Credit Default Swaps in General Equilibrium: Spillovers, Credit Spreads, and Endogenous Default*

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Abstract

This paper highlights two new effects of credit default swap markets (CDS) in a general equilibrium setting. First, when firms' cash flows are correlated, CDSs impact the cost of capital–credit spreads–and investment for all firms, even those that are not CDS reference entities. Second, when firms internalize the credit spread changes, the incentive to issue safe rather than risky bonds is fundamentally altered. Issuing safe debt requires equity holders to transfer profits from good states to bankruptcy states to ensure full repayment. Risky bonds allow equity holders to increase good state profits as much as possible because of limited liability in default. Higher credit spreads lead to more safe rather than risky debt because equity value in good states, when debts are repaid, falls. Symetrically, lower credit spreads lead to more risky rather than safe debt. CDS affect the credit spread at which firms issue risky debt, and ultimately whether firms issue defaultable securities. The direction of the tradeoff between credit spreads and firm default depends on whether CDS are used to speculate or hedge against credit risk.

1 Introduction

The global financial crisis of 2007-2008 underscored the need to better understand how financial market participants price and take risk. Credit default swaps (CDSs) are a particular type of financial instrument that market participants used with increasing regularity in the build-up to the crisis. According to the Bank for International Settlements (BIS) the notional size of the CDS market (value of all outstanding contracts) at its peak before the market crash in 2007 was $57 trillion. While that number has abated mainly as a result of the multilateral netting of contracts, its size as of the first half of 2015 was $15 trillion. Clearly the CDS market remains large and active, and continues to engender a variety of research efforts aiming to better understand the effects of CDSs on capital markets.

We investigate the general equilibrium impact of CDS markets on credit spreads[1] and the *endogenous* investment and default decisions for multiple firms, some of whom are CDS reference entities and others are not.[2] We begin by allowing firms, whose expected cash flows differ, to choose the amount of debt they wish to issue to debt financiers. Specifically, small amounts of debt can be repaid in all states and bonds are equivalent to safe assets–credit spreads will be zero. Alternatively, large amounts of debt cannot be honored in all states–credit spreads will be positive. Firm managers maximize the equity value of the firm by choosing to issue safe or risky debt depending on associated credit spread. We then introduce a market for CDSs on firm debt. The CDS market gives firm creditors access to new securities whose payouts are more aligned with their subjective beliefs. The market price of CDS contracts and the market price of debt are linked through a non-arbitrage relationship. CDSs affect the credit spread when firms issue risky debt, but not firms issue risk-free debt. Our model shows that the credit spreads of all firms, even those firms who are not CDS reference entities, are impacted by CDS trading. More importantly, we show that the CDS market alters firm incentives to issue risky rather than safe bonds depending on how CDSs affect equilibrium credit spreads.

[1]Our model has an asset with risk-free rate normalized to zero. The credit spread between a zero-coupon risky bond and the risk-free asset is the difference between the assets' yield-to-maturity. The yield-to-maturity of the risk-free asset is equal to zero. Therefore, the credit spread is equivalent to the yield-to-maturity on a risky bond.

[2]Using data from either Markit or the Depository Trust and Clearing Corporation (DTCC), there are only around 3000 firms for which a CDS contract ever exists. We do not attempt to endogenize CDS issuance in this paper; we simply take as given that the market is active only for a portion of the firms in the economy.

We build on the work by by Fostel and Geanakoplos (2012) and subsequently by Che and Sethi (2015), by showing that the collateral used to back CDS contracts re-allocates capital from the bond market to the CDS market, which changes the demand for bonds and the equilibrium credit spread required to issue risky debt. Thus, CDS trading affects real outcomes in the economy. We use the re-allocation channel to show that in equilibrium, when assets (firm production in our model) have different but correlated cash flows, an optimistic marginal buyer prices firm bonds so that the relative expected returns to investing in either bond will be equal. The equivalent expected bond returns imply that when CDSs alter the demand for one type of bond, they also alter the demand for the other, resulting in credit spread changes for all firms seeking risky-debt financing. Firms endogenously respond to lower (higher) credit spreads by increasing (decreasing) the amount of debt they issue. Thus, CDSs may increase or decrease the expected value of equity when firms issue risky-debt.

The endogenous investment and credit spreads, in our model, lead to interesting debt-financing decisions regarding the choice to issue debt that is safe rather than risky. Specifically, as credit spreads fall, the benefit to equity holders of issuing risky debt rises. Essentially, limited liability protects equity holders from default losses when cash-flows are low while preserving their return on equity when cash-flows are high and debts are repaid. The commitment to repay debt in full, in all states, can only be achieved when raising small amounts of debt, which limits production. Shareholders' alternative to safe debt is to issue risky debt. The shareholders ignore the bankruptcy states because debt holders become the residual claimants. Therefore, shareholders issue as much debt as can be repaid in states where cash flows are good, which generates higher production and profits. The cost to the firm of issuing risky debt is the credit spread. Creditors demand a positive credit spread in order to hold an asset that may default. CDS alter the credit spread that investors require to hold risky debt, and thereby alter the cost of issuing risky rather than safe debt.

We adopt a one-period, static, general equilibrium model with two states characterized by a commonly known aggregate productivity shock. The shock takes on high values in the up state and low values in the down state. For simplicity, we consider two firm types endowed with different production technologies. The technology shock is common to both firms–the production technologies generate high cash flows in the up state and low cash flows in the down state. To produce, each

firm endogenously issues non-contingent, collateralized debt. The firms know the true probability over the high and low cash-flow states, but this is unobservable.[3] We assume a portion of the cash flows from production are not pledgeable so that production generates positive profits and control rents. Debt financing comes from investors with heterogeneous beliefs about the probability of the future states. We first solve the baseline model without CDSs for equilibrium bond prices, firm investment, and characterize the states in which firms choose to issue risky or safe debt. Similarly to Che and Sethi (2015), we then extend the model to incorporate CDS contracts, where CDS purchasers must also own the underlying bond, which we call covered CDS Economies. This restriction on CDS ownership is then removed so investors are free to purchase CDS without owning the underlying bond, in what we call naked CDS economies.[4]

We begin by characterizing equilibrium based on whether the debt contracts are safe or risky. Low values of the technology shock, resulting in low expected cash flows, always result in debt contracts that are risky. Financing investment with risky debt allows the firm to produce much more and increase equity value in the high cash-flow state. Equity holders, by committing to issue safe debt, must take into account the worst case cash-flow outcome, which also limits output in the high cash-flow state. The result of lower investment also limits the residual equity claims in the high cash-flow state. For intermediate values of the technology shock, safe debt contracts are only possible when the likelihood of generating high cash-flow is sufficiently low. The greater the probability of the up-state and high cash-flow, the more equity holders wish to move their consumption to the up state. Financing investment with risky debt allows equity holders to maximize the value of equity in the up state by ignoring the down state. The remaining uninteresting case is when technology shocks are sufficiently high that debt contracts are always safe because the cash flows in the down state are always sufficient to repay debt.

We then introduce covered CDSs to study how credit spreads and the decision to

[3]Alternatively, one could assume that the state probabilities reveal a non-verifiable or non-contractible signal about cash flows.

[4]Norden and Radoeva (2013) document that there is clear firm heterogeneity in the size of the CDS market relative to the size of the underlying bond market supporting the CDS contracts. One natural way to interpret the covered CDS economy is as a CDS market that is "small" relative to the size of the underlying bond market whereby a sufficient level of investor capital remains available to purchase bonds. Naked CDS economies in our framework would correspond to very large outstanding CDS markets relative to the underlying bond market supporting those trades.

issue safe rather than risky debt change. First, consistent with Che and Sethi (2015), covered CDSs raise bond prices and lower credit spreads. Optimistic investors can hold more credit risk when using their cash as collateral to sell CDSs because of the implicit leverage embedded in CDSs.[5] The concentration of credit risk held by optimists increases the remaining supply of capital that can then finance other firm investment needs. Our analysis thus extends this mechanism to show that even when CDSs trade only on one bond, both firms' debt contracts are affected.[6] More interestingly, because investment is *endogenous* to credit spreads, firms respond by issuing more risky debt, and are more likely to default. With *fixed* investment, firms default in *fewer* states as in Che and Sethi (2015). The difference between their model and ours is that we consider not only the decision of how much debt to issue, but also whether to issue debt that is defaultable. CDSs do not trade if the firm issues safe debt. Thus, for a given technology shock in which a firm is indifferent between issuing safe or risky debt in an economy without CDSs, the firm strictly prefers to issue risky debt at lower credit spreads in the economy with covered CDSs.

The restriction that CDS buyers must own the underlying bonds is then removed and naked CDS positions are the permitted. Naked CDSs also generate spillovers even when only one firm type serves as the CDS reference entity. Pessimists increase the demand for CDSs, which raises CDS prices. Higher CDS prices reduce the amount of their own capital CDS sellers have to post to sell CDSs. The lower collateral requirement increases the embedded leverage that optimist receive and attracts more capital into derivative markets from natural bond buyers. Less investor capital is then available to fund debt irrespective of firm type. Consequently, credit spreads rise, which makes issuing risky debt less attractive compared to safe debt. The higher credit spreads reduce investment and the equity value of the firm in high cash-flow states, bringing the value of equity more in line with its value when production is financed with safe debt. The tradeoff between credit spreads and whether bonds contain credit risk, again, stands in contrast to Che and Sethi (2015). Thus, both types of CDS contracts lead to a tradeoff between credit spreads and

[5]As we later argue, the leverage in CDS is actually higher than that which can be obtained via buying bonds on margin due. The initial margins required to buy and sell CDS are substantially lower than the initial margins required to buy corporate bonds on margin or sell short.

[6]We do not endogenize which firm is the CDS reference entity. Oehmke and Zawadowski (2016b) provide one rationale for why CDS emerge on certain firms and not others, such as debt covenants and fragmentation. We take this fact as given, and explore the general equilibrium financing implications. The qualitative results of our model hold regardless of which firm serves as the CDS reference entity because the mechanism is symmetric.

the probability of default . Taken together, the model predicts that CDS markets have "unintended" consequences on corporate debt markets that are novel to the theoretical CDS literature.

The best way to think about why a firm–over certain parameters–would issue safe debt rather than risky debt, and forgo the limited liability option, is as follows. Equity holders consume in both high and low cash-flow states when bonds are safe, compared to consuming only in high cash-flow states when bonds are risky. Moreover, the high cash-flow state consumption level financed with risky debt must be greater than the high cash-flow state consumption level financed with safe debt. Otherwise, firms would always issue safe debt. The "cost" of issuing safe debt is that equity holders must forgo consumption in high cash-flow states in order to repay creditors in low cash-flow states. As the value of the technology shock rises, firms are more productive on the margin irrespective of whether production is financed with risky or safe debt. However, consumption in low-cash flow states increases only when the firm issues safe debt. Thus, as technology shocks reach a certain level, the incremental consumption gain in low cash-flow states that is associated with safe debt financing outweighs the incremental consumption gain in high-cash flow states that is associated with risky debt financing. Additionally, for any given value of cash-flow in the down state, equity holders are more likely to issue risky bonds as the probability of the high cash-flow state increases precisely because risky bonds do not require shifting potential high cash-flow consumption to the low cash-flow states to repay debts.

The growing body of theoretical CDS literature examines how CDSs affect bond, equity, and sovereign debt markets (see Augustin et. al (2014) for a complete and thorough survey on the broad literature). Our work is most closely related to a class of heterogeneous agent models developed by Fostel and Geanakoplos (2012, 2016). Fostel and Geanokoplos (2012) show that in an endowment economy, financial innovation in credit derivative markets alters asset collateral capacities, and asset prices. Fostel and Geanakoplos (2016) study the effect that credit derivatives have on whether agents engage in production, showing that credit derivatives can lead to investment beneath the first best level obtained in an Arrow-Debreu economy and can robustly destroy equilibrium. Our model is distinguishable from their models because production in our model has different expected cash-flow realizations, and we consider the decision to issue safe or risky debt. These innovations allow us to

characterize spillovers and the tradeoff between credit spreads and default risk. Che and Sethi (2015) study how CDS affect borrowing costs for a representative firm with a random output draw that raises an exogenous amount of capital. Our model adds several relevant features by explicitly modeling an endogenous production environment with different firm types. Our model also gives rise to a more in-depth discussion of the investment and default decisions because of the general equilibrium setting. Oehmke and Zawadowski (2015a) study the effects CDSs have on bond market pricing when investors have not only heterogeneous beliefs, but also heterogeneous trading frequencies. Their model is more suited to studying the effect of CDSs on secondary bond market activity. The authors do not consider investment in production or default. Common to all of these models, however, is the underlying mechanism through which CDS generate effects on the real economy. Mainly, the embedded leverage in the derivative contracts alters investor use of collateral that would otherwise be used to purchase bonds.

A different strand of CDS literature highlights the effect of CDS on the creditor-debtor relationship. Bolton and Oehmke (2011) show how CDS lead to an empty creditor problem in a model with limited commitment. Lenders' incentives to rollover loans are reduced, leading to increased bankruptcy and default risk. Firms internalize the difficulty of renegotiation their debts when creditors are protected with CDS, leading to stronger ex ante firm commitment to repay debts. Parlour and Winton (2013) show that CDS can reduce the incentives to monitor loans, thereby increasing default and credit risk. Similarly, Morrison (2005) shows that banks ability to sell off the credit risk of their loan portfolio leads firms to substitute away from monitored bank lending and into issuing risky public debt. Danis and Gamba (2015) study the trade-offs between higher ex ante commitment to debt repayment and higher ex post probability of default in a dynamic model with debt and equity issuance. They calibrate the model to U.S. data and find the positive benefits of lower CDS spreads stemming from higher repayment commitment dominate and increase welfare. The implicit assumption in all of these models is that CDS are used to hedge credit risk, which is equivalent to our covered CDS economy. All told, the increased default risk when covered CDSs trade in our model is, thus, complementary. We propose a new mechanism, though. Quite simply, lower credit spreads and limited liability raise the incentives to issue risky bonds. This is distinct from the default risk implicit in the empty creditor problem. Empty creditors may arise when firms face debt refinancing needs that are not met when creditors buy CDS contracts to insure against default.

Thus there is a maturity-mismatch argument underlying that story in which debts that come due before all cash-flows are realized may be renegotiated. Our model is one period with no refinancing or maturity mis-match, and debt liabilities and asset cash-flows are perfectly aligned. We show that CDS may lead to higher default because lower credit spreads reduce the "cost" of issuing defaultable bonds. Lastly, unlike our paper, none of debtor-creditor papers evaluate the effect on investment and default risk when investors may take purely speculative positions in the CDS market.

Empirically, Norden et. al (2014) find evidence of interest rate spillovers in syndicated bank lending markets. The authors attribute these spillovers to more effective portfolio risk management. Our model suggests an alternative explanation operating through how derivatives change the demand for bonds when firm cash-flows are correlated. Li and Tang (2016) find that there are leverage and investment spillovers between CDS reference firms and their suppliers. They argue that the higher the concentration of CDS reference entities is among a firm's customers, the lower supplier leverage ratios and investment levels are. The authors interpret their finding as the CDS market providing superior information about the credit quality of supplier firm customers (see Acharya and Johnson (2007) and Kim et. al. (2014) for references on how insider trading of CDSs leads to information transmission). Our model provides a complementary explanation under the interpretation that firms in our model are in the same industry. Investor demand for debt at the industry level is altered when firms in the industry become named CDS reference entities. Introducing CDSs changes the demand for bonds of other firms in the industry, altering bond prices and investment levels.

Our paper also provides the following new testable implication: The effect on corporate default risk of trading CDS depends on whether the CDS buyer has an insurable interest in the underlying reference entity. Current empirical studies on the effect of CDSs on default risk stem from the predictions of the empty creditor problem which require the effects to operate through hedging credit risk (see Subrahmanyam, Wang, and Tang (2014); Kim (2013); and Shan, Tang, and Winton (2015)). However, these studies cannot distinguish between covered and naked CDS positions.[7] Furthermore, our model's spillover implications call into question the widely employed method of propensity score matching used to control for the

[7]This is an ongoing project we and co-authors are undertaking.

endogeneity of CDS issuance. Firm borrowing costs in matched samples will not be exogenous to CDS introduction if CDS trading alters the cost of capital for non-CDS reference firms whose cash flows are correlated with the CDS obligors.

The organization of the paper is as follows: In Section 2, we describe firms, debt contracts and investors. We then solve the baseline economy with no CDS contracts, and describe the relevant comparative statics. In Section 3, we introduce covered CDSs. In section 4 we allow for naked CDS trading. In Section 5 we close with discussion and concluding remarks.

2 Non-CDS economy

2.1 Model

2.1.1 Time and Uncertainty

The model is a two-period general equilibrium model, with time $t = \{0, 1\}$. Uncertainty is represented by a tree, $S = \{0, U, D\}$, with a root, $s = 0$, at time 0 and two states of nature, $s = \{U, D\}$, at time 1. Without loss of generality we assume there is no time discounting. There is one durable consumption good–risk-free asset–in this economy that is also the numeraire good. We will refer to this good as cash throughout the paper.

2.1.2 Agents

Firms

There are two firms, $i = \{G, B\}$, in the economy where firm G is the "good" type and firm B is the "bad" type. Each firm is owned and operated by a manager with access to a production technology. The managers operate the firms and consume from firm profits. One could think of the positive profits earned in equilibrium as non-pledgeable control rents with which equity owners are compensated to invest in production. We use the terms *control rents* and *profits* interchangeably.[8] The only difference between the two firms is the production technology at the respective managers' disposal. The firms use the durable consumption good as an input at time 0 and produce more of this good for consumption at time 1. The respective firms have

[8]We abstract away from any agency problem between equity holders and firm managers. Alternatively, one could think of the managers as operating the firm and being paid through equity.

standard, decreasing returns to scale, production functions given by $f_i\left(I_i;\alpha_i,A^s\right)=A^s I_i^{\alpha_i}$ with the following properties: $f_i^{'}>0$, $f_i^{''}<0$. Firm G is more productive than firm B; that is, $I^{\alpha_g}>I^{\alpha_b}$, $\forall\, 0<I<1$.

The technology shock, A^s, takes on binary values at time 1, with $A^U>A^D$. The technology parameter is identical for both firms. Consequently, the only type of uncertainty in our model is aggregate.[9] Idiosyncratic risk would not have any impact in a model with two firms and two states, because agents would be able to perfectly insure themselves. The only way idiosyncratic risk would have an effect is if we considered more than two states (in which case there would still be non-aggregate risk remaining even after agents trade with each other).[10]

For simplicity we normalize the technology shock, A^U, to 1. Both firms have identical knowledge about the quality of their production process, where each firm knows that $s=U$ arrives with probability γ and $s=D$ with probability $(1-\gamma)$. Lastly, firms are competitive price takers in the market for the durable consumption good.

Investors

We consider a continuum of risk-neutral, heterogeneous investors, distributed according to $h\in H\sim U(0,1)$, who do not discount the future. The absence of time discounting allows us to focus on credit spreads without loss of generality. Investors are characterized by linear utility for the single consumption good, x_s, at time 1. Each investor is endowed with one unit of the consumption good, $e^h=1$, and assigns probability h to the up state, U, and $(1-h)$ to the down state, D. Thus, a higher h denotes more optimism. Agents agree to disagree about their subjective state probabilities. The von Neumann–Morgenstern expected utility function for investor h is given by

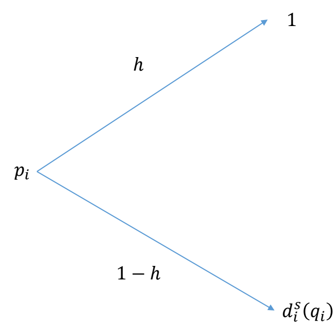
$$U^h(x_U,x_D)=hx_U+(1-h)x_D. \tag{1}$$

We assume a uniform distribution for tractability. The results will hold in general as long as the beliefs are continuous and monotonic in h. In terms of investor preferences, we assume risk-neutrality, but the results are also qualitatively preserved with homogeneous beliefs and state contingent endowments.

[9]It may be natural to think of the model as one of intra-industry debt financing in which two firms in the same industry are equally affected by a industry specific technology shock.

[10]We leave this to future research.

Figure 1: Bond payout



2.1.3 Firm financing

We assume the firms use future output as collateral and optimally raise capital from investors by issuing debt. Our aim is to better understand the effect of credit derivatives on the debt-issuance decision. The effect of credit derivatives on a more general capital allocation problem is an interesting natural extension of the model. At time 0, firms issue debt contracts that specify a fixed repayment amount (bonds) and are collateralized using the pledgeable proceeds from output, which we refer to as cash flows. The lender (investor) has the right to seize the collateral up to the face value of debt, but no more. This enforcement mechanism ensures that the firm will not simply default on its commitment at time 1.

Each bond, priced p_i at time 0, promises a face value of 1 upon maturity. The two firms issue bonds denoted by q_i at time 0. In the up state, each bond returns full face value.[11] In the down state, bonds pay creditors according to the deliverable function, $d_i^s = \min\left[1, \frac{A^s I_i^{\alpha_i}}{q_i}\right]$. If debt obligations are not honored, creditors become the residual claimants of the firm and consume from the cash flows generated from production. Firm borrowing costs are denoted by r_i, and equal to the difference in what the firm owes on maturity and the amount of capital they receive at the time of issuance, $r_i = 1 - p_i$. Figure 1 depicts the bond payouts.

[11]We assume the bonds repay in full at $s = U$ to make the model interesting; otherwise, the firm does not invest in production.

2.1.4 Firm maximization problem

Each firm chooses an investment amount, I_i, given the market price of bonds to maximize expected profits. Firms must issue debt in order to produce since they have no initial endowment. Hence $I_i = p_i q_i$. Let π_i^s denote state-contingent firm profits. Firms solve the following program:

$$\left\{ \begin{array}{c} \max_{I_i} E\left[\pi_i\right] \equiv \Pi_i = \left\{\gamma\left[A^U I_i^{\alpha_i} - q_i\right] + (1-\gamma)\left[A_i^D I_i^{\alpha_i} - q_i d_i^D\left(q_i\right)\right]\right\} \\ s.t.\, I_i = p_i q_i \end{array} \right\} \tag{2}$$

2.1.5 Investor Maximization Problem

We can now characterize each agents' budget set. Given bond prices p_i, each investor, $h \in H$, chooses cash holdings, $\left\{x_0^h\right\}$, and bond holdings, $\left\{q_i^h\right\}$, at time 0 to maximize utility given by (1) subject to the budget set defined by:

$$\begin{aligned} B^h\left(p_i\right) &= \Big\{\left(x_0^h, q_i^h, x_s^h\right) \in R_+ \times R_+ \times R_+ : \\ & x_0^h + \sum_i p_i q_i^h = e^h, \\ & x_s^h = \left(1 - \sum_i p_i q_i^h\right) + \sum_i d_i^s q_i^h \Big\},\ s = \{U, D\}. \end{aligned}$$

Each investor consumes from two potential sources in either state of nature: consumption based on risk-less cash holdings and consumption based on their total bond portfolio. In the up state, consumption from bond holdings is equal to the quantity of bonds an investor owns in his portfolio because each bond has a face value of 1. In the down state, firms may default on their debt, in which case investors take ownership of the firm and consume from the firms' available assets on a per-bond basis.

We rule out short sales of bonds by assuming $q_i \in R_+$. It is frequently argued that CDS are inherently redundant assets if one allows for assets to be perfectly leveraged and short short sales are not constrained. It is true that selling a CDS and buying a bond with leverage span the same asset payoffs. Buying a naked CDS and short selling the asset also span the same asset payoffs. However, what this argument does not consider is that taking a long position on credit risk by selling a CDS allows an investor to take *more* leverage than buying the bond on margin because the

initial margin requirements are different.[12] For example, according to the Financial Industry Regulator Authority (FINRA), the initial margin requirement for selling a 5yr CDS with a spread over LIBOR less than 100 bps is 4% of the notional amount of the CDS contract. Conversely, the regulatory minimum to purchase an investment grade bond on margin–assuming that the spread over LIBOR for said bond is also less than 100 bps–is 10% of the *market value* of the purchase.[13] Thus, the return to every dollar of collateral posted against a CDS transaction is higher than the return of a dollars worth of collateral posted as initial margin in the bond market. The same logic holds for naked CDS purchases and short sales. Investors have to take a 2% haircut with their broker to borrow a corporate bond, in addition of the 50% margin required by Regulation T.[14] According to FINRA, CDS purchasers are required to post 50% of initial margin required of CDS sellers, which is 2% in the example given above.[15]

2.1.6 Equilibrium

An equilibrium in the non-CDS economy is a collection of bond prices, firm investment decisions, investor cash holdings, bond holdings and final consumption decisions, $p_i, I_i, (x_0, q_i, x_s)_{h \in H} \in R_+ \times R_+ \times (R_+ \times R_+ \times R_+)$ such that the following are

[12]This is a different argument than suggesting that trading cost between the cash and synthetic markets are different, which tends to also be the case, as measured that bid-ask spreads (see Biswas, Nikolova, and Stahel (2015).

[13]The FINRA initial margins can be found here:
http://finra.complinet.com/en/display/display_main.html?rbid=2403&element_id=8412.
The corporate bond margins can be found here:
https://www.interactivebrokers.com/en/index.php?f=marginnew&p=bonds.

[14]See Asquith et. al. (2013).

[15]Furthermore, in the spirit of Banerjee and Graveline (2014) derivative pricing in our model contains no noise, as will be clear in the following section, because the investors know technology fundamentals. Banerjee and Graveline show in proposition 6 that imposing a short-sale ban will have no effect on bond prices when investors can trade in derivatives with no noise.

satisfied:

$$1. \int_0^1 x_0^h dh + \sum_i \int_0^1 p_i q_i^h dh = \int_0^1 e^h dh$$

$$2. \sum_i \int_0^1 q_i^h d_i^s dh + \sum_i \pi_i^s = \sum_i A^s I_i^{\alpha_i}, \; s = \{U, D\}$$

$$3. \, I_i = \int_0^1 p_i q_i^h dh$$

$$4. \, \pi_i \left(I_i\right) \geq \pi_i(\hat{I}_i), \, \forall \hat{I}_i \geq 0 \text{ for } i = \{G, B\}$$

$$5. \, \left(x_0^h, q_i^h, x_s^h\right) \in B^h \left(p_i\right) \Rightarrow U^h \left(x\right) \leq U^h \left(x^h\right), \; \forall h$$

Condition (1) says that at time 0 the initial aggregate cash endowment is held by investors for consumption or used to purchase bonds issued by firms. Condition (2) says the goods market clears at time 1 such that all firm output is consumed either by firm managers via profits or by creditors via bond payments. Condition (3) corresponds to the bond market clearing conditions. Condition (4) says that firms choose investment to maximize expected profits, and condition (5) states that investors choose optimal portfolios given their budget sets.

2.2 Equilibrium bond pricing

We begin by characterizing equilibrium bond pricing and credit spreads. This characterization will lay the foundation for analyzing the borrowing cost spillover effects in subsequent sections. We then analyze the firm investment decision that involves a choice between issuing a limited number of safe bonds or risky bonds that carry a positive credit spread. This then allows us to investigate how credit derivatives affect the firm debt issuance decision.

Equilibrium in heterogeneous belief models is characterized by marginal investors pricing debt. In equilibrium, as a result of linear utilities, the continuity of utility in h, and the connectedness of the set of agents, $H = (0, 1)$, there will be marginal buyers, $h_1 > h_2$, at state $s = 0$. Every agent $h > h_1$ will buy bonds issued by firm B, every agent $h_2 < h < h_1$ will purchase type-G bonds, and every agent $h < h_2$ will remain in cash. This regime is shown in figure 2. The marginal buyer indifferent between the payouts of the various assets available in the economy will be crucial in subsequent sections for understanding why introducing derivatives affects the credit spread for all debt contracts in the economy, irrespective of debt contracts

are referenced by CDS. With this, we now characterize the relationship between the respective firms' bond prices.

Proposition 1 *In any equilibrium without derivatives where both firms issue risky debt with positive credit spreads, type G bonds are priced higher than type B bonds.*

Proof. See appendix A.

The intuition is that the production value of the more productive firm is always higher than the less productive firm. Debt holders will therefore be willing to pay a higher price for those bonds, *ceteris paribus*, since they are the residual claimants of the firm output given default. Since both bonds have equal face value and pay in full at $s = U$, more optimistic investors will prefer firm-B bonds, the cheaper of the two assets. The two marginal investor indifference equations can be written as:

$$\frac{h_1 + (1 - h_1)\, d_g^D}{p_g} = \frac{h_1 + (1 - h_1)\, d_b^D}{p_b} \tag{3}$$

$$h_2 + (1 - h_2)\, d_g^D = p_g. \tag{4}$$

Equation (3) says that the more optimistic marginal buyer, h_1, will be indifferent between the expected payouts from type-B and -G bonds. Equation (4) says that the less optimistic marginal buyer, h_2, will be indifferent between the cash flows on type-G bonds and cash. The bond market clearing conditions for type-B and -G bonds require that the two bond prices be determined by the endowment available to the respective sets of investors buying the bonds. That is $\frac{1-h_1}{p_b} = q_b$ for type-B bonds and $\frac{h_1 - h_2}{p_g} = q_g$ for type-G bonds. This characterization clearly shows that bond prices are jointly determined based on how the marginal investors view the respective payouts. A change in the price of one bond necessarily changes the yield on the two assets. For example, an exogenous decrease in the price of bond B makes it more attractive and means it will become strictly preferred to the relatively more expensive bond G unless there is a corresponding change in the bond G's cash flows. In equilibrium, as more capital moves to finance bond B, bond G 's price must also fall and will be priced by a less optimistic investor.

2.3 Debt Financing Regime

We now turn to the issue of when it is optimal for the firm, given market prices, to issue risky debt with positive credit spreads versus issuing safe debt. The firm

maximization problem given by (2) boils down to choosing an investment, I_i, that corresponds to either full repayment or default at $s = D$. Bonds that fully repay in either state have zero credit spread: $p_i^f = 1$ where the super-script f denotes default-free pricing. Risky bonds will carry a positive credit spread, $p_i^\rho < 1$, where the super-script ρ denotes a positive credit spread. Let the corresponding expected profit levels from each of these investment decisions be denoted by Π_i^f and Π_i^ρ. The firm thus chooses $I_i^*\left(\alpha_i, \gamma, A^D\right) \equiv \arg\max_{I_i}\left[\Pi_i^f, \Pi_i^\rho\right]$. The first-order conditions for the two investment levels are

$$I_i^\rho : \alpha_i I_i^{\alpha_i - 1} = \frac{1}{p_i^\rho} \tag{5}$$

$$I_i^f : \alpha_i I_i^{\alpha_i - 1} = \frac{1}{p_i^f}\left(\frac{1}{\gamma + (1-\gamma) A^D}\right). \tag{6}$$

These are the standard marginal products of capital that must equal the marginal costs of capital conditions. The default-free condition in (6) takes into account that all debt is fully repaid at $s = U, D$ through the expected value of the technology shock, $\gamma + (1-\gamma) A^D$. Using (5) and $I_i^{\rho *} = p_i^{\rho *} q_i^{\rho *}$, the risky bond repayment function, $d_i^s\left(q_i^{\rho *}\right)$, given $s = D$ can be written as

$$d_i^D\left(q_i\right) = \frac{A^D}{\alpha_i}. \tag{7}$$

Intuitively, recoverable firm asset values are proportional to the technology shock parameter, $0 < A^D < 1$, and the productivity parameter, $0 < \alpha_i < 1$. The recovery value also places a fundamental restriction on the relationship between the two parameters. Specifically, for $A^D < \alpha_i$ firms will issue risky debt in equilibrium. Since $\alpha_g < \alpha_b$ there are different values of A^D for which there is fundamental default risk for the two firms, $0 < A_G^D < \alpha_G < A_B^D < \alpha_B < 1$. As we show in the subsequent discussion, for a given (α_G, α_B)-pair, firms' decision to issue risky or safe debt depends on the given $\left(A^D, \gamma\right)$-tuple. If firms always repay in all states, $p_i^{f*} = 1$, the credit spread is zero, and the optimal investment is given by (6): $I_i^{f*} = \left[\alpha_i\left(\gamma + (1-\gamma) A^D\right)\right]^{\frac{1}{(1-\alpha_i)}} = q_i^{f*}$. Note that the default-free bond repayment function, $d_i^s\left(q_i^{f*}\right) = \min\left[1, \frac{A^s\left(I_i^{f*}\right)^{\alpha_i}}{q_i^{f*}}\right] = 1$, implies that $\frac{A^D\left(I_i^{f*}\right)^{\alpha_i}}{q_i^{f*}} \geq 1$. Using the optimal investment level and $I_i^{f*} = q_i^{f*}$ gives a relationship between the good state probability, γ, and the value of the technology shock, A_i^D, for which issuing default-free debt is *possible* even with fundamental default risk, $\alpha_i > A^D$. Formally,

Proposition 2 *For any given state probability γ, there exists a threshold value of the technology shock, $\underline{A}_i^D(\gamma)$ where realizations of A_i^D greater than this threshold can support safe debt financing. The threshold value of $\underline{A}_i^D(\gamma)$ is an increasing function of γ.*

Proof. See appendix A.

The intuition underlying proposition 2 is that the cash flows from production must be sufficiently high in bad states (high A_i^D) for the firm to always honor its debt obligations. The "cost" of issuing safe debt is that investment will be limited and the potential up state profits are reduced in order to promise creditors full repayment given poor cash flow realizations. The higher are bad state cash flows, the less firms need to limit debt-financed investment and potential returns in good states in order to ensure creditors are always repaid, hence the "cost" of issuing safe debt is falling as A_i^D rises. Additionally, the more likely it is that good states will arrive (high γ) the more firms invest because doing so increases their expected profts. This means that cash flows in the down state must be even higher as γ increases if the firm is to fully honor its debt obligations. Thus, $\underline{A}_i^D(\gamma)$ is increasing in γ.

We must still determine what the optimal debt financing regime is for any set of parameters. This is done by solving for the equilibrium $I_i^*\left(\alpha_i, \gamma, A^D\right)$ based on (5) and (6) to $\max\limits_{I_i}\left\{\Pi_i^f\left(I_i^{f*}\right), \Pi_i^\rho\left(I_i^{\rho*}\right)\right\}$. Plugging $q_i^{f*} = I_i^{f*}$ into (2), we obtain the risk free financing expected profit function

$$\Pi_i^{f*} = \frac{(1-\alpha_i)}{\alpha_i} q_i^{f*}\left(\underline{A}_i^D(\gamma), \gamma\right).$$

As debt is always repaid in default-free regime, expected profits are only affected by state probabilities, γ, through the affect on bond quantities, q_i^{f*}. Moreover, by proposition 2, an equilibrium in which default-free bond pricing exists must be characterized by $A_i^D > \underline{A}_i^D(\gamma)$. If $A_i^D < \underline{A}_i^D(\gamma)$, safe debt financing is not profit maximizing for the firm. The expected profits from risky debt financing are obtained using the optimality condition (5) along with (2):

$$\Pi_i^{\rho*} = \gamma \times \frac{(1-\alpha_i)}{\alpha_i} q_i^{\rho*}\left(A^D\right).$$

Note from (5) that good state probabilities, γ, do not influence risky debt financing investment levels, $I_i^{\rho*}$, because of limited liability. The firms invest as if $\gamma = 1$ because

they default at $s = D$; the good state probability, γ, affects only the expected profits of the firms for a risky-debt regime. Comparing the two expected profit functions, it is clear that if the bond issuances correspond to the same face value at maturity $\left(q_i^{f*} = q_i^{\rho*}\right)$, firms prefer safe debt financing because credit spreads are zero. However, risky debt financing is prefered if the face value of risky-debt is greater than the face value of safe debt, and the ratio of the face values is less than the good state probability, γ: $\gamma > \frac{q_i^{f*}}{q_i^{\rho*}}$. Risky debt financing allows equity holders to maximize their profits in the good state because, unlike safe debt financing, nothing must be conceeded to the down state to ensure repayment. Therefore, the higher the good state probability, the more equity holders prefer to shift all profits to the good state. Lastly, note that the only interesting range of technology shocks for analyzing a tradeoff between the two different debt issuances is $A^D \in \left[\underline{A}_i^D, \alpha_i\right)$. All technology shocks below $\underline{A}_i^D$ result in the risky-debt regime, $q_i^* = q_i^{\rho*}$, and all shocks greater than α_i result in the default-free regime, $q_i^* = q_i^{f*}$. The following proposition characterizes the parameter regions over which firms issue the two different types of debt.

Proposition 3 *Firms issue risky bonds for $A_i^D < \underline{A}_i^D$ and safe bonds for $A_i^D \geq \alpha_i$. For $A^D \in \left[\underline{A}_i^D, \alpha_i\right)$ firms issue safe bonds if and only if $\gamma < \bar{\gamma}$.*

This result says that depending on the parameter values, firms must optimally choose between the default-free regime and the risky-debt regime. Low values of the bad state technology shock, $A_i^D < \underline{A}_i^D$, correspond to risky debt financing because safe debt financing requires reduced investment to guarentee creditors full repayment when cash flows turn out to be low. Lower investment also reduces profits when the technology shock turns out to be good. High values of the bad state technology shock, $A_i^D \geq \underline{A}_i^D$, enable safe debt financing because the amount of debt that must be curtailed to fully repay debt in all states is decreasing. Thus the benefit of the higher incremental good state output associated with risky debt financing is dominated by the ability to borrow at the risk free rate. Debt financing for intermediate values of the bad state technology shock, $A^D \in \left[\underline{A}_i^D, \alpha_i\right)$, depends also on the likelihood of good cash flows, γ. Risky debt financing allows for more consumption in good cash flow states, which means that higher γ leads to risky debt financing regimes.

Example 1 $\left\{A^D = 0.2, \gamma = 0.5, \alpha_G = 0.5, \alpha_B = 0.65\right\}$

This example gives results for credit spreads, marginal buyers, and the remainder of the endogenous variables in the economy for the listed parameters. Figure 2

Figure 2: Non-CDS economy

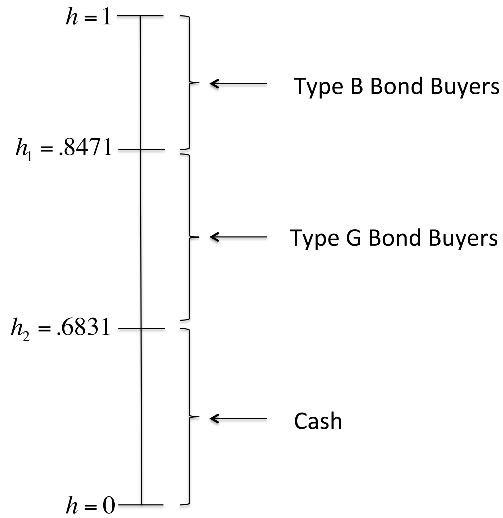


Table 1: Equilibrium values: *Non-CDS economy*

non-CDS economy		
	$i = G$	$i = B$
Price: p_i	.8099	.7973
Credit Spread: cs_i	.2347	.2542
Quantity: q_i	.2025	.1918
Investment: I_i	.1640	.1529
Output: Y_i^U	.4049	.2950
Exp.Profit: $E[\pi_i]$	.1012	.0516

is the characterization of the marginal buyer in equilibrium and table 1 gives the values of the endogenous variables. The values of the technology thresholds for determining whether default-free bonds may be issued for $\gamma = 0.5$ are $\underline{A}_G^D = .3\bar{3}$ and $\underline{A}_B^D = .4814$, and clearly $p_i < 1$, $i = G, B$. The comparative statics for bond pricing and investment are not particular to the parameters chosen so long as $A^D < \underline{A}_i^D$. Higher values of the technology shock naturally raise bonds prices, investment and profits. However, increases in the technology shock above the respective thresholds, $\underline{A}_i^D$, will change the comparative statics since the respective bond prices will be $p_i = 1$.

3 Covered CDS economy

In this section, we incorporate CDSs into the baseline model. A CDS is a financial contract in which the CDS seller compensates the buyer for losses to the value of an underlying asset for a specified credit event or default. The underlying assets in this economy are firm bonds. CDS contracts compensate buyers the difference between a bond's face value at maturity and its recovery value at the time of the credit event. Thus, CDS allow investors to hedge against idiosyncratic default risk.[16]

We first consider covered CDSs, in which buyers are required to also hold the underlying asset (that is, the bond corresponding to the CDS reference entity). We assume the seller must post enough collateral to cover the payment in the worst case scenario, but no more, to rule out any counterparty credit risk. Let q_{ic}^h be the number of CDSs that investor h can sell, and let p_{ic} be the CDS price. Therefore, the total cash that investor h holds to collateralize CDS contracts, including payments received for selling CDSs, will equal the maximum possible CDS payout in the event of firm default, times the number of CDS contracts sold:

$$1 + p_{ic}\, q_{ic} = q_{ic}^h \left(1 - d_i^s\right). \tag{8}$$

Solving for the total number of CDS contracts gives

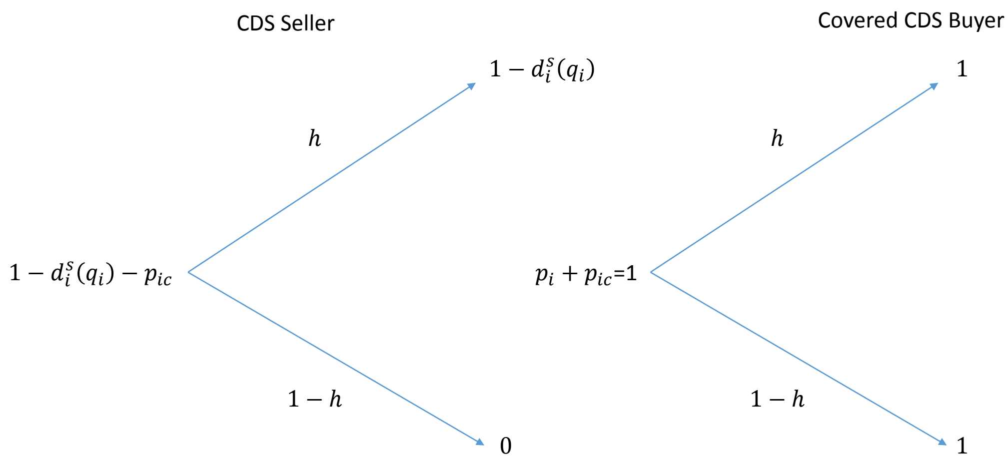
$$q_{ic}^h = \frac{1}{1 - d_i^s - p_{ic}}. \tag{9}$$

Figure 3 shows the payout to the CDS seller and buyer. At time 0, a CDS seller must post a portion of his own collateral, $1 - d_i^s - p_{ic}$, to insure each CDS. At $s = U$, the CDS seller consumes the posted collateral, because the bond pays in full. At $s = D$, all of the posted collateral is used to pay the CDS buyer. Thus, selling a CDS contract is equivalent to buying an Arrow-Up security because it pays out only when $s = U$.[17]

[16] CDS do not allow investors to insure away aggregate risk.

[17] Selling a CDS is cash-flow equivalent to leveraging the bond, but the initial margin differences between the two transactions allows investors to take more leverage through the CDS market relative to the bond market.

Figure 3: Covered CDS payout



3.1 Investor maximization problem

Given bond and CDS prices, (p_i, p_{ic}), each investor h decides on cash, bond, and CDS holdings, $\{x_0^h, q_i^h, q_{ic}^h\}$, to maximize utility (1) subject to the following budget set:

$$
\begin{aligned}
B^h(p_i, p_{ic}) \;=\; & \Big\{ \left(x_0^h, q_i^h, q_{ic}^h, x_s^h\right) \in R_+ \times R_+ \times R \times R_+ : \\
& x_0^h + \sum_i p_i q_i^h + \sum_i p_{ic} q_{ic}^h = e^h, \\
& x_s^h = x_0^h - \sum_i p_{ic} q_{ic}^h + \sum_i q_i^h d_i^s + \sum_i q_{ic}^h \left(1 - d_i^s\right), \; s = \{U, D\} \\
& max\left\{0, q_{ic}^h\right\} \le q_i^h \Big\}.
\end{aligned}
$$

The first two equations are analogous to the investor budget set in the non-CDS economy. The third equation states that since CDS buyers are required to hold the underlying asset, the maximum number of CDS contracts that can be purchased cannot exceed the number of bonds owned. Note that there is no sign restriction on q_{ic}^h. Selling CDS implies that $q_{ic}^h < 0$, while $q_{ic}^h > 0$ implies purchasing CDS. Short selling of bonds is still ruled out by the restriction $q_i \in R_+$ as in the non-CDS economy.

3.2 Equilibrium

An equilibrium in the covered-CDS economy is a collection of bond prices, CDS prices, firm investment decisions, investor cash holdings, bond holdings, CDS holdings and final consumption decisions,

$p_i, I_i, (x_0, q_i, q_{ic}, x_s)_{h\in H} \in R_+ \times R_+ \times (R_+ \times R_+ \times R \times R_+)$, such that the following are satisfied:

1. $\int_0^1 x_0^h dh + \sum_i \int_0^1 p_i q_i^h dh = \int_0^1 e^h dh$
2. $\sum_i \int_0^1 q_i^h d_i dh + \sum_i \pi_i^s = \sum_i A^s I_i^{\alpha_i},\ s = \{U, D\}$
3. $\int_0^1 q_{ic}^h = 0$
4. $I_i = \int_0^1 p_i q_i^h dh$
5. $\pi_i\left(I_i\right) \geq \pi_i(\hat{I}_i),\ \forall \hat{I}_i \geq 0$
6. $\left(x_0^h, q_i^h, q_{ic}^h, x_s^h\right) \in B^h\left(p_i, p_{ic}\right) \Longrightarrow U^h\left(x\right) \leq U^h\left(x^h\right),\ \forall h$

Condition (1) states that all of the endowment is either held by investors or used to purchase bonds. Condition (2) says the goods market clears where total firm output is consumed by firm managers via profits and used to repay bond holders. Condition (3) says that the CDS market is in zero net supply, while (4) states that the bond market clears. Condition (5) says firms choose investment to maximize profits. Lastly, (6) states that investors choose a portfolio that maximizes their utility, given their budget set.

We make use of the following lemma to characterize equilibrium in the covered CDS economy.

Lemma 1 *If* $0 < d_i^D\left(q_i\right) < 1$*, any risky bond that is a CDS reference entity will not be purchased without the corresponding CDS.*

Proof: See appendix B.

The intuition behind lemma 1 is that any investor optimistic enough to buy a bond without a CDS will be better off selling CDSs on that bond. Additionally, if the recovery value of the bond is zero, then CDSs and bonds have identical payouts

in either state, thus making CDSs redundant assets. Finally, if the recovery value of the bond is 1, then bonds are risk free and no CDSs will trade in equilibrium.

3.3 Borrowing costs and spillovers

We introduce CDSs only on one firm's debt to establish the borrowing cost spillover result. For expositional purposes, let investors issue CDSs on firm-B debt.[18] As in the non-CDS economy, there will be *marginal buyers*, $h_1 > h_2$. In equilibrium, every agent $h > h_1$ will sell CDSs on type-B debt, every agent $h_2 < h < h_1$ will purchase type-G debt, and every agent $h < h_2$ is indifferent to holding covered CDS and cash. More specifically, agents $h < h_2$ hold a portfolio of covered positions on type-B debt and CDS, and cash.[19] Compared with the non-CDS economy, covered CDSs lower borrowing costs for the firm on which CDSs are traded–firm B, because the derivative allows the optimists who believe firms will always repay their debt to hold all of the credit risk. Because CDS sellers only have to hold a portion of their collateral to cover the expected loss given default, which is always less than the price of the bond, each individual CDS seller can hold more credit risk than they could when just buying bonds. The resulting marginal buyers are therefore more optimistic than in the economy without CDSs. The marginal buyer indifference equations are now

$$\frac{h_1\left(1-d_b^D\right)}{1-p_{bc}-d_b^D} = \frac{h_1+(1-h_1)\,d_g^D}{p_g} \tag{10}$$

$$h_2+(1-h_2)\,d_g^D = p_g. \tag{11}$$

The aggregate endowment CDS sellers have at their disposal to price CDSs is now $\frac{(1-h_1)}{1-p_{bc}-d_b^D} = q_b$. The available endowment of type-G bond buyers takes the same form as the non-CDS economy: $\frac{h_1-h_2}{p_g} = q_g$, with different marginal buyers. Lastly, because some agents hold a portfolio of covered CDS and cash, a non-arbitrage pricing condition between the CDS and bond market links the price of credit risk in

[18] The qualitative results for borrowing cost spillovers and endogenous positive credit spreads are not particular to the firm's debt from which the CDSs derive their value. Endogenizing CDS issuance is an interesting question, but beyond the scope of this paper. Work by Banerjee and Graveline (2014), Bolton and Oehmke (2011), and Oehmke and Zawadowski (2015b) provides interesting reference points for thinking about why we see CDSs emerge on some firms and not others.

[19] In equilibrium, every agent $h < h_2$ will be indifferent between cash and a covered position and each agent will hold a portfolio consisting of 30.43% in covered positions and the rest in cash, for the same parameters chosen in example 1. To compute this portfolio allocation we simply divide the number of type B bonds investors hold by their total cash endowment, $\frac{q_b}{h_2}$.

the CDS market with the positive credit spread in the bond market: $p_{bc} + p_b = 1$. Consistent with Che and Sethi (2015), but with an additional dimension in terms of borrowing cost spillovers, we have the following bond pricing implications:

Proposition 4 *Covered CDSs raise bond pricing for the CDS reference entity.*

Proof. See appendix B.

More interestingly, because our model has an additional risky asset with a different bond pay-out structure, there are general equilibrium borrowing cost implications for firm G even though firm-G debt is not referenced by a CDS.

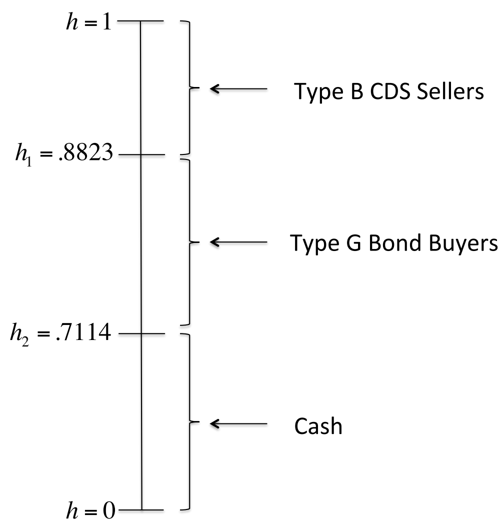
Corollary 1 *The credit spread for a non-CDS reference entity is affected by CDSs trading on other reference entities with correlated cash flows.*

Proof. See appendix B.

This result follows directly from proposition 4 because the marginal CDS seller who implicitly prices firm B debt is more optimistic than the corresponding marginal bond buyer in the non-CDS economy. This is because each CDS seller uses his endowment solely for the purpose of buying credit risk in the amount equal to the loss given default of the underlying bond contract. Thus a given set of investors of size $(1 - h_1)$ in the non-CDS economy have increased ability to purchase credit risk in the covered CDS economy. Furthermore, the number of CDS that can be sold is restricted to the number of outstanding firm B bonds. The combination of increased ability to buy credit risk and restriction of buying CDS means that the marginal buyer pricing Firm G debt is more optimistic than in the non-CDS economy. This increase in the set of investors financing both firms' debt issuances also raises bond prices for firm G.

The fact that cash flows are correlated across good and bad states is also important. Take the limiting example in which firm G's cash flows are high at $s = D$ and low at $s = U$. The firm would then find it optimal to issue debt to those investors most willing to pay for an asset whose payouts are closely correlated with their subjective beliefs. Firm G would then prefer to issue debt to pessimists who would pay a higher price for an asset that pays more at $s = D$ with certainty than to optimists who would pay high prices for high payouts in the opposing state. It is this interpretation of the model that leads naturally to an intra-industry or geographically

Figure 4: Type-B covered CDS economy



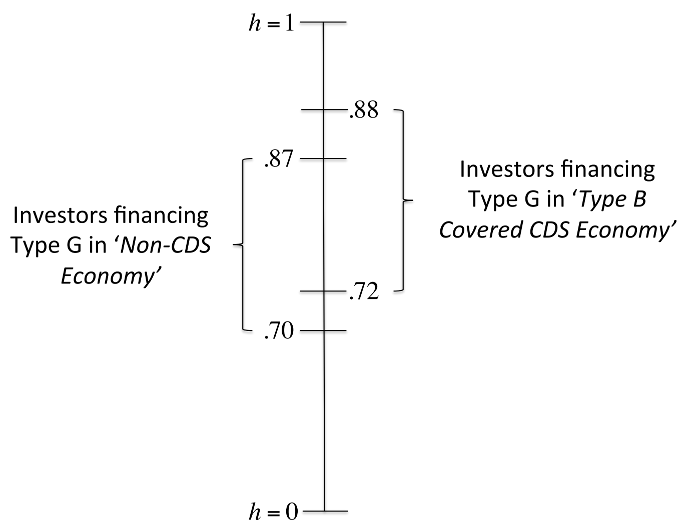
limited model of debt financing. Firms in the same industry or geographic area may be more likely to default in similar states than firms in completely different industries or regions. This sentiment is echoed by what Lang and Stulz (1992) coined as the contagion effect. They argue that bankruptcy announcements of one firm impacts the equity values of firms in the same industry. The bankruptcy contains information about the cash-flow characteristics of similar firms. This is precisely what we have in mind by the correlated cash flows in our model. The idiosyncratic nature of firm cash flows in our model is represented by different expected values rather than state probabilities. Additionally, Jorion and Zhang (2007) find evidence that the credit contagion effect of bankruptcy events are strongly associated with industry characteristics. Moreover, the recent empirical findings in Li and Tang (2016) on the leverage and borrowing of intra-industry firms are consistent with this interpretation of the model.

Figure 4 and table 2 show the corresponding marginal buyer regimes and endogenous variables in the economy for the same parameters as in example 1. Note that even though the CDS trades on firm B, the credit spread on firm G debt falls from $cs_G = .2347$ to $cs_G = .2069$. For the same parameters as in example 1, Figure 5 shows the equilibrium shift in the set of investors financing firm G's debt issuance in the non-CDS economy and the covered CDS economy with CDSs only trading on firm B.

Table 2: Equilibrium values: *Type-B covered CDS economy*

Type B Covered CDS Economy		
	$i = G$	$i = B$
Price: p_i	.8269	.8511
Credit Spread: cs_i	.2069	.1750
Quantity: q_i	.2067	.2165
Investment: I_i	.1709	.1843
Output: Y_i^U	.4134	.3331
Exp.Profit: $E[\pi_i]$	.1034	.0583

Figure 5: Covered CDS economy: *Type-G positive spillover*



3.4 CDSs and credit risk

In this section we show that the increase in bond pricing attributed to CDS trading also induces firms to issue debt with positive credit spreads for a larger set of down state probabilities than in the non-CDS economy. As shown in appendix A for proposition 2, the threshold values of $\underline{A}_i^D$, above which issuing default-free debt is profit maximizing, are determined only by parameters γ and α_i. CDSs do not change any of the fundamentals of the economy, so the threshold values are the same as in the non-CDS economy. What does change, however, is the credit spread associated with issuing risky debt.

To see how CDSs affect the firm's decision to issue risky bonds, recall that the necessary and sufficient condition to issue debt with a positive credit spread is that

the risky debt must yield higher profits to equity claimants. Expected profits can be simply expressed through the optimal debt issuance levels of the two debt contracts: $\gamma\hat{q}_i^{\rho*} > q_i^{f*}$, where the hat corresponds to the non-CDS economy quantities. The left-hand side is the expected profit associated with issuing risky debt and the right-hand side is the expected profit from issuing safe debt. Since the safe debt contract is purely a function of fundamentals, the right-hand side of the inequality remains unchanged. From proposition 4 we know that firms borrow at lower borrowing costs and issue more bonds when CDSs trade. Thus, if there is a $\bar{\gamma} < 1$ for which $\bar{\gamma}\hat{q}_i^{\rho*} = q_i^{f*}$ in the non-CDS economy, then it must be that $\bar{\gamma}\tilde{q}_i^{\rho*} > q_i^{f*}$ in the covered CDS economy and $\bar{\gamma} > \tilde{\gamma} \equiv \gamma\tilde{q}_i^{\rho*} = q_i^{f*}$, where the tildes denote covered CDS economy variables. The decrease in the threshold γ in covered CDS economies, above which firms issue risky debt, establishes the first result of the trade-off between credit spreads and default risk in CDS economies.

Proposition 5 *CDS lower credit spreads and may increase the likelihood of equilibrium default. For any value of $A_i^D \in \left[\underline{A}_i^D, \alpha_i\right)$ firms will issue safe debt contracts if and only if $\gamma \leq \tilde{\gamma} < \bar{\gamma}$.*

Endogenous default and investment helps establish this result. In Che and Sethi (2015), covered CDS also lower borrowing costs, but they increase the ability to repay debt for any given investment level. The positive relationship between covered CDSs and ability to repay debt would be true in our model if I_i were fixed. When endogenized however, firms respond to lower credit spreads by issuing more debt. Lower credit spreads raise the profits associated with risky debt financing even though fundamentals remain unchanged. If, in expectation, there was a state probability in the non-CDS economy for which the firm was indifferent between issuing either debt contract, the risky debt contract would be strictly preferred in the CDS economy. The resulting fall in credit spreads and associated increase in investment correspond to an increase in default incidence at $s = D$.

4 Naked CDSs

In this section, we extend the model by allowing investors to hold naked CDS positions: investors do not need to hold the underlying asset to purchase a CDS. A naked CDS buyer expects to receive the difference between the face value of the bond and its value at the time of default. The naked CDS payout structure is given in

Figure 6: Naked CDS payout

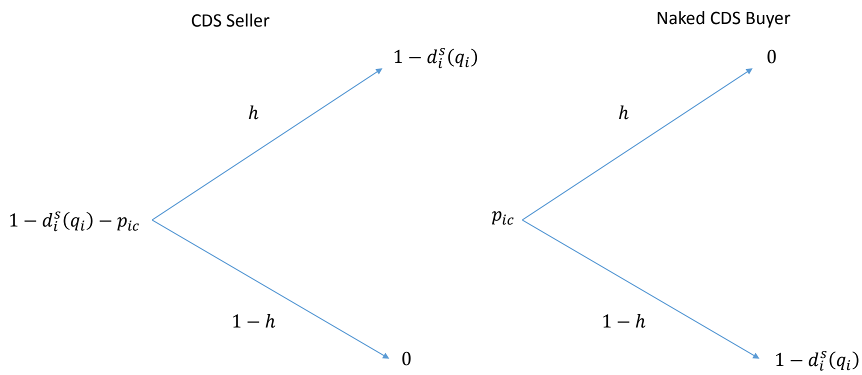


figure 6. Furthermore, notice that buying a naked CDS is equivalent to buying the Arrow-down security, since it pays out only when $s = D$.

At time 0, an investor can purchase a naked CDS by paying the premium payment, p_{ic}. The buyer believes with probability h that the up-state will occur at time 1, the firm will not default, and the CDS will not payout. The buyer believes with probability $(1 - h)$ that the down-state will occur at time 1. In this case, the buyer expects to receive the difference between the face value of the bond and its recovery value, $(1 - d_i^s)$. We continue to assume that CDS sellers post enough collateral to cover payment in the worst-case scenario, but no more. Therefore the maximum CDS payout carries over from previous economies and is given by equation (9). Moreover, the implications of lemma 1 still hold and thus risky-bonds that are CDS reference entities will not be purchased without its corresponding CDS.

4.1 Investor maximization problem

Given bond and CDS prices (p_i, p_{ic}), each investor chooses cash, bond, and CDS holdings, $\left\{x_0^h, q_i^h, q_{ic}^h\right\}$, to maximize utility (1) subject to the budget set:

$$
\begin{aligned}
B^h\left(p_i, p_{ic}\right) \;\; = \;\; & \Big\{\left(x_0^h, q_i^h, q_{ic}^h, x_s^h\right) \in R_+ \times R_+ \times R \times R_+\Big\} : \\
& x_0^h + \sum_i p_i q_i^h + \sum_i p_{ic} q_{ic}^h = e^h, \\
& x_s^h = x_0^h - \sum_i p_{ic} q_{ic}^h + \sum_i q_i^h d_i^s + \sum_i q_{ic}^h \left(1 - d_i^s\right)\Big\}.
\end{aligned}
$$

The investor's budget set is identical to the one described earlier in the covered CDS economy except that investors can now buy CDS without holding the underlying asset. Hence there is no restriction tying the maximum number of CDS contracts bought to the number of bonds held.

Equilibrium existence and institutional investor

Fostel and Geanakoplos (2016) identify an existence problem in production economies wherein collateral equilibrium may breakdown in the presence of naked CDS. In an earlier working paper version of their paper, Che and Sethi (2010) assume that a retail investor will always demand bonds so that equilibrium does not break down. We follow this assumption to compute equilibrium. Lastly, the presence of the institutional investor would not affect equilibrium in the non-CDS and covered-CDS economies. We restrict the institutional investor's ability to be exposed to any credit risk. This prevents the institutional investor to purchase any risky bondswithout CDS protection. Thus the investor will remain in cash in an economy without CDS. Moreover, in the covered CDS economy, agents who did not sell CDS held a portfolio of cash and covered CDS positions. The institutional investor could just as easily replace the agents' covered CDS positions, leaving them only holding cash and the equilibrium unperturbed.[20]

4.2 Equilibrium

An equilibrium in the naked-CDS economy is a collection of bond prices, CDS prices, firm investment decisions, and investor consumption decisions,

[20] A formal presentation of the institutional investor problem is presented in appendix C.

$p_i, p_{ic}, I_i, (x_0, q_i, q_{ic}, x_s)_{h\in H}, \left(x_0^M, x_s^M\right) \in R_+ \times R_+ \times R_+ \times (R_+ \times R_+ \times R \times R_+) \times (R_+ \times R_+)$,
such that the following are satisfied:

1. $\int_0^1 x_0^h dh + x_0^M + \sum_i p_i q_i^M + \sum_i p_{ic} q_{ic}^M = \int_0^1 e^h dh + e^M$
2. $\sum_i q_i^M + \sum_i \pi_i^s = \sum_i A^s I_i^{\alpha_i}, \ s = \{U, D\}$
3. $\int_0^1 q_{ic}^h dh + q_i^M = 0$
4. $I_i = p_i q_i^M$
5. $\pi_i\left(I_i\right) \geq \pi_i(\hat{I}_i), \ \forall \hat{I}_i \geq 0,$
6. $\left(x_0^h, q_i^h, q_{ic}^h, x_s^h\right) \in B^h\left(p_i, p_{ic}\right) \Rightarrow U^h\left(x\right) \leq U^h\left(x^h\right), \ \forall h$
7. $\left(c_i^M\right) \in B^M\left(p_i, p_{ic}\right) \Rightarrow U^M\left(c\right) \leq U^h\left(c^M\right)$

Condition (1) states that all endowments, including the institutional investor*'s,* can be allocated to one of three purposes: (a) held as collateral to issue CDSs, (b) held by the institutional investor for consumption, or (c) used by the institutional investor to purchase bonds and covered CDSs. Condition (2) says the goods market clears such that total firm output is consumed by firm managers in the form of profits and is used to repay bond holders. Condition (3) says that the CDS market is in zero net supply because all CDSs purchased as naked investments or by the retail investor as covered investments will be equal to all of the CDS issued. Condition (4) is the bond market clearing condition. Condition (5) says firms choose investment to maximize expected profits. Condition (6) states that investors choose a portfolio that maximizes their utility given their budget sets, and condition (7) says that the institutional investor holds a portfolio that maximizes his utility given his budget set.

4.3 Borrowing costs and spillovers revisited

In this section, we show that the borrowing cost spillovers are also present in the naked CDS economy. To keep the examples comparable across economies, we continue to examine an economy for which CDSs trade only on firm-B debt. As before, there will be *marginal buyers*, $h_1 > h_2$. In equilibrium, every agent $h > h_1$ will sell CDSs on type B debt, every agent $h_2 < h < h_1$ will purchase bonds issued by

type G, and every agent $h < h_2$ will buy naked CDSs on type-B debt. Naked CDSs raise type B's borrowing costs because pessimists are able to purchase Arrow-Down securities created through the CDS contract, which increases the price a CDS seller receives for selling such contracts. As the CDS price increases, investors can further leverage their cash endowments because they have to hold less of their own capital to issue CDSs. The resulting marginal CDS sellers are now more pessimistic than the bond buyers in the non-CDS economy. The marginal buyer indifference equations in the naked CDS economy are:

$$\frac{h_1\left(1-d_b^D\right)}{p_b-d_b^D}=\frac{h_1+(1-h_1)d_g^D}{p_g} \tag{12}$$

$$\frac{(1-h_2)\left(1-d_b^D\right)}{1-p_b}=\frac{h_2+(1-h_2)d_g^D}{p_g}. \tag{13}$$

Note here that firm-G bonds are priced by two separate investors. The more optimistic investor, h_1, is indifferent between selling a CDS on firm-B debt and buying firm-G bonds in (12). The more pessimistic investor, h_2, is indifferent between buying a CDS on firm-B debt and buying firm-G bonds in (13). The market-clearing conditions that establish the total purchasing power investors have to clear the markets are $\frac{(1-h_1)}{1-p_{bc}-d_b^D}=\left(q_b+\frac{h_2}{p_{bc}}\right)$ for the CDS market on firm B and $\frac{(h_1-h_2)}{p_g}=q_g$ for the bond market on firm G . Lemma 1 from the covered CDS economy continues to hold, implying that no bonds on which CDS are traded will be purchased without CDS. Furthermore, we derive the following lemma to solve for equilibrium in the naked CDS economy.

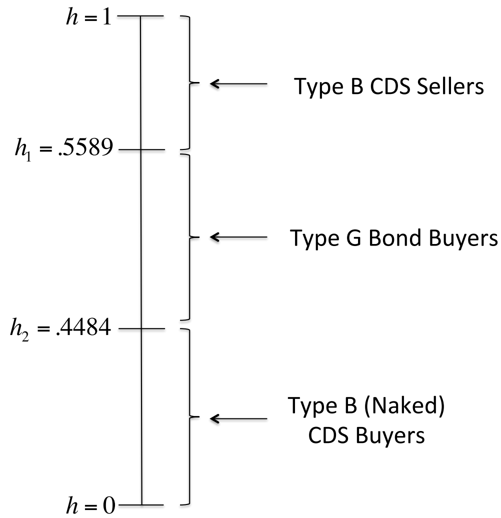
Lemma 2 *In a Naked CDS Economy, the institutional investor is the only agent that holds cash or covered assets in equilibrium.*

Proof. See appendix A. ■

The intuition for optimists not holding bonds carries over from lemma 1. For pessimists, any investor pessimistic enough to remain in cash will be better off buying a naked CDS. Thus, only the institutional investor holds risk-free assets. The borrowing cost implications for the economy are similar to those in Fostel and Geanakoplos (2012 and 2016) and Che and Sethi (2015) and are summarized in the following proposition.

Proposition 6 *Naked CDS raise credit spreads for the CDS reference entity.*

Figure 7: Type-B naked CDS economy



Proof. See appendix B.

In a naked CDS economy, investors are no longer required to hold the underlying bond to buy a CDS. This allows them to utilize their total endowment for purchasing Arrow-Down securities. The increase in demand for CDS contract raises CDS prices and attracts more optimists to sell CDS, resulting in a reallocation of capital away from bond markets and into derivative markets. This "capital reallocation channel" is common to this class of heterogeneous agent models used to analyze CDS and bond market pricing (see Fostel and Geanakoplos (2012 and 2016), Che and Sethi (2015), and Oehmke and Zawadowski (2015) for other examples). What is new in our model in terms of borrowing cost implications, is, once again, the fact that firm G will also be affected by the movement of capital from bond market purchases to collateralizing derivative contracts because firm G's cash flows from production are correlated with firm B's. Figure 7 and table 3 show the marginal buyer characterization of equilibrium and the values of the endogenous variables for the same parameters used in the non-CDS and covered CDS Economies. Figure 8 shows the shift in marginal investors financing firm-G debt in the naked CDS and covered CDS economies.

Figure 8: Naked CDS economy: *Good firm negative spillover*

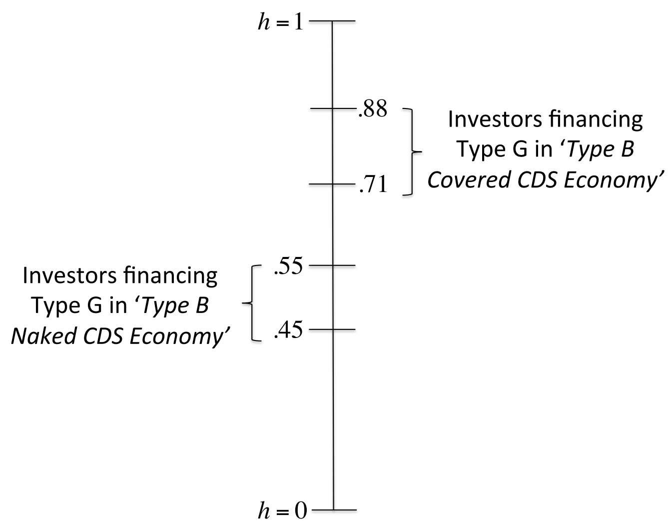


Table 3: Equilibrium values: *Type-B naked CDS economy*

Type B Naked CDS Economy		
	$i = G$	$i = B$
Price: p_i	.6341	.6381
Credit Spread: cs_i	.5770	.5672
Quantity: q_i	.1585	.1268
Investment: I_i	.1005	.0809
Output: Y_i^U	.3170	.1950
Exp.Profit: $E[\pi_i]$	.0793	.0341

4.4 CDS and default risk revisited

As with the covered CDS economy, the default-free contract is unchanged. The debt levels that are sustainable and permit firms to fully honor debt obligations in all states are driven by economy fundamentals, $\left(\alpha_i, \gamma, A^D\right)$. Using proposition 6, we know that credit spreads for risky debt financing increase in the naked CDS economy and firms respond by issuing less risky debt, $\dot{q}_i^\rho < \hat{q}_i^\rho$, where hats denote the risky bond quantity in the non-CDS economy and dots denote the equivalent in the naked CDS economy. Thus, for a given state realization in which $\bar{\gamma}\hat{q}_i^\rho = q_i^{f*}$ holds in the non-CDS economy, $\bar{\gamma}\dot{q}_i^\rho < q_i^{f*}$ will hold in the naked CDS economy.

Proposition 7 *CDS raise the credit spreads of the underlying risky bonds which may*

decrease the likelihood of default in equilibrium. For any value of $A_i^D \in \left[\underline{A}_i^D, \alpha_i\right)$, *firms will issue safe debt contracts if and only if* $\gamma \geq \dot{\gamma} > \bar{\gamma}$.

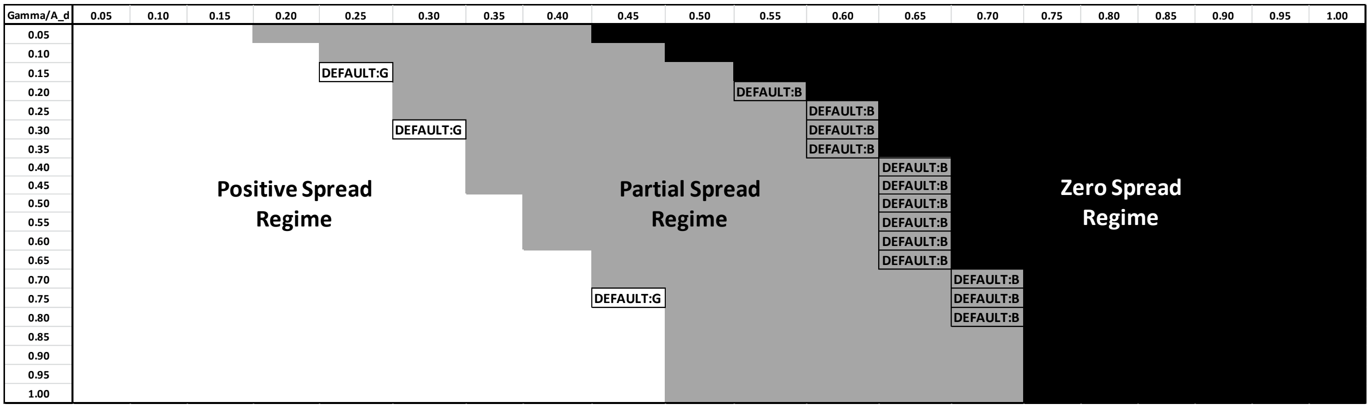
Propositions 5 and 7 show that the effect of CDS trading on credit spreads and endogenous investment can be counterbalanced by the incentives to issue risky rather than safe debt. On the one hand, CDS allow the leveraging of cash, which changes the demand for bonds and the credit spreads at which firms issue risky debt. Safe debt financing, on the other hand, is determined exclusively by economy fundamentals and is unaffected by derivative trading. The fact that derivatives affect credit spreads when firms issue risky debt implies that the relative incentives to choose between risky and safe debt financing are altered even when fundamentals remain unchanged. Thus, financial innovation through derivative trading may effect, not only credit spreads, but also the incidence of equilibrium default in equilibrium.

5 Discussion and concluding remarks

Credit derivatives can alter the tradeoff between firm borrowing costs and default probability. Lower credit spreads incentivize the firm to invest beyond the default-free level. Alternatively, for a given productivity shock, higher credit spreads are more likely to induce firms to issue default-free debt. Financial instruments that effect credit spreads can alter the tradeoff between risky and safe debt financing, even when economy fundamentals are unchanged. In terms of our model, the set of parameters, $\left(\gamma,\, A^D\right)$, in the different economies for which each firm chooses risky debt changes as CDS are introduced.

Figure 9 illustrates the effect covered CDSs have on the risky-debt regime relative to the non-CDS economy. The parameters chosen for example 1 apply throughout the paper. The diagram shows the combination of debt issuance regimes for the two firms. The area labeled "positive (credit) spread regime" is the parameter region for which both firms default when $s = D$. Similarly, "partial spread regime" corresponds to the region where only type-B defaults, and neither firm defaults in the "zero spread regime." The individual cells outlined and labeled "default G" and "default B" highlight the impact of introducing covered CDSs. For example, relative to the non-CDS economy, all of the default B cells indicate that firm B defaults on its debt obligations. Likewise, relative to the non-CDS economy, all of the default G cells indicate that firm G defaults on its debt obligations. The likelihood of $s = U$ given

Figure 9: Increased likelihood of default: *non-CDS versus covered CDS*



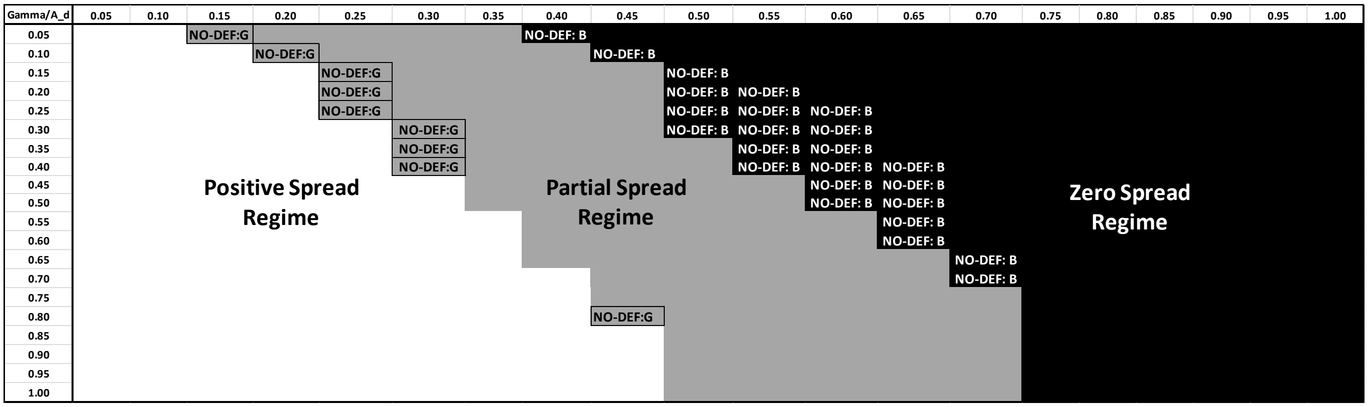
by γ is on the vertical axis (increasing γ moving downward) and the value of the technology shock, A^D, on the horizontal (increasing A^D going from left to right).

For a given A_i^D, where type B may issue safe debt, firm B defaults in the down state for greater values of γ in the covered CDS economy (the gray default B cells would be black in the non-CDS economy).[21] One may note that the associated increase in default likelihood in the covered CDS economy is consistent with the empty creditor narrative. However, our mechanism is novel and does not rely on maturity mis-match between firm assets and liabilities, rollover risk, or coordination problems. Covered CDS can increase default likelihood through the pricing incentives to issue debt that defaults in bad times even when assets and liabilities are perfectly aligned. Additionally, the model shows that the implications of default can be very different depending on whether the demand for CDS is for the purpose of hedging (covered) or for speculating (naked). Current empirical studies cannot distinguish between covered and naked positions at the CDS-counterparty level and thus can only test the covered story of the empty creditor problem.

Figure (10) shows the equilibrium comparing the covered CDS and naked CDS economies for the parameters used in example 1. It is the analogue to figure 9 in that it shows the effect of allowing naked CDSs when CDSs already exist. We make this comparison to show the effect on default of banning naked CDS; presumably, covered CDS would still be allowed. The cells labeled “No Def G” indicate that firm

[21] The default effect on firm B is more pronounced in this example because the derivative is introduced on firm B. Still, there is a spillover default effect on firm G as well.

Figure 10: Decreased likelihood of default: *covered CDS versus naked CDS*



G switches from defaulting on debt obligations in covered CDS economies to fully repaying when naked CDS are introduced. Similarly, the cells labeled "No Def B" indicate that firm B switches from defaulting on debt obligations in covered CDS economies to fully repaying when naked CDS are introduced. In contrast to the previous example, there are fewer values of γ for which the firms default. These results highlight the tradeoff between borrowing costs and default likelihood not identified in the extant CDS literature.

We further show, through the capital reallocation channel, that there are spillover effects to non-CDS reference entities when CDS are introduced in general equilibrium. The spillover effects include changes in bond pricing, firm investment, and default likelihood. Thus, our model provides a new mechanism through which to think about the empirical results on the leverage and investment externalities of CDS trading identified by Li and Tang (2016).

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A Appendix

A.1 Partial-Risk

A.1.1 Non CDS Equilibrium

This equilibrium is characterized by the fact that Firm G is always able to fully repay its debt while the bad firm defaults when $s = D$. This equilibrium will be characterized as a combination of the *Risk-Free Regime* presented in the preceding section and the *Full-Risk Regime* described in Section 2.2.

The system of equations that characterizes equilibrium in the *non-CDS economy* includes eight endogenous variables $(p_i, q_i, I_i, h_1, h_2)$ and is as follows:

$$
\begin{aligned}
&1. h_1 + (1 - h_1)\, d_b^D = p_b \\
&2. h_2 + (1 - h_2) = p_g \\
&3. \alpha_g I_g^{\alpha_g - 1} = \frac{1}{[\gamma A^U + (1-\gamma)\, A^D]} \\
&4. \alpha_b I_b^{\alpha_b - 1} = \frac{1}{p_b} \\
&5. I_i = p_i q_i \\
&6. 1 - h_1 = p_b q_b \\
&7. h_1 - h_2 = p_g q_g
\end{aligned}
$$

Equations (1) & (2) are used to determine bond prices in equilibrium. Equation (3) corresponds to type G's optimizing decision that takes into account firm profitability in the down-state while (4) corresponds to type B's optimizing decision by considering profits exclusively in the up-state. Equation (5) says that each firms'

bond issuance q_i will be in accordance with the desired investment level given bond market prices. Finally, equations (6) & (7) correspond to the bond market clearing conditions for the respective firms.

A.1.2 Covered CDS Equilibrium

CDS allow investors to sell insurance on firm default. Note that in the *Covered-CDS Economy,* CDS buyers are required to hold the underlying asset. In equilibrium, CDS will only be traded on type B debt since type G does not default in the *Partial-Risk Regime.*

The system of equations that characterizes equilibrium in the *Covered-CDS Economy* includes ten endogenous variables $(p_i, p_{bc,} q_i, q_{bc,} I_i, h_1, h_2)$ and is as follows:

$$
\begin{aligned}
&1. \frac{h_1\left(1-d_b^D\right)}{p_b-d_b^D}=1 \\
&2. h_2+\left(1-h_2\right)=p_g \\
&3. \alpha_g I_g^{\alpha_g-1}=\frac{1}{p_g\left[\gamma A^U+(1-\gamma)A^D\right]} \\
&4. \alpha_b I_b^{\alpha_b-1}=\frac{1}{p_b} \\
&5. I_i=p_i q_i \\
&6. 1-h_1=p_b q_b-q_b d_b^D \\
&7. h_1-h_2=p_g q_g \\
&8. p_b+p_{bc}=1 \\
&9. q_{bc}=\frac{1}{1-p_{bc}-d_b^D}
\end{aligned}
$$

Equations (1) & (2) are used to determine bond prices in equilibrium. Equations (3) & (4) corresponds to the firms' optimizing decisions. Equation (5) says that each firms' bond issuance q_i will be in accordance with the desired level given market price for bonds. Equations (6) & (7) correspond to the CDS market and bond market clearing conditions respectively. Equation (8) is a non-arbitrage pricing implication of assuming the return to holding a covered CDS position is equivalent to holding cash and will be priced accordingly. Equation (9) solves for the number of CDS contracts an individual investor can sell given their initial endowment.

A.1.3 Naked CDS Equilibrium

In the *Naked-CDS Economy* CDS buyers are not required to hold the underlying asset. In equilibrium, CDS will only be traded on type B debt since type G does not default in the *Partial-Risk Regime*.

The system of equations that characterizes equilibrium in the *Naked-CDS Economy* includes ten endogenous variables $(p_i, p_{bc}, q_i, q_{bc}, I_i, h_1, h_2)$ and is as follows.

$$1. \frac{h_1\left(1-d_b^D\right)}{p_b-d_b^D} = \frac{(1-h_1)\left(1-d_b^D\right)}{1-p_b}$$

$$2. h_2 + (1-h_2) = p_g$$

$$3. \alpha_g I_g^{\alpha_g-1} = \frac{1}{\left[\gamma A^U + (1-\gamma)A^D\right]}$$

$$4. \alpha_b I_b^{\alpha_b-1} = \frac{1}{p_b}$$

$$5. I_i = p_i q_i$$

$$6. (1-h_1) + p_{bc} q_b + h_2 = \left(q_b + \frac{h_2}{p_{bc}}\right)\left(1-d_b^D\right)$$

$$7. h_1 - h_2 = p_g q_g$$

$$8. p_b + p_{bc} = 1$$

$$9. q_{bc} = \frac{1}{1-p_{bc}-d_b^D}$$

Equations (1) & (2) are used to determine bond prices in equilibrium. Equations (3) & (4) corresponds to the firms' optimizing decisions. Equation (5) says that each firms' bond issuance q_i will be in accordance with the desired level given market price for bonds. Equations (6) & (7) correspond to the CDS market and bond market clearing conditions respectively. Equation (8) is a non-arbitrage pricing implication of assuming the return to holding a covered CDS position is equivalent to holding cash and will be priced accordingly. Equation (9) solves for the number of CDS contracts an individual investor can sell given their initial endowment.

B Appendix Omitted Proofs

Proposition 1. **Proof.** The two bond delivery functions are given by $d_i^s\left(q^i\right), i = G, B$. Note that $d_g^U\left(q_g\right) = d_b^U\left(q_b\right)$, and $d_g^D\left(q_g\right) = \frac{A^D}{\alpha_g} > \frac{A^D}{\alpha_b} = d_b^D\left(q_g\right)$. This means that the bonds deliver the same in the up-state at time 1, and the type g bond returns strictly more than the type b bond in the down state, hence $d_1^g\left(q_0^g\right) > d_1^b\left(q_0^b\right)$. If $p_g = p_b$ every investor would strictly prefer the type g bond because the expected return would be higher and for no additional cost. Thus firm b would not receive debt financing. Moreover, for any investor to be indifferent between the two assets, $p_g > p_b$ because $d_1^g\left(q_0^g\right) > d_1^b\left(q_0^b\right)$. ■

Proposition 2. **Proof.** Plugging in $I_i^{f*} = q_i^{f*}$ from (6) into the default-free bond delivery function $d_i^D\left(q_i^{f*}\right)$: $A_i^D\left(q_i^{f*}\right)^{\alpha_i - 1} \geq 1$ gives

$$\left\{\left[\alpha_i\left(\gamma + (1-\gamma)A_i^D\right)\right]^{\frac{1}{1-\alpha_i}}\right\}^{\alpha_i - 1} \geq 1.$$

Solving through for A_i^D yields the following expression:

$$\frac{\alpha_i \gamma}{1 - \alpha_i(1-\gamma)} \leq A_i^D. \tag{14}$$

Let $\underline{A}_i^D \equiv \frac{\alpha_i\gamma}{1-\alpha_i(1-\gamma)} = A_i^D$. Therefore, any $A_i^D > \underline{A}_i^D(\gamma)$ will allow the firm to issue default-free bonds when $\Pi^{f*} > \Pi^{\rho*}$. It is straightforward to verify $\frac{\partial \underline{A}_i^D}{\partial\gamma} > 0$ for $\alpha_i < 1$. ■

Proposition 3. **Proof.** The extreme cases where $A_i^D < \underline{A}_i^D$ implies $p_i < 1$ and $A_i^D \geq \alpha_i$ implies $p_i = 1$ follow directly from the discussion is the text. For $A_i^D \in \left[\underline{A}_i^D, \alpha_i\right)$ we know from Proposition 2 that $\underline{A}_i^D(\gamma)$ is increasing in γ. Let $\underline{\gamma}$ correspond to a value that determines a threshold of $\underline{A}_i^D\left(\underline{\gamma}\right)$, so that realizations of A_i^D above this can support $p_i = 1$. Now choose a value of $\hat{A}_i^D \in \left(\underline{A}_i^D\left(\underline{\gamma}\right), \alpha_i\right)$. The corresponding $\hat{\gamma}$ for which $\hat{A}_i^D$ is also a threshold for determining whether default-free debt is permissible must also be higher: $\hat{\gamma} > \underline{\gamma}$. This means that if $\hat{A}_i^D(\hat{\gamma})$ corresponds to default-free borrowing for $\hat{\gamma}$ it will also correspond to default-free borrowing for $\underline{\gamma}$ and $\forall\gamma \leq \hat{\gamma}$. Alternatively, if a value $\overline{\gamma}$ only corresponds to default-free borrowing for A_i^D such that $\alpha_i < A_i^D \leq \hat{A}_i^D < \bar{A}_i^D$, then the tuple $\left(\hat{A}_i^D, \bar{\gamma}\right)$ will not be default-free, nor will any tuple $\left(\hat{A}_i^D, \gamma\right)$ with $\gamma \geq \bar{\gamma}$. Thus if borrowing risk free is possible for any given $A_i^D \in \left[\underline{A}_i^D\left(\underline{\gamma}\right), \alpha_i\right)$, it will be the case that there is an upper-bound on the probability of good news $\underline{\gamma} < \overline{\gamma}$ that will push the firm into

issuing bonds with a positive credit spread. ■

Lemma 1. **Proof.** Suppose to the contrary that bonds are purchased unprotected. Then it must be the case that the utility of the agent who buys the unprotected bond is given by $u^b(h_1) = \frac{h_1+(1-h_1)d_b^D}{p_b} > 1$, which can be written as $\frac{h_1\left(1-d_b^D\right)+d_b^D}{p_b} > 1$. Note that the utility of the CDS seller is given by $u^s(h_{cds}) = \frac{h_{cds}\left(1-d_b^D\right)}{p_b-d_b^D}$. Now suppose that the investor h_1 who purchases the bad bond unprotected instead writes the CDS. His utility would be given by $u^s(h_1) = \frac{h_1\left(1-d_b^D\right)}{p_b-d_b^D}$. To finish the proof it suffices to show that h_1 prefers to write CDS over buying unprotected bonds. Let $h_1\left(1-d_b^D\right) = X$, $p_b = Y$, and $d_b^D = \Lambda$. We can then rewrite the utilities in the following way: $u^b(h_1) = \frac{X+\Lambda}{Y}$ and $u^s(h_1) = \frac{X}{Y+\Lambda}$. If $u^s(h_1) > u^b(h_1)$ then,

$$
\begin{aligned}
&\Longrightarrow \quad \frac{X+\Lambda}{Y} < \frac{X}{Y-\Lambda} \\
&\Longrightarrow \quad (X+\Lambda)(Y-\Lambda) < XY. \\
&\Longrightarrow \quad -X\Lambda + \Lambda Y - \Lambda^2 < 0 \\
&\Longrightarrow \quad (X - Y + \Lambda) > 0.
\end{aligned}
$$

Substituting back in for X, Y, and Λ we see that $h_1\left(1-d_b^D\right) - p_b + d_b^D > 0$, which is the same as $\frac{h_1+(1-h_1)d_b^D}{p_b} > 1$. Thus, any agent who would buy unprotected bonds would be better off selling CDS. ■

Proposition 4. **Proof.** This follows directly from Che & Sethi (2014) with two differences. 1) the marginal investor indifferent between selling CDS and having exposure to another risky asset does not directly price the bond for which the CDS is sold. The investor prices the return to the CDS relative to the return on the alternative risky asset. 2) investment is endogenous so that supply of bonds q_i is not fixed.

Suppose to the contrary that $\tilde{p}_i \leq \hat{p}_i, i = G, B$, where $\tilde{p}_i$ is firm is bond price in an economy with covered CDS and $\hat{p}_i$ is the bond price in a *Baseline Economy.* The expected return any given investor h places on selling a CDS is given by

$$
\frac{h\left(1-d_i^D\right)}{\tilde{p}^i - d_i^D}.
$$

In the up state, a CDS seller retains the value of the contract, $\left(1-d_i^D\right)$. The CDS seller must hold enough collateral to sell this contact. That collateral value is equal to $1-d_i^D-p_i^c$. The fundamental recovery value of the bond d_i^D is not insured, and

investors receive a price for selling CDS that they can use as collateral for additional contracts p_i^c. This is a decreasing function of bond prices because higher bond prices mean lower CDS prices. Higher bond prices mean investors need to post more of their own cash as collateral to sell a CDS, which lowers the expected return to their collateral. Thus, as bond prices decrease, the marginal investor indifferent to selling a CDS and buying an alternative risky asset is less optimistic because the returns to selling a CDS are increasing relative to the return on holding the alternative asset. Therefore, $h^1\left(\tilde{p}_i\left(\tilde{q}_i\right), d_i^s\right) \leq h^1\left(\hat{p}_i\left(\hat{q}_i\right), d_i^s\right)$. The market clearing conditions in the *Baseline Economy* and *Covered CDS Economy* for the firm for whom CDS are written are, respectively, given by

$$1 - h^1\left(\hat{p}_i\left(\hat{q}_i\right), d_i^s\right) = \hat{p}_i \hat{q}_i \tag{15}$$

$$1 - h^1\left(\tilde{p}_i\left(\tilde{q}_i\right), d_i^s\right) = \tilde{p}_i \tilde{q}_i - \tilde{q}_i d_i^s. \tag{16}$$

Note that bond quantities are different across economies as well. However, it can be shown that $q_i = \beta_i \times (p_i)^{-\Gamma_i}$ where $\beta_i \equiv \left(\frac{1}{\alpha_i}\right)^{\frac{1}{\alpha_i - 1}}$ and $\Gamma_i \equiv \frac{\alpha_i}{\alpha_i - 1} < 0$. This means that $\frac{\partial q_i}{\partial p_i} > 0$ and p_i and q_i move in the same direction. Thus the left hand side of (16) is weakly greater than that of (15) while the right hand side is strictly less. A contradiction. Hence $\tilde{p}_i > \hat{p}_i, i = G, B$. ■

Corollary 1. **Proof.** This follows directly from Proposition 4 in that $h^1\left(\tilde{p}_B\left(\tilde{q}_B\right), d_B^D\right) > h^1\left(\hat{p}_B\left(\hat{q}_B\right), d_B^D\right)$ where the tilde corresponds to bond pricing in the Covered Economy and the hat corresponds to bond pricing in the Non-CDS economy. The higher is the bond price, the lower is the CDS price, which lowers the return to "buying the Arrow-Up." The resulting marginal CDS seller in the Covered CDS Economy must therefore be more optimistic than the marginal bond buyer in the non-CDS economy. Because the set of agents is connected, the next most optimistic agent h_-^1 in the covered CDS economy will be more optimistic than h_-^1 in the non-CDS economy. ■

Proposition 5. **Proof.** From Proposition 3 we know that if default-free borrowing exists in the range of A_i^D for which $p_i = 1$ is possible, it will only occur if γ is below the threshold $\gamma < \bar{\gamma} < 1$. From Proposition 4, we know that $\frac{\partial q_i}{\partial p_i} > 0$ so that $\tilde{q}_i^{\rho *} > \hat{q}_i^{\rho *}$, bond quantities rise in the covered CDS economy. This therefore implies that $\bar{\gamma}\tilde{q}_i^{\rho *} > \bar{\gamma}\hat{q}_i^{\rho *} = q_i^{f*}$. Lastly, we know that for the same A_i^D from the $\left(\bar{\gamma}, A_i^D\right)$-tuple that made the two debt contracts equivalent in the Non-CDS economy, if issuing default-free is chosen in the covered CDS economy, there must be another $\tilde{\gamma} < \bar{\gamma}$ for

which the two debt contracts yield the same profits; otherwise, the firm always issues risky debt in which case default always occurs at A_i^D. ■

Lemma 2: **Proof.** Suppose to the contrary that h_1 holds cash. It must be the case then that the investor prefers holding cash to any other instrument int the economy. Thus we can say

$$h_1 + (1 - h_1)\, d_b^D < p_b, \tag{17}$$

and

$$1 > \frac{(1 - h_1)\left(1 - d_b^D\right)}{1 - p_b}. \tag{18}$$

Inserting (17) into the denominator of the *r.h.s* of (18) we do not perturb the inequality

$$1 > \frac{(1 - h_1)\left(1 - d_b^D\right)}{1 - \left[h_1 + (1 - h_1)\, d_b^D\right]}.$$

Rearranging and regrouping we get

$$(1 - h_1)\left(1 - d_b^D\right) > (1 - h_1)\left(1 - d_b^D\right) \ \otimes$$

a contradiction. ■

Proposition 6. **Proof.** To see this consider the re-written equilibrium market clearing condition for firm i when naked CDS are permitted where dots above variables indicate the naked CDS values:

$$1 - h^1\left(\dot{p}_i\left(\dot{q}_i\right), d_i^D\right) = \dot{p}_i \dot{q}_i - \dot{q}_i d_i^D + \left(\dot{p}_i - d_i^D\right)\frac{h^2}{1 - \dot{p}_i}. \tag{19}$$

The only functional form difference between (19) and (16) is the final term on the right hand side, $\left(\dot{p}_i - d_i^D\right)\frac{h^2}{1-\dot{p}_i}$. This term is always greater than zero because $\dot{p}_i > d_i^D$ whenever a funding equilibrium exists. The left hand sides are the same form. Consider to the contrary that $\dot{p}_i \geq \hat{p}_i$ in which case the left hand side of (19) is less than or equal to the left hand side of (15). The right hand side of (19) is a monotonically decreasing function of d_i^D. Suppose the upper-limiting case $d_i^D = \dot{p}_i$. Then the right hand side of (19) is equal to zero and $h^1\left(\dot{p}_i\left(\dot{q}_i\right), d_i^D\right) = 1$ meaning there is no equilibrium funding for firm i. Suppose the lower-limiting case where $d_i^D = 0$, in which case the right hand side of (19) is strictly greater than that of (15), a contradiction. Thus $\dot{p}_i < \hat{p}_i$ for $\forall d_i^D \in \left[0, p_i^D\right)$. ■

Proposition 7. **Proof.** The logic is the same as Proposition 5. Use $\dot{q}_i^{\rho*} < \hat{q}_i^{\rho*}$ from

Proposition 6 and $\frac{\partial q_i}{\partial p_i} > 0$ from Proposition 3. ■

C Institutional Investor

Let his utility be given by $U^M(x_s, c_s)\epsilon$, $\epsilon > 0$, where x^s is time 1 consumption and c_i is a covered CDS package made up of a bond and CDS: $c_i = q_i + q_{ic}$.[22] Since he holds no risk, a CDS contract will accompany every bond purchased. Furthermore, $U^{M'}(c_i) > 0$ so that the investor always prefers to invest when the opportunity exists. We can now characterize the *institutional investor's* budget set:

$$\begin{aligned} B^M(p_i, p_{ic}) \;=\; & \Big\{\left(x_0^M, q_i^M, q_{ic}^M, x_s^M\right) \in R_+ \times R_+ \times R_+ \times R_+ : \\ & x_0^M + \sum_i (p_i + p_{ic})\, c_i^M = e^M, \\ & x_s^M = x_0^M + \sum_i c_i^M \Big\}. \end{aligned}$$

The investor uses his endowment to either purchase bonds with covered CDS or for consumption. The institutional investor consumes the same amount in either state since covered CDS positions have identical payouts in both states. Final consumption is the sum of cash carried forward and the number of covered CDS packages purchased.

[22] The micro-foundations of the investor's preferences are not explicitly modeled, though it is easy to do so. For example, one could assume the investor is a fund manager who invests in covered CDS for which he receives a fee given by ϵ.