The Dynamics of Price Jumps in the Stock Market: an Empirical Study on Europe and U.S.*

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Abstract

We study the bivariate jump process involving the S&P 500 and the Euro Stoxx 50 with jumps extracted from high frequency data using non-parametric methods. Our analysis, based on a generalized Hawkes process, reveals the presence of self-excitation in the jump activity which is responsible for jump clustering but has a very small persistence in time. Concerning cross-market effects, we find statistically significant co-jumps occurring when both markets are simultaneously operating but no evidence of contagion in the jump activity, suggesting that the role of jumps in volatility transmission is negligible. Moreover, we find a negative relationship between the jump activity and the continuous volatility indicating that jumps are mostly detected during tranquil market conditions rather than in periods of stress. Importantly, our empirical results are robust under different jump detection methods.

1 Introduction

Prices of traded assets are sometime subject to sudden movements which are hardly described by a continuous process. Such events are commonly referred as "*Jumps*" in order to emphasize their instantaneous impact on the asset prices. They are commonly associated with a sudden flow of new information but there is no general consensus on which kind of market events

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can more likely generate discontinuous price reactions. Calcagnile et al. (2015) find that such discontinuities are only partially related to scheduled news announcements and their occurrence is largely unpredictable, Bajgrowicz et al. (2016) instead claim that jumps are rare and mostly related to news announcements. Aït-Sahalia, Cacho-Diaz, and Laeven (2015) (ADL henceforth) suggest that jump are also a vehicle of contagion across worldwide markets. This happens if asset price jumps spread from an originating region to a different one.

Nowadays, the financial literature considers the presence of jumps as a cardinal component of the asset price dynamics. As highlighted by Ait-Sahalia (2004) the study of jumps is extremely relevant for investors in terms of asset allocation and portfolio optimization as large price movements may generate significant losses and encourage the demand for higher risk premia, see Liu et al. (2003), Wright and Zhou (2009), Bates (2008), Bollerslev and Todorov (2011), Ait-Sahalia and Hurd (2015) among others. For risk management purposes jumps are important because they can generate fat tails with a significant impact on the Value at Risk (see Duffie and Pan 1997 and Pan and Duffie 2001). For asset pricing, jumps are also extremely relevant since they are responsible for market incompleteness with the implication that the jump risk cannot be perfectly hedged (Duffie et al. 2000, Eraker et al. 2003 among many others). The literature on jumps has also largely benefited from the increasing availability of high frequency data fostering a copious scientific production in the field of jump detection. Some seminal contributions include Barndorff-Nielsen and Shephard (2004, 2006), Huang and Tauchen (2005), Andersen et al. (2007b) (ABD henceforth), Andersen et al. (2010), Bollerslev et al. (2013). In these papers jumps are identified thanks to non-parametric techniques which rely on the comparison between two realized measures of volatility, one determined by continuous price changes and another one including also jumps. At a later stage, several alternative volatility measures that are also robust to jumps have been proposed: Mancini (2009) suggests a threshold based estimator, Corsi et al. (2010) (CPR henceforth) introduce a modified version of the bipower variation of Barndorff-Nielsen and Shephard (2004) and Barndorff-Nielsen and Shephard (2006) (BNS), while Andersen et al. (2012) develop new measures based on the nearest neighbor truncation. Finally, Christensen et al. (2010) introduce a quantile based estimator that generalizes the approach in Andersen et al. (2012) and it is also robust to market microstructure noise.

This paper partially follows ADL who introduced a reduced-form model for asset returns that is able to capture the time clustering of jump events within the same market (self-excitation) as well as the transmission across markets (cross-excitation). These features of jumps are of particular concern for investors, regulators and policymakers: given that jumps are an important source of risk, the study of their dynamics at the multivariate level can shed some light on their simultaneous occurrence (co-jumps) as well as on their possible transmission across different markets. However, we differ from ADL contribution and we add to the existing literature in several ways. First of all our study is based on high frequency data and modern non-parametric jump detection methods, instead of using daily returns. We also investigate jumps using multivariate Hawkes processes based on a generalized version of the method proposed by Bowsher (2007) which allows to introduce some additional explanatory variables. Interestingly, our conclusions are substantially different: we find no cross-excitation while the self-excitation is significant but exhibit a very low persistence compared to ADL. In this regard, our results are also complementary to the novel contribution of Bajgrowicz et al. (2016) who find no significant jump clustering effect at the daily time scale: according to our estimates such effects are characterized by a short persistence and can be measured only at short time scales once that the exact intraday jump times are identified. We stress the importance of taking into account the continuous volatility that, according to our evidence, exhibits an inverse relation with the jumps intensity. This indicates that the number of jumps detected during low-volatility periods is higher than in period of stress. Our results show a clear and robust evidence of the decreasing relative contribution of jumps to the total price variation during the sub-prime and the Euro Sovereign crisis. This is in stark contrast with the dynamics assumed by ADL in their parametric model where price jumps characterize periods of market turmoil. We find that the continuous intraday volatility for the S&P 500 measured from high frequency data reached an annualized level above 120% just a few days before the Lehman bankruptcy. A possible explanation for this discrepancy is that the ADL parametric approach does not take into account the potential effects generated by fast volatility changes and cross-market volatility spillovers which may substantially affect their results.

Our findings are also relevant to provide an accurate mathematical description of stock index returns, which represents a fundamental task in finance. The jump clustering as well as the absence of cross-excitation effects in the jump activity have serious consequences for portfolio optimization, risk management and option pricing. For example, if jumps propagate across markets as in the ADL model, a risk manager has to take into account the transmission mechanism to mitigate the jump risk and rational investors would require a risk premium to face the risk of contagion. Also jump clustering is remarkably important as it implies a higher probability to observe multiple jumps within the same trading day compared to a standard Poisson processes, so affecting the shape of the return distribution.

The rest of this article is organized as follows. Section 2 describes the data and Section 3 reviews the most common approaches for jump identification and test. Section 4 presents the multivariate Hawkes framework which is adopted to model the asset price dynamics and examines the presence of market co-jumps, while Section 5 concludes. Technical aspects are relegated to the two Appendixes.

2 Data description

Our data set comes from Olsen data and contains information on the S&P 500 and the Euro Stoxx 50 indexes in between 2007-09-13 and 2014-04-30; the two indexes are traded at the NYSE and at the Frankfurt Stock Exchange, respectively. The 7-year period covered by our analysis includes the sub-prime crisis leading to the bankruptcy of Lehman Brothers on September 15$^{\text{th}}$ 2008 and the subsequent European sovereign crisis in 2011. For both markets, we compute the total return from prices reported every 5 minutes. This frequency is widely recognized to offer a reasonable balance between a fine sampling frequency on the one hand and robustness to market microstructure noise on the other (see for instance Andersen et al., 2010). The NYSE and the Frankfurt Stock Exchange normally operate respectively form 9:30 to 16:00 and from 9:00 to 17:30 in local times. The first price is observed 5 minutes after the opening time. Each ordinary trading day has respectively 77 intraday returns for the S&P 500 and 100 returns for the Euro Stoxx 50. For the Euro Stoxx 50 we decide to ignore the first 10 minutes of activity due to a remarkably higher price variability compared to the rest of the day. This choice is consistent with most of the empirical literature where the first observations are usually excluded due to the potentially erratic price behavior produced by market opening procedures. We also exclude from the data set an extremely small number of days containing an anomalous number of price observations. At the end of the data cleaning process, our sample consists of 1674 trading days for the S&P 500 and 1691 for the Euro Stoxx 50. The cumulated log-return and the intraday annualized volatility measured from high frequency returns are reported in Figure 1 where we can observe the highest volatility peaks during the Sub-prime and the Euro Sovereign crisis.

3 Jumps Identification

A detailed study of the statistical features of jumps requires the identification of their occurrence in a framework that is free from any specific parametric assumption on the price evolution. The field of jump testing developed significantly in the last decade, starting from the seminal work of BNS who developed a non-parametric method which relies on the comparison between two realized measures of volatility: the bipower variation and the quadratic variation. The former is driven exclusively by continuous price changes while the latter also includes jumps. Afterwards, several alternative tests have been proposed. In order to summarize the most relevant contributions, we conveniently distinguish two main families. The first one is the BNS family which includes all the tests that are constructed by using the bipower variation or an alternative robust to jumps volatility measure . The second important family has been introduced by Lee and Mykland (2008) (LM henceforth) and is based on

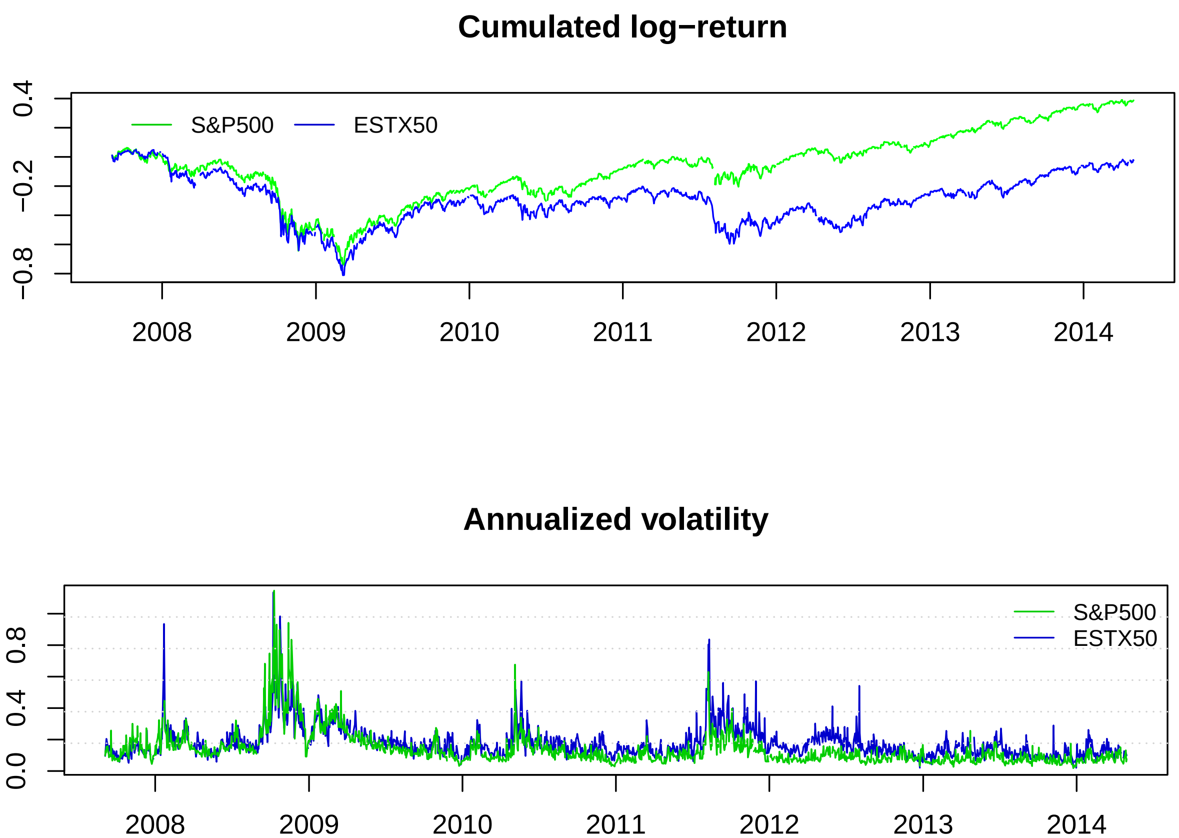

Figure 1: The top panel exhibits the time series of cumulated log-returns (including the overnight period). The bottom panel reports the time series of the annualized volatility.

the idea that jumps can be identified when a return exceeds a certain threshold determined adaptively according to the instantaneous volatility. The BNS family of tests includes the contributions of Corsi, Pirino, and Renò (2010), Andersen, Dobrev, and Schaumburg (2012) and Podolskij and Ziggel (2010). Conversely, the proposals of Andersen, Bollerslev, and Diebold (2007b), Bollerslev et al. (2013) and Bormetti et al. (2015) belong to the LM family and they differentiate on the basis of the methodology employed to determine the volatility and the threshold level.

In a recent study, Schwert (2010) empirically shows that the applications of alternative identification methods can generally lead to substantially different conclusions on the presence of jumps. Therefore, the choice of a specific identification test may potentially drive our results on the statistical properties of the detected jumps. Unfortunately, none of the identification methods is generally preferable to the others: the recent simulation studies of

Dumitru and Urga (2011) and Gilder et al. (2014) among others show that the performances of the various tests in finite samples are related to the features of the data generating process as well as to the time frequency of prices observations. In view of this, we recognize that the choice of a specific technique could substantially affect the identification of jumps, therefore we consider three alternative sets of jump events obtained selecting one test from the BNS family and a second one from the LM family; the third is derived as the intersection of the previous two. According to Dumitru and Urga (2011), the intersection of two jump tests generally leads to a substantial reduction of the effective size compared to the nominal one The technical motivations driving our choices will be largely discussed in the next subsection.

3.1 Jump Tests

Assume, as usual, that prices follow a continuous-time semi-martingale and let the log-price p_t be described by the stochastic differential equation

$$p_t = \int_0^t \mu_s \, ds + \int_0^t \sigma_s dW_s + \int_0^t J_s dN_s \tag{1}$$

where the drift μ_t has finite variation, the volatility σ_t is càdlàg, W_t is a standard Brownian motion, N_t is a finite activity counting process with possibly stochastic intensity λ_t and J_t is the random jump size. The stochastic processes encompassed by equation 1 exclude infinite activity jumps. However, the class of models covered is widely recognized to be flexible enough to capture the main features of financial time series at high frequency (see for instance Andersen et al. 2007a, Andersen et al. 2010).

Assume also that each trading day t has duration 1 and $M+1$ log-prices $p_{t,0}, \ldots, p_{t,M+1}$ are observed at equally spaced times. The intraday log-returns are indicated as $r_{t,i} = p_{t,i+1} - p_{t,i}$ for $i = 1, \ldots, M$ or alternatively with a single index r_i to denote the i-th log-return in the entire time series: $i = 1, \ldots, M \cdot T$ where T is the total number of trading days.

The BNS family of tests

The following volatility metrics is essential to the computation of the test statistics:

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2 \tag{2}$$

RV_t is the realized variance that converges in probability to the quadratic variation as $M \rightarrow \infty$:

$$p\lim_{M\to\infty} RV_t = QV_t = \int_{t-1}^{t} \sigma_s^2 ds + \int_{t-1}^{t} J_s{}^2 dN_s \tag{3}$$

and in absence of jumps the quadratic variation corresponds to the integrated variance:

$$QV_t = IV_t = \int_{t-1}^{t} \sigma_s^2 ds \tag{4}$$

To separate the contribution to the realized variance due to continuous price variation from the contribution of jumps, BNS introduce the bipower variation:

$$BPV_t \equiv \mu_1^{-2} \left(\frac{M}{M-1} \right) \sum_{i=2}^{M} |r_{t,i-1}| \, |r_{t,i}| \tag{5}$$

where $\mu_\gamma = \mathbb{E}\left(|u|^\gamma\right)$ and $u \sim N(0,1)$. In the asymptotic limit

$$p\lim_{M\to\infty} BPV_t = \int_{t-1}^{t} \sigma_s^2 ds$$

moreover, in absence of jumps, under other regularity conditions, the joint asymptotic distribution of RV_t and IV_t is normally distributed

$$\sqrt{M} \begin{pmatrix} RV_{t,M} - IV \\ BV_{t,M} - IV \end{pmatrix} \xrightarrow{D} N\left(0, \begin{bmatrix} 2 & 2 \\ 2 & 2.62 \end{bmatrix} IQ_t \right) \tag{6}$$

where $IQ_t = \int_{t-1}^{t} \sigma_s^4 ds$ is the integrated quarticity. BNS propose some alternative statistics to compute the test, the most common being based on the relative jump measure:

$$RJ_t \equiv \frac{RV_t - \hat{IV}_t}{RV_t} \tag{7}$$

where $\hat{IV}_t$ denotes some jump robust measure of the integrated variance. We generally define the test statistics for BNS family as

$$Z_t \equiv \frac{RJ_t}{\sqrt{\frac{1}{M}\left(v_{\hat{IV}_t} - v_{RV_t}\right) \frac{\hat{IQ}_t}{\hat{IV}_t^2}}} \tag{8}$$

where $\hat{IQ}_t$ is a consistent estimate of the integrated quarticity, $v_{\hat{IV}}$ and v_{RV} are constant such that

$$Var\left[\hat{IV}_t\right] = \frac{v_{\hat{IV}_t}}{M} IQ_t + O\left(M^{-2}\right) \quad \text{and} \quad Var\left[RV_t\right] = \frac{v_{RV_t}}{M} IQ_t$$

therefore $v_{RV} = 2$ while $v_{\hat{IV}}$ depends on the estimator $\hat{IV}$. The test statistics Z_t converges asymptotically to a standard normal random variable. A jump is detected with the confidence level $1-\alpha$ when $Z_t > \Phi^{-1}_{1-\alpha}$ being $\Phi^{-1}_{1-\alpha}$ the inverse standard normal distribution evaluated at $1-\alpha$. In the original proposal of BNS, $\hat{IV}_t$ coincides with BPV_t and $\hat{IQ}_t = \max\left(\hat{IV}^2_t, QP_t\right)$ being QP_t the quad-power quarticity. The large diffusion of this test statistics is due to its suitable finite sample properties highlighted by Huang and Tauchen (2005).

An interesting alternative volatility measure is the corrected threshold bipower variation of Corsi et al. (2010):

$$C-TBPV_t \equiv \mu_1^{-2} \sum_{i=2}^{M} Z_1\left(r_{t,i-1}, c_\theta^2 \hat{v}_{t,i-1}; c_\theta\right) Z_1\left(r_{t,i}, c_\theta^2 \hat{v}_{t,i}; c_\theta\right) \tag{9}$$

where

$$Z_\gamma\left(x, y; c_\theta\right) \equiv \begin{cases} |x|^\gamma & \text{if } x^2 \leq y \\ \frac{1}{2\Phi(-c_\theta)\sqrt{\pi}} \left(\frac{2}{c_\theta^2} y\right)^{\gamma/2} \Gamma\left(\frac{\gamma+1}{2}, \frac{c_\theta^2}{2}\right) & \text{if } x^2 > y \end{cases}$$

Φ is the cumulative standard normal distribution and $\Gamma(\alpha, x)$ is the upper incomplete gamma function. The $C-TBPV_t$ replaces the absolute returns exceeding the threshold by their conditional expected value under the normality assumption:

$$\mathbb{E}\left[|r_{t,i}|^\gamma \middle| r_{t,i}^2 > c_\theta^2\right] = Z_\gamma\left(r_{t,i}, c_\theta^2 \hat{v}_{t,i}; c_\theta\right)$$

The corrected threshold tripower quarticity is analogously defined as:

$$C-TTPV_t \equiv \mu_1^{-2} \sum_{i=3}^{M} Z_{4/3}\left(r_{t,i-2}, c_\theta^2 \hat{v}_{t,i-2}; c_\theta\right) Z_{4/3}\left(r_{t,i-1}, c_\theta^2 \hat{v}_{t,i-1}; c_\theta\right) Z_{4/3}\left(r_{t,i}, c_\theta^2 \hat{v}_{t,i}; c_\theta\right) \tag{10}$$

Asymptotically, the $C-TBPV_t$ and the $C-TTPV_t$ behave analogously to the bipower variation and the tripower quarticity in absence of jumps. In presence of jumps instead the upward bias which usually affects the multipower variation measures of Barndorff-Nielsen et al. (2006) is drastically reduced, with positive effects on the power of the test. The simulation study of Corsi et al. (2010) shows that the gain is particularly relevant in presence of consecutive jumps when the bias affecting the multipower variations can become extremely large with detrimental effects on jump detection.

Andersen et al. (2012) introduce two jump robust volatility measures based on nearest neighbor truncation that can be regarded as special cases of the quantile-based realized volatility measures of Christensen et al. (2010). The bias generated in finite samples by the presence of jumps and stochastic volatility is generally small for these estimators but they have a lower asymptotic efficiency compared to the bipower and the threshold bipower variation. However, the bias may become very large in presence of consecutive jumps: this is of particular concern in our study given that our analysis reveals quite often the presence of consecutive jumps due to jump clustering effects.The test proposed by Podolskij and Ziggel (2010) belongs also to the BNS family but it is constructed on the threshold estimator of Mancini (2009) removing returns larger then a certain size. It has the advantage of being efficient but it is strongly dependent on the threshold level. Such a dependence entails a serious risk of retaining jumps when the threshold is too high or removing continuous returns when the threshold is too low.

Given the considerations above, the testing methodology of Corsi et al. (2010) appears as the most appropriate within the BNS family for the purposes of our empirical analysis. As a proxy for the instantaneous volatility we adopt the estimator defined by equation 14 in the next paragraph. Remarkably, while the LM type of tests described below require substantial restriction to the volatility process, those belonging to the BNS family are more robust and remaining consistent even in presence of volatility jumps, although their power and their size in finite samples can be negatively affected by violent volatility shocks.

Identification of intraday jump times All the tests belonging to the BNS family are designed to reveal the presence of at least one jump over a certain time period, typically a single trading day. To analyse the statistical properties of jumps we need to classify each single intraday return as a jump or alternatively as a continuous price fluctuation. We follow the iterative procedure described in Andersen et al. (2010) based on the iterative application of the BNS test removing at each step the contribution of the largest absolute return from the realized variance. However, we adopt this method with some important modifications:

1. The BNS test is replaced by the methodology of Corsi et al. (2010) since it produces a smaller bias in presence of jumps.

2. The test is calculated after rescaling high frequency returns to remove the intraday periodicity of volatility as recommended by Rognlie (2010). This procedure reduces the intraday variability of volatility which induces the downward bias in the bipower variation and increases the spurious detection rate.

3. To reduce the impact of stale quotes, we remove zero intraday returns from the sample before computing the realized measures. The presence of an isolated null return annihilates two consecutive blocks in the bipower variation and only one in the realized variance. The negative bias affecting the bipower is therefore removed with the preliminary exclusion of stale quotes from the calculation.

4. Following Gilder et al. (2014) we classify as a jump as the largest absolute log return after adjusting for the intraday volatility pattern.

5. Differently from Andersen et al. (2010), we recalculate at each step the threshold bipower variation and the tripower quarticity to guarantee the removal of the upward bias in case of jumps.

6. To deal with multiple hypotheses testing and to limit the effects on the size of the test we apply the conservative Holm-Bonferroni correction. Note that without such a correction the maximum number of jumps detected on a single trading day increases remarkably : for the S&P 500 index it passes from 5 to 8, for the Euro Stoxx 50 from 7 to 11.

The LM Family of Tests

The LM test is based on a measurement of the instantaneous volatility σ_t. Such a measurement is feasible with asymptotically infinite precision only if the drift μ_t and σ_t itself change "slowly" in time (see Lee and Mykland, 2008 for further details). Such a restriction is the major limitation for these tests since their consistency is not guaranteed in the presence of volatility jumps. The tests statistics within this family is generally defined as follows:

$$z_{t,i} = \frac{|r_{t,i}|}{\sqrt{\hat{V}_{t,i}}} \tag{11}$$

where $\hat{V}_{t,i}$ is an estimator of the instantaneous volatility and $z_{t,i}$ is the normalized absolute return. As the sampling frequency increases, $\hat{V}_{t,i}$ converges to the unobserved instantaneous volatility and $z_{t,i}$ distributes as the absolute value of a standard normal random variable. A jump is detected whenever $z_{t,i}$ exceeds a predetermined threshold θ. The various LM tests proposed in the literature differ for the methodology used to determine the threshold level and for the estimation of the instantaneous volatility. Concerning the threshold, as the test is applied for every intraday return, the issue of false discovery rate (FDR) arising in the context of multiple hypotheses testing must be properly taken into account. The simplest solution (proposed by Andersen et al., 2007b) consists in the application of the Šidák approach: given a certain daily size α, the corresponding size for each intraday test is $\beta = 1 - (1 - \alpha)^{1/M}$ and the associated threshold level is $\theta = \Phi^{-1}_{1-\beta/2}$. However, finite sample volatility is always measured with an error and the Šidák approach often leads to over-reject the null. LM propose to calculate critical values from the limiting distribution of the maximum of the test statistics: as $M \to \infty$ the quantity

$$\xi_M = \frac{\max_i (z_{t,i}) - C_M}{S_M}$$

with $C_M = (2\log M)^{1/2} - [\log\pi + \log(\log M)] / 2\sqrt{2\log M}$ and $S_M = (2\log M)^{-1/2}$, distributes as a Gumbel random variable. This method is more conservative and reduces the probability of detecting spurious jumps.

With regard to the estimators used for the instantaneous volatility, LM propose the bipower variation calculated over a time window of size K depending on the sampling frequency[1]. Andersen et al. (2007b) use instead the bipower variation calculated over the entire trading day. It is important to remark that both of them are upward biased in case of jumps and may substantially lose accuracy when the instantaneous volatility moves rapidly. These issues are extremely relevant for our purposes: the former reduces the detection power of the test, especially when multiple jumps occur closely in time, and it may also influences the observed clustering pattern; the latter increases the the error affecting our local volatility estimates and therefore the probability of spurious jump detection.

To remove the bias, Bormetti et al. (2015) construct an estimator similar to LM which is based on the threshold bipower variation: the past information is weighted through an exponential moving average. It is well known that volatility is generally higher at the beginning and at the end of the trading day, following a U-shaped intraday pattern that is largely documented in the literature (see for instance Bollerslev et al. 2013 and Gilder et al. 2014). This pattern can be taken into account to improve volatility estimates. In details, let $\tilde{r}_{t,i}$ denote the log-return scaled by a proper factor to remove the intraday periodicity: $\tilde{r}_{t,i} = r_{i,t}/\zeta_i$. The local volatility estimator is defined as

$$\tilde{V}_i^{BEW} = \frac{\alpha}{\mu_1^2} |\tilde{r}_{j'}| |\tilde{r}_j| + (1-\alpha)\, \tilde{V}_{i-1}^{BEW} \quad i = 1, \ldots, M \cdot T \tag{12}$$

$$\hat{V}_{t,k}^{BEW} = \zeta_k \, \tilde{V}_{t,k}^{BEW} \qquad t = 1, \ldots, T \quad k = 1, \ldots, M \tag{13}$$

where $\tilde{V}$ indicates the estimated volatility purified by the intraday pattern (technical details on the estimation of the scaling factor ζ_i are reported in Appendix A) , $j < j' \leq i - 1$, $|\tilde{r}_j| / \sqrt{\hat{V}_j^{BEW}} \leq \theta$ and $|\tilde{r}_l| / \sqrt{\hat{V}_l^{BEW}} > \theta \quad \forall\, j < l < j'$. This estimator is a moving average weighted bipower variation excluding all the observation that exceed the threshold θ. However, the inaccuracy in presence of fast volatility changes still remains a critical issue to deal with. We address the problem as follows: i) to correct the intraday volatility patterns we follow the method proposed by Boudt et al. (2011) which ensures efficiency

[1]They recommend $K = \sqrt{252\, M}$ where 252 is the number of trading days in a year

and consistency in presence of jumps; ii) we determine the moving average parameter α minimizing the autocorrelation of the standardized squared returns ; iii) to improve the accuracy in presence of sharp volatility changes, we also consider the estimator $\hat{V}^{FEW}$ based on forward information and defined exactly as $\hat{V}^{BEW}$ but on the time reversed series (i.e. the series obtained substituting the index i with $M \cdot T - i + 1$). Our new estimator is

$$\hat{V}_i^{SEW} = \frac{1}{2}\left(\hat{V}_i^{BEW} + \hat{V}_i^{FEW}\right) \tag{14}$$

which is symmetric in time, i.e. it equally weighs past and future information.

An extensive simulation study available upon request shows that this approach has remarkably more power compared to the original specification proposed by LM because our local volatility estimator is not only able to reduce the bias due to price jumps but is also more responsive to quick changes in volatility. It is quite evident that measuring volatility accurately is crucial for a correct identification of jumps. An increasing empirical evidence pointing toward volatility jumps (Todorov and Tauchen 2011, Jacod and Todorov 2010, Corsi and Renò 2012, Wei 2012, Christensen et al. 2014, Bandi and Renò 2016) which can jeopardize the effectiveness of our jump identification methods. In presence of an upward volatility jump, for instance, the backward estimator tends to underestimate volatility and it is likely to signal spurious price jumps. Combining backward and forward information we can at least reduce such effects.

3.2 Results

Table 1 reports the summary statistics for the three alternative sets of jumps: the first set obtained from our modified version of the LM test (m-LM henceforth), the second set from sequential version of the CPR test (s-CPR), while the third set comes as the intersection of the previous two. The content of this table can be linked to Figures 2 and 3 which display the time series of jumps identified by the three detection methods. By referring to Table 1, we first note that there are significant differences between the outcomes of m-LM and the s-CPR methods: the m-LM test always detects more jumps (almost twice of those detected under the s-CPR test). According to our simulation analysis we also find that the LM-type of tests have generally more power than the BNS test confirming the results of Dumitru and Urga (2011) and Gilder et al. (2014). We observe that all the jump detection methods benefit significantly from the intraday volatility pattern correction which largely reduces the size in finite samples while the market microstructure noise has minor effects at 5 minutes. Importantly, the outcomes of the m-LM and the s-CPR tests are not independent: the size of the intersection is larger than the the product of their individual size. The intersection however ensures a large decrease of the actual size making the detection mechanism much more severe. Bajgrowicz et al. (2016) claim that the role of price jumps in the literature is probably overstated because a large fraction of jumps detected non-parametrically are spurious. To avoid drawing wrong conclusions about the dynamics of jumps they suggest to control the Family Wise Error Rate (FWER) or the FDR. The expected false discovery rates on Table 1 are calculated using the size estimated on our numerical simulation.

Empirical Results

	S&P 500			ESTX 50		
	m-LM	s-CPR	m-LM ∩ s-CPR	m-LM	s-CPR	m-LM ∩ s-CPR
days with jumps	236	167	106	465	286	222
total jumps	315	196	119	637	351	255
max. jumps per day	6	5	3	5	7	3
contrib. price var.	3.7%	2.3%	1.8%	6.5%	4.0%	3.5%
average jump size	0.44%	0.40%	0.48%	5.0%	5.2%	5.9%
max. jump size	3.6%	2.6%	2.6%	3.4%	3.4%	3.4%
min. jump size	0.09%	0.08%	0.1%	0.1%	0.04%	0.1%
FDR	7.6%	14.5%	3.2%	3.9%	9.0%	1.5%

Table 1: Summary statistics for jumps detected under different methods. The m-LM and the s-CPR tests are applied with a nominal confidence level equal to 99%. The contribution of jumps to the total price variance is calculated as the sample average of the ratio between the sum of squared detected jumps and the realized quadratic variation on each trading day (overnight returns are excluded from the denominator).

The relative contribution of jumps to the total price variance is calculated here as the sample average over all trading days of the ratio between the sum of squared jumps and the realized quadratic variation. Huang and Tauchen (2005) instead consider the sample mean of RJ_t (see equation 7) calculated using $\hat{IV}_t = BPV_t$ and they find that about 7.3% of the quadratic variation on the S&P 500 is due to jumps. The same calculation performed on our data with $\hat{IV} = C - TBPV_t$ gives an average ratio of 8.0% for the S&P 500 and 9.3% for the Euro Stoxx 50. Note that these estimates differ significantly from the results of Table 1. Interestingly, the mean of RJ_t on days where no jumps are detected according to the s-CPR test at the 99% confidence level is respectively 4.5% and 4.6% for the two indexes. These result can be interpreted in two different ways that are not mutually exclusive: 1) our choice of the confidence level is too severe to effectively remove the majority of jumps; 2) even after the corrections we adopted to take into account the intraday volatility pattern, the threshold bipower variation is still seriously downward biased. Christensen et al. (2014) find that the contribution of jumps to total price variance extracted from 5 minutes data is

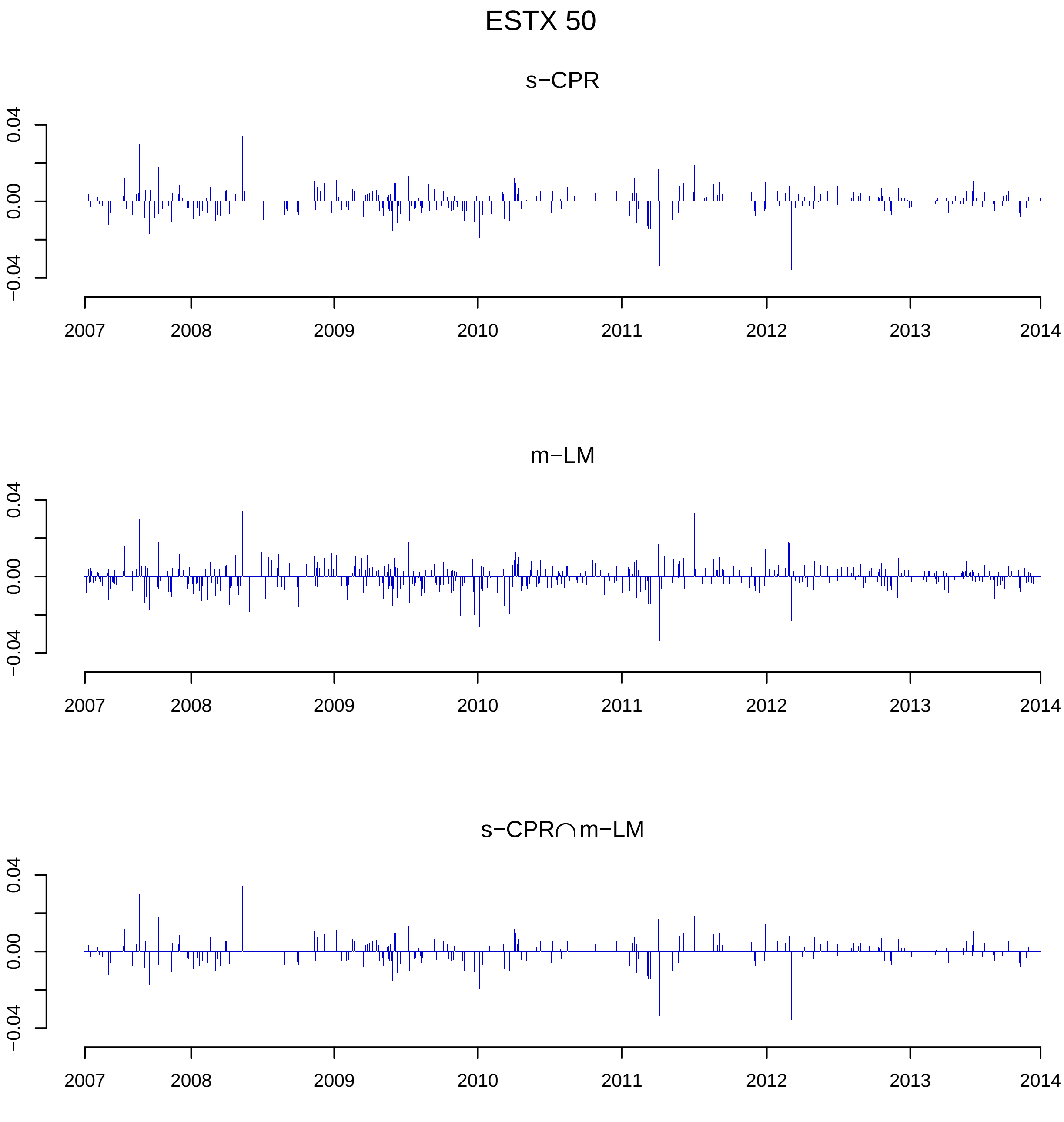

Figure 2: Jumps in the Euro Stoxx 50 identified under different detection methods

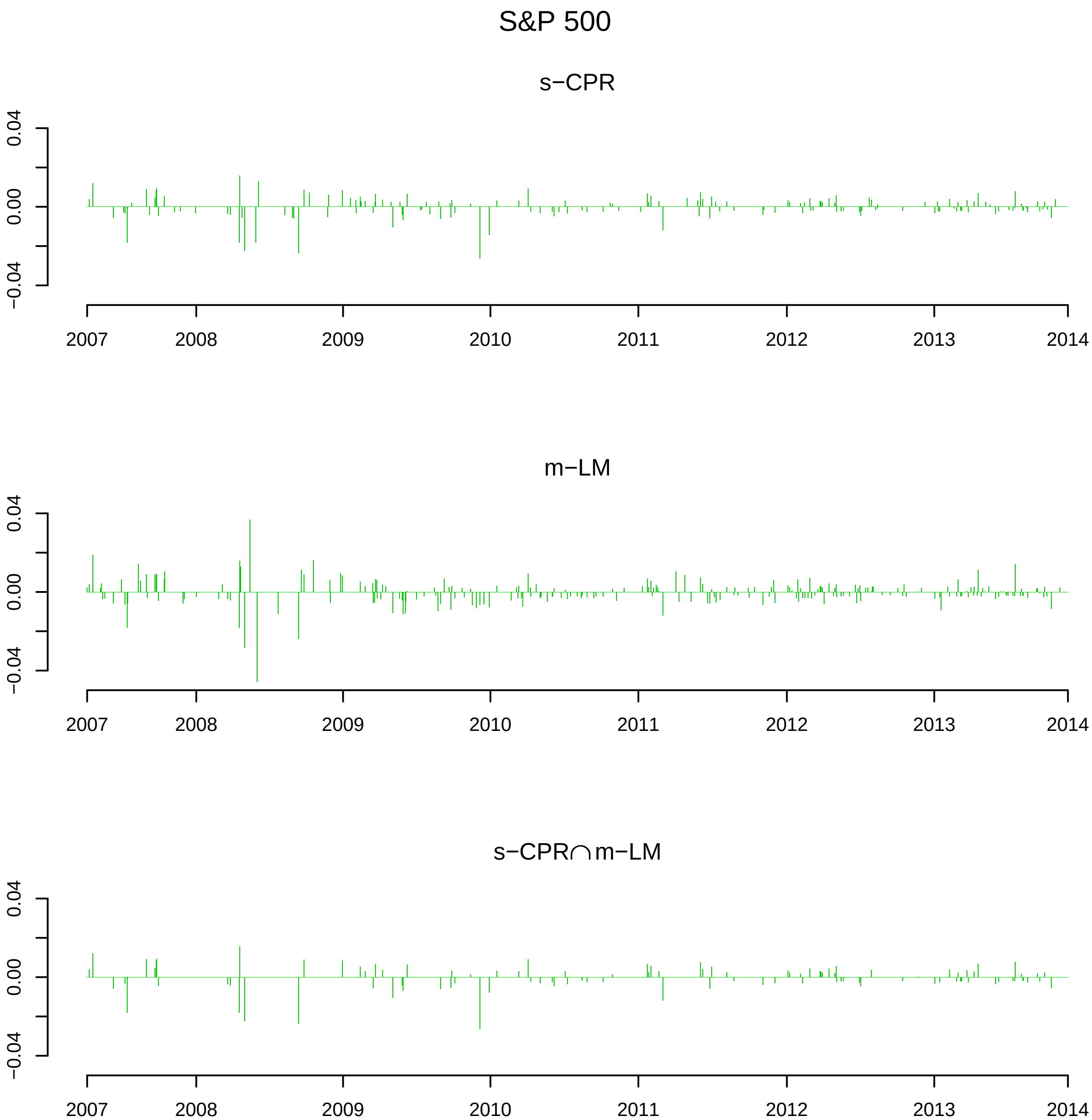

Figure 3: Jumps in the S&P 500 identified under different detection methods

usually overestimated and intraday volatility bursts[2] are often misclassified as jumps. Using data sampled at a higher frequency and applying specific corrections for the microstructure noise, they find that the contribution of jumps is much smaller (around 1% for the equity market). Such results are also confirmed by Bajgrowicz et al. (2016). We therefore retain our confidence level at 99% to avoid an excessive spurious detection rate. Moreover, we emphasize that under all identification procedures, the number of jumps as well as their size and their relative contribution to total price variance are smaller for the U.S. index compared to the Euro Stoxx 50, which can be plausibly related to the lower diversification of the European index.

Figures 4 and 5 show the intraday distribution of jump times. Even if we do not dispose of a detailed data set reporting all the relevant news announcements, the pattern of intraday jumps clearly suggests that at least a significant part of detected jumps can be related to macroeconomic releases and other scheduled announcements. For the U.S. market we notice a peak at about 30 minutes after the market opening (less pronounced under the s-CPR method) which corresponds to the macroeconomic announcements scheduled around 10 o'clock (see Gilder et al. 2014). A second and more evident peak on the U.S. market is located around 14:00 corresponding to the time at which the Federal Fund Target Rate is publicly communicated after the FOMC meeting. For the European market we observe a large number of jumps located within 14:30 and 14:35 in local time, corresponding to the start of the pre-negotiation at the NYSE. A second and smaller peak is visible 1 hour and a half later, in correspondence of the U.S. macroeconomic announcements previously mentioned. This evidence suggests some cross market dependences of the jump activity in the European market due to news on the U.S. economy. This type of information may generate simultaneous reactions in both markets which are usually referred as co-jumps. A more detailed analysis of this topic is deferred to Section 4.

Figures 6 and 7 report the intraday annualized volatility measured by the square root of the quadratic variation (thus also including the contribution of jumps). The average

[2]The recent paper of Christensen et al. (2016) shows that flash crashes are indeed characterized by drift bursts with a continuous path. Such events cannot be clearly distinguished from jumps at 5 minutes but require a higher frequency to be properly investigated.

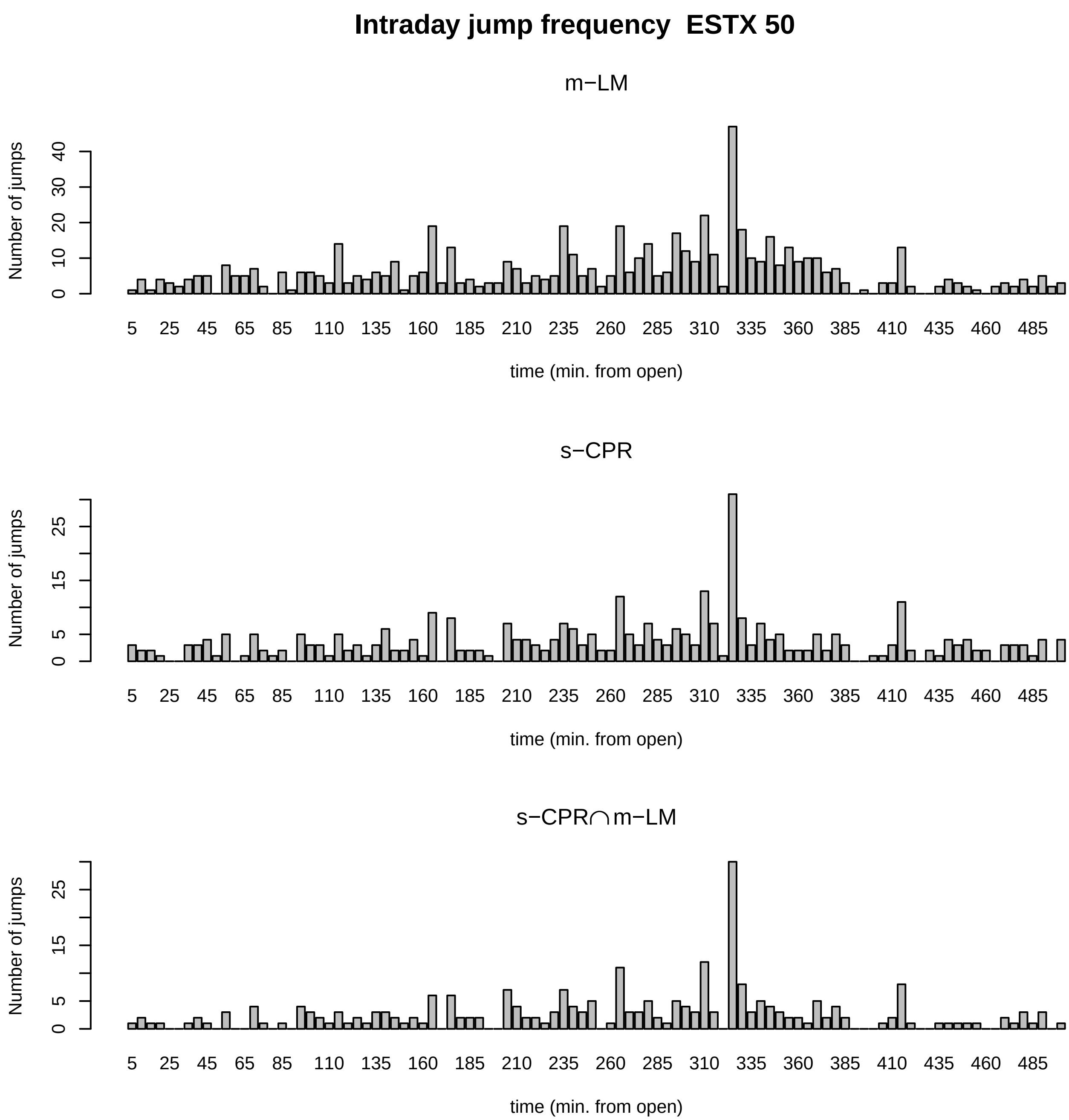

Figure 4: Distribution of intraday jump times for the Euro Stoxx 50.

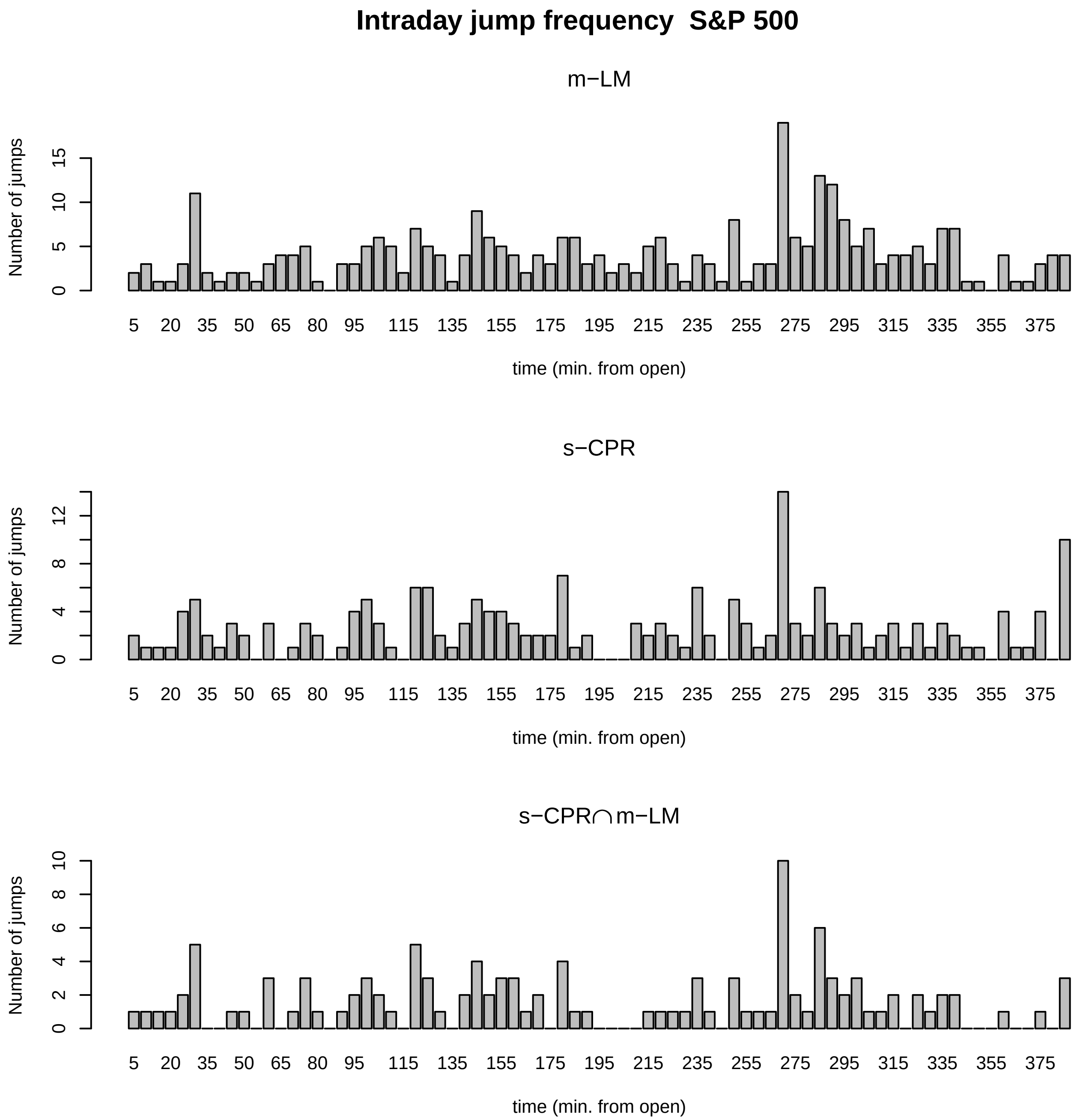

Figure 5: Distribution of intraday jump times for the S&P 500.

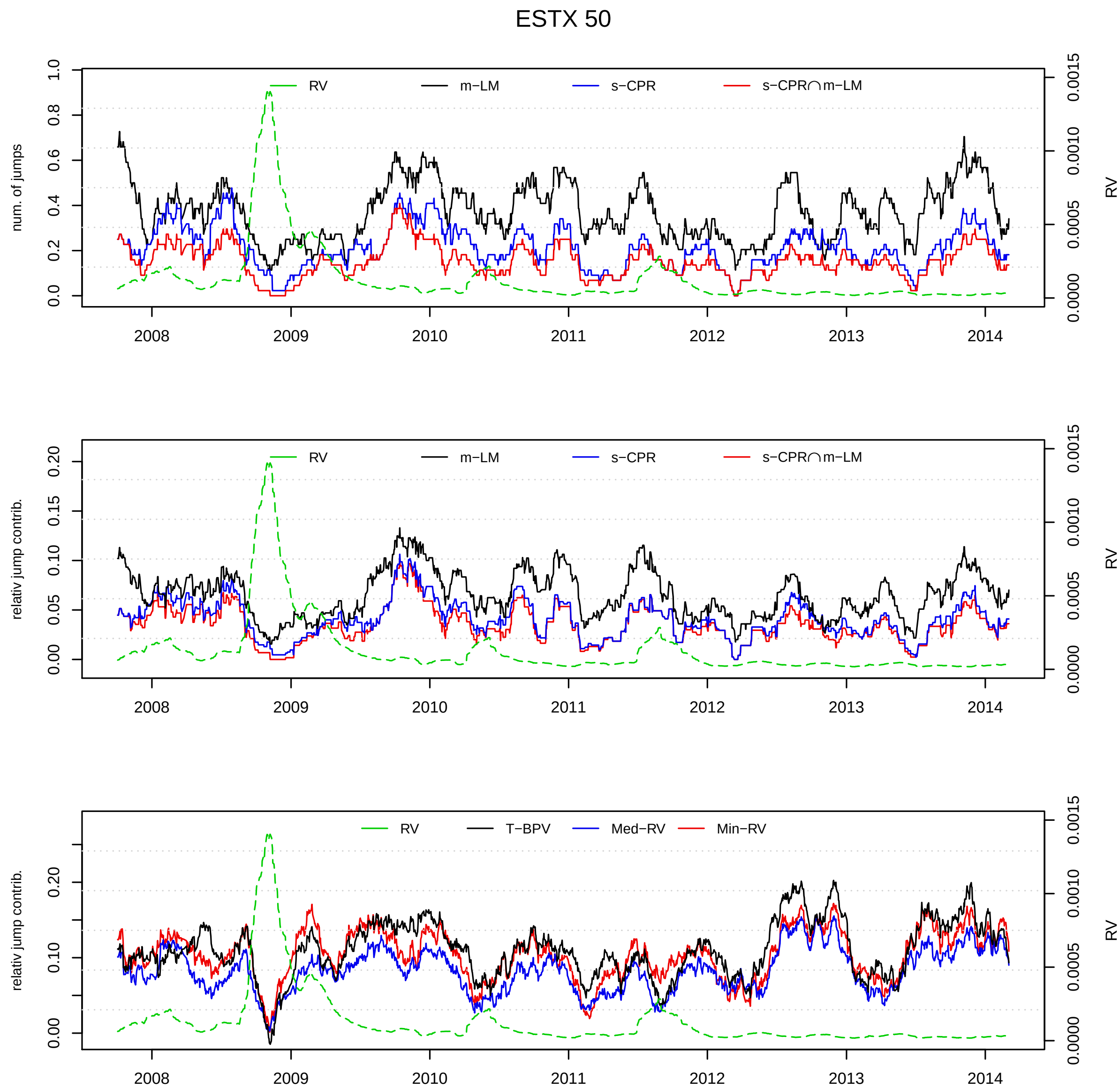

Figure 6: The top panel reports the average number of jumps, the central and the bottom panels show the average contribution of jumps to total price variance calculated respectively from detected jumps and from jump robust realized measures. The dashed line is the intraday realized volatility. All these quantities are averaged over a rolling window of two months centered on the reference date.

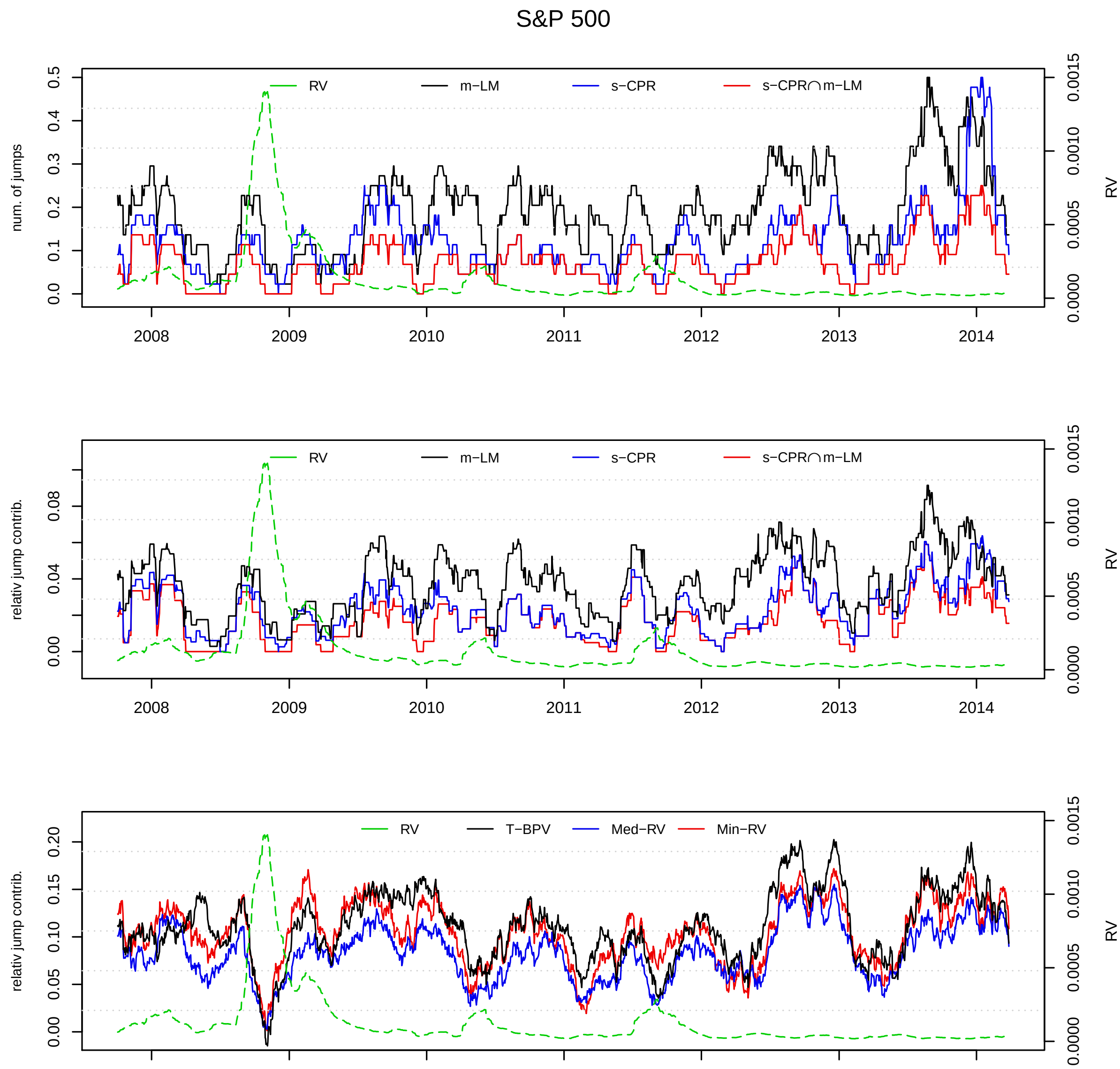

Figure 7: The top panel reports the average number of jumps, the central and the bottom panels show the average contribution of jumps to total price variance calculated respectively from detected jumps and from jump robust realized measures. The dashed line is the intraday realized volatility. All these quantities are averaged over a rolling window of two months centered on the reference date.

number of jumps and the average relative contribution of jumps to the quadratic variation are also reported. All figures show that jumps occur more frequently during low volatility periods. Remarkably, also the relative contribution of jumps is larger when the volatility is lower, regardless of the identification method. Note that the inverse relation between the volatility level and the relative contribution of jumps is even more pronounced when the continuous volatility is measured by the threshold bipower variation or using the $MinRV_t$ and the $MedRV_t$ measures proposed by Andersen et al. (2012). While the detection of a decreasing number of jumps when volatility rises can be determined by a deterioration of the power of the tests, the evidence concerning a diminished relative contribution of the jump component to the total price variance calculated on non-parametric measures of the integrated volatility is much more striking. In presence of a quickly changing volatility for instance, the bipower variation is downward biased and the relative contribution of jumps is generally overestimated. Thus the effects reported on Figures 6 and 7 cannot be induced by this finite sample bias. Therefore, we can conclude that, according to the empirical evidence, both the sub-prime and the Euro Sovereign crisis have been characterized by a large upward volatility shocks while jumps played a only minor role.

4 Modelling Jumps with Multivariate Hawkes Processes

Hawkes processes belong to the class of multivariate point processes. They have been originally introduced by Hawkes (1971b) and Hawkes (1971a) and widely adopted to model earthquakes. The use of Hawkes processes in finance has been proposed by Bowsher (2007) to describe security market transactions, by ADL to model contagion through jump cascades involving multiple markets, by Bormetti et al. (2015) to analyze the multivariate dynamics of jumps in the Italian stock market and by Granelli and Veraart (2016) to study the variance risk premium on an index whose constituents are subject to contagion.

In this study, we use the Hawkes processes to describe the evolution of the jump intensities $\lambda_{l,\tau}$ where l takes the value 1 for the Euro Stoxx 50 index and 2 for the S&P 500. We will use the notation $\tau(t,i)$ to denote the time corresponding to the i^{th} interval on day t. The standard specification for the jumps intensities is

$$\lambda_{l,\tau} = \theta_l + \sum_{l'=1}^{2} \int_{-\infty}^{\tau} g_{l,l'}(\tau - s)\, dN_{l',s} \qquad l = 1,2 \tag{15}$$

where $N_{l,\tau}$ is the counting process for market l and the function $g_{l,l'}$ (usually a negative exponential), measures the effect that an event on market l' generates on the intensity of market l. This model is able to produce jump clustering, because past jumps increase the current intensity whenever $g_{l,l} > 0$, as well as cross-excitation effects from l' to l when $g_{l,l'} > 0$ for $l \neq l'$. We use some simple variants of this model to describe the dynamics of jumps detected from high frequency data: our applications require to take into account that the NYSE and the FSX operate at different times with modest overlaps of the trading activity (normally 2 hours). Moreover trading and non-trading days can differ across countries due to specific national holidays. When the market is closed the jump intensity must be zero, nevertheless the information coming from other operating markets can possibly affect the jump intensity on the next trading day. Equation 15 is appropriate to describe each market during its operating time and when the market l' is closed we can have only self-excitation effects in market l given that no jumps can occur on l'. The missing part of the dynamics is the overnight evolution of λ_l which obviously requires some specific assumptions. Let $o_{l,t}$ and $c_{l,t}$ denote the opening and closing time of market l measured according to some time convention (for instance UTC); for non-trading days we simply assume $o_{l,t} = c_{l,t}$. We consider the following recursive evolution:

$$\lambda_{l,\tau} = \begin{cases} \theta_l + \sum_{l'=1}^{2} K_{l,l'} \int_{-\infty}^{\tau} e^{-\gamma_{l,l'}(\tau-s)} dN_{l',s} & \tau \in [o_{l,t}, c_{l,t}) \\ 0 & \tau \in [c_{l,t}, o_{l,t+1}) \end{cases} \tag{16}$$

where γ controls the speed of mean reversion while $K_{l,l'}$ establishes the size of self and mutual excitations. In principle different coefficients could be introduced for intraday and overnight periods at the cost of making the equations more complicated but according to our analysis (estimates not reported but available at request) the improvement of the fit is negligible.

Table 2 reports the maximum likelihood estimates of our model progressively including single elements of the dynamics. Model 1 is a simple Poisson process with constant intensity, obtained imposing $K_{l,l'} = 0$ for $l, l' = 1, 2$. Model 2 is a univariate Hawkes process that includes self-excitation: the restrictions are $K_{l,l'} = 0$ for $l \neq l'$. The effect of jumps on future intensity exhibits a very short persistence: for the S&P 500 the half-life time ranges from 21 min to 1 hour, for the ESTX from 21 min to 1 hour and half. For both markets the jump intensity is more persistent under the s-CPR method. When allowing for self-excitation, we obtain a remarkable increase of the likelihood under all jump identification methods that that it represents a relevant feature of the jump dynamics. When a jump occurs, its impact on the intensity is remarkably large, generally one order of magnitude larger then the baseline intensity level θ. These results are very similar to those obtained by Bormetti et al. (2015) analyzing the Italian stock market.

Model 3 also includes spillovers in the jump activity (cross-excitation). According to our results cross-excitations are always insignificant (Table 2): jumps detected from high frequency data do not seem to play a relevant role in volatility transmission across markets.

Importantly this result is robust under the different jump identification methods.

To extend our analysis, we also explore some additional features of the jump process: the dependence from continuous volatility and the role of the jump size. To this purpose we move from the standard Hawkes processes to the class of generalized Hawkes processes whose properties are discussed in details by Bowsher (2007): the generalized specification allows the deterministic component θ to be time dependent and the impact of jumps to depend on the normalized jump size. The full model is specified as follows:

$$\lambda_{l,\tau} = \begin{cases} \theta_{l,\tau} + \sum_{l'=1}^{2} K_{l,l'} \int_{-\infty}^{\tau} e^{-\gamma_{l,l'}(\tau-s)} \left| \frac{J_{l',s}}{\sqrt{v_{l',s}}} \right|^{\alpha} dN_{l',s} & \tau \in [o_{l,t}, c_{l,t}) \\ 0 & \tau \in [c_{l,t}, o_{l,t+1}) \end{cases} \tag{17}$$

where $\alpha \geq 0$. We consider the following parametrization for the deterministic time dependent component $\theta_{l,\tau}$ used to accommodate an explicit dependence on the continuous volatility level:

$$\theta_{l,\tau} = \exp\left(a_l + b_l \log v_{l,\tau}\right) \quad a_l,\ b_l \in \mathbb{R} \tag{18}$$

where our volatility proxy is

$$v_{l,\tau(t,i)} = IV_t\, \zeta_i^2 / M \tag{19}$$

IV_t is the integrated volatility on day t , ζ_i is the intraday volatility corrector described in Appendix A, and M is the number of intraday returns. Equation 19 is a proxy for the instantaneous volatility on a specific time interval i on day t. To avoid any endogeneity bias in the measurement of integrated volatility, we use a forecast of the integrated volatility built on the information available up to day $t-1$ and based on a bivariate extension of the HAR-type regressions of Corsi and Renò (2012). Our approach (described in Appendix B) has an extremely high in-sample forecasting power, meaning that the forecast values represent a good proxy for the realized volatility. At the same time, we observe significant volatility spillovers from U.S. to Europe with a time lag of one day. Interestingly, in the opposite direction we find significant lagged cross-leverage effects that are unprecedented in the literature to the best of our knowledge: negative returns in the Euro Stoxx 50 affect the volatility of the S&P 500 Probably such effects are mostly generated during the Euro Sovereign crisis.

The dependence on the jump size is introduced in our generalized Hawkes model when $\alpha > 0$ and it is determined by the absolute size of the jump normalized by the instantaneous volatility: the idea is that the impact of a jump is proportional to its size compared to the typical size of continuous returns on the same period. The results for the alternative specifications are reported in Table 3 where the distinctive features of the generalized process are gradually introduced. Model 4 extends the univariate Model 2 and also introduces the volatility dependence under the constraints $\alpha = 0$, $K_{l,l'} = 0$. Importantly, Table 3 shows that all estimates confirm a significant inverse dependence on the volatility level. Moreover, this result is consistent with the analysis of Wei (2012) who finds that the volatility is on average lower on trading days with jumps. This result confirms what we qualitatively observe in Figures 6 and 7: jumps mostly characterize mostly tranquil market conditions rather than periods of turmoil. The inverse dependence that we measure could also reflect the difficulty of our non-parametric tests to detect jumps when the volatility is high. This may be the case for instance if jump are i.i.d.: in presence of high volatility levels, the magnitude of continuous price fluctuations observed at a fixed sampling frequency may become close to the magnitude of jumps. The detection of discontinuities would then require a finer time resolution which is usually not achievable in practice due to the presence of the microstructure noise.

With regard to cross-market effects, according to further analysis not reported here for brevity, none of the cross-excitation coefficients is statistically significant when taking into account the continuous volatility, regardless of the method used to detect jumps. Concerning the role played by the jump size, we see that there is no agreement across the different detection methods: under the m-LM procedure, large jumps seem to have a larger impact on the intensity with a convex response ($\alpha > 1$); this effect disappears under the s-CPR method and for the intersection set. A possible motivation for this difference would be the presence of volatility jumps: contrary to the s-CPR method that is asymptotically robust to these events, the m-LM method is subject to an increase of the false detection rate. A rapid increase in volatility may be erroneously identified as a jump under the m-LM method which may lead to the kind of result that we have observed.

		ESTX			S&P 500		
		m-LM	s-CPR	Intersection	m-LM	s-CPR	Intersection
model 1	θ	$7.44 \cdot 10^{-4***}$	$4.16 \cdot 10^{-4***}$	$3.02 \cdot 10^{-4***}$	$4.90 \cdot 10^{-4***}$	$3.05 \cdot 10^{-4***}$	$1.85 \cdot 10^{-4***}$
		$(2.20 \cdot 10^{-5})$	$(1.38 \cdot 10^{-5})$	$(1.21 \cdot 10^{-5})$	$(1.81 \cdot 10^{-5})$	$(1.58 \cdot 10^{-5})$	$(9.68 \cdot 10^{-6})$
	$\log L$	-4147.614	-2518.953	-1911.414	-2208.438	-1467.042	-950.0483
model 2	θ	$6.32 \cdot 10^{-4***}$	$3.65 \cdot 10^{-4***}$	$2.81 \cdot 10^{-4***}$	$3.96 \cdot 10^{-4***}$	$2.37 \cdot 10^{-4***}$	$1.69 \cdot 10^{-4***}$
		$(2.86 \cdot 10^{-5})$	$(1.63 \cdot 10^{-5})$	$(1.86 \cdot 10^{-5})$	$(2.41 \cdot 10^{-5})$	$(1.85 \cdot 10^{-5})$	$(1.07 \cdot 10^{-5})$
	γ	$3.05 \cdot 10^{-2***}$	$7.47 \cdot 10^{-3***}$	$3.25 \cdot 10^{-2*}$	$2.90 \cdot 10^{-2***}$	$1.15 \cdot 10^{-2**}$	$3.30 \cdot 10^{-2*}$
		$(7.62 \cdot 10^{-3})$	$(1.68 \cdot 10^{-3})$	$(1.30 \cdot 10^{-2})$	$(6.67 \cdot 10^{-3})$	$(4.21 \cdot 10^{-3})$	$(1.38 \cdot 10^{-2})$
	$K_{l,l}$	$4.68 \cdot 10^{-3***}$	$1.21 \cdot 10^{-3***}$	$2.27 \cdot 10^{-3*}$	$5.93 \cdot 10^{-3***}$	$1.57 \cdot 10^{-3**}$	$3.06 \cdot 10^{-3*}$
		$(1.10 \cdot 10^{-3})$	$(2.63 \cdot 10^{-4})$	$(9.43 \cdot 10^{-4})$	$(1.37 \cdot 10^{-3})$	$(5.21 \cdot 10^{-4})$	$(1.43 \cdot 10^{-3})$
	$\log L$	-4062.034	-2489.863	-1895.516	-2122.551	-1448.232	-934.451
model 3	θ	$5.82 \cdot 10^{-4***}$	$3.48 \cdot 10^{-4***}$	$2.69 \cdot 10^{-4***}$	$3.76 \cdot 10^{-4***}$	$2.06 \cdot 10^{-4***}$	$1.64 \cdot 10^{-4***}$
		$(4.27 \cdot 10^{-5})$	$(1.90 \cdot 10^{-5})$	$(1.96 \cdot 10^{-5})$	$(2.91 \cdot 10^{-5})$	$(3.87 \cdot 10^{-5})$	$(1.55 \cdot 10^{-5})$
	$\gamma_{l,l}$	$3.06 \cdot 10^{-2***}$	$7.95 \cdot 10^{-3***}$	$3.24 \cdot 10^{-2*}$	$2.87 \cdot 10^{-2***}$	$1.19 \cdot 10^{-2**}$	$3.27 \cdot 10^{-2*}$
		$(7.70 \cdot 10^{-3})$	$(1.80 \cdot 10^{-3})$	$(1.33 \cdot 10^{-2})$	$(5.57 \cdot 10^{-3})$	$(4.34 \cdot 10^{-3})$	$(1.36 \cdot 10^{-2})$
	$K_{l,l}$	$4.68 \cdot 10^{-3***}$	$1.24 \cdot 10^{-3***}$	$2.53 \cdot 10^{-3*}$	$5.87 \cdot 10^{-3***}$	$1.56 \cdot 10^{-3**}$	$3.03 \cdot 10^{-3*}$
		$(1.11 \cdot 10^{-3})$	$(2.75 \cdot 10^{-4})$	$(9.47 \cdot 10^{-4})$	$(1.35 \cdot 10^{-3})$	$(5.24 \cdot 10^{-4})$	$(1.41 \cdot 10^{-3})$
	$\gamma_{l,l'}$	$3.74 \cdot 10^{-4}$	$3.26 \cdot 10^{-4}$	$5.62 \cdot 10^{-4}$	$5.65 \cdot 10^{-3}$	$1.39 \cdot 10^{-4}$	$1.27 \cdot 10^{-3}$
		$(4.57 \cdot 10^{-4})$	$(2.65 \cdot 10^{-4})$	$(5.45 \cdot 10^{-4})$	$(4.47 \cdot 10^{-3})$	$(1.50 \cdot 10^{-3})$	$(1.31 \cdot 10^{-2})$
	$K_{l,l'}$	$2.39 \cdot 10^{-4}$	$1.16 \cdot 10^{-4}$	$2.26 \cdot 10^{-4}$	$1.6 \cdot 10^{-4}$	$7.70 \cdot 10^{-5}$	$3.00 \cdot 10^{-5}$
		$(2.16 \cdot 10^{-5})$	$(9.6 \cdot 10^{-5})$	$(2.15 \cdot 10^{-4})$	$(7.94 \cdot 10^{-4})$	$(4.12 \cdot 10^{-5})$	$(4.33 \cdot 10^{-4})$
	$\log L$	-4057.068	-2488.735	-1893.671	-2121.027	-1444.632	-934.156

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 2: Estimates for the standard Hawkes process: time is measured in minutes and standard errors are reported in parenthesis.

		ESTX			S&P 500		
		m-LM	s-CPR	Intersection	m-LM	s-CPR	Intersection
model 4	a	$-1.22 \cdot 10^{1***}$	$-1.11 \cdot 10^{1***}$	$-1.20 \cdot 10^{1***}$	$-12.1 \cdot 10^{1***}$	$-1.08 \cdot 10^{1***}$	$-1.30 \cdot 10^{1***}$
		$(8.37 \cdot 10^{-1})$	$(1.00 \cdot 10^{-1})$	$(1.08 \cdot 10^{-1})$	$(9.30 \cdot 10^{-1})$	$(9.00 \cdot 10^{-1})$	(1.12)
	b	$-3.41 \cdot 10^{-1***}$	$-2.23 \cdot 10^{-1**}$	$-2.71 \cdot 10^{-1***}$	$-2.94 \cdot 10^{-1***}$	$-1.77 \cdot 10^{-1**}$	$-3.01 \cdot 10^{-1**}$
		$(5.86 \cdot 10^{-2})$	$(7.09 \cdot 10^{-2})$	$(7.63 \cdot 10^{-2})$	$(6.44 \cdot 10^{-2})$	$(6.13 \cdot 10^{-2})$	$(7.80 \cdot 10^{-2})$
	γ	$3.18 \cdot 10^{-2***}$	$5.74 \cdot 10^{-2***}$	$3.29 \cdot 10^{-2*}$	$3.07 \cdot 10^{-2***}$	$1.22 \cdot 10^{-2**}$	$3.56 \cdot 10^{-2*}$
		$(8.47 \cdot 10^{-3})$	$(1.70 \cdot 10^{-3})$	$(1.70 \cdot 10^{-2})$	$(7.78 \cdot 10^{-3})$	$(4.32 \cdot 10^{-3})$	$(1.62 \cdot 10^{-2})$
	$K_{l,l}$	$4.63 \cdot 10^{-3***}$	$1.05 \cdot 10^{-3***}$	$2.20 \cdot 10^{-3*}$	$6.07 \cdot 10^{-3***}$	$1.57 \cdot 10^{-3**}$	$3.18 \cdot 10^{-3*}$
		$(1.12 \cdot 10^{-3})$	$(2.42 \cdot 10^{-3})$	$(9.49 \cdot 10^{-4})$	$(1.50 \cdot 10^{-3})$	$(5.22 \cdot 10^{-4})$	$(1.56 \cdot 10^{-3})$
	$\log L$	-4039.128	-2483.786	-1888.9	-2111.314	-1445.279	-929.475
model 5	a	$-1.21 \cdot 10^{1***}$	$-1.11 \cdot 10^{1***}$	$-1.19 \cdot 10^{1***}$	$-1.20 \cdot 10^{1***}$	$-1.08 \cdot 10^{1***}$	$-1.31 \cdot 10^{1***}$
		$(8.29 \cdot 10^{-1})$	$(1.00 \cdot 10^{-1})$	$(9.35 \cdot 10^{-1})$	$(9.30 \cdot 10^{-1})$	(1.09)	(1.28)
	b	$-3.35 \cdot 10^{-1***}$	$-2.23 \cdot 10^{-1**}$	$-2.71 \cdot 10^{-1***}$	$-2.92 \cdot 10^{-1***}$	$-1.77 \cdot 10^{-1*}$	$-3.04 \cdot 10^{-1***}$
		$(5.82 \cdot 10^{-2})$	$(7.09 \cdot 10^{-2})$	$(6.65 \cdot 10^{-2})$	$(6.43 \cdot 10^{-2})$	$(7.49 \cdot 10^{-2})$	$(8.81 \cdot 10^{-2})$
	γ	$3.22 \cdot 10^{-2***}$	$5.74 \cdot 10^{-2***}$	$346 \cdot 10^{-2*}$	$3.00 \cdot 10^{-2***}$	$1.27 \cdot 10^{-2}$	$4.00 \cdot 10^{-2*}$
		$(7.87 \cdot 10^{-3})$	$(1.70 \cdot 10^{-3})$	$(1.73 \cdot 10^{-3})$	$(7.68 \cdot 10^{-3})$	$(7.66 \cdot 10^{-3})$	$(1.96 \cdot 10^{-2})$
	$K_{l,l}$	$1.90 \cdot 10^{-4}$	$1.05 \cdot 10^{-3***}$	$6.41 \cdot 10^{-4*}$	$2.91 \cdot 10^{-4}$	$9.40 \cdot 10^{-4}$	$5.40 \cdot 10^{-4}$
		$(1.27 \cdot 10^{-4})$	$(2.42 \cdot 10^{-3})$	$(8.35 \cdot 10^{-4})$	$(2.19 \cdot 10^{-4})$	$(9.20 \cdot 10^{-4})$	$(8.55 \cdot 10^{-4})$
	α	1.78^{***}	–	$6.89 \cdot 10^{-1}$	1.68^{***}	$3.39 \cdot 10^{-1}$	1.00
		$(3.06 \cdot 10^{-1})$	–	$(6.27 \cdot 10^{-1})$	$(3.49 \cdot 10^{-1})$	$(5.60 \cdot 10^{-1})$	$(6.82 \cdot 10^{-1})$
	$\log L$	-4023.449	-2483.786	-1888.446	-2096.817	-1445.112	-928.376

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 3: Estimates for the extended univariate Hawkes process: time is measured in minutes and standard errors are reported in parenthesis. Note that for model 5 for under the s-CPR method the constraint $\alpha \geq 0$ is binding.

Co-jumps

Hawkes processes are designed to capture jump clustering effects across markets but they fail to capture the simultaneous occurrence of jump events that are commonly referred to as co-jumps. The occurrence of co-jumps for stocks traded on the same market has been largely documented by Gilder et al. (2014), Bormetti et al. (2015), Calcagnile et al. (2015) among others and specific statistical tests have been recently designed for their detection (see Jacod and Todorov 2010 and Caporin et al. 2014). To the best of our knowledge, co-jumps involving market indexes located on different geographical regions have not been studied by the extant literature. One reason is that different markets are often operating asynchronously and the overlap of their activity may represent just a small portion of the trading day.

A systematic study of co-jumps is definitely beyond the purposes of this paper, nevertheless we highlight that the simultaneous occurrence of jumps is sometimes observed in our sample. To keep our approach simple, we base our investigation on the co-exceedance criterion whereby a co-jump is detected whenever jumps are detected in the price of both indexes within the same time interval. According to this rule the number of co-jumps under the m-LM, the s-CPR and their intersection is respectively equal to 18, 13 and 7. Given the small number of events it is natural to question whether they can be generated by random statistical fluctuations. In this regard, we first note that all the identified co-jumps always have the same sign in both markets, suggesting that they are really originated by the same type of information. However, to establish their significance from a statistical perspective we need to perform a formal test. Assume Δt is a small positive time interval, under the null hypothesis that the two processes are independent with intensities $\lambda_{1,t}$ and $\lambda_{2,t}$, the probability of observing a co-jump in the interval $[t,\, t+\Delta t]$ is approximately $1-\exp\left(-\lambda_{1,t}\cdot\lambda_{2,t}\cdot\Delta t^2\right)$. The test statistics is represented by the total number of co-jumps n that under the null is a Poisson random variable with mean $\Lambda=\sum_{i=1}^{T}\lambda_{1,i}\cdot\lambda_{2,i}\cdot\Delta t^2$ where we assume $\lambda_{l,t}=0$ when market l is closed. To calculate the test some reliable estimates of the jump intensity on both markets are needed. To this purpose, we use estimates from Model 5 for the m-LM method (since in this case the coefficient α is significant) and Model 4 in all other cases. To check the reliability of these estimates we first apply the specification tests proposed by Bowsher (2007). According to our results, the null hypothesis of correct model specification is generally not rejected at the 5% significance level for all the jump detection methods adopted. Conversely, the hypothesis that all the co-jumps observed are generated by random statistical fluctuations is always rejected at the 0.1% significance level.

5 Conclusions

We studied the statistical properties of the bivariate jump process involving the Euro Stoxx 50 and the S&P 500 index. Our analysis reveals important features characterizing jump events with relevant implications for asset price modeling, derivatives pricing, risk management and portfolio optimization. The empirical evidence is not compatible with a constant intensity Poisson process but reveals significant jump clustering effects that are well captured by the self-exciting Hawkes processes. The time persistence of such self-excitations is extremely short and therefore unable to produce measurable effects across different trading days. Concerning cross-market effects, our estimates exclude the presence of significant spillovers in the jump activity. This result is complementary to the evidence provided by Corradi et al. (2012) who document significant spillovers across markets affecting the continuous volatility and the absence of cross-market effects triggered by jumps. Interestingly, our results appear in contrast with those of ADL and in our view this discrepancy is likely determined by the different time scale at which our analyses are performed and by the specific parametric assumptions of ADL: the daily returns that they classify as jumps do not necessarily correspond to days characterized by an intense jump activity at a high frequency. Indeed, our findings suggest that jumps are more likely to be detected when the continuous volatility is low. The inverse dependence between the jump intensity and the continuous volatility can be related to a mere identification problem: when the volatility is high, the power of the test is reduced because jumps are more difficult to distinguish from continuous returns at a given sampling frequency. However, our results seem to exclude the possibility that jumps played prominent role during the sub-prime and the Euro Sovereign crisis in in 2008-2009 and 2011-2012. These periods are instead characterized by high volatility peaks which seem to overwhelm the jump component in terms of relative contribution to the total price variance. Finally, we disclose the presence of a few but statistically significant co-jumps occurring when both markets are simultaneously operating.

Our analysis provides several directions for future research. Interestingly, our supplementary regressions used to forecast volatility in Appendix B show significant cross-leverage effect between Europe and U.S. that suggest a volatility transmission mechanism which, to the best of our knowledge, is still unexplored. Our results suggest also that volatility shocks could have played a role that is much more relevant than the role of price jumps in the period 2007-2014. This point also may deserve further investigations.

A The Intraday Volatility Pattern

It is well established that the stock market volatility tends to be higher at the beginning and at the end of each trading day (see Harris 1986, Wood et al. 1985 for seminal contributions). Therefore, it is essential to take into account the intraday volatility pattern for the purpose of jumps identification (see also Boudt et al. 2011). Several methods have been proposed in the literature to estimate the intraday volatility correction factor. As an example, Taylor and Xu (1997) use the simple estimator $\hat{\zeta}^2_{TX,i}$ based on the realized volatility measure:

$$\hat{\zeta}^2_{TX,i} = \frac{M \sum_{t=1}^{T} r^2_{t,i}}{\sum_{t=1}^{T} \sum_{i=1}^{M} r^2_{t,i}} \tag{20}$$

Andersen and Bollerslev (1997) propose a more sophisticated technique called flexible Fourier function (FFF) that is based on the following regression:

$$\log |r_{t,i}| - c = \mathbf{x}_i' \theta + \epsilon_{t,i} \tag{21}$$

where c corresponds to the mean of the log absolute value of a standard normal random variable and

$$\begin{aligned} \mathbf{x}_i'\theta \;=\; & \sum_{q=0}^{Q} \sigma_t^q \Bigg[\mu_{0,q} + \mu_{1,q} \frac{i}{N_1} + \mu_{1,q} \left(\frac{i}{N_2} \right)^2 \\ & + \sum_{l=1}^{D} \lambda_{l,q} \, I_{\{i=d_l\}} + \sum_{p=1}^{P} \left(\gamma_{p,q} \cos \frac{p \cdot i \cdot 2\pi}{M} + \kappa_{p,q} \sin \frac{p \cdot i \cdot 2\pi}{M} \right) \Bigg] \end{aligned} \tag{22}$$

where $\theta = [\mu_{0,q}, \mu_{1,q}, \lambda_{l,q}, \gamma_{p,q}, \kappa_{p,q}]$ is a parameter vector, σ_t is a measure of the daily volatility level, $N_1 = (N+1)/2$ and $N_2 = (N+1)(N+2)/6$. The regression is estimated by OLS and the intraday volatility corrector is obtained as

$$\hat{\zeta}^2_{FFF,i} = \frac{M \, \exp(2\mathbf{x}_i'\theta)}{\sum_{i=M}^{M} \exp(2\mathbf{x}_i'\theta)} \tag{23}$$

Importantly, neither the estimators described above nor the method proposed by Bormetti et al. (2015) are robust to the presence of price jumps. So, if price discontinuities are concentrated in specific periods within the trading day, they may induce some distortions in the estimate of the intraday volatility corrector and consequently in the instantaneous volatility measurement. The problem is discussed by Boudt et al. (2011) who propose alternative parametric and non-parametric estimators. Let us first consider the standardized returns defined as follows:

$$\bar{r}_{t,i} = \frac{r_{t,i}}{\sqrt{BV_t/M}}$$

the shortest half scale estimator is

$$ShortH_i = 0.741 \min \left\{ \bar{r}_{(h),i} - \bar{r}_{(1),i}, \ldots, \bar{r}_{(T),i} - \bar{r}_{(T-h+1),i} \right\}$$

where T is the total number of observations, $h = \lfloor T/2 \rfloor$ is the floor of $T/2$ and $\bar{r}_{(j),i}$ are the order statistics of $\bar{r}_{j,i}$. Then the corresponding correction factor is

$$\hat{\zeta}^2_{ShotH,i} = \frac{M\, ShortH_i^2}{\sum_{i=1}^{M} ShortH_i^2}$$

A more efficient non-parametric corrector is the weighted standard deviation estimator that assigns no weight to the largest observations after scaling by $\hat{\zeta}_{ShortH,i}$:

$$WSD_i^2 = 1.081 \frac{M \sum_{t=1}^{T} \omega_{t,i}\, \bar{r}_{t,i}^2}{\sum_{i=1}^{M} \omega_{t,i}}$$

where $\omega_{t,i} = \omega \left(\tilde{r}_{t,i} / \hat{\zeta}_{ShortH,i} \right)$ and $\omega(z) = 1$ if $z^2 \leq 6.635$ and 0 otherwise[3]

$$\hat{\zeta}^2_{WSD,i} = \frac{M \cdot WSD_i^2}{\sum_{i=1}^{M} WSD_i^2} \tag{24}$$

The parametric method proposed by Boudt et al. (2011) represents a modification of the FFF estimator. To see this, consider the residuals

$$e_{t,i}^{WSD} = \log |r_{t,i}| - c - \log \hat{\zeta}_{WSD,i}$$

and define the negative likelihood function as

$$\rho_{ML}(z) = -0.5 \log \left(\frac{2}{\pi} \right) - z - c + 0.5 \exp \{ 2(z + c) \}$$

and the weights as

$$\omega_{t,i} = \begin{cases} 1 & \text{if } \rho_{ML}\left(e_{t,i}^{WSD}\right) \leq 3.36 \\ 0 & \text{otherwise.} \end{cases}$$

[3] The threshold for z corresponds to the 99° percentile of a chi squared distribution with one degree of freedom.

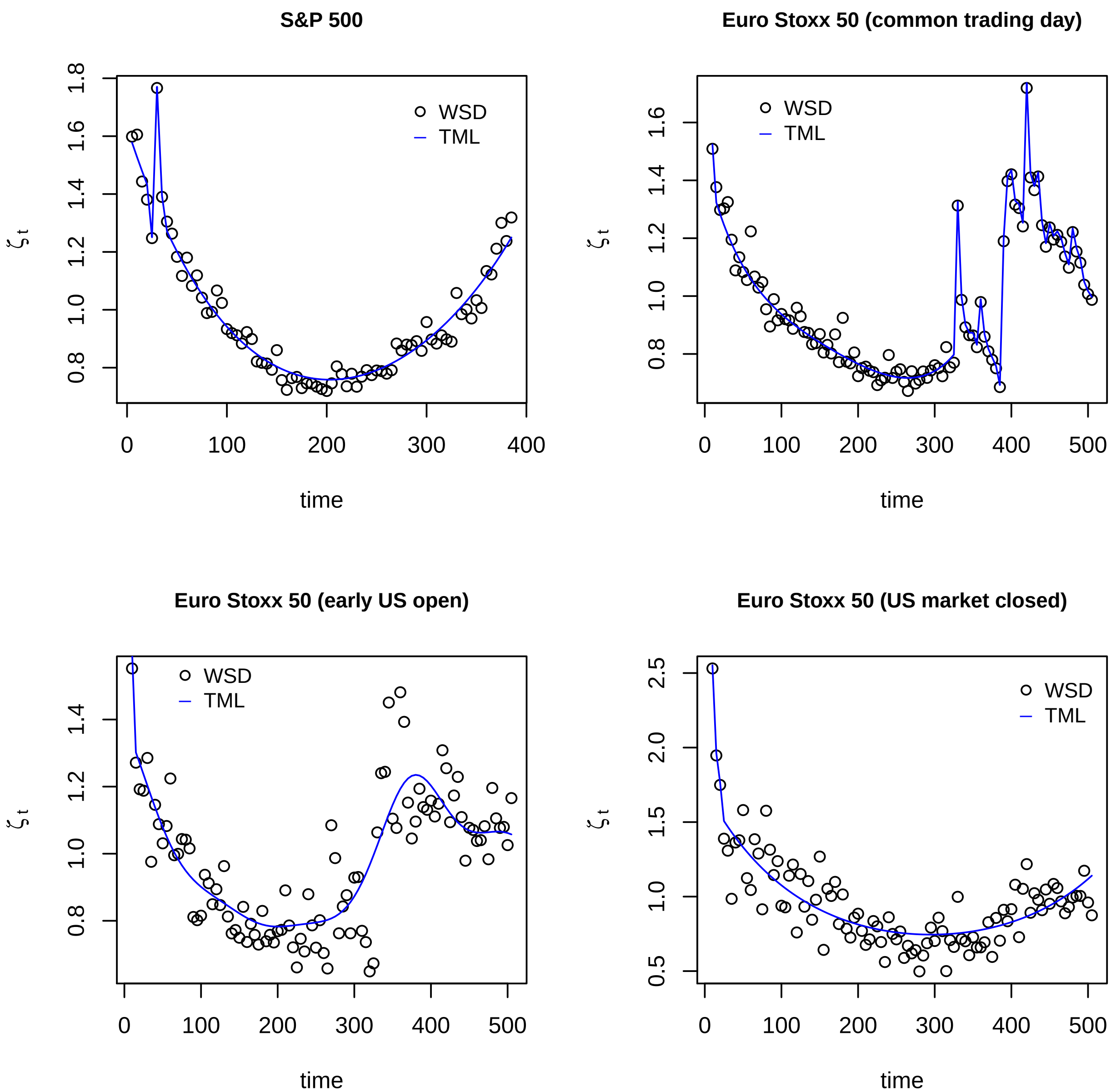

Figure 8: The blue line labeled as TML is the correction adopted in this study and is constructed using a mixed approach: for the points lying in the neighborhood of discontinuities the TML estimator is substituted by the WSD.

Then the maximum likelihood parameters are estimated as

$$\theta_{ML} = \min_{\theta} \frac{\sum_{t,i} \omega_{t,i} \rho_{ML} \left(\epsilon_{t,i} \right)}{\sum_{t,i} \omega_{t,i}}$$

where $\epsilon_{t,i}$ is calculated from a regression of the type 21 and the truncated maximum likelihood (TML) corrector is given by equation 23.

Figure 8 shows the WSD and the TML volatility correctors computed on our sample. Generally speaking, the TML estimator is more efficient and also generates smoother patterns, but it fails to capture discontinuities. Therefore, we disregard the TML corrector for some specific time intervals as the one around 10:00 EST where the S&P 500 deviates from a standard U-shaped pattern and exhibits a spike due to the documented news announcement effect mentioned in Section 3.2; analogously the TML corrector is replaced around 14:25 CET when the Euro Stoxx 50 intraday volatility spikes because of the beginning of pre-negotiations in the U.S.. In general, the intraday volatility of the Euro Stoxx 50 exhibits a strong dependence on the market activity in U.S. and Figure 8 illustrates the three different patterns observed depending on the operating time of the NYSE. The right top panel displays the pattern for a common trading day, i.e. when the Frankfurt Stock Exchange (FSX) electronic trading starts at 9:00 CET and closes at 17:30, while the NYSE opens at 15:30 CET and closes at 22:00 CET (respectively 9:30 and 16:00 EST). The overlapping period between these markets can vary based on changes resulting from the non-simultaneous adoption of the daylight saving time which generates short periods characterized by a different volatility pattern. In the left bottom panel we represent the shape of the volatility correctors for periods when the NYSE opens one hour earlier w.r.t. the CET due to the different adoption of the daylight saving time in the two regions. This anomaly involves a small number of trading days (117 out of 1691) and the WSD estimates are extremely noisy, thus we fully rely on the TML estimates except for the first 5 minutes interval when volatility is extremely large. Finally, there is a small set of days in which the FSX market is operating normally but the NYSE remains closed (only 44 days in the whole sample including some long weekends and national holidays in U.S.). The volatility pattern of the Euro Stoxx appears L-shaped during these periods as displayed by the right bottom panel in Figure 8.

B Forecasting the Integrated Volatility

In this Section we discuss some details on the volatility proxy used in Section 4. The objective is to provide accurate estimates of the integrated volatility on each trading day t using the previously available information. The aim of this approach is to circumvent some endogeneity issues which may arise if the integrated volatility is calculated subtracting the contribution of jumps from the quadratic variation introducing therefore some dependence between the volatility estimates and the price jumps occurred in the same period.

The dependent variables that we want to model are the integrated volatility of the Euro Stoxx 50 ($IV_{EU,t}$) and of the S&P 500 ($IV_{EU,t}$). Let $r_{l,t}$ denote the close to close log-return on day t while $J_{l,t}$ is the absolute contribution of jumps to the quadratic variation. More specifically the integrated volatility is calculated as the average variance of the intraday log-returns that are not identified as jumps:

$$IV_{l,t} = \frac{M}{M - \sum_{i=1}^{M} Jump_{t,i}} \sum_{i=1}^{M} r_{t,i}^2 \left(1 - Jump_{t,i}\right)$$

where $Jump_{t,i}$ is the jump indicator taking the value 1 each time that a jump is identified and zero otherwise. Clearly, this method produces distinct volatility measures for each different jump identification method. Our approach is based on a bivariate extension of the LHAR-C-CJ regression of Corsi and Renò (2012), i.e. a parsimonious regression of the HAR type (introduced by Corsi, 2009) which also includes lagged leverage effects and jumps. We propose a straightforward bivariate extension that is able to capture the cross market effects. As an example, for the Euro Stoxx 50, the regression reads as follows:

$$\begin{aligned}
\log IV_{EU,t+1} = \; & c + \beta_1 \log IV_{EU,t} + \beta_2 \log IV_{EU,t}^{(5)} + \beta_3 \log IV_{EU,t}^{(22)} + \beta_4 \log\left(1 + J_{EU,t}\right) + \beta_5 \log\left(1 + J_{EU,t}^{(5)}\right) \\
& + \beta_6 \log\left(1 + J_{EU,t}^{(22)}\right) + \beta_7 r_{EU,t}^{-} + \beta_8 r_{EU,t}^{(5)-} + \beta_9 r_{EU,t}^{(22)-} + \beta_{10} \log IV_{US,t} \\
& + \beta_{11} \log IV_{US,t}^{(5)} + \beta_{12} \log IV_{US,t}^{(22)} + \beta_{13} \log\left(1 + J_{US,t}\right) + \beta_{14} \log\left(1 + J_{US,t}^{(5)}\right) \\
& + \beta_{15} \log\left(1 + J_{US,t}^{(22)}\right) + \beta_{16} r_{US,t} + \beta_{17} r_{US,t}^{(5)-} + \beta_{18} r_{US,t}^{(22)-}
\end{aligned}$$

where for a generic observable X we have:

$$\begin{aligned}
X^{-} &= \min(X, 0) \\
X_t^{(h)} &= \frac{1}{h}\sum_{j=1}^{h} X_{t-h+1} \\
X^{(h)-} &= \min\left(X^{(h)}, 0\right) \\
X^{-(h)} &= \frac{1}{h}\sum_{j=1}^{h} \min(X_{t-h+1}, 0)
\end{aligned}$$

with h representing the order of the LHAR-C-CJ component.

The results shown in Table 4 are very similar under the alternative jump identification schemes adopted. For both indexes the strong volatility persistence is confirmed: the coefficients relative to the daily, weekly and monthly components are positive and significant. The persistence of the leverage effects is also relevant, especially for the Euro Stoxx 50 index. The main differences with respect to Corsi and Renò (2012) are found in the impact of jumps on continuous volatility which is generally insignificant in our regressions. This is probably due to the prevalence of crisis periods in our sample, when the effect of jumps is overwhelmed by a large continuous volatility component as already documented in the rest of this paper. Concerning volatility spillovers, we note a strong effect from U.S. to Europe with a lag of 1 day while the weekly component has a weak negative effect and the monthly component is not statistically significant. Importantly, we also notice a marked cross-leverage effect between Europe and U.S. that to the best of our knowledge is unprecedented in the literature and suggests a possible direction for future research. The interdependence in volatility stems also from the cross correlation of the residuals that is over 40% under all jump identification methods.

	ESTX			S&P 500		
	m-LM	s-CPR	m-LM ∩ s-CPR	m-LM	s-CPR	m-LM ∩ s-CPR
c	-1.33^{***}	-1.44^{***}	-1.48^{***}	-1.53^{***}	-1.72^{***}	-1.73^{***}
	(0.20)	(0.21)	(0.20)	(0.20)	(0.20)	(0.20)
IV_{US}	0.22^{***}	0.24^{***}	0.23^{***}	0.44^{***}	0.39^{***}	0.39^{***}
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
$IV_{US}^{(5)}$	-0.10	-0.09	-0.09	0.24^{***}	0.27^{***}	0.27^{***}
	(0.05)	(0.06)	(0.06)	(0.05)	(0.05)	(0.06)
$IV_{US}^{(22)}$	-0.07	-0.10^{*}	-0.10^{*}	0.25^{***}	0.27^{***}	0.27^{***}
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
J_{US}	11.21	373.41	475.28	32.60	188.37	526.30
	(367.65)	(355.44)	(381.62)	(312.84)	(606.05)	(609.65)
$J_{US}^{(5)}$	374.63	495.97	299.76	186.61	-28.31	-505.50
	(647.56)	(1204.55)	(1276.56)	(804.68)	(1412.76)	(1472.98)
$J_{US}^{(22)}$	-1309.15	-3032.51	-2223.71	-1299.17	355.15	1418.30
	(1436.40)	(2858.22)	(2953.36)	(1439.30)	(2871.98)	(2913.64)
r_{US}^{-}	-1.05	-0.78	-0.97	-6.70^{***}	-7.26^{***}	-7.44^{***}
	(1.61)	(1.76)	(1.74)	(1.96)	(2.07)	(2.08)
$r_{US}^{(5)-}$	0.79	-0.20	0.53	-12.19	-12.70	-12.81
	(5.61)	(6.06)	(6.01)	(6.41)	(6.72)	(6.57)
$r_{US}^{(22)-}$	25.98	26.96	29.41^{*}	8.89	19.18	22.56
	(15.47)	(15.15)	(14.82)	(17.04)	(17.98)	(18.09)
J_{EU}	-299.77	-288.77	-297.08	-530.12^{*}	-450.34	-400.91
	(264.21)	(265.17)	(273.65)	(259.97)	(311.68)	(339.78)
$J_{EU}^{(5)}$	-657.64	-722.77	-707.31	-1015.27	-1334.81	-1486.38
	(727.60)	(728.93)	(790.90)	(849.13)	(888.68)	(909.46)
$J_{EU}^{(22)}$	-352.97	-264.96	-108.15	-178.40	-569.12	-360.00
	(1252.21)	(1308.38)	(1318.49)	(1367.05)	(1455.09)	(1492.77)
IV_{EU}	0.25^{***}	0.19^{***}	0.19^{***}	-0.01	-0.02	-0.02
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
$IV_{EU}^{(5)}$	0.35^{***}	0.38^{***}	0.38^{***}	0.00	0.02	0.02
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
$IV_{EU}^{(22)}$	0.22^{***}	0.24^{***}	0.23^{***}	-0.07	-0.09	-0.09
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
r_{EU}^{-}	-12.60^{***}	-13.60^{***}	-13.26^{***}	-10.52^{***}	-10.28^{***}	-10.05^{***}
	(1.64)	(1.73)	(1.75)	(1.67)	(1.76)	(1.75)
$r_{EU}^{(5)-}$	-23.10^{***}	-22.90^{***}	-23.44^{***}	-14.58^{*}	-16.91^{**}	-16.72^{**}
	(5.01)	(5.20)	(5.19)	(6.18)	(6.32)	(6.31)
$r_{EU}^{(22)-}$	-34.97^{**}	-35.51^{**}	-36.17^{**}	-31.36^{*}	-37.80^{**}	-37.79^{**}
	(12.04)	(12.46)	(12.43)	(13.90)	(14.53)	(14.38)
R^2	0.81	0.79	0.79	0.85	0.84	0.84
obs.	1669	1669	1669	1652	1652	1652
RMSE	0.40	0.43	0.42	0.44	0.46	0.46

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 4: Estimates for the bivariate LHAR-C-CJ, robust to heteroskedasticity and autocorrelation. Standard errors in parentheses.

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