Variance swap payoffs, risk premia and extreme market conditions

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Abstract

This paper estimates the Variance Risk Premium (VRP) directly from synthetic variance swap payoffs. Since variance swap payoffs are highly volatile, we extract the VRP by using signal extraction techniques based on a state-space representation of our model in combination with a simple economic constraint. Our approach, only requiring option implied volatilities and daily returns for the underlying, provides measurement error free estimates of the part of the VRP related to normal market conditions, and allows constructing variables indicating agents' expectations under extreme market conditions. The latter variables and the VRP generate different return predictability on the major US indices. A factor model is proposed to extract a market VRP which turns out to be priced when considering Fama and French portfolios.

Keywords: Variance risk premium; Variance swaps; Return predictability; Factor Model, Kalman filter, CAPM.
JEL Classification: C12, C22, G12, G13

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1 Introduction

Financial markets trade several products with pure exposure to the volatility of a given underlying asset. In particular, variance swaps, i.e. contracts in which one party pays a fixed amount at a given maturity in exchange for a payment equal to the sum of squared daily returns of the underlying asset occurring until that maturity, have become increasingly popular to trade variance.[1] The prices and payoffs of variance swaps contain useful information on the variance risk premium (VRP), which is defined as the difference between the risk neutral and physical expectations of an asset's total return variation. The empirical features of the VRP are used for validation and development of new asset pricing models, and seem to generate market return predictability.[2] Though clear conceptually, the estimation of the VRP requires multiple sources of data as well as assumptions on the latent volatility processes, rendering its dynamic properties difficult to pinpoint.

This paper estimates the VRP directly from synthetic variance swap payoffs. Variance swap payoffs are highly volatile series, with time varying variance levels and extreme payoffs during volatile market conditions. To extract the VRP from the ex-post variance swap payoff realizations, we use signal extraction techniques based on a state-space representation of the model and the Kalman-Hamilton filter. Moreover, since we know from basic financial theory that the VRP is positive, i.e. risk adverse agents dislike the fact that variance is stochastic, we impose this economic constraint when estimating the model. This approach allows us to obtain measurement error free estimates of the VRP.

[1] Other popular instruments when the underlying is the S&P500 index are futures and options on the VIX, iPath S&P500 VIX Short-Term Futures ETNs being an important example. See Bardgett, Gourier, and Leippold (2015) for the use of this type of data for estimating an affine asset pricing model.

[2] Ait-Sahalia, Karaman, and Mancini (2015) estimate no-arbitrage term structure models for the VRP using proprietary variance swap data and show that the expectation hypothesis does not hold. Dew-Becker, Giglio, Le, and Rodriguez (2017) show that the empirical features of proprietary and synthetic variance swap data are difficult to reconcile with existing structural models for the VRP.

We are also able to separate the part of the premium related to normal market conditions, and construct variables indicating agents' expectations under extreme market conditions. Our framework only requires data on option implied volatility, e.g. the VIX index for the S&P500, and daily returns for the underlying, the sources of which are free and readily available for many assets. Note also that the model can be easily used to extract a market variance risk premium from multiple index specific variance swap payoffs.

When options are available for an asset, Carr and Wu (2009) show that the fixed leg of the variance swap can be replicated using out-of-the money options and that the fair variance swap strike equals the option implied variance expectations. Dew-Becker, Giglio, Le, and Rodriguez (2017) studying the quality of proprietary and synthetic variance swaps demonstrate that the two are identical at least for short to medium horizons. For the sake of replicability and ease of extension to assets or markets for which traded swaps are not immediately available, we build our dataset of variance swap payoffs by rolling on a weekly basis a synthetic contract with a one month maturity traded at the fair variance swap rate.

Our framework focusses on the VRP by directly modelling the variance swap payoffs, and therefore we can leave the dynamic model for both the option implied and the realized variance unspecified. We do not need to assume any underlying price process, as would be the case in a full parametric approach, e.g. Andersen, Fusari, and Todorov (2015), Ait-Sahalia, Karaman, and Mancini (2015), Bardgett, Gourier, and Leippold (2015), and Gruber, Tebaldi, and Trojani (2015).[3] In order to estimate equity and variance risk premia, the latter approach typically requires a complete specification of the price, volatility and jump processes, along with assumptions about the stochastic discount factor. Furthermore, rich datasets of high frequency returns and option panels are imperative to infer the parameters with advanced estimation and filtering methods.

[3]Bollerslev, Gibson, and Zhou (2011) and Wu (2011) directly model risk premia using implied and realized variance data without requiring assumptions on the price process.

The literature has proposed two alternative ways to approximate the VRP directly. First, the variance swap payoff itself has been used as a proxy for the VRP, as e.g. in Carr and Wu (2009), Fournier and Jacobs (2015) and Fan, Imerman, and Dai (2016). Although this is a model free VRP estimate it does not constitute an ex-ante expectation as a risk premium should be. Second, the VRP has been estimated as the difference from one period to the next between a squared option implied volatility index and an expected realized variance computed with high frequency historical returns and filtered with a particular choice of dynamic model, e.g. see Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011). However, different variance model assumptions can profoundly impact the VRP times series as shown by Bekaert and Hoerova (2014). While both ways to compute VRP estimates are simple to implement, their drawback stands in a resulting VRP time series extremely noisy and violating the positivity constraint too often to be genuine risk premia.

Using a regime switching model, we decompose the variance payoff into the VRP, which is the ex-ante conditional expectation embedded in it, and the ex-post realized shock. We allow state dependent dynamics which account for normal and extreme market conditions. Our model construction relates to the expectation hypothesis regressions in Carr and Wu (2009). While in their case the dynamics of the variance risk premium reduces to an affine transformation of the risk neutral variance expectations, we allow for idiosyncratic dynamics and stochastic behaviour for the VRP. Combined with an economic constraint on the premium, see Campbell and Thompson (2008) and Pettenuzzo, Timmermann, and Valkanov (2014) for analogous positivity constraints on the equity premium forecasts, our approach allows to precisely estimate the VRP associated with normal market activity and generates positive and smooth VRPs. The difference between the variance swap payoff and the estimated VRP gives, together with the identification of unusual and extreme episodes of market conditions, rise to two variables related to fear and surprise.

Separating the smooth part of the VRP from the part related to extreme market conditions is first done by Bollerslev and Todorov (2011). They decompose the VRP in a component that reflects compensation for continuous price moves and another component that is related to compensation for disaster risk. Extending the work of Bollerslev, Tauchen, and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014) on aggregate return predictability, Bollerslev, Todorov, and Xu (2015) provide evidence that essentially the second extreme component contributes to explain future return variation. The reported results are for the S&P500 index only and require a large panel of liquidly traded options to estimate nonparametrically jump tails and intraday high frequency future prices to obtain realized variation measures and the VRP. Our method, on the other hand, only requires option implied volatilities and daily returns for the underlying.

In our empirical application we consider four major US indices, the S&P500, DJIA, NASDAQ and RUSSELL, for which the necessary data is readily available. For all indices, our proposed model provides a good fit to the data, clearly identifies regimes with low and high volatility accounting for heteroskedasticity and correctly identifies the rare and short-lived extreme market events. The filtered smooth part of the VRP from our model is slowly moving, above its mean in volatile periods and below its mean in periods of calm financial markets, and has a high degree of persistence for all indices. Moreover, for all indices the estimated VRPs move closely together and the episodes of fear and surprise largely overlap in terms of occurrences and duration, and they correspond to and clearly align with major events such as the global financial crisis and the US debt downgrade.

In addition to providing reasonable VRPs and to identifying periods of market turmoil the outcome of our proposed methodology has important implications for market return predictability and for asset pricing in general. In particular, though the VRP significantly predicts future market returns at shorter horizons, across all four indices sizeable increases in predictability are found when the agent's reactions to extreme events, the fear and surprise indicators, are included in the predictive regressions. Though predictability is improved for all horizons, the largest improvements are found at longer horizons of up to one year, horizons for which return predictability is always a challenge. Finally, we use all four series to filter out a common factor which we interpret as a market variance risk premium (MVRP). The MVRP shares the desirable properties that the individual VRPs have and allows identifying common extreme events. When compared to other well-known asset pricing factors, the MVRP is significantly correlated only with the market factor and is priced when considering the returns on most of the five Fama and French (2015) portfolio sorts.

The rest of the paper is organized as follows. Section 2 states basic definitions and describes the data. Section 3 details the model. Section 4 provides estimation results for four US indices. Section 5 documents predictive return regressions. Section 6 estimates a joint model for retrieving the market VRP. Section 7 concludes.

2 Definitions and data

In this section, we first define the variance risk premium (VRP) from realized variances and explain how it relates to variance swap payoffs. Second, we describe the data to be used in this paper and present results from a preliminary analysis of the realized variances and variance swap payoffs.

2.1 Realized variance, VRP and variance swap payoffs

The realized variance between t and $t+\tau$ of a financial asset is computed as

$$RV_{t,t+\tau} = \sum_{i=1}^{I} \left(\ln(S_{t+i}/S_{t+i-1}) \right)^2, \tag{1}$$

with S_t the price level at time t, and I the number of observations between t and $t+\tau$. This quantity represents the sum of the spot variance and the sum of squared price discontinuities, see Andersen, Bollerslev, Diebold, and Ebens (2001) and Andersen, Bollerslev, Diebold, and Labys (2001). The VRP at time t for a given maturity τ is defined as

$$\Pi_{t,t+\tau} = E_t^Q[RV_{t,t+\tau}] - E_t^P[RV_{t,t+\tau}], \tag{2}$$

where the conditional expectations of the realized variance in (1) are under the risk neutral (Q) and physical (P) measures respectively, see Drechsler and Yaron (2011). Although the VRP is often defined as the negative of $\Pi_{t,t+\tau}$, see for example Carr and Wu (2009), for ease of exposition, we follow the notation used in Bollerslev, Tauchen, and Zhou (2009) so that $\Pi_{t,t+\tau}$ as defined in (2) is positive for all t.

The VRP defined in (2) is proportional to the price that risk averse agents are willing to pay to hedge against future variance fluctuations. As such, it represents the expected loss to the short side of an artificial variance swap with fixed leg $E_t^Q[RV_{t,t+\tau}]$ and floating leg $RV_{t,t+\tau}$, entered into at time t and held until maturity at time $t+\tau$, see Ait-Sahalia, Karaman, and Mancini (2015) for a brief description of variance swaps. The contract generates a payoff proportional to the difference

$$\begin{aligned} \mathrm{P}_{t,t+\tau} &= E_t^Q[RV_{t,t+\tau}] - RV_{t,t+\tau} \\ &= \Pi_{t,t+\tau} + (E_t^P[RV_{t,t+\tau}] - RV_{t,t+\tau}), \end{aligned} \tag{3}$$

where $E_t^P[RV_{t,t+\tau}] - RV_{t,t+\tau}$ is the prediction error under the physical measure.[4]

The term $E_t^Q[RV_{t,t+\tau}]$ is the fair strike of the artificial variance swap, see Bakshi and Madan (2000), Britten-Jones and Neuberger (2000) and Jiang and Tian (2005). As the variance swap can be replicated using a portfolio of European call and put options with

[4]The proportionality is with respect to a variance notional which without loss of generality we normalise to one.

weights inversely proportional to the square of the options' strike prices, the fair variance swap strike can be written as

$$\frac{2}{B_{t,\tau}}\left(\int_0^{\mathcal{F}_t}\frac{P_{t,\tau}(K)}{K^2}dK+\int_{\mathcal{F}_t}^{\infty}\frac{C_{t,\tau}(K)}{K^2}dK\right)+\mathcal{O}\left(\left(\frac{d\mathcal{F}_t}{\mathcal{F}_{t-}}\right)^3\right), \tag{4}$$

where $B_{t,\tau}$ the price of a time t zero-coupon bond maturing at time $t+\tau$, $\mathcal{F}_t$ the forward price, and $P_{t,\tau}(K)$ and $C_{t,\tau}(K)$, respectively, the prices of put and call options with strike price K. The first term in (4) represents the risk neutral variance expectations in absence of price discontinuities. The second term is the compensator of the discontinuous component, see Carr and Wu (2009) for details. A well known discretization of (4), which computes $E_t^Q[RV_{t,t+\tau}]$ based on quoted option prices, is

$$2e^{r_\tau}\left[\sum_{K_i\leq K_0}\frac{\Delta K_i}{K_i^2}P_{t,\tau}(K_i)+\sum_{K_i\geq K_0}\frac{\Delta K_i}{K_i^2}C_{t,\tau}(K_i)\right]-\left[\frac{\mathcal{F}_t}{K_0}-1\right]^2, \tag{5}$$

where r_τ is the risk free rate and K_0 is the strike price immediately below the forward price, see CBOE (2015). This formula encompasses the family of volatility indices trademarked by CBOE in 1993, which includes the VIX (S&P500), VXD (Dow Jones Industrial Average), VXN (NASDAQ) and RVX (RUSSELL 2000), among many others.

Thus, the payoff generated by a variance swap contract entered at t and held to maturity $t+\tau$ is computed, for instance for the S&P500 index, as

$$\mathrm{P}_{t,t+\tau}=VIX_{t,t+\tau}^2-RV_{t,t+\tau}, \tag{6}$$

where the fixed leg $VIX_{t,t+\tau}^2$ represents the option based estimate of the risk neutral expectation of the variance over the swap horizon calculated at inception according to (5) and the floating leg $RV_{t,t+\tau}$ is realized variance calculated at maturity as defined in (1).

2.2 Data

We consider four US stock market indices: S&P500, Dow Jones Industrial Average (DJIA), NASDAQ and RUSSELL 2000 (RUSSELL). For each market, we compute variance swap payoffs for monthly maturities using realized variance computed from daily squared returns, and we obtain the risk neutral variance expectations in (5) which we denote respectively by $VIX^2_{t,t+\tau}$, $VXD^2_{t,t+\tau}$, $VXN^2_{t,t+\tau}$ and $RVX^2_{t,t+\tau}$ from CBOE. In this paper, we consider τ to be equal to the one month horizon. Following Gruber, Tebaldi, and Trojani (2015) and Andersen, Fusari, and Todorov (2016), the data is sampled weekly every Wednesday and starts according to availability on February 1, 1990, for S&P500, November 26, 1997, for DJIA, August 26, 2003, for NASDAQ, and February 3, 2004, for RUSSELL. The sample ends on July 29, 2016, for all indices. A weekly frequency allows for new information to update the measures, especially the RV, and avoids local over-smoothing due to the rolling of the variance swap contract over overlapping 30-day windows.

Table 1 reports descriptive statistics. We find that on average the risk neutral variance expectation is higher than the realized variance implying a positive average variance swap payoff, the latter ranging between 8.33 for the NASDAQ and 10.73 for the S&P500, respectively. All time series are highly volatile given their high standard deviations, in particular the payoff series have standard deviations which are at least three times larger than their respective averages. In fact, variance swap payoffs have a range of at least 600, an example being the RUSSELL with a minimum of -432.66 and a maximum 216.58. The dynamics in all time series is characterised by a high persistence as measured by the estimated first order autocorrelation coefficient. In particular, the highest autocorrelation can be found in the realized variance series, equal to about 0.96 for all indices, which is expected given the overlapping nature of the data.

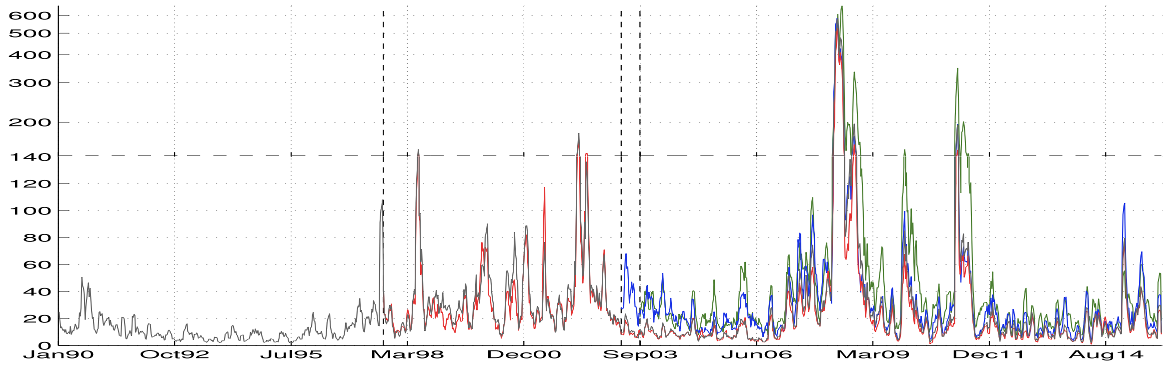
Figure 1 displays realized variances, risk neutral variance expectations, and variance swap payoffs respectively over the available time span for each market. From February 2004, when data for all indices is available, we see that the time series share the same patterns. The realized variances have upward peaks around the same dates for all indices. The risk neutral variance expectation series also have large positive jumps around the same

Table 1: Properties of the risk-neutral, physical variances and unconditional VRP

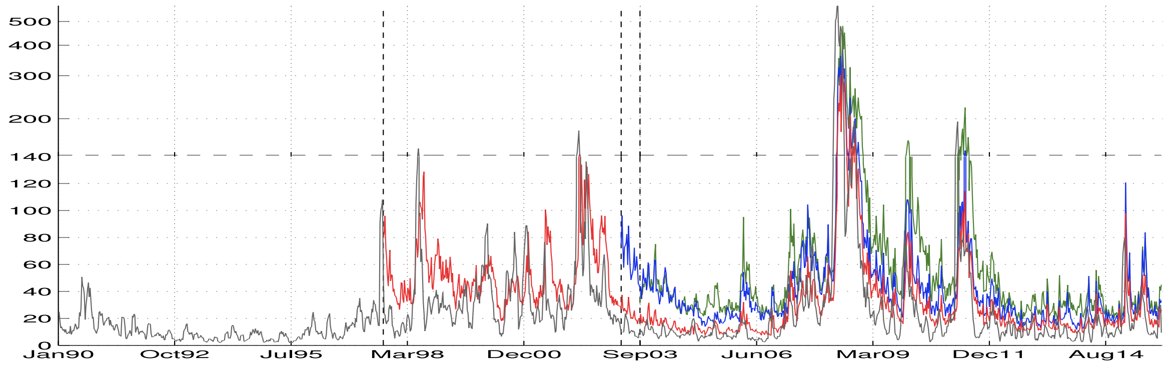
Index	Variable	Mean	Stddev	Minimum	Maximum	AR(1)	N
S&P500	$\text{VIX}^2_{t,t+\tau}$	37.184	35.781	7.926	376.04	0.919	1354
	$RV_{t-\tau,t}$	26.452	47.928	1.850	579.09	0.959	1354
	$\text{P}_{t,t+\tau}$	10.733	34.465	-450.81	212.2	0.852	1354
DJIA	$\text{VXD}^2_{t,t+\tau}$	37.823	35.04	7.418	324.77	0.929	956
	$RV_{t-\tau,t}$	28.264	47.309	0.986	526.15	0.955	956
	$\text{P}_{t,t+\tau}$	9.559	34.722	-413.31	194.77	0.863	956
NASDAQ	$\text{VXN}^2_{t,t+\tau}$	45.536	44.127	11.796	383.53	0.932	664
	$RV_{t-\tau,t}$	37.207	61.084	3.759	582.02	0.960	664
	$\text{P}_{t,t+\tau}$	8.3286	43.851	-466.71	190.16	0.859	664
RUSSELL	$\text{RVX}^2_{t,t+\tau}$	59.703	58.773	17.52	478.62	0.935	641
	$RV_{t-\tau,t}$	51.226	81.697	5.9827	661.34	0.960	641
	$\text{P}_{t,t+\tau}$	8.4777	52.065	-432.66	216.58	0.822	641

Notes: This table reports descriptive statistics for implied variance, e.g. $\text{VIX}^2_{t,t+\tau}$, realized variance, $RV_{t-\tau,t}$, and variance swap payoff $\text{P}_{t,t+\tau}$ with τ equal to one month. Stddev means standard deviation, AR(1) is the sample autocorrelation of order one and N denotes the number of observations. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSS. The sample ends on July 29, 2016 for all indices.

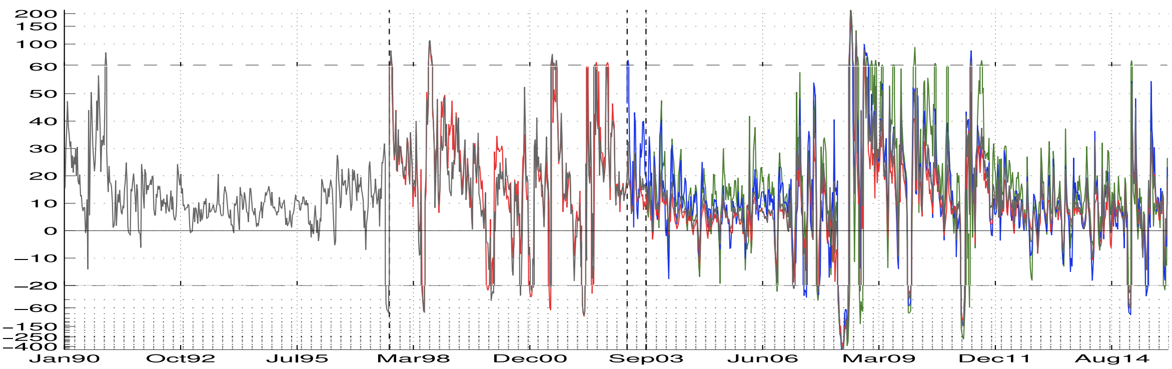
periods of the realized variance but at different dates as can be seen from the payoff series in Figure 1. Interestingly, the payoff series have peaks up and down with similar amplitude noting that the left tail events of an absolute size larger than the observed maximum occur with frequency lower than one percent. These tail episodes coincide mainly with events related to the peak of the global financial crisis. To stress the weight of these extreme negative points on the overall sample, we re-compute the mean and standard deviation of the variance swap-payoffs excluding the months of October and November 2008. We obtain respectively (standard deviation between brackets), S&P500 12.52 (23.01), DJIA 11.08 (25.57), NASDAQ 11.56 (26.18), RUSSELL 13.37 (21.31), i.e. a sensible increase in the average coupled with a striking reduction of the standard deviation.



(a) $RV_{t,t+\tau}$



(b) $VIX^2_{t,t+\tau}$



(c) $\mathrm{P}_{t,t+\tau}$

Figure 1: $RV_{t,t+\tau}$, $VIX^2_{t,t+\tau}$ and $\mathrm{P}_{t,t+\tau}$. S&P500 (grey), DJIA (red), NASDAQ (blue) and RUSSELL (green). The vertical dashed lines mark the beginning of the sample of each market.

A natural question to ask is how the volatility and the variance swap payoff variables are related to future market performance. It is well known that while volatility does not deliver return predictability, the impact of the VRP on future returns is found to be significant at medium term horizons (about four months). See for example Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) who demonstrate empirically and theoretically that, in addition to consumption risk, volatility risk plays an important role in generating returns at medium term horizons.

To illustrate this return predictability, we implement regressions using as explanatory variable $X_{t,t+\tau}$ equal to either $RV_{t,t+\tau}$ or $\mathrm{P}_{t,t+\tau}$ (as a proxy for the VRP), for respectively the four indices. Predictability is measured by the adjusted R^2 from regressions of the following type

$$\frac{1}{h}\sum_{j=1}^{h} r_{t+\tau+j} = a_0(h) + a_1(h)X_{t,t+\tau} + u_{t+h,t}, \tag{7}$$

where h denotes the horizon, r_t denotes the excess return for week t. Using Datastream, we construct the weekly aggregated market returns in excess of the three-month T-bill rate over horizons from one week (h=1) up to one year (h=52).

Table 2 provides coefficient estimates as well as adjusted R^2 for the considered horizons. In terms of return predictability generated by volatility, as measured by the realized variance $RV_{t,t+\tau}$, the S&P500 shows no significant $a_1(h)$ coefficients at any horizons, with corresponding R^2s below one percent. Also DJIA, NASDAQ and RUSSELL have no significant slope coefficients up to the nine month horizon. However, as the respective data span shortens for these indices, we find positive significant slope coefficients associated with R^2s increasing with the horizon, e.g. up to five percent for RUSSELL at the one year horizon.

In terms of return predictability generated by the variance swap payoffs, the S&P500 shows significant positive $a_1(h)$ coefficients between the one month and six month horizons, with an inverse U-shape R^2s as a function of the horizon, up to 3.19 percent at four

Table 2: Predictive return regressions

Horizon	1	4	8	12	16	20	24	36	52
					S&P500				
$RV_{t,t+\tau}$	-0.145	-0.131	-0.079	-0.111	-0.104	-0.070	-0.015	0.028	0.044
	(0.233)	(0.145)	(0.089)	(0.081)	(0.075)	(0.078)	(0.062)	(0.042)	(0.028)
R^2	0.02	0.27	0.18	0.68	0.81	0.41	-0.05	0.05	0.36
$P_{t,t+\tau}$	0.306	0.361	0.202	**0.265**	**0.278**	**0.260**	**0.176**	0.069	0.016
	(0.342)	(0.198)	(0.142)	(0.077)	(0.060)	(0.069)	(0.064)	(0.046)	(0.046)
R^2	0.14	1.28	0.78	2.16	3.19	3.41	1.77	0.33	-0.05
					DJIA				
$RV_{t,t+\tau}$	-0.030	-0.037	-0.030	-0.106	-0.129	-0.091	-0.029	0.046	**0.085**
	(0.229)	(0.137)	(0.100)	(0.096)	(0.081)	(0.078)	(0.062)	(0.049)	(0.030)
R^2	-0.10	-0.08	-0.07	0.57	1.30	0.75	-0.01	0.26	1.72
$P_{t,t+\tau}$	0.119	0.172	0.092	**0.216**	**0.288**	**0.284**	**0.215**	**0.100**	0.026
	(0.338)	(0.204)	(0.158)	(0.092)	(0.076)	(0.073)	(0.070)	(0.050)	(0.052)
R^2	-0.07	0.18	0.07	1.41	3.65	4.40	2.88	0.84	-0.03
					NASDAQ				
$RV_{t,t+\tau}$	-0.267	-0.165	-0.034	-0.045	-0.016	0.020	0.077	**0.112**	**0.118**
	(0.243)	(0.175)	(0.107)	(0.075)	(0.073)	(0.085)	(0.067)	(0.049)	(0.032)
R^2	0.25	0.48	-0.10	-0.02	-0.14	-0.12	0.60	2.45	4.18
$P_{t,t+\tau}$	0.512	**0.480**	0.198	**0.247**	**0.239**	**0.225**	0.137	0.022	-0.028
	(0.323)	(0.153)	(0.121)	(0.075)	(0.074)	(0.068)	(0.081)	(0.068)	(0.062)
R^2	0.62	2.59	0.74	1.88	2.41	2.67	1.09	-0.12	-0.05
					RUSSELL				
$RV_{t,t+\tau}$	-0.096	-0.109	-0.026	-0.061	-0.024	0.034	0.087	**0.102**	**0.103**
	(0.249)	(0.184)	(0.103)	(0.098)	(0.092)	(0.093)	(0.068)	(0.040)	(0.028)
R^2	-0.09	0.22	-0.12	0.21	-0.08	0.03	1.29	3.18	5.01
$P_{t,t+\tau}$	0.143	0.396	0.152	**0.268**	**0.261**	0.196	0.088	0.022	-0.031
	(0.439)	(0.291)	(0.171)	(0.088)	(0.077)	(0.110)	(0.087)	(0.052)	(0.036)
R^2	-0.10	1.87	0.44	2.72	3.67	2.47	0.45	-0.11	0.01

Notes: This table reports estimated slope coefficients and adjusted R^2's in percentages for predictive return regressions as defined in Equation (7). Coefficients significant at five percent are put in boldface. Newey-West standard errors are in brackets. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

months (h=16). The DJIA has a similar pattern with R^2s up to 4.40 percent at the five month horizon. The shorter series NASDAQ and RUSSELL reveal the same relationship of positive significant coefficients at medium term horizons, and with insignificant coefficients at the very short end and at the longer horizons.

3 Model

We present next the model that we use to extract, from variance swap payoffs, the VRP as well as variables related to extreme market conditions. For ease of exposition, the notation refers to the S&P500 market but this extends easily to the other indices considered here and, more generally, to any other type of asset.

3.1 Swap payoff and VRP

As explained in Section 2, the payoff generated at $t+\tau$ by a variance swap contract entered at t is

$$\mathrm{P}_{t,t+\tau} = VIX^2_{t,t+\tau} - RV_{t,t+\tau}. \tag{8}$$

The variance swap payoff contains relevant information about the latent ex-ante variance risk premium $\Pi_{t,t+\tau}$ with differences identifiable as the prediction error in computing variance expectations under the physical measure. In fact, we can rewrite the preceding equation as

$$\begin{aligned} \mathrm{P}_{t,t+\tau} &= VIX^2_{t,t+\tau} - RV_{t,t+\tau} \\ &= \mathrm{E}^Q_t[RV_{t,t+\tau}] - \mathrm{E}^P_t[RV_{t,t+\tau}] + (\mathrm{E}^P_t[RV_{t,t+\tau}] - RV_{t,t+\tau}) \\ &= \Pi_{t,t+\tau} + e_{t+\tau}, \end{aligned} \tag{9}$$

where $e_{t+\tau} = \mathrm{E}^P_t[RV_{t,t+\tau}] - RV_{t,t+\tau}$ is a zero mean innovation term. Unbiasedness of the variance expectations imply that the unconditional expectation of $\mathrm{P}_{t,t+\tau}$ is equal to the unconditional expectation of $\Pi_{t,t+\tau}$, which we denote $\overline{\Pi}$, the latter quantity being extensively documented by Carr and Wu (2009).

Our interest lies in the dynamics of the variance risk premium which we assume to be an autoregressive stochastic process

$$\Pi_{t,t+\tau} = \overline{\Pi} + \phi(\Pi_{t-1,t+\tau-1} - \overline{\Pi}) + \epsilon_{t+\tau}, \tag{10}$$

with $\epsilon_{t+\tau}$ a zero mean innovation term. Despite the fact that the ex-post payoff, $\mathrm{P}_{t,t+\tau}$, is negative for some t, as given by financial theory we require $\Pi_{t,t+\tau}$ to be positive for all t. This type of constraint is analogous to Campbell and Thompson (2008) and Pettenuzzo, Timmermann, and Valkanov (2014) who impose positivity on their equity premium forecasts. The main idea is that in periods where data are very noisy it becomes too hard to extract the underlying quantity of interest. Imposing economic constraints alleviates this identification problem.

An advantage of working directly with variance swap payoffs, as e.g. in Fournier and Jacobs (2015) and Fan, Imerman, and Dai (2016), is that we can leave the model for $\mathrm{E}_t^P[RV_{t,t+\tau}]$ unspecified. In fact, a rough way to compute the VRP, according to its definition, is subtracting risk netural variance expectation from model implied physical variance expectations on a period by period basis. Modelling the dynamics of $RV_{t,t+\tau}$ and its impact on the VRP has been studied extensively in Bekaert and Hoerova (2014). They show that different model assumptions can profoundly impact the VRP times series.

3.2 Fear and surprise

The variance swap is a financial instrument designed to hedge against sudden variance fluctuations. As shown in Figure 1, its payoff, while stable and slow moving in periods of calm markets, exhibits large and short lasting positive/negative peaks when extreme variance events occur over the life span of the contract. Hence, the variance swap payoff reflects the extent of fear or surprise generated by an extreme shock. When a sudden and abnormal price and/or variance shock occurs over the period from t to $t+\tau$, the ex-post realized variance can be decomposed as

$$RV_{t,t+\tau} = CV_{t,t+\tau} + JV_{t,t+\tau}, \tag{11}$$

where $CV_{t,t+\tau}$ represents the smooth, or continuous, component of the variance while $JV_{t,t+\tau}$ represents the realized jump, or extreme component, of it. We argue that short lasting expected or unexpected sudden extreme variance events can heavily distort the estimation of the VRP if not adequately identified and measured, giving raise to some contradicting results found in the literature, see e.g. Bekaert and Hoerova (2014) for a discussion. Also, the detection of extreme variance events and their measurement can help understanding heterogeneity of pricing of risk in different states of the market. For example, Bollerslev, Todorov, and Xu (2015) show that much of the return predictability previously ascribed in the literature to the variance risk premium is effectively coming from the part of the VRP related to how agents gauge extreme variance events.

Using similar arguments to obtain (11), depending on agents' reaction to observed shocks and perception of future states of the market, the risk neutral variance expectations, inferred from option prices, can be decomposed into the sum of a smooth part and a jump part as $\mathrm{E}_t^Q[CV_{t,t+\tau}] + \mathrm{E}_t^Q[JV_{t,t+\tau}]$. Redefining $\Pi_{t,t+\tau} = \mathrm{E}_t^Q[CV_{t,t+\tau}] - \mathrm{E}_t^P[CV_{t,t+\tau}]$ as the premium associated with the risk of variance fluctuations under normal market activity, the variance swap payoff can be expressed as

$$\begin{aligned}
\mathrm{P}_{t,t+\tau} &= VIX^2_{t,t+\tau} - RV_{t,t+\tau} \\
&= \mathrm{E}_t^Q[CV_{t,t+\tau}] + \mathrm{E}_t^Q[JV_{t,t+\tau}] - (CV_{t,t+\tau} + JV_{t,t+\tau}) \\
&= \Pi_{t,t+\tau} + (\mathrm{E}_t^Q[JV_{t,t+\tau}] - JV_{t,t+\tau}) + (\mathrm{E}_t^P[CV_{t,t+\tau}] - CV_{t,t+\tau}) \\
&= \Pi_{t,t+\tau} + \mathrm{FS}_{t,t+\tau} + e_{t+\tau},
\end{aligned} \tag{12}$$

where the term $\text{FS}_{t,t+\tau} = \text{E}_t^Q[JV_{t,t+\tau}] - JV_{t,t+\tau}$ represents the extent of fear (> 0) or surprise (< 0) that a realized extreme variance event generates.

The first case, i.e. fear, refers to the situation where agents expect an extreme variance event to occur. This can be caused by sudden rumors, sharply increasing market instability or as a reaction to a large unanticipated shock. This scenario, which reflects overly conservative expectations with respect to the size of the realized shock, relates to the fear effect elaborated in Bollerslev and Todorov (2011). The second case, which we call surprise, refers to an unexpected or underestimated price or variance extreme shock which hits the market in the period between t and $t + \tau$. In this sense, $\text{FS}_{t,t+\tau}$ represents the compensation (< 0) or the extra cost (> 0) with respect to the 'normal' price of hedging against variance fluctuations for the long side of the variance swap contract. In other words, the term $\text{FS}_{t,t+\tau}$ provides an overall measure of the degree of conservativeness of the expectations and fear of incoming large shocks, or otherwise of the unanticipated portion of a realized shock and thus the extent of the surprise it generates.

To disentangle the total VRP into a smooth part (premium associated to normal market conditions) and jump part (premium associated to extreme market events), the term $\text{FS}_{t,t+\tau}$ can be further decomposed as

$$\text{FS}_{t,t+\tau} \quad = \quad (\text{E}_t^Q[JV_{t,t+\tau}] - \text{E}_t^P[JV_{t,t+\tau}]) + \varepsilon_{t+\tau},$$

where $\text{E}_t^Q[JV_{t,t+\tau}] - \text{E}_t^P[JV_{t,t+\tau}]$ represents the jump risk premium and $\varepsilon_{t+\tau} = (\text{E}_t^P[JV_{t,t+\tau}] - JV_{t,t+\tau})$ represents the jump prediction error under the physical dynamics. To exploit this level of decomposition in practice, we face limitations due to the heterogeneity, rarity and sparsity of the extreme variance events making it difficult to identify the dynamic properties of the jump risk premium. Furthermore, the relatively short data span in our empirical applications renders also difficult to preserve the zero lower bound and to assess whether the jump component is effectively predictable without systematic bias under the physical measure, i.e. $\mathrm{E}[\varepsilon_{t+\tau}] = 0$. For these reasons, we opt for a simple approach which assumes $\mathrm{FS}_{t,t+\tau}$ to be a constant (μ) plus a zero mean noise process ($\eta_{t+\tau}$) with occurrence driven by the realization of a two-state Markov chain $s_{t+\tau}$ with $\Pr(s_{t+\tau} = n | s_{t+\tau-1} = j) = p_{nj}$, n and j denoting normal market state and jump state respectively.

Finally, it is well known that variance swap payoffs are themselves subject to different volatility states. To account for this heteroskedasticity, we separate the 'normal' market state in a low and a high volatility state. This is achieved by extending the Markov chain to three-states $s_{t+\tau} \in [l \text{ (low)}, h \text{ (high)}, j \text{ (jump)}]$ with conformable transition probability matrix, by introducing dependence of the variance of $\epsilon_{t+\tau}$ and $e_{t+\tau}$ from $s_{t+\tau}$. The full model, including heteroskedasticity and discontinuities in the payoff due to expectations/surprises, can be written as

$$\mathrm{P}_{t,t+\tau} = \Pi_{t,t+\tau} + \mathrm{I}_{s_t=j}\mu + \tilde{e}_{t+\tau} \tag{13}$$

$$\Pi_{t,t+\tau} = \overline{\Pi} + \phi(\Pi_{t-1,t+\tau-1} - \overline{\Pi}) + \epsilon_{t+\tau}, \tag{14}$$

where $\mathrm{I}_{s_t=j}$ is an indicator function which takes value one if $s_t = j$ and zero otherwise. In the measurement equation (13) $\tilde{e}_{t+\tau} = e_{t+\tau} + \mathrm{I}_{s_t=j}\eta_{t+\tau}$ is a noise with state dependent variance $\sigma^2_{\tilde{e}} = \mathrm{I}_{s_t=l}\sigma^2_{e,l} + (\mathrm{I}_{s_t=h} + \mathrm{I}_{s_t=j})\sigma^2_{e,h} + \mathrm{I}_{s_t=j}\sigma^2_{\eta}$ and in the state-propagation equation (14) the state dependent variance is $\sigma^2_{\epsilon} = \mathrm{I}_{s_t=l}\sigma^2_{\epsilon,l} + (\mathrm{I}_{s_t=h} + \mathrm{I}_{s_t=j})\sigma^2_{\epsilon,h}$, under the identifying restriction $\sigma^2_{e,h} > \sigma^2_{e,l}$ and $\sigma^2_{\epsilon,h} > \sigma^2_{\epsilon,l}$, respectively. Note that the variance of $\mathrm{FS}_{t,t+\tau}$ is identified as a marginal increase from the high volatility regime.

4 Estimation results

The 3-regime Markov switching model described by (13) - (14) is formulated as a linear state-space form. It is estimated by quasi-maximum likelihood using the Kalman-Hamilton filter, see Kim (1994) and Kim and Nelson (1999) for more details.[5] The economic constraint on $\Pi_{t,t+\tau}$ is implemented by bounding the signal to noise ratio such that the filtered state satisfies the positivity constraint point-wise.

Table 3 provides parameter estimates for the four indices. The heterogeneity in the prediction error deriving from heteroskedasticity is clear for all the indices. In the high volatility regime, the estimated standard errors are homogenous among indices and about four times higher than the base regime for the S&P500, DJIA and NASDAQ, and two times higher for the RUSSELL. The large marginal increase in the measurement error variance between the second and third regime, measured by σ_η, together with transition probabilities which imply relatively short expected durations for the third regime, proves that the latter identifies rare and short-lived extreme market events.[6]

More precisely, the implied expected durations for extreme payoff events range between 1.36 (RUSSELL) and 2.86 (DJIA) weeks. The transition probability matrix, however, reveals a persistent low volatility regime, with expected durations of 5.68, 6.74, 3.95 and 6.43 weeks, for the four indices respectively. The high volatility regime is more heterogenous, with expected durations ranging between 2.39 weeks for the S&P500 and 6.06 weeks for the RUSSELL, and acts in most cases as the layer of transition between the low and extreme volatility regimes. In fact, except for the RUSSELL, the probability p_{lj} is virtually zero. This evidence is less striking in the opposite direction with p_{jl} smaller than one percent only for DJIA and RUSSELL.

We estimate the occurrence of the extreme variance events as the observations for which the jump state posterior probability is the highest. Using the notation in Kim and Nelson (1999), the latter probability is computed as $\max\Big(P(s_{t+\tau}|\psi_{t+\tau})\Big) = P(j|\psi_{t+\tau})$,

[5] See also Egloff, Leippold, and Wu (2010) for a similar estimation technique to fit a VRP term structure model.

[6] The parameter μ is not reported in Table 3 as it turns out to be insignificant for all indices. This indicates that fears and surprises tend to compensate on average.

Table 3: Quasi-maximum likelihood estimates of (13) - (14)

Parameter	S&P500	DJIA	NASDAQ	RUSSELL
$\sigma_{e,l}$	2.71	3.56	3.28	7.34
$\sigma_{e,h}$	11.27	12.00	12.14	14.06
σ_{η}	43.27	69.27	52.64	51.41
$\sigma_{\epsilon,l}$	1.66	0.25	1.38	2.31
$\sigma_{\epsilon,h}$	1.87	1.41	1.54	3.03
$\overline{\Pi}$	13.72	8.09	14.03	16.79
ϕ	0.94	0.97	0.97	0.94
p_{ll}	0.82	0.85	0.75	0.84
p_{lh}	0.18	0.15	0.25	[0.01]
p_{hh}	0.58	0.71	0.76	0.84
p_{hj}	0.25	0.13	0.13	0.10
p_{jj}	0.36	0.65	0.31	0.27
p_{jh}	0.29	0.34	0.35	0.73
LLF	-2.8261	-2.9326	-3.1656	-3.2929
N	1354	956	664	641
Steady-state probability				
l	0.55	0.45	0.37	0.24
h	0.32	0.40	0.53	0.62
j	0.13	0.15	0.10	0.14
Expected duration				
l	5.68	6.74	3.95	6.43
h	2.39	3.42	4.20	6.06
j	1.56	2.86	1.44	1.36

Notes: LLF denotes the average loglikelihood and N the number of observations. Steady state probabilities are computed according to (4.49) in Kim and Nelson (1999). Expected duration is computed as $1/(1 - p_{ii})$, $i \in$ [l (low), h (high), j (jump)]. The [] brackets indicate insignificant parameters. The parameter μ is set to zero as it turns out to be insignificant for all indices. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

where $P(s_{t+\tau}|\psi_{t+\tau})$ is the posterior probability of the state $s_{t+\tau} \in [l, h, j]$ and $\psi_{t+\tau}$ is the information set up to $t + \tau$. Table 4 provides details on the major events that generated extreme variance swap payoffs. The longest lasting event is the global financial crisis with an estimated length of about seven months, with average variance swap payoff favouring the long side of the contract for all indices, particularly so for the S&P500 with average gains of 158 times the notational. The global financial crisis thus constitutes an example of a period where the effect of surprises dominate over fears. The collapse in oil prices in August 2015 is an example of relatively short lasting episode of 1.5 months. Regarding this event, three out of the four indices exhibit gains for the long position up to 33.4 times the notational, with the exception of the RUSSELL for which the average payoff favours the short side of the swap contract with average gains of 8.45 times the notational.

Table 3 shows that the latent state $\Pi_{t,t+\tau}$ has an unconditional level in line with Section 2.2 and a high degree of persistence for all indices. The estimated autoregressive coefficient ϕ is substantially higher than the one estimated on the raw payoff data, stressing the downward bias due to the presence of extreme payoff realisations. Figure 2 displays variance swap payoffs (grey), the smooth part of the VRP (red), and the detected extreme variance events (vertical grey lines) for the four indices. As indicated by the parameter estimates in Table 3, $\Pi_{t,t+\tau}$ is slowly moving and above its mean in volatile periods and below its mean in periods of calm financial markets. Over the period for which we have data for all indices, we see from Figure 3 that the respective $\Pi_{t,t+\tau}$ estimates are moving closely together. In fact, the pairwise correlations between the variance premia is the highest between S&P500 and DJIA (90 percent) and the lowest between the NASDAQ and the RUSSELL (76 percent). Figure 3 also shows that episodes of extreme variance events, indicated by the shaded grey areas, largely overlap both in terms of occurrences and duration.

Once endowed with the filtered VRP and the posterior probabilities associated to the states s_t, we infer the occurrence of abnormal variance swap payoffs and determine whether

Table 4: Extreme variance clusters

Event	Market	Start	End	N	$\bar{P}_{t,t+1}$	$\bar{\Pi}_{t,t+1}$
Asian Crisis	S&P500	1997-10-29	1997-12-24	8	0.58	16.29
Russian Crisis	S&P500	1998-08-12	1998-12-02	14	23.35	14.99
	DJIA	1998-09-02	1998-12-09	13	27.86	13.23
09/11	S&P500	2001-09-26	2001-12-05	10	45.50	17.45
	DJIA	2001-09-19	2001-12-12	11	23.08	15.00
Dot-com bubble burst	S&P500	2002-07-24	2002-11-27	13	-8.83	9.57
	DJIA	2002-07-24	2002-12-11	16	5.22	8.49
Global financial crisis	S&P500	2008-09-17	2009-05-27	28	-158.54	10.35
	DJIA	2008-09-17	2009-05-27	30	-40.34	3.59
	NASDAQ	2008-09-24	2009-05-27	27	-49.05	15.08
	RUSSELL	2008-09-17	2009-05-27	30	-80.04	7.26
Flash crash	S&P500	2010-05-12	2010-07-28	8	6.72	15.31
	DJIA	2010-05-12	2010-06-02	4	-28.54	14.83
	NASDAQ	2010-05-12	2010-07-07	6	-16.30	18.92
	RUSSELL	2010-05-05	2010-07-07	8	-12.15	17.73
US debt downgrade	S&P500	2011-08-10	2011-11-02	8	-46.38	9.76
	DJIA	2011-08-10	2011-11-02	7	-43.79	9.10
	NASDAQ	2011-08-10	2011-11-02	11	-19.38	6.68
	RUSSELL	2011-08-10	2011-11-02	8	-118.68	9.70
Collapse in oil prices	S&P500	2015-08-26	2015-09-30	6	-20.46	7.89
	DJIA	2015-08-26	2015-09-30	6	-18.63	7.39
	NASDAQ	2015-08-26	2015-09-30	6	-33.44	6.53
	RUSSELL	2015-08-26	2015-09-23	5	8.45	6.65

Notes: This table reports extreme variance swap payoff clusters over the period January, 1990, to September, 2015. N denotes the number of extreme variance weeks in the period between Start and End. The variables $\bar{P}_{t,t+1}$ and $\bar{\Pi}_{t,t+1}$ are respectively the average payoff and smooth VRP over the extreme variance weeks. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

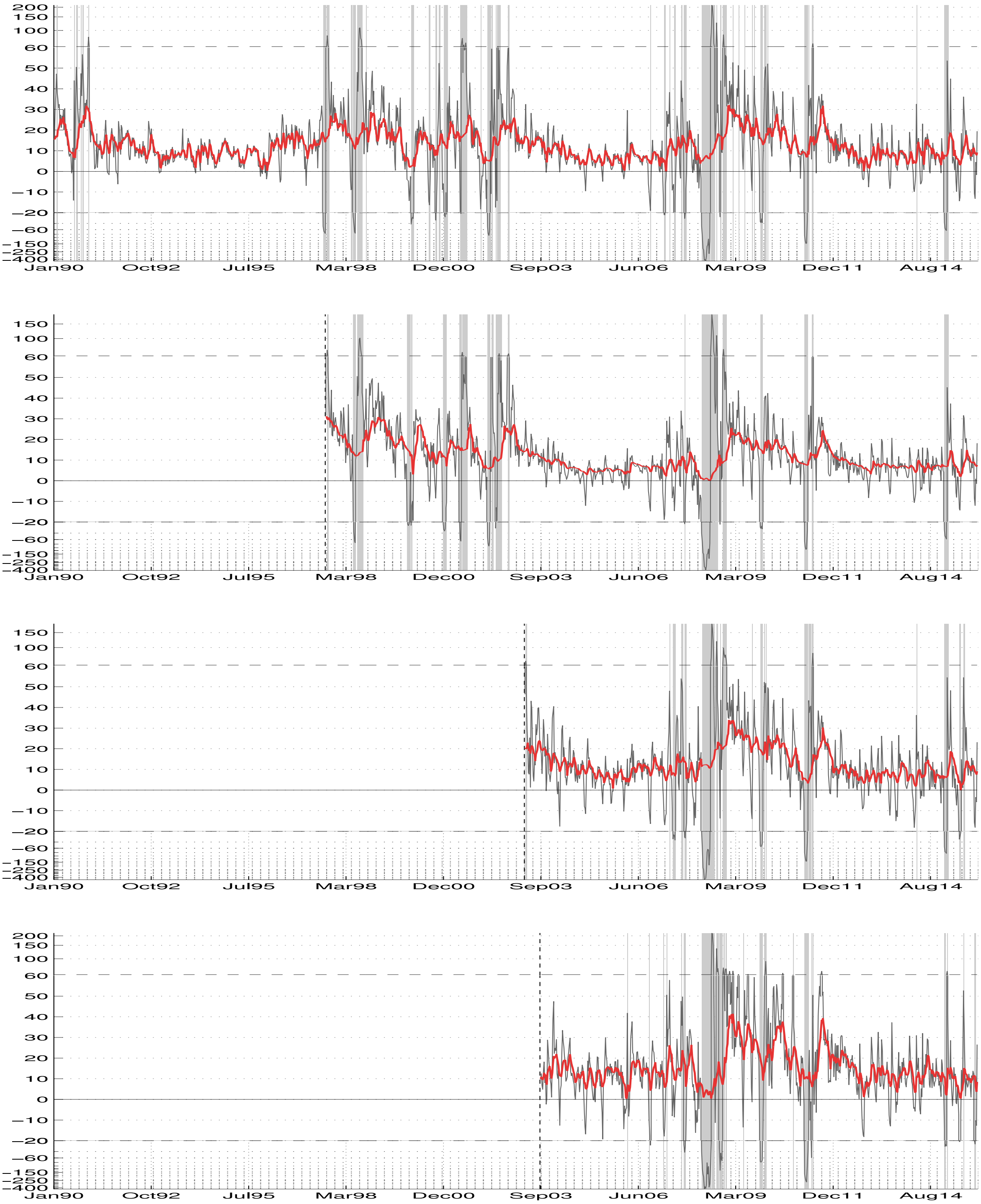


Figure 2: Variance swap payoffs (grey), $P_{t,t+\tau}$, the smooth part of the VRP (red), $\Pi_{t,t+\tau}$, and detected extreme variance events (vertical grey areas for the four indices. From the top we plot S&P500, DJIA, NASDAQ and RUSSELL, respectively.

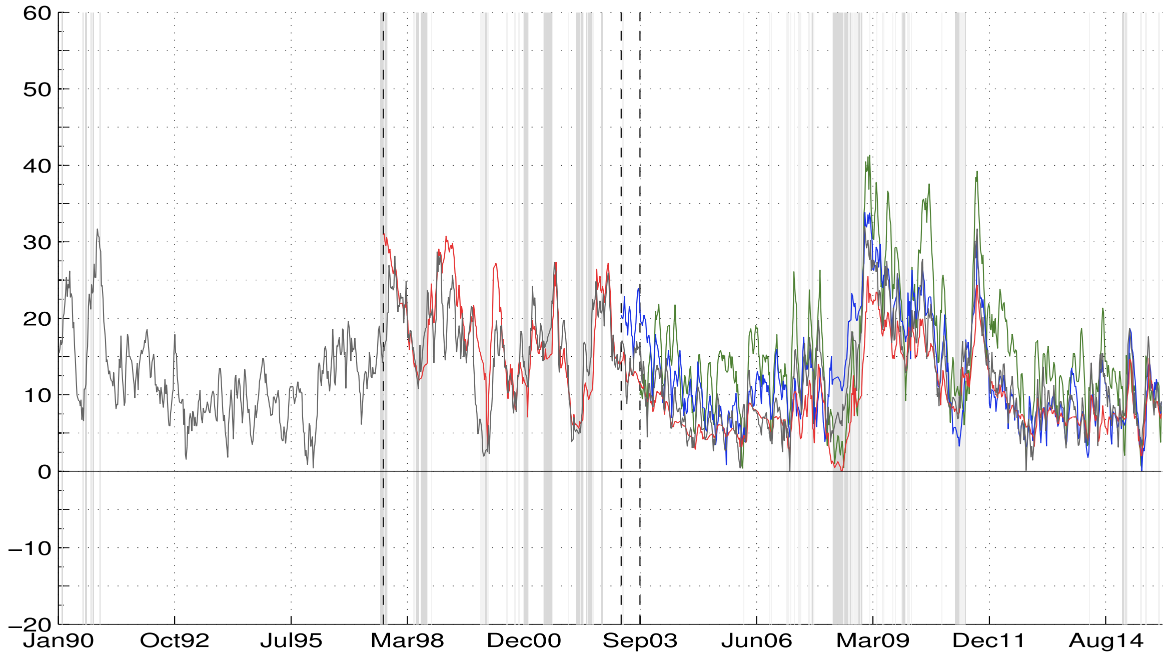


Figure 3: The smooth part of the VRP, $\Pi_{t,t+\tau}$, for all for indices together. S&P500 (grey), DJIA (red), NASDAQ (blue) and RUSSELL (green). The vertical dashed lines mark the beginning of the sample of each market. The dark grey areas indicate extreme variance events detected in the four markets. The light grey areas indicate extreme variance events detected in 3 markets or less.

and to what extent they are caused by fears or surprises. Figure 4 displays the difference between the payoff and the estimated premium, i.e. $P_{t,t+\tau} - \Pi_{t,t+\tau}$, for all indices. This quantity represents the measurement error for regimes l and h, while it represents the sum of measurement error and the extent of the fear/surprise for regime j. The difference in magnitude between normal and extreme regimes (indicated by the shaded areas) is striking as the magnitude of $FS_{t,t+\tau}$ dominates the measurement error. We see that positive deviations, associated to fears (red), mirror negative deviations, associated to surprises (green) both in absolute size and magnitude, with the exception of the unique events coinciding with the peak of the financial crisis in September and October of 2008.

To isolate the extent of fears and surprises, we offset the normal regime by intersecting

Figure 4: Fear (red), $F_{t,t+\tau}$, surprise (green), $S_{t,t+\tau}$, and detected extreme variance events (vertical grey lines for the four indices. From the top we plot S&P500, DJIA, NASDAQ and RUSSELL, respectively.

Table 5: Fear and surprise variables associated with extreme market conditions

		Mean	Std. dev.	Minimum	Maximum	N
Fear ($F_{t,t+\tau}$)	S&P500	45.716	33.351	18.813	203.88	73
	DJIA	50.040	33.587	19.655	192.695	51
	NASDAQ	54.137	34.529	24.562	177.491	33
	RUSSELL	73.889	53.625	33.313	214.128	21
Surprise ($S_{t,t+\tau}$)	S&P500	87.161	98.015	21.613	458.156	69
	DJIA	87.923	86.096	23.574	414.373	53
	NASDAQ	119.10	121.79	32.085	478.893	36
	RUSSELL	138.05	121.42	22.830	434.604	41

Notes: Fears and surprises are computed as $\mathrm{F}_{t,t+\tau} = (\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau})\mathrm{I}_{\{s_t=j \cap (\mathrm{P}_{t,t+\tau}-\Pi_{t,t+\tau})>0\}}$ and $\mathrm{S}_{t,t+\tau} = |\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau}|\mathrm{I}_{\{s_t=j \cap (\mathrm{P}_{t,t+\tau}-\Pi_{t,t+\tau})<0\}}$ respectively. N is the number of nonzero values over which the descriptive statistics are computed.

$\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau}$ with the indicator $\mathrm{I}_{s_t=j}$. The resulting variable represents the extent of the fear (> 0) or surprise (< 0) generated by a realized extreme shock on the market occurred in the period between $t - \tau$ and t. More precisely, we define fear as $\mathrm{F}_{t,t+\tau} = (\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau})\mathrm{I}_{\{s_t=j \cap (\mathrm{P}_{t,t+\tau}-\Pi_{t,t+\tau})>0\}}$ and surprise as $\mathrm{S}_{t,t+\tau} = |\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau}|\mathrm{I}_{\{s_t=j \cap (\mathrm{P}_{t,t+\tau}-\Pi_{t,t+\tau})<0\}}$. We switch the sign of the surprise effect in the following so that its coefficient in the predictive regression analysis below represents the direction of the pricing of the risk factor.

Table 5 gives descriptive statistics for the $\mathrm{F}_{t,t+\tau}$ and $\mathrm{S}_{t,t+\tau}$ variables. Fears and surprises show a similar number of occurrences, as seen in Figure 4, with the exception of RUSSELL where surprises are twice as frequent as fears. The fear variable is on average between four and five times larger than the estimated unconditional level of the VRP reported in Table 3, and at the minimum at least twice as large. The surprise variable exhibits in general a more extreme behaviour. As discussed in Section 2.2, these statistics are largely affected by a handful of extremely negative variance swap payoffs observed during the peak of the global financial crisis.

5 Predictive return regressions

In this section, we use the smooth component of the VRP, i.e. $\Pi_{t,t+1}$, and the occurrence and size of fears and surprises to empirically assess their ability to predict the equity premium. We disentangle the portion of return predictability of the variance swap payoff shown in Section 2.2 stemming from $\Pi_{t,t+1}$ from that of rare and extreme events and the reaction of agents to those events.

5.1 Smooth component of the VRP

The importance of the VRP as a predictor for future aggregate market returns has been pointed out by many authors, see Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011), Bollerslev, Marrone, Xu, and Zhou (2014), and Bekaert and Hoerova (2014) among others.

Table 6 provides insights on the contribution of $\Pi_{t,t+\tau}$ to future market returns by estimating the predictive regression model

$$\frac{1}{h}\sum_{j=1}^{h} r_{t+j} = a_0(h) + a_1(h)\Pi_{t,t+1} + u_{t+h,t}. \tag{15}$$

In general, we find that signs of return predictability emerges at rather short horizons and peaks at the two-three months mark. For the S&P500, we find positive significant slope coefficients $a_1(h)$ up to three months with the highest adjusted R^2 of 2.12 percent at the two month horizon. For the DJIA, R^2s reach 3.50 percent at the three month horizon, and remain significant up to the six month horizon. The NASDAQ has significant slope coefficients only from the 3 month horizon, with the largest R^2 of 3.06 percent at the five month horizon. The RUSSELL has only significant R^2s at the two and three month horizons.

Comparing Table 6 with the return predictability from the direct use of the variance swap payoffs as reported in Table 2, a different pattern emerges. For the S&P500 and

DJIA, when using $\Pi_{t,t+\tau}$ as a predictor rather than the variance swap payoff, the slope coefficients are significant from shorter horizons and remain so up to six months. The significant coefficients are associated with slightly lower R^2s, though when considering these as a function of the horizon the inverse U-shape is preserved. Similarly, the NASDAQ and RUSSELL also show significant slope coefficients from shorter horizons onwards when compared to the results in Table 2. However, their corresponding R^2s are in some instances higher.

Table 6: Smooth VRP predictive regressions

Horizon	1	4	8	12	16	20	24	36	52
					S&P500				
$\Pi_{t,t+\tau}$	**1.951**	**2.148**	**1.785**	**1.384**	0.933	0.665	0.536	0.277	0.224
	(0.975)	(0.844)	(0.691)	(0.593)	(0.508)	(0.457)	(0.439)	(0.335)	(0.275)
R^2	0.21	1.51	2.12	1.92	1.14	0.67	0.48	0.14	0.11
					DJIA				
$\Pi_{t,t+\tau}$	1.982	**2.086**	**2.024**	**1.719**	**1.361**	**1.133**	**0.919**	0.363	0.268
	(1.104)	(0.959)	(0.785)	(0.662)	(0.560)	(0.493)	(0.461)	(0.348)	(0.291)
R^2	0.23	1.50	3.03	3.50	3.07	2.60	1.94	0.36	0.26
					NASDAQ				
$\Pi_{t,t+\tau}$	0.863	0.982	1.535	**1.707**	**1.549**	**1.567**	**1.428**	0.887	**0.686**
	(1.274)	(1.081)	(0.888)	(0.800)	(0.726)	(0.655)	(0.611)	(0.482)	(0.322)
R^2	-0.10	0.12	1.10	2.13	2.37	3.06	2.99	1.76	1.55
					RUSSELL				
$\Pi_{t,t+\tau}$	1.875	2.203	**2.307**	**2.104**	1.235	0.878	0.713	0.317	0.236
	(1.581)	(1.278)	(1.045)	(0.879)	(0.740)	(0.667)	(0.576)	(0.445)	(0.299)
R^2	0.09	1.35	3.15	4.05	1.87	1.09	0.78	0.14	0.08

Notes: Estimation results for predictive return regressions $\frac{1}{h}\sum_{j=1}^{h} r_{t+j} = a_0(h) + a_1(h)\Pi_{t,t+1} + u_{t+h,t}$, where h denotes the horizon. Newey-West standard errors are in brackets. Coefficients significant at five percent are in boldface. Adjusted R^2s in percentages. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

5.2 Expectations and surprises

Besides the smooth VRP ($\Pi_{t,t+1}$), our model infers the agents' reaction to realized extreme variance events, namely fears ($F_{t,t+1}$) and surprises ($S_{t,t+1}$). The contribution of these variables is expected to generate significantly larger return predictability in periods when such events occur. To test this hypothesis, we augment the predictive regression model in (15) as follows

$$\frac{1}{h}\sum_{j=1}^{h} r_{t+j} = a_0(h) + a_1(h)\Pi_{t,t+1} + a_2(h)\mathrm{F}_{t,t+1} + a_3(h)\mathrm{S}_{t,t+1} + u_{t+h,t}. \tag{16}$$

Table 7 reports estimates for the parameters $a_1(h)$, $a_2(h)$ and $a_3(h)$ of the model in (16) as well as their associated adjusted R^2s. The smooth VRP slope estimates, $a_1(h)$, hardly change with respect to Table 6 for all indices, confirming the orthogonal nature of the regressors. The parameter estimates for $a_2(h)$ associated with the fear measure is positive and significant only at long horizons. For instance, the S&P500 has a significant coefficient of 0.202 at the one year horizon but insignificant coefficients at shorter horizons. This suggests that fear for large volatility shifts constitutes a risk factor that the agents are exposed to, to which they do not react immediately, but that they distinctly price for long periods. The parameter estimates for the surprise variable, $a_3(h)$, are typically significant and negative at the intermediate horizons but they switch sign at the one year horizon. This occurs for all indices, except the S&P500. By adding fear and surprise predictors, the return predictability sensibly increases, mildly at the short horizons but substantially from the medium to long horizons. In sum, our results show evidence of systematic longer lasting response of the expected average returns triggered by unexpected (at least in their magnitude) realized large shocks and fear of future extreme shocks.

We have shown that fears and surprises, i.e., direction and size of agents' reaction to extreme shocks to the market, have a relevant effect on future market performances. We argue that such an effect is likely to be asymmetric and systematically related to the current

Table 7: Premium, fear & surprise predictive regression

Horizon	1	4	8	12	16	20	24	36	52
					S&P500				
$\Pi_{t,t+\tau}$	**1.828**	**2.010**	**1.746**	**1.278**	0.796	0.527	0.446	0.248	0.223
	(0.920)	(0.809)	(0.677)	(0.584)	(0.495)	(0.441)	(0.428)	(0.333)	(0.274)
$F_{t,t+\tau}$	-0.207	-0.232	-0.388	0.129	0.283	0.304	0.306	0.226	**0.202**
	(0.594)	(0.350)	(0.476)	(0.291)	(0.206)	(0.167)	(0.163)	(0.121)	(0.092)
$S_{t,t+\tau}$	-0.317	-0.356	-0.181	**-0.193**	**-0.219**	**-0.217**	-0.114	0.006	0.051
	(0.406)	(0.205)	(0.105)	(0.065)	(0.066)	(0.083)	(0.068)	(0.043)	(0.027)
R^2	0.24	2.36	2.87	2.70	2.96	2.97	1.69	0.60	0.81
					DJIA				
$\Pi_{t,t+\tau}$	1.866	**1.883**	**1.917**	**1.643**	**1.251**	**1.035**	**0.905**	0.447	0.398
	(1.013)	(0.906)	(0.764)	(0.655)	(0.543)	(0.469)	(0.453)	(0.354)	(0.292)
$F_{t,t+\tau}$	-0.336	-0.508	-0.539	-0.007	0.229	0.304	0.385	**0.424**	**0.380**
	(0.693)	(0.428)	(0.500)	(0.319)	(0.247)	(0.197)	(0.224)	(0.141)	(0.091)
$S_{t,t+\tau}$	-0.095	-0.181	-0.040	-0.105	**-0.199**	**-0.198**	-0.099	0.023	**0.010**
	(0.394)	(0.184)	(0.104)	(0.081)	(0.085)	(0.090)	(0.066)	(0.056)	(0.035)
R^2	0.07	1.86	3.74	3.53	4.46	4.72	3.69	2.79	3.81
					NASDAQ				
$\Pi_{t,t+\tau}$	0.752	0.829	1.487	**1.625**	**1.459**	**1.476**	**1.354**	0.855	**0.659**
	(1.265)	(1.077)	(0.887)	(0.794)	(0.715)	(0.644)	(0.599)	(0.472)	(0.314)
$F_{t,t+\tau}$	-0.462	0.347	-0.049	0.477	**0.668**	**0.673**	**0.714**	**0.529**	**0.414**
	(0.527)	(0.299)	(0.344)	(0.268)	(0.177)	(0.171)	(0.160)	(0.128)	(0.120)
$S_{t,t+\tau}$	-0.662	**-0.533**	**-0.223**	**-0.180**	**-0.144**	-0.122	0.004	**0.090**	**0.120**
	(0.389)	(0.140)	(0.113)	(0.067)	(0.050)	(0.068)	(0.065)	(0.046)	(0.031)
R^2	0.65	2.67	1.68	3.51	4.94	6.07	6.08	5.04	5.64
					RUSSELL				
$\Pi_{t,t+\tau}$	1.112	1.664	**1.962**	**1.932**	1.123	0.816	0.798	0.520	0.478
	(1.379)	(1.146)	(0.995)	(0.889)	(0.739)	(0.641)	(0.571)	(0.448)	(0.290)
$F_{t,t+\tau}$	**-1.607**	-0.227	-0.694	0.114	0.398	0.365	0.394	**0.464**	**0.403**
	(0.628)	(0.426)	(0.604)	(0.400)	(0.299)	(0.238)	(0.250)	(0.135)	(0.120)
$S_{t,t+\tau}$	-0.356	-0.444	-0.164	-0.180	**-0.187**	-0.009	0.072	0.117	**0.001**
	(0.456)	(0.315)	(0.144)	(0.094)	(0.088)	(0.135)	(0.102)	(0.057)	(0.032)
R^2	0.73	2.94	4.48	4.76	3.97	2.68	1.64	2.85	4.54

Notes: Estimation results for predictive return regressions $\frac{1}{h}\sum_{j=1}^{h} r_{t+j} = a_0(h) + a_1(h)\Pi_{t,t+1} + a_2(h)F_{t,t+1} + a_3(h)S_{t,t+1} + u_{t+h,t}$, where h denotes the horizon. Newey-West standard errors are in brackets. Coefficients significant at five percent are in boldface. Adjusted R^2s in percentages. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

market conditions at the moment the shock occurs or to the type and size of the shock that triggers them. To test this hypothesis, we further extend the predictive regression model by discriminating fears and surprises according to the signed jump variation defined as the difference between positive and negative realized semivariances, i.e., the differential between partial sums of squared negative and positive jumps, see Barndorff-Neilsen, Kinnebrouk, and Shephard (2010). This hypothesis, which relates to the concept of good versus bad volatility developed in Patton and Sheppard (2015), allows us to capture the asymmetry with respect to the type of the fear/surprise triggering shock. The extended predictive regression model becomes

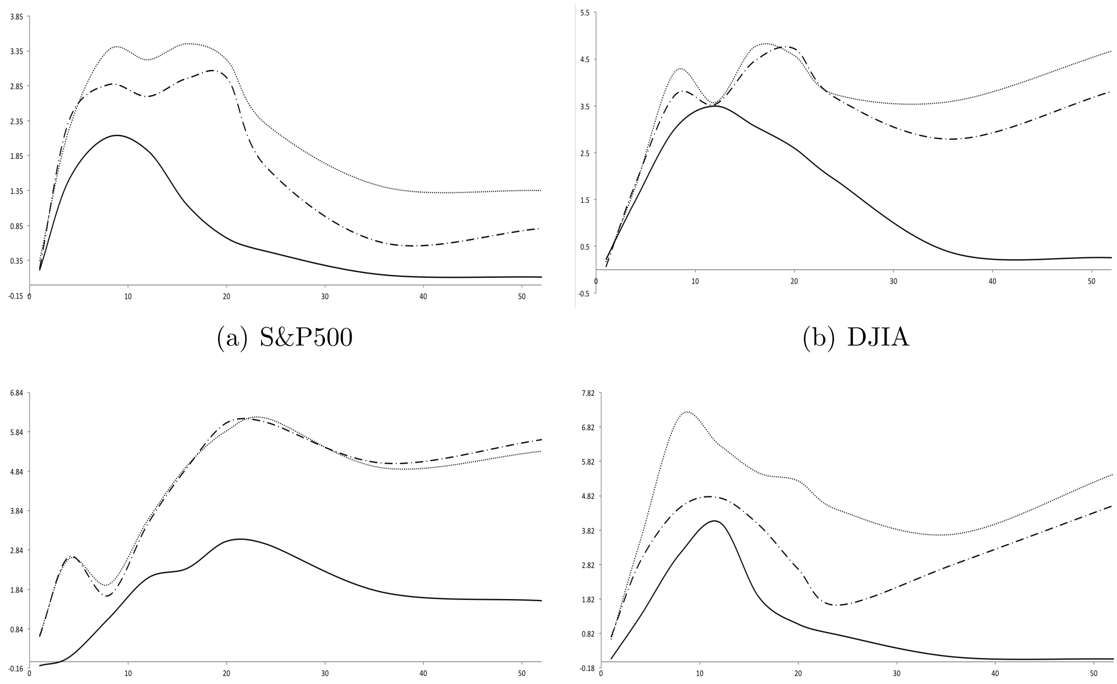
$$\begin{aligned} \frac{1}{h}\sum_{j=1}^{h} r_{t+j} &= a_0(h) + a_1(h)\Pi_{t,t+1} + a_2^{+}(h)\mathrm{F}^{+}_{t,t+1} + a_2^{-}(h)\mathrm{F}^{-}_{t,t+1} \\ &\quad + a_3^{+}(h)\mathrm{S}^{+}_{t,t+1} + a_3^{-}(h)\mathrm{S}^{-}_{t,t+1} + u_{t+h,t}. \end{aligned} \tag{17}$$

Table 8 reports the results for the model defined in (17). It turns out that when conditioning on the sign of the jump variation more fear and surprise variables coefficients become significant. In case of the S&P500, the fear variable associated with negative jump variation, i.e. $\mathrm{F}^{-}_{t,t+\tau}$, is significant and positive from the four month horizon onwards while in contrast for none of the horizons $\mathrm{F}^{+}_{t,t+\tau}$ is significant. This result is in line with the notion of leverage effect and its implication in terms intertemporal risk-return tradeoff. The surprise variable related to the positive jump variation, $\mathrm{S}^{+}_{t,t+\tau}$, is significant and negative between the two and six month horizon, while $\mathrm{S}^{-}_{t,t+\tau}$ is significant and negative between the three and five month horizon. The inverse U-shape pattern for the R^2s over the horizon becomes milder because the long horizon R^2s stay as high as, or sometimes even higher than, the medium horizon R^2s. The latter can also be seen from Figure 5 which plots the adjusted R^2 against the future returns aggregation horizon for the three predictive regression models.

Table 8: Premium, signed fear & surprise (signed jump variation) predictive regression

Horizon	1	4	8	12	16	20	24	36	52
					S&P500				
$\Pi_{t,t+\tau}$	**1.870**	**2.013**	**1.745**	**1.281**	0.798	0.527	0.448	0.247	0.222
	(0.927)	(0.811)	(0.678)	(0.584)	(0.496)	(0.441)	(0.429)	(0.333)	(0.275)
$F^{+}_{t,t+\tau}$	-0.601	-0.233	-0.531	-0.008	0.168	0.226	0.201	0.146	0.139
	(0.619)	(0.324)	(0.468)	(0.256)	(0.179)	(0.156)	(0.148)	(0.132)	(0.100)
$F^{-}_{t,t+\tau}$	1.731	-0.219	0.288	0.779	**0.830**	**0.669**	**0.807**	**0.600**	**0.498**
	(1.844)	(0.727)	(0.535)	(0.438)	(0.298)	(0.263)	(0.216)	(0.205)	(0.193)
$S^{+}_{t,t+\tau}$	0.040	-0.102	**-0.977**	**-0.759**	**-0.703**	**-0.596**	**-0.566**	-0.519	-0.309
	(1.078)	(0.375)	(0.454)	(0.287)	(0.255)	(0.299)	(0.264)	(0.436)	(0.374)
$S^{-}_{t,t+\tau}$	-0.327	-0.365	-0.154	**-0.173**	**-0.203**	**-0.204**	-0.098	0.012	**0.064**
	(0.421)	(0.210)	(0.110)	(0.067)	(0.073)	(0.091)	(0.073)	(0.048)	(0.024)
R^2	0.35	2.23	3.36	3.22	3.45	3.22	2.30	1.40	1.35
					DJIA				
$\Pi_{t,t+\tau}$	1.939	**1.858**	**1.962**	**1.664**	**1.278**	**1.045**	**0.923**	0.477	0.424
	(1.011)	(0.903)	(0.767)	(0.656)	(0.545)	(0.470)	(0.453)	(0.352)	(0.290)
$F^{+}_{t,t+\tau}$	-0.520	-0.348	-0.688	-0.080	0.135	0.258	0.310	**0.302**	**0.266**
	(0.666)	(0.367)	(0.493)	(0.263)	(0.217)	(0.200)	(0.229)	(0.132)	(0.081)
$F^{-}_{t,t+\tau}$	0.272	-1.005	-0.060	0.230	0.530	0.450	**0.621**	**0.812**	**0.741**
	(2.070)	(0.915)	(0.574)	(0.568)	(0.361)	(0.234)	(0.246)	(0.180)	(0.113)
$S^{+}_{t,t+\tau}$	-0.609	**-0.261**	-0.258	**-0.204**	**-0.315**	**-0.216**	**-0.154**	**-0.057**	**0.048**
	(0.489)	(0.068)	(0.141)	(0.049)	(0.050)	(0.028)	(0.036)	(0.023)	(0.023)
$S^{-}_{t,t+\tau}$	0.262	-0.129	0.114	-0.035	-0.117	-0.185	-0.059	0.081	**0.139**
	(0.696)	(0.335)	(0.185)	(0.126)	(0.120)	(0.150)	(0.117)	(0.088)	(0.052)
R^2	0.18	1.80	4.24	3.57	4.76	4.57	3.75	3.60	4.67
					NASDAQ				
$\Pi_{t,t+\tau}$	0.773	0.820	1.474	**1.614**	**1.454**	**1.470**	**1.350**	0.862	**0.662**
	(1.279)	(1.080)	(0.886)	(0.793)	(0.715)	(0.645)	(0.598)	(0.472)	(0.314)
$F^{+}_{t,t+\tau}$	-0.730	0.139	-0.283	0.313	**0.543**	**0.612**	**0.593**	**0.558**	**0.429**
	(0.526)	(0.229)	(0.286)	(0.225)	(0.155)	(0.173)	(0.143)	(0.082)	(0.077)
$F^{-}_{t,t+\tau}$	0.697	1.238	**0.958**	**1.180**	**1.206**	**0.931**	**1.215**	0.410	0.352
	(2.041)	(0.752)	(0.309)	(0.295)	(0.168)	(0.245)	(0.193)	(0.571)	(0.512)
$S^{+}_{t,t+\tau}$	**-1.168**	**-0.600**	**-0.260**	**-0.181**	**-0.187**	**-0.088**	-0.008	0.016	**0.080**
	(0.453)	(0.101)	(0.074)	(0.037)	(0.029)	(0.039)	(0.030)	(0.060)	(0.041)
$S^{-}_{t,t+\tau}$	-0.457	**-0.504**	-0.207	-0.178	-0.126	-0.135	0.010	0.120	**0.136**
	(0.609)	(0.205)	(0.165)	(0.093)	(0.072)	(0.086)	(0.089)	(0.065)	(0.041)
R^2	0.69	2.62	1.96	3.61	4.99	5.87	6.19	4.92	5.35
					RUSSELL				
$\Pi_{t,t+\tau}$	1.124	1.671	**1.964**	**1.936**	1.127	0.824	0.806	0.524	0.484
	(1.386)	(1.149)	(1.000)	(0.894)	(0.743)	(0.644)	(0.572)	(0.449)	(0.291)
$F^{+}_{t,t+\tau}$	-1.194	-0.43	**-1.086**	-0.108	0.210	0.237	0.222	**0.350**	**0.314**
	(0.729)	(0.296)	(0.352)	(0.297)	(0.210)	(0.200)	(0.152)	(0.090)	(0.083)
$F^{-}_{t,t+\tau}$	**-3.531**	0.8205	1.196	**1.193**	**1.310**	**1.014**	**1.232**	**1.006**	**0.822**
	(1.691)	(0.757)	(0.729)	(0.368)	(0.268)	(0.253)	(0.299)	(0.188)	(0.145)
$S^{+}_{t,t+\tau}$	-0.052	0.580	0.746	0.503	0.383	**0.715**	**0.718**	0.331	0.364
	(1.313)	(1.057)	(0.596)	(0.415)	(0.436)	(0.332)	(0.305)	(0.342)	(0.304)
$S^{-}_{t,t+\tau}$	-0.375	-0.494	-0.212	**-0.216**	**-0.217**	-0.182	-0.047	0.058	**0.105**
	(0.484)	(0.319)	(0.148)	(0.093)	(0.080)	(0.127)	(0.096)	(0.053)	(0.024)
R^2	0.66	3.56	7.14	6.29	5.48	5.25	4.41	3.71	5.45

Notes: Estimation results for predictive return regressions $\frac{1}{h}\sum_{j=1}^{h} r_{t+j} = a_0(h) + a_1(h)\Pi_{t,t+1} + a_2^{+}(h)F^{+}_{t,t+1} + a_2^{-}(h)F^{-}_{t,t+1} + a_3^{+}(h)S^{+}_{t,t+1} + a_3^{-}(h)S^{-}_{t,t+1} + u_{t+h,t}$ where h denotes the horizon. Newey-West standard errors are in brackets. Coefficients significant at five percent are in boldface. Adjusted R^2s in percentages. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.



(a) S&P500

(b) DJIA

(c) NASDAQ

(d) RUSSELL

Figure 5: Predictive regressions R^2s for the models with smooth VRP (solid line), adding expectations and surprises (dotted line), and adding signed jump variation (dashed line). See respectively the models defined in (15), (16) and (17).

6 Market VRP and CAPM regressions

Our choice of the four US stock market indices is not coincidental. Although heterogenous in terms of size, composition and degree of diversification, they all ultimately convey information about the aggregate US stock market. From the previous analysis, we find a high degree of similarity among the variance swap payoffs of the four indices. Besides the high degree of correlation, a principal component analysis reveals that the first component explains 95 percent of the total payoff variation.[7]

The previous evidence suggests existence of a common and dominant source of variance

[7]To avoid the impact of the extreme payoff realisations, we perform the principal component analysis on the filtered VRP's and we find that the relative weight of the first component amounts to 88 percent.

risk driving the four indices, i.e. a market variance risk premium (MVRP). We estimate this common variance risk factor by building a joint model that exploits the intra-market cross-sectional dimension. The state space form and the Kalman filter provide advantage because they allow to exploit information coming from multiple measurements to improve the estimation accuracy of a common latent factor. In a CAPM exercise, we test how agents price the market variance risk premium.

Defining $\mathrm{P}^i_{t,t+\tau}$ the variance swap payoff for the market index i =S&P500, DJIA, NASDAQ and RUSSELL, the model in (13) in Section 3, can be written as

$$\begin{aligned} \mathrm{P}^i_{t,t+\tau} &= \mathrm{E}^Q_t[RV^i_{t,t+\tau}] - RV^i_{t,t+\tau} \\ &= \Pi^i_{t,t+\tau} + \mathrm{I}_{s_t=j}\mu_i + e^i_{t+\tau} + \mathrm{I}_{s_t=j}\eta^i_{t+\tau}. \end{aligned} \tag{18}$$

The variance premium $\Pi^i_{t,t+\tau}$ can be written as the sum of an affine transformation (centered and rescaled) of a common factor $G_{t,t+\tau}$ and an idiosyncratic i.i.d. factor $\varepsilon^i_{t+\tau}$, i.e. $\Pi^i_{t,t+\tau} = \overline{\Pi}^i + \beta_i G_{t,t+\tau} + \varepsilon^i_{t+\tau}$. Then the measurement equation above becomes

$$\mathrm{P}^i_{t,t+\tau} = \overline{\Pi}^i + \beta_i G_{t,t+\tau} + \mathrm{I}_{s_t=j}\overline{\mathrm{FS}}_i + \tilde{e}^i_{t+\tau} \tag{19}$$

where $\tilde{e}^i_{t+\tau} = \varepsilon^i_{t+\tau} + e^i_{t+\tau} + \mathrm{I}_{s_t=j}\eta^i_{t+\tau}$ is a noise with state dependent variance $\sigma^2_{\tilde{e}^i} = (\sigma^2_{\varepsilon^i} + \sigma^2_{e^i} + \mathrm{I}_{s_t=j}\sigma^2_{\eta^i})$. Note that the variances of $\varepsilon^i_{t+\tau}$ (idiosynctratic component) and $e^i_{t+\tau}$ (prediction error) are identified in the sum, which is sufficient for the purpose of our model, but not individually. Also, identification restrictions on the variance of the common factor require $\beta_{S\&P500} = 1$. The common factor $G_{t,t+\tau}$, denoting the MVRP, is assumed to evolve as a first order autoregressive process, i.e. $G_{t,t+\tau} = \phi G_{t-1,t+\tau-1} + \epsilon_{t+\tau}$ with variance of $\epsilon_{t+\tau}$ equal to σ^2_ϵ.[8] Heteroskedasticity is accounted for in the same fashion as in Section 3.2.

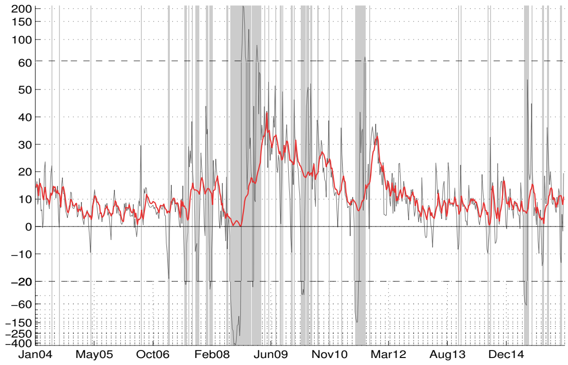
Table 9 reports quasi-maximum likelihood parameter estimates for the common factor model. Compared to the S&P500, the estimated common factor β_i loads less than pro-

[8]The MVRP is not a proper VRP since it is centered and normalized. However, up to the affine transformation $\overline{\Pi}^i + \beta_i G_{t,t+\tau}$ it becomes the linear predictor for the VRP of index i.

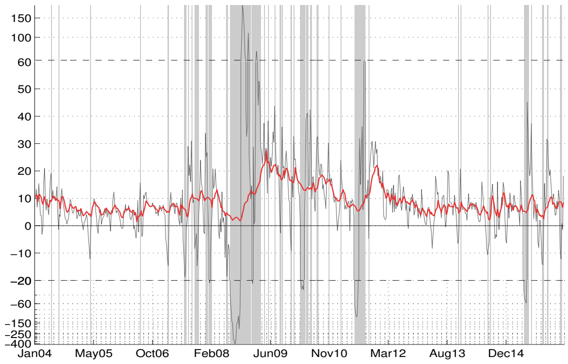
Table 9: Quasi-maximum likelihood estimates of the common factor model

Parameter	Common	S&P500	DJIA	NASDAQ	RUSSELL
$\sigma_{e^i,l}$		1.84	2.06	5.27	5.13
$\sigma_{e^i,h}$		11.46	6.38	9.04	12.51
σ_{η^i}		39.00	35.15	31.03	44.75
$\overline{\Pi}^i$		11.53	5.79	11.86	14.50
β_i		1.00	0.62	0.93	0.88
$\sigma_{\epsilon,l}$	0.74				
$\sigma_{\epsilon,h}$	1.01				
ϕ	0.98				
p_{ll}	0.94				
p_{lh}	[0.04]				
p_{hh}	0.63				
p_{hj}	0.23				
p_{jj}	0.52				
p_{jh}	0.48				
LLF	-11.0547				
T	641				
Steady-state prob.		l		h	j
		0.62		0.24	0.14
Expected durations		l		h	j
		17.11		2.68	2.07

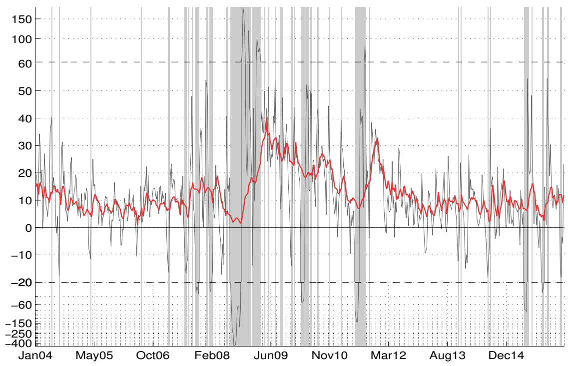
Notes: LLF denotes the average loglikelihood and T the number of observations. Steady state probabilities are computed according to (4.49) in Kim and Nelson (1999). Expected duration is computed as $1/(1-p_{ii})$, $i \in$ [l (low), h (high), j (jump)]. Identification restrictions on the variance of the common factor require $\beta_{S\&P500} = 1$. The [] brackets indicate insignificant parameters. The parameter $\overline{\text{FS}}$ is set to zero as it turns out to be insignificant. The sample frequency is weekly between February 3, 2004 and July 29, 2016.



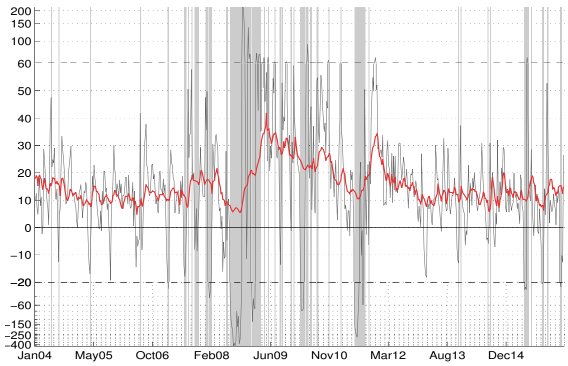
(a) S&P500



(b) DJIA



(c) NASDAQ



(d) RUSSELL

Figure 6: VRP's implied by the common factor model.

portionally on the other indices. The estimated persistence, measured by ϕ, amounts to 0.98 confirming slowly evolving VRP's for all indices, see also Figure 6 which shows the VRP's implied by the common factor model. The expected durations for the three regimes are respectively 17.11 , 2.68 and 2.07 weeks. Figure 6 shows that the regime representing episodes of fear and surprises, with a steady state probability equal to 0.14, exhibits three long clusters associated with respectively the global financial crisis, the flash crash and the US debt downgrade. The remaining events are in general sparse and less pronounced.

In an intertemporal CAPM framework of Merton (1973), we next test whether the estimated MVRP contains relevant information about the perceived level of variance risk that is actually priced in financial assets, see e.g. Bali and Zhou (2016). The three factor model of Fama and French (1993) and the five factor model of Fama and French (2015) extend the CAPM of Sharpe (1964) and Lintner (1965) to describe patterns in the return variation left unexplained by the market risk factor. The model is designed to capture the relation between the average return and factors like size (market capitalisation), price ratios like book-to-market, profitability (the difference between the returns on diversified portfolios of stocks with robust and weak profitability) and investment (the difference between the returns on diversified portfolios of the stocks of low and high investment firms).

Although not being a risk factor immediately comparable in essence to the five Fama-French factors, as it is not the return on a tradable portfolio, the MVRP represents the level of agents' volatility risk aversion and it is directly proportional to the cost of hedging against volatility risk. Correlations between the five Fama-French factors and the MVRP, show that while the Fama-French factors are substantially correlated among themselves, the MVRP is significantly correlated, i.e. 28%, only with the market factor. Hence, the MVRP contains novel information to capture the variation in the expected return of financial securities and portfolios which should be accounted for. Thus, despite being a powerful asset pricing model, the Fama-French model is incomplete in the sense that none of the factors contains relevant information about exposure to market variance risk.

Following Fama and French (2015), we consider returns on five portfolio sorts based on the following four characteristics: beta, variance, residual variance and net share issues. Details on the construction of the portfolios and data are available from Kennet R. French's data library.[9] Being standardised by construction, adding the MVRP in the Fama-French regression preserves on the one hand comparability with Fama and French (2015) and does not introduce distortion in the estimations of the intercepts, on the other hand it allows

[9]See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

to capture the correlation, and thus the exposure of the expected portfolio return to the market variance risk. Following practice in the CAPM literature, we resample the data at a monthly frequency.

Table 10 and 11 report parameter estimates of the Fama-French five-factor model plus the MVRP. The latter contributes significantly to explain the portfolios return variations for the sorts based on beta, variance and residual variance, and only marginally for the ones based on net share issues. More specifically, Table 10 Panel A, reports results for the portfolio sorted with respect to the individual components' beta. Portfolios of stocks with large betas, thus more sensitive to market fluctuations, show positive exposure to variance risk with all loadings significant at 90% confidence level at least. For a given level of perceived variance risk, portfolios more exposed to the market incorporate a higher remuneration. Conversely, portfolios with market exposure smaller than one, react in the opposite direction showing the existence of a negative premium (and thus preference) for stability. Panel B of Table 10 shows that inclusion of the MVRP in the pricing model is beneficial. As expected, for the five portfolios sorted according to the assets' variance, the exposure to variance risk increases as we move from low to high. Positive exposure to volatility risk reflects the compensation required to face the higher cost of hedging against such risk. Table 11 Panel A shows that similar patterns hold for portfolios sorts based on residual variance. The results in Table 11 Panel B show a weaker link between the expected return of portfolios based on growth, measured by net share issues, and the MVRP. Although, we would expect high growth portfolios to be more sensitive to volatility risk, we find that the MVRP loads significantly only on the fourth quintile portfolio.

Table 10: CAPM regressions

***Panel A*: Portfolio sorts based on βs**

Coefficients

	CST	MKT	SMB	HML	RMW	CMA	MVRP	R^2
Low	0.043	0.805	-0.266	-0.047	0.174	0.254	-0.034	0.88
2	0.096	0.982	-0.044	0.120	0.153	-0.192	-0.015	0.97
3	-0.006	1.059	0.060	0.097	0.053	-0.164	0.034	0.97
4	-0.049	1.203	0.231	0.124	-0.128	-0.320	0.050	0.95
High	-0.107	1.292	0.406	-0.093	-0.551	0.070	0.089	0.90

t-statistics

Low	0.427	25.653	-5.393	-0.700	2.668	2.856	-2.574
2	1.537	41.924	-1.340	3.330	3.220	-3.406	-1.751
3	-0.085	35.296	1.704	1.872	0.818	-2.148	3.328
4	-0.377	26.644	4.034	1.388	-1.562	-3.908	3.436
High	-0.462	17.366	4.246	-0.506	-3.554	0.435	3.328

***Panel B*: Portfolio sorts based on Variance**

Coefficients

	CST	MKT	SMB	HML	RMW	CMA	MVRP	R^2
Low	0.130	0.786	-0.204	-0.067	0.136	0.248	-0.011	0.91
2	0.030	1.043	0.037	-0.008	0.097	-0.051	-0.002	0.97
3	-0.106	1.207	0.089	0.044	-0.014	-0.172	0.004	0.94
4	-0.065	1.309	0.326	0.148	-0.152	-0.281	0.040	0.91
High	-0.246	1.308	0.610	0.384	-0.765	-0.748	0.049	0.88

t-statistics

Low	2.006	33.74	-4.700	-1.261	2.575	3.100	-1.143
2	0.421	54.41	0.949	-0.195	2.282	-0.704	-0.181
3	-1.060	33.85	1.683	0.448	-0.148	-1.112	0.282
4	-0.420	23.94	3.131	1.435	-1.116	-2.173	2.039
High	-1.180	12.97	5.430	2.335	-4.516	-2.828	2.470

Notes: Estimation results CAPM regressions. MKT is the market return in excess of the risk-free interest rate, SMB is small minus big, HML is high minus low, RMW is robust minus weak and CMA is conservative minus aggressive. The β sorts are based on univariate market beta. The variance sorts are based on individuals assets' variance. The sample frequency is monthly between February, 2004 and July, 2016. The t-statistics are based on Newey-West standard errors.

Table 11: CAPM regressions

***Panel A*: Portfolio sorts based on Residual variance**

Coefficients

	CST	MKT	SMB	HML	RMW	CMA	MVRP	R^2
Low	0.061	0.853	-0.200	-0.044	0.130	0.259	-0.020	0.94
2	0.035	1.034	0.046	0.087	0.034	-0.041	0.007	0.96
3	-0.082	1.134	0.108	0.059	-0.067	-0.305	0.012	0.95
4	0.030	1.269	0.238	0.061	-0.169	-0.450	0.031	0.91
High	-0.299	1.304	0.630	0.144	-0.539	-0.535	0.078	0.87

t-statistics

Low	1.070	46.15	-5.769	-1.130	2.583	3.959	-2.071
2	0.492	46.99	1.064	2.101	0.861	-0.701	0.660
3	-0.996	34.90	1.940	1.072	-0.842	-2.955	0.853
4	0.195	25.85	2.981	0.464	-1.658	-3.216	1.609
High	-1.317	16.29	5.493	0.687	-2.730	-2.221	2.405

***Panel B*: Portfolio sorts based on Net Share Issues**

Coefficients

	CST	MKT	SMB	HML	RMW	CMA	MVRP	R^2
Low	0.031	0.952	-0.090	-0.001	-0.081	0.044	0.023	0.91
2	0.047	0.998	0.111	-0.062	-0.042	0.011	-0.008	0.92
3	0.185	1.089	0.126	-0.095	-0.011	-0.302	0.004	0.92
4	0.000	1.079	0.213	-0.232	-0.133	-0.349	0.068	0.91
High	0.063	1.015	0.118	0.098	-0.670	-0.377	0.010	0.93

t-statistics

Low	0.285	29.447	-1.713	-0.010	-0.889	0.455	1.580
2	0.370	23.686	2.058	-0.800	-0.429	0.112	-0.529
3	1.480	29.497	1.544	-1.295	-0.098	-2.295	0.263
4	0.003	21.593	3.553	-2.717	-1.299	-3.225	3.522
High	0.400	24.123	1.768	1.096	-5.684	-3.864	0.526

Notes: Estimation results CAPM regressions. MKT is the market return in excess of the risk-free interest rate, SMB is small minus big, HML is high minus low, RMW is robust minus weak and CMA is conservative minus aggressive. The residual variance sorts are based on variance of the residuals from the Fama-French three-factor model. The Net Share Issues sorts are shares' growth rate. The sample frequency is monthly between February, 2004 and July, 2016. The t-statistics are based on Newey-West standard errors.

7 Conclusion

This paper estimates the Variance Risk Premium (VRP) directly from synthetic variance swap payoffs. Our approach provides measurement error free estimates of the part of the VRP related to normal market conditions, and allows constructing variables indicating agents' expectations under extreme market conditions. Our proposed methodology has implications for market return predictability and for asset pricing in general. In particular, though the VRP significantly predicts future market returns at shorter horizons, across the S&P500, DJIA, NASDAQ and RUSSELL indices, sizeable increases in predictability are found when the agents' reactions to extreme events are included in the predictive regressions. Finally, we filter out a common factor interpretable as a market variance risk premium (MVRP). The MVRP shares the properties that the individual VRPs have and allows identifying common extreme events. When compared to other well-known asset pricing factors, the MVRP is significantly correlated only with the market factor and it is priced when considering the returns on most of the five Fama and French (2015) portfolio sorts.

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