Size and Value Matter, But Not The Way You Thought

Marie Lambert[1], Boris Fays[2], and Georges Hübner[3]

[1] Marie Lambert holds the Deloitte Chair of Financial Management and Corporate Valuation at University of Liège, HEC Liège, Belgium. She is Research Associate at EDHEC Risk Institute.
[2] Boris Fays is PhD candidate at University of Liège, HEC Liège, Belgium.
[3] Georges Hübner is Professor of Finance at University of Liège, HEC Liège, Belgium. He is Associate Professor at School of Business and Economics, Maastricht University, the Netherlands and Chief Scientific Advisor, Gambit Financial Solutions, Belgium.
Corresponding author: Marie Lambert, Tel: (+32) 4 2327432. Fax: (+32) 4 2327240. E-mail: marie.lambert@ulg.ac.be. Mailing address: HEC Liège, 14 rue Louvrex, B-4000 Liège, Belgium.
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Abstract

We propose a fundamental methodological change to Fama and French (1993) factor construction procedure. Consistent with Lambert and Hübner (2013) sequential sorting procedure to classify stocks, our methodology controls ex ante for pricing errors produced by multifactor models. Our size and value factors deliver less specification errors when used to price passive portfolios including the new portfolio sorts of Fama and French (2015a, b). The sequential model delivers excellent pricing performance for corner portfolios such as low size and high B/M stocks. Furthermore, this alternative framework generates much stronger "turn-of-the-year" size and "through-the-year" book-to-market effects than conventionally documented.

Pricing anomalies related to size (Banz (1981)), value (Basu (1983)), and momentum (Jegadeesh and Titman (1993)) effects on the US stock market have been documented since the early 1980s. First related to mispricing over the Capital Asset Pricing Model, these effects have been widely recognized as priced factors since the influential work of Fama and French (1993). The size premium captures the outperformance of small capitalization stocks over large capitalizations, and Fama and French (1993) associate it with a proxy for (lack of) liquidity. The outperformance of value stocks (i.e. stocks with high book value with regard to their market value) over growth stocks has been related by the same authors to market distress (see also, Fama and French (1995)). Their paper develops a set of heuristics enabling the inference of size and book-to-market effects in the US market. The resulting so-called "Fama-French three-factor model" (hereafter, FF) has become a core version of empirical asset pricing models.

While the original factor construction algorithm developed by Fama and French (1993) has become standard, there are those who suggest that the premiums obtained with the FF technique could be misspecified. Recent studies have fueled the debate by challenging the significance of the value effect (Fama and French (2015a, 2015b), Hou, Xue and Zhang (2014, 2015)). Using the dividend discount model, Fama and French (2015a) show that a company's profitability and its investment policy might significantly affect its market value, one component of the book-to-market. The investment factor CMA (Conservative minus Aggressive) defines the return spread between firms that invest the least and the most. The profitability factor RMW (Robust minus Weak) represents the return spread between firms with the highest and the lowest operating profitability. The new factors totally subsume the significance of the value factor[1] for the specific period used in Fama and French (2015a). This evidence is further supported - over another time

[1] Despite being economically significant within the US stocks universe with an average monthly return of 0.38% and a *t*-stat above 3 for the period 1927-2015.

period - by Hou, Xue and Zhang (2014, 2015) with their *q*-factor model. Their profitability (ROE) and investment (I/A) factors are shown to outperform FF's five-factor model.

Such an inflation of the number of variables needed to explain the cross-section of stock returns can be interpreted in two very different ways. It could uncover a complexity in the return generation process that had been formerly ignored, and thus represent a real advance in empirical asset pricing. The very recent work of Fama and French (2015c) goes in that direction. Their new five-factor model fails to price all market anomalies. Considering both the factors and their small leg, they propose an ad hoc selection of factors according to the anomaly to be priced for keeping the model parsimonious. Alternatively, the need to increase the number of factors could just represent an admission of weakness in the quest for parsimony in factor models, because the right way to understand the universe of systematic risk exposures has not been adequately found. If the latter explanation is true, and this is clearly the perspective adopted in this paper, then researchers should keep on attempting to improve factor construction methodologies to show that having recourse to supplementary risk premiums become superfluous when the original ones are properly determined. Before moving to five-, six- or seven-factor models, one should first do whatever is possible to reject all potential explanations of deficiencies of the original three-factor asset-pricing model. This is the major objective of our paper, and we believe that it contributes to reinforcing a parsimonious approach to asset pricing.

Our main argument relies on the fact that FF independent sorting methodology might lead to an inconsistent definition of value stocks. The correlation between the underlying variables, namely the market capitalization and the book-to-market equity, is mechanically negative (by definition). Under the original Fama and French framework, this effect is further transposed into a negative correlation between the rankings of stocks forming portfolios. Low market capitalization stocks might have spurious high book-to-market equity leading value stocks to be strongly classified into micro-capitalizations.

To cure for these effects, we follow the sequential methodology proposed by Lambert and Hübner (2013), used to isolate fundamental risks into portfolio returns. Such a framework controls for the negative correlation between the underlying characteristic variables and allocate stocks into value portfolios controlling for the level of market value. By removing contamination effects at the early stage, micro-cap stocks are not mechanically tilted to value stocks under the sequential sorting. The second major change relies on a pre-conditioning of the sort on momentum. Our objective is to control for business cycle such as pointed out by Hou and Van Dijk (2008) who disentangle the size effect from profitability shocks and by Novy-Marx (2015) who relates part of the value effect to earnings surprise and momentum.

Our empirical premiums shed new light on the relative importance of the size and book-to-market effects in the US market over an extended period (1963-2014). We demonstrate the existence of a strong value effect, albeit not in the way Fama and French measure it. Our definition of the value effect does not refer to the original interpretation of Fama and French (1993). The value factor might capture part of default risk as distress is more likely to be found in small value stocks. However, it does not constitute a proxy for default risk (as pointed out by Vassalou and Xing (2004)). Our new factor is rather consistent with Zhang (2005), Campbell and Vuolteenaho (2004), and Campbell, Polk and Vuolteenaho (2010), and it associates the value effect with greater sensitivity of a firm's earnings to economic conditions.

We differentiate ourselves from the cited literature challenging the existence of size and value effects by controlling ex ante for external factors rather than a posteriori. This adjustment leads to a stronger "turn-of the year" (January) size effect as well as a permanent, "through-the-year" value premium over time. The figures are impressive: a long/short strategy of investing the Long leg in the Small portfolio and a short leg in the Large portfolio only in January every year, staying out of the market for the remaining 11 months, would yield an average yearly return of 4.77% and monthly standard deviation of 4.78%, which represents a yearly Sharpe ratio of 3.48 sustained over 52 years. The seasonal January size effect is so pronounced that

the mean return of the size factor from February till December each year even becomes negative, although insignificant, over the 52 years under study.

Specification tests of the sequential size and value factors reveal that the change in factor building methodology largely mitigates the need for additional risk premiums to explain stock returns. The factors deliver less specification errors when used to price passive portfolios, especially regarding the "small angels" (low size but high book-to-market stocks) which had come out, to date, as a puzzling, unresolved residual effect. Neither the Fama and French five-factor model, nor the *q*-factor model were able to outperform an alternative, yet equally parsimonious, version of the original Fama and French model (augmented or not with a momentum factor) defined under a sequential approach. Our factors not only outperform the Hou, Xou and Zhang (2014) *q*-factor for pricing the list of low-turnover market anomalies defined by Novy-Marx and Velikov (2015), but they also prove to be more efficient than the original size and value factors when used in a four-factor Carhart model or in the five-factor model from Fama and French (2015a, b) for pricing passive portfolios (including portfolios sorted on alternative market anomalies).

The rest of the paper is organized as follows. Section I reviews the daunting challenges about the size and market anomalies. Section II presents the drawbacks related to the independent sorting procedure performed in the original Fama and French methodology. Section III describes the sequential methodology for constructing mimicking portfolios based on size, book-to-market, and momentum. Section IV performs a workhorse of the properties for the two competing sets of the original size and book-to-market factors. Section V tests the significance of the sequential factors with its three competing sets of factors. Section VI compares the specification power of the four competing sets of factors to price passive portfolios and estimates the alpha bias produced by Fama and French 3-factor model (as well as the correction brought by

the sequential framework) following Fama and French (2010)'s bootstrap procedure. Section VII checks the robustness of our results. Section VIII concludes by summarizing the main insights of this research.

I. The Size and Book-to-Market Anomalies

Mispricing with regard to the original Capital Asset Pricing Model (Sharpe (1964), Lintner (1965)) due to factors such as the size and value effects has been documented in the US stock market since the early 1980s. The three-factor model (Fama and French (1993)) that captures these effects has been strongly challenged in the literature. Using mutual fund data, Huij and Verbeek (2009) point out a strong value effect but no small firm effect. They further show that the original value factor might be overestimated under the Fama and French framework. According to Li, Brooks and Miffre (2009), the portfolios underlying the value premium are not well diversified and as a consequence, the value effect is related to idiosyncratic risk. Finally, Cremers, Petajisto and Zitzewitz (2012) reveal a failure in the Fama and French methodology which leads to overestimate the size and value effects: their original work allocates the same weight to small and large sized portfolios although value effects are stronger for smaller stocks.

A weak size effect has been claimed by the literature (please refer to van Dijk (2011) and Asness et al. (2015) for a full discussion). As also shown by Horowitz, Loughran, and Savin (2000), the size effect is concentrated into microcap stocks. Using the discounted cash flow model, Berk (1997) relates the size effect to the intrinsic negative relationship between price and expected returns. Berk further documents the absence of the size effect when measured with accounting indicators (like sales). Arnott, Hsu and Moore (2005) point out that noise in stock prices due to trading or microstructure might also create the size and value effects. Vassalou and Xing (2004) provide further evidence that both effects only indirectly proxy for default risk as default is more likely to be found in small value stocks.

From Table 1, which casts some doubt about the persistence of the size and value effects through the year, the size premium appears to be only significant in January. Reinganum (1981), Roll (1981), and Keim (1983) had already identified a calendar anomaly for the size premium known as the "turn-of-the-year effect". Further evidence can be found in Jacobsen, Mamun and Visaltanachoti (2005) and Moller and Zinca (2008). Asness et al. (2015) show that after controlling ex post for quality/junk, the size effect stays significant across the whole sample period and is not only concentrated during the month of January. Hou and van Dijk (2008) reach similar conclusions after adjusting firms' returns for profitability shocks.

Table I

FF's Original Size and Value Premiums over Time

The table displays descriptive statistics for FF size (SMB_{ff}) and book-to-market (HML_{ff}) premiums over time. Both stock factors are obtained from K. French's website. Time-series mean, standard deviation (S.D.), *t*-stat for the bilateral test of time series mean equals to zero, as well as the number of observations considered are displayed. The analysis covers the original period used in Fama and French (1993) – that is from January 1963 to December 2014. It performs the analysis for January and February-December separately over the same sample period.

	SMB_{ff}				HML_{ff}			
	Mean (%)	S.D. (%)	*t*-stat	Freq	Mean (%)	S.D. (%)	*t*-stat	Freq
Total sample	0.24	3.09	1.93	624	0.38	2.85	3.3	624
January	1.97	3.41	4.16	52	1.38	3.57	2.79	52
Feb. - Dec.	0.08	3.02	0.64	572	0.29	2.76	2.47	572

Although significant throughout the year, the Fama and French value premium is almost 5 times bigger in January for the last 25 years. According to Fama and French (1993, 1995), the book-to-market factor proxies for market distress but this interpretation has recently been challenged in the literature. Recent

evidence has emphasized the need for a profitability factor rather than a distress factor for modeling the cross-sections of stock returns (Novy-Marx (2013), Ball et al. (2015)). Zhang (2005) and Petkhova and Zhang (2005) propose another explanation for the value effect: value companies are stocks that generate strong cash flows but might suffer during periods of recession. The rationale is that these "asset-in-place" firms have larger difficulties to scale down their investments during squeezed economic contexts and they ultimately end up forced to run their business with unproductive assets. Value stocks are expected to deliver a higher expected return to compensate for this risk.

II. Background: Correlation Bias in the FF (1993) Ranking Methodology

The Fama and French (1993) three-factor model and its extension for momentum (authored by Carhart (1997)) have become a benchmark of empirical asset pricing. Using a dataset from the Center for Research in Security Prices (CRSP), Fama and French consider two independent methods of scaling US stocks, i.e. an annual two-way sort on market equity and an annual three-way sort on book-to-market according to New York Stock Exchange (NYSE) breakpoints (quantiles). They then construct six value-weighted (two-dimensional) portfolios at the intersections of the annual rankings (performed each June of year *y* according to the fundamentals displayed in December of year *y*-1). The size or SMB factor (Small minus Big) measures the return differential between the average small cap and the average big cap portfolios, while the book-to-market or HML factor (High minus Low) measures the return differential between the average value and the average growth portfolios.

Under such an independent sorting, the six portfolios will have approximately the same number of stocks only if size and book-to-market are unrelated characteristics; that is, if there is no significant correlation between the risk fundamentals. However, market capitalization and book-to-market are correlated. The study of Fama and French (1993) even points out that "*using independent size and book-to-*

market sorts of NYSE stocks to form portfolios means that the highest book-to-market/market equity quintile is tilted toward the smallest stocks" (Fama and French, 1993, pp. 12).

The independent rankings defined under the FF framework induce a significant (average) correlation of about -28% (at the 1% confidence level) over the period ranging from January 1963 to December 2014. Polychoric correlation between ordered-category variables provides a way to separately quantify association and similarities between the two-way size and three-way book-to-market rankings. By using this polychoric correlation instead of a Pearson correlation between the underlying fundamentals of the portfolios, we examine if the relationship between the underlying variables (market equity and book-to-market equity) impacts how a stock falls into one of the categories, i.e. either small or big size and either low, medium or high book-to-market. The analysis is performed on each portfolio rebalancing date, i.e. June of each year *y*.

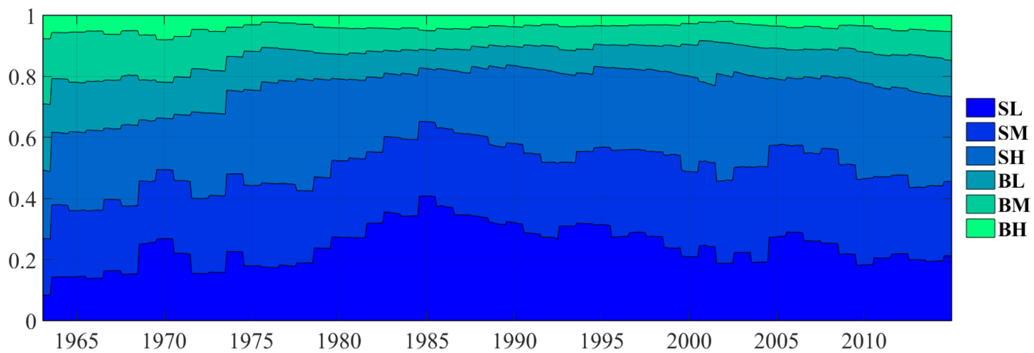
This negative correlation bias and the use of NYSE breakpoints create an imbalance between the numbers of stocks within the six portfolios composing the premiums. The imbalance within the distribution of stocks amongst the six portfolios indicates that the size effect cannot be equivalently diversified across book-to-market sorted portfolios and could therefore contaminate the value effect derived from these portfolios and vice versa.

Figure 1 illustrates the imbalance in the stock partitioning across portfolios over our sample period.

Figure I

Relative Stock Distribution among the 2x3 Characteristics Portfolios

The figure displays the total percentage stock repartition among the FF 2x3 characteristic-sorted portfolios on size (small and big) and book-to-market (low, medium and high) from January 1963 to December 2014.



In the working paper version of Cremers, Petajisto and Zitzewitz (2012), the authors already state that "*only the largest cap decile is clearly negatively correlated with SMB; the midcaps (size deciles 6-8) are positively correlated with SMB despite being included among big stocks, which should mechanically induce a negative correlation*". This quote supports our argument that using NYSE breakpoints, FF are mixing small- with mid-caps which should have an impact on the size premium. As a consequence, FF's methodology might not price accurately the incremental return of pure small cap stocks.

III. An Alternative to the Fama and French Procedure: the Sequential Sorting

To correct for the correlation bias of FF's original ranking methodology, we replace the independent sort by a sequential sorting procedure. We demonstrate that this technique leads to a substantial purification of risk factors, as it ensures stocks homogeneity in each portfolio constructed on all three fundamental risk dimensions (i.e. size, book-to-market, and momentum). Pre-conditioning the portfolio sort on momentum controls for the business cycle which might affect the level of the value and size premium (Hou and Van Dijk (2008), Novy-Marx (2015)). This is also similar in spirit with Gerakos and Linnainmaa (2016) who control ex post for the impact of change in firm size (market equity) on the value premium. Before presenting our alternative sorting technique, we describe the data used to form the aforementioned fundamental risk dimensions.

A. Data

Since the purpose of this paper is to propose a robust comparison framework to the original Fama and French approach, we strictly follow their stock selection methodology to construct our risk factors. The period ranges from January 1963 (as in Fama and French (1993)) to December 2014 and comprises all NYSE, AMEX, and NASDAQ stocks collected from the merge between the Center for Research in Security Prices (CRSP) and COMPUSTAT databases. The analysis covers 624 monthly observations. The market risk premium corresponds to the value-weighted return on all US stocks minus the one-month T-Bill rate from Ibbotson Associates. We consider stocks that fully match the following lists of filtering criteria: a CRSP share code (SHRCD) of 10 or 11 at the beginning of month t, an exchange code (EXCHCD) of 1, 2 or 3, available shares (SHROUT) and price (PRC) data at the beginning of month t, available return (RET) data for month t, at least two years of listing on COMPUSTAT to avoid the survival bias (Fama and French

(1993)) and a positive book-equity value at the end of December of year *y*-1. Our sample is thus varying over time: for instance, from a total of 5,612 stocks available as of December 2014, our conditions restrict our sample to 3,271 stocks (for 2014).

As in Fama and French (1993), we define the book value of equity as the stockholders' equity reported by Compustat (SEQ). If not available, we substitute this value by either the sum of the book value of stockholders' equity (CEQ) and the book value of preferred stock (PSTK) or the difference between a firm total assets (AT) and its total liabilities (LT). To have the book common equity, we add the balance-sheet deferred taxes and investment tax credit (TXDITC) and the book value of preferred stock (PSTK), or the liquidation value (PSTKL) or the redemption value (PSTKRV), in that order.

The book-to-market equity is the ratio between the book common equity for the fiscal year ending in calendar year *t*-1, divided by market equity of December *t*-1.

Market equity is defined as the price (PRC) of the stock times the number of shares outstanding (SHROUT) at the end of June *y* to construct the size factor and at the end of December of year *y*-1 to construct the value factor.

Carhart (1997) completes the Fama and French three-factor model by computing a momentum (i.e. a *t*-2 until *t*-12 cumulative prior-return) or UMD (Up minus Down) factor that reflects the return differential between the highest and the lowest prior-return portfolios.

B. Factor Construction Approach

The modified factor construction approach differs from the original Fama and French methodology on a number of points. Firstly, our methodology comprises a comprehensive framework that analyses the three

empirical risk dimensions altogether (size, book-to-market, and momentum). Each form of risk is equally considered.

Secondly, we impose a pre-conditioning on momentum for two main reasons: first, we aim to control for cyclical effects into the size and value premiums that arise from momentum in returns (as already pointed out), and second we intend to neutralize the trend following pattern intrinsic to traditional value-weighted portfolios. Thirdly, the modified methodology proposes a consistent and systematic sort on all listed stocks, while Fama and French only perform a heuristic split of US stocks according to the New-York Stock Exchange (NYSE) as reference point.

Finally, our sequential (dependent) sort avoids spurious cross-effects in risk factors as our methodology controls for the correlations in the underlying company fundamentals. The following subsections detail the construction of the sequential premiums.

B.1. The Sequential Sorting Procedure

In designing the sorting procedure, our objective is to detect whether, when controlling for two out of the three risk dimensions, there is still enough return variation related to the third risk criterion. Therefore, we substitute the Fama and French "independent" sort with a "sequential" or "conditional" sort, i.e. a multi-stage sorting procedure. More specifically, we successively perform three sorts. The first two sorts operate on "control risk" dimensions, followed by the risk dimension to be priced. We use the momentum effect documented by Jegadeesh and Titman (1993) and introduced in empirical asset pricing models by Carhart (1997) as one of the control risk dimensions, and then use either the size or book-to-market as the second control, depending on whether one wants to isolate the book-to-market or size risk premium.

The sequential sorting produces 27 (3x3x3) portfolios capturing the return relating to a low, medium, or a high level of the risk factor, conditional on the levels registered on the two control risk dimensions.

Taking the simple average of the differences between the portfolios' highest and lowest scores on the risk dimension to be priced whilst scoring at the same level for both control risk dimensions, we are able to obtain the return variation related to the risk under consideration. This procedure is similar to that of Lambert and Hübner (2013). To obtain the risk premium corresponding to dimension X, after sequentially controlling for dimensions Y and Z, the factor can be computed as follows,

$$X_{Y,Z,t} = \frac{1}{9}\left[\sum_{b=H,M,L}\sum_{c=H,M,L} R_t(HX|bY|cZ) - \sum_{b=H,M,L}\sum_{c=H,M,L} R_t(LX|bY|cZ)\right] \quad (1)$$

where $R_t(aX|bY|cZ)$ represents the return of a portfolio of stocks ranked *a* on dimension X, among the basket of stocks ranked *b* on dimension Y, themselves among the basket of stocks ranked *c* on dimension Z. Dimensions X, Y and Z stand for respectively the factor to be priced and its control while H, M and L stand for high, medium and low, respectively. When dimension X corresponds to market cap, the premium is defined as LX minus HX.

In contrast to an independent sorting, the sequential one ensures a balanced distribution of stocks in all 27 portfolios and hence, equivalent level of diversification.

We illustrate our methodology with the construction of the HML factor. We start by breaking up the NYSE, AMEX, and NASDAQ stocks universe into three groups according to the momentum criterion (first control). We then successively scale each of the three momentum-portfolios into three classes according to their market capitalization (second control). Splitting each of these nine portfolios once again to form three new sub-portfolios according to their book-to-market fundamentals (variable to be priced), we end up with 27 value-weighted portfolios. The rebalancing is performed on an annual basis at the end of June of year *y*. An analogy could be made to a cubic construction: each year, any stock integrates one slice, then one row,

then one cell of a cube and thus enters one and only one portfolio. The stock specific value-weighted return for each month following the yearly ranking is then related to the reward gained through the risks incurred in this portfolio.

Amongst the 27 portfolios inferred from the sequentially sorted risk factors, we retrieve only the 18 that score at a high or a low level on the risk dimension (corresponding to the last sort performed), i.e. value/growth. The nine self-financing portfolios are then created from the difference between high- and low-scored portfolios displaying the same ranking on the size and momentum dimensions (used as control variables). Finally, the HML risk factor is computed as the arithmetic average of these nine portfolios. Note that each premium can be defined in two different ways within this conditional framework[2].

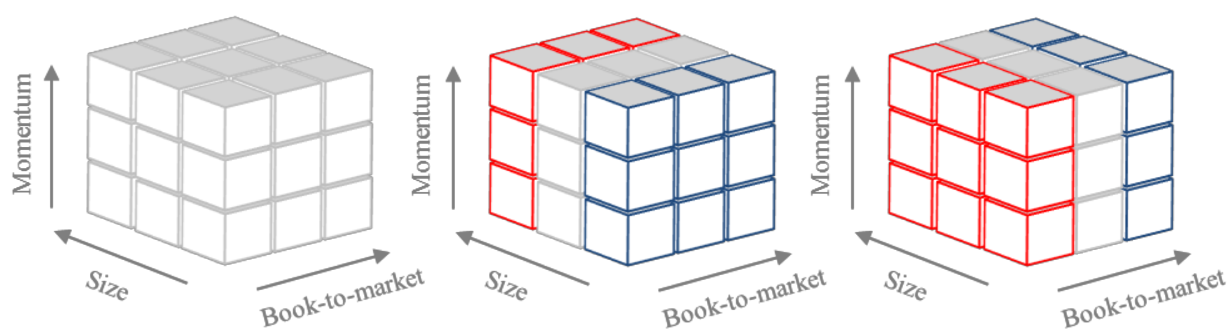
Illustrations of the sequential premiums constructions are displayed in Figure 2.

[2] The paths for the sequential SMB and HML factors used are respectively momentum, book-to-market and size and momentum, size, book-to-market. The alternative paths were tested and lead to the same conclusion. Results are available in Appendix and commented in the Robustness Tests (Section VII).

Figure II

Representative Sequential Construction of the 3x3x3 Characteristics Portfolios

The three figures display the cubic sequential methodology construction of the 3x3x3 characteristics portfolios. The left-hand figure shows the split of the stock universe when applying the sequential sorting procedure. The middle figure displays the portfolios used to construct the size premium by first sorting on momentum, then book-to-market and finally size: blue (resp. red) squares represent small (resp. large) capitalization stock portfolios. The right-hand figure displays the portfolios used to construct the value premium by first sorting on momentum, then size and finally book-to-market: blue (resp. red) squares represent high (resp. low) book-to-market ratio stock portfolios.



B.2. Implications of a Three-Way Sort

We split the sample according to three levels of size, book-to-market, and momentum. Two breakpoints are used for all fundamentals (1/3rd and 2/3rd percentiles). Instead of the original six portfolios, this method leads to a set of 27 baskets of stocks. The breakpoints are based on all US markets, not only on NYSE stocks. The finer size classification also contributes to balance the proportion between the small/value, small/growth, large/value and large/growth portfolios. It also provides a better distinction between small and large cap stocks. Sorting stocks into portfolios according to whole sample breakpoints rather than NYSE

stocks might exacerbate the tilt toward NASDAQ stocks into the small cap portfolios. The representation of NASDAQ stocks is quite important among the 9 portfolios, which fall under the low market capitalization. For illustrative purposes, the proportion amounts on average to 57% for the HML factor. However, this issue is also present in Fama and French framework, although it uses NYSE breakpoints, with on average 31% of NASDAQ stocks composing the three portfolios of low market capitalization.

We also acknowledge that our construction leads size breaks for most momentum stocks to be below the NYSE median. But using a triple sort on both market capitalization and book-to-market gives one third of the weight of the new HML premium to tiny companies, one third to small companies and one third to medium and large caps. This three-way split might appear more extreme than Fama and French's 50% weight on small stocks but is however aligned with their recent intention to show the outperformance of the value premium only defined using small capitalization stocks (Fama and French (2015c)).

It is finally important to note that when pricing the size factor, a book-to-market ratio of 0.5 may put a stock in the high B/M portfolio in one momentum-size portfolio, in the medium B/M in another, and in the low B/M in the third. What matters is that among those sub-portfolios, stocks with low market capitalization outperform stocks with large market cap. Doing so, we ensure that the "small angels" stocks, i.e. stocks with high B/M but tiny market cap, do not drive the HML premium. Moreover, a stock whose own characteristics remain unchanged may move to another B/M classification – even if the full B/M cross-section does not change in a year. This could happen for instance if the stock return follow an upward trend, which would inflate its market value and affects wrongly its B/M ratio. An independent sorting would miss this information and conclude wrongly to a low B/M. Such flexibility for stock migration is certainly a core element of our procedure since we advocate that the classification for one of the priced variable (e.g. book-to-market) should not be affected by another one (e.g. market equity).

IV. The Sequential Approach: Curing for the Ranking Correlation Issue

Section II of this paper presented preliminary evidence regarding the correlation bias inherent in the Fama and French factor construction. This section examines the empirical impact of the methodological changes introduced above, the first of which was the sequential sort procedure[3].

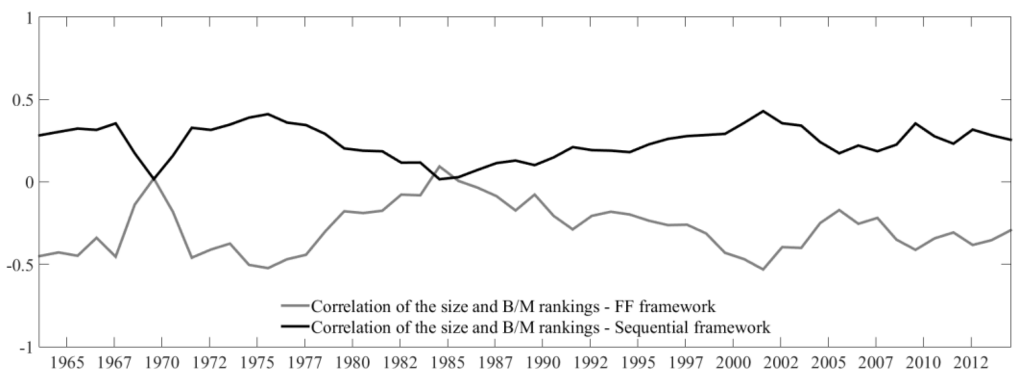
The average polychoric correlation between the sequential rankings for size and book-to-market, respectively, from the SMB' premium and the HML' premium is 24% (significant at 1% level). Contrary to the negative correlation among the ranking provided by an independent sorting, we found a significant positive correlation. This reversal effect between the rankings is illustrated over the sample period ranging from January 1963 to December 2014 in Figure 3.

[3] Our objective being to review the original construction of the size and value premiums while controlling for additional sources of risk, momentum has only been introduced into the analysis as an additional control variable. For consistency purposes, it has been defined using an annual rebalancing contrary to the original approach of Carhart (1997).

Figure III

Correlations between the Independent and Sequential Rankings over Time

This figure shows the evolution of the polychoric correlations between the independent rankings of the sequential methodology for size (*SMB*) and book-to-market (*HML*) premiums among the 3x3x3 characteristic-sorted portfolios (black line) and the FF methodology for size and book-to-market premiums among the 2x3 characteristic-sorted portfolios (gray line) from January 1963 to December 2014.



We observe a quasi-symmetrical effect between the rankings correlations produced by the original and sequential empirical risk factors. To better understand this reversal effect, Table 2 presents the historical frequencies for a low, medium or high size stock to be classified as either low, medium or high book-to-market. For the sequential procedure, the probability of a small (resp. big) capitalization to be ranked high book-to-market is the lowest (resp. highest), i.e. 24% (resp. 47%). The results challenge Fama and French's procedure, in which a small (resp. big) capitalization has the highest (resp. lowest) likelihood to be ranked high book-to-market 38% (resp. 16%).

Table II

Frequencies of the Independent FF and Sequential Rankings

This table reports, for each level of size (based on the independent ranking or the final ranking step of the sequential sort), the average repartition of stocks into the different book-to-market levels (based on the independent ranking or the final ranking of the sequential sort). The weights displayed on each line sum up to one for each panel. Under the Fama and French model, only two size classifications (i.e. low or high) may be allocated low, medium or high book-to-market. Under the sequential framework, three size classification (i.e. low, medium, and high) may be allocated either low, medium or high book-to-market. The period analyzed ranges from January 1963 and December 2014.

	Panel A: Sequential Rankings			Panel B: Fama and French Rankings		
B/M→	Low	Medium	High	Low	Medium	High
Size panel ↓						
Low	45%	31%	24%	28%	34%	38%
Medium	30%	41%	30%		N.A.	
High	26%	27%	47%	46%	38%	16%

The positive correlation[4] resulting from the sequential procedure induces that an income stock is more likely to constitute a large capitalization company under the sequential framework. Such findings are consistent with the concept of value/income generation. Our framework for pricing the value effect differs from Fama and French (1993) as it does not relate to market distress which is only present in small value stocks (Vassalou and Xing (2004)) but is close to concepts of cost reversibility and earnings risk explained by Zhang (2005).

[4] Almost nil polychoric correlations between the rankings (i.e. July, 1969 and 1985) are due to a reduction of the market equity for the overall market which lowers the tilts for the classification. Results are available on request.

The difference in frameworks evidenced above has direct implications on the distribution of stocks into portfolios. The expected size premium is defined in both frameworks as follows:

$$E(R) = \sum_{b=H,M,L} P(bY \cap SX) \times \big(R(bY \cap SX) - R(bY \cap BX)\big) \quad (2)$$

where SX $(\text{resp.}\, BX)$ represents the return of a portfolio of stocks ranked *small* (resp. *big*) on dimension *X*, i.e. size; bY represents a portfolio of stocks ranked b (i.e. low, medium or high) on dimension *Y*, i.e. book-to-market; *P* stands for probability and *E*(*R*) for expected return.

The sequential construction forces $P(bY \cap SX)$ to be equal to 1/9:

$$P(bY \cap SX) = P(SX|bY) \times P(bY) = 1/3 \times 1/3 \quad (3)$$

Table 2 displays the alternative conditional probabilities $P(bY|SX)$ induced by the two alternative paths (one that ends with B/M dimension or the one that ends with market capitalization) for the sequential framework (in Panel A) but conditional probabilities induced by the independent ranking procedure, i.e. $P(bY \cap SX)$ from the FF framework scaled so that the sum of the probabilities equals to 1 (in Panel B).

Knowing that $P(bY \cap SX) = P(bY|SX) \times P(SX)$, we uncover the implied distribution of small caps among the portfolios sorted on book-to-market. *P*(*SX*) is equal to 25% (resp. 36%, 46%), i.e. 1/9 divided by rounded 45% (resp. 31%, 24%), for low (resp. medium, high) B/M. The sequential framework adjusts the distribution of small stocks for the positive correlation between market capitalization and book value so that the sorting does not overestimate the premium driven by small stocks and puts exactly the same weight on the nine return spreads.

Under the independent framework, *P*(*SX*) is set to about 50% using the NYSE breakpoints which implies that $P(bY \cap SX)$ is respectively set to 14%, 17% and 19% for low to high B/M and low size portfolios and respectively to 23%, 19% and 8% for large portfolios. Contrary to the sequential framework, the FF

independent sorting does not equally balance the six portfolios forming the premium and overweighs small/value and large/growth portfolios.[5]

The use of sequential breakpoints favors better allocations of stocks into portfolios. Stocks are homogeneously distributed with an average of 122 stocks and a standard deviation of only 4 stocks per portfolio over the whole sample period. We control tilts of the 27 portfolios toward small stocks as the correlations between the rankings on the priced dimension and on the control variables produced by the sequential approach are almost independent (correlation close to zero). From January 1963 to December 2014, the annual ranking correlations range between -0.18% and 0.14% for the first control variable and -0.32% and 0.17% for the second control variable. The correlation among the rankings is constrained by the sequential sorting itself since the alternative construction path leads to inverting the correlations for the control rank.

Table 3 analyses how these ranking correlations condition the final (Pearson) correlation between the size and value factors. The bottom-left corner displays the cross-correlations between the two sets of premiums. The SMB' and HML' factors are correlated at 81% and 88% with their Fama and French counterparts, respectively. These levels indicate that, even though the original and the modified size and value premiums are intended to price the same risk, approximately 19% to 11% of their variation provides different information. We relate those differences to the aforementioned purification effect of the sequential sort.

[5] The same exercise can unfortunately not be done for the expected HML premium for respectively value and growth stocks as the sequential framework considers three levels of size contrary to Fama and French (1993).

Table III

Correlation Matrix of the Empirical Risk Premiums

The table reports the paired correlations (in %) among the modified (sequential) and the original FF empirical risk premiums over the period ranging from January 1963 to December 2014, as well as across these two sets of factors. Tests for significance of the pair-wise correlations are performed: *, **, and *** indicate statistical significance at the 0.1, 0.05 and 0.01 levels, respectively.

	SMB'	*HML'*	*UMD'*	*SMB$_{ff}$*	*HML$_{ff}$*	*UMD$_{ff}$*
SMB'	1					
HML'	-0.34***	1				
UMD'	-0.08**	-0.13**	1			
SMB$_{ff}$	0.81***	-0.33***	-0.08**	1		
HML$_{ff}$	-0.21***	0.88***	-0.30***	-0.23***	1	
UMD$_{ff}$	0.06	-0.06	0.63***	0.00	-0.16***	1

The momentum premium displays a lower correlation with the UMD$_{ff}$ factor. Contrary to the SMB$_{ff}$ and HML$_{ff}$ factors, the original momentum premium does not exactly follow Fama and French's (1993) methodology: the premium is rebalanced monthly rather than annually. It differs from our momentum premium with regard to the breakpoints used for the rankings and the annual rebalancing scheme used in the sequential sorting procedure.

In Table 3, the bottom-right corner presents the intra-correlations between the Fama and French premiums. The SMB$_{ff}$ and HML$_{ff}$ factors are negatively correlated over the period (-23%). The top-left corner presents the intra-correlations among the sequential premiums: the signs are the same as those displayed by FF premiums, but the correlation between SMB' and HML' factors decrease to -34% over the sample period. The sequential size effect is confined to a January effect and negative although not significant

over the rest of the year while the sequential value is positive and persistent all over the year. This induces the negative correlation, which is not related to the construction method.[6] Table 4 shows that the correlation between the original FF size and value factors is significantly positive (resp. negative) during January (resp. February-December). This further supports the tilt of the value stocks to small caps which outperform in January.

Table IV

Correlation Matrix of the Empirical Risk Premiums: January Effect

The table reports the paired correlations (in %) among the modified (sequential) and the original FF empirical risk premiums over the period ranging from January 1963 to December 2014, as well as across these two sets of factors. Tests for significance of the pair-wise correlations are performed: *, **, and *** indicate statistical significance at the 0.1, 0.05 and 0.01 levels, respectively. Panel A reports the correlations only in January and Panel B displays the correlations between February and December.

	Panel A : Only January				Panel B : February to December			
	SMB'	*HML'*	*SMB_{ff}*	*HML_{ff}*	*SMB'*	*HML'*	*SMB_{ff}*	*HML_{ff}*
SMB'	1				1			
HML'	-0.09	1			-0.40***	1		
SMB_{ff}	0.78***	0.02	1		0.83***	-0.28***	1	
HML_{ff}	0.23	0.88***	0.27*	1	-0.39***	0.88***	-0.32***	1

[6] The strong "turn-of-the-year" effect of the size factor and "through-of-the-year" effect of the value factor induce this deeper negative correlation (see infra).

V. Factor Persistence, Significance and Seasonality: Sequential vs Independent Framework

Seasonality in the size factor has already been documented in Section 1. Yet, the January effect in the value factor (1.38% average return) revealed by Table 1 has not been examined in the literature. Table 1 also showed a strong FF value premium during the month of January. However, considering the sequential premiums over the same subsample periods, investor risk aversion to value effect is persistent over time with significant average return at a 99% confidence level (Panel B of Table 5). From Table 5, no particular effect can be reported during January. The rationale is that in the FF framework, HML_{ff} is contaminated by the size "turn-of-the-year" effect. Curing for correlation among rankings mitigates this contamination.

Table V

Sequential Size and Value Premiums over Time

The table displays descriptive statistics for FF size (*SMB_{ff}*) and book-to-market (*HML_{ff}*) premiums as well as the sequential size (*SMB')* and book-to-market (*HML')* premiums over four periods of time. Both *SMB_{ff}* and *HML_{ff}* are obtained from K. French's website. Time-series mean, standard deviation, *t*-stat for the bilateral test of time series mean equals to 0, as well as the number of observations considered are displayed as well as the percentage of positive return over the sample periods (Pos. Obs.). The analysis covers the original period used in Fama and French (1993) – that is from January 1963 to December 2014. We consider the periods January and February to December separately over the same sample period.

Panel A: Fama and French Factors										
	SMB_{ff}					HML_{ff}				
	Mean (%)	S. D. (%)	*t*-stat	Freq	Pos. Obs. (%)	Mean (%)	S. D. (%)	*t*-stat	Freq	Pos. Obs. (%)
Total sample	0.24	3.09	1.93	624	51	0.38	2.85	3.30	624	57
January	1.97	3.41	4.16	52	67	1.38	3.57	2.79	52	75
Feb. - Dec.	0.08	3.02	0.64	572	50	0.29	2.76	2.47	572	56

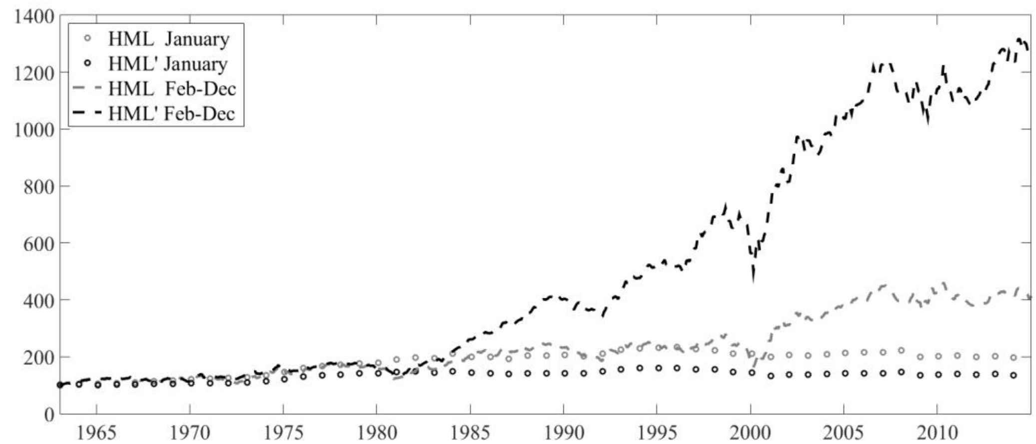
Panel B : Sequential Factors										
	SMB'					HML'				
	Mean (%)	S. D. (%)	*t*-stat	Freq	Pos. Obs. (%)	Mean (%)	S. D. (%)	*t*-stat	Freq	Pos. Obs. (%)
Total sample	0.23	4.18	1.40	624	47	0.48	2.33	5.16	624	60
January	4.77	4.78	7.20	52	87	0.63	2.97	1.54	52	62
Feb. - Dec.	-0.18	3.88	-1.10	572	44	0.47	2.27	4.93	572	60

In Figure 4, we illustrate the evolution of the value premium over the sample period and demonstrate that no "turn-of-the-year" effect applies under the sequential framework.

Figure IV

Value Calendar Effect

This figure shows respectively the evolution of $100 invested from January 1963 to December 2014 in which the full capital is invested in the sequential (black) or Fama and French (gray) value premium only in January (dotted lines) and if the initial $100 are invested in the value premium excluding the month of January (dashed lines).

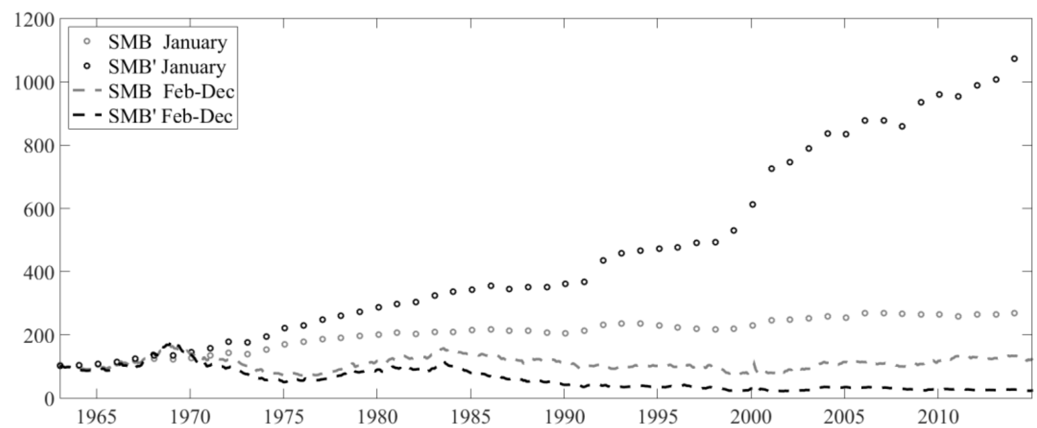


Echoing Cremers, Petajisto and Zitzewitz (2012) on the overestimation of the size effect under the Fama and French framework, our sequential framework reduces the size factor (SMB') to a January effect (which more than doubles compared to SMB_{ff} with a 4.77% monthly average return versus 1.97% for the latter). Controlling ex ante for the momentum and value effect, the size factor does not reveal to be significant outside the month of January. It is even found to be negative, although not significant, over the 52 years of the sample period when only the remaining 11 months of the year are considered. Figure 5 shows the evolution of the size premium over the sample period and reports the significant "turn-of-the-year" effect under both frameworks, the effect being even more pronounced under the sequential framework.

Figure V

Size Calendar Effect

This figure shows respectively the evolution of $100 invested from January 1963 to December 2014 in which the full capital is invested in the sequential (black) or Fama and French (gray) size premium only in January (dotted lines) and if the initial $100 are invested in the size premium excluding the month of January (dashed lines).



We perform six regression models in which the value (or resp. size) premium is first regressed on RM_{ff}, SMB (or resp. HML), and a combination of recent factors that have recently shown to compete with the Fama and French (1993) model, i.e. the Fama and French (2015a) factors, the quality factor of Asness et al. (2015) as well as the Hou, Xue and Zhang *q*-factors (2014).

Results for the original FF and sequential value premium are displayed in Table 6, Panel A and B respectively. Considering that this paper aims at reviewing the misspecification of the three-factor model,

we do not redefine the momentum factor[7]. The profitability (RMW) and the investment (CMA) factors are available on French's website. For the quality factor (Quality minus Junk), we refer to the AQR library.

Panel A of Table 6 confirms Fama and French's (2015a) evidence. The value factor is not proved significant (*t*-stat=0.97) when the profitability (RMW) and the investment (CMA) factors are introduced in the regression model. Yet, the HML_{ff} factor reappears significant (*t*-stat=3.44) when we control ex post for quality/junk stocks. Panel B of Table 6 substitutes the original FF size and book-to-market factors with our new set of sequential premiums. Strikingly, the HML' factor is persistent in any method of factor construction, suggesting that our methodology is able to control ex ante for quality (and not ex post such as in the Asness et al.'s framework).

We also assess the persistence of the value anomaly in the presence of Hou, Xue and Zhang (2014) profitability (ROE) and investment (I/A) factors. In their paper, they demonstrate the outperformance of their *q*-factor model against the recent FF five-factor model. Panel A supports this evidence as their *q*-factor model fully explains the original FF value risk premium, delivering an insignificant alpha of 0.08 (*p*-value=0.39)[8]. Yet, the sequential value factor persists against the *q*-factors with an alpha of 0.28 (*t*-stat=3.52).

To conclude, contrary to the original FF value factor, the sequential HML factor is not redundant under the *q*-factor model or the FF five-factor model.

[7] Tests substituting the original momentum factor (Carhart (1997)) with the sequential premium, which includes annual portfolio rebalancing (June of each year), provide similar results. Results are available upon request.
[8] It is worth to mention that Fama and French (2015a), Hou, Xue and Zhang (2014) and us are using different time periods.

Table VI

Significance Tests on HML (High minus Low) Factor

The table reports regression results for the Fama and French value premium (Panel A) and the sequential value premium (Panel B) on the factors, that is RM_{ff}, the size factor (SMB_{ff}), the momentum (UMD_{ff}), the profitability (*RMW),* and the investment (*CMA)*. We also add *QMJ* (Quality minus Junk) factor from Asness, et al. (2015), and the profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014). The period used to perform the regressions ranges from January 1963 to December 2014. Figures underlined in light grey are regressed from July 1967 to December 2014 since the *ROE* and *I/A* factors are only made available for this time period.

Panel A : Regression on HML_{ff}

	Int	RM_{ff}	SMB_{ff}	*HML'*	UMD_{ff}	*RMW*	*CMA*	*QMJ*	*ROE*	*I/A*	*R*
Coef	-0.11			1.01							0.77
t-statistic	-1.97			45.28							
p-value	**0.05**			0.00							
Coef	0.59	-0.18	-0.14		-0.13						0.15
t-statistic	5.43	-7.00	-3.76		-5.13						
p-value	0.00	0.00	0.00		0.00						
Coef	0.58	-0.18	-0.14		-0.13	-0.02					0.15
t-statistic	5.25	-7.03	-3.60		-5.05	-0.36					
p-value	0.00	0.00	0.00		0.00	0.72					
Coef	0.08	0.00	-0.04		-0.12	0.20	1.00				0.54
t-statistic	0.97	-0.04	-1.46		-6.20	4.72	23.09				
p-value	**0.33**	0.97	0.15		0.00	0.00	0.00				
Coef	0.27	-0.13	-0.15		-0.06	0.63	0.97	-0.68			0.62
t-statistic	3.44	-5.90	-5.33		-3.54	11.68	24.58	-11.30			
p-value	0.00	0.00	0.00		0.00	0.00	0.00	0.00			
Coef	0.08	-0.05	-0.09		-0.09				-0.13	1.02	0.53
t-statistic	0.86	-2.35	-2.82		-4.00				-3.03	20.44	
p-value	**0.39**	0.02	0.00		0.00				0.00	0.00	

(continued)

Table VI-(*continued*)

Panel B : Regression on *HML'*											
	Int	RM_{ff}	*SMB'*	HML_{ff}	UMD_{ff}	*RMW*	*CMA*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.20			0.76							0.77
t-statistic	4.07			45.28							
p-value	0.00			0.00							
Coef	0.68	-0.17	-0.16		-0.09						0.21
t-statistic	7.49	-8.23	-8.10		-4.30						
p-value	0.00	0.00	0.00		0.00						
Coef	0.65	-0.17	-0.15		-0.09	0.04					0.22
t-statistic	7.07	-8.05	-6.73		-4.35	0.80					
p-value	0.00	0.00	0.00		0.00	0.43					
Coef	0.25	-0.01	-0.10		-0.08	0.22	0.80				0.55
t-statistic	3.47	-0.76	-5.74		-5.14	5.88	21.26				
p-value	0.00	0.45	0.00		0.00	0.00	0.00				
Coef	0.38	-0.10	-0.16		-0.05	0.48	0.78	-0.43			0.59
t-statistic	5.32	-5.12	-8.54		-2.97	9.81	21.67	-7.81			
p-value	0.00	0.00	0.00		0.00	0.00	0.00	0.00			
Coef	0.28	-0.06	-0.16		-0.06				-0.10	0.80	0.53
t-statistic	3.52	-3.34	-8.15		-3.11				-2.53	18.62	
p-value	0.00	0.00	0.00		0.00				0.01	0.00	

We perform a similar analysis for the size factor. Table 7 displays the intercepts and factor betas as well as their *t*-statistics and *p*-values of a multifactor model performed on the original size factor (Panel A) and on the sequential size factor (Panel B). As pointed out by Asness et al. (2015), SMB_{ff} is only significant – at a confidence level of 99% – when controlling for profitability or quality. However, the sequential size factor appears to be consistently significant in any method of factors construction, whether or not controlling ex post for profitability or quality. Relative to the FF model, a sequential methodology proves to control ex ante rather than ex post for spurious noise (*t*-stats always significant).

Table VII

Significance Tests on SMB (Small minus Big) Factor

The table reports regression results for the Fama and French size premium (Panel A) and the sequential size premium (Panel B) on the factors that is RM_{ff}, the value factor (HML_{ff}), the momentum (UMD_{ff}), the profitability (*RMW),* and the investment (*CMA)*. We also add the *QMJ* (Quality minus Junk) factor from Asness et al. (2015), and the profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014). The period used to perform the regressions ranges from January 1963 to December 2014. Figures underlined in light grey are regressed from July 1967 to December 2014 since the *ROE* and *I/A* factors are only made available for this time period.

Panel A : Regression on SMB_{ff}											
	Int	RM_{ff}	*SMB'*	HML_{ff}	UMD_{ff}	*RMW*	*CMA*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.08		0.57								0.67
t-statistic	1.12		35.67								
p-value	**0.26**		0.00								
Coef	0.21	0.18		-0.17							0.12
t-statistic	1.73	6.63		-3.88							
p-value	**0.08**	0.00		0.00							
Coef	0.20	0.18		-0.16	0.01						0.12
t-statistic	1.65	6.54		-3.76	0.18						
p-value	**0.10**	0.00		0.00	0.86						
Coef	0.33	0.14		-0.15	0.03	-0.52					0.24
t-statistic	2.88	5.23		-3.60	1.01	-9.80					
p-value	0.00	0.00		0.00	0.31	0.00					
Coef	0.36	0.13		-0.08	0.03	-0.54	-0.14				0.24
t-statistic	3.11	4.50		-1.46	1.24	-9.89	-1.68				
p-value	0.00	0.00		0.15	0.22	0.00	0.09				
Coef	0.57	-0.06		-0.30	0.07	0.15	0.07	-0.90			0.35
t-statistic	5.25	-1.95		-5.33	2.88	1.84	0.92	-10.42			
p-value	0.00	0.05		0.00	0.00	0.07	0.36	0.00			
Coef	0.48	0.11		-0.16	0.16				-0.59	-0.14	0.29
t-statistic	3.92	4.10		-2.82	5.11				-11.38	-1.61	
p-value	0.00	0.00		0.00	0.00				0.00	0.11	

(continued)

Table VII-*(continued)*

Panel B : Regression on *SMB'*											
	Int	RM_{ff}	SMB_{ff}	*HML'*	UMD_{ff}	*RMW*	*CMA*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.00		1.17								0.67
t-statistic	-0.02		35.67								
p-value	**0.98**		0.00								
Coef	0.54	0.04		-0.58							0.12
t-statistic	3.12	0.90		-8.18							
p-value	0.00	0.37		0.00							
Coef	0.55	0.03		-0.58	-0.01						0.12
t-statistic	3.10	0.84		-8.10	-0.27						
p-value	0.00	0.40		0.00	0.78						
Coef	0.70	-0.02		-0.45	0.04	-0.91					0.29
t-statistic	4.39	-0.64		-6.73	0.99	-12.33					
p-value	0.00	0.52		0.00	0.32	0.00					
Coef	0.68	-0.01		-0.51	0.03	-0.88	0.11				0.29
t-statistic	4.24	-0.31		-5.74	0.86	-11.08	1.03				
p-value	0.00	0.76		0.00	0.39	0.00	0.30				
Coef	1.01	-0.29		-0.69	0.11	0.15	0.25	-1.33			0.43
t-statistic	6.86	-6.98		-8.54	3.29	1.32	2.55	-12.17			
p-value	0.00	0.00		0.00	0.00	0.19	0.01	0.00			
Coef	0.86	-0.04		-0.67	0.24				-0.98	0.21	0.35
t-statistic	5.18	-1.01		-8.15	5.72				-13.94	1.84	
p-value	0.00	0.31		0.00	0.00				0.00	0.07	

We carry out the same analysis on the competing factors from the FF five-factor model as well as on the *q*-factors or Hou, Xue, Zhang (2014). We test whether exposures to the recent FF factors of profitability (RMW) and investment (CMA) still provide explanatory power under the new sequential framework. We run the test twice, first with the original size and value factors (Panel A), and secondly with the sequential version of the factors (Panel B). Results for RMW are displayed in Table 8. The profitability factor appears significant under all regression constructions (FF and sequential) with a confidence of 99%, except when the quality factor is included. The significant positive loadings of the QMJ factor suggest that both factors share a similar source of risk, consistent with how the determinants of the QMJ factor are selected.

Once performed on CMA, results show that under the FF framework (Panel A of Table 9), the investment factor is significant when combined with any additional factor. This is, however, not true once implemented with the sequential factors. Under the sequential framework, CMA is subsumed by HML' and RMW (last regression of Panel B). The *t*-statistics of the intercept are not significant when we control ex ante and/or ex post for quality. In our framework, the HML premium takes into account the outperformance of firms with low investment rates, which corresponds to our value stocks in period of expansion. Value stocks can indeed expand thanks to their excess capacity and unused capital from crisis period.

This section demonstrates the outperformance of the sequential factors over the original Fama and French (1993) size and value factor and over the Fama and French (2015a) investment factor. RMW still persists after controlling for the sequential value factor but it does not under Hou, Xue, Zhang (2015) framework or once the QMJ factor has been added to the sequential model. Showing that the *q*-factors of Hou, Xue, Zhang (2014) do not explain our size and value factors, we need further tests of complementarity that will be performed in the next section[9].

[9] None of our multi-factor combinations were able to fully explain the *ROE* and *I/A* factors. Results available on request.

Table VIII

Significance Tests on RMW (Robust minus Weak) Factor

The table reports regression results for the profitability factor *RMW* on the set of factors comprising the Fama and French premiums (Panel A), i.e. the market (RM_{ff}), the size (SMB_{ff}), the value (HML_{ff}), the momentum (UMD_{ff}), and the investment (*CMA*). In Panel B, we substitute the original size and value factors with their respective sequential versions. We also add the *QMJ* (Quality minus Junk) factor from Asness et al. (2015), and the profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014). The period used to perform the regressions ranges from January 1963 to December 2014. Figures underlined in light grey are regressed from July 1967 to December 2014 since the *ROE* and *I/A* factors are only made available for this time period.

Panel A : Regressions with FF factors

	Int	RM_{ff}	SMB_{ff}	HML_{ff}	UMD_{ff}	*CMA*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.31		-0.28							0.16
t-statistic	3.97		-10.95							
p-value	0.00		0.00							
Coef	0.22			0.06						0.01
t-statistic	2.59			2.13						
p-value	0.01			0.03						
Coef	0.30	-0.04	-0.26	-0.01	0.04					0.18
t-statistic	3.71	-1.99	-9.80	-0.36	2.23					
p-value	0.00	0.05	0.00	0.72	0.03					
Coef	0.37	-0.07	-0.25	0.18	0.06	-0.41				0.25
t-statistic	4.71	-3.87	-9.89	4.72	3.16	-7.47				
p-value	0.00	0.00	0.00	0.00	0.00	0.00				
Coef	-0.10	0.13	0.04	0.29	-0.02	-0.35	0.85			0.68
t-statistic	-1.77	8.94	1.84	11.68	-1.43	-9.93	29.07			
p-value	**0.08**	0.00	0.07	0.00	0.15	0.00	0.00			
Coef	0.04	-0.03	-0.08	0.16	-0.13			0.67	-0.21	0.57
t-statistic	0.64	-1.89	-3.52	5.61	-7.90			22.12	-4.50	
p-value	**0.52**	0.06	0.00	0.00	0.00			0.00	0.00	

Panel B : Regressions with sequential factors

	Int	RM_{ff}	*SMB'*	*HML'*	UMD_{ff}	*CMA*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.31		-0.23							0.23
t-statistic	4.13		-13.60							
p-value	0.00		0.00							
Coef	0.16			0.18						0.04
t-statistic	1.89			5.36						
p-value	**0.06**			0.00						
Coef	0.29	-0.06	-0.22	0.03	0.05					0.26
t-statistic	3.67	-3.15	-12.33	0.80	2.76					
p-value	0.00	0.00	0.00	0.43	0.01					
Coef	0.32	-0.09	-0.19	0.24	0.06	-0.41				0.33
t-statistic	4.31	-5.28	-11.08	5.88	3.49	-8.33				
p-value	0.00	0.00	0.00	0.00	0.00	0.00				
Coef	-0.11	0.12	0.02	0.29	-0.02	-0.30	0.78			0.67
t-statistic	-1.95	7.85	1.32	9.81	-1.65	-8.57	24.75			
p-value	**0.05**	0.00	0.19	0.00	0.10	0.00	0.00			
Coef	0.02	-0.03	-0.06	0.22	-0.12			0.63	-0.22	0.59
t-statistic	0.29	-2.38	-3.57	6.57	-7.34			20.46	-5.10	
p-value	**0.77**	0.02	0.00	0.00	0.00			0.00	0.00	

Table IX

Significance Tests on CMA (Conservative minus Aggressive) Factor

The table reports regression results for the investment factor *CMA* on the set of factors composed of the Fama and French premiums (Panel A), i.e. the market (RM_{ff}), the size (SMB_{ff}), the value (HML_{ff}), the momentum (UMD_{ff}), and the profitability (*RMW*). In Panel B, we substitute the original size and value factors with their respective sequential versions. We also add the *QMJ* (Quality minus Junk) factor from Asness et al. (2015), and the profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014). The period used to perform the regressions ranges from January 1963 to December 2014. Figures underlined in light grey are regressed from July 1967 to December 2014 since the *ROE* and *I/A* factors are only made available for this time period.

Panel A : Regressions with FF factors										
	Int	RM_{ff}	SMB_{ff}	HML_{ff}	UMD_{ff}	*RMW*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.35		-0.12							0.03
t-statistic	4.37		-4.57							
p-value	0.00		0.00							
Coef	0.14			0.49						0.49
t-statistic	2.46			24.38						
p-value	0.01			0.00						
Coef	0.17	-0.09	0.02	0.47	0.04					0.53
t-statistic	2.85	-6.33	1.10	22.22	2.83					
p-value	0.00	0.00	0.27	0.00	0.00					
Coef	0.23	-0.10	-0.03	0.47	0.05	-0.21				0.57
t-statistic	4.05	-7.19	-1.68	23.09	3.61	-7.47				
p-value	0.00	0.00	0.09	0.00	0.00	0.00				
Coef	0.13	-0.03	0.02	0.51	0.03	-0.40	0.30			0.60
t-statistic	2.22	-2.10	0.92	24.58	2.28	-9.93	6.42			
p-value	0.03	0.04	0.36	0.00	0.02	0.00	0.00			
Coef	-0.01	-0.04	0.03	0.10	0.03			-0.11	0.86	0.86
t-statistic	-0.38	-4.77	2.96	6.27	3.67			-6.82	35.64	
p-value	**0.71**	0.00	0.00	0.00	0.00			0.00	0.00	

Panel B : Regressions with sequential factors										
	Int	RM_{ff}	*SMB'*	*HML'*	UMD_{ff}	*RMW*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.33		-0.04							0.01
t-statistic	4.12		-2.28							
p-value	0.00		0.02							
Coef	0.07			0.53						0.43
t-statistic	1.16			21.48						
p-value	**0.25**			0.00						
Coef	0.08	-0.09	0.07	0.53	0.03					0.49
t-statistic	1.32	-6.33	5.03	19.92	1.78					
p-value	**0.19**	0.00	0.00	0.00	0.08					
Coef	0.16	-0.10	0.02	0.53	0.04	-0.25				0.54
t-statistic	2.60	-7.67	1.03	21.26	2.78	-8.33				
p-value	0.01	0.00	0.30	0.00	0.01	0.00				
Coef	0.09	-0.06	0.04	0.56	0.03	-0.36	0.18			0.55
t-statistic	1.45	-3.68	2.55	21.67	1.82	-8.57	3.70			
p-value	**0.15**	0.00	0.01	0.00	0.07	0.00	0.00			
Coef	-0.02	-0.03	0.02	0.09	0.03			-0.12	0.88	0.86
t-statistic	-0.58	-4.26	2.16	4.98	3.52			-7.23	37.32	
p-value	**0.56**	0.00	0.03	0.00	0.00			0.00	0.00	

VI. Specification Tests: The Small Angel Effect

In this section, we first examine the pricing properties for the four competing sets of factors – namely, the Fama and French (1993) size and value factors, the Fama and French (2015a) investment and profitability factors, the q-factors of Hou, Xue and Zhang (2014), the sequential factors as well as the quality factor of Asness et al. (2015) – by implementing an efficiency test similar to Cremers, Petajisto and Zitzewitz (2012) and Fama and French (2012, 2015a). Fama and French (2012, 2015a) show that the original size and value premiums provide significant positive misspecification for small value stocks. We refer to these stocks as the "small angels" effect. We then evaluate and compare the bias produced by the three-factor model when evaluating the 5x5 size/BTM portfolios under both the original and sequential frameworks. Our objective is to assess whether the sequential specification of the factors is able to price passive investment portfolios without specification errors.

A. The 5x5 Specification Error Matrix

The 5x5 portfolios are constructed on the basis of a 5x5 sort into size and book-to-market[10]. Table 10 displays the specification errors (α_p) of the 25 portfolios as well as their t-statistics and p-values produced by the original FF three-factor model (Panel A), the four-factor Carhart model (Panel B) and its extensions to Fama and French (2015a) – with and without the QMJ factor of Asness et al. (2015) – and Hou, Xue and Zhang (2014) (Panel C to G). Results for the sequential approach are shown in Table 11.

Table 10 demonstrates that the Fama and French original model is misspecified for each of the four corners (Panel A), namely: the small growth stocks, the large growth stocks, the small value stocks and the

[10] Downloaded from K. French's library in January 2015.

large value stocks. Yet, using the new set of (sequential) premiums helps to cure part of the mispricing on growth stocks (Table 11, Panel A). We might relate the residual negative significant alpha in small growth stocks to the work of Campbell, Giglio, Polk, and Turley (2015) who show that growth stocks display hedging properties against declining stock returns and volatility. Large growth stocks (being the least risky) offer however a residual positive alpha under the sequential model as shown in Table 11 Panel B which might reveal some mispricing.

More importantly, the "small angels" effect, which is distinguishable in the top right corner, totally vanishes under the sequential approach (Table 11, Panel A). The abnormal performance of these extreme risk portfolios in the FF framework suggests misspecified risk factors rather than simply missing factors. Besides, controlling Fama and French model (1993) ex post with new factors from Fama and French five-factor model or the *q*-factor model of Hou, Xue and Zhang (2014) or even adding the QMJ factor of Asness et al. (2015), never cures the "small angels" effect in the FF framework. There is always one intercept of the upper right corner remaining significant (*t*-stat=2.29 and *p*-value=0.02) at a confidence interval of 95% (Table 10, Panel E). Only a control ex ante (as performed by the sequential approach) manages to cure for noise.

Table 11 shows that adding FF profitability factor (RMW) to the three-factor sequential model augmented with the Carhart momentum does not improve the specification. Besides, introducing the investment factor (CMA) leads to more significant specification errors. Only four intercepts are significant at a 90% confidence interval as shown in Panel C, whereas the amount of significant coefficients reaches six in Panel D. The investment factor (CMA) thus brings a greater amount of noise to the model. This result remains in line with our previous observation, highlighted in Section 5, where CMA is subsumed by HML' and RMW and is also consistent with the existent literature. Indeed, as pointed out by Campbell and Vuolteenaho (2004), value stocks displayed post-1963 very low "good beta" (i.e. sensitivity to equity risk

premium) but very high bad beta which measures the impact of variation in market cash flows. This evidence explains why value stocks are more affected by economic downturns. In bad times, value stocks suffer from costly reversibility as they cannot easily reduce their unproductive capital. However, in good times they have low investment rate as they benefit from the capacity of their previously unproductive equipment (Campbell, Polk and Vuolteenaho (2010)). This suggestion does not only confirm the risk definition of value stocks embedded in our construction method but also provides an explanation to the low significance displayed by the investment factors of Fama and French (2015a). The superior return offered by low investment firms are fully captured under our framework into the HML factor. One additional evidence supports this: under both the five-factor Fama and French model and the *q*-factor model, the other investment premium (I/A) of Hou, Xue and Zhang (2014) is only partly significant across the 5x5 portfolios[11].

We also perform a horse race between the *q*-factor model of Hou, Xue and Zhang (2014) and our sequential empirical model. Table 10 first displays the 5x5 regression results for the original *q*-factor model with the size (ME), profitability (ROE) and investment (I/A). Using the Hou, Xue and Zhang (2014) factor model still leaves 8 out 25 significant alphas (at a confidence level of 95%) in which we find the "small angels" effect. By comparison, the sequential three-factor model augmented with the momentum only deliver 3 significant alphas at the 5% level.

We further test the joint pricing power of the *q*-factors and our sequential factors. Panel F of Table 11 introduces the *q*-factors into the three-factor sequential model and Panel G augments the model with the momentum Carhart factor. Contrary to Hou, Xue and Zhang (2015), the HML factor keeps significant in the *q*-factor model for all but one portfolios when defined under the sequential framework[12]. Under both

[11] Results are available upon request.
[12] Results on factors are available upon request.

models, we observe not less than 7 out of 25 portfolios with significant alphas (at a confidence level of 95%). Joining our sequential sorting procedure to the *q*-factor model does not prove to improve the simple four-factor model we advocate in this paper. We reject the hypothesis of moving to a five-, or six-factor model.

Table X

Specification Errors (α) of the 25 Portfolios under the Original FF Framework

This table exhibits specification errors (α) for the 25 portfolios produced by the extended empirical CAPM models. Panels A to F display the specification errors (α) for the 25 portfolios using the Fama and French (1993) approach as well as the respective *t*-statistics and *p*-values for the different factor model combinations over the sample period (from January 1963 to December 2014). RM_{ff}, SMB_{ff}, HML_{ff}, UMD_{ff}, *RMW* and *CMA* time-series are available on K. French's library. *QMJ* (Quality minus Junk) is obtained from Asness et al. (2015). As for the size (ME), profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014) are only made available for a sample period ranging from July 1967 to December 2014.

$$R(t) - R_F(t) = \alpha + b[R_M(t) - R_F(t)] + \sum_i k_i \, F_i(t) + e(t)$$

B/M→	Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM_{ff}. SMB_{ff}. and HML_{ff}															
	α					*t(α)*					*P-value*				
Small	-0.48	-0.01	-0.01	0.14	0.13	-5.10	-0.11	-0.10	2.59	2.23	**0.00**	0.91	0.92	**0.01**	**0.03**
2	-0.18	-0.05	0.10	0.06	-0.05	-2.76	-0.87	1.80	1.14	-0.87	**0.01**	0.38	0.07	0.25	0.38
3	-0.06	0.05	0.00	0.05	0.11	-1.01	0.69	0.03	0.82	1.53	0.31	0.49	0.98	0.42	0.13
4	0.13	-0.10	-0.04	0.06	-0.08	2.17	-1.47	-0.58	0.92	-1.05	**0.03**	0.14	0.56	0.36	0.30
Big	0.17	0.03	-0.06	-0.11	-0.16	3.64	0.57	-0.84	-1.94	-1.75	**0.00**	0.57	0.40	**0.05**	0.08
Panel B: Four-factor intercepts: RM_{ff}. SMB_{ff}. HML_{ff}. and UMD_{ff}															
	α					*t(α)*					*P-value*				
Small	-0.43	0.00	-0.01	0.13	0.16	-4.48	0.01	-0.13	2.25	2.71	**0.00**	1.00	0.90	**0.02**	**0.01**
2	-0.14	0.00	0.11	0.06	-0.04	-2.04	-0.08	1.96	1.09	-0.77	**0.04**	0.93	**0.05**	0.28	0.44
3	-0.03	0.07	0.03	0.06	0.14	-0.41	1.10	0.38	0.87	1.93	0.68	0.27	0.70	0.38	**0.05**
4	0.13	-0.06	0.01	0.07	-0.03	2.05	-0.84	0.10	1.06	-0.43	**0.04**	0.40	0.92	0.29	0.67
Big	0.18	0.02	-0.06	-0.09	-0.12	3.77	0.38	-0.83	-1.50	-1.32	**0.00**	0.70	0.41	0.13	0.19
Panel C: Five-factor intercepts: RM_{ff}. SMB_{ff}. HML_{ff}. UMD_{ff}. and *RMW*															
	α					*t(α)*					*P-value*				
Small	-0.34	0.08	-0.02	0.10	0.13	-3.78	1.13	-0.30	1.73	2.18	**0.00**	0.26	0.76	0.08	**0.03**
2	-0.13	-0.06	0.05	0.00	-0.08	-1.89	-1.08	0.86	0.01	-1.41	0.06	0.28	0.39	0.99	0.16
3	-0.02	0.01	-0.05	0.01	0.08	-0.33	0.10	-0.72	0.19	1.08	0.74	0.92	0.47	0.85	0.28
4	0.15	-0.12	-0.04	0.06	-0.06	2.30	-1.71	-0.61	0.88	-0.76	**0.02**	0.09	0.54	0.38	0.45
Big	0.12	-0.02	-0.05	-0.11	-0.08	2.55	-0.42	-0.64	-1.79	-0.87	**0.01**	0.68	0.52	0.07	0.38

(continued)

Table X-*(continued)*

Panel D: Six-factor intercepts: RM_{ff}. SMB_{ff}. HML_{ff}. UMD_{ff}. *RMW*. and *CMA*															
	α					*t(α)*					*P-value*				
Small	-0.31	0.08	-0.03	0.08	0.11	-3.39	1.14	-0.60	1.34	1.81	**0.00**	0.25	0.55	0.18	0.07
2	-0.10	-0.08	0.05	-0.02	-0.08	-1.43	-1.34	0.85	-0.41	-1.37	0.15	0.18	0.39	0.68	0.17
3	0.03	0.00	-0.05	-0.01	0.06	0.51	0.07	-0.81	-0.11	0.80	0.61	0.94	0.42	0.91	0.42
4	0.16	-0.18	-0.07	0.05	-0.05	2.42	-2.49	-1.02	0.67	-0.61	**0.02**	**0.01**	0.31	0.51	0.54
Big	0.13	-0.08	-0.08	-0.11	-0.05	2.79	-1.33	-1.11	-1.82	-0.49	**0.01**	0.18	0.27	0.07	0.62
Panel E: Seven-factor intercepts: RM_{ff}. SMB_{ff}. HML_{ff}. UMD_{ff}. *RMW*. *CMA*. and *QMJ*															
	α					*t(α)*					*P-value*				
Small	-0.17	0.10	-0.05	0.07	0.14	-1.84	1.43	-0.86	1.13	2.29	0.07	0.15	0.39	0.26	**0.02**
2	-0.05	-0.12	0.03	-0.07	-0.10	-0.76	-1.93	0.54	-1.27	-1.58	0.45	**0.05**	0.59	0.21	0.11
3	0.05	0.01	-0.04	-0.02	0.04	0.73	0.09	-0.66	-0.34	0.57	0.46	0.93	0.51	0.73	0.57
4	0.18	-0.14	-0.04	0.04	0.00	2.72	-1.92	-0.49	0.61	0.00	**0.01**	0.06	0.62	0.54	1.00
Big	0.06	-0.05	-0.03	-0.12	0.00	1.21	-0.78	-0.40	-1.84	-0.04	0.22	0.44	0.69	0.07	0.97
Panel F: Five-factor intercepts: RM_{ff}. *ME*. HML_{ff}. *ROE and I/A*															
	α					*t(α)*					*P-value*				
Small	-0.22	0.25	0.06	0.18	0.22	-2.19	3.37	0.99	2.71	2.90	**0.03**	**0.00**	0.32	**0.01**	**0.00**
2	-0.07	-0.03	0.04	0.01	-0.01	-0.93	-0.41	0.58	0.15	-0.21	0.35	0.68	0.56	0.88	0.83
3	0.06	-0.04	-0.11	-0.04	0.14	0.93	-0.50	-1.50	-0.54	1.65	0.35	0.62	0.14	0.59	0.10
4	0.18	-0.23	-0.14	0.00	-0.08	2.63	-2.90	-1.70	-0.06	-0.90	**0.01**	**0.00**	0.09	0.95	0.37
Big	0.10	-0.09	-0.14	-0.19	-0.01	2.00	-1.40	-1.73	-2.83	-0.08	**0.05**	0.16	0.08	**0.00**	0.94
Panel G: Six-factor intercepts: RM_{ff}. *ME*. HML_{ff}. UMD_{ff}. *ROE and I/A*															
	α					*t(α)*					*P-value*				
Small	-0.23	0.24	0.06	0.18	0.22	-2.24	3.29	0.97	2.68	2.92	**0.03**	**0.00**	0.33	**0.01**	**0.00**
2	-0.06	-0.02	0.04	0.01	-0.01	-0.85	-0.27	0.72	0.24	-0.20	0.40	0.79	0.47	0.81	0.84
3	0.07	-0.02	-0.10	-0.03	0.15	0.99	-0.34	-1.37	-0.45	1.74	0.32	0.73	0.17	0.65	0.08
4	0.18	-0.21	-0.12	0.00	-0.07	2.61	-2.81	-1.57	0.00	-0.80	**0.01**	**0.01**	0.12	1.00	0.42
Big	0.11	-0.09	-0.13	-0.19	-0.01	2.14	-1.34	-1.71	-2.75	-0.08	**0.03**	0.18	0.09	**0.01**	0.93

Table XI

Specification Errors (α) of the 25 Portfolios under the Sequential Framework

Table 15 exhibits specification errors (α) for the 25 portfolios produced by the extended empirical CAPM models. Panels A to F display the specification errors (α) for the 25 portfolios as well as the respective *t*-statistics and *p*-values using the modified Fama and French (1993) size and value factors in the different factor model combinations over the sample period (from January 1963 to December 2014). The paths for the sequential *SMB* and *HML* factors used are respectively momentum, book-to-market and size, and momentum, size, book-to-market. The alternative paths were tested and lead to the same conclusions (see Appendix). Results are available upon request. RM_{ff}, UMD_{ff}, *RMW* and *CMA* are available on K. French's library. *QMJ* (Quality minus Junk) is obtained from Asness et al. (2015). As for the profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014) are only made available for a sample period ranging from July 1967 to December 2014.

$$R(t) - R_F(t) = \alpha + b[R_M(t) - R_F(t)] + \sum_i k_i\, F_i(t) + e(t)$$

B/M→	Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM_{ff}. *SMB'*. *and HML'*															
	α					*t(α)*					*P-value*				
Small	-0.36	0.01	-0.06	0.00	-0.08	-4.30	0.08	-0.80	0.04	-1.10	**0.00**	0.94	0.43	0.97	0.27
2	-0.04	-0.02	0.05	-0.04	-0.16	-0.38	-0.18	0.58	-0.51	-1.57	0.71	0.86	0.56	0.61	0.12
3	0.06	0.03	-0.05	-0.03	0.01	0.69	0.38	-0.64	-0.39	0.15	0.49	0.71	0.53	0.70	0.88
4	0.23	-0.11	-0.10	-0.02	-0.18	3.03	-1.37	-1.23	-0.32	-1.79	**0.00**	0.17	0.22	0.75	0.07
Big	0.24	-0.01	-0.13	-0.19	-0.22	4.43	-0.09	-1.81	-2.59	-1.94	**0.00**	0.93	0.07	**0.01**	**0.05**
Panel B: Four-factor intercepts: RM_{ff}. *SMB'*. *HML'*. *and* UMD_{ff}															
	α					*t(α)*					*P-value*				
Small	-0.31	0.03	-0.04	0.02	0.00	-3.62	0.38	-0.45	0.32	-0.03	**0.00**	0.71	0.65	0.75	0.98
2	-0.01	0.04	0.09	-0.01	-0.10	-0.09	0.41	0.97	-0.12	-0.99	0.93	0.68	0.33	0.91	0.32
3	0.08	0.07	0.00	0.01	0.09	0.79	0.79	-0.06	0.12	0.93	0.43	0.43	0.96	0.91	0.35
4	0.20	-0.05	-0.03	0.02	-0.08	2.60	-0.69	-0.33	0.26	-0.81	**0.01**	0.49	0.74	0.80	0.42
Big	0.23	-0.01	-0.12	-0.13	-0.13	4.09	-0.22	-1.59	-1.76	-1.15	**0.00**	0.83	0.11	0.08	0.25

(continued)

Table XI-*(continued)*

Panel C: Five-factor intercepts: RM_{ff}. *SMB'*. *HML'*. UMD_{ff}. and *RMW*															
	α					*t(α)*					*P-value*				
Small	-0.29	0.07	-0.07	-0.02	-0.05	-3.45	0.83	-0.80	-0.25	-0.71	**0.00**	0.41	0.42	0.80	0.48
2	0.00	0.00	0.05	-0.04	-0.11	-0.03	0.01	0.55	-0.42	-0.99	0.98	0.99	0.58	0.68	0.32
3	0.09	0.03	-0.04	0.00	0.06	0.95	0.38	-0.51	0.01	0.61	0.34	0.71	0.61	0.99	0.54
4	0.23	-0.09	-0.05	0.04	-0.08	2.87	-1.15	-0.58	0.49	-0.81	**0.00**	0.25	0.56	0.62	0.42
Big	0.15	-0.05	-0.09	-0.13	-0.07	2.86	-0.86	-1.24	-1.73	-0.66	**0.00**	0.39	0.21	0.08	0.51
Panel D: Six-factor intercepts: RM_{ff}. *SMB'*. *HML'*. UMD_{ff}, *RMW*. *and CMA*															
	α					*t(α)*					*P-value*				
Small	-0.29	0.06	-0.09	-0.04	-0.08	-3.41	0.71	-1.08	-0.51	-1.03	**0.00**	0.48	0.28	0.61	0.31
2	0.03	-0.01	0.05	-0.06	-0.14	0.28	-0.15	0.48	-0.65	-1.26	0.78	0.88	0.63	0.52	0.21
3	0.15	0.04	-0.06	-0.03	0.02	1.64	0.45	-0.68	-0.39	0.15	0.10	0.65	0.50	0.70	0.88
4	0.26	-0.13	-0.08	0.01	-0.12	3.31	-1.71	-1.01	0.12	-1.20	**0.00**	0.09	0.31	0.91	0.23
Big	0.17	-0.08	-0.12	-0.17	-0.12	3.24	-1.41	-1.65	-2.33	-1.05	**0.00**	0.16	0.10	**0.02**	0.29
Panel E: Seven-factor intercepts: RM_{ff}. *SMB'*. *HML'*. UMD_{ff}., *RMW*, *CMA* and *QMJ*															
	α					*t(α)*					*P-value*				
Small	-0.14	0.14	-0.04	0.01	0.02	-1.67	1.45	-0.50	0.13	0.31	0.10	0.15	0.62	0.89	0.76
2	0.13	0.05	0.15	0.02	0.03	1.20	0.49	1.60	0.21	0.28	0.23	0.63	0.11	0.83	0.78
3	0.21	0.13	0.06	0.08	0.15	2.17	1.44	0.74	0.93	1.46	**0.03**	0.15	0.46	0.36	0.14
4	0.28	-0.03	0.05	0.11	0.07	3.51	-0.41	0.59	1.38	0.72	**0.00**	0.68	0.55	0.17	0.47
Big	0.04	-0.06	-0.06	-0.09	0.05	0.72	-0.99	-0.76	-1.25	0.41	0.47	0.32	0.45	0.21	0.68
Panel F: Five-factor coefficients: RM_{ff}. *SMB'*. *HML'*. *ROE and I/A*															
	α					*t(α)*					*P-value*				
Small	-0.32	0.14	-0.09	-0.02	-0.08	-3.33	1.39	-1.05	-0.26	-1.03	**0.00**	0.17	0.30	0.80	0.31
2	0.00	-0.03	-0.02	-0.08	-0.12	-0.01	-0.30	-0.23	-0.82	-1.05	0.99	0.77	0.82	0.41	0.29
3	0.13	-0.04	-0.14	-0.10	0.06	1.26	-0.41	-1.55	-1.16	0.51	0.21	0.68	0.12	0.25	0.61
4	0.26	-0.22	-0.17	-0.05	-0.16	3.02	-2.63	-2.03	-0.68	-1.47	**0.00**	**0.01**	**0.04**	0.50	0.14
Big	0.16	-0.10	-0.16	-0.23	-0.06	2.71	-1.49	-2.01	-2.97	-0.52	**0.01**	0.14	**0.04**	**0.00**	0.60
Panel G: Six-factor intercepts: RM_{ff}. *SMB'*. *HML'*. UMD_{ff}, *ROE and I/A*															
	α					*t(α)*					*P-value*				
Small	-0.32	0.13	-0.09	-0.02	-0.07	-3.31	1.36	-1.02	-0.24	-0.97	**0.00**	0.17	0.31	0.81	0.33
2	0.00	-0.03	-0.02	-0.07	-0.12	0.02	-0.25	-0.19	-0.79	-1.04	0.99	0.81	0.85	0.43	0.30
3	0.13	-0.03	-0.13	-0.10	0.06	1.26	-0.35	-1.49	-1.13	0.56	0.21	0.72	0.14	0.26	0.58
4	0.25	-0.22	-0.16	-0.05	-0.15	2.99	-2.59	-1.97	-0.66	-1.42	**0.00**	**0.01**	**0.05**	0.51	0.16
Big	0.16	-0.09	-0.16	-0.22	-0.06	2.79	-1.47	-2.01	-2.94	-0.51	**0.01**	0.14	**0.05**	**0.00**	0.61

B. The alpha bias

This sub-section evaluates the bias produced by the original Fama-French three-factor model when used to price passive portfolios. We then analyze the correction brought by the sequential size and value factors.

We follow an approach similar to Fama and French (2010): they compare the actual cross-section of mutual funds' alphas to a bootstrapped alpha cross-section in a world of true zero alpha. The procedure is transposed here to passive portfolios such as the 25 portfolios sorted on size and book-to-market available on K. French's data library. The first step consists in estimating the actual alphas of the passive portfolios using the FF three-factor model, as follows:

$$R_{i,t} - Rf_t = \alpha_i^{FF} + \beta_i[Rm - Rf] + \beta_i\, SMB^{FF} + \beta_i\, HML^{FF} + \varepsilon_{i,t}^{FF} \tag{4}$$

where $R_{i,t} - Rf_t$ (hereafter $R_{i,t}^e$) stands for the i^{th} 5x5 size and value portfolios' excess return. We also assume that $\varepsilon_{i,t} \sim N(0, \sigma^2)$. We then subtract the estimated α_i^{FF} for each of the 5x5 portfolios in equation (4) from the excess return ($R_{i,t}^e$) to construct the returns on zero-alpha portfolios.

As in Fama and French (2010), we jointly[13] resample the 25 zero-alpha portfolios, i.e.$\left(R_{i,t}^e - \alpha^{FF}\right)$, with the factors returns ($Rm - Rf,\ SMB^{FF} and\ HML^{FF}$). We bootstrap 10,000 runs of 25 size/value zero-alpha portfolios (denoted by the subscript *b*) and estimate for each portfolio the alpha in a world in which true alpha is zero:

$$\left(R_{i,t}^e - \alpha_i^{FF}\right)_b = \hat{\alpha}_{i,b}^0 + \widehat{\beta_{\iota,b}}[Rm - Rf]_{t,b} + \widehat{\beta_{\iota,b}}\, SMB_{t,b}^{FF} + \widehat{\beta_{\iota,b}}\, HML_{t,b}^{FF} + \hat{\varepsilon}_{i,t,b}^{FF} \tag{5}$$

[13] The bootstrap procedure is a random selection of monthly observations for all portfolios with replacement. The conditional resampling is performed to capture the cross-sectional correlation between portfolio returns constituting our sample. Such as in Harvey and Liu (2016), our bootstrap preserves cross-section and time-series dependence.

Across the simulations, we average the alphas and their t-statistics ($t(\alpha)$) estimates at the same percentile in order to construct an empirical cumulative density function (CDF) of the cross-sectional zero-alphas ($\hat{\alpha}_{i,b}^{0}$) under the original Fama-French framework (as defined above) and then repeat the exercise on the sequential factors. We display simulated CDFs of $t(\alpha)$ for both the independent and sequential frameworks in Figure 6 (grey lines).

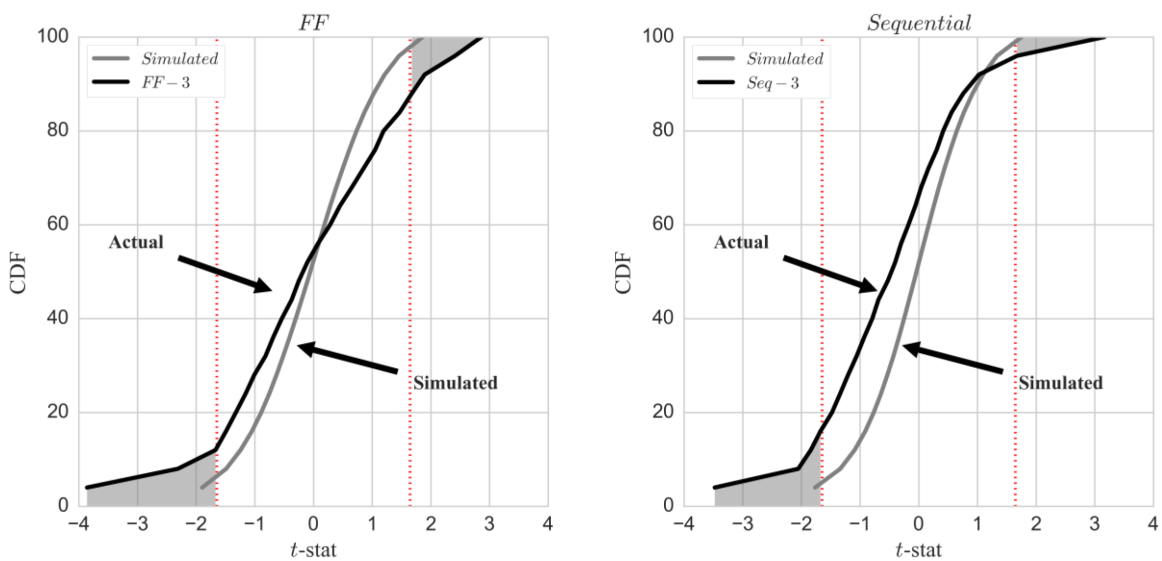
On the same graphs, we plot the $t(\alpha)$ estimates for the actual 25 portfolios using the FF and sequential three-factor models. Since we only have 25 portfolios, we use 20 years rolling windows[14] to smooth the CDF of the actual $t(\alpha)$ estimates.

[14] The $t(\alpha)$ for the simulated benchmark are also estimated with samples size of 20 years to match the length of the actual $t(\alpha)$ estimates. So, one run provides 25 cross-sectional alphas estimated from 240 monthly returns selected randomly out of the actual sample. The use of window of 240 months allows appropriate degrees of freedom when estimating the intercept of the model.

Figure VI

Cumulative Density Function of the Three-Factor Models $t(\alpha)$ for size/value portfolios

Figures illustrate the CDF of $t(\alpha)$ estimates on 5x5 passive portfolios sorted on size and book-to-market under the three-factor model. The simulated CDF of $t(\alpha)$ estimates for zero-alpha portfolios is represented by the grey line. The black line is the CDF of the $t(\alpha)$ estimates for actual portfolios using the three-factor models (using the original FF factors on the left and the sequential factors on the right). The red dotted lines represent the interval of insignificant t-stat at 90% confidence level. The sample period is from January 1963 to December 2014.



Compared to their respective (zero-alpha CDF) benchmark, the actual CDF derived from the FF model presents a centered $t(\alpha)$-distribution around zero whereas the sequential CDF has been shifted to the left. The consequence is that, under the sequential framework, a larger proportion of alphas fall into the 90% confidence interval (red dotted lines) than in the original framework. Compared to Fama-French original framework, the sequential model produces less highly significant alphas. Extreme significant positive and

negative alphas are more likely to be found under the original model as shown by comparing the left-hand and right-hand grey areas. The surface contained into the grey areas amounts to respectively 2.83 % (FF) and 2.08% (sequential)[15] supporting the evidence of fatter tails in the Fama and French density. In Table 12, we report these empirical distributions of the $t(\alpha)$ estimates for the simulated and actual three-factor models.

[15] Areas are estimated using a trapezoidal rule.

Table XII

Percentiles of $t(\alpha)$ estimates for actual and simulated size and value portfolios returns

The table replicates Table III of Fama and French (2010) for 5x5 passive portfolios. It shows values of $t(\alpha)$ at selected percentiles (Pct) of the distribution of $t(\alpha)$ estimates for actual (Act) passive 5x5 portfolio returns. Actual $t(\alpha)$ estimates are based on a 240-month rolling windows. Sim is the average value of $t(\alpha)$ at the selected percentiles from the simulations (based on 240-month rolling windows). Column "% < Act" represents the fraction of the 10,000 simulated $t(\alpha)$-estimates which are lower than the actual $t(\alpha)$ for equivalent percentiles. The period is January 1963 to December 2014 and results are shown for the FF and sequential three-factor models, respectively in Panel A and B.

	Panel A: FF Framework			Panel B: Sequential Framework		
Pct	Sim	Act	%<Act	Sim	Act	%<Act
3-Factor Gross Returns						
4	-1.90	-3.86	0.22	-1.76	-3.48	0.86
8	-1.49	-2.31	5.04	-1.34	-2.05	11.29
12	-1.24	-1.67	16.27	-1.09	-1.84	10.59
16	-1.05	-1.49	14.57	-0.91	-1.67	10.21
20	-0.89	-1.32	14.25	-0.77	-1.48	11.87
24	-0.75	-1.16	15.30	-0.64	-1.34	12.48
28	-0.63	-1.01	16.44	-0.53	-1.20	13.11
32	-0.52	-0.82	20.74	-0.43	-1.06	14.81
36	-0.41	-0.69	22.51	-0.34	-0.93	15.84
40	-0.30	-0.54	26.00	-0.25	-0.79	17.96
44	-0.20	-0.37	31.36	-0.16	-0.69	18.93
48	-0.10	-0.25	33.58	-0.08	-0.53	22.37
52	-0.01	-0.10	39.08	0.00	-0.40	25.15
56	0.09	0.08	48.57	0.09	-0.30	25.94
60	0.19	0.28	60.65	0.17	-0.17	28.14
64	0.29	0.45	67.73	0.26	-0.06	30.12
68	0.39	0.66	76.98	0.35	0.04	30.87
72	0.50	0.85	83.17	0.44	0.17	32.66
76	0.61	1.05	87.81	0.54	0.31	35.39
80	0.73	1.19	88.39	0.65	0.42	36.34
84	0.87	1.47	93.00	0.77	0.56	37.56
88	1.03	1.68	93.65	0.92	0.76	41.42
92	1.21	1.89	93.70	1.10	1.03	47.73
96	1.46	2.41	96.94	1.33	1.67	74.76
100	1.86	2.86	95.20	1.75	3.16	98.15

VII. Robustness tests

Having shown the superiority of the sequential risk factors to price size and value anomalies, we investigate the robustness of our results for the alternative sequential path, for a taxonomy of market anomalies (Novy-Marx and Velikov (2015)), and for a set of 150 portfolios sorted on market capitalization, book-to-market, investment and profitability dimensions as defined in Fama and French (2015a, b).

A. Alternative Sequential Path

As a first robustness test, we check the consistency of our results under the alternative path for constructing the SMB and HML factors. We re-build Panel B of Tables 6, 7 and 9 and Table 11 under the alternative sequential path. Tables can be found in Appendix and are denoted with a *. The tables challenge the redundancy of the value (HML) factor under the sequential framework and emphasize the strong significance of the sequential size (SMB) factor as well as the redundancy of the investment (CMA) factor in a sequential four-factor Carhart model. In particular, Table 11* confirms the superiority of the sequential factors constructed under the alternative path for pricing the size/value anomalies.

B. Market Anomalies of Novy-Marx and Velikov (2015)

Lewellen, Nagel and Shaken (2013) recommend four different methods to avoid misleading conclusion on empirical factor superiority, among them they suggest testing competing factor models not only on the size and value sorted portfolios but also on alternative market anomalies. In this section, we compare the performance of the sequential four-factor model to its FF equivalent and its extensions for a list of ten market

anomalies with low turnover defined by Novy-Marx and Velikov (2015). Data have been downloaded from Novy-Marx's website[16].

Table 13 confirms the superiority of the simple four-factor Carhart model over the *q*-factor for pricing market anomalies. The Fama and French extension to a six-factor model though displays less specification errors. For both the original model and its extension, we can observe the outperformance of the sequential factors for a group of market anomalies, namely gross profitability, accruals, net issuance and value/profitability. For the other anomalies, both frameworks (independent and sequential) provide similar pricing power. Yet, the original four-factor model offers the lowest pricing error across market anomalies. Besides, Table 13 challenges the redundancy of the HML factor, which stays strongly significant under the sequential framework.

[16] The period covered in this dataset is from January 1963 to December 2013.

Table XIII

Specification Errors (α) of Low Turnover Market Anomalies

This table reports the number of significant specification errors (α) and HML loadings (β) for Novy-Marx and Velikov's (2015) market anomalies ranked in deciles (gross return). Results are displayed for a 10% confidence interval. We consider 10 low turnover strategies, i.e. Gross Profitability, ValProf, Accruals, Net Issuance (A), Asset Growth, Investment, Piotroski's F-score, Asset Turnover, Gross Margins, and Ohlson's O-score. Each market anomalies are composed of ten long-only portfolios for which a description is given by Novy-Marx and Velikov (2015) at the following address: http://rnm.simon.rochester.edu/data_lib/ToAatTC/signals_details.pdf. The regressions model are defined as follows: $C4_{ff}$ is the traditional Carhart (1997) four factor, $FF6_{ff}$ is the recent Fama-French 5-factor model augmented with a momentum premium (UMD_{ff}), HXZ_{ff} is the Hou Xue and Zhang (2015) *q*-factors model with Fama-French Value premium (HML_{ff}). We provide for each of these models, an alternative definition where Fama-French Size and Value premium are substituted by their respective sequential twins.

		Low Turnover Strategies (deciles)									
		Gross Prof.	ValProf	Acc.	NI (A)	Asset Growth	Invest.	F-score	Asset Turn.	Gross Margins	O-score
$C4_{ff}$	$\#\alpha_{10\%}$	6	7	4	5	2	4	4	2	4	5
	$\#\beta_{10\%}$	9	8	7	8	9	7	6	3	10	9
$C4_{seq}$	$\#\alpha_{10\%}$	2	5	2	4	2	3	4	2	5	5
	$\#\beta_{10\%}$	10	8	7	7	9	7	6	4	10	9
$FF6_{ff}$	$\#\alpha_{10\%}$	2	4	5	6	1	4	3	1	4	4
	$\#\beta_{10\%}$	8	8	7	7	5	6	6	6	9	7
$FF6_{seq}$	$\#\alpha_{10\%}$	2	4	4	4	2	4	3	1	5	4
	$\#\beta_{10\%}$	10	8	6	6	5	8	4	6	9	9
HXZ_{ff}	$\#\alpha_{10\%}$	4	5	7	4	1	4	5	1	3	2
	$\#\beta_{10\%}$	7	7	7	9	7	5	7	6	7	9
HXZ_{seq}	$\#\alpha_{10\%}$	1	5	6	5	1	5	6	1	5	2
	$\#\beta_{10\%}$	10	7	4	9	7	9	5	6	7	8

C. Alpha bias of FF and sequential 3-factor model for characteristics-sorted portfolios

As a final robustness test, we repeat the alpha bias test from Section 6-B on 150 characteristics-sorted portfolios available on K. French's website, that is size/book-to-market, size/investment, size/profitability, book-to-market/investment, book-to-market/profitability, and profitability/investment. We report the CDF of the cross-sectional $t(\alpha)$ estimates for the simulated and actual[17] three-factor models and plot them in Figure 8.

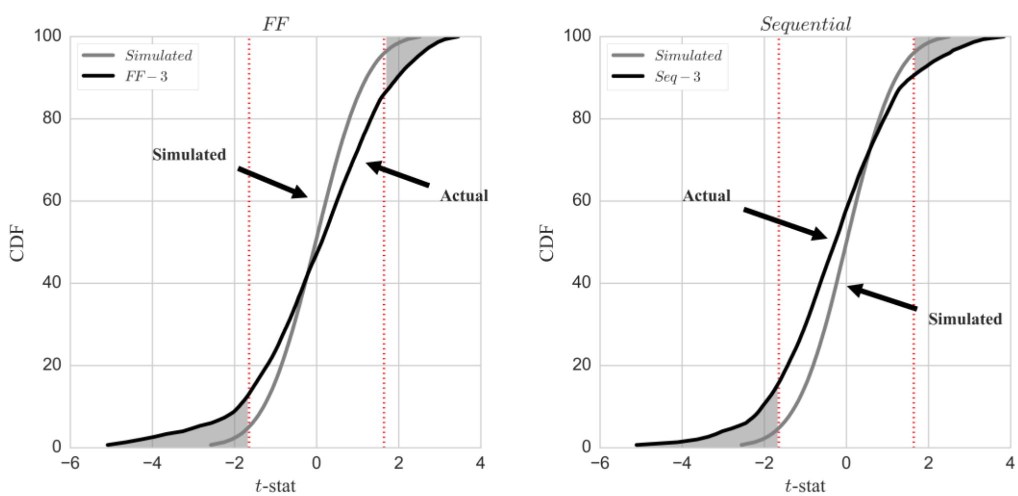
Repeating the test on the cross-section of 150 portfolios, we confirm the results found in Section 6.B. The sequential framework produces a larger proportion of insignificant alphas at the 90% confidence level as shown by the left-skewed CDF. The Fama-French model delivers more extreme (negative and positive) t-statistics: the sum of the grey areas amounts to 2.48 % and 2.21% of the total graph surface for FF and the sequential model, respectively.

[17] We also use 20 years rolling windows to smooth the CDF of the actual $t(\alpha)$ estimates as in section 6.B.

Figure VII

Cumulative Density Function of the Three-Factor Models *t*(α) for characteristics- sorted portfolios

Figures illustrate the CDF of *t*(α) estimates for a set of passive portfolios (5x5) sorted on size/value, size/investment, profitability/investment, size/profitability, book-to-market/investment, book-to-market/profitability, using a three-factor model. The 150 portfolios are downloaded from Ken French's website. The simulated zero-alpha CDF is represented by the grey line. The black line is the CDF of the actual *t*(α) estimates from the respective three-factor models (FF versus sequential). The red dotted lines represent the interval of insignificant *t*-stats at 90% confidence level. The sample period is from January 1963 to December 2014.



VIII. Conclusion

Arnott (2005, p. 14) indicates that "*when we separate the size effect from the value-versus-growth effect, we find that size as measured by market capitalization is far less powerful than is generally believed. And, reciprocally, the value effect — because some of its efficacy has been siphoned off by the mislabeled size effect — is far more powerful and more consistent than is generally believed.*" This paper presents a factor construction methodology that is able to capture this empirical evidence pointed out by the professional community.

We revisit the size and book-to-market effects in the US market over the 1963-2014 sample period using the sequential approach to factor construction of Lambert and Hübner (2013). It avoids the contamination of the premiums from the fundamentals' correlation structure on which the sorting steps are operated. While the independent sorting induces a negative correlation between a stock ranking on size and book-to-market, the sequential sort induces a positive correlation. This means that the sequential framework indeed relates, on average, value stocks to large capitalization stocks with high level of fixed assets. In times of recession, such value stocks are left with unproductive capital that induces high operating costs and further deepens the effect of recession. Yet, in good economic periods, these firms benefit from excess capital capacity and low reinvestment needs which further increase the generation of cash flows. Such interpretation does not hold under the Fama and French framework as value stocks are mostly found among small size stocks. Besides, in good time value stocks have low investment rate as they benefit from the capacity of their previously unproductive equipment (Campbell, Polk and Vuolteenaho (2010)). The superior returns offered by low investment firms are, as a consequence, fully captured under our framework into the HML factor. It therefore absorbs the information driven by the risk factor CMA (Fama and French (2015a)'s investment factor), but not the other way around.

The main innovations of our methodology involve a finer size classification and a conditional sorting of stocks into portfolios. We consider three risk dimensions (size, value and momentum) with a pre-conditioning on momentum in order to control for the business cycle, earnings surprise and profitability shocks. The conditional sorting procedure addresses the question of whether return variation related to the third risk criterion still exists even after having controlled for two other risk dimensions. The sorting procedure involves performing a sequential sort in three stages: the first two sorts are performed on control risks, followed by the risk dimension to be priced. Alternative paths have been tested and support entirely our results.

Compared to an independent sorting method, our factor construction methodology captures more accurately the return spread associated with the source of risk to be priced. The conditional sorting and the finer size classification both contribute to better balancing the small/large value/growth stocks into portfolios. The most significant improvement of the new method lies in the reduction of specification errors when pricing passive benchmark investment portfolios. Overall, this modified Fama and French methodology enables us to deliver a set of risk premiums that better price the extreme risks involved in portfolios displaying small market capitalization but strong book-to-market characteristics. The Fama and French (1993) model with and without the Carhart (1997) momentum, the five-factor Fama and French (2015a) model as well as the *q*-factor model of Hou, Xue and Zhang (2015) all fail to price these extreme portfolios.

Using our sequential methodology to derive the book-to-market factor, we do not witness anymore a tilt toward small value stocks (which drives the value factor in FF framework and its relation with a distress factor) and discover a remarkably steady and significant value effect across the year and every business cycle, and all market capitalizations. Our value factor indeed associates the value effect with greater

sensitivity of a firm's earnings to the economic conditions. In bad times, value stocks suffer from costly reversibility, as they cannot easily reduce their unproductive capital.

Our findings are close to those of Asness et al. (2015) where the size factor resurrects after ex-post controlling for noise. Contrary to them however, we control ex ante for noise in the estimation of the factor itself. It appears that using our methodology, both the size and book-to-market factors remain significant across various model specifications. Our study documents a strong "through-the-year" value effect and only a "turn-of-the-year" size effect.

The critical stance of our paper made it necessary to explore the potential improvements offered by the new sequential procedure over the original Fama and French (1993) method but also over the new sets of competing factors and market anomalies that have recently florished in the literature. These robustness checks deliver clear insights with regards to the key drivers of the alternative approach's pricing performance. A conditional sorting procedure purifies the size and value risk factors, so that our version of the traditional Carhart model outperforms an extended empirical model (such as a five-, or six- or seven-factor model) to explain market anomalies.

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APPENDICES

Table VI*

Significance Tests on HML' Alternative (High minus Low) Factor

The table reports regression results for the sequential value premium on the factors, that is RM_{ff}, the size factor (SMB_{ff}), the momentum (UMD_{ff}), the profitability (*RMW),* and the investment (*CMA)*. We also add *QMJ* (Quality minus Junk) factor from Asness, et al. (2015), and the profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014). The period used to perform the regressions ranges from January 1963 to December 2014. Figures underlined in light grey are regressed from July 1967 to December 2014 since the *ROE* and *I/A* factors are only made available for this time period.

Panel A: Regression on *HML' alternative*											
	Int	RM_{ff}	$\boldsymbol{SMB'_2}$	HML_{ff}	UMD_{ff}	*RMW*	*CMA*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.18			0.76							0.77
t-statistic	3.69			45.66							
p-value	0.00			0.00							
Coef	0.64	-0.17	-0.16		-0.09						0.21
t-statistic	7.09	-8.36	-7.85		-4.08						
p-value	0.00	0.00	0.00		0.00						
Coef	0.63	-0.17	-0.15		-0.09	0.02					0.21
t-statistic	6.93	-8.21	-6.75		-4.10	0.42					
p-value	0.00	0.00	0.00		0.00	0.68					
Coef	0.24	-0.02	-0.10		-0.08	0.20	0.78				0.53
t-statistic	3.34	-1.10	-5.76		-4.77	5.23	20.64				
p-value	0.00	0.27	0.00		0.00	0.00	0.00				
Coef	0.38	-0.11	-0.16		-0.04	0.47	0.75	-0.46			0.58
t-statistic	5.32	-5.77	-8.79		-2.46	9.78	21.16	-8.44			
p-value	0.00	0.00	0.00		0.01	0.00	0.00	0.00			
Coef	0.29	-0.06	-0.15		-0.06				-0.10	0.76	0.50
t-statistic	3.59	-3.46	-7.65		-2.72				-2.66	17.54	
p-value	0.00	0.00	0.00		0.01				0.01	0.00	

Table VII*

Significance Tests on SMB' Alternative (Small minus Big) Factor

The table reports regression results for the sequential size premium on the factors that is RM_{ff}, the value factor (HML_{ff}), the momentum (UMD_{ff}), the profitability (*RMW),* and the investment (*CMA)*. We also add the *QMJ* (Quality minus Junk) factor from Asness et al. (2015), and the profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014). The period used to perform the regressions ranges from January 1963 to December 2014. Figures underlined in light grey are regressed from July 1967 to December 2014 since the *ROE* and *I/A* factors are only made available for this time period.

Panel A: Regression on *SMB' alternative*											
	Int	RM_{ff}	SMB_{ff}	$\boldsymbol{HML'_2}$	UMD_{ff}	*RMW*	*CMA*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.00		1.17								0.67
t-statistic	-0.02		35.67								
p-value	**0.98**		0.00								
Coef	0.54	0.04		-0.58							0.12
t-statistic	3.12	0.90		-8.18							
p-value	0.00	0.37		0.00							
Coef	0.55	0.03		-0.58	-0.01						0.12
t-statistic	3.10	0.84		-8.10	-0.27						
p-value	0.00	0.40		0.00	0.78						
Coef	0.70	-0.02		-0.45	0.04	-0.91					0.29
t-statistic	4.39	-0.64		-6.73	0.99	-12.33					
p-value	0.00	0.52		0.00	0.32	0.00					
Coef	0.68	-0.01		-0.51	0.03	-0.88	0.11				0.29
t-statistic	4.24	-0.31		-5.74	0.86	-11.08	1.03				
p-value	0.00	0.76		0.00	0.39	0.00	0.30				
Coef	1.01	-0.29		-0.69	0.11	0.15	0.25	-1.33			0.43
t-statistic	6.86	-6.98		-8.54	3.29	1.32	2.55	-12.17			
p-value	0.00	0.00		0.00	0.00	0.19	0.01	0.00			
Coef	0.86	-0.04		-0.67	0.24				-0.98	0.21	0.35
t-statistic	5.18	-1.01		-8.15	5.72				-13.94	1.84	
p-value	0.00	0.31		0.00	0.00				0.00	0.07	

Table IX*

Significance Tests on CMA (Conservative minus Aggressive) Factor

The table reports regression results for the investment factor *CMA* on the set of factors composed of the alternative "sequential" Fama and French premiums, i.e. the market (RM_{ff}), the size (SMB'_2), the value (HML'_2), the momentum (UMD_{ff}), and the profitability (*RMW*). We also add the *QMJ* (Quality minus Junk) factor from Asness et al. (2015), and the profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014). The period used to perform the regressions ranges from January 1963 to December 2014. Figures underlined in light grey are regressed from July 1967 to December 2014 since the *ROE* and *I/A* factors are only made available for this time period.

Panel A : Regression with alternative sequential factors										
	Int	RM_{ff}	***SMB'₂***	***HML'₂***	*UMD*	*RMW*	*QMJ*	*ROE*	*I/A*	*R*
Coef	0.33		-0.04							0.01
t-statistic	4.13		-2.30							
p-value	0.00		0.02							
Coef	0.08			0.53						0.42
t-statistic	1.32			21.29						
p-value	**0.19**			0.00						
Coef	0.10	-0.09	0.07	0.53	0.02					0.48
t-statistic	1.57	-6.26	4.75	19.56	1.56					
p-value	**0.12**	0.00	0.00	0.00	0.12					
Coef	0.17	-0.10	0.01	0.53	0.03	-0.24				0.52
t-statistic	2.80	-7.54	0.96	20.64	2.50	-7.89				
p-value	0.01	0.00	0.34	0.00	0.01	0.00				
Coef	0.10	-0.06	0.04	0.56	0.02	-0.36	0.20			0.54
t-statistic	1.55	-3.35	2.61	21.16	1.47	-8.48	4.01			
p-value	**0.12**	0.00	0.01	0.00	0.14	0.00	0.00			
Coef	-0.02	-0.03	0.02	0.09	0.03			-0.12	0.89	0.86
t-statistic	-0.62	-4.22	2.26	4.95	3.34			-7.05	38.14	
p-value	**0.53**	0.00	0.02	0.00	0.00			0.00	0.00	

Table XI*

Specification Errors (α) of the 25 Portfolios under the Alternative Sequential Framework.

The Table exhibits specification errors (α) for the 25 portfolios produced by the extended empirical CAPM models. Panels A to F display the specification errors (α) for the 25 portfolios as well as the respective *t*-statistics and *p*-values using the modified Fama and French (1993) size and value factors in the different factor model combinations over the sample period (from January 1963 to December 2014). The paths for the sequential *SMB* and *HML* factors used are respectively book-to-marke, momentum and size, and size, momentum and book-to-market. RM_{ff}, UMD_{ff}, *RMW* and *CMA* are available on K. French's library. *QMJ* (Quality minus Junk) is obtained from Asness et al. (2015). As for the profitability (*ROE*) and investment (*I/A*) from Hou, Xue, Zhang (2014) are only made available for a sample period ranging from July 1967 to December 2014.

$$R(t) - R_F(t) = \alpha + b[R_M(t) - R_F(t)] + \sum_i k_i\, F_i(t) + e(t)$$

B/M→	Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High
Panel A: Four-factor intercepts: RM_{ff}. SMB'_2. HML'_2. and UMD_{ff}															
	α					*t(α)*					*P-value*				
Small	-0.36	0.03	-0.04	0.05	0.00	-4.08	0.35	-0.56	0.64	0.01	**0.00**	0.73	0.57	0.52	1.00
2	-0.07	-0.01	0.08	-0.02	-0.10	-0.63	-0.12	0.87	-0.22	-1.04	0.53	0.91	0.38	0.83	0.30
3	0.02	0.04	-0.02	-0.02	0.09	0.23	0.40	-0.27	-0.30	0.89	0.82	0.69	0.79	0.76	0.37
4	0.17	-0.10	-0.06	0.00	-0.08	2.21	-1.28	-0.80	-0.04	-0.87	**0.03**	0.20	0.42	0.97	0.39
Big	0.21	-0.02	-0.12	-0.13	-0.15	3.90	-0.30	-1.69	-1.84	-1.33	**0.00**	0.76	0.09	0.07	0.18
Panel B: Five-factor intercepts: RM_{ff}. SMB'_2. HML'_2. UMD_{ff} and *RMW*															
	α					*t(α)*					*P-value*				
Small	-0.33	0.06	-0.09	0.00	-0.05	-3.65	0.66	-1.09	-0.06	-0.75	**0.00**	0.51	0.28	0.95	0.45
2	-0.06	-0.06	0.02	-0.05	-0.11	-0.57	-0.59	0.26	-0.58	-1.11	0.57	0.55	0.80	0.56	0.27
3	0.03	-0.01	-0.07	-0.04	0.06	0.35	-0.08	-0.89	-0.49	0.58	0.73	0.94	0.37	0.62	0.56
4	0.20	-0.14	-0.09	0.02	-0.09	2.47	-1.84	-1.20	0.24	-0.88	**0.01**	0.07	0.23	0.81	0.38
Big	0.14	-0.06	-0.10	-0.13	-0.08	2.73	-1.00	-1.33	-1.80	-0.74	**0.01**	0.32	0.18	0.07	0.46

(continued)

Table XI*-*(continued)*

Panel C: Six-factor intercepts: *RM$_{ff}$. SMB'$_2$. HML'$_2$. UMD$_{ff}$, CMA, and RMW*															
	α					*t(α)*					*P-value*				
Small	-0.31	0.06	-0.09	-0.02	-0.06	-3.39	0.71	-1.19	-0.23	-0.87	**0.00**	0.48	0.24	0.82	0.38
2	-0.01	-0.05	0.03	-0.06	-0.13	-0.05	-0.50	0.35	-0.69	-1.27	0.96	0.62	0.73	0.49	0.20
3	0.12	0.02	-0.08	-0.06	0.02	1.23	0.17	-0.93	-0.71	0.22	0.22	0.86	0.35	0.48	0.83
4	0.24	-0.16	-0.11	0.00	-0.12	3.03	-2.16	-1.48	-0.04	-1.23	**0.00**	**0.03**	0.14	0.97	0.22
Big	0.16	-0.09	-0.12	-0.17	-0.12	3.11	-1.51	-1.70	-2.39	-1.10	**0.00**	0.13	0.09	**0.02**	0.27
Panel D: Seven-factor intercepts: *RM$_{ff}$. SMB'$_2$. HML'$_2$. UMD$_{ff}$, CMA, RMW, and QMJ*															
	α					*t(α)*					*P-value*				
Small	-0.20	0.09	-0.10	-0.01	-0.01	-2.12	0.98	-1.19	-0.09	-0.20	**0.03**	0.33	0.24	0.93	0.84
2	0.06	-0.03	0.10	-0.02	0.00	0.52	-0.32	1.08	-0.19	-0.03	0.61	0.75	0.28	0.85	0.98
3	0.14	0.08	0.02	0.02	0.14	1.42	0.85	0.22	0.28	1.33	0.16	0.39	0.82	0.78	0.18
4	0.25	-0.09	-0.01	0.08	0.06	3.09	-1.13	-0.14	1.03	0.61	**0.00**	0.26	0.89	0.31	0.54
Big	0.03	-0.07	-0.06	-0.09	0.04	0.58	-1.08	-0.78	-1.30	0.37	0.56	0.28	0.44	0.19	0.71
Panel E: Five-factor intercepts: *RM$_{ff}$. SMB'$_2$. HML'$_2$. I/A, and ROE*															
	α					*t(α)*					*P-value*				
Small	-0.29	0.16	-0.08	0.00	-0.07	-3.07	1.62	-0.95	0.05	-0.89	**0.00**	0.11	0.34	0.96	0.38
2	0.01	-0.02	-0.01	-0.06	-0.11	0.09	-0.23	-0.07	-0.65	-1.03	0.93	0.82	0.94	0.52	0.30
3	0.13	-0.03	-0.14	-0.08	0.06	1.28	-0.31	-1.58	-0.92	0.52	0.20	0.76	0.11	0.36	0.60
4	0.26	-0.22	-0.17	-0.04	-0.14	3.11	-2.61	-2.00	-0.51	-1.37	**0.00**	**0.01**	**0.05**	0.61	0.17
Big	0.15	-0.10	-0.16	-0.21	-0.06	2.64	-1.57	-2.06	-2.75	-0.53	**0.01**	0.12	**0.04**	**0.01**	0.59
Panel F: Six-factor intercepts: *RM$_{ff}$. SMB'$_2$. HML'$_2$. UMD$_{ff}$, I/A, and ROE*															
	α					*t(α)*					*P-value*				
Small	-0.29	0.16	-0.08	0.01	-0.06	-3.05	1.60	-0.92	0.07	-0.83	**0.00**	0.11	0.36	0.94	0.41
2	0.01	-0.02	0.00	-0.06	-0.11	0.11	-0.18	-0.02	-0.62	-1.01	0.91	0.85	0.98	0.54	0.31
3	0.13	-0.02	-0.13	-0.07	0.06	1.28	-0.25	-1.54	-0.89	0.58	0.20	0.80	0.13	0.38	0.56
4	0.26	-0.21	-0.16	-0.04	-0.14	3.08	-2.57	-1.96	-0.49	-1.32	**0.00**	**0.01**	**0.05**	0.63	0.19
Big	0.16	-0.10	-0.16	-0.21	-0.06	2.70	-1.54	-2.05	-2.72	-0.52	**0.01**	0.12	**0.04**	**0.01**	0.60