Dealer Trading at the Fix

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Abstract

This paper develops a model of dealer conduct – and misconduct – at the London 4 pm fix, a major currency market benchmark. The analysis clarifies the dealers' incentives and strategies, explains why price dynamics appear unchanged despite reforms, and provides insights relevant to benchmark design. Fix prices will be unusually volatile without collusion. Collusion is profitable because it shuts down a form of free-riding in which dealers front-run each other. The price trend decelerates less as the fix moment approaches under collusion than under independent trading. Statistical tests detect this shift around 2008, when major banks admit their dealers began colluding.

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Extreme exchange rate volatility at around 4 pm in London, when a key benchmark price is calculated, has long been familiar to market participants (see Figure 1). Widespread suspicions that this reflects dealer misconduct gained support in 2013 when *Bloomberg* reported that foreign exchange dealers were colluding in private electronic chatrooms. After extensive legal investigations, the major dealing banks pleaded guilty to collusion and paid fines and settlements in excess of $11 billion (Maton and Gramhir, 2015). Despite institutions reforms in 2015, price dynamics at 4 pm in London remain qualitatively unchanged (Ito and Yamada, 2017b), which highlights the importance of understanding trader behavior around price benchmarks. Such an understanding could also shed light on documented misconduct around benchmarks in other settings including precious metals, Treasury securities, interest rate derivatives, NDFs, and the Tokyo 10 am fix.[1] The literature provides little guidance, however.

This paper develops a model of dealer trading around a price benchmark and examines market equilibrium under three competitive conditions: independent trading, sharing confidential information about customer orders, and collusive trading.[2] The model shows that collusion maximizes dealer profits, as might be expected, but the source of collusive profits is unrelated to traditional market power. Instead, collusion shuts down the dealers' efforts to free-ride on each other, a strategy that involves front running. Perhaps surprisingly, sharing information about customer orders reduces the dealers' average profitability relative to independent trading; it does so by intensifying free-riding. The model suggests that volatility will be high before the calculation of a benchmark price even when dealers trade independently, and in consequence high volatility at a fix cannot reliably signal collusion. The model also provides an explanation for the absence of qualitative changes in price dynamics around the London 4 pm fix after institutional reforms (Ito and Yamada, 2017b). Our analysis will be of interest to academics and to practitioners in at least three settings: authorities investigating reported misconduct; bank compliance officers; and regulators evaluating benchmark calculation methodologies.

The London 4 pm fix was originally intended to facilitate the valuation of cross-border portfolios and is currently used in constructing most major international equity- and bond-market indexes, including

[1] See FCA (2014a) for precious metals; Stempel (2015) for Treasury securities; Leising and van Voris (2014) for interest rate derivatives; Armstrong (2013) for Asian NDFs; and Ito and Yamada (2017a) for the Tokyo fix.

[2] This paper does not advocate prohibited behaviors. It provides a positive rather than a normative analysis of dealer behavior around OTC fixes.

those of MSCI. Asset managers whose performance is evaluated relative to these indexes can reduce tracking risk by trading foreign exchange at the fix price itself, which they achieve by placing fill-at-fix orders with their dealers. Dealers require these orders to be received well in advance of the fix and manage the associated risks by trading in the interbank market. Dealers in the model trade with other dealers during two periods before the fix and one period after the fix. Their order flow has a linear impact on price consistent with abundant evidence (e.g., Evans and Lyons (2002)) and with other models of optimal execution (Bertsimas and Lo, 1998; Almgren, 2012; Obizhaeva and Wang, 2013).

The time delay between order arrival and execution gives dealers an opportunity to influence the fix price; slippage (price impact) gives them the power to do so. The model's rational, risk-neutral dealers exploit these advantages in two ways. First, they accumulate the inventory required to serve customers before the fix. Second, they front-run, meaning they open a proprietary position before trading for their customers (Comerton-Forde and Putniņš, 2011). Front-running is illegal in most markets though not in foreign exchange. It increases volatility before the fix and generates retracements after the fix.

Fix orders are driven by economy-wide forces such as returns in foreign equity markets (Melvin and Prins, 2015) and are thus positively correlated across dealers. Each fix dealer rationally uses his own net fix order to anticipate the net orders of other dealers. On that basis he expects the other dealers to trade in his same direction throughout the pre-fix period. A rational dealer front-runs the other dealers by taking a larger initial proprietary position than otherwise. This additional trading, which amounts to a form of free-riding, is not neutral to the other dealers: upon liquidation it reduces their profits.

Information sharing and collusion each have important but relatively subtle implications for optimal dealer strategies and fix-price dynamics. When dealers share information they free-ride on each other more aggressively, taking a larger proprietary position in period 1 and liquidating more of that position before the fix. Relative to independent trading this reduces average dealer profits while moderating pre-fix volatility and post-fix retracements. Collusion has the opposite effects: It shuts down free-riding and raises average dealer profits relative to independent trading. Collusive dealers make fewer liquidation trades before the fix and more after the fix so pre-fix volatility and post-fix retracements both become more pronounced.

Information sharing and collusion also influence another dimension of fix-price dynamics: the acceleration or deceleration of the pre-fix price trend. Under risk neutrality free-riding dealers trade progressively less as the fix moment approaches so the pre-fix price trend can be expected to decelerate. Information-sharing intensifies free-riding and thus the expected deceleration. Collusion eliminates free-riding and reduces the expected deceleration.

Beginning around 2008 pre-fix price movements tended not to decelerate but instead to accelerate as the fix moment approached (Figure 1), a property we label convexity. To explain convexity we enhance the model's realism in two ways. First, we allow a trade's price impact to rise with order flow, consistent with the order-driven structure of most interdealer trading (Osler, 2009). Second, we introduce risk aversion among fix dealers. Under both of these modifications dealers have an incentive to "bang-the-close," meaning to concentrate their trading just before the fix moment (Comerton-Forde and Putniņš, 2011; Saakvitne, 2016). Banging-the-close generates a convex price path.

The model suggests that it would be difficult to make a strong case for collusion based exclusively on volatility and retracements around a fix. Prices will be volatile before the fix even when dealers trade independently, for the simple reason that many orders will be executed within a short time frame. Retracements after the fix reflect proprietary trading, some of which will occur regardless of whether dealers collude. A *rise* in volatility or retracements over time would be equally uninformative with regard to dealer strategies. These features of price dynamics also respond to a number of variables, some of which are known to change over time such as the variance of non-fix order flow. Convexity, by contrast, is influenced almost exclusively by dealer strategies and its other determinants are generally stable. We develop a measure of convexity and apply it to high-frequency exchange-rate data covering the years 1996 through 2013 for seven major currencies vis-à-vis the US dollar: EUR, JPY, GBP, CHF, CAD, NZD, and DKK. Binomial and bootstrap tests do not reject the hypothesis that convexity path increased around 2008 when fix dealers reportedly began colluding in earnest (Department of Justice, 2017).

The stability of price dynamics around the London fix (Ito and Yamada, 2017b) is striking given the magnitude of institutional reforms since 2015: the fix-price calculation window was extended from one to five minutes, dealers were prohibited from chatroom conversations, and banks began to process fix orders via automated algorithms. Our model suggests that price dynamics were sustained for two

reasons. First, the incentives for an agent with information about fix orders have not changed. Second, non-dealers can now glean information about fix orders from the price trend immediately following 3:45, given the dealers' reliance on execution algorithms. Non-dealers with that information can adopt the optimal strategy identified by the model for an informed dealer: front-run the rest of the market by opening a speculative position immediately after 3:45; liquidate that position partly before and partly after the fix. This strategy would help sustain the volatility and retracements previously observed at the fix. SmartFix, a new software package marketed to active traders, helps non-dealers profit from fix trading (Albinus, 2016).

The model also reveals that volatility and retracements at the fix could potentially represent a sustainable equilibrium even though do not fit the traditional image of an efficient market. Given the institutional investors avoid tracking risk by placing fill-at-fix orders, asset managers, larger dealers, and smaller dealers may be connected in a web of strategic complementarities. The asset managers' fix orders lead to heightened exchange rate volatility and proprietary trading by dealers. The proprietary trades further increase volatility, which encourages asset managers to place more fix orders. The heightened volatility also discourages fix trading by smaller dealers, who have less information about the market's net fix order and thus face greater risk in executing their own orders. Greater concentration of fix orders at large dealers brings yet more volatility.

The potential relevance of our analysis to the dramatic price movements observed around benchmarks in other OTC markets is highlighted by regulator comments. According to the director of FINMA, for example, "[t]he behaviour patterns in precious metals were somewhat similar to the behaviour patterns in foreign exchange" (Harvey, 2014). Our analysis may be less applicable to LIBOR manipulation because the LIBOR is not based on prices set to match supply and demand; further, LIBOR manipulation appears to have been intended to support bank health rather than the profits and bonuses of individual dealers (Abrantes-Metz et al., 2012; Gandhi et al., 2016).

There is at present limited academic research on financial benchmarks, so far as we are aware. Duffie et al. (2014) use a search model to show that benchmarks in OTC markets can, on net, improve market performance by enhancing transparency. These benefits certainly pertain to thin markets, such as those for corporate or municipal bonds, where end-users have little information about an asset's fair value. In

the highly-liquid foreign exchange market, by contrast, market prices are freely available intraday and fix prices are primarily important for portfolio valuation. Our model indicates that the strategic management of fix orders by dealers creates a gap between the fix and the asset's fair value. The transparency benefits demonstrated by Duffie et al. could thus be partially offset by costs from lower market efficiency and higher volatility.

Two papers develop models focused on benchmark trading. Onur and Reiffen (2016) use their model to examine issues that are orthogonal to this paper, specifically futures-market settlement rules and the distribution of trading between floor and electronic markets. Evans's (2016) model of fix dealing implies that dealers do not manage fix orders until after the fix price has been set. This has two notable implications: dealers do not exploit private information about fix orders by trading before the fix and dealers do not front run customer orders even in the absence of penalties for doing so. The practical relevance of this analysis may be constrained by the lack of alignment between these implications and core finance principles as well as confirmed dealer behavior. Evans points to collusion as the source of high volatility around the London 4 pm fix given the contrast between his model's predictions and actual fix-price dynamics. That inference essentially relies on the behavior of volatility and retracements to signal dealer strategies, a methodology that our model suggests may not be robust.

Front-running can be interpreted as a form of trade-based manipulation (Allen and Gale, 1992), about which there is an extensive literature (Hart, 1977; Jarrow, 1992; Kumar and Seppi, 1992; Aggarwal and Wu, 2006; Vitale, 2000). That literature generally considers a single trader who exploits the market power that arises naturally when agents are asymmetrically informed (Kyle, 1985; Glosten and Milgrom, 1985). The one related model of front-running also focuses on a single informed trader with market power (Pagano and Röell, 1993). Our model assumes that dealers have market power, consistent with these others, but our key findings involve the way dealers interact strategically.

Fix prices and closing prices have much in common, and concerns about closing prices are nothing new.[3] A tendency for U.S. equity prices to rise at the end of the day was documented in the mid-1980s (Harris, 1986) and a number of explanations have been suggested that involve misconduct. Carhart et al.

[3] Cordi et al. (2016) provide a brief survey.

(2002) suggest that equity fund managers may intentionally inflate quarter-end mutual fund values; Hillion and Suominen (2004) suggest that equity brokers may intentionally inflate their apparent skill. These ideas seem unlikely to be important for the London fix given that the chatroom conversations included no fund managers and foreign exchange trades are handled on a principal basis rather than an agency basis. Saakvitne (2016) develops a model that predicts a form of banging-the-close. This model includes orders analogous to fill-at-fix orders but does not compare legal trading with collusion because it assumes dealers have full information about orders. The implications of Saakvitne's model depend critically on the assumption that the price impact of order flow is entirely temporary, an assumption that is inconsistent with the core microstructure prediction that order flow will have a permanent price impact (Kyle, 1985; Glosten and Milgrom, 1985) and with evidence that consistently supports that prediction (this evidence is summarized in Osler (2009)). Notably, the price impact of order flow around the London Fix is largely permanent, as indicated by Figure 1.

Cushing and Madhavan (2000), who examine price dynamics at the close for Russell 1000 stocks, suggest that high volatility and post-close retracements in the late 1990s were unrelated to misconduct and instead reflected the common tendency among market makers to temporarily lower (raise) prices when they hold excess (insufficient) inventory. We view such price pressures as a possible source of retracements that is complementary to the front-running and excess trading predicted by our model.

I. The model

This section outlines the model and highlights how it incorporates critical insights from microstructure while reflecting the structure of trading at the time financial benchmarks are calculated.

Fix dealers: Customer fix orders are managed by $2 \leq N+1 < \infty$ identical OTC dealers. The asset in question could be a security, a commodity, or a base currency. The price is quoted in terms of a numeraire currency. When not holding the fix asset the dealer holds the numeraire asset, the return to which is normalized to zero. The number of dealers involved in setting financial benchmarks is consistently low. In forex the top four banks account for over 50% of spot dealing (*Euromoney*, 2016) and the management of fix orders is yet more concentrated because small and regional banks generally pass

fix orders to the dominant dealers. In gold bullion just five banks set the fix price prior to reforms (Harvey, 2014).

Fix orders: Before fix trading begins each dealer receives a random set of customer fix orders. Representative dealer d matches off his buy and sell orders to the extent possible and manage the remaining amount, F_d, in the interbank market. F_d includes a component shared by all other dealers, Φ, and a dealer-specific deviation, $\eta_d : F_d = \Phi + \eta_d$. The terms Φ and η_d are i.i.d. and mutually uncorrelated with mean zero and variances $\sigma_\Phi^2 > 0$ and $\sigma_\eta^2 > 0$, respectively. The correlation in fix orders is denoted $\rho : 0 < \rho = \sigma_\phi^2 / (\sigma_\phi^2 + \sigma_\eta^2) < 1$. We assume for convenience that dealer d's customers are buyers, $F_d > 0$, so dealer d himself is a buyer in the interdealer market.

The model takes customer fix orders as exogenous but their origin in reality is well understood. International equity funds worth \$9 trillion are benchmarked to the MSCI indexes and another \$2 trillion are benchmarked to the Citi World Government Bond Index, and all of these indexes are marked to market with the WM/Reuters Closing Spot Rates (Cochrane, 2015). These institutions have a high incentive to avoid tracking risk, which they can do by trading exactly at the fix price. Fix trading is concentrated at month end, when firms set aside employee retirement income and international investing institutions adjust their currency hedge positions. Melvin and Prins (2015) provide evidence that month-end portfolio hedging influences trading and prices at the London 4 pm fix. They also show that hedging trades are influenced by recent returns to foreign equity markets, which motivates our assumption that fix orders are positively correlated across dealers.

Inventory management: The pre-fix trading interval has two trading periods, periods 1 and 2, during which representative dealer d trades quantities D_{1d} and D_{2d} at prices P_1 and P_2, respectively. After the fix is calculated dealers trade with each other once more at price P_3 to restore inventories to their initial levels. The difference between the amount a dealer trades before the fix and his fix order is denoted X_d:

$$X_d \equiv D_{1d} + D_{2d} - F_d \quad . \tag{1}$$

The fix: The fix price is set equal to the period-2 interdealer price, $P_F = P_2$. Given the generality of this pricing structure, our analysis should be relevant to benchmarks calculated with a variety of methodologies. Indeed, foreign exchange dealers appear to have behaved similarly at both the London 4

pm fix and the ECB fix, which occurs at 2:15 European Central time, though the methodologies for determining these benchmarks are distinct. Until December, 2014, the WM/Reuters 4 pm fix used (roughly) the median of traded interdealer prices sampled once per second over the 60-second interval centered on the hour. The ECB fix is set according to a central bank "concertation procedure," the details of which are distinct but not published. Dealers also behaved similarly around fixes in precious metals (Harvey, 2014).

Dealer objectives: Dealers are risk-neutral profit maximizers. Dealer d's revenues comprise $P_F F_d$ from selling to customers at the fix plus $P_3 X_d$ from liquidating any net excess trading after the fix. His costs come from purchasing inventory in periods 1 and 2: $P_1 D_{1d} + P_2 D_{2d}$. Interest expense is irrelevant because fix trading occurs intraday. Following the literature we abstract from the cost of bank capital as well as the potential costs of violating laws or regulations. Dealer d's profits, π_d, are:

$$\pi_d = P_F F_d + P_3 X_d - P_1 D_{1d} - P_2 D_{2d} \ . \tag{2}$$

Throughout our analysis we rely on the following restatement of profits:

$$\pi_d = D_{1d}\left(P_2 - P_1\right) + X_d\left(P_3 - P_2\right). \tag{3}$$

The first term on the right represents the period-2 gain or loss on inventory accumulated in period 1. The second term on the right captures the period-3 gain or loss on the dealer's net position after period 2. In equilibrium the first term is positive in expectation and the second term is negative in expectation. The profitability of fix trading thus derives entirely from the interaction between early inventory accumulation (D_{1d}) and later returns ($P_2 - P_1$).

Price generating process: When executing fix trades the dealers trade against each other and against an atomistic of fringe smaller dealers. We assume that the atomistic fringe extracts the expected information content of order flow and in consequence fix trades have a linear contemporaneous price impact proportional to $\theta > 0$ (in this we follow closely-related research, e.g., Bertsimas and Lo (1998), Cushing and Madhavan (2000)). Returns are also driven public information and other factors orthogonal to the fix that generate order flow shocks, ε_t, which are i.i.d. with zero mean and variance $\sigma_\varepsilon^2 > 0$:

$$P_t - P_{t\text{-}1} = \theta\left(D_{td} + \sum_N D_{tn} + \varepsilon_t\right), \qquad t = \{1,2,3\}.^4 \tag{4}$$

Equation (4) shows that the price follows a random walk, $P_t - P_{t-1} = \theta\varepsilon_t$, outside of the fix trading interval, with one-period return variance $\theta^2\sigma_\varepsilon^2$.

Equation (4) is empirically well-grounded. Microstructure research shows that order flow has a permanent price impact for all major asset classes.[5] Consistent with this, the foreign exchange market's response to fix trades is largely permanent, as shown in Figure 1. Equation (4) is also theoretically well-grounded. A permanent price impact of order flow is implied by models of asymmetric information (Kyle, 1985; Glosten and Milgrom, 1985), and asymmetric information is an integral feature of the forex market. Larger banks are generally better informed than smaller banks (Bjønnes et al., 2009), and smaller dealers actively filter the order flow of large dealers to extract information (Menkhoff and Schmeling, 2010). Much of a dealing banks' private information comes from customer order flow, as when dealers observe hedge funds trading in anticipation of macro news (Rime et al., 2010). The trades of financial customers tend to carry information while those of commercial customers and individuals do not (Menkhoff et al., 2016).

II. Independent trading

We begin by examining dealer strategies and fix-price dynamics under independent trading. Representative dealer d choose D_{1d} in period 1 and X_d (or equivalently D_{2d}) in period 2. To choose D_{1d}, however, he must first identify how that choice will influence his future choice of X_d.

A. Period-2 decision

Let α_1 represent the share of dealer d's net fix order that he trades in period 1, $\alpha_1 \equiv D_{1d} / F_d$, and x represent net excess trading as a share of his net fix order, $x \equiv X_d/F_d$. A hat will denote dealer d's expected average value of a parameter for the other dealers: e.g., $\hat{\alpha}_1 \equiv E_{2d}\{\sum_N D_{1n}\}/ E_{2d}\{\sum_N F_n\}$. In period 2 dealer d takes α_1 and $\hat{\alpha}_1$ as given and chooses x to solve:

[4] P_0 is left unspecified.

[5] See, e.g., Shleifer (1986) for equities, Evans and Lyons (2002) for foreign exchange, and Simon (1991) for bonds.

$$\underset{x}{Max}\;\; E_{2d}\{\pi_d\} = \alpha_1 F_d \theta \left\langle [(1-\alpha_1)F_d + (1-\hat{\alpha}_1)E_{2d}\{\sum_N F_n\}] \right\rangle + F_d(\alpha_1 - x)\theta(xF_d + E_{2d}\{\sum_N X_n\}). \quad (5)$$

Optimal x depends on dealer d's expected value of the other dealers' proprietary trading, $E_{2d}\{\sum_N X_n\}$:

$$x = \frac{\alpha_1}{2} - \frac{E_{2d}\{\sum_N X_n\}}{F_d}. \quad (6)$$

The rational-expectations solution for x, derived in the Appendix, is:

$$x = \frac{\alpha_1}{2+\rho N} \equiv q\alpha_1, \qquad 0 < q \equiv \frac{1}{2+\rho N} < \frac{1}{2}. \quad (7)$$

B. Period-1 decision

Given this solution for x, dealer d chooses α_1 to solve

$$\underset{\alpha_1}{Max}\;\; E_{1d}\{\pi_d\} = \alpha_1 F_d \theta \left[F_d(1-\alpha_1) + (1-\hat{\alpha}_1)\rho N F_d\right] + \alpha_1 F_d (1-q)\theta\left[q\alpha_1 F_d + E_{1d}\{\sum_N X_n\}\right] \quad (8)$$

The first-order condition shows that α_1 depends on the other dealers' expected behavior, $\hat{\alpha}_1$ and $\hat{q}$:

$$\alpha_1 = \frac{(1+\rho N) - \hat{\alpha}_1 \rho N[1-\hat{q}(1-\hat{q})]}{2[1-q(1-q)]}. \quad (9)$$

Dealer symmetry implies that $\alpha_1 = \hat{\alpha}_1$ and $q = \hat{q}$ in market equilibrium, which closes the model. Lemma 1 describes equilibrium trading:

Lemma 1: When risk-neutral dealers trade independently, dealer *d*'s trades are proportional to his net fix order in every period:

$$D_{1d} = \frac{(2+\rho N)(1+\rho N)}{(2+\rho N)^2 - (1+\rho N)} F_d \equiv \alpha_1 F_d, \qquad \frac{2}{3} < \alpha_1 < 1; \quad (10a)$$

$$D_{2d} = \left[1 - \left(\frac{1+\rho N}{2+\rho N}\right)\alpha_1\right]F_d \equiv (1-\alpha_2)F_d, \qquad \frac{1}{3} < \alpha_2 = \alpha_1 - x < 1; \quad (10b)$$

$$X_d \equiv -D_{3d} = \left(\frac{1}{2+\rho N}\right)\alpha_1 F_d = xF_d, \qquad 0 < x < \frac{1}{3}. \quad (10c)$$

From Lemma 1 we learn dealer trading at the fix has five critical properties: customer inventory is accumulated before the fix; pre-fix trading is distributed across both pre-fix periods; dealers front-run their customers; dealers free-ride on each other and in consequence pre-fix trading is concentrated in period 1; dealers earn positive profits.

Inventory accumulation before the fix: Profit-maximizing fix dealers trade before the fix to acquire the inventory required to serve customers, allowing slippage (θ) to move the price in a profitable direction. This is only possible because the price of a fix trade is set after the dealer and customer agree on the quantity to be traded. On a regular OTC trade, by contrast, prices and quantities are both agreed up front. In this case slippage cannot affect the customer's price and it increases the dealer's cost of acquiring the inventory.

Distributed trading: Profit-maximizing fix dealers trade in both periods before the fix. As shown in Equation (3), profits from trading at the fix are the product of the dealer's period-1 inventory accumulation and the period-2 return. Without period-2 purchases any inventory purchased in period 1 does not appreciate. Without period-1 purchases there is no inventory to appreciate in period 2.

Front-running: Profit-maximizing fix dealers trade more than required to service their customer fix orders: $X_d = xF_d > 0$. More importantly, fix dealers front-run their customers. Dealer d uses his own fix order to extract a signal of the market's likely overall net fix order, $E_{1d}\{F_d + \sum_N F_n\} = (1+\rho N)\, F_d$, which allows him to forecast the pre-fix trend. In period 1 he exploits this forecast by opening a proprietary position proportional to $(1+\rho N)\, F_d$. In period 2 he purchases the full amount of his net customer fix order, which – together with the other dealers' net purchases – raises P_2 and appreciates his initial proprietary position. To realize profits from this appreciation the dealer liquidates the fraction $(1+\rho N)/(2+\rho N)$ of his proprietary position in period 2 and liquidates the remaining amount, X_d, in period 3. These liquidating trades depress P_2 and P_3; the allocation of the liquidating trades between periods 2 and 3 minimizes the expected total liquidation loss.

Front-running client orders is illegal in most well-regulated financial markets. It is not explicitly prohibited in currency markets, however, in part because there is no institution with the authority to enforce trading rules worldwide. In addition, currencies are neither securities nor financial instruments so they are not covered by Europe's MIFID or similar legislation elsewhere.[6]

[6] Practitioners acknowledge the lack of regulation in forex: "As for front-running, … it's simply impossible to regulate. Consequently there is a general perception that front-running is not illegal, and any abuse in this regard can only be discouraged on reputational grounds" (Kaminska, 2013). Regulators attempt to exploit the leverage associated with reputation, as illustrated by the FX Global Code, a set of guidelines for the wholesale forex market (Bank for

The Swiss financial authority reports front-running at the silver fix (Harvey, 2014) and the Bank of England reports that it was common at the London fix prior to reforms:

> Traders increased the volume traded by them at the fix in the desired direction in excess of the volume necessary to manage the risk associated with the firm's fix position. Traders have referred to this process as "overbuying" or "overselling" (Grabiner, 2014, p. 11).

In forex, the amounts traded in excess of customer orders were sometimes substantial. CFTC transcripts report one forex dealer telling another, "haha i [sic] sold a lot up there and over sold by 100" (CFTC, 2015), meaning 100 million of either USD or EUR.

Free-riding: Dealer d adjusts his trading to exploit the anticipated trades of other dealers. To understand this form of free-riding it is helpful to contrast dealer d's trading strategy with uncorrelated (ρ = 0) and correlated ($\rho > 0$) fix orders. With $\rho = 0$ dealer d and the other dealers know nothing about each others' orders and there are no strategic interactions. With $\rho > 0$, by contrast, each dealer can forecast the other dealers' orders and thus their trades, albeit imperfectly, and will forecast that they trade in his same direction. On this basis he anticipates a stronger period-2 trend than he would with $\rho = 0$, which motivates him to open a larger proprietary position in period 1: $\partial D_{1d}/\partial\rho > 0$. The added proprietary position is essentially a front-running trade against the other dealers, the liquidation of which reduces their profits. Liquidation in period 2 reduces the fix price. Liquidation in period 3 reduces P_3, which causes the other dealers to loss more when they liquidate their net excess trading.

Free-riding also influences how much of his proprietary position the dealer liquidates in period 2. With $\rho > 0$ dealer d expects a stronger period-3 trend reversal than with $\rho = 0$, so he liquidates a smaller amount after the fix, $\partial x/\partial\rho < 0$. For later reference we note that the larger proprietary position and smaller net excess trading with $\rho > 0$, in combination, imply that dealer d increases the amount liquidated before the fix by more than the increase in his proprietary position.

The net excess trading predicted by this model, X_d, like the trade-based manipulation studied by Allen and Gale (1992) and others, undermines market efficiency by creating a divergence between the benchmark price and the asset's fair value. Here the fair value is the price that would be achieved if dealers only traded the amount of their fix orders or P_3. A necessary condition for dealers to create a

International Settlements, 2016). Legal authorities have also begun to treat front-running in forex as a form of fraud (Matthews, 2016).

divergence is that their order flow affects the price, a phenomenon that is well documented (Evans and Lyons, 2002) and well understood (Kyle, 1985; Glosten and Milgrom, 1985). This condition is not sufficient, however, and in normal OTC trading an uninformed proprietary position would be unprofitable due to the "unravelling problem": the dealer would buy at a high price and sell at a low price because his purchases (sales) drive the price up (down). Trade-based manipulation can be profitable if there are lags or instabilities in the expectation formation process (e.g., Hart, 1977; Aggarwal and Wu, 2006), but these are not critical for profits at the fix.

Positive profits: A risk-neutral fix dealer's expected profits are positive:

$$E_0\left\{\pi^{Indep}\right\} = \theta\sigma_F^2[\alpha_1(1-\alpha_1) + x(\alpha_1 - x)] > 0\,, \tag{11}$$

where $\sigma_F^2 \equiv Cov(F_D, \sum_{N+1} F_n) = (N+1)\sigma_\phi^2 + \sigma_\eta^2$ **.** Profits are positive because the customer's price is set after the quantity has been agreed. The first term on the right, $\theta\sigma_F^2\alpha_1(1-\alpha_1)$, captures the gains from distributing trades across both periods 1 and 2 if the dealer trades just the customer's fix order. The second term, $\theta\sigma_F^2 x(\alpha_1 - x)$, captures the gains from trading more than the customer's order.

Proposition 1 summarizes key features of equilibrium when dealers trade independently:

Proposition 1: When risk-neutral d**ealers trade independently equilibrium trading exhibits the following key features:**

a. Trading before the fix: The dealer accumulates the inventory required to fulfill his customer orders before the fix;

b. Distributed trading: The dealer trades in both pre-fix periods;

c. Front-running: The dealer opens a proprietary position before trading for his customers and liquidates it in periods 2 and 3;

d. Free-riding: The dealer's initial proprietary trade includes a component intended to front-run the other dealers;

e. Positive profits: The dealer earns positive profits because he distributes his trading over both pre-fix periods and because he takes a proprietary position.

C. Fix price dynamics when dealers trade independently

The model under independent trading predicts two of the striking features of fix-price dynamics evident in Figure 1: high pre-fix volatility and partial post-fix trend reversals. This section takes a close look at volatility, retracements, and a third feature not yet discussed: the shape of the pre-fix trend.

Volatility before the fix, denoted **Σ**, is measured as the variance of returns from P_0 to $P_F = P_2$:

$$\Sigma^{Indep} \equiv E\left\{(P_F - P_0)^2\right\} = 2\theta^2\sigma_\varepsilon^2 + \theta^2(N+1)\sigma_F^2 + \theta^2(N+1)\sigma_F^2 x(2+x) . \qquad (12)$$

The first term on the right, $2\theta^2\sigma_\varepsilon^2$, captures the two-period volatility that would be observed under regular OTC trading outside the fix interval, driven entirely by non-fix trading shocks. Pre-fix price volatility will exceed this level because fix dealers rationally trade before the fix to accommodate customer orders. This effect is captured by the second term, $\theta^2(N+1)\sigma_F^2$. By implication high volatility before the fix need not reflect dealer misconduct. Nonetheless, misconduct could be a contributor: the third term, $\theta^2(N+1)\sigma_F^2 x(2+x)$, shows that the dealers' net excess trading intensifies volatility.

Post-fix retracements: Negative return autocorrelation around the fix, denoted Λ**,** is measured as the coefficient from a regression of post-fix returns, $P_3 - P_F$, on pre-fix returns, $P_F - P_0$:

$$\Lambda^{Indep} = \frac{-\theta^2(N+1)\sigma_F^2(1+x)x}{\Sigma} = \frac{-(N+1)\sigma_F^2(1+x)x}{2\sigma_\varepsilon^2 + (N+1)\sigma_F^2 + (N+1)\sigma_F^2 x(2+x)} < 0. \qquad (13)$$

Post-fix retracements are due entirely to the dealers' net excess trading: if $x = 0$ then $\Lambda^{Indep} = 0$.

Cushing and Madhavan (2000) suggest that retracements at the NASDAQ close had a different source in the late 1990s: price pressures, meaning the common tendency for dealers with excess (insufficient) inventory to lower (raise) prices. Though they do not evaluate this hypothesis rigorously, this phenomenon is well-documented for equity markets (e.g., Hendershott and Menkveld, 2014). In currency markets, by contrast, studies generally find no evidence for such price shading (e.g., Bjønnes and Rime, 2005). This absence is generally attributed to a simple cost-benefit analysis: price shading can be costly because it communicates information about a dealer's position that other dealers can exploit. Price shading has limited benefits in forex because an aggressive trade in the interdealer market is fast and inexpensive. Indeed, existing evidence for currency markets supports a positive rather than negative response of price to lagged order flow, because small dealers tend to imitate the trading of large dealers (Menkhoff and Schmeling, 2010). Nonetheless, we view price pressures and excess trading as complementary explanations for negative autocorrelation around fixes.

Convexity: The model has implications for the shape of the pre-fix price path. If the trend accelerates (decelerates) up to the fix we say the path is convex (concave) and measure convexity, or Π, as the ratio of the expected period-2 return to the expected period-1 return:

$$0 \le \Pi^{Indep} \equiv \frac{E\{P_2 - P_1 \mid \sum_{N+1} F_n\}}{E\{P_1 - P_0 \mid \sum_{N+1} F_n\}} = \frac{\theta(1-\alpha_2)\sum_{N+1} F_n}{\theta\alpha_1 \sum_{N+1} F_n} = \frac{(1-\alpha_2)}{\alpha_1} = \frac{1}{1+\rho N} \ . \tag{14}$$

If dealers know nothing about each others' orders ($\rho = 0$) they trade 2/3 of their customer fix order in both periods 1 and 2; in consequence $\alpha_1 = (1-\alpha_2)$ and $\Pi^{Indep}=1$. The free-riding that arises with positively correlated orders creates a strictly concave pre-fix price path ($\Pi^{Indep} < 1$) in two ways. First, each dealer opens larger proprietary position in period 1, which increases α_1 and strengthens the period-1 price trend. Second, each dealer liquidates more of that proprietary position in period 2, which reduces (1-α_2) and weakens the period-2 price trend.

Proposition 2 summarizes the model's implications for fix-price dynamics when dealers trade independently:

Proposition 2: When risk-neutral d**ealers trade independently, price dynamics will display the following three features:**

a. **High pre-fix volatility: Volatility will be high before the fix for two reasons: first, dealers accumulate the inventory required to serve customer orders before the fix; second, dealers take proprietary positions;**

b. **Retracements: The dealers' proprietary trading will cause partial retracements of the pre-fix trend after the fix;**

c. **Strict concavity of the pre-fix price path: Dealer free-riding will cause the pre-fix trend to decelerate, on average, as the fix approaches.**

III. Information sharing and collusion

This section examines market equilibrium when dealers share confidential information about customer orders or collude outright.

A. Information sharing

Sharing information about customer orders with a dealer at another bank is considered unethical because it puts the customer at risk of manipulation. Forex dealers know this because bank compliance

officers regularly remind them regularly. Nonetheless, transcripts of private chat-room conversations show that forex dealers were accustomed to sharing information about customer fix orders (see, e.g., FCA (2014b)). To model such behavior we assume that each fix dealer gives accurate information to the other fix dealers about his customer net fix order before fix trading begins. (Later discussion considers the possibility that dealers are not fully truthful.) Each dealer analyzes his period-2 decision before making his period-1 decision, as described in Section II.

Lemma 2, which summarizes dealer d's fix trading under information sharing, shows that it shares the five critical features identified previously trading: customer inventory is accumulated before the fix; pre-fix trading is distributed across both pre-fix periods; dealers front-run their customers; dealers free-ride by front-running each other; dealers earn positive profits. Nonetheless, the market's average net fix order, $\bar{F} \equiv (F_d + \sum_N F_n)/(N+1)$, plays a critical role. Shares of this quantity in pre-fix trading are denoted δ_t, $t = 1, 2$; the share of this quantity traded after the fix is denoted χ.

Lemma 2: In equilibrium when risk-neutral dealers share information about customer fix orders, dealer d's trading, as a function of his own and the average fix order, is:

a. $$D_{1d} = \left(\frac{(1+N)(2+N)}{(2+N)^2-(1+N)}\right)\bar{F} \equiv \delta_1^{InfoShare}\bar{F}\,, \qquad \frac{6}{7} \le \delta_1^{InfoShare} < 1\,; \tag{15a}$$

b. $$D_{2d} = F_d - \left(\frac{1+N}{2+N}\right)\delta_1^{InfoShare}\bar{F} \equiv F_d - \delta_2\bar{F}\,, \qquad \frac{4}{7} \le \delta_2^{InfoShare} = \delta_1^{InfoShare} - \chi^{InfoShare} < 1\,; \tag{15b}$$

c. $$X_d \equiv -D_{3d} = \left(\frac{1}{2+N}\right)\delta_1^{InfoShare}\bar{F} \equiv \chi^{InfoShare}\bar{F}\,, \qquad 0 < \chi^{InfoShare} \le \frac{2}{7}\,. \tag{15c}$$

In equilibrium under information sharing, dealer d's proprietary trading is determined by the average customer fix order, instead of his own customer order, F_d. In both cases the trade is a proportion of his forecast for the period-2 return, but that forecast is more accurate when he knows the market's net fix order, $(1+N)\bar{F}$, than when he has to estimate it based on his own order. The fraction of this position liquidated in period 2, $(1+N)/(2+N)$, exceeds the corresponding fraction under independent trading.

Free-riding intensifies under information sharing: a dealer with no fix orders of his own, or with net fix orders in the opposite direction to the majority, nonetheless takes the same inventory position in period 1 as the other dealers. By contrast, recall that under independent trading every dealer trades a fraction of

his own fix order in every period. If he has no fix order he does not trade; if his customers are buying when all the other customers are selling, he nonetheless buys.

Proposition 3 summarizes the key properties of trading under information sharing:

Proposition 3: In equilibrium when risk-neutral dealers share information about customer fix orders,

a. Free-riding is more pronounced than under independent trading. On average dealers take larger proprietary positions in period 1 and liquidate more of those positions in period 2:

$$\alpha_1 < \delta_1^{InfoShare} \text{ and } \quad \alpha_2 < \delta_2^{InfoShare} . \tag{16a}$$

b. The additional free-riding undermines dealer profits relative to independent trading:

$$E_0\left\{\pi^{InfoShare}\right\} = \theta\sigma_F^2[\delta_1(1-\delta_1) + \chi(\delta_1 - \chi)] < E_0\left\{\pi^{Indep}\right\} = \theta\sigma_F^2[\alpha_1(1-\alpha_1) + x(\alpha_1 - x)]. \tag{16b}$$

c. On average across dealers, net excess trading is lower than under independent trading:

$$0 < \chi^{InfoShare} = \left(\frac{1+N}{(2+N)^2 - (1+N)}\right) \leq x^{Indep} = \left(\frac{1+\rho N}{(2+\rho N)^2 - (1+\rho N)}\right) \leq \frac{1}{3} \ . \tag{16c}$$

The finding that dealers are better off sharing zero information than sharing truthfully about their fix orders may seem counterintuitive, but it has direct parallels in the analysis of information sharing among oligopolistic firms. Clarke (1983), whose model of oligopoly is closest to ours, shows that in an oligopolistic market with stochastic elements the firms' welfare is minimized if firms pool their information. Nonetheless, a key choice variable for fix dealers, trade timing, has no parallel in a traditional model of the firm. This could help explain a notable difference between our findings and those of traditional models: Competition among fix dealers is closest to Cournot competition, insofar as dealers choose quantities not prices, but traditional oligopoly models show that information sharing is beneficial to firms under Cournot competition and costly otherwise (Vives, 1990).

The conclusion that information sharing is costly to dealers may be robust to the possibility that dealers are not fully truthful. Gal-Or (1985) analyzes such a model and concludes that in Nash equilibrium firms still do worse by sharing information. Clarke (1983) finds that firms do best if they not only share information but also collude on trading strategies. We examine that possibility in the next section.

Proposition 4 summarizes how information sharing affects fix price dynamics:

Proposition 4: In equilibrium when risk-neutral dealers share information about customer fix orders the additional free-riding means that

a. Pre-fix volatility is less pronounced than under independent trading though it still exceeds volatility during non-fix periods,

$$\Sigma^{InfoShare} \equiv E\left\{(P_F - P_0)^2\right\} = \theta^2 2\sigma_\varepsilon^2 + \theta^2 (N+1)\sigma_\phi^2 (1+\chi^{InfoShare})^2 < \Sigma^{Indep} ; \tag{17a}$$

b. Post-fix trend retracements are less pronounced than under independent trading,

$$\Lambda^{Indep} < \Lambda^{InfoShare} = -\frac{(N+1)\sigma_F^2 (1+\chi^{InfoShare})\chi^{InfoShare}}{2\sigma_\varepsilon^2 + (N+1)\sigma_F^2 (1+\chi^{InfoShare})^2} < 0 \text{ ; and} \tag{17b}$$

b. The average pre-fix price path is *more concave* than under independent trading,

$$0 < \Pi^{InfoShare} = \frac{1}{1+N} < \Pi^{Indep} = \frac{1}{1+\rho N} < 1 . \tag{17c}$$

B. Collusion

Collusion violates antitrust laws, standard regulatory limits, and bank policies; the penalties can include jail time. One might therefore wonder why dealers who already share information might also collude. This section shows that collusion raises average dealer profits because it shuts down free-riding.

The dealers could organize their collusive activity in a variety of different ways. At the London 4 pm fix, dealers in the self-described "cartel" apparently assigned a single dealer to control all customer fix trading for the group (FCA, 2014). If dealers trust each other they could execute the same collusive strategy by sharing the agreed trading responsibilities. For convenience we assume that a single dealer has full control and that the other cartel members do not cheat. We also assume that all dealers join just one cartel. The outcome with K separate cartels is isomorphic to independent trading with $N = K$ -1.

The profit-maximizing collusive strategy depends entirely on the market's total fix orders, $F_{Tot} \equiv \sum_{N+1} F_n$. Otherwise the strategy is identical to the strategy of a dealer who knows nothing about the other dealers' orders ($\rho = 0$). Lemma 3 summarizes this outcome using D_1, D_2, and $X \equiv \chi^{Collude} F_{Tot}$ to denote pre-fix inventory accumulation and excess trading under collusion.

Lemma 3: With risk-neutral dealers a cartel will trade 2/3 of total fix orders in both periods 1 and 2.:

$$\frac{D_1}{F_{Tot}} = \frac{D_2}{F_{Tot}} = \frac{2}{3}, \qquad \frac{X}{F_{Tot}} \equiv \chi^{Collude} = \frac{1}{3} . \tag{18}$$

In effect the cartel opens a proprietary position in period 1 equal to 2/3 of the market's net fix order. Because colluding dealers do not free-ride this proprietary position is smaller, on average, than the

proprietary positions of dealers trading independently or sharing information. The cartel liquidates equal amounts of its proprietary position in periods 2 and 3.

Proposition 5: When risk-neutral dealers collude,

a. **Dealers do not free-ride;**

b. **Expected profits are higher than under information sharing or independent trading,**

$$E_0\left\{\overline{\pi}^{Collude}\right\} = \theta\sigma_F^2 \big/ 3 > E_0\left\{\overline{\pi}^{Indep}\right\} > E_0\left\{\overline{\pi}^{InfoShare}\right\}; \tag{19a}$$

c. **Net excess trading is higher than under information sharing or independent trading,**

$$\chi^{Collude} = 1/3 > x^{Indep} > \chi^{InfoShare} \tag{19b}$$

The source of collusive fix profits is necessarily unrelated to traditional monopoly power because collusive fix dealers do not agree on a specific price. They agree instead on strategy for the amount and timing of a trade sequence intended to influence the price. Collusion at the fix does share some notable features with a different microeconomic phenomenon, “shrouding.” Originally introduced by Gabaix and Laibson (2006), shrouding applies to producers who advertise a low price for a headline product while hiding the high price of an additional component. Manufacturers of home printers, for example, advertise low prices for the printers while hiding the high price of ink.

The shrouding concept may be relevant to fix dealing insofar as liquidity, the product sold by the dealers, has multiple dimensions. For normal-sized forex trades (up to roughly $25 million), immediacy is the entire product and its price is the bid-ask spread. Larger trades are generally handled on a best-efforts basis, which means that the product is a combination of immediacy and the dealer’s skill at minimizing price impact. In consequence, spreads are typically wider on large trades. Though fix orders are normally large, prior to reforms the dealers generally set a bid-ask spread of zero on such orders (Levine, 2015): in effect, the price on the headline product, immediacy, was zero. The liquidity provided with fix orders was not costless, however, because the banks maximized their total price impact instead of minimizing it. This cost was shrouded from the customers’ view by the opacity of the interdealer market and by the dealers’ decision to communicate through private chat rooms.

Because net excess trading is larger under collusion than under independent trading or information sharing, there will generally be a larger divergence between the fix price and the asset’s fair value and the

market will be less efficient. Likewise, pre-fix volatility and post-fix retracements will be more pronounced.[7] The one feature of the market that is less pronounced under collusion is the concavity of the pre-fix price path: under collusion that path will be linear rather than strictly concave because dealers trade equal amounts in both periods before the fix.

Proposition 6: In equilibrium when risk-neutral dealers collude,

a. Pre-fix volatility is higher than under independent trading or information sharing:

$$\Sigma^{Collude} = 2\theta^2\sigma_\varepsilon^2 + \theta^2(N+1)\sigma_\phi^2(1+\chi^{Collude})^2 > \Sigma^{Indep} > \Sigma^{InfoShare}; \quad (20a)$$

b. Post-fix retracements are more pronounced than under independent trading or information sharing:

$$\Lambda^{Collude} = -\frac{(N+1)\sigma_F^2\chi^{Collude}(1+\chi^{Collude})}{2\sigma_\varepsilon^2 + (N+1)\sigma_F^2(1+\chi^{Collude})^2} < \Lambda^{Indep} < \Lambda^{InfoShare} < 0; \quad (20b)$$

c. The pre-fix path will be linear rather than strictly concave:

$$0 < \Pi^{InfoShare} < \Pi^{Indep} < \Pi^{Collude} = 1. \quad (20c)$$

In sum, information sharing and collusion have opposing effects on per-dealer profits, proprietary trading, net excess trading, pre-fix volatility, post-fix retracements, free-riding, and convexity of the pre-fix price path. Similarly opposing effects of collusion and information sharing arise in Clarke's (1983) analysis of oligopolistic producers in a stochastic environment.

Dynamic collusion: The possibility of cheating by cartel members cannot reasonably be ruled out given the dealing banks' admission that dealers violated bank ethical standards and anti-trust laws. Indeed, given the strong incentives for dealers to cheat on each other identified by the model, fix trading was isomorphic to a prisoner's dilemma and cheating should perhaps be expected. A cooperative dealer would have colluded as outlined above; a non-cooperative dealer could have lied about his fix orders and trading those orders independently, or traded for his own account separately from the cartel, liquidating those trades earlier than the rest of the cartel and depressing their profits.

[7] Evans (2016) develops a model in which heightened volatility should arise after but not before the fix under independent trading. If so, volatility before the fix could be a sign of collusion. This result depends on a critical feature of that model – dealers do not immediately adjust prices to the information provided in customer order flow – that appears to be inconsistent with a core principle of modern finance. This finding also implies that dealers do not front-run, a conclusion that is inconsistent with existing evidence and with standard reasoning microstructure (e.g., Comerton-Forde and Putniņš, 2011).

As a repeated game, fix trading can also be analyzed in terms of dynamic collusion. If forex demand and supply functions are known with certainty, equilibrium cartel behavior would be determined by the fact that cheating can be identified unambiguously. In reality, however, forex dealers face many sources of uncertainty and signals of cheating would be noisy. Green and Porter (1984) and Abreu et al. (1986) show that Bertrand competitors facing such uncertainty can rationally adopt both carrots and sticks. They cooperate if and only if the price remains within a certain range, but if the price breaches that range they perceive a high likelihood of cheating. In response they retaliate for a finite number of rounds and then revert to collusion. In fix trading the trigger for retaliation could have been an observed price path that was inconsistent with the expected path under collusion. The price rise might begin or end earlier than expected, for example. Retaliation itself need not take place within the context of fix trading and could take many forms. A suspicious dealer could quote slower or wider prices in direct trading, accommodate smaller amounts, or engage in social exclusion.

Given randomness in the price process, the cartel theories of Green and Porter (1984) and Abreu et al. (1986) predict an irregular cycle of cheating, retaliation, and renewed cooperation which, in our contest, could generate irregularities in fix-price dynamics. Empirical research on cartels does not entirely support this theoretical perspective, however. Cartels typically survive for at least a few years – many last beyond ten years – and price wars are less frequent and less intense than theory predicts (Levenstein and Suslow, 2006). The empirical research also highlights conditions under which collusion tends to thrive, at least two of which were met by forex dealing at the fix. First, in successful cartels compensation is responsive to market conditions, which helps them avoid disagreement over how to adjust collusive rents (Levenstein and Suslow, 2011). In the fix cartel, the dominant dealer for the day – a role that apparently rotated according to the relative magnitude of each dealer’s fix orders – earned the day’s entire gains which were determined by that day’s net fix order, that day’s market conditions, and his own skill. Second, members of a successful cartel will typically apply a low discount rate to the future (Levenstein and Suslow, 2016). In the fix cartel, dealers were secure within their respective banks and interest rates were generally low.

IV. Banging-the-close and convexity

So far the model has clarified how strategic interactions among dealers can generate two striking features of price dynamics around the London 4 pm fix: high volatility before the fix and retracements after the fix, regardless of whether dealers collude. However, the model does not capture another notable phenomenon associated with benchmark prices: banging-the-close or equivalently "concentrating orders in the moments before and during the 60-second window" surrounding the calculation of the London fix (Vaughan, Finch, and Choudhury, 2013). Evans (2016) provides evidence of this around the London 4 pm fix and the CFTC has identified banging-the-close by specific traders in the markets for palladium and platinum in 2007 and 2008 (Doering and Rampton, 2010). If dealers bang-the-close the pre-fix price path should be convex, consistent with the pattern in forex since roughly 2008 (Figure 1).

This section examines two natural extensions of the model that produce banging-the-close and strict convexity of the pre-fix price path: sensitivity of price impact to order flow and dealer risk aversion.[8]

A. Quantity-sensitive price impact

Cushing and Madhavan (2000) provide evidence that order flow has a stronger impact on NASDAQ prices just prior to the close than at other times. As discussed in Comerton-Forde and Putniņš (2011), this could reflect the dealers' intentional efforts to maximize price impact. Traders in a limit-order market know they can minimize price impact by dividing a large trade into smaller transactions distributed over time. This allows depth at the best quotes to be replenished between each trade (Bertsimas and Lo, 1998). By symmetry, traders in a limit-order market know they can maximize price impact by trading a large amount all at once, exhausting available depth at the best quote and many price levels beyond. These observations imply that instantaneous price impact is sensitive to order flow.

Let total order flow in a given period be $Q_t \equiv \sum_{N+1} D_{tn} + \varepsilon_t$, t = {1,2,3}. To capture the effect of order flow on price we assume that θ is an increasing function of Q_t: $\theta_t = \theta(Q_t)$, $\theta'(Q_t)>0$ and for tractability we

[8] It would also be possible, in theory, to introduce lagged effects of order flow, where a negative effect could reflect price pressures (Hendershott and Menkveld, 2014) and a positive effect could reflect imitation trading. Imitation t rading has substantial documentation in forex (Menkhoff and Schmeling, 2010) and would enhance the profitability of proprietary trading, but any lagged effect of order flow reduces the model's transparency and renders it intractable.

assume constant elasticity: $\theta'(Q_t)Q_t/\theta(Q_t) \equiv e \geq 0$. The baseline model becomes a special case of the current model with $e = 0$.

The sequence of analysis once again begins with representative dealer d analyzing his period-2 choice of excess trading, X_d, as a function of period-1 variables and then choosing his period-1 trade. Under independent trading each dealer must estimate the other dealers' orders and trades. Under information sharing dealers know each others' orders but not their trading strategies. Under collusion one dealer controls all orders and trading. Lemmas 4 through 6 summarize dealer trading in these three competitive settings: derivations are provided in the Appendix. Period-2 trading is not reported because is fully determined by fix orders, initial proprietary positions, and net excess trading.

Lemma 4: When price impact responds to order flow with constant elasticity $e>0$ and risk-neutral dealers trade independently, dealer d trades as follows:

a. $$D_{1d} = \frac{(2+e+\rho N)(1+\rho N)}{(2+e+\rho N)^2 - (1+e)(1+\rho N)\,{}^{\theta_2}\!/\!{}_{\theta_3}} F_d \equiv \alpha_1 F_d \ , \tag{21a}$$

b. $$X_d \equiv -D_{3d} = \frac{(1+e)\,{}^{\theta_2}\!/\!{}_{\theta_3}}{(2+e+\rho N)} \alpha_1 F_d \equiv x F_d \ . \tag{21b}$$

Lemma 5: When price impact is sensitive to order flow with constant elasticity $e>0$ and risk-neutral dealers share information, dealer d trades as follows:

a. $$D_{1d} = \left(\frac{(1+N)(2+e+N)}{(2+e+N)^2 - (1+N)(1+e)\,{}^{\theta_2}\!/\!{}_{\theta_3}} \right) \overline{F} \equiv \delta_1^{InfoShare} \overline{F} \ , \tag{22a}$$

b. $$X_d \equiv -D_{3d} = \left(\frac{(1+e)\,{}^{\theta_2}\!/\!{}_{\theta_3}}{(2+e+N)} \right) \delta_1^{InfoShare} \overline{F} \equiv \chi^{InfoShare} \overline{F} \ . \tag{22b}$$

Lemma 6: When price impact is sensitive to order flow with constant elasticity $e>0$ and risk-neutral dealers collude, dealers will collectively trade as follows:

a. $$D_1 = \frac{(2+e)(1+N)}{(2+e)^2 - (1+e)\,{}^{\theta_2}\!/\!{}_{\theta_3}} \overline{F} \equiv \delta_1^{Collude} (1+N) \overline{F} \ , \tag{23a}$$

b. $$X \equiv -D_3 = \frac{(1+e)\,{}^{\theta_2}\!/\!{}_{\theta_3}}{(2+e)} \delta_1^{Collude} \overline{F} \equiv \chi^{Collude} (1+N) \overline{F} \ . \tag{23b}$$

These solutions are highly non-linear and cannot be expressed in closed form: the trading shares for each period depend on the ratio of expected price impacts, θ_2/θ_3 , which depends non-linearly on the trading shares. Nonetheless, comparative statics reveal that banging-the-close becomes more likely because a rise in e brings a shift of trading from period 1 to period 2 unless parameters are extreme.

A pair of thought experiments clarifies why banging-the-close is more likely when $e > 0$. Assume the market begins in equilibrium with $e = 0$ and now let a dealer consider of shifting one unit of trading from period 1 to period 2. With $e = 0$ this would increase the appreciation of every unit of period-1 inventory but the period-1 inventory itself would be smaller and the gains would exactly offset the losses. With $e > 0$ the gains also include an increase in the period-2 price impact and the dealer's profits rise.

Now let a dealer consider adding one unit of excess trading in period 2 to be liquidated in period 3. With $e = 0$ the benefits of a larger period-2 return exactly offset the additional loss from liquidating the extra trading in period 3. With $e > 0$, the period-2 price impact exceeds the period-3 price impact before the shift because period-2 trading exceeds period-3 trading. The dealer's profits will once again increase.

Given the non-linearity of this equilibrium we examine it with simulations. Equilibrium is determined by three exogenous parameters: N, ρ, and e. Figure 2A plots $(1-\alpha_2)/\alpha_1$ for representative values of e and N with ρ=0.5. Additional solutions, not reported, show that the equilibrium is not highly sensitive to ρ. Period-2 trading can be a multiple of period-1 trading, consistent with banging-the-close, and the expected price path can be strictly convex. These outcomes occur if dealers collude (N=0) or if competition is limited. More specifically, period-2 trading exceeds period-1 trading if $\rho N<e$ under independent trading and if $N<e$ under information sharing.

When price impact is sensitive to order flow, pre-fix volatility and retracements become more pronounced. The analysis is considerably more complex than with $e = 0$, however, because price impact – and thus volatility – changes even if a given quantity of pre-fix trading shifts between periods 1 and 2. The variance-based measure used in Sections II and III now involves third and fourth moments of all distributions. To assess how dealer strategies affect volatility we therefore rely on "relative volatility," V, defined as the ratio of (a) the price change under strategic behavior to (b) the price change in the absence

of strategic or manipulative behavior, meaning when dealers trade only the amount of their own fix orders divided evenly across pre-fix periods:

$$V \equiv \frac{E_0\left\{\theta(.)\left[\alpha_1\sum_{N+1}F_n+\varepsilon_1\right]+\theta(.)\left[\alpha_2\sum_{N+1}F_n+\varepsilon_2\right]|\sum_{N+1}F_n\right\}}{E_0\left\{\theta(.)\left[\sum_{N+1}F_n\Big/2+\varepsilon_1\right]+\theta(.)\left[\sum_{N+1}F_n\Big/2+\varepsilon_2\right]|\sum_{N+1}F_n\right\}} \quad . \tag{24}$$

If $e = 0$ then $V_{e=0}^{Indep} = 1 + x_{e=0}$. If $e > 0$ then V has no closed-form solution so we apply a Taylor series expansion: $V_{e>0}^{Indep} \approx 2^e\left[\alpha_1^{1+e} + (1-\alpha_2)^{1+e}\right]$. Figure 2B, which plots V for representative values of e and N with ρ=0.5, shows that relative volatility is increasing with e, as one might expect.

B. Risk aversion

In our second extension of the model fix dealers have mean-variance utility with risk aversion $\gamma/2$:

$$E\{\pi_d\} - \frac{\gamma}{2}Var(\pi_d). \tag{25}$$

As before, we begin by analyzing representative dealer d who trades independently and who analyzes period 2 before period 1. His expected profits, conditional on period-1 information, remain as shown in Equation (5). The conditional variance of those profits is determined by the non-fix trade shocks, ε_2 and ε_3; by dealer d's error in forecasting other dealers' fix orders, $\vartheta_d \equiv \sum_N F_n - E_{2d}\{\sum_N F_n\}$; and by his error in forecasting other dealers' excess trading, $\mu_d \equiv \sum_N X_n - E_{2d}\{\sum_N X_n\}$. These risk terms have variance σ_ϑ^2>0 and σ_μ^2>0, respectively, and covariance $Cov(\vartheta_d, \mu_d)$. The Appendix, which derives equilibrium trading shares, highlights three critical combinations of the model's risk primitives, σ_ϕ^2, σ_η^2, and σ_ε^2: $Var_1(\pi_d)$ or the variance of profits in period 1, $R_d \equiv \gamma\theta[2\sigma_\varepsilon^2 + \sigma_\mu^2 + (1-\hat{\alpha}_1)Cov(\mu_d, \vartheta_d)]$, and $R_x \equiv \gamma\theta(\sigma_\varepsilon^2 + \sigma_\mu^2)$.

Lemma 7: When dealers are risk averse and trade independently, dealer *d* will trade as follows:

a. $$D_{1d} = \frac{(1+\rho N)}{(2+\rho N)[1-q(1-q)]+\gamma\theta Var_1(\pi_d)}F_d \equiv \alpha_1 F_d, \qquad \frac{1}{2} < \alpha_1 < 1, \tag{26a}$$

b. $$X_d = \frac{1+R_d}{2+R_x+\rho N}\alpha_1 F_d \equiv q\alpha_1 F_d \equiv xF_d \quad , \qquad 0<x<\alpha_1< 1 \tag{26b}$$

This solution is highly non-linear; period-1 trading as a share of fix orders, α_1, depends on the risk terms R_x and R_d as well as q; these, in turn, depend non-linearly on each other as well as α_1. Equilibrium trading shares cannot be expressed in closed form and comparative statics are inconclusive.

Simulations for a wide variety of parameters, presented in Figures 3A through 3D, confirm that proprietary trading is inversely related to risk aversion, which is logical because proprietary trades are the only source of a dealer's risk exposure. For many parameter combinations the proprietary position shrinks sufficiently that period-1 trading is smaller than expected period-2 trading, though the incentive to free-ride is still operative, and the price path is strictly convex. Similar shifts accompany a rise in any of the market's primitive risk variables or an increase in N. The latter effect arises because higher N brings a higher variance of total fix orders.

When dealers share information about customer fix orders they no longer face uncertainty about the orders of other dealers. This means $\sigma_\vartheta^2 = \sigma_\mu^2 = Cov(\vartheta, \mu) = 0$, which streamlines the analysis considerably. The variance of profits for the dealer's period-2 decision becomes $\theta^2\sigma_\varepsilon^2(X_d^2 + D_{1d}^2)$ and equilibrium trading shares can be expressed in closed form.

Lemma 8: When risk-averse dealers share information about customer orders, dealer *d* adopts the following strategy:

a. $$D_{1d} = \left(\frac{(1+N)(2+N+\theta\gamma\sigma_\varepsilon^2)}{(2+N+\theta\gamma\sigma_\varepsilon^2)^2 - (1+N)}\right)\overline{F} \equiv \delta_1^{InfoShare}\overline{F}\ , \qquad 0 < \delta_1^{InfoShare} < 1 \tag{27a}$$

b. $$X_d \equiv -D_{3d} = \left(\frac{1+N}{2+N+\theta\gamma\sigma_\varepsilon^2}\right)\delta_1^{InfoShare}\overline{F} \equiv \chi^{InfoShare}\overline{F}\ , \qquad 0 < \chi^{InfoShare} < 1 \tag{27b}$$

Risk-averse dealers who share information, like risk averse dealers trading independently, take smaller proprietary positions in period 1: $\partial\delta_1^{InfoShare}/\partial\gamma < 0$. Expected period-2 trading will exceed period 1 trading, consistent with banging-the-close, and the price path will be strictly convex, if $N(3+2N) < 2\gamma\theta\sigma_\varepsilon^2$. Intuitively, a dealer's proprietary trading position is smaller if he is highly risk averse, if risk is high, or if competition is limited. Note that the relation between convexity and N changes sign between independent trading and information sharing. Under information sharing convexity is

inversely related to N because fix dealers face no uncertainty about orders so the influence of free-riding, which rises with the number of competitors, dominates.

When dealers collude, equilibrium trading strategies can again be expressed in closed form. As shown in Figures 3A-3D the effects of collusion are largely the same under risk aversion and risk neutrality. Increased risk aversion unambiguously reduces the aggregate proprietary position, as noted previously. A risk-averse dealer also liquidates more of his proprietary position in period 2, since that position is his sole source of risk and the longer he holds it the more risk he bears.

Lemma 9: When risk-averse dealers collude, dealers will bang-the-close and the average pre-fix price path is strictly convex:

a. $$D_1 = \left(\frac{(2+\theta\gamma\sigma_\varepsilon^2)(1+N)}{(2+\theta\gamma\sigma_\varepsilon^2)^2 - 1} \right) \overline{F} \equiv \delta_1^{Collude}(1+N)\overline{F}, \qquad 0 < \delta_1^{Collude} < 1, \tag{28a}$$

b. $$X = \left(\frac{1+N}{2+\theta\gamma\sigma_\varepsilon^2} \right) \delta_1^{Collude} \overline{F} \equiv \chi^{Collude}(1+N)\overline{F}, \qquad 0 < \chi^{Collude} < \frac{1}{3} \tag{28c}$$

c. $$\frac{\partial \delta_1^{Collude}}{\partial \gamma} < 0, \ \frac{\partial (1-\delta_2^{Collude})}{\partial \gamma} > 0. \tag{28c}$$

In sum: If price impact is sensitive to order flow or if dealers are risk-averse the model predicts both banging-the-close and convexity of the pre-fix price path under collusion. Banging-the-close can also be optimal when dealers trade independently if competition is limited.

V. Identifying collusion from price dynamics

Our analysis of fix trading has implications for an important practical question: Could one extract reliable evidence for collusion from high volatility before the fix or pronounced retracements after the fix? The paper's findings are not encouraging in this regard. The model implies that volatility will inevitably be high just before the fix: dealers rationally accumulate the required inventory before the fix, not after, and thus a large amount of customer orders will be executed within a short time frame. Retracements could reflect price shading by non-fix dealers whose inventories have shifted in response to trading by fix dealers (Hendershott and Menkveld, 2014). Volatility and retracements could be more pronounced if dealers make proprietary trades or bang-the-close, but these strategies are not explicitly prohibited in some markets and would be attractive regardless of whether dealers collude.

Collusion is no easier to identify based on *changes over time* in volatility or retracements at the fix. As shown in Equations 12, 17a, and 20a, these changes could reflect variation in factors other than collusion including the volatility of non-fix order flow, σ_ε^2; the variance of aggregate fix orders , $(N+1)\sigma_F^2$; the correlation of fix orders across dealers, ρ; and the number of fix dealers, $N+1$. Some of these factors, such as σ_F^2, are difficult to measure so it would be challenging to distinguish the contribution of collusion from the contributions of these other factors to any change in volatility or retracements.

According to the model, changes in convexity are subject to fewer ambiguities. Convexity is influenced by just two factors beyond the way dealers compete – the number of dealers and the correlation of orders across dealers – both of which could be fairly stable over time. Every version of the model considered above implies a more convex price path under collusion than under independent trading or information sharing. We make no claim that convexity could provide evidence for collusion sufficiently reliable to resolve a legal matter. Nonetheless, an analysis of convexity could help evaluate the validity of our model. Given that regulators claim, and dealing banks have admitted, that collusion among dealers began around 2007-2008, the model would not be supported by the absence of any rise in convexity.

Convexity of a price path has not traditionally been a focus of economic analysis so we develop a measure. We then use two statistical tests to evaluate whether convexity rose to a significant extent around 2008 using data for seven currencies vis-à-vis the U.S. dollar.

A. Measuring Convexity

Our data comprise tick-by-tick OTC quotes among interbank dealers from the Reuters Dealing platform. They begin in February 1996 for JPY, GBP, CHF, CAD, NZD, and DKK and in January, 1999 for EUR. Data for all currencies end in December 2013 but for CHF we drop all observations after October, 2011, when the Swiss National Bank began to support of a floor on the exchange rate between the franc and the euro. Following Melvin and Prins (2015) we focus exclusively on end-of-month trading days. We use mid-quotes sampled at the end of every minute and rely on minute-by-minute signed returns: $(p_{\tau,m} - p_{\tau,m-1})I_\tau$ where $p_{\tau,m}$ is the log price at the end of minute m on day τ and I_τ is an indicator variable; $I_\tau=1$ (-1) if the exchange rate rises (falls) between 3:45 and 4:00 on day τ.

We measure the convexity of a price path in two steps (see Figure 4A). We begin by taking the difference between two areas: (a) the area between the actual average price path and the path assuming the price never changes, and (b) the corresponding area if the path had linearly connected the original beginning and ending price levels. We then divide this difference by the area under the path (b).

We apply this measure to the London 4 pm fix using data from 3:45 to 4:00 pm. As shown in Figure 4B, convexity at the London fix was generally rising during our sample period, consistent with the model. The first observation for each series is convexity of the average month-end price path over the period from the beginning of the sample through December, 2002. Each subsequent observation shows convexity of the average month-end price path using a progressively longer sample. Early in the sample convexity was negative for many currencies, consistent with the concave price path predicted by free-riding. By the end of the sample convexity was positive for all currencies, consistent with the hypothesis that collusion curtailed free-riding and encouraged banging-the-close, both of which would have raised convexity.

We use two approaches to test the statistical significance of this apparent rise in convexity. Our null hypothesis is that convexity was unchanged after 2007; the alternative hypothesis is that convexity was higher after 2007.

B. Binomial test for rising convexity

As shown in Table 1, which presents convexity of the average month-end price path before and after the end of 2007, measured convexity is higher after 2007 for all seven currencies. To test the statistical significance of this result we first note that, informally speaking, the null hypothesis implies that convexity should have a roughly-even chance of rising after 2008. Each currency thus represents a single Bernoulli trial with probability of a rise in the vicinity of 50%; further, the number of currencies with rising convexity should have a binomial distribution so long as convexity is independent across currencies.

The independence of convexity across currencies might initially seem unlikely, given that daily exchange-rate returns are tied by triangular arbitrage. However, convexity is calculated from individual returns sampled at a much higher frequency than once per day. Further, convexity is many steps removed from returns: it is a complex property of the underlying return process calculated from an average of price series drawn from different months over many years. To examine more carefully whether independence

is a reasonable assumption we calculate the convexity of each end-month pre-fix path for each currency using the full sample of data. This gives 208 convexity values for each of GBP, JPY, CAD, NOK, and DKK; 191 values for CHF; and 173 values for EUR. Of the 21 bilateral correlations across these convexity series, only eight are positive and their average is just -0.005. Independence appears to be plausible.

The binomial test indicates a rejection of the null hypothesis of no change in convexity. Under the assumption the distribution of convexity is symmetrical and the probability of a rise is exactly 50%, the likelihood of observing higher convexity after 2007 for all seven currencies is just 0.008. We are not aware of any reason the likelihood of a rise would differ from 50% but little is known about the distribution so we examine the possibility by considering a binomial distribution with $n = 7$ and allowing the probability of a rise to exceed 50%. This experiment reveals that failing to reject the null requires an extremely asymmetric distribution of convexity under the null. The null hypothesis of no change in convexity is still rejected at the 5% level if the probability of a rise in a currency's convexity is as high as 65%.

C. Bootstrap tests for rising convexity

To evaluate the null hypothesis of stable convexity on a currency-by-currency basis we use a bootstrap test. The bootstrap is a non-parametric approach that makes no assumptions about the distribution of the variable to be tested – in this case convexity – but instead relies on the data to characterize that distribution. The procedure is complex so we begin with the following summary: We use data from the period before 2008 to identify shocks to the return process. We then use those shocks, sampled with replacement and adjusted appropriately for volatility and autocorrelation, to create simulated end-month price paths. From those we calculate 1,000 convexity values the distribution of which should be consistent with the null. We finish by comparing convexity of the average post-2008 price path to this distribution.

To identify return shocks we first estimate an AR(1) on one-minute returns over 3:45 to 4:00 pm:

$$p_{\tau,m+1} - p_{\tau,m} = \beta_0 + \beta_1(p_{\tau,m} - p_{\tau,m-1}) + \zeta_{\tau t,m+1} \ . \tag{29}$$

The residuals from Equation (29) will reflect time-varying volatility, to control for which we estimate a GARCH(1,1) using non-overlapping samples of 15-minute returns during all trading hours on all trading

days.[9] For each end-month day we select the fitted standard deviation for the 3:45 to 4:00 pm interval, κ_τ. We then create standardized return shocks, $\chi_{\tau,m}$, as follows:

$$\chi_{\tau,m} = \zeta_{\tau,m}/\kappa_\tau . \tag{30}$$

For each simulated end-month price path we first set the 3:45 log price, $p_{3:45}$, at $ln(100)$. To create $p_{3:46}$ we need to a simulated return between 3:45 and 3:46, $\hat{r}_{3:46}$. According to Equation (29), this requires a lagged return, so we sample with replacement one of the original 3:45 returns, $\hat{r}_{3:45}$. Creating $\hat{r}_{3:46}$ also requires a simulated return shock, $\hat{\zeta}_{3:46}$, to create which we sample with replacement one of the standardized return shocks, $\hat{\chi}_{3:46}$, and sample with replacement one of the fix-interval standard deviations, $\hat{\kappa}_{3:46}$, and multiply them: $\hat{\zeta}_{3:46} = \hat{\kappa}_{3:46}\hat{\chi}_{3:46}$. Combining $\hat{r}_{3:45}$ and $\hat{\zeta}_{3:46}$ according to the estimated AR(1) process gives $\hat{r}_{3:46}$:

$$r_{3:46} = \hat{\beta}_0 + \hat{\beta}_1 \hat{r}_{3:45} + \hat{\zeta}_{3:46} \quad , \tag{31a}$$

Adding $\hat{r}_{3:46}$ to $p_{3:45}$ gives $p_{3:46}$:

$$p_{3:46} = p_{3:45} + r_{3:46} \quad . \tag{31b}$$

To finish creating the first simulated pre-fix price path we sample 14 more values of $\chi_{\tau,m}$ and κ_τ to create 14 more volatility-adjusted return shocks, $\hat{\zeta}_j = \hat{\kappa}_j \hat{\chi}_j$, j = 3:47, 3:48, … 4:00. We apply these sequentially as shown in Equations (31a) and (31b) to generate simulated log prices for 3:47 to 4:00. Taking their antilog gives a simulated pre-fix price path consistent with the null hypothesis that convexity did not change after 2007. We do not calculate the convexity of this price path, however.

For a given currency, observed convexity for the period after 2007 is calculated from the average of 72 price paths, one for each end-month date between January 2008 and December 2013. The number of price paths included in that average is likely to influence the convexity measure. Therefore, before calculating convexity we create 71 more simulated price series and take the average of the full set of 72 price paths.[10] This average price path provides our first observation of convexity under the null. We then

[9] Trading days were defined to exclude weekends and major holidays.
[10] For CHF, 48 pre-fix price paths are included in the average from which we calculate convexity, not 72.

create 999 more sets of 72 simulated price paths, calculate the average price path for each set, and calculate the convexity of each average path. The resulting 1,000 simulated convexity values capture the distribution of post-2007 convexity under the null hypothesis that convexity did not change after 2007.

For each currency we evaluate whether post-2007 convexity is anomalous relative to the pre-2008 period by comparing observed post-2007 convexity with the 1,000 simulated convexities. The marginal significance of a test for consistency with the null is simply the percent of the 1,000 simulated convexity values that exceed observed post-2007 convexity.

These tests, like the binomial test, generally indicate rejection of the null that convexity was unchanged after 2007. The marginal significance levels shown in Table 1 imply rejection of the null for five of the seven currencies at the 5% significance level. For the remaining two currencies, CAD and JPY, the marginal significance levels are 7% and 14%, respectively. The lack of significance for JPY may not be surprising since fix trading in JPY is roughly evenly divided between the London fix and another fix, specifically the Tokyo fix at 10 am Tokyo time (Ito and Yamada, 2017a).

VI. Discussion

Before closing we step back to consider whether fix-price dynamics could be consistent with an efficient market and to discuss insights from the model relevant to the fix calculation methodology.

A. Price dynamics and market efficiency

Evans (2016) suggests that high pre-fix volatility and predictable retracements are inconsistent with market efficiency. Predictable price dynamics often disappear in the long run and it is commonly assumed that they will inevitably be competed away in an efficient market. This may not occur, however, if the behaviors generating the patterns are strategic complements, meaning each agent's behavior raises profits or reduces risk for other agents (Bulow et al., 1985).

Our model raises the possibility that the behaviors generating fix price dynamics are multi-dimensional strategic complements. Customers are strategic complements to each other: fix orders from one customer increase pre-fix volatility and thus the potential for tracking error if a fund does not trade at the fix, which increases the funds' incentive to place fix orders. The customers are also strategic

complements with the dealers: fix orders motivate dealers to trade in excess and/or collude, which further increases fix volatility and the funds' incentive to place fix orders.

One might expect that other banks would be strategic substitutes for the big banks and compete for fix business, given the profits from fix dealing. However, the risks involved were sufficiently severe that small and mid-sized dealers began to pass their fix orders on to the big banks. Instead of competing away fix profits these other banks effectively introduced an additional layer of strategic complementarities. By passing orders on they increased the concentration of fix orders at the big banks. Those banks then had more about the market's net fix order and faced less in managing their fix orders, which could have increased their incentive to take proprietary positions. Additional proprietary trading, in turn, would have increased volatility, increased the incentive for funds to place fix orders, and increased the incentive for small and mid-sized banks to drop out of fix dealing.

These strategic complementarities exist because of three features of financial markets that are excluded from standard models: (i) professionally-managed funds need a benchmark for portfolio valuation; (ii) once such a benchmark price exists, funds have a strong incentive to trade at that benchmark; and (iii) forex is a quote-driven market so most orders to trade at a benchmark price will be handled by dealer who trades as a principal.

B. Fix regulation

Our model can explain why fix-price dynamics are qualitatively unchanged since 2015 despite notable reforms (Ito and Yamada, 2017b) and provides other insights regarding fix regulation.

The London 4pm fix is now calculated over five minutes rather than one minute, effectively increasing the sample for selecting the median price from 61 to 301 prices. In the model the extended time interval should act like an increase in the variance of non-fix trading, σ_ε^2. If dealers are risk averse this could reduce their incentive to take proprietary positions and encourage them to shift trading towards the end of the fix period, as shown in Section IV. Otherwise this modification does not change the incentives that drive the behavior identified in this paper.

Most banks now require their dealers to process fix orders via automated algorithms that essentially eliminate dealer autonomy and distribute the trades over the pre-fix interval. These banks also prohibit

chat-room conversations among dealers. This appears to preclude dealer misconduct, though curiously it might support dealer profits by eliminating free-riding. More importantly, the predictability of trading by automated algorithms could allow non-dealers to replace dealers in driving fix-price dynamics. Because the dealers' execution of fix orders follows a somewhat predictable pattern, non-dealers can interpret the return immediately following 3:45 as a signal of the upcoming pre-fix price trend. Based on this information a non-dealer can rationally font-run the dealers' fix trading. SmartFix, that facilitates fix-based speculative strategies among non-dealers (Albinus, 2016). Like the dealer trading it replaces, this non-dealer trading could generate excess volatility and retracements.

The model provides little reason for optimism with respect to another proposed reform, the implementation of a clearing auction for fix orders. This could provide an equilibrium price for all fix buy and sell orders that can be matched off. However, it would provide no match for the fix order imbalance and, according to the model, it is the imbalance that drives the striking fix-price dynamics. The trouble is that fix orders have zero price elasticity — customers have instructed their banks to trade a certain amount regardless of the market price. Non-fix trading is price elastic, so one solution is to incorporate non-fix trading into the execution of fix orders. However, this solution amounts to the situation that already exists.

To moderate fix-price dynamics one could try to elicit non-fix orders to match the net fix order imbalance. The NASDAQ once attempted to do something along these lines by publishing market-on-close imbalances shortly before the close (Cushing and Madhavan, 2000). Our analysis suggests that instead of dampening fix-price dynamics this could intensify them because rational non-dealers, when informed of the dealers' net order imbalance, will engage in excess trading. The NASDAQ dropped this approach and instituted a closing call in 2004.

The foregoing analysis has an awkward implication: fill-at-fix orders have negative externalities. Their execution is associated with high volatility relative to non-fix times even in the absence of dealer misconduct; this raises the risks faced by dealers and thus execution costs for end-users. In addition, fix volatility can be associated with distortions in this critical benchmark rate that reduce asset values for underlying investors and can compromise economic analysis. This implication is awkward because it highlights a trade-off. On the one hand, fill-at-fix orders are a natural and ex-ante reasonable solution to

challenges faced by buy-side customers. On the other hand, long-established economic logic points to a specific remedy for negative externalities: calibrated disincentives for the behavior that generates them. Even if some sort of calibrated disincentive seemed worthy of consideration, decades of experience with financial market regulation shows that every regulatory policy has its own set of negative externalities, the magnitude of which are difficult to identify ex ante.

VII. Conclusions

This paper examines dealer behavior at the London 4 pm fix in foreign exchange, a benchmark relevant to index funds valued at $11 trillion (Cochrane, 2015). Funds that choose to trade at the fix must inform their dealers well in advance, which means dealers have an opportunity to engage in profitable misconduct. Observers long wondered whether misconduct explained the high volatility and retracements around 4 pm, and these suspicions have been confirmed by investigations that brought fines and guilty pleas from major banks (Maton and Gambhir, 2015). Similar price dynamics have been observed around fixes in other markets, most notably precious metals.

This paper examines a model of dealer trading at the fix in which dealers choose among strategies that including front-running, sharing confidential information about customer orders, banging-the-close, and collusion. The model's assumptions about price formation and the structure of trading are based on core theories and evidence in microstructure. A critical finding is that dealers rationally manage fill-at-fix orders before the fix price is set. High volatility becomes inevitable as the bulk of fill-at-fix orders are executed during a short time interval. The model also shows, however, that dealers will rationally trade for their own account in advance of customer orders. This front-running intensifies pre-fix volatility and introduces post-fix retracements. Front-running, volatility, and retracements are most pronounced under collusion dealers and least pronounced when dealers share information about customer orders. Collusion raises average dealer profits through a mechanism that is unrelated to traditional market power. Profits rise because collusion shuts down the dealers' normal attempts to front-run each others' fix trading.

We extend the model by introducing dealer risk aversion and by allowing price impact to respond to order flow. These versions of the model capture two additional dimensions of the London Fix. First,

banging-the-close, meaning dealers concentrate fix trading near the moment of fix calculation. Second, convexity or equivalently an acceleration of the price trend as the fix moment approaches.

All versions of the model imply that convexity of the pre-fix price path should be higher when dealers collude. We test the model by examining convexity before and after December 2007, when collusive trading reportedly began in earnest (Department of Justice, 2017), using tick data for seven major currencies vis-à-vis the US dollar: GBP, EUR, JPY, CHF, CAD, NZD, and DKK during 1996-2013. We develop a new measure of convexity and show that it was higher after 2007 for all seven currencies. Two different statistical tests reject the hypothesis that convexity did not increase after 2007.

The model can explain why excess volatility and price retracements continue to characterize fix-price dynamics despite recent reforms. Now that dealer trading at the fix is automated, agents with no fix orders can use price movements around 3:45 pm as a signal of dealer fix orders. Given this information, rational speculative strategies will generate excess volatility and retracements. These price dynamics may not be traded away because fix orders set in motion a nexus of mutually reinforcing behaviors among fix dealers, asset managers seeking to avoid tracking error, and smaller dealers.

Future theoretical research on the London 4 pm fix could usefully endogenize customer fix orders within the context of this model. Future empirical research could usefully examine the relative contributions of price pressures and proprietary trading to post-fix retracements.

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Appendix: Solution details

A.1. Rational expectations equilibrium under risk neutrality and independent trading

Dealer d's period-2 expectation of the other dealers' proprietary trading, $E_{2d}\{\sum_N X_n\}$, necessarily depends on his period-2 information set, $\Omega_{2d} \equiv \{F_d, D_{1d}, P_1 - P_0\}$. To identify the functional form of this expectation, assume that it is linear and apply the method of undermined coefficients:

$$E_{2d}\{\sum_N X_n\} = A(P_1 - P_0) + BD_{1d} + CF_d \,. \tag{A.1}$$

The coefficients A, B, and C are identified from rationality constraints. The first is that dealers should expect their own proprietary trading, as a share of their net fix order, to be neither more nor less than the unconditional expected value of that share:

$$E_{1d}\{X_d\}/F_d = E_0\left\{\sum_{N+1} X_n \Big/ \sum_{N+1} F_n\right\}. \tag{A.2}$$

This implies the following equality which can only be satisfied if A = 0:

$$\frac{E_{1d}\{X_d\}}{F_d} = \frac{\alpha_1}{2}\left[1 - B - \frac{A}{\theta}(1+\rho N)\right] = \frac{\alpha_1}{2}\left[1 - B - \frac{A}{\theta}(1+N)\right] = E_0\left\{\frac{\sum_{N+1} X_n}{\sum_{N+1} F_n}\right\}. \tag{A.3}$$

A second rationality constraint is that dealers should not make predictable forecast errors, or:

$E_{1d}\{\Psi_d\} \equiv E_{1d}\left\{\sum_N X_n - E_{2d}\left\{\sum_N X_n\right\}\right\} = 0.$ With A = 0, this implies:

$$E_{1d}\{\Psi_d\} = E_{1d}\left\{\left(\frac{1-B}{2}\right)\sum_N D_{1n} - BD_{1d} - \frac{C}{2}\left(\sum_N F_n + 2F_d\right)\right\} = 0\,. \tag{A.4}$$

This can be solved for B and C by considering (a) the model's symmetry, which implies that $\alpha_{1n} = \alpha_{1m}$ for all n, and (b) the structure of fix orders, which implies $E_{1d}\{F_n\} = \rho F_c$ where $\rho \equiv \sigma_\phi^2 \big/ (\sigma_\phi^2 + \sigma_\eta^2)$. Equation (A.4) becomes $E_{1d}\{\Psi_d\} = 0 = \alpha_1[B(2+\rho N) - \rho N] - C(2+\rho N)$ or

$$C = \alpha_1\left(\frac{\rho N}{(2+\rho N)} - B\right). \tag{A.5}$$

Applying this to Equation (A.1) reveals that $E_{2d}\left\{\sum_N X_n\right\}$ depends only on D_{1d}:

$$E_{2d}\left\{\sum_N X_n\right\} = \frac{\rho N}{2+\rho N} D_{1d} \qquad . \tag{A.6}$$

Thus $B = \rho N/(2+\rho N)$ and C =0. In combination with Equation (5), this implies:

$$X_d = \frac{1}{2+\rho N} D_{1d} \equiv q D_{1d}\,. \tag{A.7}$$

Comparative statics:

$$\frac{\partial \alpha_1}{\partial \rho N} = \frac{3+2\rho N}{Y^2} > 0, \text{ where } Y \equiv (2+\rho N)^2 - (1+\rho N)\,. \tag{A.8}$$

$$\frac{\partial \alpha_2}{\partial \rho N} = -\frac{\partial \alpha_1}{\partial \rho N} + \frac{\partial x}{\partial \rho N} < 0. \tag{A.9}$$

$$\frac{\partial x}{\partial \rho N} = -\frac{-\rho N(2+\rho N)}{Y^2} < 0. \tag{A.10}$$

A.2 Rational expectations equilibrium under independent trading if dealers are risk neutral and price impact is sensitive to order flow.

Assume $\theta = \theta(Q_t)$, $\theta'(Q_t)>0$, $\theta'(Q_t)Q_t/\theta(Q_t) \equiv e$, where Q_t represents order flow. Representative dealer d's period-2 optimal excess trading is:

$$X_d = D_{1d}\left(\frac{1+e}{2+e}\right)\frac{\theta_2}{\theta_3} - E_{2d}\{\sum_N X_n\}\frac{1}{2+e}. \tag{A.11}$$

Using the method of undetermined coefficients, under the two rationality constraints outlined in Section A.1, it can be shown that $E_{2d}\{\sum_N X_n\} = BD_{1d}$ with $B \equiv \rho N\left(\frac{1+e}{2+e+\rho N}\right)\frac{\theta_2}{\theta_3}$, (using $\theta_t \equiv E_{1d}\{\theta(Q_t)\}$),

and thus $X_d = \left(\frac{1+e}{2+e+\rho N}\right)\frac{\theta_2}{\theta_3}D_{1d}$. The rest of the solution follows.

Comparative statics:

$$\frac{\partial D_{1d}}{\partial e} = \frac{(1+N)\left[(1+N)^2 \theta_2/\theta_3 - (2+e+N)^2\right]}{Z^2} \tag{A.12}$$

where $Z \equiv (2+e+N)^2 - (1+N)(1+e)\theta_2/\theta_3$.

$$\frac{\partial X_d}{\partial e} = \frac{(1+N)(2+e+N)\theta_2/\theta_3\,(N-e)}{Z^2}. \tag{A.13}$$

A.3 Rational expectations equilibrium with risk aversion under independent trading

Dealer d evaluates expected profits and the variance of profits for the period-1 and period-2 trading decisions. Unexpected profits for the period-2 decision are:

$$\pi_d - E_{2d}\{\pi_d\} = D_{1d}\theta(1-\hat{\alpha}_1)(\vartheta+\varepsilon_2+\varepsilon_3) + (D_{1d}-X_d)\theta(\mu-\varepsilon_3). \tag{A.14}$$

The variance of profits conditional on period-2 information is:

$$\begin{aligned} Var_2(\pi_d) = {} & \theta^2\left\langle \alpha_1^2 F_d^2\left[2\sigma_\varepsilon^2 + (1-\hat{\alpha}_1)^2\sigma_\vartheta^2\right] + (\alpha_1 F_d - X_d)^2(\sigma_\varepsilon^2+\sigma_\mu^2)\right\rangle \\ & + \theta^2\left\langle 2\alpha_1 F_d(\alpha_1 F_d - X_d)\left[(1-\hat{\alpha}_1)Cov(\mu,\vartheta) - \sigma_\varepsilon^2\right]\right\rangle. \end{aligned} \tag{A.15}$$

This depends on the variance of non-fix trading shocks, σ_ε^2 and on the variance and covariance of dealer d's prediction errors, σ_μ^2 and σ_ϑ^2, and $Cov(\mu,\vartheta)$, respectively, where $\vartheta \equiv \sum_N F_n - E_{2d}\{\sum_N F_n\}$ and $\mu \equiv \sum_N X_n - E_{2d}\{\sum_N X_n\}$. The prediction-error properties are partially endogenous because they depend on the dealer's proprietary trading. In equilibrium these errors, as well as their variances and covariance, depend on the three underlying sources of randomness: ϕ, η, and ε.

To identify utility-maximizing period-2 trading, dealer d applies Equations (4) and (29) to his overall optimization problem, Equation (27). The first-order condition for X_d implies:

$$X_d = D_{1d}\frac{(1+R_d)}{2+R_x} - E_{2d}\{\sum_N X_n\}\frac{1}{2+R_x}. \tag{A.16}$$

The new risk terms, R_d and R_x, are defined in the text. Dealer d's proprietary trading is again linear in his period-1 trading and that the proportionality coefficient, q, now depends non-linearly on risk. To identify $E_{2d}\{\sum_{n\neq d} X_n\}$ we again assume agnostically that it is linear in the dealer's information:
$E_{2d}\{\sum_{n\neq d} X_n\} = A(P_1 - P_0) + BD_{1d}$ (this excludes F_d based on the results of Section II).[11] We once again infer that $A = 0$ from the rational expectation constraint that a dealer expects his overtrading, as a share of his fix orders, to equal the unconditional average share of overtrading. B can once again be identified from the rational expectation constraint that the dealer's period-2 expectation error should have expected value of zero conditional on period-1 information: $B = \rho N(1+R_d)/(2+R_x+\rho N)$.

Applying this to the Equation (A.16), above, gives the following solution for excess trading:

$$X_d = \frac{1+R_d}{2+R_x+\rho N}\alpha_1 F_d \equiv q\alpha_1 F_d \ , \quad q = \frac{1+R_d}{2+R_x+\rho N} \tag{A.17}$$

Dealer d next identifies the variance of profits from the perspective of period 1:

$$Var_1(\pi_d) = \sigma_\varepsilon^2\left[2 - q(1-q)\right] + \sigma_\vartheta^2[(1-\hat{\alpha}_1)^2 + (1-q)^2 q^2 \hat{\alpha}_1^2 + 2(1-\hat{\alpha}_1)(1-q)q\hat{\alpha}_1]. \tag{A.18}$$

The period-1 trading strategy in Lemma 5 solves the dealer's period-1 optimization problem:

$$\underset{\alpha_1}{Max} \ \alpha_1 F_d \theta[F_d(1-\alpha_1) + \sum_N F_n(1-\hat{\alpha}_1)] + \alpha_1 F_d(1-q)\theta\left[q\alpha_1 F_d + \sum_N X_n\}\right] - \frac{\gamma}{2}(\alpha_1 F_d)^2\theta^2 Var_1(\pi_d). \tag{A.19}$$

Comparative statics:

$$\frac{\partial \delta_1}{\partial \gamma} = \frac{-(1+N)\left[(2+N+\gamma\theta\sigma_\varepsilon^2)^2 + (1+N)\right]}{W^2} < 0 \ , \tag{A.20}$$

$$\frac{\partial \delta_2}{\partial \gamma} = \frac{-(1+N)\left[(2+N+\gamma\theta\sigma_\varepsilon^2)(N+\gamma\theta\sigma_\varepsilon^2) + (1+N)\right]}{W^2} < 0 \ , \tag{A.21}$$

$$\frac{\partial x}{\partial \gamma} = \frac{-2(1+N)(2+N+\gamma\theta\sigma_\varepsilon^2)}{W^2} < 0 \ , \tag{A.22}$$

where $W \equiv (2+N+\gamma\theta\sigma_\varepsilon^2)^2 - (1+N)$.

[11] The irrelevance of F_d is confirmed in unreported analysis.

Figure 1: Exchange rate dynamics around the London 4 pm fix

Mean price path from 60 minutes before to 60 minutes after the London 4 pm fix using tick-by-tick quotes from Reuters Dealing, and interbank trading platform, for EUR-USD, GBP-USD, USD-JPY, USD-CHF, CAD-USD, NZD-USD, and DKK-USD. The series begin in February 1, 1996, except EUR-USD, which begins January 1, 1999. All series end on December 31, 2013 except CHF-USD, which ends in October, 2011. All series are indexed to 100 at 3:45 pm. Declining prices have trends reversed for the average.

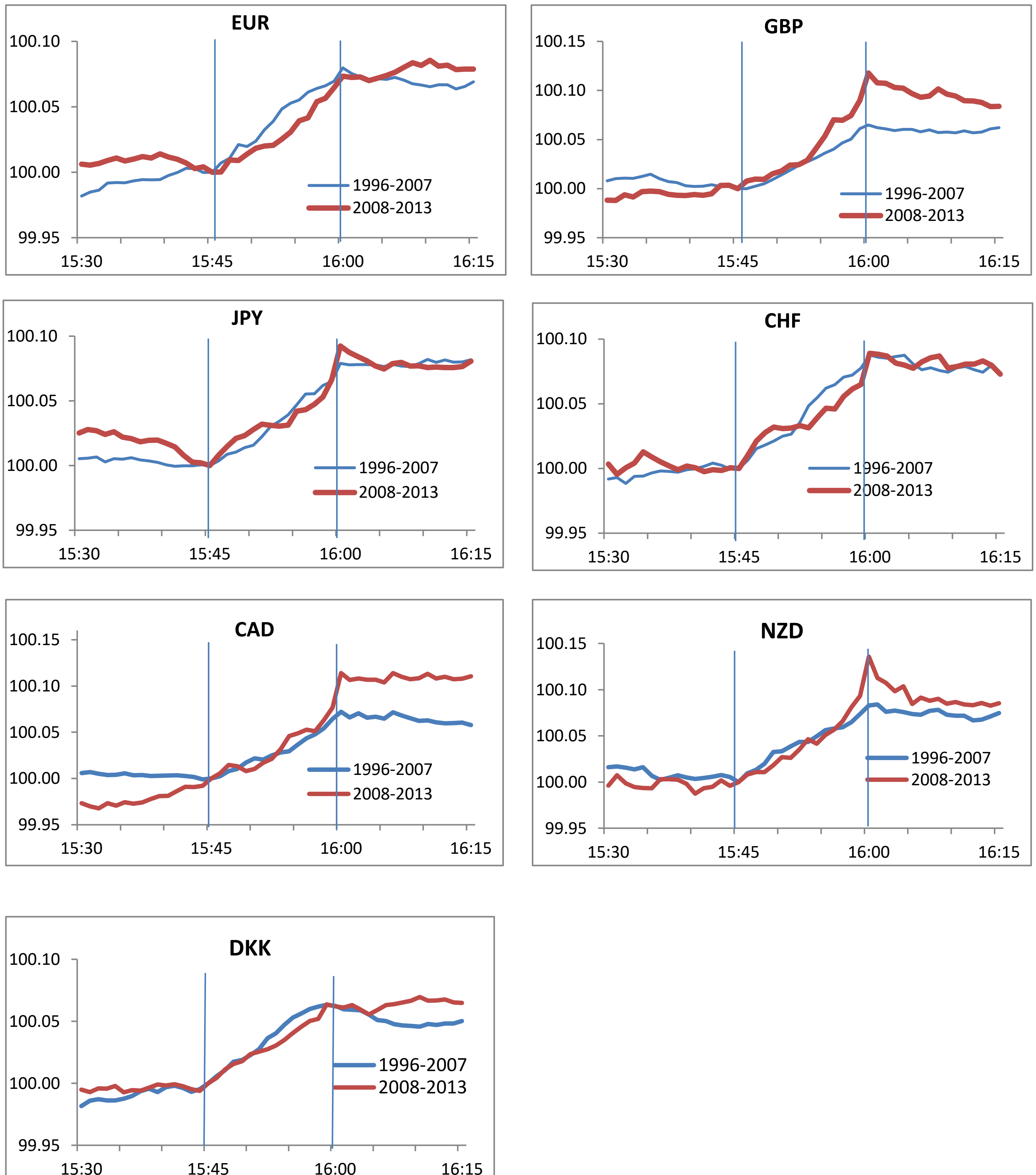
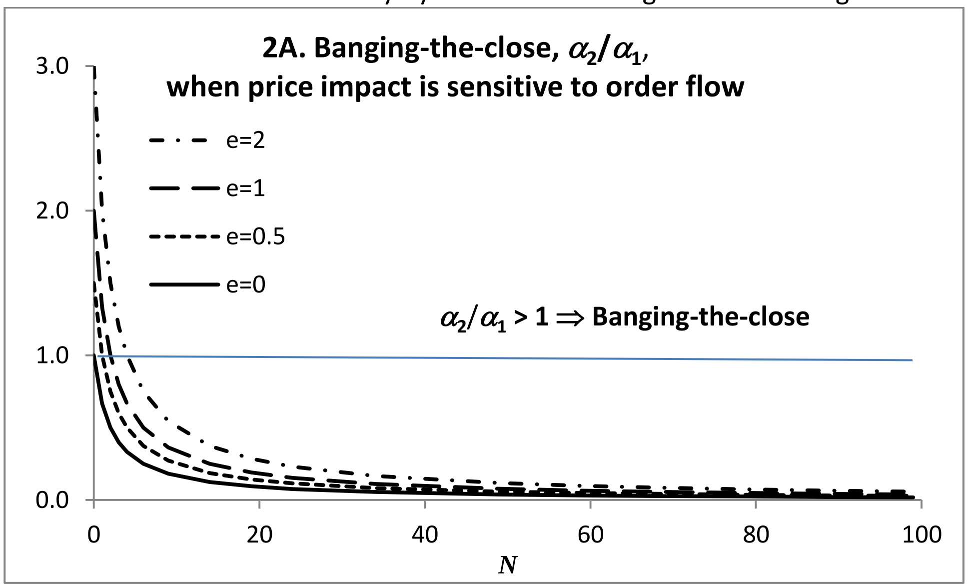


Figure 2: Volatility and convexity when price impact is sensitive to order flow.

For all simulations ρ=0.5. When $N \geq 1$ dealers trade independently; when N=0 dealers collude.

2A: Figure shows the ratio of period-2 trading to period-1 trading. If the ratio exceeds unity the pre-fix price path is convex. If the ratio exceeds unity by a substantial margin dealers "bangs-the-close."



2B. Figure shows simulated values of relative volatility under independent trading. Relative volatility is the ratio of volatility with strategic dealing to volatility if dealers engage in zero proprietary trading and distribute trades evenly across the pre-fix periods.

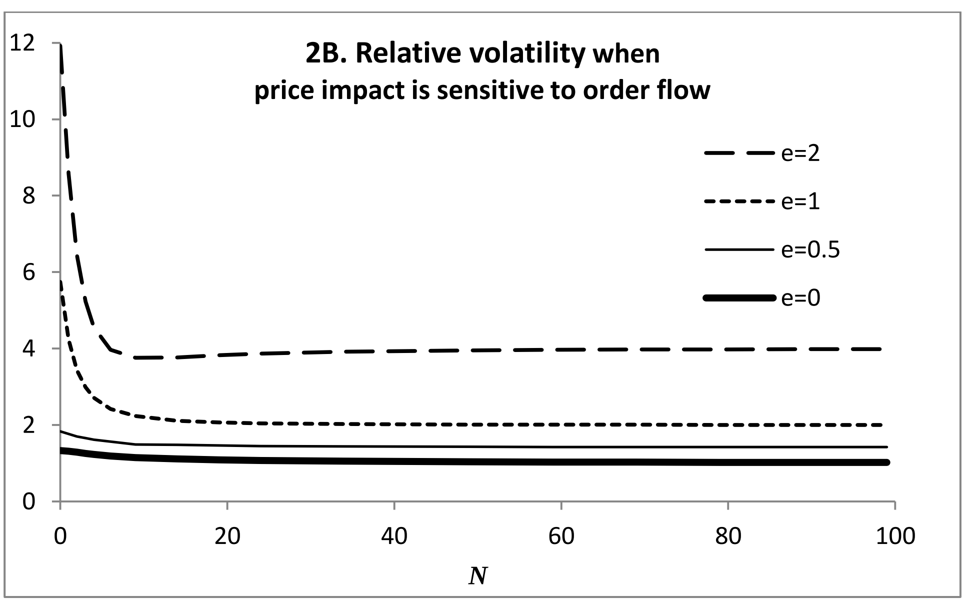


Figure 3: Banging-the-close under risk aversion

Charts show simulated levels of proprietary trading and period-2 trading relative to period-1 trading for N risk-averse dealers trading independently ($N\geq1$) or colluding (N=0). For all simulations $\rho = 0.5$ and $\sigma_{\varepsilon}^{2} = 1$. Charts end at $N = 9$ to ensure differences at N=0 are readily apparent. With high and moderate risk aversion $\gamma\theta = 0.5$and $\gamma\theta = 0.25$, respectively. With high fix-orders risk $\sigma_{\eta}^{2} = \sigma_{\varepsilon}^{2} = 1$. With low fix-orders risk $\sigma_{\eta}^{2} = \sigma_{\varepsilon}^{2} = 0.1$.

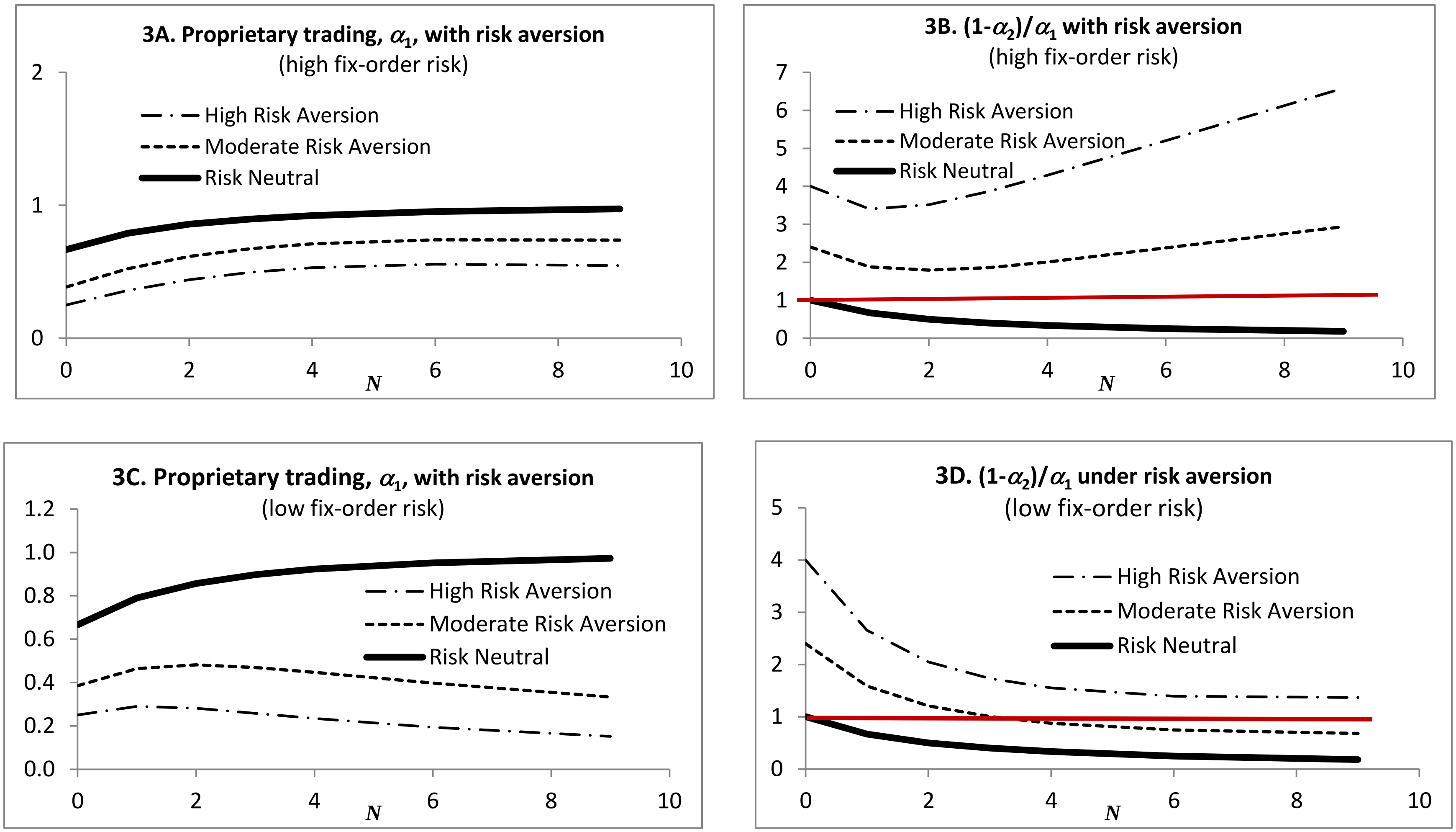
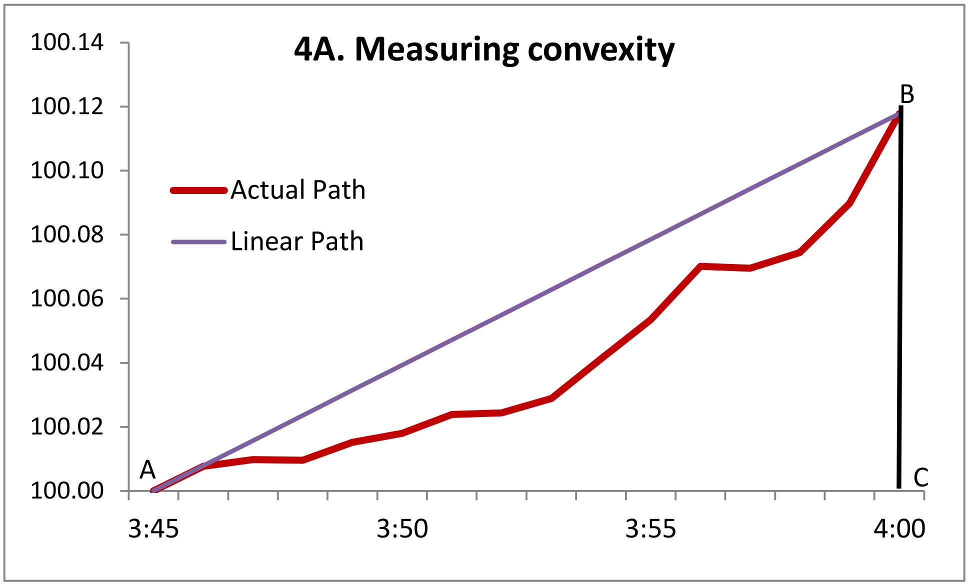


Figure 4. Testing for a change in convexity

4A. Measuring convexity. We calculate (1) the area of the triangle ABC with (2) the area lying between segment AB and the actual average price path. Specifically we measure convexity as the ratio (2)/(1). The actual path shown below is the average end-of-month path for GBP-USD from January, 2008 through December, 2013.



4B. Convexity: Chart shows convexity of the average month-end price path over 3:45-4:00. Data for each observation span the beginning of the sample through the end of December in the year specified. One-minute returns calculated from tick-by-tick data from Reuters Dealing beginning January, 1996, through May, 2013 except EUR which begins January, 1999, and CHF which ends October, 2011.

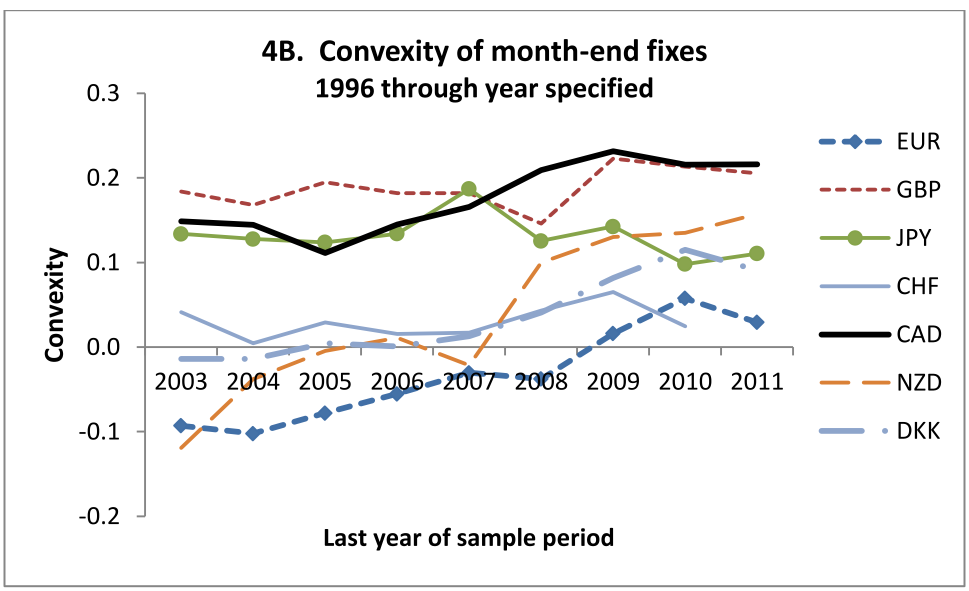


Table 1. Convexity of pre-fix price path

Table shows convexity of average pre-fix price path for end-month days from January,2008 through December, 2013 except for CHF, where the sample ends in October, 2011. Marginal significance is based on the bootstrapped distribution of convexity for the average pre-fix path in the earlier part of the sample period. Sample begins February 1996 for all currency pairs but EUR, which begins January, 1999. Data are tick-by-tick interdealer dealer OTC quotes from the Reuters Dealing platform for exchange rates vis-à-vis USD. ** indicates significance at the 5% level.

	Convexity, 1996-2007	**Convexity, 2008-2013**	**Marginal significance**
EUR	0.0226	0.1875**	0.017
GBP	-0.1810	0.3063**	0.032
JPY	-0.1527	0.2370	0.138
CHF	-0.0233	0.1282**	0.039
CAD	0.1658	0.3715*	0.066
NZD	-0.0214	0.3482**	0.037
DKK	0.0128	0.2864**	0.012