

# The Effects of Oil Inventories on Growth Prospects, Futures Markets, and Risk Premia <sup>\*</sup>

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## Abstract

This paper studies the effects of introducing storable inputs into a general equilibrium model of endogenous growth. We explicitly account for an occasionally binding non-negativity constraint on storage. To solve the model, we rely on global non-linear solution methods, allowing us to make conditional statements on endogenous variables. We find a substantial nonlinear impact of inventory levels on growth prospects in the economy. The state of commodity holdings has an important impact on utilization of production capacity. In an economy with endogenous growth risks, firms target an optimal level of inventories to be able to increase oil usage in boom times. In an economy with transitory shocks only, the opposite is true. Macroeconomic dynamics implied by the model explain major stylized fact of financial markets, such as the term structure of futures premia and a negative co-movement of oil inventories with the risk-free rate.

**Keywords:** Commodities · storage · futures markets · endogenous growth

**JEL:** D24 · D51 · G12

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# 1 Introduction

There is a vast literature on the interaction of the oil sector and macroeconomic dynamics. At the same time, only little work has been done, reconciling this relationship in general equilibrium. We want to answer the question: *What determines the demand for storage in a general equilibrium framework?* And what are the implications of commodity storage for the economy? Our general equilibrium framework, facilitates sustained endogenous growth and commodities as an input factor. We explicitly allow for the storage of commodities. Commodities – where we think of oil – are an important input factor for production. The level of oil inventories should thus influence growth prospects in the economy. This in turn underlines the importance of the storage decision, which is driven by economic fundamentals in our model. Risk premia, inferred from the models dynamics, reflect equilibrium mechanics and offer a rational for patterns found in empirical data.

In more detail, production of the final good in our model requires standard physical capital, oil, labor, and patents, where we follow [Kung and Schmid \(2015\)](#) for the latter. We allow for transitory shocks to labor augmenting productivity which induce endogenous and prolonged growth patterns via R&D. Households have recursive [Epstein and Zin \(1991\)](#) preferences. Since our focus is on oil, we introduce an additional oil sector which extracts oil, using a linear technology. The extracted oil can be used for production, and it can be stored for future use. Finally, we explicitly account for the non-negativity constraint inherent to oil inventories.

Oil usage is positively correlated to the business cycle. [Kim and Loungani \(1992\)](#) document a correlation of 0.72 for annual U.S. data of energy use and output.<sup>1</sup> [Kilian \(2009\)](#) show in a VAR-estimation that aggregate demand shocks drive real activity and the real price of oil. The results also show that oil production takes about six months to adjust to higher demand. Increased oil usage can thus only be matched by depleting inventories. Prices experience prolonged growth, while real activity does not seem to further profit from

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<sup>1</sup>Energy use is measured as consumption of fossil fuels: petroleum, coal, and natural gas.

the positive shock. We postulate, as inventories decline, further expansion of real activity is dampened via the price channel. As documented by [Hamilton \(2003\)](#), oil prices have a negative impact on future levels of GDP, when controlling for lagged realizations. The analysis of [Kilian \(2009\)](#) shows that the importance of oil supply shocks is neglectable compared to demand shocks when explaining real price fluctuation. Therefore, to focus on the mechanism laid out above, we exclude shocks to the supply side. Nevertheless, the proposed model is already able to rationalize important stylized facts related to the oil sector.

The model setup allows us to analyze the interplay of production, oil prices, and oil inventories. We show that it helps to explain several major empirical findings. Our main results can be summarized as follows. First, a high storage level increases the profitability of R&D activity and positively affects economic growth. Second, very low levels of inventory let the risk-free rate surge up. With oil production held at a normal level, low levels of inventories indicate low output and consumption today and a close to normal level of the two variables in the subsequent period. Low output stalls investment. To compensate this effect and encourage savings, the risk-free rate needs to go up. Third, the production side of the economy has an impact on futures markets. We observe a term structure of futures prices which is in backwardation most of the time. The term structure of futures premia is generally upward sloping, but can change signs for low inventory levels. Intuitively, high levels of inventories buffer price volatility, and by that exposure of short-term futures to risk in the economy, i.e. premia for short-term futures are lower than premia for long-term futures. Oppositely, when inventories are low, the spot price is more strongly affected by innovations in productivity, and thus, premia on short term futures rise. Finally, the convenience yield is negatively related to inventory levels.

In our model, the commodity sector differs from standard physical capital in two ways. First, oil wells need to be installed before producing oil for the subsequent period. This essentially results in a *two-period-to-build* characteristic for oil. Second, we allow the agents to hold excess storage, i.e. agents can build up inventories to react to changes in economic conditions. To highlight the mechanisms driving our results, we solve two nested models

and stepwise shut down characteristics of the commodity input. In the first step, we restrain the agent from holding excess storage. In the second step, oil extracted by the oil wells can (and must) be used immediately. This allows us to isolate the effect of the non-negativity constraint. In an additional calibration, we show that oil usage in the case of only transitory shocks is countercyclical as opposed to procyclical in the full model featuring endogenous growth propagation. It also confirms that we need endogenous growth for investors to hold excess storage. Agents are more inclined to keep sufficient inventories when the risk of a stock-out affects (long-run) growth prospects in the economy than when it is confined to (short-run) business cycle fluctuations.

The paper contributes to the literature by fully endogenizing the storage decision in general equilibrium. To the best of our knowledge, we are the first to solve an endogenous growth economy with an explicit constraint on inventory holdings. We do not resort to exogenously imposing a cost function on low levels of storage. Instead, we capture the cost of a stock-out on a grid and deploy spectral basis methods to improve convergence and reduce computational burden. This allows us to reproduce non-linearities in oil prices already emphasized by [Deaton and Laroque \(1992\)](#).

The remainder of this paper is organized as follows. Section (2) gives a brief literature review. In Section (3), we introduce the model setup. In Section (4), we present the results. Section (5) analyzes the implications of the non-negativity constraint and the growth channel. Section (6) concludes.

## 2 Related Literature

Our paper contributes to the literature, which links macroeconomic fundamentals to asset prices in a general equilibrium model with production. In particular, we contribute to the literature which considers oil as a production input.

**Partial equilibrium models** Beginning with the classical *Theory of Storage* of Kaldor (1939), Working (1949), Telser (1958), and Brennan (1958), a rich literature evolved concerned with the pricing of storable and perishable goods. Deaton and Laroque (1992) elaborate on the role of non-negative storage and emphasize its non-linear impact in a framework of utility maximizing rational inventory holders. Routledge, Seppi, and Spatt (2000) build on this competitive rational expectations model of storage and study the impact of the state of inventories on the term structure of spot and forward prices, and volatilities in a two-period, partial equilibrium framework. As a consequence of the nonnegativity constraint on inventories, the spot has an embedded timing option that is absent in the future. This in turn characterizing the difference to forward contracts in the ownership of equity. The value of the timing option can be related to levels of inventory. Litzenberger and Rabinowitz (1995) characterize oil wells as call options in a multi-period framework. Their model generates backwardation in oil futures markets, i.e. a negative slope of the futures curve, as an endogenous outcome associated with the option feature of oil reserves in the ground. Casassus and Collin-Dufresne (2005) construct a three-factor model of commodity spot prices, which allows convenience yields to depend on spot prices and interest rates. In line with the *Theory of Storage*, they find for crude oil, that convenience yields are significantly increasing in spot commodities prices. Additionally, convenience yields significantly increase in interest rates. Gorton, Hayashi, and Rouwenhorst (2013) build a simple two-period model integrating the *Theory of Storage* and the *Theory of Normal Backwardation* to analyze the relationship between the state of inventories and risk premia of individual commodity futures. Their model features inventory holding, speculators, and hedgers who are assumed to have *mean-variance* preferences. In line with their model, they empirically find a negative, nonlinear relationship between convenience yields and level of inventories. In addition, they find the state of inventories to carry information on futures risk premia.

By now, motivated by the fruitful results of these partial equilibrium models, there is a small but growing literature on the role of oil as input to real production, part of a consumption bundle, or both of the above.

**General equilibrium models** Casassus, Collin-Dufresne, and Routledge (2009) assume that oil investment is irreversible and occurs at a cost including a fixed component. This implies oil drilling to be periodic and lumpy. The model generates two regimes which implies a state dependent risk premium. The model matches the term structure of futures prices and volatilities reasonably well. Additionally, on average it generates a backwardated term structure. In contrast to our model there is no storage, oil reserves are used at a constant rate, and are no contemporaneous control variable. Moreover, as we model investment in oil related capital as continuous process, our model is in line with empirical results by Anderson, Kellogg, and Salant (2014), who provide evidence that oil drilling takes place all the time.

Casassus, Liu, and Tang (2013) extend the *Theory of Storage* to a multi-commodity level, and find that the convenience yield of a commodity depends on its relative scarcity with respect to other related commodities. The implied feedback effect between commodities is necessary to replicate the upward-sloping correlation term structure of futures returns observed for related commodities.

Casassus and Higuera (2013) propose an equilibrium model with exogenous pricing kernel and exogenous oil price process to disentangle industry from business cycle effects for oil on stock returns. Oil is considered as input factor to production. They find, that the value of all non-oil industries decreases upon an oil price shock. This decrease is attributed to the significant negative effect of oil on growth opportunities.

In Ready (2015) the economy is subject to long-run risks in productivity. The representative agent has recursive preferences. The oil good can either be consumed or used in production. Model results show that a decrease in the ability of oil supply to respond quickly to changes in prices and a corresponding increase in uncertainty about long-run prices, can explain changes in the behavior of futures prices and returns from 2005 to 2012. High oil prices create an externality which reduces productivity growth. While in his setup oil supply is exogenous, we model it explicitly as a variable of choice and thereby account for storage in a tractable manner. Furthermore, in contrast to the exogenously given relation between economic growth and oil prices, in our model this relation is an endogenous outcome.

[Hitzemann \(2014\)](#) studies a two sector economy. One sector produces a general consumption good, while the other sector produces the oil consumption good. He shows that the interplay of exogenous oil productivity shocks and endogenous fluctuations in oil demand, resulting from macro shocks, generates stylized facts of oil futures prices and volatilities. In his model small but persistent movements in the growth rate and oil inventory are necessary to produce a sizeable risk premium for short term oil futures and an upward-sloping term structure of risk premia. In contrast to our model, he considers oil as a good in the consumption bundle of the representative investor and by that abstracts from its direct and significant impact on production. Moreover, he does not explicitly account for the non-negativity constraint inherent in a model with commodity storage, but imposes a cost function. In addition, in his model, long-run risks are exogenous, while our contribution is embedded into a framework of endogenous growth, which goes back to [Romer \(1990\)](#). Recent studies by [Kung and Schmid \(2015\)](#) and [Kung \(2015\)](#) motivate the modelling of the long-term growth prospects of a production economy endogenously by innovation in R&D. The authors show that in equilibrium, R&D endogenously drives a small but persistent component in productivity. By that, they provide an equilibrium foundation of long-run risks in the spirit of [Bansal and Yaron \(2004\)](#) and [Croce \(2014\)](#). For the relevance of recursive preferences and temporal resolution of uncertainty see [Kreps and Porteus \(1978\)](#). Recursive preferences make growth prospects relevant for the decision of the household. First attempts to induce endogenous long-run risks in consumption growth in a general equilibrium framework are conducted by [Kaltenbrunner and Lochstoer \(2010\)](#).

### 3 Model Setup

Our model embeds the decision on commodities storage into a general equilibrium model with endogenous growth. We follow [Kung and Schmid \(2015\)](#) and [Comin and Gertler \(2006\)](#) who motivate sustained growth via investment in an R&D sector rather than by exogenously

specifying the growth rate of production technology and long-run risks in the expected growth rate as proposed by [Croce \(2014\)](#). Different from most models, oil enters the production process instead of being part of the consumption bundle. The agents can invest in normal capital (used in the production of final goods) and in oil related capital (used in the extraction of oil). Furthermore, the managers of a representative firm in the oil sector have to decide, on how much of the commodity to sell to the production sector and how much to store for future transactions. Finally, we follow the long-run risks literature and assume that the households have [Epstein and Zin \(1991\)](#) recursive preferences.

As for the endogenous growth part, as well as preference wise our work remains close to the literature, we will start by describing the problem of the manager operating in the sector producing the final consumption good. Subsequently, we introduce an oil related sector and lay out the way commodity is used in the process of production. We then present the R&D sector and the way uncertainty enters the economy. We close the model by describing the households.

### **3.1 Production**

Our model comprises four production sectors. The final consumption good is produced using physical capital, oil, labor and patented goods as in [Comin and Gertler \(2006\)](#) and [Kung and Schmid \(2015\)](#). For the sake of brevity and without loss of generality, we set the labor input to 1 in the following as agents do not suffer disutilities from working. The market for the final good is perfectly competitive. Patented goods are produced by firms with market power, and patents are generated in a perfectly competitive R&D sector. Monopolistic competition in the market for patented goods generates sustained endogenous growth.

Uncertainty enters the model via stationary shocks to the labor augmenting productivity. As a positive shock to productivity makes the production of the final consumption good more efficient, this shocks can be interpreted as a demand shock for the oil market. The utilization of the production capacity in the final good sector than depends on the level of oil inventory.



For high and intermediate storage levels, the final good producer can immediately react to improved economic condition, i.e. a positive transitory level shock, by reducing inventory. In contrast, low levels of oil inventory restrict the impact of positive shocks in the period of occurrence.

### 3.1.1 The Final Good

As there is perfect competition in the market for the final good, we can assume one representative price-taking firm. The final consumption good is produced by this representative firm with a constant returns to scale production function:

$$Y_t = (\mathcal{K}\mathcal{O}_t^\alpha (A_t N_t)^{1-\alpha})^{1-\omega_p} G_t^{\omega_p} \quad (1)$$

$\mathcal{K}\mathcal{O}_t$  denotes a composite of physical capital and oil specified below,  $G_t$  denotes aggregated patents (also specified below),  $N_t$  is labor used in the production process, and  $A_t$  is labor augmenting technology. The exponent  $\alpha$  is the share of capital,  $\omega_p$  is the share of patents. Physical capital and oil are aggregated by a constant elasticity of substitution function, and  $\mathcal{K}\mathcal{O}_t$  is given by

$$\mathcal{K}\mathcal{O}_t = f(K_t^k, O_t) = \left( (1 - \omega_o)^{\frac{1}{\xi_{ko}}} (K_t^k)^{1 - \frac{1}{\xi_{ko}}} + \omega_o^{\frac{1}{\xi_{ko}}} O_t^{1 - \frac{1}{\xi_{ko}}} \right)^{\frac{1}{1 - \frac{1}{\xi_{ko}}}}. \quad (2)$$

$K_t^k$  denotes physical capital, and  $O_t$  oil in production. The parameters  $\omega_o$  and  $\xi_{ko}$  are the weight of oil and the elasticity of substitution between physical capital and oil, respectively. Here, we allow the elasticity of substitution between physical capital and oil to be different from 1, as suggested by empirical findings. For  $\xi_{ko} = 1$  the function converges to a standard Cobb-Douglas function, and  $\xi_{ko} = \infty$  results in the Leontief production function. This aggregation is in line with [Kim and Loungani \(1992\)](#), who deploy this aggregation of oil and physical capital to formalize production.

Patents are aggregated following [Dixit and Stiglitz \(1977\)](#).  $G_t$  is defined as

$$G_t = \left[ \int_{i=0}^{K_t^p} X_{i,t}^\nu di \right]^{\frac{1}{\nu}}, \quad (3)$$

where  $X_{i,t}$  is the patent of type  $i$ , and where the elasticity of substitution between patents is given by  $\frac{1}{1-\nu}$ ,  $\nu$  denoting the inverse markup. Note that we do not aggregate over a unit mass, but over the interval from 0 to  $K_t^p$ , i.e. over the set of all available patents.

The manager of the final goods firm invests into capital, buys oil and patents, and pays wages to his employees. Thus, the dividends of the representative firm are given by

$$D_t = Y_t - I_t^k - P_t^o O_t - W_t - \int_{i=0}^{K_t^p} P_{i,t}^p X_{i,t} di, \quad (4)$$

where  $I_t^k$  is investment in the future capital stock.  $P_t^o$  and  $P_{i,t}^p$  are the prices of oil and patent  $i$ , respectively.  $W_t$  denotes overall wages, where we have set  $N_t = 1$ .

The capital law of motion is specified according to [Jermann \(1998\)](#)

$$K_{t+1}^k = (1 - \delta_k) K_t^k + \phi \left( \frac{I_t^k}{K_t^k} \right) K_t^k. \quad (5)$$

The rate of depreciation for physical capital is given by  $\delta_k$ .  $\phi \left( \frac{I_t^k}{K_t^k} \right)$  is the adjustment cost function which is given by

$$\phi \left( \frac{I_t^k}{K_t^k} \right) = a_{0,k} + \frac{a_{1,k}}{1 - \frac{1}{\xi_k}} \left( \frac{I_t^k}{K_t^k} \right)^{1 - \frac{1}{\xi_k}}, \quad (6)$$

where  $\xi_k$  is the elasticity of substitution between old and new capital, i.e. the adjustment cost parameter.<sup>2</sup>

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<sup>2</sup>The parameters  $a_{0,k}$  and  $a_{1,k}$  are chosen such that the adjustment costs in the deterministic steady state,  $\mu_{ss}$ , are zero.  $\phi \left( \frac{I_{ss}^k}{K_{ss}^k} \right) = \frac{I_{ss}^k}{K_{ss}^k}$  and  $\phi' \left( \frac{I_{ss}^k}{K_{ss}^k} \right) = 1$ . That leaves us with  $a_{1,k} = (e^{\mu_{ss}} - 1 + \delta_k)^{\frac{1}{\xi_k}}$  and  $a_{0,k} = \frac{1}{1 - \xi_k} (e^{\mu_{ss}} - 1 + \delta_k)$ .

Given the above setup, the manager of the firm maximizes the present value of dividends taken prices and wages as well as the pricing kernel  $\mathcal{M}_{t,t+h}$  of the household as given.

$$\mathcal{J}_t = \max_{\{I_{t+h}^k, K_{t+h}^k, O_{t+h}, \{X_{i,t+h}\}_{i \in [0, K_{t+h}^p]}\}_{0 \leq h \leq \infty}} E_t \left[ \sum_{h=0}^{\infty} \mathcal{M}_{t,t+h} D_{t,t+h} \right] \quad (7)$$

$\mathcal{J}_t$  denotes the present value of the firm. The pricing kernel is the outcome of the households problem as described in Section 3.2. The pricing kernel of the household is stated in Equation (24).

The final good sector is subject to shocks to labor augmenting productivity. We assume that shocks are transient and specify  $A$  exogenously via

$$A_t = e^{a_t} \quad (8)$$

$$a_t = (1 - \rho)a^* + \rho a_{t-1} + \sigma \varepsilon_t^a. \quad (9)$$

The parameter  $\rho$  denotes the autocorrelation of log-productivity, and innovations to productivity are  $\varepsilon_t^a \stackrel{iid}{\sim} \mathcal{N}(0, 1)$ .

### 3.1.2 The Oil Sector

Firms in the oil sector can invest in oil producing capital, i.e oil wells, rigs etc. In subsequent periods oil extraction rate is assumed to be constant and simply depends on capital invested in the oil producing sector. The oil sector is again subject to perfect competition, so that we can assume one price-taking representative firm. Oil production is given by

$$\phi^o(K_t^o) = \tau K_t^o, \quad (10)$$

where  $\tau$  is the extraction rate and  $K_t^o$  capital in the oil producing sector, which includes e.g. oil wells or rigs. For simplicity, we abstract from labor. The law of motion for oil producing

capital follows the same law of motion as standard physical capital<sup>3</sup>

$$K_{t+1}^o = (1 - \delta_o) K_t^o + \phi \left( \frac{I_t^o}{K_t^o} \right) K_t^o. \quad (11)$$

The adjustment costs are

$$\phi \left( \frac{I_t^o}{K_t^o} \right) = a_{0,o} + \frac{a_{1,o}}{1 - \frac{1}{\xi_o}} \left( \frac{I_t^o}{K_t^o} \right)^{1 - \frac{1}{\xi_o}}, \quad (12)$$

where  $\delta_o$  is the rate of depreciation for oil producing capital and  $\xi_o$  is the adjustment cost parameter.

We assume oil extracted in  $t$  to be available in the subsequent period at  $t + 1$ . Once in inventories, the oil producer can sell the extracted oil to the final goods firm, or he can save the oil for subsequent periods. The stock of inventories evolves according to

$$S_{t+1} = (1 - \delta_s) (S_t - O_t) + \phi^o (K_t^o), \quad (13)$$

where  $\delta_s$  are the percentage cost of storage for oil. To keep the model as simple as possible, we refrain from a more detailed decomposition of the cost into depreciation and cost of storage capacity etc.

The cash flows for the oil producing firm are given by

$$D_t^o = P_t^o O_t - I_t^o. \quad (14)$$

The manager of the firm maximizes the present value of these cash-flows, given the price for

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<sup>3</sup>Here we abstract from the fact of heterogeneity between various types of oil wells, such as on and off shore drilling. Particularly, the planning horizon in the oil sector may differ from other investments. [Casassus, Collin-Dufresne, and Routledge \(2009\)](#) further argue that inventories are rarely close to a stock-out. But deem this fact unproblematic, as inventories are difficult to measure. Additionally, the complexity of storage technology cannot be captured in equilibrium models. Our model framework is thus only stylized representation of reality.

oil and the households' pricing kernel

$$\mathcal{J}_t^o = \max_{\{I_{t+h}^o, K_{t+h}^o, O_{t+h}\}_{\{0 \leq h \leq \infty\}}} E_t \left[ \sum_{h=0}^{\infty} \mathcal{M}_{t,t+h} D_{t,t+h}^o \right], \quad (15)$$

The optimization is subject to the restriction that he can never sell more oil than the current inventory level:

$$O_t \leq S_t. \quad (16)$$

We denote the Lagrange multiplier associated with this occasionally binding constraint as  $\lambda_t$ , which is the shadow price of owning one unit more oil.

### 3.1.3 The Sector for Patented Goods

The production of final goods depends on the use of patents. The owners of these patents have a monopoly for their specific patent  $i$  and thus have the power to set prices. They know the demand schedule  $X_{i,t}(P_{i,t}^p)$  for patent  $i$  which follows from the optimization of the producer of the final good. The patent producer  $i$ 's problem is then to maximize the firms profits in period  $t$

$$\Pi_{i,t}^p = \max_{P_{i,t}^p} X_{i,t}(P_{i,t}^p) \cdot P_{i,t}^p - X_{i,t}(P_{i,t}^p) \cdot 1, \quad (17)$$

where we assume that the patent producer can turn one unit of consumption into one new patent of type  $i$ . The markup over the marginal cost of production depends on the elasticity of substitution between patents faced by the final goods' producer, and is given by  $1/\nu$ .

The value of holding one patent  $i$  is accordingly defined recursively as

$$\mathcal{J}_{i,t}^P = \Pi_{i,t}^p + (1 - \delta_p) E_t [\mathcal{M}_{t,t+1} \mathcal{J}_{i,t+1}^P]. \quad (18)$$

As patents can be replaced by newer patents, tomorrow's value is diminished by the probability  $\delta_p$  that this happens.

### 3.1.4 The R&D Sector

The market for patents is perfectly competitive and has free entry. Entrepreneurs conduct research activity and can enter the market upon successfully having developed a new patent. Accordingly, the law of motion for the measure of available patents is given by

$$K_{t+1}^p = \chi_t I_t^p + (1 - \delta_p) K_t^p, \quad (19)$$

where  $I_t^p$  is activity in R&D and  $\chi_t$  represents the efficiency of the R&D sector. It captures technological spill-over as in [Comin and Gertler \(2006\)](#) and takes the form

$$\chi_t = \frac{\bar{\chi} K_t^p}{(I_t^p)^{1-\eta} (K_t^p)^\eta}. \quad (20)$$

For high levels of investment relative to the level of the R&D sector making new inventions becomes more and more difficult. For  $\eta = 0$  the amount of new patents solely depends on the level of  $K_t^p$ , and for  $\eta = 1$  the amount of new patents is only driven by R&D activity.

Note that the entrepreneurs in the R&D-sector do not internalize their effect on the aggregate efficiency of R&D activity, but take  $\chi_t$  as given. New patents in place induce further growth. This leads to an endogenous growth rate in the economy different from zero.

## 3.2 The Household

The households have identical preferences and share their risks perfectly, such that we can assume one representative investor, who has [Epstein and Zin \(1991\)](#) recursive preferences

$$\mathcal{U}_t = \left( (1 - \beta) C_t^{1-\frac{1}{\psi}} + \beta \mathcal{R}_t^{1-\frac{1}{\psi}} \right)^{\frac{1}{1-\frac{1}{\psi}}}, \quad (21)$$

where  $C_t$  is today's consumption and  $\mathcal{R}_t$  the certainty equivalent of future consumption,

$$\mathcal{R}_t = E_t [\mathcal{U}_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}. \quad (22)$$

$\mathcal{U}_t$  denotes the continuation utility. The parameters  $\gamma$  and  $\psi$  are the agents' relative risk aversion and intertemporal elasticity of substitution, respectively. When  $\gamma \neq 1/\psi$  state variables are priced and the agent cares about growth prospects in the economy. For  $\gamma > 1/\psi$  he has a preference for early resolution of uncertainty. The household's budget constraint is

$$C_t + P_t \mathcal{K}_{t+1}^m + B_{t+1} = W_t + (P_t + D_t) \mathcal{K}_t^m + e^{r_{t,t+1}^f} B_t, \quad (23)$$

where  $P_t$  is the price of one share of aggregate equity and  $\mathcal{K}_t^m$  denotes aggregate equity holdings. Further, equity shares pay a dividend denoted by  $D_t$ . Investments into one-period bonds are denoted by  $B_t$ , and  $r_{t,t+1}^f$  is the risk-free rate.

### 3.3 Model Solution

#### 3.3.1 Equilibrium

**Pricing kernel** The pricing kernel of the household is defined as the marginal rate of substitution between consumption today and tomorrow. For recursive preferences, we have

$$\mathcal{M}_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{\mathcal{U}_{t+1}}{\mathcal{R}_t} \right)^{\frac{1}{\psi} - \gamma}. \quad (24)$$

For every asset, its return  $R_{t,t+1}$  has to meet the Euler equation

$$1 = E_t [\mathcal{M}_{t,t+1} R_{t,t+1}]. \quad (25)$$

Plugging in the return on physical capital and oil producing capital determines the optimal investment into these two kinds of capital.

**Return on physical capital** For physical capital, we rely on the standard result from [Restoy and Rockinger \(1994\)](#) who show that the return on investment is equal to the market return of capital when the production and adjustment functions are homogeneous of degree

one. The return is given by

$$R_{t+1}^k = \frac{\frac{\partial Y_{t+1}}{\partial K_{t+1}^k} + \lambda_{t+1}^k \left[ (1 - \delta_k) - \phi' \left( \frac{I_{t+1}^k}{K_{t+1}^k} \right) \frac{I_{t+1}^k}{K_{t+1}^k} + \phi \left( \frac{I_{t+1}^k}{K_{t+1}^k} \right) \right]}{\lambda_t^k}, \quad (26)$$

where  $\lambda_t^k$  is the Lagrange multiplier of the law of motion for physical capital, i.e. the shadow price of capital.

**Return on oil related capital** The return on oil related capital is a little more intricate, as we have to account for the case of running into a stock-out. The return is given by

$$R_{t+1}^o = \frac{\tau \frac{P_{t+1}^o - \lambda_{t+1}^o}{1 - \delta_s} + \lambda_{t+1}^o \left[ (1 - \delta_o) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right]}{\lambda_t^o}, \quad (27)$$

where  $\lambda_t^o$  is the Lagrange multiplier of the law of motion for oil related capital, i.e. its shadow price.

The optimal amount of oil used in the production of the final good follows from equating the marginal productivity of oil and its market price:

$$\frac{\partial Y_t}{\partial O_t} = P_t^o. \quad (28)$$

The amount of newly installed oil capital then follows from plugging  $R_{t+1}^o$  into the Euler equation. Equation (27) implies that it depends on tomorrow's cost of installing further capital and also on tomorrow's oil price times the extraction rate.

The manager of the oil firm also decides on the stock of oil inventories. The associated return on oil inventories is given by

$$R_{t+1}^s = \frac{P_{t+1}^o}{\lambda_t^s}, \quad (29)$$

where  $\lambda_t = \frac{\partial Y_t}{\partial O_t} - (1 - \delta_s) \lambda_t^s$  accounts for the shadow price of oil.



**Return on R&D** The return on investing into R&D depends on the value of one patent and determines economic growth in our economy. As all patents are symmetric we can drop the indices, and the return is

$$R_{t+1}^p = \frac{\mathcal{J}_{t+1}^p (K_{t+1}^p - (1 - \delta_p) K_t^p)}{I_t^p}. \quad (30)$$

The final good producers inverse demand for patent  $i$  is determined by its marginal product:

$$P_{i,t}^p = \frac{\partial Y_t}{\partial X_{i,t}} = \frac{\partial Y_t}{\partial G_t} \frac{\partial G_t}{\partial X_{i,t}} \quad (31)$$

$$= \omega_p (\mathcal{K}\mathcal{O}_t^\alpha A_t^{1-\alpha})^{1-\omega_p} (\bar{X}_t (K_t^p)^{1/\nu})^{(\omega_p-\nu)} X_{i,t}^{\nu-1} \quad (32)$$

Again, symmetry implies that we can drop the index  $i$ .

$$\Pi_t^p = \left( \frac{1}{\nu} - 1 \right) \bar{X}_t. \quad (33)$$

Plugging in the price,  $\bar{X}_t$  simplifies to

$$\bar{X}_t = \left( \nu \omega_p (\mathcal{K}\mathcal{O}_t^\alpha A_t^{1-\alpha})^{1-\omega_p} (K_t^p)^{(\omega_p-\nu)/\nu} \right)^{\frac{1}{1-\omega_p}}. \quad (34)$$

Following [Kung and Schmid \(2015\)](#), we furthermore assume  $\alpha = 1 - \frac{\omega_p - \omega_p}{1 - \omega_p}$  to ensure stable growth. The production function then becomes linearly homogeneous in the composite  $\mathcal{K}\mathcal{O}_t$  of oil and capital and the level  $A_t$  of productivity. We have

$$Y_t = \mathcal{K}\mathcal{O}_t^\alpha Z_t^{1-\alpha} \bar{A} \quad (35)$$

$$Z_t = e^{a_t} K_t^p, \quad (36)$$

where  $\bar{A} \equiv (\omega_p \nu)^{\frac{\omega_p}{1-\omega_p}}$ . The overall demand for patents becomes

$$X_t = (\omega_p \nu)^{\frac{1}{1-\omega_p}} e^{a_t(1-\alpha)} \left( \frac{\mathcal{K}\mathcal{O}_t}{K_t^p} \right)^\alpha K_t^p. \quad (37)$$

Finally, the trend growth is given by

$$\mu_t = \log \left( 1 - \delta^p + \chi_t \frac{I_t^p}{K_t^p} \right) = \log \left( \frac{K_{t+1}^p}{K_t^p} \right), \quad (38)$$

where  $\mu_t$  is the  $t$ -measurable growth rate of the stock of patents from period  $t$  to  $t + 1$ .

**Market clearing** Goods markets clear with

$$C_t = Y_t - I_t^k - I_t^p - I_t^o - X_t. \quad (39)$$

$X_t = \bar{X}_t K_t^p$  captures the cost of producing patented goods.

### 3.3.2 The Risk-free Rate and Equity Premia

**Risk-free rate** The risk-free rate is calculated as the inverse of the expected pricing kernel. This gives us

$$e^{r_{t,t+h}^f} = E_t [\mathcal{M}_{t,t+h}]^{-h}, \quad (40)$$

where  $h$  denotes the maturity.

**Equity premia** We approximate the risk premia with the expected difference between the realized return on capital and the  $t$ -measurable risk-free rate. This gives

$$ep_t^m = E_t \left[ r_{t+1}^m - r_{t,t+1}^f \right] \quad (41)$$

$$ep_t^k = E_t \left[ r_{t+1}^k - r_{t,t+1}^f \right], \quad (42)$$

where  $r_{t+1}^m = \ln R_{t+1}^m$  is the log-return on the market, and  $r_t^k = \ln R_{t+1}^k$  is the log-return on physical capital. The return on the market is given by

$$R_{t+1}^m = \frac{P_{t+1}^m + D_{t+1}^m}{P_t^m}. \quad (43)$$

Dividends on the market are defined as  $D_t^m = Y_t - I_t^k - I_t^p - I_t^o - W_t - \bar{X}_t K_t^p$ .

Note that prices and returns for the market claim must be approximated, as we do not have a closed form expression like for the return on capital.

### 3.3.3 Futures, Futures Premia, and Convenience Yields

**Futures prices** Futures prices are calculated such that the value of the futures contract is zero. The futures price for a time to maturity of one period is thus given by

$$0 = E_t [\mathcal{M}_{t,t+1} (P_{t+1} - F_t^1)]. \quad (44)$$

For longer times to maturity, the futures prices follow by

$$0 = E_t [\mathcal{M}_{t,t+1} (F_t^T - F_{t+1}^{T-1})], \quad (45)$$

where  $F_t^T$  is the futures price for maturity  $T$ . We define the log returns on futures by

$$r_{t+1}^{F^T} = f_{t+1}^{T-1} - f_t^T = \log \left( \frac{F_{t+1}^{T-1}}{F_t^T} \right), \quad (46)$$

where  $f$  denotes the log of the futures price. As usual, we interpret the return of the futures as a premium, as it does not require any initial investment.

Finally, the convenience yield  $CY_t$  is defined as in [Gorton, Hayashi, and Rouwenhorst \(2013\)](#),

$$CY_t = \underbrace{P_t^o e^{r_{t,t+T}^f} - F_t^T}_{\text{adj. basis}} + \delta_s. \quad (47)$$

The basis, i.e. the difference  $P_t^o - F_t^T$  between the spot price and the futures price, determines whether the market is in contango or in backwardation. For  $P_t^o > F_t^T$  the market is in backwardation and for  $P_t^o < F_t^T$  the market is in contango.

### 3.4 Solution Method

To account for the non-linearities arising close to the occasionally binding constraint for oil inventories, the model is solved using a global non-linear solution algorithm. As in [Judd \(1992\)](#), we use projection methods to approximate the policy functions for investment  $(I_t^k, I_t^o, I_t^p)$  and optimal oil usage  $(O_t)$ . Further, we approximate the households continuation utility  $(\mathcal{U}_t)$ , as well as the value of a patent  $(\mathcal{J}_t)$  and the shadow price of one more unit of oil in inventory  $(\lambda_t^s)$ . We thus determine

$$\{I_t^k, I_t^o, I_t^p, O_t, \mathcal{U}_t, \mathcal{J}_t^P, \lambda_t^s\}_{\{0 \leq t \leq \infty\}} \quad (48)$$

when we solve for the equilibrium.

Due to recursive preference, a very small discount rate, persistent growth patterns, and of course, the non-negativity constraint on commodities storage, convergence proved to be very hard to achieve. Even though piece-wise approximation would usually be the suitably approach for problems involving occasionally binding constraints, we choose Chebyshev polynomials as our set of basis functions due to their nice properties of convergence. Ultimately, we solve the equilibrium conditions listed in [Appendix A.3](#) directly with MATLAB's *fmincon* solver, which proves to be extremely stable. To reduce the computational burden, we use orthogonal collocation, which allows us to have exactly as many points on our approximating grid as our degree of approximation. Finally, *fmincon* allows parallel function evaluation. We detrend all variables with growth of the R&D stock and are thus left with 4 state variables in our stationary equilibrium. The vector of state variables in logs is given by

$$\{a_t, \hat{k}_t^k, \hat{k}_t^o, \hat{s}_t\}_{\{0 \leq t \leq \infty\}}, \quad (49)$$

which completely describes the state of our economy at any point in time. We find, that 4<sup>th</sup> order approximations suffice for detrended physical capital  $(\hat{k}_t^k)$  and productivity  $(a_t)$ . In contrast, policy functions become increasingly non-linear up to possibly being kinked in

the directions of oil producing capital and the stock of inventories. Consequentially, the approximating degree in those two directions is of order 9 to obtain a sufficiently good fit to the equilibrium. We find that increasing the degree of approximation for the states  $a$  and  $\hat{k}^k$  does not pose a significant burden on the solver, while  $\hat{k}^o$  and  $\hat{s}$  typically take more iteration steps and can only be increased jointly. Table I reports Euler equation errors for investments into the three types of capital in the model. We compute these errors on a grid with 50 points in each dimension. Euler equation errors are then reported as maximum and mean errors on the grid. As a third measure, we use a long time series of 10,000 periods and evaluate the average absolute deviation over the whole sample path. Overall, errors tend to be higher for investment into the stock of inventories than for the investment decisions. Figure 1 depicts absolute Euler equation errors for all four dimensions.

The interval bounds for the exogenous state variables are  $\pm 3.25$  times the unconditional volatility, such that they cover 99.95% of probability mass. The bounds for the endogenous state variables are chosen such that they will not be violated during the simulations. Expectations are computed with a 5-node *Gauss-Hermite* quadrature.

We simulate the model quarterly over 80 years (320 periods) and use an additional burn-in period of 500 quarters. The results are based on 10,000 paths. The technology process is highly persistent, and thus we rely on Tauchen (1986) and the extension of Floden (2008) for its simulation. The exogenous process is discretized with 31 points. The model is solved using the algorithm developed in Branger, Gräber, and Schumacher (2016).

Finally, in a second step, we approximate conditional financial variables, such as futures prices, the price of the aggregate market claim, the associated premia, and the risk-free rate in the same fashion as we have solved for the dynamic equilibrium.

## 4 Results

A key finding of our model is that the level of inventories in oil has an important effect on investment and expected growth in the economy as well as returns, futures prices, and futures premia. All the above are well documented facts about the oil market and underline its importance in the process of production. Even further, the model produces a behavior of the risk-free rate very close to what we observe in the data, when being seen in connection with inventory levels. Especially, non-linear effects close to the occasionally binding constraint of inventory levels are hereby an important factor, which can even turn signs for variables.

### 4.1 Calibration

The calibration of the model is depicted in Table II. To remain comparable with respect to the growth part of the model we choose the decision horizon (quarterly) and key parameters similar to [Kung and Schmid \(2015\)](#).

**Investors Preferences** Risk aversion is set to  $\gamma = 10$  and the intertemporal elasticity of substitution  $\psi$  equals 1.5. The subjective discount factor, a parameter with significant impact on steady state growth of the economy, is set to 0.986% annually in order to match the risk-free rate in the data.

**Final good production and patent sector** The share of capital in production is set to  $\alpha = 0.35$ . The patent share,  $\omega_p$ , is at 0.5, chosen as in [Comin and Gertler \(2006\)](#). To restrict the economy to grow at a stable path the inverse markup,  $\nu$ , is set at 0.61. Patent obsolescence rate  $\delta_p$  is according to [Kung and Schmid \(2015\)](#) chosen at 15% annually. For the elasticity of substitution between new inventions and existing R & D,  $\eta$ , capturing spill over effects in innovation, we again follow [Kung and Schmid \(2015\)](#) and set  $\eta = 0.83$ . This parameter governs the value of holding patent  $i$ . By that, it affects the profitability of investments in the R & D sector and has strong effects on the model's performance in

generating sustained growth patterns. The chosen value is in a reasonable range of what is estimated in [Griliches \(1990\)](#). The scale parameter  $\bar{\chi}$  is calibrated at 0.325 to match the average growth rate of the U.S. economy.

**The oil sector** To isolate the effect of storage, we choose rates of depreciation ( $\delta_k, \delta_o$ ) and adjustment cost ( $\xi_k, \xi_o$ ) alike at 8% annually, 1.43 respectively. The cost of storage for inventories in oil ( $\delta_s$ ) is also set to 8% annually. The extraction rate of oil ( $\tau$ ) is equal to 1. This makes the numerical solution of the model much easier, than for values smaller than 1, but does not alter it's intuition, as it is just a scale parameter. Finally, the weight of oil in the aggregation with capital ( $\omega_o$ ) is 0.5. Elasticity of substitution between the two inputs ( $\xi_{ko}$ ) is set to 0.4. Estimates range from 0.09 ([Backus and Crucini \(2000\)](#)) to 0.7 ([Kim and Loungani \(1992\)](#)), other estimates even being above 1.<sup>4</sup> For the stochastic process of productivity we follow [Kung and Schmid \(2015\)](#) and choose the annualized autocorrelation to be consistent with the autocorrelation of R&D intensity at 0.95 and it's volatility to be 3.4% in order to match the volatility of output growth.

**Calibration strategy** For macroeconomic moments we rely on the sample reported in [Kung and Schmid \(2015\)](#) (post-Great Depression sample - 1930 to 2008). As [Kung and Schmid \(2015\)](#) do not report empirical counterparts to their model outcomes for standard asset pricing moments, we rely on post war data (1953 to 2008) taken from [Kung \(2015\)](#). Investment data for oil related, aggregate and research related investment are taken from NIPA-table 5.3.3. (Real Private Fixed Investment by Type, Quantity Indexes). For data on futures prices we use *CME Group (NYMEX) Light Sweet Crude Oil Futures* obtained via Thomson Reuters Eikon. We use maturities ranging from 1 to 18 months. Due to availability of futures with long maturities our sample is restricted from 1990 to 2014. The 1-month future is used to approximate the spot price as in [Litzenberger and Rabinowitz \(1995\)](#). For

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<sup>4</sup>There exists a strand of literature commenting on complementarity and substitutability of the two inputs. See for example [Berndt and Wood \(1979\)](#) and [Jorgensen \(1986\)](#).

the risk-free rate, we use the 90 day treasury bill and compute the ex ante risk-free rate as in [Beeler and Campbell \(2009\)](#).

## 4.2 The Macroeconomy

### 4.2.1 Macroeconomic Quantities

Table [III](#) reports all relevant moments for macroeconomic variables. As reported in panel A and B output grows at an annualized rate of 1.87% (1.90% in empirical data). Standard deviation of output growth is 2.38% (2.33%). For consumption growth the model implies a standard deviation of 1.36% (1.42%). Finally turning our attention to the volatilities of investment growth, the model implied investment volatility for physical capital is 2.98% and 3.68% for investment in R&D (6.22%, 4.89%). While the mean output growth and the volatility of output growth are close to the target values taken from [Kung and Schmid \(2015\)](#), the model implied volatilities of consumption growth and investment in R&D are slightly below the empirical counterparts. The model implied volatility of investment in physical capital is at 2.98% below what is observed in the data. We conclude that even without assuming heterogeneity with respect to adjustment cost and rates of depreciation, our model generates major stylized facts close to the data. Allowing for more flexibility with respect to the choice of parameters would potentially ease moment matching. But at the same time loosening this parameters restriction would obliterate the interpretation of the inventory channel.

Panel B further displays the volatility of output growth, as well as volatility-ratios of consumption to output, investment in capital to output, and investment in R&D to output. Expectedly the consumption-to-output ratio is at 0.57 very close to the data (0.61). Investment-to-output ratio for physical capital is at 1.25 clearly below the target (2.67). This is a standard problem of general equilibrium models. Finally, the investment-to-output ratio for R&D is at 1.54 and thus also does not match the data exactly.



Panel C reports autocorrelations for growth in consumption, output and investment in physical capital. As opposed to [Kung and Schmid \(2015\)](#), our model implied autocorrelations are not as high as can be supported by the data. Still, we report an autocorrelation of about 12% for consumption, and 3% for output and investment.

Panel D reports selected oil related volatility-ratios, namely investment in oil related capital over aggregate investment and oil usage over investment in oil related capital. Choosing capital characteristics alike, unsurprisingly the ratio of variation in investment in oil related capital and aggregate investment is at 0.88 very low when being compared to the data (3.73). As emphasized above, all stocks are parameterized alike and thus display similar volatilities. Accordingly, all types of investment react procyclically, and thus, investment volatility on the aggregate level is not reduced by diversification, as it is the case in the data. The volatility ratio for oil usage over investment is at 0.13, being about one half of its empirical counterpart.

Table IV depicts correlations for macroeconomic and oil related variables. First of all, we observe a positive correlation for almost all variables over the growth cycle. The most strongly correlated couple is investment to output growth being almost to 1, closely followed by consumption and output at 0.99. The stock of inventories and oil in production are positively correlated with output growth, yet only weakly at about 0.08 and 0.07. This reflects a tradeoff between a rising demand opposed to long-run investment in oil producing capital. In the short-run, oil is depleted in the process of production, when times are good, indicating a positive correlation of  $\Delta y$  and  $\Delta o$  and a negative correlation of  $\Delta y$  and  $\Delta s$ . In the long-run, with stocks variables ( $K^k$ ,  $K^o$ ,  $K^p$ ,  $S$ ) only adjusting gradually over the growth path,  $\Delta y$  and  $\Delta s$  do also co-move positively.<sup>5</sup> Growth in oil prices is strongly pro-cyclical but negatively correlated to inventory growth. This being in line with economic intuition. Normalized levels of oil inventories are counter-cyclical and negatively correlated with all

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<sup>5</sup>More formally, Equation (13), the law of motion for inventories, implies a lead-lag relationship of inventories and oil usage. In other words, the level of inventories ( $s$ ) is reduced only one period after oil usage ( $o$ ) goes up. A low level of inventories in turn causing oil usage to go down as well. As the former effect has a one period lag, it will not be captured by contemporaneous measures of correlation of the growth rates.

growth rates except the oil price, reflecting co-movement of variables over the business cycle. Lastly, the ratio of oil used in production to storage levels at the beginning of the period is also negatively correlated to all variables except oil usage and consumption growth. This indicates a higher intensity of oil usage over the business cycle.

#### 4.2.2 A Static Equilibrium Assessment

As documented by [Kung and Schmid \(2015\)](#), the equations driving investment in R&D and by that economic growth are (30), for the return on investment in R&D, and (18), stating the recursive value of the firm producing patent  $i$ . *The intuition is as follows:* Due to market power, monopolists generate profits and by that induce a positive value for holding the patent. This in turn makes increased levels of investment in the R&D sector profitable and invokes positive sustained growth for the economy. The general mechanism inducing long-run growth in the model of [Kung and Schmid \(2015\)](#) turns positive transitory shocks into long-run growth by temporarily increasing patent value. An increased patent value will attract new investors to enter the R&D sector. As a consequence the sector grows and by that temporarily increases growth in the economy above steady state value. All other state variables, positively affecting profits, will have similar effects.

**Macroeconomic Reactions** The same mechanism holds true in our model. Figure 2 depicts investment, patent value, economic growth, and the oil price, as functions of the state variables, namely productivity level ( $a$ ), physical capital ( $k^k$ ), oil producing capital ( $k^o$ ), and oil inventory ( $s$ ). First, turning to patent value ( $\log(\mathcal{J}^p)$ ) as a function of *productivity*, we see, that it is steeply increasing in the level of productivity. As a consequence economic growth for the subsequent period ( $\mu_t$ ) rises as well, which is in line with the general intuition laid out above. Investigating the impact of *physical capital* on investment, we observe investment in physical capital to be decreasing, and investment in oil producing capital increasing in the stock of physical capital. Also, as might be expected, patent value and thereby economic

growth are increasing in the capital stock. For *oil producing capital*, effects - except for prices - are similar to what was reported for physical capital. Results are less pronounced. This is due to the buffering effect of storage and the *two period time-to-build* characteristic of oil supply. Oil producing capital does not immediately enter the process of production, but supplies a good (commodity) that in turn can be used up in the following period or stored for subsequent periods. Clearly, the effect on prices is very pronounced, as an increased supply of oil producing capital raises oil inventory and drives storage away from the optimal level. Prices drop and the increased production output of the final good can be consumed or invested in capital stock. Turning our attention to *oil in inventories*, we see macroeconomic variables as functions of storage levels at the beginning of the period - meaning before being depleted in the process of production. The price of oil ( $p^o$ ) increases in decreasing levels of inventories. The slope of the price becomes steeper, for low levels of inventories, as the probability of a stock-out increases. Patent value and economic growth fall in decreasing inventories. Being in the stock-out, i.e. in this case, for inventory levels smaller than the deterministic steady-state level of inventory, this effect is accelerated, as shown by a steepening of the slope. This is equipollent with the finding of high oil prices dampening economic growth, a well documented empirical fact firstly highlighted by [Hamilton \(1983\)](#). Recent evidence is provided by [Casassus and Higuera \(2012\)](#) and [Kilian \(2008\)](#).

**State dependent oil usage** Oil usage as a production input establishes the key innovation to a standard endogenous growth economy. Figure 3 displays oil usage ( $o$ ) as a function of the four state variables. We vary the level of inventories ( $s$ ), to analyze conditional reactions with respect to the other three state variables. The upper panel depicts policy functions for low levels of inventories, the middle for intermediate levels, and the lower panel for high levels of inventories.<sup>6</sup> For low levels of inventories the non-negativity constraint of oil storage stated in Equation (16) is always binding. It is always optimal for the agent to exhaust all inventories. As a result, policy functions are flat. For very high levels of inventories Equation

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<sup>6</sup>Plots for oil in inventories are all the same across rows and only differ with respect to scaling.

(16) is never binding, and the agent can adjust oil usage to the state variables, optimally. Oil usage increases in the level of productivity ( $a$ ), meaning that oil usage reacts positively to good economic conditions. For positive growth opportunities an investor with a high EIS will temporarily increase oil usage and by that deplete inventories. Increased output can be consumed and reinvested in physical and oil producing capital and patents. Sure enough, oil usage is also increasing in the level of physical capital ( $k^k$ ) in place - the same logic applies. The intuition changes for oil producing capital ( $k^o$ ): Oil wells extract new oil from the ground and by that replenish inventories in the subsequent period. Therefore, oil usage increases in level of oil producing capital.

For intermediate levels of inventories Equation (16) is occasionally binding and by that a combination of the low and high inventory scenarios. Reactions for productivity and physical capital go in the same direction as for high levels of inventories, but are less pronounced. As above, oil usage increases in the level of oil producing capital, as more oil producing capital today constitutes more additional oil inventories tomorrow. This is eventually hindered by insufficient levels of contemporaneous inventories. Consequentially, the function is increasing for low levels and is flat for high levels of oil producing capital.

Lastly, in line with economic intuition, oil usage, as a function of oil in inventories, is monotonically increasing and flattens out to the right end. Holding all other variables constant, Equation (16) tends to be binding for low levels of inventories.<sup>7</sup>

**Variables in the  $\{k^o, s\}$ -space** Figure 4 depicts control variables as functions of the level oil producing capital and levels of inventories. The kink in oil usage and the oil price is clearly visible, and depends on the relative levels of inventories and oil producing capital. Investment in physical capital and oil producing capital seem to react rather smooth to the constraint. On the other hand, the value of a patent, and consequently, research intensity (investment in patents) seems to be more strongly effected by the non-negativity constraint

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<sup>7</sup>For certain combinations of values for state variables, oil usage can be decreasing in the level of productivity. Particularly, this is true for *relatively* low levels of inventories. As long as the agent is not in the constraint region, he will use *good* times to replenish inventories to reach an optimal, i.e. higher level.

of oil holdings. Research intensity reacts to the present value of patents, defined in Equation (18). Depending on the level of oil producing capital, oil usage will increase in levels of inventory and by that increase profits ( $\Pi^p$ ). In the *non-constrained* region high oil usage will, ceteris paribus, lead to lower levels of inventories in subsequent periods. The ratio of oil producing capital and inventories thus revert to an optimal level. Lower levels of inventories in turn indicate less oil in production and thus lower profits in the patent sector. Consistently, for very low levels of storage, patent value is a decreasing function of the level of oil producing capital. Oil usage is decreased to build up inventories, and as a result, profits in the patent sector will temporarily go down.

### 4.2.3 Dynamics in the Macroeconomy

After the static analysis of the policy functions presented in Figure 2 it is useful to deepen the understanding of the dynamic implications of the model setup as displayed by the stochastic impulse response functions in Figure 5. Stochastic impulse response functions are computed as log deviations from the stochastic steady state after a one-standard deviation innovation to productivity ( $a_t$ ).

**Production inputs and stocks** First, looking at growth in the R&D sector ( $\mu_t$ ), we see persistent growth over subsequent periods. Investment growth stays positive with growth in the R&D sector. This translates to persistent growth in both capital stocks. As adjustment cost and rates of depreciation for both types of capital ( $k^k, k^o$ ) are the same, both stocks grow at the same rate. Due to oil in inventories, oil usage can react immediately at the event of the shock. Hence, oil price growth caused by a rise in productivity is dampened. Caused by the two-period to build characteristic of oil in inventories, storage levels are depleted in the period after the shock. This in turn causing negative growth in oil usage for this period. These dynamics are also reflected in the ratios of oil utilization with respect to inventories and physical capital. Which determines the marginal products for production inputs. Oil usage relative to storage levels jumps up immediately after the shock and remains high over

the growth cycle. Oil usage relative to physical capital installed first jumps and then jumps down again followed by a persistent negative trend over the growth cycle.

**The evolution of prices** These movements play together to determine the evolution of oil price growth following the shock to productivity. With storage, oil prices act as nexus between intertemporal (Equation (25) and (29)) and intratemporal (Equation (28)) optimality of oil usage. In the dimension of intratemporal optimality high productivity pushes up marginal product, driving up demand of the final good producer. At the same time, induced growth will increase demand for oil in future periods. The equilibrium price will thus balance contemporaneous and future demand for oil. In standard business cycle models prices would exhibit fast mean-reversion, meaning initial positive growth followed by negative growth. This is different for our model. Prices jump up initially, but display persistent growth as the ratio of oil related variables (inventories, usage and capital) drop over the growth cycle.

**Output, consumption, and oil usage** Figure 6 depicts growth of consumption, output and oil usage. We see, that all three variables are positively correlated over all phases of the growth cycle. In the second period output growth experiences a dent caused by the drop in oil usage. By that, depletion of inventories eventually dampens growth forces via the price channel.

The upper panel of Figure 7 depicts generic time series for the ratio between *oil used* and *inventory levels* at the beginning of the period, and the ratio of *physical capital* and *oil producing capital*. Even though the ratio between the two stocks of capital remains quite stable over time, the deployment of oil varies over time. Further oil inventories are almost depleted each period as the ratio hovers only barely below 1. This is in line with the finding of [Litzenberger and Rabinowitz \(1995\)](#), who point out that large amounts of inventories can be attributed to oil in transit, i.e. pipeline, tankers and refineries. Additionally, given that the model does not account for supply shocks, agents in the economy have little incentive to

hold excessively high levels of precautionary inventories. The absence of risk in oil extraction supports the tendency to invest in oil producing capital to mitigate productivity risks. Conclusively, the majority of storage can be attributed to antagonizing the *two-period-to-build* characteristic of the oil market as holding inventories is costly while installed capital supplies a recurrent yield. The lower panel, shows that compared with innovations in output and consumption growth oil usage barely changes over time but rather traces long-run movements. This is strongly in line with above logic and is further supported by the low ratio of variation in oil usage to variation in investment of 13%. Oil supplied and used in the process of production is closely linked to oil related capital in place and thus exhibits only little variation as commonly associated with capital stocks.

## 4.3 Asset Prices

### 4.3.1 Asset Pricing Quantities

Table V, displays first and second moments for the risk-free rate, return on market, return on capital, and returns of the futures contracts on oil. Looking at panel A, the risk-free rate implied by the model is at 1.63% with a standard deviation of 0.24%. The equity premium for the (unleveled) aggregate market is at a sizeable level of 1.52%. Given the volatility of 3.02%, the Sharpe-ratio, i.e. the market price of risk, is 0.50 and by that overstates the data (0.33 %). Nevertheless, excess returns and volatilities are only one fourth of what we observe in the data. The reason for that can quite evidently be found in a mediocre autocorrelation and low standard deviation of consumption growth in conjunction with low dividend volatilities, as reported in Table III. As we do not use a generic dividend claim this is neither surprising nor atypical for returns in a production-economy. The excess return on physical capital is very low.

Moments for the futures market are reported in panel B of the table. The short-term (3-month) oil future earns a premium of 2.34%, thus accounting for about one half of the observed futures premium in the data. The model generates a term structure of futures risk

premia that is on average upward sloping, but changes signs at maturity of 9-month. The slope of the futures price curve for the 9-month future, relative to the 3-month contract, is 1.014 and by that gives a good approximation to what is observed in the data (1.009). Recursive preferences together with endogenous growth in conjunction with the cost inherent to a zero lower bound give us a major insight, on what drives premia in the futures market. For the Epstein-Zin investor covariation of an assets' cash flow with consumption growth and the state of the economy matter for asset prices. As reported above, consumption and oil price growth co-vary positively. Covariation with the state is ambiguous. High productivity, a good state, indicates high prices, yielding a positive correlation of oil prices and the discount factor. High levels of inventories, a good state as well, indicate low prices and thus a negative correlation with the discount factor. Bad growth prospects (reflected in high oil prices), make holding oil or short-term futures a hedge against low levels of inventories. Agents with recursive preferences care about continuation utility of the underlying state of the economy, and act accordingly. When productivity goes up, prices, growth prospects, and consumption rise. Short-term futures thus command a positive risk-premium. For longer maturities, this effect is reenforced by a future tightening of oil inventories, pushing up prices even further. This effect determines the term-structure of futures premia, and is closely related to the price dynamics described in section 4.2.3. A weakness of the model can be identified by very low volatility in the futures market. As it does not account for supply shocks, it leaves out an important driver of uncertainty inherent to commodities markets.

### 4.3.2 Mechanics of Asset Prices

Additional insights about the dynamics of the model are offered in Figure 8, which depicts interest rates, futures prices, futures premia, and excess return on market as functions of physical capital ( $k^k$ ) and oil in inventories ( $s$ ), respectively.



**Futures prices and risk-free rates** For *oil in inventories*, we observe a fairly linear reaction of risk free-rates for medium and high levels of storage. The inverse is true for futures prices. While futures prices fall in the level of oil in inventories, expecting oil prices to plummet due to an oversupply of oil, interest rates rise as a reaction to improved growth prospects. For low inventories, underlying mechanics are mostly driven by an increase of risk in the economy. In endowment economies the logic is as follows: Low expected growth rates and high volatility will result in a *flight-to-quality* causing high prices for the risk-free asset. In other words, a low risk-free rate. In the production set up the same logic applies for moderate and high levels of inventories. However, when the agent faces extremely low levels of inventories, and by that low output today, bad growth prospects, and high expected volatilities, an adverse reaction can be observed. The agent decides to no longer save but consume as much as possible. This finding is further supported by an intensified decrease in overall investment. In more detail, oil producing capital being held at steady state, inventories tomorrow will be replenished to be at an acceptable level again. With this in mind, it is optimal to consume as much as possible today. As a result short-term interest rates soar up. Low investment expenditures yield bad growth prospects and by that dampen this effect for longer maturities. The negative correlation of inventory levels and the risk-free rate is documented by [Casassus and Collin-Dufresne \(2005\)](#) and [Brown and Yücal \(2002\)](#), who rationalize the reaction in interest rates by greater near-term than long term effect of the supply side. Again, our model facilitates exactly this effect for the 3 month and 9 month risk-free rate. [Brown and Yücal \(1999\)](#) even link parts of the rise in the US federal funds rate in 1999 and 2000 to precisely this effect.

**Risk premia** The lower panels of Figure 8 display expected excess returns for futures. For *physical capital*, short-term futures premia react more pronounced to increases in the capital stock, than long-term futures. This can be rationalized by a ceteris paribus increased physical to oil related capital ratio, causing high oil prices in the near future and convergence of stocks in the long-run, lowering oil prices. Consequently, the return of short-term futures

is affected more strongly. At the same time futures prices rise in the level of physical capital as high levels of capital nurture economic growth. Finally, for *oil in inventory*, expected premia for market capital increase in the level of inventories. We attribute this to the massive increase in interest rates close to the stock-out. This effect will not translate to longer maturities as the risk-free rate for longer maturities is a lot less exposed to inventory risk. The figure further illustrates a change in the slope of the term structure of futures premia. Low levels of inventories, yield a downward sloping term structure and high levels an upward sloping term structure. The intuition behind this mechanism goes as follows: For low levels of inventories oil prices (and by that short-term futures) are more exposed to short-run risks and by that command a higher risk-premium. Recent evidence for sign changes in the slope of the term structure is provided by [Hamilton and Wu \(2014\)](#). This result can also be interpreted, as reflecting the equilibrium forces in the economy highlighted in section [4.2.2](#). Low inventories are undesirable for agents in the economy. As oil inventories dwindle, the spot or short-term futures need to pay a higher premium, in order to attract new investment in the stock of inventories. As discussed above, in equilibrium the economy tends to have high levels of inventories such that the term structure is on average upward sloping. As the deterministic steady state level of inventories is significantly lower than for the stochastic steady state, evaluating the term structure at the deterministic steady state level will result in an downward sloping term structure, as discussed above. A further illustration for maturities from 3 to 12 months is given in the mid panel of [Figure 9](#).

**The convenience yield** The right panel of [Figure 9](#) presents the convenience yield, implied by the model. The convenience yield rises in low levels of inventories. This is the major result implied by the theory of storage. To name only a few, this relationship is empirically assessed by [Casassus and Collin-Dufresne \(2005\)](#), [Routledge, Seppi, and Spatt \(2000\)](#), and [Litzenberger and Rabinowitz \(1995\)](#). The impulse response for the convenience yield in [Figure 10](#) supports this evidence, as the convenience yield goes up over the growth cycle. High levels of productivity, together with sustained low levels of inventories relative

to oil utilization cause an increased convenience yield over its stochastic steady state. This goes in log-step with the dynamics of price levels for the oil spot as discussed in subsection 4.2.

**Term structures** For a visualization of the term structure of futures for maturities from 3 to 18 months we turn to Figure 11, which displays the average term structure of futures prices, futures premia, and futures return volatility, as implied by the model and empirical data. Firstly, the term structure of futures prices is mainly downward sloping, which is in line with [Casassus, Collin-Dufresne, and Routledge \(2009\)](#), who find that the term structure of futures is in backwardation 63% of the time. Secondly, and most noteworthy, as stated above, futures premia are on average increasing in time to maturity, and most noteworthy, display a hump for the 9-month future. This is true for both, model and data. Finally, the term structure of volatility is downward sloping on average for futures returns. Nevertheless, our model exhibits vastly too little volatility.

To understand, why the term structure of futures premia is upward sloping and hump shaped, we can again turn to Figure 5 and the dynamics of oil prices over the growth cycle. Persistent growth of prices over the growth cycle indicates more priced risk on the long end of the futures curve, i.e a higher covariation with the pricing kernel. This intuition finds support in Table VI. Covariances are increasing in the maturities of the futures contracts. Finally, long-run futures are effected by mean-reversion properties of the growth-cycle and thus display lower volatility and decreasing premia. Empirical counterparts are reported in Table VII. Lastly, Figure 12 plots generic time series for the weak backwardation and futures prices with maturities of 3 and 12 months. The model generates the co-movement of the weak backwardation with the price of the spot as underlined by [Litzenberger and Rabinowitz \(1995\)](#).

## 5 Inspecting the Mechanism

In order to work out the fine differences to a model without storage we additionally solve two models, in which we shut off the storage channel in two steps. First, we consider a model where we do not allow any excess storage (henceforth, TtB). This meaning that Equation (16) is now always binding. Closing this channel, still leaves us with the *time-to-build* constraint, as oil producing capital needs to be installed first, leaving us with oil being extracted in period  $t + 1$ , making oil available on the market in  $t + 2$ . Second, we consider another model, where we additionally shut off the aforementioned two period *time-to-build* constraint (henceforth, NoS). In the context of our model this implies one capital good producing the oil input just in time. Particularly, with our calibration, setting the extraction rate  $\tau = 1$  and homogeneous laws of motion, this boils down to a standard production model with two capital inputs. We also present a business cycle calibration of our model to clarify the impact of the endogenous growth channel, i.e. shutting off the innovation and patent sector.

**Tightening the constraint** Table VIII compares macroeconomic quantities of the three models. At first glance, shutting off channels of storage does not seem to alter model results significantly. Thus, in a model allowing for excess storage standard general equilibrium implications seem to subsist. However, technically speaking, moving from Full to *TtB* model, we turn a constraint with slack to a constraint with no slack. This in turn tightening the opportunity set. We see this reflected in: firstly, a higher volatility of consumption growth for the constraint model, secondly, lower volatility in overall investment growth, and particularly for R&D. Thirdly, this going hand in hand with lower expected output growth for the *TtB* model. Finally, moving to a standard model as in NoS the rational remains unchanged. The effect of the change in the constraint is visualized in Figure 13. In the model without the occasionally binding constraint all displayed variables change from kinked to fairly linear functions of inventory levels. This in fact indicates, that the non-negativity constraint introduces important nonlinearities into the model.

**Term structure implications** Figures 11, 14, and 15 depict term structures for the three models. Starting the analysis for the *NoS* model term structures for futures prices and volatilities remain unchanged. Then again, the term structure of futures premia is right out downward sloping. This is a standard result for the majority of asset pricing models. Moving to the *TtB* model, the term structure of futures premia is humped at  $T = 6$ , caused by the following rationale. When the exogenous level of productivity in the economy changes, physical capital can be adjusted for the following period, while adjustment of oil in production takes two periods. Keeping this in mind, the price effect of a positive shock will thus be amplified by faster adjusting physical capital in  $T = 3$ . The relative usage of oil compared to physical capital decreases, driving up the premium of the 6-month future. For the *Full*-model excess inventories re-enforce this effect as discussed in Section (4.2.3). Table IX depicts asset pricing implications for the additional calibrations.

**The growth channel** Table X compares moments of the Full model and the business cycle calibration. Holding all parameters constant, the business cycle calibration generates a higher volatility of consumption growth relative to the volatility of output growth. As we want to analyze the impact on asset pricing quantities, we adjust the volatility of labor augmenting productivity downward, to match the consumption growth volatility of the Full model and the data. Secondly, we set exogenous growth to match the growth rate, implied by the Full model. The most notable difference to the Full model is, firstly, a lower volatility of the growth rate of oil usage relative to the volatility output growth. This means that oil usage reacts more pronounced to economic conditions in the Full model. Note that output is only about half as volatile, as in the Full model (1.40%, 2.38%). Secondly, oil usage is procyclical in the Full model and countercyclical in the business cycle calibration. Finally, autocorrelation of consumption growth is zero. The last two indicate that for the Full model, oil is used to boost the growth cycle, while in the business cycle calibration oil is rather used as a hedge against economic downturns. Table XI displays asset pricing quantities for both models. Firstly, risk premia for capital in the business cycle calibration are essentially zero.

Secondly, futures premia are in the same ballpark as in the model exhibiting endogenous growth. Surprising at first sight, this can be explained by the high risk-free rate. As long as oil prices are procyclical, their expected rate of return must be above the return on a risk-free asset. Lastly, futures premia do not reveal any systematic pattern.

**Dynamic responses of all four models** Important differences in all four models are displayed in Figure 16. The TtB model cannot adjust oil usage immediately, but requires two periods to adjust oil usage, as it is equal to the level of storage. The NoS model can adjust after only one period. As oil is characterized like standard physical capital, their ratio does not change over time. Finally, in the business cycle calibration oil usage is reduced upon a positive innovation to productivity. Oil usage relative to inventories remains below its steady state level as the productivity shock fades out. As described above, the reverse is true for the Full model.

## 6 Conclusion

We embed a decision of commodities storage into a general equilibrium framework with endogenous growth mechanisms in the spirit of [Comin and Gertler \(2006\)](#), and [Kung and Schmid \(2015\)](#). Transitory shocks to technology root sustained economic growth. Agents endowed with recursive preferences care about long-term growth prospects. The non-negativity constraint, inherent in modelling commodities storage, is modeled explicitly and accounted for by solving the model with a non-linear global solution algorithm. Stock-out costs arise endogenously as a result of the constraint.

The model reproduces and rationalizes several empirical findings of the literature. The main findings can be summarized as follows. Firstly, oil price behaviour is explained as a function of inventories. We find that low levels of inventories negatively affect growth prospects through this price channel. Secondly, for asset prices, the negative co-movement of the risk-free rate and oil inventories can be reproduced. Moreover, the term structure of

futures prices, and premia, and the behavior of the convenience yield can be traced back to first order principles: *preferences, technology and resource constraints* as promoted by [Lucas \(1976\)](#). In models prohibiting excess storage this result cannot be replicated. Further, the growth channel and associated risks are vital to induce a motive for excess storage in a representative agents framework. Identifying important risk factors and their effects at the intersection of the oil sectors and the rest of the economy in a structural model gives important insights for empirical research and the conduct of economic policy.

For tractability, we only model demand shocks and omit shocks to the supply side of oil. Even though this already gives many new insights, it would still be interesting to additionally capture the effects of supply shocks. Finally, a huge strand of literature discusses the connection of unemployment and oil prices, which gives rise to including a decision on labor into this framework, see for example [Brown and Yücal \(2002\)](#), [Loungani and Yücel \(2000\)](#), and [Carruth, Hooker, and Oswald \(1998\)](#). Both of the above go beyond the scope of this paper and we leave these endeavours up to future research.

# A Model Derivation

## A.1 Households

The Agent has Epstein Zin Preferences

$$\mathcal{U}_t = \left[ (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta \mathcal{R}_t^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \quad (\text{A.1})$$

$$\mathcal{R}_t = E_t \left[ \mathcal{U}_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \quad (\text{A.2})$$

Budget Constraint

$$C_t + P_t \times \mathcal{K}_{t+1}^m + B_{t+1} = W_t \times L_t + (P_t + D_t) \times \mathcal{K}_t^m + e^{r \cdot \Delta t} B_t \quad (\text{A.3})$$

Resulting Pricing Kernel

$$\mathcal{M}_{t,t+1} = \frac{\frac{\partial \mathcal{U}_t}{\partial C_{t+1}}}{\frac{\partial \mathcal{U}_t}{\partial C_t}} \quad (\text{A.4})$$

$$= \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{\mathcal{U}_{t+1}}{\mathcal{R}_t} \right)^{\frac{1}{\psi} - \gamma} \quad (\text{A.5})$$

Households inter-temporal Optimality

$$1 = E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^i \right] \quad (\text{A.6})$$

## A.2 Firms

As this does not alter results, derivation are performed for one representative firm.

### A.2.1 Final Good

**Final consumption good** Goods  $Y_t$  produced in perfectly competitive market. Cobb-Douglas Production Technology for composite of capital and oil  $\mathcal{K}\mathcal{O}_t$ , labor  $N_t$  and patents  $G_t$ .

$$Y_t = \left( \mathcal{K}\mathcal{O}_t^\alpha (A_t N_t)^{1-\alpha} \right)^{1-\omega_p} G_t^{\omega_p} \quad (\text{A.7})$$

$$G_t = \left[ \int_{i \in K_t^p} X_{i,t}^\nu di \right]^{\frac{1}{\nu}} \quad (\text{A.8})$$

$X_{i,t}$  quantity of patent  $i \in [0, K_t^p]$



## Productivity

$$A_t = e^{a_t} \quad (\text{A.9})$$

$$a_t = (1 - \rho)a^* + \rho a_{t-1} + \sigma_{t-1} \varepsilon_t^a \quad (\text{A.10})$$

$$\varepsilon_t^a \stackrel{iid}{\sim} \mathcal{N}(0, 1) \quad (\text{A.11})$$

**Aggregating Capital and Oil in Production** Firms use capital and oil in the process of production:

$$\mathcal{K}\mathcal{O}_t = f(K_t^k, O_t) = \left( \left( (1 - \omega_o)^{\frac{1}{\xi_{ko}}} (K_t^k)^{1 - \frac{1}{\xi_{ko}}} + \omega_o^{\frac{1}{\xi_{ko}}} O_t^{1 - \frac{1}{\xi_{ko}}} \right)^{\frac{1}{1 - \frac{1}{\xi_{ko}}}} \right) \quad (\text{A.12})$$

With oil in production being less or equal the level of inventories

$$O_t \leq S_t \quad (\text{A.13})$$

## Problem of the Manager (Aggregate Firm)

$$\mathcal{J}_t = \max_{\{O_t, I_t^k, I_t^o, K_{t+1}^k, K_{t+1}^o, S_{t+1}, X_{j,t}\}_{i \geq 0, j \in [0, K_t^p]}} E_t [\sum_{i=0}^{\infty} \mathcal{M}_{t,t+i} D_{t+i}] \quad (\text{A.14})$$

Dividends are defined in the usual way

$$D_t = Y_t - I_t^k - I_t^o - W_t L_t - \int_{i \in K_t^p} P_{i,t}^p X_{i,t} di \quad (\text{A.15})$$

## Laws of Motion :

Capital law of motion:

$$K_{t+1}^k = (1 - \delta_k) K_t^k + \Phi \left( \frac{I_t^k}{K_t^k} \right) K_t^k \quad (\text{A.16})$$

$$\Phi_k \left( \frac{I_t^k}{K_t^k} \right) = \alpha_{2,k} + \frac{\alpha_{1,k}}{1 - \frac{1}{\delta_k}} \left( \frac{I_t^k}{K_t^k} \right)^{1 - \frac{1}{\delta_k}} \quad (\text{A.17})$$

Oil producing capital law of motion:

$$K_{t+1}^o = (1 - \delta_o) K_t^o + \Phi \left( \frac{I_t^o}{K_t^o} \right) K_t^o \quad (\text{A.18})$$

$$\Phi_o \left( \frac{I_t^o}{K_t^o} \right) = \alpha_{2,o} + \frac{\alpha_{1,o}}{1 - \frac{1}{\delta_o}} \left( \frac{I_t^o}{K_t^o} \right)^{1 - \frac{1}{\delta_o}} \quad (\text{A.19})$$

Inventories in stock evolve according to:

$$S_{t+1} = (1 - \delta_s) (S_t - O_t) + \tau K_t^o \quad (\text{A.20})$$

## A.2.2 Intangible Good Sector

Monopolists produce patents and solve

$$\Pi_{j,t}^p = \max_{P_{j,t}} P_{j,t} X_{j,t}(P_{j,t}) - X_{j,t}(P_{j,t}) \quad (\text{A.21})$$

Present value of a firm holding the patent i:

$$\mathcal{J}_t^{(p,j)} = \Pi_{j,t}^p + (1 - \delta_p) E_t \left[ \mathcal{M}_{t,t+1} \mathcal{J}_{t+1}^{(p,j)} \right] \quad (\text{A.22})$$

## A.2.3 R & D Sector

Develop patents and sell to monopolist patent producers

**Law of motion for capital in R & D Sector :**

$$K_{t+1}^p = \chi_t I_t^p + (1 - \delta_p) K_t^p \quad (\text{A.23})$$

$$\chi_t = \frac{\bar{\chi} K_t^p}{(I_t^p)^{1-\eta} (K_t^p)^\eta} \quad (\text{A.24})$$

## A.3 The Competitive Equilibrium

Lagrange multiplier  $\lambda_t^k$ ,  $\lambda_t^o$ ,  $\lambda_t^s$ , and  $\lambda_t$  : First Order Conditions and Constraints

$$\mathcal{J}_t^p : \quad \mathcal{J}_t^p - \Pi_t^p - (1 - \delta_p) E_t \left[ \mathcal{M}_{t,t+1} \mathcal{J}_{t+1}^p \right] \stackrel{!}{=} 0 \quad (\text{A.25})$$

$$I_t^p : \quad I_t^p - E_t \left[ \mathcal{M}_{t,t+1} \mathcal{J}_{t+1}^p \right] (K_{t+1}^p - (1 - \delta_p) K_t^p) \stackrel{!}{=} 0 \quad (\text{A.26})$$

$$I_t^k : \quad \phi^{k'} \left( \frac{I_t^k}{K_t^k} \right) \lambda_t^k - 1 \stackrel{!}{=} 0 \quad (\text{A.27})$$

$$I_t^o : \quad \phi^{o'} \left( \frac{I_t^o}{K_t^o} \right) \lambda_t^o - 1 \stackrel{!}{=} 0 \quad (\text{A.28})$$

$$O_t : \quad \frac{\partial Y_t}{\partial O_t} - (1 - \delta_s) \lambda_t^s - \lambda_t \stackrel{!}{=} 0 \quad (\text{A.29})$$

$$K_{t+1}^k : \quad E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^k \right] - \lambda_t^k \stackrel{!}{=} 0 \quad (\text{A.30})$$

$$K_{t+1}^o : \quad E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^o \right] - \lambda_t^o \stackrel{!}{=} 0 \quad (\text{A.31})$$

$$S_{t+1} : \quad E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^s \right] - \lambda_t^s \stackrel{!}{=} 0 \quad (\text{A.32})$$

$$\lambda_t^k : \quad - \left( K_{t+1}^k - K_t^k (1 - \delta_k) - \phi^k \left( \frac{I_t^k}{K_t^k} \right) K_t^k \right) \stackrel{!}{=} 0 \quad (\text{A.33})$$

$$\lambda_t^o : \quad - \left( K_{t+1}^o - K_t^o (1 - \delta_o) - \phi^o \left( \frac{I_t^o}{K_t^o} \right) K_t^o \right) \stackrel{!}{=} 0 \quad (\text{A.34})$$

$$\lambda_t^s : \quad - (S_{t+1} - (S_t - O_t)(1 - \delta_s) - \phi^s(K_t^o)) \stackrel{!}{=} 0 \quad (\text{A.35})$$

Kuhn-Tucker Condition:

$$\lambda_t : \lambda_t(\theta, S)(O_t - S_t) = 0 \quad (\text{A.36})$$

Returns:

$$R_{t+1}^k = \frac{\frac{\partial f(K_{t+1}^k, O_{t+1}, G_{t+1}, A_{t+1})}{\partial K_{t+1}^k} + \lambda_{t+1}^k \left( (1 - \delta_k) - \phi^{k'} \left( \frac{I_{t+1}^k}{K_{t+1}^k} \right) \frac{I_{t+1}^k}{K_{t+1}^k} + \phi^k \left( \frac{I_{t+1}^k}{K_{t+1}^k} \right) \right)}{\lambda_t^k} \quad (\text{A.37})$$

$$R_{t+1}^o = \frac{\lambda_{t+1}^s \frac{\partial \phi^s(K_{t+1}^o)}{\partial K_{t+1}^o} + \lambda_{t+1}^o \left( (1 - \delta_o) - \phi^{o'} \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi^o \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right)}{\lambda_t^o} \quad (\text{A.38})$$

$$R_{t+1}^s = \frac{\lambda_{t+1}^s(1 - \delta_s) + \lambda_{t+1}}{\lambda_t^s} \quad (\text{A.39})$$

The corresponding derivatives are

$$\phi^{k'} \left( \frac{I_t^k}{K_t^k} \right) = a_1^k \left( \frac{I_t^k}{K_t^k} \right)^{-\frac{1}{\xi_k}} \quad (\text{A.40})$$

$$\phi^{o'} \left( \frac{I_t^o}{K_t^o} \right) = a_1^o \left( \frac{I_t^o}{K_t^o} \right)^{-\frac{1}{\xi_o}} \quad (\text{A.41})$$

$$\frac{\partial \phi^s(K_t^o)}{\partial K_t^o} = \tau \quad (\text{A.42})$$

$$\frac{\partial Y_t}{\partial K_t^k} = \alpha(1 - \omega_p) \frac{Y_t}{\mathcal{K}O_t} \left( \frac{\omega_o}{K_t^k} \mathcal{K}O_t \right)^{\frac{1}{\xi_{ko}}} \quad (\text{A.43})$$

$$\frac{\partial Y_t}{\partial O_t^k} = \alpha(1 - \omega_p) \frac{Y_t}{\mathcal{K}O_t} \left( \frac{1 - \omega_o}{O_t} \mathcal{K}O_t \right)^{\frac{1}{\xi_{ko}}} \quad (\text{A.44})$$

$$\mathcal{K}O_t = \left( (1 - \omega_o)^{\frac{1}{\xi_{ko}}} (K_t^k)^{1 - \frac{1}{\xi_{ko}}} + \omega_o^{\frac{1}{\xi_o}} O_t^{1 - \frac{1}{\xi_{ko}}} \right)^{\frac{1}{1 - \frac{1}{\xi_{ko}}}} \quad (\text{A.45})$$

## A.4 Equilibrium

Combining this we obtain the collected equilibrium conditions. We assume  $\alpha = 1 - \frac{\omega_p - \omega_p}{1 - \omega_p}$  for stable growth.

$$\bar{A} \equiv (\omega_p \nu)^{\frac{\omega_p}{1 - \omega_p}} \quad (\text{A.46})$$

$$a_t = (1 - \rho)a^* + \rho a_{t-1} + \sigma \varepsilon_t^a \quad (\text{A.47})$$

$$Z_t = e^{a_t} K_t^p \quad (\text{A.48})$$

$$\mathcal{M}_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{\mathcal{U}_{t+1}}{\mathcal{R}_t} \right)^{\frac{1}{\psi} - \gamma} \quad (\text{A.49})$$

$$\mathcal{U}_t = \left( (1 - \beta) C_t^{1 - \frac{1}{\gamma}} + \beta \mathcal{R}_t^{1 - \frac{1}{\gamma}} \right)^{\frac{1}{1 - \frac{1}{\gamma}}} \quad (\text{A.50})$$

$$\mathcal{R}_t = E_t \left[ \mathcal{U}_{t+1}^{1 - \gamma} \right]^{\frac{1}{1 - \gamma}} \quad (\text{A.51})$$

$$Y_t = \mathcal{K} \mathcal{O}_t^\alpha Z_t^{1 - \alpha} \bar{A} \quad (\text{A.52})$$

$$C_t = Y_t - I_t^k - I_t^o - I_t^p - X_t \quad (\text{A.53})$$

$$\Pi_t^p = \left( \frac{1}{\nu} - 1 \right) \bar{X}_t \quad (\text{A.54})$$

$$\bar{X}_t = \omega_p \nu e^{a_t(1 - \alpha)} \left( \frac{\mathcal{K} \mathcal{O}_t}{K_t^p} \right)^\alpha \quad (\text{A.55})$$

$$\mathcal{J}_t^p = \Pi_t^p + (1 - \delta^p) E \left[ \mathcal{M}_{t,t+1} \mathcal{J}_{t+1}^p | \mathcal{F}_t \right] \quad (\text{A.56})$$

$$I_t^p = E_t \left[ \mathcal{M}_{t,t+1} \mathcal{J}_{t+1}^p \right] (K_{t+1}^p - (1 - \delta_p) K_t^p) \quad (\text{A.57})$$

$$\mathcal{K} \mathcal{O}_t = \left( (1 - \omega_o)^{\frac{1}{\xi_{ko}}} (K_t^k)^{1 - \frac{1}{\xi_{ko}}} + \omega_o^{\frac{1}{\xi_{ko}}} O_t^{1 - \frac{1}{\xi_{ko}}} \right)^{\frac{1}{1 - \frac{1}{\xi_{ko}}}} \quad (\text{A.58})$$

$$\frac{\partial Y_t}{\partial K_t^k} = \alpha (1 - \omega_p) \frac{Y_t}{\mathcal{K} \mathcal{O}_t} \left( \frac{\omega_o}{K_t^k} \mathcal{K} \mathcal{O}_t \right)^{\frac{1}{\xi_{ko}}} \quad (\text{A.59})$$

$$\frac{\partial Y_t}{\partial O_t^k} = \alpha (1 - \omega_p) \frac{Y_t}{\mathcal{K} \mathcal{O}_t} \left( \frac{1 - A_o}{O_t} \mathcal{K} \mathcal{O}_t \right)^{\frac{1}{\xi_{ko}}} \quad (\text{A.60})$$

$$K_{t+1}^k = K_t^k (1 - \delta_k) + \phi^k \left( \frac{I_t^k}{K_t^k} \right) K_t^k \quad (\text{A.61})$$

$$K_{t+1}^o = K_t^o (1 - \delta_o) + \phi^o \left( \frac{I_t^o}{K_t^o} \right) K_t^o \quad (\text{A.62})$$

$$K_{t+1}^p = \chi_t I_t^p + (1 - \delta_p) K_t^p \quad (\text{A.63})$$

$$\chi_t = \frac{\bar{\chi} K_t^p}{(I_t^p)^{1 - \eta} (K_t^p)^\eta} \quad (\text{A.64})$$

$$S_{t+1} = (S_t - O_t)(1 - \delta_s) + \phi^s (K_t^s) \quad (\text{A.65})$$

$$1 = E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^k \right] \quad (\text{A.66})$$

$$1 = E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^o \right] \quad (\text{A.67})$$

$$1 = E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^s \right] \quad (\text{A.68})$$

$$\phi^{k'} \left( \frac{I_t^k}{K_t^k} \right) = a_1^k \left( \frac{I_t^k}{K_t^k} \right)^{-\frac{1}{\xi_k}} \quad (\text{A.69})$$

$$\phi^{o'} \left( \frac{I_t^o}{K_t^o} \right) = a_1^o \left( \frac{I_t^o}{K_t^o} \right)^{-\frac{1}{\xi_o}} \quad (\text{A.70})$$

$$\frac{\partial \phi^s (K_t^s)}{\partial K_t^s} = \tau \quad (\text{A.71})$$

$$\lambda_t^k = \frac{1}{\phi^{k'}\left(\frac{I_t^k}{K_t^k}\right)} \quad (\text{A.72})$$

$$\lambda_t^o = \frac{1}{\phi^{k'}\left(\frac{I_t^o}{K_t^o}\right)} \quad (\text{A.73})$$

$$\lambda_t = \frac{\partial Y_t}{\partial O_t} - (1 - \delta_s)\lambda_t^s \quad (\text{A.74})$$

$$R_{t+1}^k = \frac{\frac{\partial Y_{t+1}}{\partial K_{t+1}^k} + \lambda_{t+1}^k \left( (1 - \delta_k) - \phi^{k'}\left(\frac{I_{t+1}^k}{K_{t+1}^k}\right) \frac{I_{t+1}^k}{K_{t+1}^k} + \phi^k\left(\frac{I_{t+1}^k}{K_{t+1}^k}\right) \right)}{\lambda_t^k} \quad (\text{A.75})$$

$$R_{t+1}^o = \frac{\lambda_{t+1}^s \frac{\partial \phi^s(K_{t+1}^o)}{\partial K_{t+1}^o} + \lambda_{t+1}^o \left( (1 - \delta_o) - \phi^{o'}\left(\frac{I_{t+1}^o}{K_{t+1}^o}\right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi^o\left(\frac{I_{t+1}^o}{K_{t+1}^o}\right) \right)}{\lambda_t^k} \quad (\text{A.76})$$

$$R_{t+1}^s = \frac{\lambda_{t+1}^s(1 - \delta_s) + \lambda_{t+1}}{\lambda_t^s} \quad (\text{A.77})$$

Detrending and logs

$$\bar{a} \equiv \left( \frac{\omega_p}{1 - \omega_p} \right) \log((\omega_p \nu)) \quad (\text{A.78})$$

$$a_t = (1 - \rho)a^* + \rho a_{t-1} + \sigma_{t-1} \varepsilon_t^a \quad (\text{A.79})$$

$$\bar{z}_t = a_t \quad (\text{A.80})$$

$$e^{\mu_t} = \chi_t \hat{I}_t^p - (1 - \delta_p) \quad (\text{A.81})$$

$$\mathcal{M}_{t,t+1} = \beta e^{-\frac{1}{\psi} \Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right) (g_{t+1} + \log(\hat{U}_{t+1}) - \log(\hat{\mathcal{R}}_t))} \quad (\text{A.82})$$

$$\hat{U}_t = \left( (1 - \beta) \hat{C}_t^{1 - \frac{1}{\gamma}} + \beta \hat{\mathcal{R}}_t^{1 - \frac{1}{\gamma}} \right)^{\frac{1}{1 - \frac{1}{\gamma}}} \quad (\text{A.83})$$

$$\mathcal{R}_t = E \left[ e^{(1 - \gamma)(\mu_t + \log(\hat{U}_{t+1}))} | \mathcal{F}_t \right]^{\frac{1}{1 - \gamma}} \quad (\text{A.84})$$

$$\hat{Y}_t = \mathcal{K} \hat{\mathcal{O}}_t^\alpha \hat{Z}_t^{1 - \alpha} \bar{A} \quad (\text{A.85})$$

$$\hat{C}_t = \hat{Y}_t - \hat{I}_t^k - \hat{I}_t^o - \hat{I}_t^p \hat{X}_t \quad (\text{A.86})$$

$$\hat{\Pi}_t^p = \left( \frac{1}{\nu} - 1 \right) \hat{X}_t \quad (\text{A.87})$$

$$\hat{X}_t = \omega_p \nu e^{a_t(1 - \alpha)} \mathcal{K} \hat{\mathcal{O}}_t^\alpha \quad (\text{A.88})$$

$$\mathcal{J}_t^p = \Pi_t^p + (1 - \delta^p) E [\mathcal{M}_{t,t+1} \mathcal{J}_{t+1}^p] \quad (\text{A.89})$$

$$I_t^p = E [\mathcal{M}_{t,t+1} \mathcal{J}_{t+1}^p] (K_{t+1}^p - (1 - \delta_p) K_t^p) \quad (\text{A.90})$$

$$\mathcal{K} \hat{\mathcal{O}}_t = \left( (1 - \omega_o)^{\frac{1}{\xi_{ko}}} (\hat{K}_t^k)^{1 - \frac{1}{\xi_{ko}}} + A_o^{\frac{1}{\xi_{ko}}} \hat{O}_t^{1 - \frac{1}{\xi_{ko}}} \right)^{\frac{1}{1 - \frac{1}{\xi_{ko}}}} \quad (\text{A.91})$$

$$\frac{Y_t}{\partial K_t^k} = \alpha (1 - \omega_p) \frac{\hat{Y}_t}{\mathcal{K} \hat{\mathcal{O}}_t} \left( \frac{\omega_o}{\hat{K}_t^k} \mathcal{K} \hat{\mathcal{O}}_t \right)^{\frac{1}{\xi_{ko}}} \quad (\text{A.92})$$

$$\frac{\partial Y_t}{\partial O_t^k} = \alpha (1 - \omega_p) \frac{\hat{Y}_t}{\hat{K}O_t} \left( \frac{1 - \omega_o}{\hat{O}_t} \hat{K}O_t \right)^{\frac{1}{\varepsilon_{ko}}} \quad (\text{A.93})$$

$$\hat{k}_{t+1}^k = \hat{k}_t^k + \log \left( (1 - \delta_k) + \phi^k \left( \frac{\hat{I}_t^k}{\hat{K}_t^k} \right) \right) - \mu_t \quad (\text{A.94})$$

$$\hat{k}_{t+1}^o = \hat{k}_t^o + \log \left( (1 - \delta_o) + \phi^o \left( \frac{\hat{I}_t^o}{\hat{K}_t^o} \right) \right) - \mu_t \quad (\text{A.95})$$

$$\hat{\lambda}_t = \frac{\bar{\lambda}}{(\hat{I}_t^n)^{1-\eta}} \quad (\text{A.96})$$

$$\hat{s}_{t+1} = \log \left( (\hat{S}_t - \hat{O}_t)(1 - \delta_s) + \phi^s \left( \hat{K}_t^o \right) \right) - \mu_t \quad (\text{A.97})$$

$$1 = E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^k \right] \quad (\text{A.98})$$

$$1 = E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^o \right] \quad (\text{A.99})$$

$$1 = E_t \left[ \mathcal{M}_{t,t+1} R_{t+1}^s \right] \quad (\text{A.100})$$

$$\phi^{k'} \left( \frac{\hat{I}_t^k}{\hat{K}_t^k} \right) = a_1^k \left( \frac{\hat{I}_t^k}{\hat{K}_t^k} \right)^{-\frac{1}{\varepsilon_k}} \quad (\text{A.101})$$

$$\phi^{o'} \left( \frac{\hat{I}_t^o}{\hat{K}_t^o} \right) = a_1^o \left( \frac{\hat{I}_t^o}{\hat{K}_t^o} \right)^{-\frac{1}{\varepsilon_o}} \quad (\text{A.102})$$

$$\frac{\partial \phi^s \left( \hat{K}_t^o \right)}{\partial \hat{K}_t^o} = \tau \quad (\text{A.103})$$

$$\lambda_t^k = \frac{1}{\phi^{k'} \left( \frac{\hat{I}_t^k}{\hat{K}_t^k} \right)} \quad (\text{A.104})$$

$$\lambda_t^o = \frac{1}{\phi^{o'} \left( \frac{\hat{I}_t^o}{\hat{K}_t^o} \right)} \quad (\text{A.105})$$

$$\lambda_t = \frac{\partial \hat{Y}_t}{\partial \hat{O}_t} - (1 - \delta_s) \lambda_t^s \quad (\text{A.106})$$

$$R_{t+1}^k = \frac{\frac{\partial Y_{t+1}}{\partial K_{t+1}^k} + \lambda_{t+1}^k \left( (1 - \delta_k) - \phi^{k'} \left( \frac{I_{t+1}^k}{K_{t+1}^k} \right) \frac{j_{t+1}^k}{K_{t+1}^k} + \phi^k \left( \frac{I_{t+1}^k}{K_{t+1}^k} \right) \right)}{\lambda_t^k} \quad (\text{A.107})$$

$$R_{t+1}^o = \frac{\lambda_{t+1}^s \frac{\partial \phi^s(K_{t+1}^o)}{\partial K_{t+1}^o} + \lambda_{t+1}^o \left( (1 - \delta_o) - \phi^{o'} \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{j_{t+1}^o}{K_{t+1}^o} + \phi^o \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right)}{\lambda_t^o} \quad (\text{A.108})$$

$$R_{t+1}^s = \frac{\lambda_{t+1}^s (1 - \delta_s) + \lambda_{t+1}}{\lambda_t^s} \quad (\text{A.109})$$

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	Model		
	Full	TtB	NoS
<b>Approximation Order</b>	4.4.9.9	4.4.9.9	4.4.7
<b>Physical Capital</b>			
$\log_{10}(\ EEE\ _{\infty})$	-3.07	-5.89	-6.50
$\log_{10}(\ EEE\ _1)$	-4.04	-6.43	-6.79
$\log_{10}(\ EEE\ _1^{simul})$	-4.05	-6.70	-6.73
<b>Oil Producing Capital</b>			
$\log_{10}(\ EEE\ _{\infty})$	-2.91	-5.72	-6.44
$\log_{10}(\ EEE\ _1)$	-3.94	-6.11	-6.75
$\log_{10}(\ EEE\ _1^{simul})$	-3.89	-6.24	-6.73
<b>Oil in Inventories</b>			
$\log_{10}(\ EEE\ _{\infty})$	-1.92	-4.40	-
$\log_{10}(\ EEE\ _1)$	-3.01	-4.78	-
$\log_{10}(\ EEE\ _1^{simul})$	-2.98	-4.83	-

**Table I:** Euler Equation Errors (All Growth Models)

This table depicts Euler equation errors for all three growth models. Full is the benchmark model, TtB the *time-to-build* model, and NoS the *no storage* model. The top panel depicts Euler equation errors for the return on physical capital. The mid panel depicts Euler equation errors for the return on oil related capital. The bottom panel depicts Euler equation errors for the return on oil in inventories. Euler equation errors are reported with respect to the  $\mathcal{L}^1$ -norm for the mean error on the grid and simulation. Errors reported are in order, the maximum error on the grid ( $\log_{10}(\|EEE\|_{\infty})$ ), average error on the grid ( $\log_{10}(\|EEE\|_1)$ ), and average error for a sample path of 10,000 periods ( $\log_{10}(\|EEE\|_1^{simul})$ ).

Parameter	Description	Value	Target/Reference
$\beta^4$	Subjective discount rate	0.986	Risk-free rate
$\psi$	EIS	1.5	Bansal and Yaron (2004)
$\gamma$	Risk aversion	10	Kung and Schmid (2015)
$\alpha$	Capital share	0.35	Kung and Schmid (2015)
$\xi_k$	Adjustment cost for capital	1.43	$\sigma(\Delta i^k)$
$\delta_k \times 4$	Depreciation rate of capital	8%	Kung and Schmid (2015)
$\omega_p$	Patent share	0.5	Comin and Gertler (2006)
$\nu$	Inverse markup	0.61	Kung and Schmid (2015)
$\delta_p \times 4$	Patent obsolescence rate	15%	Kung and Schmid (2015)
$\eta$	Elasticity of new patents wrt R&D	0.83	Griliches (1990)
$\bar{\chi}$	Scale parameter	0.325	$E[\mu]$
$\tau$	Oil extraction rate	1	
$\omega_{ko}$	Weight of oil in production	0.5	Backus and Crucini (2000)
$\xi_{ko}$	Elasticity of substitution capital/oil	0.4	Kim and Loungani (1992)
$\delta_s \times 4$	Cost of storage on commodity	8%	Focus on storage
$\xi_o$	Adjustment cost for oil capital	1.43	Focus on storage
$\delta_o \times 4$	Depreciation rate of oil capital	8%	Focus on storage
$\rho^4$	Autocorrelation	0.95	$\rho(I^p/K^p)$
$\sigma \times \sqrt{4}$	Volatility of technology shock	3.4%	$\sigma(\Delta y)$

**Table II:** Parameters (Full Model)

This table depicts all parameter values characterizing the Full model calibration. All parameters are annualized. The upper panel describes the preferences of the household. The following panels describe in order parameters governing the production side of the economy, growth, the oil related sector, and the stochastic process introducing uncertainty to the economy. The right column states the parameters source.

Variable	Description	Data	Model		
			Mean	5%	95%
<b>Panel A</b>		Expected Growth Rate and Volatilities			
$E[\Delta y]$	Output	1.90%	1.87%	0.64%	3.15%
$\sigma[\Delta c]$	Consumption	1.42%	1.36%	1.24%	1.50%
$\sigma[\Delta i^k]$	Investment in capital	6.22%	2.98%	2.78%	3.18%
$\sigma[\Delta i^o]$	Investment in oil capital	14.91%	3.07%	2.86%	3.28%
$\sigma[\Delta i^p]$	Investment in R&D	4.89%	3.68%	3.43%	3.93%
$\sigma[\Delta d^m]$	Dividend on market	–	1.29%	1.17%	1.44%
$\sigma[\Delta d^k]$	Dividend on capital	–	0.52%	0.35%	0.76%
<b>Panel B</b>		Volatility Ratios			
$\sigma[\Delta y]$		2.33%	2.38%	2.22%	2.54%
$\sigma[\Delta c]/\sigma[\Delta y]$		0.61	0.57	0.54	0.61
$\sigma[\Delta i^k]/\sigma[\Delta y]$		2.67	1.25	1.22	1.28
$\sigma[\Delta i^p]/\sigma[\Delta y]$		2.10	1.54	1.51	1.57
<b>Panel C</b>		Autocorrelation			
$AC(\Delta c)$		0.40	0.12	-0.01	0.26
$AC(\Delta y)$		0.37	0.03	-0.07	0.14
$AC(\Delta i^k)$		0.25	0.03	-0.07	0.12
<b>Panel D</b>		Oil Volatility Ratios			
$\sigma[\Delta i^o]/\sigma[\Delta i^p + \Delta i^k + \Delta i^o]$		3.73	0.88	0.86	0.91
$\sigma[\Delta o]/\sigma[\Delta i^o]$		0.33	0.13	0.08	0.20

**Table III:** Macroeconomic Moments (Full Model)

This table depicts all the first and second moments for the most relevant macroeconomic quantities for the Full Model. Values are stated on an annualized basis. The right two columns report values for the 5- and 95%-quantiles for the distribution of the reported variables. Panel A states expected output growth and volatilities for the growth rates of consumption, investment in physical capital and patents, and dividends for the aggregate claim and dividends on holding physical capital. Panel B states ratios for volatilities of consumption and investment growth for physical capital and patents relative to the volatility of output growth. Panel C depicts the autocorrelation for the growth rates of consumption, output and investment in physical capital. For panels A-C empirical moments reported are taken from [Kung and Schmid \(2015\)](#) (post-Great Depression sample - 1930 to 2008). For panel D oil related empirical moments are based on data from NIPA tables for investment, and EIA for data on oil product supplied. Due to availability of data the sample ranges from 1990 until 2014.

	$\Delta c$	$\Delta y$	$\Delta o$	$\Delta i^k$	$\Delta i^o$	$\Delta i^p$	$\Delta p^o$	$\Delta d^m$	$\Delta d^k$
$\Delta c$	1.00	0.99	0.21	0.98	0.98	0.97	0.95	1.00	0.24
$\Delta y$	0.99	1.00	0.07	1.00	1.00	1.00	0.98	0.98	0.09
$\Delta s$	0.22	0.08	1.00	0.04	0.04	0.01	-0.10	0.24	0.92
$s$	-0.16	-0.04	-0.86	-0.01	-0.01	0.01	0.11	-0.17	-0.67
$o - s$	0.05	-0.17	0.53	-0.03	-0.04	-0.05	-0.11	-0.17	-0.67

**Table IV:** Correlation of Macroeconomic Quantities (Model Implied - Full Model)

This table reports correlations for macroeconomic quantities implied by the Full Model. The upper three rows report correlations for growth rates. The lower two rows report correlations between the stock of inventories and oil usage relative to oil in inventories (o-s). aValues are rounded to the second decimal place.

Variable	Description	Data	Model		
			Mean	5%	95%
<b>Panel A</b>		Asset Pricing			
$E[r_{t,t+3}^f]$	Short-rate	1.61%	1.63%	1.09%	2.18%
$\sigma(r_{t,t+3}^f)$		0.67%	0.24%	0.16%	0.35%
$E[r^m - r_{t,t+3}^f]$	Return on market	5.84%	1.52%	1.40%	1.64%
$\sigma(r^m)$		17.87%	3.02%	2.78%	3.26%
$\frac{E[r^m - r_{t,t+3}^f]}{\sigma(E[r^m - r_{t,t+3}^f])}$		0.33	0.50	0.30	0.71
$E[r^k - r^f]$	Return on physical capital	–	1.07%	1.01 %	1.13%
$\sigma(r^k)$		–	2.13%	1.99%	2.28%
$\frac{E[r^k - r_{t,t+3}^f]}{\sigma(E[r^k - r_{t,t+3}^f])}$		–	0.52	0.30	0.73
<b>Panel B</b>		Futures Market			
$E[r^{F3}]$	Return on 3-mon future	4.80%	2.34%	1.43%	3.14%
$E[r^{F6}]$	Return on 6-mon future	7.34%	2.77%	1.99%	3.56%
$E[r^{F9}]$	Return on 9-mon future	7.78%	2.92%	1.89%	3.93%
$E[r^{F12}]$	Return on 12-mon future	7.19%	2.77%	1.76%	3.79%
$\sigma[r^{F3}]$	S.d. return on 3-mon future	37.65%	2.39%	2.23%	2.54%
$\sigma[r^{F6}]$	S.d. return on 6-mon future	34.12%	2.44%	2.28%	2.60%
$\sigma[r^{F9}]$	S.d. return on 9-mon future	30.66%	2.36%	2.21%	2.52%
$\sigma[r^{F12}]$	S.d. return on 12-mon future	28.01%	2.31%	2.15 %	2.46%
$E[F^3/F^9]$	Slope of futures price curve	1.009	1.014	1.010	1.018
$E[F^9/F^{18}]$	Slope of futures price curve	1.014	1.021	1.014	1.028
$\sigma(r^{F3})/\sigma(r^{F9})$	Vol-ratio of futures returns	1.228	1.012	1.002	1.021
$\sigma(r^{F9})/\sigma(r^{F18})$	Vol-ratio of futures returns	1.251	1.075	1.072	1.080
$E[r^{F9} - r^{F3}]$	Slope of futures risk premia	2.97%	0.60%	0.30%	0.90%
$E[r^{F18} - r^{F9}]$	Slope of futures risk premia	-1.35%	-0.20%	-0.30%	-0.10%

**Table V:** Asset Pricing Moments (Full Model)

This table depicts all the first, second moments and Sharpe-ratios for the most relevant return quantities for all three models. Full is the benchmark model, TtB the *time-to-build* model, and NoS the *no storage* model. Values are stated on an annualized basis. Panel A focusses on standard asset pricing quantities. The lower panel focusses on quantities of the futures market. For panel A moments reported are taken from [Kung \(2015\)](#) (post war sample 1953 to 2008). For panel B oil related moments are computed with data from Thomson Reuters Eikon. Due to availability of oil futures prices for all maturities the sample ranges from 1990 to 2014.

	$r^f$	$r^{F_3}$	$r^{F_6}$	$r^{F_9}$	$r^{F_{12}}$	$r^{F_{15}}$	$r^{F_{18}}$
$Cov(m, \cdot)$	-	-0.44	-0.70	-0.79	-0.70	-0.69	-0.67
$Cov(\Delta c, \cdot)$	4.0E-06	7.9E-05	8.1E-05	7.9E-05	7.7E-05	7.5E-05	7.4E-05
$\rho(\Delta c, \cdot)$	0.47	0.97	0.97	0.98	0.98	0.98	0.98

**Table VI:** Covariance and Correlation Returns (Model Implied - Full Model)

This table depicts covariances and correlations of aggregate returns and futures for contracts with maturities of 3 to 18 months. The upper panel displays covariances and the lower panel correlations. Values are rounded to the second decimal place.

	$r^f$	$r^{F_3}$	$r^{F_6}$	$r^{F_9}$	$r^{F_{12}}$	$r^{F_{15}}$	$r^{F_{18}}$
$Cov(\Delta c)$	0.0E+00	3.0E-06	3.4E-05	3.8E-05	4.4E-05	4.3E-05	3.9E-05
$\rho(\Delta c)$	0.23	0.00	0.03	0.04	0.05	0.06	0.06

**Table VII:** Covariance and Correlation Returns (Empirical Data)

This table depicts covariances and correlations of aggregate return and futures for contracts with maturities of 3 to 12 months for the empirical timeseries of 1990 to 2014. The upper row displays covariances and the lower row correlations. Values are rounded to the second decimal place.



Variable	Description	Data	Model		
			Full	TtB	NoS
<b>Panel A</b>		Expected Growth Rate and Volatilities			
$E[\Delta y]$	Output	1.90%	1.87%	1.81%	1.85%
$\sigma[\Delta c]$	Consumption	1.42%	1.36%	1.43%	1.43%
$\sigma[\Delta i^k]$	Investment in capital	6.22%	2.98%	2.91%	2.95%
$\sigma[\Delta i^p]$	Investment in R&D	4.89%	3.68%	3.63%	3.62%
$\sigma[\Delta d^m]$	Dividend on market	–	1.29%	1.37%	1.37%
$\sigma[\Delta d^k]$	Dividend on capital	–	0.52%	0.50%	0.51%
<b>Panel B</b>		Volatility Ratios			
$\sigma[\Delta y]$		2.33%	2.38%	2.38%	2.38%
$\sigma[\Delta c]/\sigma[\Delta y]$		0.61	0.57	0.60	0.60
$\sigma[\Delta i^k]/\sigma[\Delta y]$		2.67	1.25	1.22	1.24
$\sigma[\Delta i^p]/\sigma[\Delta y]$		2.10	1.54	1.53	1.52
<b>Panel C</b>		Autocorrelation			
$AC(\Delta c)$		0.40	0.12	0.11	0.11
$AC(\Delta y)$		0.37	0.03	0.03	0.04
$AC(\Delta i^k)$		0.25	0.03	0.02	0.02
<b>Panel D</b>		Oil Volatility Ratios			
$\sigma[\Delta i^o]/\sigma[\Delta i^p + \Delta i^k + \Delta i^o]$		3.73	0.88	0.87	0.86
$\sigma[\Delta o]/\sigma[\Delta i^o]$		0.33	0.13	0.13	2.05

**Table VIII:** Macroeconomic Moments (All Growth Models)

This table depicts all the first and second moments for the most relevant macroeconomic quantities for all three models. Full is the benchmark model, TtB the *time-to-build* model, and NoS the *no storage* model. Values are stated on an annualized basis. Panel A states expected output growth and volatilities for the growth rates of consumption, investment in physical capital and patents, and dividends for the aggregate claim and dividends on holding physical capital. Panel B states ratios for volatilities of consumption and investment growth for physical capital and patents relative to the volatility of output growth. Panel C depicts the autocorrelation for the growth rates of consumption, output and investment in physical capital. For panels A-C empirical moments reported are taken from [Kung and Schmid \(2015\)](#) (post-Great Depression sample - 1930 to 2008). For panel D oil related empirical moments are based on data from NIPA tables for investment, and EIA for data on oil product supplied. Due to availability of data the sample ranges from 1990 until 2014.

Variable	Description	Data	Model		
			Full	TtB	NoS
<b>Panel A</b>		Asset Pricing			
$E[r_{t,t+3}^f]$	Short-rate	1.61%	1.63%	1.68%	1.69%
$\sigma(r_{t,t+3}^f)$		0.67%	0.24%	0.25%	0.25%
$E[r^m - r_{t,t+3}^f]$	Return on market	5.84%	1.52%	1.45%	1.46%
$\sigma(r^m)$		17.87%	3.02%	2.92%	2.93%
$\frac{E[r^m - r_{t,t+3}^f]}{\sigma(E[r^m - r_{t,t+3}^f])}$		0.33	0.50	0.51	0.51
$E[r^k - r^f]$	Return on physical capital	–	1.07%	1.03 %	1.05%
$\sigma(r^k)$		–	2.13%	2.08%	2.11%
$\frac{E[r^k - r_{t,t+3}^f]}{\sigma(E[r^k - r_{t,t+3}^f])}$		–	0.52	0.51	0.51
<b>Panel B</b>		Futures Market			
$E[r^{F3}]$	Return on 3-mon future	4.80%	2.34%	2.83%	2.85%
$E[r^{F6}]$	Return on 6-mon future	7.34%	2.77%	2.85%	2.82%
$E[r^{F9}]$	Return on 9-mon future	7.78%	2.92%	2.82%	2.79%
$E[r^{F12}]$	Return on 12-mon future	7.19%	2.77%	2.80%	2.77%
$\sigma[r^{F3}]$	S.d. return on 3-mon future	37.65%	2.39%	2.38%	2.38%
$\sigma[r^{F6}]$	S.d. return on 6-mon future	34.12%	2.44%	2.44%	2.32%
$\sigma[r^{F9}]$	S.d. return on 9-mon future	30.66%	2.36%	2.38%	2.27%
$\sigma[r^{F12}]$	S.d. return on 12-mon future	28.01%	2.31%	2.32 %	2.21%
$E[F^3/F^9]$	Slope of futures price curve	1.009	1.014	1.015	1.015
$E[F^9/F^{18}]$	Slope of futures price curve	1.014	1.021	1.021	1.021
$\sigma(r^{F3})/\sigma(r^{F9})$	Vol-ratio of futures returns	1.228	1.012	1.000	1.049
$\sigma(r^{F9})/\sigma(r^{F18})$	Vol-ratio of futures returns	1.251	1.075	1.079	1.080
$E[r^{F9} - r^{F3}]$	Slope of futures risk premia	2.97%	0.60%	0.00%	-0.10%
$E[r^{F18} - r^{F9}]$	Slope of futures risk premia	-1.35%	-0.20%	-0.10%	-0.10%

**Table IX:** Asset Pricing (All Growth Models)

This table depicts all the first, second moments and Sharpe-ratios for the most relevant return quantities for all three models. Full is the benchmark model, TtB the *time-to-build* model, and NoS the *no storage* model. Values are stated on an annualized basis. Panel A focusses on standard asset pricing quantities. The lower panel focusses on quantities of the futures market. For panel A moments reported are taken from Kung (2015) (post war sample 1953 to 2008). For panel B oil related moments are computed with data from Thomson Reuters Eikon. Due to availability of oil futures prices for all maturities the sample ranges from 1990 to 2014.

	Data	Growth Cycle Model	Business Cycle Model
<b>Panel A</b> Expected Growth Rate and Volatilities			
$E[\Delta y]$	1.90%	1.87%	1.87%
$\sigma[\Delta c]$	1.42%	1.36%	1.33%
$\sigma[\Delta i^k]$	6.22%	2.98%	1.54%
$\sigma[\Delta i^p]$	4.89%	3.68%	-
$\sigma[\Delta d^m]$	-	1.29%	0.57%
$\sigma[\Delta d^k]$	-	0.52%	0.00%
<b>Panel B</b> Volatility Ratios			
$\sigma[\Delta y]$	2.33%	2.38%	1.40%
$\sigma[\Delta c]/\sigma[\Delta y]$	0.61	0.57	0.95
$\sigma[\Delta i^k]/\sigma[\Delta y]$	2.67	1.25	1.10
$\sigma[\Delta i^p]/\sigma[\Delta y]$	2.10	1.54	-
$\sigma[\Delta o]/\sigma[\Delta y]$	-	0.17	0.10
<b>Panel C</b> Autocorrelation and Correlation			
$AC(\Delta c)$	0.40	0.12	0.00
$AC(\Delta y)$	0.37	0.03	-0.01
$AC(\Delta i^k)$	0.25	0.03	-0.01
$\rho(\Delta y, \Delta o)$	-	0.07	-0.03
$\rho(\Delta c, \Delta o)$	-	0.21	-0.04

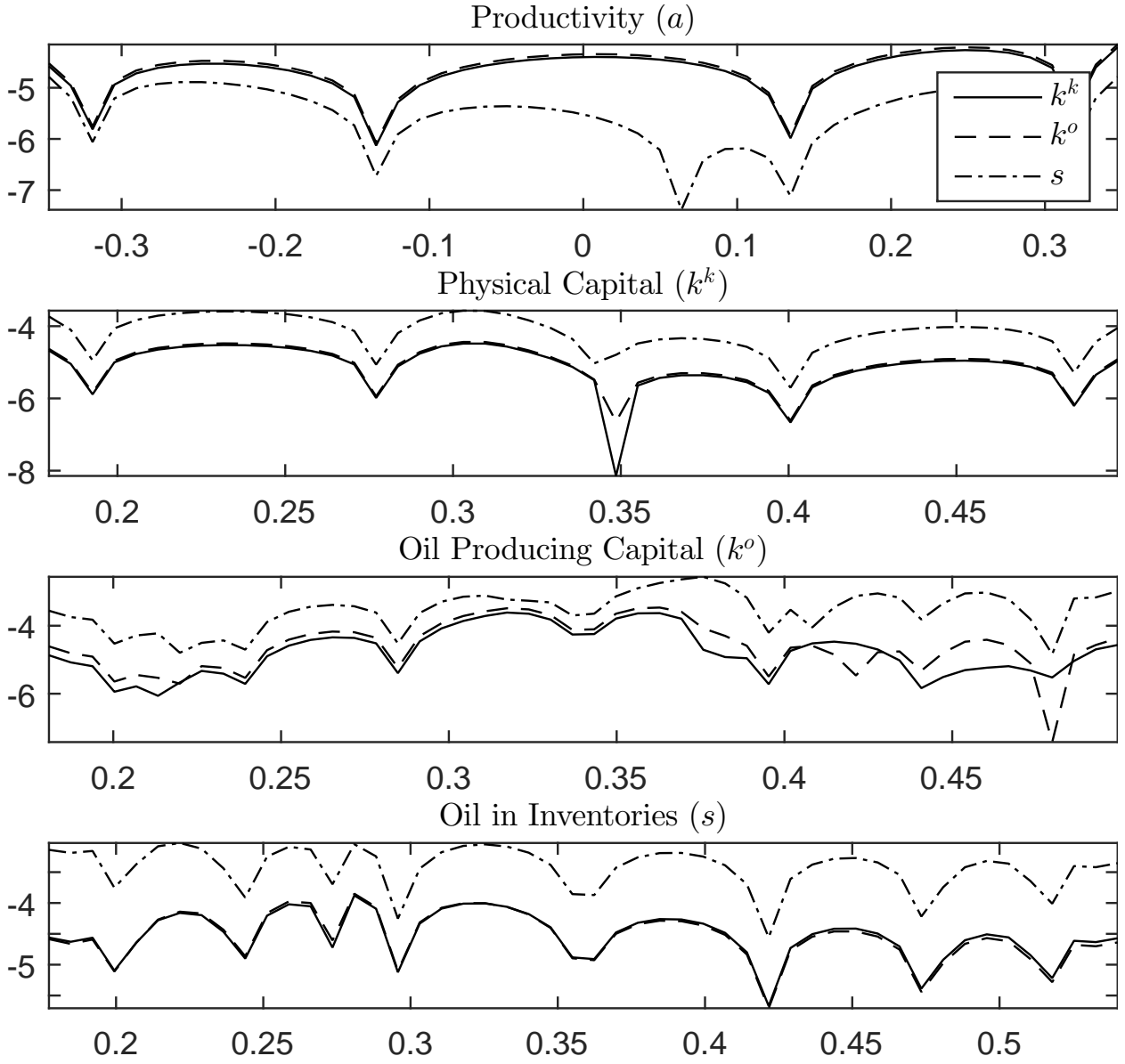
**Table X:** Comparison Growth and Business Cycle Model (Macro)

This table depicts all the first and second moments for the most relevant macroeconomic quantities for the Full model and the business cycle calibration. Values are stated on an annualized basis. Panel A states expected output growth and volatilities for the growth rates of consumption, investment in physical capital and patents, and dividends for the aggregate claim and dividends on holding physical capital. Panel B states ratios for volatilities of consumption and investment growth for physical capital and patents relative to the volatility of output growth. Panel C depicts the autocorrelation for the growth rates of consumption, output and investment in physical capital. For panels A-C empirical moments reported are taken from [Kung and Schmid \(2015\)](#) (post-Great Depression sample - 1930 to 2008). For panel D oil related empirical moments are based on data from NIPA tables for investment, and EIA for data on oil product supplied. Due to availability of data the sample ranges from 1990 until 2014.

		Growth Cycle Model	Business Cycle Model
<b>Panel A</b>		Asset Pricing	
$E[r_{t,t+3}^f]$	1.61%	1.63%	2.67%
$\sigma(r_{t,t+3}^f)$	0.67%	0.24%	0.03%
$E[r^m - r_{t,t+3}^f]$	5.84%	1.52%	0.03%
$\sigma(r^m)$	17.87%	3.02%	1.12%
$\frac{E[r^m - r_{t,t+3}^f]}{\sigma(E[r^m - r_{t,t+3}^f])}$	0.33	0.50	0.03
$E[r^k - r^f]$	–	1.07%	0.03%
$\sigma(r^k)$	–	2.13%	1.11%
$\frac{E[r^k - r_{t,t+3}^f]}{\sigma(E[r^k - r_{t,t+3}^f])}$	–	0.52	0.03
<b>Panel B</b>		Futures Market	
$E[r^{F3}]$	4.80%	2.34%	2.93%
$E[r^{F6}]$	7.34%	2.77%	2.43%
$E[r^{F9}]$	7.78%	2.92%	2.70%
$E[r^{F12}]$	7.19%	2.77%	2.69%
$\sigma[r^{F3}]$	37.65%	2.39%	1.40%
$\sigma[r^{F6}]$	34.12%	2.44%	1.40%
$\sigma[r^{F9}]$	30.66%	2.36%	1.37%
$\sigma[r^{F12}]$	28.01%	2.31%	1.33%
$E[F^3 / F^9]$	1.009	1.014	1.013
$E[F^9 / F^{18}]$	1.014	1.021	1.020
$\sigma(r^{F3}) / \sigma(r^{F9})$	1.228	1.012	1.025
$\sigma(r^{F9}) / \sigma(r^{F18})$	1.251	1.075	1.095
$E[r^{F9} - r^{F3}]$	2.97%	0.60%	-0.20%
$E[r^{F18} - r^{F9}]$	-1.35%	-0.20%	0.00%

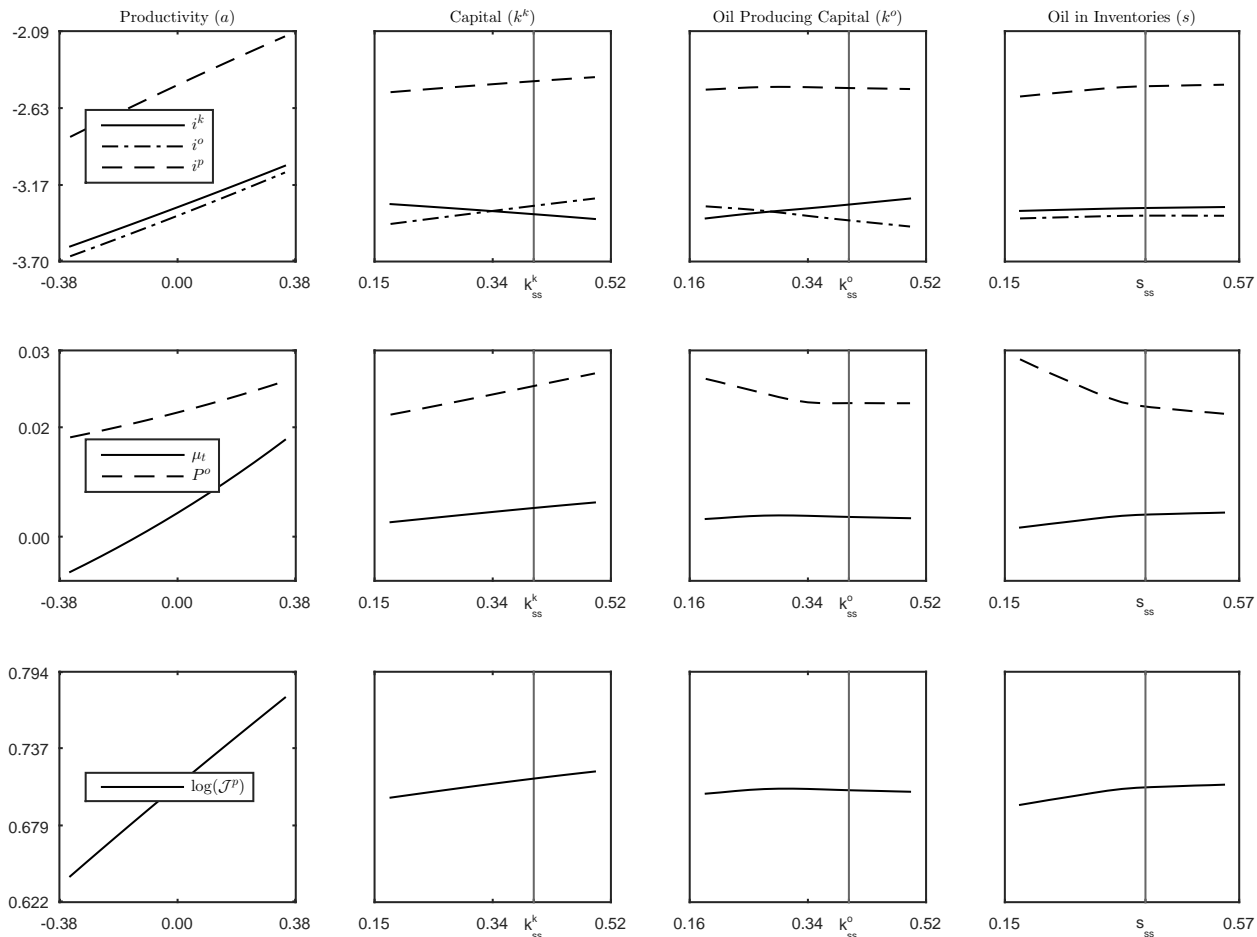
**Table XI:** Comparison Growth and Business Cycle Model (Asset Pricing)

This table depicts all the first, second moments and Sharpe-ratios for the most relevant return quantities for the Full model and the business cycle calibration. Values are stated on an annualized basis. Panel A focusses on standard asset pricing quantities. The lower panel focusses on quantities of the futures market. For panel A moments reported are taken from [Kung \(2015\)](#) (post war sample 1953 to 2008). For panel B oil related moments are computed with data from Thomson Reuters Eikon. Due to availability of oil futures prices for all maturities the sample ranges from 1990 to 2014.



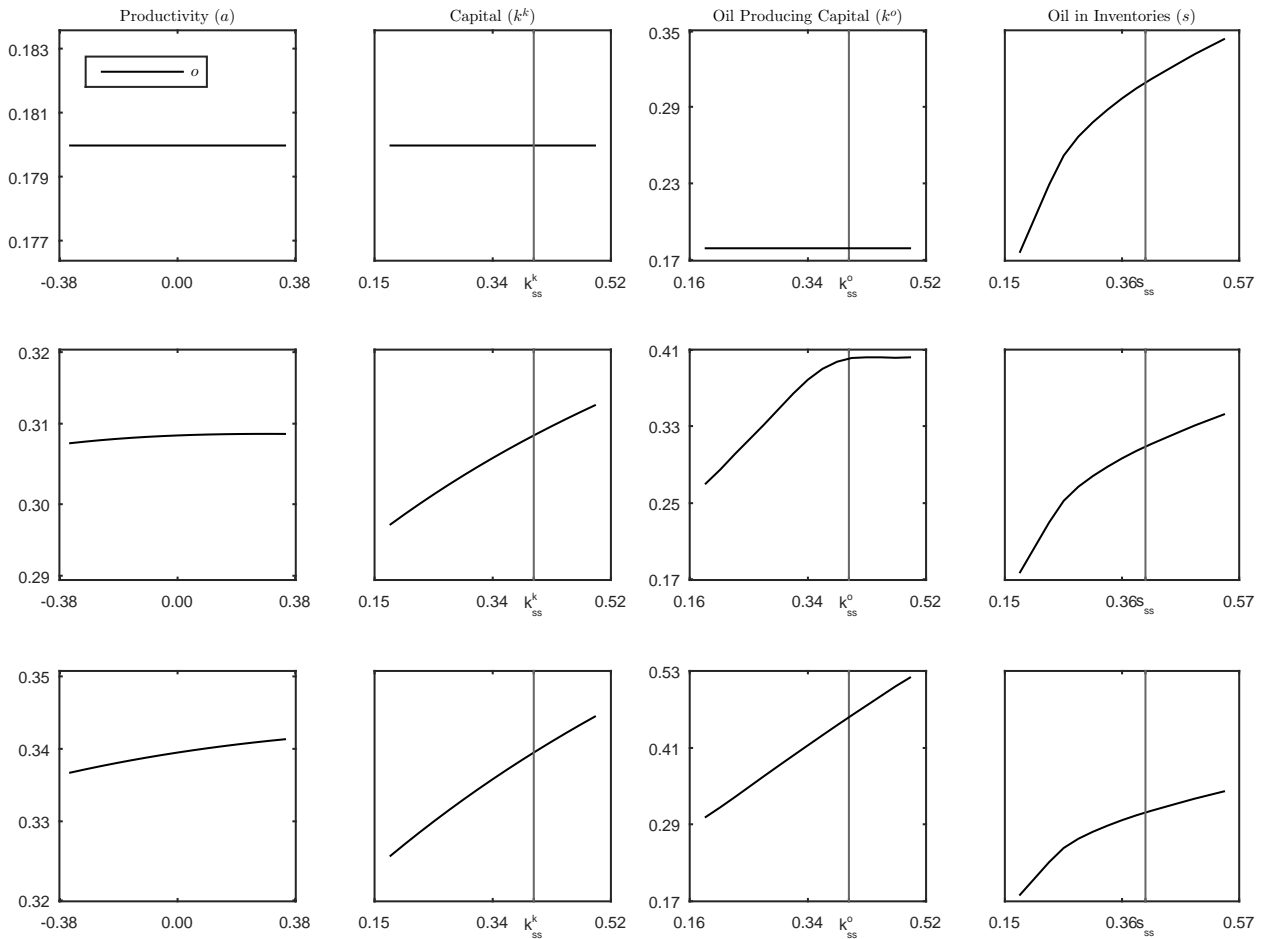
**Figure 1:** Euler Equation Errors (Full Model)

This figure depicts Euler equation errors as  $\log_{10}(|EEE|)$  for the return on physical capital  $k^k$ , oil producing capital  $k^o$ , and oil in inventories  $s$  for the Full model. Euler equation errors are computed for a grid of 50 points in each direction. The upper panel depicts Euler errors for the grid of productivity ( $a$ ). The second panel depicts Euler errors for the grid of physical capital ( $k^k$ ). The third panel depicts Euler errors for the grid of oil producing capital ( $k^o$ ). The fourth panel depicts Euler errors for the grid of oil in inventories ( $s$ ). For productivity level upper and lower bounds are at  $\pm 3.25$  standard deviations.



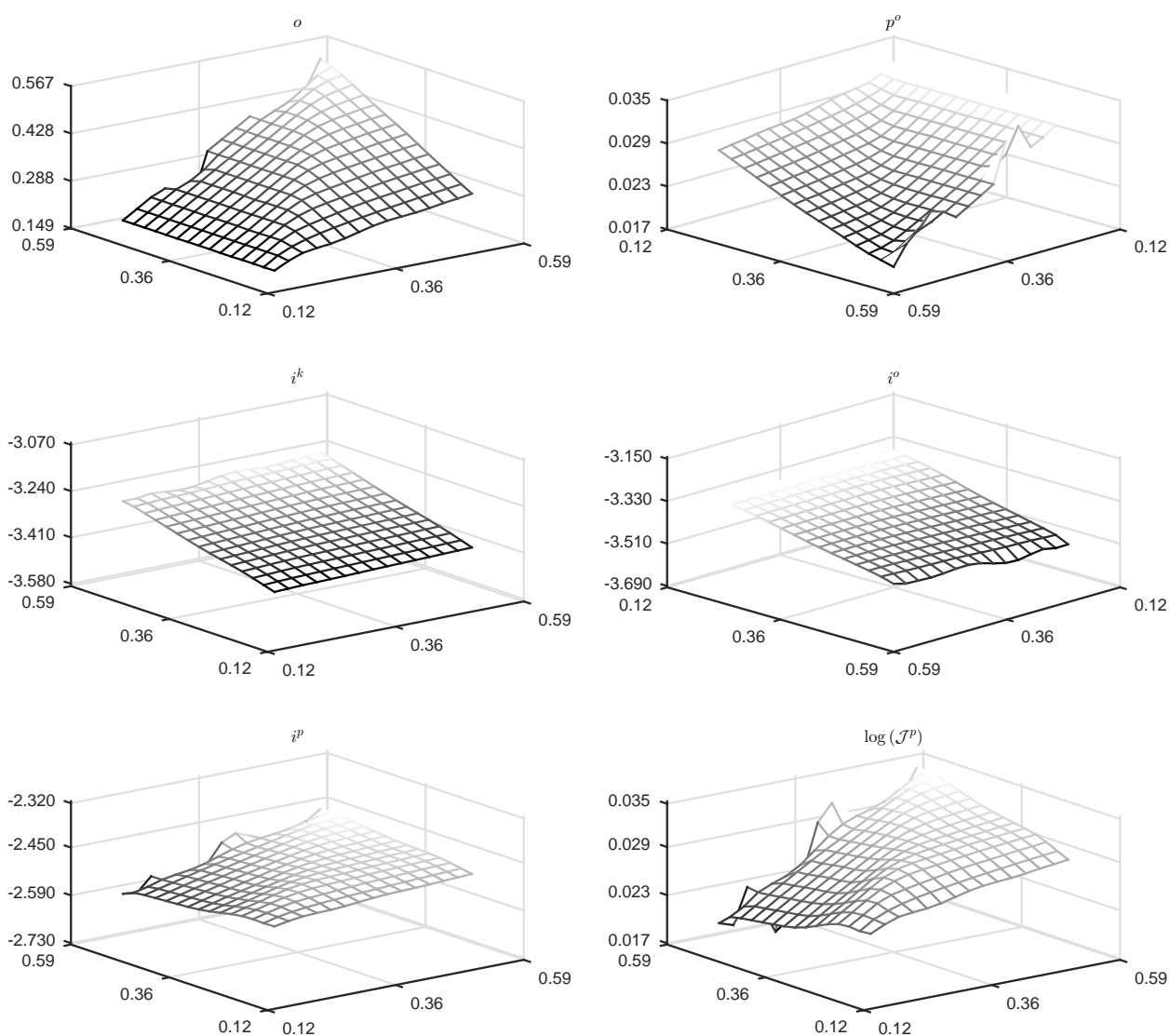
**Figure 2:** Macroeconomic Control Variables (Full Model)

This figure depicts relevant macroeconomic policy functions as functions of the four state variables productivity, physical capital, oil related capital and oil in inventories for the Full model. The first row of panels depicts investment in physical capital ( $i^k$ ), oil related capital ( $i^o$ ), and the R&D sector ( $i^p$ ). The second row of panels depicts growth of the R&D sector ( $\mu_t$ ) and oil prices ( $P^o$ ). The last row of panels depicts the value of a patent ( $\log(\mathcal{J}^p)$ ). Bounds for state variables are chosen such that they match the bounds of the grid of approximation. For productivity level upper and lower bounds are at  $\pm 3.25$  standard deviations. For all endogenous state variables the grey line represents the deterministic steady state.



**Figure 3:** Oil Usage as a Function of State Variables (Full Model)

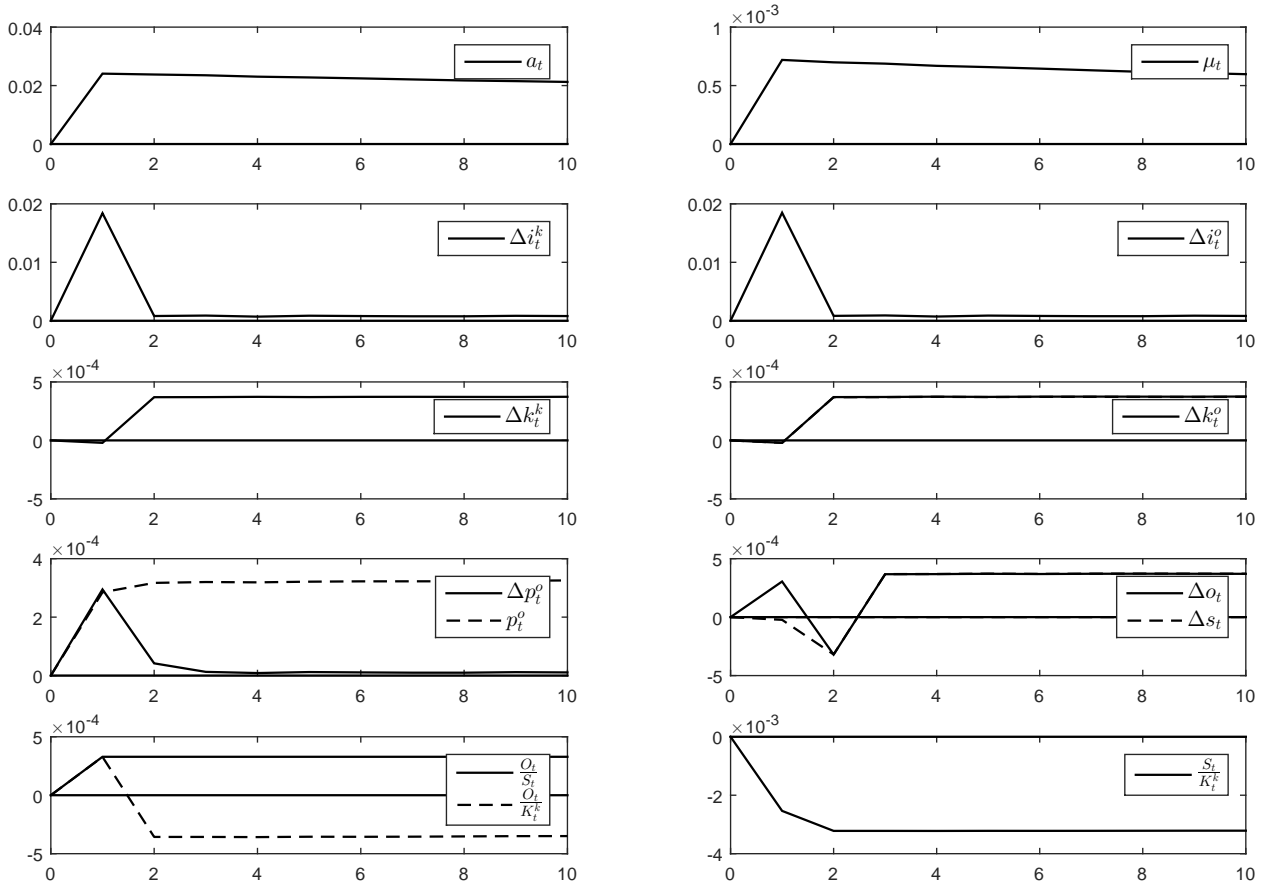
This figure depicts oil usage as functions of the four state variables productivity, physical capital, oil related capital and oil in inventories for the Full model. In the first row of panels state variables are chosen, such that the non-negativity constraint is always binding in the space of productivity, physical capital and oil producing capital. In the second row of panels state variables are chosen, such that the non-negativity constraint is occasionally binding in the space of productivity, physical capital and oil producing capital. In the last row of panels state variables are chosen, such that the non-negativity constraint is never binding in the space of productivity, physical capital and oil producing capital. Bounds for state variables are chosen such that they match the bounds of the grid of approximation. For productivity level upper and lower bounds are at  $\pm 3.25$  standard deviations. For all endogenous state variables the grey line represents the deterministic steady state.



**Figure 4:** Macroeconomic Control Variables (Mesh Plot - Full Model)

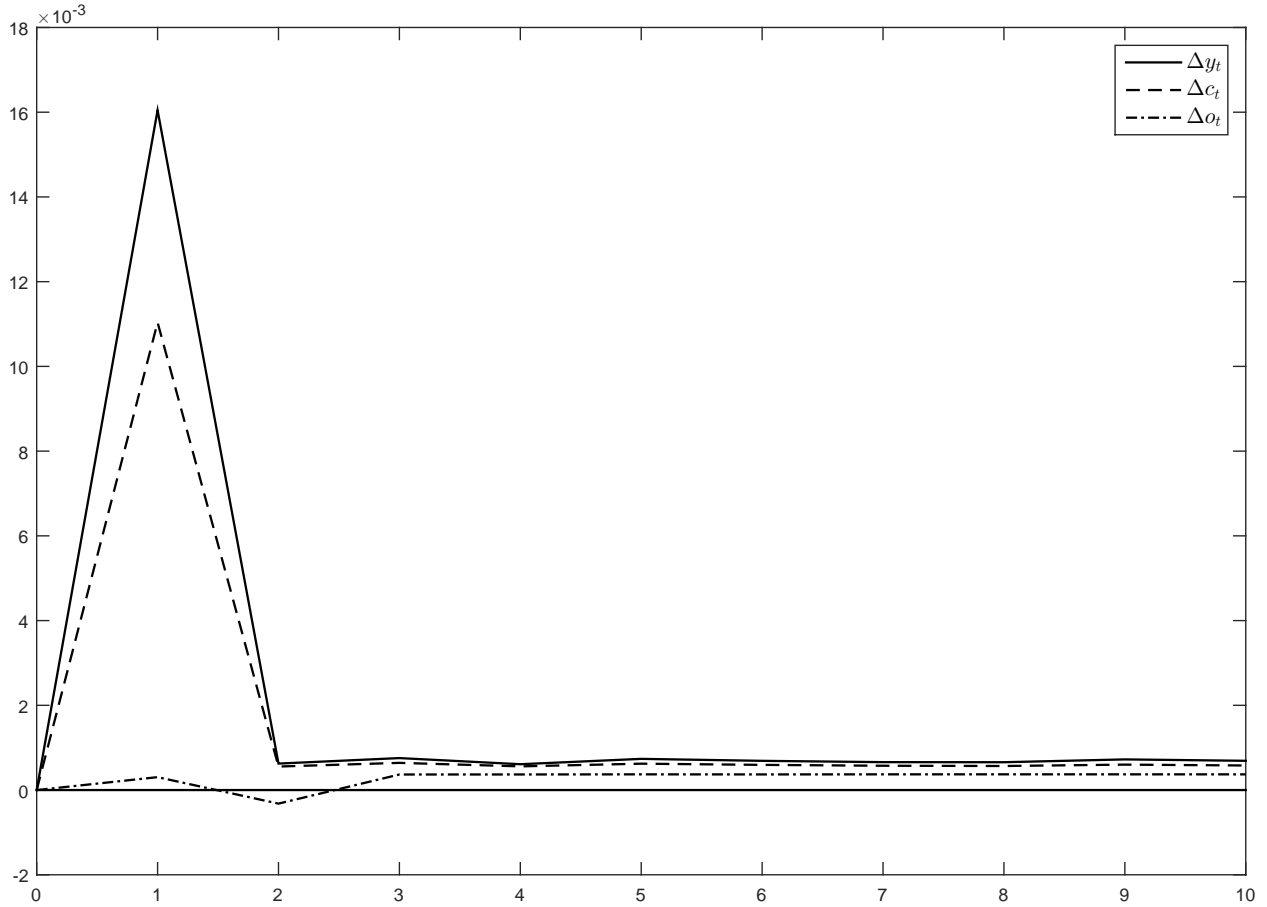
This figure depicts relevant macroeconomic policy functions as functions of oil related capital and oil in inventories for the Full model. The first row of panels depicts the amount of oil used in the process of production ( $o$ ) and the oil prices ( $P^o$ ). The second row of panels depicts investment in physical capital ( $i^k$ ), investment in oil producing capital ( $i^o$ ), and the R&D sector ( $i^p$ ). The last row of panels depicts the value of a patent ( $\log(\mathcal{J}^p)$ ). Bounds for state variables are chosen such that they match the bounds of the grid of approximation. For oil usage, investment in physical capital, investment in patents and patent value, the left axis contains oil producing capital and the right axis oil in inventories. For oil price and investment in oil producing capital the left axis contains oil in inventories and the right axis oil producing capital.





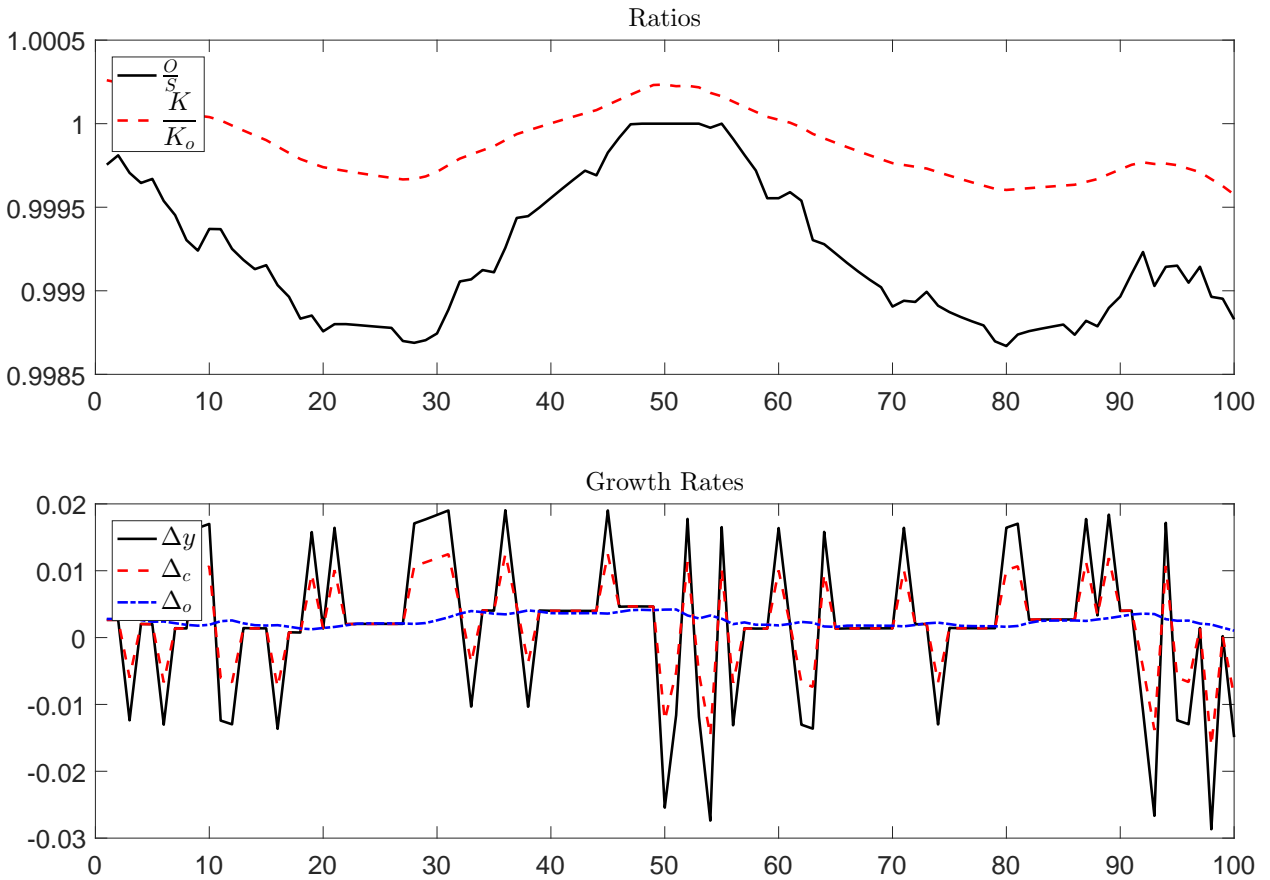
**Figure 5:** Stochastic Impulse Response Functions for Macroeconomic Variables (Full Model)

This Figure depicts stochastic impulse response functions for major economic quantities endogenous to the Full model. Graphs are log deviations from the stochastic steady state. Impulse responses are computed as the average of 50,000 sample paths after being exposed to a one-standard deviation positive shock to the level of productivity. The plots depict in order the level of labor augmenting technology ( $a_t$ ), growth in the R&D sector ( $\mu_t$ ), growth in investment in physical capital ( $\Delta i_t^k$ ), growth in investment in oil related capital ( $\Delta i_t^o$ ), growth in oil usage ( $\Delta o_t$ ), growth in the stock of physical and oil related capital ( $\Delta k_t^k, \Delta k_t^o$ ), growth and level of the oil price ( $\Delta p_t^o, p_t^o$ ), growth in oil usage and the log change in the stock of inventories ( $\Delta o_t, \Delta s_t$ ), the ratios of oil usage over stocks in inventories and oil usage over the the stock of physical capital ( $O_t/S_t, O_t/K_t^k$ ), and the ratio of oil in stock over physical capital ( $S_t/K_t^k$ ).



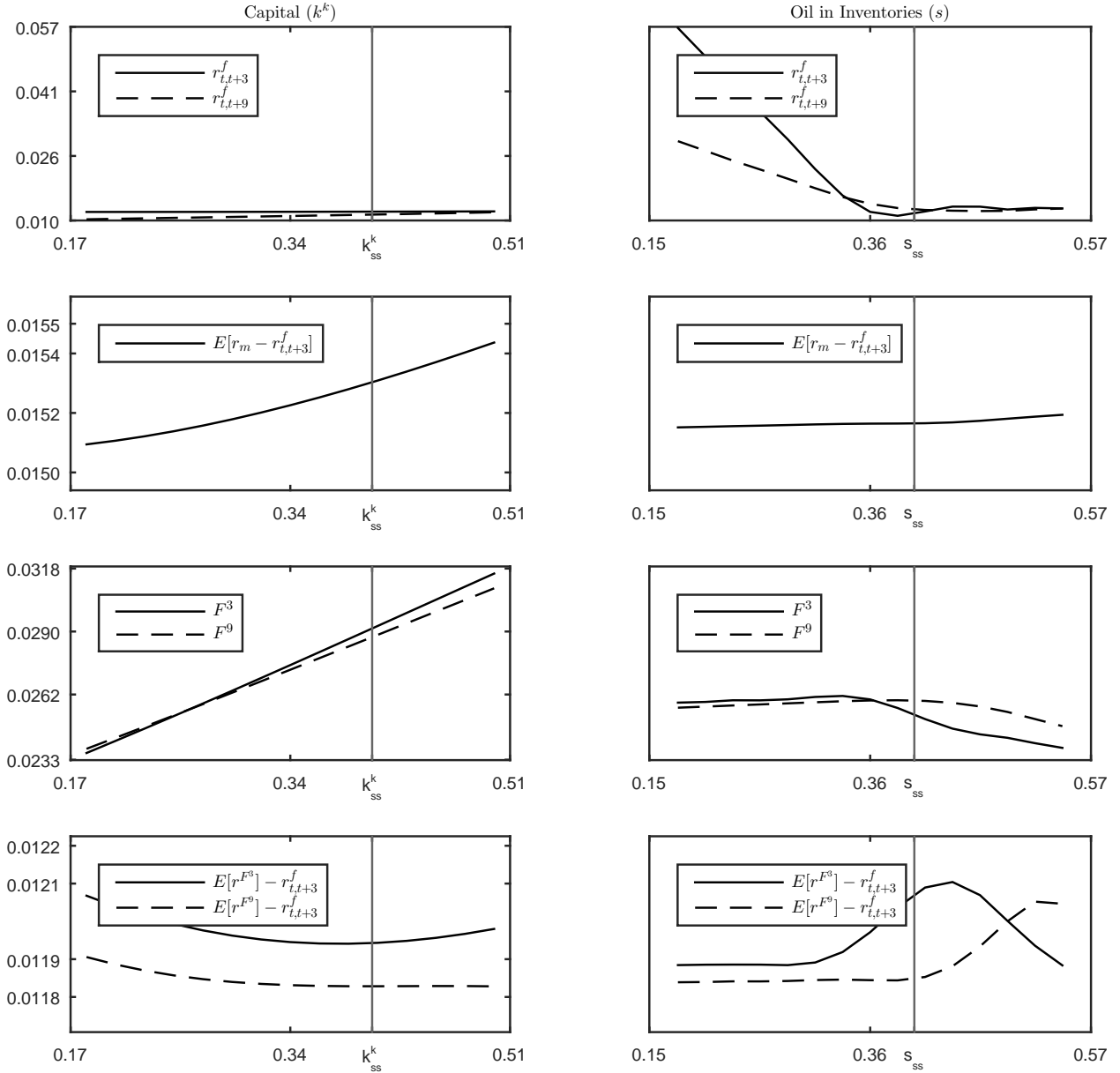
**Figure 6:** Stochastic Impulse Response Functions C,Y,O (Full Model)

This figure depicts stochastic impulse response functions for consumption, output and oil usage endogenous to the Full model. Graphs are log deviations from the stochastic steady state. Impulse responses are computed as the average of 50,000 sample paths after being exposed to a one-standard deviation positive shock to the level of productivity. The graph includes growth rates for output ( $\Delta y_t$ ), consumption ( $\Delta c_t$ ), and oil supplied to the process of production ( $\Delta o_t$ )



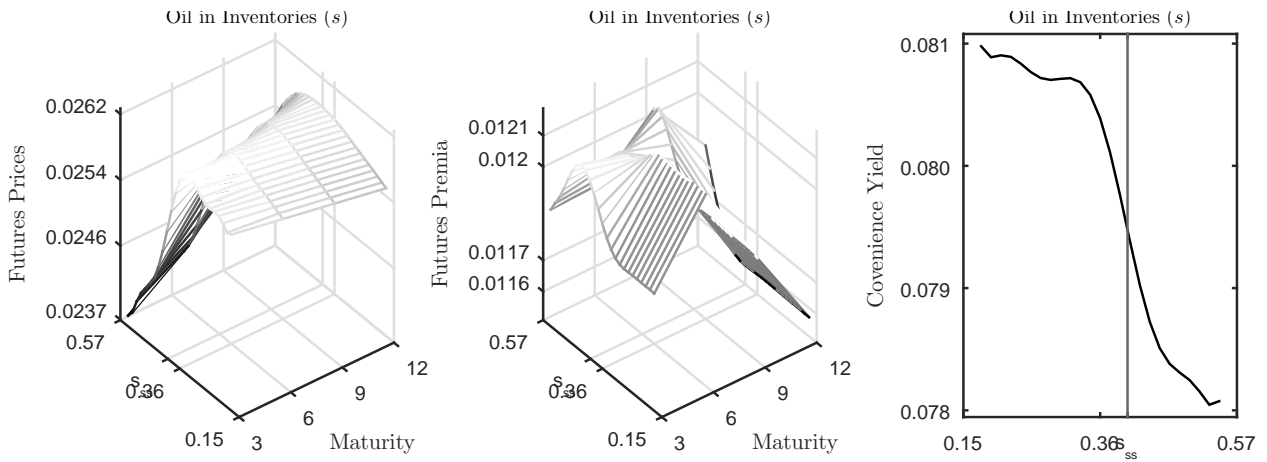
**Figure 7:** Timeseries - Macroeconomic Quantities (Full Model)

This figure displays generic timeseries for the Full model. It displays graphs for the ratio of oil usage over oil in inventories ( $\frac{Q}{S}$ ) and the ratio of physical capital over oil related capital ( $\frac{K^k}{K^o}$ ) in the upper panel. The lower panel displays growth rates of output ( $\Delta y_t$ ), consumption ( $\Delta c_t$ ), and oil usage ( $\Delta o_t$ ). The time series contains 100 subsequent periods randomly selected from simulated sample paths.



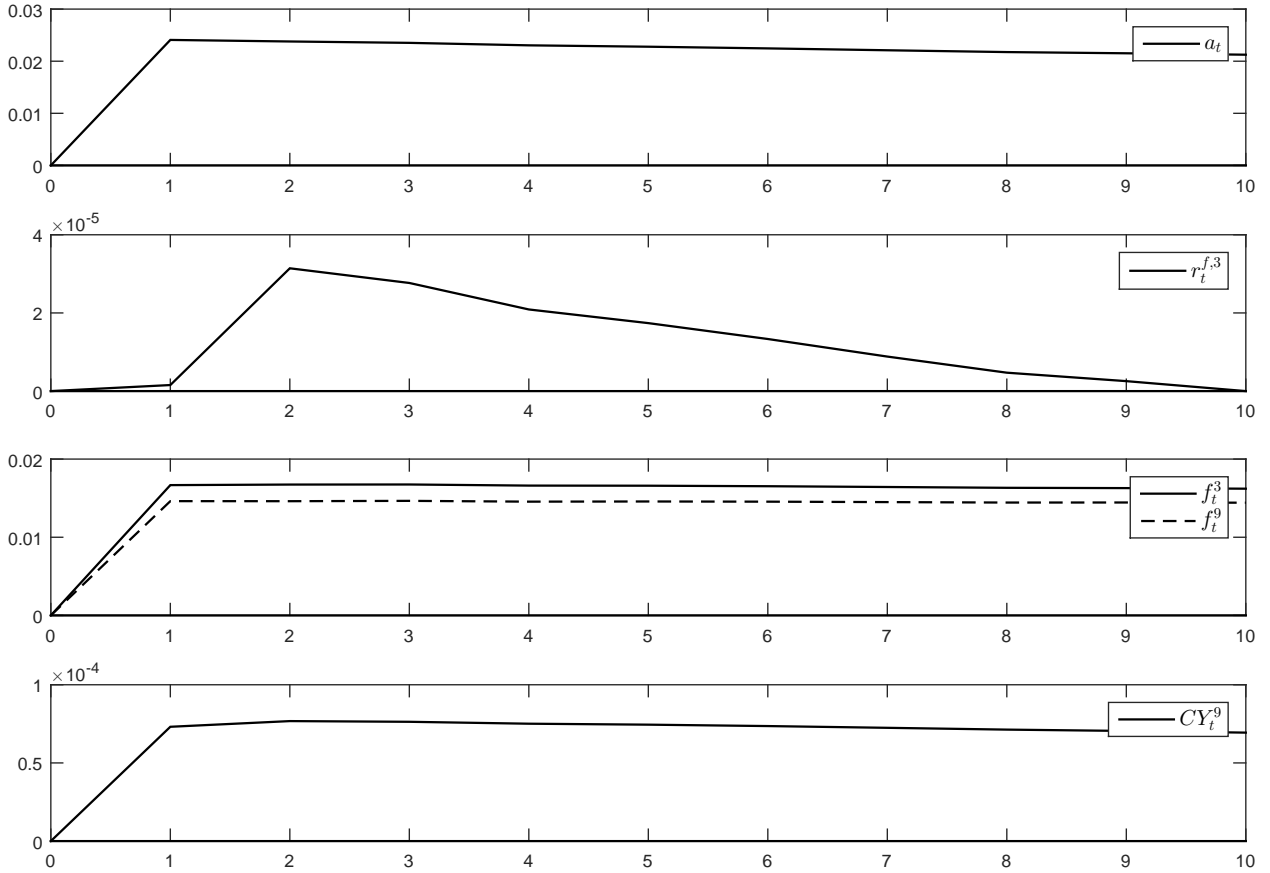
**Figure 8:** Asset Pricing Control Variables (Full Model)

This figure depicts relevant asset pricing quantities as functions of the state variables physical capital, and oil in inventories for the Full model. The upper two panels contain the 3-month risk-free rate ( $r_{t,t+3}^f$ ), and 9-month risk-free rate ( $r_{t,t+9}^f$ ). The second row depicts the return on market ( $E[r^m - r_{t,t+3}^f]$ ). The third row depicts the 3-month futures price ( $F_t^3$ ), and 9-month futures price ( $F_t^9$ ). The lower two panels contain premia for 3-month future ( $E[r^{F^3}]$ ), and 9-month future ( $E[r^{F^9}]$ ). Bounds for state variables are chosen such that they match the bounds of the grid of approximation. The deterministic steady state is indicated by the grey vertical line.



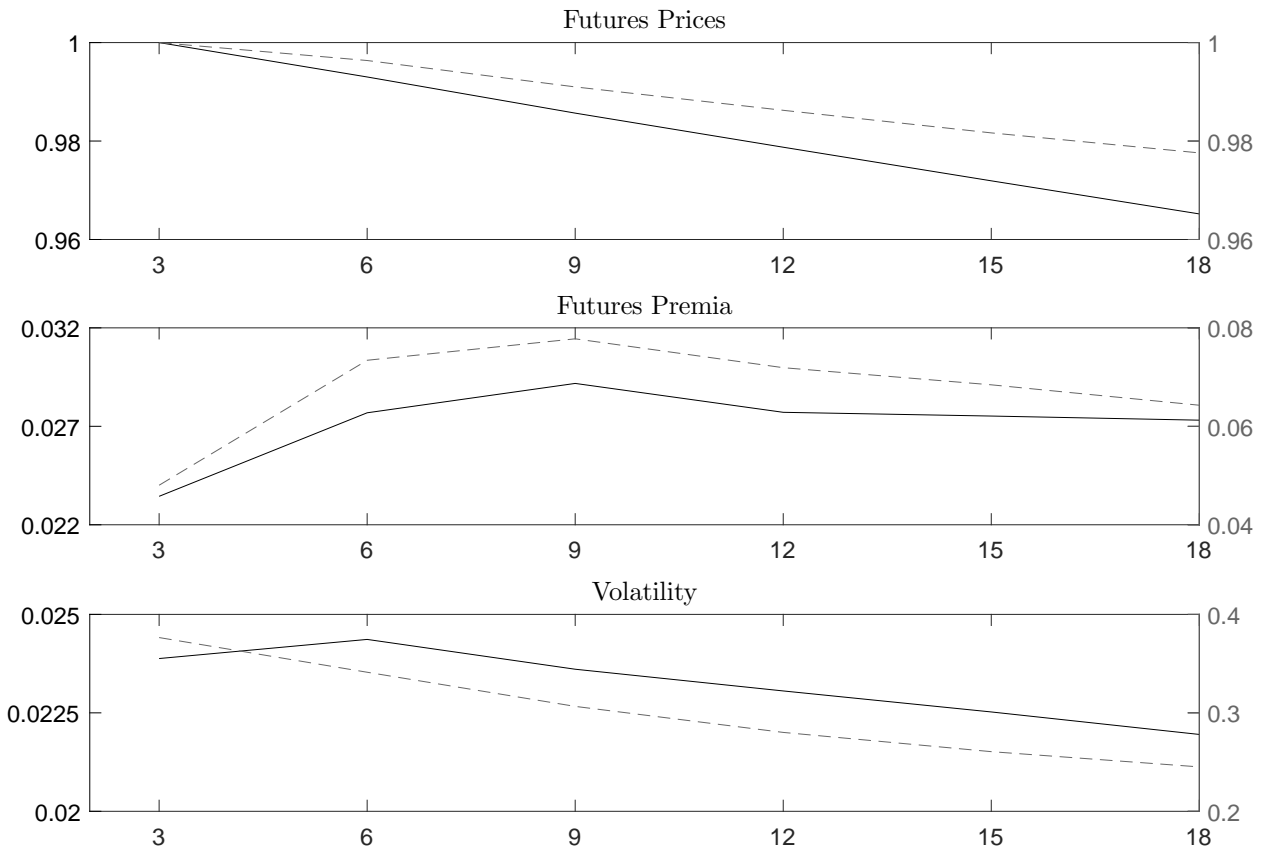
**Figure 9:** Futures and Convenience Yields (Full Model)

This figure depicts the term structure of futures, futures premia and the convenience yield as functions of oil in inventories for the Full model. Bounds for the state variable are chosen such that they match the bounds of the grid of approximation. The left panel depicts the term structure of futures prices for maturities of 3 - 12 months, the mid panel depicts the term structure of futures premia for maturities of 3 - 12 months, and the right panel the convenience yield. The deterministic steady state is indicated by the grey vertical line.



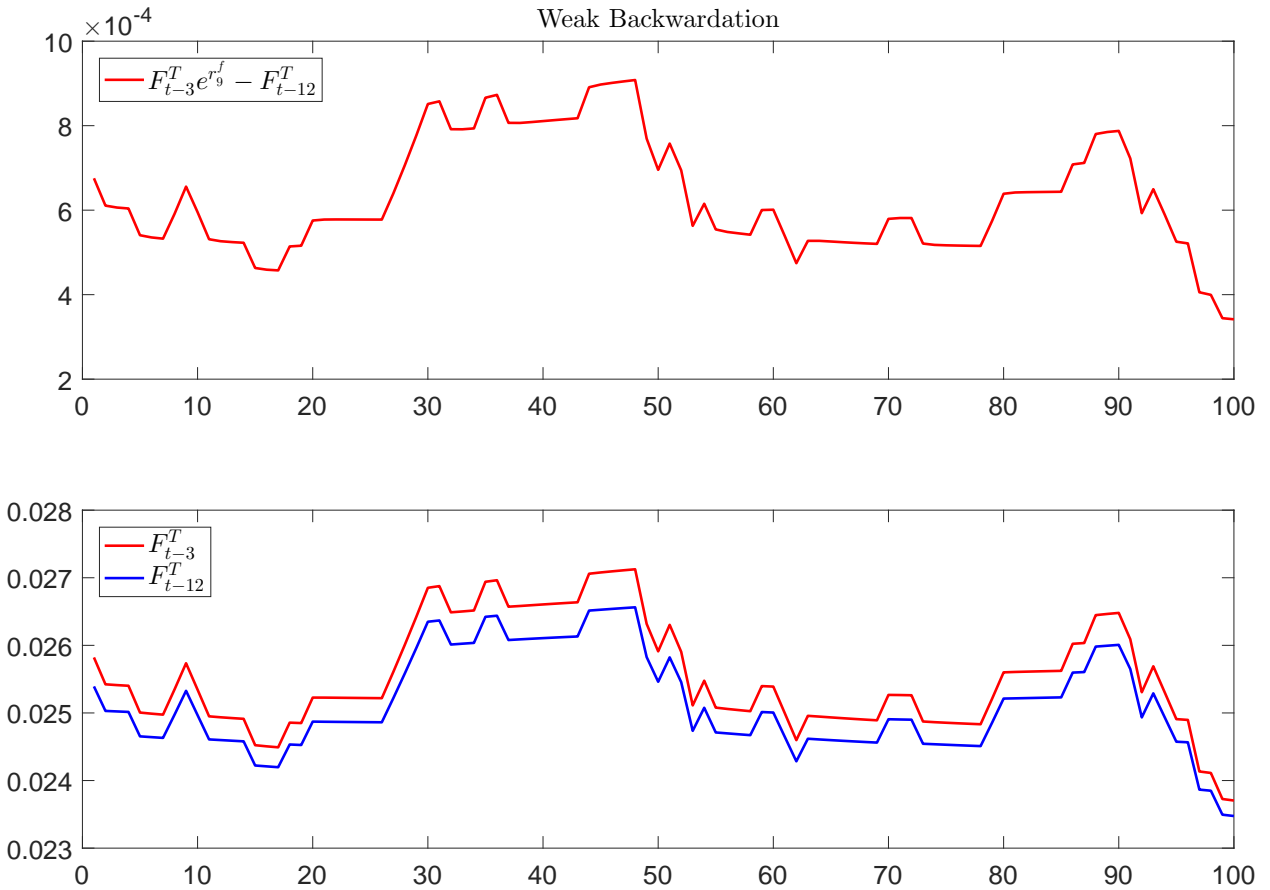
**Figure 10:** Stochastic Impulse Response Functions Asset Pricing Quantities (Full Model)

This figure depicts stochastic impulse response functions for major asset pricing quantities endogenous to the for the Full model. Graphs are log deviations from the stochastic steady state. Impulse responses are computed as the average of 50,000 sample paths after being exposed to a one-standard deviation positive shock to the level of productivity. The plots depict in order technology ( $a_t$ ), risk-free rate ( $r_t^{f,3}$ ), 3-month log futures price ( $f_t^3$ ), 12-month log futures price ( $f_t^{12}$ ), and 9-month convenience yield ( $CY_t^9$ ).



**Figure 11: Average Term Structures (Full Model)**

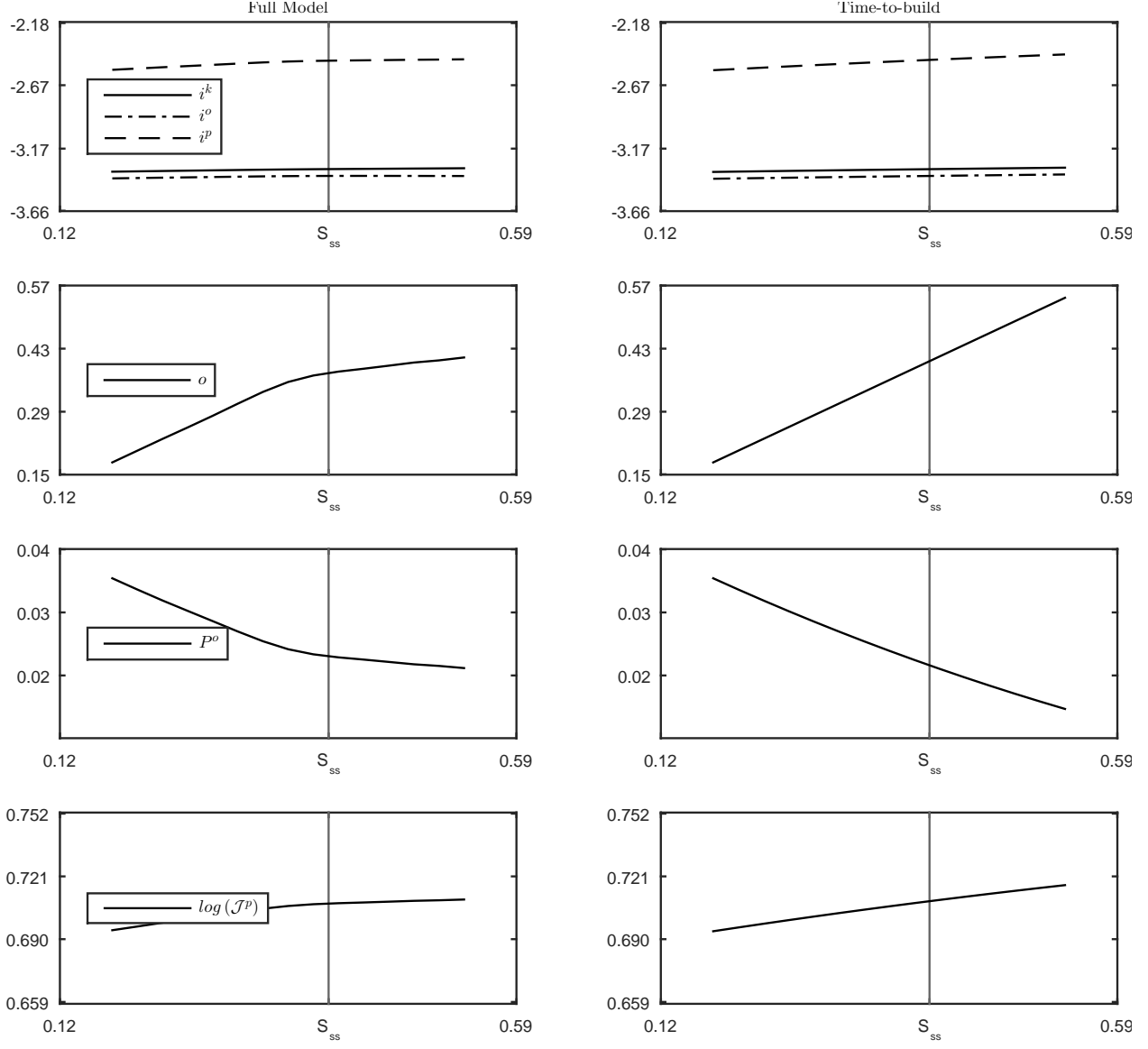
This figure depicts term structures for the futures market for maturities from 3 to 18 months for the Full model. The upper panel displays the term structure of futures prices. The middle panel displays the term structure of futures premia. The lower panel displays the term structure of return volatilities. Futures prices are normalized by the price for the 3-month futures contract. The solid black line is the term structure implied by the model. The dashed grey line depicts term structures implied by the data.



**Figure 12:** Timeseries - Adjusted Basis and Oil Prices

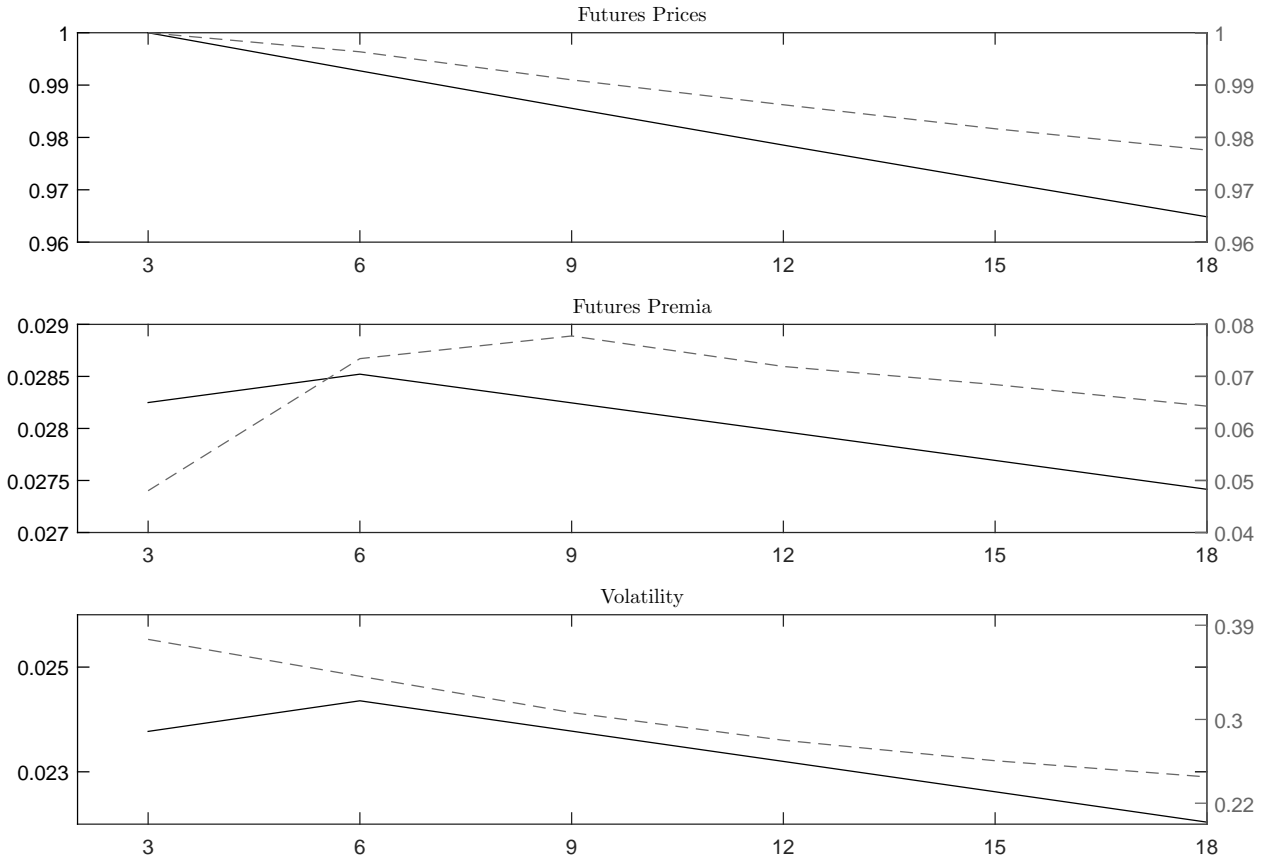
The upper panel of this figure displays the adjusted basis for a 12-month future. Oil prices are approximated with the nearest future as in [Litzenberger and Rabinowitz \(1995\)](#). The lower panel displays time series of futures with maturities of 3 and 12 months. The time series contains 100 subsequent periods randomly selected from simulated sample paths.





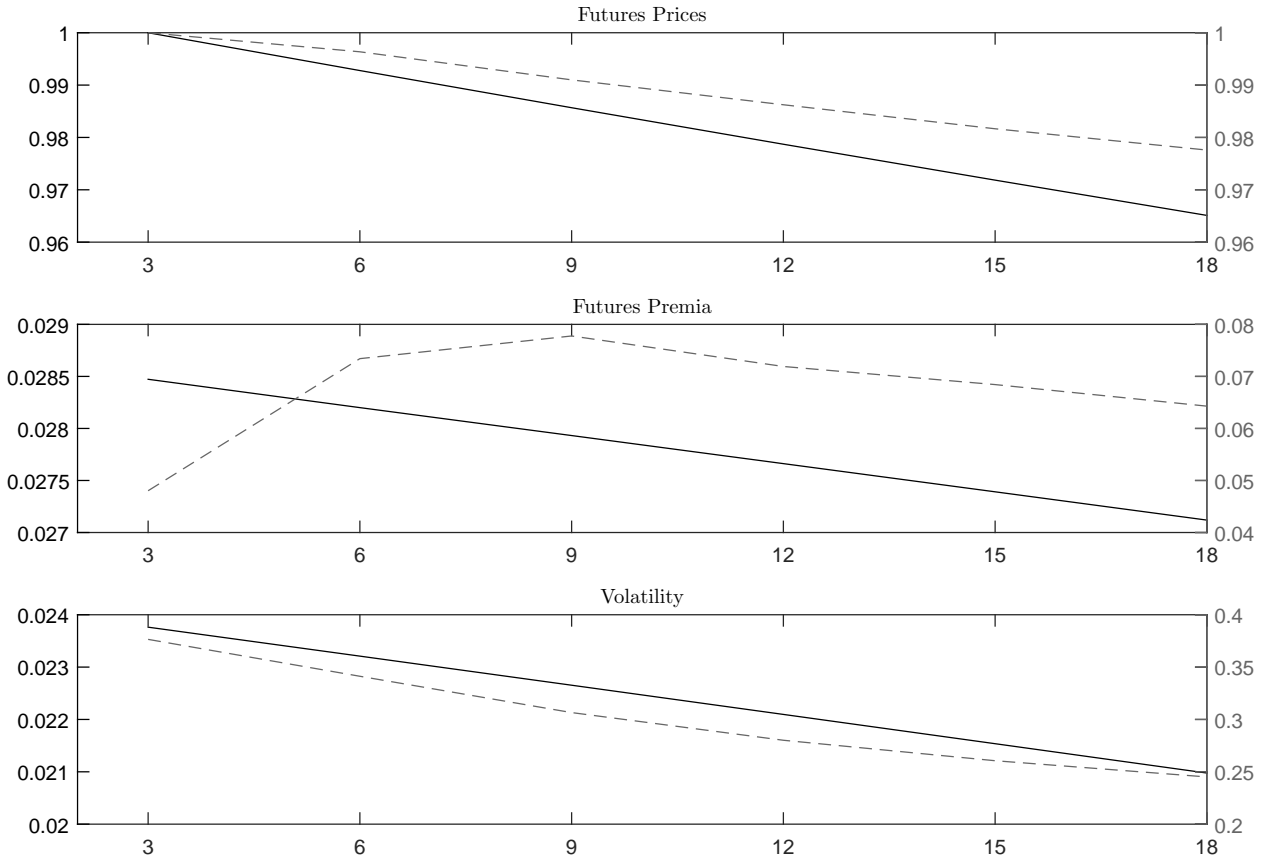
**Figure 13:** Macroeconomic Control Variables - Full vs TtB

This figure depicts relevant macroeconomic policy functions as functions of oil in inventories for the Full model solution and the *time-to-build* model. The first row of panels depicts investment in physical capital ( $i^k$ ), oil related capital ( $i^o$ ), and the R&D sector ( $i^p$ ). The second row of panels depicts the amount of oil used in the process of production ( $o$ ). The third row of panels depicts oil prices ( $P^o$ ). The last row of panels depicts the value of a patent ( $\log(\mathcal{J}^p)$ ).



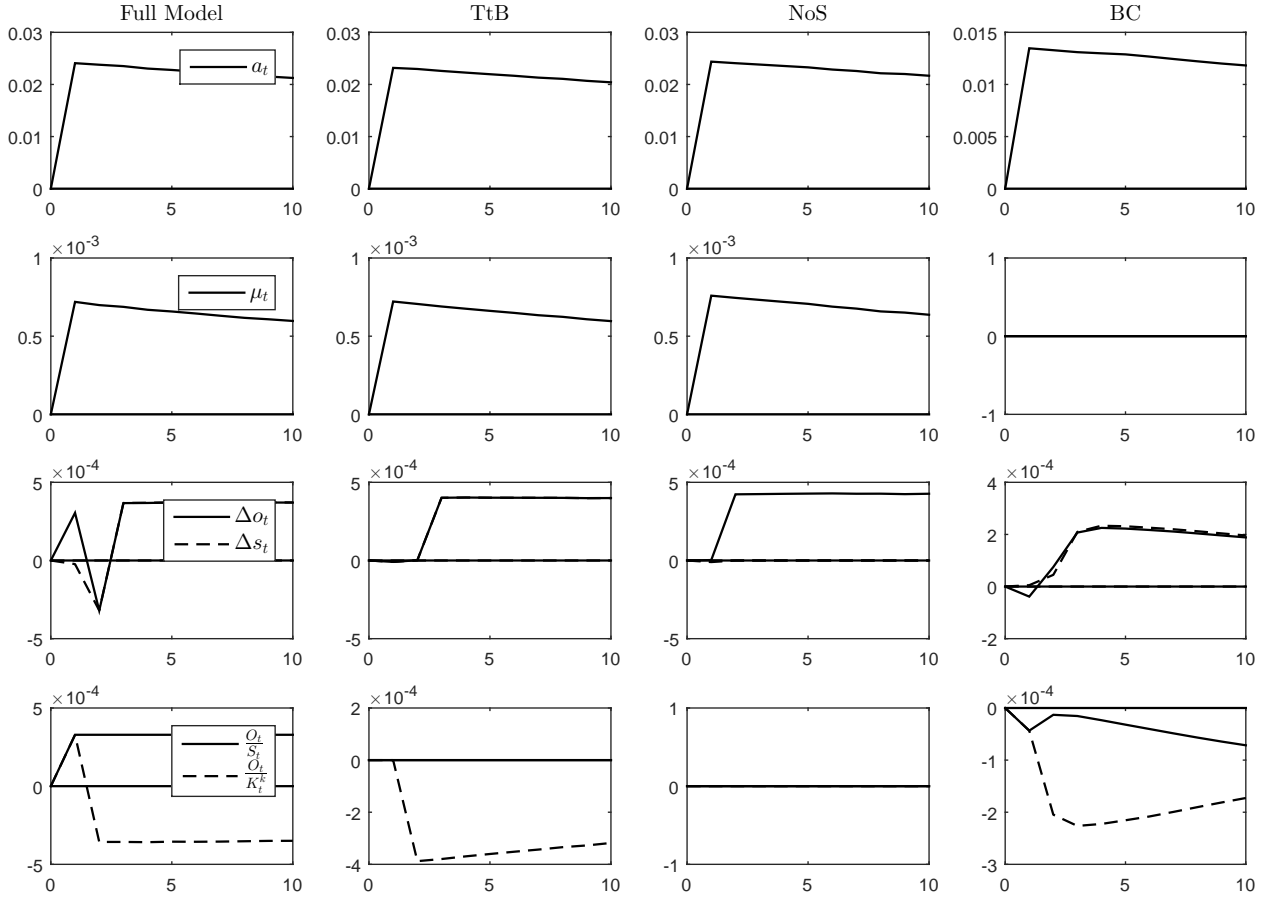
**Figure 14:** Average Term Structures (Time-to-build Constraint)

This figure depicts term structures for the futures market for maturities from 3 to 18 months for the TtB model. The upper panel displays the term structure of futures prices. The middle panel displays the term structure of futures premia. The lower panel displays the term structure of return volatilities. Futures prices are normalized by the price for the 3-month futures contract. The solid black line is the term structure implied by the model. The dashed grey line depicts term structures implied by the data.



**Figure 15: Average Term Structures (No Storage)**

This figure depicts term structures for the futures market for maturities from 3 to 18 months for the NoS model. The upper panel displays the term structure of futures prices. The middle panel displays the term structure of futures premia. The lower panel displays the term structure of return volatilities. Futures prices are normalized by the price for the 3-month futures contract. The solid black line is the term structure implied by the model. The dashed grey line depicts term structures implied by the data.



**Figure 16:** Stochastic Impulse Response (all Models)

This figure depicts stochastic impulse response functions for major economic quantities for all four model calibrations. From left to right models displayed are: Full, time-to build, no storage, and the business cycle calibration. Graphs are depicted as log deviations from the stochastic steady state. Impulse responses are computed as the average of 50,000 sample paths after being exposed to a one-standard deviation positive shock to the level of productivity. The plots depict in order from top to bottom the level of labor augmenting productivity ( $a_t$ ), growth in the R&D sector ( $\mu_t$ ), growth in oil usage ( $\Delta o_t$ ) and log change in inventories ( $\Delta s_t$ ), and the ratios of oil usage over stocks in inventories and oil usage over the the stock of physical capital ( $O_t/S_t$ ,  $O_t/K_t^k$ ).