

IDIOSYNCRATIC JUMP RISK MATTERS: EVIDENCE FROM EQUITY RETURNS AND OPTIONS*

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Abstract

The recent literature provides conflicting empirical evidence on the relationship between idiosyncratic risk and equity returns. This paper sheds new light on this relationship by exploiting the richness of option data. We disentangle four risk factors that potentially contribute to the equity risk premium: systematic Gaussian risk, systematic jump risk, idiosyncratic Gaussian risk, and idiosyncratic jump risk. First, we find that while systematic risk factors explain the greater part of the risk premium on a stock, idiosyncratic factors explain more than 40% of the average premium. Second, we show that the contribution of idiosyncratic risk to the equity risk premium arises exclusively from the jump risk component. Tail risk thus plays a central role in the pricing of idiosyncratic risk.

JEL Classification C51, C58, G12, G13

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1 Introduction

An investor should be rewarded for bearing systematic risk. One of the key insights of Sharpe (1964) and Lintner's (1965b) CAPM, building on the seminal work of Markowitz (1952), is that idiosyncratic risk, however, should not carry a risk premium as it can be diversified away. Since the CAPM, numerous asset pricing models have been developed building on the premise that idiosyncratic risk is not priced.¹ However, the recent literature has strongly challenged this notion.² Although the channel through which idiosyncratic risk could be priced is still a matter of debate, it is now widely accepted that, given market incompleteness, idiosyncratic risk can be priced.³ While previous studies are informative about the relative importance of idiosyncratic risk in explaining expected stock returns, they do not attempt to identify whether the importance of idiosyncratic risk arises from its diffusive or tail components. Thus, little is known on the relative contribution of systematic and idiosyncratic diffusive and tail risk in explaining the equity premium.

Our study departs from existing work by decomposing stocks' systematic and idiosyncratic shocks into a Gaussian and a jump components. Our approach offers an ideal framework to study the relative importance of each factors in explaining expected excess returns on equity. In particular, our study is the first to uncover the central role of idiosyncratic tail risk in explaining expected stock returns. Indeed, we find that idiosyncratic risk explains more than 40% of expected excess equity returns and, more importantly, that this is exclusively due to the jump risk component. Idiosyncratic Gaussian risk is not priced. This finding is consistent with the idea that investors have a hard time to hedge idiosyncratic tail risk and, thus, require a premium to bear their exposure to this risk.

We exploit the richness of stock option data to extract the expected risk premium associated with

¹Notably, Merton's (1973) ICAPM extends the insights of the CAPM to an intertemporal setup. The arbitrage pricing theory of Ross (1976) shows that any common return factor is a potential asset pricing factor. Fama and French (1992, 1993, 2015) and Carhart (1997), for instance, identify such potential factors, but diversifiable idiosyncratic risk is still assumed not to carry any risk premium.

²Concerns about the pricing of idiosyncratic risk dates back to Douglas (1969) and Lintner (1965a). Goyal and Santa-Clara (2003) contributed to putting this debate back at the forefront of the asset pricing literature by providing empirical evidence that idiosyncratic matters. Among others, Ang, Hodrick, Xing, and Zhang (2006) find that stocks with high idiosyncratic volatilities had "abysmally" low average returns, lower than what could be explained by their exposure to aggregate volatility.

³Goyal and Santa-Clara (2003) highlight that a possible channel is background risk; investors hold nontraded assets (e.g. human capital or private businesses) which add background risk to their traded portfolio decisions. Jacobs and Wang (2004) provide evidence that idiosyncratic consumption risk is a priced factor in the cross section of stock returns. Hence, the average idiosyncratic stock variance being a proxy for idiosyncratic consumption risk could explain why idiosyncratic risk is priced. Consistent with this insight, Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2014) provide evidence linking the average idiosyncratic volatility to income risk faced by households. Alternatively, Stambaugh, Yu, and Yuan (2015) argue that the negative relationship between idiosyncratic volatility and stock returns could be driven by arbitrage asymmetry, as buying could be easier than shorting for many equity investors.

each risk factor, thereby avoiding the exclusive use of noisy realizations of historical equity returns. To this end, we develop a GARCH-jump model in which a firm's systematic and idiosyncratic risk have both a Gaussian and a tail component.

Our pricing kernel is such that each risk factor can potentially be priced. The model offers quasi-closed form solutions for the price of European options. We estimate the model on 260 firms that are or were part the S&P 500 index between 1996 and 2015, using equity returns and option prices of the market index and of each individual firm.⁴ To our knowledge, this is the most comprehensive joint estimation analysis of option-pricing models conducted in the literature.

Our empirical analysis highlights three new results. First, systematic risk accounts for only 59.8% of the average equity risk premium (ERP) on a stock, only one third of which is due to systematic normal risk. Second, and most importantly, we find that the 40.2% contribution of idiosyncratic risk to the ERP is essentially due to idiosyncratic jump risk only. That is, the Gaussian component of idiosyncratic risk, which is easily diversifiable, is not priced once other sources of risk are accounted for.⁵ Consistent with Bates (2008), jump and normal risks are priced differently by investors. While the results of Christoffersen, Jacobs, and Ornathanalai (2012) and Ornathanalai (2014) already supported this view at the market level, our results document that both sources of risk have drastically different impact on the expected return of individual stocks. When estimating a nested version of the model in which idiosyncratic jump risk is omitted, idiosyncratic normal risk appears to be priced. For the great majority of stocks, the nested variant of the model appears to be misspecified, however, since it offers a significantly worse fit to equity returns and options than the model with idiosyncratic jumps. This result is of significant interest as most of the literature on idiosyncratic risk assumes conditional normality.

Our third empirical finding is that idiosyncratic jump risk shares a strong commonality across firms. Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2014) document that idiosyncratic (total) variances have a strong factor structure. Based on the idiosyncratic volatilities of 20,000 CRSP stocks over 85 years, they document that a single factor explains 35% of the time variation firm-level idiosyncratic risk. In light of these results, our model of stock variance allows for two sources of commonality: one arising from commonality in idiosyncratic normal risk, the other from commonality in idiosyncratic jump risk. Over the 20 years in our sample, 260 firm-by-firm regressions of total idiosyncratic variance

⁴We considered all 1,000 stocks that were part of the index during this period; neglected stocks were set aside only because not enough options were liquidly traded over at least a consecutive 5-year window.

⁵Note that, while the expected stock return is not affected by idiosyncratic Gaussian risk, an option's vega is still positive and affected by total volatility.

on these two sources of commonality yield an average R^2 of 73.4%; regressing on the commonality in idiosyncratic jump risk yields an average R^2 of 31.8%. Our results thus extend the findings of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2014) in that we document that idiosyncratic tail risk explains a large fraction of the commonality in idiosyncratic variance. Already hard to hedge by nature, tail risk becomes virtually undiversifiable in times of turmoil, which justifies the risk premium attached to it.

Our study is the first to conduct a joint estimation, based on equity returns and options, of an option-valuation model to disentangle the four risk premiums associated with systematic and idiosyncratic, normal and tail (jump) risk. It is, however, closely related to several contemporaneous papers.

Christoffersen, Fournier, and Jacobs (2013, hereafter CFJ) document a strong factor structure in equity options. Consequently, building on Heston (1993), they develop a stochastic volatility model in which a firm total variance is decomposed into a systematic and an idiosyncratic components. The authors study the effect of firm beta and market variance to explain the cross-sectional variations of equity options. Among others, their model predicts that stocks with higher betas have higher implied volatilities and steeper smiles, consistent with the empirical findings of Duan and Wei (2009).

Our framework extends that of CFJ in that we allow for a jump component both in market returns and in the idiosyncratic part of stock returns. Moreover, our joint estimation methodology builds on those of Christoffersen, Jacobs, and Ornathanalai (2012) and Ornathanalai (2014), and allows us to quantify how the equity risk premium is affected by the four sources of risk affecting stocks in our setup. Our model and pricing kernel nest those of Elkamhi and Ornathanalai (2010) who complement the analysis in CFJ and quantify the impact of market jump risk on equity options. They find that firms with a larger return compensation for systematic normal risk have a higher option-implied volatility level, while firms with a larger return compensation for systematic jump risk have steeper option-implied volatility slope. However, stocks in their framework do not exhibit idiosyncratic jump risk, and they do not study the pricing of idiosyncratic risk. Along the same lines, Babaoğlu (2015) further document that a “jump beta” is needed to adequately explain equity returns, market risk exposures, and equity option prices.

Bloorforoosh (2014) extends the CFJ model by allowing for idiosyncratic normal risk to be priced. He finds strong empirical support for the hypothesis that idiosyncratic risk is indeed priced. Bloorforoosh (2014) also documents that idiosyncratic volatilities exhibit a factor structure virtually as strong as that of total volatilities, consistent with Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2014). Using a similar model, Xiao and Zhou (2014) study the same four risk factors as we do, but relying exclusively on realizations of historical equity returns.

Closer to our study, Gouri er (2014) further extends CFJ and estimates a continuous-time jump-diffusion model using a two-stage estimation procedure based on equity returns, options and intraday data observed on 29 stocks between 2006 and 2012. Gouri er’s (2014) framework allows her to study the important role played by total (normal and jump) idiosyncratic risk in the equity and, most importantly, the variance risk premium. She finds that compensation for idiosyncratic risk represents, on average, 50% of the equity risk premium and 80% of the variance risk premium. Although the models, datasets and estimation methods in our studies differ along several dimensions, the results that are common to our two studies are consistent and our analyses complement one another. In particular, Gouri er (2014) provides strong empirical evidence that idiosyncratic risk is a key determinant of the equity risk premium; we provide strong empirical evidence that tail risk is actually at the core of the relationship between idiosyncratic risk and the equity risk premium.

This paper is organized as follows. Section 2 presents our model for the market and the individual stocks. Section 3 presents the data and discusses the estimation methodology. Then, Section 4 presents our empirical analysis. Section 5 concludes.

2 The Model

We develop a model in which, in the spirit of the CAPM, stocks are exposed to systematic risk. Unlike the traditional one-factor CAPM, however, market and stock return are not solely driven by a diffusive component. The market can crash, or more generally jump, and the stocks in our model are exposed to this systematic jump risk, as well as to idiosyncratic normal and jump risk. As such, our model falls under the framework of Kraus and Litzenberger (1976), but extends it in various directions.

2.1 Stock Returns

Returns on the market index, M_t , and a given stock, S_t , are modeled as follows:

$$R_{M,t+1} \equiv \log\left(\frac{M_{t+1}}{M_t}\right) = \mu_{M,t+1} - \xi_{M,t+1}^{\mathbb{P}} + z_{M,t+1} + y_{M,t+1}, \quad (2.1)$$

$$R_{S,t+1} \equiv \log\left(\frac{S_{t+1}}{S_t}\right) = \mu_{S,t+1} - \xi_{S,t+1}^{\mathbb{P}} + \beta_{S,z}z_{M,t+1} + \beta_{S,y}y_{M,t+1} + z_{S,t+1} + y_{S,t+1} \quad (2.2)$$

where stock returns are driven by the stock’s exposure to systematic Gaussian and jump risk, $z_{M,t+1}$ and $y_{M,t+1}$, as well as stock-specific innovations $z_{S,t+1}$ and $y_{S,t+1}$, respectively capturing idiosyncratic normal

and jump risk.

For $u \in \{M, S\}$, $\xi_{u,t+1}^{\mathbb{P}}$ is the convexity correction associated with the Gaussian, $z_{u,t+1}$, and the normal inverse Gaussian (NIG) innovations, $y_{u,t+1}$.⁶ Hence,⁷

$$\mathbb{E}_t^{\mathbb{P}}[M_{t+1}] = M_t \exp(\mu_{M,t+1}) \text{ and } \mathbb{E}_t^{\mathbb{P}}[S_{t+1}] = S_t \exp(\mu_{S,t+1}).$$

That is, $\mu_{u,t+1} - r_{t+1}$ can be interpreted as the instantaneous equity risk premium on the index and the stock, given the risk-free rate r_{t+1} .⁸

Before discussing the exact form of the instantaneous risk premiums (Section 2.2), we further characterize the distribution of the shock processes. The Gaussian innovations are given by

$$z_{u,t+1} = \sqrt{h_{u,z,t+1}} \varepsilon_{u,t+1}, \quad u \in \{M, S\}$$

where the $\varepsilon_{u,t+1}$ are serially independent standard normal random variables and the conditional variance of $z_{u,t+1}$ follows a GARCH dynamics. Indeed, the market conditional variance is

$$\begin{aligned} h_{M,z,t+1} &= w_{M,z} + b_{M,z} h_{M,z,t} + \frac{a_{M,z}}{h_{M,z,t}} (z_{M,t} - c_{M,z} h_{M,z,t})^2. \\ &= \sigma_{M,z}^2 + b'_{M,z} (h_{M,z,t} - \sigma_{M,z}^2) + \frac{a_{M,z}}{h_{M,z,t}} (z_{M,t}^2 - h_{M,z,t} - 2c_{M,z} h_{M,z,t} z_{M,z,t}), \end{aligned} \quad (2.3)$$

where $\sigma_{M,z}^2 = \frac{w_{M,z} + a_{M,z}}{1 - b'_{M,z}}$, is the unconditional level of the market variance, and $b'_{M,z} = b_{M,z} + a_{M,z} c_{M,z}^2$, is the variance persistence.

The specification of the stock's conditional variance is inspired from the literature on component

⁶The convexity correction, $\xi_{M,t}^{\mathbb{P}} = \xi_{z_{M,t}}^{\mathbb{P}}(1) + \xi_{y_{M,t}}^{\mathbb{P}}(1)$, is based on the cumulant generating function of z_M and y_M (cf. Appendix A). The same holds for $\xi_{S,t+1}^{\mathbb{P}} = \xi_{z_{S,t}}^{\mathbb{P}}(\beta_{S,z}) + \xi_{y_{S,t}}^{\mathbb{P}}(\beta_{S,y}) + \xi_{z_{S,t}}^{\mathbb{P}}(1) + \xi_{y_{S,t}}^{\mathbb{P}}(1)$.

⁷The filtration is generated by the market noise terms as well as the stock noise terms, that is $\mathcal{F}_t^{\mathbb{S}} = \sigma\{z_{M,\tau}, y_{M,\tau}, z_{S,\tau}, y_{S,\tau}; S \in \mathbb{S}\}_{\tau=1}^t$. $\mathbb{E}_t^{\mathbb{P}}[S_{t+1}]$ is a shorthand for $\mathbb{E}^{\mathbb{P}}[S_{t+1} | \mathcal{F}_t^{\mathbb{S}}]$. Since all innovation time series are independent, $\mathbb{E}^{\mathbb{P}}[M_{t+1} | \mathcal{F}_t^{\mathbb{S}}] = \mathbb{E}^{\mathbb{P}}[M_{t+1} | \mathcal{F}_t^M]$ where $\mathcal{F}_t^M = \sigma\{z_{M,\tau}, y_{M,\tau}\}_{\tau=1}^t$ and we still use $\mathbb{E}_t^{\mathbb{P}}[\cdot]$ to represent both conditional expectations.

⁸Over a short period of time, $\mu_{M,t+1}$ and r_{t+1} are close to zero, such that

$$\mathbb{E}_t^{\mathbb{P}}[M_{t+1}/M_t] - \mathbb{E}_t^{\mathbb{Q}}[M_{t+1}/M_t] = \exp(\mu_{M,t+1}) - \exp(r_{t+1}) \approx \mu_{M,t+1} - r_{t+1}.$$

volatility models,⁹ that is

$$\begin{aligned} h_{S,z,t+1} &= q_{S,z,t+1} + b_{S,z}(h_{S,z,t} - q_{S,z,t}) + \frac{a_{S,z}}{h_{S,z,t}}(z_{S,t}^2 - h_{S,z,t} - 2c_{S,z}h_{S,z,t}z_{S,t}) \\ &= \kappa_{S,z}h_{M,z,t+1} + b_{S,z}(h_{S,z,t} - \kappa_{S,z}h_{M,z,t}) + \frac{a_{S,z}}{h_{S,z,t}}(z_{S,t}^2 - h_{S,z,t} - 2c_{S,z}h_{S,z,t}z_{S,t}). \end{aligned} \quad (2.4)$$

However, rather than varying around a long-run volatility component of its own, the conditional variance of a particular stock loads on market variance through $\kappa_{S,z}h_{M,z,t+1}$, while the idiosyncratic variance in excess of this central tendency, $h'_{S,z,t+1} = h_{S,z,t+1} - \kappa_{S,z}h_{M,z,t+1}$, has a GARCH structure. In the spirit of Martin and Wagner (2016), we refer to $h'_{S,z,t+1}$ as the excess idiosyncratic variance.¹⁰

The jumps, $y_{u,t+1}$, have a NIG distribution with location parameter set at 0, a tail heaviness parameter α_u and an asymmetry parameter δ_u . Following Ornathanalai (2014), the time-homogeneous scale parameter of the distribution is allowed to vary and is denoted by $h_{u,y,t+1}$.¹¹ We refer to $h_{u,y,t+1}$ as the jump intensity process.¹² The jump intensities of the market and the stock exhibit GARCH dynamics alike those of their variance counterparts, but with separate parameters:¹³

$$h_{M,y,t+1} = w_{M,y} + b_{M,y}h_{M,y,t} + \frac{a_{M,y}}{h_{M,y,t}}(z_{M,t} - c_{M,y}h_{M,y,t})^2, \quad (2.5)$$

$$h_{S,y,t+1} = \kappa_{S,y}h_{M,y,t+1} + b_{S,y}(h_{S,y,t} - \kappa_{S,y}h_{M,y,t}) + \frac{a_{S,y}}{h_{S,y,t}}(z_{S,t}^2 - h_{S,y,t} - 2c_{S,y}h_{S,y,t}z_{S,t}). \quad (2.6)$$

As for the variance of Gaussian shocks, idiosyncratic jump intensity has a central tendency $\kappa_{S,y}h_{M,y,t+1}$ and excess idiosyncratic intensity $h'_{S,y,t+1} = h_{S,y,t+1} - \kappa_{S,y}h_{M,y,t+1}$.

Conditional moments of the market and stock returns are derived in the Online Appendix OA.2. In

⁹On GARCH component models, see, among others, Engle and Lee (1999), Christoffersen, Jacobs, Ornathanalai, and Wang (2008), Engle and Rangel (2008), and Engle, Ghysels, and Sohn (2013).

¹⁰The variance process in Gouriéroux (2014) has a similar structure and she refers to the analogue of $h'_{S,z,t+1}$ as residual idiosyncratic variance.

¹¹Earlier drafts of this paper featured Poisson rather than NIG jumps. While the main results were qualitatively similar, the estimated jump parameters were much less stable. In particular, the Poisson-jump version of the model had a harder time accommodating the positive jumps during and after the Great Recession.

¹²Strictly speaking, $h_{u,y,t+1}$ is not an intensity as it does not parameterize the number of jumps observed over a period Δt . However, the normal-inverse Gaussian distributions is closed under convolution in the sense that, given α_u and δ_u , the sum of two NIG shocks with scale parameters h_1 and h_2 would have a scale parameter of $h_1 + h_2$. Hence, the NIG jump as specified here is observationally equivalent to a compound Poisson process with i.i.d. NIG increments whose intensity would be time-varying (cf. Online Appendix OA).

¹³Christoffersen, Jacobs, and Ornathanalai (2012) compare, on market data, a model in which a single factor drives normal variance and jump intensity to a model akin to ours. They find the model with separate variance and intensity dynamics to dominate its counterpart in terms of fitting the data.

particular, total market and stock variances are given by

$$\text{Var}_t^{\mathbb{P}} [R_{M,t+1}] = h_{M,z,t+1} + \frac{\alpha_M^2}{(\alpha_M^2 - \delta_M^2)^{3/2}} h_{M,y,t+1}, \quad (2.7)$$

$$\text{Var}_t^{\mathbb{P}} [R_{S,t+1}] = \underbrace{\beta_{S,z}^2 h_{M,z,t+1} + \beta_{S,y}^2 \frac{\alpha_M^2}{(\alpha_M^2 - \delta_M^2)^{3/2}} h_{M,y,t+1}}_{\text{total systematic variance}} + \underbrace{h_{S,z,t+1} + \frac{\alpha_S^2}{(\alpha_S^2 - \delta_S^2)^{3/2}} h_{S,y,t+1}}_{\text{total idiosyncratic variance}}. \quad (2.8)$$

Following the literature, we define idiosyncratic variance as the variance of the residuals obtained after accounting for systematic risk factors, here normal and jump market risk. In sum, our model of market returns is essentially the NIG variant of the model considered in Ornathanalai (2014).¹⁴ We simply extend his framework to allow stocks (i) to have systematic normal and jump risk exposure and (ii) to exhibit idiosyncratic normal and jump risk. In particular, our model remains in the affine class of models, which is key to obtaining a closed-form solution for the price of European options on the market index and individual stocks (cf. Section 2.3). This solution generalizes those of Elkamhi and Ornathanalai (2010) and Babaoğlu (2015) by adding idiosyncratic jumps in stock returns.

2.2 Pricing Kernel and Risk Premiums

In an incomplete market setup, the pricing kernel, m_{t+1} , is potentially affected by untraded sources of risk. As highlighted in the literature on modeling the pricing kernel, in our context, it suffices to work with the projection of the pricing kernel on the observed sources of risk. Indeed, if p_t is the time t price of an asset with a time $t + 1$ cash flow x_{t+1} that depends on the realization of $\{z_{M,t+1}, y_{M,t+1}, z_{S,t+1}, y_{S,t+1}\}$,¹⁵ then

$$p_t = \mathbb{E}_t^{\mathbb{P}} [m_{t+1} x_{t+1}] = \mathbb{E}_t^{\mathbb{P}} \left[\mathbb{E}_t^{\mathbb{P}} \left[m_{t+1} x_{t+1} \mid z_{M,t+1}, y_{M,t+1}, z_{S,t+1}, y_{S,t+1} \right] \right] = \mathbb{E}_t^{\mathbb{P}} [\tilde{m}_{t+1} x_{t+1}]$$

where $\tilde{m}_{t+1} = \mathbb{E}_t^{\mathbb{P}} [m_{t+1} \mid z_{M,t+1}, y_{M,t+1}, z_{S,t+1}, y_{S,t+1}]$. If $z_{S,t+1}$ and $y_{S,t+1}$ are orthogonal to the pricing kernel, then they do not matter in the pricing and the projection is simply $\tilde{m}_{t+1} = \mathbb{E}_t^{\mathbb{P}} [m_{t+1} \mid z_{M,t+1}, y_{M,t+1}]$.

The recent literature, however, highlights that firm-specific (or idiosyncratic) risk can be correlated with risk factors that do enter the pricing kernel. In line with much of the option pricing literature, we take a reduced-form approach to modeling the pricing kernel and assume an exponentially affine

¹⁴In Ornathanalai's 2014 study, the NIG variant of the model offers the best fit to market data when compared to variants with Merton jumps, variance gamma jumps, or CGMY jumps (Carr, Geman, Madan, and Yor 2002).

¹⁵See, for instance, Rubinstein (1975), Brennan (1979), Ait-Sahalia and Lo (1998) Jackwerth (2000) and Bakshi, Madan, and Panayotov (2010) for analyses of pricing kernels dependent on market returns.

Radon-Nikodym derivative (RND)

$$e^{r_{t+1}} \tilde{m}_{t+1} = \frac{\frac{dQ}{dP} \Big|_{\mathcal{F}_{t+1}^S}}{\frac{dQ}{dP} \Big|_{\mathcal{F}_t^S}} = \frac{\exp(-\Lambda_M z_{M,t+1} - \Gamma_M y_{M,t+1} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t+1} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t+1})}{E_{\mathcal{F}_t^S}^P [\exp(-\Lambda_M z_{M,t+1} - \Gamma_M y_{M,t+1} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t+1} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t+1})]}, \quad (2.9)$$

where \mathbb{S} is the set of firms in the economy. Implicitly, Λ_S and Γ_S are related to the projection of the pricing kernel on $z_{S,t+1}$ and $y_{S,t+1}$. In particular, if the firm-specific risk factors are not priced, that is $\Lambda_S = \Gamma_S = 0$, then our RND is equivalent to the one used by Christoffersen, Jacobs, and Ornathanalai (2012). As they point out, their RND is consistent with the pricing kernel studied by Bates (2008).¹⁶

Risk Premiums

As in Christoffersen, Jacobs, and Ornathanalai (2012) and Ornathanalai (2014), the pricing kernel in (2.9) yields an equity risk premium, $\mu_{M,t} - r_t$ which admits a decomposition in terms of a normal and a jump risk premium, that is

$$\mu_{M,t} - r_t = \lambda_M h_{M,z,t} + \gamma_M h_{M,y,t} \quad (2.10)$$

where the mappings between λ_M and γ_M and their pricing kernel counterparts Λ_M and Γ_M are given in Appendix C. Note that if a market price of risk in the RND is zero (e.g. $\Gamma_M = 0$), then the associated risk premium is zero (e.g. $\gamma_M = 0$).

Appendix C further establishes that the equity risk premium on a stock, $\mu_{S,t} - r_t$, can be decomposed in four risk premiums: the normal and jump market risk premiums, as well as the idiosyncratic normal and jump premiums:

$$\mu_{S,t} - r_t = \underbrace{\beta_{S,z} \lambda_M h_{M,z,t} + \gamma_{M,S}(\beta_{S,y}) h_{M,y,t}}_{\text{systematic}} + \underbrace{\lambda_S h_{S,z,t} + \gamma_S h_{S,y,t}}_{\text{idiosyncratic}} \quad (2.11)$$

Once again, if a market price of risk in the RND is zero (e.g. $\Gamma_S = 0$), then the associated risk premium is zero (e.g. $\gamma_S = 0$). Besides, note that although the model is affine, the premium associated with systematic jump depends non-linearly on the jump beta, $\beta_{S,y}$, and the market price of jump risk, γ_M , through function $\gamma_{M,S}(\cdot)$, which has a single root at 0. More details are provided in Appendix C.

To illustrate the difference between our framework and a standard conditional CAPM framework,

¹⁶Similar pricing kernels are studied in continuous-time setups by, among others, Bates (1991, 2006), Liu, Pan, and Wang (2005), and Eraker (2008).

consider the familiar

$$\beta_{S,t+1}^{\text{CAPM}} = \frac{\text{Cov}_t^{\mathbb{P}}(r_{S,t+1}, r_{M,t+1})}{\text{Var}_t^{\mathbb{P}}(r_{M,t+1})} = \frac{\text{Cov}_t^{\mathbb{P}}(e^{R_{S,t+1}-r_{t+1}}, e^{R_{M,t+1}-r_{t+1}})}{\text{Var}_t^{\mathbb{P}}(e^{R_{M,t+1}-r_{t+1}})}, \quad (2.12)$$

where $r_{u,t+1} \equiv \frac{u_{t+1}-u_t}{u_t}$ are simple returns. In the context of our model, a first-order approximation of this total beta yields

$$\beta_{S,t+1}^{\text{CAPM}} \simeq \frac{\text{Cov}_t^{\mathbb{P}}(\beta_{S,z} z_{M,t+1} + \beta_{S,y} y_{M,t+1}, z_{M,t+1} + y_{M,t+1})}{\text{Var}_t^{\mathbb{P}}(z_{M,t+1} + y_{M,t+1})} = \frac{\beta_{S,z} h_{M,z,t+1} + \beta_{S,y} \frac{\alpha_M^2}{(\alpha_M^2 - \delta_M^2)^{3/2}} h_{M,y,t+1}}{h_{M,z,t+1} + \frac{\alpha_M^2}{(\alpha_M^2 - \delta_M^2)^{3/2}} h_{M,y,t+1}} \quad (2.13)$$

and, in a CAPM setting, the risk premium on the stock would be

$$\mu_{S,t+1}^{\text{CAPM}} - r_{t+1} = \beta_{S,t+1}^{\text{CAPM}} (\mu_{M,t} - r_t) = \beta_{S,t+1}^{\text{CAPM}} (\lambda_M h_{M,z,t+1} + \gamma_M h_{M,y,t+1}). \quad (2.14)$$

Contrasting equations (2.14) and (2.11) highlights two features of our model. First, in our model, stocks can have different sensitivities to normal and jump risk. Second, λ_S and γ_S are not assumed to be null, but are jointly estimated from past returns and option data.

2.3 Option Prices

The model, once risk-neutralized, remains within the affine class of models (see Appendix D). Hence, we build on the work of Heston and Nandi (2000) and obtain a closed-form solution for the price of European index and stock options.¹⁷ For $u_t \in \{M_t, S_t\}$, the price of an European call option is

$$C_t(u_t, K, T) = u_t P_{1,t,T} - K e^{-r_{t,T}(T-t)} P_{2,t,T} \quad (2.15)$$

where $r_{t,T} = \frac{1}{T-t} \sum_{j=1}^{T-1} r_{t+j}$, in which r_{t+j} is the deterministic risk-free rate at time $t+j$. The conditional probabilities $P_{1,t,T}$ and $P_{2,t,T}$ are given by

$$\begin{aligned} P_{1,t,T} &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[\frac{1}{\phi i} \exp(-i\phi \log \tilde{K}_{t,T}) \varphi_{t,T}^{\mathbb{Q}}(\phi i + 1) \right] d\phi \\ P_{2,t,T} &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[\frac{1}{\phi i} \exp(-i\phi \log \tilde{K}_{t,T}) \varphi_{t,T}^{\mathbb{Q}}(\phi i) \right] d\phi \end{aligned}$$

¹⁷Heston and Nandi (2000) relies on an inversion similar to the one of Gil-Pelaez (1951).

where i is the imaginary number,

$$\tilde{K}_{t,T} = \frac{K e^{-r_{t,T}(T-t)}}{u_t}$$

and the conditional moment generating function $\varphi_{t,T}^{\mathbb{Q}}(\phi) = E_t^{\mathbb{Q}} \left[\exp \left(\phi \sum_{j=1}^{T-t} \tilde{R}_{u,t+j} \right) \right]$ of the aggregated excess returns $\sum_{j=1}^{T-t} \tilde{R}_{u,t+j} = \sum_{j=1}^{T-t} (R_{u,t+j} - r_{t+j})$ over the period $[t, T]$ satisfies

$$\varphi_{t,T}^{\mathbb{Q}}(\phi) = \exp \left(\begin{aligned} &\mathcal{A}_{u,T-t}(\phi) + \mathcal{B}_{u,T-t}(\phi) h_{M,z,t+1}^* + C_{u,T-t}(\phi) h_{M,y,t+1}^* \\ &+ \mathcal{D}_{u,T-t}(\phi) h_{S,z,t+1}^* + \mathcal{E}_{u,T-t}(\phi) h_{S,y,t+1}^* \end{aligned} \right).$$

The deterministic functions $\mathcal{A}_u, \mathcal{B}_u, C_u, \mathcal{D}_u, \mathcal{E}_u$ are calculated based on the recursion in Appendix E. In particular, $\mathcal{D}_{M,T-t} = \mathcal{E}_{M,T-t} = 0$.

3 Joint Estimation Using Returns and Option Prices

Relying on a joint estimation procedure is of particular importance to our study. Indeed, the risk premium parameters we aim to study are relatively poorly identified under the physical measure. However, these parameters play a crucial role in the pricing kernel and, as such, are key to reconcile the price of the options and the underlying returns.¹⁸ Moreover, in the absence of jumps, a deep-out-of-the money option would be almost worthless, especially if the option is relatively short-dated. These options will thus improve our ability to estimate the likelihood of jumps. Hence, the richness of stock option data plays a key role in allowing us to extract the expected risk premium associated each risk factor.

3.1 Data

To estimate the model, we use the returns and prices of options on the S&P 500, as proxy for the market, and on 260 stocks that are or were part of the index since 1996. These stocks were selected based on whether their options had been actively traded over at least a consecutive 5-year window. Daily index and stock returns, from January 1996 to August 2015, are obtained from the Center for Research in Security Prices (CRSP).¹⁹ To compute the corresponding daily excess log-returns (henceforth, returns), we use one-month Treasury bill rate (from Ibbotson Associates) as extracted from Kenneth French's

¹⁸See, among others, Chernov and Ghysels (2000), Pan, 2002, Chernov (2003), Eraker (2004), Santa-Clara and Yan (2010), Christoffersen, Jacobs, and Ornathanalai (2012), and Ornathanalai (2014).

¹⁹In fact, we extract returns starting from January 1986. Returns between January 1986 and December 1995 are used to warmup the variance process; their likelihood, however, does not impact the estimation of the parameters. A similar procedure is used for individual stocks.

data library.

The prices of options on the SPX and the stocks, between January 1996 to August 2015, are obtained from OptionMetrics.²⁰ We restrict our analysis to out-of-the-money monthly options with at least one week and at most one year to maturity. Observations for which the ask price is lower than the bid price are excluded. The price of the option is defined as the mid point between the ask and the bid, and options with a price lower than the bid-ask spread are excluded. Moreover, the open interest and the volume must be strictly positive. We further remove options that violate the common arbitrage conditions. For options on individual stocks, we follow Broadie, Chernov, and Johannes (2007) and de-Americanize the option prices.²¹ Finally, among the remaining options, we select the three most liquid puts and three most liquid calls on each Wednesday, for each maturity available.²² This leaves us with a total of 44,267 option prices on the SPX, and 2,975,839 on the 260 stocks.²³ Tables 1 and 2 summarize the option data sets. Figure 1 provides an overview of how implied volatilities vary through time as the S&P 500 evolves. As evidenced in the lower panel of the figure, while implied volatilities on stocks comove with implied volatilities on S&P 500 options, the former are significantly larger than the latter.

3.2 Joint Estimation

Following Christoffersen, Jacobs, and Ornthanalai (2012) and Ornthanalai (2014), the model's parameters are estimated by maximizing the weighted joint log-likelihood function

$$L_u(\Theta_u) = \frac{T_u + N_u}{2} \left(\frac{L_{u,\text{returns}}(\Theta_u)}{T_u} + \frac{L_{u,\text{options}}(\Theta_u)}{N_u} \right), \quad (3.16)$$

where, $u \in \{M, S\}$, T_u is the number of returns observed, N_u is the total number of option observations, and Θ_u represents the parameter set of the model.

We opt for a two-stage estimation approach. That is, we first maximize the joint likelihood L_M

²⁰The zero-coupon term structure is also extracted from OptionMetrics and used for option pricing. The rate corresponding to an option's maturity is obtained through linear interpolation whenever necessary.

²¹Specifically, for each American option, OptionMetrics uses a Cox, Ross, and Rubinstein (1979) binomial tree to derive the option's implied volatility, accounting for dividends. Given this implied volatility and dividends extracted from OptionMetrics, we compute the price of the corresponding European option.

²²We follow the literature and use Wednesday data because it is the least likely day to be a holiday and it is least likely to be affected by weekend effects. For more details on the advantages of using Wednesday data, see Dumas, Fleming, and Whaley (1998). If markets are closed on a given Wednesday (e.g. Christmas, January 1, Independence day or 9/11) we use the previous business day.

²³In total, we considered options on the 1,000 different firms that were part of the S&P 500 at any point in our sample. Our selection procedure discarded 738 firms. Two additional firms (tickers BEN and NEE) were further discarded because they experimented very extreme returns that caused numerical problems in the particle filter; we are currently working on improving the importance sampling step in order to reintroduce these firms.

with respect to Θ_M and then, turn to maximizing L_S for each stock taking the results for the market as given. Although it has inconvenients, this approach is crucial to keeping the estimation procedure tractable in our settings. Indeed, in opposition to typical GARCH processes in which the noise term is fully determine once we condition on observed returns and the initial variance, the presence of jumps implies, focusing on the market model, that the Gaussian component $z_{M,t}$ and the jump part $y_{M,t}$ of time t innovation cannot be separated. Consequently, as pointed out by Durham, Geweke, and Ghosh (2015), the conditional variance $h_{M,z,t}$ and intensity $h_{M,y,t}$ remain uncertain, even with the observed returns up to time t .²⁴ However, both $h_{M,z,t+1}$ and $h_{M,y,t+1}$ can be fully recovered from the initial conditions $h_{M,z,1}$, $h_{M,y,1}$, the returns $R_{M,1:t} = \{R_{M,s}\}_{s=1}^t$ and the jump innovations $y_{M,1:t} = \{y_{M,s}\}_{s=1}^t$. In this spirit, we propose a particle filter that infers the average (filtered) $z_{M,t}$, $y_{M,t}$, $h_{M,z,t}$ and $h_{M,y,t}$, while accounting for the uncertainty with respect to the conditional variance and the jump intensity. Appendix F further describes the particle filter used to compute the log-likelihood $L_{M,\text{returns}}(\Theta_M)$. The same procedure is applied to each stock in a second estimation stage, keeping the market parameters and latent variables fixed. Conceptually, the particle filter could be extended to deal with a one-stage estimation of the market and the 260 firms; numerically, however, this would be absolutely intractable. Our alternative is computationally efficient and, given the richness of the index option data, we are confident that the two-stage estimation procedure yields parameter estimates for the market model that are more accurate than the ones that could be obtained from a poorly behaved one-stage procedure.

Following the option-pricing literature, the log-likelihood of the option fit, $L_{u,\text{options}}(\Theta_u)$, is based on relative implied volatility pricing errors.²⁵ In particular, if $IV_{u,k}^{\text{MKT}}$ is the Black and Scholes (1973) implied volatility associated with the market price of option k on underlying $u \in \{M, S\}$ and $IV_{u,k}^{\text{MODEL}}$ the implied volatility inverted from the corresponding model price, then the relative implied volatility error is

$$e_{u,k} = \frac{IV_{u,k}^{\text{MODEL}} - IV_{u,k}^{\text{MKT}}}{IV_{u,k}^{\text{MKT}}}.$$

Assuming that the relative implied volatility error is normally distributed, $e_{u,k} \sim \mathcal{N}(0, \sigma_e^2)$, and

²⁴Technically speaking, $\mathcal{G}_t^M = \sigma\{R_{M,\tau}\}_{\tau=1}^{t-1}$ is the σ -field generated by the returns process which is coarser than the σ -field $\mathcal{F}_t^M = \sigma\{z_{M,\tau}, y_{M,\tau}\}_{\tau=1}^{t-1}$ generated by the innovation terms. The conditional variance $h_{M,z,t}$ and the jump intensity $h_{M,y,t}$ are both \mathcal{F}_{t-1}^M measurable, but they are not \mathcal{G}_{t-1}^M -measurable.

²⁵This criterion, or variants thereof, is used by Bakshi, Carr, and Wu (2008), Christoffersen, Jacobs, and Ornthanalai (2012), and Ornthanalai (2014). Renault (1997) offers an interesting discussion on the benefits of using IVRMSE when comparing option pricing models. Alternatively, some authors will consider vega-weighted RMSE (VWRMSE) since VWRMSE and IVRMSE take very similar value, while the former have the advantage of being faster to compute than the latter. See for instance Carr and Wu (2007) and Trolle and Schwartz (2009). Note that using relative implied volatility errors has the advantage of not assigning excessive weighting to option prices observed during the financial crisis.

uncorrelated with shocks in returns, we obtain

$$L_{u,\text{options}}(\Theta_u) = -\frac{1}{2} \sum_{k=1}^{N_u} \left(\log(2\pi\sigma_e^2) + \frac{e_{u,k}^2}{\sigma_e^2} \right).$$

Note that σ_e is identified using the sample standard deviation of $\{e_{u,k}\}_{k=1}^{N_u}$.

4 Empirical Results

4.1 Market

Although the focus of our study is the pricing of idiosyncratic risk, we first briefly discuss results obtained at the market level. Overall, these results are very close to those in the option pricing literature. In particular, our results are much in line with those reported by Ornathanalai (2014) for the NIG variant of his model, which is essentially our market model. Parameters, reported in Table 3, are largely similar, except maybe for $a_{M,y}$, the parameter governing the variance of jump intensity, which is much larger for us than it was for Ornathanalai. This difference could be due to our sample covering more of the Great Recession and its aftermath.

For each subset of option O , Table 4 reports two metrics

$$\text{IVRMSE} = \sqrt{\frac{1}{N} \sum_{k \in O} (IV_k^{\text{MODEL}} - IV_k^{\text{MKT}})^2} \quad \text{and} \quad \text{RIVRMSE} = \sqrt{\frac{1}{N} \sum_{k \in O} \left(\frac{IV_k^{\text{MODEL}} - IV_k^{\text{MKT}}}{IV_k^{\text{MKT}}} \right)^2}. \quad (4.17)$$

The first, IVRMSE, provides an absolute measure of the implied-volatility pricing errors. The latter, a relative measure that is probably more informative when comparing pricing errors through time. By both measures, our market fit to the option data, as detailed in Panel A of Table 4 compares favourably to the results in the option-pricing literature. This is true through time and across maturities and moneyness levels. As documented by Ornathanalai, the NIG jumps in our model allow for particularly large levels of (negative) skewness and excess kurtosis (cf. Table 3). This theoretical feature of the model explains its particularly good fit across maturities and moneyness levels. Moreover, the NIG jumps properly capture, empirically, the nonnormal innovations in returns; consequently, the filtered conditionally standard normal innovations, $\varepsilon_{M,t}$, have skewness and excess kurtosis that are close to zero, as it should be. Figure 2 plots the filtered normal innovations $z_{M,t}$ (top panel), jumps (middle panel) and volatility components (bottom panel). Again, results are qualitatively similar to those of Ornathanalai.

Table 3 also reports risk premiums based on the average and median level of normal, $\lambda_M h_{M,z,t}$, and jump, $\gamma_M h_{M,y,t}$ components of the conditional equity risk premium (ERP). The median levels of the premiums are respectively 1.72% and 3.04%, for a median ERP of 4.76%. These numbers are comparable to those of Ornathanalai, who reports an annualized normal risk premium of 1.43% and a jump risk premium of 3.22%, for a total of 4.65%, based on the unconditional level of variance and jump intensity. Hence, although Table 3 reports that the jump component of variance, $h_{M,y}$, explains on average only 26.7% of total variance, $h_{M,z} + \frac{\sigma_M^2}{(\sigma_M^2 - \delta_M^2)^{3/2}} h_{M,y}$, the jump risk premium outweighs its normal counterpart.

The average ERP is higher than the median at 6.17%, and decomposes into an average normal premium of 2.32% and an average jump premium of 3.85%. Naturally, the average is more sensitive than the median (or any measure based on unconditional GARCH levels) to extreme values of the premiums observed during periods of turmoil. The top panel of Figure 3 reports how the premium unfolds through time. At its peak, in November 2008, the estimated ERP reaches 40.16%. While this number may appear high, Martin's (2016) measure of the ERP, as extracted from one-month-to-maturity options alone, rises to more than 50% around the same time, while its three-month counterpart flirts with the 40% level. Using a panel of options with median time-to-expiration of 14 business days, Bollerslev and Todorov (2011) find that the jump component of the ERP rises above 40% during the same period.

The bottom panel of Figure 3 reports, on a daily basis, the ratio of the ERP that is explained by the jump component. This ratio is at its lowest during periods of turmoil, when the normal risk carries a higher than usual premium. When the ERP is particularly low, which coincides with periods of low volatility on the market, the jump risk premium explains up to 80% of the total ERP. Hence, while Figure 2 documents that, as Bates (2008) pointed out, jump risk is countercyclical, the relative importance of jump risk in the ERP appears to be mildly cyclical.

In sum, our results at the market level are consistent with the literature.

4.2 Idiosyncratic Jump Risk Matters

We now turn to our paper's main empirical contribution. Namely, while our results are consistent with the literature highlighting that idiosyncratic does matter for the equity risk premium, we provide evidence that idiosyncratic jump risk is at the center of this empirical phenomenon.

Table 5 reports summary statistics on the parameters associated with the 260 stocks under consideration. While there is substantial cross-sectional variation, the average value of the parameters of

the variance and intensity processes are comparable to the parameters obtained for the market model. Remarkably, more than 75% of the firms exhibit less negative skewness and excess kurtosis than the market. This is consistent with Bakshi, Kapadia, and Madan (2003), who document that the option-implied skewness of individual stocks is typically much less negative than that of the market index.²⁶

Of particular interest, the normal and jump betas are on average 0.914 and 1.090 respectively. The normal beta ranges from 0.206 to 1.770, but 50% of firms under consideration have a normal beta between 0.688 and 1.120. In comparison, the jump beta ranges from 0.202 to 3.942, while 50% of firms under consideration have a normal beta between 0.815 and 1.291. Interestingly, the correlation between firms normal and jump beta is only of 0.195 (cf. Table 7). That is, there is a positive correlation, but firms with large normal betas do not necessarily have large jump betas, and the other way around.

Table 7 further reports the correlation between $\beta_{S,v}$, $v \in \{z, y\}$, and the firm-by-firm time series average of the systematic normal, $\beta_{S,z} \lambda_M h_{M,z,t}$, and jump, $\gamma_{M,S}(\beta_{S,y}) h_{M,y,t}$, risk premiums. Unsurprisingly, the correlation between $\beta_{S,v}$ and the corresponding systematic premium is high but imperfect.²⁷ Consistent with the modest correlation between $\beta_{S,z}$ and $\beta_{S,y}$, the correlation between the normal (jump) beta and the systematic jump (normal) premium are positive but modest at 0.191 (0.190). These results highlight the importance of accounting for separate systematic premiums on both types of risk, as emphasized by Elkamhi and Ornathanalai (2010) and Babaoğlu (2015).

Figure 4 decomposes, for each of the 260 stocks in our sample, the stock's equity risk premium in terms of the premiums associated with the four different risk factors in the model: (i) systematic normal, $\beta_{S,z} \lambda_M h_{M,z,t}$, (ii) systematic jump, $\gamma_{M,S}(\beta_{S,y}) h_{M,y,t}$, (iii) idiosyncratic normal, $\lambda_S h_{S,z,t}$, and (iv) idiosyncratic jump, $\gamma_S h_{S,y,t}$. Making this decomposition possible is the key econometric contribution of our paper. The empirical results are striking. First, consistent with financial theory, we find that systematic risk is priced and explains an important part (59.8%) of the equity risk premium. Normal systematic risk explains 20.3% of the total equity risk premium; systematic jump risk, 39.5%. Consistent with the discussion on the betas, the proportion of the systematic premium explained by its jump component ($\frac{(ii)}{(i)+(ii)}$), varies largely, from 14.7% to 67.7%.

²⁶ Albuquerque (2012) develops and empirically supports a model in which conditional asymmetric stock return correlations and negative skewness in aggregate returns are caused by cross-sectional heterogeneity in firm announcement events.

²⁷ For the normal premium, the time series average

$$\frac{1}{T_S} \sum_{t \in \mathcal{T}_S} \beta_{S,z} \lambda_M h_{M,z,t} = \beta_{S,z} \lambda_M \bar{h}_{M,z,\mathcal{T}_S}, \quad S \in \mathbb{S}, \quad (4.18)$$

is linear in $\beta_{S,z}$ which makes the imperfect correlation puzzling at first sight. However, the firm-specific window of available data, \mathcal{T}_S , introduces cross-sectional variation in $\bar{h}_{M,z,\mathcal{T}_S}$.

However, the most striking result illustrated in Figure 4 is that the premium associated with idiosyncratic risk account for a 40.2% fraction of the total premium ($\frac{(iii)+(iv)}{(i)+(ii)+(iii)+(iv)}$). Moreover, and perhaps most importantly, the normal component of idiosyncratic risk, which is easily diversifiable, is not priced once other sources of risk are accounted for. This is consistent with the average value of λ_S being very small at 5.079×10^{-5} ; Figure 4 shows that this leads to idiosyncratic normal risk premium that are economically insignificant. While it is now widely accepted that, given market incompleteness, idiosyncratic risk can be priced, we find that idiosyncratic jump risk, alone, matters in the equity risk premium. As shown in Figure 4, the proportion of the equity risk premium explained by the premium on the jump idiosyncratic risk factor varies significantly from firm to firm, but idiosyncratic normal risk virtually does not matter for any of the 260 firms in our sample.

Averages across industries

Figure 5 presents, for the eight largest Global Industry Classification Standard (GICS) industries covered by our sample, the evolution through time of the component of the industry's average firm's equity risk premium that is due to exposure to idiosyncratic jumps.²⁸ Note that all firms load, through their normal and jump betas, on the systematic risk premiums reported in Figure 3. Hence, the idiosyncratic jump risk premium (solid line) reported in Figure 5 adds to the premium arising from the firms' exposure to systematic risk factors (grey '+' marks).

For all industries, jump risk premiums increase around both recessions in our sample. In fact, the increase is relatively mild around the first recession for all industries, except Information Technology who had just been hit by the burst of the dot-com bubble. On the other hand, idiosyncratic jump risk premiums increase markedly for all industries around the Great Recession. Interestingly, the crisis peak in idiosyncratic jump risk premium for Financials is not as high as that experienced by Consumer Discretionary or Materials, for instance. However, as reported Table 6, firms from the Financial sector are, on average, the ones exhibiting the second highest normal beta and the second highest jump beta. Hence their total premium (summing the solid line with the grey '+') raises significantly during the crisis.

²⁸We do not report results for Telecommunication Services (2 firms) and Utilities (3 firms) as we do not have enough firms from these sectors.

4.3 Commonality in Idiosyncratic Jump Risk

Following the literature, we defined idiosyncratic variance as the variance of the residuals obtained after accounting for risk factors, here normal and jump market risk. In particular, a stock's idiosyncratic variance and jump intensity are defined as (eq. (2.4) and (2.6))

$$\begin{aligned} h_{S,z,t+1} &= \kappa_{S,z} h_{M,z,t+1} + b_{S,z} (h_{S,z,t} - \kappa_{S,z} h_{M,z,t}) + \frac{a_{S,z}}{h_{S,z,t}} (z_{S,t}^2 - h_{S,z,t} - 2c_{S,z} h_{S,z,t} z_{S,t}), \\ h_{S,y,t+1} &= \kappa_{S,y} h_{M,y,t+1} + b_{S,y} (h_{S,y,t} - \kappa_{S,y} h_{M,y,t}) + \frac{a_{S,y}}{h_{S,y,t}} (z_{S,t}^2 - h_{S,y,t} - 2c_{S,y} h_{S,y,t} z_{S,t}). \end{aligned}$$

In order to account for the documented strong commonality in idiosyncratic variances (Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2014), henceforth HKLV), each process evolves around a dynamic level $\kappa_{S,v} h_{M,v,t+1}$, $v \in \{z, y\}$. Table 5 report that the normal kappa is on average 0.971, further supporting the commonality documented in HKLV. Our results extend those of HKLV by documenting a strong commonality in jump risk: the average jump kappa is 0.513. Hence, the commonality in jump risk is less important than that documented in variances, but is still sizable.

Firm-by-firm regressions (untabulated) of total idiosyncratic variance (cf. equation 2.8) on $\kappa_{S,z} h_{M,z,t}$ and $\kappa_{S,y} h_{M,y,t}$ yield an average R^2 of 73.4%; regressing on the $\kappa_{S,y} h_{M,y,t}$ alone yields an average R^2 of 31.8%. Our results thus extend the finding of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2014) in that we document that idiosyncratic tail risk explain a large fraction of the commonality in idiosyncratic variance. As such, tail risk, which is already hard to hedge by nature, becomes virtually undiversifiable in times of turmoil, which justifies the risk premium attached to it.

Interestingly, Table 7 reports that the three significant components of risk premiums are positively correlated with the firms' average level of total idiosyncratic variance (IVAR). The same holds for the excess idiosyncratic components of normal, $\widetilde{\text{IVAR}}_{S,z}$, and jump risk, $\widetilde{\text{IVAR}}_{S,y}$, which are respectively the average of the following time series: $h_{S,z,t} - \kappa_{S,z} h_{M,z,t}$, and $h_{S,y,t} - \kappa_{S,y} h_{M,y,t}$. This result is consistent with the results of Martin and Wagner (2016), who find that stocks exhibiting higher (lower) than average idiosyncratic volatility command higher (lower) expected excess returns.

4.4 On the Importance of Accounting for Equity-Specific Jumps

Financial theory tells us that diversifiable risk should not be priced. In most models, idiosyncratic risk is simply normal risk. As this normal risk should be easily diversified away, conditionally normal models imply that idiosyncratic risk should not be priced. Our results confirm that idiosyncratic normal risk

indeed is not priced. Idiosyncratic jump risk, on the other hand, is difficult to diversify by nature. As such, it can bear a risk premium, and it does.

When one estimates a conditionally normal model on actual returns, the filtered “normal” innovations are all but normal. They typically have a very large kurtosis; in a misspecified normal model, the supposedly normal innovations are also capturing jumps. Given the importance of the premium on these idiosyncratic jumps in our model, we conjecture that, in conditionally normal models, the risk premium on idiosyncratic normal risk originates from the model’s misspecification.

To validate this conjecture, we estimate a nested version of our model in which the market model remains unchanged, but the stock model does not exhibit idiosyncratic jump risk. That is,

$$\begin{aligned} R_{S,t+1} &= \mu_{S,t+1} - \xi_{S,t+1}^{\mathbb{P}} + \beta_{S,z} z_{M,t+1} + \beta_{S,y} y_{M,t+1} + z_{S,t+1}, \\ \mu_{S,t+1} - r_{t+1} &= \beta_{S,z} \lambda_M h_{M,z,t+1} + \gamma_{M,S} (\beta_{S,y}) h_{M,y,t+1} + \lambda_S h_{S,z,t+1}, \\ \xi_{S,t+1}^{\mathbb{P}} &= \xi_{z_{M,t}}^{\mathbb{P}} (\beta_{S,z}) + \xi_{y_{M,t}}^{\mathbb{P}} (\beta_{S,y}) + \xi_{z_{S,t}}^{\mathbb{P}} (1), \end{aligned} \quad (4.19)$$

where market innovations have separate normal variance and jump intensity, as specified in Section 2, and $z_{S,t+1}$ is simply assumed to be conditionnaly normal, with GARCH variance as specified in equation (2.4).

Figure 6 shows that the composition of the total risk premium is drastically different once idiosyncratic jumps are neglected. The systematic components are very similar to those reported in Figure 4. The premium associated with idiosyncratic normal risk, however, is now more important than the sum of premiums associated with systematic risk. The expected excess return on an average stock (not tabulated) rises significantly, from 10.3% (5.9% systematic / 4.4% idiosyncratic) in the full model to 13.3% (6.8% / 6.5%) in the nested model of equation (4.19). Both Christoffersen, Jacobs, and Ornathanalai (2012 – 22.15%, Table 6) and Ornathanalai (2014 – 15.50%, Table 3) document, at the market level, that the equity risk premium levels implied by conditionally normal model are unreasonably high. The difference is not as marked at the stock level, perhaps due to the presence of systematic jump risk. Yet, it appears that ignoring idiosyncratic jump risk also leads to a severe misspecification at the stock level.

Panel A of Table 8 provides further evidence of this misspecification. In particular, the filtered $\varepsilon_{S,t}$, which are supposed to be conditionnaly standard normal innovations under the model of equation (4.19), exhibit levels of excess kurtosis that are much too high. While the theoretical level should be 0, the median level reported in Panel A of Table 8 is 9.14. In comparison, the corresponding median is 0.74 in

Table 5. While the filtered “normal” innovations exhibit skewness and kurtosis, they theoretically don’t, which reduces the ability of the stock model to properly fit the stocks’ implied volatility smile through time. Indeed, the entire RIVRMSE distribution reported in Panel A of Table 8 is shifted to the right when compared with the full model’s RIVRMSE in Table 5. That is, although the model of equation (4.19) still allow for relatively high levels of (negative) skewness and kurtosis, thanks to systematic jumps, the normality assumption at the idiosyncratic level clearly deteriorates the fit to options.

In sum, the contrast between the results obtained when considering or neglecting idiosyncratic jumps highlights the importance of accounting for these equity-specific jumps.

4.5 Realized Premium based Portfolios of Stocks

In a typical study of factor models, the market prices associated with the different risk factors are estimated from the panel of returns. Here, they are identified from returns and option prices. A potential issue, if stock and option markets are partly segmented, is that the estimated risk premiums might reflect, for instance, option market makers’ shadow price of equity. While this concern is partly mitigated by the use of stock returns in the joint estimation, it still might affect the estimated magnitude of the premium. This criticism apply to any study using options prices to learn about the equity risk premium or even the physical distribution.²⁹

To appraise whether this criticism indeed points to a weakness of our framework, we here analyze portfolios formed according to the model-implied risk premium associated with idiosyncratic jump risk. First, on each day t out of the 4,951 days in our sample, we sort available stocks according to the expected excess return associated to idiosyncratic jumps:

$$RP_{S,y,t} = E_t^{\mathbb{P}} \left[\exp \left\{ \gamma_S h_{S,y,t+1} - \xi_{yS,t+1}^{\mathbb{P}} + y_{S,t+1} \right\} \right] = e^{\gamma_S h_{S,y,t+1}}. \quad (4.20)$$

We then divide these stocks in five quintile portfolios P1 to P5, from lowest to highest expected return; stocks within a portfolio are weighted according to their market capitalization on day t . Finally, we create a long-short portfolio with a long position in the portfolio with the highest expected return, P5, and a short position in that with the lowest expected return, P1. For comparison, the same procedure is used to create a long-short portfolio on the basis of $RP_{S,z,t} = e^{\lambda_S h_{S,z,t+1}}$.

Table 9 reports regressions of the returns of these long-short portfolios on some of the most prevalent

²⁹For instance, Martin (2016) and Martin and Wagner (2016) infer the equity risk premium directly from option prices. Ross (2015) and Jensen, Lando, and Pedersen (2016) infer the \mathbb{P} distribution from the evolution of the \mathbb{Q} distribution.

factors in empirical asset pricing. First, the regression labelled FF3 is based on the Fama and French (1993) 3-factor models: market (MKT), small minus big (SMB), high minus low (HML). The regression labelled FF5 extends the set of regressors to those the Fama and French (2015) 5-factor model, adding the robust minus weak (RMW), and conservative minus aggressive (CMA) factors. The regression labelled CF4 considers the Carhart (1997) 4-factor model, essentially adding a momentum (MOM) factor to FF3. The regression labelled AHXZ is inspired by the work of Ang, Hodrick, Xing, and Zhang (2006) and extends FF3 by adding the innovation on the CBOE Volatility Index (VIX) to the set of regressors.³⁰ Finally, we consider a kitchen sink regression, labelled All, in which we control for all the aforementioned factors, that is:

$$r_t^{P5-P1} = \alpha + \beta_{\text{MKT}} \text{MKT}_t + \beta_{\text{SMB}} \text{SMB}_t + \beta_{\text{HML}} \text{HML}_t + \beta_{\text{RMW}} \text{RMW}_t + \beta_{\text{CMA}} \text{CMA}_t + \beta_{\text{MOM}} \text{MOM}_t + \beta_{\Delta \text{VIX}} \Delta \text{VIX}_t + \epsilon_t, \quad (4.21)$$

where r_t^{P5-P1} is the day- t simple excess return on the long-short portfolio formed on the basis of the model-implied risk premiums, $\text{RP}_{S,\cdot,t}$, and $\Delta \text{VIX}_t = \text{VIX}_t - \text{VIX}_{t-1}$ is the innovation on the VIX. The alphas of the regressions are reported in annualized percentage terms.

Table 9 is divided in two panels. In the first five columns of Panel A, the long-short portfolio is formed according to the quintiles of the risk premium associated with idiosyncratic Gaussian risk in our model, $\text{RP}_{S,z,t}$. Regression coefficients are in bold whenever they are significant at the 5% level or in italics if they are at the 10% level; t-stats are based on robust Newey and West (1987) standard errors. In particular, only two of the alphas in these five regressions are significant at the 10% level. Panel A contains a second set of regressions. In these regressions, the long-short portfolio is constructed considering the risk-premium in excess of the common component. That is, to obtain the long-short portfolio used in the last five columns of Panel A, we sort stocks into quintiles of $\text{RP}'_{S,z,t} = e^{\lambda_S h'_{S,z,t+1}}$, where $h'_{S,z,t+1}$ is the predicted excess idiosyncratic Gaussian variance introduced in Section 2.1. None of the alphas in these five regressions are significant. Consistent with our results in Section 4.2, idiosyncratic Gaussian risk does not appear to be priced.

In Panel B, we repeat the same analysis, but forming the long-short portfolio on the basis of the risk premiums associated with (excess) idiosyncratic jump risk, $\text{RP}_{S,y,t}$ ($\text{RP}'_{S,y,t}$). The alphas of the first five regressions are highly significant, both statistically and economically. They vary between 8.3% to 17.8%

³⁰This regression is akin to their ex post regression (6), with the difference that we use ΔVIX rather than a factor mimicking aggregate volatility risk.

annually, depending on the regression considered. It is worth noting that, while the metric used to sort stocks into portfolios is inferred in part from option prices, the alphas reported here are obtained trading only stocks. These results thus confirm that idiosyncratic jump risk is priced in stock markets.

Besides, the alphas on the $RP'_{S,y,t}$ long-short portfolios are just as significant, statistically and economically, as the alphas on the portfolio based on the total idiosyncratic jump risk premium. These results show that idiosyncratic jump risk is not just priced through its common component; excess idiosyncratic jump risk matters. This result is consistent with the intuition one could build from the theoretical analysis in Martin and Wagner (2016). Unfortunately, to the best of our knowledge, it is not possible to assign different prices of risk to the common and excess components of idiosyncratic jump risk and remain in an affine option-pricing framework.

Characteristics of the Quintile Portfolios and Double-Sort Portfolios

In our model, the only systematic risk factors are the Gaussian and jump innovations on the market. Accounting for more factors, like the Fama and French (2015) or Carhart (1997) factors in the option-pricing model would have required postulating a dynamics for each of the factors, introducing many more parameters. It further would have forced estimation only from returns, at least for these factors, as options are not traded on these factors.

Yet, in Table 9, the loadings on these factors are almost all significant. One can thus conjecture that neglecting these factors in the the model led to idiosyncratic risk proxies that are partly driven by these factors. Consistent with this intuition, Table 10 shows that the quintile portfolios obtained based on the idiosyncratic jump risk premium display near monotonic market betas, market capitalizations (ME), book-to-market ratios (BE/ME), operating profitability (OP) investment levels, trailing 12-month returns, and the volatility betas.³¹ See Appendix G for more details on these variables.

The alphas in Panel B of Table 9 are significant even after linearly controlling for the factors corresponding to these variables. The patterns observed in Table 10 could nonetheless raise concerns that the alphas are somehow nonlinearly related to the fundamentals of the stocks in the top and bottom quintile portfolios used in the long-short strategy. To alleviate these concerns, we perform a double sort. On each day t , we first sort stocks into quintiles (Q1 to Q5) based on their market beta over the past year. Then,

³¹The market and volatility betas here are obtained by performing the following regression

$$r_{S,t-k} = \alpha + \beta_{MKT,t} MKT_{t-k} + \beta_{\Delta VIX,t} \Delta VIX_{t-k} + \epsilon_{t-k}, \quad k = 0, \dots, 252.$$

This regression corresponds to the pre-formation regression of Ang, Hodrick, Xing, and Zhang (2006), over the past year (rather than month, as in AHXZ) of data.

within each market-beta quintile, we sort stocks into terciles based on $RP_{S,y,t}$ (first five columns; the last five columns are based on $RP'_{S,y,t}$). We then take a long position in a cap-weighted portfolio of the top-tercile stocks, and a short position in a cap-weighted portfolio of the bottom-tercile stocks. This leaves us with five long-short portfolios, each of which is composed of stocks with homogenous market betas. The first row of Table 11 reports the alphas of regression (4.21) for each of these long-short portfolios. The procedure is repeated for the other six variables in Table 10.

The alphas on the long-short portfolios built from (excess) idiosyncratic jump risk premiums are positive and significant, both statistically and economically, in 30 (31) of the 35 regressions. When they are not statistically significant, they are still positive; the lack of significance is mainly due to the large standard errors on some of these portfolios. There are no clear patterns in the alphas across the quintiles of most variables, the exception potentially being volatility betas. In sum, no single one of these seven variables appears to be driving the main result of our paper: idiosyncratic jump risk carries a positive risk premium.

5 Conclusion

In this study, we shed new light on the relationship between idiosyncratic risk and equity returns. We develop a model allowing us to disentangle the contribution of four different risk factors to the equity risk premium: systematic and idiosyncratic risk are both decomposed in their normal and jump components. Using 20 years of returns and options on the S&P 500 and more than 250 stocks, we find that normal and jump risk have a drastically different impact on the expected return on individual stocks.

While our pricing kernel is such that each risk factor can potentially be priced, we find that the normal component of idiosyncratic risk, which is easily diversifiable, is not priced once other sources of risk are accounted for. Firm-specific jump risk, however, is priced and justifies more than 40% of the expected excess return on an average stock. Given the recent conflicting empirical evidence regarding how idiosyncratic risk affects expected returns, these findings might provide new guidance for future studies.

Our focus in this paper is on the relationship between jump risk and the equity risk premium. Given the strong links between the equity risk premium and the variance risk premium, it is natural to wonder whether our findings extend to the variance risk premium; the results of Gouriéroux (2014) certainly suggest they do. Hence, it appears that properly accounting for jump risk is crucial in any attempts to study the

risk premiums associated with idiosyncratic risk.

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Figure 1: S&P 500 and ATM Implied Volatilities

The upper panel of this figure presents the level of the S&P 500 index between January 1996 to August 2015; gray-shaded regions highlight NBER-dated recessions. The middle panel reports S&P 500 index excess returns over the same period. The lower panel reports the weekly at-the-money implied volatility from the nearest-to-maturity SPX options as extracted from OptionMetrics, along with the average of the weekly at-the-money implied volatility across the firms.

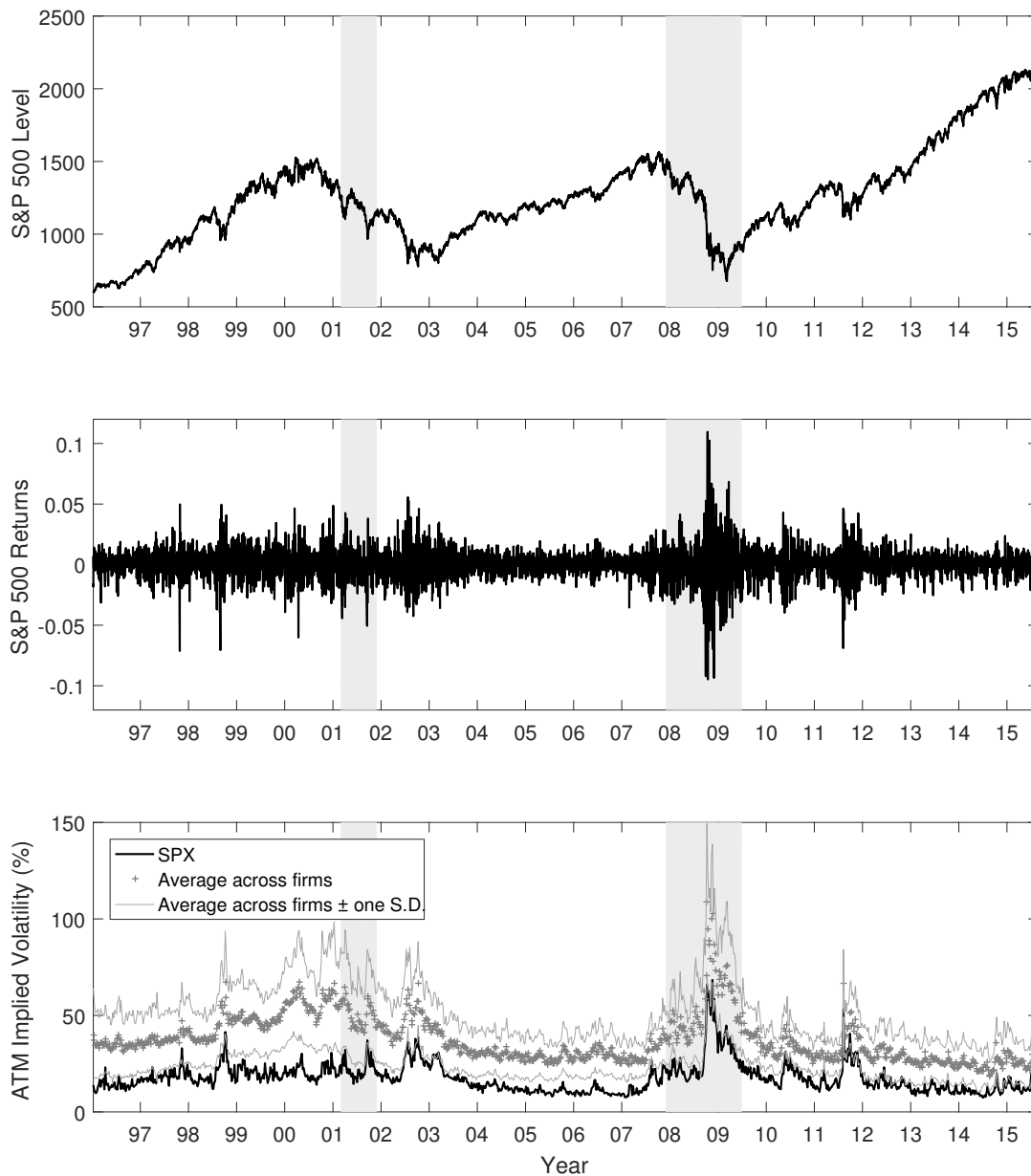


Figure 2: Filtered Innovations and Variances for the Market Model

This figure presents the states for the the market model, filtered using the parameters in Table 3. The top panel reports the filtered Gaussian innovations. The middle panel reports the filtered jump components (superposed on returns). The dashed line in the bottom panel reports, in annualized terms, the contribution of the jump component to the returns' conditional volatility, $\sqrt{252 \frac{\alpha_M^2}{(\alpha_M^2 - \delta_M^2)^{3/2}} h_{M,y,t}}$; the solid line reports the total volatility, $\sqrt{252(h_{M,z,t} + \frac{\alpha_M^2}{(\alpha_M^2 - \delta_M^2)^{3/2}} h_{M,y,t})}$.

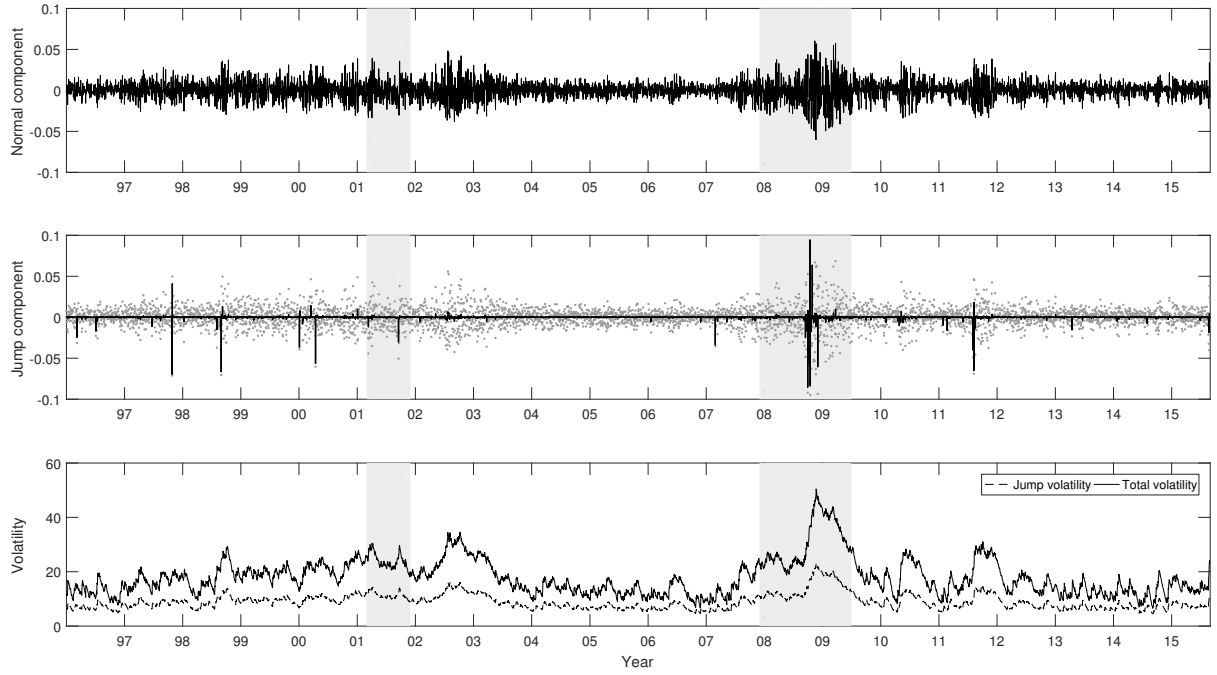


Figure 3: Annualized Normal and Jump Risk Premiums for the Market Model

The top panel of this figure reports the annualized equity risk premium, $252(\lambda_M h_{M,z,t} + \gamma_M h_{M,y,t})$, and the component due to jump risk, $252\gamma_M h_{M,y,t}$. The lower panel reports, on a daily basis, the proportion of the total premium explained by the jump component.

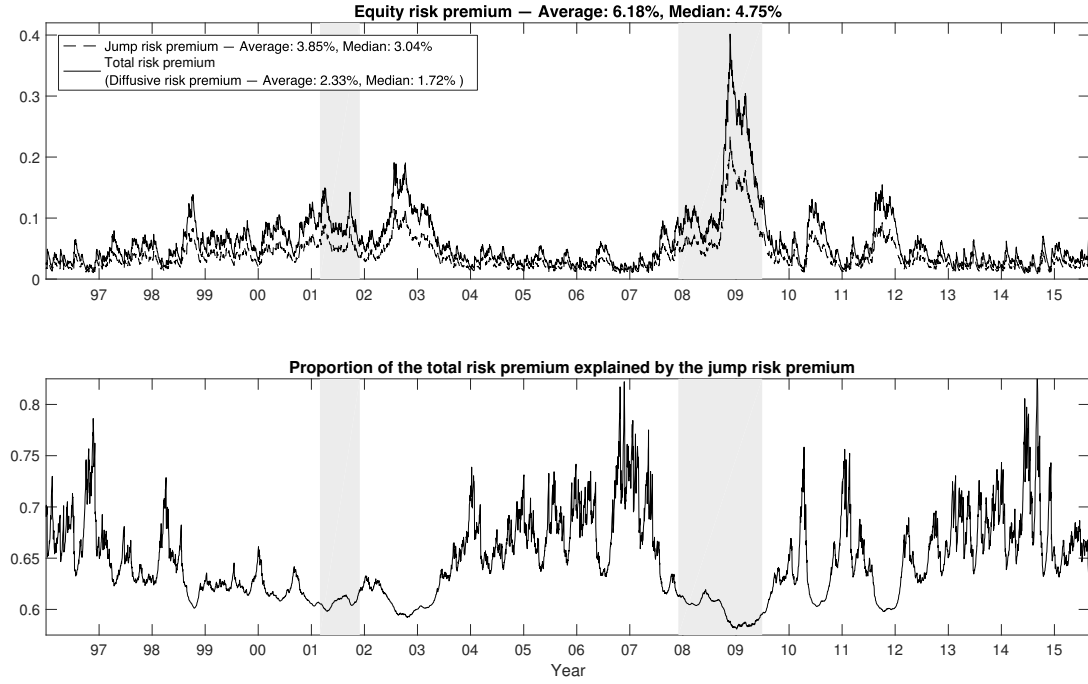


Figure 4: Decomposition of the Equity Risk Premium by Firm

This figure presents, for each of the 260 stocks, the decomposition of its equity risk premium in terms of the premiums associated with the four different risk factors in the model: (i) systematic normal, (ii) systematic jump, (iii) idiosyncratic normal, and (iv) idiosyncratic jump. On average, systematic normal risk accounts for 20.3% of the total premium, systematic jump risk for 39.5%, and idiosyncratic jump risk 40.2%. Firms are grouped by industry, based on the Global Industry Classification Standard (GICS). Results for telecommunication services and utilities are not reported since they concern only five firms.

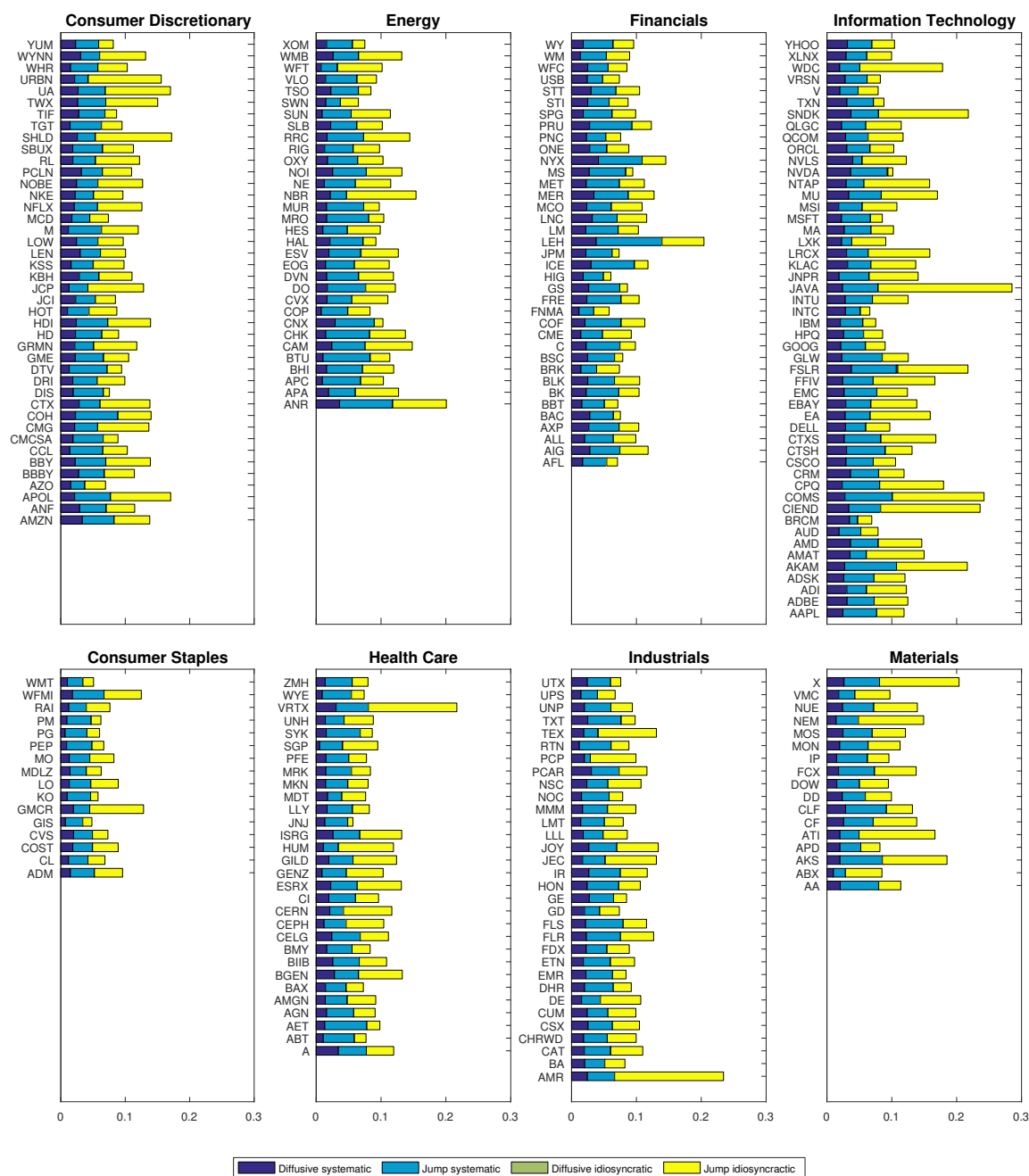


Figure 5: Time Series Decomposition of the Average Equity Risk Premium by Industry

This figure presents, for each of the eight largest GICS industries covered by our sample, the evolution through time of (i) the component of the industry's average equity risk premium that is due to exposure to idiosyncratic jumps (solid line), and (ii) the premium arising from the firms' exposure to systematic risk factors (grey '+' marks).

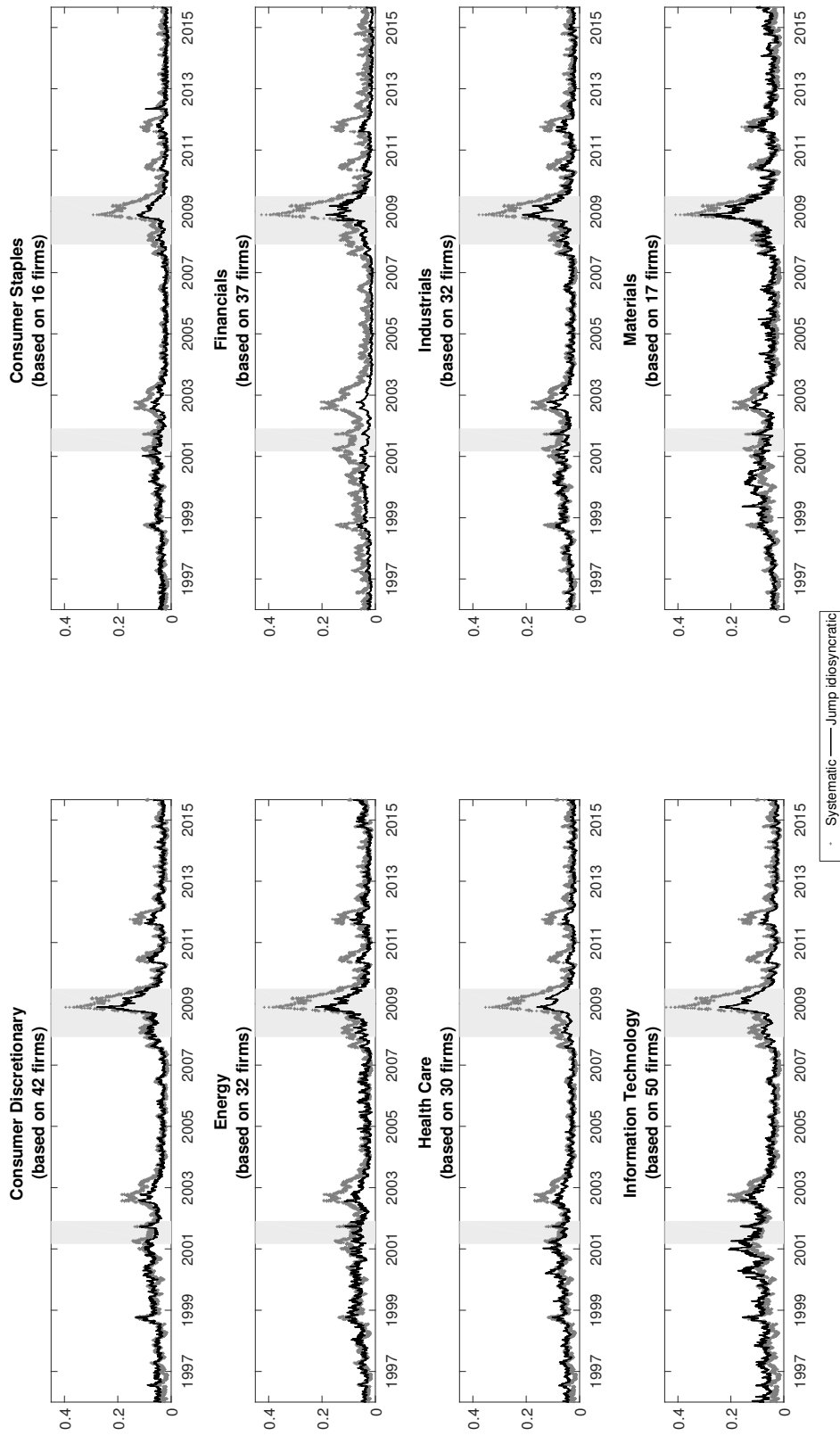


Figure 6: Decomposition of the Equity Risk Premium by Firm

This figure presents, for each of the 260 stocks, the decomposition of its equity risk premium in terms of the premiums associated with the three different risk factors in the nested model of equation (4.19): (i) systematic normal, (ii) systematic jump, and (iii) idiosyncratic normal. On average, systematic normal risk accounts for 15.0% of the total premium, systematic jump risk for 36.1%, and idiosyncratic normal risk 48.9%. Firms are grouped by industry, based on the Global Industry Classification Standard (GICS). Results for telecommunication services and utilities are not reported since they concern only five firms.

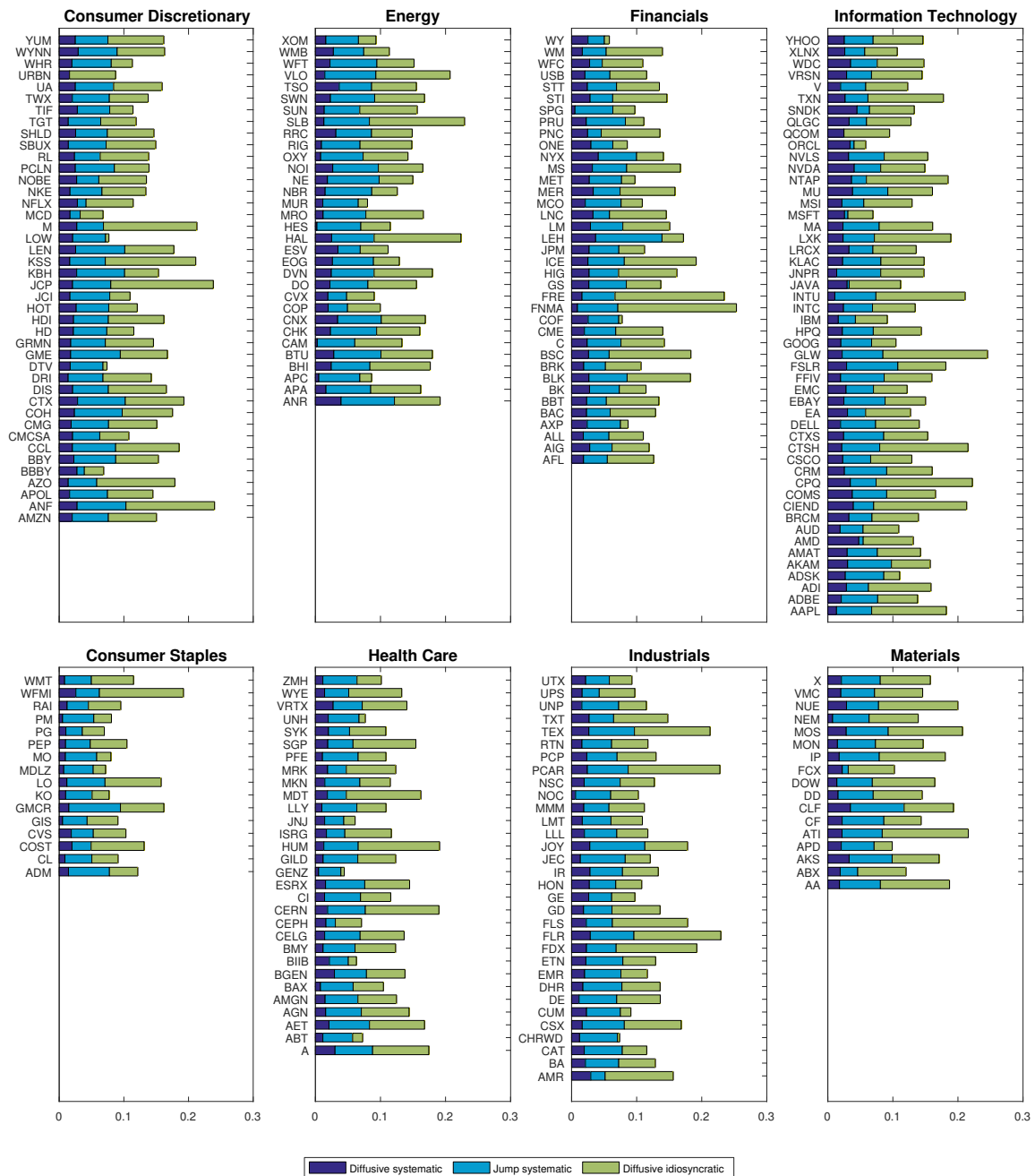


Table 1: Description of the SPX Index Option Data (1996-2015).

Moneyness is defined as K/F , where F is the forward price of the index and K is the option's strike price. DTM stands for days to maturity. Our final option dataset contains 44,267 observations.

Panel A: Number of option contracts.						
	DTM ≤ 30	30 < DTM ≤ 90	90 < DTM ≤ 180	180 < DTM ≤ 250	DTM > 250	All
0.80 < $K/F \leq 0.85$	256	1,666	1,046	715	487	4,170
0.85 < $K/F \leq 0.90$	505	1,999	1,258	861	639	5,117
0.90 < $K/F \leq 0.95$	789	2,220	1,410	1,031	828	6,278
0.95 < $K/F \leq 1.00$	1,278	2,505	1,530	1,193	884	7,390
1.00 < $K/F \leq 1.05$	2,234	4,379	2,008	1,101	973	10,695
1.05 < $K/F \leq 1.10$	484	2,432	1,517	908	723	6,064
1.10 < $K/F \leq 1.15$	83	972	871	624	512	3,062
1.15 < $K/F \leq 1.20$	18	288	380	373	287	1,346
All	5,647	16,461	10,020	6,806	5,333	44,267
Panel B: Average option prices.						
	DTM ≤ 30	30 < DTM ≤ 90	90 < DTM ≤ 180	180 < DTM ≤ 250	DTM > 250	All
0.80 < $K/F \leq 0.85$	1.19	4.18	12.00	21.45	29.92	11.92
0.85 < $K/F \leq 0.90$	1.61	7.06	18.45	30.56	41.24	17.26
0.90 < $K/F \leq 0.95$	2.93	13.05	27.94	43.18	55.69	25.69
0.95 < $K/F \leq 1.00$	8.71	26.05	45.48	62.61	78.04	39.19
1.00 < $K/F \leq 1.05$	7.29	19.61	38.88	60.88	77.47	30.17
1.05 < $K/F \leq 1.10$	1.92	6.78	16.99	31.11	44.74	17.12
1.10 < $K/F \leq 1.15$	1.34	3.68	9.16	18.44	28.45	12.32
1.15 < $K/F \leq 1.20$	1.22	3.17	6.17	10.57	17.08	9.00
All	5.65	13.50	25.84	39.90	53.11	24.12
Panel C: Average implied volatility.						
	DTM ≤ 30	30 < DTM ≤ 90	90 < DTM ≤ 180	180 < DTM ≤ 250	DTM > 250	All
0.80 < $K/F \leq 0.85$	0.4123	0.3094	0.2781	0.2586	0.2478	0.2920
0.85 < $K/F \leq 0.90$	0.3252	0.2676	0.2509	0.2364	0.2294	0.2594
0.90 < $K/F \leq 0.95$	0.2515	0.2320	0.2117	0.2186	0.2172	0.2290
0.95 < $K/F \leq 1.00$	0.1830	0.1968	0.2049	0.2024	0.2018	0.1976
1.00 < $K/F \leq 1.05$	0.1482	0.1574	0.1720	0.1844	0.1866	0.1636
1.05 < $K/F \leq 1.10$	0.1910	0.1595	0.1609	0.1669	0.1701	0.1647
1.10 < $K/F \leq 1.15$	0.2757	0.1879	0.1691	0.1679	0.1696	0.1778
1.15 < $K/F \leq 1.20$	0.3725	0.2367	0.1884	0.1687	0.1657	0.1909
All	0.2046	0.2057	0.2043	0.2024	0.1996	0.2040

Table 2: Description of Firms Option Data (1996-2015).

Moneyness is defined as K/F , where F is the forward price of the underlying and K is the option's strike price. DTM stands for days to maturity. Our final option dataset contains 2,975,839 observations.

Panel A: Number of option contracts.					
	DTM ≤ 30	30 < DTM ≤ 90	90 < DTM ≤ 180	180 < DTM ≤ 250	DTM > 250
0.80 < $K/F \leq 0.85$	6,791	53,859	75,543	46,305	14,036
0.85 < $K/F \leq 0.90$	16,416	100,294	101,743	56,700	15,120
0.90 < $K/F \leq 0.95$	42,747	168,293	120,217	61,422	15,549
0.95 < $K/F \leq 1.00$	100,267	216,362	123,431	60,425	14,575
1.00 < $K/F \leq 1.05$	103,111	245,827	160,306	85,404	20,169
1.05 < $K/F \leq 1.10$	36,508	169,034	146,104	80,884	19,519
1.10 < $K/F \leq 1.15$	13,119	90,264	110,095	67,035	17,543
1.15 < $K/F \leq 1.20$	5,390	47,073	73054	47,754	14,333
All	324,349	1,091,006	910,493	505,929	130,844
Panel B: Average option prices.					
	DTM ≤ 30	30 < DTM ≤ 90	90 < DTM ≤ 180	180 < DTM ≤ 250	DTM > 250
0.80 < $K/F \leq 0.85$	1.04	1.42	2.16	2.96	4.05
0.85 < $K/F \leq 0.90$	1.06	1.54	2.56	3.55	5.00
0.90 < $K/F \leq 0.95$	1.12	1.79	3.26	4.52	6.22
0.95 < $K/F \leq 1.00$	1.45	2.52	4.48	5.98	8.11
1.00 < $K/F \leq 1.05$	1.47	2.50	4.36	5.85	8.03
1.05 < $K/F \leq 1.10$	1.16	1.79	3.08	4.21	6.03
1.10 < $K/F \leq 1.15$	1.14	1.61	2.51	3.34	4.77
1.15 < $K/F \leq 1.20$	1.16	1.53	2.26	2.98	4.03
All	1.33	2.03	3.25	4.31	5.87
Panel C: Average implied volatility.					
	DTM ≤ 30	30 < DTM ≤ 90	90 < DTM ≤ 180	180 < DTM ≤ 250	DTM > 250
0.80 < $K/F \leq 0.85$	0.7919	0.5436	0.4274	0.3872	0.3515
0.85 < $K/F \leq 0.90$	0.6392	0.4623	0.3851	0.3585	0.3352
0.90 < $K/F \leq 0.95$	0.4903	0.3869	0.3537	0.3398	0.3238
0.95 < $K/F \leq 1.00$	0.3618	0.3388	0.3386	0.3318	0.3169
1.00 < $K/F \leq 1.05$	0.3504	0.3258	0.3214	0.3162	0.3066
1.05 < $K/F \leq 1.10$	0.4675	0.3617	0.3217	0.3092	0.3017
1.10 < $K/F \leq 1.15$	0.5917	0.4244	0.3435	0.3168	0.2993
1.15 < $K/F \leq 1.20$	0.7147	0.4897	0.3802	0.3407	0.3072
All	0.4252	0.3819	0.3514	0.3334	0.3163
					0.4569
					0.4183
					0.3785
					0.3418
					0.3268
					0.3453
					0.3703
					0.4017
					0.3661

Table 3: Index Parameters Estimated Using Returns and Option Data

The index parameters are estimated using daily index returns and weekly cross-sections of out-of-the-money options, from January 1996 to August 2015. Parameters are estimated using multiple simplex search method optimizations (fminsearch in Matlab). Robust standard errors are calculated from the outer product of the gradient at the optimal parameter values. The average volatility of volatilities are annualized and are given in percentage. They are computed by multiplying the square root of the average of $\text{Var}_{t-1} [h_{M,\cdot,t+1}]$ by $100 \times 252^{3/2}$.

	Normal	Jump
λ_M / γ_M	0.824 (2.40E-10)	0.701 (3.55E-10)
$w_{M,v}$	-1.63E-06 (2.50E-08)	-4.05E-07 (1.88E-08)
$a_{M,v}$	2.42E-06 (3.36E-09)	4.44E-06 (7.83E-09)
$b_{M,v}$	0.940 (6.94E-10)	0.934 (4.58E-09)
$c_{M,v}$	144.19 (2.86E-11)	140.50 (1.30E-11)
α_M		11.856 (1.31E-08)
δ_M		-7.018 (3.01E-10)
Average risk premium	2.33	3.85
Median risk premium	1.72	3.04
Persistence	0.991	
Percent of annual variance	74.0	26.0
Avg. volatility of volatility	3.25	7.78
Average volatility (%)	18.01	
Average skewness	-6.25	
Average excess kurtosis	383.70	
Skewness of innovations, $\varepsilon_{M,t}$	-0.12	
Ex. kurtosis of innovations, $\varepsilon_{M,t}$	-0.07	
Return log-likelihood	78,395	
Option log-likelihood	12,788	
Total log-likelihood	91,183	
RIVRMSE	14.39	

Table 4: Valuation Errors on the Options Used for the Estimation

We use the joint MLE estimates of Tables 3 and 5 to compute implied volatility root mean squared errors (IVRMSE) and relative implied volatility root mean squared errors (RIVRMSE) for various moneyness, maturity, and year bins. We then average IVRMSE and RIVRMSE for each moneyness, maturity and year bin across firms. IVRMSEs and RIVRMSEs are given in percentage.

Panel A: Valuation Errors on the Options Used for the Estimation of the Market Model

Overall IVRMSE and RIVRMSE			Sorting by year			
	IVRMSE	RIVRMSE		IVRMSE	RIVRMSE	
All	3.086	14.391	1996	2.326	15.834	
Sorting by maturity			1997	3.844	17.178	
			1998	5.318	20.129	
			1999	4.625	17.786	
	IVRMSE	RIVRMSE	2000	2.419	10.508	
	DTM ≤ 30	3.364	16.185	2001	2.756	11.757
30 < DTM ≤ 90	3.108	14.606	2002	2.192	9.345	
90 < DTM ≤ 180	2.936	13.590	2003	2.313	11.214	
180 < DTM ≤ 270	3.016	13.607	2004	1.260	8.067	
270 < DTM ≤ 365	3.076	14.137	2005	2.307	17.593	
Sorting by moneyness			2006	2.327	17.721	
			2007	2.002	14.031	
			2008	5.086	13.635	
	IVRMSE	RIVRMSE	2009	4.018	15.546	
	0.80 < $K/F \leq 0.85$	3.815	11.663	2010	3.724	17.210
	0.85 < $K/F \leq 0.90$	3.533	11.818	2011	3.416	13.978
	0.90 < $K/F \leq 0.95$	3.334	12.543	2012	2.615	13.138
	0.95 < $K/F \leq 1.00$	3.006	13.421	2013	1.255	8.961
	1.00 < $K/F \leq 1.05$	2.684	16.316	2014	1.543	11.914
	1.05 < $K/F \leq 1.10$	2.729	16.577	2015	2.018	16.756
1.10 < $K/F \leq 1.15$	2.808	15.384				
1.15 < $K/F \leq 1.20$	3.037	15.134				

Panel B: Average Valuation Errors on the Options Used for the Estimation of the Firm Model

Overall average IVRMSE and RIVRMSE			Sorting by year			
	IVRMSE	RIVRMSE		IVRMSE	RIVRMSE	
All	6.241	13.483	1996	3.914	11.750	
Sorting by maturity			1997	4.492	11.879	
			1998	5.669	11.865	
			1999	5.627	11.229	
	IVRMSE	RIVRMSE	2000	7.343	12.146	
DTM ≤ 30	9.195	17.130	2001	6.411	11.793	
30 < DTM ≤ 90	6.296	13.720	2002	6.638	13.457	
90 < DTM ≤ 180	5.253	12.125	2003	4.624	11.786	
180 < DTM ≤ 270	5.201	12.355	2004	3.131	9.881	
270 < DTM ≤ 365	5.307	12.719	2005	3.473	12.214	
Sorting by moneyness			2006	3.482	11.293	
			2007	4.315	12.787	
			2008	10.121	14.176	
	IVRMSE	RIVRMSE	2009	8.138	14.595	
	0.80 < K/F ≤ 0.85	7.359	12.184	2010	4.097	11.882
	0.85 < K/F ≤ 0.90	6.837	12.252	2011	4.656	11.911
	0.90 < K/F ≤ 0.95	6.249	12.670	2012	4.149	11.888
	0.95 < K/F ≤ 1.00	5.948	14.023	2013	3.062	10.215
	1.00 < K/F ≤ 1.05	5.613	14.239	2014	3.595	12.737
	1.05 < K/F ≤ 1.10	5.925	13.469	2015	3.718	12.907
1.10 < K/F ≤ 1.15	6.318	13.413				
1.15 < K/F ≤ 1.20	6.982	13.826				

Table 5: Firm Parameters Estimated Using Returns and Option Data

The index parameters are estimated using daily index returns and weekly cross-sections of out-of-the-money options, from January 1996 to August 2015. Parameters are estimated using multiple simplex search method optimizations (fminsearch in Matlab). Robust standard errors are calculated from the outer product of the gradient at the optimal parameter values. For firms, we report statistics on the joint MLE estimates obtained for the 260 individual stocks in our sample. Q1 and Q3 report the 25th and 75th percentiles of the estimates.

	Average	S.D.	Min	Q1	Median	Q3	Max
$\beta_{S,z}$	0.908	0.297	0.206	0.676	0.925	1.116	1.756
$\beta_{S,y}$	1.085	0.417	0.202	0.809	1.034	1.283	3.926
λ_S	0.000	0.002	-0.015	0.000	0.000	0.000	0.010
γ_S	0.953	0.300	0.083	0.791	0.974	1.086	2.092
$\kappa_{S,z}$	0.957	0.705	0.083	0.509	0.776	1.200	4.665
$a_{S,z}$	2.06E-06	1.23E-06	2.25E-07	1.31E-06	1.73E-06	2.55E-06	9.07E-06
$b_{S,z}$	0.992	0.003	0.978	0.991	0.993	0.994	0.999
$c_{S,z}$	103.90	46.85	-90.17	69.23	115.46	139.98	207.42
$\kappa_{S,y}$	0.518	0.291	0.091	0.329	0.471	0.627	2.212
$a_{S,y}$	4.82E-06	3.41E-06	4.54E-07	2.25E-06	4.22E-06	6.88E-06	2.43E-05
$b_{S,y}$	0.927	0.057	0.303	0.907	0.927	0.961	0.997
$c_{S,y}$	125.08	51.86	-252.57	107.15	132.38	147.54	325.27
α_S	10.757	4.880	0.744	8.353	10.587	12.294	43.452
δ_S	-6.239	2.209	-15.255	-7.316	-6.572	-5.128	-0.302
Avg. volatility (%)	39.21	13.57	18.90	30.56	36.39	45.67	162.41
Avg. skewness	-6.49	15.03	-132.05	-4.58	-2.81	-1.61	-0.11
Avg. excess kurtosis	1024.24	4359.87	1.28	52.63	114.23	231.93	42494.19
Skewness of innovations. $\varepsilon_{S,t}$	0.10	0.35	-3.60	0.06	0.12	0.18	0.86
Ex. kurtosis of innovations. $\varepsilon_{S,t}$	1.68	8.06	-0.07	0.49	0.74	1.04	100.40
RIVRMSE	13.48	2.73	8.87	11.93	12.96	14.59	39.63

Table 6: Firm Parameters Estimated Using Returns and Option Data for each of the eight largest GICS industries

The index parameters are estimated using daily index returns and weekly cross-sections of out-of-the-money options, from January 1996 to August 2015. Parameters are estimated using multiple simplex search method optimizations (fminsearch in Matlab). Robust standard errors are calculated from the outer product of the gradient at the optimal parameter values. For firms, we report statistics on the joint MLE estimates obtained for the 260 individual stocks in our sample across the eight largest GICS industries covered in our sample. Q1 and Q3 report the 25th and 75th percentiles of the estimates.

	Average	S.D.	Min	Q1	Median	Q3	Max
Consumer Discretionary (based on 42 firms)							
$\beta_{S,z}$	0.937	0.241	0.477	0.762	0.963	1.092	1.421
$\beta_{S,y}$	1.025	0.321	0.500	0.779	0.983	1.259	1.945
$\kappa_{S,z}$	1.095	0.727	0.226	0.616	0.847	1.411	4.062
$\kappa_{S,y}$	0.617	0.379	0.185	0.378	0.511	0.754	2.212
Consumer Staples (based on 16 firms)							
$\beta_{S,z}$	0.567	0.181	0.289	0.435	0.541	0.711	0.851
$\beta_{S,y}$	0.832	0.215	0.582	0.647	0.780	0.946	1.338
$\kappa_{S,z}$	0.920	1.040	0.167	0.477	0.639	0.877	4.665
$\kappa_{S,y}$	0.341	0.156	0.143	0.201	0.300	0.482	0.575
Energy (based on 32 firms)							
$\beta_{S,z}$	0.735	0.263	0.326	0.576	0.695	0.930	1.472
$\beta_{S,y}$	1.331	0.442	0.536	1.069	1.248	1.619	2.503
$\kappa_{S,z}$	1.355	0.642	0.360	0.891	1.253	1.645	3.228
$\kappa_{S,y}$	0.529	0.230	0.097	0.422	0.529	0.650	1.155
Financials (based on 36 firms)							
$\beta_{S,z}$	1.015	0.266	0.445	0.849	1.007	1.176	1.756
$\beta_{S,y}$	1.206	0.578	0.495	0.898	1.179	1.378	3.926
$\kappa_{S,z}$	0.713	0.391	0.124	0.466	0.649	0.865	1.886
$\kappa_{S,y}$	0.441	0.152	0.175	0.351	0.442	0.500	1.011
Health Care (based on 30 firms)							
$\beta_{S,z}$	0.734	0.282	0.206	0.576	0.662	0.904	1.400
$\beta_{S,y}$	0.996	0.283	0.500	0.862	1.018	1.090	1.913
$\kappa_{S,z}$	0.660	0.510	0.130	0.326	0.584	0.832	2.880
$\kappa_{S,y}$	0.419	0.253	0.091	0.273	0.345	0.512	1.352
Industrials (based on 32 firms)							
$\beta_{S,z}$	0.896	0.189	0.516	0.749	0.868	1.039	1.316
$\beta_{S,y}$	0.985	0.304	0.202	0.802	0.984	1.146	1.661
$\kappa_{S,z}$	0.785	0.483	0.197	0.483	0.682	1.011	2.766
$\kappa_{S,y}$	0.472	0.273	0.140	0.329	0.419	0.542	1.444
Information Technology (based on 50 firms)							
$\beta_{S,z}$	1.184	0.218	0.765	0.999	1.199	1.326	1.568
$\beta_{S,y}$	1.083	0.423	0.275	0.814	0.987	1.282	2.416
$\kappa_{S,z}$	1.018	0.869	0.083	0.389	0.706	1.409	3.503
$\kappa_{S,y}$	0.635	0.330	0.125	0.423	0.565	0.777	1.674
Materials (based on 17 firms)							
$\beta_{S,z}$	0.864	0.208	0.430	0.732	0.864	1.035	1.236
$\beta_{S,y}$	1.173	0.440	0.435	0.856	1.159	1.545	1.943
$\kappa_{S,z}$	1.215	0.661	0.463	0.743	1.013	1.615	3.086
$\kappa_{S,y}$	0.550	0.290	0.091	0.359	0.569	0.698	1.134

Table 7: Correlation Between the Parameters of Stocks

This table reports the correlation between a subset of the parameters associated with the 260 stocks under consideration. IVAR stands for idiosyncratic variance; IVAR, $\widehat{\text{IVAR}}_{S,z}$, and $\widehat{\text{IVAR}}_{S,y}$ are respectively the firm-by-firm average of the following time series: $h_{S,z,t} + \frac{\sigma_S^2}{(\sigma_S^2 - \delta_S^2)^{3/2}} h_{S,y,t}$, $h_{S,z,t} - \kappa_{S,z} h_{M,z,t}$, and $h_{S,y,t} - \kappa_{S,y} h_{M,y,t}$. The $\text{RP}_{u,v}$ are the firm-by-firm average of the risk premiums associated with source of risk v_u , $v \in \{z, y\}$, $u \in \{M, S\}$.

	$\beta_{S,y}$	γ_S	$\kappa_{S,z}$	$\kappa_{S,y}$	IVAR	$\widehat{\text{IVAR}}_{S,z}$	$\widehat{\text{IVAR}}_{S,y}$	$\text{RP}_{M,z}$	$\text{RP}_{M,y}$	$\text{RP}_{S,y}$
$\beta_{S,z}$	0.195	-0.105	0.042	0.332	0.270	0.370	0.166	0.989	0.193	0.239
$\beta_{S,y}$		-0.058	0.015	-0.016	0.066	0.159	0.216	0.190	0.982	0.069
γ_S			-0.006	0.002	-0.070	0.051	-0.114	-0.106	-0.059	0.489
$\kappa_{S,z}$				0.429	0.347	0.336	0.266	0.048	0.014	0.367
$\kappa_{S,y}$					0.280	0.504	0.145	0.346	-0.012	0.609
IVAR						0.480	0.375	0.273	0.076	0.314
$\widehat{\text{IVAR}}_{S,z}$							0.490	0.401	0.188	0.609
$\widehat{\text{IVAR}}_{S,y}$								0.177	0.222	0.576

Table 8: Firm Parameters Estimated Using Returns and Option Data: Neglecting Idiosyncratic Jumps

The index parameters are estimated using daily index returns and weekly cross-sections of out-of-the-money options, from January 1996 to August 2015. Parameters are estimated using multiple simplex search method optimizations (fminsearch in Matlab). Robust standard errors are calculated from the outer product of the gradient at the optimal parameter values. For firms, we report statistics on the joint MLE estimates obtained for the 260 individual stocks in our sample. Q1 and Q3 report the 25th and 75th percentiles of the estimates.

	Average	S.D.	Min	Q1	Median	Q3	Max
$\beta_{S,z}$	0.908	0.335	0.101	0.684	0.908	1.141	2.051
$\beta_{S,y}$	1.365	0.550	0.000	0.985	1.398	1.698	3.961
λ_S	0.765	0.440	0.030	0.397	0.727	1.016	2.242
$\kappa_{S,z}$	0.874	0.874	0.045	0.309	0.649	1.200	5.690
$a_{S,z}$	2.66E-06	2.25E-06	9.29E-08	1.12E-06	2.12E-06	3.63E-06	2.30E-05
$b_{S,z}$	0.995	0.014	0.906	0.997	0.998	0.999	1.000
$c_{S,z}$	53.30	84.76	-282.82	7.99	52.37	95.89	437.76
Avg. volatility (%)	36.56	9.50	18.33	29.41	35.10	43.24	64.32
Avg. skewness	-2.92	2.33	-13.32	-4.26	-2.45	-1.03	0.00
Avg. excess kurtosis	149.03	150.39	0.00	35.04	107.46	216.51	1014.66
Skewness of innovations, $\varepsilon_{S,t}$	-0.37	1.20	-9.34	-0.50	-0.08	0.12	2.97
Ex. kurtosis of innovations, $\varepsilon_{S,t}$	16.78	31.96	0.69	5.00	9.14	15.34	349.03
RIVRMSE	15.57	6.42	9.94	12.24	14.19	17.01	80.62

Table 9: Excess Returns of Portfolios Based on Idiosyncratic Jump Risk Premiums

Each day, we compute the conditional model-implied risk premium associated with each stock's (i) Panel A: idiosyncratic diffusive risk, $RP_{S,z,t}$ ($RP'_{S,z,t}$ for the last five columns), and (ii) Panel B: idiosyncratic jump risk, $RP_{S,y,t}$ ($RP'_{S,y,t}$ for the last five columns). Stocks are then sorted into quintile portfolios from the lowest (P1) to the highest (P5) level of $RP_{S,\cdot,t}$; stocks in the portfolios are weighted according to market capitalization. A long-short portfolio is created from taking long position in (P5) and a short position in (P1). The daily returns of the long-short portfolio are then regressed on (subsets of) the following seven variables: the Fama-French market (MKT), small minus big (SMB), high minus low (HML), robust minus weak (RMW), and conservative minus aggressive (CMA) factors, the momentum (MOM) factor, and returns on the CBOE volatility index (ΔVIX). The regression constant is reported in annualized percentage points ($\Delta t = 1/252$).

Panel A: Portfolios Based on the Idiosyncratic Diffusive Risk Premium										
	Idiosyncratic Diffusive Risk Premium					In Excess of the Common Component				
	FF3	FF5	MOM	AHXZ	All	FF3	FF5	MOM	AHXZ	All
$Cst \times \frac{100}{\Delta t}$	1.748 (0.71)	4.220 (1.74)	2.599 (1.05)	1.534 (0.63)	4.449 (1.83)	1.287 (0.53)	3.559 (1.46)	2.082 (0.84)	1.011 (0.41)	3.657 (1.50)
MKT	0.119 (6.80)	0.049 (2.50)	0.097 (5.57)	0.144 (6.28)	0.048 (2.05)	0.089 (5.10)	0.024 (1.22)	0.069 (3.89)	0.122 (5.24)	0.034 (1.39)
SMB	0.095 (3.72)	0.053 (2.25)	0.103 (3.88)	0.093 (3.63)	0.058 (2.46)	0.085 (3.25)	0.051 (2.02)	0.092 (3.44)	0.082 (3.13)	0.055 (2.18)
HML	-0.542 (-18.22)	-0.374 (-12.03)	-0.587 (-18.76)	-0.538 (-18.32)	-0.405 (-11.59)	-0.561 (-18.93)	-0.396 (-11.37)	-0.603 (-18.97)	-0.556 (-19.07)	-0.422 (-11.14)
RMW		-0.202 (-4.97)			-0.192 (-4.67)		-0.167 (-4.14)			-0.157 (-3.88)
CMA		-0.424 (-7.46)			-0.399 (-6.95)		-0.426 (-6.99)			-0.404 (-6.57)
MOM			-0.094 (-4.29)		-0.045 (-2.31)			-0.087 (-3.96)		-0.039 (-1.93)
ΔVIX				0.023 (1.76)	0.005 (0.34)				0.030 (2.22)	0.013 (0.95)
Adj. R^2	22.0%	26.2%	22.9%	22.0%	26.4%	21.1%	25.0%	21.9%	21.2%	25.1%

Panel B: Portfolios Based on the Idiosyncratic Jump Risk Premium										
	Idiosyncratic Jump Risk Premium					In Excess of the Common Component				
	FF3	FF5	MOM	AHXZ	All	FF3	FF5	MOM	AHXZ	All
$Cst \times \frac{100}{\Delta t}$	8.412 (2.28)	16.317 (4.54)	11.070 (3.06)	8.294 (2.25)	17.840 (4.97)	9.513 (2.50)	16.201 (4.35)	12.148 (3.31)	9.488 (2.52)	17.807 (4.91)
MKT	0.473 (15.59)	0.259 (11.92)	0.406 (15.76)	0.487 (10.79)	0.179 (5.96)	0.487 (14.76)	0.299 (12.29)	0.421 (14.80)	0.490 (9.72)	0.214 (6.56)
SMB	0.401 (10.16)	0.231 (6.09)	0.424 (10.84)	0.399 (10.06)	0.253 (6.95)	0.285 (6.76)	0.173 (4.38)	0.309 (7.11)	0.285 (6.75)	0.196 (5.05)
HML	-0.428 (-5.25)	0.044 (1.00)	-0.568 (-7.04)	-0.426 (-5.28)	-0.073 (-1.56)	-0.115 (-1.34)	0.346 (5.87)	-0.254 (-3.38)	-0.115 (-1.34)	0.223 (4.06)
RMW		-0.756 (-12.90)			-0.731 (-13.87)		-0.536 (-9.41)			-0.510 (-8.71)
CMA		-1.142 (-11.39)			-1.057 (-11.96)		-1.170 (-12.70)			-1.082 (-12.14)
MOM			-0.292 (-5.52)		-0.161 (-4.01)			-0.290 (-5.92)		-0.168 (-4.17)
DVIX				0.013 (0.52)	-0.049 (-2.41)				0.003 (0.10)	-0.053 (-2.21)
Adj. R^2	29.1%	44.2%	32.6%	29.1%	45.2%	24.1%	37.4%	27.7%	24.0%	38.6%

Table 10: Description of Quintile Portfolios

This table describes the quintile portfolios obtained in Table 9. Each day, the following variables are recorded for the firms in each quintile portfolio: market beta, log of market capitalisation, book-to-market ratio, operating profitability (OP), investment, trailing 12-month return, and the volatility beta. This table reports the time-series average of these variables for each of the quintile portfolios obtained using the total idiosyncratic risk premium (first five columns) or its component in excess of the common component (last five columns). Standard deviations are reported within square brackets.

	Idiosyncratic Jump Risk Premium					In Excess of the Common Component				
	P1	P2	P3	P4	P5	P1	P2	P3	P4	P5
Market beta	0.97 [0.11]	1.00 [0.09]	1.06 [0.12]	1.11 [0.15]	1.26 [0.25]	0.98 [0.13]	0.99 [0.12]	1.05 [0.11]	1.09 [0.14]	1.21 [0.23]
log(ME)	25.40 [0.28]	24.96 [0.38]	24.61 [0.46]	24.30 [0.52]	23.79 [0.54]	25.14 [0.38]	25.17 [0.40]	24.86 [0.42]	24.49 [0.45]	23.99 [0.51]
BE/ME	1.76 [0.53]	1.42 [0.56]	1.47 [0.75]	1.24 [0.83]	1.20 [1.08]	1.43 [0.58]	1.54 [0.55]	1.53 [0.93]	1.55 [1.33]	1.56 [1.90]
OP (%)	5.74 [1.30]	5.46 [1.35]	5.93 [1.36]	6.10 [1.60]	5.42 [1.85]	5.80 [1.54]	5.79 [1.40]	5.58 [1.45]	5.74 [1.50]	5.70 [1.71]
Investment (%)	14.63 [18.44]	15.86 [19.06]	19.31 [19.78]	26.96 [45.46]	35.51 [77.07]	20.46 [33.20]	16.30 [15.94]	15.37 [16.53]	18.98 [23.66]	28.43 [49.08]
12-month return (%)	15.25 [15.54]	15.72 [16.84]	17.84 [18.21]	19.27 [19.74]	20.11 [29.61]	19.11 [16.43]	16.57 [15.81]	15.12 [17.32]	14.79 [19.20]	16.73 [27.06]
Volatility beta (%)	0.29 [3.07]	0.50 [3.44]	0.53 [4.30]	-1.11 [6.29]	-2.65 [9.71]	-0.32 [3.97]	0.46 [3.44]	0.83 [4.16]	-0.06 [5.44]	-2.37 [9.13]

Table 11: Double Sort: Excess Returns of Portfolios Based on Idiosyncratic Jump Risk Premiums

On each day t , we first sort stocks into quintiles (Q1 to Q5) based on their market beta over the past year. Then, within each market-beta quintile, we sort stocks into terciles based on $RP_{S,y,t}$ (first five columns; the last five columns are based on $RP'_{S,y,t}$). We then take a long position in a cap-weighted portfolio of the top-tercile stocks, and a short position in a cap-weighted portfolio of the bottom-tercile stocks. This leaves us with five long-short portfolios, each of which is composed of stocks with homogenous market betas. The first row of this table reports the alphas of regression (4.21) for each of these long-short portfolios. The procedure is repeated for six other variables: log of market capitalisation, book-to-market ratio, operating profitability (OP), investment, trailing 12-month return, and the volatility beta.

	Idiosyncratic Jump Risk Premium					In Excess of the Common Component				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Market beta	10.408 (2.71)	8.957 (2.44)	16.837 (4.43)	15.242 (3.62)	10.024 (1.43)	11.783 (3.23)	11.368 (3.04)	20.495 (5.35)	15.627 (3.60)	10.759 (1.89)
log(ME)	11.629 (2.46)	7.435 (2.05)	8.791 (2.73)	17.906 (5.77)	10.027 (3.80)	11.283 (2.30)	9.990 (2.57)	9.429 (2.84)	18.620 (5.99)	8.780 (3.02)
BE/ME	20.747 (4.63)	13.477 (2.89)	9.252 (2.21)	14.811 (3.50)	14.381 (3.47)	15.653 (2.66)	18.488 (3.92)	12.847 (3.12)	17.991 (4.33)	15.290 (3.72)
OP (%)	9.027 (1.61)	10.796 (2.26)	11.430 (2.46)	7.035 (1.47)	24.957 (5.02)	18.828 (3.46)	12.194 (2.71)	14.171 (3.03)	10.364 (2.15)	13.506 (2.70)
Investment (%)	13.785 (2.97)	12.780 (3.39)	15.409 (3.98)	23.275 (5.13)	11.985 (2.36)	16.072 (3.42)	11.650 (3.07)	22.329 (5.86)	16.914 (3.64)	5.305 (1.01)
12-month return (%)	17.567 (3.42)	5.529 (1.46)	12.164 (3.36)	14.953 (3.66)	12.720 (2.57)	14.197 (2.85)	13.406 (3.46)	13.994 (3.89)	13.975 (3.40)	4.232 (0.88)
Volatility beta (%)	3.451 (0.68)	12.885 (2.97)	14.973 (3.95)	14.183 (3.67)	15.743 (3.18)	7.568 (1.49)	14.425 (3.27)	12.403 (3.39)	16.668 (4.35)	16.876 (3.21)

APPENDIX

A Innovations' Cumulant Generating Functions

A.1 Continuous Component

For any $z_{u,t} \in \{z_{M,t}, z_{S,t} : S \in \mathbb{S}\}$, the conditional cumulant generating function of $z_{u,t}$ satisfies

$$\xi_{z_{u,t}}^{\mathbb{P}}(\phi) = \log E_{\mathcal{F}_{t-1}^{\mathbb{S}}}^{\mathbb{P}} [\exp(\phi z_{u,t})] = \frac{\phi^2}{2} h_{u,z,t}.$$

A.2 Jump Component

The conditional cumulant generating function of $y_{u,t} \in \{y_{M,t}, y_{S,t} : S \in \mathbb{S}\}$ is

$$\xi_{y_{u,t}}^{\mathbb{P}}(\phi) = \log E_{\mathcal{F}_{t-1}^{\mathbb{S}}}^{\mathbb{P}} [\exp(\phi y_{u,t})] = \Pi_u(\phi) h_{u,y,t}$$

where

$$\Pi_u(\phi) = \left(\sqrt{\alpha_u^2 - \delta_u^2} - \sqrt{\alpha_u^2 - (\delta_u + \phi)^2} \right). \quad (\text{A.1})$$

B Innovations' Risk Neutral Cumulant Generating Functions

Lemma 1 For any $\varepsilon_{u,t} \in \{\varepsilon_{M,t}, \varepsilon_{S,t} : S \in \mathbb{S}\}$, the conditional cumulant generating function of $\varepsilon_{u,t}$ under \mathbb{Q} is

$$\xi_{\varepsilon_{u,t}}^{\mathbb{Q}}(\phi) = \log E_{t-1}^{\mathbb{Q}} [\exp(\phi \varepsilon_{u,t})] = \frac{1}{2} \phi^2 - \Lambda_u \sqrt{h_{u,z,t}} \phi$$

which corresponds to the cumulant generating function of a Gaussian random variable of expectation $-\Lambda_u \sqrt{h_{u,z,t}}$ and variance 1. To obtain a risk neutral sequence of standard normal innovations, we must set

$$\varepsilon_{u,t}^* = \varepsilon_{u,t} + \Lambda_u \sqrt{h_{u,z,t}}. \quad (\text{B.2})$$

Sketch of the proof.

$$\begin{aligned} \xi_{\varepsilon_{u,t}}^{\mathbb{Q}}(\phi) &= \log E_{t-1}^{\mathbb{Q}} [\exp(\phi \varepsilon_{u,t})] \\ &= \log E_{t-1}^{\mathbb{P}} \left[\frac{\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})}{E_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})]} \exp(\phi \varepsilon_{u,t}) \right] \\ &= \xi_{\varepsilon_{u,t}}^{\mathbb{P}}(\phi - \Lambda_u \sqrt{h_{u,z,t}}) - \xi_{\varepsilon_{u,t}}^{\mathbb{P}}(-\Lambda_u \sqrt{h_{u,z,t}}). \end{aligned}$$

Note that, given that $h_{u,z,t}$ is $\mathcal{F}_{t-1}^{\mathbb{S}}$ -measurable, the conditional cumulant generating function of $z_{u,t}$ under \mathbb{Q} can easily be show (by replicating the above) to be that of a normal variable of expectation $-\Lambda_u h_{u,z,t}$ and variance $h_{u,z,t}$. In other words, consistent the results in Christoffersen, Elkamhi, Feunou, and Jacobs (2010), the risk-neutral $\mathcal{F}_{t-1}^{\mathbb{S}}$ -conditional variance of $z_{u,t}$, $h_{u,z,t}^*$, is equal to its physical counterpart, $h_{u,z,t}$.

Lemma 2 For any $y_{u,t} \in \{y_{M,t}, y_{S,t} : S \in \mathbb{S}\}$, the conditional cumulant generating function of $y_{u,t}$ under \mathbb{Q} is

$$\xi_{y_{u,t}}^{\mathbb{Q}}(\phi) = \log E_{t-1}^{\mathbb{Q}} [\exp(\phi y_{u,t})] = \Pi_u^*(\phi) h_{u,y,t}^* \quad (\text{B.3})$$

where

$$\Pi_u^*(\phi) = \sqrt{\alpha_u^2 - (\delta_u - \Gamma_u)^2} - \sqrt{\alpha_u^2 - (\delta_u - \Gamma_u + \phi)^2} \quad (\text{B.4})$$

The proof uses a similar argument as for $\xi_{\varepsilon u,t}^{\mathbb{Q}}(\phi)$. Details are provided in the Online Appendix OB.

The risk neutral jump component is still a NIG random variable with no location parameter, the tail heaviness parameter $\alpha_u^* = \alpha_u$ is not affected by the change of measure, the asymmetry parameter becomes $\delta_u^* = \delta_u - \Gamma_u$ and the scale variable is $h_{u,y,t}^* = h_{u,y,t}$.

C Risk Premiums

Lemma 3 *The mappings between λ_M and γ_M and their pricing kernel counterparts Λ_M and Γ_M are*

$$\lambda_M = \Lambda_M \text{ and } \gamma_M = \Pi_u(1) - \Pi_u^*(1).$$

For the stock parameters λ_S and γ_S , the relation is

$$\lambda_S = \Lambda_S, \gamma_{M,S}(\beta_{S,y}) = \Pi_M(\beta_{S,y}) - \Pi_M^*(\beta_{S,y}), \gamma_S = \Pi_S(1) - \Pi_S^*(1)$$

where $\Pi_u(\cdot)$ and $\Pi_u^*(\cdot)$ are defined at equations (A.1) and (B.4).

Proof of Lemma 3. Since the proof for the market component is similar, the focus is put on the stock specific parameters. More details are available in the Online Appendix.

Since the discounted stock price should behave as a \mathbb{Q} -martingale,

$$\begin{aligned} 1 &= \mathbb{E}_{t-1}^{\mathbb{Q}} \left[\frac{\exp(-r_t) S_t}{S_{t-1}} \right] \\ &= \mathbb{E}_{t-1}^{\mathbb{P}} \left[\frac{\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})}{\mathbb{E}_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})]} \exp(R_{S,t} - r_t) \right]. \end{aligned}$$

Replacing the excess return using (2.2) and the cumulant generating functions, we get

$$1 = \exp \left(\begin{aligned} &\mu_{S,t}^{\mathbb{P}} - r_t - \xi_{z_{M,t}}^{\mathbb{P}}(\beta_{S,z}) - \xi_{y_{M,t}}^{\mathbb{P}}(\beta_{S,y}) - \xi_{z_{S,t}}^{\mathbb{P}}(1) - \xi_{y_{S,t}}^{\mathbb{P}}(1) \\ &+ \xi_{z_{M,t}}^{\mathbb{P}}(\beta_{S,z} - \Lambda_M) + \xi_{y_{M,t}}^{\mathbb{P}}(\beta_{S,y} - \Gamma_M) + \xi_{z_{S,t}}^{\mathbb{P}}(1 - \Lambda_S) + \xi_{y_{S,t}}^{\mathbb{P}}(1 - \Gamma_S) \\ &- \xi_{z_{M,t}}^{\mathbb{P}}(-\Lambda_M) - \xi_{y_{M,t}}^{\mathbb{P}}(-\Gamma_M) - \xi_{z_{S,t}}^{\mathbb{P}}(-\Lambda_S) - \xi_{y_{S,t}}^{\mathbb{P}}(-\Gamma_S) \end{aligned} \right).$$

Because,

$$\begin{aligned} -\xi_{z_{M,t}}^{\mathbb{P}}(\beta_{S,z}) + \xi_{z_{M,t}}^{\mathbb{P}}(\beta_{S,z} - \Lambda_M) - \xi_{z_{M,t}}^{\mathbb{P}}(-\Lambda_M) &= -\Lambda_M \beta_{S,z} h_{M,z,t}, \\ -\xi_{z_{S,t}}^{\mathbb{P}}(1) + \xi_{z_{S,t}}^{\mathbb{P}}(1 - \Lambda_S) - \xi_{z_{S,t}}^{\mathbb{P}}(-\Lambda_S) &= -\Lambda_S h_{S,z,t}, \\ -\xi_{y_{M,t}}^{\mathbb{P}}(\beta_{S,y}) + \xi_{y_{M,t}}^{\mathbb{P}}(\beta_{S,y} - \Gamma_M) - \xi_{y_{M,t}}^{\mathbb{P}}(-\Gamma_M) &= -h_{M,y,t} \gamma_{M,S}(\beta_{S,y}) \\ -\xi_{y_{S,t}}^{\mathbb{P}}(1) + \xi_{y_{S,t}}^{\mathbb{P}}(1 - \Gamma_S) - \xi_{y_{S,t}}^{\mathbb{P}}(-\Gamma_S) &= -h_{S,y,t} \gamma_S, \end{aligned}$$

we conclude that

$$1 = \exp(\mu_{S,t}^{\mathbb{P}} - r_t - \Lambda_M \beta_{S,z} h_{M,z,t} - h_{M,y,t} \gamma_{M,S}(\beta_{S,y}) - \Lambda_S h_{S,z,t} - h_{S,y,t} \gamma_S).$$

Therefore,

$$\mu_{S,t}^{\mathbb{P}} = r_t + \Lambda_M \beta_{S,z} h_{M,z,t} + h_{M,y,t} \gamma_{M,S} (\beta_{S,y}) + \Lambda_S h_{S,z,t} + h_{S,y,t} \gamma_S.$$

D Risk Neutral Conditional Variances and Jump Intensities

Lemma 4 *Let*

$$\eta_t^* = \left[1 \quad h_{M,z,t}^* \quad h_{M,y,t}^* \quad h_{S,z,t}^* \quad h_{S,y,t}^* \quad (\varepsilon_{M,t}^*)^2 \quad \sqrt{h_{M,z,t}^*} \varepsilon_{M,t}^* \quad (\varepsilon_{S,t}^*)^2 \quad \sqrt{h_{S,z,t}^*} \varepsilon_{S,t}^* \right]'$$

Then, for any $u \in \{M, S\}$ and $v \in \{z, y\}$,

$$h_{u,v,t+1}^* = \pi_{u,v} \eta_t^* \quad (\text{D.5})$$

where $\pi_{u,v}$ is a 1×9 vector of constants satisfying

$$\begin{aligned} \pi_{M,z,1} &= w_{M,z} & \pi_{M,z,7} &= -2a_{M,z} (c_{M,z} + \Lambda_M) \\ \pi_{M,z,2} &= b_{M,z} + a_{M,z} (c_{M,z} + \Lambda_M)^2 & \pi_{M,z,i} &= 0 \text{ for } i \in \{3, 4, 5, 8, 9\} \\ \pi_{M,z,6} &= a_{M,z} \\ \pi_{S,z,1} &= \kappa_{S,z} \pi_{M,z,1} - a_{S,z} & \pi_{S,z,6} &= \kappa_{S,z} \pi_{M,z,6} \\ \pi_{S,z,2} &= \kappa_{S,z} (\pi_{M,z,2} - b_{S,z}) & \pi_{S,z,7} &= \kappa_{S,z} \pi_{M,z,7} \\ \pi_{S,z,4} &= b_{S,z} + a_{S,z} (2c_{S,z} + \Lambda_S) \Lambda_S & \pi_{S,z,8} &= a_{S,z} \\ \pi_{S,z,i} &= 0 \text{ for } i \in \{3, 5\} & \pi_{S,z,9} &= -2a_{S,z} (c_{S,z} + \Lambda_S) \\ \pi_{M,y,1} &= w_{M,y} & \pi_{M,y,6} &= a_{M,y} \\ \pi_{M,y,2} &= a_{M,y} (c_{M,y} + \Lambda_M)^2 & \pi_{M,y,7} &= -2a_{M,y} (c_{M,y} + \Lambda_M) \\ \pi_{M,y,3} &= b_{M,y} & \pi_{M,y,i} &= 0 \text{ for } i \in \{4, 5, 8, 9\} \\ \pi_{S,y,1} &= \kappa_{S,y} \pi_{M,y,1} - a_{S,y} & \pi_{S,y,6} &= \kappa_{S,y} \pi_{M,y,6} \\ \pi_{S,y,2} &= \kappa_{S,y} \pi_{M,y,2} & \pi_{S,y,7} &= \kappa_{S,y} \pi_{M,y,7} \\ \pi_{S,y,3} &= \kappa_{S,y} (\pi_{M,y,3} - b_{S,y}) & \pi_{S,y,8} &= a_{S,y} \\ \pi_{S,y,4} &= a_{S,y} (2c_{S,y} + \Lambda_S) \Lambda_S & \pi_{S,y,9} &= -2a_{S,y} (c_{S,y} + \Lambda_S) \\ \pi_{S,y,5} &= b_{S,y} \end{aligned}$$

Proof of Lemma 4. The risk neutral market conditional variance $h_{M,z,t+1}^*$ and jump intensity variable $h_{M,y,t+1}^*$ are obtained by replacing (B.2) in (2.3) and (2.5).

In the case of the stocks, for any $v \in \{z, y\}$,

$$\begin{aligned} & (\varepsilon_{S,t}^2 - 1 - 2c_{S,v} \sqrt{h_{S,z,t}^*} \varepsilon_{S,t}) \\ &= \left(\varepsilon_{S,t}^* - \Lambda_S \sqrt{h_{S,z,t}^*} \right)^2 - 1 - 2c_{S,v} \sqrt{h_{S,z,t}^*} \left(\varepsilon_{S,t}^* - \Lambda_S \sqrt{h_{S,z,t}^*} \right) \\ &= (2c_{S,v} + \Lambda_S) \Lambda_S h_{S,z,t}^* + \left((\varepsilon_{S,t}^*)^2 - 1 - 2(c_{S,v} + \Lambda_S) \sqrt{h_{S,z,t}^*} \varepsilon_{S,t}^* \right). \end{aligned} \quad (\text{D.6})$$

where the first equality arises from (B.2). Replacing back in the conditional variance (2.4) and the jump intensity process (2.6) leads to their risk neutral versions.

E Moment Generating Function of Risk-Neutral Excess Returns

Lemma 5 For $u \in \{M, S\}$, the conditional moment generating function of the excess returns satisfies

$$\varphi_{\tilde{R},t,T}^{\mathbb{Q}}(\phi) = \exp\left(\mathcal{A}_{u,T-t}(\phi) + \mathcal{B}_{u,T-t}(\phi)h_{M,z,t+1}^* + \mathcal{C}_{u,T-t}(\phi)h_{M,y,t+1}^* + \mathcal{D}_{u,T-t}(\phi)h_{S,z,t+1}^* + \mathcal{E}_{u,T-t}(\phi)h_{S,y,t+1}^*\right)$$

where the coefficients are found using a backward recursion over time. Indeed, $\varphi_{u,0}^{\mathbb{Q}}(\phi) = 1$ implies that

$$\mathcal{A}_{u,0}(\phi) = \mathcal{B}_{u,0}(\phi) = \mathcal{C}_{u,0}(\phi) = \mathcal{D}_{u,0}(\phi) = \mathcal{E}_{u,0}(\phi) = 0.$$

For $i \in \{0, 1, \dots, 9\}$, let

$$\zeta_{u,T-t-1,i}(\phi) = \mathcal{B}_{u,T-t-1}(\phi)\pi_{M,z,i} + \mathcal{C}_{u,T-t-1}(\phi)\pi_{M,y,i} + \mathcal{D}_{u,T-t-1}(\phi)\pi_{S,z,i} + \mathcal{E}_{u,T-t-1}(\phi)\pi_{S,y,i},$$

where the π are as provided in Appendix D. If $\zeta_{s,6}(\phi) < \frac{1}{2}$ and $\zeta_{s,8}(\phi) < \frac{1}{2}$ for any $s \in \{t+1, \dots, T\}$, then

$$\begin{aligned}\mathcal{A}_{u,T-t}(\phi) &= \mathcal{A}_{u,T-t-1}(\phi) + \zeta_{u,T-t-1,1}(\phi) - \frac{1}{2} \log(1 - 2\zeta_{u,T-t-1,6}(\phi)) - \frac{1}{2} \log(1 - 2\zeta_{u,T-t-1,8}(\phi)), \\ \mathcal{B}_{u,T-t}(\phi) &= \zeta_{u,T-t-1,2}(\phi) - \frac{1}{2}\beta_{u,z}^2\phi + \frac{1}{2} \frac{(\zeta_{u,T-t-1,7}(\phi) + \beta_{u,z}\phi)^2}{1 - 2\zeta_{u,T-t-1,6}(\phi)}, \\ \mathcal{C}_{u,T-t}(\phi) &= \zeta_{u,T-t-1,3}(\phi) - \Pi_M^*(\beta_{u,y})\phi + \Pi_M^*(\beta_{u,y}\phi), \\ \mathcal{D}_{u,T-t}(\phi) &= \zeta_{u,T-t-1,4}(\phi) - \frac{1}{2}(\beta'_{u,z})^2\phi + \frac{1}{2} \frac{(\zeta_{u,T-t-1,9}(\phi) + \beta'_{u,z}\phi)^2}{1 - 2\zeta_{u,T-t-1,8}(\phi)}, \\ \mathcal{E}_{u,T-t}(\phi) &= \zeta_{u,T-t-1,5}(\phi) - \Pi_S^*(\beta'_{u,y})\phi + \Pi_S^*(\beta'_{u,y}\phi).\end{aligned}$$

where for the market case, $\beta_{M,z} = \beta_{M,y} = 1$ and $\beta'_{M,z} = \beta'_{M,y} = 0$ while for the stock, $\beta'_{S,z} = \beta'_{S,y} = 1$.

As the proof is strongly inspired from the existing literature, we refer to the Online Appendix OE.

F Particle Filter

In the following, whenever the subscript M and S have been dropped, the approach is applicable to both market and stock data.

The filter is based on pure jump particle paths $y_{1:T}^{(i)} = \{y_1^{(i)}, y_2^{(i)}, \dots, y_T^{(i)}\}$, $i \in \{1, \dots, N\}$ and the sequential importance resampling (SIR) of Gordon, Salmond, and Smith (1993) is implemented.³² A single step of the SIR is now described.

Assume that N jump paths $y_{1:t-1}^{(i)}$, $i \in \{1, 2, \dots, N\}$ are available up to time $t-1$. As a by-product, the conditional variance $h_{z,t}^{(i)}$ and the jump scale variable $h_{y,t}^{(i)}$ are recovered.

³²Throughout the paper, $N = 25,000$ particles are used.

1. For $i \in \{1, 2, \dots, N\}$, the time t jump $y_t^{(i)}$ is simulated from the proposal distribution³³

$$f\left(\cdot \mid y_{1:t-1}^{(i)}, R_{1:t-1}\right) = f_{NIG}\left(\cdot; \alpha, \delta, h_{y,t}^{(i)}\right).$$

2. For $i \in \{1, 2, \dots, N\}$, update the importance weights (up to a normalizing constant) to reflect how likely the simulated particles are with respect to the time t information R_t :

$$\bar{\omega}_t^{(i)} = f\left(R_t \mid R_{1:t-1}, y_{1:t}^{(i)}\right).$$

More precisely, from equations (2.1) and (2.10), the market returns satisfy³⁴

$$R_{M,t} = r_t + \left(\lambda_M - \frac{1}{2}\right) h_{M,z,t} + (\gamma_M - \Pi_M(1)) h_{M,y,t} + z_{M,t} + y_{M,t}.$$

Therefore, conditionally on a simulated path $y_{M,1:t}^{(i)}$ and on the past returns $R_{M,1:t-1}$, the time t market return $R_{M,t}$ is normally distributed with expectation

$$m_{M,t}^{(i)} = r_t + \left(\lambda_M - \frac{1}{2}\right) h_{M,z,t}^{(i)} + (\gamma_M - \Pi_M(1)) h_{M,y,t}^{(i)} + y_{M,t}^{(i)}$$

³³More precisely,

$$\begin{aligned} f_{NIG}(x; \alpha, \delta, h) &= \frac{\alpha h K_1(\alpha \sqrt{h^2 + x^2})}{\pi \sqrt{h^2 + x^2}} \exp(h \sqrt{\alpha^2 - \delta^2} + \delta x) \\ K_1(x) &= \int_0^\infty \exp(-x \cosh(t)) \cosh(t) dt. \end{aligned}$$

³⁴Similarly, the stock returns

$$\begin{aligned} R_{S,t+1} &\cong r_t + \left(\beta_{S,z} \lambda_M - \frac{1}{2} \beta_{S,z}^2\right) \bar{h}_{M,z,t} + [\gamma_{M,S}(\beta_{S,y}) - \Pi_M(\beta_{S,y})] \bar{h}_{M,y,t} + \beta_{S,z} \bar{z}_{M,t+1} + \beta_{S,y} \bar{y}_{M,t+1} \\ &\quad + \left(\lambda_S - \frac{1}{2}\right) h_{S,z,t} + [\gamma_S - \Pi_S(1)] h_{S,y,t} + z_{S,t+1} + y_{S,t+1} \end{aligned}$$

where $\bar{h}_{M,z,t}, \bar{h}_{M,y,t}, \bar{z}_{M,t+1}, \bar{y}_{M,t+1}$ are the filtered value obtained from the estimation of the market model. Therefore,

$$f\left(R_{S,t} \mid R_{S,1:t-1}, y_{S,1:t}^{(i)}, \bar{h}_{M,z,1:t}, \bar{h}_{M,y,1:t}, \bar{z}_{M,1:t+1}, \bar{y}_{M,1:t+1}\right) = \phi\left(R_{S,t}; m_{S,t}^{(i)}, h_{S,z,t}^{(i)}\right)$$

with

$$\begin{aligned} m_{S,t}^{(i)} &= r_t + \left(\beta_{S,z} \lambda_M - \frac{1}{2} \beta_{S,z}^2\right) \bar{h}_{M,z,t} + [\gamma_{M,S}(\beta_{S,y}) - \Pi_M(\beta_{S,y})] \bar{h}_{M,y,t} + \beta_{S,z} \bar{z}_{M,t+1} + \beta_{S,y} \bar{y}_{M,t+1} \\ &\quad + \left(\lambda_S - \frac{1}{2}\right) h_{S,z,t}^{(i)} + [\gamma_S - \Pi_S(1)] h_{S,y,t}^{(i)} + y_{S,t+1}^{(i)} \end{aligned}$$

and variance $h_{S,z,t}^{(i)}$.

and variance $h_{M,z,t}^{(i)}$,

$$f\left(R_{M,t} \mid R_{M,1:t-1}, y_{M,1:t}^{(i)}\right) = \frac{1}{\sqrt{2\pi h_{M,z,t}^{(i)}}} \exp\left\{-\frac{1}{2} \frac{\left(R_{M,t} - m_{M,t}^{(i)}\right)^2}{h_{M,z,t}^{(i)}}\right\}.$$

3. For $i \in \{1, 2, \dots, N\}$, compute the normalized weights

$$\omega_t^{(i)} = \frac{\bar{\omega}_t^{(i)}}{\sum_{k=1}^N \bar{\omega}_t^{(k)}}.$$

4. For $i \in \{1, 2, \dots, N\}$, update the conditional variance and the jump scale variable. For the market, based on (2.3) and (2.5),

$$\begin{aligned} h_{M,z,t+1}^{(i)} &= w_{M,z} + b_{M,z} h_{M,z,t}^{(i)} + \frac{a_{M,z}}{h_{M,z,t}^{(i)}} \left(z_{M,t}^{(i)} - c_{M,y} h_{M,z,t}^{(i)}\right)^2 \\ h_{M,y,t+1}^{(i)} &= w_{M,y} + b_{M,y} h_{M,y,t}^{(i)} + \frac{a_{M,y}}{h_{M,z,t}^{(i)}} \left(z_{M,t}^{(i)} - c_{M,y} h_{M,z,t}^{(i)}\right)^2 \end{aligned}$$

where $z_{M,t}^{(i)} = R_{M,t} - m_{M,t}^{(i)}$.³⁵

5. From normalized importance weights, compute the filtered variables

$$\begin{aligned} \tilde{z}_{M,t} &= \sum_{i=1}^N z_{M,t}^{(i)} \omega_t^{(i)}, & \tilde{h}_{M,z,t+1} &= \sum_{i=1}^N h_{M,z,t+1}^{(i)} \omega_t^{(i)}, \\ \tilde{y}_{M,t} &= \sum_{i=1}^N y_{M,t}^{(i)} \omega_t^{(i)}, & \tilde{h}_{M,y,t+1} &= \sum_{i=1}^N h_{M,y,t+1}^{(i)} \omega_t^{(i)}. \end{aligned}$$

6. Resample the particles using the continuous sampling of Malik and Pitt (2011).³⁶

(a) Draw N particles from the current particle set from a smoothed empirical cdf as proposed in Malik and Pitt (2011) and let $\{h_{M,z,t+1}^{(j_i)}\}_{i=1}^N$ and $\{h_{M,y,t+1}^{(j_i)}\}_{i=1}^N$ denotes the resulting conditional variances and the jump intensity variables once the resampling is accomplished.³⁷

³⁵For the stock,

$$\begin{aligned} h_{S,z,t+1}^{(i)} &= \kappa_{S,z} \tilde{h}_{M,z,t+1} + b_{S,z} \left(h_{S,z,t}^{(i)} - \kappa_{S,z} \tilde{h}_{M,z,t}\right) + a_{S,z} \left(\left(h_{S,z,t}^{(i)}\right)^{-1} \left(z_{S,t}^{(i)}\right)^2 - 1 - 2c_{S,y} z_{S,t}^{(i)}\right) \\ h_{S,y,t+1}^{(i)} &= \kappa_{S,y} \tilde{h}_{M,y,t+1} + b_{S,y} \left(h_{S,y,t}^{(i)} - \kappa_{S,y} \tilde{h}_{M,y,t}\right) + a_{S,y} \left(\left(h_{S,z,t}^{(i)}\right)^{-1} \left(z_{S,t}^{(i)}\right)^2 - 1 - 2c_{S,y} z_{S,t}^{(i)}\right) \end{aligned}$$

where $z_{S,t}^{(i)} = R_{S,t} - m_{S,t}^{(i)}$.

³⁶As argued in Creal (2012), basic resampling methods are ill-suited for maximum likelihood estimation.

³⁷Note that when the number of resampled particles is small, we use importance sampling to increase it. To this end, the jump intensity variable is artificially increased and a weight correction is applied accordingly.

(b) Replace the current conditional variance and jump intensity with their resampled values:

$$h_{M,z,t+1}^{(i)} \leftarrow h_{M,z,t+1}^{(j_i)}, \quad \text{and} \quad h_{M,y,t+1}^{(i)} \leftarrow h_{M,y,t+1}^{(j_i)}.$$

The log-likelihood is obtained as a by-product of the particle filter. Indeed,

$$L_{M,\text{returns}}(\Theta_M) = \sum_{t=1}^T \log \left(\sum_{i=1}^N \bar{\omega}_t^{(i)} \right).$$

G Stock Fundamentals

The market and the volatility betas are obtained by regressing a stock's excess returns on the S&P 500 excess returns and daily changes on the VIX using the past year of data:

$$r_{S,t-k} = \alpha + \beta_{\text{MKT},t} \text{MKT}_{t-k} + \beta_{\Delta \text{VIX},t} \Delta \text{VIX}_{t-k} + \epsilon_{t-k}, \quad k = 0, \dots, 252.$$

The betas are considered missing if less than 63 data points are available over the past year.

The market equity (ME) is obtained by multiplying the number of outstanding shares by the close price for each stock.

The book equity (BE) is computed as the difference between the total assets of a firm (ATQ in Compustat) and its liabilities. The latter are defined as the sum of the debt in current liabilities (DLCQ) and half of the long-term debt (DLTTQ) as in Bharath and Shumway (2008). Both the debt in current liabilities and the long-term debt are linearly interpolated between quarterly data points to obtain daily estimates. BE is considered missing when negative.

The operating profitability (OP) is defined as the quarterly revenue at time t (REVTQ), minus the cost of goods sold at time t (COGSQ), the interest expense at time t (XINTQ), and selling, general, and administrative expenses at time t (XSGAQ), divided by book equity for the last year (i.e. at t minus 1 year). All the fundamental values used to compute OP were linearly interpolated from quarterly data.

The investment level is obtained from the book value of assets. Specifically, it is computed as the change in total assets over the previous year (from t minus 1 year to t), divided by the total assets at the end of the previous year (i.e. at time t minus 1 year). The values of the assets are also linearly interpolated from quarterly data to obtain daily estimates.

Finally, the trailing twelve-month return is obtained by taking the sum of daily excess returns over

the last year (i.e. 252 previous business days, when available).

ONLINE APPENDIX

(not part of the paper)

OA NIG

The jump $y_{u,t+1}$ have a NIG distribution with location parameter 0, a scale parameter $h_{u,y,t+1}$, an asymmetry parameter δ_u and a tail heaviness parameter α_u . The first standardized moments are

$$\mathbb{E}_t^{\mathbb{P}} [y_{u,t+1}] = \frac{\delta_u}{\sqrt{\alpha_u^2 - \delta_u^2}} h_{u,y,t+1}, \quad \text{Var}_t^{\mathbb{P}} [y_{u,t+1}] = \frac{\alpha_u^2}{(\sqrt{\alpha_u^2 - \delta_u^2})^3} h_{u,y,t+1}, \quad \text{Skew}_t^{\mathbb{P}} [y_{u,t+1}] = \frac{3\delta_u}{\alpha_u (\alpha_u^2 - \delta_u^2)^{\frac{1}{4}}} \frac{1}{\sqrt{h_{u,y,t+1}}}$$

and the excess kurtosis is

$$\text{ExKurt}_t^{\mathbb{P}} [y_{u,t+1}] = 3 \left(1 + \frac{4\delta_u^2}{\alpha_u^2} \right) \frac{1}{\sqrt{\alpha_u^2 - \delta_u^2}} \frac{1}{h_{u,y,t+1}}.$$

The moment generating function is

$$\varphi_{y_{u,t+1}}(\phi) = \exp \left(\left(\sqrt{\alpha_u^2 - \delta_u^2} - \sqrt{\alpha_u^2 - (\delta_u + \phi)^2} \right) h_{u,y,t+1} \right)$$

OA.1 Interpretation of the Jump Intensity Parameter

For comparison, let N_{t+1} be a Poisson random variable of intensity λ_{t+1} , and consider the compound Poisson random variable $\sum_{j=0}^{N_t} J_j$ where the jumps J_j are independent NIG(0, h' , δ' , α') random variables.

The moment generating function of $\sum_{j=0}^{N_t} J_j$ is

$$\begin{aligned} \varphi_{\sum_{j=0}^{N_t} J_j}(\phi) &= \exp(-\lambda_t) + \sum_{j=1}^{\infty} \exp \left(j \left(\sqrt{(\alpha')^2 - (\delta')^2} - \sqrt{(\alpha')^2 - (\delta' + \phi)^2} \right) h' \right) \exp(-\lambda_t) \frac{\lambda_t^j}{j!} \\ &= \exp(-\lambda_t) \sum_{j=0}^{\infty} \left[\lambda_t \exp \left(\left(\sqrt{(\alpha')^2 - (\delta')^2} - \sqrt{(\alpha')^2 - (\delta' + \phi)^2} \right) h' \right) \right]^j \frac{1}{j!} \\ &= \exp \left(\lambda_t \left[\exp \left(\left(\sqrt{(\alpha')^2 - (\delta')^2} - \sqrt{(\alpha')^2 - (\delta' + \phi)^2} \right) h' \right) - 1 \right] \right) \\ &\cong \exp \left(\lambda_t \left(\sqrt{(\alpha')^2 - (\delta')^2} - \sqrt{(\alpha')^2 - (\delta' + \phi)^2} \right) h' \right) \end{aligned}$$

where the last approximation holds from a first order Taylor expansion, provided that h' is close to zero.

Letting $\alpha' = \alpha_u^2$, and $\delta' = \delta_u$, a direct comparison between $\varphi_{\sum_{j=0}^{N_t} J_j}(\phi)$ and $\varphi_{y_{u,t+1}}(\phi)$ implies that

$$h_{u,y,t+1} \cong \lambda_t h',$$

that is $h_{u,y,t+1}$ may be interpreted as a scaled version of the jump intensity.

OA.2 Returns' Conditional Moments

The conditional moment generation function of $az_{M,t+1} + by_{M,t+1} + c(z_{S,t+1} + y_{S,t+1})$ is

$$\begin{aligned} & \mathbb{E}_t^{\mathbb{P}} [\exp(\phi (az_{M,t+1} + by_{M,t+1} + c(z_{S,t+1} + y_{S,t+1})))] \\ &= \mathbb{E}_t^{\mathbb{P}} [\exp(\phi az_{M,t+1})] \mathbb{E}_t^{\mathbb{P}} [\exp(\phi by_{M,t+1})] \mathbb{E}_t^{\mathbb{P}} [\exp(\phi cz_{S,t+1})] \mathbb{E}_t^{\mathbb{P}} [\exp(\phi cy_{S,t+1})] \\ &= \exp\left(\frac{a^2\phi^2}{2} h_{M,z,t} + \Pi_M(b\phi) h_{M,y,t} + \frac{c^2\phi^2}{2} h_{S,z,t} + \Pi_S(c\phi) h_{S,y,t}\right) \end{aligned}$$

where

$$\Pi_u(\phi) = \sqrt{\alpha_u^2 - \delta_u^2} - \sqrt{\alpha_u^2 - (\delta_u + \phi)^2}.$$

Note that

$$\begin{aligned} \frac{\partial \Pi_u}{\partial \phi}(\phi) &= \frac{(\delta_u + \phi)}{\sqrt{\alpha_u^2 - (\delta_u + \phi)^2}} & \frac{\partial^2 \Pi_u}{\partial \phi^2}(\phi) &= \frac{\alpha_u^2}{(\alpha_u^2 - (\delta_u + \phi)^2)^{\frac{3}{2}}} \\ \frac{\partial^3 \Pi_u}{\partial \phi^3}(\phi) &= 3 \frac{\alpha_u^2(\delta_u + \phi)}{(\alpha_u^2 - (\delta_u + \phi)^2)^{\frac{5}{2}}} & \frac{\partial^4 \Pi_u}{\partial \phi^4}(\phi) &= 3\alpha_u^2 \frac{\alpha_u^2 + 4(\delta_u + \phi)^2}{(\alpha_u^2 - (\delta_u + \phi)^2)^{\frac{7}{2}}} \end{aligned}$$

The cumulant generating function is therefore

$$\xi(\phi; a, b, c) = \frac{a^2\phi^2}{2} h_{M,z,t} + \Pi_M(b\phi) h_{M,y,t} + \frac{c^2\phi^2}{2} h_{S,z,t} + \Pi_S(c\phi) h_{S,y,t}$$

Note that

$$\begin{aligned}
\frac{\partial \xi}{\partial \phi}(\phi; a, b, c) &= a^2 \phi h_{M,z,t} + b \frac{\partial \Pi_M}{\partial \phi}(b\phi) h_{M,y,t} + c^2 \phi h_{S,z,t} + c \frac{\partial \Pi_S}{\partial \phi}(c\phi) h_{S,y,t}, \\
\frac{\partial^2 \xi}{\partial \phi^2}(\phi; a, b, c) &= a^2 h_{M,z,t} + b^2 \frac{\partial^2 \Pi_M}{\partial \phi^2}(b\phi) h_{M,y,t} + c^2 h_{S,z,t} + c^2 \frac{\partial^2 \Pi_S}{\partial \phi^2}(c\phi) h_{S,y,t}, \\
\frac{\partial^3 \xi}{\partial \phi^3}(\phi; a, b, c) &= b^3 \frac{\partial^3 \Pi_M}{\partial \phi^3}(b\phi) h_{M,y,t} + c^3 \frac{\partial^3 \Pi_S}{\partial \phi^3}(c\phi) h_{S,y,t}, \\
\frac{\partial^4 \xi}{\partial \phi^4}(\phi; a, b, c) &= b^4 \frac{\partial^4 \Pi_M}{\partial \phi^4}(b\phi) h_{M,y,t} + c^4 \frac{\partial^4 \Pi_S}{\partial \phi^4}(c\phi) h_{S,y,t}.
\end{aligned}$$

The first moment of the market and stock returns are

$$\begin{aligned}
E_t^{\mathbb{P}}[R_{M,t+1}] &= \mu_{M,t+1}^{\mathbb{P}} - \xi_{M,t+1}^{\mathbb{P}} + \frac{\partial \xi}{\partial \phi}(0; 1, 1, 0), \\
E_t^{\mathbb{P}}[R_{S,t+1}] &= \mu_{S,t+1}^{\mathbb{P}} - \xi_{S,t+1}^{\mathbb{P}} + \frac{\partial \xi}{\partial \phi}(0; \beta_{S,z}, \beta_{S,y}, 1).
\end{aligned}$$

Their variances correspond to

$$\begin{aligned}
\text{Var}_t^{\mathbb{P}}[R_{M,t+1}] &= \text{Var}_t^{\mathbb{P}}[z_{M,t+1} + y_{M,t+1}] = \frac{\partial^2 \xi}{\partial \phi^2}(0; 1, 1, 0) \\
\text{Var}_t^{\mathbb{P}}[R_{S,t+1}] &= \text{Var}_t^{\mathbb{P}}[\beta_{S,z} z_{M,t+1} + \beta_{S,y} y_{M,t+1} + z_{S,t+1} + y_{S,t+1}] = \frac{\partial^2 \xi}{\partial \phi^2}(0; \beta_{S,z}, \beta_{S,y}, 1).
\end{aligned}$$

Similarly, since the third cumulant corresponds to the third centered moment, the third standardized moment are respectively

$$\text{Skew}_t^{\mathbb{P}}[R_{M,t+1}] = \frac{\frac{\partial^3 \xi}{\partial \phi^3}(0; 1, 1, 0)}{\left(\frac{\partial^2 \xi}{\partial \phi^2}(0; 1, 1, 0)\right)^{\frac{3}{2}}} \text{ and } \text{Skew}_t^{\mathbb{P}}[R_{S,t+1}] = \frac{\frac{\partial^3 \xi}{\partial \phi^3}(0; \beta_{S,z}, \beta_{S,y}, 1)}{\left(\frac{\partial^2 \xi}{\partial \phi^2}(0; \beta_{S,z}, \beta_{S,y}, 1)\right)^{\frac{3}{2}}}.$$

Finally, the excess kurtosis are

$$\begin{aligned} \mathbb{E}_t^{\mathbb{P}} \left[\left(\frac{R_{M,t+1} - \mathbb{E}_t^{\mathbb{P}} [R_{M,t+1}]}{\sqrt{\text{Var}_t^{\mathbb{P}} [R_{M,t+1}]}} \right)^4 \right] - 3 &= \frac{\frac{\partial^4 \xi}{\partial \phi^4} (0; 1, 1, 0)}{\left(\frac{\partial^2 \xi}{\partial \phi^2} (0; 1, 1, 0) \right)^2}, \\ \mathbb{E}_t^{\mathbb{P}} \left[\left(\frac{R_{M,t+1} - \mathbb{E}_t^{\mathbb{P}} [R_{M,t+1}]}{\sqrt{\text{Var}_t^{\mathbb{P}} [R_{M,t+1}]}} \right)^4 \right] - 3 &= \frac{\frac{\partial^4 \xi}{\partial \phi^4} (0; \beta_{S,z}, \beta_{S,y}, 1)}{\left(\frac{\partial^2 \xi}{\partial \phi^2} (0; \beta_{S,z}, \beta_{S,y}, 1) \right)^2}. \end{aligned}$$

OA.3 Conditional Variance and Jump intensity Variable Moments

Lemma 6

$$\begin{aligned} \text{Var}_{t-1}^{\mathbb{P}} [h_{M,z,t+1}] &= 2a_{M,z}^2 (1 + 2c_{M,z}^2 h_{M,z,t}), \\ \text{Var}_{t-1}^{\mathbb{P}} [h_{M,y,t+1}] &= 2a_{M,y}^2 (1 + 2c_{M,y}^2 h_{M,y,t}), \\ \text{Var}_{t-1}^{\mathbb{P}} [h_{S,z,t+1}] &= \kappa_{S,z}^2 \text{Var}_{t-1}^{\mathbb{P}} [h_{M,z,t+1}] + 2a_{S,z}^2 (1 + 2c_{S,z}^2 h_{S,z,t}), \\ \text{Var}_{t-1}^{\mathbb{P}} [h_{S,y,t+1}] &= \kappa_{S,y}^2 \text{Var}_{t-1}^{\mathbb{P}} [h_{M,y,t+1}] + 2a_{S,y}^2 (1 + 2c_{S,y}^2 h_{S,y,t}). \end{aligned}$$

Proof. Recall that the market conditional variance is

$$h_{M,z,t+1} = w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} (\varepsilon_{M,t} - c_{M,z} \sqrt{h_{M,z,t}})^2.$$

Therefore, $\mathbb{E}_{t-1}^{\mathbb{P}} [h_{M,z,t+1}] = w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} (1 + c_{M,z}^2 h_{M,z,t})$ and

$$\begin{aligned} \text{Var}_{t-1}^{\mathbb{P}} [h_{M,z,t+1}] &= a_{M,z}^2 \mathbb{E}_{t-1}^{\mathbb{P}} \left[\left((\varepsilon_{M,t} - c_{M,z} \sqrt{h_{M,z,t}})^2 - (1 + c_{M,z}^2 h_{M,z,t}) \right)^2 \right] \\ &= a_{M,z}^2 \mathbb{E}_{t-1}^{\mathbb{P}} \left[\varepsilon_{M,t}^4 - 4c_{M,z} \sqrt{h_{M,z,t}} \varepsilon_{M,t}^3 + 2(2c_{M,z}^2 h_{M,z,t} - 1) \varepsilon_{M,t}^2 + 4c_{M,z} \sqrt{h_{M,z,t}} \varepsilon_{M,t} + 1 \right] \\ &= a_{M,z}^2 (3 + 2(2c_{M,z}^2 h_{M,z,t} - 1) + 1) \\ &= 2a_{M,z}^2 (1 + 2c_{M,z}^2 h_{M,z,t}). \end{aligned}$$

The market jump scale parameter is

$$h_{M,y,t+1} = w_{M,y} + b_{M,y} h_{M,y,t} + a_{M,y} (\varepsilon_{M,t} - c_{M,y} \sqrt{h_{M,y,t}})^2.$$

Hence $E_{t-1}^{\mathbb{P}} [h_{M,y,t+1}] = w_{M,y} + b_{M,y}h_{M,y,t} + a_{M,y}(1 + c_{M,y}^2 h_{M,z,t})$ and

$$\begin{aligned} \text{Var}_{t-1}^{\mathbb{P}} [h_{M,y,t+1}] &= a_{M,y}^2 E_{t-1}^{\mathbb{P}} \left[\left((\varepsilon_{M,t} - c_{M,y} \sqrt{h_{M,z,t}})^2 - (1 + c_{M,y}^2 h_{M,z,t}) \right)^2 \right] \\ &= 2a_{M,y}^2 (1 + 2c_{M,y}^2 h_{M,z,t}). \end{aligned}$$

The stock conditional variance satisfies

$$h_{S,z,t+1} = \kappa_{S,z} h_{M,z,t+1} + b_{S,z} (h_{S,z,t} - \kappa_{S,z} h_{M,z,t}) + a_{S,z} (\varepsilon_{S,t}^2 - 1 - 2c_{S,z} \sqrt{h_{S,z,t}} \varepsilon_{S,t}).$$

Therefore, $E_{t-1}^{\mathbb{P}} [h_{S,z,t+1}] = \kappa_{S,z} E_{t-1}^{\mathbb{P}} [h_{M,z,t+1}] + b_{S,z} (h_{S,z,t} - \kappa_{S,z} h_{M,z,t})$ and

$$\begin{aligned} \text{Var}_{t-1}^{\mathbb{P}} [h_{S,z,t+1}] &= E_{t-1}^{\mathbb{P}} \left[\left(\kappa_{S,z} (h_{M,z,t+1} - E_{t-1}^{\mathbb{P}} [h_{M,z,t+1}]) + a_{S,z} (\varepsilon_{S,t}^2 - 1 - 2c_{S,z} \sqrt{h_{S,z,t}} \varepsilon_{S,t}) \right)^2 \right] \\ &= \kappa_{S,z}^2 E_{t-1}^{\mathbb{P}} \left[(h_{M,z,t+1} - E_{t-1}^{\mathbb{P}} [h_{M,z,t+1}])^2 \right] + a_{S,z}^2 E_{t-1}^{\mathbb{P}} \left[(\varepsilon_{S,t}^2 - 1 - 2c_{S,z} \sqrt{h_{S,z,t}} \varepsilon_{S,t})^2 \right] \\ &\quad + 2\kappa_{S,z} a_{S,z} E_{t-1}^{\mathbb{P}} \left[(h_{M,z,t+1} - E_{t-1}^{\mathbb{P}} [h_{M,z,t+1}]) (\varepsilon_{S,t}^2 - 1 - 2c_{S,z} \sqrt{h_{S,z,t}} \varepsilon_{S,t}) \right] \\ &= \kappa_{S,z}^2 \text{Var}_{t-1}^{\mathbb{P}} [h_{M,z,t+1}] + 2a_{S,z}^2 (1 + 2c_{S,z}^2 h_{S,z,t}) \\ &\quad + 2\kappa_{S,z} a_{S,z} E_{t-1}^{\mathbb{P}} \left[(\varepsilon_{M,t}^2 - 1 - 2c_{M,z} \sqrt{h_{M,z,t}} \varepsilon_{M,t}) (\varepsilon_{S,t}^2 - 1 - 2c_{S,z} \sqrt{h_{S,z,t}} \varepsilon_{S,t}) \right] \\ &= \kappa_{S,z}^2 \text{Var}_{t-1}^{\mathbb{P}} [h_{M,z,t+1}] + 2a_{S,z}^2 (1 + 2c_{S,z}^2 h_{S,z,t}) \\ &\quad + 2\kappa_{S,z} a_{S,z} \left(E_{t-1}^{\mathbb{P}} [(\varepsilon_{M,t}^2 - 1)] E_{t-1}^{\mathbb{P}} [(\varepsilon_{S,t}^2 - 1)] - 2c_{S,z} \sqrt{h_{S,z,t}} E_{t-1}^{\mathbb{P}} [(\varepsilon_{M,t}^2 - 1)] E_{t-1}^{\mathbb{P}} [\varepsilon_{S,t}] \right. \\ &\quad \left. - 2c_{M,z} \sqrt{h_{M,z,t}} E_{t-1}^{\mathbb{P}} [(\varepsilon_{M,t}) (\varepsilon_{S,t}^2 - 1)] E_{t-1}^{\mathbb{P}} [(\varepsilon_{M,t}) (\varepsilon_{S,t}^2 - 1)] \right. \\ &\quad \left. + 4c_{M,z} c_{S,z} \sqrt{h_{M,z,t}} \sqrt{h_{S,z,t}} E_{t-1}^{\mathbb{P}} [\varepsilon_{M,t}] E_{t-1}^{\mathbb{P}} [\varepsilon_{S,t}] \right) \\ &= \kappa_{S,z}^2 \text{Var}_{t-1}^{\mathbb{P}} [h_{M,z,t+1}] + 2a_{S,z}^2 (1 + 2c_{S,z}^2 h_{S,z,t}). \end{aligned}$$

Finally,

$$h_{S,y,t+1} = \kappa_{S,y} h_{M,y,t+1} + b_{S,y} (h_{S,y,t} - \kappa_{S,y} h_{M,y,t}) + a_{S,y} (\varepsilon_{S,t}^2 - 1 - 2c_{S,y} \sqrt{h_{S,y,t}} \varepsilon_{S,t})$$

implies that $E_{t-1}^{\mathbb{P}} [h_{S,y,t+1}] = \kappa_{S,y} E_{t-1}^{\mathbb{P}} [h_{M,y,t+1}] + b_{S,y} (h_{S,y,t} - \kappa_{S,y} h_{M,y,t})$ and

$$\begin{aligned} \text{Var}_{t-1}^{\mathbb{P}} [h_{S,y,t+1}] &= E_{t-1}^{\mathbb{P}} \left[\left(\kappa_{S,y} (h_{M,y,t+1} - E_{t-1}^{\mathbb{P}} [h_{M,y,t+1}]) + a_{S,y} (\varepsilon_{S,t}^2 - 1 - 2c_{S,y} \sqrt{h_{S,y,t}} \varepsilon_{S,t}) \right)^2 \right] \\ &= \kappa_{S,y}^2 \text{Var}_{t-1}^{\mathbb{P}} [h_{M,y,t+1}] + 2a_{S,y}^2 (1 + 2c_{S,y}^2 h_{S,y,t}). \end{aligned}$$

OB Detailed Proofs of Appendix B's results

Proof of Lemma 1. The conditional cumulant generating function of $\varepsilon_{u,t}$ under \mathbb{Q} is

$$\begin{aligned}
\xi_{\varepsilon_{u,t}}^{\mathbb{Q}}(\phi) &= \log E_{t-1}^{\mathbb{Q}} [\exp(\phi \varepsilon_{u,t})] \\
&= \log E_{t-1}^{\mathbb{P}} \left[\frac{\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})}{E_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})]} \exp(\phi \varepsilon_{u,t}) \right] \\
&= \log E_{t-1}^{\mathbb{P}} \left[\frac{\exp(-\Lambda_u z_{u,t})}{E_{t-1}^{\mathbb{P}} [\exp(-\Lambda_u z_{u,t})]} \exp(\phi \varepsilon_{u,t}) \right] \\
&= \xi_{\varepsilon_{u,t}}^{\mathbb{P}}(\phi - \Lambda_u \sqrt{h_{u,z,t}}) - \xi_{\varepsilon_{u,t}}^{\mathbb{P}}(-\Lambda_u \sqrt{h_{u,z,t}}) \\
&= \left(\frac{1}{2} (\phi - \Lambda_u \sqrt{h_{u,z,t}})^2 - \frac{1}{2} (-\Lambda_u \sqrt{h_{u,z,t}})^2 \right) \\
&= \left(\frac{1}{2} \phi^2 - \Lambda_u \sqrt{h_{u,z,t}} \phi \right).
\end{aligned}$$

■

Proof of Lemma 2. For any $y_{u,t} \in \{y_{M,t}, y_{S,t} : S \in \mathbb{S}\}$, the conditional cumulant generating function of $y_{u,t}$ under \mathbb{Q} is

$$\begin{aligned}
\xi_{y_{u,t}}^{\mathbb{Q}}(\phi) &= \log E_{t-1}^{\mathbb{Q}} [\exp(\phi y_{u,t})] \\
&= \log E_{t-1}^{\mathbb{P}} \left[\frac{\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})}{E_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})]} \exp(\phi y_{u,t}) \right] \\
&= \log E_{t-1}^{\mathbb{P}} \left[\frac{\exp(-\Gamma_u y_{u,t})}{E_{t-1}^{\mathbb{P}} [\exp(-\Gamma_u y_{u,t})]} \exp(\phi y_{u,t}) \right] \\
&= \xi_{y_{u,t}}^{\mathbb{P}}(\phi - \Gamma_u) - \xi_{y_{u,t}}^{\mathbb{P}}(-\Gamma_u) \\
&= \Pi_u(\phi - \Gamma_u) h_{u,y,t} - \Pi_u(-\Gamma_u) h_{u,y,t} \\
&= \left(\sqrt{\alpha_u^2 - (\delta_u - \Gamma_u)^2} - \sqrt{\alpha_u^2 - (\delta_u + \phi - \Gamma_u)^2} \right) h_{u,y,t}
\end{aligned}$$

which is the cumulant generating function of a NIG of parameter $\mu_u^* = \mu_u = 0$, $\alpha_u^* = \alpha_u$, $\delta_u^* = \delta_u - \Gamma_u$ and $h_{u,y,t}^* = h_{u,y,t}$.

OC Detailed Proofs of Appendix C's results

OC.1 Market Drift under \mathbb{P}

Recall that

$$\ln \frac{M_t}{M_{t-1}} = R_{M,t} = \mu_{M,t}^{\mathbb{P}} - \xi_{M,t}^{\mathbb{P}} + z_{M,t} + y_{M,t}$$

Since the discounted stock price should behave as a \mathbb{Q} -martingale,

$$\begin{aligned} 1 &= \mathbb{E}_{t-1}^{\mathbb{Q}} \left[\frac{\exp(-r_t) M_t}{M_{t-1}} \right] \\ &= \mathbb{E}_{t-1}^{\mathbb{Q}} [\exp(R_{M,t} - r_t)] \\ &= \mathbb{E}_{t-1}^{\mathbb{P}} \left[\frac{\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})}{\mathbb{E}_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})]} \exp(R_{M,t} - r_t) \right] \\ &= \mathbb{E}_{t-1}^{\mathbb{P}} \left[\frac{\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t})}{\mathbb{E}_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t})]} \exp(\mu_{M,t}^{\mathbb{P}} - \xi_{M,t}^{\mathbb{P}} + z_{M,t} + y_{M,t} - r_t) \right] \\ &= \mathbb{E}_{t-1}^{\mathbb{P}} \left[\frac{\exp(\mu_{M,t}^{\mathbb{P}} - r_t - \xi_{M,t}^{\mathbb{P}} + (1 - \Lambda_M) z_{M,t} + (1 - \Gamma_M) y_{M,t})}{\mathbb{E}_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t})]} \right] \\ &= \exp \left(\begin{aligned} &\mu_{M,t}^{\mathbb{P}} - r_t - \xi_{M,t}^{\mathbb{P}} (1) - \xi_{y_{M,t}}^{\mathbb{P}} (1) \\ &+ \xi_{z_{M,t}}^{\mathbb{P}} (1 - \Lambda_M) + \xi_{y_{M,t}}^{\mathbb{P}} (1 - \Gamma_M) - \xi_{z_{M,t}}^{\mathbb{P}} (-\Lambda_M) - \xi_{y_{M,t}}^{\mathbb{P}} (-\Gamma_M) \end{aligned} \right). \end{aligned}$$

Because,

$$-\xi_{z_{M,t}}^{\mathbb{P}} (1) + \xi_{z_{M,t}}^{\mathbb{P}} (1 - \Lambda_M) - \xi_{z_{M,t}}^{\mathbb{P}} (-\Lambda_M) = -\frac{1}{2} h_{M,z,t} + \frac{1}{2} h_{M,z,t} (1 - \Lambda_M)^2 - \frac{1}{2} h_{M,z,t} \Lambda_M^2 = -\Lambda_M h_{M,z,t}$$

and

$$-\xi_{y_{M,t}}^{\mathbb{P}} (1) + \xi_{y_{M,t}}^{\mathbb{P}} (1 - \Gamma_M) - \xi_{y_{M,t}}^{\mathbb{P}} (-\Gamma_M) = -h_{M,y,t} \gamma_M$$

where

$$\begin{aligned} \gamma_M &= \sqrt{\alpha_M^2 - \delta_M^2} - \sqrt{\alpha_M^2 - (\delta_M + 1)^2} + \sqrt{\alpha_M^2 - (\delta_M + 1 - \Gamma_M)^2} - \sqrt{\alpha_M^2 - (\delta_M - \Gamma_M)^2} \\ &= \Pi_M(1) - \Pi_M^*(1), \end{aligned}$$

we conclude that

$$1 = \exp(\mu_{M,t}^{\mathbb{P}} - r_t - \Lambda_M h_{M,z,t} - h_{M,y,t} \gamma_M).$$

Therefore,

$$\mu_{M,t}^{\mathbb{P}} = r_t + \Lambda_M h_{M,z,t} + \gamma_M h_{M,y,t}.$$

OC.2 Stock Drift under \mathbb{P}

Recall that

$$\ln \frac{S_t}{S_{t-1}} = R_{S,t} = \mu_{S,t}^{\mathbb{P}} - \xi_{S,t}^{\mathbb{P}} + \beta_{S,z} z_{M,t} + \beta_{S,y} y_{M,t} + z_{S,t} + y_{S,t}$$

Since the discounted stock price should behave as a \mathbb{Q} -martingale,

$$\begin{aligned} 1 &= \mathbb{E}_{t-1}^{\mathbb{Q}} \left[\frac{\exp(-r_t) S_t}{S_{t-1}} \right] \\ &= \mathbb{E}_{t-1}^{\mathbb{Q}} [\exp(R_{S,t} - r_t)] \\ &= \mathbb{E}_{t-1}^{\mathbb{P}} \left[\frac{\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})}{\mathbb{E}_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})]} \exp(R_{S,t} - r_t) \right] \\ &= \mathbb{E}_{t-1}^{\mathbb{P}} \left[\frac{\frac{\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})}{\mathbb{E}_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S y_{S,t})]}}{\exp(\mu_{S,t}^{\mathbb{P}} - \xi_{S,t}^{\mathbb{P}} + \beta_{S,z} z_{M,t} + \beta_{S,y} y_{M,t} + z_{S,t} + y_{S,t} - r_t)} \right] \\ &= \mathbb{E}_{t-1}^{\mathbb{P}} \left[\frac{\exp(\mu_{S,t}^{\mathbb{P}} - r_t - \xi_{S,t}^{\mathbb{P}} + (\beta_{S,z} - \Lambda_M) z_{M,t} + (\beta_{S,y} - \Gamma_M) y_{M,t} + (1 - \Lambda_S) z_{S,t} + (1 - \Gamma_S) y_{S,t})}{\mathbb{E}_{t-1}^{\mathbb{P}} [\exp(-\Lambda_M z_{M,t} - \Gamma_M y_{M,t} - \Lambda_S z_{S,t} - \Gamma_S y_{S,t})]} \right] \\ &= \exp \left(\begin{aligned} &\mu_{S,t}^{\mathbb{P}} - r_t - \xi_{z_{M,t}}^{\mathbb{P}} (\beta_{S,z}) - \xi_{y_{M,t}}^{\mathbb{P}} (\beta_{S,y}) - \xi_{z_{S,t}}^{\mathbb{P}} (1) - \xi_{y_{S,t}}^{\mathbb{P}} (1) \\ &+ \xi_{z_{M,t}}^{\mathbb{P}} (\beta_{S,z} - \Lambda_M) + \xi_{y_{M,t}}^{\mathbb{P}} (\beta_{S,y} - \Gamma_M) + \xi_{z_{S,t}}^{\mathbb{P}} (1 - \Lambda_S) + \xi_{y_{S,t}}^{\mathbb{P}} (1 - \Gamma_S) \\ &- \xi_{z_{M,t}}^{\mathbb{P}} (-\Lambda_M) - \xi_{y_{M,t}}^{\mathbb{P}} (-\Gamma_M) - \xi_{z_{S,t}}^{\mathbb{P}} (-\Lambda_S) - \xi_{y_{S,t}}^{\mathbb{P}} (-\Gamma_S) \end{aligned} \right). \end{aligned}$$

Because,

$$\begin{aligned} -\xi_{z_{M,t}}^{\mathbb{P}} (\beta_{S,z}) + \xi_{z_{M,t}}^{\mathbb{P}} (\beta_{S,z} - \Lambda_M) - \xi_{z_{M,t}}^{\mathbb{P}} (-\Lambda_M) &= -\frac{1}{2} h_{M,z,t} \beta_{S,z}^2 + \frac{1}{2} h_{M,z,t} (\beta_{S,z} - \Lambda_M)^2 - \frac{1}{2} h_{M,z,t} \Lambda_M^2 \\ &= -\Lambda_M \beta_{S,z} h_{M,z,t}, \\ -\xi_{z_{S,t}}^{\mathbb{P}} (1) + \xi_{z_{S,t}}^{\mathbb{P}} (1 - \Lambda_S) - \xi_{z_{S,t}}^{\mathbb{P}} (-\Lambda_S) &= -\frac{1}{2} h_{S,z,t} + \frac{1}{2} h_{S,z,t} (1 - \Lambda_S)^2 - \frac{1}{2} h_{S,z,t} \Lambda_S^2 \\ &= -\Lambda_S h_{S,z,t}, \\ -\xi_{y_{M,t}}^{\mathbb{P}} (\beta_{S,y}) + \xi_{y_{M,t}}^{\mathbb{P}} (\beta_{S,y} - \Gamma_M) - \xi_{y_{M,t}}^{\mathbb{P}} (-\Gamma_M) &= (-\Pi_M (\beta_{S,y}) + \Pi_M (\beta_{S,y} - \Gamma_M) - \Pi_M (-\Gamma_M)) h_{M,y,t} \\ &= \Pi_M^* (\beta_{S,y}) - \Pi_M (\beta_{S,y}) = -\gamma_{M,S} (\beta_{S,y}) h_{M,y,t} \\ -\xi_{y_{S,t}}^{\mathbb{P}} (1) + \xi_{y_{S,t}}^{\mathbb{P}} (1 - \Gamma_S) - \xi_{y_{S,t}}^{\mathbb{P}} (-\Gamma_S) &= \Pi_S^* (1) - \Pi_S (1) = -\gamma_S h_{S,y,t}, \end{aligned}$$

we conclude that

$$1 = \exp\left(\mu_{S,t}^{\mathbb{P}} - r_t - \Lambda_M \beta_{S,z} h_{M,z,t} - h_{M,y,t} \gamma_{M,S}(\beta_{S,y}) - \Lambda_S h_{S,z,t} - h_{S,y,t} \gamma_S\right).$$

where

$$\gamma_{M,S}(\beta_{S,y}) = \Pi_M(\beta_{S,y}) - \Pi_M^*(\beta_{S,y}) \text{ and } \gamma_S = \Pi_S(1) - \Pi_S^*(1).$$

Therefore,

$$\mu_{S,t}^{\mathbb{P}} = r_t + \Lambda_M \beta_{S,z} h_{M,z,t} + h_{M,y,t} \gamma_{M,S}(\beta_{S,y}) + \Lambda_S h_{S,z,t} + h_{S,y,t} \gamma_S.$$

OD Calculation Associated with Appendix D

Replacing (D.6) in the market conditional variance leads to

$$\begin{aligned} h_{M,z,t+1} &= w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} \left(\varepsilon_{M,t} - c_{M,z} \sqrt{h_{M,z,t}} \right)^2 \\ &= w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} \left(\left(\varepsilon_{M,t}^* - \Lambda_M \sqrt{h_{M,z,t}} \right) - c_{M,z} \sqrt{h_{M,z,t}} \right)^2 \\ &= w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} \left(\varepsilon_{M,t}^* - (c_{M,z} + \Lambda_M) \sqrt{h_{M,z,t}} \right)^2 \\ &= w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} \left(\left(\varepsilon_{M,t}^* \right)^2 - 2(c_{M,z} + \Lambda_M) \sqrt{h_{M,z,t}} \varepsilon_{M,t}^* + (c_{M,z} + \Lambda_M)^2 h_{M,z,t} \right) \\ &= w_{M,z} + \left(b_{M,z} + a_{M,z} (c_{M,z} + \Lambda_M)^2 \right) h_{M,z,t} + a_{M,z} \left(\varepsilon_{M,t}^* \right)^2 - 2a_{M,z} (c_{M,z} + \Lambda_M) \sqrt{h_{M,z,t}} \varepsilon_{M,t}^*. \end{aligned}$$

A similar argument leads to the stock conditional variance:

$$\begin{aligned} h_{S,z,t+1} &= \kappa_{S,z} h_{M,z,t+1} + b_{S,z} (h_{S,z,t} - \kappa_{S,z} h_{M,z,t}) + a_{S,z} \left(\varepsilon_{S,t}^2 - 1 - 2c_{S,z} \varepsilon_{S,t} \right) \\ &= \kappa_{S,z} h_{M,z,t+1}^* + b_{S,z} \left(h_{S,z,t}^* - \kappa_{S,z} h_{M,z,t}^* \right) \\ &\quad + a_{S,z} \left((2c_{S,z} + \Lambda_S) \Lambda_S h_{S,z,t}^* + \left(\left(\varepsilon_{S,t}^* \right)^2 - 1 - 2(c_{S,z} + \Lambda_S) \sqrt{h_{S,z,t}^*} \varepsilon_{S,t}^* \right) \right) \\ &= \kappa_{S,z} h_{M,z,t+1}^* + b_{S,z} \left(h_{S,z,t}^* - \kappa_{S,z} h_{M,z,t}^* \right) + a_{S,z} (2c_{S,z} + \Lambda_S) \Lambda_S h_{S,z,t}^* \\ &\quad + a_{S,z} \left(\varepsilon_{S,t}^* \right)^2 - a_{S,z} - 2a_{S,z} (c_{S,z} + \Lambda_S) \sqrt{h_{S,z,t}^*} \varepsilon_{S,t}^* \\ &= \pi_{S,z,1} + \pi_{S,z,2} h_{M,z,t}^* + 0h_{S,y,t}^* + \pi_{S,z,4} h_{S,z,t}^* + 0h_{S,y,t}^* \\ &\quad + \pi_{S,z,6} \left(\varepsilon_{M,t}^* \right)^2 + \pi_{S,z,7} \sqrt{h_{M,z,t}^*} \varepsilon_{M,t}^* + \pi_{S,z,8} \left(\varepsilon_{S,t}^* \right)^2 + \pi_{S,z,9} \sqrt{h_{S,z,t}^*} \varepsilon_{S,t}^*. \end{aligned}$$

The risk-neutral market jump scale parameter is

$$\begin{aligned}
h_{M,y,t} &= w_{M,y} + b_{M,y}h_{M,y,t} + a_{M,y}(\varepsilon_{M,t} - c_{M,y}\sqrt{h_{M,z,t}})^2 \\
&= w_{M,y} + a_{M,y}c_{M,y}^2h_{M,z,t} + b_{M,y}h_{M,y,t} + a_{M,y}(\varepsilon_{M,t}^2 - 2c_{M,y}\sqrt{h_{M,z,t}}\varepsilon_{M,t}) \\
&= \left(\begin{aligned} &w_{M,y} + a_{M,y}c_{M,y}^2h_{M,z,t} + b_{M,y}h_{M,y,t} \\ &+ a_{M,y}\left((\varepsilon_{M,t}^* - \Lambda_M\sqrt{h_{M,z,t}})^2 - 2c_{M,y}\sqrt{h_{M,z,t}}(\varepsilon_{M,t}^* - \Lambda_M\sqrt{h_{M,z,t}})\right) \end{aligned} \right) \\
&= w_{M,y} + a_{M,y}(c_{M,y} + \Lambda_M)^2h_{M,z,t} + b_{M,y}h_{M,y,t} + a_{M,y}\left((\varepsilon_{M,t}^*)^2 - 2(c_{M,y} + \Lambda_M)\sqrt{h_{M,z,t}}\varepsilon_{M,t}^*\right) \\
&= w_{M,y} + a_{M,y}(c_{M,y} + \Lambda_M)^2h_{M,z,t}^* + b_{M,y}h_{M,y,t}^* + a_{M,y}\left((\varepsilon_{M,t}^*)^2 - 2(c_{M,y} + \Lambda_M)\sqrt{h_{M,z,t}^*}\varepsilon_{M,t}^*\right) \\
&= \pi_{M,y,1} + \pi_{M,y,2}h_{M,z,t}^* + \pi_{M,y,3}h_{M,y,t}^* + 0h_{S,z,t}^* + 0h_{S,y,t}^* \\
&\quad + \pi_{M,y,6}(\varepsilon_{M,t}^*)^2 + \pi_{M,y,7}\sqrt{h_{M,z,t}^*}\varepsilon_{M,t}^* + 0(\varepsilon_{S,t}^*)^2 + 0\sqrt{h_{S,z,t}^*}\varepsilon_{S,t}^*
\end{aligned}$$

Similarly, the risk-neutral stock jump scale parameter is

$$\begin{aligned}
h_{S,y,t} &= \kappa_{S,y}h_{M,y,t+1} + b_{S,y}(h_{S,y,t} - \kappa_{S,y}h_{M,y,t}) + a_{S,y}(\varepsilon_{S,t}^2 - 1 - 2c_{S,y}\varepsilon_{S,t}) \\
&= \kappa_{S,y}h_{M,y,t+1}^* + b_{S,y}(h_{S,y,t}^* - \kappa_{S,y}h_{M,y,t}^*) \\
&\quad + a_{S,y}\left((2c_{S,y} + \Lambda_S)\Lambda_S h_{S,z,t}^* + ((\varepsilon_{S,t}^*)^2 - 1 - 2(c_{S,y} + \Lambda_S)\sqrt{h_{S,z,t}^*}\varepsilon_{S,t}^*)\right) \\
&= \kappa_{S,y}h_{M,y,t+1}^* - a_{S,y} - b_{S,y}\kappa_{S,y}h_{M,y,t}^* + b_{S,y}h_{S,y,t}^* + a_{S,y}(2c_{S,y} + \Lambda_S)\Lambda_S h_{S,z,t}^* \\
&\quad + a_{S,y}(\varepsilon_{S,t}^*)^2 - 2a_{S,y}(c_{S,y} + \Lambda_S)\sqrt{h_{S,z,t}^*}\varepsilon_{S,t}^* \\
&= \pi_{S,y,1} + \pi_{S,y,2}h_{M,z,t}^* + \pi_{S,y,3}h_{M,y,t}^* + \pi_{S,y,4}h_{S,z,t}^* + \pi_{S,y,5}h_{S,y,t}^* \\
&\quad + \pi_{S,y,6}(\varepsilon_{M,t}^*)^2 + \pi_{S,y,7}\sqrt{h_{M,z,t}^*}\varepsilon_{M,t}^* + \pi_{S,y,8}(\varepsilon_{S,t}^*)^2 + \pi_{S,y,9}\sqrt{h_{S,z,t}^*}\varepsilon_{S,t}^*
\end{aligned}$$

OE Detailed Proofs of Appendix E's results

Lemma 7 if ε represents a standard normal random variable, then

$$E[\exp(a\varepsilon^2 + b\varepsilon)] = \exp\left(-\frac{1}{2}\ln(1 - 2a) + \frac{1}{2}\frac{b^2}{(1 - 2a)}\right)$$

provided that $a < \frac{1}{2}$.

Proof of Lemma 7.

$$\begin{aligned}
& \mathbb{E}[\exp(a\varepsilon^2 + b\varepsilon)] \\
&= \int \exp(a\varepsilon^2 + b\varepsilon) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\varepsilon^2\right) d\varepsilon \\
&= \int \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(1-2a)}{2}\left(\varepsilon^2 - 2\frac{b}{(1-2a)}\varepsilon\right)\right) d\varepsilon \\
&= \exp\left(\frac{1}{2}\frac{b^2}{(1-2a)}\right) \sqrt{\frac{1}{(1-2a)}} \int \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{1}{(1-2a)}}} \exp\left(-\frac{1}{2}\frac{\left(\varepsilon - \frac{b}{(1-2a)}\right)^2}{\frac{1}{(1-2a)}}\right) d\varepsilon.
\end{aligned}$$

If $1 - 2a > 0$, then the integral is one since it corresponds to the area under the density function of a $\mathcal{N}\left(\frac{b}{(1-2a)}, \frac{1}{(1-2a)}\right)$ random variable. Hence,

$$\mathbb{E}[\exp(a\varepsilon^2 + b\varepsilon)] = \exp\left(\ln \sqrt{\frac{1}{(1-2a)}} + \frac{1}{2}\frac{b^2}{(1-2a)}\right).$$

OE.1 Moment Generating Function

For $u \in \{M, S\}$, The risk neutral returns process is

$$\begin{aligned}
\log \frac{u_{t+1}}{u_t} &= R_{u,t+1} = r_{t+1} - \xi_{u,t+1}^{\mathbb{Q}} + \beta_{u,z} z_{M,t+1}^* + \beta_{u,y} y_{M,t+1}^* + \beta'_{u,z} z_{S,t+1}^* + \beta'_{u,y} y_{S,t+1}^* \\
z_{u,t+1}^* &= \sqrt{h_{u,z,t+1}^*} \varepsilon_{u,t+1}^*, \quad \varepsilon_{u,t+1}^* \sim \mathcal{N}(0, 1) \\
y_{u,t+1}^* &\sim NIG(0, \alpha_u^*, \delta_u^2, h_{u,z,t+1}^*)
\end{aligned}$$

where the convexity correction³⁸ is

$$\xi_{u,t}^{\mathbb{Q}} = \xi_{z_{M,t}^*}^{\mathbb{Q}}(\beta_{u,z}) + \xi_{y_{M,t}^*}^{\mathbb{Q}}(\beta_{u,y}) + \xi_{z_{S,t}^*}^{\mathbb{Q}}(\beta'_{u,z}) + \xi_{y_{S,t}^*}^{\mathbb{Q}}(\beta'_{u,y}).$$

For the market case, $\beta_{M,z} = \beta_{M,y} = 1$ and $\beta'_{M,z} = \beta'_{M,y} = 0$. For the stock, $\beta'_{S,z} = \beta'_{S,y} = 1$.

³⁸Appendix B shows that

$$\xi_{z_{u,t}^*}^{\mathbb{Q}}(\phi) = \frac{\phi^2}{2} h_{u,z,t}^* \text{ and } \xi_{y_{u,t}^*}^{\mathbb{Q}}(\phi) = \Pi_u^*(\phi) h_{u,y,t}^*$$

where $\Pi_u^*(\phi) = \exp\left(\frac{1}{2}\delta_u^2\phi^2 + \alpha_u^*\phi\right)$ and $\alpha_u^* = \alpha_u - \delta_u^2\Gamma_u$.

Proof of Lemma 5. For $u \in \{M, S\}$, let $\tilde{R}_{u,t+j}$ denotes the excess return. Its risk neutral dynamics is

$$\begin{aligned}\tilde{R}_{u,t+j} &= R_{u,t+j} - r_{t+j} \\ &= -\frac{1}{2}\beta_{u,z}^2 h_{M,z,t+j}^* - \Pi_M^*(\beta_{u,y}) h_{M,y,t+j}^* - \frac{1}{2}(\beta'_{u,z})^2 h_{S,z,t+j}^* - \Pi_S^*(\beta'_{u,y}) h_{S,y,t+j}^* \\ &\quad + \beta_{u,z} z_{M,t+j}^* + \beta_{u,y} y_{M,t+j}^* + \beta'_{u,z} z_{S,t+j}^* + \beta'_{u,y} y_{S,t+j}^*.\end{aligned}$$

For the market case, $\beta_{M,z} = \beta_{M,y} = 1$ and $\beta'_{M,z} = \beta'_{M,y} = 0$. For the stock, $\beta'_{S,z} = \beta'_{S,y} = 1$. The proof is based on a backward recursion over time. Indeed, the moment generating function of $\sum_{j=1}^{T-t} \tilde{R}_{u,t+j}$ given \mathcal{F}_t^S is

$$\begin{aligned}& \varphi_{\tilde{R},t,T}^Q(\phi) \\ &= E_t^Q \left[\exp \left(\phi \sum_{j=1}^{T-t} \tilde{R}_{u,t+j} \right) \right] \\ &= E_t^Q \left[\exp(\phi \tilde{R}_{u,t+1}) E_{t+1}^Q \left[\exp \left(\phi \sum_{j=1}^{T-t-1} \tilde{R}_{u,t+1+j} \right) \right] \right] \\ &= E_t^Q \left[\exp \left(\phi \left(-\frac{1}{2}\beta_{u,z}^2 h_{M,z,t+1}^* - \Pi_M^*(\beta_{u,y}) h_{M,y,t+1}^* - \frac{1}{2}(\beta'_{u,z})^2 h_{S,z,t+1}^* - \Pi_S^*(\beta'_{u,y}) h_{S,y,t+1}^* \right. \right. \right. \\ &\quad \left. \left. \left. + \beta_{u,z} z_{M,t+1}^* + \beta_{u,y} y_{M,t+1}^* + \beta'_{u,z} z_{S,t+1}^* + \beta'_{u,y} y_{S,t+1}^* \right) \right) \right] \\ &\quad \times \exp \left(\begin{aligned} &\mathcal{A}_{u,T-t-1}(\phi) + \mathcal{B}_{u,T-t-1}(\phi) h_{M,z,t+2}^* + \mathcal{C}_{u,T-t-1}(\phi) h_{M,y,t+2}^* \\ &+ \mathcal{D}_{u,T-t-1}(\phi) h_{S,z,t+2}^* + \mathcal{E}_{u,T-t-1}(\phi) h_{S,y,t+2}^* \end{aligned} \right) \end{aligned}$$

(from the induction hypothesis).

Therefore,

$$\begin{aligned}
& \varphi_{\bar{R},t,T}^{\mathbb{Q}}(\phi) \\
&= \mathbb{E}_t^{\mathbb{Q}} \left[\exp \left(\phi \left(-\frac{1}{2} \beta_{u,z}^2 h_{M,z,t+1}^* - \Pi_M^* (\beta_{u,y}) h_{M,y,t+1}^* - \frac{1}{2} (\beta'_{u,z})^2 h_{S,z,t+1}^* - \Pi_S^* (\beta'_{u,y}) h_{S,y,t+1}^* \right. \right. \right. \\
&\quad \left. \left. \left. + \beta_{u,z} z_{M,t+1}^* + \beta_{u,y} y_{M,t+1}^* + \beta'_{u,z} z_{S,t+1}^* + \beta'_{u,y} y_{S,t+1}^* \right) \right) \right] \\
&= \mathbb{E}_t^{\mathbb{Q}} \left[\exp \left(\begin{aligned} & \mathcal{A}_{u,T-t-1}(\phi) + \zeta_{u,T-t-1,1}(\phi) + \zeta_{u,T-t-1,2}(\phi) h_{M,z,t+1}^* + \zeta_{u,T-t-1,3}(\phi) h_{M,y,t+1}^* \\ & + \zeta_{u,T-t-1,4}(\phi) h_{S,z,t+1}^* + \zeta_{u,T-t-1,5}(\phi) h_{S,y,t+1}^* \\ & + \zeta_{u,T-t-1,6}(\phi) (\varepsilon_{M,t+1}^*)^2 + \zeta_{u,T-t-1,7}(\phi) \sqrt{h_{M,z,t+1}^*} \varepsilon_{M,t+1}^* \\ & + \zeta_{u,T-t-1,8}(\phi) (\varepsilon_{S,t+1}^*)^2 + \zeta_{u,T-t-1,9}(\phi) \sqrt{h_{S,z,t+1}^*} \varepsilon_{S,t+1}^* \end{aligned} \right) \right] \\
&= \exp \left(\begin{aligned} & \mathcal{A}_{u,T-t-1}(\phi) + \zeta_{u,T-t-1,1}(\phi) \\ & + (\zeta_{u,T-t-1,2}(\phi) - \frac{1}{2} \beta_{u,z}^2 \phi) h_{M,z,t+1}^* + (\zeta_{u,T-t-1,3}(\phi) - \Pi_M^* (\beta_{u,y}) \phi) h_{M,y,t+1}^* \\ & + (\zeta_{u,T-t-1,4}(\phi) - \frac{1}{2} (\beta'_{u,z})^2 \phi) h_{S,z,t+1}^* + (\zeta_{u,T-t-1,5}(\phi) - \Pi_S^* (\beta'_{u,y}) \phi) h_{S,y,t+1}^* \end{aligned} \right) \\
&\quad \mathbb{E}_t^{\mathbb{Q}} \left[\exp \left(\begin{aligned} & + \zeta_{u,T-t-1,6}(\phi) (\varepsilon_{M,t+1}^*)^2 + (\zeta_{u,T-t-1,7}(\phi) + \beta_{u,z} \phi) \sqrt{h_{M,z,t+1}^*} \varepsilon_{M,t+1}^* + \beta_{u,y} \phi y_{M,t+1}^* \\ & + \zeta_{u,T-t-1,8}(\phi) (\varepsilon_{S,t+1}^*)^2 + (\zeta_{u,T-t-1,9}(\phi) + \beta'_{u,z} \phi) \sqrt{h_{S,z,t+1}^*} \varepsilon_{S,t+1}^* + \beta'_{u,y} \phi y_{S,t+1}^* \end{aligned} \right) \right].
\end{aligned}$$

But the moment generating function of the risk-neutral jump component (B.3) gives $\mathbb{E}_{\mathcal{F}_t^S}^{\mathbb{Q}} [\exp(\beta \phi y_{u,t+1}^*)] = \exp(\Pi_u^* (\beta \phi) h_{u,y,t+1}^*)$. Therefore,

$$\mathbb{E}_{\mathcal{F}_t^S}^{\mathbb{Q}} [\exp(\beta_{u,y} \phi y_{M,t+1}^*)] = \exp(\Pi_M^* (\beta_{u,y} \phi) h_{M,y,t+1}^*)$$

and

$$\mathbb{E}_{\mathcal{F}_t^S}^{\mathbb{Q}} [\exp(\beta'_{u,y} \phi y_{S,t+1}^*)] = \exp(\Pi_S^* (\beta'_{u,y} \phi) h_{S,y,t+1}^*).$$

Moreover, Lemma 7 implies that

$$\begin{aligned}
& \mathbb{E}_t^{\mathbb{Q}} \left[\exp \left(\zeta_{u,T-t-1,6}(\phi) (\varepsilon_{M,t+1}^*)^2 + (\zeta_{u,T-t-1,7}(\phi) + \beta_{u,z} \phi) \sqrt{h_{M,z,t+1}^*} \varepsilon_{M,t+1}^* \right) \right] \\
&= \exp \left(-\frac{1}{2} \ln(1 - 2\zeta_{u,T-t-1,6}(\phi)) + \frac{1}{2} \frac{(\zeta_{u,T-t-1,7}(\phi) + \beta_{u,z} \phi)^2}{(1 - 2\zeta_{u,T-t-1,6}(\phi))} h_{M,z,t+1}^* \right)
\end{aligned}$$

if $\zeta_{u,T-t-1,6}(\phi) < \frac{1}{2}$ and

$$\begin{aligned} & \mathbb{E}_t^{\mathbb{Q}} \left[\exp \left(\zeta_{u,T-t-1,8}(\phi) (\varepsilon_{S,t+1}^*)^2 + (\zeta_{u,T-t-1,9}(\phi) + \beta'_{u,z}\phi) \sqrt{h_{S,z,t+1}^*} \varepsilon_{S,t+1}^* \right) \right] \\ &= \exp \left(-\frac{1}{2} \ln(1 - 2\zeta_{u,T-t-1,8}(\phi)) + \frac{1}{2} \frac{(\zeta_{u,T-t-1,9}(\phi) + \beta'_{u,z}\phi)^2}{(1 - 2\zeta_{u,T-t-1,8}(\phi))} h_{S,z,t+1}^* \right) \end{aligned}$$

provided that $\zeta_{u,T-t-1,8}(\phi) < \frac{1}{2}$. Therefore,

$$\begin{aligned} \varphi_{\tilde{R},t,T}^{\mathbb{Q}}(\phi) &= \exp \left(\begin{aligned} & \mathcal{A}_{u,T-t-1,T}(\phi) + \zeta_{u,T-t-1,1}(\phi) \\ & + (\zeta_{u,T-t-1,2}(\phi) - \frac{1}{2}\beta_{u,z}^2\phi) h_{M,z,t+1}^* + (\zeta_{u,T-t-1,3}(\phi) - \Pi_M^*(\beta_{u,y})\phi) h_{M,y,t+1}^* \\ & + (\zeta_{u,T-t-1,4}(\phi) - \frac{1}{2}(\beta'_{u,z})^2\phi) h_{S,z,t+1}^* + (\zeta_{u,T-t-1,5}(\phi) - \Pi_S^*(\beta'_{u,y})\phi) h_{S,y,t+1}^* \end{aligned} \right) \\ & \exp \left(-\frac{1}{2} \ln(1 - 2\zeta_{u,T-t-1,6}(\phi)) + \frac{1}{2} \frac{(\zeta_{u,T-t-1,7}(\phi) + \beta_{u,z}\phi)^2}{(1 - 2\zeta_{u,T-t-1,6}(\phi))} h_{M,z,t+1}^* \right) \exp(\Pi_M^*(\beta_{u,y}\phi) h_{M,y,t+1}^*) \\ & \exp \left(-\frac{1}{2} \ln(1 - 2\zeta_{u,T-t-1,8}(\phi)) + \frac{1}{2} \frac{(\zeta_{u,T-t-1,9}(\phi) + \beta'_{u,z}\phi)^2}{(1 - 2\zeta_{u,T-t-1,8}(\phi))} h_{S,z,t+1}^* \right) \exp(\Pi_S^*(\beta'_{u,y}\phi) h_{S,y,t+1}^*). \end{aligned}$$

A comparison with

$$\varphi_{\tilde{R},t,T}^{\mathbb{Q}}(\phi) = \exp \left(\begin{aligned} & \mathcal{A}_{u,T-t}(\phi) + \mathcal{B}_{u,T-t}(\phi) h_{M,z,t+1}^* + \mathcal{C}_{u,T-t}(\phi) h_{M,y,t+1}^* \\ & + \mathcal{D}_{u,T-t}(\phi) h_{S,z,t+1}^* + \mathcal{E}_{u,T-t}(\phi) h_{S,y,t+1}^* \end{aligned} \right)$$

leads to the recursive formulation of the coefficients.

OE.2 European Call Option Price

Given the moment generating function (MGF) $\varphi_{\tilde{R},t,T}^{\mathbb{Q}}(\phi)$ of the risk-neutral excess returns, we build on the work of Heston and Nandi (2000) and obtain a closed-form solution for the price of European index and stock options. More precisely, for $u_t \in \{M_t, S_t\}$, the price of an European call option is

$$\begin{aligned} C_t(u_t, K, T) &= e^{-r_t, T(T-t)} \mathbb{E}_t^{\mathbb{Q}} [\max(u_T - K, 0)] \\ &= e^{-r_t, T(T-t)} \left(\mathbb{E}_t^{\mathbb{Q}} [u_T \mathcal{I}\{u_T > K\}] - K \mathbb{E}_t^{\mathbb{Q}} [\mathcal{I}\{u_T > K\}] \right), \end{aligned}$$

where $r_{t,T} = \frac{1}{T-t} \sum_{j=1}^{T-1} r_{t+j}$, in which r_{t+j} is the deterministic risk-free rate at time $t+j$ and $\mathcal{I}\{A\}$ is the indicator function that worth 1 if the event A is realized and 0 otherwise. Since

$$u_T = u_t \exp \left(\sum_{j=1}^{T-t} R_{u,t+j} \right) = u_t \exp(r_{t,T}(T-t)) \exp \left(\sum_{j=1}^{T-t} \tilde{R}_{u,t+j} \right),$$

then

$$\begin{aligned} P_{2,t,T} &= \mathbb{E}_t^{\mathbb{Q}} [\mathcal{I}\{u_T > K\}] \\ &= \mathbb{Q} \left[u_t e^{r_{t,T}(T-t)} \exp \left(\sum_{j=1}^{T-t} \tilde{R}_{u,t+j} \right) > K \right] \\ &= \mathbb{Q} \left[\sum_{j=1}^{T-t} \tilde{R}_{u,t+j} > \ln \tilde{K}_{t,T} \right] \\ &= 1 - F_{\tilde{R},t,T}^{\mathbb{Q}} (\log(\tilde{K}_{t,T})) \end{aligned}$$

where $\tilde{K}_{t,T} = \frac{K e^{-r_{t,T}(T-t)}}{u_t}$, and $F_{\tilde{R},t,T}^{\mathbb{Q}}$ is the cumulative distribution function associated with the MGF $\varphi_{\tilde{R},t,T}^{\mathbb{Q}}(\phi)$. Using results from Feller (1971) and Kendall and Stuart (1977),

$$P_{2,t,T} = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \operatorname{Re} \left[\frac{1}{\phi i} e^{-i\phi \log \tilde{K}_{t,T}} \varphi_{\tilde{R},t,T}^{\mathbb{Q}}(\phi i) \right] d\phi.$$

Moreover,

$$\mathbb{E}_t^{\mathbb{Q}} [u_T \mathcal{I}\{u_T > K\}] = u_t e^{r_{t,T}(T-t)} \mathbb{E}_t^{\mathbb{Q}} \left[\exp \left(\sum_{j=1}^{T-t} \tilde{R}_{u,t+j} \right) \mathcal{I}\{u_T > K\} \right] = u_t e^{r_{t,T}(T-t)} P_{1,t,T}$$

where

$$P_{1,t,T} = \mathbb{E}_t^{\mathbb{Q}} \left[\exp \left(\sum_{j=1}^{T-t} \tilde{R}_{u,t+j} \right) \mathcal{I}\{u_T > K\} \right] = \int_{\log(\tilde{K}_{t,T})}^{\infty} \exp(x) f_{\tilde{R},t,T}^{\mathbb{Q}}(x) dx = \int_{\log(\tilde{K})}^{\infty} \tilde{p}(x) dx$$

where $f_{\tilde{R},t,T}^{\mathbb{Q}}$ is the density function of the excess returns and the last equality stands by letting $\tilde{p}(x) = \exp(x) f_{\tilde{R},t,T}^{\mathbb{Q}}(x)$. Note that \tilde{p} is a well-defined distribution since $\exp(x)$ is always positive and

$$\int_{-\infty}^{\infty} \tilde{p}(x) dx = \int_{-\infty}^{\infty} \exp(x) p(x) dx = \varphi_{\tilde{R},t,T}^{\mathbb{Q}}(1) = 1,$$

given that $f_{t,T}^{\mathbb{Q}}(1)$ is the gross expected excess return under \mathbb{Q} , that is

$$\int_{-\infty}^{\infty} \tilde{p}(x) dx = E_t^{\mathbb{Q}} \left[\exp \left(\sum_{j=1}^{T-t} \tilde{R}_{u,t+j} \right) \right] = E_t^{\mathbb{Q}} \left[\exp(-r_{t,T}(T-t)) u_T \mid u_t = 1 \right] = 1.$$

Hence, following Heston and Nandi (2000), we note that MGF corresponding to \tilde{p} is simply

$$\tilde{\varphi}_{t,T}^{\mathbb{Q}}(\phi) = \int_{-\infty}^{\infty} \exp(\phi x) \tilde{p}(x) dx = \int_{-\infty}^{\infty} \exp(\phi x) \exp(x) \varphi_{\tilde{R},t,T}^{\mathbb{Q}}(x) dx = \varphi_{t,T}^{\mathbb{Q}}(\phi + 1)$$

and thus

$$\begin{aligned} P_{1,t,T} &= \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \operatorname{Re} \left[\frac{1}{\phi i} e^{-i\phi \log \tilde{K}} \tilde{\varphi}_{t,T}^{\mathbb{Q}}(\phi i) \right] d\phi. \\ &= \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \operatorname{Re} \left[\frac{1}{\phi i} e^{-i\phi \log \tilde{K}} \varphi_{t,T}^{\mathbb{Q}}(\phi i + 1) \right] d\phi. \end{aligned}$$

Finally, we have that

$$C_t(u_t, K, T) = u_T P_{1,t,T} - K e^{-r_{t,T}(T-t)} P_{2,t,T}.$$