Heterogeneous Intermediaries in a Production Economy

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Abstract

We study a production economy where financial intermediaries face Value-at-Risk constraints. Intermediaries with Epstein–Zin preferences and an intertemporal elasticity of substitution above one save more and accumulate wealth faster than households, amplifying investment and growth. When constraints bind, this channel breaks down: investment falls and long-run growth drops substantially. Log-utility specifications—standard in production economies—mask these effects almost entirely, eliminating cross-agent differences in risk aversion and fixing savings rates. Extending to heterogeneous intermediaries reinforces our findings. Preference specification is not a technical convenience: it determines whether financial regulation appears consequential or irrelevant for the real economy.

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1 Introduction

Financial intermediaries are central to aggregate capital allocation. When they face constraints on their risk-taking — through regulation or market pressure — the consequences extend beyond asset prices to investment and economic growth. Yet most of the theoretical intermediary asset pricing literature analyzes these effects in endowment economies, where real quantities are fixed by assumption, or in production economies with linear or logarithmic preferences. We study a production economy with endogenous investment and growth and agents who have recursive preferences. We ask how binding portfolio constraints on intermediaries affect long-run economic activity.

We build a continuous-time general equilibrium model with financial intermediaries and households. Intermediaries are less risk-averse and more productive than households, reflecting their access to a broader set of investment opportunities. All agents have Epstein–Zin preferences with the same intertemporal elasticity of substitution greater than one. Intermediaries face a Value-at-Risk (VaR) constraint on their portfolio volatility, following Kargar (2021). We extend the baseline model to allow for two types of intermediaries — less risk-averse funds and more risk-averse banks — alongside households.

The paper makes three contributions.

Real effects of portfolio constraints. When the VaR constraint binds, intermediaries reduce their demand for risky capital. Because intermediaries are more productive than households, this shift of capital lowers aggregate productivity. Investment falls, the price of capital declines, and long-run growth drops. These effects are absent in endowment economies and emerge precisely because investment is endogenous.

Log utility masks the real effects. A common simplification in production economies is to assume log utility, which admits more tractable solutions. We show that this assumption dramatically understates the real effects of portfolio constraints. Log utility combines two changes that matter independently: it equalizes risk aversion across agents and restricts the intertemporal elasticity of substitution to one. The first removes the differential leverage channel: with equal risk aversion, portfolio shares are less dispersed and intermediaries are less leveraged even without constraints. The second removes the wealth accumulation channel: with a unit IES, consumption-wealth ratios are constant and identical across agents, so intermediaries no longer save more than households or grow their wealth share over time. With both channels shut down, constraints bind in an economy that is already less dynamic, producing negligible real effects.

Heterogeneous intermediaries. Introducing funds and banks alongside households reinforces the above

results. Under Epstein–Zin preferences, the VaR constraint binds primarily for funds — the most productive intermediaries — which amplifies the growth decline relative to the homogeneous case. Banks partially absorb risk, but not enough to offset the distortion. In the three-agent economy, the growth effects of the constraint are much smaller under log utility than under Epstein–Zin preferences.

Related literature. The intermediary asset pricing literature studies how intermediaries' marginal valuations affect equilibrium returns. He and Krishnamurthy (2012, 2013) show that a binding equity capital constraint drives large swings in risk premia during crises in an endowment economy. Kargar (2021) introduces heterogeneous intermediaries subject to a VaR constraint and shows that their responses to shocks differ by risk aversion — consistent with empirical evidence on the behavior of different types of financial institutions. We build directly on Kargar (2021) by placing this framework in a production economy, which reveals real effects that an endowment economy cannot capture.

Several papers study intermediary frictions in production economies. Brunnermeier and Sannikov (2014) show that nonlinear amplification can generate volatile crisis episodes. Gertler and Kiyotaki (2015) demonstrate that the anticipation of bank runs depresses output even when runs do not occur. He and Krishnamurthy (2019) explain crisis dynamics using intermediary capital shocks under log utility. Elenev, Landvoigt, and Van Nieuwerburgh (2021) assess the welfare effects of capital restrictions in a model with a government sector. Maxted (2023) introduces diagnostic expectations, and Krishnamurthy and Li (2024) match credit spreads and output using log utility and sentiment. We differ from all of these papers in our use of Epstein–Zin preferences and in showing that preference specification determines whether financial constraints have real effects at all.

Empirically, Adrian, Etula, and Muir (2014) construct an intermediary stochastic discount factor from broker-dealer leverage shocks, and He, Kelly, and Manela (2017) show that primary dealer capital shocks are priced in the cross-section of asset classes. Related work includes Haddad and Muir (2021), Baron and Muir (2022), Du, Hébert, and Huber (2023), and Bauer, Bernanke, and Milstein (2023). Dumas (1989) studies how differences in risk aversion shape the distribution of wealth and the dynamics of the capital stock in a production economy. We examine how a portfolio constraint changes investment and growth when intermediaries and households also differ in how productively they use capital.

The remainder of the paper is organized as follows. Section 2 presents the model and its solution. Section 3 reports the results. Section 4 concludes.

2 Model

The economy has two types of agents, financial intermediaries (type a) and households (type c). We present this two-agent economy in full and describe the three-agent extension in Section 2.6. Table 1 summarizes the notation used in the presentation of the model.

2.1 Agents

Agents are indexed by $i \in \{a, c\}$. They have Epstein–Zin preferences with heterogeneous risk aversion and a common intertemporal elasticity of substitution. We assume $\gamma_a < \gamma_c$ and $\psi_a = \psi_c = \psi > 1$. The agents also differ in productivity, measured as output per unit of capital. We assume that intermediaries are more productive than households ($A_a > A_c$), reflecting their access to a broader set of investment opportunities.

Each agent chooses consumption $C_{i,t} \geq 0$, a portfolio weight $\pi_{i,t} \geq 0$ on the risky asset (capital), and an investment rate $\iota_{i,t} \geq 0$. Short positions in capital are ruled out; borrowing at the riskless rate r_t is permitted. The budget constraint is

$$\frac{dW_{i,t}}{W_{i,t}} = \pi_{i,t}\, dr^K_{i,t} + (1 - \pi_{i,t})\, r_t\, dt - c_{i,t}\, dt, \tag{1}$$

where $dr^K_{i,t}$ is the return per unit of capital held by agent i and $c_{i,t} \equiv C_{i,t}/W_{i,t}$ is the consumption-wealth ratio. Each agent maximizes the respective value function

$$V_{i,t} = \max_{\{C_{i,s},\, \pi_{i,s},\, \iota_{i,s}\}_{s \geq t}} \mathbb{E}_t \left[\int_t^\infty f_i(C_{i,s}, V_{i,s})\, ds \right], \tag{2}$$

with the Epstein–Zin aggregator

$$f_i(C_i, V_i) = \frac{1 - \gamma_i}{1 - \frac{1}{\psi_i}} V_i \left[\left(\frac{C_i}{\left[(1 - \gamma_i) V_i\right]^{\frac{1}{1-\gamma_i}}} \right)^{1 - \frac{1}{\psi_i}} - (\rho_i + \kappa) \right]. \tag{3}$$

and the subjective discount rate ρ_i.

To maintain a stationary wealth distribution, we use an overlapping-generations structure with birth and death rate κ. Newborn intermediaries and households arrive with wealth shares $\bar{x}$ and $1 - \bar{x}$, respectively.

We reduce the state space by defining the intermediary's wealth share at time t

$$x_t \equiv \frac{W_{a,t}}{W_{a,t} + W_{c,t}} \in [0, 1]. \tag{4}$$

Its dynamics are given in Equation (14) below.

2.2 Production

Aggregate capital evolves as

$$dK_t = (\iota_t - \delta) K_t \, dt + \sigma_K K_t \, dZ_t, \tag{5}$$

where δ is the depreciation rate, σ_K is capital volatility, and Z_t is a standard Brownian motion. The price of per unit of installed capital p_t follows

$$\frac{dp_t}{p_t} = \mu_{p,t} \, dt + \sigma_{p,t} \, dZ_t, \tag{6}$$

where $\mu_{p,t}$ and $\sigma_{p,t}$ are equilibrium objects determined below. Aggregate wealth equals the value of the capital stock:

$$W_{a,t} + W_{c,t} = p_t K_t. \tag{7}$$

Each agent optimally chooses an investment rate by maximizing the net profit from investing in capital subject to quadratic adjustment costs $\Phi(\iota) \equiv \frac{\chi}{2}(\iota - \delta)^2$:

$$\max_{\iota_{i,t}} p_t \iota_{i,t} - \iota_{i,t} - \Phi(\iota_{i,t}). \tag{8}$$

Since p_t and δ are common across agents, the optimal investment rate is the same for all agents:

$$\iota_t^* = \frac{p_t - 1}{\chi} + \delta. \tag{9}$$

Each agent produces output $Y_{i,t} = A_i K_{i,t}$. Aggregate output is $Y_t = A_t K_t$ with

$$A_t = \pi_{a,t} \, x_t \, A_a + \pi_{c,t} (1 - x_t) \, A_c. \tag{10}$$

A larger intermediary wealth share, or greater intermediary leverage, raises aggregate productivity. The return per unit of capital held by agent i is

$$dr_{i,t}^K = \left[\frac{A_i - \iota_t^* - \Phi(\iota_t^*)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t}\right] dt + (\sigma_K + \sigma_{p,t})\, dZ_t. \tag{11}$$

2.3 Market Clearing

Capital and goods market clearing require, respectively:

$$\pi_{a,t}\, x_t + \pi_{c,t}(1 - x_t) = 1 \tag{12}$$

$$p_t \left[c_{a,t}\, x_t + c_{c,t}(1 - x_t)\right] + \iota_t^* + \Phi(\iota_t^*) = A_t. \tag{13}$$

Bond market clearing is implied by the capital market condition.[1]

2.4 Equilibrium

Applying Itô's lemma to x_t and using the budget constraint and capital return, the state variable dynamics are

$$\begin{aligned} dx_t = {} & \kappa(\bar{x} - x_t)dt \\ & + x_t(1 - x_t)\Bigg[(\pi_{a,t} - \pi_{c,t})(\iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t) \\ & - (\pi_{a,t} - \pi_{c,t})(x_t \pi_{a,t} + (1 - x_t)\pi_{c,t})(\sigma_K + \sigma_{p,t})^2 \\ & + \left(\pi_{a,t} \frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{c,t} \frac{A_c - \iota_t^* - \Phi(\iota_t^*)}{p_t}\right) + c_{c,t} - c_{a,t}\Bigg] dt \\ & + \Big[x_t(1 - x_t)(\pi_{a,t} - \pi_{c,t})(\sigma_K + \sigma_{p,t})\Big] dZ_t. \end{aligned} \tag{14}$$

The drift and diffusion of p_t are linked to derivatives of the price function via

$$\mu_{p,t} = \frac{p_{t,x}}{p_t}\, \mu_{x,t} + \frac{1}{2} \frac{p_{t,xx}}{p_t}\, \sigma_{x,t}^2 \tag{15}$$

$$\sigma_{p,t} = \frac{p_{t,x}}{p_t}\, \sigma_{x,t}. \tag{16}$$

Substituting the optimal investment rate and the state dynamics into the agents' Bellman equation, the

[1] Since the riskless asset is in zero net supply, the bond market clearing condition $(1 - \pi_{a,t})x_t + (1 - \pi_{c,t})(1 - x_t) = 0$ follows directly from the capital market clearing condition.

HJB for agent i (derived in Appendix A.2) is

$$\begin{aligned}\rho_i + \kappa \;=\; & \max_{\pi_{i,t}} \Bigg\{ G_{i,t} + (1 - \tfrac{1}{\psi_i}) \left(r_t + \pi_{i,t} \left(\frac{A_i - \iota_t^* - \Phi(\iota_t^*)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t \right) - G_{i,t} \right) \\ & - \frac{1}{\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \mu_{x,t} - \frac{1}{2} (1 - \frac{1}{\psi_i}) \gamma_i \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 \\ & - \frac{1}{2\psi_i} \left[\left(\frac{1-\gamma_i}{1-\psi_i} - 1 \right) \frac{(G_{i,t,x})^2}{G_{i,t}^2} + \frac{G_{i,t,xx}}{G_{i,t}} \right] \sigma_{x,t}^2 \\ & - \frac{1-\gamma_i}{\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \pi_{i,t} \sigma_{x,t} (\sigma_K + \sigma_{p,t}). \end{aligned} \tag{17}$$

where $G_{i,t} \equiv G_i(x_t)$ denotes the transformed value function, which coincides with the consumption-wealth ratio. The first-order condition for the portfolio weight gives

$$\pi_{i,t} = \underbrace{\frac{1}{\gamma_i} \frac{\mu_{r,t} - r_t}{(\sigma_K + \sigma_{p,t})^2}}_{\text{myopic demand}} + \underbrace{\frac{1}{\gamma_i} \frac{1-\gamma_i}{1-\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \frac{\sigma_{x,t}}{\sigma_K + \sigma_{p,t}}}_{\text{hedging demand}}, \tag{18}$$

where $\mu_{r,t} - r_t$ is the excess return on capital. The myopic term is common to log-utility investors; the hedging term captures demand for changes in the state x_t and vanishes under log utility.

2.5 Value-at-Risk Constraint

Intermediaries face an upper bound on portfolio return volatility. Under locally normal innovations this reduces to a bound on portfolio weight times total volatility:

$$\pi_{a,t} \left(\sigma_K + \sigma_{p,t} \right) \leq \bar{\sigma}_{\text{FI}}. \tag{19}$$

Whether the constraint binds in equilibrium depends on the state of the economy and the volatility bound.

2.6 Three-Agent Extension

We extend the model by splitting the financial sector into funds (type a) and banks (type b), with households (type c) unchanged. The ordering of risk aversion and productivity is $\gamma_a < \gamma_b < \gamma_c$ and $A_a \geq A_b > A_c$. The state space expands to $(x_t,\, y_t)$, where

$$x_t \equiv \frac{W_{a,t} + W_{b,t}}{W_{a,t} + W_{b,t} + W_{c,t}} \quad \text{(financial sector wealth share)}, \tag{20}$$

$$y_t \equiv \frac{W_{a,t}}{W_{a,t} + W_{b,t}} \quad \text{(share of funds within the financial sector)}. \tag{21}$$

All other equations carry over with minor modifications. The details are presented in Appendix B.

2.7 Logarithmic Utility

As a benchmark, we solve the model under log utility ($\gamma_i = \psi_i = 1$ for all i). Under this specification risk aversion is equalized across agents and the hedging demand vanishes. Appendix C derives the three-investor equilibrium under log utility.

2.8 Solution

The model is solved numerically. Linearity of all equilibrium conditions in K_t implies that the value function G_i depends only on x (two-agent model) or x and y (three-agent model), reducing the state space accordingly. The equilibrium price function $p_t(x)$ is obtained by iteration: given a guess for p_t, we solve the HJB equations, check market clearing and update p_t until convergence. Details are in Appendix D.

3 Quantitative Results and Mechanisms

We study the real and financial implications of VaR constraints on financial intermediaries across four model specifications: the two-investor economy with Epstein–Zin (EZ) and log preferences, and the three-investor economy with EZ and log preferences. Welfare is assessed via certainty equivalent consumption. We also examine two intermediate preference cases that isolate the roles of risk aversion and the intertemporal elasticity of substitution.

3.1 Parameters

The parameters are given in Table 2. Risk aversion is $\gamma_a = 2.5$ for funds, $\gamma_b = 5.5$ for banks, and $\gamma_c = 8.5$ for households. The IES is $\psi = 1.5$ for all agents, the subjective discount rate is set to $\rho = 3\%$. The birth-and-death rate is $\kappa = 0.017$, with newborn wealth shares $\bar{x} = 0.05$ and $\bar{y} = 0.20$. For capital dynamics, we set the deprication rate to $\delta = 0.1$, the adjustment cost parameter χ is equal to 3, and the volatility of capital growth is $\sigma_K = 0.12$. The productivities for the different agents are set to $A_a = 0.18$, $A_b = 0.17$, and $A_c = 0.165$. These parameters target a benchmark growth rate of approximately 1.8%.

All simulation-based moments are computed from five paths of 50,000 monthly steps each. We discard the first 5,000 months of each path, leaving 45,000 months (3,750 years) per path. We use the same shock sequence for each corresponding path across specifications. To quantify Monte Carlo uncertainty in a

growth comparison, we compute the difference in annualized capital growth within each matched path and construct a pointwise 95% Student-t interval from the five path-level differences. These intervals describe simulation uncertainty conditional on the model and its calibration.

3.2 Two Investors with EZ Preferences

Panel A of Table 3 reports average capital growth rates under the benchmark two-investor EZ economy ($\gamma_a = 2.5$, $\gamma_c = 8.5$, $\psi = 1.5$) for different constraint levels $\bar{\sigma}_{\text{FI}}$.

Growth and investment. In the unrestricted economy, the annualized capital growth rate is 1.78%. As the constraint tightens, growth first rises slightly to 1.92% before falling sharply to 1.03% under the tightest restriction $\bar{\sigma}_{\text{FI}} = 0.12$. The initial increase occurs because the constraint reduces wealth share volatility, which raises investment incentives at the margin; once the constraint binds tightly, however, the direct restriction on intermediary leverage dominates.

The percentage of states in which the constraint binds is not a sufficient statistic for its real impact. For $\bar{\sigma}_{\text{FI}} \leq 0.15$, the constraint binds in virtually all states, yet growth differs substantially across these cases because the permitted leverage ceiling differs.

Panel A of Table 4 reports further results for the case of two agents with EZ preferences. Average productivity falls from 17.97% in the unrestricted case to 17.45% under $\bar{\sigma}_{\text{FI}} = 0.12$. The investment rate follows the price of capital (see Equation (9)): it ranges from 12.71% unrestricted to 11.91% under the tightest constraint, and the price of capital moves from 1.0814 to 1.0573 accordingly.

Wealth distribution. Starting from a newborn share of $\bar{x} = 5\%$, intermediaries accumulate wealth over time through higher productivity, leverage, and expected returns. Their average wealth share is 78.24% unrestricted, rises to 81.14% for $\bar{\sigma}_{\text{FI}} = 0.15$, and falls to 63.48% under the tightest restriction. Wealth share volatility also drops to nearly zero under $\bar{\sigma}_{\text{FI}} = 0.12$, since the constraint pushes portfolio weights together across agents (see Equation (14)).

Asset pricing. The risk-free rate is 1.39% in the unrestricted and mildly restricted cases, but falls sharply to -6.04% under $\bar{\sigma}_{\text{FI}} = 0.12$. The equity risk premium declines more modestly, from 7.76% to 7.10%.

Mechanism. Figure 2 shows that the risk premium is decreasing in the intermediary wealth share x: when households dominate ($x \approx 0$), risk aversion is high and the premium must be large to clear the

capital market; when intermediaries dominate ($x \approx 1$), lower risk aversion depresses the premium.

The portfolio weights in Figure 3 follow directly. At the boundaries $x = 0$ and $x = 1$, the respective agent holds the entire capital stock and their weight equals one. Intermediaries take on increasing leverage as x falls, exploiting the high premium set by more risk-averse households. The household's weight is not monotone: it exhibits a U-shape driven by the interplay of myopic and hedging demand (see Figure 4). At low values of x, myopic demand dominates and declines with x; at intermediate x, hedging demand turns negative because the risky asset co-moves positively with x, paying off when investment opportunities worsen. At high x, hedging demand rises again as households seek exposure to the productive intermediary-dominated economy.

When the VaR constraint binds, intermediary portfolio weights are capped for low values of x. This limits leverage and, since intermediaries are more productive, reduces average productivity $\bar{A}$ (see the left graph in Figure 5).

Lower productivity reduces the price of capital and thus the investment rate (see Figure 6), depressing growth. Price volatility σ_p peaks at intermediate x and is lower under tighter restrictions, consistent with the constraint compressing portfolio weight dispersion (see Figure 7).

With $\psi > 1$, the substitution effect dominates the income effect: agents save more when investment opportunities improve. Less risk-averse intermediaries therefore have lower consumption rates, accumulate wealth faster, and maintain a larger share of the economy. Tighter constraints reduce investment opportunities and raise consumption rates, further lowering growth (see Figure 8). The aggregate dividend yield, which equals the wealth-weighted average consumption rate (see Equation (13)), is a decreasing function of x: it is high when risk-averse households dominate and low when intermediaries dominate. Tighter restrictions shift the distribution of x downward and raise the aggregate consumption rate, which in turn lowers the price of capital.

Welfare. We assess welfare using certainty equivalent consumption (CE), defined as the constant consumption flow that yields the same lifetime utility as the equilibrium allocation, i.e., CE is implicitly defined via

$$\frac{\mathrm{CE}_i}{W_i} = G_i(x)^{\frac{1}{1-\psi_i}}. \tag{22}$$

Table 5 reports the estimated certainty equivalents. For intermediaries, CE rises from 503.0 in the unrestricted case to 574.6 at $\bar{\sigma}_{\mathrm{FI}} = 0.135$ before falling to 532.7 under the tightest restriction. For households, CE falls from 232.1 to 194.2 over the same range before partially recovering to 205.6 under the tightest

restriction. The small changes at the loosest bound are consistent with portfolio rebalancing, risk transfer across investors, and equilibrium price adjustments cushioning the direct effect of the constraint. As the bound tightens, intermediaries have less scope to absorb risk. At the tightest bound, the intermediary CE falls from its peak and the household CE remains below its unrestricted level.

Intermediaries have higher CE per unit of wealth than households despite equal IES. Lower risk aversion means intermediaries absorb more aggregate risk and earn higher expected returns, with only a small risk-disutility penalty; households apply a much larger risk adjustment to similar payoffs.

Summary. Binding VaR constraints limit intermediary leverage, reduce average productivity and the price of capital, lower investment rates, and depress long-run growth. The effects on investment and growth are modest at the loosest bound and more pronounced at the tightest bound.

3.3 Two Investors with Log Utility

Under log utility, the HJB has a closed-form solution, which is why this specification is widely used. Compared to the EZ benchmark, all agents now have unit risk aversion and unit IES, so preference heterogeneity is eliminated; differences in behavior are driven solely by productivity differences ($A_a > A_c$).

Under log utility, annualized capital growth falls from 1.14% in the unrestricted case to 0.87% under the tightest constraint (Panel B of Table 3). The decline is 0.27 percentage points, with a paired 95% Monte Carlo interval of $[-0.33, -0.21]$ percentage points. Under EZ preferences, the same comparison yields a decline from 1.78% to 1.03%, or 0.75 percentage points, with an interval of $[-0.82, -0.68]$. The growth cost of the tightest VaR constraint is therefore substantially smaller under log utility.

Panel B of Table 4 shows that the average intermediary wealth share is 22.6% — roughly 55 percentage points below the EZ benchmark. Two forces drive this difference. First, with equal risk aversion, leverage is motivated solely by productivity differences: intermediary portfolio weights remain below 200% (see Figure 9), substantially below the levels of more than 250% reached in the EZ case (see Figure 3). Second, with unit IES, consumption-wealth ratios are constant and equal to $\rho + \kappa = 4.5\%$ for both agents, eliminating the differential savings channel that allows intermediaries to grow their wealth share in the EZ economy.

3.4 A Comparison of EZ and Log Utility

3.4.1 EZ with $\psi \approx 1$

Moving from the EZ benchmark to log utility sets both risk aversion and the IES to one. To distinguish their effects, we first reduce the IES while retaining benchmark risk aversion, then examine the effect of equalizing risk aversion.

The unit-IES limit. The normalized Epstein–Zin aggregator has a finite limit as $\psi_i \to 1$ for fixed $\gamma_i \neq 1$. When we set $\gamma_i = \psi_i = 1.001$ for both investors, the numerical solution agrees with the log-utility solution. This agreement supports approximating the unit-IES equilibrium along a locally regular branch at $\psi = 1.001$. Solving the model at $\psi = 1$ would require using the limiting aggregator.

Throughout, $\psi \approx 1$ denotes the numerical specification with $\psi = 1.001$.

Panel A of Tables 6 and 7 reports the results for $\psi \approx 1$ with benchmark risk aversion ($\gamma_a = 2.5$, $\gamma_c = 8.5$). In the unrestricted economy, annualized capital growth is 1.81%, compared with 1.78% in the EZ benchmark. As the IES approaches one, each investor's consumption-to-wealth ratio approaches $\rho_i + \kappa$. In the near-unit-IES simulation, the investment rate is 12.77% and the price of capital is 1.0830, both slightly above their benchmark values.

Under the tightest constraint, annualized capital growth falls to 0.87%, the same reported rate as under log utility. Portfolio weights remain similarly dispersed across investors in the near-unit-IES and benchmark EZ economies (Figures 10 and 3). The decline in growth from the unrestricted case to the tightest constraint is larger with a near-unit IES than in the EZ benchmark.

3.4.2 EZ with Identical γ

We next equalize risk aversion ($\gamma_a = \gamma_c = 2.5$) and retain $\psi_a = \psi_c = 1.5$. In Panel B of Table 6, annualized capital growth falls from 1.78% in the unrestricted economy to 1.58% under the tightest constraint. Equalizing risk aversion reduces dispersion in portfolio weights. The intermediary's portfolio weight peaks at roughly 140% (Figure 11), compared with more than 250% in the benchmark. With less intermediary leverage, the constraint has a smaller effect on productivity and growth.

The risk-free rate rises from 1.39% in the unrestricted benchmark to about 4.7%–4.9% across restriction levels. Lower aggregate risk aversion reduces demand for the riskless asset, raising the return required to clear the bond market. The equity risk premium declines from 7.76% to around 6.6%–7.0%. Capital-price volatility is about one-tenth of its benchmark level (Figures 12 and 7).

When we also reduce the IES to $\psi \approx 1$, annualized capital growth falls to 1.20% in the unrestricted economy and to 0.87% under the tightest constraint (Panel C of Table 6). Relative to the equal-risk-aversion case with $\psi_a = \psi_c = 1.5$, portfolio weights remain similar, while investment and the price of capital fall and consumption rises. In the unrestricted economy, aggregate consumption as a share of output increases from 25.6% to 29.1%. The investment rate is 12.10% and the price of capital is 1.0629. The risk-free rate is about 2.6% across restriction levels.

Equalizing risk aversion reduces intermediary leverage and dampens the growth response to the VaR constraint. Moving the IES close to one changes saving and investment, and its effect on unrestricted growth depends on whether risk aversion differs across investors. Under log utility, the tightest constraint lowers growth by less than in the EZ benchmark.

3.5 EZ Preferences: Three Investors

We now introduce heterogeneity within the financial sector by splitting intermediaries into funds (type a, $\gamma_a = 2.5$, $A_a = 0.18$) and banks (type b, $\gamma_b = 5.5$, $A_b = 0.17$), following the intermediary market structure in Kargar (2021), but now in a production economy.

In the three-investor EZ economy, annualized capital growth falls from 2.08% in the unrestricted case to 0.89% under the tightest constraint, $\bar{\sigma}_{\text{FI}} = 0.12$ (Panel C of Table 3). The change is -1.19 percentage points, with a paired 95% Monte Carlo interval of $[-1.21, -1.17]$ percentage points. The growth loss is larger than the 0.75 percentage point decline in the two-investor EZ economy. Average productivity and investment respond in the same direction as in the two-investor benchmark.

In the unrestricted economy, funds hold 72.0% of total wealth (xy), while banks hold 1.5% and households 28.0%. Under the tightest constraint, funds' share falls to 47.4% and households' to 26.7%, while banks' share rises to 25.9%. When funds are constrained, banks — less risk-averse than households and more productive — absorb capital and invest approximately 200% of their wealth. This partial risk absorption means aggregate productivity falls less than it would if households had to absorb the capital instead. However, banks consume more than funds would have absent constraints, reducing resources available for investment and lowering the equilibrium price of capital, which in turn reduces investment and growth through equation (9).

The main qualitative conclusions from the two-investor model are robust: VaR constraints on the most productive intermediaries depress investment and long-run growth. The decline in growth from the unrestricted case to the tightest bound is larger when banks are included.

3.6 Log Utility: Three Investors

In the three-investor model, the tightest VaR constraint reduces long-run growth under both preference specifications (Table 3). The growth loss is smaller under log utility than under EZ preferences, so the comparison from the two-investor model carries over when banks are included.

4 Conclusion

This paper analyzes the role of portfolio restrictions on financial intermediaries in a continuous-time production economy with heterogeneous agents and Epstein–Zin preferences. Our model incorporates financial intermediaries and households, with intermediaries generally being less risk-averse and more productive. We introduce a Value-at-Risk constraint that limits the risk-taking of these intermediaries. We demonstrate that this regulatory friction substantially impacts investment, depresses capital prices, and significantly alters the growth rate of aggregate consumption, often exhibiting an inversely U-shaped response to the constraint's tightness.

Our comparison of preference specifications highlights that log-utility economies, frequently used for tractability, substantially underestimate these effects and can lead to misleading conclusions about the relevance of regulation. Extending the model to a heterogeneous intermediary sector confirms that the main mechanism remains intact: restrictions on the most productive intermediaries depress growth even when other intermediaries can absorb part of the risk.

This work bridges the intermediary asset pricing literature with models of real economic activity, offering insights for both theoretical modeling and policy design. It provides new insights into the macroeconomic consequences of financial regulation and suggests that policy measures constraining intermediaries' risk-taking should carefully weigh the trade-off between financial stability and long-run growth.

Table 1: Notation

Symbol	Type	Description
γ_i	constant	risk aversion
ψ_i	constant	intertemporal elasticity of substitution
ρ_i	constant	subjective discount rate
A_i	constant	productivity
$\pi_{i,t}$	function	share of wealth invested into capital
$c_{i,t}$	function	consumption-wealth ratio
r_t	function	risk-free rate
κ	constant	agents' birth and death rate
$\bar{x}$	constant	share of newborn intermediaries
$\bar{y}$	constant	share of newborn type a-agents within intermediaries
x_t	variable	total wealth share of intermediaries
y_t	variable	wealth share of tpe b- agents within intermediary sector
δ	constant	depreciation rate of capital
σ_K	constant	volatility of capital growth
K_t	function	amount of capital
p_t	function	price of capital
$\mu_{p,t}$	function	drift coefficient of p_t
$\sigma_{p,t}$	function	diffusion coefficient of p_t
$\iota_{i,t}$	function	investment rate
χ	constant	investment adjustment costs
$\overline{A}_t$	function	aggregate productivity
$\bar{\sigma}_{\mathrm{FI}}$	constant	upper bound on portfolio volatility

Quantities indexed by i are agent-specific. All functions have as arguments the state variables x and y in the three agent model and the state varaible x in the two-agent version.

Preferences	
ψ_a, ψ_b, ψ_c	1.5
γ_a	2.5
γ_b	5.5
γ_c	8.5
ρ	0.03
Production and demography	
δ	0.10
σ_K	0.12
χ	3
κ	0.017
$\bar{x}$	0.05
$\bar{y}$	0.20
A_a	0.18
A_b	0.17
A_c	0.165

Table 2: Parameterization of the model

The table gives the parameters in the base case for the model with three investors. In the model with two investors, only agents of type a (now called 'financial intermediaries') and type c (households) are present.

Restriction	μ_K (%)	μ_C (%)	μ_Y (%)	prob. restricted
Panel A: Two agents, EZ preferences				
—	1.78	1.78	1.78	0.00
0.3	1.79	1.79	1.79	0.01
0.2	1.87	1.87	1.87	0.21
0.15	1.92	1.92	1.92	0.99
0.135	1.73	1.73	1.73	1.00
0.12	1.03	1.03	1.03	1.00
Panel B: Two agents, logarithmic preferences				
—	1.14	1.14	1.14	0.00
0.3	1.14	1.14	1.14	0.00
0.2	1.12	1.12	1.12	0.82
0.15	1.00	1.00	1.00	1.00
0.135	0.93	0.93	0.93	1.00
0.12	0.87	0.87	0.87	1.00
Panel C: Three agents, EZ preferences				
—	2.08	2.08	2.08	0.00
0.3	2.09	2.09	2.09	0.00
0.2	2.13	2.13	2.13	0.08
0.15	2.06	2.06	2.06	0.76
0.135	1.84	1.84	1.84	1.00
0.12	0.89	0.89	0.89	1.00
Panel D: Three agents, logarithmic preferences				
—	0.97	0.97	0.97	0.00
0.3	0.97	0.97	0.97	0.00
0.2	0.95	0.95	0.95	0.94
0.15	0.82	0.82	0.82	1.00
0.135	0.78	0.78	0.78	1.00
0.12	0.75	0.75	0.75	1.00

Table 3: Growth rates

The table presents the average growth rates of capital (μ_K), consumption (μ_C), and output (μ_Y). The column 'prob. restricted' shows the probability that the restriction is binding.

Restriction	$\bar{A}$	ι	price	r_f	$E[r]-r_f$	x	
Panel A: Two investors, EZ preferences							
—	0.1797	0.1271	1.0814	0.0139	0.0776	0.7824	
	(0.0002)	(0.0018)	(0.0053)	(0.0098)	(0.0015)	(0.0662)	
0.3	0.1797	0.1272	1.0817	0.0140	0.0776	0.7836	
	(0.0002)	(0.0017)	(0.0051)	(0.0100)	(0.0015)	(0.0652)	
0.2	0.1797	0.1280	1.0840	0.0124	0.0774	0.7929	
	(0.0004)	(0.0016)	(0.0048)	(0.0142)	(0.0011)	(0.0577)	
0.15	0.1787	0.1282	1.0845	-0.0135	0.0753	0.8114	
	(0.0006)	(0.0013)	(0.0040)	(0.0115)	(0.0001)	(0.0348)	
0.135	0.1776	0.1262	1.0785	-0.0415	0.0736	0.7836	
	(0.0003)	(0.0008)	(0.0024)	(0.0035)	(0.0001)	(0.0210)	
0.12	0.1745	0.1191	1.0573	-0.0604	0.0710	0.6348	
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
Panel B: Two investors, logarithmic preferences							
—	0.1707	0.1202	1.0607	0.0506	0.0674	0.2260	
	(0.0026)	(0.0022)	(0.0065)	(0.0013)	(0.0021)	(0.1213)	
0.3	0.1707	0.1202	1.0607	0.0506	0.0674	0.2260	
	(0.0026)	(0.0022)	(0.0065)	(0.0013)	(0.0021)	(0.1213)	
0.2	0.1705	0.1200	1.0601	0.0503	0.0672	0.2250	
	(0.0025)	(0.0021)	(0.0062)	(0.0014)	(0.0020)	(0.1112)	
0.15	0.1690	0.1188	1.0564	0.0486	0.0658	0.2135	
	(0.0009)	(0.0007)	(0.0022)	(0.0002)	(0.0007)	(0.0476)	
0.135	0.1682	0.1182	1.0546	0.0481	0.0652	0.1926	
	(0.0004)	(0.0003)	(0.0009)	(0.0000)	(0.0003)	(0.0222)	
0.12	0.1675	0.1176	1.0527	0.0478	0.0646	0.1659	
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0008)	
Panel C: Three investors, EZ preferences							
—	0.1792	0.1299	1.0896	0.0274	0.0737	0.7204	0.9792
	(0.0003)	(0.0015)	(0.0044)	(0.0042)	(0.0004)	(0.0773)	(0.0085)
0.3	0.1792	0.1300	1.0900	0.0275	0.0737	0.7227	0.9794
	(0.0003)	(0.0014)	(0.0043)	(0.0044)	(0.0004)	(0.0762)	(0.0079)
0.2	0.1794	0.1305	1.0914	0.0269	0.0738	0.7386	0.9806
	(0.0004)	(0.0015)	(0.0044)	(0.0075)	(0.0005)	(0.0692)	(0.0066)
0.15	0.1790	0.1300	1.0899	0.0122	0.0734	0.7793	0.9821
	(0.0007)	(0.0017)	(0.0050)	(0.0143)	(0.0001)	(0.0453)	(0.0041)
0.135	0.1779	0.1278	1.0834	-0.0153	0.0725	0.7871	0.9747
	(0.0005)	(0.0012)	(0.0035)	(0.0071)	(0.0001)	(0.0304)	(0.0036)
0.12	0.1737	0.1181	1.0542	-0.0322	0.0701	0.7327	0.6464
	(0.0001)	(0.0003)	(0.0010)	(0.0024)	(0.0000)	(0.0253)	(0.0300)

Panel D: Three investors, logarithmic preferences							
—	0.1688	0.1185	1.0556	0.0497	0.0657	0.1780	0.6689
	(0.0024)	(0.0020)	(0.0060)	(0.0011)	(0.0020)	(0.1043)	(0.1675)
0.3	0.1688	0.1186	1.0559	0.0496	0.0656	0.1779	0.6687
	(0.0024)	(0.0020)	(0.0061)	(0.0011)	(0.0020)	(0.1043)	(0.1675)
0.2	0.1685	0.1184	1.0552	0.0494	0.0654	0.1742	0.6836
	(0.0021)	(0.0018)	(0.0053)	(0.0010)	(0.0017)	(0.0910)	(0.1298)
0.15	0.1670	0.1171	1.0513	0.0484	0.0641	0.1382	0.6270
	(0.0006)	(0.0005)	(0.0014)	(0.0001)	(0.0005)	(0.0358)	(0.0506)
0.135	0.1664	0.1167	1.0500	0.0483	0.0637	0.1169	0.5501
	(0.0002)	(0.0002)	(0.0006)	(0.0000)	(0.0002)	(0.0205)	(0.0308)
0.12	0.1660	0.1163	1.0490	0.0482	0.0633	0.1003	0.4606
	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0001)	(0.0126)	(0.0560)

Table 4: Real and financial quantities from model simulation

The table gives the mean and the standard deviation (in parentheses) of key asset pricing moments and real quantities. The parameters for the model are given in Table 2. The column labeled 'Restriction' gives the value of the restriction parameter $\bar{\sigma}_{\mathrm{FI}}$.

	Two agents		Two agents, identical γ		Three agents		
Restriction	a	c	a	c	a	b	c
—	503.00	232.14	698.81	533.69	611.04	337.44	291.84
0.3	505.25	231.69	698.81	533.69	613.59	337.56	290.96
0.2	528.14	225.60	698.81	533.69	630.15	331.84	279.45
0.15	574.84	200.54	696.23	533.62	632.31	312.30	240.87
0.135	574.62	194.18	687.29	534.10	603.52	312.77	218.07
0.12	532.66	205.64	673.36	534.78	532.77	327.17	217.15

Table 5: Certainty Equivalents

The table shows the certainty equivalents computed according to Equation (22).

Restriction	μ_K (%)	μ_C (%)	μ_Y (%)	Prob. restricted
Panel A: $\psi_a = \psi_c \approx 1$, $\gamma_a = 2.5$, $\gamma_c = 8.5$				
—	1.81	1.81	1.81	0.00
0.3	1.81	1.81	1.81	0.02
0.2	1.81	1.81	1.81	0.46
0.15	1.65	1.65	1.65	1.00
0.135	1.37	1.37	1.37	1.00
0.12	0.87	0.87	0.87	1.00
Panel B: $\psi_a = \psi_c = 1.5$, $\gamma_a = \gamma_c = 2.5$				
—	1.78	1.78	1.78	0.00
0.3	1.78	1.78	1.78	0.00
0.2	1.79	1.79	1.79	0.69
0.15	1.75	1.75	1.75	1.00
0.135	1.68	1.68	1.68	1.00
0.12	1.58	1.58	1.58	1.00
Panel C: $\psi_a = \psi_c \approx 1$, $\gamma_a = \gamma_c = 2.5$				
—	1.20	1.20	1.20	0.00
0.3	1.20	1.20	1.20	0.00
0.2	1.20	1.20	1.20	0.00
0.15	1.14	1.14	1.14	1.00
0.135	1.00	1.00	1.00	1.00
0.12	0.87	0.87	0.87	1.00

Table 6: Growth rates for alternative preference specifications

The table gives the expected growth rates of capital (μ_K), consumption (μ_C), and output (μ_Y). The column 'Prob. restricted' shows the probability that the restriction is binding.

Restriction	$\bar{A}$	ι	price	r_f	$E[r]-r_f$	x
Panel A: $\gamma_a = 2.5$, $\gamma_c = 8.5$, $\psi_a = \psi_c \approx 1$						
—	0.1797	0.1277	1.0830	0.0072	0.0781	0.7495
	(0.0005)	(0.0004)	(0.0013)	(0.0133)	(0.0009)	(0.0875)
0.3	0.1798	0.1277	1.0832	0.0073	0.0781	0.7510
	(0.0005)	(0.0004)	(0.0013)	(0.0137)	(0.0009)	(0.0871)
0.2	0.1800	0.1276	1.0827	0.0011	0.0780	0.7544
	(0.0016)	(0.0008)	(0.0023)	(0.0191)	(0.0013)	(0.0748)
0.15	0.1771	0.1255	1.0765	-0.0376	0.0739	0.7059
	(0.0008)	(0.0007)	(0.0020)	(0.0086)	(0.0007)	(0.0495)
0.135	0.1735	0.1226	1.0677	-0.0584	0.0702	0.5267
	(0.0005)	(0.0004)	(0.0012)	(0.0007)	(0.0004)	(0.0310)
0.12	0.1675	0.1175	1.0526	-0.0602	0.0646	0.1670
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0008)
Panel B: $\gamma_a = \gamma_c = 2.5$, $\psi_a = \psi_c = 1.5$						
—	0.1722	0.1270	1.0810	0.0487	0.0700	0.3330
	(0.0021)	(0.0025)	(0.0074)	(0.0009)	(0.0019)	(0.1208)
0.3	0.1722	0.1270	1.0810	0.0487	0.0700	0.3330
	(0.0021)	(0.0025)	(0.0074)	(0.0009)	(0.0019)	(0.1208)
0.2	0.1722	0.1270	1.0811	0.0486	0.0699	0.3376
	(0.0021)	(0.0024)	(0.0071)	(0.0010)	(0.0019)	(0.1128)
0.15	0.1714	0.1264	1.0793	0.0473	0.0680	0.3622
	(0.0009)	(0.0010)	(0.0030)	(0.0001)	(0.0007)	(0.0505)
0.135	0.1707	0.1257	1.0770	0.0470	0.0670	0.3475
	(0.0004)	(0.0005)	(0.0015)	(0.0000)	(0.0003)	(0.0257)
0.12	0.1697	0.1246	1.0737	0.0468	0.0658	0.3164
	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0014)

Restriction	$\bar{A}$	ι	price	r_f	$E[r]-r_f$	x
Panel C: $\gamma_a = \gamma_c = 2.5$, $\psi_a = \psi_c \approx 1$						
—	0.1716	0.1210	1.0629	0.0263	0.0691	0.3611
	(0.0010)	(0.0008)	(0.0025)	(0.0004)	(0.0009)	(0.0615)
0.3	0.1716	0.1210	1.0629	0.0263	0.0691	0.3611
	(0.0010)	(0.0008)	(0.0025)	(0.0004)	(0.0009)	(0.0615)
0.2	0.1716	0.1210	1.0629	0.0263	0.0691	0.3611
	(0.0010)	(0.0008)	(0.0025)	(0.0004)	(0.0009)	(0.0615)
0.15	0.1708	0.1203	1.0610	0.0260	0.0682	0.3283
	(0.0009)	(0.0007)	(0.0022)	(0.0002)	(0.0008)	(0.0514)
0.135	0.1691	0.1189	1.0567	0.0259	0.0663	0.2494
	(0.0004)	(0.0003)	(0.0009)	(0.0000)	(0.0003)	(0.0228)
0.12	0.1675	0.1176	1.0527	0.0262	0.0646	0.1661
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0008)

Table 7: Economic quantities from the simulation for alternative preference specifications

The table gives the mean and the standard deviation (in parentheses) of key asset pricing moments and real values in the model with two investors for selected scenarios regarding risk aversion and IES. The other parameters are given in Table 2.

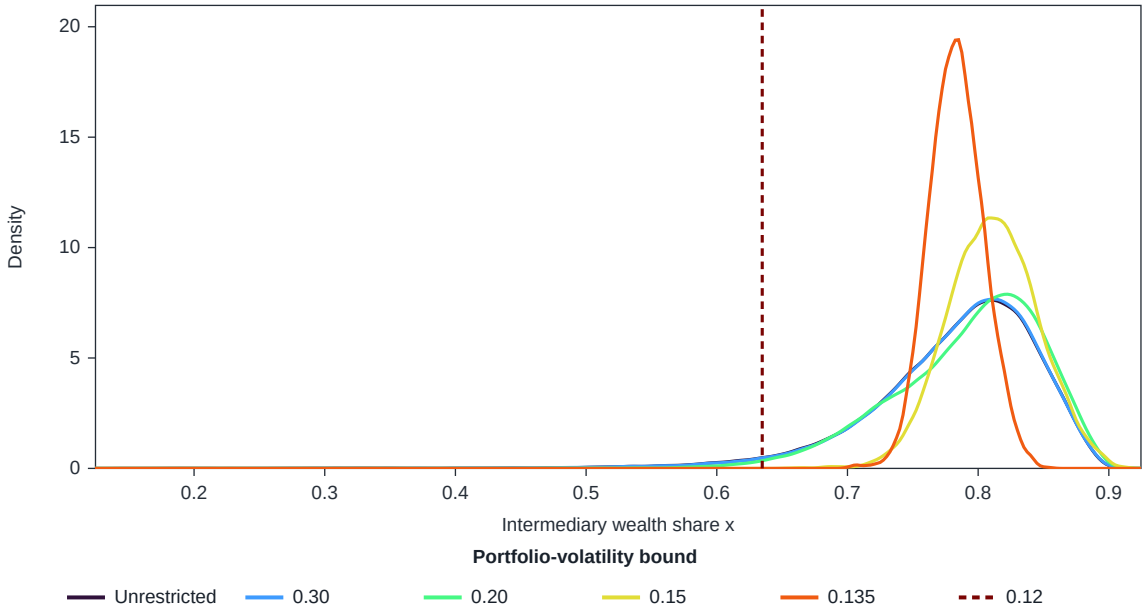


Figure 1: Stationary densities of x, two agents, EZ preferences

The figure shows stationary densities of the wealth share of the financial intermediary sector x from the simulation of the model with two agents featuring Epstein-Zin preferences. The densities are displayed for different values of the restriction $\bar{\sigma}_{\mathrm{FI}}$. The blue line refers to the unrestricted case. The model parameters are given in Table 2.

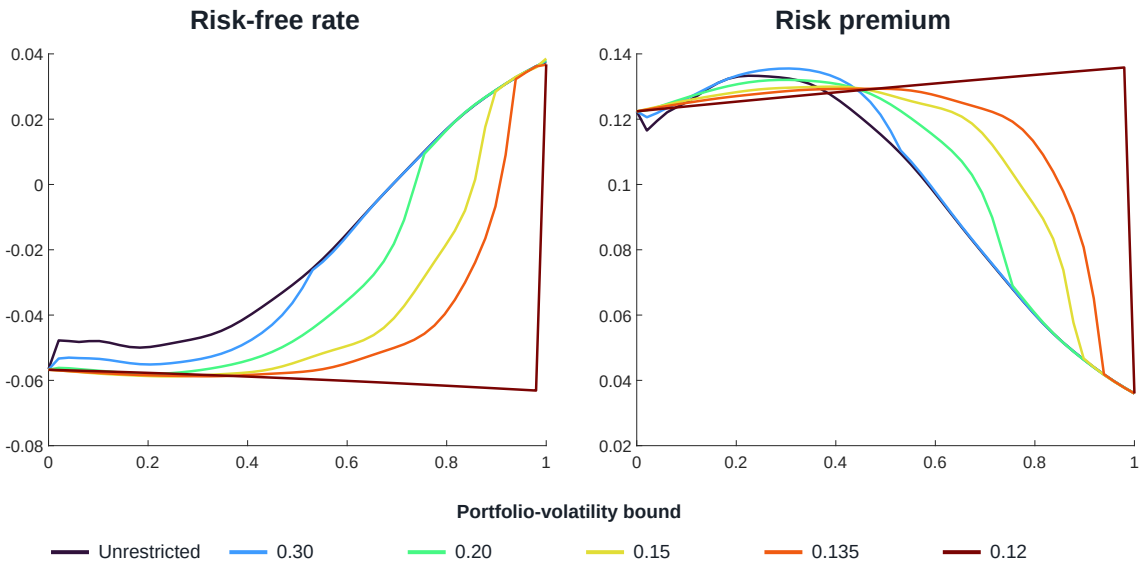


Figure 2: Risk-free rate and risk premium on consumption claim, two agents, EZ preferences

The figure gives the risk-free rate (left graph) and the expected excess return (i.e., the risk premium) on the consumption claim (right graph) in the base case model with two investors featuring EZ preferences, both represented as functions of the intermediary wealth share x. The quantities are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case. The parameters for the model are given in Table 2.

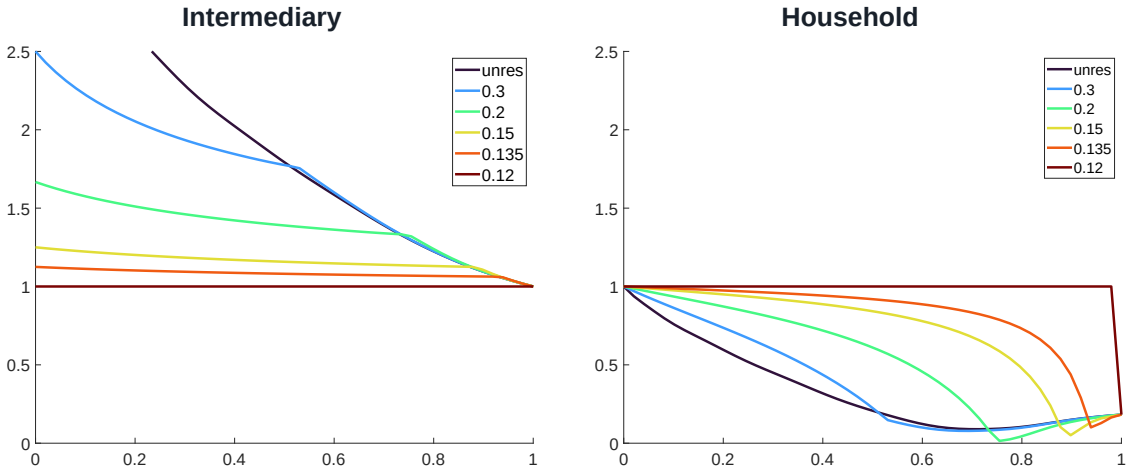


Figure 3: Portfolio weights

The figure gives the portfolio weights for the financial intermediary (left graph) and the household (right graph) in the base case model with two investors featuring EZ preferences, both as functions of the intermediary wealth share x. The weights are displayed for different values of the restriction parameter $\bar{\sigma}_{\mathrm{FI}}$. The blue line refers to the unrestricted case. The parameters for the model are given in Table 2.

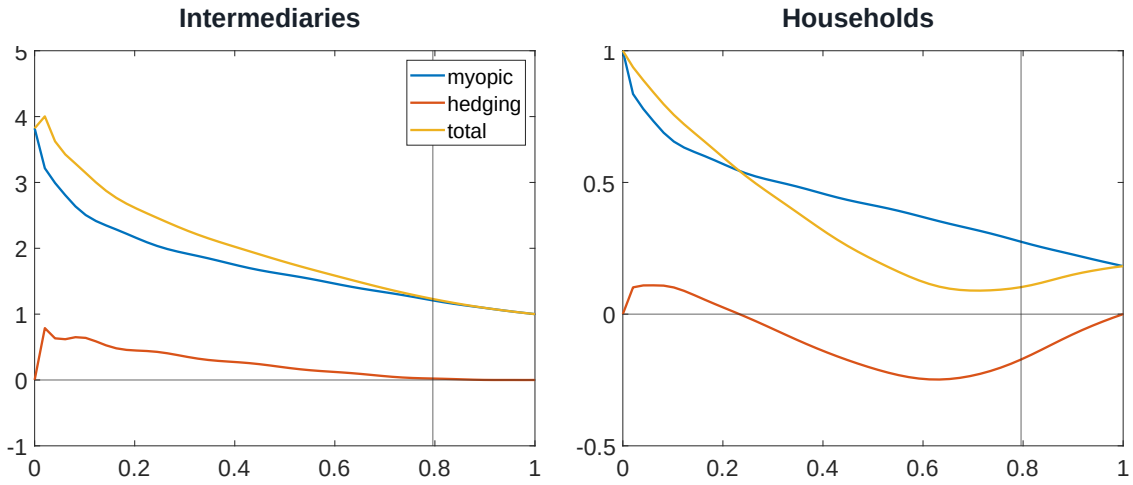


Figure 4: Demand decomposition

The figure shows the demand composition for the financial intermediary (left graph) and the household (right graph) in the base case model with two investors with EZ preferences, without a restriction on the portfolio weights. The blue line describes the myopic demand, the red line the hedging demand and the yellow line the total demand. The parameters for the model are given in Table 2.

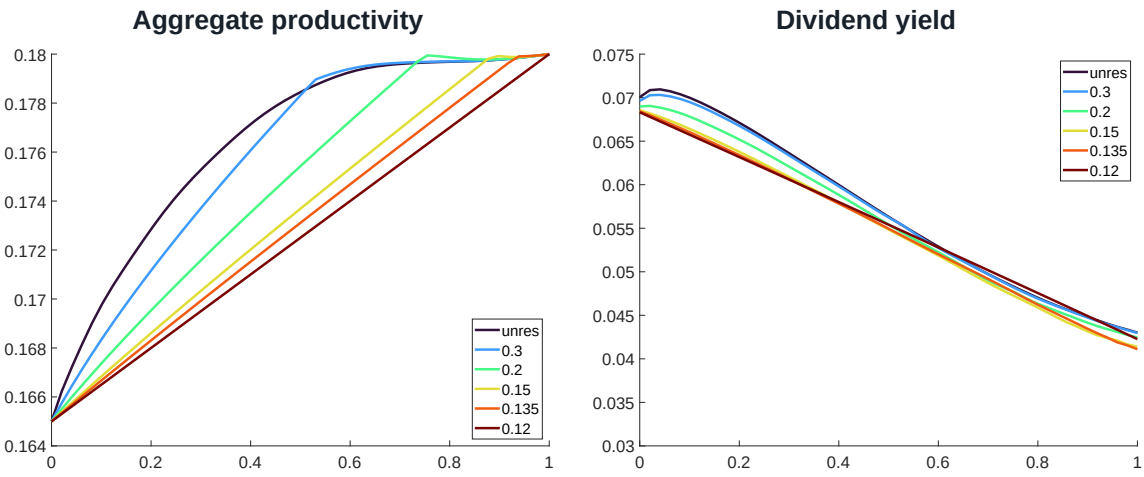


Figure 5: Aggregate productivity and dividend yield

The figure gives aggregate productivity (left graph) and the dividend yield $\frac{\bar{A}_t - \iota^* - \Phi(\iota^*)}{p_t}$ (right graph) in the base case model with two investors with EZ preferences, both as functions of the intermediary wealth share x. The quantities are displayed for different values of the restriction $\bar{\sigma}_{\text{FI}}$. The blue line represents the unrestricted case. The parameters for the model are given in Table 2.

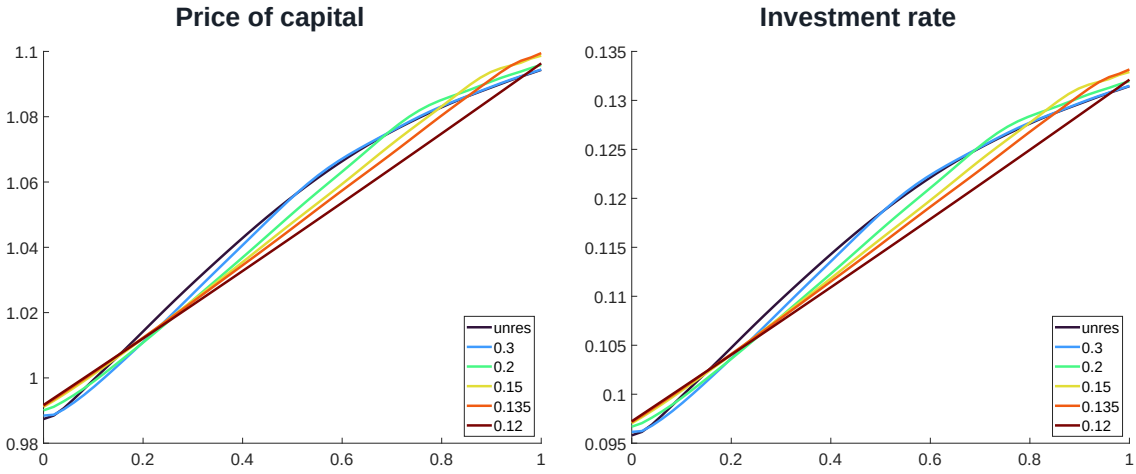


Figure 6: Price of capital and investment rate

The figure gives the price of capital (left graph) and the investment rate (right graph) in the base case model with two investors with EZ preference, both as functions of the intermediary wealth share x. The quantities are displayed for different values of the restriction parameter $\bar{\sigma}_{\mathrm{FI}}$, where the blue line refers to the unrestricted case. The parameters for the model are given in Table 2.

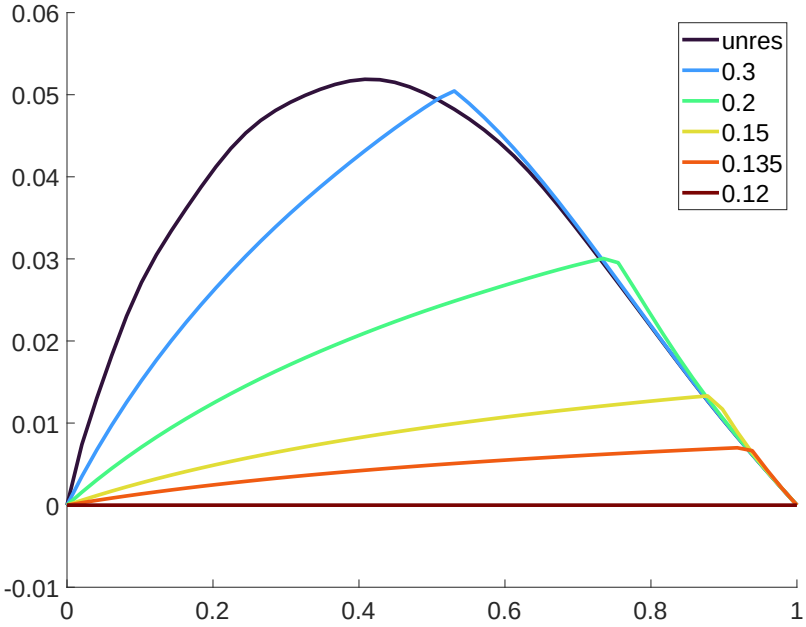


Figure 7: Volatility of price of capital

The figure gives the volatility of the price of capital in the base case model with two investors with EZ preferences as a funtion of the intermediary wealth share x. The volatility is displayed for different values of the restriction $\bar{\sigma}_{\text{FI}}$, where the blue line refers to the unrestricted case. The parameters for the model are given in Table 2.

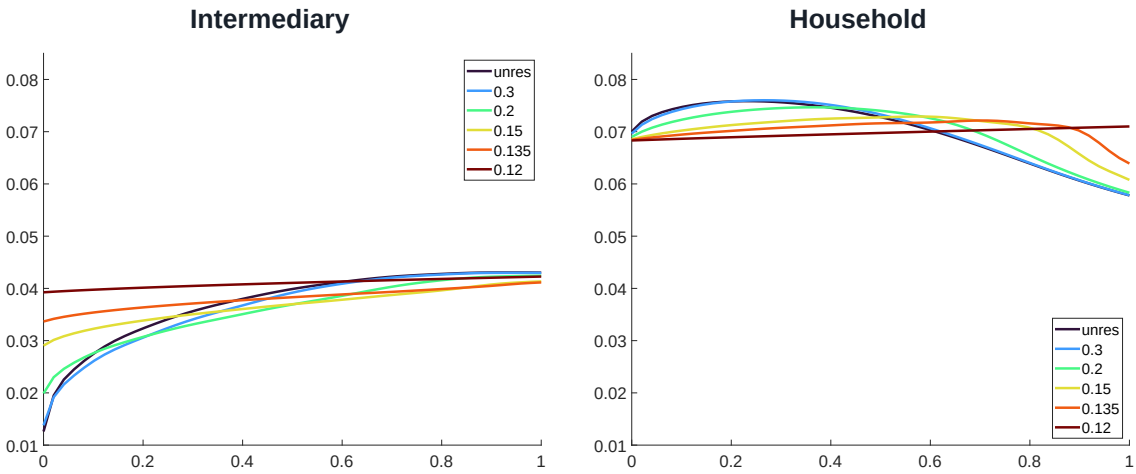


Figure 8: Consumption–wealth ratios

The figure gives the consumption–wealth ratios for the financial intermediary (left graph) and the household (right graph) in the base case model with two investors with EZ preferences, both as functions of the intermediary wealth share x. The ratios are displayed for different values of the restriction parameter $\bar{\sigma}_{\mathrm{FI}}$, where the blue line refers to the unrestricted case. The parameters for the model are given in Table 2.

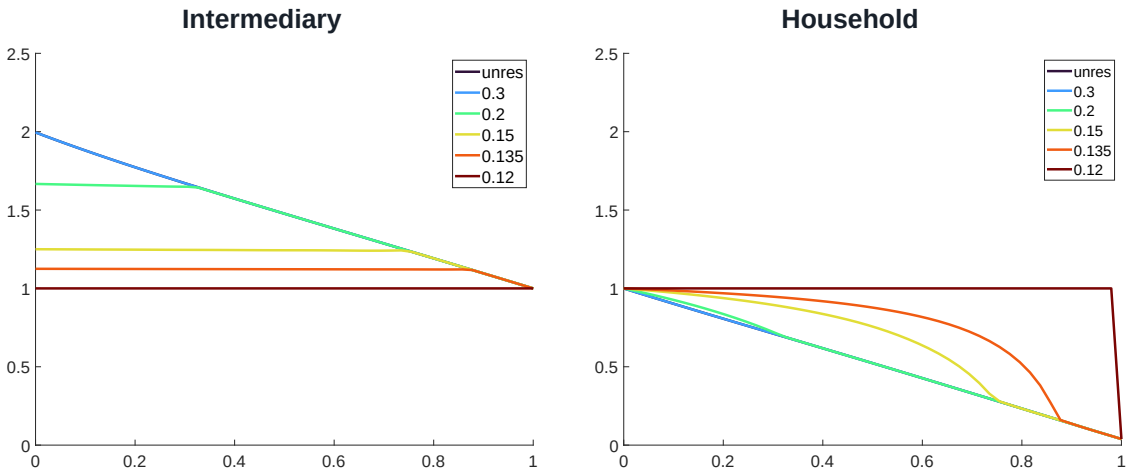


Figure 9: Portfolio weights (log utility)

The figure gives the portfolio weights of the financial intermediary (left graph) and the household (right graph) in the model with two investors and log utility, both as functions of the intermediary wealth share x. The weights are displayed for different values of the restriction $\bar{\sigma}_{\mathrm{FI}}$, where the blue line refers to the unrestricted case. The parameters for the model are given in Table 2.

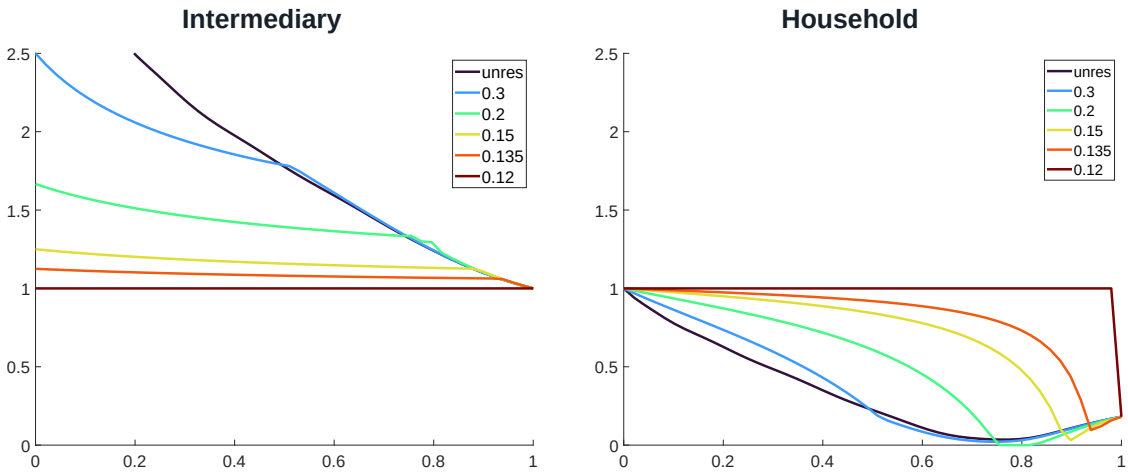


Figure 10: Portfolio weights for EZ with $\psi \approx 1$

The figure gives the portfolio weights for the financial intermediary (left graph) and the household (right graph) in the model with two investors with EZ preferences where $\psi = 1.001$ for both investors. The weights are displayed as functions of the intermediary wealth share x for different values of the restriction parameter $\bar{\sigma}_{\text{FI}}$, where the blue line refers to the unrestricted case.The parameters for the model are given in Table 2.

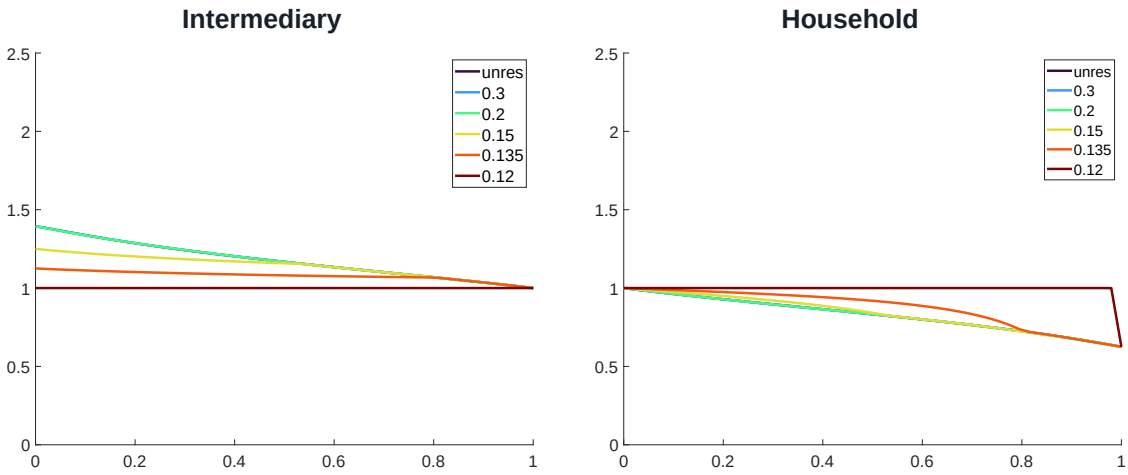


Figure 11: portfolio weights for EZ with $\gamma_a = \gamma_c = 2.5$ and $\psi_a = \psi_c = 1.5$

The figure gives the portfolio weights for the financial intermediary (left graph) and the household (right graph) in the model with two investors with EZ preferences where $\gamma_a = \gamma_c = 2.5$ and $\psi_a = \psi_c = 1.5$. The weights are displayed as functions of the intermediary wealth share x and for different values of the restriction parameter $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case. The parameters for the model are given in Table 2.

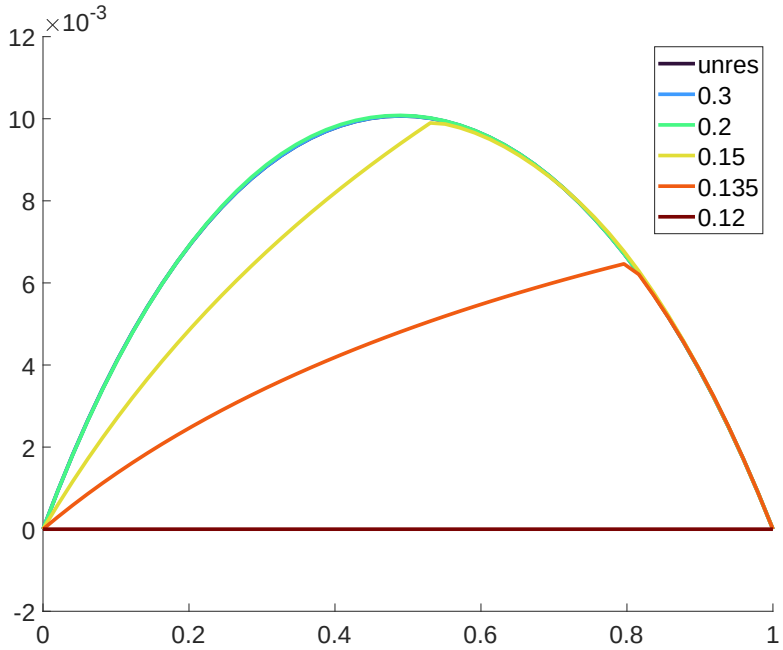


Figure 12: volatility of price σ_p for EZ with $\gamma_a = \gamma_c = 2.5$ and $\psi_a = \psi_c = 1.5$

The figure gives the volatility of the price of capital in the model with two investors with EZ preferences where $\gamma_a = \gamma_c = 2.5$ and $\psi_a = \psi_c = 1.5$. The volatility is displayed as a function of the intermediary wealth share x for different values of the restriction parameter $\bar{\sigma}_{\text{FI}}$, where the blue line refers to the unrestricted case. The parameters for the model are given in Table 2.

A The model with two investors

A.1 The law of motion of the state variables

We compute the dynamics of the wealth share of the bank using Ito's lemma:

$$\begin{aligned}
dx_t =& d\frac{W_{a,t}}{W_{a,t}+W_{c,t}} \\
=& \frac{1}{W_{a,t}+W_{c,t}} dW_{a,t} - \frac{W_{a,t}}{(W_{a,t}+W_{c,t})^2} d(W_{a,t}+W_{c,t}) \\
& - \frac{1}{(W_{a,t}+W_{c,t})^2} dW_{a,t} d(W_{a,t}+W_{c,t}) + \tfrac{1}{2} \frac{2W_{a,t}}{(W_{a,t}+W_{c,t})^3} \left(d(W_{a,t}+W_{c,t})\right)^2
\end{aligned}$$

We plug in the dynamics of wealth (Equation (2)) and the market clearing condition for capital (Equation (13)). To account for the overlapping generations, we have to add the birth and death rates $\kappa(\bar{x}-x_t)$. Then, simplifying yields

$$\begin{aligned}
dx_t =& \kappa(\bar{x}-x_t)dt \\
& + x_t(1-x_t)\Bigg[\big(\pi_{a,t}-\pi_{c,t}\big)\big(\iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t\big) \\
& \quad - \big(\pi_{a,t}-\pi_{c,t}\big)\big(x_t\pi_{a,t} + (1-x_t)\pi_{c,t}\big)\big(\sigma_K + \sigma_{p,t}\big)^2 \\
& \quad + \left(\pi_{a,t}\frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{c,t}\frac{A_c - \iota_t^* - \Phi(\iota_t^*)}{p_t}\right) + c_{c,t} - c_{a,t} \Bigg] dt \\
& + \Big[x_t(1-x_t)(\pi_{a,t}-\pi_{c,t})(\sigma_K + \sigma_{p,t}) \Big] dZ_t.
\end{aligned}$$

A.2 Deriving the HJB

The HJB-approach implies to solve the maximization problem in Equation 2:

$$\begin{aligned}
0 =& \max_{c_i,\pi_i} \{ f(C_i,V_i) + E(dV_i) \} \\
=& \max_{c_i,\pi_i} \Bigg\{ f(C_i,V_i) + \frac{\partial J}{\partial W} E\left[dW\right] + \frac{\partial V_i}{\partial x} E\left[dx\right] + \frac{1}{2}\left(\frac{\partial^2 V_i}{\partial W^2} E\left[(dW)^2\right] + \frac{\partial^2 V_i}{\partial x^2} E\left[(dx)^2\right] \right) \\
& \quad + \frac{\partial^2 V_i}{\partial W \partial x} E\left[dW\,dx\right] \Bigg\} \\
=& \max_{c_i,\pi_i} \Bigg\{ f(C_i,V_i) + \frac{\partial V_i}{\partial W} W_{i,t} \left[r_t + \pi_{i,t}\left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_{p,t} - r_t \right) - \tfrac{C_{i,t}}{W_{i,t}} \right] \\
& \quad + \frac{\partial V_i}{\partial x}\mu_x + \frac{1}{2}\left(\frac{\partial^2 V_i}{\partial W^2} W_{i,t}^2 \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 + \frac{\partial^2 V_i}{\partial x^2}\sigma_x^2 \right) + \frac{\partial^2 V_i}{\partial W \partial x} W_{i,t}\pi_{i,t}\sigma_x(\sigma_K + \sigma_{p,t}) \Bigg\}
\end{aligned}$$

We use the usual guess for the indirect utility:

$$V_i = \frac{W_i^{1-\gamma_i}}{1-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} \tag{a.1}$$

where $G_i(x,K)$ describes the Value function of agent i. Plugging this in and simplifying gives

$$\begin{aligned}\rho_i + \kappa = \max_{\pi_{i,t}, c_{i,t}} \Bigg\{ & G_{i,t} + (1-\tfrac{1}{\psi_i}) \left(r_t + \pi_{i,t} \left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t \right) - c_{i,t} \right) \\ & - \tfrac{1}{\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \mu_{x,t} - \tfrac{1}{2}(1-\tfrac{1}{\psi_i}) \gamma_i \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 \\ & - \tfrac{1}{2}\tfrac{1}{\psi_i} \left((\tfrac{1-\gamma_i}{1-\psi_i} - 1) \frac{(G_{i,t,x})^2}{G_{i,t}^2} + \frac{G_{i,t,xx}}{G_{i,t}} \right) \sigma_{x,t}^2 - \frac{1-\gamma_i}{\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \pi_{i,t} \sigma_{x,t} (\sigma_K + \sigma_{p,t}) \Bigg\}.\end{aligned}$$

The first-order condition gives the optimality of the consumption:

$$\frac{\partial f}{\partial C} = \frac{\partial V_i}{\partial W} \tag{a.2}$$

Deriving the Epstein-Zinn aggregator

$$f(C_i, V_i) = \frac{1-\gamma_i}{1-\frac{1}{\psi_i}} V_i \left[\left(\frac{C_i}{[(1-\gamma_i)V_i]^{\frac{1}{1-\gamma_i}}} \right)^{1-\frac{1}{\psi_i}} - \rho_i \right] \tag{a.3}$$

$$= \frac{C_i^{1-\frac{1}{\psi_i}}}{1-\frac{1}{\psi_i}} [(1-\gamma_i)V_i]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} - \frac{1-\gamma_i}{1-\frac{1}{\psi_i}} V_i \rho_i \tag{a.4}$$

with respect to the consumption yields

$$f_C(C_i, V_i) = [(1-\gamma_i)V_i]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} C_i^{-\frac{1}{\psi_i}}$$

and the derivative with respect to W_i of our guess in Equation (b.27) for the indirect utility is

$$\frac{\partial V_i}{\partial W} = W_i^{-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}}. \tag{a.5}$$

Hence we can plug in everything in (a.2) and simplify to get the optimal consumption:

$$\left[(1-\gamma_i) \frac{W_i^{1-\gamma_i}}{1-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} \right]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} C_i^{-\frac{1}{\psi_i}} = W^{-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} \tag{a.6}$$

$$C_i = W_i G_i(x) \tag{a.7}$$

Hence, the value function is also the consumption wealth ratio. The aggregator becomes

$$\begin{aligned} f(C_i, V_i) = & \frac{1}{1-\frac{1}{\psi_i}} (W_i G_i(x))^{1-\frac{1}{\psi_i}} W_i^{(1-\gamma_i)\left(1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}\right)} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}\left(1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}\right)} \\ & - \frac{\rho_i}{1-\frac{1}{\psi_i}} W_i^{1-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} \\ = & \frac{1}{1-\frac{1}{\psi_i}} W_i^{1-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} (G_i(x) - \rho_i) \end{aligned}$$

Plugging this in and simplifying, we get

$$\rho_i + \kappa = \max_{\pi_{i,t}} \Bigg\{ G_i + (1 - \tfrac{1}{\psi_i}) \left(r_t + \pi_i \left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_{p,t} - r_t \right) - \tfrac{C_i}{W_i} \right) \\ - \tfrac{1}{\psi_i} \frac{G_{i,x}}{G_i} \mu_x - \tfrac{1}{2}(1 - \tfrac{1}{\psi_i}) \gamma_i \pi_i^2 (\sigma_K + \sigma_{p,t})^2 - \tfrac{1}{2} \tfrac{1}{\psi_i} \left((\tfrac{1-\gamma_i}{1-\psi_i} - 1) \frac{(G_{i,x})^2}{G_i^2} + \frac{G_{i,xx}}{G_i} \right) \sigma_x^2 \\ - \frac{1 - \gamma_i}{\psi_i} \frac{G_{i,x}}{G_i} \pi_i \sigma_x (\sigma_K + \sigma_{p,t}) \Bigg\} \tag{a.8}$$

Deriving this equation with respect to $\pi_{i,t}$ and solving for that unknown gives the first-order condition for the portfolio weights:

$$\pi_{i,t} = \frac{1}{\gamma_i} \frac{\left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_t^p - r_t \right)}{(\sigma_K + \sigma_t^p)^2} + \frac{1}{\gamma_i} \frac{1 - \gamma_i}{1 - \psi_i} \left(\frac{G_{i,x}}{G_i} \frac{(\sigma_K + \sigma_t^p)\sigma_x}{(\sigma_K + \sigma_t^p)^2} \right) \tag{a.9}$$

A.3 Certainty-equivalent consumption

In order to compare the well-fare of the agents in the different model specifications, we calculate the certainty-equivalent consumption.

The certainty-equivalent (CE_i) consumption is defined as the constant consumption level that gives the same lifetime utility as the actual consumption stream[2]. Therefore, we need to solve

$$V_i(W_i, x) = \frac{CE_i^{1-\gamma_i}}{1 - \gamma_i} \tag{a.10}$$

Plugging in the guess for the indirect utility (Equation (a.1)) and simplifying gives

$$CE_i = W_i G_i(x)^{\frac{1}{1-\psi_i}}. \tag{a.11}$$

B Calculations to solve the model with three investors

B.1 Description of the three-agent model

Agents. The economy is populated by three agents indexed by

$$i \in \{a, b, c\}, \tag{b.12}$$

where type a agents correspond to *Funds*, type b agents to *Bank Holding Companies* and type c agents to *Households*. So the type a and type b agents make up the *bank* (type a) in the two-agent model. Again, agents exhibit the following ordering for the *risk aversion* parameter γ_i such that

$$\gamma_a < \gamma_b < \gamma_c. \tag{b.13}$$

Again, all agents exhibit the same *intertemporal elasticity of substitution*, i.e. $\psi_a = \psi_b = \psi_c = \psi$.

As before, agents have different productivity levels, and we assume

$$A_a \geq A_b > A_c. \tag{b.14}$$

The policy functions and optimization problem of each individual agent are similar to before (see Equation (2)).

[2]We can *ignore* the continuation value and the impact of the IES in this case because there are no consumption surprises and no intertemporal substitution of the consumption is constant.

Production Sector. The production sector (described in Section 2.2) remains the same, with the only exceptions that Equation (7) now becomes

$$W_{a,t} + W_{b,t} + W_{c,t} = p_t K_t, \tag{b.15}$$

and Equation (10) changes to

$$\overline{A}_t = \pi_{a,t} x_t y_t A_a + \pi_{b,t} x_t (1 - y_t) A_b + \pi_{c,t} (1 - x_t) A_c. \tag{b.16}$$

State Variables. Analogously to above, each agent is fully characterized by the joint *wealth* levels $W_{a,t}, W_{b,t}$ and $W_{c,t}$. Agents jointly interact through market clearing so that they need to know the decisions of all other agents as via market clearing this will affect their own decisions.

Following Kargar (2021, p. 513), we use two wealth shares as state variables:

$$x_t \equiv \frac{W_{a,t} + W_{b,t}}{W_{a,t} + W_{b,t} + W_{c,t}} \qquad\qquad y_t \equiv \frac{W_{a,t}}{W_{a,t} + W_{b,t}} \tag{b.17}$$

such that x_t is the overall wealth share of the a and b agents, which we can think of as the share of the total financial sector in the economy, and y_t is the share of the type a agents within the financial sector. Thus, the wealth share of type a agents is given by $x_t y_t$, the wealth share of type b agents by $x_t(1 - y_t)$ and that of type c agents by $(1 - x_t)$. The laws of motion of the state variables will be shown below.

Applying Ito's Lemma on (b.17) and using Equations (2) and (11) yields

$$\begin{aligned}
dx_t =& \kappa(\bar{x} - x_t)dt \\
&+ x_t(1 - x_t)\Bigg[\left(\iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t - (\sigma_K + \sigma_{p,t})^2\right)\left(y_t \pi_{a,t} + (1 - y_t)\pi_{b,t} - \pi_{c,t}\right) \\
&\quad - \left(y_t c_{a,t} + (1 - y_t) c_{b,t} - c_{c,t}\right) \\
&\quad + \left(y_t \pi_{a,t} \frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} + (1 - y_t)\pi_{b,t} \frac{A_b - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{c,t} \frac{A_c - \iota_t^* - \Phi(\iota_t^*)}{p_t} \right) \Bigg] dt \\
&+ x_t(1 - x_t)\left[y_t \pi_{a,t} + (1 - y_t)\pi_{b,t} - \pi_{c,t}\right](\sigma_K + \sigma_{p,t}) dZ_t,
\end{aligned} \tag{b.18}$$

where the drift coefficient is denoted by $\mu_{x,t}$ and the diffusion coefficient by $\sigma_{x,t}$.

Analogously, for y_t it follows that

$$\begin{aligned}
dy_t =& \kappa(\bar{y} - y_t)dt \\
&+ y_t \Bigg[(1 - y_t)\Big[\left(\pi_{a,t} - \pi_{b,t}\right)\left(\iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t\right) \\
&\quad - \left(\pi_{a,t} - \pi_{b,t}\right)\left(y_t \pi_{a,t} + (1 - y_t)\pi_{b,t}\right)\left(\sigma_K + \sigma_{p,t}\right)^2 \Big] \\
&\quad + (1 - y_t)\left(\pi_{a,t} \frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{b,t} \frac{A_b - \iota_t^* - \Phi(\iota_t^*)}{p_t} \right) + c_{b,t} - c_{a,t} \Bigg] dt \\
&+ \Big[y_t(1 - y_t)(\pi_{a,t} - \pi_{b,t})(\sigma_K + \sigma_{p,t}) \Big] dZ_t.
\end{aligned} \tag{b.19}$$

Likewise, the drift coefficient is denoted by $\mu_{y,t}$ and the diffusion coefficient by $\sigma_{y,t}$. The derivations of dx_t and dy_t are in the appendix in section B.2.

However, in x_t and y_t the drift ($\mu_{p,t}$) and diffusion ($\sigma_{p,t}$) coefficient of the price p_t is still unknown. As everything, apart from the constants, is a function of the state variables x_t and y_t, we use this fact to

obtain the following for $\mu_{p,t}$ and $\sigma_{p,t}$

$$\frac{dp_t(x,y)}{p_t(x,y)} = \underbrace{\left(\frac{p_{t,x}}{p}\mu_{x,t} + \frac{p_{t,y}}{p_t}\mu_{y,t} + \frac{p_{t,xy}}{p_t}\sigma_{x,t}\sigma_{y,t} + 0.5\frac{p_{t,xx}}{p_t}\sigma_{x,t}^2 + 0.5\frac{p_{t,yy}}{p_t}\sigma_{y,t}^2\right)}_{=\mu_{p,t}} dt \tag{b.20}$$
$$+ \underbrace{\left(\frac{p_{t,x}}{p_t}\sigma_{x,t} + \frac{p_{t,y}}{p_t}\sigma_{y,t}\right)}_{=\sigma_{p,t}} dZ_t.$$

Market Clearing Analogously to the two-agent model, we can write the market clearing conditions in terms of the state variables x and y. For the capital market, the condition becomes

$$\pi_{a,t}x_t y_t + \pi_{b,t}x_t(1-y_t) + \pi_{c,t}(1-x_t) = 1. \tag{b.21}$$

The bond market clears if the following holds

$$(1-\pi_{a,t})x_t y_t + (1-\pi_{b,t})x_t(1-y_t) + (1-\pi_{c,t})(1-x_t) = 0. \tag{b.22}$$

Finally, the goods market clears when the following condition is satisfied:

$$\overline{A}_t = p_t\big[c_{a,t}x_t y_t + c_{b,t}x_t(1-y_t) + c_{c,t}(1-x_t)\big] + \iota^* + \Phi(\iota^*). \tag{b.23}$$

HJB equations. Using (b.18) and (b.19) on (2), the HJB for each agent is given by the following where $G_{i,t} \equiv G_i(t, x_t, y_t)$ is the Value Function of agent i (The exact derivation is in the appendix in section B.3)

$$\begin{aligned}
\rho_i + \kappa = \max_{\pi_{i,t}, c_{i,t}} \Bigg\{ & G_{i,t} + (1-\tfrac{1}{\psi_i})\left(r_t + \pi_{i,t}\left(\frac{A_i - \iota_t^* - \Phi(\iota_t^*)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K\sigma_{p,t} - r_t\right) - c_{i,t}\right) \\
& - \tfrac{1}{\psi_i}\left(\frac{G_{i,t,x}}{G_{i,t}}\mu_{x,t} + \frac{G_{i,t,y}}{G_{i,t}}\mu_{y,t}\right) - \tfrac{1}{2}(1-\tfrac{1}{\psi_i})\gamma_i\pi_{i,t}^2(\sigma_K + \sigma_{p,t})^2 \\
& - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i} - 1)\frac{(G_{i,t,x})^2}{G_{i,t}^2} + \frac{G_{i,t,xx}}{G_{i,t}}\right)\sigma_{x,t}^2 \\
& - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i} - 1)\frac{(G_{i,t,y})^2}{G_{i,t}^2} + \frac{G_{i,t,yy}}{G_{i,t}}\right)\sigma_{y,t}^2 \\
& - \tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i} - 1)\frac{G_{i,t,x}G_{i,t,y}}{G_{i,t}^2} + \frac{G_{i,t,xy}}{G_{i,t}}\right)\sigma_{x,t}\sigma_{y,t} \\
& - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,t,x}}{G_{i,t}}\pi_{i,t}\sigma_{x,t}(\sigma_K + \sigma_{p,t}) - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,t,y}}{G_{i,t}}\pi_{i,t}\sigma_{y,t}(\sigma_K + \sigma_{p,t})\Bigg\},
\end{aligned} \tag{b.24}$$

and optimal investment is substituted from (9).

The first-order condition for the portfolio weights gives the following form:

$$\pi_{i,t} = \frac{1}{\gamma_i}\frac{\left(\frac{A_i - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K\sigma_t^p - r_t\right)}{(\sigma_K + \sigma_t^p)^2} + \frac{1}{\gamma_i}\frac{1-\gamma_i}{1-\psi_i}\left(\frac{G_{i,x}}{G_i}\frac{(\sigma_K + \sigma_t^p)\sigma_x}{(\sigma_K + \sigma_t^p)^2} + \frac{G_{i,y}}{G_i}\frac{(\sigma_K + \sigma_t^p)\sigma_y}{(\sigma_K + \sigma_t^p)^2}\right) \tag{b.25}$$

The portfolio weight has the myopic and intertemporal hedging components of Merton (1973), as in Kargar (2021, p. 515). The first term is the myopic demand for capital which is the same for investors with logarithmic preferences. The second term is the hedging demand for changes in the state variables.

B.2 Law of motions of the state variables

We calculate the dynamics of the wealth share of intermediaries using Ito's Lemma:

$$\begin{aligned}
dx_t =& d\,\frac{W_{a,t}+W_{b,t}}{W_{a,t}+W_{b,t}+W_{c,t}} \\
=& \frac{1}{W_{a,t}+W_{b,t}+W_{c,t}} d(W_{a,t}+W_{b,t}) - \frac{W_{a,t}+W_{b,t}}{(W_{a,t}+W_{b,t}+W_{c,t})^2} d(W_{a,t}+W_{b,t}+W_{c,t}) \\
& - \frac{1}{(W_{a,t}+W_{b,t}+W_{c,t})^2} d(W_{a,t}+W_{b,t}+W_{c,t})\, d(W_{a,t}+W_{b,t}) \\
& + \tfrac{1}{2} \frac{2(W_{a,t}+W_{b,t})}{(W_{a,t}+W_{b,t}+W_{c,t})^3} \left((W_{a,t}+W_{b,t}+W_{c,t})\right)^2
\end{aligned}$$

The next step is to plug in the wealth dynamics (i.e., the budget constraint in Equation (2)) and use the market clearing condition in Equation (12). Adding $\kappa(\bar{x}-x_t)$ to account for the death and birth in the overlapping generations set-up and simplifying yields

$$\begin{aligned}
dx_t =& \kappa(\bar{x}-x_t)dt \\
& + x_t(1-x_t)\Bigg[\left(\iota_t^* - \delta + \mu_{p,t} + \sigma_K\sigma_{p,t} - r_t - (\sigma_K+\sigma_{p,t})^2\right)\left(y_t\pi_{a,t} + (1-y_t)\pi_{b,t} - \pi_{c,t}\right) \\
& \quad - \left(y_t c_{a,t} + (1-y_t)c_{b,t} - c_{c,t}\right) \\
& \quad + \left(y_t\pi_{a,t}\frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} + (1-y_t)\pi_{b,t}\frac{A_b - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{c,t}\frac{A_c - \iota_t^* - \Phi(\iota_t^*)}{p_t}\right)\Bigg] dt \\
& + x_t(1-x_t)\left[y_t\pi_{a,t} + (1-y_t)\pi_{b,t} - \pi_{c,t}\right](\sigma_K+\sigma_{p,t})dZ_t,
\end{aligned} \tag{b.26}$$

We calculate the dynamics of the wealth share within the intermediaries using Ito's Lemma:

$$\begin{aligned}
dy_t =& d\frac{W_{a,t}}{W_{a,t}+W_{b,t}} \\
=& \frac{1}{W_{a,t}+W_{b,t}} dW_{a,t} - \frac{W_{a,t}}{(W_{a,t}+W_{b,t})^2} d(W_{a,t}+W_{b,t}) \\
& - \frac{1}{(W_{a,t}+W_{b,t})^2} dW_{a,t} d(W_{a,t}+W_{b,t}) + \tfrac{1}{2}\frac{2W_{a,t}}{(W_{a,t}+W_{b,t})^3}\left(d(W_{a,t}+W_{b,t})\right)^2
\end{aligned}$$

Similar to the dynamics of x above, we plug in the dynamics of the wealth (Equation (2)) and the market clearing condition for capital (Equation (12)). To account for the overlapping generations, we have to add the birth and death rates $\kappa(\bar{y}-y_t)$. Then, simplifying yields

$$\begin{aligned}
dy_t =& \kappa(\bar{y}-y_t)dt \\
& + y_t(1-y_t)\Bigg[\left((\pi_{a,t}-\pi_{b,t})(\iota_t - \delta + \mu_p + \sigma_K\sigma_{p,t} - r_t)\right. \\
& \quad \left. - (\pi_{a,t}-\pi_{b,t})(y\pi_{a,t} + (1-y)\pi_{b,t})(\sigma_K+\sigma_{p,t})^2\right) \\
& \quad + \left(\pi_{a,t}\frac{A_a - \iota - \Phi(\iota)}{p_t} - \pi_{b,t}\frac{A_b - \iota - \Phi(\iota)}{p_t}\right) + c_b - c_a\Bigg] dt \\
& + y_t(1-y_t)(\pi_{a,t}-\pi_{b,t})(\sigma_K+\sigma_{p,t})dZ_t
\end{aligned}$$

B.3 Derivation of the HJB

The HJB-approach is used to solve the maximization problem in Equation (2):

$$
\begin{aligned}
0 =& \max_{c_i,\pi_i} \{f(C_i,V_i) + E(dV_i)\} \\
=& \max_{c_i,\pi_i} \Bigg\{ f(C_i,V_i) + \frac{\partial J}{\partial W} E\left[dW\right] + \frac{\partial V_i}{\partial x} E\left[dx\right] + \frac{\partial V_i}{\partial y} E\left[dy\right] \\
&+ \frac{1}{2}\left(\frac{\partial^2 V_i}{\partial W^2} E\left[(dW)^2\right] + \frac{\partial^2 V_i}{\partial x^2} E\left[(dx)^2\right] + \frac{\partial^2 V_i}{\partial y^2} E\left[(dy)^2\right] \right) \\
&+ \frac{\partial^2 V_i}{\partial W \partial x} E\left[dW dx\right] + \frac{\partial^2 V_i}{\partial W \partial y} E\left[dW dy\right] + \frac{\partial^2 V_i}{\partial x \partial y} E\left[dx dy\right] \Bigg\} \\
=& \max_{c_i,\pi_i} \Bigg\{ f(C_i,V_i) + \frac{\partial V_i}{\partial W} W_{i,t} \left[r_t + \pi_{i,t}\left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_{p,t} - r_t \right) - \tfrac{C_{i,t}}{W_{i,t}} \right] \\
&+ \frac{\partial V_i}{\partial x}\mu_x + \frac{\partial V_i}{\partial y}\mu_y + \frac{1}{2}\left(\frac{\partial^2 V_i}{\partial W^2} W_{i,t}^2 \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 + \frac{\partial^2 V_i}{\partial x^2}\sigma_x^2 + \frac{\partial^2 V_i}{\partial y^2}\sigma_y^2 \right) \\
&+ \frac{\partial^2 V_i}{\partial W \partial x} W_{i,t}\pi_{i,t}\sigma_x(\sigma_K + \sigma_{p,t}) + \frac{\partial^2 V_i}{\partial W \partial y} W_{i,t}\pi_{i,t}\sigma_y(\sigma_K + \sigma_{p,t}) + \frac{\partial^2 V_i}{\partial x \partial y}\sigma_x\sigma_y \Bigg\}
\end{aligned}
$$

We use the usual guess for the indirect utility:

$$
V_i = \frac{W_i^{1-\gamma_i}}{1-\gamma_i} G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}} \tag{b.27}
$$

where $G_i(x,y,K)$ describes the Value function of agent i. Plugging this in gives

$$
\begin{aligned}
\rho_i + \kappa = \max_{\pi_{i,t}, c_{i,t}} \Bigg\{ & G_{i,t} + (1 - \tfrac{1}{\psi_i}) \left(r_t + \pi_{i,t}\left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t \right) - c_{i,t} \right) \\
& - \tfrac{1}{\psi_i}\left(\frac{G_{i,t,x}}{G_{i,t}}\mu_{x,t} + \frac{G_{i,t,y}}{G_{i,t}}\mu_{y,t} \right) - \tfrac{1}{2}(1 - \tfrac{1}{\psi_i})\gamma_i \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 \\
& - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i} - 1)\frac{(G_{i,t,x})^2}{G_{i,t}^2} + \frac{G_{i,t,xx}}{G_{i,t}} \right)\sigma_{x,t}^2 \\
& - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i} - 1)\frac{(G_{i,t,y})^2}{G_{i,t}^2} + \frac{G_{i,t,yy}}{G_{i,t}} \right)\sigma_{y,t}^2 \\
& - \tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i} - 1)\frac{G_{i,t,x}G_{i,t,y}}{G_{i,t}^2} + \frac{G_{i,t,xy}}{G_{i,t}} \right)\sigma_{x,t}\sigma_{y,t} \\
& - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,t,x}}{G_{i,t}}\pi_{i,t}\sigma_{x,t}(\sigma_K + \sigma_{p,t}) - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,t,y}}{G_{i,t}}\pi_{i,t}\sigma_{y,t}(\sigma_K + \sigma_{p,t}) \Bigg\}.
\end{aligned} \tag{b.28}
$$

The first-order condition gives the optimality of the consumption:

$$
\frac{\partial f}{\partial C} = \frac{\partial V_i}{\partial W} \tag{b.29}
$$

Deriving the Epstein-Zinn aggregator

$$f(C_i,V_i) = \frac{1-\gamma_i}{1-\frac{1}{\psi_i}}V_i\left[\left(\frac{C_i}{[(1-\gamma_i)V_i]^{\frac{1}{1-\gamma_i}}}\right)^{1-\frac{1}{\psi_i}} - \rho_i\right] \tag{b.30}$$

$$= \frac{C_i^{1-\frac{1}{\psi_i}}}{1-\frac{1}{\psi_i}}\left[(1-\gamma_i)V_i\right]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} - \frac{1-\gamma_i}{1-\frac{1}{\psi_i}}V_i\rho_i \tag{b.31}$$

with respect to the consumption yields

$$f_C(C_i,V_i) = \left[(1-\gamma_i)V_i\right]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} C_i^{-\frac{1}{\psi_i}}$$

and the derivative with respect to W_i of our guess in Equation (b.27) for the indirect utility is

$$\frac{\partial V_i}{\partial W} = W_i^{-\gamma_i}G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}}. \tag{b.32}$$

Hence we can plug in everything in (b.29) and simplify to get the optimal consumption:

$$\left[(1-\gamma_i)\frac{W_i^{1-\gamma_i}}{1-\gamma_i}G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}}\right]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} C_i^{-\frac{1}{\psi_i}} = W^{-\gamma_i}G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}} \tag{b.33}$$

$$C_i = W_iG_i(x,y) \tag{b.34}$$

Hence, the value function is also the consumption, wealth ratio. The aggregator becomes

$$\begin{aligned} f(C_i,V_i) =& \frac{1}{1-\frac{1}{\psi_i}}(W_iG_i(x,y))^{1-\frac{1}{\psi_i}}W_i^{(1-\gamma_i)\left(1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}\right)}G_i^{\frac{1-\gamma_i}{1-\psi_i}\left(1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}\right)} \\ & - \frac{\rho_i}{1-\frac{1}{\psi_i}}W_i^{1-\gamma_i}G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}} \\ =& \frac{1}{1-\frac{1}{\psi_i}}W_i^{1-\gamma_i}G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}}(G_i-\rho_i) \end{aligned}$$

Plugging this in and simplifying, we get

$$\begin{aligned} \rho_i+\kappa = \max_{\pi_{i,t}}\Bigg\{ & G_i + (1-\tfrac{1}{\psi_i})\left(r_t + \pi_i\left(\frac{A_i-\iota-\Phi(\iota)}{p_t}+\iota-\delta+\mu_p+\sigma_K\sigma_{p,t}-r_t\right) - \tfrac{C_i}{W_i}\right) \\ & - \tfrac{1}{\psi_i}\left(\frac{G_{i,x}}{G_i}\mu_x + \frac{G_{i,y}}{G_i}\mu_y\right) - \tfrac{1}{2}(1-\tfrac{1}{\psi_i})\gamma_i\pi_i^2(\sigma_K+\sigma_{p,t})^2 \\ & - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i}-1)\frac{(G_{i,x})^2}{G_i^2} + \frac{G_{i,xx}}{G_i}\right)\sigma_x^2 - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i}-1)\frac{(G_{i,y})^2}{G_i^2} + \frac{G_{i,yy}}{G_i}\right)\sigma_y^2 \\ & - \tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i}-1)\frac{G_{i,x}G_{i,y}}{G_i^2} + \frac{G_{i,xy}}{G_i}\right)\sigma_x\sigma_y \\ & - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,x}}{G_i}\pi_i\sigma_x(\sigma_K+\sigma_{p,t}) - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,y}}{G_i}\pi_i\sigma_y(\sigma_K+\sigma_{p,t})\Bigg\} \end{aligned} \tag{b.35}$$

Deriving this equation with respect to $\pi_{i,t}$ and solving for that unknown gives the first-order condition

for the portfolio weights:

$$\pi_{i,t} = \frac{1}{\gamma_i} \frac{\left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_t^p - r_t\right)}{(\sigma_K + \sigma_t^p)^2} + \frac{1}{\gamma_i} \frac{1-\gamma_i}{1-\psi_i} \left(\frac{G_{i,x}}{G_i} \frac{(\sigma_K + \sigma_t^p)\sigma_x}{(\sigma_K + \sigma_t^p)^2} + \frac{G_{i,y}}{G_i} \frac{(\sigma_K + \sigma_t^p)\sigma_y}{(\sigma_K + \sigma_t^p)^2} \right) \tag{b.36}$$

C Log Utility

Motivation. This section presents the model in terms of log utility. Log utility is the most commonly used utility function in asset pricing models. The popularity of log utility is due to the fact that a separation approach can be taken so that the value function doesn't need to be solved explicitly. However, these computational gains come at a cost, as log utility suffers from the well-known drawbacks of the equity premium and risk-free rate puzzle due to the entanglement of risk aversion and the intertemporal elasticity of substitution (willingness to substitute consumption over time) while also not incorporating a preference for early resolution of uncertainty.

To emphasize the contribution of Epstein-Zin preferences, we also solve our model under log utility. Since log utility is a degenerate parameterization of Epstein-Zin, this entails that we must write a separate modified algorithm to solve the model.

HJB. Since the model is the same, we will only lay out the lifetime utility maximisation problem. This is now given by

$$V_i = \max_{\{C_{i,s}, \pi_{i,s}, \iota_{i,s}\}_{s \in [t,\infty)}} \int_t^\infty e^{-\rho(s-t)} log\big(C(s)\big) ds \quad \text{s.t.} \quad \frac{dW_{i,t}}{W_{i,t}} = \pi_{i,t} dr_{i,t}^K + (1 - \pi_{i,t}) r_t dt - \frac{C_{i,t}}{W_{i,t}} dt.$$

For the stationary solution, the HJB for each investor is given by

$$\rho_i J(W_i, x, y) = \max_{c_i, \pi_i} \Bigg\{ \log(C_i) + J_W E[dW_i] + J_x E[dx] + J_y E[dy] + 0.5 J_{WW} E[(dW)^2] + 0.5 J_{xx} E[(dx)^2] + 0.5 J_{yy} E[(dy)^2] + J_{Wx} E[dW dx] + J_{Wy} E[dW dy] + J_{xy} E[dx dy] \Bigg\}.$$

The *initial guess* for the HJB is to separate out the level of wealth so that we postulate

$$J_i = \frac{1}{\rho_i} \log(W_i) + G_i(x, y),$$

which then yields

$$\log(W_i) + \rho_i G_i = \max_{c_i, \pi_i} \Bigg\{ \log(C_i) + \frac{1}{\rho_i} \left(r + \pi_i \big[\mu_{r_i^K} - r\big] - \frac{C_i}{W_i} \right) + G_{i,x} \mu_x + G_{i,y} \mu_y - 0.5 \frac{1}{\rho_i} \pi_i^2 (\sigma_K + \sigma_p)^2 + 0.5 G_{i,xx} \sigma_x^2 + 0.5 G_{i,yy} \sigma_y^2 + G_{i,xy} \sigma_x \sigma_y \Bigg\},$$

where

$$\mu_{r_i^K} = \frac{A_i - \iota_t^* - \Phi(\iota_t^*)}{p} + \iota^* - \delta + \mu_p + \sigma_K \sigma_p. \tag{c.37}$$

Notice that $\mu_{r_i^K}$, as in the Epstein-Zin case, depends on the unknown price function $p(x, y)$.

Optimal Policies. The first-order condition for *optimal consumption* implies

$$C_i^* = \rho_i W_i, \tag{c.38}$$

so that agents always consume a constant fraction of wealth.

The first-order condition for the *optimal portfolio weight* is

$$\pi_i^* = \frac{\mu_{r_i^K} - r}{(\sigma_K + \sigma_p)^2}. \tag{c.39}$$

Notice that ι_t^* remains unchanged and is given by (9).

The first order conditions make the computational advantage of log-utility apparent: the optimal policies can be computed without any knowledge of the value function G_i. Both for CRRA or Epstein-Zin preferences can the individual level of wealth can be substituted out, but the optimal policies will depend explicitly on the value function.

Hence, the only unknown function to be solved for is the price function which we will also achieve numerically via the same projection algorithm.

Numerical Solution. We use the same set of gridpoints as for the Epstein-Zin case. Additionally, we set $p(x, y)$ as a Chebyshev Polynomial, i.e.

$$p(x, y) = \sum_{i=1}^{N} \sum_{j=1}^{N} \theta_{i,j} T_i(x) T_j(y). \tag{c.40}$$

With log-utility, we can instantly solve for the general equilibrium as the following will show.

To obtain the policies, we need to solve the system of first order conditions along with the market clearing conditions. This yields the following system of five equations with the five unknowns π_a^*, π_b^*, π_c^*, σ_p, and $mu_p - r$.

$$\begin{aligned}
\pi_a^* &= \frac{A_a}{p(\sigma_K + \sigma_p)^2} + \frac{\iota^* - \delta + (\mu_p - r) + \sigma_K \sigma_p}{(\sigma_K + \sigma_p)^2} \\
\pi_b^* &= \frac{A_b}{p(\sigma_K + \sigma_p)^2} + \frac{\iota^* - \delta + (\mu_p - r) + \sigma_K \sigma_p}{(\sigma_K + \sigma_p)^2} \\
\pi_c^* &= \frac{A_c}{p(\sigma_K + \sigma_p)^2} + \frac{\iota^* - \delta + (\mu_p - r) + \sigma_K \sigma_p}{(\sigma_K + \sigma_p)^2} \\
1 &= xy\pi_a^* + x(1-y)\pi_b^* + (1-x)\pi_c^* \\
\sigma_p &= \frac{p_x}{p}\left[x\left(y\pi_a^* + (1-y)\pi_b^* - 1\right)(\sigma_K + \sigma_p)\right] + \frac{p_y}{p}\left[y(1-y)(\pi_a^* - \pi_b^*)(\sigma_K + \sigma_p)\right].
\end{aligned}$$

We solve the system by root-finding at all N^2 gridpoints by inserting (c.40).

We found it to be numerically more stable to solve for σ_p as a function of the other four unknowns and insert the following expression into the other four equations:

$$\sigma_p = \frac{\left[\frac{p_x}{p} x\left[y\pi_a^* + (1-y)\pi_b^* - 1\right] + \frac{p_y}{p} y(1-y)(\pi_a^* - \pi_b^*)\right]}{1 - \left(\frac{p_x}{p} \cdot x\left[y\pi_a^* + (1-y)\pi_b^* - 1\right] + \frac{p_y}{p} \cdot y(1-y)(\pi_a^* - \pi_b^*)\right)} \sigma_K.$$

Once this system has been solved, it is straightforward to solve for μ_x, σ_x, μ_y, σ_y, and μ_p. Knowing μ_p and $\mu_p - r$ then directly yields the riskless rate r. We then update the price function based on the goods market clearing condition and iterate until the parameters of the price function converge.

D Solution Algorithm

State Variables. Due to a separation of wealth approach, we can restrict the set of state variables to x_t and y_t.[3] In other words, the Value Function G_i of each agents only depends on x_t and y_t.

Auxiliary State Variable. However, a problem is that the dynamics of x_t and y_t depend on the unknown price function (through $\mu_{p,t}$ and $\sigma_{p,t}$) and the dynamics of the price function depend on x_t and y_t. This is why we will guess a price function and update the guess while simultaneously updating the guess for the value functions G_i until the program converges to the general equilibrium. Therefore, the price function $p_t(x_t, y_t)$ will serve as an auxiliary state variable.

[3] This is also shown in the derivation of the HJB equation in the appendix.

Stationary Solution. Due to the infinite time horizon, we will solve for a stationary solution so that we can drop the time dependency.

Projection Algorithm. We opt for a projection algorithm and approximate the value functions $G_a(x,y), G_b(x,y), G_c(x,y)$ and the price function $p(x,y)$ with Chebyshev polynomials. Thus, we use

$$G_i(x,y) = \sum_{k=1}^{N}\sum_{j=1}^{N} c_{k,j}^{(i)} T_k(x) T_j(y) \qquad \text{and} \qquad p(x,y) = \sum_{k=1}^{N}\sum_{j=1}^{N} \theta_{k,j} T_k(x) T_j(y),$$

where $T_k(\cdot)$ is the k-th Chebyshev polynomial of the first kind. The advantage of this functional form is that we can directly compute all derivatives. The goal is to solve for the unknown coefficients $c_{k,j}^{(i)}$ and $\theta_{k,j}$ that solve the model at a certain set of points.

We use collocation so that there are as many gridpoints as there are unknown coefficients. The collocation points are given by a simple meshgrid. Firstly, we compute the Chebyshev nodes scaled into the unit interval for both x and y individually. The meshgrid is then simply constructed by the mesh of the individual grids for x and y.

Optimality Conditions The coefficients $c_{k,j}^{(i)}$ for the value function have to satisfy the individual HJB shown in Equation (17)). The price has to satisfy the goods market clearing condition (Equation (13)).

Initialization. The initial coefficients $c_{k,j}^{(i)}$ are set to ρ_i for $k = j = 1$ and zero otherwise. The price coefficients are initialized as 1 for $k = j = 1$ and zero otherwise.

Overview Numerical Solution. The model is solved iteratively in MatLab with the *fsolve*-function until the coefficients $c_{k,j}^{(i)}$ and $\theta_{k,j}$ satisfy the optimality conditions for the value functions G_i and the price function p_t (see above). Each iteration is as follows

1. Take the price and its derivatives from the previous iteration as given. Calculate the optimal investment rate ι_t^* as in Equation (9).

2. Take the coefficients for the value functions as given from the previous iteration. Evaluate the value functions $G_i(x,y)$ and its derivatives on the gridpoints.

3. The next step is to solve for the optimal portfolio weights by using the Matlab function *fsolve*. As initialization we use $\pi_{a,t} = \pi_{b,t} = \pi_{c,t} = (\mu_{p,t} - r_t) = 1$. For $\pi_{b,t}$ and $\pi_{c,t}$, we consider the exponential of the input to ensure that the values are always positive.

   With the initial guesses for $\pi_{a,t}, \pi_{b,t}, \pi_{c,t}$ and $(\mu_{p,t} - r_t)$, we calculate $\sigma_{p,t}$ by analytically solving Equation (b.20) with the expressions for $\sigma_{x,t}$ and $\sigma_{y,t}$ from Equations (b.18) and (b.19). This gives

$$\sigma_{p,t} = \frac{\frac{p_x}{p} x(y\pi_a + (1-y)\pi_b - 1) + \frac{p_y}{p} y(1-y)(\pi_a - \pi_b)}{1 - \left(\frac{p_x}{p} x(y\pi_a + (1-y)\pi_b - 1) + \frac{p_y}{p} y(1-y)(\pi_a - \pi_b)\right)} \sigma_K \tag{d.41}$$

   We take the first-order conditions (b.25) for each investor and the market clearing for capital in Equation (12) with the expressions for $\sigma_{x,t}$ and $\sigma_{y,t}$ from Equations (b.18) and (b.19). This gives

the following system of five equations with

$$\pi_{c,t} = \frac{1}{\gamma_c}\frac{(\iota - \delta + \mu_p + \sigma_K\sigma_{p,t} - r_t)}{(\sigma_K + \sigma_{p,t})^2} + \frac{1}{\gamma_c}\frac{A_a - \iota - \Phi(\iota)}{p_t(\sigma_K + \sigma_{p,t})^2} \tag{d.42}$$
$$+ \frac{1}{\gamma_c}\frac{1-\gamma_c}{1-\psi_c}\left(\frac{G_{c,x}}{G_c}x_t\left[y\pi_{a,t} + (1-y)\pi_{b,t} - 1\right] + \frac{G_{c,y}}{G_c}y_t(1-y_t)(\pi_{a,t} - \pi_{b,t})\right)$$

$$\pi_{b,t} = \frac{1}{\gamma_b}\frac{(\iota - \delta + \mu_p + \sigma_K\sigma_{p,t} - r_t)}{(\sigma_K + \sigma_{p,t})^2} + \frac{1}{\gamma_b}\frac{A_b - \iota - \Phi(\iota)}{p_t(\sigma_K + \sigma_{p,t})^2} \tag{d.43}$$
$$+ \frac{1}{\gamma_b}\frac{1-\gamma_b}{1-\psi_b}\left(\frac{G_{b,x}}{G_b}x_t\left[y\pi_{a,t} + (1-y)\pi_{b,t} - 1\right] + \frac{G_{b,y}}{G_b}y_t(1-y_t)(\pi_{a,t} - \pi_{b,t})\right)$$

$$\pi_{a,t} = \frac{1}{\gamma_a}\frac{(\iota - \delta + \mu_p + \sigma_K\sigma_{p,t} - r_t)}{(\sigma_K + \sigma_{p,t})^2} + \frac{1}{\gamma_a}\frac{A_c - \iota - \Phi(\iota)}{p_t(\sigma_K + \sigma_{p,t})^2} \tag{d.44}$$
$$+ \frac{1}{\gamma_a}\frac{1-\gamma_a}{1-\psi_a}\left(\frac{G_{a,x}}{G_a}x_t\left[y\pi_{a,t} + (1-y)\pi_{b,t} - 1\right] + \frac{G_{a,y}}{G_a}y_t(1-y_t)(\pi_{a,t} - \pi_{b,t})\right)$$

$$1 = xy\pi_{a,t} + x(1-y)\pi_{b,t} + (1-x)\pi_{c,t} \tag{d.45}$$

with $\pi_{a,t}$, $\pi_{b,t}$, $\pi_{c,t}$ and $(\mu_{p,t} - r_t)$ as unknowns. $\sigma_{p,t}$ is given as an additional output while solving the system of equations.

4. If the restriction is considered, we check if the condition in Equation (19) holds for each grid point. If this is not the case for a gridpoint, we set $\pi_{a,t} = \frac{\bar{\sigma}_{FI}}{(\sigma_K + \sigma_{p,t})}$ on that combination of x and y and solve the system of equations from the previous step again without considering Equation (d.44).

5. With the solution from the last step, we can calculate $\mu_{x,t}$, $\sigma_{x,t}$, $\mu_{y,t}$ and $\sigma_{y,t}$ (see Equations (b.18) and (b.19)). Now, we can calculate $\mu_{p,t}$ as given in Equation (b.20).

6. The risk-free rate is obtained by subtracting the solution for $(\mu_{p,t} - r_t)$ from step 3 from $\mu_{p,t}$ as calculated in the previous step.

7. Calculate the violation of the optimality conditions.

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