Labor Rigidity, Inflation Risk and Bond Returns

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Abstract

This paper exploits information from the variance-ratios of macroeconomic variables to infer about the short and long-run components of dividend risk and inflation risk. While labor rigidity shifts dividend risk towards the short horizon, it also reveals –by means of labor-share variation– the component of inflation risk which is correlated with fundamentals. A simple general equilibrium model with labor rigidity can explain how inflation interacts with the real growth and the labor-share, as well as many patterns of the term-structures of real and nominal bond yields. The model is robust to many properties of equity returns.

Keywords: labor rigidity · inflation · term-structure · interest rates · equilibrium asset pricing

JEL Classification: D51, E21, G12

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1 Introduction

Asset pricing models usually have difficulty to jointly reconcile the predictions about equity and bonds with the empirical evidence. Namely, leading models have different rationale but share the property that priced risk obtains from long-horizon discounted cash-flows. In turn, those models generate counter-factual upward-sloping and downward-sloping term-structures for equity premia and real bond yields respectively. Moreover, these models are not consistent with the timing of risk of macroeconomic variables –i.e. variance or variance-ratios of growth rates computed at different horizons. The latter provide information about how prices are determined in equilibrium. Although disregarded in most of the literature, such an information offers additional testable implications and can help us to understand the macroeconomic determinants of asset prices.[1]

How does the timing of risk of real macroeconomic variables and inflation look like? how does the timing of risk in fundamentals transmit to the term-structures of real and nominal interest rates? This paper empirically and theoretically addresses these questions. An empirical analysis provides guidance to design the connection between inflation risk and real growth in a simple general equilibrium model which rationalizes many properties of real and nominal yields and reconciles them with the main patterns of equity and dividend strip returns. The model is consistent with the idea that expected inflation has a non-neutral and negative effect on future real growth, which leads to a positive inflation risk premium in nominal bonds (Fama and Schwert, 1977; Fama, 1981; Bekaert and Wang, 2010).

Lettau and Wachter (2007, 2011) provide the intuition about the opposite slopes of equity premia and real interest rates in a partial equilibrium setting. Marfè (2013b) provides a macroeconomic foundation: labor rigidity lowers the loading of wages on transitory risk and, in turn, gives rise to operating leverage that explains the downward-sloping term-structure of dividend risk. This property is inherited by equity returns, enhances the price of short-run risk and generates the upwards-sloping term-structure of real yields. This paper builds on Marfè (2013b) and analyses the timing of inflation risk and its interaction with real growth. Namely, a permanent shock uncorrelated with fundamentals generates upward-sloping inflation risk, whereas a small transitory shock generates the link with real growth. The latter helps explain many patterns of real and nominal yields and their correlation, when labor rigidity is strong enough –i.e. markets compensate short-run risk.[2]

An empirical investigation supports the main model mechanism. First, I review some

[1]See van Binsbergen, Brandt, and Koijen (2012), van Binsbergen, Hueskes, Koijen, and Vrugt (2013) and van Binsbergen and Koijen (2015) about the term-structures of asset prices. See Belo, Collin-Dufresne, and Goldstein (2015) and Marfè (2013a,b) about the term-structures of macroeconomic fundamentals.

[2]The role of short-run risk (e.g. business cycle) has been recently emphasized in Greenwald, Lettau, and Ludvigson (2014) and Koijen, Lusting, and van Nieuwerburgh (2014).

properties about the timing of macroeconomic risk and the role of labor rigidity. The variance ratios of wages, output and dividends are respectively markedly upward-sloping, slightly upward-sloping and markedly downward-sloping. An explicit or implicit mechanism of income insurance from shareholders to workers rationalizes these facts and is consistent with many empirical patterns –i.e. macroeconomic variables are subject to both permanent and transitory shocks (Lettau and Ludvigson, 2014) but are co-integrated (Lettau and Ludvigson, 2005), income insurance within the firm exists but does not concern permanent shocks (Guiso, Pistaferri, and Schivardi, 2005; Menzio, 2005) and the labor-share is counter-cyclical (Ríos-Rull and Santaeulàlia-Llopis, 2010).

Second, I document that the term-structure of inflation risk is markedly upward-sloping –that is the upward-sloping effect of the permanent component of inflation dominates the downward-sloping effect of the transitory component.

Third, a decomposition of real growth rates of output, wages and dividends as well as inflation rates allows to infer about their short-run and long-run components. Namely, the analysis consists of the joint use of a principal component analysis and the term-structures of variance ratios. Namely the former is used to recover orthogonal latent factors, the latter are used to detect whether the principal components contribute to long-run risk (upward-sloping variance ratios above unity) or short-run risk (downward-sloping variance ratios below unity). Finally, the short-run and long-run components of the original variables are obtained by making use separately of the principal components associated either to long-run risk or to short-run risk.

Fourth, the short-run component of inflation, although small, is important because it represents the channel through which inflation is linked to real growth. Namely, inflation is moderately positively correlated with real growth. However, I document that the short-run components of real growth in ouput, wages and dividends are almost perfectly correlated with the short-run component of inflation. Instead, the long-run components of real growth and the long-run component of inflation are essentially uncorrelated.

Fifth, I verify the link between the timing of inflation risk and the timing of risk in real growth. If the latter is mainly driven by labor rigidity, then the dynamics of the labor-share should reveal the transitory component of real growth. Namely, the labor-share should be a decreasing function of such a component. In addition, I have documented that the short-run component of inflation is almost perfectly correlated with the short-run components of real growth in ouput, wages and dividends. Thus, expected inflation should also be a decreasing function of the transitory component of real growth. In turn, the current level of the labor-share should positively forecast realized inflation at some future horizon. I document that this intertemporal relationship is statistically significant in the actual data. The relationship between labor-share and inflation has been investigated in the macroeconomic literature:

Sbordone (1999), Gali and Gertler (1999) and Gali, Gertler, and Lopez-Salido (2001) provide international evidence that resources devoted to workers' remuneration positively lead the shape of post-war inflation.[3]

The empirical analysis provides guidance to build a simple general equilibrium model. The main ingredients are the following. First, similarly to Marfè (2013b) total resources are subject to both a permanent shock with time-varying growth (Bansal and Yaron, 2004) and a transitory shock. Wages have a low exposition to transitory risk and generate operating leverage on dividends, such that co-integration is preserved but dividend risk is downward-sloping as a result of labor rigidity. Shareholders feature recursive utility and act as a representative agent on the financial markets because of limited market participation (Berk and Walden, 2013; Greenwald, Lettau, and Ludvigson, 2014). Inflation is modelled by means of a permanent shock and a transitory shock. The former dominates, such that inflation risk is upward-sloping, but it is uncorrelated with real quantities, whereas the latter is perfectly correlated with the transitory component of real growth. In turn, the labor-share is positively correlated with expected inflation. These four stylized facts about inflation are in line with the empirical findings documented by this paper. Finally, equilibrium quantities –such as real and nominal bond yields, dividend strips and equity prices– are obtained with analytical solutions up to a standard log-linearization of the wealth process around its endogenous steady-state (Benzoni, Collin-Dufresne, and Goldstein, 2011).

The model can generate upward-sloping real and nominal yields and downward-sloping premia on dividend strip returns under standard preferences. Among many other predictions, the model leads to two peculiar testable implications: i) a positive correlation between real and nominal yields which increases with the horizon, and ii) a term-spread due to inflation (i.e. the difference between nominal and real term-spreads) which is positive and upward-sloping. I document that these stylized facts are in line with the data and crucially depend on the joint role of labor rigidity and transitory risk. Indeed, the real yield's loading on the transitory shock is negative and monotone decreasing with maturity, whereas the nominal yield loading can be either positive or negative and non-monotone with maturity when transitory risk in real growth and inflation are positively correlated. This effect is due to a positive nominal risk premium, which is sizeable for labor rigidity strong enough. Such a disconnect in the dynamics of the short-maturity real and nominal yields gives rise to the above mentioned stylized facts.

The model calibration exploits information from the term-structures of variance-ratios of real consumption, wages, dividends as well as inflation. This allows to infer about the strength of both the operating leverage due to labor rigidity and the link between transitory risk in inflation and real growth. The baseline calibration reconciles the timing of macroeconomic

[3] The analysis of this paper contributes to this literature by providing evidence of a similar relationship on a longer sample (1929-2013) and by investigating the related asset pricing implications.

risk with many patterns of real and nominal yields as well as equity and dividend strip returns under standard preferences. Namely, the model captures eight stylized facts about real and nominal bonds: *(i)* both real and nominal yields are positive and increasing; *(ii)* nominal yields are higher than real yields; *(iii)* both real and nominal yield volatilities are decreasing; *(iv)* nominal yield volatility is higher than real yield volatility; *(v)* the correlation between real and nominal yields increases with the maturity; *(vi)* both real and nominal term-spreads are increasing; *(vii)* the nominal term-spread is higher than the real term-spread; *(viii)* the term-spread due to inflation is increasing. In addition, the model generates a low and smooth risk-free rate, a sizeable equity premium, equity excess volatility and downward-sloping dividend strip return volatility and risk premia.

Related literature. The paper complements the large literature on equilibrium models for equity and bond returns. This paper diverges from long-run risk frameworks (Eraker, 2008; Piazzesi and Schneider, 2006; Hasseltoft, 2012; Bansal and Shaliastovich, 2013), since the evidence about the timing of macroeconomic risk emphasizes the joint role of permanent and transitory shocks.[4] Some contributions investigate the role of habit and time-varying risk aversion, such as Wachter (2006) and Buraschi and Jiltsov (2007). Other papers focus on the dynamics of correlation between equity and bonds (Hasseltoft, 2009; Hasseltoft and Burkhardt, 2012; Song, 2014). Differently from the previous literature, this paper focuses on the link between the timing of macroeconomic risk and the term-structures of real and nominal yields.

The paper is also related to works that investigate the role of labor markets on asset prices. Income insurance within the firm provides a rationale for the high equity premium (Danthine and Donaldson, 1992, 2002) and the negative slope of dividend risk (Marfè, 2013b). Labor market frictions can generate similar results (Kuehn, Petrosky-Nadeau, and Zhang, 2012; Favilukis and Lin, 2015). Labor markets are also important for cross-sectional returns and the value premium dynamics (Donangelo, Gourio, and Palacios, 2015; Marfè, 2015).

Recently, a few papers have proposed different economic channels that can explain the negative slope of risk premia for dividend strips (Ai, Croce, Diercks, and Li, 2015; Marfè, 2013b; Belo, Collin-Dufresne, and Goldstein, 2015; Croce, Lettau, and Ludvigson, 2015; Hasler and Marfè, 2015). This paper complements this literature by investigating the interaction between inflation risk and real growth and focusing on the term-structures of real and nominal yields.

The paper is organized as follows. Section 2 provides empirical support to the main model mechanism. Section 3 describes the economy and derives the equilibrium asset pricing predictions. Model calibration and results are in Section 4. Section 5 concludes.

[4]Note that the Bansal and Yaron (2004)'s model obtains as a subcase: a counter-factual prediction of many long-run risk models is that real yields are downward-sloping.

2 Empirical Support

2.1 The Timing of Macroeconomic Risk

This section focuses on the timing of risk of output, wages and dividends. The variance-ratios (VR's) of their growth rates –i.e. the variance of growth rates computed over an horizon of τ years relative to τ times the variance of growth rates computed over a one year– tell whether their risk increases linearly with the horizon. Indeed, VR's larger than one and increasing identify long-horizon risk, whereas VR's lower than one and decreasing mean that risk concentrates at short horizons.

Marfè (2013b) documents that the VR's of output are approximatively flat or slightly upward-sloping, while the VR's of wages and dividends are respectively markedly upward-sloping and downward-sloping. These VR's from the sample 1929-2013 are reported in the left panel of Figure 1. Marfè (2013b) provides empirical support that the slopes of the VR's can be largely imputed to labor rigidity. Namely, output, wages and dividends are co-integrated in levels. Thus, these variables have to face the same permanent shock, whereas they have heterogeneous exposition to transitory risk. In turn, their VR's are heterogeneous too. Marfè (2013b) argues that a mechanism of income insurance from shareholders to workers takes place within the firm. Then, after a negative (positive) transitory shock, the share of resources devoted to workers remuneration increases (decreases) relative shareholder's remuneration. Wages load less (more) on transitory risk relative to permanent risk and their VR's increase (decrease) because the upward-sloping effect due to permanent risk dominates. Vice-versa dividends load more (less) on transitory risk relative to permanent risk and their VR's decrease (increase) because the downward-sloping effect due to transitory risk dominates.

This economic mechanism is consistent with a number of stylized facts documented in the literature: Lettau and Ludvigson (2005) and Lettau and Ludvigson (2014) document that macroeconomic variables are co-integrated and subject to both permanent and transitory shocks; Ríos-Rull and Santaeulàlia-Llopis (2010), among others, document the implication that the labor-share is counter-cyclical; finally, Guiso, Pistaferri, and Schivardi (2005) document that income insurance within the firm exists and does not concern permanent shocks. Moreover, Marfè (2013b) explicitly tests the above economic mechanism: variation in the labor-share strongly correlates with variation in the gap between the VR's of wages and VR's of dividends. Consistently with the hypothesis of labor rigidity, such a relation is positive and robust to many macroeconomic controls. Intuitively, the term-structure effect of labor rigidity is depicted in the right panel of Figure 1: the VR's of the remainder of output minus wages are markedly downward-sloping and almost recover the shape of the VR's of dividends.

In a nutshell, labor rigidity is the main driver of the timing of macroeconomic risk and explains the negative slope of dividend risk. In turn, variation of the labor-share reveals the

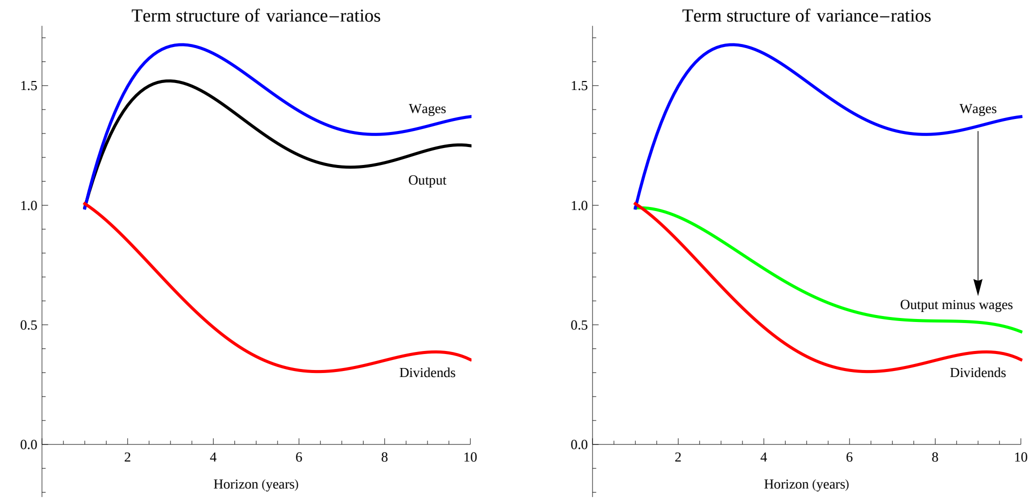


Figure 1: The Timing of Macroeconomic Risk and Labor Rigidity

Left panel: Variance-ratios of GDP (black), wages (blue) and dividends (red) real growth rates as a function of the horizon. Right panel: Variance-ratios of wages (blue), dividends (red) and output minus wages (green) as a function of the horizon. Data are yearly on the sample 1930:2013. The variance ratio procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations.

dynamics of short-run macroeconomic risk.

2.2 The Timing of Inflation Risk

Although a huge empirical literature, little is known about the timing of inflation risk. Figure 2 reports the VR's of log changes in the price index from the sample 1929-2013. The term-structure of VR's is markedly upward-sloping. Namely, these VR's imply that growth rates computed over 5 and 10 years are respectively about twice and more than three times riskier than one year growth rates.

The following analysis investigates the sources of the timing of inflation risk and its short-run and long-run components. I start by performing a Principal Component Analysis (PCA) of the growth rates of output (Δy), wages (Δw), dividends (Δd) as well as price index (Δp). Panel A of Table 1 reports the VR's for these variables and Panel B reports the estimates of the PCA. Namely, we observe that the first principal component (PC) explains a large part of total variance, about 80%, whereas the second and the third PC's explain respectively about 15% and 5% of total variance and the fourth PC is almost residual. Panel C of Table 1 reports the VR's of the PC's. The first and the fourth PC's feature markedly downward-sloping term-structures, whereas the second and the third PC's are characterized by markedly upward-sloping term-structures. Therefore, the first and the fourth PC's capture the short-run variation in macroeconomic data and the second and the third PC's capture long-run variation. To recover the short-run and long-run orthogonal components of the original variables, it is

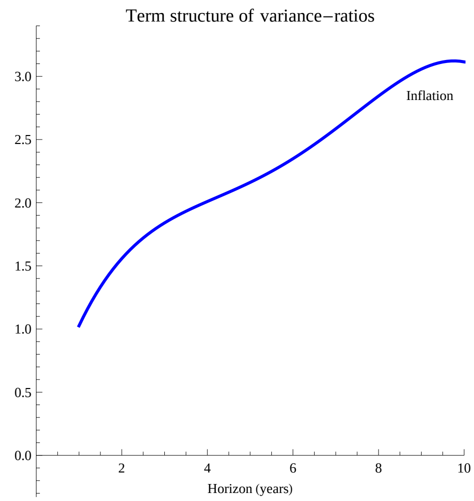


Figure 2: The Timing of Inflation Risk

Left panel: Variance-ratios of price index log changes as a function of the horizon. Data are yearly on the sample 1930:2013. The variance ratio procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations.

sufficient to use respectively the 1st and 4th PC's and the 2nd and 3rd PC's and the inverse of the weighting matrix (Λ). The short-run components are given by:

$$\begin{pmatrix} \Delta y^{SR} \\ \Delta w^{SR} \\ \Delta d^{SR} \\ \Delta p^{SR} \end{pmatrix} = \left[\Lambda' \cdot \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \right] \begin{pmatrix} \text{1st PC} \\ \text{2nd PC} \\ \text{3rd PC} \\ \text{4th PC} \end{pmatrix}.$$

The long-run components are given by:

$$\begin{pmatrix} \Delta y^{LR} \\ \Delta w^{LR} \\ \Delta d^{LR} \\ \Delta p^{LR} \end{pmatrix} = \left[\Lambda' \cdot \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \right] \begin{pmatrix} \text{1st PC} \\ \text{2nd PC} \\ \text{3rd PC} \\ \text{4th PC} \end{pmatrix}.$$

And they sum up to the original variables (up to the unconditional mean):

$$\begin{pmatrix} \Delta y \\ \Delta w \\ \Delta d \\ \Delta p \end{pmatrix} = \begin{pmatrix} \mathbb{E}[\Delta y] \\ \mathbb{E}[\Delta w] \\ \mathbb{E}[\Delta d] \\ \mathbb{E}[\Delta p] \end{pmatrix} + \begin{pmatrix} \Delta y^{SR} \\ \Delta w^{SR} \\ \Delta d^{SR} \\ \Delta p^{SR} \end{pmatrix} + \begin{pmatrix} \Delta y^{LR} \\ \Delta w^{LR} \\ \Delta d^{LR} \\ \Delta p^{LR} \end{pmatrix}.$$

Panel D of Table 1 reports the variance-ratios of both the short-run and long-run components of the original variables. As expected, the former and the latter are characterized by respectively

Table 1: Variance-Ratios and Principal Component Analysis

Panel A of the table reports the variance-ratios of the real growth rates of output (Δy), wages (Δw), dividends (Δd) and inflation (Δp) over the horizon from one to 10 years. Panel B reports the PCA weighting matrix and the explained variance of the PC's from the real growth rates and inflation. Panel C reports the variance-ratios of the PC's over the horizon from one to 10 years. Panel D reports the variance-ratios of the short-run and long-run decompositions of the real growth rates and inflation over the horizon from one to 10 years. The data sample is 1929-2013. The variance ratios procedure accounts for heteroscedasticity and overlapping observations (Campbell, Lo, and MacKinlay, 1997, pp. 48-55).

Panel A – Variance Ratios of growth rates

Horizon (years)	1	2	3	4	5	6	7	8	9	10
Δy	1.00	1.39	1.53	1.46	1.33	1.19	1.15	1.19	1.23	1.24
Δw	1.00	1.47	1.67	1.64	1.54	1.36	1.29	1.31	1.35	1.36
Δd	1.00	0.86	0.68	0.46	0.37	0.30	0.33	0.38	0.34	0.37
Δp	1.00	1.60	1.88	1.94	2.09	2.38	2.68	2.82	2.98	3.15

Panel B – Principal Component Analysis

	Λ, weights on:				explained
	Δy	Δw	Δd	Δp	variance (%)
1st PC	0.135	0.657	-0.101	-0.735	79.3
2nd PC	0.127	0.709	-0.147	0.678	15.2
3rd PC	0.982	-0.189	-0.009	0.012	5.2
4th PC	0.042	0.171	0.984	0.026	0.3

Panel C – Variance Ratios of principal components

Horizon (years)	1	2	3	4	5	6	7	8	9	10
1st PC	1.00	0.88	0.69	0.46	0.34	0.26	0.29	0.34	0.31	0.33
2nd PC	1.00	1.48	1.69	1.73	1.70	1.60	1.54	1.54	1.60	1.60
3rd PC	1.00	1.61	1.99	2.21	2.44	2.81	3.12	3.20	3.35	3.53
4th PC	1.00	1.13	1.06	0.81	0.61	0.47	0.38	0.37	0.39	0.36

Panel D – Variance Ratios of short-run and long-run growth rates

Horizon (years)	1	2	3	4	5	6	7	8	9	10
Δy^{SR}	1.00	0.95	0.78	0.53	0.37	0.30	0.33	0.37	0.33	0.33
Δw^{SR}	1.00	0.86	0.68	0.46	0.37	0.26	0.26	0.32	0.31	0.34
Δd^{SR}	1.00	0.89	0.70	0.46	0.35	0.26	0.29	0.34	0.31	0.34
Δp^{SR}	1.00	0.88	0.69	0.46	0.35	0.26	0.28	0.34	0.31	0.34
Δy^{LR}	1.00	1.42	1.66	1.74	1.72	1.63	1.57	1.56	1.61	1.62
Δw^{LR}	1.00	1.43	1.68	1.77	1.75	1.67	1.61	1.60	1.64	1.66
Δd^{LR}	1.00	1.40	1.60	1.63	1.59	1.50	1.44	1.44	1.50	1.51
Δp^{LR}	1.00	1.57	1.84	1.94	2.13	2.46	2.76	2.87	3.03	3.19

downward-sloping and upward-sloping variance ratios.

The joint use of PCA and VR's allows to infer additional information from the data and provides a novel tool that can help to understand how macroeconomic variables interact with each other. In the following, I verify how short-run and long-run components of inflation interact with the real growth. Finally, I show that such results are consistent with and supportive of the term-structure effect of labor rigidity commented in the previous section.

Inflation Risk and Real Growth

Table 2 reports the first order autocorrelation of inflation which is large and positive: 63%. The short- and long-run components of inflation are instead characterized by respectively negative (-32%) and positive (63%) autocorrelations, which are both highly significant. This result is consistent with the decomposition made by the joint use of PCA and VR's.

Table 2 reports the contemporaneous correlation between inflation (Δp) and real growth for output, wages and dividends ($\Delta y, \Delta w$ and Δd). Those correlations are positive but relatively small: about 20%. An instructive exercise consists of computing the correlation between

Table 2: Inflation Risk and Real Growth

The table reports the correlations between the yearly growth rates of the price index (Δp) and the yearly real growth rates of output (Δy), wages (Δw) and dividends (Δd). The table also reports the correlations among the short-run and long-run components of the same variables computed as in Table 1. The last column denotes the first order autocorrelations. The data sample is 1929-2013. The p-values are reported in parenthesis. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

	Δy	Δy^{SR}	Δy^{LR}	Δw	Δw^{SR}	Δw^{LR}	Δd	Δd^{SR}	Δd^{LR}	AC(1)
Δp	.24**	.15	.19*	.21*	.15	.16	.13	.16	-.31***	.63***
	(.03)	(.19)	(.08)	(.06)	(.17)	(.14)	(.24)	(.15)	(.00)	(.00)
Δp^{SR}	.41***	.93***	.00	.38***	.96***	.00	.99***	.99***	.00	-.32***
	(.00)	(.00)	(1.00)	(.00)	(.00)	(1.00)	(.00)	(.00)	(1.00)	(.00)
Δp^{LR}	.18	.00	.20*	.15	.00	.17	-.03	.00	-.31***	.63***
	(.11)	(1.00)	(.07)	(.17)	(1.00)	(.13)	(.81)	(1.00)	(.00)	(.00)

the short- and long-run components of inflation and respectively the short- and long-run components of real growth. Table 2 shows that the long-run components of inflation and real growth are barely uncorrelated, whereas the short-run components are extremely correlated:

$$\text{corr}(\Delta p^{SR}, \Delta y^{SR}) = 93\%, \qquad \text{corr}(\Delta p^{SR}, \Delta w^{SR}) = 96\%, \qquad \text{corr}(\Delta p^{SR}, \Delta d^{SR}) = 99\%.$$

This result is quite interesting since it means that the link between inflation risk and fundamentals obtains by means of a latent short-run factor. Such a factor enters both short-run inflation and short-run real growth with the same sign. In turn, the positive relation between the whole inflation and real growth should be likely imputed to such a common short-run factor.

Inflation Risk and Labor Rigidity

The previous sections have shown two results. On the one hand, labor rigidity is at the heart of the timing of risk of real quantities and mainly explains the negative slope of dividend risk. On the other hand, inflation risk is upward-sloping but its relation with the real growth is

mainly due to short-run risk and obtains because their short-run components are driven by a common latent factor.

In order to test whether these stylized facts are consistent with each other, I look at the dynamics of the labor-share. Indeed, accordingly with Marfè (2013b), variation in the labor-share provides information about transitory risk. Namely, labor rigidity implies that the labor-share is negatively driven by the transitory shock in fundamentals. Since short-run inflation is strongly positively correlated with short-run real growth, I expect that the current level of the labor-share positively predicts future inflation growth.

To test this economic mechanism, I regress the cumulative inflation over 1 to 7 years on the current labor-share. Table 3 reports the estimates from the sample 1929-2013. Consistently

Table 3: Inflation Risk and Labor Rigidity

The table reports the coefficient, Newey-West t-statistics, standardized coefficient and adjusted-R^2 of the regressions of the cumulative inflation rates over the horizon n of 1, 2, 3, 5 and 7 years on the current labor-share:

$$\frac{1}{n}\sum\nolimits_{i=1}^{n} \Delta p_{t+i} = a + b\, W/Y_t + \epsilon_t$$

The data sample is 1929-2013. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Horizon (years)	1	2	3	5	7
Coefficient	.70*	.54**	.59**	.50**	.40**
t-stat	(1.92)	(2.08)	(2.20)	(2.25)	(1.91)
economic significance	.35	.42	.47	.49	.45
adj-R^2	.11	.17	.22	.23	.19

with the above reasoning, we observe a positive and highly significant relation between the labor-share and future inflation growth over several horizons. This relation strengthens with the horizon: the adjusted R^2 ranges from about 11% to 23% . The economic significance ranges from about 35% to 49%. These results are consistent with the international evidence by Gali, Gertler, and Lopez-Salido (2001) in postwar-data.

2.3 Summary

Overall, the empirical analysis documents a number of stylized facts. First, inflation risk is upward-sloping as suggested by VR's. Second, inflation risk is linked to fundamentals by means of a latent short-run factor. Third, consistently with the idea that labor rigidity is at the heart of the timing of macroeconomic risk, labor-share variation forecasts future inflation.

These stylized facts provide new empirical guidance to design an equilibrium model that aims to jointly explain the term-structures of fundamentals, those of equity returns as well

as those of both real and nominal interest rates. Section 3 and 4 propose a closed-form and parsimonious general equilibrium model that rationalizes these stylized facts.

3 Model

3.1 Economy

The economy has the following structure. A representative firm produces a cash-flows stream, which represents total output minus investments: $C = Y - I$. Such cash-flows constitute the total resources shared by workers and shareholders: the former receive wages (W) and the latter receive dividends (D). The resource constraint is $C = W + D$. Consistently with Berk and Walden (2013), I assume limited market participation such that workers do not access the financial markets and consume their wages. Consequently, shareholders act as a representative agent on the stock market and consume dividends.[5]

Shareholders feature recursive preferences in spirit of Kreps and Porteus (1979), Epstein and Zin (1989) and Weil (1989). I assume their continuous time counterpart, as in Duffie and Epstein (1992), for the sake of tractability. These preferences allow for the separation between the coefficient of relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS). Given a consumption process C, the utility at each time t is defined as

$$J_t = \mathbb{E}_t \int_{u \geq t} f(C_u, J_u) du, \qquad \text{with} \qquad f(C, J) = \beta \chi J \big(\tfrac{C^{1-1/\psi}}{((1-\gamma)J)^{1/\chi}} - 1 \big), \tag{1}$$

where $\chi = \frac{1-\gamma}{1-1/\psi}$, β is the time-discount rate, ψ is EIS and γ is RRA. Power utility obtains for $\psi \to 1/\gamma$.

Aggregate consumption dynamics are modelled as the product of a permanent and a transitory shock as in Marfè (2013b). The former, X, features time-variation in expected growth, as in the long-run risk literature, and induces an upward-sloping effect on the term-structure of risk. The latter, Z, is usually used in the real business cycle literature and induces a downward-sloping effect. Thus, the two shocks jointly lead to flexibility at modelling the timing of risk. Namely, the dynamics of aggregate consumption $C = XZ$ satisfy:

$$d \log X_t = dx_t = (\mu_t - \sigma_x^2/2)dt + \sigma_x dB_{x,t}, \tag{2}$$

$$d\mu_t = \lambda_\mu(\bar{\mu} - \mu_t)dt + \sigma_\mu dB_{\mu,t}, \tag{3}$$

$$d \log Z_t = dz_t = -\lambda_z z_t dt + \sigma_z dB_{z,t}, \tag{4}$$

[5]While the endogenous determination of market participants goes beyond the scope of this paper, Berk and Walden (2013) show that labor markets provide risk-sharing to workers, such that their consumption endogenously equals their wages and limited market participation obtains.

I assume homoscedasticity and independent Brownian shocks just for the sake of exposition and tractability.

Consumption, wages and dividends

Workers and shareholders receive income from two sources, wages and dividends, and where the mix between these two sources of income varies over time. However, aggregate consumption and dividends are co-integrated in levels. Then, the excess volatility of dividends over consumption obtains by a levered exposition $(1 + \phi)$ to the transitory shock:

$$D_t = \alpha X_t Z_t^{1+\phi}, \tag{5}$$

where $X_t \equiv e^{x_t}$, $Z_t \equiv e^{z_t}$, $\alpha \in (0, 1)$ and $\phi \geq 0$. The steady-state dividend-share is $D/C = \alpha$. Such a specification of dividends allows to capture not only co-integration and excess volatility but also other features of the data: i) the slightly upward-sloping risk of consumption and the downward-sloping risk of dividends; ii) the pro-cyclical dynamics of the fraction of total resources devoted to shareholders' remuneration; iii) the negative correlation between the dividend- and the labor-share; iv) the markedly upward-sloping risk of wages.

The above dynamics can be understood in terms of "distributional risk" between shareholders and workers, due to labor rigidity. Denote wages as the reminder of consumption $W = C - D$, then we have:

$$W_t = X_t Z_t - \alpha X_t Z_t^{1+\phi} = C_t(1 - \alpha Z_t^{\phi}), \tag{6}$$

and, therefore, the dividend- and labor-shares, D_t/C_t and W_t/C_t, are respectively a convex and a concave function of the transitory shock z_t. Let denote the labor-share as[6]

$$\omega(z_t) = 1 - \alpha e^{\phi z_t}, \tag{7}$$

The non-linearity is governed by the parameter ϕ, which can be interpreted as the degree of labor rigidity. In presence of a negative transitory shock ($z_t < 0$), workers are partially insured because the fraction of total resources devoted to wages increases, whereas shareholders suffer more since the dividend-share decreases. Such an insurance mechanism has two main implications. The first is a *cyclicality and leverage effect*: insurance makes dividends riskier in bad times and, hence, provides a rationale for a high equity premium. The second effect is a *term-structure effect*: jointly with co-integration, insurance induces high short-term risk

[6]As long as z_t has Ornstein-Uhlenbeck dynamics, $\omega(z_t)$ is not bounded in $(0, 1)$. However under plausible parameters the probability that $\omega(z_t) < 0$ is negligible since α is small and z_t is smooth. Hence, hereafter I assume $1 > \alpha e^{\phi z_t}$, $\forall t$.

to dividends and high long-run risk to wages. In turn the former and the latter feature term-structures of risk respectively decreasing and increasing with the horizon.[7] Under labor rigidity, the instantaneous volatility of consumption, wages and dividends growth rates satisfy

$$\sigma_{W,t} < \sigma_{C,t} < \sigma_{D,t}. \tag{8}$$

Define the moment generating function of log consumption, wages and dividends as $\mathbb{C}_t(\tau, n) = \mathbb{E}_t[C^n_{t+\tau}]$, $\mathbb{W}_t(\tau, n) = \mathbb{E}_t[W^n_{t+\tau}]$ and $\mathbb{D}_t(\tau, n) = \mathbb{E}_t[D^n_{t+\tau}]$. Then, the term-structures of expected growth rates are given by

$$g_C(t,\tau) = \frac{1}{\tau}\log\left(\frac{\mathbb{C}_t(\tau,1)}{\mathbb{C}_t(0,1)}\right), \quad g_W(t,\tau) = \frac{1}{\tau}\log\left(\frac{\mathbb{W}_t(\tau,1)}{\mathbb{W}_t(0,1)}\right), \quad g_D(t,\tau) = \frac{1}{\tau}\log\left(\frac{\mathbb{D}_t(\tau,1)}{\mathbb{D}_t(0,1)}\right). \tag{9}$$

Similarly, the term-structures of growth rates variance are given by

$$\sigma^2_C(t,\tau) = \frac{1}{\tau}\log\left(\frac{\mathbb{C}_t(\tau,2)}{\mathbb{C}_t(\tau,1)^2}\right), \quad \sigma^2_W(t,\tau) = \frac{1}{\tau}\log\left(\frac{\mathbb{W}_t(\tau,2)}{\mathbb{W}_t(\tau,1)^2}\right), \quad \sigma^2_D(t,\tau) = \frac{1}{\tau}\log\left(\frac{\mathbb{D}_t(\tau,2)}{\mathbb{D}_t(\tau,1)^2}\right). \tag{10}$$

Long-run growth μ_t and transitory shock z_t induce respectively an upward- and a downward-sloping effect to all the term-structures of variances. Interestingly, labor rigidity alters the slopes of wage and dividend risk. The stronger labor rigidity, the larger in magnitude the (positive) slope of wage risk and the (negative) slope of dividend risk:

$$\frac{\partial^2}{\partial\phi\,\partial\tau}\sigma^2_W(t,\tau) > 0, \qquad \frac{\partial^2}{\partial\phi\,\partial\tau}\sigma^2_D(t,\tau) < 0. \tag{11}$$

Therefore, labor rigidity shifts wage risk toward the long-horizon and dividend risk toward the short horizon. A natural metric to measure such an effect is the term-structure of variance ratios, that is the ratio of horizon-τ growth rates variances relative to short horizon growth rates variances:

$$VR_C(t,\tau) = \frac{\sigma^2_C(t,\tau)}{\sigma^2_C(t,\tau_0)}, \quad VR_W(t,\tau) = \frac{\sigma^2_W(t,\tau)}{\sigma^2_W(t,\tau_0)}, \quad VR_D(t,\tau) = \frac{\sigma^2_D(t,\tau)}{\sigma^2_D(t,\tau_0)},$$

where τ_0 usually denotes one-quarter or one year. For $\phi > 0$, the variance ratios of wages and dividends lie respectively above and below those of aggregate consumption.

[7] Consistently with the model, Menzio (2005) theoretically shows and empirically supports the idea that labor market frictions lead to wages that are rigid with respect to transitory shocks but not with respect to permanent shocks.

Inflation

The price index, Π, is given by the product of a permanent and a transitory component:

$$\Pi_t = I_t Z_t^{\varphi}.$$

In accord with the empirical findings of Section 2, the permanent component, I, is uncorrelated with fundamentals and has a smooth dynamics:

$$\begin{aligned} d\log I_t &= i_t dt, \\ di_t &= \lambda_i(\bar{i} - i_t)dt + \sigma_i dB_{i,t}. \end{aligned}$$

Instead, the transitory component is a function of the same transitory shock shared by fundamentals. Thus, the instantaneous inflation rate satisfies:

$$\pi_t \equiv d\log \Pi_t = i_t dt + \varphi dz_t.$$

Define the moment generating function of the log price index as $\mathbb{\Pi}_t(\tau, n) = \mathbb{E}_t[\Pi_{t+\tau}^n]$. Then, the term-structures of the expected inflation and the inflation risk are given by

$$g_{\Pi}(t,\tau) = \tfrac{1}{\tau}\log\left(\tfrac{\mathbb{\Pi}_t(\tau,1)}{\mathbb{\Pi}_t(0,1)}\right), \tag{12}$$

$$\sigma_{\Pi}^2(t,\tau) = \tfrac{1}{\tau}\log\left(\tfrac{\mathbb{\Pi}_t(\tau,2)}{\mathbb{\Pi}_t(\tau,1)^2}\right). \tag{13}$$

Long-run price index growth i_t and transitory shock z_t induce respectively an upward- and a downward-sloping effect to the term-structure of inflation risk. Thus, they can be interpreted as the model counterparts of the long-run and short-run components of inflation from the empirical analysis of Section 2.

The coefficient φ determines the relative importance of the two effects and, hence, can be set to match the positive slope of inflation risk from actual data:

$$\tfrac{\partial^2}{\partial\varphi\,\partial\tau}\sigma_{\Pi}^2(t,\tau) < 0.$$

Expected inflation interacts with real growth through the transitory shock. The correlations

$$\operatorname{corr}(g_{\Pi}(t,\tau),\, g_C(t,\tau)), \quad \operatorname{corr}(g_{\Pi}(t,\tau),\, g_W(t,\tau)), \quad \text{and} \quad \operatorname{corr}(g_{\Pi}(t,\tau),\, g_D(t,\tau)),$$

are positive for $\varphi > 0$ and negative for $\varphi < 0$. Thus, a positive coefficient φ allows to reproduce the sign of those correlations in actual data.

Interestingly, a positive φ also guarantees that the current level of the labor-share positively

predicts inflation:

$$\text{corr}(\omega(z_t),\, g_\Pi(t,\tau)) \gtrless 0 \qquad \text{if} \qquad \varphi \gtrless 0.$$

Such a model implication captures the positive intertemporal relation between variation in the labor-share, due to labor rigidity (i.e. $\phi > 0$), and the realized future inflation.

3.2 Equilibrium and Asset Prices

Equilibrium State-Price Density

The exponential affine dynamics of shareholders' consumption and preferences in Eq. (1) guarantee a model solution which emphasizes the role of long-run growth μ_t and transitory shock z_t in the price formation and preserves tractability. A first order approximation of the shareholders' consumption-wealth ratio around its (endogenous) steady-state, e^{cq}, provides closed form solutions for prices and return moments up to such approximation. In particular, I follow Benzoni, Collin-Dufresne, and Goldstein (2011).

Under limited market participation, the shareholders' marginal utility is the valid state-price density (Duffie and Epstein (1992)):

$$\xi_{t,u} = e^{\int_t^u f_J(D_\tau, J_\tau)d\tau} \frac{f_C(D_u, J_u)}{f_C(D_t, J_t)}, \qquad \forall u \geq t. \tag{14}$$

Thus, the real price of a payoff stream $\{F_u, u \in (t,\infty)\}$ equals $\mathbb{E}_t[\int_t^\infty \xi_{t,u} F_u du]$. The economy leads to a stationary equilibrium, although the equilibrium state-price density is an involved function of the integrated process D_t. Stationarity is a necessary condition to produce realistic testable implications. The equilibrium state price density has dynamics given by

$$\tfrac{d\xi_{0,t}}{\xi_{0,t}} = \tfrac{df_C}{f_C} + f_J dt = -r(t)dt - \theta_x(t)dB_{x,t} - \theta_\mu(t)dB_{\mu,t} - \theta_z(t)dB_{z,t}, \tag{15}$$

where the instantaneous risk-free rate satisfies

$$r(t) = r_0 + r_\mu \mu_t + r_z z_t, \tag{16}$$

with r_0 derived in the Appendix, $r_\mu = \frac{1}{\psi}$, $r_z = -\frac{\lambda_z}{\psi}(1+\phi)$, and the instantaneous prices of

risk are given by

$$\theta_x(t) = \quad -\frac{\partial_X f_C}{f_C} X_t \sigma_x \quad = \gamma\, \sigma_x, \tag{17}$$

$$\theta_\mu(t) = \quad -\frac{\partial_\mu f_C}{f_C} \sigma_\mu \quad = \frac{\gamma - 1/\psi}{e^{cq} + \lambda_\mu} \sigma_\mu, \tag{18}$$

$$\theta_z(t) = \quad -\frac{\partial_z f_C}{f_C} \sigma_z \quad = \left(\gamma - \frac{\lambda_z(\gamma - 1/\psi)}{e^{cq} + \lambda_z} \right) (1+\phi)\sigma_z. \tag{19}$$

The risk-free rate is affine in μ_t and z_t. Coefficients r_μ and r_z are respectively positive and negative and both decrease in magnitude with ψ, as usual under recursive preferences. Moreover, r_z increases in magnitude with reversion λ_z and labor rigidity ϕ.

The price of risk due to the permanent shock, $\theta_x(t)$, has the usual form and is a price for the contribution of x_t to the instantaneous volatility of shareholders' consumption. The price of risk due to long-run growth, , is similar in long-run risk models and is a price for the contribution of μ_t to the variation in the continuation utility value. This term vanishes under power utility ($\psi \to \gamma^{-1}$), increases with the preference for the early resolution of uncertainty, $\gamma - 1/\psi$, and decreases in magnitude with the reversion in long-run growth. The price of risk due to the transitory shock, $\theta_z(t)$, has two components. The first, $\gamma(1+\phi)\sigma_z$, is a positive price for the contribution of z_t to the instantaneous volatility of shareholders' consumption. The second, $\theta_z(t) - \gamma(1+\phi)\sigma_z$, is a price for the contribution of z_t to the variation in the continuation utility value. The latter term is negative and increases with the reversion of z_t under preferences for the early resolution of uncertainty, whereas it disappears under power utility. Both components of $\theta_z(t)$ increase in magnitude with ϕ: *the risk of owning capital due to labor rigidity is priced in equilibrium.* Under the usual parametrization $\gamma > \psi > 1$, z_t has a positive price for its effect on the current shareholders' consumption and a negative price for its effect on the evolution of the utility process.

Equilibrium Bond Yields

The equilibrium real price of the non-defaultable bond with maturity τ is given by

$$\mathscr{B}_{t,\tau} = \mathbb{E}_t \left[\xi_{t,t+\tau} \right] = e^{b_0(\tau) + b_\mu(\tau)\mu_t + b_z(\tau) z_t}, \tag{20}$$

where b_0, b_μ and b_z are deterministic functions of the maturity derived in the Appendix. Hence, the real bond yield $\mathscr{y}(t,\tau)$ is state-dependent but its volatility inherits the homoscedasticity of the states:

$$\mathscr{y}(t,\tau) = -b_0(\tau)/\tau + \frac{r_\mu \left(1 - e^{-\tau\lambda_\mu}\right)}{\tau\lambda_\mu} \mu_t + \frac{r_z \left(1 - e^{-\tau\lambda_z}\right)}{\tau\lambda_z} z_t.$$

The short- and long-run limits of the term-structure of real bond yields lead to the steady state term-spread:

$$\mathcal{y}(t,\infty) - \mathcal{y}(t,0) = -\frac{\sigma_\mu r_\mu \left(2\theta_\mu \lambda_\mu + \sigma_\mu r_\mu\right)}{2\lambda_\mu^2} - \frac{\sigma_z r_z \left(2\theta_z \lambda_z + \sigma_z r_z\right)}{2\lambda_z^2},$$

which can be either positive or negative for $\gamma > \psi > 1$, since $r_\mu \theta_\mu > 0$ and $r_z \theta_z < 0$. Thus, for labor rigidity strong enough, the term-structure of real interest rates is increasing.

The numeraire-adjusted state-price density is given by

$$\xi_{0,t}^\star = \xi_{0,t} \Pi_t^{-1},$$

and satisfies

$$\frac{d\xi_{0,t}^\star}{\xi_{0,t}^\star} = \frac{\Pi_t^{-1} d\xi_{0,t} + \xi_{0,t} d(\Pi_t^{-1}) + \langle \xi_{0,t}, \Pi_t^{-1} \rangle dt}{\xi_{0,t} \Pi_t^{-1}}.$$

The equilibrium nominal price of the non-defaultable bond with maturity τ is given by

$$\mathcal{B}_{t,\tau}^\star = \mathbb{E}_t \left[\xi_{t,t+\tau}^\star \right] = e^{k_0(\tau) + k_\mu(\tau)\mu_t + k_z(\tau) z_t + k_i(\tau) i_t}, \tag{21}$$

where k_0, k_μ, k_z and k_i are deterministic functions of the maturity derived in the Appendix. Hence, the nominal bond yield $\mathcal{y}^\star(t,\tau)$ is state-dependent but its volatility inherits the homoscedasticity of the states:

$$\mathcal{y}^\star(t,\tau) = -k_0(\tau)/\tau + \frac{r_\mu \left(1 - e^{-\tau\lambda_\mu}\right)}{\tau \lambda_\mu} \mu_t + \frac{\left(r_z \left(1 - e^{-\tau\lambda_z}\right) + e^{-\tau\lambda_z} \varphi \lambda_z\right)}{\tau \lambda_z} z_t + \frac{1 - e^{-\lambda_i \tau}}{\tau \lambda_i} i_t.$$

Notice that, since $r_z < 0$, for $\varphi > 0$ the instantaneous correlation between real and nominal yields is increasing with the maturity:

$$\tfrac{\partial}{\partial \tau} \operatorname{corr}\left(\mathcal{y}(t,\tau),\ \mathcal{y}^\star(t,\tau)\right) > 0.$$

The short- and long-run limits of the term-structure of nominal bond yields lead to the steady state term-spread:

$$\begin{aligned} \mathcal{y}^\star(t,\infty) - \mathcal{y}^\star(t,0) &= -\frac{\sigma_\mu r_\mu \left(2\theta_\mu \lambda_\mu + \sigma_\mu r_\mu\right)}{2\lambda_\mu^2} - \frac{\sigma_z r_z \left(2\theta_z \lambda_z + \sigma_z r_z\right)}{2\lambda_z^2} + \frac{1}{2} \varphi \sigma_z \left(\varphi \sigma_z + 2\theta_z\right) - \frac{\sigma_i^2}{2\lambda_i} \\ &= \mathcal{y}(t,\infty) - \mathcal{y}(t,0) + \underbrace{\frac{1}{2} \varphi \sigma_z \left(\varphi \sigma_z + 2\theta_z\right) - \frac{\sigma_i^2}{2\lambda_i}}_{\text{nominal term-spread due to inflation}}, \end{aligned} \tag{22}$$

which can be either positive or negative. Permanent inflation risk induces a negative effect,

whereas transitory inflation risk induces a positive effect as long as it positively correlates with real growth ($\varphi > 0$). Labor rigidity affects the nominal term-spread by means of two channels: first, the stronger labor rigidity, the larger the real spread; second, the stronger labor rigidity the larger the equilibrium price of transitory risk (θ_z): in turn, the larger is the nominal term-spread, as long as transitory inflation risk positively correlates with real growth (as documented in the empirical analysis of Section 2).

Equilibrium Dividend Strips and Market Asset

The equilibrium price of the market dividend strip with maturity τ is given by

$$P_{t,\tau} = \mathbb{E}_t \left[\xi_{t,t+\tau} D_{t+\tau} \right] = X_t e^{A_0(\tau) + A_\mu(\tau)\mu_t + A_z(\tau) z_t}, \tag{23}$$

where A_0, A_μ and A_z are deterministic functions of the maturity derived in the Appendix. The instantaneous volatility and premium on the dividend strip with maturity τ are given by

$$\sigma_P(t,\tau) = \sqrt{\sigma_x^2 + \tfrac{(1-e^{-\lambda_\mu \tau})^2(\psi-1)^2}{\lambda_\mu^2 \psi^2}\sigma_\mu^2 + \tfrac{e^{-2\lambda_z \tau}(\psi + e^{\lambda_z \tau} - 1)^2(1+\phi)^2}{\psi^2}\sigma_z^2}, \tag{24}$$

$$(\mu_P - r)(t,\tau) = \gamma \sigma_x^2 + \tfrac{(1-e^{-\lambda_\mu \tau})(1-1/\psi)(\gamma - 1/\psi)}{\lambda_\mu (e^{cq} + \lambda_\mu)}\sigma_\mu^2 + \tfrac{\left(\frac{1}{\psi}(1-e^{-\lambda_z \tau}) + e^{-\lambda_z \tau}\right)(\psi \gamma e^{cq} + \lambda_z)(1+\phi)^2}{\psi (e^{cq} + \lambda_z)}\sigma_z^2. \tag{25}$$

The price of the dividend strip is exponential affine in x_t, μ_t and z_t. Thus, the price relative to the dividend level is a stationary function of the states. The functions $A_\mu(\tau)$ and $A_z(\tau)$ are the semi-elasticities of the price:

$$A_\mu(\tau) = \partial_\mu \log P_{t,\tau} = \frac{(1 - e^{-\lambda_\mu \tau})(1 - 1/\psi)}{\lambda_\mu}, \tag{26}$$

$$A_z(\tau) = \partial_z \log P_{t,\tau} = \left((1 - e^{-\lambda_z \tau})/\psi + e^{-\lambda_z \tau} \right)(1 + \phi). \tag{27}$$

Notice that $A_\mu(\tau)$ and $A_z(\tau)$ respectively increases and decreases with ψ. Moreover, $A_z(\tau)$ increases with labor rigidity ϕ. Thus, prices inherit the leverage effect on dividends: namely, the labor rigidity effect is amplified for $\psi < 1$ and vice-versa.

The permanent shock as well as the states μ_t and z_t contribute to the return volatility and command a premium. Permanent shocks do not lead to excess volatility. Instead, the loadings on $B_{\mu,t}$ and $B_{z,t}$, are proportional to the fundamentals' volatilities σ_μ and σ_z, but also depend on the horizon τ, the elasticity of intertemporal substitution and the persistence of the states. Namely, the loading on long-run growth is increasing in ψ and decreasing in reversion λ_μ. The reverse holds for the loading on the transitory shock, which is decreasing in ψ and increasing in λ_z. The latter is also amplified by the coefficient ϕ, which captures the leverage effect due to labor rigidity.

The dividend strip premium is the sum of three compensations. The permanent shock compensation has the usual form: $\gamma\sigma_x^2$. Instead, the compensations associated to the states μ_t and z_t depend also on the persistence of the states, the elasticity of intertemporal substitution as well as the horizon τ. The premium due to long-run growth is decreasing with reversion λ_μ and increasing with ψ:

$$\frac{\gamma - 1/\psi}{\lambda_\mu(e^{cq} + \lambda_\mu)} A_\mu(\tau)\sigma_\mu^2. \tag{28}$$

Such a term is a compensation for long-run risk and is positive if shareholders have preference for the early resolution of uncertainty and the intertemporal substitution effect dominates the wealth effect –i.e. the usual parametrization $\gamma > \psi > 1$. The premium due to the transitory shock z_t is the sum of two terms:

$$\gamma A_z(\tau)(1+\phi)\sigma_z^2, \qquad \text{and} \qquad -\frac{\lambda_z(\gamma - 1/\psi)}{e^{cq} + \lambda_z} A_z(\tau)(1+\phi)\sigma_z^2. \tag{29}$$

The former term is always positive and decreases with ψ. The latter term decreases with ψ and is negative (positive) under preference for the early (late) resolution of uncertainty. Finally, both terms increase in magnitude with the degree of labor rigidity, ϕ, and depend on the horizon τ.

The slopes of the term-structures of dividend strips' volatility and premia are given by

$$\frac{\partial}{\partial \tau}\sigma_P^2(t,\tau) = \frac{2(\psi-1)}{\psi^2}\left(e^{-2\lambda_\mu\tau}(e^{\lambda_\mu\tau}-1)(\psi-1)\frac{\sigma_\mu^2}{\lambda_\mu} \; - \; e^{-2\tau\lambda_z}(e^{\tau\lambda_z}-1+\psi)\lambda_z(1+\phi)^2\sigma_z^2\right), \tag{30}$$

$$\frac{\partial}{\partial \tau}(\mu_P - r)(t,\tau) = \frac{\psi-1}{\psi^2}\left(\frac{(\gamma-1/\psi)e^{-\lambda_\mu\tau}\sigma_\mu^2}{e^{cq}+\lambda_\mu} \; - \; \frac{e^{-\tau\lambda_z}\lambda_z(e^{cq}\gamma\psi+\lambda_z)(1+\phi)^2\sigma_z^2}{e^{cq}+\lambda_z}\right). \tag{31}$$

The slope of the term-structure of volatility depends on two terms, due respectively to the states μ_t and z_t. The former is always positive and, hence, induces an upward sloping effect. Instead, the latter term is negative if the intertemporal substitution effect dominates the wealth effect and vice-versa. Therefore, the term-structure of volatility is monotone upward sloping if $\psi < 1$, whereas it is not necessarily monotone if $\psi > 1$. A non-monotone (e.g. U-shaped) term-structure of risk obtains if labor rigidity leads to a leverage effect, ϕ, strong enough to outweigh the upward sloping effect due long-run growth, for some horizons τ.

Also the slope of the term-structure of premia depends on two terms, due respectively to the states μ_t and z_t. The former is positive if shareholders have preferences for the early resolution of uncertainty and the intertemporal substitution effect dominates the wealth effect. The latter term is negative if the intertemporal substitution effect dominates the wealth effect and vice-versa. Under the usual parametrization $\gamma > \psi > 1$, long-run growth leads to an upward-sloping effect, whereas transitory shocks lead to a downward-sloping effect. Therefore, the term-structure of equity premia is not necessarily monotone since both permanent and

transitory shocks enter the model.

Provided $\gamma > \psi > 1$, (i) the stronger labor rigidity, the smallest the slope of the term-structure of equity premia:

$$\tfrac{\partial^2}{\partial\phi\partial\tau}(\mu_P - r)(t,\tau) < 0;$$

(ii) for labor rigidity strong enough, the slope of the term-structure of equity premia is negative

$$\exists\, \phi > 0 : \tfrac{\partial}{\partial\tau}(\mu_P - r)(t,\tau) < 0.$$

Thus, labor rigidity can explain in equilibrium the recent empirical findings by van Binsbergen et al. (2012), given standard preferences.

The equilibrium price of the market asset is given by

$$P_t = \mathbb{E}_t\left[\int_t^\infty \xi_{t,u} D_u du\right] = X_t \frac{\alpha}{\beta} e^{(u_0 + u_\mu \mu_t + (u_z + \chi(1+\phi))z_t)/\chi} \tag{32}$$

where u_0, u_μ and u_z are derived in the Appendix. The instantaneous volatility and premium on the market asset are given by

$$\sigma_P(t) = \sqrt{\sigma_x^2 + \left(\frac{1-1/\psi}{e^{cq}+\lambda_\mu}\right)^2 \sigma_\mu^2 + \left(1 - \frac{(1-1/\psi)\lambda_z}{e^{cq}+\lambda_z}\right)^2 (1+\phi)^2\sigma_z^2}, \tag{33}$$

$$(\mu_P - r)(t) = \gamma\sigma_x^2 + \frac{(1-1/\psi)(\gamma-1/\psi)}{(e^{cq}+\lambda_\mu)^2}\sigma_\mu^2 + \frac{(\gamma\psi e^{cq}+\lambda_z)(\psi e^{cq}+\lambda_z)}{(e^{cq}+\lambda_z)^2\psi^2}(1+\phi)^2\sigma_z^2. \tag{34}$$

The market price is the time integral of the dividend strip price over the infinite horizon: $P_t = \int_0^\infty P_{t,\tau} d\tau$. Therefore, the market price-dividend ratio is a stationary function of μ_t and z_t. Namely, prices increase with μ_t as long as the intertemporal substitution effect dominates the wealth effect and increase with z_t for any preference setting: $\partial_\mu P_t \gtreqless 0$ if $\psi \gtreqless 1, \forall\gamma$ and $\partial_z P_t > 0,\ \forall\psi,\gamma$.

Return volatility has three components due to the three shocks of the model. The permanent shock x_t does not lead to excess volatility. Instead, the price loadings on $B_{\mu,t}$ and $B_{z,t}$ depend on the preference parameters and are respectively increasing and decreasing in ψ. Interestingly, labor rigidity contributes to excess volatility since $\partial_z \log P_t$ increases in magnitude with ϕ.

Also the equity premium is given by three components due to the three shocks of the model. The permanent shock x_t leads to the usual positive premium $\gamma\sigma_x^2$. Long-run growth requires a compensation, which is positive if shareholders have preference for the early resolution of uncertainty and the intertemporal substitution effect dominates the wealth effect. Instead, z_t commands a premium which is always positive, increasing with γ and decreasing with ψ. Moreover, such a compensation term increases with the degree of labor rigidity ϕ.

4 Discussion

4.1 Model Calibration and Inflation Risk

The model calibration consists of three steps. First, I set the cash-flows parameters of real quantities by minimizing some moment conditions from the real growth rates of consumption, wages and dividends –including their variance-ratios. Second, given the parameters associated to the dynamics of z_t, I set the residual parameters that govern the price index by minimizing some moment conditions about inflation risk. Third, preference parameters are set to fit a number of asset pricing quantities.

Consumption, wages and dividends

The calibration procedure uses information from the term-structures of variance-ratios of cash-flows. Namely, I exploit analytical solutions to set the cash-flows parameters. The model dynamics for C, W and D consist of eight parameters $\Theta = \{\bar{\mu}, \sigma_x, \lambda_\mu, \sigma_\mu, \lambda_z, \sigma_z, \alpha, \phi\}$. Thus, I choose eight moment conditions: the relative error between the empirical and the model long-run growth of consumption (g_C), yearly volatility of consumption ($\sigma_C(1)$) and dividends ($\sigma_D(1)$) growth rates, average level of the dividend-share $(1-\omega)$ and its volatility ($\sigma_{1-\omega}$):

$$
\begin{aligned}
m_1(\theta) &= \tfrac{|g_C^{\text{empirical}} - g_C|}{g_C^{\text{empirical}}}, \\
m_2(\theta) &= \tfrac{|\sigma_C(1)^{\text{empirical}} - \sigma_C(1)|}{\sigma_C(1)^{\text{empirical}}}, \\
m_3(\theta) &= \tfrac{|\sigma_D(1)^{\text{empirical}} - \sigma_D(1)|}{\sigma_D(1)^{\text{empirical}}}, \\
m_4(\theta) &= \tfrac{|(1-\omega)^{\text{empirical}} - (1-\omega)|}{(1-\omega)^{\text{empirical}}}, \\
m_5(\theta) &= \tfrac{|\sigma_{1-\omega}^{\text{empirical}} - \sigma_{1-\omega}|}{\sigma_{1-\omega}^{\text{empirical}}},
\end{aligned}
$$

and three additional conditions that capture the relative error between the empirical and the model term-structures of variance ratios of consumption, wages and dividends over a ten years horizon:

$$
\begin{aligned}
m_6(\theta) &= \sum_{\tau=2}^{10} \tfrac{|VR_C^{\text{empirical}}(\tau) - VR_C(\tau)|}{VR_C^{\text{empirical}}(\tau)}, \\
m_7(\theta) &= \sum_{\tau=2}^{10} \tfrac{|VR_W^{\text{empirical}}(\tau) - VR_W(\tau)|}{VR_W^{\text{empirical}}(\tau)}, \\
m_8(\theta) &= \sum_{\tau=2}^{10} \tfrac{|VR_D^{\text{empirical}}(\tau) - VR_D(\tau)|}{VR_D^{\text{empirical}}(\tau)}.
\end{aligned}
$$

The variance-ratios of growth rates are computed as $VR_i(\tau) = \frac{\sigma_i^2(\tau)}{\sigma_i^2(1)}$ for $i = \{C, W, D\}$. The latter three moment conditions capture the timing of macroeconomic risk and, in particu-

lar, the *term-structure effect* of labor rigidity. Finally, I obtain the parameter vector Θ by minimizing the average-relative-error:

$$\Theta = \arg\min_{\theta} ARE(\Theta) = \arg\min_{\theta} \tfrac{1}{8} \sum\nolimits_{i=1}^{8} m_i(\theta).$$

The empirical moments are as follows: I set the long-run growth rate of consumption to 2% and the volatility of consumption to 2.5%, which are the usual values from the literature; the volatility of dividends is set to 15%, which is the value reported in Belo et al. (2015); the average value of the dividend-share is set to 4.8% and its volatility to 1.8%, which are computed using the ratio of dividends over the sum of dividends and wages (as in the model). These numbers are close to the values considered in Longstaff and Piazzesi (2004), Lettau and Ludvigson (2005) and Santos and Veronesi (2006). The variance-ratios of wages and dividends are computed as in Section 2: wage risk increases from one to about 1.5 over a 10 years horizon, whereas dividend risk decreases from one to about 0.35. Finally, consumption variance-ratios are computed from the growth rates in Beeler and Campbell (2012) and increase from one to about 1.5 over a 10 years horizon.

Table 4 reports the model parameters and Table 5 reports both the empirical and the model-implied moments of cash-flows. The left panel of Figure 3 shows the model implied

Table 4: Calibration of Real Quantities – Model Parameters

	Symbol	Value
volatility of permanent shock	σ_x	.009
steady-state long-run growth	$\bar{\mu}$	.020
reversion of long-run growth	λ_μ	.617
volatility of long-run growth	σ_μ	.022
reversion of transitory shock	λ_z	.189
volatility of transitory shock	σ_z	.033
steady-state dividend-share	α	.041
labor rigidity	ϕ	3.92

term-structures of variance-ratios for both aggregate consumption, wages and dividends, as well as their empirical counterparts. The model accurately captures both rise and decline of respectively wage and dividend risk with the horizon. Therefore, the calibration procedure does a good job at recovering the whole shape of the empirical term-structures and, hence, the timing of consumption, wages and dividend risk. The size of such risks is shown in the right panel of Figure 3, which plots the term-structures of the corresponding volatilities. The decline in the timing of dividend risk is due to the levered exposition of dividends to the transitory component of consumption, z_t. Namely, the operating leverage parameter $\phi = 3.92$ allows for both i) the correct slope of the term-structure of dividends volatility, and ii) the excess volatility of dividends over consumption in yearly growth rates (i.e. 15% versus 3%).

Table 5: Calibration of Real Quantities – Empirical and Model Moments

	Symbol	Data	Model
long-run growth	g_C	.020	.020
one year consumption volatility	$\sigma_C(1)$	.025	.033
one year dividends volatility	$\sigma_D(1)$	.150	.150
dividend-share average (D/(D+W))	$1-\omega$	.048	.041
dividend-share volatility	$\sigma_{1-\omega}$	.018	.009

Variance ratios of consumption $VR_C(\tau)$

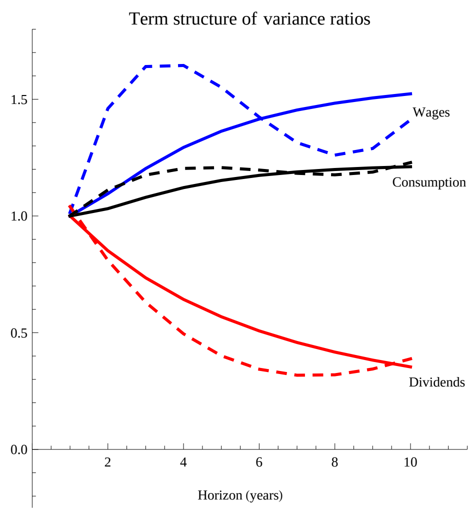
τ	2	3	4	5	6	7	8	9	10
Data	1.11	1.17	1.20	1.20	1.20	1.19	1.19	1.20	1.21
Model	1.03	1.08	1.12	1.15	1.17	1.19	1.20	1.21	1.21

Variance ratios of wages $VR_W(\tau)$

τ	2	3	4	5	6	7	8	9	10
Data	1.50	1.67	1.63	1.52	1.39	1.31	1.30	1.33	1.37
Model	1.09	1.20	1.29	1.36	1.41	1.45	1.48	1.50	1.52

Variance ratios of dividends $VR_D(\tau)$

τ	2	3	4	5	6	7	8	9	10
Data	0.85	0.66	0.49	0.37	0.31	0.31	0.35	0.39	0.35
Model	0.85	0.73	0.64	0.57	0.51	0.46	0.417	0.38	0.35



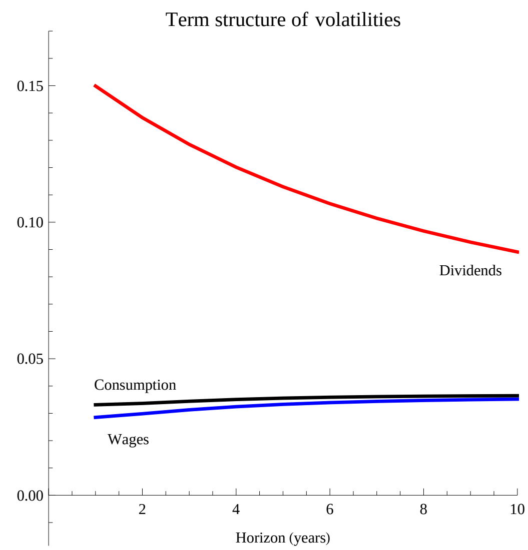


Figure 3: Term-structures of real consumption, wages and dividends

Left panel: Variance-ratios of wages (blue), consumption (black) and dividends (red) as a function of the horizon. Dashed lines denote empirical data. Right panel: Volatility of wages (blue), consumption (black) and dividends (red) as a function of the horizon. Cash-flows parameters are from Table 4.

At the same time, the model leads to smooth wage growth rates (i.e. 2%).[8]

[8]Note that the model dynamics are too simple and parsimonious to account for the double hump in the term-structure of wage VR's. However, such a characteristic (which disappears in post-war data) is not relevant to asset pricing. Indeed, the term-structure effect of labor rigidity leads to essentially monotone downward-sloping dividend risk. The latter is relevant to asset pricing and is almost perfectly captured by the model. Indeed, both empirical and model-implied long horizon VR's of dividends are about three times smaller than long-horizon VR's of wages.

Inflation risk

Given the parameters concerning the dynamics of z_t, the dynamics for inflation are characterized by four additional parameters: $\{\bar{i}, \lambda_i, \sigma_i\}$, which govern the permanent component, and the coefficient φ, which governs the effect of z_t and, thus, captures the link with the macroeconomic fundamentals. These parameters are set by minimizing the relative error about the yearly mean and volatility of inflation as well as the term-structure of variance-ratios:

$$\begin{aligned} m_1(\theta) &= \tfrac{|g_\Pi^{\text{empirical}} - g_\Pi|}{g_\Pi^{\text{empirical}}}, \\ m_2(\theta) &= \tfrac{|\sigma_\Pi(1)^{\text{empirical}} - \sigma_\Pi(1)|}{\sigma_\Pi(1)^{\text{empirical}}}, \\ m_3(\theta) &= \sum_{\tau=2}^{10} \tfrac{|VR_\Pi^{\text{empirical}}(\tau) - VR_\Pi(\tau)|}{VR_\Pi^{\text{empirical}}(\tau)}. \end{aligned}$$

The empirical moments are from Section 2: yearly inflation is 3.05% with 4.00% volatility; the variance-ratios are markedly increasing and range from one to 3.15 over a 10 years horizon.

The model produces an almost perfect fit of both the first two moments of inflation as well as its variance-ratios. Table 6 reports the model parameters and Table 7 reports both the empirical and the model-implied moments. Figure 4 shows the model implied variance-ratios

Table 6: Calibration of Inflation – Model Parameters

	Symbol	Value
steady-state long-run inflation	$\bar{i}$	.029
reversion of permanent shock	λ_i	.497
volatility of permanent shock	σ_i	.040
coefficient transitory shock	φ	1.15

for inflation and their empirical counterparts. Consistently with the data, the calibration implies that the upward-sloping effect of the permanent component of inflation, i_t, dominates the downward sloping effect of the transitory component, φz_t.

Table 7: Calibration of Inflation – Empirical and Model Moments

	Symbol	Data	Model
long-run inflation	g_Π	.030	.030
one year inflation volatility	$\sigma_\Pi(1)$	.040	.040

Variance ratios of inflation $VR_\Pi(\tau)$

τ	2	3	4	5	6	7	8	9	10
Data	1.55	1.84	2.01	2.16	2.34	2.58	2.84	3.05	3.12
Model	1.33	1.69	2.03	2.31	2.54	2.73	2.88	3.01	3.12

Section 2 documents a few additional stylized facts. While the permanent component of inflation risk generates upward-sloping variance ratios, inflation is correlated with fundamen-

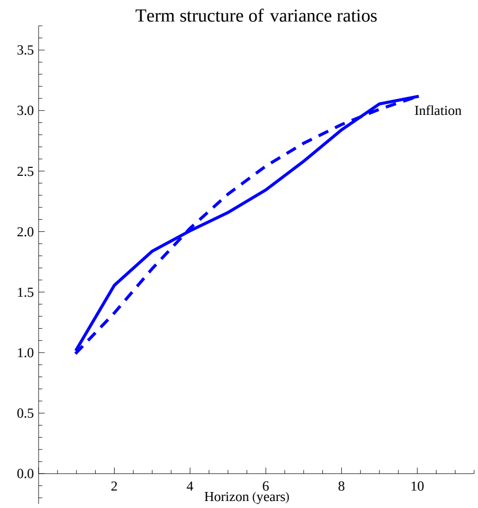


Figure 4: Term-structure of inflation risk

Variance-ratios of inflation as a function of the horizon. Dashed line denotes empirical data. Cash-flows parameters are from Table 4 and 6.

tals only through its short-run component. Moreover, such a short-run component is behind the intertemporal relationship between labor-share variation and future inflation.

The model captures these stylized facts. The positive coefficient φ recovers the positive relationship between inflation and real growth due to transitory risk z_t. The model-implied expected growth for consumption, wages and dividends are plotted as a function of expected inflation for several horizons in Figure 5. Moreover, the model calibration recovers a positive

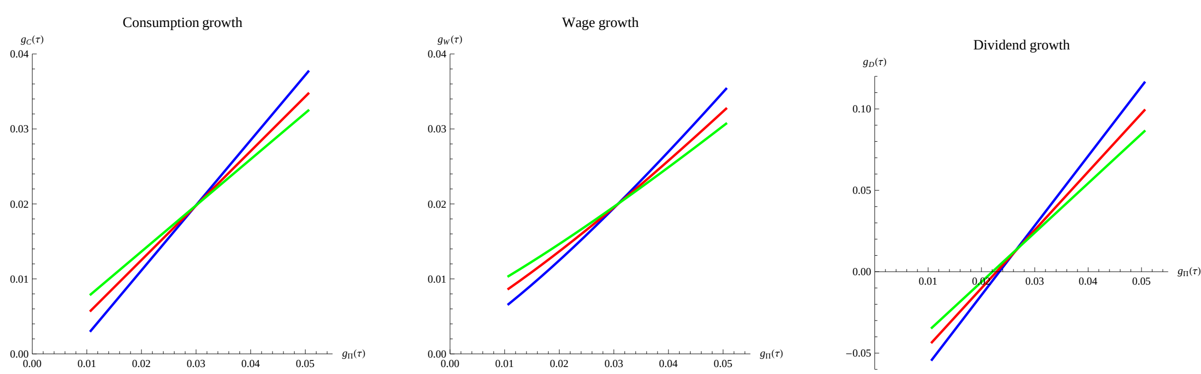


Figure 5: Inflation and real growth

The right, middle and left panels show the expected consumption, wages and dividend growth for several horizons as a function of one year expected inflation. Blue, red and green lines denote respectively 1, 3 and 5 years horizon. Cash-flows parameters are from Table 4 and 6.

relationship between the current level of the labor share and expected inflation at several horizons, as shown in Figure 6. The results of Figure 4, 5 and 6 are consistent with the empirical findings of Section 2 and point out the importance of modelling both permanent

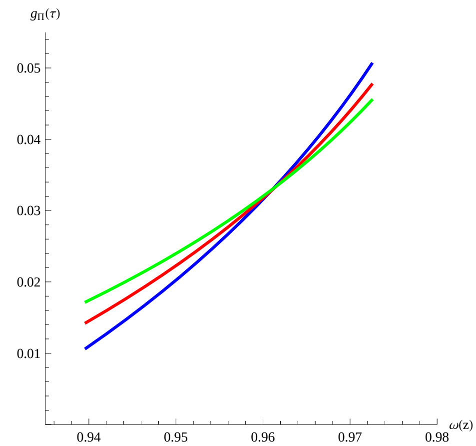


Figure 6: Inflation risk and labor rigidity

The figure shows expected inflation for several horizons as a function of the current labor-share. Blue, red and green lines denote respectively 1, 3 and 5 years horizon. Cash-flows parameters are from Table 4 and 6.

and transitory components of macroeconomic variables as well as their interaction.

Preferences

A number of preference settings are investigated in the usual range of values considered in the literature. I focus on the case in which the intertemporal substitution effect dominates the wealth effect ($\psi > 1$) and the investor has preference for the early resolution of uncertainty ($\gamma > 1/\psi$). Indeed, in such a case the model produces increasing term-structures for both real and nominal interest rates and, simultaneously, decreasing term-structures for both equity volatility and premia.

The baseline calibration considers a moderate elasticity of intertemporal substitution above unity ($\psi = 1.25$) with time-discount rate ($\beta = 3.5\%$) and realtive risk aversion ($\gamma = 10$) set to generate a low risk-free rate and high equity premium.

4.2 Real and Nominal Bond Yields

The baseline calibration leads to real and nominal bond yields that are increasing with the maturity. The left panel of Figure 7 shows the term-structures of real and nominal bond yields. The right panel shows the term-spreads: $y(t,\tau) - y(t,0)$ and $y^\star(t,\tau) - y^\star(t,0)$. Consistently with the actual data both the term-structure are upward-sloping. This result obtains because labor rigidity enhances the equilibrium price of short-run risk. In turn, the demand for short maturity bonds, as hedge instruments, increases and the yields at short maturities decreases. This leads to positive and increasing term-spreads.

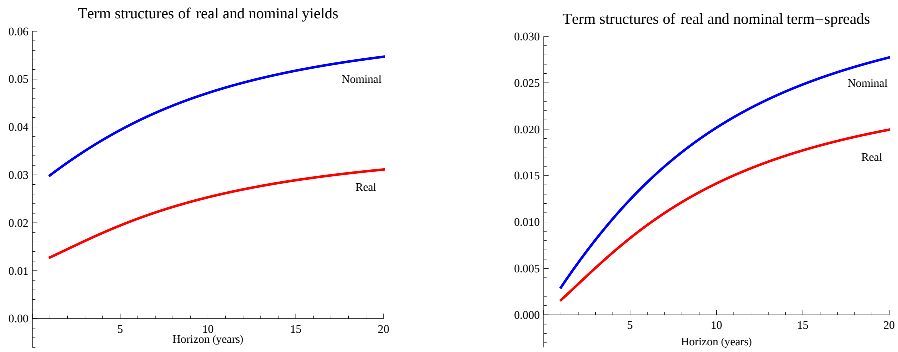


Figure 7: Term-structures real and nominal interest rates

Left and right panels show the term structures of respectively real (red) and nominal (blue) yields and real (red) and nominal (blue) term-spreads. Cash-flows parameters are from Table 4 and 6; preference parameters are: $\psi = 1.25, \gamma = 10$ and $\beta = 3.5\%$.

The upper, middle and lower panels of Figure 8 show the effect of a shift respectively in the elasticity of intertemporal substitution (ψ), in labor rigidity (ϕ) and in the loading on transitory risk in inflation (φ). The smaller the elasticity of intertemporal substitution, the lower the level of short-maturity yields and the steeper the positive slope of the yield curve. Most of risk in the economy is transitory and its equilibrium price is decreasing with ψ. Thus, the lower ψ, the stronger the hedging property of real bonds and, in turn, the lower the short-maturity yields. Notice that the opposite mechanism obtains in long-run risk models in which upward-sloping real yields are at odds with a high equity compensation.

The stronger labor rigidity, the lower the level of short-maturity yields and the steeper the positive slope of the yield curve. Since labor rigidity shifts dividend risk toward the short horizon, the larger ϕ, the stronger the hedging property of real bonds and, in turn, the lower the short-maturity yields. The economic mechanism is inherited by nominal bonds. While many asset pricing models imply negative and decreasing real yields, labor rigidity helps to reconcile equilibrium predictions with the actual data.

The more inflation loads on transitory risk, the lower level of short-maturity yields and the steeper the positive slope of the yield curve. Since the nominal state-price density is decreasing in the price index, the larger φ, the stronger the hedging property of nominal bonds and, in turn, the lower the short-maturity yields.

Figure 9 shows the state-dependent behavior of interest rates. Namely real and nominal yields are plotted as a function of the maturity and the transitory shock z_t. Consistent with the findings of Buraschi and Jiltsov (2007), the slope of the term structure of real yields is counter-cyclical in the model. After a negative transitory shock, consumption growth is expected to rise implying a small demand for short-maturity bonds. Thus, short-maturity bonds are cheap and their yields are high. The opposite holds after a positive transitory shock. Long-maturity bonds are instead less sensitive to z_t, since transitory risk does not

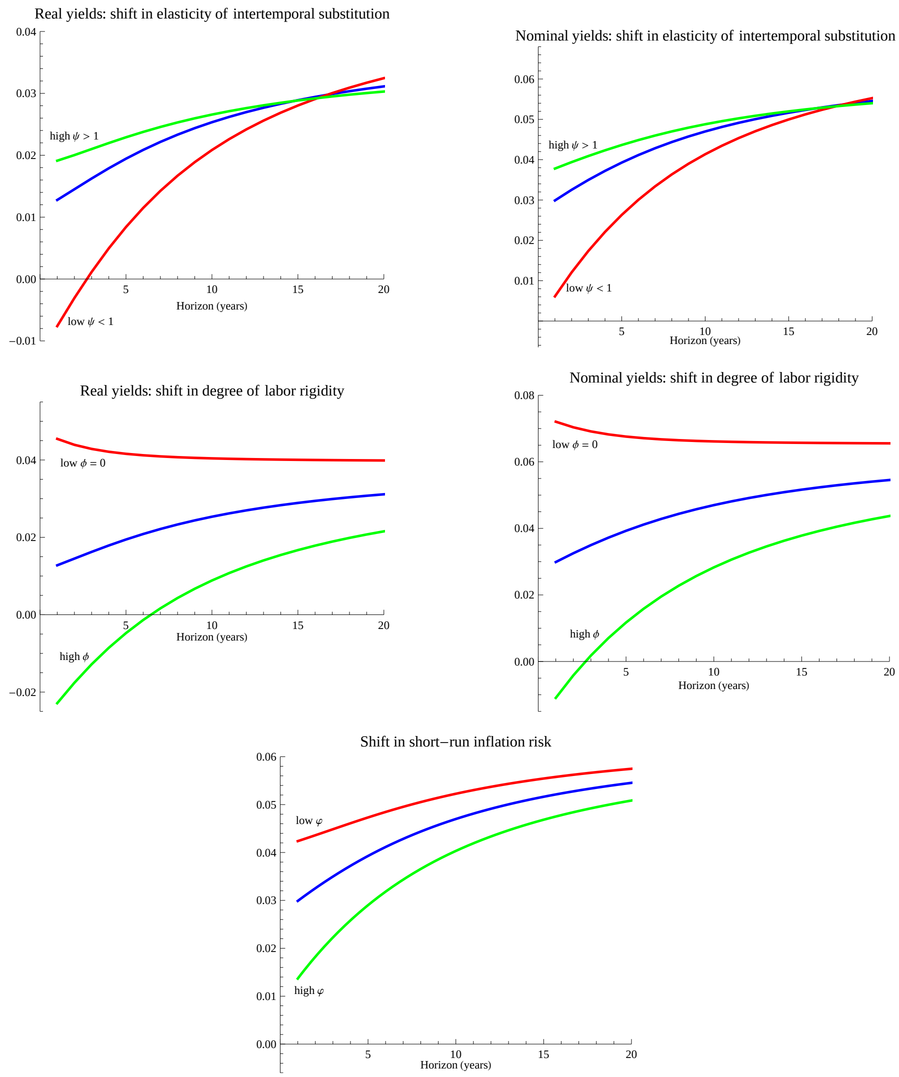


Figure 8: Real and nominal interest rates: comparative static

Upper panels: term-structures of real (left) and nominal (right) yields for several values of the elasticity of intertemporal substitution. Middle panels: term-structures of real (left) and nominal (right) yields for several values of the degree of labor rigidity. Lower panel: term-structure of nominal yields for several values of the loading on transitory risk in inflation. Cash-flows parameters are from Table 4 and 6; preference parameters are: $\psi = 1.25, \gamma = 10$ and $\beta = 3.5\%$.

affect long-horizon consumption growth. Thus, the long-maturity yields are about flat. This mechanism is enhanced by the degree of labor rigidity, since it induces a large equilibrium price for transitory risk. Nominal yields inherit the cyclical properties of real yields apart at the very short maturity. Short-maturity nominal yields are indeed affected by the fact that the nominal state-price density is a negative function of the price index. The latter is increasing with z_t for $\varphi > 0$ (as suggested by actual data). In turn, short-maturity nominal yields move positively with the transitory shock. Consequently, for $\varphi > 0$ the correlation between real and nominal yields increases with the maturity, which is also the case in the actual data.

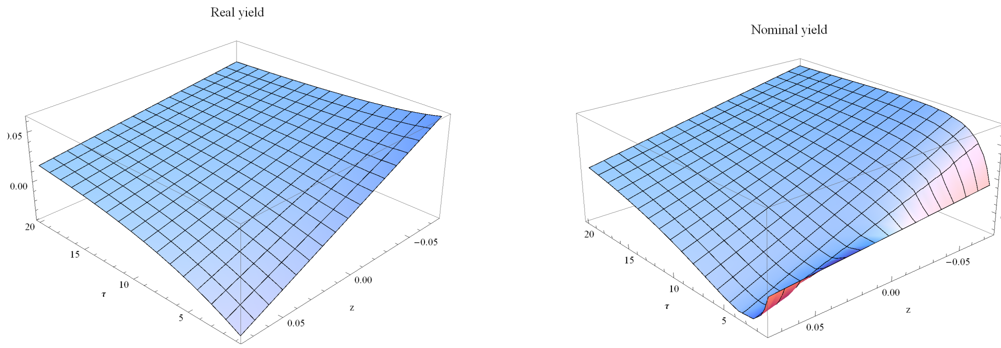


Figure 9: Real and nominal interest rates and transitory risk

Left and right panels show the term structures of respectively real and nominal yields as a function of the maturity and the transitory shock. Cash-flows parameters are from Table 4 and 6; preference parameters are: $\psi = 1.25, \gamma = 10$ and $\beta = 3.5\%$.

The model has also predictions about the volatility of real and nominal yields. The term-structures of yield volatilities are shown in Figure 10. Consistently with the actual data, these

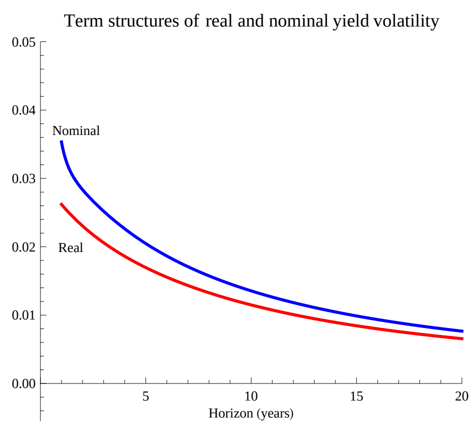


Figure 10: Term-structures of real and nominal yield volatility

The figure shows term-structures of real (red) and nominal (blue) yield volatility. Cash-flows parameters are from Table 4 and 6; preference parameters are: $\psi = 1.25, \gamma = 10$ and $\beta = 3.5\%$.

term-structures are downward-sloping and the volatility of the nominal yield is slightly larger than the volatility of the real yield.

Section 3.2 shows that the component of the nominal term-spread due to inflation risk (see Eq. (22)) can be either positive or negative. Namely, the transitory component of inflation contributes positively (for $\varphi > 0$) and the permanent component of inflation contributes negatively. Moreover, the former effect increases with the degree of labor rigidity. Figure 11 shows that the nominal term-spread due to inflation risk –computed as

$$\big(\underset{\sim}{y}^\star(t,\tau) - \underset{\sim}{y}^\star(t,0)\big) - \big(\underset{\sim}{y}(t,\tau) - \underset{\sim}{y}(t,0)\big)$$

for each maturity τ– is positive and upward-sloping in the baseline calibration. Notice that this model prediction is quite peculiar to the present framework. Indeed, it depends on the relative importance of transitory and permanent risks in inflation and their connection with transitory and permanent risks in real growth and the way these risks are priced in equilibrium. Namely, a positive and increasing nominal term-spread due to inflation risk obtains for φ and ϕ positive and large enough. This result is in line with the actual data and further supports

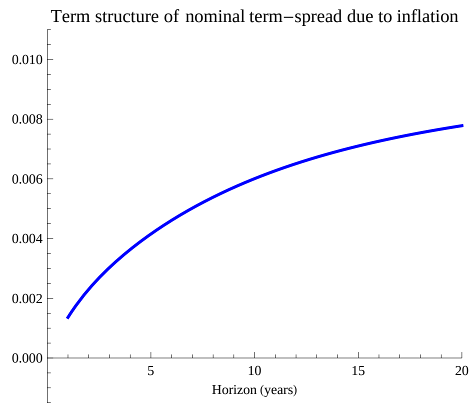


Figure 11: Term-spread due to inflation risk

The figure shows the term-structure of the term-spread due to inflation risk. Cash-flows parameters are from Table 4 and 6; preference parameters are: $\psi = 1.25, \gamma = 10$ and $\beta = 3.5\%$.

the main model mechanism.

Table 8 shows the real and nominal treasury yields from actual data. The sample is 2003-2015, which is the only time sample for which treasury real yields are available. Maturity ranges from 5 to 20 years.[9] We observe some stylized facts: *(i)* both real and nominal yields are positive and increasing; *(ii)* nominal yields are higher than real yields; *(iii)* both real and nominal yield volatilities are decreasing; *(iv)* nominal yield volatility is higher than real yield volatility; *(v)* the correlation between real and nominal yields increases with the maturity; *(vi)* both real and nominal term-spreads are increasing; *(vii)* the nominal term-spread is higher than the real term-spread; *(viii)* the term-spread due to inflation –i.e. the nominal term-spread minus the real one– is increasing.

The analysis of the model predictions shows that the baseline calibration captures all of the eight stylized facts listed above. Thus, the model reconciles many features of the equilibrium real and nominal bond returns with the macroeconomy: namely, the timing of risk for consumption, wages and dividends (mainly due to labor rigidity) as well as the connection between inflation risk and real quantities (mainly driven by the short-run component of inflation

[9]Since the time sample and the maturities available for TIPs are limited (www.treasury.gov), data in Table 8 is used to compare the qualitative model predictions with the empirical stylized facts. However, data in Table 8 is not intended to be a quantitative target of the model calibration. The latter is indeed based on the much longer sample 1929-2013 of data about consumption, wages, dividends and inflation.

Table 8: Empirical real and nominal yields

The table reports the average and standard deviation of real and nominal treasury yields with maturity of 5, 7, 10 and 20 years from daily data on the sample 2003-2015. The term-spreads are defined as the yields with maturity of 7, 10 and 20 years minus the yields with maturity of 5 years. The term-spread due to inflations is defined as the nominal term-spread minus the real term-spread.

Maturity (years)	5	7	10	20
real yield	.007	.010	.012	.015
real yield volatility	.011	.010	.009	.008
nominal yield	.026	.030	.034	.040
nominal yield volatility	.013	.011	.010	.009
real and nominal yield correlation	.889	.895	.918	.914
Maturity (years)		7-5	10-5	20-5
real term spread		.003	.006	.009
nominal term spread		.004	.008	.014
term spread due to inflation risk		.001	.002	.006

and real growth).

4.3 Dividend Strips and Market Asset

The aim of this section is to verify whether the baseline calibration leads to model predictions about equity which are consistent with the data.

Table 9 reports a number of model statistics about equity for several specifications of preference parameters. The baseline calibration ($\psi = 1.25, \gamma = 10, \beta = 3.5\%$) provides a good fit of the main traditional asset pricing patterns. The risk-free rate is about 1.1% with moderate volatility of 3.0%. The equity premium is sizeable and about 5.1%. Equity volatility is relatively high and about 13.6%. The levels of the Sharpe ratio and log price-dividend ratio are about 37.2% and 3.42, quite in line with the actual data. Overall, the model largely solves the risk-free rate and equity premium puzzle and, thus, preserves the result of Danthine and Donaldson (2002) about operating leverage due to labor rigidity. Notice that these results obtain in an economy without stochastic volatility and jumps.

Table 9 also shows how model statistics about equity change with the elasticity of intertemporal substitution and the relative risk aversion. Since labor rigidity leads to a high price for transitory risk, the equity compensation is decreasing with ψ. However, the lower the elasticity of intertemporal substitution, the higher the equity premium but the larger the risk-free rate volatility. Thus, the latter can be used to impose discipline on the setting of preference parameters. The baseline calibration ($\psi = 1.25, \gamma = 10, \beta = 3.5\%$) seems the

Table 9: Equity returns

Data									
Sample		r	σ_r	$\mu_P - r$	σ_P	SR	$\log P/D$	$\sigma_{\log P/D}$	
1931-2009		0.006	0.030	0.062	0.198	0.313	3.38	0.450	
1947-2009		0.010	0.027	0.063	0.176	0.358	3.47	0.429	
Baseline calibration									
ψ	γ	r	σ_r	$\mu_P - r$	σ_P	SR	$\log P/D$	$\sigma_{\log P/D}$	slope
1.25	10	0.011	0.030	0.051	0.136	0.372	3.42	0.137	< 0
Alternative preference settings									
ψ	γ	r	σ_r	$\mu_P - r$	σ_P	SR	$\log P/D$	$\sigma_{\log P/D}$	slope
0.75	10	-0.013	0.051	0.093	0.209	0.444	3.26	0.223	> 0
	7.5	0.001	0.051	0.079	0.209	0.382	3.24	0.223	> 0
	5	0.015	0.051	0.066	0.209	0.317	3.22	0.222	> 0
1.00	10	0.003	0.038	0.065	0.164	0.397	3.35	0.000	$= 0$
	7.5	0.013	0.038	0.054	0.164	0.332	3.35	0.000	$= 0$
	5	0.024	0.038	0.044	0.164	0.267	3.35	0.000	$= 0$
1.25	10	0.011	0.030	0.051	0.136	0.372	3.42	0.137	< 0
	7.5	0.019	0.030	0.041	0.136	0.305	3.43	0.138	< 0
	5	0.028	0.030	0.032	0.136	0.239	3.45	0.138	< 0
1.50	10	0.015	0.025	0.042	0.117	0.359	3.47	0.231	< 0
	7.5	0.022	0.025	0.034	0.117	0.289	3.49	0.232	< 0
	5	0.030	0.025	0.026	0.117	0.221	3.52	0.232	< 0

Empirical data are from Constantinides and Ghosh (2011). Slope denotes dividend strips risk premia.

preferable choice.

Since labor rigidity alters the timing of macroeconomic risk, it is important to verify whether the baseline calibration leads to term-structures of equity claim (i.e. dividend strips) volatility and risk premia consistent with the actual data. The left panel of Figure 12 shows these term-structures. Both equity volatility and risk premia are decreasing with the horizon. Negative slopes obtain for values of elasticity of intertemporal substitution larger than one and labor rigidity strong enough. Here, the operating leverage due to labor rigidity is calibrated to match the empirical gap between the upward-sloping risk of wages and the downward-sloping risk of dividends. This result is in line with the empirical findings of van Binsbergen et al. (2012), van Binsbergen et al. (2013) and van Binsbergen and Koijen (2015) and confirms the theoretical explanation of Marfè (2013b). The right panel of Figure 12 shows the term-structure of equity volatility and those of dividends and consumption growth volatility. The model leads to "long-horizon" excess volatility of equity over fundamentals in line with Beeler and Campbell (2012).

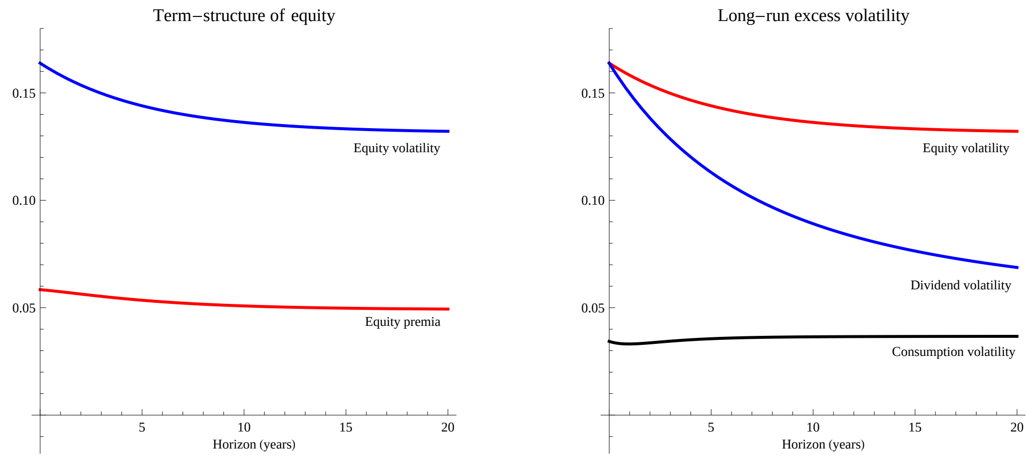


Figure 12: Term-structures of equity volatility and premia

The left panel shows the term-structures of equity claim (i.e. dividend strips) volatility and risk premia as a function of the horizon. The right panel shows the term-structures of equity claim (i.e. dividend strips) volatility, dividend growth volatility and consumption growth volatility as a function of the horizon. Cash-flows parameters are from Table 4 and 6; preference parameters are: $\psi = 1.25, \gamma = 10$ and $\beta = 3.5\%$.

5 Conclusion

This paper documents that the timing of inflation risk is upward-sloping and that the link between inflation risk and real growth is due to a small transitory shock.

A simple general equilibrium model with labor rigidity allows to rationalize the timing of risk of real macroeconomic variables, the timing of inflation risk as well as many patterns concerning the term-structures of real and nominal interest rates, the term-structures of dividend strips return volatility and risk-premia and some standard properties of equity.

Namely, future real growth is negatively related with expected inflation and, hence, leads to a positive risk premium on nominal bonds. The latter is sizeable because labor rigidity shifts dividend risk toward the short horizon and enhances the pricing of short-run risk. In turn, real and nominal bond yields are upward-sloping and dividend strips premia are downward-sloping. Thus, the model provides a macroeconomic foundation to the partial equilibrium setting by Lettau and Wachter (2007, 2011).

Appendix

For the easy of notation, throughout the appendix I define $d_0 \equiv \log \alpha$ and $d_z \equiv 1 + \phi$, such that $\log D_t = x_t + d_0 + d_z(z_t - \bar{z})$ (with $\bar{z} \equiv \mathbb{E}(z_t)$). Moreover, I denote with $C_{s,t}$ and $Q_{s,t}$ shareholders' consumption and wealth respectively. Recall that under limited market participation in equilibrium $C_{s,t} = D_t$ and $Q_{s,t} = P_t$.

Equilibrium State-Price Density. Under the infinite horizon, the utility process J satisfies the following Bellman equation: $\mathcal{D}J(X, \mu, z) + f(C_s, J) = 0$, where $\mathcal{D}$ denotes the differential operator.

Then we have

$$0 = J_X \mu X + \tfrac{1}{2} J_{X,X} \sigma_x^2 X^2 + J_\mu \lambda_\mu (\bar{\mu} - \mu) + \tfrac{1}{2} J_{\mu,\mu} \sigma_\mu^2 + J_z \lambda_z (\bar{z} - z) + \tfrac{1}{2} J_{z,z} \sigma_z^2 + f(C_s, J).$$

Guess a solution of the form $J(X, \mu, z) = \frac{1}{1-\gamma} X^{1-\gamma} g(\mu, z)$. The Bellman equation reduces to

$$0 = \mu - \tfrac{1}{2}\gamma\sigma_x^2 + \tfrac{g_\mu}{g} \tfrac{\lambda_\mu(\bar{\mu}-\mu)}{1-\gamma} + \tfrac{1}{2} \tfrac{g_{\mu,\mu}}{g} \tfrac{\sigma_\mu^2}{1-\gamma} + \tfrac{g_z}{g} \tfrac{\lambda_z(\bar{z}-z)}{1-\gamma} + \tfrac{1}{2} \tfrac{g_{z,z}}{g} \tfrac{\sigma_z^2}{1-\gamma} + \tfrac{\beta}{1-1/\psi} \left(g^{-1/\chi} e^{(1-1/\psi)(d_0+d_z z)} - 1 \right). \tag{A1}$$

Under limited market participation ($C_{s,t} = D_t$) and stochastic differential utility, the pricing kernel has dynamics given by

$$d\xi_{0,t} = \xi_{0,t} \tfrac{df_C}{f_C} + \xi_{0,t} f_J dt = -r(t)\xi_{0,t} - \theta_x(t)\xi_{0,t} dB_{x,t} - \theta_\mu(t)\xi_{0,t} dB_{\mu,t} - \theta_z(t)\xi_{0,t} dB_{z,t}, \tag{A2}$$

where, by use of Itô's Lemma and Eq. (A1), we get

$$r(t) = -\tfrac{\partial_X f_C}{f_C} \mu X - \tfrac{1}{2} \tfrac{\partial_{X,X} f_C}{f_C} \sigma_x^2 X^2 - \tfrac{\partial_\mu f_C}{f_C} \lambda_\mu (\bar{\mu} - \mu) - \tfrac{1}{2} \tfrac{\partial_{\mu,\mu} f_C}{f_C} \sigma_\mu^2 - \tfrac{\partial_z f_C}{f_C} \lambda_z (\bar{z} - z) - \tfrac{1}{2} \tfrac{\partial_{z,z} f_C}{f_C} \sigma_z^2 - f_J,$$

$$\theta_x(t) = -\tfrac{\partial_X f_C}{f_C} \sigma_x X, \qquad \theta_\mu(t) = -\tfrac{\partial_\mu f_C}{f_C} \sigma_\mu, \qquad \theta_z(t) = -\tfrac{\partial_z f_C}{f_C} \sigma_z,$$

An exact solution for $g(\mu, z)$ satisfying Eq. (A1) does not exist for $\psi \neq 1$. Therefore, I look for a solution of $g(\mu, z)$ around the unconditional mean of the consumption-wealth ratio. Aggregate wealth is given by

$$Q_{s,t} = \mathbb{E}_t \left[\int_t^\infty \xi_{t,u} C_{s,u} du \right],$$

and, applying Fubini's Theorem and taking standard limits, the consumption-wealth ratio satisfies

$$\tfrac{C_{s,t}}{Q_{s,t}} = r(t) - \tfrac{1}{dt} \mathbb{E}_t \left[\tfrac{dQ}{Q} \right] - \tfrac{1}{dt} \mathbb{E}_t \left[\tfrac{d\xi}{\xi} \tfrac{dQ}{Q} \right]. \tag{A3}$$

Guess

$$Q_{s,t} = C_{s,t} \beta^{-1} \left(g(\mu_t, z_t) e^{(\gamma-1)(d_0 + d_z z_t)} \right)^{1/\chi}$$

and apply Itô's Lemma to get $\frac{dQ}{Q}$. Then, plug in the wealth dynamics, the risk-free rate and the pricing kernel into Eq. (A3): after tedious calculus you can recognize that the guess solution is correct. Notice that the consumption-wealth ratio approaches to β when $\psi \to 1$ as usual.

Denote $cq = \mathbb{E}[\log C_{s,t} - \log Q_{s,t}]$, hence, a first-order approximation of the consumption-wealth ratio around cq produces

$$\tfrac{C_{s,t}}{Q_{s,t}} = \beta g(\mu_t, z_t)^{-1/\chi} e^{(1-1/\psi)(d_0 + d_z z_t)} \approx e^{cq} \left(1 - cq + \log\beta - \tfrac{1}{\chi} \big(\log g(\mu_t, z_t) + (\gamma - 1)(d_0 + d_z z_t) \big) \right).$$

Using such approximation in the Bellman equation (A1) leads to

$$\begin{aligned} 0 = \mu - \tfrac{1}{2}\gamma\sigma_x^2 + \tfrac{g_\mu}{g} \tfrac{\lambda_\mu(\bar{\mu}-\mu)}{1-\gamma} + \tfrac{1}{2} \tfrac{g_{\mu,\mu}}{g} \tfrac{\sigma_\mu^2}{1-\gamma} + \tfrac{g_z}{g} \tfrac{\lambda_z(\bar{z}-z)}{1-\gamma} + \tfrac{1}{2} \tfrac{g_{z,z}}{g} \tfrac{\sigma_z^2}{1-\gamma} \\ + \tfrac{1}{1-1/\psi} \left(e^{cq} \left(1 - cq + \log\beta - \tfrac{1}{\chi} \log g(\mu, z) + (1 - 1/\psi)(d_0 + d_z z) \right) - \beta \right), \end{aligned}$$

which has exponentially affine solution $g(\mu, z) = e^{u_0 + (1-\gamma)d_0 + u_\mu \mu + (u_z + (1-\gamma)d_z)z}$, where u_0, u_μ and u_z have explicit solutions and the endogenous constant cq satisfies $cq = \log\beta - \chi^{-1}(u_0 + u_\mu \bar{\mu} + u_z \bar{z})$

(recall $\bar{z} = \mathbb{E}[z_t] = 0$). The risk-free rate and the prices of risk take the form:

$$\begin{aligned}
r_0 = &\frac{1}{2}\Bigg(\frac{2\big(\beta\psi^2(\gamma-1)^2+e^{cq}((1-\psi)u_0+\psi(cq-1-\log(\beta))(\gamma-1))(\psi\gamma-1)\big)}{\psi(1-\psi)(\gamma-1)} + \frac{2\bar{z}(u_z-\psi(u_z-d_z(\gamma-1))\gamma)\lambda_z}{\psi(-1+\gamma)} \\
&- \frac{2\bar{\mu}u_\mu(\psi\gamma-1)\lambda_\mu}{\psi(\gamma-1)} - \gamma\,(1+\gamma)\,\sigma_x^2 - \frac{(u_z-\psi(u_z-d_z(\gamma-1))\gamma)^2\sigma_z^2}{\psi^2(\gamma-1)^2} - \frac{u_\mu^2(\psi\gamma-1)^2\sigma_\mu^2}{(\psi-\psi\gamma)^2}\Bigg), \\
r_\mu = &\frac{\psi(\gamma-1)\gamma+u_\mu(\psi\gamma-1)(e^{cq}+\lambda_\mu)}{\psi(\gamma-1)}, \\
r_z = &\frac{-\psi d_z(\gamma-1)\gamma\lambda_z+u_z(\psi\gamma-1)(e^{cq}+\lambda_z)}{\psi(\gamma-1)},
\end{aligned}$$

$$\theta_x(t) = \gamma\sigma_x, \qquad \theta_\mu(t) = \frac{u_\mu\left(\gamma-\frac{1}{\psi}\right)}{1-\gamma}\sigma_\mu, \qquad \theta_z(t) = \left(d_z\gamma + \frac{u_z(1-\psi\gamma)}{\psi(\gamma-1)}\right)\sigma_z,$$

and the results in the text easily follow.

Proposition A. *The following conditional expectation has exponential affine solution:*

$$\mathscr{M}_{t,\tau}(\bar{c}) = \mathbb{E}_t[e^{c_0+c_1\log\xi_{0,t+\tau}+c_2x_{t+\tau}+c_3\mu_{t+\tau}+c_4z_{t+\tau}}] = \xi_{0,t}^{c_1}X_t^{c_2}e^{\ell_0(\tau,\bar{c})+\ell_\mu(\tau,\bar{c})\mu_t+\ell_z(\tau,\bar{c})z_t}, \tag{A4}$$

where $\bar{c} = (c_0, c_1, c_2, c_3, c_4)$, model parameters are such that the expectation exists finite and ℓ_0, ℓ_μ and ℓ_z are deterministic functions of time.

Proof of Proposition A: Consider the following conditional expectation:

$$\mathscr{M}_{t,\tau}(\bar{c}) = \mathbb{E}_t[e^{c_0+c_1\log\xi_{0,t+\tau}+c_2\log X_{t+\tau}+c_3\mu_{t+\tau}+c_4z_{t+\tau}}] \tag{A5}$$

where $\bar{c} = (c_0, c_1, c_2, c_3, c_4)$ is a coefficient vector such that the expectation exists. Guess an exponential affine solution of the kind:

$$\mathscr{M}_{t,\tau}(\bar{c}) = e^{c_1\log\xi_{0,t}+c_2\log X_t+\ell_0(\tau,\bar{c})+\ell_\mu(\tau,\bar{c})\mu_t+\ell_z(\tau,\bar{c})z_t}, \tag{A6}$$

where $\ell_0(\tau,\bar{c}), \ell_\mu(\tau,\bar{c})$ and $\ell_z(\tau,\bar{c})$ are deterministic functions of time. Feynman-Kac gives that $\mathscr{M}$ has to meet the following partial differential equation

$$\begin{aligned}
0 = &\mathscr{M}_t - \mathscr{M}_\xi(r_0 + r_\mu\mu + r_z z) + \tfrac{1}{2}\mathscr{M}_{\xi,\xi}(\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2) + \mathscr{M}_X(\mu X) + \tfrac{1}{2}\mathscr{M}_{X,X}\sigma_x^2X^2 \\
&+ \mathscr{M}_\mu\lambda_\mu(\bar{\mu}-\mu) + \tfrac{1}{2}\mathscr{M}_{\mu,\mu}\sigma_\mu^2 + \mathscr{M}_z\lambda_z(\bar{z}-z) + \tfrac{1}{2}\mathscr{M}_{z,z}\sigma_z^2 - \mathscr{M}_{\xi,X}\theta_x(t)\sigma_xX \\
&- \mathscr{M}_{\xi,\mu}\theta_\mu(t)\sigma_\mu - \mathscr{M}_{\xi,z}\theta_z(t)\sigma_z,
\end{aligned}$$

where the arguments have been omitted for ease of notation. Plugging the resulting partial derivatives from the guess solution into the pde and simplifying gives a linear function of the states μ and z. Hence, we get three ordinary differential equations for $\ell_0(\tau,\bar{c}), \ell_\mu(\tau,\bar{c})$ and $\ell_z(\tau,\bar{c})$:

$$\begin{aligned}
0 = &\ell_0'(\tau,\bar{c}) - c_1r_0 + \tfrac{1}{2}c_1(c_1-1)(\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2) + \tfrac{1}{2}c_2(c_2-1)\sigma_x^2 + \ell_\mu(\tau,\bar{c})\lambda_\mu\bar{\mu} \\
&+ \tfrac{1}{2}\ell_\mu(\tau,\bar{c})^2\sigma_\mu^2 + \ell_z(\tau,\bar{c})\lambda_z\bar{z} + \tfrac{1}{2}\ell_z(\tau,\bar{c})^2\sigma_z^2 - c_1c_2\theta_x(t)\sigma_x - c_1\ell_\mu(\tau,\bar{c})\theta_\mu(t)\sigma_\mu \\
&- c_1\ell_z(\tau,\bar{c})\theta_z(t)\sigma_z \\
0 = &\ell_\mu'(\tau,\bar{c}) - c_1r_\mu + c_2 - \ell_\mu(\tau,\bar{c})\lambda_\mu \\
0 = &\ell_z'(\tau,\bar{c}) - c_1r_z - \ell_z(\tau,\bar{c})\lambda_z
\end{aligned}$$

with initial conditions: $\ell_0(0,\bar{c}) = c_0, \ell_\mu(0,\bar{c}) = c_3$ and $\ell_z(0,\bar{c}) = c_4$. Explicit solutions are avaliable.

□

Dividends. The conditional moment generating function $\mathbb{D}_t(\tau, n)$ is exponential affine and obtains as a special case of $\mathscr{M}_{t,\tau}(\bar{c})$ with $\bar{c} = (nd_0, 0, n, 0, nd_z)$. Therefore,

$$\sigma_D^2(t,\tau) = v_{D,\tau,x}\sigma_x^2 + v_{D,\tau,\mu}\sigma_\mu^2 + v_{D,\tau,z}\sigma_z^2,$$

where the coefficients are given by

$$v_{D,\tau,x} = 1, \quad v_{D,\tau,\mu} = \tfrac{4e^{-\lambda_\mu \tau} - e^{-2\lambda_\mu \tau} + 2\lambda_\mu \tau - 3}{2\lambda_\mu^3 \tau}, \quad v_{D,\tau,z} = \tfrac{e^{-\lambda_z \tau}\sinh(\lambda_z \tau) d_z^2}{\lambda_z \tau}.$$

The moment generating functions and the term-structures of volatility for wages, and total consumption are computed in a similar way.

Dividend Strips. The equilibrium price of the market dividend strip with maturity τ of Eq. (23) obtains as a special case of $\mathscr{M}_{t,\tau}(\bar{c})$ with $\bar{c} = (d_0, 1, 1, 0, d_z)$. Therefore, it is given by $P_{t,\tau} = \xi_{0,t}^{-1}\mathscr{M}_{t,\tau}(\bar{c}) = X_t e^{A_0(\tau) + A_\mu(\tau)\mu_t + A_z(\tau) z_t}$ with

$$\begin{aligned}
A_0(\tau) = \ell_0(\tau, \bar{c}) = \tfrac{1}{4}\Big(&-\tfrac{4e^{-\tau\lambda_\mu}\bar{\mu}(-1+r_\mu)(1+e^{\tau\lambda_\mu}(-1+\tau\lambda_\mu))}{\lambda_\mu} + \tfrac{e^{-2\tau(\lambda_z+\lambda_\mu)}}{\lambda_z^3\lambda_\mu^3} \\
&\times \Big(-e^{2\tau\lambda_\mu}\left(r_z + d_z\lambda_z\right)^2\lambda_\mu^3\sigma_z^2 + 4e^{\tau(\lambda_z+2\lambda_\mu)}\left(r_z + d_z\lambda_z\right)\lambda_\mu^3\left(-\bar{z}\lambda_z^2 + \sigma_z\left(\theta_z\lambda_z + r_z\sigma_z\right)\right) \\
&-e^{2\tau\lambda_z}\left(-1 + r_\mu\right)^2\lambda_z^3\sigma_\mu^2 + 4e^{\tau(2\lambda_z+\lambda_\mu)}\left(-1 + r_\mu\right)\lambda_z^3\sigma_\mu\left(\theta_\mu\lambda_\mu + \left(-1 + r_\mu\right)\sigma_\mu\right) \\
&+e^{2\tau(\lambda_z+\lambda_\mu)}\Big(\lambda_\mu^3\Big(4\lambda_z^2\left(\bar{z}\left(r_z + \left(d_z - \tau r_z\right)\lambda_z\right) + \lambda_z\left(d_0 - \tau\left(r_0 + \theta_x\sigma_x\right)\right)\right) - 4\theta_z\lambda_z\left(r_z + \left(d_z - \tau r_z\right)\lambda_z\right)\sigma_z \\
&+\left(d_z^2\lambda_z^2 - 2d_z r_z\lambda_z + r_z^2\left(2\tau\lambda_z - 3\right)\right)\sigma_z^2\Big) + 4\left(r_\mu - 1\right)\theta_\mu\lambda_z^3\lambda_\mu\left(\tau\lambda_\mu - 1\right)\sigma_\mu + \left(r_\mu - 1\right)^2\lambda_z^3\left(2\tau\lambda_\mu - 3\right)\sigma_\mu^2\Big)\Big)\Big), \\
A_\mu(\tau) = \ell_\mu(\tau, \bar{c}) = {} & \tfrac{1 + e^{-\tau\lambda_\mu}(-1+r_\mu) - r_\mu}{\lambda_\mu}, \\
A_z(\tau) = \ell_z(\tau, \bar{c}) = {} & \tfrac{-r_z + e^{-\tau\lambda_z}(r_z + d_z\lambda_z)}{\lambda_z},
\end{aligned}$$

and $A_0(0) = d_0$, $A_\mu(0) = 0$ and $A_z(0) = d_z$. Itô's Lemma gives the dynamics of the market dividend strip price:

$$dP_{t,\tau} = [\cdot]dt + \partial_x P_{t,\tau}\sigma_x dB_{x,t} + \partial_\mu P_{t,\tau}\sigma_\mu dB_{\mu,t} + \partial_z P_{t,\tau}\sigma_z dB_{z,t}.$$

Therefore the return volatility and premium are given by

$$\begin{aligned}
\sigma_P(t,\tau) &= P_{t,\tau}^{-1}\sqrt{(\partial_x P_{t,\tau}\sigma_x)^2 + (\partial_\mu P_{t,\tau}\sigma_\mu)^2 + (\partial_z P_{t,\tau}\sigma_z)^2} = \sqrt{\sigma_x^2 + (A_\mu(\tau)\sigma_\mu)^2 + (A_z(\tau)\sigma_z)^2}, \\
(\mu_P - r)(t,\tau) &= -\tfrac{1}{dt}\left\langle \tfrac{d\xi_{0,t}}{\xi_{0,t}}, \tfrac{dP_{t,\tau}}{P_{t,\tau}} \right\rangle = \theta_x(t)\sigma_x + \theta_\mu(t)A_\mu(\tau)\sigma_\mu + \theta_z(t)A_z(\tau)\sigma_z.
\end{aligned}$$

The slopes of the return volatility and premium for the market dividend strip obtain by standard calculus.

Market Asset and Equity Premium. Under the assumption of limited market participation, the shareholders act as a representative agent on the financial markets and, hence, the equilibrium price of the market asset is equal to the shareholders' wealth. Therefore, using the previous results, the market asset price can be written as

$$P_t = Q_{s,t} = C_{s,t}e^{-cq_t} = X_t e^{-\log\beta + u_0\chi^{-1} + d_0 + u_\mu\chi^{-1}\mu_t + (u_z\chi^{-1} + d_z)z_t}.$$

The dynamics of the market asset price obtains by applying Itô's Lemma to P_t:

$$dP_t = [\cdot]dt + \partial_x P_t\sigma_x dB_{x,t} + \partial_\mu P_t\sigma_\mu dB_{\mu,t} + \partial_z P_t\sigma_z dB_{z,t}.$$

Therefore the return volatility and premium are given by

$$\sigma_P(t) = P_t^{-1}\sqrt{(\partial_x P_t \sigma_x)^2 + (\partial_\mu P_t \sigma_\mu)^2 + (\partial_z P_t \sigma_z)^2} = \sqrt{\sigma_x^2 + (u_\mu \chi^{-1}\sigma_\mu)^2 + ((u_z\chi^{-1} + d_z)\sigma_z)^2},$$
$$(\mu_P - r)(t) = -\frac{1}{dt}\left\langle \frac{d\xi_{0,t}}{\xi_{0,t}}, \frac{dP_t}{P_t}\right\rangle = \theta_x(t)\sigma_x + \theta_\mu(t)u_\mu\chi^{-1}\sigma_\mu + \theta_z(t)\big(u_z\chi^{-1} + d_z\big)\sigma_z.$$

Inflation, Real Bond Yields and Nominal Bond Yields. The conditional moment generating function $\sqcap_t(\tau, n)$ is exponential affine. Namely,

$$\sqcap_t(\tau, n) = \mathbb{E}_t[I_{t+\tau}^n]\mathscr{M}_{t,\tau}(\bar{c})$$

where $\bar{c} = (0, 0, 0, 0, n\varphi)$ and

$$\mathbb{E}_t[I_{t+\tau}^n] = I_t e^{h_0(\tau,n)+h_i(\tau,n)i_t}$$

with

$$h_0(\tau, n) = -\frac{n\left(4\bar{i}\lambda_i^2\left(-\tau\lambda_i - e^{-\tau\lambda_i} + 1\right) + n\sigma_i^2\left(-2\tau\lambda_i - 4e^{-\tau\lambda_i} + e^{-2\tau\lambda_i} + 3\right)\right)}{4\lambda_i^3},$$
$$h_i(\tau, n) = \frac{n - ne^{-\tau\lambda_i}}{\lambda_i}.$$

The equilibrium price of the real bond with maturity τ obtains as a special case of $\mathscr{M}_{t,\tau}(\bar{c})$ with $\bar{c} = (0, 1, 0, 0, 0)$. Then, it is given by $\mathscr{B}_{t,\tau} = \xi_{0,t}^{-1}\mathscr{M}_{t,\tau}(\bar{c})$. Thus, $b_0(\tau) = \ell_0(\tau, \bar{c}), b_\mu(\tau) = \ell_\mu(\tau, \bar{c})$ and $b_z(\tau) = \ell_z(\tau, \bar{c})$.
The equilibrium price of the nominal bond with maturity τ is computed as

$$\mathscr{B}_{t,\tau}^\star = \mathbb{E}_t[\xi_{t,\tau}^\star] = \xi_{0,t}^{-1}\mathscr{M}_{t,\tau}(\bar{c})I_t\mathbb{E}_t[I_{t+\tau}^{-1}] = \xi_{0,t}^{-1}\mathscr{M}_{t,\tau}(\bar{c})e^{h_0(\tau,-1)+h_i(\tau,-1)i_t}$$

where $\bar{c} = (0, 1, 0, 0, -\varphi)$. Thus, $k_0(\tau) = \ell_0(\tau, \bar{c}) + h_0(\tau, -1), k_\mu(\tau) = \ell_\mu(\tau, \bar{c}), k_z(\tau) = \ell_z(\tau, \bar{c})$ and $k_i(\tau) = h_i(\tau, -1)$.

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