\boldsymbol{w}$ represents the weights corresponding to each sector in the aggregate dynamic.

B2 A Constrained GMM Framework

We now introduce a framework to enforce the constraints Eq. (B8) and Eq. (B10) while estimating the regression system. The primary reference for techniques in this section can be found in chapters 8 and 13 of Hansen (2022).

To illustrate how to enforce constraints across equations, we use the equity sector, as specified in Eq. (B1), as an example. The goal is to stack the equations into a single regression model, such that:

$$
\begin{bmatrix} \underset{T\times1}{\boldsymbol{y_1}} (r_{t+1,\ t=1\dots T}) \\ \underset{T\times1}{\boldsymbol{y_2}} (\Delta d_{t+1,\ t=1\dots T}) \\ \underset{T\times1}{\boldsymbol{y_3}} (da_{t+1,\ t=1\dots T}) \end{bmatrix} = \begin{bmatrix} \underset{T\times2}{\boldsymbol{x_1}} & 0 & 0 \\ 0 & \underset{T\times2}{\boldsymbol{x_2}} & 0 \\ 0 & 0 & \underset{T\times2}{\boldsymbol{x_3}} \end{bmatrix} \begin{bmatrix} \underset{2\times1}{\boldsymbol{\beta_r}} \\ \underset{2\times1}{\boldsymbol{\beta_d}} \\ \underset{2\times1}{\boldsymbol{\beta_\phi}} \end{bmatrix} + \begin{bmatrix} \underset{T\times1}{\boldsymbol{\epsilon_1}} \\ \underset{T\times1}{\boldsymbol{\epsilon_2}} \\ \underset{T\times1}{\boldsymbol{\epsilon_3}} \end{bmatrix}. \tag{B11}
$$

We define:

$$
\underset{3T\times1}{\boldsymbol{Y}} = \begin{bmatrix} \underset{T\times1}{\boldsymbol{y_1}} (r_{t+1,\ t=1\dots T}) \\ \underset{T\times1}{\boldsymbol{y_2}} (\Delta d_{t+1,\ t=1\dots T}) \\ \underset{T\times1}{\boldsymbol{y_3}} (da_{t+1,\ t=1\dots T}) \end{bmatrix},
$$

which is a vector containing a vertical stack of the three time-series to be predicted. The matrix:

$$
\underset{3T\times(3\times2)}{\boldsymbol{X}} = \begin{bmatrix} \underset{T\times2}{\boldsymbol{x_1}} & 0 & 0 \\ 0 & \underset{T\times2}{\boldsymbol{x_2}} & 0 \\ 0 & 0 & \underset{T\times2}{\boldsymbol{x_3}} \end{bmatrix}
$$

contains the predictor $\boldsymbol{x_i}$ for each equation. Here, $\boldsymbol{x_i}$ is the vector containing a column of 1 and the time-series of dp_t for all equations. Similarly, $\boldsymbol{\beta}$ represents a vertical stack of $\boldsymbol{\beta_i}$ for each equation (including the intercept and slope), and $\boldsymbol{\epsilon}$ is the vertical stack of residuals for all equations.

It is straightforward to see that running an ordinary least squares (OLS) estimation using the stacked regression model in Eq. (B11) is equivalent to estimating the equations individually:

$$
\widehat{\boldsymbol{\beta}}_{\text{ols}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y} = \begin{bmatrix} (\boldsymbol{x}_1'\boldsymbol{x_1})^{-1}\boldsymbol{x}_1'\boldsymbol{y_1} \\ (\boldsymbol{x}_2'\boldsymbol{x_2})^{-1}\boldsymbol{x}_2'\boldsymbol{y_2} \\ (\boldsymbol{x}_3'\boldsymbol{x_3})^{-1}\boldsymbol{x}_3'\boldsymbol{y_3} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\beta_1} \\ \boldsymbol{\beta_2} \\ \boldsymbol{\beta_3} \end{bmatrix}.
$$

The variance of the OLS estimator, as derived in standard textbooks, depends on the sample moment conditions of this regression model:

$$
\hat{\boldsymbol{G}} = \boldsymbol{X}'\hat{\boldsymbol{\epsilon}} = \begin{bmatrix} \boldsymbol{x_1}'\hat{\boldsymbol{\epsilon}}_1 \\ \boldsymbol{x_2}'\hat{\boldsymbol{\epsilon}}_2 \\ \boldsymbol{x_3}'\hat{\boldsymbol{\epsilon}}_3 \end{bmatrix}.
$$

One can compute a variance estimator $\boldsymbol{VAR(\widehat{\beta}_{ols})}$ (the covariance matrix of $[\boldsymbol{x_1}'\boldsymbol{\epsilon_1}, \boldsymbol{x_2}'\boldsymbol{\epsilon_2}, \boldsymbol{x_3}'\boldsymbol{\epsilon_3}]$), incorporating heteroskedasticity and serial correlation, by using a Newey-West variance estimator of $\boldsymbol{G}$:

$$
\hat{\boldsymbol{\Omega}} = \boldsymbol{VAR(X'\hat{\epsilon})},
$$

and the variance of the OLS estimator is given by:

$$
\hat{\boldsymbol{V}}_\beta = \boldsymbol{VAR(\widehat{\beta}_{ols})} = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\hat{\boldsymbol{\Omega}}\left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}.
$$

It is important to note that a GMM estimator, which also uses $\hat{\boldsymbol{G}}$ and a weight matrix $\boldsymbol{W}$ of moments, is equivalent to the OLS estimator:

$$
\widehat{\boldsymbol{\beta}}_{\text{gmm}} = \left(\boldsymbol{X}'\boldsymbol{X}\boldsymbol{W}\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\left(\boldsymbol{X}'\boldsymbol{X}\boldsymbol{W}\boldsymbol{X}'\boldsymbol{Y}\right) = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y} = \widehat{\boldsymbol{\beta}}_{\text{ols}},
$$

because this is an exactly-identified linear model. Furthermore, the GMM estimator that uses the weight matrix

$$
\boldsymbol{W} = \hat{\boldsymbol{\Omega}}^{-1},
$$

is the feasible efficient estimator and has the same variance, $\hat{\boldsymbol{V}}_\beta$.

The derivations above represent classic textbook results, with the only addition being the inclusion of a multi-equation system. This system still remains an exactly-identified linear regression model, meaning that the weight matrix $\hat{\boldsymbol{\Omega}}$ in the efficient GMM estimator affects only the variance, not the estimator itself. Once we introduce constraints across the equations, the system becomes over-identified, making the estimation of $\hat{\boldsymbol{\Omega}}$ crucial for the performance of the constrained estimator. Therefore, we introduce a constrained GMM estimator that uses $\hat{\boldsymbol{\Omega}}$ to balance the covariance between the equations while simultaneously enforcing the constraints imposed across the equations. This allows the weight matrix to account for the relationships between the equations and the impact of the constraints, improving the overall efficiency of the estimator.

The constrained GMM estimator is defined as:

$$\widehat{\beta}_{\text{cgmm}} = \arg\min J(\boldsymbol{\beta}) = \frac{1}{T}\hat{\boldsymbol{g}}'\boldsymbol{W}\hat{\boldsymbol{g}}, \quad \text{s.t.} \quad \boldsymbol{R}'\widehat{\boldsymbol{\beta}} - \boldsymbol{c} = 0, \tag{B12}$$

where $\hat{\boldsymbol{g}}$ is the sample average of the moment conditions $\hat{\boldsymbol{G}} = \boldsymbol{X}'\hat{\boldsymbol{\epsilon}}$, and the constraints across the equations are summarized by the linear relationship $\boldsymbol{R}'\widehat{\boldsymbol{\beta}}_{\text{cgmm}} - \boldsymbol{c} = 0$. The constrained estimator can be solved in closed form as:

$$\widehat{\boldsymbol{\beta}}_{\text{cgmm}} = \widehat{\boldsymbol{\beta}}_{\text{ols}} - \left(\boldsymbol{X}'\boldsymbol{X}\boldsymbol{W}\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{R}\left(\boldsymbol{R}'\left(\boldsymbol{X}'\boldsymbol{X}\boldsymbol{W}\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{R}\right)^{-1}\left(\boldsymbol{R}'\widehat{\boldsymbol{\beta}}_{\text{ols}} - \boldsymbol{c}\right). \tag{B13}$$

Further, if we apply the efficient GMM weight matrix such that:

$$\boldsymbol{W} = \hat{\boldsymbol{\Omega}}^{-1} = \boldsymbol{VAR}(\boldsymbol{X}'\boldsymbol{\epsilon})^{-1},$$

and recall that:

$$\hat{\boldsymbol{V}}_{\beta} = \boldsymbol{VAR}(\widehat{\boldsymbol{\beta}}_{\boldsymbol{ols}}) = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\hat{\boldsymbol{\Omega}}\left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1},$$

then the constrained estimator simplifies to:

$$\widehat{\boldsymbol{\beta}}_{\text{cgmm}} = \widehat{\boldsymbol{\beta}}_{\text{ols}} - \hat{\boldsymbol{V}}_{\beta}\boldsymbol{R}\left(\boldsymbol{R}'\hat{\boldsymbol{V}}_{\beta}\boldsymbol{R}\right)^{-1}\left(\boldsymbol{R}'\widehat{\boldsymbol{\beta}}_{\text{ols}} - \boldsymbol{c}\right). \tag{B14}$$

The constrained estimator, by its construction, must satisfy the linear constraints embedded in the GMM, such that,

$$\boldsymbol{R}'\widehat{\boldsymbol{\beta}}_{\text{cgmm}} - \boldsymbol{c} = 0.$$

In contrast, the OLS estimator (unconstrained) is the optimal least squares estimator, minimizing the GMM objective function in Eq. (B12) to reach a value of zero. From the equation above, it is evident that this constrained GMM strikes a balance between the optimal least squares estimator and the imposed constraints. The second term in the equation,

$$\hat{\boldsymbol{V}}_{\beta}\boldsymbol{R}\left(\boldsymbol{R}'\hat{\boldsymbol{V}}_{\beta}\boldsymbol{R}\right)^{-1}\left(\boldsymbol{R}'\widehat{\boldsymbol{\beta}}_{\text{ols}} - \boldsymbol{c}\right),$$

adjusts the OLS estimator to ensure compliance with the constraints, while minimizing the cost in terms of efficiency and deviation from the optimal least squares estimator.

Moreover, this framework enables a test of the linear constraints by comparing the distance between the constrained and unconstrained estimators, as indicated by the objective function in Eq. (B12). Specifically, we have:

$$J(\boldsymbol{\beta}_{\text{cgmm}}) - J(\boldsymbol{\beta}_{\text{ols}}) \sim \chi^2(q), \tag{B15}$$

where q represents the number of enforced constraints.

With the $\boldsymbol{R}$ and $\boldsymbol{c}$ given, the constrained estimator in Eq. (B14) only depends on the estimation of

$$\hat{\boldsymbol{W}} = \hat{\boldsymbol{\Omega}}^{-1} = \boldsymbol{VAR}(\boldsymbol{X}'\hat{\boldsymbol{\epsilon}})^{-1}.$$

Similar to the case without constraints, the weight matrix is computed using an iterative-GMM approach. The process starts with an initial guess for the weight matrix, which is then used to estimate $\widehat{\boldsymbol{\beta}}_{\text{cgmm}}$. With this initial estimate of $\widehat{\boldsymbol{\beta}}_{\text{cgmm}}$, we calculate the residuals $\boldsymbol{\epsilon}$, and update the estimate of $\hat{\boldsymbol{\Omega}}$, and thus $\boldsymbol{W}$. The updated $\boldsymbol{W}$ is then used in the solution of the constrained estimator in Eq. (B14). This process is then repeated iteratively, updating $\boldsymbol{W}$ and re-estimating $\widehat{\boldsymbol{\beta}}_{\text{cgmm}}$, until $\widehat{\boldsymbol{\beta}}_{\text{cgmm}}$ converges to a stable value.

B3 B.Cs and A.B.C in the Constrained Estimation

In general, we can extend the three-equation system in Eq. (B1) to K multiple equations ($i = 1 \dots K$) as follows:

$$\underset{T\times 1}{\boldsymbol{y_i}} = \underset{T\times M}{\boldsymbol{x_i}} \underset{M\times 1}{\boldsymbol{\beta_i}} + \underset{T\times 1}{\boldsymbol{\epsilon_i}}, \; i = 1 \dots K.$$

We introduce constraints across each equation to establish a relationship among β_i. Each equation has M right-hand-side predictors and thus M moments. The system in matrix format becomes:

$$\underset{TK\times 1}{\boldsymbol{Y}} = \underset{TK\times MK}{\boldsymbol{X}} \underset{MK\times 1}{\boldsymbol{\beta}} + \underset{TK\times 1}{\boldsymbol{\epsilon}},$$

where $\boldsymbol{Y}$ is the vertical stack of the K left-hand-side variables, $\underset{T\times 1}{\boldsymbol{Y_i}}$, and the right-hand side is:

$$\underset{TK\times MK}{\boldsymbol{X}} = \text{diag}(\underset{T\times M}{\boldsymbol{x_i}}).$$

According to our predictive regression system, we set $K = 10$ to include ten variables in $\boldsymbol{Y}$, such that:

$$\boldsymbol{Y} = [\boldsymbol{r_{t+1}},\; t = 1 \dots T;\; \boldsymbol{\Delta cf_{t+1}},\; t = 1 \dots T;\; \boldsymbol{ratio_{t+1}},\; t = 1 \dots T;\; cw_{t+1},\; t = 1 \dots T],$$

where $\boldsymbol{Y}$ is a vertical stack of ten time-series, including returns, cash flow growth, ratios for the three sectors, and a consumption-wealth ratio to enforce the aggregate budget constraint (A.B.C) in Eq. (A26).

We further set predictors using a common set of time-series:

$$\boldsymbol{x_i} = [\boldsymbol{ratio_t},\; t = 1 \dots T; \boldsymbol{control_t},\; t = 1 \dots T],\; \forall i = 1 \dots K,$$

where we include three valuation ratios and other control variables, including a column of ones, as presented in the empirical results.

Now, we use the equity sector estimation in Eq. (B1) as an example to illustrate how to specify $\boldsymbol{R}$ and $\boldsymbol{c}$ for the sector budget constraint (B.C.). The predictor for three equations is a $T \times M$ matrix, with the coefficients for each equation defined as a $M \times 1$ vector. Specifically, the budget constraint (B.C.) for the equity sector in Eq. (A12) implies that:

$$(1-\rho^E)\boldsymbol{\beta(r^E)}_{M\times 1} - (1-\rho^E)\boldsymbol{\beta(\Delta d)}_{M\times 1} + \rho^E \boldsymbol{\beta(dp)}_{M\times 1} = [1; \boldsymbol{0}_{(M-1)\times 1}].$$

Here, only the first element in the constant matrix equals one since it maps to the first predictor in $\boldsymbol{x}$, which is dp.

Since we vertically stack $\boldsymbol{\beta(r^E)}_{M\times 1}$, $\boldsymbol{\beta(\Delta d)}_{M\times 1}$, and $\boldsymbol{\beta(dp)}_{M\times 1}$ into a $3M\times 1$ vector, a linear relationship that presents the constraint in the format of $\boldsymbol{R'}\widehat{\boldsymbol{\beta}} = \boldsymbol{c}$ is given by:

$$\begin{bmatrix} (1-\rho^E)\boldsymbol{I}(M) & -(1-\rho^E)\boldsymbol{I}(M) & \rho^E \boldsymbol{I}(M) \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta(r^E)}_{M\times 1} \\ \boldsymbol{\beta(\Delta d)}_{M\times 1} \\ \boldsymbol{\beta(dp)}_{M\times 1} \end{bmatrix} = [1; \boldsymbol{0}_{(M-1)\times 1}]. \tag{B16}$$

Extending this to the full system, where we have $K = 10$ equations, the coefficients are vertically stacked as:

$$\boldsymbol{\beta}_{10M\times 1} = [\boldsymbol{\beta(r^E)}; \boldsymbol{\beta(r^B)}; \boldsymbol{\beta(r^X)}; \boldsymbol{\beta(\Delta d)}; \boldsymbol{\beta(\Delta ns)}; \boldsymbol{\beta(\Delta nm)}; \boldsymbol{\beta(dp)}; \boldsymbol{\beta(nsb)}; \boldsymbol{\beta(nma)}; \boldsymbol{\beta(cw)}].$$

Since we have M right-hand-side variables, a constraint similar to the equity sector in Eq. (A12) requires M equations to hold in the system. A linear relation to present the budget constraint of the equity sector in Eq. (A12) can be written as:

$$\boldsymbol{R_{BC1}}'\boldsymbol{\beta} = \boldsymbol{c_{BC1}},$$

with

$$\boldsymbol{R_{BC1}}' = \left[(1-\rho^E)\boldsymbol{I}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), -(1-\rho^E)\boldsymbol{I}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), \rho^E\boldsymbol{I}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), \boldsymbol{0}(M)\right],$$

and, $\boldsymbol{c_{BC1}} = [1; \boldsymbol{0}_{(M-1)\times 1}]$. Similarly, the budget constraint for the public sector in Eq. (C12) can be expressed as:

$$\boldsymbol{R_{BC2}}'\boldsymbol{\beta} = \boldsymbol{c_{BC2}},$$

with

$$\boldsymbol{R_{BC2}}' = \left[\boldsymbol{0}(M), (1-\rho^B)\boldsymbol{I}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), -(1-\rho^B)\boldsymbol{I}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), \rho^B\boldsymbol{I}(M), \boldsymbol{0}(M), \boldsymbol{0}(M)\right],$$

and $\boldsymbol{c_{BC2}} = [0; 1; \boldsymbol{0}_{(M-2)\times 1}]$. The budget constraint for the external sector in Eq. (A15) is given by:

$$\boldsymbol{R_{BC3}}'\boldsymbol{\beta} = \boldsymbol{c_{BC3}},$$

with

$$\boldsymbol{R_{BC3}}' = \left[\boldsymbol{0}(M), \boldsymbol{0}(M), (1-\rho^X)\boldsymbol{I}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), -(1-\rho^X)\boldsymbol{I}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), \rho^X\boldsymbol{I}(M), \boldsymbol{0}(M)\right],$$

and $\boldsymbol{c_{BC3}} = [0; 0; 1; \boldsymbol{0}_{(M-3)\times 1}]$. In addition, the aggregate budget constraint in Eq. (A26) can be expressed as:

$$cw_t = \alpha_{cw} + w_1 dp_t + w_2 nsb_t + w_3 nma_t,$$

which translates to: $\boldsymbol{R_{ABC}}'\boldsymbol{\beta} = \boldsymbol{c_{ABC}}$, with

$$\boldsymbol{R_{ABC}}' = \left[\boldsymbol{0}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), \boldsymbol{0}(M), -w_1\boldsymbol{I}(M), -w_2\boldsymbol{I}(M), -w_3\boldsymbol{I}(M), \boldsymbol{I}(M)\right], \ \boldsymbol{c_{ABC}} = \boldsymbol{0}_{M\times 1}.$$

The constraints for the entire system are thus consolidated as:

$$\begin{bmatrix} \boldsymbol{R_{BC1}}' \\ \boldsymbol{R_{BC2}}' \\ \boldsymbol{R_{BC3}}' \\ \boldsymbol{R_{ABC}}' \end{bmatrix} \boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{c_{BC1}} \\ \boldsymbol{c_{BC2}} \\ \boldsymbol{c_{BC3}} \\ \boldsymbol{c_{ABC}} \end{bmatrix}. \tag{B17}$$

With the constraints specified, applying the general solution in Eq. (B14) and an iterative-GMM approach allows us to estimate the entire predictive regression system. However, due to the relatively high-dimensional nature of this system, estimating the covariance matrix of the moments $\hat{\boldsymbol{\Omega}}$ could lead to numerical instability in the estimation process. Specifically, each left-hand side variable is predicted by M common variables, resulting in $K \times M$ moments. To address the challenges posed by high dimensionality, we simplify the assumptions regarding the covariances between the equations, reducing the number of covariance parameters that need to be estimated in the iterative GMM process. We set unnecessary covariance terms to zero, retaining only those terms that are economically meaningful.

First, we retain the covariance of moments within each equation, as this portion of the covariance is essential for the separate estimation of each equation. When conducting a joint estimation of all equations, we require the co-movement between them to achieve efficiency and better enforce constraints.

For moments across equations, we keep those associated with the same type of variables (returns, cash flows, ratios) to capture their co-movement across different sectors. Intuitively, this can be supported by common cyclical factors affecting asset returns, cash flows, and ratios. Conversely, we specify that moments from different variables of any two sectors, such as equity returns and government surplus, are uncorrelated.

Furthermore, in relation to the budget constraints enforced in each sector, we maintain the covariance of moments related to returns and ratios within the same sector. This approach enhances our ability to efficiently enforce budget constraints. Lastly, based on the aggregate budget constraint, we retain the covariance of moments related to the three ratios and the consumption-wealth ratio. This facilitates a more efficient enforcement of the aggregate budget constraint.

In summary, due to the nature of our framework, we require a precise estimation of the covariance of moments $\hat{\boldsymbol{\Omega}}$, which is high-dimensional and likely sparse, with many of its elements being close to zero. Therefore, we employ an economically meaningful truncation estimator to filter out these close-to-zero components (see, for example, the overview by Fan et al. (2016)). Additionally, we apply a method from Higham (1988) that utilizes singular value decomposition to ensure the positive definiteness of $\hat{\boldsymbol{\Omega}}$ during the iterative GMM process. Overall, our specification of the covariance $\hat{\boldsymbol{\Omega}}$ aims to achieve a parsimonious and stable implementation of the constrained GMM estimation.

B4 Monte Carlo simulation and benefits of constrained GMM

With the constrained GMM framework established, a natural question arises: what are the advantages of using joint estimation to enforce constraints across equations compared to separate estimation using OLS? We contend that the benefits can be summarized in two key aspects:

1. Under ideal conditions, where the constraints hold over a sufficiently long sample period, the constrained estimator demonstrates greater efficiency, resulting in lower standard errors.
2. In cases where the constraints are mis-specified, leading to random deviations in the data and consequently biased OLS estimators, enforcing the constraints can effectively mitigate this bias.

We utilize simulation results to verify these arguments and provide theoretical intuitions to support our findings.

Now, let's introduce the simulation algorithm based on our framework. For each sector, we start with $t = 1$, setting the initial value equal to the first observation in the data and simulating the time series in a loop:

$$r^i_{t+1} = [ratio^{i=1\ldots N}_t; control_t]\boldsymbol{b^r_i} + \epsilon(r)^i_{t+1},$$

$$\Delta cf^i_{t+1} = [ratio^{i=1\ldots N}_t; control_t]\boldsymbol{b^{cf}_i} + \epsilon(cf)^i_{t+1}.$$

The shock to the ratios is implied from the aggregate budget constraint (ABC):

$$\rho^i\epsilon(ratio)^i_{t+1} = (1-\rho^i)\epsilon(\Delta cf)^i_{t+1} - (1-\rho^i)\epsilon(r)^i_{t+1}. \tag{B18}$$

The ratios are also implied to follow the sector budget constraints, such that:

$$ratio^i_{t+1} = [ratio^{i=1\ldots N}_t; control_t]\boldsymbol{\phi_i} + \epsilon(ratio)^i_{t+1}.$$

The loop then repeats from $t = 1$ to $t = T$. The algorithm above ensures that the budget constraint holds perfectly for each sector:

$$(1-\rho^i)r^i_{t+1} = (1-\rho^i)\Delta cf^i_{t+1} + ratio^i_t - \rho^i ratio^i_{t+1}.$$

The aggregate budget constraint is then expressed as:

$$cw_{t+1} = \boldsymbol{w_2}'\boldsymbol{ratio_{t+1}}. \tag{B19}$$

For simplicity, we set the covariance of shocks in each equation to follow i.i.d normal distribution. We utilize the estimates obtained from the multivariate OLS as the null parameters for generating simulation samples. Without loss of generality, we adjust ϕ_i for all i downward to eliminate potential outliers associated with values of ϕ that are close to one. Consequently, we retain $\boldsymbol{b^r_i}$ unchanged and modify $\boldsymbol{b^{cf}_i}$ to ensure that the accounting identity is maintained for each sector.

This simulation demonstrates an ideal case where the constraints hold without randomness, meaning that knowing any two out of the three key variables (return, cash flow, ratio) of a sector implies the third. Additionally, the OLS estimator must satisfy the linear constraints:

$$\boldsymbol{R}'\widehat{\boldsymbol{\beta}}_{\text{ols}} - \boldsymbol{c} = 0.$$

Thus, according to the solution in Eq. (B14), $\widehat{\boldsymbol{\beta}}_{\text{cgmm}} = \widehat{\boldsymbol{\beta}}_{\text{ols}}$. Under these ideal conditions, one could estimate the system equation by equation. Moreover, the constraints reduce the dimensionality of the system. For example, Cochrane (2008) estimates coefficients only for r^E_{t+1} and dp_{t+1}, using the budget constraints to imply the coefficients for Δd_{t+1}.

Building on our observations regarding the ideal conditions, we aim to further investigate the impact of random deviations from the budget constraints on the estimation process. This exploration will deepen our understanding of the asymptotic properties of constrained and unconstrained estimators. To generate deviations from the budget constraints, we introduce a random mean-zero noise to the ratios $ratio^{i=1\dots N}$ and the consumption-wealth ratio cw, defined as follows:

$$\widetilde{ratio}^i_{t+1} = ratio^i_{t+1} + u^i_{t+1},$$
$$\widetilde{cw}_{t+1} = cw_{t+1} + u^{cw}_{t+1},$$

where u^i_{t+1} and u^{cw}_{t+1} are i.i.d. noise terms that introduce deviations from the original ratios and consumption-wealth ratio. This adjustment allows us to analyze how these random perturbations affect the performance of the estimators and their alignment with the imposed constraints.

We generate 10,000 simulated samples to estimate the predictive regression systems, setting the length of each sample to match our quarterly data length ($T = 280$). This allows us to compare the small-sample properties of the OLS and constrained-GMM estimators. Figure B.3 presents the four coefficients of primary interest: the coefficients of dp_t predicting three variables in the equity sector—$r^E(dp)$, $\Delta d(dp)$, and $dp(dp)$—along with our core finding, the coefficients of nsb predicting equity returns, $r^E(nsb)$. The dashed lines in the figure represent the null parameters used in the simulations. We compare three estimators: OLS, constrained GMM with sector budget constraints (BC), and constrained GMM with sector budget constraints and the aggregate budget constraint (BC+ABC). The figure shows the average values and confidence intervals ($+/-2$ standard deviations) of these three estimators across 10,000 simulations, allowing for a detailed comparison of their performance.

From this figure, we draw two consistent conclusions as previously discussed. First, the constrained estimator has narrower confidence intervals, demonstrating higher efficiency. Additionally, because we introduced extra noise into the predictors in the simulations, the OLS Estimator shows bias. Specifically, since the OLS coefficients for using X to explain Y follow the format of $Cov(X,Y)/Var(X)$, the added volatility in $\widetilde{ratio}^i_{t+1}$ drives the absolute value of the coefficients downward, leading to bias. Interestingly, the constrained estimation mitigates this bias, yielding results closer to the null parameters.

We now discuss the intuition behind these two observations. Taking the equity sector as an example, separate estimation requires three left-hand-side variables to estimate the coefficients in three equations. However, by incorporating the budget constraint in the joint estimation, one set of coefficients becomes redundant. As a result, the estimator effectively uses the three left-hand-side variables to estimate only two sets of coefficients, leading to a gain in efficiency. To make this intuition more concrete, we compute the variance of $\widehat{\boldsymbol{\beta}}_{\text{cgmm}}$ as follows:

$$\boldsymbol{V}_{\text{cgmm}} = \boldsymbol{V}_{\beta} - \boldsymbol{V}_{\beta}\boldsymbol{R}\left(\boldsymbol{R}'\boldsymbol{V}_{\beta}\boldsymbol{R}\right)^{-1}\boldsymbol{R}'\boldsymbol{V}_{\beta}. \tag{B20}$$

It is straightforward to see that $\boldsymbol{V}_{\text{cgmm}}$ is smaller than $\boldsymbol{V}_{\beta}$ overall, as the difference in variance between the two estimators,

$$\boldsymbol{V}_{\beta} - \boldsymbol{V}_{\text{cgmm}} = \boldsymbol{V}_{\beta}\boldsymbol{R}\left(\boldsymbol{R}'\boldsymbol{V}_{\beta}\boldsymbol{R}\right)^{-1}\boldsymbol{R}'\boldsymbol{V}_{\beta},$$

must be a positive-definite matrix. On the other hand, why can the constrained estimator help reduce bias? We argue that, suppose one can specify the correct covariance structure of shocks in the equation system, then enforcing constraints helps identify mis-specification, e.g., perturbations added to valuation ratios in our simulations, and hence reduce the associated bias.

We further extend our simulation algorithm to generate longer time series in order to validate our findings in an asymptotic setting. Figure B.4 shows the behavior of our key focus—the coefficients of nsb predicting equity returns, $r^E(nsb)$—across 10,000 simulated samples. As the length of the simulated time series increases, the variance of all estimators decreases. More importantly, for any given sample size, the constrained estimator consistently outperforms OLS by demonstrating smaller estimation errors and providing estimates that are closer to the null parameters.

One limitation of the simulation analysis above is that the variance estimator depends on the sample estimate of the covariance matrix $\hat{\mathbf{\Omega}}$, which can partially diminish the efficiency of the constrained estimator in practice. Consequently, the actual benefits of the constrained estimator rely on specific implications, as documented in our empirical results.

Appendix C Derivations in the Theoretical Model

For tractability, assume i.i.d. growth in raw output as in Jiang et al. (2024a):

$$\frac{Y_{t+1}^r}{Y_t^r} = \exp\left(g - \frac{\sigma^2}{2} + \sigma\varepsilon_{t+1}\right). \tag{C1}$$

C1 Government's Optimal Tax and Spending

The government takes $\{Y_t^r\}$ and the pricing kernel as given and chooses tax and spending shares to maximize the representative household's indirect utility

$$F_0 = \max_{\{C_t\}} \mathbb{E}_0\Big[\sum_{t=0}^{\infty} e^{-\beta t} U(C_t)\Big].$$

In this setup, maximizing F_0 is equivalent to maximizing household wealth,

$$W_0 = \mathbb{E}_0\Big[\sum_{s=1}^{\infty} M_s C_s\Big] = \mathbb{E}_0\Big[\sum_{s=1}^{\infty} M_s\big(Y_s^r - Y_s^r\theta(TY_s^r, GY_s^r) - Y_s^r GY_s^r\big)\Big], \tag{C2}$$

where $T_t = TY_t^r \times Y_t^r$ and $G_t = GY_t^r \times Y_t^r$.

The quadratic distortion function in Eq. (21) is

$$\theta_\tau(TY^r) = \frac{1}{2}c^\tau\left(\frac{T}{Y^r}\right)^2, \qquad \theta_g(GY^r) = \frac{1}{2}c^g\left(\frac{G}{Y^r}\right)^2,$$

so that

$$\theta(TY^r, GY^r) = \frac{c^\tau}{2}(TY^r)^2 - \frac{c^g}{2}(GY^r)^2,$$

and

$$\frac{\partial\theta}{\partial TY^r} = c^\tau TY^r, \qquad \frac{\partial\theta}{\partial GY^r} = -c^g GY^r. \tag{C3}$$

As shown below, $-c^\tau$ and c^g map into the output effects (multipliers) of taxes and spending.

The government chooses $\{TY_{t+1}^r, GY_{t+1}^r\}$ subject to the intertemporal budget constraint

$$B_0 \leq \mathbb{E}_0\Big[\sum_{s=1}^{\infty} \frac{M_s}{M_0}(T_s - G_s)\Big], \tag{C4}$$

with debt dynamics

$$B_{t+1} = B_t(1 + R_{t+1}^B) - T_{t+1} + G_{t+1}.$$

The government treats debt valuation as exogenous (hence R_{t+1}^B adjusts with the value of debt).

We assume the government maximizes Eq. (C2) augmented by a quadratic "private benefit" term, $\frac{s^\tau}{2}Y_{t+1}^r(TY_{t+1}^r)^2$, which captures non-benevolent motives (e.g., private benefits/credibility; Barro, 1973). Relative to a standard tax-smoothing setup with exogenous spending, this term helps match empirical magnitudes of tax/spending effects on output and induces a joint determination of optimal taxes and spending.

The Bellman equation is

$$P(B_t, Y_t^r) = \max_{TY_{t+1}^r, GY_{t+1}^r} \mathbb{E}_t\Big[m_{t+1}\Big(Y_{t+1}^r - Y_{t+1}^r\theta(TY_{t+1}^r, GY_{t+1}^r) - Y_{t+1}^r GY_{t+1}^r + \frac{s^\tau}{2} Y_{t+1}^r (TY_{t+1}^r)^2 + P(B_{t+1}, Y_{t+1}^r)\Big)\Big]. \tag{C5}$$

With i.i.d. Y^r we can use guess-and-verify to obtain a steady state with constant TY^r and GY^r and a stable debt-to-output ratio. The optimized value function is homogeneous: $P^*(B, Y^r) = p^*(B/Y^r)Y^r$, implying a constant marginal value of debt,

$$P_B^* = \frac{\partial P^*}{\partial B} = p'(B/Y^r) = \text{constant}.$$

Optimal fiscal shares satisfy the (steady-state) first-order conditions

$$-1 - \frac{\partial \theta}{\partial GY^r} = -p'(B/Y^r), \qquad -\frac{\partial \theta}{\partial TY^r} + s^\tau TY^r = p'(B/Y^r).$$

As a parsimonious baseline we set $s^\tau = 2c^\tau$ and focus on calibrating c^τ and c^g (the fiscal multipliers). The main prediction—negative predictability of the surplus-to-debt ratio for equity returns—is not sensitive to s^τ, and this baseline yields empirically reasonable multipliers (Blanchard and Perotti, 2002; Romer and Romer, 2010; Christiano et al., 2011; Auerbach and Gorodnichenko, 2012; Ilzetzki et al., 2013; Ramey and Zubairy, 2018). Using Eq. (C3), the FOCs imply

$$c^\tau TY^r + c^g GY^r = 1.$$

In addition, under i.i.d. Y^r, the steady-state surplus-to-output ratio is pinned down by initial debt:

$$B_0/Y_0^r \propto TY^r - GY^r.$$

Together, these conditions determine the optimal fiscal policy.

When output distortions and private benefits are shut down ($s^\tau = c^\tau = c^g = 0$), the government optimally sets $G = 0$ (since spending does not raise output here) and fiscal policy becomes neutral for consumption-wealth dynamics:

$$W_0 = \mathbb{E}_0\Big[\sum_{s=1}^{\infty} M_s C_s\Big] = \mathbb{E}_0\Big[\sum_{s=1}^{\infty} M_s Y_s^r\Big], \qquad P^*(B, Y^r) \propto Y^r.$$

The government is then indifferent among all tax/spending paths satisfying Eq. (C4), restoring Ricardian equivalence.

C2 Asset Valuation

Fiscal policy determines household consumption and after-tax dividends:

$$C_t = Y_t - G_t = Y_t^r\big(1 - \theta(TY_t^r, GY_t^r)\big) - G_t = Y_t^r - Y_t^r\theta(TY_t^r, GY_t^r) - Y_t^r GY_t^r, \tag{C6}$$

$$D_t = Y_t - T_t = Y_t^r\big(1 - \theta(TY_t^r, GY_t^r)\big) - T_t = Y_t^r - Y_t^r\theta(TY_t^r, GY_t^r) - Y_t^r TY_t^r. \tag{C7}$$

With constant TY^r and GY^r, all cash flows are proportional to Y_t^r and share the same growth rate:

$$\frac{NS_{t+1}}{NS_t} = \frac{D_{t+1}}{D_t} = \frac{C_{t+1}}{C_t} = \frac{Y_{t+1}^r}{Y_t^r} = \exp\left(g - \frac{\sigma^2}{2} + \sigma\varepsilon_{t+1}\right).$$

The risk-free rate under the CRRA pricing kernel is

$$r = s - \log \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right] = s + \gamma g - \frac{\gamma(\gamma+1)}{2}\sigma^2. \tag{C8}$$

Dividends can be written as $D_t = Y_t^r(1-\theta-TY^r)$. Define $\lambda = \gamma\sigma^2$ and $\delta = r+\lambda-g$. Then

$$E_t\left[\frac{M_{t+1}}{M_t}Y_{t+1}^r\right] = Y_t^r e^{-\delta}, \qquad E_t\left[\frac{M_{t+k}}{M_t}Y_{t+k}^r\right] = Y_t^r e^{-k\delta},\ \forall k. \tag{C9}$$

Equity value and return are therefore

$$A_t = \mathbb{E}_t\left[\sum_{u=t+1}^{\infty}\frac{M_u}{M_t}(1-\theta-TY^r)\,Y_u^r\right] = \frac{e^{-\delta}}{1-e^{-\delta}}D_t, \tag{C10}$$

$$R_{t+1}^E = \frac{A_{t+1}+D_{t+1}}{A_t} = \exp\left(r+\lambda-\frac{\sigma^2}{2}+\sigma\varepsilon_{t+1}\right), \qquad \mathbb{E}_t\left[R_{t+1}^E\right] = e^{r+\lambda}, \tag{C11}$$

with risk premium $\lambda = \gamma\sigma^2$. Under i.i.d. growth,

$$dp_t = \delta = r+\lambda-g, \quad \rho^E = e^{-\delta}, \quad \mathbb{E}_t r_{t+1}^E = r+\lambda, \quad \mathbb{E}_t\Delta d_{t+1} = g.$$

The log-linearized budget constraint (as used in the main text) reduces to

$$(1-\rho^E)\mathbb{E}_t r_{t+1}^E - (1-\rho^E)\mathbb{E}_t\Delta d_{t+1} = (1-e^{-\delta})\delta = dp_t - \rho^E\mathbb{E}_t dp_{t+1}.$$

Benefiting from the constant-growth setting, we use a common discount factor throughout:

$$\rho = \rho^E = \rho^B = e^{-\delta}.$$

Similarly,

$$\frac{NS_t}{B_t} = \frac{1-e^{-\delta}}{e^{-\delta}} = \frac{1-\rho}{\rho}. \tag{C12}$$

The public sector also satisfies

$$(1-\rho^B)\mathbb{E}_t r_{t+1}^B - (1-\rho^B)\mathbb{E}_t\Delta ns_{t+1} = (1-e^{-\delta})\delta = nsb_t - \rho^B\mathbb{E}_t nsb_{t+1}.$$

C3 Response of Cash Flows to Fiscal Shocks

Given static within-sector risk-return relations, we adopt a comparative-static exercise (as in Barro, 1979). Consider a small, temporary deviation in expected fiscal shares:

$$\tilde{TY}_{t+1}^r = TY_{t+1}^r e^{u_{t+1}}, \qquad \tilde{GY}_{t+1}^r = GY_{t+1}^r e^{u_{t+1}^g}, \qquad u_{t+1}^g = \alpha u_{t+1}. \tag{C13}$$

Then

$$\Delta\tilde{t}_{t+1} - \Delta t_{t+1} = u_{t+1}, \qquad \Delta\tilde{g}_{t+1} - \Delta g_{t+1} = \alpha u_{t+1}.$$

For any variable x, define its (semi-elasticity) response coefficient

$$U_x = \frac{\tilde{x}_{t+1} - x_{t+1}}{u_{t+1}}. \tag{C14}$$

Using the public-sector log-linearization,

$$\frac{TY^r}{TY^r - GY^r} = \frac{T}{T-G} = \frac{1}{1-\beta}, \qquad \Delta ns_{t+1} = \frac{1}{1-\beta}\Delta t_{t+1} - \frac{\beta}{1-\beta}\Delta g_{t+1},$$

we obtain

$$\Delta\tilde{ns}_{t+1} - \Delta ns_{t+1} = U_{\Delta ns}u_{t+1} = \frac{1-\alpha\beta}{1-\beta}u_{t+1}.$$

For dividends, a first-order expansion of Eq. (C7) implies

$$\tilde{D}_{t+1} = D_{t+1} - \frac{\partial\theta}{\partial TY^r}(e^{u_{t+1}} - 1)Y^r_{t+1} - \frac{\partial\theta}{\partial GY^r}(e^{\alpha u_{t+1}} - 1)Y^r_{t+1} - (e^{u_{t+1}} - 1)TY^r Y^r_{t+1}.$$

Using $(e^\epsilon - 1) \approx \epsilon$ and $1 - \epsilon \approx e^{-\epsilon}$ for small ϵ gives

$$\tilde{D}_{t+1} \approx D_{t+1} \exp\left[\left(-\frac{T}{D} - \frac{\partial\theta}{\partial TY^r}\frac{Y^r}{D} - \alpha\frac{\partial\theta}{\partial GY^r}\frac{Y^r}{D}\right)u_{t+1}\right].$$

With Eq. (C3), the dividend-growth response is

$$\Delta\tilde{d}_{t+1} - \Delta d_{t+1} = U_{\Delta d}u_{t+1} = \left(-\frac{T}{D} - c^\tau\frac{T}{D} + \alpha c^g\frac{G}{D}\right)u_{t+1}. \tag{C15}$$

Analogously, from Eq. (C6) the consumption-growth response is

$$\Delta\tilde{c}_{t+1} - \Delta c_{t+1} = U_{\Delta c}u_{t+1} = \left(-\alpha\frac{G}{C} - c^\tau\frac{T}{C} + \alpha c^g\frac{G}{C}\right)u_{t+1}. \tag{C16}$$

C4 Response of Prices to Fiscal Shocks and Return Predictability

Let u_{t+1} have mean μ_u and variance σ_u^2. Since

$$r = s - \log\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right],$$

the shock-induced change in the risk-free rate is

$$\tilde{r} - r = \gamma U_{\Delta c}\mu_u + \frac{1}{2}(\gamma U_{\Delta c})^2\sigma_u^2.$$

Now suppose the fiscal shock persists for K periods, with $\{u_{t+k}\}_{k=1}^K$ i.i.d. The shock affects the bond value through the first K cash flows:

$$\tilde{B}_t = B_t + \mathbb{E}_t\sum_{k=1}^{K}\left[\frac{M_{t+k}}{M_t}NS_{t+k}\left(e^{(-\gamma U_{\Delta c}+U_{\Delta ns})u_{t+k}} - 1\right)\right].$$

Using Eq. (C9) and Eq. (C12), this simplifies to

$$\tilde{B}_t = B_t\left(1 + (1-\rho^K)\,\mathbb{E}_t\left[e^{(-\gamma U_{\Delta c}+U_{\Delta ns})u_{t+1}} - 1\right]\right) \approx B_t\left(1 + (1-\rho^K)\log\mathbb{E}_t\left[e^{(-\gamma U_{\Delta c}+U_{\Delta ns})u_{t+1}}\right]\right),$$

hence

$$\tilde{b}_t = b_t + (1-\rho^K)\log\mathbb{E}_t\left[e^{(-\gamma U_{\Delta c}+U_{\Delta ns})u_{t+1}}\right].$$

To obtain return responses, use the (log-linearized) budget constraints:

$$\mathbb{E}_t r^B_{t+1} = \mathbb{E}_t\Delta ns_{t+1} + \frac{1}{1-\rho}nsb_t - \frac{\rho}{1-\rho}\mathbb{E}_t nsb_{t+1}, \qquad \mathbb{E}_t r^E_{t+1} = \mathbb{E}_t\Delta d_{t+1} + \frac{1}{1-\rho}da_t - \frac{\rho}{1-\rho}\mathbb{E}_t da_{t+1}.$$

First,

$$\mathbb{E}_t\left(\Delta\tilde{n}s_{t+1} - \Delta ns_{t+1}\right) = \log\mathbb{E}_t\left[e^{U_{\Delta ns}u_{t+1}}\right] = U_{\Delta ns}\mu_u + \frac{1}{2}(U_{\Delta ns})^2\sigma_u^2.$$

With $nsb_t \approx \kappa + (1-\rho)(\log ns_t - \log b_t)$, we have

$$\frac{1}{1-\rho}(\tilde{nsb}_t - nsb_t) = -(\tilde{b}_t - b_t) = -(1-\rho^K)\log\mathbb{E}_t\left[e^{(-\gamma U_{\Delta c}+U_{\Delta ns})u_{t+1}}\right],$$

and

$$\frac{\rho}{1-\rho}\mathbb{E}_t(\tilde{nsb}_{t+1} - nsb_{t+1}) = \rho\,\mathbb{E}_t(\Delta\tilde{ns}_{t+1} - \Delta ns_{t+1}) - (\rho - \rho^K)\log\mathbb{E}_t\Big[e^{(-\gamma U_{\Delta c} + U_{\Delta ns})u_{t+1}}\Big].$$

Accounting for the remaining $K-1$ periods of shocks implies

$$\mathbb{E}_t(\tilde{nsb}_{t+1} - nsb_{t+1}) = (1-\rho)\mathbb{E}_t(\Delta\tilde{ns}_{t+1} - \Delta ns_{t+1}) + \frac{1-\rho^{K-1}}{1-\rho^K}(\tilde{nsb}_t - nsb_t).$$

Reorganizing yields the expected return responses:

$$\mathbb{E}_t(\tilde{r}^B_{t+1} - r^B_{t+1}) = (1-\rho)(\tilde{r} - r) + (1-\rho)\gamma U_{\Delta c}U_{\Delta ns}\sigma_u^2,$$

$$\mathbb{E}_t(\tilde{r}^E_{t+1} - r^E_{t+1}) = (1-\rho)(\tilde{r} - r) + (1-\rho)\gamma U_{\Delta c}U_{\Delta d}\sigma_u^2.$$

To focus on the risk-premium channel (consistent with evidence that expected returns are more predictable than cash-flow growth), we normalize the cash-flow mean effect to zero:

$$\mathbb{E}_t(\Delta\tilde{ns}_{t+1} - \Delta ns_{t+1}) = U_{\Delta ns}\mu_u + \frac{1}{2}(U_{\Delta ns})^2\sigma_u^2 = 0 \quad\Rightarrow\quad \mu_u = -\frac{1}{2}U_{\Delta ns}\sigma_u^2.$$

Under this normalization, the induced variation in $\tilde{r} - r$ is small (and equals zero when $\gamma U_{\Delta c} = U_{\Delta ns}$). Retaining only the risk-premium channel gives

$$\mathbb{E}_t(\tilde{r}^E_{t+1} - r^E_{t+1}) = (1-\rho)\gamma U_{\Delta c}U_{\Delta d}\sigma_u^2, \qquad \tilde{nsb}_t - nsb_t = (1-\rho)(1-\rho^K)\gamma U_{\Delta c}U_{\Delta ns}\sigma_u^2.$$

Hence the single-period predictive coefficient is

$$b(nsb_t, r^E_{t+1}) = \frac{\mathbb{E}_t(\tilde{r}^E_{t+1} - r^E_{t+1})}{\tilde{nsb}_t - nsb_t} = \frac{1}{1-\rho^K}\cdot\frac{U_{\Delta d}}{U_{\Delta ns}}.$$

Since shocks are i.i.d. over the K-period episode, the long-horizon change in expected returns is

$$\mathbb{E}_t\left(\sum_{k=1}^{K}\rho^{k-1}\tilde{r}^E_{t+k} - \sum_{k=1}^{K}\rho^{k-1}r^E_{t+k}\right) = (1-\rho^K)\gamma U_{\Delta c}U_{\Delta d}\sigma_u^2,$$

so the long-horizon predictive coefficient equals

$$\frac{1-\rho^K}{1-\rho}\cdot b(nsb_t, r^E_{t+1}).$$

Finally, under a $(K-1)$-period shock episode starting at $t+1$,

$$\tilde{nsb}_{t+1} - nsb_{t+1} = (1-\rho)(1-\rho^{K-1})\gamma U_{\Delta c}U_{\Delta ns}\sigma_u^2 \quad\Rightarrow\quad \mathbb{E}_t(\tilde{nsb}_{t+1} - nsb_{t+1}) = \frac{1-\rho^{K-1}}{1-\rho^K}(\tilde{nsb}_t - nsb_t).$$

These expressions complete the proof of the return-predictability proposition in the main text. As a useful benchmark, when $c^\tau = c^g = 0$ and $\alpha = 0$, we have $U_{\Delta c} = 0$ and fiscal shocks do not affect risk pricing (Ricardian equivalence).

C5 Calibration of Output Distortion

We compute steady-state ratios using sample averages and set $\beta = \rho = 0.999$ (per the public-sector present-value identity). Since

$$c^\tau TY^r + c^g GY^r = (c^\tau + c^g\beta)\,TY^r = 1,$$

combining this restriction with Eq. (26) yields

$$c^{\tau} = -\frac{1}{1+\alpha} - b(nsb_t, r^E_{t+1})\frac{(1-\rho^K)(1-\alpha\beta)}{(1-\beta)(1+\alpha)}\frac{D}{T} + \frac{\alpha}{1+\alpha}(TY^r)^{-1}, \tag{C17}$$

$$c^{g} = \left(\frac{1}{1+\alpha}(TY^r)^{-1} + \frac{1}{1+\alpha} + b(nsb_t, r^E_{t+1})\frac{(1-\rho^K)(1-\alpha\beta)}{(1-\beta)(1+\alpha)}\frac{D}{T}\right)\Big/\beta. \tag{C18}$$

Since T/Y is observable but Y^r is not, write

$$TY^r = \frac{T}{Y}\cdot\frac{Y}{Y^r},$$

and solve for Y/Y^r from the net-distortion definition:

$$\frac{Y}{Y^r} = 1 - \frac{1}{2}c^{\tau}\left(\frac{T}{Y^r}\right)^2 + \frac{1}{2}c^{g}\left(\frac{G}{Y^r}\right)^2 = 1 - \frac{1}{2}c^{\tau}\left(\frac{T}{Y}\cdot\frac{Y}{Y^r}\right)^2 + \frac{1}{2}c^{g}\left(\frac{G}{Y}\cdot\frac{Y}{Y^r}\right)^2,$$

which implies the quadratic equation

$$\frac{1}{2}\left(c^{\tau}\left(\frac{T}{Y}\right)^2 - c^{g}\left(\frac{G}{Y}\right)^2\right)\left(\frac{Y}{Y^r}\right)^2 + \frac{Y}{Y^r} - 1 = 0. \tag{C19}$$

Substituting Eqs. (C17)–(C18) into Eq. (C19) yields a quadratic in Y/Y^r. Define

$$Ar = (1+\beta)\left[-\frac{1}{1+\alpha} - b(nsb_t, r^E_{t+1})\frac{(1-\rho^K)(1-\alpha\beta)}{(1-\beta)(1+\alpha)}\frac{D}{T}\right]\left(\frac{T}{Y}\right)^2, \qquad Br = \frac{1}{2}\cdot\frac{\alpha-\beta}{1+\alpha}\cdot\frac{T}{Y} + 1.$$

Then

$$\frac{Y}{Y^r} = \frac{-Br + \sqrt{Br^2 + 2Ar}}{Ar}. \tag{C20}$$

C6 Fiscal Multipliers

Analogous to the response of dividends and consumption to fiscal policy shocks, output growth also reflects these changes. Specifically, the distortion in output growth induced by fiscal policy is given by:

$$\Delta\tilde{y}_{t+1} - \Delta y_{t+1} = \left(-c^{\tau}\frac{T}{Y}\right)u^{\tau}_{t+1} + \left(c^{g}\frac{G}{Y}\right)u^{g}_{t+1}, \tag{C21}$$

where $\Delta\tilde{y}_{t+1}$ denotes output growth under the distorted policy path, and $u^{\tau}_{t+1}, u^{g}_{t+1}$ represent tax and government spending shocks, respectively.
Suppose the economy is in a steady state at time t, and a single tax shock u^{τ}_{t+1} occurs in period $t+1$. Then,

$$\frac{\mathrm{E}_t\left(\Delta\tilde{y}_{t+1} - \Delta y_{t+1}\right)}{\mathrm{E}_t\left(\Delta\tilde{t}_{t+1} - \Delta t_{t+1}\right)} = -\frac{T}{Y}c^{\tau}. \tag{C22}$$

Note that:

$$\Delta\tilde{y}_{t+1} - \Delta y_{t+1} \approx \frac{\tilde{Y}_{t+1} - Y_{t+1}}{Y_t}, \quad \Delta\tilde{t}_{t+1} - \Delta t_{t+1} \approx \frac{\tilde{T}_{t+1} - T_{t+1}}{T_t},$$

and given that T_t/Y_t equals its steady-state value, we obtain the following expression for the tax multiplier:

$$\frac{\mathrm{E}_t\left(\tilde{Y}_{t+1} - Y_{t+1}\right)}{\mathrm{E}_t\left(\tilde{T}_{t+1} - T_{t+1}\right)} = -c^{\tau}. \tag{C23}$$

Similarly, under a government spending shock u^g_{t+1}, we obtain the spending multiplier:

$$\frac{\mathrm{E}_t\left(\tilde{Y}_{t+1} - Y_{t+1}\right)}{\mathrm{E}_t\left(\tilde{G}_{t+1} - G_{t+1}\right)} = c^g. \tag{C24}$$

Taken together, the coefficients c^τ and c^g represent forward-looking fiscal multipliers implied by the output distortion in our framework. That is, they measure the expected dollar change in output resulting from a one-dollar change in taxes or government spending, respectively. As such, the values derived in equations Eq. (C17) and Eq. (C18) can be used to infer fiscal multipliers from the predictive coefficient $b(nsb_t, r^E_{t+1})$. In Figure C.1, we further illustrate how c^τ and c^g vary with different values of $b(nsb_t, r^E_{t+1})$. For visual clarity, the tax multiplier is plotted with its sign reversed to align the direction of both series. Consistent with the intuition from Eq. (26), a more negative $b(nsb_t, r^E_{t+1})$ corresponds to a larger tax multiplier and a smaller spending multiplier, while a more positive $b(nsb_t, r^E_{t+1})$ indicates the reverse.

Appendix D Additional Figures and Tables

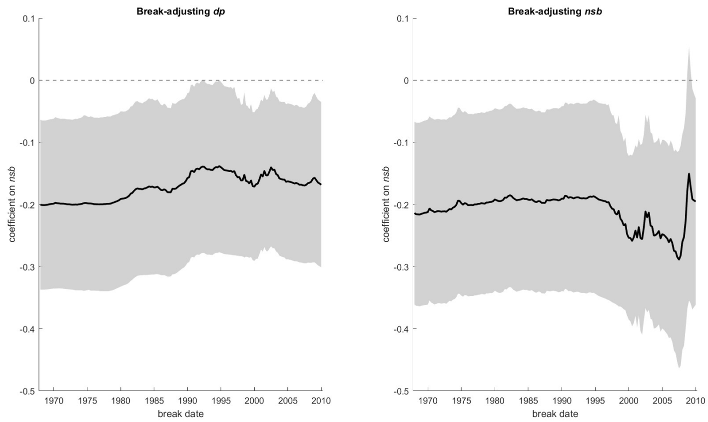
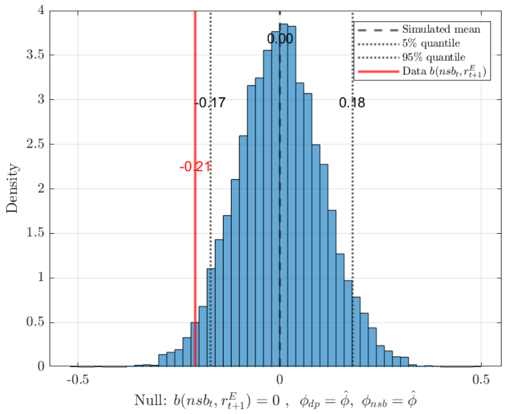
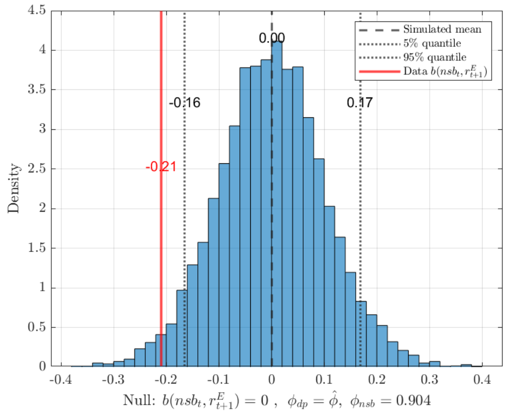


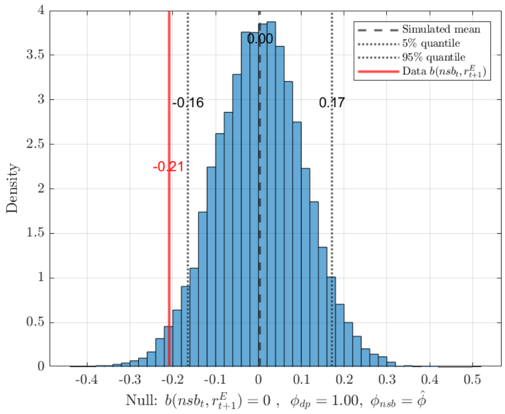
FIGURE B.1: **Break-adjusting** dp **and** nsb**.** We adjust either the log dividend-to-price ratio dp (left panel) or the log net surplus-to-debt ratio nsb (right panel) for a structural break in the mean occurring at the date on the X-axis using the procedure in Lettau and Van Nieuwerburgh (2008). Next, at each date, we estimate regressions of future one-quarter equity returns on the three valuation ratios – dp, nsb, and nma. The plots the report the resulting coefficient on nsb and its 90% confidence interval.



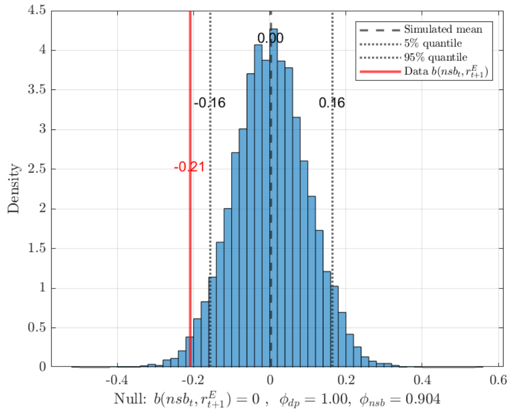
(a) Scenario A: $\phi_{dp} = \hat{\phi}$, $\phi_{nsb} = \hat{\phi}$



(b) Scenario B: $\phi_{dp} = \hat{\phi}$, $\phi_{nsb} = 0.904$



(c) Scenario C: $\phi_{dp} = 1.00$, $\phi_{nsb} = \hat{\phi}$



(d) Scenario D: $\phi_{dp} = 1.00$, $\phi_{nsb} = 0.904$

FIGURE B.2: **Distribution of $b(nsb_t, r^E_{t+1})$ under the null of zero predictability.** We simulate 10,000 samples from a data-generating process in which dp is the sole predictor of equity returns, with its predictive coefficient on r^E_{t+1} implied by the budget constraint (BC). Both dp and nsb follow AR(1) processes, with innovations drawn jointly from a multivariate normal distribution matched to estimation residuals. Persistence parameters are varied as follows: (A) $\phi_{dp} = \hat{\phi}$, $\phi_{nsb} = \hat{\phi}$; (B) $\phi_{dp} = \hat{\phi}$, $\phi_{nsb} = 0.904$; (C) $\phi_{dp} = 1.00$, $\phi_{nsb} = \hat{\phi}$; (D) $\phi_{dp} = 1.00$, $\phi_{nsb} = 0.904$. Red vertical lines mark the empirical benchmark estimate of $b(nsb_t, r^E_{t+1})$, dashed lines the simulated mean, and dotted lines the 5th and 95th percentiles.

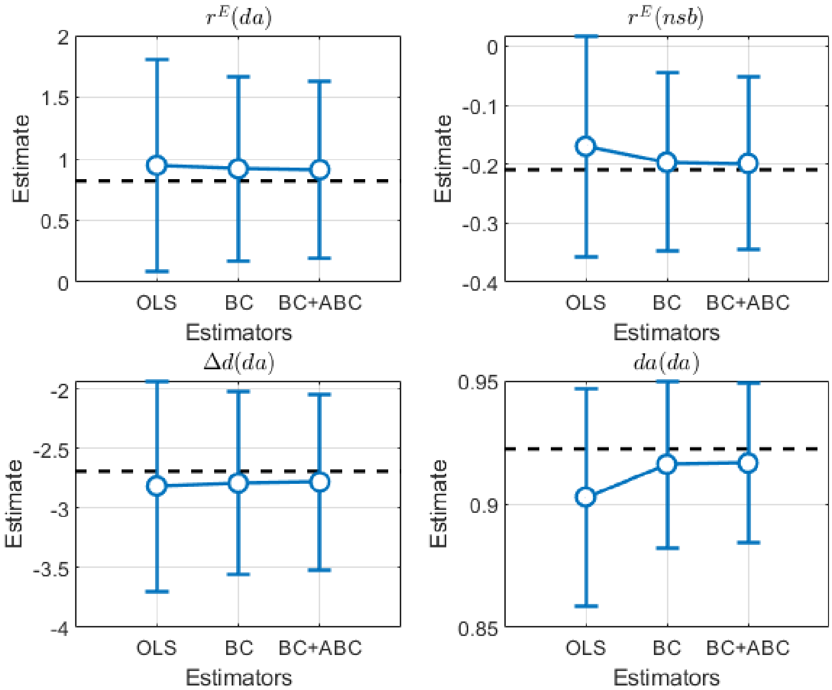


FIGURE B.3: **Simulation Results, Small Sample.** This figure presents the average estimates and confidence intervals (±2 standard deviations) for three estimators—OLS, constrained GMM with sector budget constraints (BC), and constrained GMM incorporating both sector budget constraints and the aggregate budget constraint (BC+ABC)—across 10,000 simulations. Each simulation replicates a sample length of T = 280 quarters, matching the length of our empirical data, following the Monte Carlo algorithm detailed in Appendix Section B4. The subplots display the four primary coefficients of interest: the predictive coefficients of dp_t for three equity sector variables—$r^E(dp)$, $\Delta d(dp)$, and $dp(dp)$—as well as our core result, the predictive coefficient of nsb_t for equity returns, $r^E(nsb)$. The dashed lines indicate the null parameters used in the simulations.

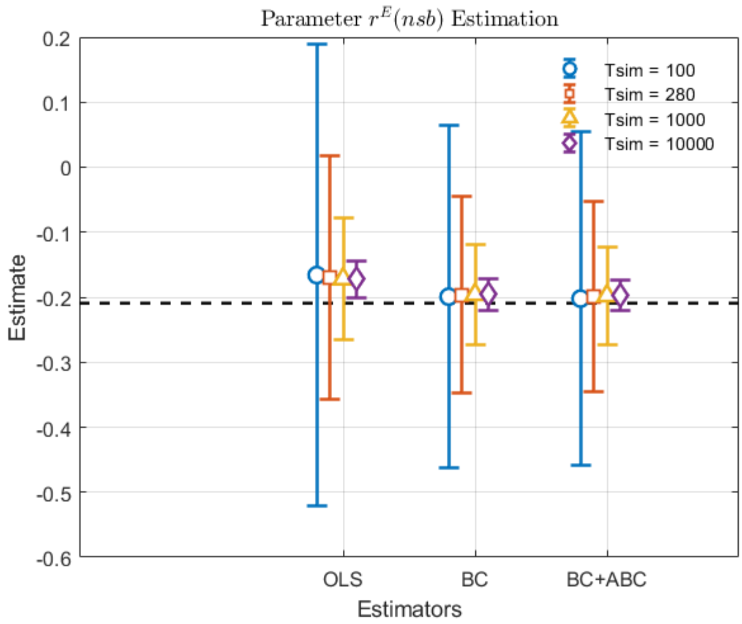


FIGURE B.4: **Simulation Results, Increasing Sample Size.** This figure illustrates the behavior of our key parameter of interest—the coefficient of nsb in predicting equity returns, $r^E(nsb)$—across 10,000 simulated samples of varying lengths: $T = 100$, $T = 280$ (matching our empirical data), $T = 1000$, and $T = 10000$. For each sample length, the figure presents the average estimates and confidence intervals (±2 standard deviations) for three estimators: OLS, constrained GMM with sector budget constraints (BC), and constrained GMM incorporating both sector budget constraints and the aggregate budget constraint (BC+ABC). The simulations are generated using the Monte Carlo algorithm described in Appendix Section B4.

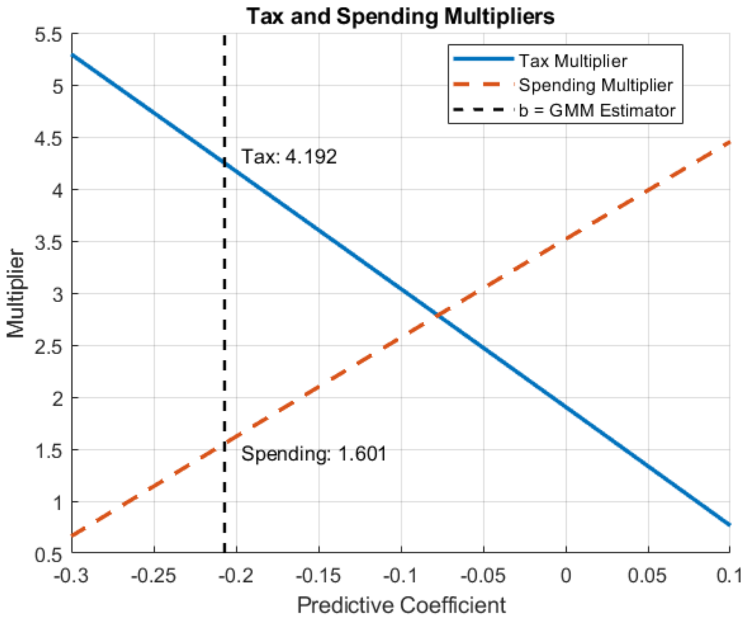


FIGURE C.1: **Output distortion changes with predictive coefficient** This figure illustrates how the implied tax and spending multipliers, as derived in Eq. (C23) and Eq. (C24), vary with the predictive coefficient of nsb_t for future equity returns, r^E_{t+1}, as specified in Eq. (26). The tax multiplier is plotted with an opposite sign to align the direction of both series. All steady-state values are calibrated using sample averages. The fiscal shock horizon is set to $K = 20$ quarters, and tax and spending shocks co-move with a calibrated coefficient $\alpha = 0.827$ from the main text. The benchmark case, where the predictive coefficient $b(nsb_t, r^E_{t+1}) = -0.205$ as reported in Panel B of Table 4, is marked by a black dashed line.