Banking Regulation and Market Making*

David A. Cimon† Corey Garriott‡

November 8, 2017

ABSTRACT

Recent banking regulation can harm bond market liquidity by motivating a shift to agency intermediation. In a model, theoretical market makers are made to satisfy balance-sheet constraints that are stylizations of the banking rules from Basel III and the U.S. Volcker Rule. The regulations cause market makers to reduce their intermediation by refusing principal positions and instead matching clients on an agency basis. As a result, asset prices exhibit greater price impact and greater price pressure. However, the regulations can improve the bid-ask spread because they induce entry by new market makers.

Keywords: market making; market microstructure; fixed income; liquidity; regulation; Basel III

JEL Classifications: G14, G20, L10

---

*For helpful comments and discussions, we are grateful to Gregory Bauer, Elif Dalkir, Amy Edwards, Jean-Sébastien Fontaine, Itay Goldstein, Tamara Gomes, Yesol Huh, Sebastian Infante, Darcey McVanel, Albert Menkveld, Christine Parlour, Joshua Slive, Francesco Trebbi, Kumar Venkataraman, Jan Wrampelmeyer and Liyan Yang. We thank conference participants at the 2017 FMA Annual Meeting, the 13th Annual Central Bank Conference on the Microstructure of Financial Markets, the 2017 SAFE Microstructure Conference, the 2017 NFA Annual Conference, the 2017 CEA Annual Meeting and the 2017 Bank of Canada–Banco de Espana workshop, as well as seminar participants at the Bank of Canada and Wilfrid Laurier University. We are grateful for the research assistance of James Pinnington. The views expressed in this paper are solely those of the authors and may not reflect those of the Bank of Canada; all errors are our own. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

†Bank of Canada, Financial Markets Department, 234 Wellington Street, Ottawa, Ontario, Canada K1A 0G9. E-mail: dcimon@bank-banque-canada.ca.

‡Bank of Canada, Financial Markets Department, 234 Wellington Street, Ottawa, Ontario, Canada K1A 0G9. E-mail: cgarriott@bank-banque-canada.ca.

The banking regulations motivated by the financial crisis have generated controversy. Critics argue that the new rules, intended to control risk-taking at banks, have the unintended consequence of harming bond market liquidity.[1] The concern is that regulation unduly constrains the balance sheet of banks and leads them to limit their market making. The unintended cost could be large, as banks are still the primary market makers in bond markets (Bao et al. 2016), yet it has proven difficult to identify the cost in the data. Liquidity metrics do not show a universal increase or decrease during the regulatory period. Some metrics such as bid-ask spreads have even improved after the regulations, whereas some metrics such as price impact have worsened.[2] It is not clear whether the unintended cost is large or even present.

In this paper, we provide a theoretical analysis of the regulation to explain why the liquidity data could appear mixed and, moreover, to show how regulation on the balance sheet affects market making. Our finding is that regulation in the style of the Basel III framework or the U.S. Volcker Rule motivates a shift toward an agency model of intermediation. In the agency model, a market maker tries to match investor clients directly and without using its balance sheet. The move to agency can reduce liquidity because investors require price concessions to motivate them to take the other side of a match.

The model we present is intentionally simple. An industry of market makers interacts with two populations of investors, buyers and sellers, who are segmented and can transact only with market makers. There is one period, one asset, no information, no agency frictions, and capital-structure irrelevance would hold if there were no regulation. One might not expect rich behaviour in such an environment, but we can generate striking results using

[1]See Duffie (2012), ESRB (2016), CGFS (2016), or "Top of Mind: A Look at Liquidity," Goldman Sachs, Global Macro Research, August 2015; "Global financial markets liquidity study," PwC, August 2015; "Addressing Market Liquidity," BlackRock, Viewpoint, October 2016.

[2]For example, compare Adrian et al. (2015) to Dick-Nielsen and Rossi (2016).

two frictions that are novel to market microstructure. The first friction is explicit corporate finance. Market makers cannot hold "negative" quantities and must explicitly finance operations using debt, equity, and securities financing, an essential tool for bond dealing (Huh and Infante 2016). The second friction is industrial organization. Market makers compete for investor business as Cournot quantity competitors, a friction that is appropriate to bond markets, which are characterized by substantial market power (O'Hara et al. 2016).

These frictions create a novel divergence in two popular liquidity metrics, the bid-ask spread and the price impact. We find spreads derive from the industrial organization, whereas price impacts derive from the corporate finance. The divergence distinguishes the model within microstructure theory, in which liquidity metrics typically co-move. The divergence can help explain data in which tight bid-ask spreads coexist with prices that are more sensitive to demands for immediacy. As the recent regulations are balance-sheet regulations and therefore constraints on corporate financing, we find they worsen price impacts. In contrast, we find bid-ask spreads can improve because regulation creates profit opportunities that lead to market-maker entry.

Using the model, we can match findings from the growing empirical literature on liquidity after Basel III and the U.S. Volcker Rule. In line with the model, U.S. market makers are reluctant to commit capital to large inventory positions in the post-regulatory period (Bessembinder et al. 2016; Schultz 2017), and at the same time price impacts after sudden trade flows have worsened (Dick-Nielsen and Rossi 2016; Bao et al. 2016; Schultz 2017). Nevertheless, the bid-ask spread for U.S. corporate bonds is largely unchanged or perhaps is slightly better (Trebbi and Xiao 2016; Bessembinder et al. 2016; Adrian et al. 2015), and there is entry of new market makers (Bao et al. 2016). The model also predicts an increase in "price pressure" (Hendershott and Menkveld 2014), which has not been examined in the context of regulation.[3]

We study three types of regulation: capital, funding liquidity, and position. The capital and liquidity regulations are stylizations of the Basel III framework. The capital regulation, representing the Basel III Leverage Ratio, is simply an upper bound on debt financing.[4] The liquidity regulations, representing the Basel III Liquidity Coverage Ratio (LCR) and Net Stable Funding Ratio (NSFR), require possession of a high-quality asset in a proportion to repo and reverse-repo exposures. Last, the position regulation is a stylization of the U.S. Volcker Rule, and it limits the absolute value of a market maker's position. Although each regulation imposes a different balance-sheet constraint, we find they lead to a common outcome. Market makers respond by rationing their balance sheet, matching buyers to sellers as an agent rather than acting as their principal counterparty. By working as an agent, market makers avoid principal positions, relaxing both capital and position limits and avoiding asset purchases obliged by liquidity regulations.

While regulation in the model makes price impact more costly, it has benefits in diminishing the likelihood of default. Liquidity regulation reduces the likelihood of dealer default by obligating the purchase of HQLA, moving dealers to take smaller net positions and also decreasing the variance of their payoffs. Leverage regulation reduces the likelihood of default by obligating a buffer of equity, which has the added benefit of limiting the maximum long positions of dealers. However, position limits are not necessarily effective in reducing default because dealers are as free as before to finance their positions with debt. While we present the benefits of regulation in terms of default probability, we do not weigh this value relative

[3] Friewald and Nagler (2016) give a methodology to measure corporate-bond price pressure.

[4] We do not study the Basel III Capital Adequacy Ratio because it reduces to a leverage ratio in a model with one asset.

to losses in market liquidity. To evaluate the welfare consequences of financial stability (and to arrive at a statement about optimal regulation) would require a macroeconomy and is beyond the scope of a paper on liquidity.

Although there are already stylized models of dealer markets, a new model is necessary to understand balance-sheet regulation. The literature on dealers relies on informational frictions (Vives 2011; Dutta and Madhavan 1997; Biais 1993; Kyle 1989), aversion to risky inventory (Laux 1995; Ho and Stoll 1983), or combinations of the two (Liu and Wang 2016) as ways to generate illiquidity. The friction of information asymmetry is less applicable to bond markets, as most information relevant for bond valuation is public (Fleming and Remolona 1999). The friction of risk aversion does not treat the balance sheet explicitly, and the topic of balance-sheet regulation demands a framework that enables frictions on corporate finance to have effects on liquidity.

We conclude that regulation on the balance sheet does decrease market liquidity. However, the cost is unlikely be measurable in bid-ask spreads, which are determined by industrial organization and not by the ease of financing inventory. Instead, the cost can be measured in new inventory premia in asset prices, such as in price pressure. Empiricsts could study liquidity under regulation further by examining price pressure in the post-regulatory period, which we predict should increase.

A. Model predictions and the empirical literature

The model makes observable predictions that can be compared to the data. However, the model's predictions change depending on which regulations are assumed to be binding on its dealers. For the purpose of comparing the model to the data, we focus narrowly on its predictions for the case that the leverage or position regulations are binding, as empirical work finds these rules to be constraining dealers' balance sheets (Bessembinder et al. 2016; Bao et al. 2016), and market participants emphasize the role of these rules in surveys.[5] The case provides a stable example because its predictions are the same regardless of which of the leverage or position regulations bind and are robust to the presence or absence of liquidity regulation.

In this case, the model has three predictions for dealer behaviour: Incumbent dealers move toward an agency basis of intermediation, they decrease the absolute size of their inventory positions, and new dealers enter the business. The data from U.S. dealers in the post-regulatory period confirms these predictions. Dealers are committing less capital to market-making positions and are rationing their capital using agency intermediation when possible.[6] In Bao et al. (2016), new dealers enter the business of market-making, and it is particularly dealers who are less regulated who step in or increase presence.

In addition to the predictions on dealer behaviour, the model also contains predictions for trading costs: Regulation exacerbates the price impact of large trade flows and "price pressure," whereas bid-ask spreads remain level or even improve mildly due to entry. Here the data partially confirms the model's predictions. Spread metrics since the reforms are observed to improve mildly (Adrian et al. 2015; Trebbi and Xiao 2016) or show no evidence of deterioration (Bessembinder et al. 2016), and either finding would accord with the model. However, Choi and Huh (2016) show that spread metrics are biased downward if dealers are engaging in agency trading, and after correcting the metrics some spreads deteriorate post-regulation. As for price impact, both Dick-Nielsen and Rossi (2016) and Bao et al. (2016) exploit events that create a need for immediate trading and find their price impact

[5] See survey in Committee on the Global Financial System 55, "Fixed Income Market Liquidity." (2016)
[6] See Adrian et al. (2017); Bessembinder et al. (2016); Schultz (2017); Bao et al. (2016)

is significantly worse in the post-regulatory period. However, Schultz (2017) finds that the price impact after such events is only larger during volatile trading conditions.

Evidence from foreign exchange (FX) prices corroborates the model's predictions on price pressure. Pinnington and Shamloo (2016) and Du et al. (2016) find large and persistent deviations in covered interest parity in FX markets, which they attribute to a binding leverage ratio. Our results support their attribution, as the deviations from parity are mostly negative in sign. This would result if borrowing is more difficult than lending, an outcome of a binding Basel III capital constraint.

I. Model

We present a model of Cournot-competitive market makers. This model has three periods: the financing period $t = 0$, the trading period $t = 1$ and the liquidation period $t = 2$. Financial institutions must pay an upfront cost in order to become market makers. In order to finance their costs, market makers must issue either debt or equity and pay a market rate of return.

A. Assets

Traded asset: There exists an asset in unlimited supply. The asset's liquidation value is uncertain until $t = 2$, at which point it delivers a value $V \sim N(v, \sigma)$ in cash.

High-quality liquid asset (HQLA): There also exists an HQLA in unlimited quantity. This HQLA can be obtained frictionlessly, in unlimited supply, at a normalized price of 1. At the end of the game, a unit of HQLA liquidates to deliver $1 + r_F$ in cash.

Repurchase agreements (repo) and reverse-repurchase agreements (reverse repo): There exist repo and reverse-repo agreements in unlimited quantity. A repo transaction is a cash loan at a rate r_R that is collateralized by equivalent value of the asset.[7] A reverse-repo transaction in our context is a specific reverse repo (Duffie 1996), a loan of the model's asset at a rate r_R that is collateralized by equivalent value of cash.

B. *Agents*

There are three types of agent: investor buyers, investor sellers, and financial institutions. Investor buyers and sellers are interested in buying or selling the asset. They can transact only with market makers and not with one another. Financial institutions are companies that can issue debt and equity and that can invest in a market-making technology. In addition, they have access to the market for HQLA and the market for repo and reverse-repo agreements.

C. *Investors*

We model representative, utility-maximizing buyers and sellers. These representative investors are unable to transact with each other and must transact with financial institutions. Investors are price sensitive and have a quadratic liquidity preference to own the asset. The investors who buy assets (sold by financial institutions) have the utility function,

$$U(S) = \frac{-\lambda_S S^2}{2} + (v + l_S - P_S)S. \tag{1}$$

The investors who sell assets (bought by financial institutions) have the utility function,

$$U(B) = \frac{-\lambda_B B^2}{2} - (v - l_B - P_B)B. \tag{2}$$

[7] We consider repos with no haircut, which occur at the traded asset's expected value v.

Investors maximize their utility by choosing the quantity B or S, which represent the total quantities that market makers can buy from (B) and sell to (S) the investors, given prices P_B and P_S. The utility functions of these investors correspond to linear inverse-demand curves at which financial institutions can buy and sell the traded asset:

$$P_B = v - l_B + \lambda_B B \tag{3}$$

$$P_S = v + l_S - \lambda_S S \tag{4}$$

The values l_B and l_S are parameters for the level of investor demand. We refer to these variables as investor demand to buy (l_S) or investor demand to sell (l_B), or together simply as investor demand. Larger values of the investor-demand parameters act to raise the investor inverse-demand curves. The values λ_B and λ_S represent the investor's liquidity preference. Larger values of the liquidity-preference parameters act to rotate the investor inverse-demand curves.

D. Financial institutions

There exist an infinite number of risk-neutral financial institutions. Each financial institution can choose to pay an exogenous cost c to invest in the market-making technology and become a market maker. Institutions who choose to pay this cost are referred to interchangeably as "market-makers" and "dealers" and are indexed $i \in \{1, N\}$. This cost must be financed in $t = 0$ through either debt issuance D_0 or equity issuance E_0. The total rate of return required on the institution's assets is exogenously set by the unmodeled macroeconomy at $1 + r_A$, which we assume is greater than $1 + r_F$. For simplicity, we assume all financial institutions choose an equal debt-equity ratio.

A market maker has a balance sheet and cannot take "negative" positions. It cannot spend cash it does not have or sell bonds it does not have. Instead, market makers must explicitly short either cash or bonds. If a market maker lacks funds to make desired purchases, then it must issue debt. Unless this additional debt is a repo, the market maker must also earn the return $1 + r_A$ on additional debt. Debt created via repo has its own rate, r_R. Any additional debt issued in period 1 is designated D_1.

Each financial institution solves three problems: (1) the entry problem in $t = 0$; (2) the corporate financing decision in $t = 0$; and (3) the market maker's problem in $t = 1$. The firm's entry problem is to choose whether to pay $D_0 + E_0 \geq c + H_i$ in order to become a market maker. Financial institution i does so if it can earn a return $1 + r_A$ on its assets. The firm's corporate financing problem is to choose initial values D_0 and E_0 in order to maximize the final value of the firm $D_2 + E_2$, given the results of market making. Last, the firm's market-making problem is to maximize profits from buying and selling to investors, given investor demands l_B and l_S. Each institution i that participates in market making expects a profit of:

$$E[\pi_i] = (v - P_B)b_i + (P_S - v)s_i - r_R v(b_i - s_i) + H_i(1 + r_F) - (1 + r_A)(c + H_i). \quad (5)$$

E. Market structure and timing

At the beginning of the model at $t = 0$, all financial institutions observe the investor demands l_B and l_S, choose whether to pay the cost c to become a market maker and choose a quantity $H_i \geq 0$ of HQLA to purchase. The firms then choose to finance through some combination of debt D_0 and equity E_0.

After the institutions have completed their financing decisions, $t = 1$ begins. Each market maker selects a quantity b_i to buy from the market and s_i to sell to the market, given aggregate quantities of $B = \sum_i b_i$ and $S = \sum_i s_i$. The markets clear at prices P_B and P_S.

Finally, at $t = 2$, the asset liquidates. Market makers who engaged in repo agreements return the cash and receive collateral from their counterparties. Market makers who engaged in reverse-repo return the collateral and receive cash from their counterparties. Finally, market makers close any outstanding positions in the traded asset at its realized value V. Then the market makers liquidate and distribute any profits $D_2 + E_2$ to their financiers.

II. Baseline equilibrium

In this section, we present the baseline, unregulated equilibrium. An equilibrium in the model consists of: (i) a solution to each market maker's profit maximization problem given a number of firms N; (ii) a solution to each financial institution's corporate finance problem; and (iii) an equilibrium number of entrants N^* such that each financial institution earns a return at least $1 + r_A$ on its assets but, were $N^* + 1$ institutions to enter, they would not.

A. *Market-making decision*

Definition 1: *Let $\gamma = 2r_R v$ be the cost of accessing the repo market.*

Given a number of entrants N, each market maker chooses b_i and s_i to maximize:

$$E[\pi_i(N)] = E\left[\left(V - v - \frac{\gamma}{2} + l_B - \lambda_B \sum_i b_i\right) b_i + \left(v - V + \frac{\gamma}{2} + l_S - \lambda_S \sum_i s_i\right) s_i\right] + (1 + r_F)H_i - (1 + r_A)(c + H_i). \tag{6}$$

The result of this unconstrained optimization problem yields the standard symmetric Cournot results, with liquidity supplies of:

$$b_i^*(N) = \frac{l_B - \frac{\gamma}{2}}{\lambda_B(N+1)} \tag{7}$$

$$s_i^*(N) = \frac{l_S + \frac{\gamma}{2}}{\lambda_S(N+1)}. \tag{8}$$

Given that $r_A > r_F$, the institution optimally holds a quantity $H_i = 0$ of the high quality asset.

B. Corporate-financing decision

The baseline equilibrium for the corporate-financing decision follows from the standard Modigliani-Miller results. For a financial institution trying to maximize its final value $V = D_2 + E_2$, any initial combination of debt and equity is equally optimal (this will not be true under regulation). The financial institution must finance $c = D_0 + E_0$ in order to become a market maker. Throughout trading, the market maker has no requirement to issue any long-term debt, but holds short-term debt equal to its repo position $D_1 = v(b_i - s_i)$, which it clears following trading.

C. Market-maker entry decision

Given a number of entrants N and optimal behaviour, the expected profit of each market maker is:

$$E[\pi_i^*(N)] = \frac{(l_B - \frac{\gamma}{2})^2}{\lambda_B(N+1)^2} + \frac{(l_S + \frac{\gamma}{2})^2}{\lambda_S(N+1)^2} - (1 + r_A)c. \tag{9}$$

Given the expected profit, financial institutions choose whether to become market makers. Institutions will continue to enter as long as $\pi_i > 0$. Since $\pi_i^*(N)$ is decreasing in N, there is a single equilibrium number of entrants N^* such that:

$$N^* \leq \sqrt{\frac{(l_B - \frac{\gamma}{2})^2}{\lambda_B(1+r_A)c} + \frac{(l_S + \frac{\gamma}{2})^2}{\lambda_S(1+r_A)c}} - 1 < N^* + 1. \tag{10}$$

Theorem 1 (Existence of a Baseline Equilibrium)**:**

(i) Given a number of entrants N, there exist unique liquidity supplies b_i and s_i for each market maker i, such that each market maker solves his optimization problem.

(ii) Given (i), there exist debt and equity values D_0 and E_0, such that each financial institution maximizes its firm value.

(iii) Given (i) and (ii), there exists a unique equilibrium number of entrants N^* such that each financial institution earns a return of at least $1 + r_A$ on its assets. Instead, were $N^* + 1$ financial institution to enter, each would earn less than $1 + r_A$ on its assets.

III. Basel III regulations on funding liquidity and capital

In this section, we subject the market maker to balance-sheet constraints styled after the Basel III framework for liquidity and capital regulation. For liquidity regulation, we model the Basel III Liquidity Coverage Ratio (LCR) as well as the Net Stable Funding Ratio (NSFR). The Basel III LCR asks institutions to hold sufficient assets deemed high-quality to cover all liabilities due in 30 days or less. We model the LCR as an obligation to hold quantities of HQLA in proportion to repo liabilities. The Basel III NSFR asks institutions to secure funding deemed stable at terms at least as long as the terms of certain investments. We model the NSFR as an obligation to possess additional long-term funding (*i.e.* not repo) in proportion to cash invested in reverse repo. The extra funding is used to purchase HQLA. As such, the two liquidity requirements are mirrored: the LCR obligates HQLA purchases for repo, and the NSFR for reverse repo.

For capital regulation, we model the Basel III capital regulations as a single leverage constraint. The two capital regulations in Basel III, the Leverage Ratio and the Capital Adequacy Ratio, are distinguished by their risk weighting, which in our context is immaterial because there is only one asset in the model. We model a leverage ratio as a numerical limit on the total value of the institution's debt $(D_0 + D_1)$.

Formally, we model the Basel III framework via three assumptions:

Assumption 1: *Financial institutions that wish to engage in a repo transaction must hold HQLA. For a repo transaction of size $b_i - s_i$, they must hold $H_i = \alpha_L(b_i - s_i)v$. Financial institutions purchase this HQLA using additional debt in $t = 0$*

Assumption 2: *Financial institutions that wish to engage in a reverse-repo transaction must issue additional long-term debt in $t = 0$, which they invest in HQLA. For a reverse-repo transaction of size $s_i - b_i$, they must hold $H_i = \alpha_N(s_i - b_i)v$.*

Assumption 3: *Financial institutions are subject to a leverage ratio constraint, represented by β. The institution may not exceed this ratio before the asset value realizes. The maximum amount of debt an institution may hold is $\beta \geq \frac{D_0+D_1}{D_0+D_1+E_0}$.*

In the case of Assumptions 1 and 2, the assumption that institutions finance all HQLA purchases using debt provides a "worst case" for results relating to risk. Financial institutions which financed HQLA purchases using equity would be less likely to default than those who finance these purchases through debt. The HQLA is purchased by market makers in $t = 0$ and thus forms part of their corporate financing problem.

A. Market-making decision

For a given a number of entrants N, each market maker chooses b_i and s_i to solve the constrained maximization problem:

$$\max_{b_i,s_i} E[\pi_i(N)] = E\left[\left(V - v - \frac{\gamma}{2} + l_B - \lambda_B \sum_i b_i\right) b_i + \left(v - V + \frac{\gamma}{2} + l_S - \lambda_S \sum_i s_i\right) s_i\right]$$
$$+ (1 + r_F)H_i - (1 + r_A)(c + H_i), \tag{11}$$

$$\text{S.T.} \quad v(b_i - s_i) \leq \Psi, \tag{12}$$

where H_i , the amount of HQLA, is equal to either $H_i = \alpha_L(b_i - s_i)v$, if the market maker accesses the repo market ($b_i > s_i$), or $H_i = \alpha_N(s_i - b_i)v$, if the market maker accesses the reverse-repo market ($s_i > b_i$); and Ψ is an upper bound on the market maker's debt D_1.

To express equilibrium behaviour, it is convenient to define three constants:

Definition 2: *Let* $\Gamma_L = 2v(r_R + \alpha_L(r_A - r_F))$ *be the net cost of accessing the repo market in the presence of the LCR.*

Definition 3: *Let* $\Gamma_N = 2v(r_R - \alpha_N(r_A - r_F))$ *be the net benefit of accessing the reverse-repo market in the presence of the NSFR.*

Definition 4: *Let* $\Lambda = \lambda_B + \lambda_S$ *be the aggregate liquidity preference.*

In equilibrium, the two liquidity regulations never bind at the same time, because the market maker cannot be both long and short in the asset. The LCR binds if the market maker uses repo to fund a net long position ($b_i > s_i$), whereas the NSFR binds if the market maker uses reverse repo to borrow assets for a net short position ($b_i < s_i$). Since repo and reverse repo are made costly by the liquidity regulations, the market maker avoids them if it is economical by taking no net position ($b_i = s_i$). Alternatively, if investor demand is sufficiently strong, it is more profitable for the market maker to use repo or reverse repo despite the regulatory cost. The binding regions are formalized in Proposition 1.

Proposition 1 (Regions of effect of the liquidity constraints)**:**

In equilibrium, for a given number of market makers N,

(i) if $\lambda_S l_B - \lambda_B l_S > \frac{\Gamma_L}{2}\Lambda$, the LCR binds, and the market maker chooses $b_i > s_i$.

(ii) if $\frac{\Gamma_L}{2}\Lambda > \lambda_S l_B - \lambda_B l_S > \frac{\gamma}{2}\Lambda$, the LCR binds, and the market maker chooses $b_i = s_i$.

(iii) if $\frac{\gamma}{2}\Lambda > \lambda_S l_B - \lambda_B l_S > \frac{\Gamma_N}{2}\Lambda$, the NSFR binds, and the market maker chooses $b_i = s_i$.

(iv) if $\frac{\Gamma_N}{2}\Lambda > \lambda_B l_S - \lambda_S l_B$, the NSFR binds, and the market maker chooses $b_i < s_i$.

As for capital regulation, the leverage ratio binds only if the market maker takes a sufficiently long position in the asset. This is because it is an upper bound on debt, which increases in repo but not in reverse repo, as repo is accounted as debt whereas reverse repo is accounted as an investment (under the two major accounting standards, GAAP and IFRS). The binding regions are formalized in Proposition 2.

Proposition 2 (Region of effect of the leverage ratio)**:**

In equilibrium, for a given number of market makers N, the leverage ratio binds only if $\lambda_S l_B - \lambda_B l_S > \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B \lambda_S (N+1)\Psi}{v}$. The market maker chooses $b_i - s_i = \frac{\Psi}{v}$.

Since the regulations bind only in certain regions of the investor demand variables l_B and l_S, the equilibrium choices of b_i and s_i are piecewise defined in the regions of l_B and l_S. In the regulated equilibrium, liquidity supply for a given N market makers is:

$$b_i(N) = \begin{cases} \frac{l_B - \frac{\Gamma_N}{2}}{\lambda_B(N+1)} & \text{if } \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_N}{2}\Lambda \\ \frac{l_B + l_S}{(N+1)(\lambda_B+\lambda_S)} & \text{if } \frac{\Gamma_N}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda \\ \frac{l_B - \frac{\Gamma_L}{2}}{\lambda_B(N+1)} & \text{if } \frac{\Gamma_L}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B\lambda_S(N+1)\Psi}{v} \\ \frac{l_B + l_S}{(N+1)\Lambda} + \frac{\lambda_S\Psi}{v\Lambda} & \text{if } \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B\lambda_S(N+1)\Psi}{v} < \lambda_S l_B - \lambda_B l_S \end{cases} \tag{13}$$

$$s_i(N) = \begin{cases} \frac{l_S + \frac{\Gamma_N}{2}}{\lambda_S(N+1)} & \text{if } \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_N}{2}\Lambda \\ \frac{l_B + l_S}{(N+1)(\lambda_B+\lambda_S)} & \text{if } \frac{\Gamma_N}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda \\ \frac{l_S + \frac{\Gamma_L}{2}}{\lambda_S(N+1)} & \text{if } \frac{\Gamma_L}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B\lambda_S(N+1)\Psi}{v} \\ \frac{l_B + l_S}{(N+1)\Lambda} - \frac{\lambda_B\Psi}{v\Lambda} & \text{if } \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B\lambda_S(N+1)\Psi}{v} < \lambda_S l_B - \lambda_B l_S. \end{cases} \tag{14}$$

We find the market maker responds to regulation by rationing its balance sheet, taking smaller net positions ($|b_i - s_i|$) than in the baseline case. It even takes no net position ($b_i = s_i$) if buyer and seller demand are relatively balanced in value, which is the case in the second segment of the functions. The market maker achieves the rationing by shifting toward an agency basis of trade, meaning it it acts more as a broker between investors than as an ultimate counterparty. For example, in the case that seller demand is stronger ($\lambda_S l_B > \lambda_B l_S$), the market maker buys less from sellers than in the baseline equilibrium ($b_i < b_i^*$), yet it sells more inventory to buyers ($s_i > s_i^*$), meaning it is brokering a greater flow of assets from sellers to buyers than before. The regulated equilibrium is worse for investors than the baseline, as some investors who would like to trade more of the asset are unable.

Next, we give the effect of regulation on prices. In the regulated equilibrium, the prices for a given N market makers are functions of equilibrium liquidity supply and hence are also defined piecewise. They are:

$$P_B(N) = \begin{cases} v - l_B + \frac{N}{N+1}(l_B - \frac{\Gamma_N}{2}) & \text{if } \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_N}{2}\Lambda \\ v - l_B + \lambda_B \frac{N}{N+1}\frac{l_B+l_S}{\lambda_B+\lambda_S} & \text{if } \frac{\Gamma_N}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda \\ v - l_B + \frac{N}{N+1}(l_B - \frac{\Gamma_L}{2}) & \text{if } \frac{\Gamma_L}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B\lambda_S(N+1)\Psi}{v} \\ v - l_B + \frac{N\lambda_B}{\Lambda}(\frac{l_B+l_S}{N+1} + \lambda_S\frac{\Psi}{v}) & \text{if } \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B\lambda_S(N+1)\Psi}{v} < \lambda_S l_B - \lambda_B l_S \end{cases} \tag{15}$$

$$P_S(N) = \begin{cases} v + l_S - \frac{N}{N+1}(l_S + \frac{\Gamma_N}{2}) & \text{if } \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_N}{2}\Lambda \\ v + l_S - \lambda_S \frac{N}{N+1}\frac{l_B+l_S}{\lambda_B+\lambda_S} & \text{if } \frac{\Gamma_N}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda \\ v + l_S - \frac{N}{N+1}(l_S + \frac{\Gamma_L}{2}) & \text{if } \frac{\Gamma_L}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B\lambda_S(N+1)\Psi}{v} \\ v + l_S - \frac{N\lambda_S}{\Lambda}(\frac{l_B+l_S}{N+1} - \lambda_B\frac{\Psi}{v}) & \text{if } \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B\lambda_S(N+1)\Psi}{v} < \lambda_S l_B - \lambda_B l_S. \end{cases} \tag{16}$$

The asset price responds to regulation by exhibiting price pressure (Hendershott and Menkveld 2014), a pricing dislocation due to the cost of holding inventory. The prices are lower than baseline if the market maker takes a long position ($b_i > s_i$) and higher than baseline if the market maker takes a short position ($b_i < s_i$). The reason price pressure appears is, again, the shift to agency intermediation. In order for the market maker to limit its net position, it must pass some of its position to investors, but they require a price concession to accept the position. To sell excess inventory, the market maker must lower the price to solicit interest from buyers; to cover excess shorts, the market maker must raise the price to solicit interest from sellers.

Table I
Size of liquidity impact of each regulation, by asset class

	LCR	NSFR	LR
Sovereign Debt	None	Low	High
Corporate Debt (Above AA-)	Low	Low	Low
Corporate Debt (BBB- to A+)	High	High	Low
Corporate Debt (Below BBB-)	Very High	Very High	Low

The results illustrate how securities financing has a stabilizing role in bond markets. Using repo, a market maker can sell an asset to investor clients by borrowing it rather than procuring it outright, which would move the price. Borrowing has less impact on prices because it increases the effective supply of the asset, as a borrowed asset is used by two different investors at the same time. This is similar to the way that lendable deposits at banks increase the effective money supply. Prices are more stable under a principal market maker because it is adjusting asset supply to investor demand, whereas an agency market maker can only adjust the price.

Table I illustrates the predicted effects of various regulations on bond markets. The LCR and NSFR have their largest effects on more risky asset classes because, in Basel III, the size of the obligated purchases of HQLA generally increases with the risk weighting of an asset. In the case of the LCR, sovereign debt is exempted, so it has no effect in this case. Unlike the LCR and NSFR, a leverage ratio impacts any market maker if they acquire asset in large volume. Given that sovereign debt trades in much larger volume than corporate debt, our model predicts that the leverage ratio is likely to cause larger distortions in sovereign debt than for corporates. However, were corporate bonds to trade in very high quantity and unidirectionally, they too would be impacted by the leverage ratio.

Last, we give the effect of regulation on the bid-ask spread. The outcome is simple. In a Cournot oligopoly model in two markets, the sole determinant of the bid-ask spread is the number of firms, as the bid-ask spread is generated via imperfect competition. If N is fixed or remains constant, the spreads in the regulated and baseline equilibria are equal: $P_S(N) - P_B(N) = P_S^*(N) - P_B^*(N)$. Alternatively, spreads fall if additional firms enter and increase if fewer firms enter.

Since the LCR and NSFR never bind at the same time, there are testable implications on liquidity outcomes conditional on the regulation. The implications summarize the results in the section.

Testable implication 1: *Market makers limit their net position by trading more on an agency basis: When market makers are already long, they buy less and sell more; when already short, they buy more and sell less.*

Testable implication 2: *There is greater price pressure: Prices exhibit greater correlation with inventory position. By implication, there is greater price impact after large or unexpected purchases.*

Testable implication 3: *The bid-ask spread does not change if the composition of participants does not change. The bid-ask spread widens if fewer participants enter and contracts if more participants enter.*

Some of these implications have already been studied. Bessembinder et al. (2016) studies implication 1 and finds that dealers are committing less capital to market making in the post-regulatory period. Dick-Nielsen and Rossi (2016), and Bao et al. (2016) study implication 2 and find that price impacts after events necessitating sudden bond sales have gotten worse in the post-regulatory period. However, there is no study specifically on inventory pressure on the asset price post-regulation. Trebbi and Xiao (2016) studies implication 3 and finds a variety of metrics related to bid-ask spreads have not changed or have even improved, as do Adrian et al. (2015).

B. Corporate financing decision

There is a loss of capital-structure irrelevance due to the leverage constraint, which limits debt at a financial institution created by its use of repo and thus limits its maximum net position in the asset. The financial institution finances $c + H_i = D_0 + E_0$ in order to become a market maker and may finance additional debt using repo, with $D_1 = v(b_i - s_i)$. Given that a market maker's profit is limited if his repo exposure is restricted, he can increase his expected profit by selecting a capital structure which maximizes the possible repo debt. The loss of irrelevance is formalized in Proposition 3.

Proposition 3 (Loss of capital-structure irrelevance)**:**

For a given number of market makers N,

(i) If there is a leverage ratio, Modigliani-Miller capital-structure irrelevance does not hold. Financial institutions are able to increase their value by choosing sufficient initial equity such that $D_0 \leq \frac{\beta E_0 - (1-\beta) v(b_i - s_i)}{1-\beta}$.

(ii) If there is no leverage ratio, even if there are liquidity regulations, Modigliani-Miller capital structure irrelevance holds.

In contrast to liquidity regulation, which always binds, the leverage ratio constraint is limited in its application. For a financial institution that optimally chooses a capital structure low in debt, the leverage ratio binds only when the market maker faces relatively heavy demand to sell. The necessary level of seller demand that causes the leverage ratio to bind is strictly greater than the level that causes the LCR to bind, as the LCR binds on any long position. Supposing the financial institution choses to finance its operation cost c with equity, the maximum repo position is of size:

$$\Psi = \frac{\beta c}{(1-\beta)(1+\alpha_L)}. \tag{17}$$

Liquidity regulation can oblige financial institutions to obtain additional funding to purchase HQLA, which aggravates the debt limit of the leverage ratio. Were institutions to equity-finance their HQLA purchases, the reverse would be true.

Proposition 4 (Liquidity regulation aggravates the binding of the leverage ratio)**:** *For a given number of market makers N, when financial institution use debt to finance HQLA purchases, the maximum additional debt a market maker can enter, Ψ, is lower under the LCR that it would be without the LCR.*

C. Market-maker entry decision

For a given number of market makers N, the expected profit function follows from the equilibrium liquidity supply functions and price functions. It is defined piecewise:

$$E[\pi_i(N)] = \begin{cases} \frac{(l_B - \frac{\Gamma_N}{2})^2}{\lambda_B (N+1)^2} + \frac{(l_S + \frac{\Gamma_N}{2})^2}{\lambda_S (N+1)^2} - (1+r_A)c & \text{if } \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_N}{2}\Lambda \\ \frac{(l_B + l_S)^2}{(N+1)^2 \Lambda} - (1+r_A)c & \text{if } \frac{\Gamma_N}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda \\ \frac{(l_B - \frac{\Gamma_L}{2})^2}{\lambda_B (N+1)^2} + \frac{(l_S + \frac{\Gamma_L}{2})^2}{\lambda_S (N+1)^2} - (1+r_A)c & \text{if } \frac{\Gamma_L}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B \lambda_S (N+1)\Psi}{v} \\ \frac{(l_B + l_S)^2}{(N+1)^2 \Lambda} + \frac{(l_B \lambda_S - l_S \lambda_B - \Lambda \frac{\Gamma_L}{2})\Psi}{v\Lambda} & \\ -\frac{N \lambda_S \lambda_B \Psi^2}{v^2 \Lambda} - (1+r_A)c & \text{if } \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B \lambda_S (N+1)\Psi}{v} < \lambda_S l_B - \lambda_B l_S. \end{cases} \tag{18}$$

The profit function determines the number of entrants in equilibrium. Under certain parameterizations there are two equilibrium values of N, which occur in the relative vicinity of $\pi_i(N) = 0$. Specifically, if seller demand is large but not too large, it is possible the leverage ratio binds for a smaller number of market makers but does not bind if the number of market makers is larger. In the figures in this paper, we parameterize the model such that multiple equilibria do not arise. In this case we denote the equilbirum number of firms under Basel III regulation by $\hat{N}$, which is the number of entrants such that

$$\pi_i(\hat{N}) \geq 0 > \pi_i(\hat{N} + 1). \tag{19}$$

A property of $\hat{N}$ is that it is increasing in l_S and also in l_B. In other words, the more business there is to be done with investors, the more financial institutions enter the market-making industry.

The equilibrium number of entrants decreases when liquidity regulation applies. Profits decline due to obligatory purchases of HQLA, so fewer financial institutions invest in market making. However, unlike the case of liquidity regulation, the effect of the leverage ratio on market maker is ambiguous. If the leverage ratio is sufficiently tight, entry by new market makers can increase. This is formalized in Proposition 5.

Proposition 5 (The change in the number of firms)**:**

For given investor demands l_B and l_S,

(i) If market makers are bound only by liquidity regulation and not by the leverage ratio, fewer firms enter in equilibrium compared to the baseline case.

(ii) If the leverage ratio is sufficiently tight (β low enough such that $(1+\alpha_L)v\sqrt{\frac{1+r_A}{c\lambda_B}} > \frac{\beta}{1-\beta}$), then for sufficiently large seller demand relative to buyer demand, more firms enter in equilibrium compared to the case with no leverage ratio.

Proposition 5 part (ii) provides conditions under which the leverage ratio stimulates entry. The reason the leverage regulation can stimulate entry is because it acts as an restriction on quantity output. If firms in quantity competition can coordinate to reduce output, it enhances their returns, and the leverage ratio acts as such a coordination device. Market makers cannot produce liquidity for buyers beyond a certain level. Still, in order to stimulate entry successfully, the ratio has to be sufficiently tight relative to the expense of being a market maker. If it is too expensive to be a market maker in terms of the entry cost c (or if the asset value v is sufficiently cheap that it is easy to finance), even a binding leverage ratio does not stimulate entry. The proposition leads to a testable implication.

Testable implication 4: *If the leverage ratio is binding, the level β is sufficiently low (as in Proposition 5 Part ii), and seller demand is sufficiently high, outside market makers enter the industry. Otherwise, entry will not increase.*

Finally, theorem 2 establishes that the number of market makers and their choices can be determined in Cournot equilibrium.

Theorem 2 (Existence of a constrained equilibrium)**:**

(i) Given a number of entrants N, there exist an equilibrium consisting of liquidity supplies b_i, s_i and debt and equity levels D_0 and E_0, consistent with the baseline equilibrium.

(ii) There exists either a single unique equilibrium, or two equilibria for the number of entrants, denoted by N^* and N_B. N^* is the equilibrium number of entrants such that the leverage ratio does not bind and each financial institution earns a return of $1 + r_A$ on its assets. N_B is the equilibrium number of entrants such that the leverage ratio does bind and each financial institution earns a return of $1 + r_A$ on its assets. In each case, were an additional firm to enter, each firm would earn less than $1 + r_A$ on its assets.

D. Illustrated results

We illustrate the equilibrium behaviour of quantities and prices in Figure 1. The first panel graphs the liquidity supply functions $b_i(\hat{N})$ and $s_i(\hat{N})$, and the second panel graphs the prices $P_B(\hat{N})$ and $P_S(\hat{N})$. The panels graph the functions against the seller demand l_B for a fixed buyer demand l_S. In other words, the graphs vary *demand for market makers to buy* holding *demand for market makers to sell* constant. Accordingly, on the left side of the graphs, the market markers are short, and on the right side of the graph, the market makers are long.

FIGURE 1 ABOUT HERE

The graph of liquidity supply in Figure 1, Panel A, deviates from liquidity supply in the baseline model. In the baseline model, a change in seller demand changes only quantity bought b_i and not quantity sold s_i. In contrast, in Figure 1, a change in seller demand does change quantity sold s_i, in the three regions marked NSFR, LCR and leverage ratio (LR). The three regions correspond to those in Propositions 1 and 2. As discussed, the Basel III regulations induce the market maker to shift its trading strategy toward an agency basis, distributing assets from sellers to buyers rather than taking them on the balance sheet. Accordingly, in the marked regions, quantity sold rises parallel with quantity bought, showing the market maker is trading on agency.

Asset prices in Figure 1, Panel B, also deviate from results in the baseline model and in the same three regions. The deviations illustrate how asset prices exhibit price pressure from inventory. The asset price in the NSFR region deviates by the cost of holding HQLA required by the NSFR, which is proportional to α_N. Similarly, the asset price in the LCR region deviates by the cost of holding HQLA for the LCR, which is proportional to α_L.

Last, the asset price has a monotonic negative relationship with seller demand when the leverage ratio binds. In each of these regions, the market maker is operating as an agency intermediary and is adjusting the price to incentivize investors to trade with one another.

We give additional figures showing the partial effects of each of the regulations in isolation. Figures 3, 4 and 5 show the individual effects of each regulation on equilibrium liquidity supply, prices and spreads respectively. In each case, we compare the individual effect of each regulation to the baseline case in which no regulation is imposed.

FIGURES 3, 4 AND 5 ABOUT HERE

E. Impact of Regulation on Financial Institution Risk

In this section we analyze the impact of regulation on financial institution risk. We rely on two definitions in order to clarify the risk-reducing nature of regulations.

Definition 5: *Let ζ_i be excess expected profit that each of the financial institutions earn due to the discrete nature of Cournot firms*

Definition 6: *Let D_0^H and D_0^c be the amounts of debt in $t = 0$ used to fund HQLA costs H_i and startup costs c, respectively.*

First, we define ζ_i as the excess expected profit earned by each firm. Were the number of firms non-discrete, this would be equal to zero. However, in equilibrium, the excess profit earned by N firms is insufficient to support the entry of $N + 1$ firms and thus each entrant earns some excess profit. For simplicity of analysis we assume this number of be equal to zero ($\zeta_i = 0$).

Second, we divide the debt issued by dealers in $t = 0$ into two parts, D_0^H and D_0^c. To derive a "worst case" for the risk-related benefits of regulation, we assume that dealers entirely debt-finance their HQLA purchases, rather than creating an additional equity buffer.

Thus, when dealers must hold HQLA for regulatory reasons, $D_0^H = H_i$. This simplification is also useful for comparing the impact of regulation to the baseline scenario given that, in both cases, $D_0^c + E_0 = c$

To derive the impact of risk, we define an institutional default as the case when a financial institution is unable to pay back the complete value of its debt in $t = 0$, equal to $(1+r_D)D_0$. The realized value of the institution's profit function for given the realized asset value V, before paying out debt and equity holders, can be shown to be equal to :

$$(V - v)(b_i - s_i) + (1 + r_A)(H_i + c) + \zeta_i \tag{20}$$

Through substitution of $\zeta_i = 0$, D_0^H and D_0^c above, it can be shown that for any realization of the traded asset value, V, default occurs when:

$$(V - v)(b_i - s_i) + (r_A - r_D)H_i < (r_D - r_A)D_0^c - (1 + r_A)E_0 \tag{21}$$

We note that, despite the risk of default, we assume r_A remains constant and determined by the unmodeled macroeconomy. While r_D could be determined as an equilibrium result of the probability of default, risk free rate and the recovery value, we instead rely on the simple assumption that r_D is weakly increasing in relative leverage ($\frac{\partial r_D}{\partial(D_0^c - E_0)} \geq 0$).

Policy Implication 1: The NSFR incentivizes financial institutions to take no risk when $\frac{\gamma}{2}\Lambda > \lambda_S l_B - \lambda_B l_S > \frac{\Gamma_N}{2}\Lambda$. Further, for a given number of entrants N, it reduces their risk when $\frac{\Gamma_N}{2}\Lambda > \lambda_S l_B - \lambda_B l_S$.

Policy Implication 2: The LCR incentivizes financial institutions to take no risk when $\frac{\Gamma_L}{2}\Lambda > \lambda_S l_B - \lambda_B l_S > \frac{\gamma}{2}\Lambda$. Further, for a given number of entrants N, it reduces their risk when $\lambda_S l_B - \lambda_B l_S > \frac{\Gamma_L}{2}\Lambda$.

Policy Implication 3: The leverage ratio creates an upper limit $(V - v)\frac{\Psi}{v}$ to the amount of risk dealers take on long positions. The LR also creates a minimum equity buffer at all times.

In this model, the regulations are able to reduce risk to financial institutions. We show that the reduction in risk depends on the regulation in question. The NSFR and LCR are effective at reducing risk in two ways. First, they stabilize the cash flows expected by financial institutions by forcing them to hold additional HQLA, increasing the left-hand side of Equation 21. Second, they reduce financial institutions incentives to hold large positions, reducing the magnitude of institutions' downside risk. The NSFR reduces the net position when $s_i > b_i$, while the LCR reduces the net position when $b_i > s_i$. The combination of the two regulations incentivizes dealers to take on no risk over the interval $\frac{\Gamma_L}{2}\Lambda > \lambda_S l_B - \lambda_B l_S > \frac{\Gamma_N}{2}\Lambda$. Further, for a fixed number of entrants N, dealers take on less risk under all combinations of $\lambda_S l_B$, $\lambda_B l_S$ outside this interval.

The leverage ratio creates additional benefits where the NSFR and LCR do not. First, it forces institutions to maintain a minimum equity buffer, E_0, compared to their total debt holdings. If institutions would otherwise fund themselves using primarily debt, this lowers the probability of default by decreasing the right-hand side of Equation 21 under all combinations of $\lambda_S l_B$, $\lambda_B l_S$. Second, the leverage ratio caps the long position an institution can take at $b_i - s_i = \frac{\Psi}{v}$. Unlike the combination of the LCR and NSFR, this second benefit from the leverage ratio only affects long-positions, and does not create a cap on risk for dealers who take short-positions. This implies that the leverage ratio is more effective at protecting financial institutions from sudden drops in asset value when they are long on an asset, than from sudden appreciations in asset value when they are short.

IV. Extension: Position limits and the Volcker Rule

In this section we cease to examine the Basel III capital and liquidity regulations and instead study a position-limit regulation styled on the Volcker Rule. The Volcker Rule, section eight of the 2012 Dodd-Frank Act in the United States, is a complex ban on proprietary trading at financial institutions with access to federal backstops. It obligates regulated institutions to satisfy an array of metrics, including a set of internal position limits for each trading desk. The risk and position limits are the primary way the regulation binds dealers, as dealers are unable to exceed this limits in response to unexpected client demand (Bao et al. 2016). Thus, we choose to stylize this aspect of the Volcker Rule in isolation of the others. In effect, we assume the asset already satisfies the other metrics of the Volcker Rule. Formally, we impose Assumption 4 and 5 (and relax the other assumptions).

Assumption 4: *Market makers have an exogenous position limit Δ, such that their net position must be below this limit following trading: $|b_i - s_i| \leq \Delta$.*

Assumption 5: *Investors have liquidity preferences equal to $\lambda_S = 1$ and $\lambda_B = 1$*

We derive equilibrium behaviour under a position limit on the absolute net position. Our finding is that it can be understood in the same way as the leverage ratio. Like the leverage ratio, a position limit is a cap on the long position of the market maker. In addition, the position limit also is a cap on the size of its short position. Thus it can be thought of as a leverage ratio plus the mirror image of the leverage ratio. However, unlike the leverage ratio, we show that the position limit does not prevent market makers from generating risk through high leverage.

A. Market-making decision

The market maker solves the constrained maximization problem given a number of market makers N,

$$\max_{b_i, s_i} \pi_i(N) = \left(-r_R v + l_B - \sum_i b_i\right) b_i + \left(r_R v + l_S - \sum_i s_i\right) s_i - (1 + r_A)c,$$
$$\text{S.T.} \quad |b_i - s_i| \leq \Delta. \tag{22}$$

The position limit binds when an imbalance in investor demand would lead the market maker to take a position larger than the limit Δ. Formally, it binds when $l_B - l_S$ is large relative to repo borrowing costs:

$$|l_B - l_S - \gamma| < (N + 1)\Delta. \tag{23}$$

Since the position limit binds only in certain regions of the investor demand variables l_B and l_S, the equilibrium choices of b_i and s_i are piecewise defined in the relevant regions of l_B and l_S. In equilibrium, liquidity supply given a number of market makers N is:

$$b_i(N) = \begin{cases} \frac{l_B + l_S}{2(N+1)} - \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma < -(N+1)\Delta \\ \frac{l_B - \frac{\gamma}{2}}{N+1} & \text{if } |l_B - l_S - \gamma| \leq (N+1)\Delta \\ \frac{l_B + l_S}{2(N+1)} + \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma > (N+1)\Delta \end{cases} \tag{24}$$

$$s_i(N) = \begin{cases} \frac{l_B+l_S}{2(N+1)} + \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma < -(N+1)\Delta \\ \frac{\frac{\gamma}{2}+l_S}{N+1} & \text{if } |l_B - l_S - \gamma| \leq (N+1)\Delta \\ \frac{l_B+l_S}{2(N+1)} - \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma > (N+1)\Delta. \end{cases} \tag{25}$$

The previous analysis of the binding leverage ratio applies here. In short, the market maker responds to a binding position limit by shifting its trading to a pure agency basis. Every security in excess of the position limit that it buys or sells must in turn be sold or bought with investors. If the position limit does not bind, the market maker supplies the same liquidity as in the baseline case.

In terms of pricing, the previous analysis of the binding leverage ratio applies. When a market maker shifts to an agency basis, it must unwind its positions by trading with investors. As buyers or sellers require a price concession to motivate trade, the asset price adjusts to motivate them to trade. The equilibrium liquidity supply and asset prices lead to a further testable implication.

Testable implication 5: *There is greater price pressure: Prices exhibit greater correlation with inventory position. By implication, there is greater price impact after large or unexpected purchases.*

B. Corporate financing decision and Market-maker entry decision

Capital-structure irrelevance holds under the position limit. Unlike the leverage ratio, in which the constraint is on debt, firms cannot improve their position limit by financing using a greater share of equity. The irrelevance result of the baseline model is therefore unaffected.

For a given number of market makers N, the profit function is similar to the leverage ratio, if Ψ is replaced with Δ. Again, as with the leverage ratio, the equilibrium number of entrants can increase if the position limit is sufficiently binding. This is obtained from Proposition 5 if Ψ is replaced with Δ, so we do not repeat the proof. The profit function is symmetric about $l_B - l_S$, and the same proof holds for large l_B instead of large l_S. Thus the Volcker Rule may lead to more entry regardless of whether there is greater buyer or greater seller pressure, if the position limit is sufficiently tight relative to the cost of market making.

Testable implication 6: *If the Volcker Rule is binding, the position limit is sufficiently small, and either seller or buyer demand is sufficiently large, outside market makers enter the industry. Otherwise, entry will not increase.*

Policy Implication 4: The Volcker creates an upper limit to the amount of risk dealers take on both long and short positions.

While the impacts of the Volcker Rule may appear qualitatively similar to the leverage ratio, there are two key differences related to risk. Unlike the leverage ratio, the Volcker Rule limits the amount of risk financial institutions may take in both long and short positions because it places a cap on the absolute value $|b_i - s_i|$. Compared to the leverage ratio, this reduces risk to financial institutions by limiting their ability to take short positions. However, a limit to the Volcker Rule is that it does not reduce the ability of financial institutions to take on excess leverage. In the context of the model, this means that the financial institutions may have little to no equity buffer, unlike in the case of the leverage ratio.

V. Conclusions

This paper studies how three types of regulation change the behaviour of market makers. The regulations we study are the Basel III standards on capital and funding liquidity and the U.S. Volcker Rule limits on position. Though the regulations are different, they all create a common incentive for market makers to trade on an agency basis: Market makers respond to regulation by matching buyers to sellers rather than acting as the ultimate counterparty. The regulations have a second and long-term consequence. If financial institutions are leverage-constrained or Volcker-constrained, outside market makers enter the market-making business to supply capital made that has been made scarce by regulation.

The outcome for market liquidity as measured by the bid-ask spread can be an improvement. The entry of new market makers intensifies competition, tightening the bid-ask spread. However, liquidity is less resilient to sudden demands to trade. The price impact of a large imbalance of trade is worse, particularly when we fix the number of market makers (no entry). The model can thus account for data that show both improving bid-ask spreads as well as worsening costs to immediacy (Trebbi and Xiao 2016; Bessembinder et al. 2016; Dick-Nielsen and Rossi 2016; Bao et al. 2016).

Last, the model affords two new predictions for future empirical work. First, inventory premia or "price pressure" should increase. If market makers relegate their trading to the matching of buyers and sellers, rather than using their capital to absorb imbalances, it is prices that must adjust to equilibriate supply and demand. Second, the Basel III reforms should affect market makers' long positions differently from short positions. Regulation that affects repo rather than reverse repo affects long positions rather than short positions, and vice versa.

REFERENCES

Adrian, T., N. Boyarchenko, and O. Shachar (2017). Dealer balance sheets and bond liquidity provision. *Journal of Monetary Economics*.

Adrian, T., M. Fleming, O. Shachar, and E. Vogt (2015). Has U.S. corporate bond market liquidity deteriorated? *New York Federal Reserve Liberty Street Economics*.

Bao, J., M. O'Hara, and X. A. Zhou (2016). The Volcker Rule and market-making in times of stress. *Available at SSRN: https://ssrn.com/abstract=2836714*.

Bessembinder, H., S. E. Jacobsen, W. F. Maxwell, and K. Venkataraman (2016). Capital commitment and illiquidity in corporate bonds. *Available at SSRN: https://ssrn.com/abstract=2752610*.

Biais, B. (1993). Price formation and equilibrium liquidity in fragmented and centralized markets. *The Journal of Finance 48*(1), 157–185.

Choi, J. and Y. Huh (2016). Customer liquidity provision in corporate bond markets.

Committee on the Global Financial System (2016). Fixed income market liquidity. *CGFS Papers* (55).

Dick-Nielsen, J. and M. Rossi (2016). The cost of immediacy for corporate bonds. *Available at SSRN: https://ssrn.com/abstract=2139403*.

Du, W., A. Tepper, and A. Verdelhan (2016). Covered interest rate parity deviations in the post-crisis world. *Available at SSRN: https://ssrn.com/abstract=2768207*.

Duffie, D. (1996). Special repo rates. *The Journal of Finance 51*(2), 493–526.

Duffie, D. (2012). Market making under the proposed Volcker Rule. *Available at SSRN: https://ssrn.com/abstract=1990472*.

Dutta, P. K. and A. Madhavan (1997). Competition and collusion in dealer markets. *The Journal of Finance 52*(1), 245–276.

European Systemic Risk Board (2016). Market liquidity and market making. *Reports*.

Fleming, M. J. and E. M. Remolona (1999). Price formation and liquidity in the US treasury market: The response to public information. *The Journal of Finance 54*(5), 1901–1915.

Friewald, N. and F. Nagler (2016). Dealer inventory and the cross-section of corporate bond returns. *Available at SSRN: https://ssrn.com/abstract=2526291*.

Hendershott, T. and A. J. Menkveld (2014). Price pressures. *Journal of Financial Economics 114*(3), 405–423.

Ho, T. S. and H. R. Stoll (1983). The dynamics of dealer markets under competition. *The Journal of finance 38*(4), 1053–1074.

Huh, Y. and S. Infante (2016). Bond market liquidity and the role of repo. *Available at SSRN: https://ssrn.com/abstract=2773678*.

Kyle, A. S. (1989). Informed speculation with imperfect competition. *The Review of Economic Studies 56*(3), 317–355.

Laux, P. A. (1995). Dealer market structure, outside competition, and the bid-ask spread. *Journal of Economic Dynamics and Control 19*(4), 683–710.

Liu, H. and Y. Wang (2016). Market making with asymmetric information and inventory risk. *Journal of Economic Theory 163*, 73–109.

O'Hara, M., Y. Wang, and X. A. Zhou (2016). The execution quality of corporate bonds. *Available at SSRN: https://ssrn.com/abstract=2836714*.

Pinnington, J. and M. Shamloo (2016). Limits to arbitrage and deviations from covered interest rate parity. *Bank of Canada Working Paper*.

Schultz, P. (2017). Inventory management by corporate bond dealers. *Available at SSRN: https://ssrn.com/abstract=2966919*.

Trebbi, F. and K. Xiao (2016). Regulation and market liquidity. *National Bureau of Economic Research Working Paper*.

Vives, X. (2011). Strategic supply function competition with private information. *Econometrica 79*(6), 1919–1966.

A Appendix

A. Baseline equilibrium

Proof of Theorem 1

Parts i, ii and iii

The proof of the baseline equilibrium is consistent with a standard problem of Cournot competition with endogenous entry. The market maker has a profit function denoted:

$$E[\pi_i] = E\left[\left(V - v - r_R v + l_B - \lambda_B \sum_i b_i\right) b_i + \left(v - V + r_R v + l_S - \lambda_S \sum_i s_i\right) s_i - (1 + r_A)c\right]. \tag{A.1}$$

The problem is solved in the standard way, first by taking the first-order conditions with respect to b_i and s_i for all i. Second, the symmetry conditions of $b_i = b_j$ and $s_i = s_j$ are imposed for all N entrants, and the expectation operator evaluated, giving equilibrium liquidity supplies of

$$b_i = \frac{l_B - r_R v}{\lambda_B(N+1)} \tag{A.2}$$

$$s_i = \frac{l_S + r_R v}{\lambda_S(N+1)}. \tag{A.3}$$

The prices at which the market maker buys and sells to the market are given by:

$$P_B = v - l_B + \frac{N}{N+1}(l_B - r_R v) \tag{A.4}$$

$$P_S = v + l_S - \frac{N}{N+1}(l_S + r_R v) \tag{A.5}$$

The equilibrium liquidity supplies and prices can then be inserted into the profit functions. Through algebraic manipulation, the equilibrium expected profit is then given by:

$$E[\pi_i] = \frac{(l_B - r_R v)^2}{\lambda_B(N+1)^2} + \frac{(l_S + r_R v)^2}{\lambda_S(N+1)^2} - (1 + r_A)c. \tag{A.6}$$

This profit function is independent of the market maker's debt and equity choice, which therefore have no effect on the firm. The optimal combination of debt and equity is any

$D_0, E_0 \geq 0$ such that $D_0 + E_0 = c$. In equilibrium, more firms will continue to enter until they earn zero profit. Setting the profit equation equal to 0, and solving for N, gives an equilibrium number of entrants equal to:

$$\hat{N} = \sqrt{\frac{(l_B - r_R v)^2}{\lambda_B(1 + r_A)c} + \frac{(l_S + r_R v)^2}{\lambda_S(1 + r_A)c}} - 1. \tag{A.7}$$

As the number of firms is discrete, N^* is the integer value directly below the preceding equation, such that $N^* \leq \hat{N} < N^* + 1$. As with the rest of the baseline equilibrium, the equilibrium number of entrants is consistent with the general properties of a Cournot equilibrium with endogenous entry.

B. Constrained equilibrium

Proof of Proposition 1

This proof is in four subparts: (1) a market maker who accesses the reverse repo market by selling more units than he buys ($s_i > b_i$); (2) a market maker who holds zero net inventory ($s_i = b_i$); (3) a market maker who accesses the repo market and must hold HQLA ($s_i < b_i$); and, (4) comparing the three previous market makers to the baseline case.

First, for a market maker who accesses the reverse repo market and must hold HQLA. In doing so, he maximizes his profit function while adding HQLA holdings equal to $\alpha_N v(s_i - bi)$. These holdings pay a return of r_F and cost a rate of r_A. By taking the first derivatives of this profit function and substituting terms for Γ_N, equilibrium supplies are shown to be:

$$b_i = \frac{l_B - \frac{\Gamma_N}{2}}{\lambda_B(N + 1)} \tag{A.8}$$

$$s_i = \frac{l_S + \frac{\Gamma_N}{2}}{\lambda_S(N + 1)}. \tag{A.9}$$

Given the equilibrium supplies, the optimal prices are given by:

$$P_B = v - l_B + \frac{N}{N + 1}(l_B - \frac{\Gamma_N}{2}) \tag{A.10}$$

$$P_S = v + l_S - \frac{N}{N + 1}(l_S + \frac{\Gamma_N}{2}). \tag{A.11}$$

Substituting these into the profit functions gives an expected total profit of:

$$E[\pi_1] = \frac{(l_B - \frac{\Gamma_N}{2})^2}{\lambda_B(N+1)^2} + \frac{(l_S + \frac{\Gamma_N}{2})^2}{\lambda_S(N+1)^2} - (1+r_A)c. \tag{A.12}$$

Since this profit function relies on the market maker who performs a net sale, this profit function is viable for $s_i > b_i$ which, in equilibrium is given by $\lambda_B l_S - \lambda_S l_B \geq -\frac{\Gamma_N}{2}(\lambda_B + \lambda_S)$

Second, a market maker may wish to hold a positive inventory, but because of the cost of holding HQLA, holds a zero inventory instead. To hold a zero inventory, a market maker sets $s_i = b_i$ and maximizes his initial profit function. This results in equilibrium supplies of:

$$b_i = s_i = \frac{l_B + l_S}{(N+1)(\lambda_B + \lambda_S)}. \tag{A.13}$$

Given this equilibrium supply, the market prices are:

$$P_B = v - l_B + \lambda_B \frac{N}{N+1} \frac{l_B + l_S}{\lambda_B + \lambda_S} \tag{A.14}$$

$$P_S = v + l_S - \lambda_S \frac{N}{N+1} \frac{l_B + l_S}{\lambda_B + \lambda_S}. \tag{A.15}$$

Substitution into the profit function yields an expected profit of:

$$E[\pi_2] = \frac{(l_B + l_S)^2}{(N+1)^2(\lambda_B + \lambda_S)} - (1+r_A)c. \tag{A.16}$$

This profit function is viable for any parameter set. By simply evaluating the reduced form profit functions, it would appear that $\pi_2 < \pi_1$. However, for all $\lambda_B l_S - \lambda_S l_B < -\frac{\Gamma_N}{2}(\lambda_B + \lambda_S)$, a market maker would optimally choose to hold $s_i < b_i$ and thus performs a strategy inconsistent with π_1.

Third, a market maker may wish to access the repo market and hold HQLA. In doing so, he maximizes his profit function while adding HQLA holdings equal to $\alpha_L v(b_i - si)$. These holdings pay a return of r_F and cost a rate of r_A. By taking the first derivatives of this profit function and substituting terms for Γ_L, equilibrium supplies are shown to be:

$$b_i = \frac{l_B - \frac{\Gamma_L}{2}}{\lambda_B(N+1)} \tag{A.17}$$

$$s_i = \frac{l_S + \frac{\Gamma_L}{2}}{\lambda_S(N+1)}. \tag{A.18}$$

Given the equilibrium supplies, the optimal prices are given by:

$$P_B = v - l_B + \frac{N}{N+1}(l_B - \frac{\Gamma_L}{2}) \tag{A.19}$$

$$P_S = v + l_S - \frac{N}{N+1}(l_S + \frac{\Gamma_L}{2}). \tag{A.20}$$

Substituting these into the profit functions gives a total profit of:

$$\pi_3 = \frac{(l_B - \frac{\Gamma_L}{2})^2}{\lambda_B(N+1)^2} + \frac{(l_S + \frac{\Gamma_L}{2})^2}{\lambda_S(N+1)^2} - (1 + r_A)c. \tag{A.21}$$

As the market maker holds HQLA only when he wishes to buy more than he sells, this strategy is not viable for any parameter set where the market maker wishes to supply $s_i > b_i$. In the baseline case, this occurs when $\lambda_B l_S - \lambda_S l_B \geq -\frac{\gamma}{2}(\lambda_B + \lambda_S)$, and thus the spaces spanned by π_3 and π_1 are mutually exclusive.

As above, by simply evaluating the reduced form π_2 and π_3 it would appear that $\pi_3 \geq \pi_2$. However, for all $\lambda_B l_S - \lambda_S l_B > -\frac{\Gamma_L}{2}(\lambda_B + \lambda_S)$, the market maker would wish to hold $s_i > b_i$ which is inconsistent with the strategy in π_3. Thus, in equilibrium, the spaces over which π_1, π_2, π_3 and their respective demand functions hold are mutually exclusive.

Finally, in the baseline equilibrium, $b_i > s_i$ when $\lambda_S l_B - \lambda_B l_S > \frac{\gamma}{2}\Lambda$. Thus, when $\lambda_S l_B - \lambda_B l_S \leq \frac{\gamma}{2}\Lambda$, the LCR has no effect on the market maker's decision. These results are reversed in the case of the NSFR.

Proof of Proposition 2

Taking the initial issuance as given, define the maximum value of repo debt that a market maker can acquire such that he does not violate his leverage ratio in $t = 1$ as Ψ:

$$\Psi \geq v(b_i - s_i) \tag{A.22}$$

Substituting the equilibrium values b_i, s_i from the case with the liquidity ratio alone, algebraic manipulation can show that the leverage constraint binds when:

$$\lambda_B l_S - \lambda_S l_B < -\frac{\lambda_B \lambda_S (N+1)\Psi}{v} - \frac{\Gamma_L}{2}(\lambda_B + \lambda_S). \tag{A.23}$$

Proof of Proposition 3

Given debt D_0 and equity E_0 from the initial issuance, the maximum value of repo exposure is equal to:

$$v(1+\alpha_L)(b_i - s_i) = \Psi = \frac{\beta E_0 - (1-\beta) D_0}{1-\beta}. \tag{A.24}$$

Profit is decreasing when repo exposure is restricted and thus the market maker optimally chooses an initial debt level that prevents the repo constraint from binding. This is given by:

$$D_0 \leq \frac{\beta E_0 - (1-\beta)(1+\alpha_L) v (b_i - s_i)}{1-\beta}. \tag{A.25}$$

As the market maker is able to increase outputs by selecting a capital structure with $D_0 \leq \frac{\beta E_0 - (1-\beta) v b_i}{1-\beta}$, then he is able to increase his firm's value through capital structure. This is driven by the assumption that the market maker is able to finance his subsequent market making costs only through additional debt (the repo market), rather than additional equity issuances. Thus, Modigliani-Miller capital structure irrelevance does not hold under these assumptions.

Proof of Proposition 4

Consider an institution which debt finances HQLA and takes a long position $b_i - s_i > 0$. The minimum debt, and maximum equity this institution can issue in the first period are $D_0 = v\alpha_L(b_i - s_i)$ and $E_0 = c$. These values correspond to a repo position of $D_1 = v(b_i - s_i)$. Substituting these values into the formula for the leverage ratio gives a position size $b_i - s_i$ which solves:

$$\frac{v(1+\alpha_L)(b_i - s_i)}{v(1+\alpha_L)(b_i - s_i) + c} = \beta \tag{A.26}$$

Evaluating this expression gives a maximum additional debt limit of:

$$v(b_i - s_i) = \Psi = \frac{\beta c}{(1-\beta)(1+\alpha_L)}. \tag{A.27}$$

This expression is strictly smaller for any positive value of α_L.

Proof of Proposition 5, Part i

The decrease in participants under liquidity regulation follows directly from the profit functions present in Proposition 1. Consider the case of the LCR, with a fixed number of entrants $\hat{N}$. Under the LCR, a market maker holds either HQLA or holds zero inventory when $b_i \geq s_i$. These two cases correspond to the equilibrium values π_2 and π_3 from Proof of Proposition 1. In equilibrium, these only occurs when $\lambda_S l_B - \lambda_B l_S > \frac{\gamma}{2}\Lambda$.

Next, consider the equilibrium value π from Proof of Theorem 1, Part iii. By simply evaluating the inequalities, $\pi \geq \pi_2$ and $\pi \geq \pi_3$, algebraic manipulation shows that these conditions would hold over the entire space where $\lambda_S l_B - \lambda_B l_S > \frac{\gamma}{2}\Lambda$ also holds. However, as market makers must hold HQLA under the LCR, π is not viable, and the market maker must choose either of the strategies resulting in π_2 or π_3. Thus, if the market maker is affected by the liquidity ratio, his profit is less than the profit he would earn in the baseline model given the same number of entrants.

All three functions π, π_2 and π_3 are decreasing in N when $N > 0$. Thus, given that $\pi \geq \pi_2$ and $\pi \geq \pi_3$ for a given $\hat{N}$ over the interval $\lambda_S l_B - \lambda_B l_S > \frac{\gamma}{2}\Lambda$, the equilibrium value number of firms N^* when the LCR is in place is lower than the equilibrium number of firms N^* from the baseline model.

The results for the NSFR, which affects market makers in equilibrium when $\lambda_S l_B - \lambda_B l_S < \frac{\gamma}{2}\Lambda$, are symmetric.

Proof of Proposition 5, Part ii

In order to give conditions under which leverage regulation can increase the number of equilibrium entrants, it is sufficient to show the profit function in the regulated equilibrium is positive for the equilibrium number of firms in the unregulated equilibirum under those conditions. This would imply that the market makers under regulation would earn positive returns above r_A, and thus too few market makers have entered to satisfy the entry condition.

In the case where the market maker holds HQLA and the leverage ratio binds, substituting the new position limit of $s_i = b_i - \frac{\Psi}{v}$ into the market maker's initial profit function gives a new profit function of:

$$E[\pi_i] = (-\frac{\Gamma_L}{2} + l_B - \lambda_B \sum_i b_i)b_i + (\frac{\Gamma_L}{2} + l_S - \lambda_S \sum_i b_i + \lambda_S \sum_i \frac{\Psi}{v})(b_i - \frac{\Psi}{v}) - (1 + r_A)c. \tag{A.28}$$

Taking the first-order condition gives equilibrium liquidity supply values of:

$$b_i = \frac{l_B + l_S}{(N+1)(\lambda_B + \lambda_S)} + \frac{\lambda_S \Psi}{v(\lambda_B + \lambda_S)} \tag{A.29}$$

$$s_i = \frac{l_B + l_S}{(N+1)(\lambda_B + \lambda_S)} - \frac{\lambda_B \Psi}{v(\lambda_B + \lambda_S)}. \tag{A.30}$$

Where the value Ψ is defined:

$$\Psi = \frac{\beta c}{(1-\beta)(1+\alpha_L)}. \tag{A.31}$$

These supply functions result in an expected profit function, for the leverage-ratio binding case, of:

$$\pi_B(N) = \frac{(l_B + l_S)^2}{(N+1)^2 \Lambda} + \frac{(l_B \lambda_S - l_S \lambda_B - \Lambda \frac{\Gamma_L}{2})\Psi}{v\Lambda} - \frac{N \lambda_S \lambda_B \Psi^2}{v^2 \Lambda} - (1 + r_A)c. \tag{A.32}$$

The equilibrium continuous-valued N^* in the LCR-binding case is such that $\pi_i(N) = 0$, where $\pi_i(N) = 0$ is given in segment three of (18). We insert this N^* as the argument to the profit function assuming the leverage constraint binds. This function is then given by:

$$\begin{aligned}\pi_B(N^*) = {} & \frac{(l_B \lambda_S - l_S \lambda_B - \Lambda \frac{\Gamma_L}{2})\Psi}{v\Lambda} + \frac{(l_B + l_S)^2 (1 + r_A)c}{(\frac{(l_B - \frac{\Gamma_L}{2})^2}{\lambda_B} + \frac{(l_S + \frac{\Gamma_L}{2})^2}{\lambda_S})\Lambda} \\ & - \left(\sqrt{\frac{(l_B - \frac{\Gamma_L}{2})^2}{\lambda_B (1 + r_A)c} + \frac{(l_S + \frac{\Gamma_L}{2})^2}{\lambda_S (1 + r_A)c}} - 1 \right) \frac{\lambda_S \lambda_B \Psi^2}{v^2 \Lambda} - (1 + r_A)c \end{aligned} \tag{A.33}$$

In the next stage, we show that under some conditions, the profit function for the leverage-ratio constrained case, using the number of entrants from the non-leverage ratio constrained case goes to infinity. This shows that entrants would earn positive profits, implying that more entrants are necessary under the leverage-ratio constrained case.

The Taylor expansion of (A.33), dropping all terms below order one in l_B, is

$$\left(\frac{\lambda_S\Psi}{\Lambda v}-\frac{\lambda_S\lambda_B\Psi^2}{\Lambda v^2}\frac{1}{\sqrt{\lambda_B(1+r_A)c}}\right)\left(l_B-l_B^0\right). \tag{A.34}$$

So the limit of (A.33) as $l_B \to \infty$ goes to positive infinity if

$$\frac{\sqrt{(1+r_A)c}}{\sqrt{\lambda_B}}>\frac{\Psi}{v}, \tag{A.35}$$

or, substituting for Ψ, the equilibrium number of entrants increases for a debt-equity ratio β if:

$$(1+\alpha_L)v\sqrt{\frac{1+r_A}{c\lambda_B}}>\frac{\beta}{1-\beta}. \tag{A.36}$$

Proof of Theorem 2

Proof of the equilibrium supply functions is contained in Propositions 1 and 2, while the equilibrium debt and equity follow from Proposition 3.

Proof of either a single or dual equilibrium in the number of entrants N follows in two parts, depending on whether the leverage constraint binds. When the leverage constraint does not bind, his profit is identical to the liquidity ratio cases π_1, π_2 and π_3, as shown in Proof of Proposition 1. When the leverage ratio does bind, substitution of the supply functions from Proof of proposition 2 results in a profit function of:

$$\pi_B=\frac{(l_B+l_S)^2}{(N+1)^2(\lambda_B+\lambda_S)}+\frac{(l_B\lambda_S-l_S\lambda_B-(\lambda_B+\lambda_S)\frac{\Gamma_L}{2})\Psi}{v(\lambda_B+\lambda_S)}-\frac{N\lambda_S\lambda_B\Psi^2}{v^2(\lambda_B+\lambda_S)}-(1+r_A)c. \tag{A.37}$$

The combination of these equations yields a total profit of:

$$\begin{aligned}\pi = &\mathbb{1}\left(\lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_N}{2}\Lambda\right) \cdot \pi_1 \\ &+ \mathbb{1}\left(\frac{\Gamma_N}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda\right) \cdot \pi_2 \\ &+ \mathbb{1}\left(\frac{\Gamma_L}{2}\Lambda < \lambda_S l_B - \lambda_B l_S \leq \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B \lambda_S (N+1)\Psi}{v}\right) \cdot \pi_3 \\ &+ \mathbb{1}\left(\frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B \lambda_S (N+1)\Psi}{v} < \lambda_S l_B - \lambda_B l_S\right) \cdot \pi_B. \end{aligned} \tag{A.38}$$

In the case where the leverage ratio does not bind, the number of entrants N^* is determined by inverting the relevant profit function. If $\hat{N}$ is the value which solves the inverted profit function, then the equilibrium number of entrants is $N^* \leq \hat{N} < N^* + 1$.

When the leverage constraint does bind, there exists $N_B^* > 0$, which solves:

$$(1 + r_A)c = \frac{(l_B + l_S)^2}{(N_B^* + 1)^2 \Lambda} + \frac{(l_B \lambda_S - l_S \lambda_B - \Lambda \frac{\Gamma_L}{2})\Psi}{v\Lambda} - \frac{N_B^* \lambda_S \lambda_B \Psi^2}{v^2 \Lambda}. \tag{A.39}$$

As before, the number of entrants N_B is such that $N_B \leq N_B^* < N_B + 1$.

While the equilibrium supply functions and prices are unique given the number of entrants, the number of entrants is not necessarily unique itself. When the market maker's leverage constraint does not bind, the number of entrants in equilibrium is identical to that in the baseline model. The number of entrants is unique if, for the values N^* and N_B above, the market maker's leverage constraint either binds when N is replaced by both N^* and N_B, or does not bind when N is replaced by both N^* and N_B. Consider the case where either N^* or N_B firms enter, and:

$$\begin{aligned}\lambda_S l_B - \lambda_B l_S &\leq \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B \lambda_S (N^* + 1)\Psi}{v} \\ &\leq \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B \lambda_S (N_B + 1)\Psi}{v}. \end{aligned} \tag{A.40}$$

If N^* firms enter, leverage constraint does not bind. Through the profit function in the baseline model each firm earns zero profit in expectation and, thus, this is an equilibrium. If N_B firms enter, the leverage constraint does not bind. Since $N_B < N^*$, substitution of the baseline liquidity supply into the profit function gives a profit greater than zero. Thus, more firms could have entered and this is not an equilibrium.

Alternatively, there is a unique equilibrium where the leverage constraint binds and N_B firms enter if:

$$\begin{aligned} \lambda_S l_B - \lambda_B l_S &> \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B \lambda_S (N^* + 1)\Psi}{v} \\ &> \frac{\Gamma_L}{2}\Lambda + \frac{\lambda_B \lambda_S (N_B + 1)\Psi}{v}. \end{aligned} \tag{A.41}$$

The uniqueness of this equilibrium is symmetric to the one above, with one alteration. If N^* firms enter, each earns less than zero profit and defaults, and if N_B firms enter, each earns zero profit. Thus, in this case, there is a unique number of entrants N_B.

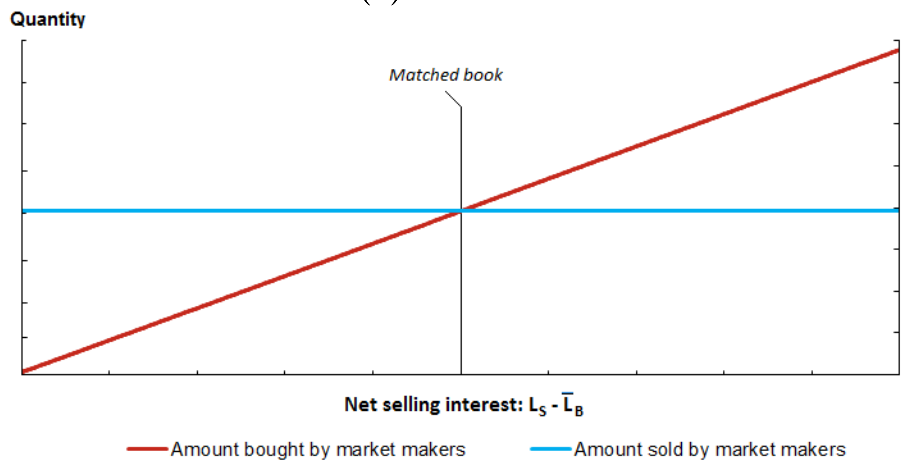
Finally, there exist two equilibria if, given equilibrium entrants N^*, market makers earn zero profit and the leverage constraint does not bind, but given entrants N_B, market makers also earn zero profit and the leverage constraint does bind. This occurs, given values N^* and N_B defined above, if:

$$N_B < \frac{v}{\lambda_S \lambda_B \Psi}(\lambda_S l_B - \lambda_B l_S - \frac{\Gamma_L}{2}\Lambda) - 1 \leq N^*. \tag{A.42}$$

Thus, given the conditions above, there exists either: (1) a unique equilibrium number of entrants N^* or N_B, or (2) two equilibria number of entrants N^* or N_B.

Figure 1
Liquidity supply, base case and regulated case

(a) Base case



(b) Regulated case

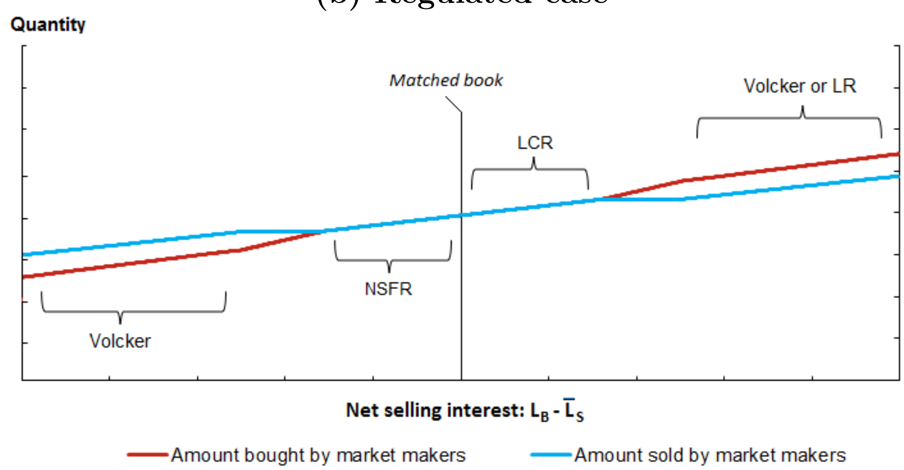
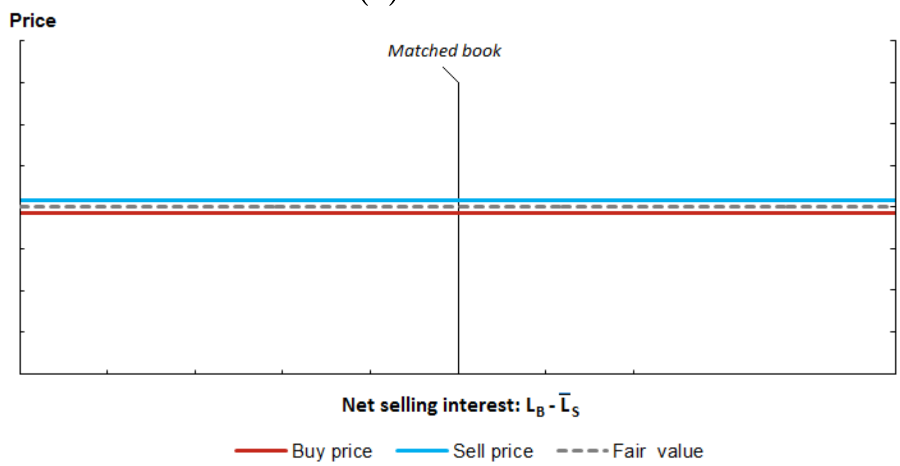


Figure 1 illustrates the quantity supplied by market makers given varying degrees of sell pressure l_B. The buying pressure l_S is fixed, so the chart shows net selling pressure. The top panel shows the base case. More seller interest causes market makers to buy more, and in the absence of regulation, it does not lead them to sell more. Liquidity regulation adds costs when market makers rely on repo transactions to fund inventory or source assets. The NSFR creates additional costs to reverse repo and causes market makers to trade on agency, which can be seen in the region marked NSFR, as the buy and sell lines are parallel (and overlap). The LCR creates additional costs to repo and similarly causes market makers to trade on agency, which can be seen in the region marked NSFR, as the buy and sell lines are parallel (and overlap). Finally, a leverage ratio or position limit restricts the total size of market makers' positions, forcing trading on agency, which can be seen in the regions marked LR and Volcker, as the buy and sell lines are parallel. The leverage ratio only binds on long positions, whereas the position limit binds on both long and short positions.

Figure 2
Bid and ask prices, base case and regulated case

(a) Base case



(b) Regulated case

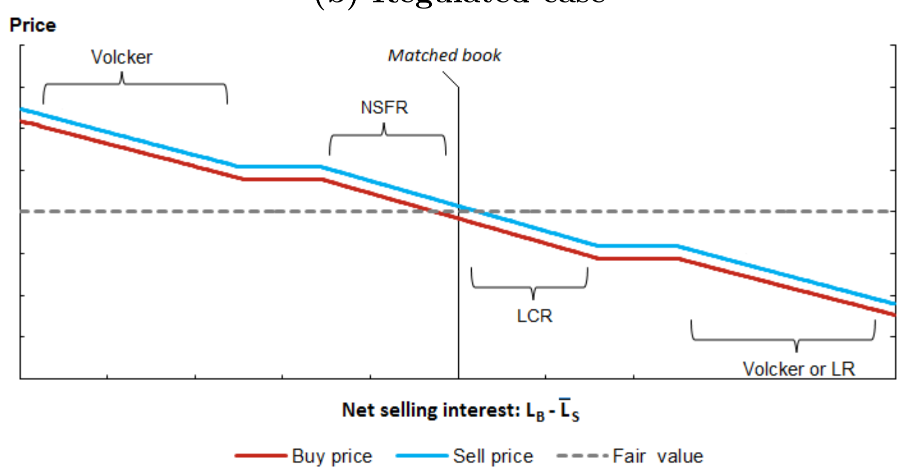


Figure 2 illustrates the equilibrium bid and ask prices given varying degrees of sell pressure l_B. The buying pressure l_S is fixed, so the chart shows net selling pressure. The top panel shows the base case. Higher or lower seller interest does not lead to visible movement in the prices. Liquidity regulation adds costs when market makers rely on repo transactions to fund inventory or source assets. Prices rise or fall when market makers trade on agency in order to incentivize the opposite side of the market to supply liquidity.

Figure 3
Liquidity supply under each regulation

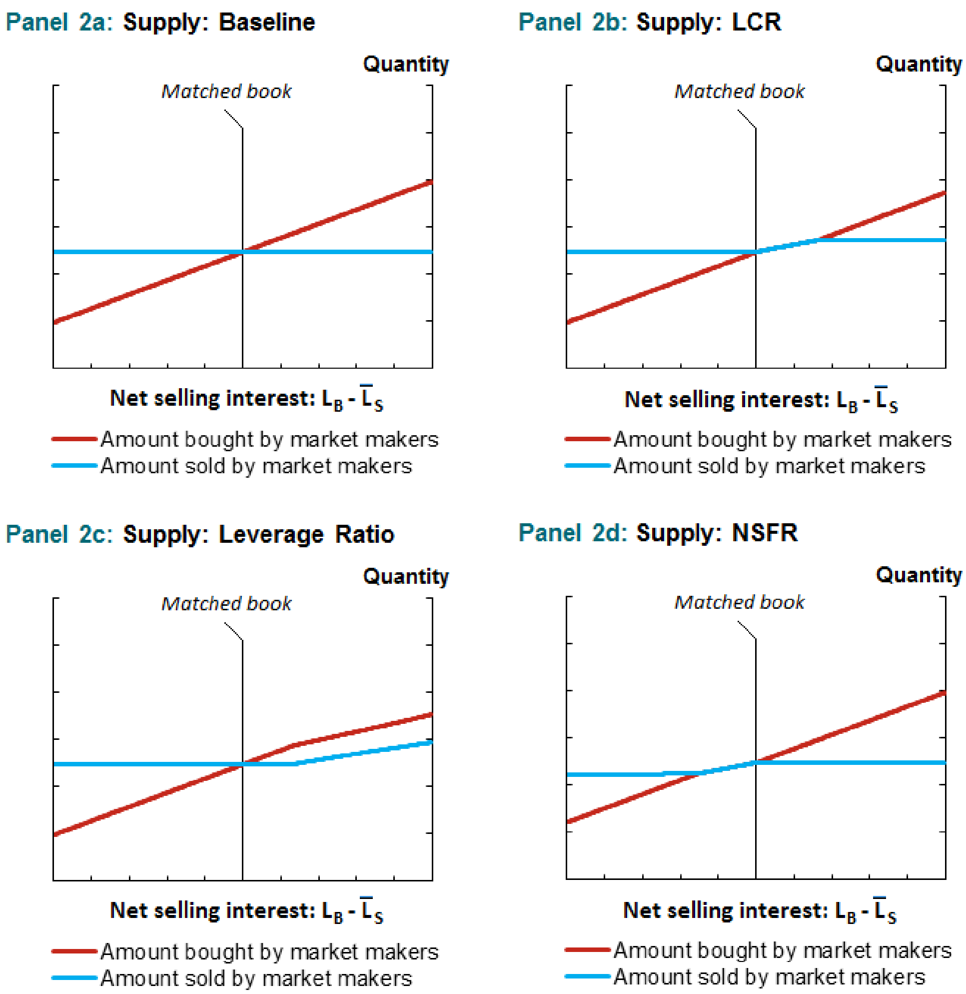


Figure 3 illustrates the quantity supplied by a market maker, given varying degrees of sell pressure l_B. The buying pressure l_S is assumed to be constant. In the baseline model, the quantity bought by the market maker increases linearly with buy pressure, while the quantity sold remains unaffected. The introduction of liquidity constraints, the LCR and the NSFR, introduce kinks into the liquidity supply functions. One kink represents the point where the market maker maintains zero net inventory in order to avoid holding HQLA, while the second kink represents the point where he begins holding HQLA. The introduction of a leverage ratio introduces a single kink in the supply function, at the point where the leverage ratio binds. At this point, the market maker begins selling a unit for every unit bought.

Figure 4
Bid and ask prices under each regulation

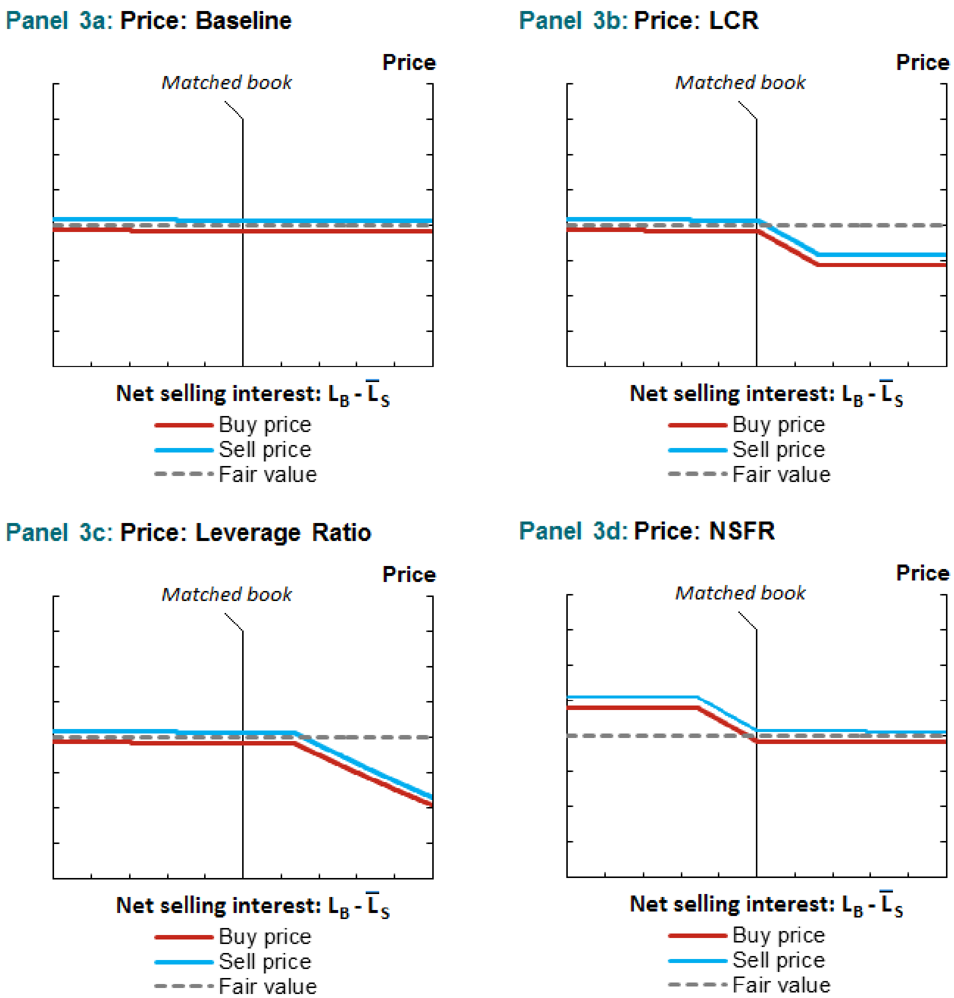


Figure 4 illustrates the prices at which market makers buy and sell the asset, given varying degrees of sell pressure l_B. The buying pressure l_S is assumed to be constant. In the baseline model, prices remain at approximately the value of the asset v, as more market makers enter when sell pressure increases. The introduction of liquidity constraints, the LCR and NSFR, introduces kinks into the price functions. When market makers must hold HQLA, their cost of marking markets in the asset changes. In the case of the LCR, it becomes cheaper for them to sell the asset than to buy it, shifting the price downward. In the case of the NSFR, it becomes cheaper for them to buy the asset than to sell it, shifting the price upwards. The introduction of a leverage ratio also introduces a kink into the pricing function at the point where the leverage ratio binds. At this point, market makers are unable to buy as much as the baseline case and must sell in excess, reducing the price.

Figure 5
Bid-ask spread under each regulation

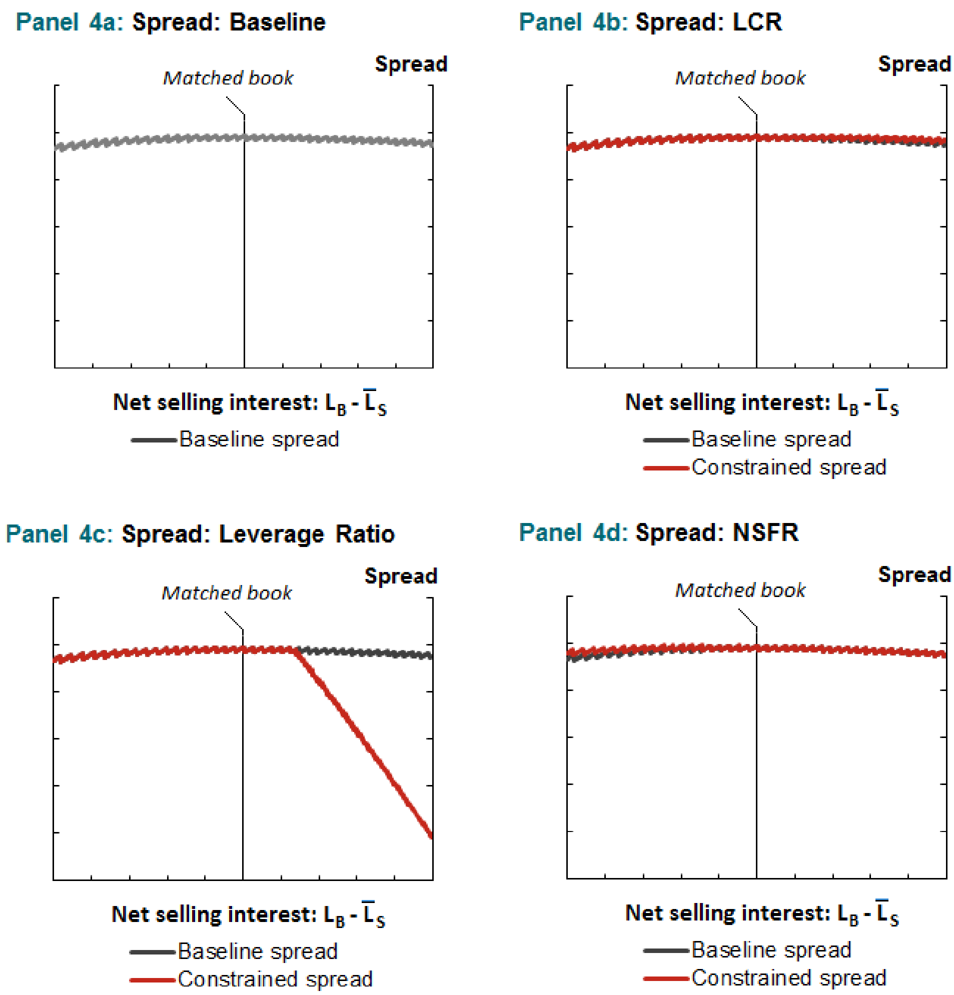


Figure 5 illustrates the bid-ask spread, given varying degrees of sell pressure l_B. The buying pressure l_S is assumed to be constant. In the baseline case, spreads slowly narrow as sell pressure increases. The entry of new market makers narrows spreads quicker than the increase in sell pressure increases them. With the introduction of liquidity constraints, spreads widen slightly, due to the cost of holding HQLA. The introduction of a leverage ratio introduces a kink in the spreads. When the leverage ratio binds, market makers are restricted in their ability to increase their amount bought and must increase their amount sold. Unlike other regulations, the effect of the leverage ratio on spreads in ambiguous. In the illustrated case, new market makers enter at a faster rate, decreasing spreads rapidly. Alternatively, in the unillustrated case, the entry of new market makers may slow, increasing spreads.