CoRisk: measuring systemic risk through default probability contagion*

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Abstract

We propose a novel systemic risk measurement model based on stochastic processes, correlation networks and conditional probabilities of default. For each country we consider three different economic sectors (sovereigns, corporates, banks) and we model each of them as a linear combination of two stochastic processes: a country-specific idiosyncratic component and a common systematic factor. Through correlation networks we derive conditional default probabilities, thus obtaining the CoRisk, which measures the variation in the probability of default due to contagion effects. Our model is applied to Eurozone countries, and the results show that the sovereign crisis has increased systemic risks more than the financial one: the two events together have caused a phase transition difficult to reverse, as risk propagation does not act as a mean for balancing inequalities across countries but, on the contrary, weakens the weakest and strengthens the strongest.

Keywords: stochastic processes, default probabilities, correlation networks, contagion effects.

*The authors thank Viral Acharya, Robert Engle and Peter Sarlin for useful comments and suggestions. We also thank participants to the 2015 Credit Risk Conference (Venice, 2015), International Dauphine-ESSEC-SMU Conference on Systemic Risk (Singapore, 2015), CFE conference (London, 2015), NYU Stern PhD Seminar Series (New York, 2016), IFABS Conference on Systemic Risk (Barcelona, 2016), IAAE Annual Conference (Milan, 2016), EFMA Annual Conference (Basel, 2016), Asian Meeting of the Econometric Society (Kyoto, 2016).

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1 Introduction

The financial crisis and, more recently, the sovereign crisis, have led to an increasing research literature on systemic risk, with different definitions and measurement models.

According to ECB (2009) ”Systemic risk is the risk of experiencing a strong systemic event, which adversely affects a number of systemically important intermediaries or markets”. This definition introduces two key elements for the study of systemic risk: as emphasised by Borio and Drehmann (2009), financial instability firstly involves the system as a whole, and not only individual institutions; secondly, it does not consider the financial system in isolation, but as ultimately linked to the real economy. While systemic risk definitions share this broad view and differ on implementation details, such as the involved agents, the kind of shocks or the analysed dynamics, measurement models are still quite divergent.

A first distinction between systemic risk models derives from the use of a cross-sectional rather than a time-dynamic perspective: while the former mostly concentrates on the relationships between agents operating in the market, the latter focuses on cause-and-effect relationships over time. As a consequence, we can distinguish between models centred on the notion of contagion, and models that aim at predicting what will happen in the nearby future, in an early-warning perspective. In addition, while contagion models identify transmission channels, thus embracing the whole system but only for descriptive purposes, time-dependent models associate a specific risk measure to individual institutions.

A second distinction originates in the identification of the risk sources, thus setting endogenous against exogenous causes, as well as idiosyncratic against systematic shocks.

A third diversity concerns the economic environment as the context in which systemic risk arises and propagates: most models concentrate either on the financial or the sovereign sector, while others include both of them.

In this work we will overcome these classifications by combining different approaches. First of all, for each country we will consider three different economic sectors: sovereigns, corporates and financials. Secondly, we will model each of them as a spread measure, derived as a linear combination of two stochastic processes: an idiosyncratic and a systematic factor. Doing so, we can disentangle, in an endogenous way, different sources of risk. Third, the spread will be used for two purposes: (a) to derive correlation networks, thus identifying contagion channels in a cross-sectional perspective; (b) to calculate the default probabilities associated to each economic sector in each country, in an early warning perspective. Last, we will combine default probabilities and correlation networks by deriving $CoRisk_{in}$ and $CoRisk_{out}$, two time-dependent measures which explain to what extent the default probability of each economic sector is affected

by ($CoRisk_{in}$) or affects ($CoRisk_{out}$) its neighbours when contagion is included.

The described strategy allows to simultaneously assign precise risk measures to the different agents operating in the system by considering, at the same time, the system as a whole. Differently from most related papers, we will allow for both positive and negative contagion, meaning that the default probability of each agent can be decreased or increased by its relationships with other nodes. Moreover, the distinction between incoming and outgoing effects enables to decouple the identification of vulnerable, rather than systemic important economic sectors. Finally, in order to overcome the micro- vs macro-based dualism recognizable in the literature, we will derive aggregate default probabilities at the country level using a bottom-up approach. As a consequence, we will obtain a synthetic risk measure for each country, that can be disentangled in all its components according to the source (economic sector) or the kind (sector-specific or contagion) of risk, and which varies along time reacting to both idiosyncratic and systematic shocks.

2 An overview of systemic risk measures

As previously introduced, the study of systemic risk is particularly problematic because of the high number of dimensions that can be included: according to this choice different perspectives have been adopted and, therefore, different statistical tools have been used and applied to a great variety of data in many geographical regions and periods. For simplicity we have chosen two main discriminant factors, thus dividing models on systemic risk into three main categories: bivariate models, causal models and cross-sectional models. While the first two explicitly deal with the time-dimension in an endogenous rather than an exogenous way, the latter focuses on the cross-sectional dimension.

Bivariate Models. From a chronological viewpoint, the first systemic risk measures have been proposed for the financial sector, in particular by Acharya et al. (2010), Adrian and Brunnermeier (2011), Brownlees and Engle (2012), Acharya et al. (2012), Dumitrescu and Banulescu (2014) and Hautsch et al. (2015). On the basis of market share prices, these models consider systemic risk as endogenously determined and calculate the quantiles of the estimated loss probability distribution of a bank conditional on an extreme event in the financial market. A similar approach has been applied by Popescu and Turcu (2014) to the sovereign sector, using bond interest rates.

The above described methodology is useful to identify the most *systemically important* institutions, since its bivariate nature allows the derivation of conditional default probabilities or losses during shock events in the reference market, possibly caused by other institutions. However, it does not address the issue of

how risks are transmitted between different institutions in a multivariate framework.

Causal Models. A different stream of research considers systemic risks as exogenous factors and has been proposed, among others, by Chong et al. (2006), Longstaff (2010) and Shleifer and Vishny (2010), who examined the impact of monetary policies on default probabilities for the banking sector, with a particular focus on crisis periods. More general causal models, proposed by Duffie et al. (2000), Lando and Nielsen (2010), Koopman et al. (2012), Betz et al. (2014) and Duprey et al. (2015), explain whether the default probability of a bank, a country, or a company depends on a set of exogenous risk sources, thus combining idiosyncratic and systematic factors. A further evolution has been proposed, among others, by Bartram et al. (2007), Ang and Longstaff (2012) and Brownlees et al. (2014): they combine idiosyncratic and systematic sources of distress through endogenous models expressed in terms of univariate stochastic processes.

While powerful from an early warning perspective, causal models, similarly to bivariate ones, concentrate on single institutions rather than on the economic system as a whole and, therefore, underestimate systemic sources of risk arising from contagion effects within the system.

Cross-sectional Models. In order to address the multivariate nature of systemic risk, researchers have recently proposed correlation network models, able to combine the rich structure of financial networks (see, e.g., Lorenz et al., 2009; Battiston et al., 2012) with a parsimonious approach based on the dependence structure among market prices. The first contributions in this framework are Billio et al. (2012) and Diebold and Yilmaz (2014), who derive connectedness measures based on Granger-causality tests and variance decompositions. Barigozzi and Brownlees (2013), Ahelegbey et al. (2015) and Giudici and Spelta (2015) extend such methodology by introducing stochastic graphical models, while Das (2015) derives a systemic risk decomposition into individual and network contributions.

Correlation network models are very useful for identifying the most important contagion channels in a cross-sectional perspective, thus identifying the most *vulnerable* institutions. However, since they are built on cross-sectional data, they can not be used as predictive models in a time-varying context. Moreover, the importance of each institution only depends on its position in the graph, and not on its specific risk.

A new combined approach. Bivariate and causal models explain whether the risk of a bank, a company, or of a country, is affected by a market crisis event or

by a set of exogenous risk factors; correlation network models explain whether the same risk depends on contagion effects. We improve all these three classes of models by introducing multivariate stochastic processes and by combining them with correlation networks: doing so, we merge the advantages of bivariate models (endogeneity and non-linearity), causal models (predictive capability) and correlation networks (contagion channels). To achieve our aim, we significantly extend the approach by Ang and Longstaff (2012) and Brownlees et al. (2014), by employing a correlated set of linear combinations of two stochastic processes (a systematic and an idiosyncratic one) rather than a single process, and by applying it to three rather than one economic sector.

In more detail, we first select three risk measures from publicly available data: (a) the spread between the cost of debt for countries (interest rates on 10-years maturity government bonds) and a benchmark rate, which gives a measure of sovereign risk; (b) the spread between the cost of debt for corporates (aggregate interest rates on bank lendings to non-financial corporates) and a benchmark rate, which gives a measure of corporate risk; (c) the spread between the funding cost of the banking system (aggregate interest rates on deposits of non-financial corporates and households) and a benchmark rate, which gives a measure of bank risk. We define three stochastic processes on the three risk measures so that, on the basis of the estimated parameters, a probability of default can be calculated, for each economic sector and within each country, independently from the others. We then estimate a correlation network model based on the estimated partial correlations between the risk measures: by so doing, we simultaneously consider both the cross-sectional and the time perspectives. In addition, we derive default probabilities conditionally on the estimated network. The difference between such conditional probabilities and the unconditional ones can be employed to assess the effect of systemic contagion: the resulting measure will be named $CoRisk$. We propose two different kinds of $CoRisk$: $CoRisk_{in}$, which measures how an economic sector is influenced by the default probability of its neighbours, thus providing a measure of its *vulnerability* (as in bivariate and econometric causal models); $CoRisk_{out}$, which measures to what extent each economic sector influences its neighbours, thus providing a measure of its *systemic importance* (as in cross-sectional models). Furthermore, since conditional default probabilities can be aggregated at the country level, we obtain a country specific default probability that can be disentangled according to all the dimensions introduced so far: time, economic sector (sovereign, corporate and bank), kind of risk (sector-specific and contagion), or source of risk (idiosyncratic and systematic).

We remark that a multivariate approach related to ours has been suggested by Gray et al. (2013), Ramsay and Sarlin (2015) and Schwaab et al. (2015). We extend these contributions by taking endogeneity into account as well as by using a proper probability metric, thus making explicit what suggested in Das

(2015): a measure of systemic risk that can be decomposed in an individual node and a network component. Other similar approaches have been recently proposed by Mezei and Sarlin (2015) and Betz et al. (2016): the former define an aggregation operator in order to jointly estimate the importance of each single node as well as contagion effects deriving from links with other nodes; the latter develop a tail risk analysis of networks in order to build a robust set of regressors for defining systemic contributions. We improve both approaches by calculating node default probabilities for three different economic sectors in each country and by deriving link measures of contagion through partial correlations between linear combinations of stochastic processes: in such a way we can (a) allow for non-linear effects through stochastic differential equations, (b) allow for contagion effects not only between, but also within each country, and (c) disentangle the idiosyncratic and the systematic, as well as the sector-specific and the systemic components for the three economic sectors in each country. In addition, our $CoRisk$ measure is allowed to be both positive or negative, meaning that the resulting default probability of each economic sector or country can be increased or decreased according to the sign of partial correlations. From an economic viewpoint, when a country is negatively related to troubled countries, its final default probability decreases because it is perceived as a flight-to-quality haven, meaning that it is positively affected by contagion effects. On the contrary, when countries are positively connected to troubled economies their default probability increases because they suffer negative contagion. Such a distinction between positive and negative contagion, to our knowledge, only appears in Grinis (2015).

Our proposed model will be applied to data from Eurozone countries in the recent time period. For descriptive purposes, we have identified four crucial time windows and we will show correlation networks and risk distributions in each of them: the pre-crisis period (2003-2006), the financial-crisis period (2007-2009), the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015).

Our main economic findings can be summarized according to three dimensions: (a) the economic sector dimension, (b) the country dimension and (c) the time dimension at the aggregate country level. Concerning (a), the corporate sector is strongly influenced by the systematic component, and this explains why it reacts to monetary policy changes more than sovereigns and banks. On the other hand, the sovereign and bank sectors have deeply suffered, respectively, the sovereign and the financial crisis, and they seem to behave quite similarly, thus confirming their "diabolic loop". In the last period, correlation networks show the creation of two distinct clusters, characterized by positive within and negative cross correlations, that clearly separate peripheral and core economies: such separation creates loop effects within each cluster, further alienating troubled and strong economies. Concerning (b), peripheral countries mostly behave as exporters, rather than importers of system risk: as a consequence, core economies

are mostly affected by contagion risk, while peripheral countries strongly suffer high sector-specific default probabilities. Concerning (c), the sovereign crisis has had a larger impact on systemic risk with respect to the financial crisis. A possible explanation consists in different ways peripheral and core economies reacted to the financial crisis: peripheral countries, with high public debts, had little fiscal space to improve balance sheets and, therefore, the financial crisis triggered their imbalances to emerge in the subsequent sovereign crisis. However, the sequence of these two events has determined an irreversible phase transition, leading to a new non-stable and non-optimal equilibrium, where instability derives from peripheral-countries trajectories diverging from core ones. This conclusion is further confirmed by the time evolution of risk distributions across Eurozone countries and by the role of risk propagation, which does not act as a mean for balancing inequalities across countries but, on the contrary, weakens the weakest and strengthens the strongest ones.

The paper is structured as follows: Sections 3 and 4 describe the proposed models, with Section 3 introducing multivariate linear combinations of interest rate spreads and correlation networks and Section 4 defining default probabilities and *CoRisk*. Section 5 presents data and the empirical evidence obtained from multivariate stochastic processes and correlation networks, while Section 6 shows the obtained default probabilities, *CoRisk* and its comparison with other centrality measures. Finally, Section 7 concludes with some closing remarks.

3 Multivariate spread processes

Consider $i = 1, ..., N$ countries which, in a first stage, have only one economic sector. We assume that the time dynamics of the liability side of each country is expressed by the evolution of the associated interest rate, which can be described by a linear combination of two stochastic processes: a common systematic component and an idiosyncratic factor. More formally, for each country $i = 1, \dots, N$:

$$Z_t^i = y_t^i - S_t, \tag{3.1}$$

where S_t stands for the systematic process, while y_t^i represents the idiosyncratic process referred to country i; the complete process Z_t^i describes the resulting time evolution of interest spreads. From an economic viewpoint, the above formulation expresses Z_t^i as the difference between the cost of a long term debt and the cost of liquidity.

Both the systematic and the idiosyncratic processes can be modelled as CIR processes (Cox et al. 1985), as follows:

$$\begin{cases} \mathrm{d}\, S_t = (a - vS_{t-1})\,\mathrm{d}\, t + b\sqrt{S_{t-1}}\,\mathrm{d}\, B_t, \\ \mathrm{d}\, y_t^i = (\theta_1^i - \theta_2^i y_{t-1}^i)\,\mathrm{d}\, t + \theta_3^i \sqrt{y_{t-1}^i}\,\mathrm{d}\, W_t, \end{cases} \tag{3.2}$$

where $\mathrm{d}\, B_t$ and $\mathrm{d}\, W_t$ are two independent Brownian motions.

We then assume the following correlation structure:

$$\begin{cases} \mathrm{Corr}[y_t^i, y_t^j] = \rho^{ij}, \\ \mathrm{Corr}[S_t, y_t^j] = \gamma^j. \end{cases} \tag{3.3}$$

Note that the first equation in (3.3) is consistent with the assumptions used in the formulation of multidimensional CIR processes (see e.g. Kalogeropoulos et al., 2011); the second one introduces a correlation between each idiosyncratic process and the systematic process S_t.

The model proposed in (3.1)-(3.3) defines a multivariate stochastic process able: (a) to capture both the systematic and the idiosyncratic components that may affect interest rate spread dynamics, using linear combinations of stochastic processes; (b) to model the correlation structure of interest rate spreads across different countries.

To exploit (b) we now derive the instantaneous covariance matrix corresponding to our proposed model. First define:

$$P = \begin{bmatrix} 1 & \rho^{12} & \dots & \rho^{1N} \\ \rho^{21} & 1 & \dots & \rho^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{N1} & \rho^{N2} & \dots & 1 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} \gamma^1 \\ \vdots \\ \gamma^i \\ \vdots \\ \gamma^N \end{bmatrix}, \tag{3.4}$$

where each element in P is the correlation coefficient between the idiosyncratic processes of any two countries, while each element of Γ is the correlation coefficient between any idiosyncratic process and the systematic process, as defined in (3.3). Let A be the instantaneous covariance matrix of the spread vector $Z = (Z^1, \dots, Z^N)$. A can be shown to be as follows:

$$A = \Phi \cdot \Theta^T, \tag{3.5}$$

where each vector of the matrices Φ and Θ^T is equal to:

$$[\Phi]^i = \left[b\sqrt{S_0}, \quad 1, \quad \sqrt{S_0 y_0^i}\, b\theta_3^i [\Gamma]^i, \quad \sqrt{y_0^i}\,\theta_3^i \sqrt{[P]^i} \right],$$

$$[\Theta^T]^j = \begin{bmatrix} b\sqrt{S_0} \\ \\ \sqrt{S_0 y_0^j} b \theta_3^j [\Gamma]^j \\ \\ 1 \\ \\ \sqrt{y_0^j}\theta_3^j \sqrt{[P]^j} \end{bmatrix}.$$

The parameters of the proposed process can be estimated by extending results available for univariate stochastic processes (see e.g. Iacus, 2008), and based on the maximization of the log-likelihood function. Through the application of the invariance principle of maximum likelihood estimators, we can compute the covariance matrix A on which networks are based. Before that, we extend the methodology proposed so far to the more realistic multi-sector situation.

For each country, let us consider the aggregate financial liabilities of sovereigns, (non-financial) corporates and banks as the idiosyncratic components in (3.1). Formally, by denoting the three economic sectors respectively with $\{1,2,3\}$, for each country $i = 1, \ldots, N$ equation (3.1) becomes the following system:

$$\begin{cases} Z_{t,1}^i = y_{t,1}^i - S_t, \\ Z_{t,2}^i = y_{t,2}^i - S_t, \\ Z_{t,3}^i = y_{t,3}^i - S_t. \end{cases} \tag{3.6}$$

In (3.6) the systematic component S_t as well as the idiosyncratic factors $y_{t,\{1,2,3\}}$ follow a CIR process:

$$\begin{cases} \mathrm{d}\, S_t = (a - vS_{t-1})\,\mathrm{d}\, t + b\sqrt{S_{t-1}}\,\mathrm{d}\, B_t, \\ \mathrm{d}\, y_{t,\{1,2,3\}}^i = [(\theta_1)_{\{1,2,3\}}^i - (\theta_2)_{\{1,2,3\}}^i y_{t-1,\{1,2,3\}}^i]\,\mathrm{d}\, t + (\theta_3)_{\{1,2,3\}}^i \sqrt{y_{t-1,\{1,2,3\}}^i}\,\mathrm{d}\, W_t. \end{cases} \tag{3.7}$$

We then assume the following correlation structure:

$$\begin{cases} \mathrm{Corr}[y_t^m; y_t^n] = \rho^{mn}, \\ \mathrm{Corr}[S_t; y_t^m;] = \gamma^m, \end{cases} \tag{3.8}$$

where $\{m,n\} \in (V \times W)$, with $V = \{1, ..., N\}$ denoting countries and $W = \{1,2,3\}$ economic sectors.

The new model in (3.6)-(3.8) defines a general multivariate stochastic process able: (a) to capture both the systematic and the sector-specific idiosyncratic components that may affect interest rate spread dynamics, using linear combinations of stochastic processes; (b) to model the correlation structure of interest rate spreads across different countries and different sectors. Note that the instantaneous covariance matrix of the new process turns out to be the same as

that in (3.5), albeit with a different dimensionality, being a $3N \times 3N$ rather than a $N \times N$ matrix.

Once the covariance matrix A has been estimated (as previously discussed), it can be employed to calculate correlation coefficients and, consistently, correlation networks between countries and economic sectors (following Billio, 2012; Ahelegbey et al., 2015; Giudici and Spelta, 2015). However, such correlations can be misleading because they take into account bivariate (marginal) relationships between interest spreads. For this reason we propose to employ conditional (partial) correlations, different from bivariate ones as they are adjusted by the presence of all the other variables in the system. Let A^{-1} be the inverse of the covariance matrix, with elements a^{mn}. The partial correlation coefficient ρ_{mnVW} between variables Z^m and Z^n, conditional on the remaining variables in $V \times W$, can be obtained as:

$$\rho_{mnVW} = \frac{-a^{mn}}{\sqrt{a^{mm}a^{nn}}}. \tag{3.9}$$

In order to better explain partial correlations and their differences with respect to marginal ones, we now report a useful and interesting property. For $\{m, n\} \in (V \times W)$, set $S = (V \times W) \setminus \{m, n\}$ and suppose to express the dependence between spread measures through multiple linear models in the following way:

$$\begin{cases} Z^m = a^m + \sum_{n \neq m} a_{mn|S} Z^n; \\ Z^n = a^n + \sum_{m \neq n} a_{nm|S} Z^m. \end{cases} \tag{3.10}$$

It can be shown that the partial correlation coefficient between Z^m and Z^n, given all the other $3N - 2$ spread measures, can be interpreted as the geometric average between the multiple linear coefficients in (3.10):

$$|\rho_{mn|S}| = |\rho_{nm|S}| = \sqrt{a_{mn|S} \cdot a_{nm|S}}. \tag{3.11}$$

Note that in case of only two components ($S = \varnothing$), equation (3.10) becomes:

$$\begin{cases} Z^m = a_m + a_{mn} Z^n \\ Z^n = a_n + a_{nm} Z^m, \end{cases} \tag{3.12}$$

from which the marginal correlation coefficient ρ_{mn} can be derived as the geometric average between the coefficients in (3.12):

$$|\rho_{mn}| = |\rho_{nm}| = \sqrt{a_{mn} \cdot a_{nm}}.$$

We propose to build a correlation network based on partial correlations rather than on marginal correlations. To achieve this aim we introduce an undirected graph $G = (P, E)$, with a vertex set $P = V \times W = \{1, ..., 3N\}$ and an edge

set $E = P \times P$. Such edge set is defined by binary elements e_{mn} that describe whether pairs of vertices are (symmetrically) linked to each other ($e_{mn} = 1$) or not ($e_{mn} = 0$), depending on whether the partial correlation coefficient between the corresponding pair of variables is different from or equal to zero.

4 Default probabilities and CoRisk

For each node $m \in V \times W$, a sector-specific probability of default, PD_t^m, can be obtained by considering the expected dynamic of debt:

$$D_{t+1}^m = (1 - PD_t^m) e^{y_t^m} D_t^m, \tag{4.1}$$

where D_{t+1}^m (D_t^m) is the total debt at time $t+1$ (t). Note that the analogous dynamic of a risk-free debt is the following:

$$D_{t+1}^m = e^{S_t} D_t^m. \tag{4.2}$$

If we (reasonably) assume to be in an arbitrage-free context, we can equate (4.1) and (4.2), thus obtaining PD_t^m:

$$PD_t^m = 1 - e^{-Z_t^m}. \tag{4.3}$$

From (4.3), a decrease in Z_t^m implies a decrease in the probability of default, consistently with the definition of the process Z_t^m as an interest rate spread.

The probability of default derived in (4.3) is sector-specific, as it is assumed independent from the default probability of other institutions: in our view this is an unrealistic assumption, since economic sectors of different countries depend on each other, as well as the default probability of each country depends on all its three economic sectors.

We thus propose to evolve the PD into a total default probability, TPD, able to incorporate both sector-specific and contagion components. To ease the exposition, in this Section we will propose an economically intuitive approach: a complete mathematical treatment is provided in Appendix A.

Let us assume to have a "global" spread process $\tilde{Z}^m$, expressed as a linear function of the "baseline" spread $Z_t^m = -ln(1 - PD_t^m)$, which depends exclusively on m, and of a further component which depends on the spread measures Z_t^n of the other nodes $n \neq m$:

$$\tilde{Z}_{t+1}^m = Z_t^m + \sum_{n \neq m} a_{mn|S} Z_t^n. \tag{4.4}$$

Assuming that the total default probability TPD can be expressed as a function of $\tilde{Z}$ as in (4.3), by replacing Z_t^m in (4.3) with the right hand side of equation (4.4), and by substituting the coefficients $a_{mn|S}$ with their geometric averages

$\rho_{mn|S}$ (obtained from the estimated process parameters) according to (3.11), we derive a new expression for the probability of default conditional on non-defaulted neighbours n in the previous time, that we name TPD:

$$TPD_{t+1}^{m} = 1 - (1 - PD_t^m) \cdot \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn|S}}. \tag{4.5}$$

From (4.5), we can define the incoming contagion effect ($CoRisk_{in}$), conditional on non-defaulted neighbours in the previous time, as the TPD component that strictly depends on neighbours $n \neq m$:

$$CoRisk_{in,t}^{m} = 1 - \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn|S}}. \tag{4.6}$$

For each agent m, $CoRisk_{in}$ is an increasing function of both PD^n (default probability of neighbours) and $\rho_{mn|S}$ (partial correlations). In other words, the worse the nodes to which m is more connected, the worse the default probability of m itself.

By combining (4.5) with (4.6) and by assuming $TPD_{t+1}^m > 0$ (a rather obvious request), $CoRisk_{in}$ can be interpreted as the percentage variation of the survival probability due to contagion:

$$CoRisk_{in,t}^{m} = \frac{(1 - PD_t^m) - (1 - TPD_{t+1}^m)}{1 - PD_t^m}. \tag{4.7}$$

Economically, $CoRisk_{in}$ measures the change in the survival probability of an agent m when potential contagion deriving from all other agents is included. According to equations (4.5) and (4.7), the total default probability TPD can be either greater or lower than PD depending on the sign of the $CoRisk_{in}$ measure: more precisely, if $CoRisk_{in} > (<) 0$, the default probability of node m increases (decreases) after the inclusion of contagion effects. This distinction comes from considering partial correlations as signed numbers rather than in absolute value, thus allowing for "beneficial" or "adverse" effects. As a consequence we will obtain *negative contagion* when an institution m is disadvantaged by positive links with its neighbours ($CoRisk_{in} > 0$ and $TPD > PD$), while *positive contagion* will occur if institution m takes advantage of negative links with its neighbours ($CoRisk_{in} < 0$ and $TPD < PD$).

In order to define outgoing contagion effects, we can calculate to what extent agent m affects its neighbours. Formally, we can define $CoRisk_{out}$, conditional on not having defaulted in the previous time, as follows:

$$CoRisk_{out,t}^{m} = 1 - \prod_{n \neq m} (1 - PD_t^m)^{\rho_{nm|S}} = 1 - (1 - PD_t^m)^{\sum_{n \neq m} \rho_{nm|S}}. \tag{4.8}$$

Note that the two definitions (4.6) and (4.8) introduce asymmetries in the model: even if the graph is undirected and, thus, symmetric, the incoming and

outgoing contagion effects are different, since each node is associated to a different default probability and, consequently, its contagion effect towards its neighbours is different from the effect it receives from them.

This distinction allows us to disjointly calculate, for each agent, its vulnerability ($CoRisk_{in}$) and its systemic importance ($CoRisk_{out}$). If the two measures coincide, the default probability of node m is equal to the geometric average of the default probabilities of its neighbours: on the contrary, if $CoRisk_{out}^{m} > CoRisk_{in}^{m}$ ($<$), the default probability of node m is greater (lower) than the geometric average of the default probabilities of its neighbours, meaning that its systemic importance is greater (lower) than its vulnerability.

As an example, consider the graphs in Figure 1, where each node is associated to its sector-specific PD and each pair of nodes is associated to the corresponding partial correlation coefficient $\rho_{mn|S}$.

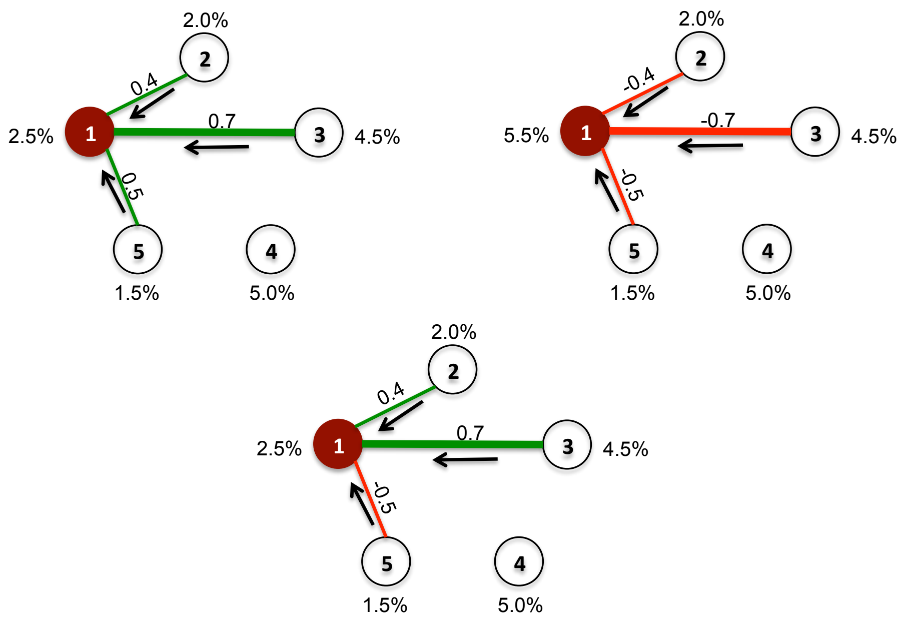

Figure 1: $CoRisk_{in}$, an illustrative example

In the first case (all positive correlations) the final $CoRisk_{in}$ value is 0.047, meaning that contagion has decreased the survival probability of node 1 by 4.7%, bringing its default probability from $PD^1 = 2.9\%$ to $TPD^1 = 7.2\%$ (negative contagion). In the second example, instead, all the correlation coefficients are negative, and the calculated $CoRisk_{in}$ becomes -0.049, meaning that contagion has increased the survival probability of node 1 by 4.9% (positive contagion). According to equation (4.5), the total TPD^1 has decreased, being equal to 0.87%. Note that in this second example the $CoRisk_{in}$ measure is not equal, in absolute value, to the one obtained in the previous example: this because the exponent ρ introduces non-linear effects in the relationship (4.6). In the last example, where both positive and negative correlations appear, the calculated $CoRisk_{in}$ measure

is equal to 0.032, meaning that contagion has decreased the survival probability of node 1 by 3.2%, reaching a total default probability $TPD^1 = 5.6\%$ (overall negative contagion).

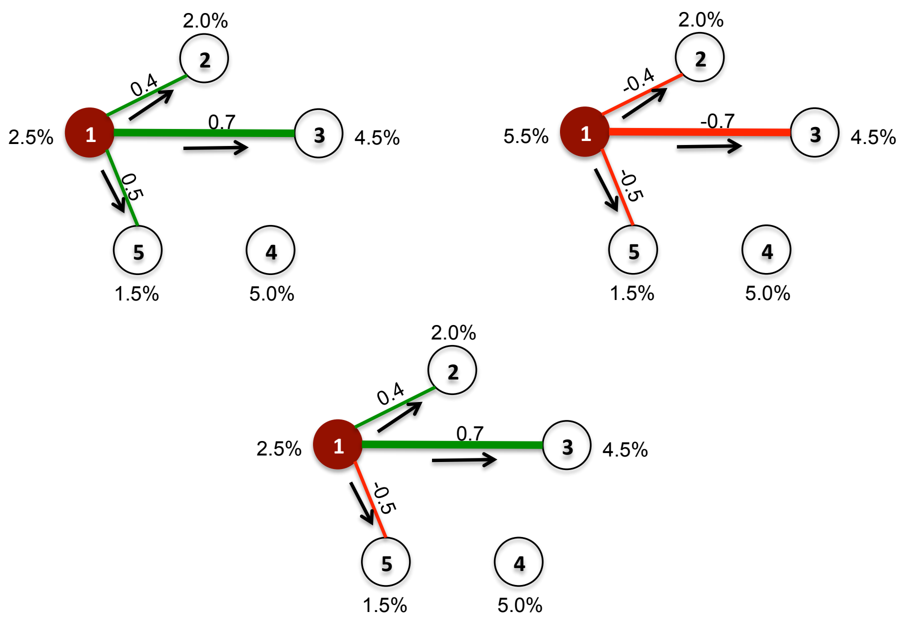

Figure 2: $CoRisk_{out}$, an illustrative example

Figure 2 reports the same graphs as in Figure 1, but we now concentrate on the outgoing effects in order to understand how node 1 affects its neighbours. In the first example, the overall $CoRisk_{out}$ is equal to 0.040: this result is lower with respect to the $CoRisk_{in}$ value because the incoming contagion is highly affected by the large default probability of node 3. Similarly, in the second situation the final $CoRisk_{out}$ is -0.095. This result is lower than the corresponding $CoRisk_{in}$ because, now, the default probability of node 1 is much bigger than the default probabilities of its neighbours: consequently, the contagion effect due to negative correlations is amplified, meaning that a negative relation with node 1 strongly decreases the default probability of the set $ne(1)$. In the last example the calculated $CoRisk_{out}$ measure is equal to 0.015, lower than $CoRisk_{in}$ as in the first example.

The total default probabilities introduced in (4.5) are defined for each economic sector within each country. However, it is important to calculate the total default probability of an entire country, obtained by aggregating the default probabilities of its economic sectors. To derive such probability we assume that a country will default if at least one of its economic sectors defaults.

Thus, denoting with $A^{3,i}$, $A^{1,i}$ and $A^{2,i}$ the sets of defaults for, respectively, the sovereign, corporate and bank sectors of country i, we are interested in deriving $P(\bigcup_{j \in W} A^{j,i} | S^i)$, where $S^i = \{A^m; \forall m \in V \times W, m \in ne(i,j), m \neq (i,j)\}$. It can be shown that such a probability, named $TPD^i_{country,t}$, is equal to:

$$TPD^{i}_{country,t} = 1 - \prod_{\substack{j,j'=1 \\ j'>j}}^{3} (1 - TPD^{j,i}_{t} | \bar{A}^{j'}), \tag{4.9}$$

where the three probabilities are the $TPDs$ derived through (4.5) by considering, respectively, all the other nodes, all the other nodes but the corporate sector of country i, all the other nodes but the corporate and bank sectors of country i.

5 Data

We focus on eleven european countries: Austria, Belgium, Finland, France, Germany and the Netherlands (core countries); Greece, Ireland, Italy, Portugal and Spain (peripheral countries). For each country and economic sector we consider the aggregate funding costs as idiosyncratic components for modeling sovereign, corporate and bank risk: (a) interest rates on 10-year maturity government bonds, (b) aggregate interest rates on bank loans to non-financial corporates, (c) aggregate interest rates on bank deposits from non-financial corporates and households. Concerning the common systematic component, there are many choices for a benchmark rate: we suggest a rate that reflects the impact of the European Central Bank monetary policy, such as the 3-months Euribor. All data are publicly available and have been selected with monthly frequencies.

Summary statistics are shown in Tables 1 and 2. In order to better describe the country-specific, sector-specific and time-evolution components of the resulting $N = 11 \times 3$ -dimensional system of interest rate spreads, data have been grouped in four different time windows: (a) the pre-crisis period (2003-2006), (b) the financial crisis period (2007-2009), (c) the sovereign crisis period (2010-2012) and (d) the post-crisis period (2013-2015). For each of them means, standard deviations as well as correlations with Euribor interest rates are reported.

[Tables 1 and 2 here]

From Tables 1 and 2 note that interest rates on loans have the highest correlation coefficients with Euribor interest rates, during all time-windows and in almost all countries. The same correlations referred to interest rates on government bonds vary over time: low during the pre-crisis period and higher afterwards (with the exception of Greece). The correlations of bank interest rates with the Euribor follow a similar pattern, being very low until 2012 in almost all countries, and strongly positive afterwards.

The time evolution of the interest rate processes for the sovereign sector can be observed in Figure 3.

[Figure 3 here]

Figure 3 shows that interest rates on government bonds were initially very similar, while in 2010 they started diverging: decreasing in core countries and increasing in peripheral countries. Greece, Ireland and Portugal present the highest volatility, especially during their sovereign crisis in 2010-2011, followed by Italy and Spain and, to a lesser extent, Belgium.

The time evolution of the interest rate processes for the corporate sector can be observed in Figure 4.

[Figure 4 here]

From Figure 4 note that interest rates on loans to non-financial corporates differ across the main european countries. In particular, Greece and Portugal have the highest values while Finland and Austria present the lowest ones. The interest curves of corporates do not show substantial overlaps: they all increase during the financial crisis of 2008 and, to a lesser extent, during the sovereign crisis of 2011. All rates show positive correlations with the Euribor dynamics. Overall, the scale of variation of corporate rates is much smaller than that of sovereign rates, especially in peripheral countries.

The time evolution of the interest rate processes for the bank sector can be observed in Figure 5.

[Figure 5 here]

Figure 5 shows an interest rate pattern substantially different with respect to sovereigns and non-financial corporates. The highest rates occur in France, Belgium and the Netherlands consistently through time, while the curves of the other countries do overlap: this is especially true for peripheral countries, affected not only by the financial crisis but also by the sovereign crisis. Overall, the scale of variation of bank rates is slightly lower than that observed for corporates.

6 Empirical Results

The first step in model estimation consists in deriving the coefficients of the stochastic processes in (3.2), for the two components (idiosyncratic and systematic) of each economic sector and country: such results are reported in Tables 3, 4 and 5. Tables 4 and 5 show that, during the two crisis periods, all the parameters (drift and volatility) are sensibly higher in peripheral countries than in core countries. In the post-crisis period, however, drift terms return to their initial values (with the exception of Greece), while volatilities remain quite high.

6.1 Correlation Networks

Our aim is now to derive the correlation network models obtained by calculating partial correlations as in (3.9), for sovereigns, corporates and banks. To achieve

this aim it is necessary to calculate, within each sector j, the 11×11 inverse correlation matrix of the spreads $Z^i_{t,j}$ for each time period t. To better interpret the results, we only show the most significant correlations: in particular, a connection between two countries will be kept or dropped on the basis of a correlation *t-tests* based on $\alpha = 0.10$. Moreover, for explanatory purposes we will just show networks referred to the four time windows previously identified. Such correlations are depicted in Figure 6: green lines stand for positive partial correlations, while red lines indicate negative partial correlations; the ticker the line, the stronger the connection.

[Figure 6 here]

Comparing the sovereign correlation networks in Figure 6, note that their pattern has substantially changed over the years: in the pre-crisis period the overall number of significant partial correlations is quite high; during the financial crisis they decrease; during the sovereign crisis they further decrease and a "clustering effect" that separates core and peripheral economies in two quite distinct subgraphs emerges. Last, in the post-crisis period the partial correlation pattern returns to the pre-crisis situation, however with a persisting clustering effect, emphasized not only by positive within subgraph correlations, but also by negative ones across the two subgraphs.

Bank correlation networks, similarly to sovereign ones, are quite connected in the first two periods, and become sparser afterwards. In this case, the clustering effect becomes evident in the last, rather than in the third period. This time delay may also be due to the different kind of data used for banks with respect to sovereigns: the latter are market-based data, characterized by quick reactions to the economic perspectives of a country; the former, instead, depend upon banks' decisions and are characterized by a degree of viscosity with respect to the external environment.

By analyzing the corporate correlation networks in Figure 6, a substantial change over time in the partial correlation pattern emerges again, further underlining the importance of the dynamic perspective. During the pre-crisis period the overall number of significant correlations is quite high, similarly to sovereign and bank ones. During the financial crisis such number substantially decreases; during the sovereign crisis significant correlations increase again, and they drop in the last period, characterized by low growth and close-to-zero Euribor interest rates. Differently from what observed in the other two economic sectors, a clustering effect between core and peripheral countries is not evident: a possible explanation is that corporate interest rates are highly and constantly correlated with Euribor rates across time and, thus, clustering effects become less significant while the systematic component, affected by monetary policies, becomes the most important risk driver.

6.2 Default probabilities and contagion

Having estimated all the parameters, as well as partial correlations, we are now able to calculate the sector-specific and time-dependent probabilities of default of each sovereign ($PD^i_{t,1}$), corporate ($PD^i_{t,2}$) and bank ($PD^i_{t,3}$) sector in each country i, based respectively on the spread measures $Z^i_{t,1}$, $Z^i_{t,2}$ and $Z^i_{t,3}$ according to equation (4.3). Using such PDs and the estimated partial correlations, we can thus calculate the total default probability of each economic sector in each country $TPD^i_{t,\{1,2,3\}}$ as in (4.5) and, by comparing them with the sector specific default probabilities, we can obtain the $CoRisk$ measures.

Summary statistics of $CoRisk_{in}$ during the four different time windows are shown in Tables 6 and 7. Their corresponding time evolutions, together with default probabilities PD and TPD, are shown in Figure 7.

[Tables 6 and 7, Figure 7 here]

Let us first consider the results referred to the sovereign sector, obtained by jointly reading Tables 6 and 7 together with Figure 7. By looking at the single sector-specific PD (top graphs), it is clear that Greece presents the most critical situation, with the highest PD values. Portugal has similar, but lower results. Ireland presents an anticipated increase in its default probability because of its deep sovereign crisis in 2011, but in the following years it starts performing quite well until reaching very low PD values in 2015. Italy and Spain show similar intermediate values, while core countries behave quite similarly to each other, with the lowest PDs across time.

The $CoRisk_{in}$ pattern (middle graphs) can be understood by looking at the networks in Figure 6: countries with high positive correlations with peripheral economies, characterized by high PDs, have a high $CoRisk_{in}$: this is the case, for example, of France and Belgium in the second period, strongly connected, respectively, to Italy and Portugal, and to Italy and Spain. Similarly, Spain presents a high $CoRisk_{in}$ during the sovereign crisis period, due to its strong positive link with Ireland, a particularly troubled country in such years. On the other hand, countries which are negatively or not connected with peripheral ones (such as, for instance, Germany in the second period and Finland and Austria in the latest years) have close to zero or negative $CoRisk_{in}$ measures. The clustering effect observed in Figure 6 in recent times implies that large peripheral countries (such as Italy and Spain) are negatively affected by positive correlations with each other (negative contagion: $CoRisk_{in} > 0$), while smaller core countries (such as Austria and Finland) take advantage of negative correlations with peripheral economies (positive contagion: $CoRisk_{in} < 0$), thus decreasing their own default probability. This result can be explained thinking at capital flows: when a country i is facing a crisis period, investors tend to shift their portfolio towards "safer" places in order to reduce risk, and such places are the countries negatively

related to i, which, therefore, show an improvement in their survival probability. This mechanism justifies the difference between *positive* and *negative contagion* derived in Section 4.

Moving to the TPD time-evolution, it appears to be a mix between the sector-specific PD and the $CoRisk_{in}$ contribution, with the former prevailing. In peripheral economies, characterized by high sector-specific PDs, the $CoRisk_{in}$ contribution should be very low; however, the rise of two distinct clusters originates a sort of "diabolic loop", by which peripheral countries become positively connected between each other and negatively connected to core ones. For this reason their total default probability TPD is strongly influenced not only by its corresponding sector-specific PD, but also by high $CoRisk_{in}$ values. Following the same (but reversed) mechanism, core economies preserve low TPD even after the inclusion of contagion effects: the only exception is France, which presents an extremely high $CoRisk_{in}$ during the financial crisis due to a positive connection with Italy. Germany lies in an intermediate situation, with its $CoRisk_{in}$ growing in the recent years, along with positive connections with the periphery, in the light of its increasing leading role in the Euro area.

The results referred to corporates show sector-specific PDs less volatile than sovereign ones, both across countries and time. They all peak during the financial crisis and decrease afterwards, remaining almost constant during the following years. In recent times, the ranking of countries reflects what has been observed for sovereign risk, with Greece presenting much higher values than all the other countries, and core economies having the lowest ones. This means that, in the Euro area, sovereign risk has become the driving risk source. The $CoRisk_{in}$ pattern shows that almost all countries suffered contagion effects during the financial crisis and, to a lesser extent, during the sovereign crisis, thus highlighting an overall negative contagion across corporates. More precisely, Italy presents the highest $CoRisk_{in}$ values because of its strong positive relationships with Portugal and Spain, as shown in Figure 6. Differently from what has been observed for sovereigns, $CoRisk_{in}$ is the prevailing effect in the calculation of the total default probability for the corporate sector (with the exception of Germany in the last two periods, because of its very low sector-specific PD values): such a conclusion is supported by Figure 6, which shows that partial correlations are much higher both in number and value with respect to the sovereign sector, and that a clustering effect is not evident.

The results for the banks reveal that the sector-specific PDs of all countries have only been influenced by the financial crisis. Consistently with Figure 5, France, Belgium and the Netherlands present the highest levels, because of their high values of interest rates on deposits. The $CoRisk_{in}$ pattern shows both negative and positive contagion effects during the second time-period, with the former regarding core countries and the latter peripheral ones. However, from

2010-2012 (when two distinct clusters start emerging, as for the sovereign sector) $CoRisk_{in}$ starts increasing both in core and peripheral economies because of highly positive partial correlations within each cluster; this effect is further amplified in peripheral countries because of their higher sector-specific default probabilities, thus generating again the self-reinforcing and "diabolic" loop previously observed. Similarly to corporates, $CoRisk_{in}$ is the prevailing component in the composition of the total default probability of banks, both across time and countries.

6.3 From economic sectors to countries

In order to understand to what extent a whole country is influenced by the others, the aggregate total default probability proposed in (4.9) can be employed to summarize contagion effects into a unique default probability at the country level. Such aggregate $TPDs$ for the euro area countries are shown in Figure 8.

[Figure 8 here]

From Figure 8 two main considerations emerge. First, the financial crisis has had a more homogenous impact across countries than the sovereign one: all the aggregated TPDs strongly increased during 2008, while in the following time-window a clear distinction between peripheral and core countries appears, with the former having higher values than the latter. Two notable exceptions to the general pattern are: (a) France, which presents high values mainly because of its positive correlations with peripheral countries during both the financial and the sovereign crisis; (b) Ireland, characterized by a deep sovereign and bank crisis in 2011 (worsened by positive links with peripheral countries), followed by strong reforms and, recently, good economic results (increased by positive relations with core economies). Second, the pre- and post- crisis periods appear to be substantially different: during the pre-crisis years, in fact, default probabilities were almost constant and stable across time, and very homogenous across countries; but after the sovereign crisis the situation has become more heterogenous, with high volatilities in all countries and a clear distinction between peripheral and core economies, with Ireland joining the latter. This effect, consistently with Figure 7, means that the sovereign crisis has had the strongest impact on the Euro area, an impact that still persists through diverging clusters.

The total PD of a country is a function of the total default probabilities of its three economic sectors which, in turn, are functions of two contributions: their sector-specific PD and the $CoRisk_{in}$ measure. Moreover, previous results confirm that the hypothesis of Corollary A.1.7 (see Appendix A) are verified. We are thus allowed to disentangle the final default probability of a country into six percentage components. The normalized results are shown in Figure 9.

[Figure 9 here]

From Figure 9 note that the sovereign contribution in peripheral countries is larger than in core ones across time; in the latter, the main component of sovereign risk is due to contagion effects, while in the former the sector-specific PD component is higher. In almost all countries the corporate contribution is stronger during "normal" times, such as before the financial crisis and in the latest period, depending on sector-specific PDs in peripheral economies and on contagion effects in core economies. Last, core economies suffered a substantial improvement in contagion effects for the bank sector during the sovereign crisis through their exposition to peripheral banks, while peripheral economies witnessed an increase in their sector-specific bank default probabilities.

Overall, combining cross-sectional and time comparisons, the distribution of risk in its six components looks quite homogenous across countries before the financial crisis while, recently, the situation has not returned back to equilibrium: strong contagion risks persist in core economies, while sector-specific default probabilities are still high, and worsened by intra-clustering effects, in peripheral countries.

6.4 Vulnerability vs Systemic Importance

While $CoRisk_{in}$ incorporates incoming effects and thus measures the *vulnerability* of each economic sector in a country, $CoRisk_{out}$ can be applied to obtain an estimation of the *systemic importance* of each economic sector in each country. A comparison between vulnerability ($CoRisk_{in}$) and systemic importance ($CoRisk_{out}$) across time and countries is shown in Figure (10).

[Figure 10 here]

By comparing $CoRisk_{out}$ and $CoRisk_{in}$ contributions for the sovereign sector, different conclusions can be deduced. First, during the pre-crisis and the financial crisis periods the two measures look very similar, meaning that sector-specific default probabilities were homogenous across countries. Important differences start emerging during the sovereign crisis period: in such years Greece is clearly more an exporter rather than an importer of risk, while the situation is reversed for Portugal and Spain. In most recent years, all peripheral countries have the highest, even if decreasing, $CoRisk_{out}$ contributions, since their sector-specific PD is significantly higher than in core economies. It is interesting to observe that there are not negative $CoRisk_{out}$ measures for the sovereign sector, meaning that all Euro countries contribute to increase the default probability of their neighbours.

The incoming and outgoing contributions for the corporate sector emphasize, once again, the difference between core and peripheral countries, with the latter

characterized by higher $CoRisk_{out}$ and the former by higher $CoRisk_{in}$. Similarly to sovereigns, the two $CoRisk$ contributions are very close during the first two time-periods, while they start diverging afterwards. Similar results can be observed for the bank sector.

Overall, peripheral (core) countries appear to be more exporters (importers) rather than importers (exporters) of systemic risk, especially after the sovereign crisis. This result can be once more explained by the emerging of clustering effects in the third period, and it is a further confirmation of the persisting, and difficult to reverse consequences of the sovereign crisis on Eurozone countries.

6.5 CoRisk as a new centrality measure

We have considered both the incoming and outgoing $CoRisk$ as risk measures, able to calculate the vulnerability or the systemic importance of an economic sector in different countries and across time. However, $CoRisk$ has been derived employing network features, and can thus be applied in more general frameworks. More precisely, differently from other centrality measures, we assign two weights to a network: (a) $\rho_{mn} \in [-1, 1]$, which measures the weight of the link between each pair of nodes; (b) $PD^m \in [0, 1]$, which measures the dimension of each node. We can thus derive two centrality measures, both based on these two weights but different for the meaning they attribute to centrality:

Incoming centrality: how much a node is affected by its neighbours, according to (a) the number and weight of links, and (b) the importance (dimension) of neighbours.

Outgoing centrality: how much a node affects its neighbours, according to (a) the number and weight of links, and (b) the importance (dimension) of the node itself.

In order to better understand the meaning of these new network centrality definitions, we have decided to compare them to other two measures, commonly used especially in the systemic risk field: the eigenvector centrality (see e.g. Furfine, 2003; Billio et al., 2012) and the weighted degree, calculated as the sum of all partial correlations (see e.g. Giudici and Spelta, 2015). We have applied them to our data, and the corresponding results are shown in Tables 8 and 9 (centrality measures) and in Tables10 and 11 (centrality ranks). In order to summarize the comparison between rankings, the Spearman correlation coefficient has been calculated: the results are shown in Table 12.

[Table 12 here]

Table 12 reveals that, overall, both $CoRisk_{in}$ and $CoRisk_{out}$ orderings are quite similar to the one obtained with the weighted degree of centrality. As previously underlined, the difference between the two lies in the inclusion of two rather than one weight in $CoRisk$: more precisely, $CoRisk_{in}$ and $CoRisk_{out}$ depend on both partial correlations and, respectively, the default probabilities (or, more generally, the dimensions) of neighbours or the default probability (dimension) of the node itself.

On the other hand, eigenvector centrality measures the importance of each node in the graph by looking at its relations with other central nodes, so that a node becomes much more important if it is connected to important ones. This mechanism, applied without considering the impact of each node on the basis of its dimension, amplifies the distance between $CoRisk$ and the eigenvector centrality measure. This effect is particularly evident during crisis periods, for both incoming and outgoing effects.

6.6 *CoRisk* as a new systemic risk measure

The incoming and outgoing $CoRisk$, as derived in (4.6) and (4.8), can be considered not only as centrality measures, but also as new early warning indicators of systemic risk, and can thus be compared to other systemic risk measures. In particular, we can analyze their differences with respect to $\Delta CoVaR_{in}$ and $\Delta CoVaR_{out}$, calculated, respectively, as the part of the of the m-th economic sector's systemic risk that can be attributed to the system, or as the part of the system's systemic risk that can be attributed to an economic sector m

$$\begin{cases} \Delta CoVaR^m_{in} = CoVaR_q^{m|VaR_q^{system}} - CoVaR^{m|VaR_{50}^{system}}, \\ \Delta CoVaR^m_{out} = CoVaR_q^{system|VaR_q^m} - CoVaR^{system|VaR_{50}^m}, \end{cases} \tag{6.1}$$

with $CoVaR_q^{m|VaR_q^{system}}$ defined as the q-th Value at Risk of the economic sector m, conditional on an extreme event in the system; $CoVaR_q^{system|VaR_q^m}$ defined as the q-th Value at Risk of the system, conditional on an extreme event in the sector m. Values at Risk and, consequently, $CoVaR$ have been calculated for the sector-specific default probability distributions (PD^m).

In order to compute $CoVar$, we have computed, In each time-period, a quantile regression of each economic sector on the system ($CoVaR_{in}$), or of the system on each economic sector ($CoVaR_{out}$): with such coefficients and the Value at Risks, we then obtained six $CoVar$ time-series: for incoming and outgoing effects, and for the three economic sectors.

The differences between $\Delta CoVar$ and $CoRisk$ are shown in Figure 11.

[Figure 11 here]

The graphs show that, in general, $\Delta CoVaR$ underestimates risk with respect to our measure $CoRisk$ (both *in* and *out*) during the first two periods. This difference may be due to the fact that $CoVaR$ is based on the correlations between extreme, and possibly unfrequent events (tails of the distributions), and does not consider the effects of an homogenous increase in the default probabilities across countries.

For this reason, $\Delta CoVaR$ does not distinguish between the pre-crisis and the sovereign-crisis periods, being almost equal to zero in both of them: $CoRisk$, on the contrary, is very low until 2006 while it increases afterwards, because it takes into account not only correlations, but also the increasing levels of the PDs. Furthermore, $\Delta CoVaR$ overestimates both incoming and outgoing risk effects for those economic sectors whose distributions considerably differ from the others: this is, for example, the case of Portuguese and Greek sovereigns during the last years. Finally, $\Delta CoVaR$ does not really differentiate incoming and outgoing contagion effects: both of them are extremely high for Greece since 2012, even if it is reasonable to assume that Greece is more an exporter, rather than an importer, of systemic risk.

In this framework, we believe that $CoRisk$ better captures contagion effects, consistently across time, across countries and economic sectors, and between incoming and outgoing effects.

7 Conclusions

In this work we have proposed a new systemic risk measurement model, based on multivariate stochastic processes, default probabilities and correlation networks. The model has been applied to the economies of the European monetary union in the recent time period. For each country we have considered three economic sectors (sovereigns, corporates and banks), and we have modelled each of them as a linear combination of two stochastic processes: a country-specific idiosyncratic component and a common systematic factor. We have built a partial correlation network within each sector, thus deriving a statistical representation of the transmission mechanisms of systemic risk that correctly takes into account interdependence effects. We have then derived the default probability of each economic sector in each country, both unconditionally and conditionally on the network structure: the comparison between them allows the definition of a novel risk indicator, $CoRisk$, that explicitly measures the contagion effect on the probability of default of each economic sector of a country.

The proposed methodology seems quite effective and efficient, particularly when compared to alternative network based measures, such as the node degree and the eigenvector centrality, and to classical systemic risk measures such as $CoVaR$.

From an applied viewpoint, our main economic findings for the Euro area can be summarized according to three dimensions: (a) the economic sector dimension, (b) the country dimension and (c) the time dimension at the aggregate country level.

Concerning (a), the corporate sector is strongly influenced by the systematic component, and this implies that it responds to monetary policy changes more than sovereigns and banks. On the other hand, the sovereign and bank sectors deeply suffered, respectively, the sovereign and the financial crisis. For both sectors, the sovereign crisis has generated two distinct clusters, characterized by positive within and negative cross correlations, clearly separating peripheral and core economies. Such separation creates loop effects within each cluster, further alienating troubled and strong economies. In a situation in which core economies benefit from positive contagion while peripheral countries suffer negative contagion, risk propagation does not act as a mean for balancing inequalities across countries; on the contrary, it weakens the weakest and strengthens the strongest countries.

Concerning (b), core countries mostly behave as importers, rather than exporters of system risk. As a consequence, core economies are mostly affected by contagion risk and are rather vulnerable than systemic important; peripheral countries, instead, strongly suffer high sector-specific default probabilities and high contagion deriving from cluster effects, so they are both vulnerable and systemic important.

Concerning (c) the sovereign crisis has had a larger impact on systemic risk with respect to the financial crisis. A possible explanation consists in different ways peripheral and core economies reacted to the financial crisis: peripheral countries, with high public debts, had little fiscal space to improve balance sheets and, therefore, the financial crisis triggered their imbalances to emerge in the subsequent sovereign crisis. The time sequence of these two events has determined an irreversible phase change, leading to a new non-stable equilibrium, where instability derives from peripheral-countries trajectories diverging from core ones.

8 Acknowledgements

We acknowledge the financial support of the PRIN MISURA project. The work is based on the DREAMT PhD thesis research of Laura Parisi, under the supervision of Paolo Giudici.

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A Appendix

A.1 TPD and $CoRisk$ derivation

Since we want to consider the default probability of each economic sector as a function of both its sector-specific PD and contagion effects coming from neighbours, the first step consists in deriving the functional form for TPD_t^m as in (4.5).

Lemma A.1.1. *Given an undirected graph $G = (P, E)$ with vertex set $P = V \times W$ and edge set $E = P \times P$, given the weights PD_t^m of each node $m \in V \times W$ at time t and a matrix P_t of partial correlation coefficients, $\rho_{mn|S}$, measuring the strength of each edge $e \in E$ at time t; the total default probability of each node m, conditional on non-defaulted neighbours in the previous time, can be expressed as a function if its weight PD_t^m, its neighbours' weights PD_t^n ($n \neq m$) and their partial correlations $\rho_{mn|S}$ as follows:*

$$TPD_{t+1}^m = 1 - (1 - PD_t^m) \cdot \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn|S}}. \tag{A.1}$$

Proof. Let m be the node for which we want to measure the contagion effect, and n be any other node in the graph G which may have an effect on m because of their partial correlation $\rho_{mn|S}$. According to (4.3), let us define the functional form $f(x^m, t) = 1 - e^{-x_t^m}$, so that $PD_t^n = f(Z^n, t)$. We can now introduce a random variable $\tilde{Z}_{t+1}^m$ such that $TPD_{t+1}^m = f(\tilde{Z}^m, t+1)$. Without loss of generality, the linear combination in (3.10) can be rewritten by substituting Z^m with $\tilde{Z}_{t+1}^m = f^{-1}(TPD^m, t+1)$ and Z^n with $f^{-1}(PD^n, t)$. Furthermore, we can consider the sector-specific contribution $a^m = f^{-1}(PD^m, t)$ as a fixed effect (baseline Z^m), and we can approximate the regression coefficients $a_{mn|S}$ with their geometric average $\rho_{mn|S}$ as a consequence of the partial correlation property (3.11). Doing so, we obtain the following system:

$$\begin{cases} \tilde{Z}_{t+1}^m = Z_t^m + \sum_{n \neq m} \rho_{mn|S} Z_t^n \\ TPD_{t+1}^m = 1 - e^{-\tilde{Z}_{t+1}^m} \\ Z_t^m = -\ln(1 - PD_t^m), \end{cases}$$

and, consequently,

$$TPD_{t+1}^m = 1 - e^{-(Z_t^m + \sum_{n \neq m} \rho_{mn|S} Z_t^n)} = 1 - (1 - PD_t^m) \cdot \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn|S}}.$$

□

After having derived the total default probability associated to each economic sector in each country, we are now interested in extracting the contagion component from it. More precisely, we want to identify a variable able to measure to what extent the total default probability of node m depends on all the other nodes $n \neq m$. This variable, named incoming contagion risk ($CoRisk^m_{in,t}$), needs to have some properties: (a) it has to depend on the neighbours $n \neq m$ and not on m itself; (b) it has to be related to TPD^m since it is part of it; (c) it has to measure contagion risk and, therefore, it has to be an increasing function of both PD^n and $\rho_{mn|S}$. As a consequence, we propose the following.

Definition A.1.2. Given the assumptions in Lemma A.1.1 and $\forall m \in V \times W$, the incoming contagion risk conditional on defaulted neighbours in the previous time, $CoRisk^m_{in,t}$, is defined as the TPD^m component which measures to what extent the default probabilities of the other nodes are transmitted to node m:

$$CoRisk^m_{in,t} = 1 - \prod_{n \neq m} (1 - PD^n_t)^{\rho_{mn|S}}. \tag{A.2}$$

The following Lemma derives the relationship between incoming contagion risk and the total default probability.

Lemma A.1.3. *The incoming contagion risk, $CoRisk^m_{in,t}$, can be interpreted as the percentage variation in the survival probability of node m due to contagion effects:*

$$CoRisk^m_{in,t} = \frac{(1 - PD^m_t) - (1 - TPD^m_{t+1})}{1 - PD^m_t}. \tag{A.3}$$

Proof. From (A.2) the following can be derived:

$$\prod_{n \neq m} (1 - PD^n_t)^{\rho_{mn|S}} = 1 - CoRisk^m_{in,t}.$$

By substituting it into (A.1) the result is

$$TPD^m_{t+1} = 1 - (1 - PD^m_t) \cdot (1 - CoRisk^m_{in,t}),$$

and, after having rearranged terms, equation (A.3) can be obtained and the Lemma is proven. □

Similarly to what has been proposed for measuring incoming contagion effects, we now focus on outgoing contagion: the objective is to jointly estimate not only to what extent node m is affected by, but also affects the other nodes in the network. For this reason we define outgoing contagion as follows.

Definition A.1.4. Given the assumptions in Lemma A.1.1 and $\forall m \in V \times W$, the outgoing contagion risk conditional on not having defaulted in the previous

time, $CoRisk^{m}_{out,t}$, measures to what extent the default probability of node m is transmitted to the other nodes $n \neq m$:

$$CoRisk^{m}_{out,t} = 1 - \prod_{n \neq m} (1 - PD^{m}_{t})^{\rho_{nm|S}} = 1 - (1 - PD^{m}_{t})^{\sum_{n \neq m} \rho_{nm|S}}. \tag{A.4}$$

Incoming and outgoing contagion risks, as well as total default probabilities, have been derived for each economic sector, without considering correlations between different economic sectors of the same country. Our aim is to build a total default probability, aggregated over different sectors and able to measure the total default probability of an entire country. Since it has to be based on contagion risk, sector-specific risk and intra-country correlations, the following can be shown.

Lemma A.1.5. *Given the sets of default events for the corporate ($A^{1,i}_{t}$), bank ($A^{2,i}_{t}$) and sovereign ($A^{3,i}_{t}$) sector for each country i at time t, and given the system set $S^{i}_{t} = \{A^{m}_{t}; \forall m \in V \times W, m \in ne(i,j), m \neq (i,j)\}$, the total default probability aggregated at the country level is:*

$$TPD^{i}_{country,t} = 1 - \prod_{\substack{j,j'=1 \\ j'>j}}^{3} (1 - TPD^{j,i}_{t} | \bar{A}^{j'}), \tag{A.5}$$

where the three probabilities $TPD^{j,i}_{t}$ are calculated through (A.1) *conditional on, respectively, all the other nodes, all the other nodes but the corporate sector of the same country i, all the other nodes but the corporate and bank sectors of the same country i.*

Proof. First, the $TPDs$ of a sector can be treated as conditional probabilities on default events of other sectors. Second, in order to consider correlations within different economic sectors belonging to the same country, we assume that a country will default if at least one of its economic sectors is in default. By combining these two issues, our objective function becomes the following probability:

$$Pr(A^{1,i}_{t} \cup A^{2,i}_{t} \cup A^{3,i}_{t} | S^{i}_{t}) = 1 - Pr(\bar{A}^{1,i}_{t} \cap \bar{A}^{2,i}_{t} \cap \bar{A}^{3,i}_{t} | S^{i}_{t}). \tag{A.6}$$

By exploiting conditional probability definition, the following holds:

$$\begin{aligned} Pr(\bar{A}^{1,i}_{t} \cap \bar{A}^{2,i}_{t} \cap \bar{A}^{3,i}_{t} | S^{i}_{t}) &= Pr(\bar{A}^{1,i}_{t} \cap \bar{A}^{2,i}_{t} | \bar{A}^{3,i}_{t}, S^{i}_{t}) \cdot Pr(\bar{A}^{3,i}_{t} | S^{i}_{t}) = \\ &= Pr(\bar{A}^{1,i}_{t} | \bar{A}^{2,i}_{t}, \bar{A}^{3,i}_{t}, S^{i}_{t}) \cdot Pr(\bar{A}^{2,i}_{t} | \bar{A}^{3,i}_{t}, S^{i}_{t}) \cdot Pr(\bar{A}^{3,i}_{t} | S^{i}_{t}). \end{aligned}$$

Recalling that A^{i} identifies defaults and $\bar{A}^{i}$ is its complementary set, (A.6) can be rewritten as follows:

$$
\begin{aligned}
TPD^i_{country} &= Pr(A^{1,i}_t \cup A^{2,i}_t \cup A^{3,i}_t | S^i_t) = \\
&= 1 - [(1 - Pr(A^{1,i}_t | \bar{A}^{2,i}_t, \bar{A}^{3,i}_t, S^i_t)) \cdot (1 - Pr(A^{2,i}_t | \bar{A}^{3,i}_t, S^i_t)) \cdot (1 - Pr(A^{3,i}_t | S^i_t))],
\end{aligned} \tag{A.7}
$$

where the conditional default probabilities can be substituted with the total default probabilities $TPD^{j,i}_t$ which are, by definition, conditional on the system S^i.

□

Lemma A.1.6. *The total survival probability aggregated at the country level, $1 - TPD^i_{country,t+1}$, can be disentangled in its components, according to the reference economic sector j and to the source of risk (sector-specific or deriving from contagion), as follows:*

$$
\ln(1 - TPD^i_{country,t+1}) = \sum_{j=1}^{3} \ln(1 - PD^{j,i}_t) + \sum_{\substack{j,j'=1 \\ j'>j}}^{3} \ln(1 - CoRisk^{j,i}_t | \bar{A}^{j'}) \tag{A.8}
$$

Proof. Lemma A.1.3 and Lemma A.1.5 provide the following system:

$$
\begin{cases}
TPD^{j,i}_{t+1} = 1 - (1 - PD^{j,i}_t) \cdot (1 - CoRisk^{j,i}_{in,t}), \\
TPD^i_{country,t+1} = 1 - [1 - (TPD^{1,i}_{t+1} | \bar{A}^{2,i}_t, \bar{A}^{3,i}_t, S^i_t)] \cdot [1 - (TPD^{2,i}_{t+1} | \bar{A}^{3,i}_t, S^i_t)] \cdot [1 - TPD^{3,i}_{t+1} | S^i_t)],
\end{cases} \tag{A.9}
$$

Remembering that $PD^{j,i}_t$ are sector-specific default probabilities and are thus independent from other sectors or countries, the solution of the system is:

$$
\begin{aligned}
1 - TPD^i_{country,t+1} =& (1 - PD^{1,i}_t)(1 - CoRisk^{1,i}_{in,t} | \bar{A}^{2,i}_t, \bar{A}^{3,i}_t, S^i_t) \cdot \\
& \cdot (1 - PD^{2,i}_t)(1 - CoRisk^{2,i}_{in,t} | \bar{A}^{3,i}_t, S^i_t) \cdot (1 - PD^{3,i}_t)(1 - CoRisk^{3,i}_{in,t}).
\end{aligned} \tag{A.10}
$$

By applying a logarithmic transformation the result is:

$$
\begin{aligned}
\ln(1 - TPD^i_{country,t+1}) =& \ln(1 - PD^{1,i}_t) + \ln(1 - PD^{2,i}_t) + \ln(1 - PD^{3,i}_t) + \\
& + \ln(1 - CoRisk^{1,i}_{in,t} | \bar{A}^{2,i}_t, \bar{A}^{3,i}_t, S^i_t) + \ln(1 - CoRisk^{2,i}_{in,t} | \bar{A}^{3,i}_t, S^i_t) + \\
& + \ln(1 - CoRisk^{3,i}_{in,t}),
\end{aligned} \tag{A.11}
$$

and the Lemma is proven.

□

Corollary A.1.7. *If a country i is not in default and if $|CoRisk_t^{i,j}| < 1$, the total default probability aggregated at the country level, $TPD^i_{country,t+1}$, can be disentangled in its components, according to the reference economic sector j and to the source of risk (sector-specific or deriving from contagion), as follows:*

$$TPD^i_{country,t+1} \approx \sum_{j=1}^{3} PD_t^{j,i} + \sum_{\substack{j,j'=1 \\ j'>j}}^{3} (CoRisk_t^{j,i} | \bar{A}^{j'}) \tag{A.12}$$

Proof. The result can be directly derived by applying a first-order Taylor expansion to the logarithmic function in (A.8). Since, by definition, default probabilities have always values in [0,1], the following constraints must be added in order to approximate logarithms with linear functions:

$$\begin{cases} TPD^i_{country,t+1} \neq 1, \\ PD_t^{j,i} \neq 1, \\ |CoRisk_t^{j,i}| < 1 \qquad \forall t, \forall i \in V, \forall j \in W. \end{cases} \tag{A.13}$$

□

Remark: Consistently with the application of this paper, Lemma A.1.5, A.1.6 and Corollary A.1.7 have been proposed for a system composed by N countries and 3 economic sectors. However, they can be easily generalized for a $N \times M$ system, with economic sectors $j \in W = \{1, ..., M\}$.

A.2 *CoRisk* properties

From a mathematical viewpoint, *CoRisk* (both *in* and *out*) is a non-linear and asymmetric function of partial correlations and default probabilities. Remembering that $\rho_{mn|S} \in [-1, 1]$ and $PD \in [0, 1]$, *CoRisk* is a function $f : \Re^2 \to \Re$ and, in particular, $CoRisk = f(x, y) : [-1, 1] \times [0, 1] \to (-\infty, 1]$.

In order to better interpret the CoRisk measure, it is important to study its limit conditions. More precisely, $CoRisk_{in}$ is equal to zero when, one of the two following conditions holds:

$$\begin{cases} PD^n = 0, \qquad \forall n \in ne(m); \\ \rho_{mn|S} = 0, \qquad \forall n \neq m. \end{cases} \tag{A.14}$$

This is consistent with the definition of $CoRisk_{in}$, meaning that the contribution to the default probability of a country m that derives from contagion effects is null (a) if all its neighbours have zero default probability, or (b) if country m is not partially related to any other country. Secondly, $CoRisk_{in}$ reaches its highest value 1 if $\exists\, n \in ne(m)$ *s.t.* $PD^n = 1$, meaning that the highest contribution to the vulnerability of node m occurs when at least one of the other nodes

n is in default. Finally, it is interesting to observe that $CoRisk_{in}$ is negative (the so-called *positive contagion*) when negative partial correlations prevail: in particular, $CoRisk_{in} \to -\infty$ if $\exists\ n \in ne(m)$ $s.t.$ the two following conditions simultaneously hold:

$$\begin{cases} PD^n \to 1, \\ \rho_{mn|S} \to -1. \end{cases} \tag{A.15}$$

Similarly, the systemic importance of node m is null ($CoRisk^m_{out} = 0$) when m is not connected to any node or when its sector-specific default probability is equal to zero. Under the hypothesis that $\sum_{n \in ne(m)} \rho_{nm|S} \neq 0$, $CoRisk^m_{out}$ reaches its maximum point when $PD^m = 1$, meaning that the highest systemic importance of node m occurs when m itself is in default and is not an isolated point. On the other hand, when negative correlations prevail node m positively affects its neighbours, overall decreasing their default probability.

B Tables and Figures

Table 1: Interest rates: pre-crisis and financial-crisis periods

Pre-crisis Period									
	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
Country	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	3.866	0.368	-0.033	4.096	0.289	0.371	3.248	0.189	-0.463
Bel	3.894	0.366	-0.041	4.525	0.225	0.171	4.117	0.251	-0.415
Fin	3.845	0.381	0.009	3.640	0.312	0.791	2.664	0.225	0.202
Fra	3.859	0.352	-0.020	4.351	0.159	0.399	3.669	0.102	-0.389
Ger	3.806	0.352	0.008	4.982	0.189	-0.065	3.142	0.274	-0.540
Gre	4.045	0.343	0.109	5.659	0.264	0.880	0.402	0.117	0.606
Ire	3.826	0.378	-0.002	4.675	0.372	0.634	2.564	0.202	0.806
Ita	4.027	0.349	0.096	4.538	0.307	0.654	3.131	0.258	0.140
Net	3.843	0.362	-0.015	4.693	0.207	0.272	3.971	0.269	-0.074
Por	3.919	0.358	0.071	4.548	0.321	0.929	3.033	0.252	0.301
Spa	3.850	0.362	-0.019	3.619	0.324	0.780	2.487	0.174	0.144

Financial-crisis Period									
	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
Country	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	4.198	0.298	0.752	4.531	0.943	0.967	3.352	0.179	0.383
Bel	4.216	0.322	0.834	4.754	0.671	0.986	4.009	0.156	0.427
Fin	4.107	0.350	0.847	4.378	1.103	0.980	2.968	0.355	0.838
Fra	4.063	0.370	0.852	4.710	0.587	0.956	3.565	0.059	0.353
Ger	3.808	0.502	0.840	4.961	0.579	0.986	2.688	0.047	0.726
Gre	4.826	0.437	-0.417	6.326	0.777	0.972	1.386	0.523	-0.175
Ire	4.686	0.481	-0.668	5.321	1.274	0.990	2.427	0.457	0.748
Ita	4.494	0.268	0.644	5.208	1.062	0.968	3.182	0.533	0.972
Net	4.067	0.352	0.846	4.751	0.797	0.994	3.758	0.053	0.153
Por	4.385	0.292	0.605	5.416	1.031	0.949	3.058	0.407	0.930
Spa	4.218	0.278	0.756	4.873	0.789	0.904	2.717	0.234	0.528

Notes: summary statistics for interest rates on government bonds ($y^i_{t,1}$), interest rates on loans to non-financial corporates ($y^i_{t,2}$) and interest rates on deposits to families and non-financial corporates ($y^i_{t,3}$) during the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009), for 11 Eurozone countries. Means, standard deviations and correlations with Euribor interest rates have been reported.

Table 2: Interest rates: sovereign-crisis and post-crisis periods

Sovereign-crisis Period									
	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
Country	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	2.972	0.587	0.586	2.845	0.232	0.934	2.297	0.104	0.256
Bel	3.565	0.660	0.877	3.460	0.169	0.920	3.182	0.177	-0.136
Fin	2.634	0.642	0.499	2.450	0.269	0.971	2.138	0.129	-0.232
Fra	2.992	0.482	0.618	3.318	0.144	0.823	3.150	0.080	0.023
Ger	2.282	0.697	0.383	3.837	0.191	0.933	2.564	0.095	0.500
Gre	15.780	6.526	0.011	5.649	0.553	0.313	2.491	0.312	-0.265
Ire	7.171	2.136	0.832	3.323	0.281	0.869	1.939	0.399	-0.182
Ita	4.984	0.891	0.305	3.505	0.332	0.221	2.784	0.441	-0.470
Net	2.638	0.622	0.469	3.436	0.192	0.991	3.801	0.139	0.026
Por	8.728	2.953	0.426	4.264	0.676	0.203	2.511	0.456	-0.172
Spa	5.179	0.815	-0.008	3.532	0.260	0.404	2.486	0.282	0.051

Post-crisis Period									
	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
Country	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	1.430	0.608	0.829	2.323	0.097	0.969	1.681	0.206	0.837
Bel	1.676	0.740	0.854	2.851	0.190	0.950	2.697	0.222	0.847
Fin	1.357	0.564	0.864	1.870	0.110	0.964	1.587	0.330	0.786
Fra	1.589	0.650	0.856	2.725	0.194	0.856	2.864	0.124	0.835
Ger	1.091	0.521	0.857	3.085	0.204	0.911	2.015	0.192	0.833
Gre	8.943	1.881	-0.293	5.442	0.366	0.921	2.161	0.971	0.891
Ire	2.485	1.158	0.816	3.095	0.071	-0.496	1.892	0.280	0.630
Ita	3.014	1.133	0.801	3.511	0.237	0.947	2.896	0.368	0.578
Net	1.387	0.612	0.850	2.908	0.174	0.953	3.560	0.170	0.820
Por	4.205	1.708	0.718	4.108	0.338	0.919	2.730	0.434	0.952
Spa	3.045	1.266	0.728	3.129	0.360	0.935	2.258	0.348	0.800

Notes: summary statistics for interest rates on government bonds ($y^i_{t,1}$), interest rates on loans to non-financial corporates ($y^i_{t,2}$) and interest rates on deposits to families and non-financial corporates ($y^i_{t,3}$) during the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015), for 11 Eurozone countries. Means, standard deviations and correlations with Euribor interest rates have been reported.

Table 3: Estimated parameters for the systematic process

	a	v	b
2003-2006	0.014	0.001	0.056
2007-2009	0.899	0.355	0.569
2010-2012	0.405	0.262	0.149
2013-2015	0.008	0.002	0.090

Notes: estimated parameters of the systematic process S_t (3-months Euribor), for the pre-crisis (2003-2006), financial-crisis (2007-2009), sovereign-crisis (2010-2012) and post-crisis (2013-2015) periods.

Table 4: Estimated parameters for the idiosyncratic processes: pre-crisis and financial-crisis periods

Pre-crisis Period

	$y_{t,1}$ (Sov)			$y_{t,2}$ (Corp)			$y_{t,3}$ (Bank)		
Country	$(\theta_1)_1$	$(\theta_2)_1$	$(\theta_3)_1$	$(\theta_1)_2$	$(\theta_2)_2$	$(\theta_3)_2$	$(\theta_1)_3$	$(\theta_2)_3$	$(\theta_3)_3$
Aus	0.319	0.085	0.073	0.365	0.092	0.029	0.101	0.035	0.009
Bel	0.351	0.093	0.075	0.481	0.108	0.034	0.143	0.039	0.015
Fin	0.348	0.093	0.079	0.053	0.014	0.032	0.237	0.093	0.018
Fra	0.388	0.103	0.077	0.828	0.191	0.041	0.998	0.285	0.033
Ger	0.390	0.105	0.079	0.305	0.063	0.012	0.173	0.065	0.021
Gre	0.433	0.109	0.076	0.006	0.001	0.028	0.012	0.001	0.035
Ire	0.333	0.090	0.077	0.190	0.041	0.036	0.011	0.001	0.042
Ita	0.392	0.099	0.074	0.252	0.057	0.030	0.075	0.041	0.039
Net	0.379	0.101	0.080	0.366	0.079	0.018	0.500	0.075	0.032
Por	0.374	0.097	0.076	0.004	0.001	0.035	0.728	0.248	0.034
Spa	0.361	0.096	0.076	0.056	0.015	0.032	0.033	0.015	0.030

Financial-crisis Period

	$y_{t,1}$ (Sov)			$y_{t,2}$ (Corp)			$y_{t,3}$ (Bank)		
Country	$(\theta_1)_1$	$(\theta_2)_1$	$(\theta_3)_1$	$(\theta_1)_2$	$(\theta_2)_2$	$(\theta_3)_2$	$(\theta_1)_3$	$(\theta_2)_3$	$(\theta_3)_3$
Aus	0.448	0.110	0.083	1.500	0.342	0.001	0.069	0.021	0.027
Bel	0.333	0.082	0.080	1.497	0.324	0.001	0.070	0.019	0.025
Fin	0.235	0.061	0.081	1.487	0.356	0.001	1.507	0.514	0.001
Fra	0.238	0.062	0.081	1.500	0.216	0.001	0.522	0.022	0.515
Ger	0.145	0.045	0.093	1.243	0.319	0.001	0.401	0.012	0.523
Gre	0.760	0.150	0.108	1.518	0.320	0.002	1.096	0.022	1.480
Ire	0.755	0.157	0.109	1.494	0.318	0.001	0.023	0.009	0.906
Ita	0.635	0.143	0.074	1.507	0.347	0.001	0.001	0.011	0.568
Net	0.231	0.061	0.080	1.483	0.351	0.001	0.001	0.006	0.620
Por	0.796	0.183	0.091	1.507	0.288	0.001	1.497	0.099	0.001
Spa	0.708	0.169	0.083	1.521	0.311	0.001	0.033	0.005	0.756

Notes: estimated parameters of the idiosyncratic processes for sovereigns $y^m_{t,1}$ (interest rates on 10-years maturity government bonds), corporates $y^m_{t,2}$ (interest rates on loans to non-financial corporates) and banks $y^m_{t,3}$ (interest rates on deposits), during the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009).

Table 5: Estimated parameters for the idiosyncratic processes: sovereign-crisis and post-crisis periods

Sovereign-crisis Period

	$y_{t,1}$ (Sov)			$y_{t,2}$ (Corp)			$y_{t,3}$ (Bank)		
Country	$(\theta_1)_1$	$(\theta_2)_1$	$(\theta_3)_1$	$(\theta_1)_2$	$(\theta_2)_2$	$(\theta_3)_2$	$(\theta_1)_3$	$(\theta_2)_3$	$(\theta_3)_3$
Aus	1.507	0.529	0.001	0.592	0.126	0.060	1.487	0.670	0.001
Bel	1.492	0.435	0.001	0.349	0.106	0.034	0.356	0.118	0.012
Fin	0.030	0.032	0.112	0.232	0.106	0.047	0.099	0.047	0.025
Fra	0.073	0.039	0.115	0.691	0.073	0.345	1.271	0.406	0.037
Ger	0.040	0.042	0.121	0.150	0.045	0.023	0.001	0.003	0.021
Gre	1.835	0.102	0.593	0.362	0.058	0.046	0.407	0.148	0.094
Ire	0.407	0.057	0.262	0.067	0.021	0.037	0.157	0.069	0.108
Ita	0.489	0.095	0.158	0.093	0.023	0.030	0.035	0.001	0.043
Net	1.477	0.597	0.001	0.001	0.035	0.027	0.212	0.054	0.021
Por	0.624	0.061	0.234	0.156	0.029	0.037	0.027	0.001	0.027
Spa	0.710	0.129	0.162	0.115	0.031	0.028	0.038	0.008	0.019

Post-crisis Period

	$y_{t,1}$ (Sov)			$y_{t,2}$ (Corp)			$y_{t,3}$ (Bank)		
Country	$(\theta_1)_1$	$(\theta_2)_1$	$(\theta_3)_1$	$(\theta_1)_2$	$(\theta_2)_2$	$(\theta_3)_2$	$(\theta_1)_3$	$(\theta_2)_3$	$(\theta_3)_3$
Aus	0.050	0.057	0.168	0.001	0.034	0.015	0.001	0.116	0.007
Bel	0.038	0.047	0.157	0.001	0.058	0.021	0.001	0.078	0.014
Fin	0.057	0.061	0.170	0.001	0.040	0.012	0.019	0.029	0.021
Fra	0.047	0.053	0.154	0.057	0.028	0.036	0.250	0.091	0.035
Ger	0.047	0.069	0.191	1.483	0.497	0.001	0.985	0.274	0.001
Gre	1.023	0.124	0.286	0.020	0.009	0.024	1.493	0.653	0.001
Ire	0.038	0.052	0.159	0.484	0.155	0.018	0.057	0.042	0.030
Ita	0.014	0.030	0.134	0.001	0.002	0.022	0.104	0.048	0.034
Net	0.039	0.049	0.165	0.001	0.046	0.029	1.486	0.430	0.001
Por	0.104	0.051	0.188	0.001	0.070	0.015	0.001	0.107	0.026
Spa	0.066	0.054	0.135	1.511	0.493	0.001	0.001	0.142	0.014

Notes: estimated parameters of the idiosyncratic processes for sovereigns $y^m_{t,1}$ (interest rates on 10-years maturity government bonds), corporates $y^m_{t,2}$ (interest rates on loans to non-financial corporates) and banks $y^m_{t,3}$ (interest rates on deposits), during the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015).

Table 6: $CoRisk_{in}$: pre-crisis and financial-crisis periods

Pre-crisis Period

	$CoRisk_{sov}$ (%)				$CoRisk_{corp}$(%)				$CoRisk_{bank}$(%)			
Country	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	1.36	0.30	1.09	2.23	1.91	0.09	1.78	2.11	1.90	0.18	1.68	2.31
Bel	1.42	0.32	1.14	2.34	0.49	0.09	0.26	0.56	0.78	0.28	0.44	1.49
Fin	-0.01	0.01	-0.03	0.01	4.28	0.32	3.97	5.16	0.52	0.13	0.39	0.84
Fra	0.60	0.14	0.47	1.01	0.92	0.09	0.73	1.07	1.11	0.31	0.84	2.01
Ger	0.44	0.13	0.32	0.79	2.01	0.23	1.56	2.33	2.20	0.55	1.75	3.87
Gre	0.96	0.17	0.82	1.45	0.99	0.18	0.83	1.51	-0.37	0.08	-0.64	-0.30
Ire	0.89	0.20	0.71	1.46	1.21	0.13	1.08	1.54	2.09	0.25	1.65	2.56
Ita	0.91	0.16	0.78	1.36	0.92	0.07	0.83	1.10	1.90	0.15	1.78	2.30
Net	0.95	0.22	0.75	1.57	1.97	0.17	1.67	2.23	0.60	0.09	0.52	0.85
Por	0.58	0.14	0.44	0.98	1.15	0.26	0.94	1.87	-0.04	0.15	-0.21	0.30
Spa	0.51	0.12	0.40	0.86	1.25	0.12	1.10	1.54	1.52	0.30	1.25	2.34

Financial-crisis Period

	$CoRisk_{sov}$ (%)				$CoRisk_{corp}$(%)				$CoRisk_{bank}$(%)			
Country	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	1.86	0.78	0.97	3.52	-0.04	0.16	-0.54	0.13	6.86	1.56	4.08	8.98
Bel	4.82	1.73	2.46	8.19	-0.20	0.36	-0.75	0.14	4.17	0.55	3.07	4.98
Fin	3.98	1.40	2.02	6.62	2.05	1.61	0.11	5.35	3.60	0.91	1.98	4.83
Fra	6.66	2.65	3.76	12.13	1.26	0.52	0.43	2.10	5.32	1.40	2.84	7.11
Ger	-2.18	0.97	-4.11	-1.00	5.94	2.49	2.02	9.40	7.79	1.76	4.32	10.06
Gre	3.19	1.34	1.71	6.00	2.12	1.28	0.58	4.63	3.19	1.15	1.24	4.58
Ire	1.28	0.69	0.34	2.29	5.44	2.34	2.56	10.83	-1.78	0.63	-2.52	-0.62
Ita	-0.06	0.67	-0.92	0.94	7.37	3.43	2.38	13.57	2.86	0.51	2.13	4.01
Net	3.10	1.10	1.32	4.40	2.06	1.30	-0.01	3.85	3.77	1.18	1.77	5.27
Por	1.96	0.71	0.74	2.80	2.70	1.25	0.75	4.79	-1.80	0.34	-2.36	-1.28
Spa	1.79	0.71	0.94	3.25	2.14	1.14	0.35	3.65	-1.07	1.05	-2.19	0.59

Notes: summary statistics of $CoRisk_{in}$ for the three economic sectors (sovereigns, corporates and banks) and during the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009). Means and standard deviations have been reported.

Table 7: $CoRisk_{in}$: sovereign-crisis and post-crisis periods

Sovereign-crisis Period

	$CoRisk_{sov}$ (%)				$CoRisk_{corp}$(%)				$CoRisk_{bank}$(%)			
Country	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	2.86	0.53	1.73	3.62	3.21	0.48	2.40	3.98	2.45	0.62	1.35	3.44
Bel	2.59	0.28	2.17	3.15	0.44	0.30	-0.16	0.87	-1.63	0.34	-2.19	-1.11
Fin	1.47	0.39	0.77	2.09	5.04	0.86	3.60	6.42	4.29	0.88	2.86	5.74
Fra	4.02	0.89	2.65	6.08	0.66	0.21	0.23	0.95	1.95	0.30	1.43	2.45
Ger	1.78	0.45	0.97	2.44	-0.40	0.28	-0.97	-0.04	-2.02	0.21	-2.40	-1.74
Gre	3.61	1.22	1.71	5.71	1.55	0.31	1.05	2.00	-1.41	0.23	-1.76	-0.97
Ire	4.99	1.29	2.78	6.72	1.70	0.28	1.32	2.17	-0.48	0.03	-0.53	-0.42
Ita	2.82	1.19	1.45	4.83	1.47	0.31	1.05	2.00	0.43	0.17	0.18	0.71
Net	1.64	0.36	0.73	2.35	2.90	0.50	2.04	3.68	0.57	0.12	0.43	0.77
Por	10.80	3.06	5.70	16.10	4.36	0.53	3.51	5.12	0.07	0.07	-0.05	0.18
Spa	8.81	1.71	6.46	12.69	3.72	0.68	2.68	4.86	1.84	0.37	1.36	2.45

Post-crisis Period

	$CoRisk_{sov}$ (%)				$CoRisk_{corp}$(%)				$CoRisk_{bank}$(%)			
Country	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	-4.60	0.82	-5.88	-3.18	2.78	0.11	2.58	2.99	0.64	0.06	0.52	0.74
Bel	6.78	1.08	4.87	8.42	0.31	0.02	0.26	0.34	2.60	0.21	2.21	2.90
Fin	-3.75	1.31	-6.79	-1.81	6.14	0.39	5.40	6.52	0.65	0.19	0.42	1.04
Fra	0.43	0.20	0.05	0.76	0.51	0.06	0.41	0.59	0.07	0.00	0.07	0.08
Ger	5.23	0.95	3.64	6.60	7.20	0.40	6.52	7.76	5.62	0.78	4.29	6.80
Gre	-0.94	0.21	-1.21	-0.53	-0.72	0.07	-0.81	-0.59	0.89	0.16	0.55	1.12
Ire	1.64	0.67	0.68	2.81	-1.26	0.05	-1.37	-1.20	0.93	0.19	0.69	1.28
Ita	1.80	0.64	0.85	2.87	-1.57	0.07	-1.72	-1.47	0.52	0.13	0.35	0.78
Net	0.82	0.24	0.46	1.19	0.62	0.11	0.45	0.79	0.18	0.24	-0.11	0.62
Por	-4.01	0.98	-5.70	-2.44	1.61	0.08	1.44	1.72	1.42	0.61	0.25	2.10
Spa	2.63	1.15	0.91	4.24	1.78	0.07	1.65	1.85	4.15	0.46	3.43	4.90

Notes: summary statistics of $CoRisk_{in}$ for the three economic sectors (sovereigns, corporates and banks) and during the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015). Means and standard deviations have been reported.

Table 8: Network centrality measures: pre-crisis and financial-crisis periods

Pre-crisis Period						
	Sovereign		Corporate		Bank	
Country	DC	Eigen.	DC	Eigen.	DC	Eigen.
Aus	0.701	0.227	0.931	0.277	1.448	0.990
Bel	0.971	0.158	0.161	0.000	0.995	0.356
Fin	0.015	0.000	2.026	1.000	0.186	0.121
Fra	0.916	0.196	0.227	0.000	0.761	0.000
Ger	0.915	0.211	1.100	0.000	1.236	1.000
Gre	1.161	1.000	0.597	0.520	-0.308	0.781
Ire	0.877	0.231	0.791	0.378	0.707	0.000
Ita	1.061	0.954	0.516	0.325	1.324	0.000
Net	1.219	0.125	1.149	0.226	0.273	0.000
Por	0.979	0.578	0.717	0.710	0.240	0.000
Spa	0.873	0.340	0.573	0.694	0.739	0.000

Financial-crisis Period						
	Sovereign		Corporate		Bank	
Country	DC	Eigen.	DC	Eigen.	DC	Eigen.
Aus	1.064	0.210	-0.106	0.099	1.610	0.894
Bel	0.721	0.010	-0.044	0.050	0.821	0.806
Fin	0.916	0.307	0.712	0.298	0.962	0.244
Fra	2.402	0.973	0.435	0.135	1.185	0.548
Ger	-0.185	1.000	1.439	0.710	1.921	0.466
Gre	1.019	0.189	0.691	0.794	1.038	0.000
Ire	0.512	0.182	1.512	0.758	0.021	0.000
Ita	0.156	0.000	2.383	0.938	0.784	0.000
Net	0.887	0.585	0.242	0.118	1.093	0.927
Por	0.451	0.277	0.639	1.000	-0.012	0.000
Spa	1.309	0.423	0.276	0.210	0.154	1.000

Notes: degree of connectivity (DC) and eigenvector centrality (Eigen.) measures referred to the three economic sectors (sovereigns, corporates and banks) during the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009).

Table 9: Network centrality measures: sovereign-crisis and post-crisis periods

Sovereign-crisis Period						
	Sovereign		Corporate		Bank	
Country	DC	Eigen.	DC	Eigen.	DC	Eigen.
Aus	1.170	0.388	1.379	0.000	1.379	0.000
Bel	0.872	0.220	-1.070	0.174	-1.070	0.000
Fin	0.651	1.000	2.592	0.698	2.592	0.000
Fra	1.663	0.000	0.911	0.000	0.911	0.000
Ger	1.009	0.862	-0.794	0.000	-0.794	0.000
Gre	0.676	0.000	-0.632	0.836	-0.632	0.217
Ire	0.514	0.000	-0.100	0.880	-0.100	0.000
Ita	0.765	0.000	0.174	0.000	0.174	0.356
Net	0.940	0.612	0.357	0.301	0.357	0.332
Por	0.813	0.000	0.047	1.000	0.047	1.000
Spa	1.614	0.000	1.204	0.516	1.204	0.482

Post-crisis Period						
	Sovereign		Corporate		Bank	
Country	DC	Eigen.	DC	Eigen.	DC	Eigen.
Aus	0.069	0.000	1.381	0.650	0.125	0.728
Bel	2.242	0.565	0.025	0.192	1.023	0.150
Fin	-0.147	0.619	2.011	1.000	0.316	0.268
Fra	0.784	0.000	0.213	0.000	-0.012	0.059
Ger	1.883	0.000	1.872	0.000	2.773	1.000
Gre	-0.705	0.000	-0.192	0.019	0.391	0.676
Ire	0.571	0.068	-0.183	0.126	0.513	0.094
Ita	1.286	0.312	0.036	0.216	0.189	0.353
Net	-0.200	0.769	0.383	0.616	-0.232	0.002
Por	0.540	1.000	0.479	0.000	0.878	0.635
Spa	0.584	0.000	0.707	0.348	1.805	0.719

Notes: degree of connectivity (DC) and eigenvector centrality (Eigen.) measures referred to the three economic sectors (sovereigns, corporates and banks) during the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015).

Table 10: Rankings comparison: pre-crisis and financial-crisis periods

Pre-crisis Period											
Sovereign				Corporate				Bank			
CoR_{in}	CoR_{out}	*DC*	*Eigen.*	CoR_{in}	CoR_{out}	*DC*	*Eigen.*	CoR_{in}	CoR_{out}	*DC*	*Eigen.*
Bel	Net	Net	Gre	Fin	Ger	Fin	Fin	Ger	Ita	Aus	Ger
Aus	Ita	Gre	Ita	Ger	Net	Net	Por	Ire	Spa	Ita	Aus
Gre	Bel	Ita	Por	Net	Fin	Ger	Spa	Ita	Ire	Ger	Gre
Net	Fra	Por	Spa	Aus	Gre	Aus	Gre	Aus	Ger	Bel	Bel
Ita	Spa	Bel	Ire	Spa	Ire	Ire	Ire	Spa	Fra	Fra	Fin
Ire	Ger	Fra	Aus	Ire	Ita	Por	Ita	Fra	Net	Spa	Ita
Fra	Aus	Ger	Ger	Por	Aus	Gre	Aus	Bel	Aus	Ire	Fra
Por	Por	Ire	Fra	Gre	Por	Spa	Net	Net	Bel	Net	Spa
Spa	Gre	Spa	Bel	Ita	Bel	Ita	Ger	Fin	Por	Por	Ire
Ger	Ire	Aus	Net	Fra	Spa	Fra	Fra	Por	Fin	Fin	Net
Fin	Fin	Fin	Fin	Bel	Fra	Bel	Bel	Gre	Gre	Gre	Por

Financial-crisis Period											
Sovereign				Corporate				Bank			
CoR_{in}	CoR_{out}	*DC*	*Eigen.*	CoR_{in}	CoR_{out}	*DC*	*Eigen.*	CoR_{in}	CoR_{out}	*DC*	*Eigen.*
Fra	Fra	Fra	Ger	Ita	Ita	Ita	Por	Ger	Aus	Ger	Spa
Bel	Bel	Spa	Fra	Ger	Ire	Ire	Ita	Aus	Ger	Aus	Net
Fin	Aus	Aus	Net	Ire	Ger	Ger	Gre	Fra	Bel	Fra	Aus
Gre	Spa	Gre	Spa	Por	Fin	Fin	Ire	Bel	Net	Net	Bel
Net	Net	Fin	Fin	Spa	Gre	Gre	Ger	Net	Fin	Gre	Fra
Por	Fin	Net	Por	Gre	Fra	Por	Fin	Fin	Gre	Fin	Ger
Aus	Gre	Bel	Aus	Net	Por	Fra	Spa	Gre	Ita	Bel	Fin
Spa	Por	Ire	Gre	Fin	Spa	Spa	Fra	Ita	Fra	Ita	Gre
Ire	Ita	Por	Ire	Fra	Bel	Net	Net	Spa	Por	Spa	Ita
Ita	Ire	Ita	Bel	Aus	Net	Bel	Aus	Ire	Ire	Ire	Ire
Ger	Ger	Ger	Ita	Bel	Aus	Aus	Bel	Por	Spa	Por	Por

Notes: rankings obtained with $CoRisk_{in}$, $CoRisk_{out}$, degree of connectivity and eigenvector centrality measures, ordered from the highest to the lowest and referred to the three economic sectors (sovereigns, corporates, banks) and to the pre-crisis period (2003-2006) and the financial-crisis period (2007-2009).

Table 11: Rankings comparison: sovereign-crisis and post-crisis periods

Sovereign-crisis Period											
Sovereign				Corporate				Bank			
CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$
Por	Gre	Fra	Fin	Fin	Spa	Fin	Por	Fin	Fin	Fin	Por
Spa	Spa	Spa	Ger	Por	Por	Aus	Ire	Aus	Spa	Aus	Spa
Ire	Por	Aus	Net	Spa	Fin	Spa	Gre	Fra	Fra	Spa	Ita
Fra	Ire	Ger	Aus	Aus	Gre	Fra	Fin	Spa	Aus	Fra	Net
Gre	Fra	Net	Bel	Net	Net	Net	Spa	Net	Net	Net	Gre
Aus	Ita	Bel	Fra	Ire	Aus	Ita	Net	Ita	Ita	Ita	Fin
Ita	Aus	Por	Spa	Gre	Ita	Por	Bel	Por	Por	Por	Aus
Bel	Bel	Ita	Por	Ita	Bel	Ire	Aus	Ire	Ire	Ire	Fra
Ger	Fin	Gre	Ita	Fra	Ire	Gre	Fra	Gre	Gre	Gre	Ire
Net	Ger	Fin	Gre	Bel	Fra	Ger	Ita	Bel	Ger	Ger	Ger
Fin	Net	Ire	Ire	Ger	Ger	Bel	Ger	Ger	Bel	Bel	Bel

Post-crisis Period											
Sovereign				Corporate				Bank			
CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$	CoR_{in}	CoR_{out}	DC	$Eigen.$
Bel	Ita	Bel	Por	Ger	Ger	Fin	Fin	Ger	Ger	Ger	Ger
Ger	Bel	Ger	Net	Fin	Fin	Ger	Aus	Spa	Spa	Spa	Aus
Spa	Ire	Ita	Fin	Aus	Aus	Aus	Net	Bel	Por	Bel	Spa
Ita	Ger	Fra	Bel	Spa	Por	Spa	Spa	Por	Bel	Por	Gre
Ire	Spa	Spa	Ita	Por	Spa	Por	Ita	Ire	Ire	Ire	Por
Net	Por	Ire	Ire	Net	Bel	Net	Bel	Gre	Fra	Gre	Ita
Fra	Fra	Por	Ger	Fra	Net	Fra	Ire	Fin	Aus	Fin	Fin
Gre	Aus	Aus	Fra	Bel	Fra	Ita	Gre	Aus	Ita	Ita	Bel
Fin	Net	Fin	Spa	Gre	Ita	Bel	Ger	Ita	Fin	Aus	Ire
Por	Fin	Net	Aus	Ire	Ire	Ire	Por	Net	Gre	Fra	Fra
Aus	Gre	Gre	Gre	Ita	Gre	Gre	Fra	Fra	Net	Net	Net

Notes: rankings obtained with $CoRisk_{in}$, $CoRisk_{out}$, degree of connectivity and eigenvector centrality measures, ordered from the highest to the lowest and referred to the three economic sectors (sovereigns, corporates, banks) and to the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015).

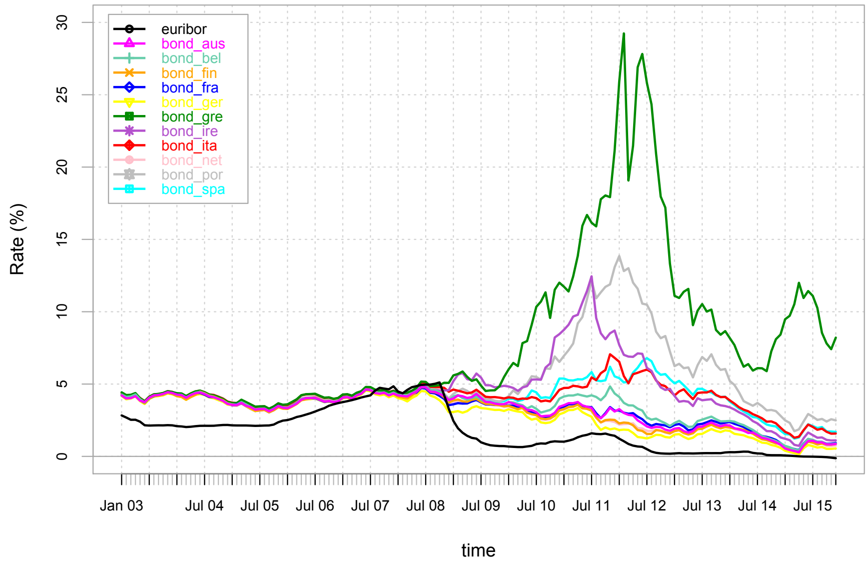
Table 12: Correlation coefficients between rankings

$CoRisk_{in}$						
	Sovereign		Corporate		Bank	
Period	*DC*	*Eigen.*	*DC*	*Eigen.*	*DC*	*Eigen.*
2003-2006	0.436	0.136	0.936	0.373	0.764	0.245
2007-2009	0.582	0.064	0.809	0.811	0.936	0.518
2010-2012	0.136	-0.736	0.691	0.655	0.982	0.345
2013-2015	0.736	0.018	0.927	0.245	0.982	0.573

$CoRisk_{out}$						
	Sovereign		Corporate		Bank	
Period	*DC*	*Eigen.*	*DC*	*Eigen.*	*DC*	*Eigen.*
2003-2006	0.527	-0.136	0.782	0.118	0.591	-0.182
2007-2009	0.791	0.109	0.982	0.709	0.755	0.318
2010-2012	-0.100	-0.818	0.445	0.618	0.973	0.445
2013-2015	0.864	0.173	0.927	0.264	0.809	0.445

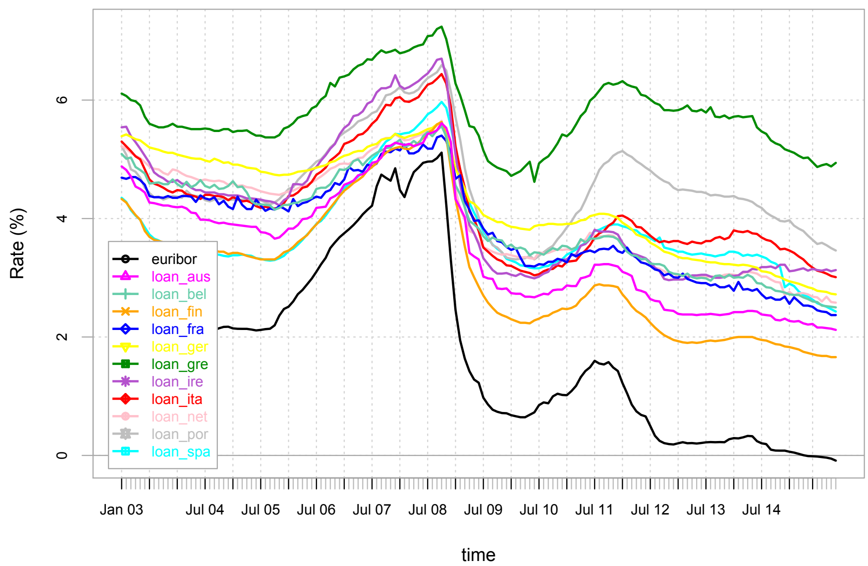
Notes: Spearman correlation coefficients between $CoRisk_{in}$ and $CoRisk_{out}$ rankings and rankings based on, respectively, degree centrality (DC) and eigenvector centrality ($Eigen.$), referred to the three economic sectors (sovereigns, corporates, banks) and to the four time-periods (pre-crisis, financial-crisis, sovereign-crisis, post-crisis).

Figure 3: Sovereigns interest rates



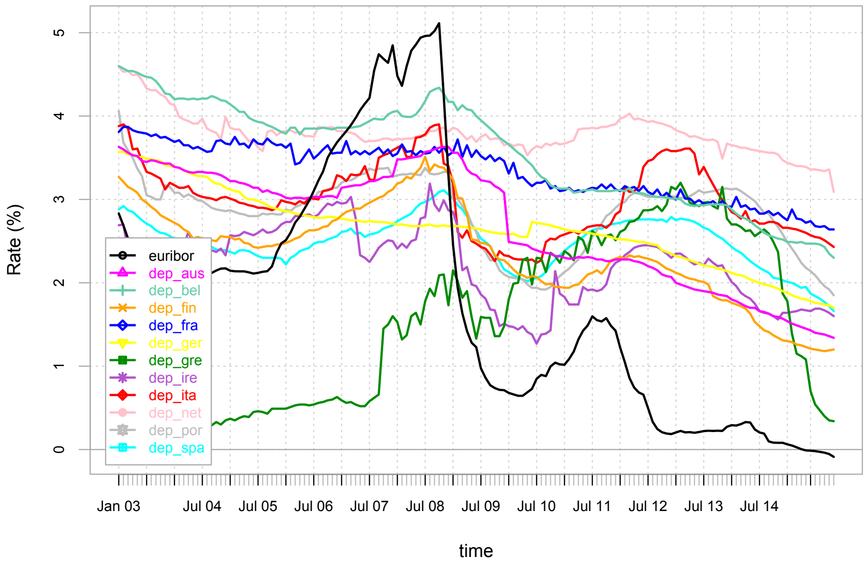
Notes: monthly time evolution of 10-years maturity bond interest rates and 3-months Euribor, from January 2003 until December 2015 and referred to 11 Eurozone countries.

Figure 4: Corporates interest rates



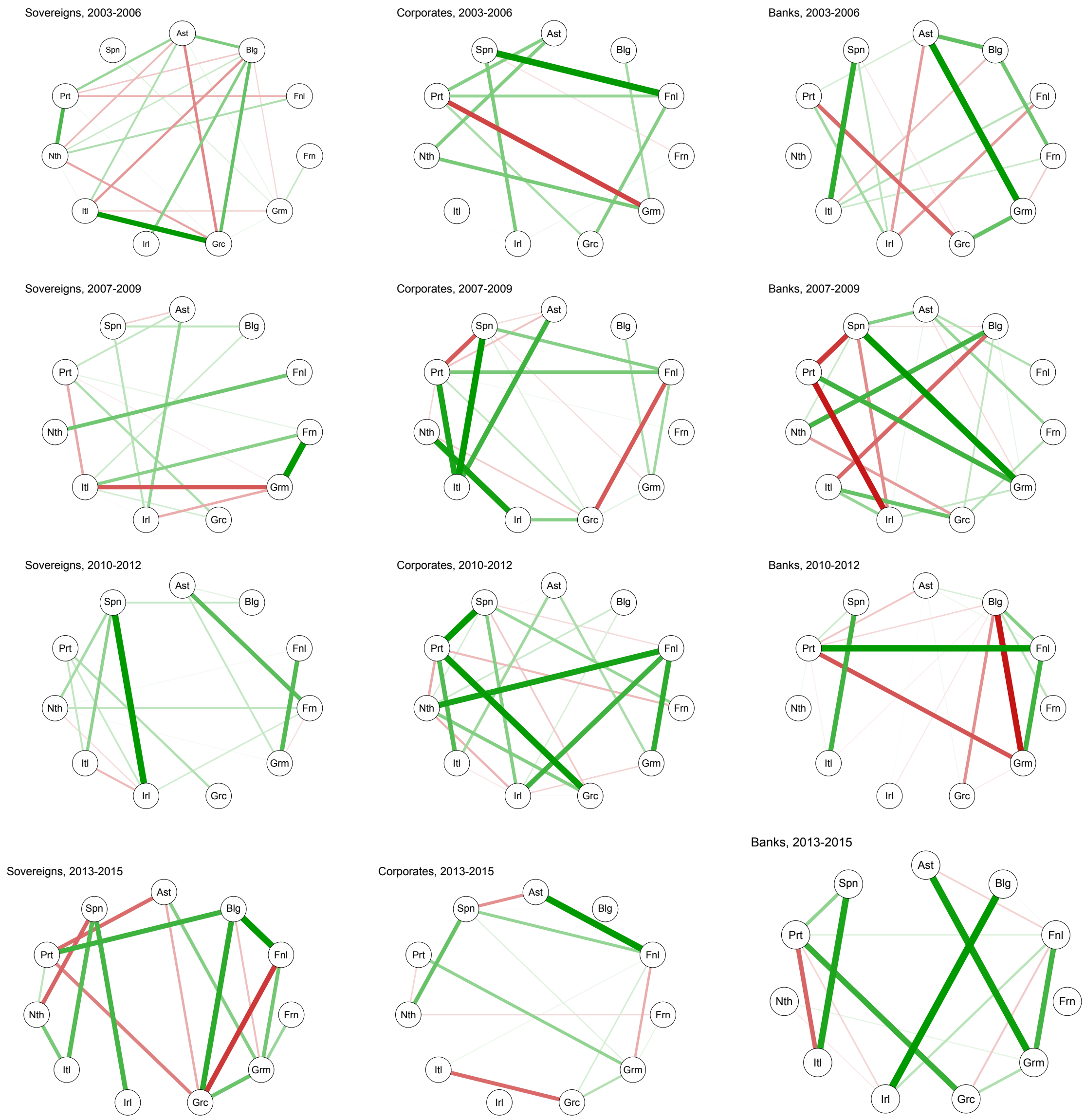
Notes: monthly time evolution of aggregate interest rates on loans to non-financial corporates and 3-months Euribor, from January 2003 until December 2015 and referred to 11 Eurozone countries.

Figure 5: Banks interest rates



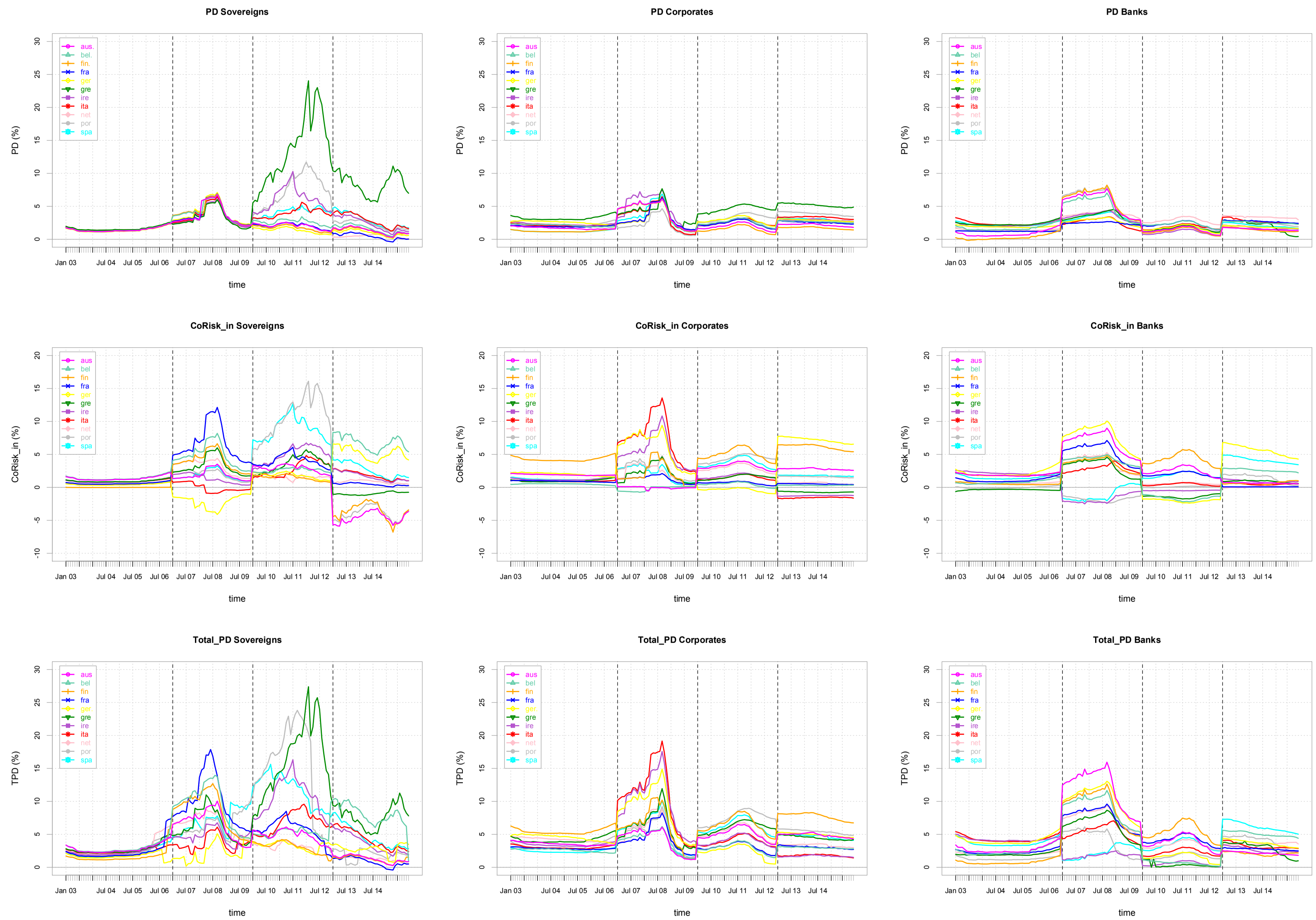
Notes: monthly time evolution of aggregate interest rates on deposits to both families and non-financial corporates, and 3-months Euribor, from January 2003 until December 2015 and referred to 11 Eurozone countries.

Figure 6: Partial correlation networks



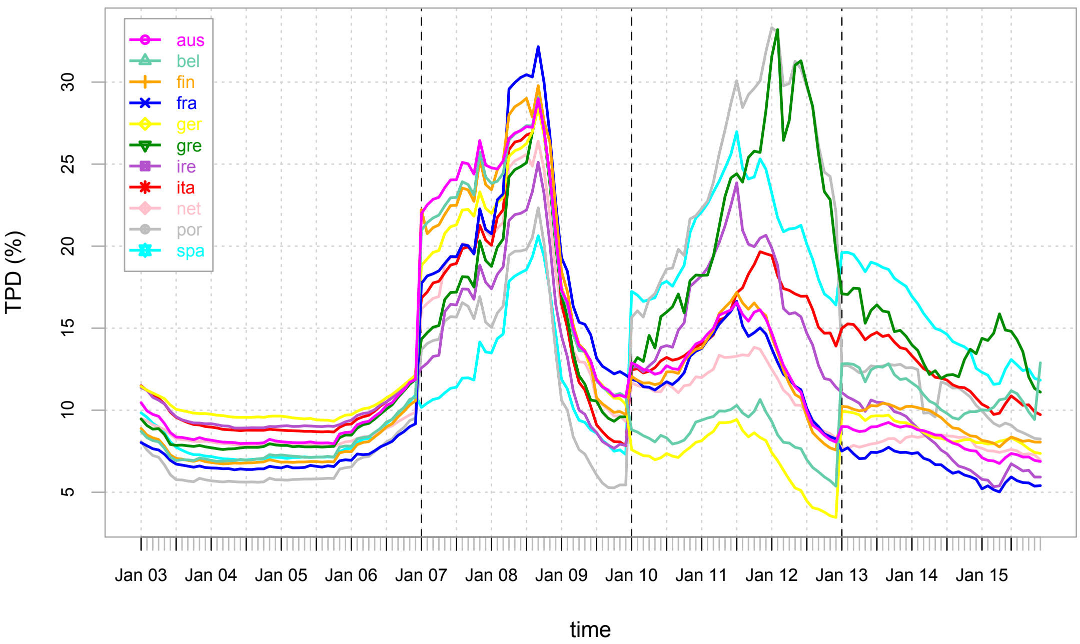
Notes: partial correlation networks for the 11 european countries considered in the sample, based on the spread measures for sovereigns $Z^i_{t,1}$ (left), corporates $Z^i_{t,2}$ (middle) and banks $Z^i_{t,3}$ (right), during the pre-crisis (first row), financial-crisis (second row), sovereign-crisis (third row) and post-crisis (fourth row) periods. Green lines stand for positive partial correlations, red lines for negative correlations; the ticker the line, the stronger the connection.

Figure 7: Sector-specific PD, $CoRisk_{in}$ and TPD



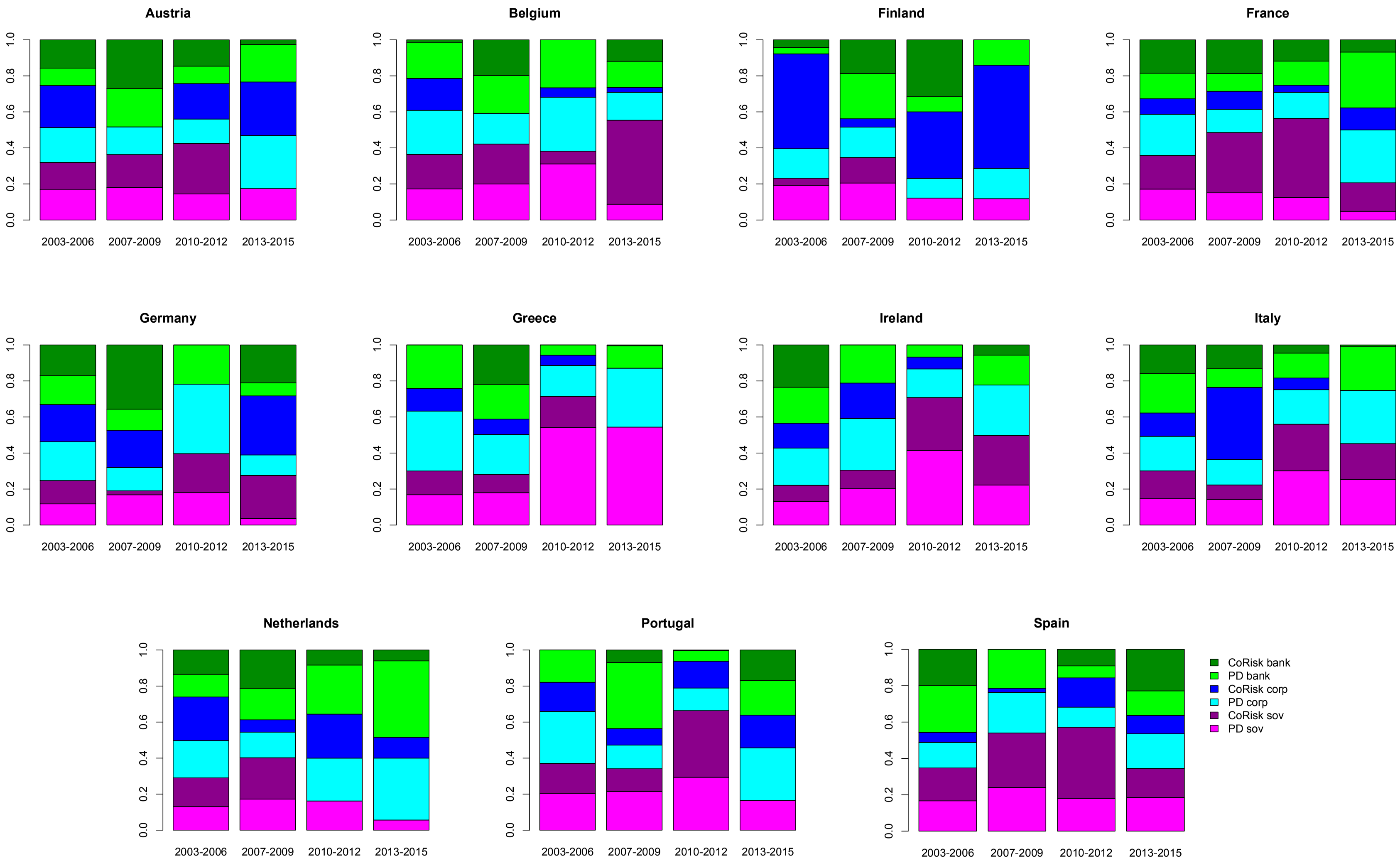
Notes: sector-specific default probabilities $PD^i_{t,\{1,2,3\}}$ (top), $CoRisk_{in}$ measures (middle) and total default probabilities $TPD^i_{t,\{1,2,3\}}$ (bottom) from 2003 until 2015, for the sovereign (left), corporate (middle) and bank (right) sectors and referred to 11 Eurozone countries.

Figure 8: Aggregate total default probabilities $TPD^i_{country}$



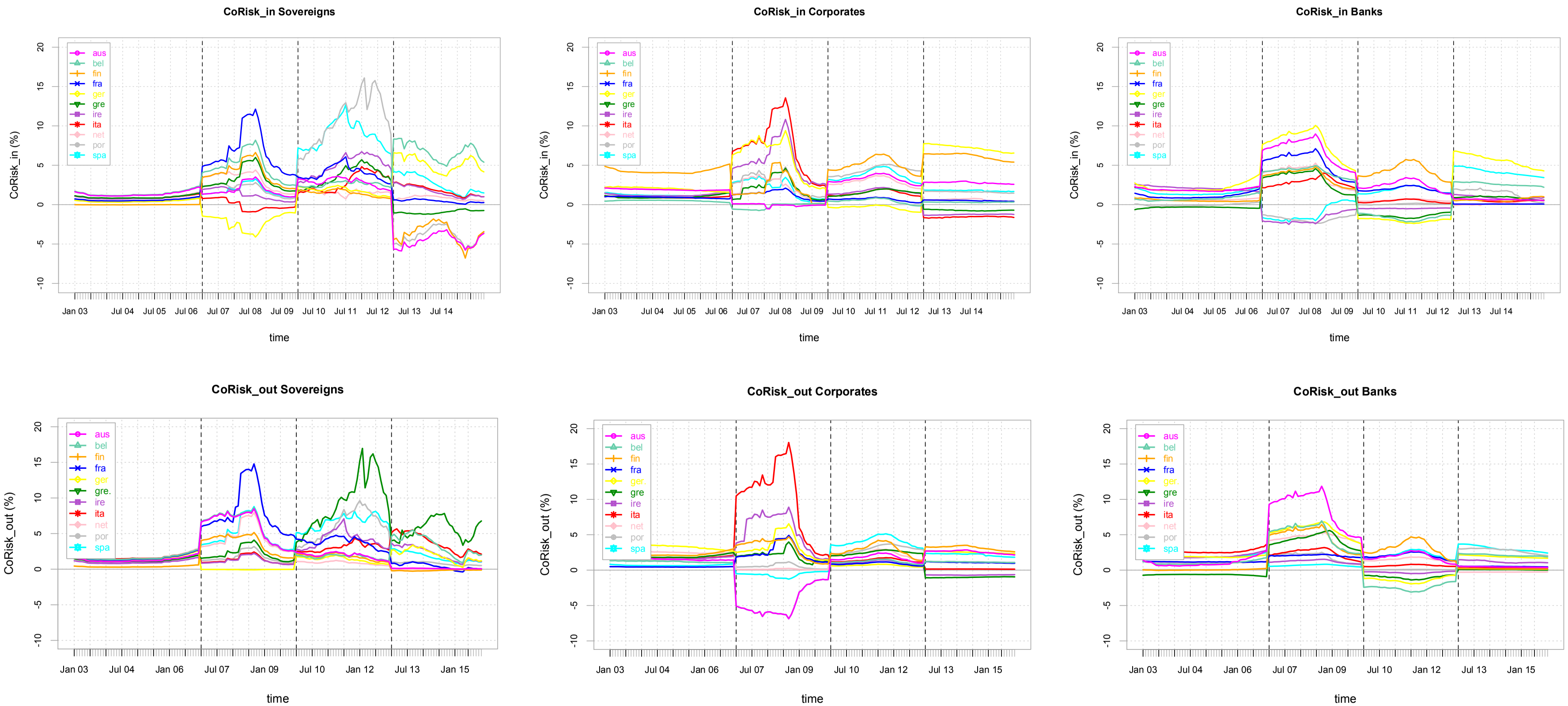
Notes: total default probabilities aggregated at the country level $TPD^i_{country}$, from 2003 until 2015 and referred to 11 Eurozone countries.

Figure 9: Risk contributions



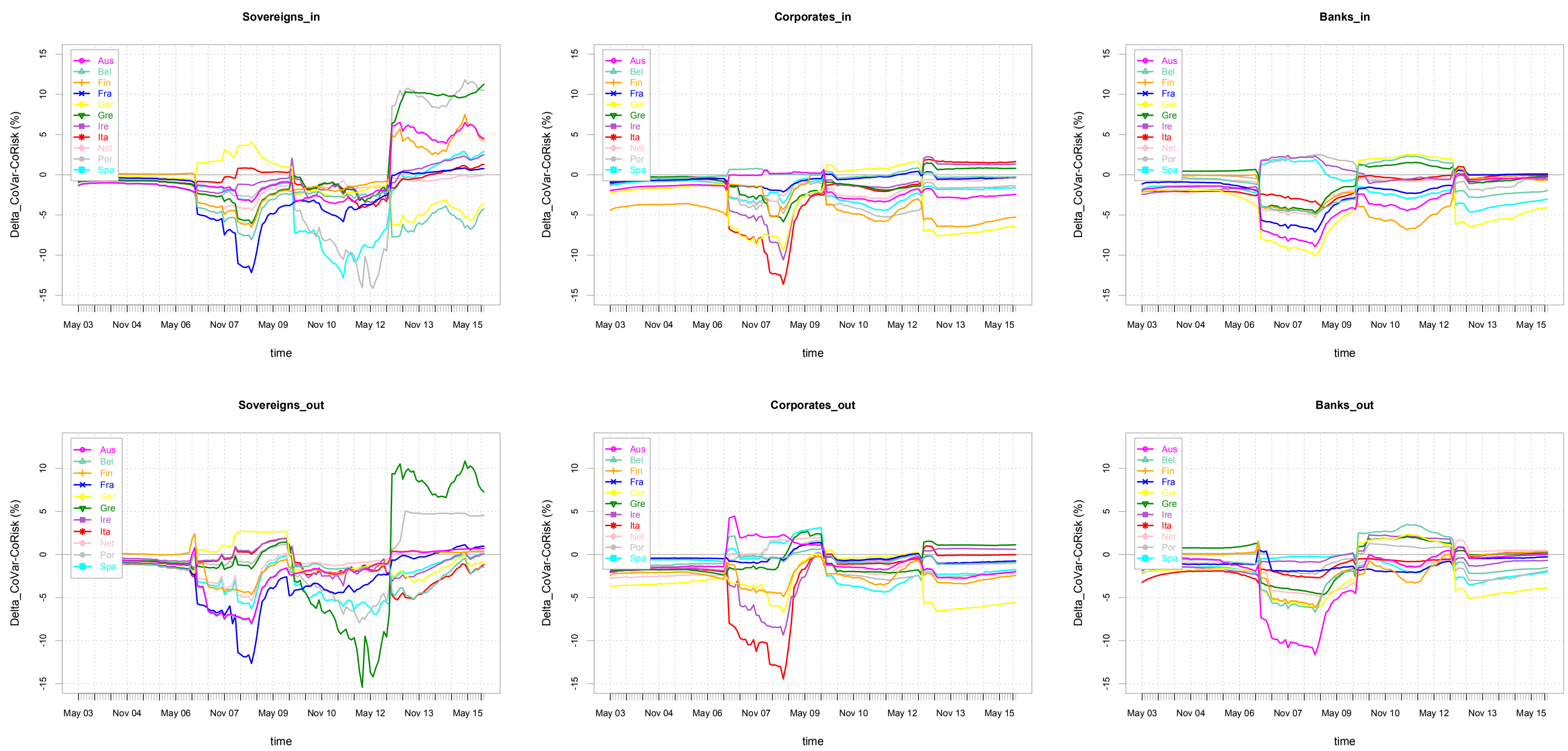
Notes: aggregate total default probabilities contributions: $CoRisk_{in}$ and PD percentage components for the three economic sectors, averaged over the four time periods (pre-crisis, financial-crisis, sovereign-crisis, post-crisis) and referred to 11 Eurozone countries.

Figure 10: $CoRisk_{in}$ vs $CoRisk_{out}$



Notes: comparison between $CoRisk_{in}$ (top) and $CoRisk_{out}$ (bottom), from 2003 until 2015 for the sovereign (left), corporate (middle) and bank (right) sectors and referred to 11 Eurozone countries.

Figure 11: $\Delta CoVar$ vs $CoRisk$



Notes: difference between $\Delta CoVaR_in$ and $CoRisk_in$ (top) and $\Delta CoVaR_out$ and $CoRisk_out$ (bottom), from 2003 until 2015 for the sovereign (left), corporate (middle) and bank (right) sectors and referred to 11 Eurozone countries ($q = 0.95$).