The Impact of Renegotiable Debt on Firms*

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ABSTRACT

This paper develops a model to investigate the impact of renegotiable debt on firms. The novel feature is that firms can renegotiate debt both in distress and outside distress, which allows us to rationalize empirical timing patterns of debt renegotiations. We show that this feature is crucial to explain the cross-section of observed credit spreads and the joint distribution of corporate events and the debt control premium. These debt pricing patterns are not captured by existing models. Incorporating both renegotiation events also generates novel testable implications for the impact of renegotiable debt on corporate policies.

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1. Introduction

Creditors play an active role in corporate governance by influencing firm decisions. The recent governance literature emphasizes debt renegotiations as the channel behind creditors' influence and identifies two conspicuous patterns of this channel (Roberts and Sufi, 2009a; Nini, Smith, and Sufi, 2012; Denis and Wang, 2014). First, debt renegotiations occur, on average, already early after contract initiation. Second, debt renegotiations commonly take place outside firm distress and not only in distress. Yet the existing theoretical literature only captures that renegotiations can occur but neglects the patterns when renegotiations occur. This void is surprising given that renegotiations crucially influence firms and, hence, understanding when renegotiations take place is key to gauge the role of debt renegotiations (Nini, Smith, and Sufi, 2012; Roberts, 2015).

This paper investigates the impact of renegotiable debt on firms. To this end, we develop a model with the novel feature that a firm can renegotiate debt both in distress and outside distress. We show that this feature is key to understanding the role of renegotiable debt for firms. Specifically, our baseline model (i) is the first that rationalizes empirical timing patterns of debt renegotiation, (ii) explains cross-sectional features of real firms' debt prices that are not captured by existing models, and (iii) generates novel corporate policy predictions.

The model is based on the corporate finance framework with distressed reorganization of a levered firm Fan and Sundaresan (2000). To incorporate cross-sectional variation in operating activities and funding needs, we also consider an investment opportunity. Similar to Hackbarth and Mauer (2012), agency costs of debt arise from the financing and timing of the investment. To mitigate this friction, firms install a covenant with initial debt that restricts the issuance of new debt. As the covenant prevents optimal financing of the investment, firms renegotiate it at investment with initial debt holders. We find that this ability to renegotiate debt outside distress perfectly disciplines firms in their ex-post financing decision, thereby mitigating the agency cost of debt. Our representation of non-distressed renegotiation is consistent with the empirical observation that many renegotiations occur outside distress and are prevalently a consequence of borrowers' desire to go beyond the restrictive initial debt contract to increase investment or alter the financial policy (Nini, Smith, and Sufi, 2009; Roberts and Sufi, 2009b; Denis and Wang, 2014; Roberts, 2015). Examples of such non-distressed renegotiations are those of Aircastle Limited's credit agreement that was initiated in December 2012. Despite a stock price increase of more than 75% until March 2016, this agreement was renegotiated four times. The first amendment occurred after only eight months and substantially increased the initial credit amount.

We first validate the baseline model by investigating its ability to rationalize empirical timing patterns of debt renegotiations. For this purpose, we match model-implied firms to the real firms sample of Nini, Smith, and Sufi (2009) and compare renegotiation patterns in simulated matched samples to their empirical counterpart. We find that by accommodating that debt renegotiations also occur outside distress and not only due to or in anticipation of distress, the baseline model matches many empirical renegotiation timing patterns. For instance, the mean model-implied loan duration to initial renegotiation or maturity is 65.5% of the initially stated debt maturity. This number reflects its empirical counterpart of around 61% very well. Conversely, we find that existing debt renegotiation models severely overestimate durations to debt renegotiation. We also develop a benchmark renegotiation model that mirrors our baseline model but ignores non-distressed renegotiation. Comparing the baseline model to this benchmark model shows that incorporating non-distressed renegotiations is primarily responsible for our model's ability to match renegotiation timing patterns.

Second, we show that accommodating when debt renegotiations occur is crucial to explain observed debt prices. For example, the benchmark model severely overestimates empirical credit spread levels and fails to reflect the positive impact of liquidation costs on credit spreads. These drawbacks are driven by equity holders' tendency to overlever the firm at investment. The novel channel of our baseline model that firms can also renegotiate debt at investment prevents this overleverage. Hence, the baseline model reflects observed credit spread levels and the key empirical patterns regarding the impact of firm characteristics associated with debt renegotiation on credit spreads reported in Davydenko and Strebulaev (2007). In addition, our model is the first to explain the joint distribution of debt control premia and corporate events. Specifically, the baseline model generates control premia time patterns that are consistent with the empirical evidence in Feldhütter, Hotchkiss, and Karakas (2016) that such premia increase before defaults and covenant violations. Overall, we find that modeling debt renegotiation constitutes a crucial step towards improving corporate finance models' ability to explain observed debt prices. Most improvements, however, materialize only when we incorporate that debt renegotiations can occur both in distress and outside distress.

Third, the baseline model rationalizes corporate policy choices that cannot be explained by models considering solely distressed reorganization. In particular, it motivates the frequent use of financing covenants in private debt (Roberts and Sufi, 2009a). The baseline model also generates novel cross-sectional predictions on covenant structures and investment timing.

We acknowledge that there are several mechanism to mitigate the agency friction besides non-distressed renegotiation. We test and discuss such alternative mechanisms. Non-distressed renegotiation implies larger firm value than, for example, the debt priority approach of Hackbarth and Mauer (2012) or installing a non-renegotiable covenant. In addition, it allows firms to remove the covenant without calling initial debt or violating the covenant, which is particularly important for firms with large restructuring costs or limited access to financial markets.

Literature review. We contribute to three veins of the literature. First, we link the theoretical to the empirical literature on debt renegotiation. The existing theoretical literature incorporates either distressed reorganization or non-distressed renegotiation.[1] Whereas this literature provides profound insights on the impact of debt renegotiation in an isolated state, it neglects the recent empirical literature showing that renegotiations occur, on average, relatively early after contract initiation and in both distressed and non-distressed states (Roberts and

[1]The main idea behind the distressed reorganization models in Giammarino (1989), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Mella-Barral (1999), Fan and Sundaresan (2000), and Sundaresan and Wang (2007) is that equity holders can reduce contractual debt obligations due to costly bankruptcy threats when firm performance deteriorates. Models of non-distressed renegotiation only consider static frameworks (Bergman and Callen, 1991; Berlin and Mester, 1992; Gorton and Kahn, 2000; Dessein, 2005; Garleanu and Zwiebel, 2009). The motive for renegotiation in these studies is investment distortions that harm debt holders.

Sufi, 2009b; Denis and Wang, 2014; Roberts, 2015). We show that capturing these empirical regularities is key to explaining how renegotiable debt affects firms.

Second, we complement existing studies on debt pricing (Fan and Sundaresan, 2000; Hackbarth, Miao, and Morellec, 2006; Almeida and Philippon, 2007; Chen, Collin-Dufresne, and Goldstein, 2009; Huang and Huang, 2012). Whereas corporate finance models are quite successful in replicating average observed credit spreads (Bhamra, Kuehn, and Strebulaev, 2010b; Chen, 2010), they still struggle to explain spread variations across firms (Eom, Helwege, and Huang, 2004; Zhang, Zhou, and Zhu, 2009). We contribute by showing that incorporating the ability to renegotiate in distressed and non-distressed states rationalizes cross-sectional debt pricing patterns. In addition, we address the notion that debt prices reflect surplus extracted by lenders, as formalized in a discrete time model by Rajan (1992). To our knowledge, we are the first to explain control premia patterns that have attracted the attention of the recent empirical literature (Feldhütter, Hotchkiss, and Karakas, 2016).

Finally, our study is related to the research on the effects of debt financing frictions on investment (Whited, 1992; Mella-Barral, 1999; Hennessy, 2004; Hugonnier et al., 2015; Favara et al., 2017). Our finding of a significant role of these frictions for investment are consistent with this literature. The novel aspect is that we incorporate the recent advances in empirical studies showing that non-distressed debt renegotiation is an important channel through which debt financing affects investment (Chava and Roberts, 2008; Denis and Wang, 2014). Explicitly modeling this channel as a specific mechanism behind the link between financing and investment allows us to derive novel cross-sectional predictions. In this regard, we also contribute to the literature on covenant structures (Bradley and Roberts, 2015; Kwan and Carleton, 2010) and the effect of creditor rights on financial policies Porta, Lopez-de-Silanes, and Shleifer (2008); Acharya, Sundaram, and John (2011); Vig (2013); Favara, Morellec, Schroth, and Valta (2017).

2. The model

Our framework is in the spirit of Mello and Parsons (1992) with the extension for investment of Hackbarth and Mauer (2012). The novel feature of the model is that we incorporate the ability to renegotiate debt both outside distress and in distress.

We first describe the firm's assets in place and the investment opportunity. Next, we discuss the financing covenant, covenant renegotiation, and the financing of investment. Finally, we present distressed reorganization.

2.1. Assumptions

Assets are continuously traded in complete and arbitrage-free markets. Investors may lend and borrow at the risk-free rate r. Corporate taxes are paid at a constant rate τ on operating cash flows and full offsets of corporate losses are allowed. Firms act in the best interest of equity holders and choose corporate policies to maximize equity value.

The firm's assets in place and investment opportunity. We consider a firm with assets in place and an investment opportunity. At each time t, assets in place generate a cash flow X_t. The cash flow X_t constitutes the exogenous state variable. Following Grossman and Hart (1986) and Bolton and Scharfstein (1996), we assume that X_t is observable, but not verifiable by courts or other outside parties. Hence, an ex-ante contract specifying, for example, financing or investment policies contingent on cash flows is not feasible because courts cannot enforce it. This assumption is standard in the debt overhang literature (e.g., Bhattacharya and Faure-Grimaud, 2001; Favara, Morellec, Schroth, and Valta, 2017). The cash flow X_t of the firm follows a geometric Brownian motion under the risk-neutral probability measure $\mathbb{Q}$

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t, \qquad X_0 > 0, \tag{1}$$

in which μ is the drift, σ the volatility, and W_t a Brownian motion under $\mathbb{Q}$.

In practice, covenants in place are often modified to accommodate borrowers' future operating and financing activities (Roberts, 2015). To generate cross-sectional variation in operating activities and funding needs, we incorporate an investment opportunity. The investment opportunity is modeled as an American call option on the firm's cash flow, analogous to Hackbarth and Mauer (2012). Specifically, if equity holders decide to invest, an investment cost I must be paid to scale all future cash flows by some factor $s > 1$. After investment, the firm consists of only invested assets. The investment decision is irreversible. In reality, investment opportunities are usually not perfectly correlated to existing assets or may occur and disappear randomly. Our investment specification is a simplification to investigate the impact of operating activities and funding needs on debt renegotiation in a tractable fashion.

Initially, the firm is financed by issuing equity and private debt of infinite maturity. Private debt is by far the most important source of external financing in OECD countries (Gorton and Winton, 2003). Whereas renegotiations also occur with public bonds in practice, they are more costly, have a higher exposure to free-rider problems, and are more difficult to coordinate than renegotiations with private debt holders (Rajan, 1992; Krishnaswami, Spindt, and Subramaniam, 1999).[2] After initial debt is issued, the firm pays a total coupon rate c_0^A to initial debt holders (A). The firm is subject to corporate taxes at a constant rate τ. If the promised debt service exceeds the cash flow, shareholders can inject funds to finance the coupon. Alternatively, shareholders have the possibility to default on their debt obligations (Leland, 1998), which leads to immediate liquidation. Debt holders enjoy absolute priority of their claims. In default, debt holders obtain the unlevered value of assets in place times the recovery rate α, whereas the investment opportunity is lost (Hackbarth and Mauer, 2012).[3]

Covenant renegotiation and the financing of investment. Following the literature on corporate finance models with investment, we assume that new debt can only be issued to fi-

[2]Changes to covenants, for example, must be approved by bondholders representing no less than two-thirds of the total principal. Such a majority approval is difficult in case of diffusely held public bonds because the Trust Indenture Act of 1939 gives the trustees in public bond issues only limited discretion during renegotiation outside bankruptcy.

[3]Alternatively, we assume that only a fraction of the investment opportunity is lost in default and that the remaining fraction of the investment opportunity may or may not be levered up in the future. These alternative assumptions have only a minor impact on our results.

nance investment so that a firm is choosing investment and financing policies simultaneously (Whited and Wu, 2006; Sundaresan and Wang, 2007; Gomes and Schmid, 2010; Hackbarth and Mauer, 2012). This assumption is consistent with empirical evidence showing that debt restructuring occurs commonly upon lumpy investment (Dudley, 2012). We allow the firm to install a covenant in the initial debt contract that prevents the issuance of new debt (financing covenant).[4] Covenants restricting new debt financing are ubiquitous in debt contracts of real firms (e.g., Smith and Warner, 1979, Bradley and Roberts, 2015) and more common than, for example, direct capital expenditure restrictions (e.g., Nini, Smith, and Sufi, 2012). A covenant specifying future debt financing contingent on cash flows is not feasible because X_t is not verifiable by courts.

Equity holders renegotiate this covenant with debt holders upon investment to finance the investment cost through both additional equity and new private debt. The financing of investment is determined as the Nash solution to the following renegotiation game. Debt holders can enforce the prevailing financing covenant and prevent equity holders from issuing additional debt, which constitutes debt holders' outside option or disagreement point. Equity holders' outside option is to invest immediately and finance the cost by issuing equity only. The surplus associated with this game stems from financing the investment cost by issuing a mix of debt and equity as opposed to financing with an equity issuance only. Initial debt holders obtain surplus through a compensation coupon c_1^A, resulting in a total coupon to initial debt holders of $c_{01}^A = c_0^A + c_1^A \geq c_0^A$. The compensation to initial debt holders includes both the full compensation for the dilution of initial debt due to the issuance of new debt and part of the renegotiation surplus. Equity holders' bargaining power is denoted by η. Hence, debt holders' bargaining power is $1 - \eta$. In this setup, the Nash solution corresponds to a combination of coupons $\left\{c_1^A, c_1^B\right\}$ given the initial coupon c_0^A. The values of new debt and new equity sum up to the investment cost I.

[4]In particular, the decision to implement the covenant is binary. Therefore, the model does not allow to analyze covenant tightness.

The model's compensation coupon reflects that debt holders can typically negotiate a change in contract terms that improves the value of their claim (Kahan and Rock, 2009; Feldhütter, Hotchkiss, and Karakas, 2016) and that changes in coupon at renegotiation are common in practice (Roberts and Sufi, 2009b). In Section 3.2, we show that alternative model assumptions on the mean of the compensation entail the same model solution, and, hence, do not impact the results.

In practice, non-distressed debt renegotiations occur for multiple reasons. Our representation of these events through renegotiation of a financing covenant at investment is consistent with the following empirical facts: (i) debt renegotiations are prevalently a consequence of borrowers' desire to go beyond the restrictive initial debt contract to increase investment or to alter the financial policy (Roberts and Sufi, 2009b; Denis and Wang, 2014; Roberts, 2015); (ii) debt renegotiations do not only serve to resolve distress but also constitute a governance channel through which creditors intervene in firm policies subject to conflicts of interest (Dichev and Skinner, 2002; Chava and Roberts, 2008; Denis and Wang, 2014; Bradley and Roberts, 2015): (iii) debt holders can often negotiate a change in contract terms that improves the value of their claim (Rajan, 1992; Kahan and Rock, 2009; Feldhütter, Hotchkiss, and Karakas, 2016); (iv) covenants and interest rates are the most frequently adapted debt contract terms at renegotiations (Roberts and Sufi, 2009b; Roberts, 2015). In sum, our representation captures observed features of real firm renegotiations, which allows us to explore the implications of firms' ability to renegotiate in different states.

Reorganization: Debt renegotiation in corporate distress. We model distressed reorganization as a debt-equity swap following Fan and Sundaresan (2000). In particular, if cash flows deteriorate, equity holders offer debt holders to swap their original debt against equity. We assume that the investment opportunity and the associated restructuring option are preserved in this case. A disagreement triggers immediate default. The firm's claim holders have an incentive to reorganize to avoid default costs and the loss of the investment opportunity. The equity fraction offered to debt holders in exchange for their debt, denoted by $1-\theta$, corresponds to the Nash solution. A distressed debt-equity swap implies that the firm becomes an all-equity firm, which results in a loss of the tax shield. Thus, whereas corporate taxes encourage debt financing by shielding part of a firm's cash flow from taxation, distressed reorganization limits the incentive to issue debt.

Figure 1 presents the time line of the model. Covenant renegotiation takes place at investment if there is no previous reorganization ("covenant renegotiation"). Reorganization can occur before or after investment. If equity holders first reorganize after firm initiation ("initial reorganization"), they may still invest later ("investment after initial reorganization"). Equity holders can again decide to reorganize this new debt ("reorganization after investment following reorganization") in case of distress.

INSERT FIGURE 1 HERE

If reorganization occurs after covenant renegotiation ("reorganization after covenant renegotiation"), initial debt holders, new debt holders, and equity holders bargain over the surplus. η_o and η_n denote the bargaining power of initial and new debt holders, respectively, with $\eta_o + \eta_n + \eta = 1$. We consider the asymmetric Nash solution to the multilateral bargaining game.[5]

The model uses a framework with one distressed and one non-distressed renegotiation. Including several non-distressed renegotiation rounds would impede closed form solutions. Our simple setup allows us to investigate the economic mechanisms behind the ability to renegotiate debt in different states and, thereby, explore the impact of creditor governance on firms.

[5] Britz, Herings, and Predtetchinski (2010) provide non-cooperative support for this asymmetric Nash bargaining solution. Specifically, they consider a non-cooperative multilateral bargaining game in which the proposing player in each round is determined by a Markov process. The authors show that stationary subgame perfect equilibrium payoffs converge to the weighted Nash bargaining solution with the stationary distribution of the Markov process as the weight vector.

3. Model solution

Figure 1 illustrates the use of subscripts in the model timeline. Value functions and parameters after firm initiation but before covenant renegotiation or initial reorganization carry a subscript 0. After covenant renegotiation or after initial reorganization, we use a subscript 1. To distinguish between these two cases, the additional subscript l (l for low) labels value functions after initial reorganization, while the subscript h (h for high) labels value functions after covenant renegotiation. Value functions and parameters after reorganization following investment and after investment following reorganization carry a subscript 2. A subscript 3 indicates that a firm has reorganized twice (reorganization after investment following initial reorganization).

The model is solved by backward induction. First, Section 3.1 presents an overview of corporate policies and value functions after initial renegotiation. Subsequently, Section 3.2 solves the bargaining game of covenant renegotiation and presents our main theoretical result. Finally, we solve for the value functions of corporate securities and corporate policies before covenant renegotiation or distressed reorganization (Section 3.3).

3.1. Value functions and corporate policies after initial renegotiation

Initial reorganization and the value functions thereafter. The value functions after investment following reorganization correspond to the case analyzed in Fan and Sundaresan (2000). Appendix A provides details. In the period after initial reorganization but before subsequent investment, the firm is all-equity financed with the option to invest and relever simultaneously. In this case, the value of equity, $e_{1l}(X)$, corresponds to the firm value. The investment threshold is denoted by U_1. We present both $e_{1l}(X)$ and U_1 in closed-form in Appendix A.

Next, we consider initial reorganization. Denote the initial reorganization boundary by S_0. The outside option is that equity holders default on their debt obligation and debt holders receive the unlevered value of assets in place net of default costs. The fraction of the unlevered asset value offered to debt holders in exchange for their claim at initial reorganization, $1-\theta_0$, is determined as the Nash solution to the initial reorganization game

$$\theta_0 \quad = \quad \arg\max_{\tilde{\theta}_0} \left\{\tilde{\theta}_0 e_{1l}\left(S_0\right) - 0\right\}^{\eta} \left\{\left(1-\tilde{\theta}_0\right) e_{1l}\left(S_0\right) - \alpha \frac{1-\tau}{r-\mu} S_0\right\}^{1-\eta}. \tag{2}$$

In Appendix A, we show that the sharing rule in this game corresponds to

$$\theta_0 = \eta\left(1 - \alpha \frac{v\left(S_0\right)}{e_{1l}\left(S_0\right)}\right). \tag{3}$$

Value functions after covenant renegotiation. S_1 is the reorganization boundary after covenant renegotiation. Multilateral bargaining entails a fraction $\theta_1 = \eta\left(1-\alpha\right)$ of total surplus to equity holders, thereby generalizing the outcome of bilateral bargaining calculated in Fan and Sundaresan (2000). The full solution to the multilateral bargaining game is presented in Appendix A. Let $d_{1h}\left(\cdot; c_{01}^A\right)$, $d_{1h}\left(\cdot; c_1^B\right)$, and $e_{1h}\left(\cdot; c_{01}^A + c_1^B\right)$ denote the value of initial debt, new debt, and equity, respectively, after covenant renegotiation. Because of the coexistence of two creditors at this stage, we write the dependence on the coupon explicitly. We provide the solutions to these value functions and the closed-form expression for equity holders' choice of the reorganization boundary S_1 in Appendix A.

3.2. Covenant renegotiation

The threshold triggering covenant renegotiation is denoted by U_0. We define the sharing rule as $\left\{c_{01}^A, c_1^B\right\}$, i.e., as the total coupon to initial debt holders and the coupon of new debt. $\left\{c_{01}^A, c_1^B\right\}$ is determined as the Nash solution to the covenant renegotiation game:

$$\begin{aligned}\left\{c_{01}^A, c_1^B\right\} \quad = \quad & \arg\max_{\left\{\tilde{c}_{01}^A, \tilde{c}_1^B\right\}} \left\{e_{1h}\left(sU_0; \tilde{c}_{01}^A + \tilde{c}_1^B\right) + d_{1h}^B\left(sU_0; \tilde{c}_1^B\right) - e_{1h}\left(sU_0; c_0^A\right)\right\}^{\eta} \\ & \cdot \left\{d_{1h}\left(sU_0; \tilde{c}_{01}^A\right) - d_{1h}\left(sU_0; c_0^A\right)\right\}^{1-\eta}\end{aligned} \tag{4}$$

The expression in the curly brackets in the first line of Eq. (4) on the right-hand side shows the surplus to equity holders in covenant renegotiation. This surplus is the value of equity at covenant renegotiation less equity holders' outside option. The value of equity at covenant renegotiation is the total value of equity given the cash flow after investment and the total coupon after covenant renegotiation, plus the issue proceeds from new debt, minus the investment cost. Equity holders' outside option corresponds to the total value of equity given the cash flow after investment but a coupon equal to the initial coupon, minus the investment cost. Similarly, the expression in the curly brackets in the second line of Eq. (4) displays the surplus to initial debt holders in covenant renegotiation. This surplus is the difference between the value of initial debt at covenant renegotiation and debt holders' outside option. The value of initial debt at covenant renegotiation considers the additional compensation coupon of $c_{01}^A - c_0^A > 0$. Conversely, debt holders' outside option only includes the initial coupon.

The following Proposition 1 presents the main theoretical result. Proofs are in the Appendix.

Proposition 1. *The Nash solution to the covenant renegotiation game in Eq. (4) entails firm-value maximizing financing of the investment opportunity.*

First-best leverage at covenant renegotiation follows from Pareto efficiency of the Nash solution. The proof of Proposition 1 also illustrates that equity holders and debt holders receive a fraction of total surplus corresponding to their respective bargaining power, as is standard in Nash bargaining frameworks.

Proposition 1 is robust to various model assumptions. In particular, the proposition remains valid with different debt priority structures determining the outside options in the reorganization game after covenant renegotiation. Similarly, considering alternative setups to the multilateral bargaining reorganization game (e.g., a coordination of creditors) does not impact the financing of investment. Finally, different means of compensation, such as a one-time compensation payment to initial debt holders, leave the first-best leverage result unaffected.

3.3. Value functions and corporate policies before renegotiation

Value functions. We present the value functions before covenant renegotiation or initial reorganization for equity and corporate debt, denoted by $e_0(X)$ and $d_0(X)$, respectively.

Proposition 2. *(i) The value of equity in the continuation region $S_0 \leq X \leq U_0$ is given by*

$$e_0(X) = A_0^{e_0} + A_1^{e_0} X + A_2^{e_0} X^{\beta_1} + A_3^{e_0} X^{\beta_2}, \tag{5}$$

in which

$$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{6}$$

$$A_0^{e_0} = -\frac{(1-\tau)\, c_0^A}{r}, \qquad A_1^{e_0} = \frac{1-\tau}{r-\mu}. \tag{7}$$

$A_2^{e_0}, A_3^{e_0}$ jointly solve the system $M \begin{bmatrix} A_2^{e_0} & A_3^{e_0} \end{bmatrix}^T = b^{e_0}$, in which

$$M = \begin{bmatrix} S_0^{\beta_1} & S_0^{\beta_2} \\ U_0^{\beta_1} & U_0^{\beta_2} \end{bmatrix}, \tag{8}$$

$$b^{e_0} = \begin{bmatrix} -A_0^{e_0} - A_1^{e_0} S_0 + \theta_0 e_{1l}(S_0) \\ \eta F_{1h} s U_0 + (1-\eta)\, e_{1h}\left(sU_0; c_0^A\right) - \eta d_{1h}\left(sU_0; c_0^A\right) - A_0^{e_0} - A_1^{e_0} U_0 \end{bmatrix}. \tag{9}$$

F_{1h} is the factor to calculate the first-best firm value ($f_{1h}(X) = F_{1h} X$), i.e.,

$$F_{1h} = \left[1 - \tau + \tau (1-\beta_2)^{\frac{1}{\beta_2}} (1-\theta)\right] \frac{1}{r-\mu}, \tag{10}$$

and $e_{1l}(\cdot)$ is given in closed form in Appendix A, Eqs. (39)-(41).

(ii) The value of corporate debt in the continuation region $S_0 \leq X \leq U_0$ is given by

$$d_0(X) = A_0^{d_0} + A_2^{d_0} X^{\beta_1} + A_3^{d_0} X^{\beta_2}, \tag{11}$$

in which $\beta_{1,2}$ are defined in Eq. (6) and $A_0^d = \frac{c_0^A}{r}$. A_2^d, A_3^d jointly solve the system $M \begin{bmatrix} A_2^d & A_3^d \end{bmatrix}^T = b^d$, in which

$$M = \begin{bmatrix} S_0^{\beta_1} & S_0^{\beta_2} \\ U_0^{\beta_1} & U_0^{\beta_2} \end{bmatrix}, \tag{12}$$

$$b^d = \begin{bmatrix} -A_0^d + (1-\theta_0)\, e_{1l}(S_0) \\ (1-\eta)\, F_{1h} s U_0 + (1-\eta)\, e_{1h}\left(sU_0; c_0^A\right) - \eta d_{1h}\left(sU_0; c_0^A\right) - A_0^d \end{bmatrix}. \tag{13}$$

F_{1h} is given in Eq. (10) and Appendix A, Eqs. (39)-(41), states $e_{1l}(\cdot)$ in closed form.

The value functions are independent of the model assumptions to which Proposition 1 is robust (see discussion in Section 3.2).

Reorganization threshold, renegotiation boundary, and capital structure. For a given initial coupon c_0^A, equity holders solve

$$\{S_0, U_0\} = \arg\max_{\tilde{S_0}, \tilde{U_0}} e_0(X). \tag{14}$$

The boundary conditions for equity are

$$e_0(S_0) = \theta_0 e_{1l}(S_0) \tag{15}$$

$$e_0(U_0) = e_{1h}(sU_0; c_1) - \left(I - d_{1h}\left(sU_0; c_1^B\right)\right). \tag{16}$$

Hence, the smooth-pasting conditions read

$$\frac{\partial}{\partial X} e_0(X)\,|_{X=S_0} = \eta \frac{\partial}{\partial X} e_{1l}(X)\,|_{X=S_0} - \eta\alpha \frac{1-\tau}{r-\mu} \tag{17}$$

$$\frac{\partial}{\partial X} e_0(X)\,|_{X=U_0} = \eta f_{1h} s + (1-\eta) \frac{\partial}{\partial X} e_{1h}\left(sX; c_0^A\right)|_{X=U_0} - \eta \frac{\partial}{\partial X} d_{1h}\left(sX; c_0^A\right)|_{X=U_0}, \tag{18}$$

in which we inserted θ_0 as given in Eq. (2). Finally, equity holders choose the initial capital structure by maximizing the value of their objective function ex-ante. Thus, they solve

$$c_0^A = \arg\max_{\tilde{c}_0^A} \left\{ e_0\left(X_0\right) + d_0\left(X_0\right) \right\}, \tag{19}$$

subject to Eqs. (17)-(18). A closed-form solution does not exist. We use numerical procedures and verify the optimality of the renegotiation and reorganization boundaries numerically.

4. Model analysis

In this section, we explain the friction that induces non-distressed renegotiation and analyze the impact of firms' ability to renegotiate also outside distress besides in distress.

4.1. Baseline parameters

We use the baseline parameter values of Hackbarth and Mauer (2012). Specifically, the risk free interest rate is $r = 6\%$, the risk-neutral growth rate of the cash flows $\mu = 1\%$, the volatility of the cash flow $\sigma = 25\%$, the tax advantage of debt $\tau = 15\%$, the recovery rate $\alpha = 75\%$, the initial cash flow $X_0 = 20$, the investment cost $I = 200$, and the scale parameter $s = 2$. The bargaining power of equity holders is set to $\eta = 0.5$, as in the base case of Fan and Sundaresan (2000). In distressed reorganization after covenant renegotiation, we assume that new and initial debt holders have equal bargaining power, i.e., $\eta_o = \eta_n = 0.25$.

These parameters imply an initial market-to-book ratio of 1.6 for our baseline firm, which closely reflects the average in our empirical sample of firms with private credit agreements from Nini, Smith, and Sufi (2009).[6]

[6] We calculate the initial market-to-book ratio of a model firm by dividing the market value of the firm by the value of invested assets.

4.2. The impact of incorporating non-distressed renegotiation

To investigate the impact of firms' ability to renegotiate debt also outside distress, we compare the baseline model to a benchmark model. The latter is akin to the baseline model, the only difference being that it ignores non-distressed renegotiation. Specifically, the benchmark model is the model of Fan and Sundaresan (2000), augmented for an investment opportunity in the spirit of Hackbarth and Mauer (2012). As in our baseline model, the investment is financed by issuing new equity and pari passu debt. Valuation and policies of benchmark firms are in Appendix D.

We first analyze in Panel A of Table 1 the friction that motivates non-distressed renegotiation. The first line shows the corporate policies and value of the benchmark firm. The second line depicts the benchmark first best firm, which exhibits investment and financing policies that maximize firm value. Comparing the benchmark firm with the first best firm shows distortions in corporate policies. Specifically, the baseline benchmark firm overlevers at investment (Column lev_U), overinvests (Column U_0), and underlevers initially (Column lev_0). The agency cost associated with this friction is 1.12% for our baseline parameters, as shown in Column AC. It is measured as the difference between the benchmark and first best firm values, expressed as a percentage of first best firm value. Whereas the distortions due to the agency conflict of debt are similar to those in the standard model without debt reorganization of Hackbarth and Mauer (2012), the agency cost in Table 1 is larger by factor two. This larger cost occurs because incorporating distressed reorganization induces firms to install higher initial leverage, which aggravates the agency conflict of debt. The magnitude of the agency cost is comparable to the shareholder-debtholder agency costs in Parrino and Weisbach (1999) and Childs, Mauer, and Ott (2005).

INSERT TABLE 1 HERE

In our baseline model, Proposition 1 states that firms' ability to renegotiate the financing covenant with initial debt holders induces first best leverage at covenant renegotiation. Line three of Panel A show that eliminating the agency conflict on this leverage choice further mitigates the policy distortions associated with the timing of covenant renegotiation and initial leverage.[7] Overall, the ability to renegotiate debt outside distress increases baseline firm value by 0.65% compared to the benchmark model in line one by way of mitigating policy distortions. We list the value of the ability to renegotiate outside distress in Column $VNDR$. Thus, reducing agency cost of debt motivates firms to implement a covenant and to renegotiate it outside distress.

In Panels B to I of Table 1, we explore the influence of parameter variation on the value of non-distressed renegotiation. In each panel, we change one parameter and report the resulting corporate policies and firm values. Panels B to E show that the value of non-distressed renegotiation remains of similar magnitude when bargaining power varies. Specifically, altering the bargaining power of equity holders or the allocation of bargaining power between initial and new debt holders hardly affects $VNDR$. In Panel F, we reduce the recovery rate to $\alpha = 0.55$, which reflects the estimate of Glover (2016). In Panel G, we use the tax rate of 0.27 in He and Xiong (2012). Panel H shows results for the volatility of 0.29 in Morellec, Nikolov, and Schrhoff (2012), and Panel I depicts those for the cash flow drift of 0.02 in Grenadier and Malenko (2011). Table 1 shows that for these parameters, $VNDR$ ranges between 0.34% and 1.31%.

4.3. Discussion of alternative mechanisms to mitigate the agency friction

We now discuss alternative approaches to mitigate the agency friction.

Debt priority. In a framework without renegotiations, Hackbarth and Mauer (2012) investigate how firms select the debt priority structure to mitigate the agency cost of debt. We

[7] Specifically, underinvestment occurs because initial debt holders capture part of the surplus associated with covenant renegotiation. Due to this remaining agency conflict, firms also somewhat underlever initially. The remaining agency cost of debt in the R-model compared to the benchmark first best amount to 0.47%. Renegotiation of or commitment to investment timing is not possible because X_t is observable, but not verifiable by courts or other outside parties.

consider the analogous approach, but additionally allow firms to reorganize debt in distress. Details are in Appendix D.

INSERT TABLE 2 HERE

With the debt priority approach, a firm selects the optimal debt priority among the three priority structures observed in practice, namely, equal priority (pari passu) debt, senior initial debt, and senior additional debt (Schwartz, 1989; Hackbarth and Mauer, 2012). The results are shown in line one of Table 2. With baseline parameters, the firm optimally places initial debt senior, as in Hackbarth and Mauer (2012). In line two, we present the results for the corresponding first best firm with senior initial debt (see Appendix D for details). Issuing new debt at investment improves initial senior debt holders' bargaining outcome in distressed reorganization after investment. Hence, issuing new debt entails a wealth transfer from equity holders to initial senior debt holders. To mitigate this wealth transfer, equity holders in line one underlever at investment (column lev_U), underinvest (column U_0), and underlever at initiation (column lev_0) compared to the first best investment and financing policies in line two. Overall, the agency cost of the policy distortions in the firm with optimal debt priority is 0.79%. Thus, whereas Hackbarth and Mauer (2012) find that optimal debt priority virtually eliminates agency cost of debt without debt renegotiation, we find that the debt priority approach induces considerable agency costs when incorporating firms' ability to reorganize in distress.

Comparing line three in Panel A of Table 1 with line one of Table 2 shows that the firm value with non-distressed renegotiation is larger than that with the optimal debt priority structure. In fact, we find that our non-distressed renegotiation mechanism dominates the debt priority approach for all parameter variations in Table 1 (not tabulated).

Callable debt. Fischer et al. (1989) show that a firm refinances to first best leverage if all existing debt is called and new debt is issued at the upper restructuring boundary. Thus, callable debt also reduces agency frictions by inducing optimal leverage at refinancing. This mechanism, however, can be costly. Fischer, Heinkel, and Zechner (1989), for example, assume a cost of recapitalization upon a debt call between one and ten percent of the new debt amount and Christensen, Flor, Lando, and Miltersen (2014) one of three percent. Empirically, Datta, Datta-Iskandar, and Patel (1999) estimate that the total cost of issuing new debt is around 3%. In addition, firms incur search frictions to find new debt financing (e.g., Hugonnier, Malamud, and Morellec, 2015). Thus, non-distressed debt renegotiation can be the preferred mechanism to mitigate the agency cost of debt, particularly for firms with limited access to capital markets.

Covenant violation. A firm could simply violate the financing covenant by issuing new debt and repay the existing debt instead of renegotiating with creditors. Violating a covenant without the consent of creditors, however, imposes serious consequences on firms' investment and financing policies, collateral requirements, monitoring and reporting frequencies, ratings, CEO turnover, and interest rate spreads (e.g., Chava and Roberts, 2008; Nini, Smith, and Sufi, 2009).

Non-renegotiable covenant. We also investigate the possibility that firms could commit to all-equity financing of the investment opportunity by way of installing a non-renegotiable financing covenant. For example, a firm may include this covenant in non-renegotiable public bond contracts. We find that it is not worthwhile to do so for realistic parameters because the firm value loss from giving up the flexibility to upstructure debt at investment is much larger than the agency cost of debt from overleverage at investment.

5. Results and empirical predictions

The recognition that firms renegotiate both in distress and upon investment allows us to explain empirically observed renegotiation timing patterns, debt prices, and corporate policies. It also yields novel testable predictions.

5.1. Renegotiation timing patterns

We first investigate our model's ability to explain empirically observed renegotiation timing. This step is important to validate that the model provides a useful representation of when real firms renegotiate their debt.

Renegotiation timing patterns are implied by the objectively realized probabilities. Hence, we need to specify a risk premium to relate risk-neutral to physical measures. Following Strebulaev (2007), we choose a risk premium on assets of 6.5%, which implies a cash flow drift under the statistical measure of $\mu^P = 1\% + 6.5\% = 7.5\%$. The corresponding risk premium of $\lambda = 0.26$ is similar to the $\lambda = 0.3$ used by Anderson and Carverhill (2012).

We start by exploring a baseline firm's probability to renegotiate a loan within four years after initiation.[8] The baseline firm's renegotiation probability is 47%. Only a fraction of 2% of the first renegotiations after contract initiation are distressed reorganizations, which suggests that non-distressed renegotiations are an important driver of renegotiation timing statistics.

To provide an intuition for the drivers of model-implied renegotiation probability, Fig. 2 plots this probability against a model firm's primary determinants of renegotiation, namely, leverage and market-to-book. We generate a range of market-to-book and leverage ratios by varying the scale factor s and the coupon rate c. The renegotiation probability increases with market-to-book because a more valuable investment opportunity incentivizes equity holders to renegotiate the covenant earlier. Notably, the renegotiation probability declines with leverage in case market-to-book is large. Debt holders have a larger outside option with higher leverage, i.e., a stronger bargaining position in covenant renegotiation, which mitigates equity holders' incentive to renegotiate the covenant. This negative effect dominates the positive impact of leverage on the distressed reorganization probability because the high market-to-book firms' renegotiation probability is mainly driven by covenant renegotiation. In contrast, the renegotiation probability increases with leverage for low market-to-book firms. As distressed reorganization is the main driver of the renegotiation probability for these firms, the positive impact of higher

[8] We obtain this renegotiation probability by simulation.

leverage on the distressed reorganization probability dominates the negative impact on the covenant renegotiation probability. Overall, Fig. 2 provides an explanation for why despite the simple intuition in Fan and Sundaresan (2000) that higher initial leverage should increase the distressed reorganization probability, empirical studies only find insignificant or weak evidence for the relation between debt renegotiations and leverage (Roberts and Sufi, 2009a; Roberts, 2015). A novel testable prediction from our analysis is that this relation should turn positive for high market-to-book firms due to non-distressed covenant renegotiations.

INSERT FIGURE 2 HERE

Fig. 2 also illustrates that it is crucial to incorporate the empirical cross-sectional distribution for deriving model-implied renegotiation timing statistics. For example, the renegotiation probability of an average model firm does not necessarily reflect the renegotiation probability in an empirical cross section for two reasons. First, the renegotiation probability in a cross-section also depends on the correlation between leverage and market-to-book ratios and not only on the averages of these ratios. Specifically, this probability is higher if market-to-book and leverage are negatively correlated, i.e., tend to lie on the conjunction between the higher left and the lower right corner of Fig. 2, than if they were positively correlated. Second, the renegotiation probability of an average model firm can deviate from that in the cross section because the relation between market-to-book or leverage and the renegotiation probability is non-linear.

To investigate the model's ability to explain empirically observed renegotiation timing patterns, we therefore adapt a simulation approach in the spirit of Strebulaev (2007). Specifically, we consider cross-sections of model-implied firms that reflect the real firms' cross-sectional distribution of market-to-book and leverage ratios. We then measure average renegotiation statistics in the model-implied cross-sections and compare them to the statistics in empirical sample of real firms. To obtain the empirical sample, we collect the 3,720 private credit agreements covered in Nini, Smith, and Sufi (2009) from their home page. We use this real firms sample as an empirical benchmark because several studies report the renegotiation timing patterns in the sample (e.g., Roberts and Sufi, 2009b; Denis and Wang, 2014). The sample consists of all private loans extended to nonfinancial public firms from 1996 to 2005 in the Loan Pricing Corporation Dealscan database, for which the original credit agreements are available in SEC filings. We merge these data with Compustat information, leaving us with 3,688 observations. Following the literature (e.g., Roberts and Sufi, 2009b, Denis and Wang, 2014), we calculate the average market-to-book and market leverage ratios over the four quarters previous to the initiation of each private credit agreement.[9] After dropping firms with any missing data over these four quarters, our final empirical sample consists of 3,070 private credit agreements. Each observation in the empirical cross-section is characterized by the average market-to-book and leverage ratios of the corresponding firm over the four previous quarters. We winsorize market-to-book and leverage ratios in the empirical cross-section at the 2.5% and 97.5% percentiles.

Next, we generate a model-implied sample of firms reflecting the cross-sectional joint distribution of market-to-book and leverage ratios in the empirical sample. Following Kuehn and Schmid (2014), we calculate the model-implied market-to-book ratio as the sum of the market value of the firm divided by the value of invested assets. Market leverage is the market value of debt net of the present value of the investment surplus to debt holders, divided by the market value of the firm.[10] We obtain a wide range of leverages and market-to-book ratios through variations in cash flow X (cf. Strebulaev, 2007) and in option scale parameter s (cf. Arnold, Wagner, and Westermann, 2013). To generate these variations, we simulate cash flows of a large universe of initially optimally financed firms with different option scale parameters for ten years ("pre-simulation").

For each firm in the empirical cross-section, we select the model-implied firm at the end of the pre-simulation with the shortest Euclidean distance with respect to leverage and the natural

[9]The market-to-book ratio is constructed as Total Assets (item 44) minus Book Equity plus Market Equity, all scaled by Total Assets. Book Equity is Total Assets minus Total Liabilities (item 54) minus Preferred Stock (item 55) plus Deferred Taxes (item 52). Market Equity is the Equity Price (item 14) times Common Shares Outstanding (item 61). The market leverage ratio is Book Debt, divided by Total Assets minus Book Equity plus Market Equity.

[10]We subtract the present value of debt holders' investment surplus from the gross debt value to improve the comparability of our model to both real firms and alternative models. First, the FASB principles require issue premiums of debt beyond the maturity amount of the related liability to be amortized (FASB, 2008). Thus, debt holders' anticipated investment surplus is generally not reflected in the balance sheet debt position of real firms. Second, the non-distressed renegotiation surplus is absent in alternative models such as our benchmark model or the model of Fan and Sundaresan (2000).

logarithm of one plus the market-to-book ratio. We obtain a sample of 3,070 model-implied matched firms. Details on simulation and matching procedures are in Appendix E.

Table 3 shows statistics for the cross-section of Nini, Smith, and Sufi (2009) and for our model-implied samples at matching. The matched samples are structurally similar to the empirical sample. The average distance at matching is 0.10 with an average cross-sectional standard deviation of 0.17. The mean and median market leverage ratios of the matched samples deviate by 0.01 or less from those in the empirical sample. Similarly, median market-to-book ratios are well matched. The mean model-implied market-to-book ratio of 1.44 is below the 1.69 in the empirical sample because model firms exercise sufficiently valuable investment opportunities, which bounds their market-to-book ratio from above. Finally, the matched samples also reflect the large negative correlation between market-to-book and leverage in the empirical sample.

INSERT TABLE 3 HERE

We now investigate the timing patterns of debt renegotiation predicted by our model. To this end, we simulate each matched sample forward with a quarterly frequency. This simulation is denoted as "post-simulation." To preserve the empirical sample properties in the simulated samples over time, we replace a reorganized or invested firm by the corresponding firm at matching. That is, each replacement uses a firm with identical investment opportunity and cash flow as at the time of matching. In addition, we reflect the average contract maturity of 46.7 months in Denis and Wang (2014) by choosing a horizon of 16 quarters for the post-simulation. For each of the 100 pre-simulations, we consider 100 post-simulations, resulting in a total of 10,000 simulations. We record renegotiation timing statistics in the post-simulation and compare these patterns to those in empirical samples.

Roberts and Sufi (2009b) and Denis and Wang (2014) report stylized facts about empirical renegotiation patterns in large, randomly selected subsamples of the sample of Nini, Smith, and Sufi (2009).[11] These studies provide statistics measuring the timing of the first renegotiation

[11]Denis and Wang (2014) only incorporate covenant renegotiations. Roberts and Sufi (2009b) consider all debt renegotiations with material changes even if they do not entail a covenant modification. A comparison of

after initiation, which can directly be compared to our model's predictions. The empirical benchmark statistics are summarized in columns one and two of Table 5. On average, slightly more than 60% of contracts are renegotiated at least once. The average effective duration of an initial contract, i.e., the mean waiting time to the first renegotiation or final maturity (in case of contracts not renegotiated) is 2.0 years in Denis and Wang (2014) and 1.5 years in Roberts and Sufi (2009b). We also obtain the effective duration in percentage of the initially stated maturity of 58% from Table 2 in Denis and Wang (2014) and of 64% from Table 1 in Roberts and Sufi (2009b).[12] In addition, the studies show that the time to the first renegotiation of renegotiated contracts is around 1.4 years.

In Table 5, we also report stylized timing patterns that additionally consider subsequent renegotiations besides the first renegotiation. Only around 18% of empirically observed debt renegotiations are associated with corporate distress.[13] The average time to any renegotiation or maturity divided by the initially stated maturity (duration % of stated maturity) is 61%. In addition, Figure 1 in the study of Denis and Wang (2014) suggests that around 37% of contracts are renegotiated only once, 22% are renegotiated twice, and close to 13% are renegotiated three times. About 53% of contracts are amended between two and five times during their lifespan. The study also shows that conditional on being renegotiated, the average contract is renegotiated 2.7 times.

INSERT TABLE 5 HERE

Next, we present the renegotiation patterns of model-implied samples. Column three of Table 5 summarizes the renegotiation patterns implied by the baseline model with debt renegotiations both in and outside distress. To assess the contribution of non-distressed renegotiations, we compare patterns implied by our model to those implied by the benchmark model in the fourth column. 48.2% of baseline firms and only 6.1% of benchmark firms renegotiate

the renegotiation timing patterns in Roberts and Sufi (2009b) and Denis and Wang (2014) suggests that this differentiation is of minor importance.

[12]When omitting right censored outcomes instead of assigning them to "no renegotiation" observations, the effective duration is 57% in Roberts and Sufi (2009b).

[13]Roberts (2015) confirms in his samples that most renegotiations are initiated by borrowers in response to changing conditions, rather than from lender interventions due to close default.

their contracts before maturity. Thus, the baseline model reflects the around 62% of renegotiated contracts in the empirical samples substantially better than the benchmark model. The observation that the renegotiation probability predicted by our model is below the empirical counterpart suggests that some renegotiations in the empirical samples are triggered by motives beyond those captured in the model. Further, the baseline model reflects durations more accurately than the benchmark model. Specifically, the effective duration of 2.6 years and the effective duration in percentage of the stated maturity of 65.5% in the baseline model are close to the empirical durations, whereas the benchmark model's durations are much higher then their empirical counterparts. The time to first renegotiation of 1.1 years for the renegotiated baseline firms is also close to the around 1.4 years in the empirical samples.[14]

We also use the model to predict timing and frequency patterns of first and subsequent renegotiation rounds.[15] The model implies an 8.7% portion of distressed reorganizations, which is consistent with the observation in the empirical sample of Roberts and Sufi (2009a) that the vast majority of debt renegotiations occur outside distress. By construction, all renegotiations are due to corporate distress in the benchmark model. The baseline model's duration in percent of stated maturity of 45.9% is closer to its empirical counterpart of 61% than that of the benchmark model of 91.8%. In addition, the baseline model reflects renegotiation frequencies quite well. For instance, it predicts that 32.4% of contracts are renegotiated just once compared to 37% in the empirical sample of Denis and Wang (2014) and 45.7% between two and five times compared to 53%. Finally, the average renegotiation frequency is 3.4 in the model compared to 2.7 in the empirical sample.[16]

The baseline model's ability to reflect empirical renegotiation timing patterns is robust to alternative specifications as shown in Table 5. We first trim the empirical sample at the 2.5%

[14] Whereas the 1.5 years in the benchmark model are close to its empirical counterpart, this number alone is of limited relevance because it only reflects the time to renegotiation in the 6.1% of contracts that are renegotiated.

[15] In the simulated samples, subsequent renegotiations occur in firms that have been replaced with matched firms at previous renegotiations. Hence, this part of the analysis is subject to the assumption that real firms start each renegotiation round with similar market-to-book and leverage ratios as at matching. We discuss the results with an alternative replacement assumption in the robustness discussion below.

[16] As above, the 2.1 average frequency of the benchmark model is of limited relevance because it only reflects the frequency in the 6.1% of contracts that are renegotiated.

and 97.5% percentiles in column two instead of winsorizing to ensure that our conjecture is not driven by outliers. In columns three to eight, we test the robustness of our results to parameter variations. Column nine shows the results when matching model-implied firms at the end of the pre-simulation to real firms with an alternative matching distance, namely, with the Euclidean distance based on the vector space of percentage deviations from target leverage and market-to-book. In column ten, we replace renegotiated firms in the simulation with optimally financed firms at initiation instead of with firms at matching. Column eleven applies a pre-simulation of 100 years instead of ten years. In column twelve, we run all simulations on a monthly (instead of quarterly) basis, and column 13 shows the results when book values of debt are used to construct model-implied leverage and market-to-book ratios as in Bhamra, Kuehn, and Strebulaev, 2010a. Throughout the alternative specifications, the statistics measuring the timing patterns to the first renegotiation are robust. Some deviations are present in timing patterns that additionally consider subsequent renegotiations besides the first renegotiation. For example, a higher bargaining power and lower recovery rate accelerate distressed reorganizations, and replacing renegotiated firms with optimally financed firms delays subsequent renegotiations.

Overall, the simulation analysis shows that the baseline model captures key timing and frequency patterns of real firm renegotiations, thereby considerably improving existing corporate finance models' representation of when debt renegotiations occur.

5.2. Debt prices

The recognition that debt renegotiations occur relatively shortly after contract initiation and in different firm states has implications for debt prices. In this section, we investigate these model implications and compare them to empirical credit spread and control premium patterns.

5.3. Decomposing debt prices

Empirically, credit spreads are usually observed from traded credit default swap (CDS) contracts. Such contracts define a constant spread that compensates the seller for covering the underlying firm's credit risk. As CDSs contain no right to renegotiate with the firm, Feldhütter et al. (2016) measure the control premium by calculating the percentage premium of debt over otherwise identical credit default swap (CDS) implied debt prices.

To compare our model predictions to empirical debt price patterns, it is crucial to conceptually adapt our calculation of model-implied credit spreads and control premia to these empirical concepts. To this end, we first calculate the value of a hypothetical no-surplus debt claim that still reflects the firm's credit risk. This claim corresponds to initial debt but obtains no renegotiation surplus from any renegotiation. Thus, it receives the recovery value of assets at distressed reorganization and only the compensation for the initial debt's dilution due to the issuance of new debt at non-distressed renegotiation. Details on the calculation of the no-surplus claim are in Appendix G.

To obtain the model-implied credit spread, we solve for the initial constant coupon that equates the value of the no-surplus debt claim. "Constant means that this coupon is not changed at non-distressed renegotiation. The credit spread then corresponds to the constant coupon divided by the value of the no-surplus debt claim, minus the risk-free interest rate. Intuitively, this credit spread is the constant yield over the risk-free rate that compensates debt holders for the firm's non-compliance with contractual (promised) debt payments. Thus, it conceptually corresponds to a traded CDS spread.

The control premium of initial debt is simply the difference between the initial debt value and the no-surplus debt value. Intuitively, it reflects the present value of the surplus that debt holders can extract from the firm through debt renegotiation, expressed as a fraction of the initial debt value.

5.3.1. Credit spreads

We now investigate the model's ability to explain observed credit spreads. To this end, we calibrate model firms to the median empirical firm in the study of Davydenko and Strebulaev

(2007), which empirically estimates the impact of firm characteristics associated with debt renegotiation on corporate credit spreads in a large sample of real firms (empirical sample).[17]

The median empirical firm has a leverage of 30.3%, a market-to-book ratio of 1.47, and an asset volatility of 20.4%. The median risk free interest rate in the empirical sample is 5.94%. We use the median empirical firm as a benchmark because the market-to-book ratio of this firm can be approximated more closely with model firms than that of the mean firm (see Section 5.1). Column one of Table 6 summarizes credit spread characteristics in the empirical sample. The median firm's long term credit spread in Panel A is 109 basis point (bps). Panel B shows the spread sensitivities to firm variables associated with debt renegotiation (strategic variables) from Table VI in Davydenko and Strebulaev (2007). Panel C summarizes the spread sensitivities to standard firm variables (non-strategic variables) from Table V in Davydenko and Strebulaev (2007).

In columns two to four of Table 6, we summarize the predictions of different models for credit spreads. We calculate the marginal impact of bargaining power as the first derivative of a typical firm's credit spread with respect to bargaining power. The effect of the bargaining friction is the credit spread of a model firm with bargaining failure minus that of a model firm without failure, following the approach in the Appendix of (Davydenko and Strebulaev, 2007).[18] For each model, we select the initial coupon and investment scale parameter to match the median empirical leverage and market-to-book ratios. We also set a volatility of 20.4% and a risk-free interest rate of 5.94%. The remaining model parameters, for which we find no correspondence in the empirical study of Davydenko and Strebulaev (2007), are set equal to our baseline parameters.

INSERT TABLE 6 HERE

[17]Davydenko and Strebulaev (2007) extract credit spreads from corporate bond yields. A concern with using bond yields instead of CDS spreads is that bond yields could also reflect a control premium. The control premium of corporate bonds, however, is quantitatively very small. Specifically, Feldhütter, Hotchkiss, and Karakas (2016) show that the medium present value of the control premium is only 32 bps, which converts to a spread annuity of only around 2 bps. In addition, Zhang, Zhou, and Zhu (2009) investigate the impact of the recovery rate and leverage on traded CDS spreads and find results that are similar to Davydenko and Strebulaev (2007).

[18]Bargaining failure implies that the firm needs to finance the investment cost by issuing equity and immediately defaults at distressed reorganization.

Column two of Table 6 shows that the baseline model reflects the key credit spread patterns of the empirical sample. Specifically, the credit spread of the median baseline firm is 113 bps. This level is very close to the 109 bps in the empirical sample. In addition, all strategic and non-strategic variables have the correct sign. We also compare the quantitative magnitudes of the model-implied spread sensitivities to strategic variables with those in the empirical sample. The sensitivity to the recovery rate is -192 bps, to bargaining power is 31 bps, and to the bargaining friction is -82 bps. The corresponding magnitudes in the empirical sample are -9 to -73 bps for the recovery rate, 23 to 115 bps for bargaining power, and -14 to -50 bps for the renegotiation friction.[19] It should be pointed out that the proxies in the empirical sample are noisy measures of the underlying strategic variables and, hence, the empirical estimates of the magnitudes only serve as a lower bound (Davydenko and Strebulaev, 2007). Though, the exercise shows that the quantitative magnitudes of the model-implied sensitivities are within a reasonable range.

Column three of Table 6 summarizes credit spread patterns in the benchmark model. The median benchmark firm has a credit spread of 249 bps. This spread considerably overestimates its empirical counterpart of 109 bps because the inability to renegotiate with debt holders outside distress induces equity holders to overlever at investment. In addition, the benchmark model fails to reflect the empirically positive impact of liquidation costs on credit spreads, as shown in Panel B. Specifically, a higher liquidation cost mitigates equity holders' incentive to overlever at investment, which explains the negative sensitivity in the benchmark model.

In column four, we analyze credit spreads in the debt priority approach of Hackbarth and Mauer (2012). We consider senior initial debt that is the optimal debt structure of the baseline parameters. Panel A implies that the model severely underestimates the empirical credit spread. Specifically, this model predicts that initial debt only carries minor credit risk because the firm places this debt senior to the junior debt that is issued at investment. In addition, the model fails to predict the empirically positive sensitivity of credit spreads to the market-to-book ratio.

[19] We consider all significant proxies of Table VI in Davydenko and Strebulaev (2007) to calculate these empirical ranges but omit the two normalized proxies for which the study does not provide the statistics necessary to calculate the magnitude of the marginal effects.

A caveat with analyzing a typical firm is that because credit spreads are nonlinear in firm characteristics, the spread of a median model firm could deviate from the median spread of an empirical cross-section of real firms (Bhamra, Kuehn, and Strebulaev, 2010b). Therefore, we use the baseline model to measure credit spreads in 1000 matched samples of the simulation in Section 5.1. We apply this approach to ensure that the model-implied matched samples, in which we measure median credit spreads, are structurally similar to the empirical cross-section of Nini, Smith, and Sufi (2009). The matched samples have a median credit spread of 207 bps, with a median market leverage of 0.39 and a median market-to-book ratio of 1.36. These numbers mirror those in the empirical cross-section of Nini, Smith, and Sufi (2009), which has a median credit spread of 198 bps with a median market leverage of 0.39 and a median market-to-book ratio of 1.43.[20] Thus, our simulation exercise confirms that the baseline model reflects the observed median credit spread of the empirical cross-section of real firms.

Overall, our results emphasize the central role of incorporating both distressed and non-distressed renegotiations to generate credit spread patterns that are consistent with their empirical counterparts. Thus, our model provides an important step in improving the ability of corporate finance models to explain the levels and cross-sectional determinants of real firms' credit spreads.

5.3.2. Control premium

To explore the control premium in our model, we first plot the average control premium in a typical post-simulation of Section 5.1 before distressed reorganizations in Figure 3 and non-distressed renegotiations in Figure 4.

INSERT FIGURES 3 AND 4 HERE

The recent empirical literature confirms that creditor control generates observable premiums in bond prices (Feldhütter, Hotchkiss, and Karakas, 2016). The time-series patterns of the joint

[20] The median interest rate spread over LIBOR in Nini, Smith, and Sufi (2009) is 150 bps and the median TED-spread in the empirical sample period is 48 bps (see https://fred.stlouisfed.org/series/TEDRATE#0).

distribution of control premia and corporate events in our model are consistent with those in observed bond prices. Specifically, Figure 3 shows that the average control premium increases as firms get closer to distressed reorganization and peaks close to the reorganization date. Figure 4 implies that the average control premium also increases before non-default renegotiation states and peaks close to these renegotiations. Feldhütter, Hotchkiss, and Karakas (2016) find the same patterns in their empirical sample for both defaults and non-default covenant violations,[21] i.e., the average control premium of observed bond prices increases towards these debt renegotiation events and peaks around the renegotiation date.

In addition to the joint distribution of average control premia and corporate events in one simulated economy, we also present statistics over all simulations of Section 5.1 in Table 7. The numbers confirm that our model rationalizes the time-series patterns of control premia.

INSERT TABLE 7 HERE

Compared to the empirical magnitudes, the control premium that we calculate in our model is relatively large because we consider private debt, which creditors can directly renegotiate with the firm. In contrast, Feldhütter, Hotchkiss, and Karakas (2016) measure the control premium of public bonds, which are difficult to renegotiate due to free-rider and coordination problems (Rajan, 1992; Krishnaswami et al., 1999). They argue that bond prices still reflect a control premium because bond holders indirectly profit from private debt renegotiations. For example, private creditors can renegotiate a corporate strategy with low risk and an increase in their private debt interest rate, which causes their control premium. Public bonds indirectly reflect a control premium because these claims' value also increases when private creditors renegotiate a low risk strategy. This premium is, however, smaller than that in private debt because public bondholders do not profit from the larger private debt interest rate.

Finally, the R-model is also consistent with the cross-sectional prediction in Feldhütter, Hotchkiss, and Karakas (2016) that control premia increase with debt holders' bargaining power.

[21] Covenant violations rarely lead to liquidation or bankruptcy but rather to debt renegotiation (Nini, Smith, and Sufi, 2012; Roberts and Sufi, 2009a). Feldhütter, Hotchkiss, and Karakas (2016) argue that control is valuable to debt holders around such non-default events because creditors attain more influence on firms.

5.4. Corporate policies

Covenant structure. In Section 4.2, we show that installing a renegotiable covenant increases firm value by way of mitigating agency cost. In contrast, installing a covenant in the benchmark model reduces firm value because the inability to renegotiate the covenant outside distress prevents the firm from upstructuring debt. Hence, our model with non-distressed renegotiation helps explain why financing covenants are ubiquitous in credit contracts (Bradley and Roberts, 2015). In addition, the model is consistent with the empirical fact that private debt is more likely to have restrictive covenants than public bonds, which are hard to renegotiate (Kahan and Tuckman, 1995; Kwan and Carleton, 2010).

Table 1 shows that lower bargaining power of equity holders entails a higher value of non-distressed renegotiation. The primary reason is that higher bargaining power of debt holders decreases the agency cost of debt, thereby increasing firm value. A resulting novel prediction is that firms should have a stronger propensity to install a financing covenant in case equity holders' bargaining power is low. In addition, the model predicts that financing covenant use increases with cash flow volatility. This cross-sectional prediction is consistent with the empirical result in Bradley and Roberts (2015) that private debt covenants are more likely for firms with higher cash flow volatility.

Investment timing. Whereas previous theories suggest that financing affects investment timing (Mauer and Triantis, 1994; Boyle and Guthrie, 2003; Whited, 2006), this extant literature remains silent on the exact mechanism of how financing affects the investment of real firms. Usually, they assume an exogenous financing friction cost or an exogenous limit on the available external financing. Recent empirical studies emphasize covenant renegotiations as an important mechanism behind the investment financing link (Chava and Roberts, 2008). We show that incorporating this mechanism is, generally, crucial to show how financing affects investment timing and, specifically, key to derive specific cross-sectional predictions on when the impact of financing on investment is more pronounced.

To highlight the investment financing link, we investigate the impact of firm characteristics associated with debt renegotiation on investment. Figure 5 first plots the baseline firm's investment boundary against bargaining power (solid line). For each η, we use the optimal initial leverage. The dashed line shows the investment boundary of the first best firm. This firm has value maximizing investment financing and investment policies. For each η, we choose the initial coupon of the baseline firm. The figure has two implication. First, the baseline firm's equity holders underinvest compared to the first best because they need to share part of the renegotiation surplus at investment with debt holders and new debt transfers wealth from equity to initial debt. Second, the tendency to underinvest decreases with bargaining power. This effect occurs because the surplus to debtholders at investment declines with bargaining power. Our results regarding the impact of bargaining power on investment generalize the finding in Favara, Morellec, Schroth, and Valta (2017) who derive the same predictions but only for distressed firms.

INSERT FIGURE 5 HERE

We also plot the investment policies for the benchmark model (solid line) and the first best benchmark model in Figure 6. As in Hackbarth and Mauer (2012), firms overinvest compared to the first best because equity holders can transfer wealth from debt holders at investment by issuing too much new equity. This tendency declines with bargaining power for two reasons. First, a higher η implies lower initial leverage, which mitigate the wealth transfer problem. Second, issuing new debt generates lower issue proceeds with higher η because of the large reorganization risk, which reduces equity holders' incentives to issue too much new debt.

INSERT FIGURE 6 HERE

Similarly, Figures 7 and 8 show that incorporating non-distressed renegotiation also generates predictions with respect to the recovery rate that are contrary to those in the benchmark model. Specifically, the tendency to underinvest in the baseline model increases with the recovery rate. A higher recovery rate induces larger initial leverage, which increases the wealth transfer from equity to initial debt at investment. In contrast, the tendency to overinvest declines with the recovery rate in the benchmark model because a higher recovery rate makes initial debt less risky, which mitigates the wealth transfer problem.

INSERT FIGURE 7 HERE

INSERT FIGURE 8 HERE

Overall, our results on investment have two implications. First, incorporating non-distressed renegotiation is crucial to capture the impact of financing on investment, leading to contrary predictions compared with models that consider only distressed reorganization. Second, factors associated with debt renegotiation are an important determinant of investment timing.

6. Conclusion

Existing corporate finance models cannot explain key features of the impact of renegotiable debt on firms. We argue that this gap is caused by neglecting that, in practice, most debt renegotiations occur already early after contract initiation and arise both in and outside distress (see, e.g., Roberts and Sufi, 2009b). We develop a model in which firms can renegotiate debt with creditors either upon investment to waive a financing covenant or at corporate distress to reorganize the capital structure. Covenant renegotiation mitigates agency costs of debt and distressed reorganization avoids bankruptcy costs.

We show that broadening the events in which firms renegotiate debt from distressed to non-distressed states is crucial to explain how renegotiable debt affects firms. The model rationalizes renegotiation timing patterns and explains a rich set of cross-sectional empirical results in terms of debt prices. It also yields novel predictions on corporate policies.

The insights motivate further empirical research. For instance, a recent literature emphasizes the role of creditor rights for distressed firms' access to finance, leverage, investment, risk taking, and debt demand (Porta, Lopez-de-Silanes, and Shleifer, 2008; Acharya, Sundaram, and John, 2011; Vig, 2013; Favara, Morellec, Schroth, and Valta, 2017). Our results on the importance of non-distressed renegotiations imply that creditor rights affect corporate policies also outside distress. Further, the creditor governance channel should have implications for how firms select their debt ownership structure.

Appendix

A. Value functions after initial reorganization or covenant renegotiation

Value functions after initial reorganization. Define S_0^ν $[U_0^\nu]$ as the first time the firm reaches the initial reorganization [investment] threshold:

$$S_0^\nu \;:=\; \inf\{t \geq 0 : X_t \leq S_0 \,|X_0 = X\}, \tag{20}$$

$$U_0^\nu \;:=\; \inf\{t \geq 0 : X_t \geq U_0 \,|X_0 = X\}. \tag{21}$$

T_0 is the first time the firm reaches initial reorganization or investment:

$$T_0 := \inf\{S_0^\nu, U_0^\nu\}. \tag{22}$$

We first consider the case in which $T_0 = S_0^\nu$, i.e., reorganization occurs before covenant renegotiation. The value functions and corporate policies after investment following initial reorganization are analogous to Fan and Sundaresan (2000). We also present them for the sake of completeness.

Unlevered firm value. The unlevered firm value $v(X)$ is given by

$$v(X) = \frac{1-\tau}{r-\mu} X. \tag{23}$$

Denote S_2 as the firm's reorganization threshold after investment following initial reorganization. Following Fan and Sundaresan (2000), the sharing rule $1-\theta_2$, i.e., the fraction of equity offered to debt holders in exchange for their claim, is determined as the Nash solution to the final reorganization game

$$\theta_2 = \arg \max_{0 \leq \tilde{\theta}_2 \leq 1-\alpha} \left\{\tilde{\theta}_2 v(S_2) - 0\right\}^\eta \left\{\left(1-\tilde{\theta}_2\right) v(S_2) - \alpha v(S_2)\right\}^{1-\eta} = \eta(1-\alpha). \tag{24}$$

The valuation of equity after investment following initial reorganization. The value of equity after investment following initial reorganization, denoted by $e_2(X)$, corresponds to the present value of the expected payoffs to shareholders until reorganization plus the payoff at reorganization:

$$e_2(X) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{S_2^{\nu}} e^{-r(u-t)}(1-\tau)(X_u - c_2)\,\mathrm{d}u + e^{-rS_2^{\nu}}\theta_2 v(S_2)\,|X_t = X\right]. \tag{25}$$

S_2^{ν} is the first time the firm reaches the reorganization threshold after investment following reorganization, i.e.,

$$S_2^{\nu} = \inf\{t \geq T_0 : X_t \leq S_2\,|T_0 = S_0^{\nu}\}, \tag{26}$$

and c_2 is the coupon of the firm after investment following reorganization. Solving the corresponding Hamilton-Jacobi-Bellman equation yields

$$e_2(X) = A_0^{e_2} + A_1^{e_2}X + A_2^{e_2}X^{\beta_2}, \tag{27}$$

in which

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{28}$$

$$A_0^{e_2} = -\frac{1-\tau}{r}c_2, \qquad A_1^{e_2} = \frac{1-\tau}{r-\mu}, \tag{29}$$

$$A_2^{e_2} = S_2^{-\beta_2}\left(\frac{1-\tau}{r}c_2 - (1-\theta_2)\frac{1-\tau}{r-\mu}S_2\right). \tag{30}$$

Equity holders' choice of the reorganization threshold after investment following initial reorganization is

$$S_2 = \frac{\beta_2}{\beta_2 - 1}\frac{r-\mu}{r}\frac{1}{1-\theta_2}c_2. \tag{31}$$

The valuation of corporate debt after investment following initial reorganization. Similarly, the value of total corporate debt after investment following initial reorganization, denoted by

$d_2(X)$, corresponds to the present value of the expected payoffs to debt holders until reorganization plus the payoff at reorganization, i.e.,

$$d_2(X) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{S_2^{\nu}} e^{-r(u-t)} c_2 \mathrm{d}u + e^{-rS_2^{\nu}} (1-\theta_2)\, v(S_2) \,|X_t = X\right]. \tag{32}$$

Solving the associated Hamilton-Jacobi-Bellman equation, we find that

$$d_2(X) = A_0^{d_2} + A_2^{d_2} X^{\beta_2}, \tag{33}$$

in which $A_0^{d_2} = \frac{c_2}{r}$ and

$$A_2^{d_2} = \left(\frac{r-\mu}{r}\right)^{-\beta_2} \left(\frac{\beta_2}{\beta_2 - 1}\right)^{-\beta_2} c_2^{1-\beta_2} \frac{1}{r} \left(\frac{1}{1-\theta_2}\right)^{1-\beta_2} \left((1-\tau)\frac{\beta_2}{\beta_2-1} - 1\right), \tag{34}$$

and β_2 is defined in Eq. (28).

Capital structure at investment after initial reorganization. The threshold for investment after initial reorganization is denoted by U_1. At investment after reorganization, equity holders choose the first-best capital structure because the firm is an all equity financed firm after initial reorganization and the issue proceeds of new debt accrue to equity holders. The optimal (first-best) capital structure maximizes firm value, which yields the coupon

$$c_2 = c_2^{fb} = \frac{r}{r-\mu} \frac{\beta_2 - 1}{\beta_2} (1-\beta_2)^{\frac{1}{\beta_2}} (1-\theta_2)\, U_1. \tag{35}$$

We calculate the corresponding first-best firm value as

$$f_2(U_1) = f_2^{fb}(U_1) = \left[1 - \tau + \tau (1-\beta_2)^{\frac{1}{\beta_2}} (1-\theta_2)\right] \frac{U_1}{r-\mu}. \tag{36}$$

The firm value can be decomposed into the unlevered asset value and the value of the tax shield, in which the latter also incorporates the loss of tax shield due to the debt-equity swap in case of final reorganization.

The valuation of equity after initial reorganization. The value of equity after initial reorganization, denoted by $e_{1l}(X)$, corresponds to the present value of the expected payoffs to shareholders until investment plus the expected value of the payoff at investment:

$$e_{1l}(X) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{U_1^\nu} e^{-r(u-t)}(1-\tau)X_u \mathrm{d}u + e^{-rU_1^\nu}(f_2(sU_1) - I)\,|X_t = X\right]. \tag{37}$$

U_1^ν is the first time the firm reaches the investment threshold after initial reorganization, i.e.,

$$S_2^\nu = \inf\{t \geq T_0 : X_t \geq U_1\,|T_0 = S_0^\nu\}, \tag{38}$$

and U_1 is the firm's investment threshold after initial reorganization. $f_2(\cdot)$ is the firm value after investment following initial reorganization as given in closed-form by Eq. (36). Solving the corresponding Hamilton-Jacobi-Bellman equation yields

$$e_{1l}(X) = A_1^{e_{1l}}X + A_2^{e_{1l}}X^{\beta_1}, \tag{39}$$

in which

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{40}$$

$$A_1^{e_{1l}} = \frac{1-\tau}{r-\mu}, \qquad A_2^{e_{1l}} = U_1^{-\beta_1}\frac{I}{\beta_1 - 1}. \tag{41}$$

Equity holders' choice of investment boundary, U_1, is given as

$$U_1 = \frac{\beta_1}{\beta_1 - 1}\frac{(r-\mu)I}{(s-1)(1-\tau) + s\tau(1-\beta_2)^{\frac{1}{\beta_2}}(1-\theta_2)}. \tag{42}$$

Initial reorganization. The initial reorganization problem reads

$$\theta_0 = \arg\max_{\tilde{\theta}_0}\left\{\tilde{\theta}_0 e_{1l}(S_0) - 0\right\}^\eta \left\{\left(1-\tilde{\theta}_0\right)e_{1l}(S_0) - \alpha\frac{1-\tau}{r-\mu}S_0\right\}^{1-\eta}. \tag{43}$$

The solution to this problem is

$$\theta_0 = \eta \left(1 - \alpha \frac{v_2(S_0)}{e_{1l}(S_0)}\right). \tag{44}$$

Value functions after covenant renegotiation. We now consider the case in which $T_0 = U_0^{\nu}$, i.e., investment occurs before initial reorganization. The value of equity after covenant renegotiation, $e_{1h}(X)$, is analogous to the case after investment following initial reorganization (Eqs. (27)-(30)). The reorganization threshold after covenant renegotiation exhibits the same functional form as the reorganization threshold after investment following initial reorganization (Eq. (31)):

$$S_1 = \frac{\beta_2}{\beta_2 - 1} \frac{r - \mu}{r} \frac{1}{1 - \theta_1} \left(c_{01}^A + c_1^B\right). \tag{45}$$

The value function of total corporate debt is analogous to the value function of corporate debt after investment following initial reorganization (Eqs. (33)- (34)).

In case of reorganization after covenant renegotiation, equity holders bargain with initial and new debt holders over the unlevered value of the assets. A sharing rule is determined by $\{\theta_1, \theta_1^o, \theta_1^n\}$, $\theta_1 + \theta_1^o + \theta_1^n = 1$, denoting the fraction of unlevered assets θ_1 $[\theta_1^o, \theta_1^n]$ to equity holders [initial debt holders, new debt holders], respectively. The asymmetric Nash solution to the multilateral bargaining game maximizes the asymmetric Nash product:

$$\{\theta_1^o, \theta_1^n\} = \arg\max_{\tilde{\theta}_1^o, \tilde{\theta}_1^n} \left(\tilde{\theta}_1 v(S_1)\right)^{\eta} \cdot \left(\tilde{\theta}_1^o v(S_1) - O^o(S_1)\right)^{\eta_o} \cdot \left(\tilde{\theta}_1^n v(S_1) - O^n(S_1)\right)^{\eta_n}, \tag{46}$$

with $\tilde{\theta}_1 = 1 - \tilde{\theta}_1^o - \tilde{\theta}_1^n$. $O^o(\cdot)$ and $O^n(\cdot)$ denote the outside option of initial and new debt holders, respectively:

$$O^o(S_1) = \frac{c_{01}^A}{c_{01}^A + c_B^1} \alpha \frac{1 - \tau}{r - \mu} S_1, \tag{47}$$

$$O^n(S_1) = \frac{c_1^B}{c_{01}^A + c_B^1} \alpha \frac{1 - \tau}{r - \mu} S_1 = \alpha v(S_1) - O^o(S_1). \tag{48}$$

The solution to problem (46) is given by

$$\theta_1^o = \eta_o(1-\alpha) + \frac{O^o(S_1)}{v(S_1)} = \eta_o(1-\alpha) + \alpha\frac{c_{01}^A}{c_{01}^A + c_B^1}, \tag{49}$$

$$\theta_1^n = \eta_n(1-\alpha) + \frac{O^n(S_1)}{v(S_1)} = \eta_n(1-\alpha) + \alpha\frac{c_1^B}{c_{01}^A + c_B^1}. \tag{50}$$

Consequently,

$$\theta_1 = \eta(1-\alpha). \tag{51}$$

The sharing rule $\{\theta_1, \theta_1^o, \theta_1^n\}$ is intuitive: Each party receives the value of its outside option and a fraction of the surplus that corresponds to the party's respective bargaining power.

The value function of initial debt holders' corporate debt is given by:

$$d_{1h}\left(X; c_{01}^A\right) = A_0^{d_{1h}^A} + A_2^{d_{1h}^A} X^{\beta_2}, \tag{52}$$

in which

$$A_0^{d_{1h}^A} = \frac{c_{01}^A}{r} \tag{53}$$

$$A_2^{d_{1h}^A} = S_1^{-\beta_2}\left(-\frac{c_A^{01}}{r} + \eta_o(1-\alpha)v(S_1) + O^o(S_1)\right), \tag{54}$$

and S_1 is given in closed form in Eq. (45). Analogously, the value function of new debt holders' corporate debt can be written as:

$$d_{1h}\left(X; c_1^B\right) = A_0^{d_{1h}^B} + A_2^{d_{1h}^B} X^{\beta_2}, \tag{55}$$

in which $A_0^{d_{1h}^B} = \frac{c_1^B}{r}$ and

$$A_2^{d_{1h}^A} = S_1^{-\beta_2}\left(-\frac{c_B^1}{r} + \eta_n(1-\alpha)v(S_1) + O^n(S_1)\right). \tag{56}$$

B. Covenant renegotiation

Proof of Proposition 1. The covenant renegotiation problem is stated in Eq. (4). Define $\tilde{c}_1 := \tilde{c}_{01}^A + \tilde{c}_1^B$. A change of variables yields

$$\begin{aligned}
& \max_{\{\tilde{c}_{01}^A, \tilde{c}_1^B\}} \left(e_{1h}\left(sU_0; \tilde{c}_{01}^A + \tilde{c}_1^B\right) + d_{1h}\left(sU_0; \tilde{c}_1^B\right) - e_{1h}\left(sU_0; c_0^A\right)\right)^{\eta} \\
& \cdot \left(d_{1h}\left(sU_0; \tilde{c}_{01}^A\right) - d_{1h}\left(sU_0; c_0^A\right)\right)^{1-\eta} \\
= & \max_{\{\tilde{c}_{01}^A, \tilde{c}_1\}} \left(e_{1h}\left(sU_0; \tilde{c}_1\right) + d_{1h}\left(sU_0; \tilde{c}_1 - \tilde{c}_{01}^A\right) - e_{1h}\left(sU_0; c_0^A\right)\right)^{\eta} \\
& \cdot \left(d_{1h}\left(sU_0; \tilde{c}_{01}^A\right) - d_{1h}\left(sU_0; c_0^A\right)\right)^{1-\eta} \\
= & \max_{\{\tilde{c}_1\}} \Bigg[\max_{\{\tilde{c}_{01}^A\}} \left(e_{1h}\left(sU_0; \tilde{c}_1\right) + d_{1h}\left(sU_0; \tilde{c}_1 - \tilde{c}_{01}^A\right) - e_{1h}\left(sU_0; c_0^A\right)\right)^{\eta} \\
& \cdot \left(d_{1h}\left(sU_0; \tilde{c}_{01}^A\right) - d_{1h}\left(sU_0; c_0^A\right)\right)^{1-\eta} \Bigg] . \qquad (57)
\end{aligned}$$

The last equality uses the fact that $\tilde{c}_1$ is bounded by the firm's debt capacity from above and by zero from below; $\tilde{c}_{01}^A$ is bounded by $\tilde{c}_1$ from above and by zero from below; hence, the range of $\left\{\tilde{c}_{01}^A, \tilde{c}_1\right\}$ is a convex and compact set in $\mathbb{R}^2$. This property further guarantees the existence of the maximum. We proceed by solving the following problem for an arbitrary, but fixed, $\tilde{c}_1$:

$$\begin{aligned}
\left\{c_{01}^A\right\} = & \arg\max_{\{\tilde{c}_{01}^A\}} \left(e_{1h}\left(sU_0; \tilde{c}_1\right) + d_{1h}\left(sU_0; \tilde{c}_1 - \tilde{c}_{01}^A\right) - e_{1h}\left(sU_0; c_0^A\right)\right)^{\eta} \\
& \cdot \left(d_{1h}\left(sU_0; \tilde{c}_{01}^A\right) - d_{1h}\left(sU_0; c_0^A\right)\right)^{1-\eta} . \qquad (58)
\end{aligned}$$

The surplus from covenant renegotiation to equity holders ($SE(\cdot)$), debt holders ($SD(\cdot)$), and total ($ST(\cdot)$), respectively, is given as

$$SE\left(U_0; c_{01}^A, c_1^B\right) = e_{1h}\left(sU_0; c_{01}^A + c_1^B\right) + d_{1h}\left(sU_0; c_1^B\right) - e_{1h}\left(sU_0; c_0^A\right), \qquad (59)$$

$$SD\left(U_0; c_{01}^A, c_1^B\right) = d_{1h}\left(sU_0; c_{01}^A\right) - d_{1h}\left(sU_0; c_0^A\right), \qquad (60)$$

$$ST\left(U_0; c_{01}^A, c_1^B\right) = f_{1h}\left(sU_0; c_{01}^A, c_1^B\right) - e_{1h}\left(sU_0; c_0^A\right) - d_{1h}\left(sU_0; c_0^A\right), \qquad (61)$$

in which $f_{1h}(\cdot)$ denotes the firm value after covenant renegotiation. For any given total coupon c_1, total firm value and total surplus are independent of the coupon to initial debt holders c_{01}^A, such that we can write

$$ST\left(U_0; c_{01}^A, c_1^B\right) = ST\left(U_0; c_1\right). \tag{62}$$

Intuitively, firm value is determined by the total coupon, but not by the allocation of the total coupon to new and initial debt holders. Hence, re-writing Eq. (58) by using this insight and the definition of total surplus in Eq. (61) yields that for any given $\tilde{c}_1$, c_{01}^A is the solution to the maximization problem

$$c_{01}^A = \arg\max_{\{\tilde{c}_{01}^A\}} \left(SE\left(U_0; \tilde{c}_{01}^A, \tilde{c}_1\right)\right)^\eta \left(ST\left(U_0; \tilde{c}_1\right) - SE\left(U_0; \tilde{c}_{01}^A, \tilde{c}_1\right)\right)^{1-\eta}. \tag{63}$$

The first order condition with respect to c_{01}^A reads:

$$\eta \frac{\frac{\partial}{\partial c_{01}^A} SE\left(U_0; c_{01}^A, \tilde{c}_1\right)}{SE\left(U_0; c_{01}^A, \tilde{c}_1\right)} + (1-\eta) \frac{-\frac{\partial}{\partial c_{01}^A} SE\left(U_0; c_{01}^A, \tilde{c}_1\right)}{ST\left(U_0; \tilde{c}_1\right) - SE\left(U_0; c_{01}^A, \tilde{c}_1\right)} = 0, \tag{64}$$

which is equivalent to

$$SE\left(U_0; c_{01}^A, \tilde{c}_1\right) = \eta ST\left(U_0; \tilde{c}_1\right). \tag{65}$$

Consequently,

$$SD\left(U_0; c_{01}^A, \tilde{c}_1\right) = (1-\eta) ST\left(U_0; \tilde{c}_1\right). \tag{66}$$

That is, each party receives a fraction of total surplus corresponding to its respective bargaining power. Plugging Eqs. (65) and (66) into Eq. (57) yields the maximization problem

$$\begin{aligned} c_1 &= \arg\max_{\{\tilde{c}_1\}} \left(SE\left(U_0; c_{01}^A, \tilde{c}_1\right)\right)^\eta \left(ST\left(U_0; \tilde{c}_1\right) - SE\left(U_0; c_{01}^A, \tilde{c}_1\right)\right)^{1-\eta} \\ &= \arg\max_{\{\tilde{c}_1\}} \left(\eta ST\left(U_0; \tilde{c}_1\right)\right)^\eta \left((1-\eta) ST\left(U_0; \tilde{c}_1\right)\right)^{1-\eta}. \end{aligned} \tag{67}$$

The first order condition with respect to c_1 is given by:

$$\eta \frac{\eta \frac{\partial}{\partial c_1} ST\left(U_0; c_1\right)}{\eta ST\left(U_0; c_1\right)} + (1-\eta) \frac{(1-\eta) \frac{\partial}{\partial c_1} ST\left(U_0; c_1\right)}{(1-\eta) ST\left(U_0; c_1\right)} = 0, \tag{68}$$

which is equivalent to

$$\frac{\partial}{\partial c_1} ST\left(U_0; c_1\right) = 0. \tag{69}$$

Due to concavity of total surplus, Condition (69) implies that the total new coupon is determined such that total surplus is maximized. Surplus maximization is equivalent to firm value maximization. Hence, the total new coupon corresponds to the first-best capital structure at U_0 and is given in closed-form as

$$c_1 = c_1^{fb} = \frac{r}{r-\mu} \frac{\beta_2 - 1}{\beta_2} \left(1-\beta_2\right)^{\frac{1}{\beta_2}} \left(1-\theta\right) sU_0, \tag{70}$$

analogous to Appendix A, Eq. (35). Thus,

$$c_1^B + c_{01}^A = c_1^{fb}. \tag{71}$$

The corresponding first-best firm value at U_0 is analogous to Eq. (36) of Appendix A:

$$f_1\left(U_0\right) = f_1^{fb} = \left[1 - \tau + \tau\left(1-\beta_2\right)^{\frac{1}{\beta_2}} \left(1-\theta\right)\right] \frac{U_0}{r-\mu}. \tag{72}$$

C. Value functions and corporate policies before covenant renegotiation or initial reorganization

Proof of Proposition 2. (i) The value of equity before investment or initial reorganization, $e_0\left(X\right)$, corresponds to the present value of expected future cash flows to equity holders:

$$\begin{aligned} e_0\left(X\right) \quad = \quad & \mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_0} e^{-r(u-t)}\left(1-\tau\right)\left(X_u - c_0^A\right) \mathrm{d}u \,|X_t = X\right] \\ & + \mathbb{E}^{\mathbb{Q}}\left[1_{T_0 = S_0^\nu} e^{-r\left(S_0^\nu - t\right)} \theta_0 e_{1l}\left(S_0\right) |X_t = X\right] \\ & + \mathbb{E}^{\mathbb{Q}}\left[1_{T_0 = U_0^\nu} e^{-r\left(U_0^\nu - t\right)} \left(e_{1h}\left(sU_0; c_1\right) - \left(I - d_{1h}\left(sU_0; c_1^B\right)\right)\right) |X_t = X\right], \end{aligned} \tag{73}$$

in which S_0^ν, U_0^ν, and T_0 are defined in Eqs. (20)-(22). c_{01}^A and c_1^B are determined by Eq. (4). The first line of Eq. (73) corresponds to the cash flow to equity holders until initial reorganization or covenant renegotiation. In case of distressed reorganization, equity holders obtain a fraction θ_0 of the unlevered firm value (second line). The third line shows the value of equity at covenant renegotiation. The associated ordinary differential equation (ODE) reads

$$\begin{cases} e_0(X) = \theta_0 e_{1l}(X) & 0 \leq X \leq S_0 \\ re_0(X) = (1-\tau)\left(X - c_0^A\right) + \mu X e_0'(X) + \frac{1}{2}\sigma^2 X^2 e_0''(X) & S_0 < X < U_0 \\ e_0(X) = e_{1h}\left(sX; c_{01}^A + c_1^B\right) - \left(I - d_{1h}\left(sX; c_1^B\right)\right) & X \geq U_0, \end{cases} \tag{74}$$

subject to the boundary conditions

$$e_0(S_0) = \theta_0 e_{1l}(S_0), \tag{75}$$

$$e_0(U_0) = e_{1h}(sU_0; c_1) - \left(I - d_{1h}\left(sU_0; c_1^B\right)\right). \tag{76}$$

Standard arguments imply the solution as stated in the main text of Section 3.3, Eqs. (5)–(10).

(ii) The value of debt before covenant renegotiation or initial reorganization, $d_0(X)$, corresponds to the present value of expected future cash flows to initial debt holders:

$$\begin{aligned} d_0(X) = {} & \mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_0} e^{-r(u-t)} c_0^A \mathrm{d}u \,|X_t = X\right] + \mathbb{E}^{\mathbb{Q}}\left[1_{T_0 = U_0^\nu} e^{-r\left(U_0^\nu - t\right)} d_{1h}\left(sU_0; c_{01}^A\right) |X_t = X\right] \\ & + \mathbb{E}^{\mathbb{Q}}\left[1_{T_0 = S_0^\nu} e^{-r\left(S_0^\nu - t\right)} (1-\theta_0)\, e_{1l}(S_0) \,|X_t = X\right]. \end{aligned} \tag{77}$$

The first term in the first line of Eq. (77) shows the value of coupon payments to debt holders until covenant renegotiation or initial reorganization. At covenant renegotiation, initial debt holders have a claim on the total coupon c_{01}^A determined by the Nash solution, which corresponds to the second term in the first line of Eq. (77). In reorganization, initial debt holders receive a fraction $(1-\theta_0)$ of the unlevered equity value, which is shown in line two of Eq. (77). The ODE for the value of initial debt is given by

$$\begin{cases} d_0(X) &= (1-\theta_0)\, e_{1l}(X) & 0 \le X \le S_0 \\ rd_0(X) &= c_0^A + \mu X d_0'(X) + \frac{1}{2}\sigma^2 X^2 d_0''(X) & S_0 < X < U_0 \\ d_0(X) &= d_{1h}\left(sX; c_{01}^A\right) & X \ge U_0, \end{cases} \tag{78}$$

subject to the boundary conditions

$$d_0(S_0) = (1-\theta_0)\, e_{1l}(S_0), \qquad d_0(U_0) = d_{1h}\left(sU_0; c_{01}^A\right). \tag{79}$$

Standard arguments imply the solution as stated in the main text of Section 3.3, Eqs. (11)–(13).

The value of a claim on non-distressed renegotiation surplus. The value of a claim on non-distressed renegotiation surplus, denoted by $d_{0nds}(X)$ solves the ODE

$$\begin{cases} d_{0nds}(X) &= 0 & 0 \le X \le S_0 \\ rd_{0nds}(X) &= \mu X d_{0nds}'(X) + \frac{1}{2}\sigma^2 X^2 d_{0nds}''(X) & S_0 < X < U_0 \\ d_{0nds}(X) &= d_{1h}\left(sX; c_{01}^A\right) - d_{1h}\left(sX; c_0^A\right) & X \ge U_0, \end{cases} \tag{80}$$

subject to the boundary conditions

$$d_{0nds}(S_0) = 0, \qquad d_{0nds}(U_0) = d_{1h}\left(sU_0; c_{01}^A\right) - d_{1h}\left(sU_0; c_0^A\right). \tag{81}$$

In particular, the second equation of Eqs. (81) reflects that the claim obtains the surplus of initial debt holders at covenant renegotiation. We solve the ODE subject to its boundary conditions using standard techniques.

D. Benchmark model

The benchmark model corresponds to the model of Hackbarth and Mauer (2012), augmented with distressed reorganization as in Fan and Sundaresan (2000). This approach to modeling distressed reorganization is similar to that in Sundaresan and Wang (2007). The only difference is that parties in the latter model renegotiate contractual coupon payments instead of a debt-equity swap. We first present the main case with equal priority debt. Subsequently, we show how to derive the solutions in case of senior initial debt. Finally, we present the first best benchmark firm, which uses firm value maximizing financial and investment policies.

Value functions after initial reorganization. There is no covenant after initial reorganization because the firm is all equity financed up to investment. Hence, the value functions and boundaries $e_{1l}(X), e_2(X), d_2(X), U_1$, and S_2 are calculated with the same formulae as in the baseline model.

Value functions after investment. At the reorganization boundary after investment (S_1), the outside options of initial and new debt holders are analogous to Hackbarth and Mauer (2012), augmented with the possibility to renegotiate debt in distress:

$$O^o(S_1) = \alpha \frac{c_A^0}{c_A^0 + c_B^1} v(S_1), \tag{82}$$

$$O^n(S_1) = \alpha \frac{c_B^1}{c_A^0 + c_B^1} v(S_1). \tag{83}$$

Analogous to the model with both distressed and non-distressed renegotiation, we consider the solution to the asymmetric Nash product, see Eq. (46). Eqs. (82)–(83) imply the sharing rule

$$\theta_1^o = \eta_o(1-\alpha) + \frac{O^o(S_1)}{v(S_1)} = \eta_o(1-\alpha) + \alpha \frac{c_A^0}{c_A^0 + c_B^1}, \tag{84}$$

$$\theta_1^n = \eta_n(1-\alpha) + \frac{O^n(S_1)}{v(S_1)} = \eta_n(1-\alpha) + \alpha \frac{c_B^1}{c_A^0 + c_B^1}. \tag{85}$$

Consequently,

$$\theta_1 = \eta(1-\alpha). \tag{86}$$

Thus, the solution for S_1, the value function of equity $e_{1h}(X)$, and the value functions of initial and new debt, $d_{1h}\left(X;c_A^{01}\right)$ and $d_{1h}\left(X;c_B^{1}\right)$, use the same functional form as in the model with both distressed and non-distressed renegotiation (see Eqs. (45), (27)-(30), (52)–(54), and (55)–(56), respectively). The difference consists of the value of the outside option, on which the values of initial and new debt depend explicitly.

Value functions before investment or initial reorganization. At the investment boundary U_0, equity holders determine the amount of new debt by maximizing the value of equity and issue proceeds of new debt:

$$c_1^B = \arg\max_{\tilde{c}_1^B} e_{1h}\left(sU_0; c_0^A + \tilde{c}_1^B\right) + d_{1h}\left(sU_0; \tilde{c}_1^B\right). \tag{87}$$

This financing choice affects the boundary condition of the ODE for the value of equity:

$$\begin{cases} e_0(X) = \theta_0 e_{1l}(X) & 0 \le X \le S_0 \\ re_0(X) = (1-\tau)\left(X - c_0^A\right) + \mu X e_0'(X) + \frac{1}{2}\sigma^2 X^2 e_0''(X) & S_0 < X < U_0 \\ e_0(X) = e_{1h}\left(sX; c_0^A + c_1^B\right) - \left(I - d_{1h}\left(sX; c_1^B\right)\right) & X \ge U_0, \end{cases} \tag{88}$$

subject to the boundary conditions

$$e_0(S_0) = \theta_0 e_{1l}(S_0), \tag{89}$$

$$e_0(U_0) = e_{1h}(sU_0; c_1) - \left(I - d_{1h}\left(sU_0; c_1^B\right)\right), \tag{90}$$

in which c_1^B is determined by Eq. (87). The functional form of the value function of equity corresponds to that presented in the model with both distressed and non-distressed renegotiation (Proposition 2), but Eq. (9) is replaced by

$$b^{e_0} = \begin{bmatrix} -A_0^{e_0} - A_1^{e_0} S_0 + \theta_0 e_{1l}(S_0) \\ e_{1h}\left(sU_0; c_0^A + c_1^B\right) + d_{1h}\left(sU_0; c_1^B\right) - I - A_0^{e_0} - A_1^{e_0} U_0 \end{bmatrix}. \tag{91}$$

The associated smooth-pasting conditions are:

$$
\begin{aligned}
\frac{\partial}{\partial X} e_0(X)|_{X=S_0} &= \theta \frac{\partial}{\partial X} e_{1l}(X)|_{X=S_0}, & (92)\\
\frac{\partial}{\partial X} e_0(X)|_{X=U_0} &= \frac{\partial}{\partial X} e_{1h}(sX;c_1)|_{X=U_0} + \frac{\partial}{\partial X} d_{1h}\left(sX;c_1^B\right)|_{X=U_0}. & (93)
\end{aligned}
$$

Finally, equity holders choose the initial capital structure by maximizing the value of their objective function ex-ante. Therefore, equity holders solve

$$
c_0^A = \arg\max_{\tilde{c}_0^A} \{e_0(X_0) + d_0(X_0)\}. \tag{94}
$$

Equity holders' problem consists of solving Eq. (94) subject to Eqs. (92)-(93). A closed-form solution does not exist. We use numerical procedures and verify the optimality of the investment and reorganization boundaries numerically.

Similar to the valuation of equity, the ODE for the value of initial debt is given by

$$
\begin{cases}
d_0(X) = (1-\theta_0)\, e_{1l}(X) & 0 \leq X \leq S_0 \\
r d_0(X) = c_0^A + \mu X d_0'(X) + \frac{1}{2}\sigma^2 X^2 d_0''(X) & S_0 < X < U_0 \\
d_0(X) = d_{1h}\left(sX;c_0^A\right) & X \geq U_0,
\end{cases} \tag{95}
$$

subject to the boundary conditions

$$
d_0(S_0) = (1-\theta_0)\, e_{1l}(S_0), \qquad d_0(U_0) = d_{1h}\left(sU_0;c_0^A\right). \tag{96}
$$

In particular, the value of initial debt at investment is still determined by initial coupon payments c_0^A, but incorporates the increase in reorganization risk due to the issuance of new debt. Hence, the functional form of the value function of initial debt corresponds to that presented in the model with both distressed and non-distressed renegotiation (Proposition 2), but Eq. (13) is replaced by

$$b^d = \begin{bmatrix} -A_0^d + (1-\theta_0)\, e_{1l}(S_0) \\ d_{1h}\left(sU_0; c_0^A\right) - A_0^d \end{bmatrix}. \tag{97}$$

Senior initial debt. Considering senior initial debt instead of equal priority debt impacts initial and new debt holders' outside option in distressed reorganization after investment:

$$O^o(S_1) = \min\left\{\alpha v(S_1), \frac{c_A^0}{r}\right\}, \tag{98}$$

$$O^n(S_1) = \max\left\{0, \alpha v(S_1) - \frac{c_A^0}{r}\right\}. \tag{99}$$

The remaining steps are analogous to the solution for equal priority debt.

First-best. The first-best benchmark firm corresponds to a benchmark firm, but with firm value maximizing investment and financing decisions. Specifically, all value-matching conditions in the valuation of corporate securities correspond to those of a benchmark firm. In the first-best, however, the smooth-pasting condition at the investment boundary and the new coupon at investment are firm value maximizing instead of equity value maximizing as in the benchmark modelmodel. Specifically, the first-best coupon c_1^{fb} (see Eq. (70)) is installed at investment instead of equity holders' coupon choice determined by condition (87). Further, we replace the smooth-pasting condition (93) by the condition that ensures firm value maximization:

$$\frac{\partial}{\partial X} e_0(X)\,|_{X=U_0} + \frac{\partial}{\partial X} d_0(X)\,|_{X=U_0} = f_{1h} s. \tag{100}$$

E. Details on the pre-simulation

This appendix provides details on the pre-simulation, which is used to generate the model-implied cross-section of firms. First, we establish a large universe of initially optimally financed model firms. Specifically, we consider different firms in our model with an initial scale parameter from $s = 1$ to $s = 2.23$ using a step size of 0.01. The upper bound $s = 2.23$ is the highest s such that the investment boundary is not higher than 1.10 (rounded to two decimals), which is our cutoff boundary. For each scale parameter s, we consider 100 initially identical firms. Thus, our universe consists of 12,400 firms.

In the benchmark model, we follow an analogous approach. Specifically, we consider firms with an investment threshold at or above 1.10, which results in a range of scale parameters from 1.0 to 2.05 using a step size of 0.01. Each firm selects its optimal capital structure. As with our model, we consider 100 initially identical firms for each s. Our universe in the benchmark case consists of 10,600 firms.

For both our model and the benchmark model, the universe is simulated forward for ten years with a quarterly frequency. To this end, we simulate cash flows of each firm in the universe. As in Strebulaev (2007), firms deviate from their initially optimal capital structure. Firms that reorganize or invest are replaced by an optimally financed firm with an identical investment opportunity and the cash flow at initiation.

To match model-implied firms with real firms, we select for each real firm the model-implied firm at the end of the pre-simulation that minimizes the Euclidean distance with respect to leverage and the log of one plus market-to-book ratio:

$$\{s, X\} = \arg\min \sqrt{\left(lev_{empirical} - lev_{model}\right)^2 + \left(\log\left(1 + mtb_{empirical}\right) - \log\left(1 + mtb_{model}\right)\right)^2}. \tag{101}$$

F. Waiting time to investment

By Bayes' rule, the conditional waiting time to renegotiation (conditional on investment and not reorganization taking place) can be calculated as

$$\mathbb{E}\left[T_0 \left| X_{T_0} = U_0, \mathcal{F}_t \right.\right] = \frac{\mathbb{E}\left[T_0 1_{X_{T_0}=U_0} \left| \mathcal{F}_t \right.\right]}{\mathbb{E}\left[1_{X_{T_0}=U_0} \left| \mathcal{F}_t \right.\right]}. \tag{102}$$

We first calculate the denominator, which we also denote as $u\left(X_t\right)$:

$$\begin{aligned}
\mathbb{E}\left[1_{X_{T_0}=U_0} \left| \mathcal{F}_t \right.\right] &= \mathbb{E}\left[1_{X_{T_0}=U_0} \left(1_{T_0 \leq t} + 1_{T_0 > t}\right) \left| \mathcal{F}_t \right.\right] &(103)\\
&= 1_{T_0 \leq t} 1_{X_{T_0}=U_0} + \mathbb{E}\left[1_{X_{T_0}=U_0} 1_{T_0 > t} \left| \mathcal{F}_t \right.\right] &(104)\\
&= 1_{T_0 \leq t} 1_{X_{T_0}=U_0} + u\left(X_t\right), &(105)
\end{aligned}$$

in which we used the Markov property in the last step. Because $\left(\mathbb{E}\left[1_{X_{T_0}=U_0} \left| \mathcal{F}_t \right.\right]\right)_t$ is a martingale and using Ito's lemma, we find that $u\left(X_t\right)$ solves the ODE

$$\mu^P X u'\left(X\right) + \frac{1}{2}\sigma^2 X^2 u''\left(X\right) = 0, \tag{106}$$

subject to the boundary conditions

$$u\left(S_0\right) = 0, \qquad u\left(U_0\right) = 1. \tag{107}$$

The solution is given by

$$u\left(X\right) = \frac{1}{U_0^\gamma - S_0^\gamma} X^\gamma + \frac{S_0^\gamma}{U_0^\gamma - S_0^\gamma}, \tag{108}$$

with

$$\gamma = 1 - \frac{2\mu^P}{\sigma}. \tag{109}$$

Next, we calculate the numerator of Eq. (102), which we denote by $z\left(X_t\right)$. Analogous arguments to the ones above imply that

$$\mathbb{E}\left[T_0 1_{X_{T_0}=U_0}\left|\mathcal{F}_t\right.\right] = T_0 1_{T_0\leq t} 1_{X_{T_0}=U_0} + tw\left(X_t\right) + u\left(X_t\right). \tag{110}$$

Using that $\left(\mathbb{E}\left[T_0 1_{X_{T_0}=U_0}\left|\mathcal{F}_t\right.\right]\right)_t$ is a martingale, Ito's lemma, and the dynamics of $u\left(X_t\right)$, we find that $w\left(X_t\right)$ solves the ODE

$$\mu^P X w'\left(X\right) + \frac{1}{2}\sigma^2 X^2 w''\left(X\right) + u\left(X\right) = 0, \tag{111}$$

subject to the boundary conditions

$$w\left(S_0\right) = 0, \qquad\qquad u\left(w_0\right) = 0. \tag{112}$$

The solution is given by

$$w\left(X\right) = w_1 X^\gamma + w_2 + \frac{2}{\gamma\sigma^2}\left(\frac{1}{\gamma} + \log\left(X\right)\right), \tag{113}$$

in which

$$w_1 = \frac{2\left(\log\left(U_0\right) - \log\left(S_0\right)\right)}{\gamma\sigma^2\left(S_0^\gamma - U_0^\gamma\right)} \tag{114}$$

$$w_2 = -w_1 U_0^\gamma - \frac{2}{\gamma\sigma^2\left(\frac{1}{\gamma} + \log\left(U_0\right)\right)}, \tag{115}$$

and γ as in Eq. (109).

G. Calculation of credit spreads and the control premium

Credit spreads. We calibrate model-implied firms to the median empirical firm as follows. For any given scale parameter s, we consider the optimal capital structure at $X_0 = 1$. We then solve for the cash flow X_l that matches the target median leverage of 30.3%. In addition, we calculate this firm's market-to-book ratio at the cash flow level X_l, denoted by $MtB\,(X_l)$. Finally, we consider all s that allow us to match target leverage and choose the firm with the s that exhibits the minimum absolute difference between the median empirical market-to-book ratio of 1.47 and $MtB\,(X_l)$.

Credit spreads in the benchmark model are calculated by first dividing the initial coupon by the initial market value of debt and then subtracting the risk free interest rate. In the baseline model, the spread between the initial debt yield and risk free rate cannot be directly used to compare credit spreads with the benchmark model for two reasons. First, the market value of initial debt in the baseline model reflects also the present value of the surplus initial debt holders obtain at covenant renegotiation. This surplus is defined in Eq. (60). Second, the initial debt's coupon in the baseline model is not constant but changes at covenant renegotiation. We calculate credit spreads in the baseline model to reflect constant coupon payments and entail no surplus. To this end, we first calculate the net debt value by subtracting the value of a claim to the surplus in non-distressed renegotiation to initial debt holders from the initial debt value. Next, we solve for the constant coupon that implies the same initial debt value as the net debt value, given corporate policies in the baseline model. Finally, we calculate credit spreads as this constant coupon divided by the net debt value and then subtract the risk free interest rate.

Control premium. To calculate the control premium, we first consider the value of a hypothetical no-surplus debt claim, given the covenant renegotiation and the initial reorganization boundaries as in the baseline model. This claim obtains no surplus from any renegotiation such that its value at the boundaries is analogous to the respective outside option of initial debt holders in the baseline model. The value of this no-surplus claim is denoted by $d_{ns}\,(X)$ and satisfies the ODE

$$\begin{cases} d_{ns}\,(X) &= \alpha \frac{1-\tau}{r-\mu} S_0 & 0 \leq X \leq S_0 \\ r d_{ns}\,(X) &= c_0^A + \mu X d'_{ns}\,(X) + \frac{1}{2}\sigma^2 X^2 d''_{ns}\,(X) & S_0 < X < U_0 \\ d_{ns}\,(X) &= d_{1h,ns}\left(sX; c_0^A\right) & X \geq U_0, \end{cases} \tag{116}$$

subject to the boundary conditions

$$d_{ns}\left(S_0\right)=\alpha\frac{1-\tau}{r-\mu}S_0, \qquad\qquad d_{ns}\left(U_0\right)=d_{1h,s}\left(sU_0;c_0^A\right). \tag{117}$$

$d_{1h,ns}\left(\cdot;c_0^A\right)$ denotes the value of a claim to a coupon stream c_0^A. In case of default after covenant renegotiation, the claim holder obtains the liquidation value $\alpha\frac{1-\tau}{r-\mu}S_1$. The value function writes

$$d_{1h,ns}\left(X;c_0^A\right)=\frac{c_0^A}{r}+S_1^{-\beta_2}\left(-\frac{c_A^0}{r}+\alpha\frac{1-\tau}{r-\mu}S_1\right)X^{\beta_2} \tag{118}$$

in which

$$S_1=\frac{\beta_2}{\beta_2-1}\frac{r-\mu}{r}\frac{1}{1-\theta_1}c_0^A. \tag{119}$$

We solve ODE (116) subject to (117) using standard techniques.

Finally, we calculate the control premium as the difference between the baseline model's debt value and the no-surplus debt claim's value, expressed as a percentage of the baseline model's debt value:

$$CP\left(X\right)=100\frac{d_0\left(X\right)-d_{ns}\left(X\right)}{d_0\left(X\right)}. \tag{120}$$

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Figures and Tables

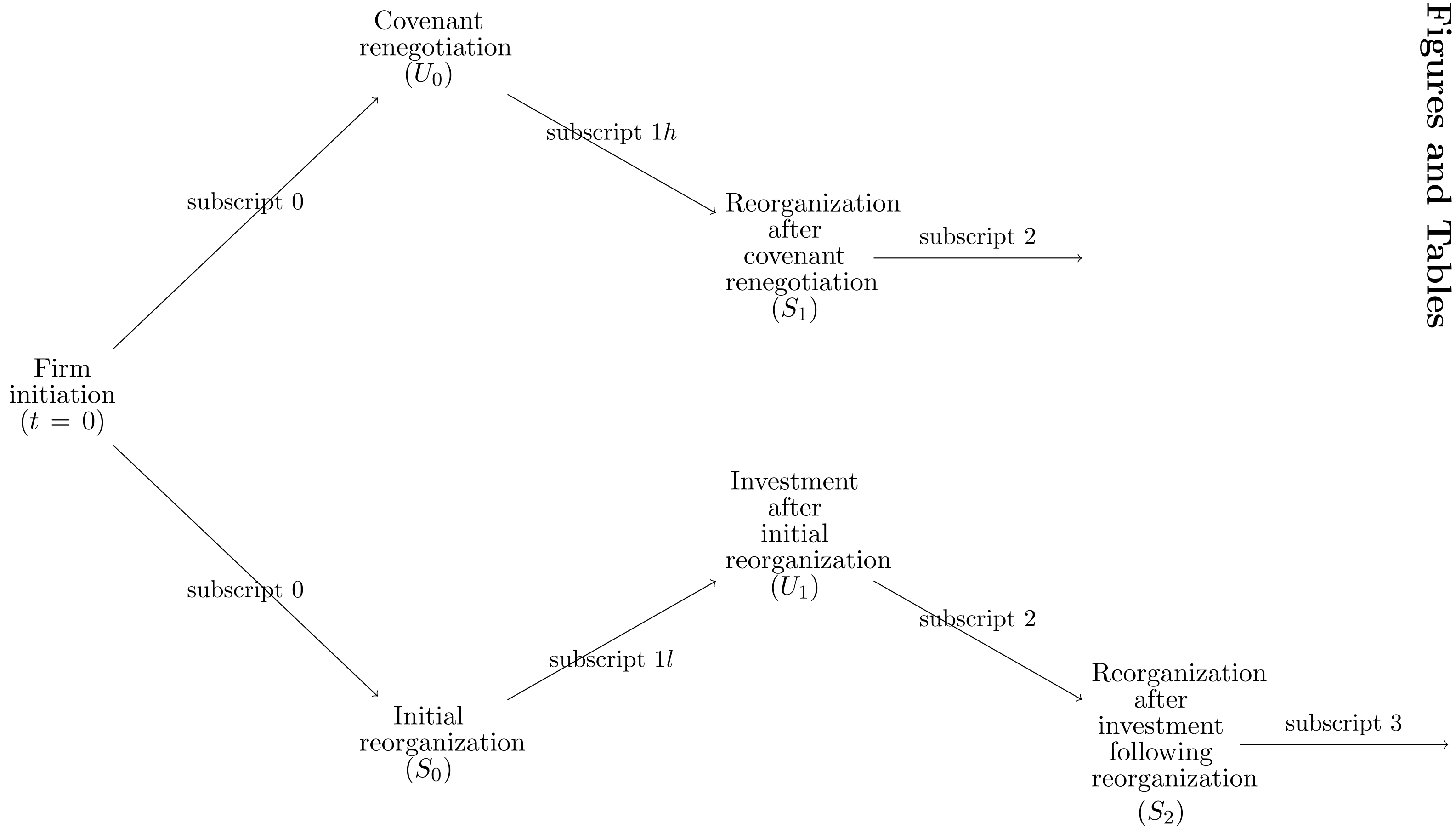

Figure 1. Timeline of the potential occurrence of debt renegotiation. This figure shows the sequence of the potential occurrence of covenant renegotiation, distressed reorganization, and investment over time. The notation for the corresponding thresholds are in parenthesis. The subscripts apply to the corresponding value functions.

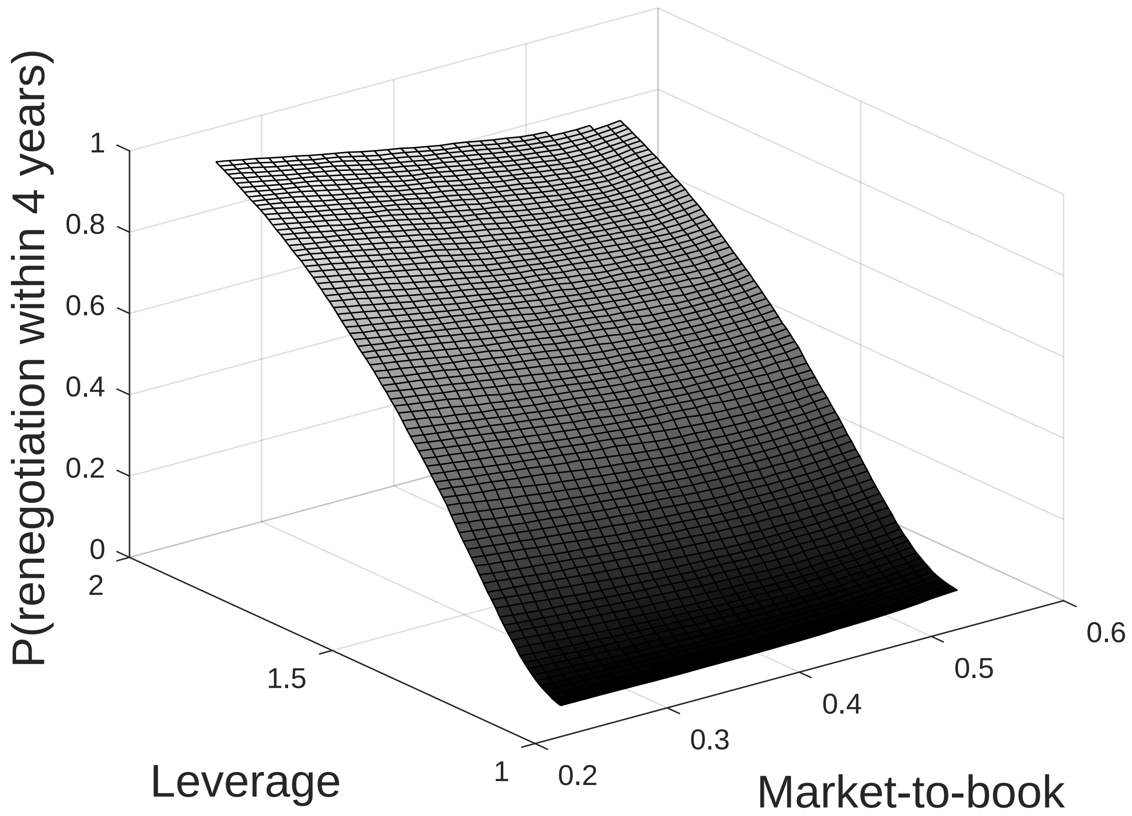

Figure 2. Probability to renegotiate within four years. This figure shows the baseline firm's probability to renegotiate debt (either in distress or outside distressed) within four years. The probability is shown as a function of leverage (x-axis) and the market-to-book ratio (y-axis). Variations in market-to-book and leverage are obtained by varying the investment scale factor s and the coupon c.

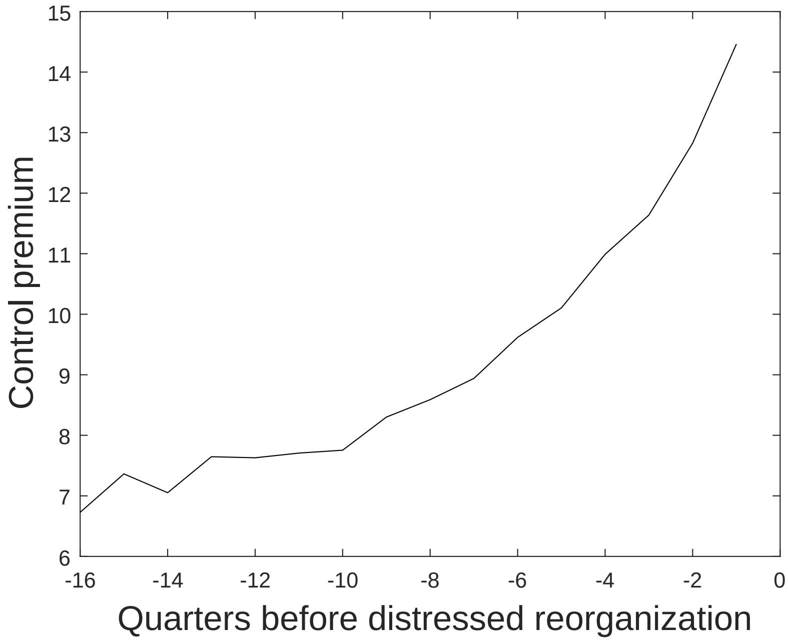

Figure 3. Control premia before distressed reorganization. This figure shows average debt control premia in a simulated economy before distressed reorganization.

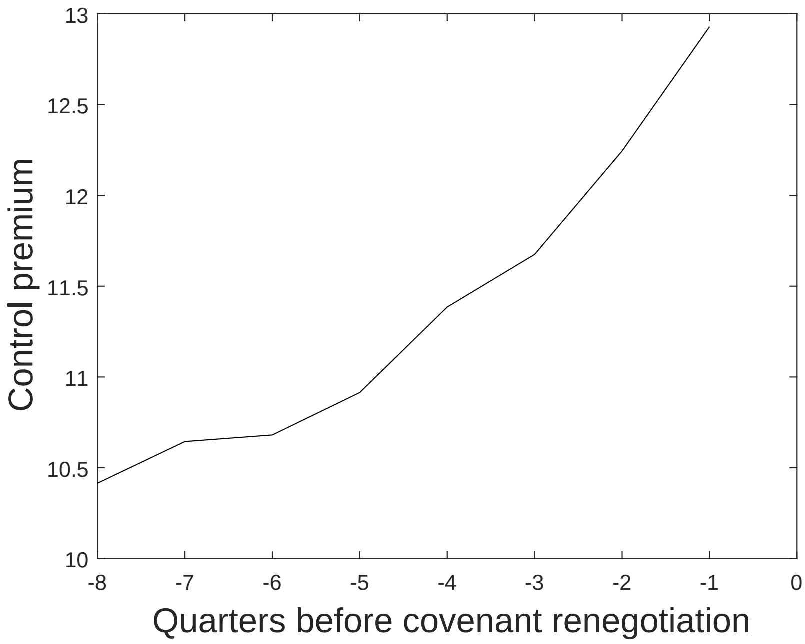

Figure 4. Control premia before covenant renegotiation. This figure shows average debt control premia in a simulated economy before covenant renegotiation.

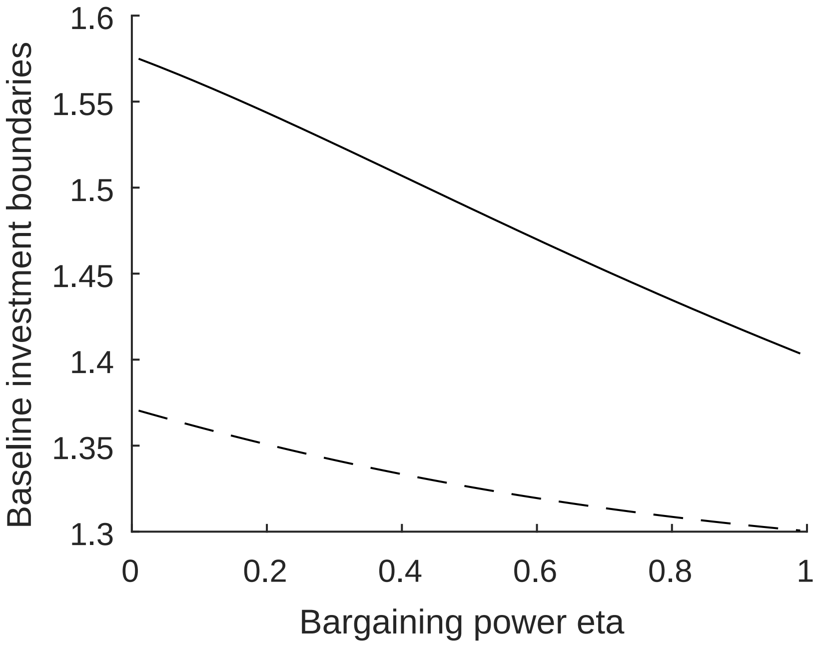

Figure 5. The impact of bargaining power on investment in the baseline model. This figure plots the investment boundary of the baseline firm (solid line) against the bargaining power of equity holders. For each bargaining power level, we apply the optimal leverage. The dashed line is the investment boundary of the first best firm, given the same coupon as the baseline firm.

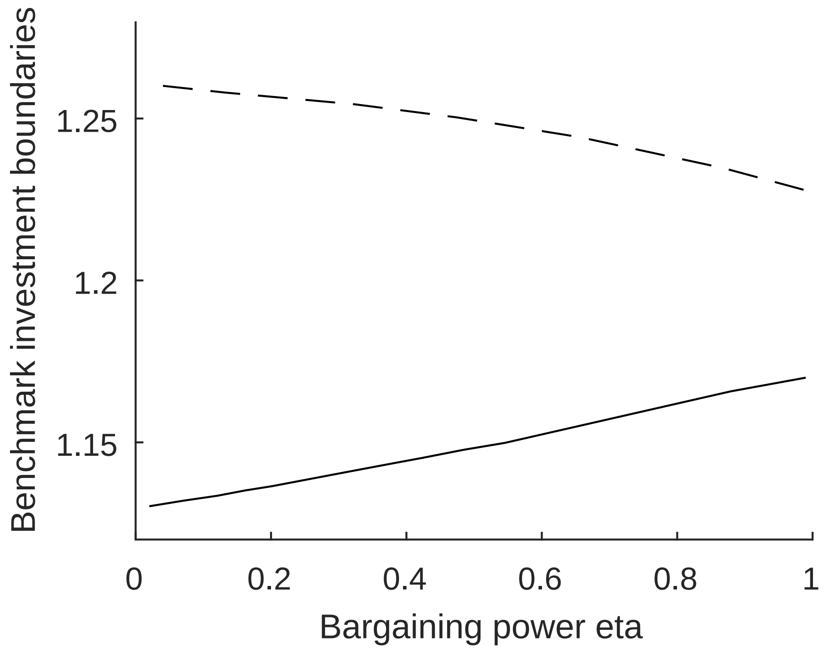

Figure 6. The impact of bargaining power on investment in the benchmark model. This figure plots the investment boundary of the benchmark firm (solid line) against the bargaining power of equity holders. For each bargaining power level, we apply the optimal leverage. The dashed line is the investment boundary of the first best firm, given the same coupon as the benchmark firm.

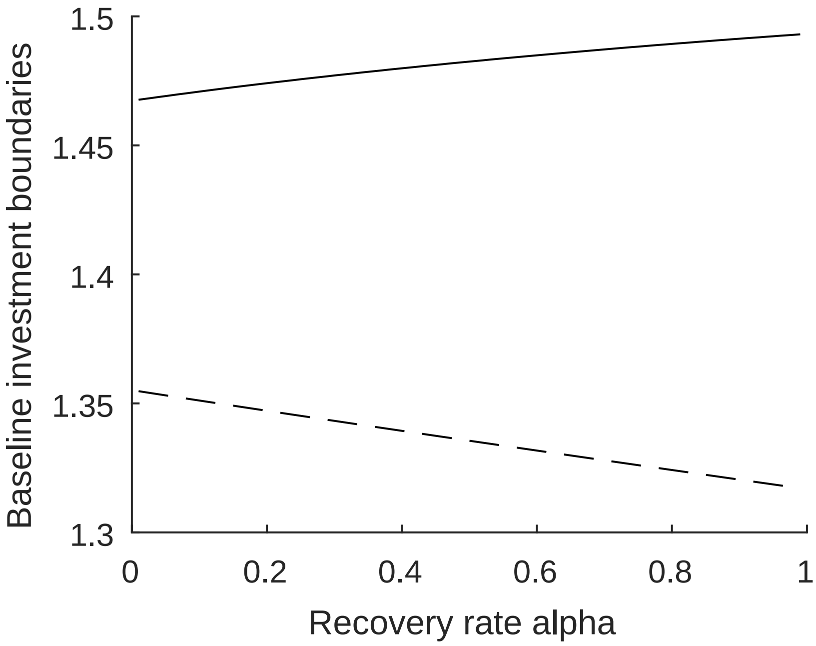

Figure 7. The impact of the recovery rate on investment in the baseline model. This figure plots the investment boundary of the baseline firm (solid line) against the recovery rate. For each recovery rate, we apply the optimal leverage. The dashed line is the investment boundary of the first best firm, given the same coupon as the baseline firm.

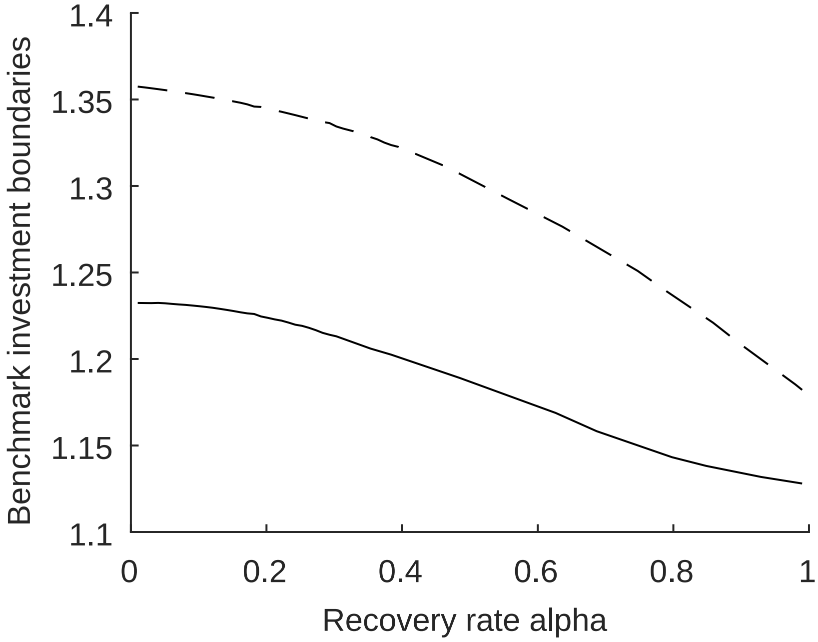

Figure 8. The impact of the recovery rate on investment in the benchmark model. This figure plots the investment boundary of the benchmark firm (solid line) against the recovery rate. For each recovery rate, we apply the optimal leverage. The dashed line is the investment boundary of the first best firm, given the same coupon as the benchmark firm.

Table 1
The value of the ability to renegotiate outside distress.

This table shows the impact of the ability to renegotiate debt outside distress on firm value and policies for different parameters. Panel A displays baseline parameter results. In Panels B–J, we change one parameter at a time. η is equity holders' bargaining power, η_o is the bargaining power of initial debt holders in reorganization after investment, α is the recovery rate in default, τ is the corporate tax rate, σ is the volatility of cash flows, and μ the cash flow drift. The benchmark firm is a firm with an investment opportunity as in Hackbarth and Mauer (2012), augmented for the ability to reorganize debt in distress as in Fan and Sundaresan (2000). Equity holders choose the financing, investment, and reorganization policies to maximize the value of their claim. In the benchmark first best, first best financing and investment policies are chosen. The baseline firm can renegotiate debt in and outside distress. In all cases, equity holders choose the distressed reorganization policy. S_0 is the reorganization boundary, U_0 is the investment boundary initiating covenant renegotiation in the R-firm, and c_0^A is the initial coupon. lev_U and lev_0 are leverage at investment and initial leverage, respectively, and f_0 is the initial firm value. AC is the agency cost of debt reported as a fraction of benchmark first best firm value and $VNDR$ is the value of the ability to renegotiate outside distress, defined as the firm value increase in the baseline firm compared to the benchmark firm, expressed as a fraction of the baseline firm value.

	S_0	U_0	c_0^A	lev_U	lev_0	f_0	AC	$VNDR$
Panel A: Baseline parameters								
Benchmark	2.91	22.93	6.48	0.71	0.17	539.75	1.12	0.00
Benchmark first best	9.57	27.25	23.83	0.68	0.51	545.88	0.00	0.00
Baseline	7.36	29.77	16.67	0.68	0.43	543.30	0.47	0.65
Panel B: Higher bargaining power of equity holders ($\eta = 0.75$)								
Benchmark	2.23	23.22	4.36	0.66	0.11	535.71	1.00	0.00
Benchmark first best	9.17	26.82	19.18	0.63	0.42	541.10	0.00	0.00
Baseline	6.78	28.86	13.18	0.63	0.35	539.10	0.37	0.63
Panel C: Lower bargaining power of equity holders ($\eta = 0.25$)								
Benchmark	3.19	22.80	7.96	0.75	0.20	543.73	1.28	0.00
Benchmark first best	9.91	27.74	28.86	0.72	0.60	550.79	0.00	0.00
Baseline	8.09	30.69	21.15	0.72	0.53	547.77	0.55	0.73
Panel D: Lower bargaining power of initial debt in reorg. after inv. ($\eta_o = 0.125$)								
Benchmark	2.31	23.61	5.03	0.65	0.13	539.73	1.12	0.00
Benchmark first best	10.72	27.43	27.13	0.68	0.56	545.82	0.00	0.00
Baseline	7.36	29.77	16.66	0.68	0.43	543.30	0.46	0.66
Panel E: Higher bargaining power of initial debt in reorg. after inv. ($\eta_o = 0.375$)								
Benchmark	3.02	25.60	6.69	0.74	0.18	540.13	1.02	0.00
Benchmark first best	8.68	27.06	21.27	0.68	0.47	545.70	0.00	0.00
Baseline	7.36	29.77	16.66	0.68	0.43	543.30	0.44	0.58
Panel F: Lower recovery rate ($\alpha = 0.55$)								
Benchmark	4.54	23.56	9.34	0.63	0.24	535.87	0.75	0.00
Benchmark first best	9.68	27.31	21.76	0.60	0.47	539.94	0.00	0.00
Baseline	7.53	29.67	15.40	0.60	0.40	537.74	0.41	0.34
Panel G: Higher tax rate ($\tau = 0.27$)								
Benchmark	5.35	24.67	12.65	0.77	0.31	493.10	2.20	0.00
Benchmark first best	9.86	31.52	24.35	0.71	0.54	504.20	0.00	0.00
Baseline	8.85	34.76	19.45	0.71	0.51	499.72	0.89	1.31
Panel H: Higher cash flow volatility ($\sigma = 0.29$)								
Benchmark	3.08	25.18	7.92	0.72	0.19	554.87	1.32	0.00
Benchmark first best	9.33	30.88	26.69	0.67	0.51	552.18	0.00	0.00
Baseline	7.55	34.27	19.54	0.67	0.46	549.47	0.49	0.83
Panel J: Higher cash flow drift ($\mu = 0.02$)								
Benchmark	2.13	21.90	5.78	0.69	0.12	721.45	1.02	0.00
Benchmark first best	9.77	26.98	30.35	0.68	0.50	728.89	0.00	0.00
Baseline	7.58	29.25	21.30	0.68	0.42	725.99	0.40	0.62

Table 2
Optimal priority structure

This table shows firm value and policies in case the firm can choose the optimal debt priority structure. The benchmark firm is a firm with an investment opportunity as in Hackbarth and Mauer (2012), augmented for the ability to reorganize debt in distress as in Fan and Sundaresan (2000). Equity holders choose the financing, debt priority, investment, and reorganization policies to maximize the value of their claim. In the benchmark first best, first best financing and investment policies are chosen. In both cases, equity holders choose the distressed reorganization policy. S_0 is the reorganization boundary, U_0 is the investment boundary initiating covenant renegotiation in the R-firm, and c_0^A is the initial coupon. lev_U and lev_0 are leverage at investment and initial leverage, respectively, and f_0 is the initial firm value. AC is the agency cost of debt reported as a fraction of benchmark first best firm value.

	Priority initial debt	S_0	U_0	c_0^A	lev_U	lev_0	f_0	$\%AC$
Benchmark	Senior	4.91	29.31	10.94	0.61	0.32	540.27	0.79
Benchmark first best	Senior	8.58	26.63	18.82	0.68	0.52	544.59	0.00

Table 3
Summary statistics of empirical and simulated samples at matching

This table shows summary statistics for the empirical cross-section from Nini, Smith, and Sufi (2009) and for the simulated baseline model-implied firm samples at matching. The market leverage in the simulated samples is the market value of debt net of the present value of the investment surplus to debt holders, divided by the market value of the firm. The model-implied market-to-book ratio is the sum of the market value of the model firm divided by the value of invested assets. *Correlation* is the correlation coefficient between market-to-book and leverage.

	Empirical cross section	*Simulated samples*
Market leverage mean	0.41	0.42
Market leverage median	0.40	0.39
Market leverage standard deviation	0.22	0.21
Market-to-book ratio mean	1.69	1.44
Market-to-book ratio median	1.39	1.38
Market-to-book ratio standard deviation	0.90	0.32
Correlation	-0.79	-0.67

Table 4
Empirical and model-implied timing patterns of debt renegotiation

This table summarizes key statistics on renegotiation timing. % renegotiated is the portion of debt contracts renegotiated before maturity. Effective duration is the mean waiting time to the first renegotiation or final maturity in years. Effective duration % of stated maturity is the average time to the first renegotiation or maturity, expressed in terms of the initially stated maturity. Time to first renegotiation is the average time in years to the first renegotiation of contracts that are renegotiated. % distressed renegotiations is the percentage of total debt renegotiations associated with corporate distress. Duration % of stated maturity is the average time to any renegotiation or to maturity divided by the initially stated maturity, including renegotiations occurring after the first renegotiation. The next four lines summarize the frequencies of renegotiation rounds. Average renegotiation frequency is the mean number of renegotiation rounds of renegotiated contracts. The empirical samples are the loan contract samples of Roberts and Sufi (2009b) and Denis and Wang (2014). In the simulated samples of the baseline model, firms can renegotiate a financing covenant outside distress and reorganize in distress. In the simulated samples of the benchmark model, firms have an investment option in the spirit of Hackbarth and Mauer (2012) plus the ability to reorganize in distress following Fan and Sundaresan (2000).

	Empirical DW 2014	*Empirical RS 2009b*	*Baseline model*	*Benchmark model*
% renegotiated	60.6	64.5	48.2	6.1
Effective duration	2.0	1.5	2.6	3.8
Eff. duration % of stated maturity	58	64	65.5	96.2
Time to first renegotiation	1.3	1.5	1.1	1.5
% distressed renegotiations		18	8.7	100.0
Duration % of stated maturity	61		45.9	91.8
% renegotiated once	37		32.4	56.7
% renegotiated twice	22		16.1	15.8
% renegotiated three times	13		11.8	9.6
% renegotiated two to five times	53		45.7	36.4
Average renegotiation frequency	2.7		3.4	2.1

Table 5: **Model-implied timing patterns of debt renegotiation, robustness**

This table summarizes key statistics on renegotiation timing for various alternative parameters and assumptions. % renegotiated is the portion of debt contracts renegotiated before maturity. Effective duration is the mean waiting time to the first renegotiation or final maturity in years. Effective duration % of stated maturity is the average time to the first renegotiation or maturity, expressed in terms of the initially stated maturity. Time to first renegotiation is the average time in years to the first renegotiation of contracts that are renegotiated. % distressed renegotiations is the percentage of total debt renegotiations associated with corporate distress. Duration % of stated maturity is the average time to any renegotiation or to maturity divided by the initially stated maturity, including renegotiations occurring after the first renegotiation. The next four lines summarize the frequencies of renegotiation rounds. Average renegotiation frequency is the mean number of renegotiation rounds of renegotiated contracts. In the simulated samples of the baseline model, firms can renegotiate a financing covenant outside distress and reorganize in distress. In each column, we vary one parameter or assumption of our approach. The second column ("outliers") trims the empirical sample instead of winsorizing it; columns three to seven show the results based on parameter variations; column eight ("matching") considers the Euclidean distance based on the vector space of percentage deviations from target leverage and market-to-book when matching model-implied to empirical firms; column 9 ("replace") replaces renegotiated firms with firms at initiation (instead of at matching); column 10 ("pre-sim") uses a pre-simulation period of 100 years.

	Baseline	*outliers*	$\eta = 0.75$	$\alpha = 0.55$	$\tau = 0.27$	$\sigma = 0.29$	$\mu = 0.02$	*matching*	*replace*	*pre-sim*	$\Delta = \frac{1}{12}$	book debt	$q = 1$
% renegotiated	48.2	44.7	48.6	51.0	45.3	46.5	48.2	46.1	48.2	48.2	48.1	45.7	47.6
Effective duration	2.6	2.8	2.6	2.5	2.7	2.7	2.6	2.7	2.6	2.6	2.6	2.7	2.6
Eff. duration % of maturity	65.5	69.7	65.4	63.4	66.9	67.5	65.2	68.2	65.5	65.5	65.6	68.3	64.9
Time to first renegotiation	1.1	1.3	1.1	1.1	1.1	1.2	1.1	1.2	1.1	1.1	1.1	1.2	1.1
% distressed renegotiations	8.7	1.9	13.1	15.6	11.9	11.8	8.1	13.3	7.9	8.6	8.6	3.0	1.2
Duration % of maturity	45.9	50.4	45.9	44.6	46.9	48.4	45.4	46.1	56.4	46.0	46.2	48.6	44.0
% renegotiated once	32.4	38.8	34.4	33.9	31.0	37.5	31.5	37.9	53.8	33.3	33.5	36.4	28.9
% renegotiated twice	16.1	16.9	16.2	16.6	16.7	16.9	16.0	18.1	19.3	16.5	16.6	16.2	14.7
% renegotiated three times	11.8	11.0	11.6	11.9	12.5	11.6	11.8	12.1	11.7	11.8	11.9	11.2	11.5
% renegotiated two to five times	45.7	43.2	45.1	46.0	47.8	45.0	45.9	46.3	42.6	45.9	46.0	43.9	45.1
Average renegotiation frequency	3.4	3.1	3.4	3.3	3.5	3.1	3.5	3.0	2.0	3.4	3.4	3.3	3.8
Market leverage mean	0.42	0.41	0.41	0.41	0.43	0.42	0.43	0.41	0.42	0.42	0.42	0.41	0.47
Market leverage median	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.40	0.40
Market leverage std	0.21	0.17	0.20	0.20	0.20	0.21	0.20	0.22	0.21	0.21	0.21	0.20	0.17
Market-to-book mean	1.44	1.44	1.42	1.43	1.53	1.47	1.46	1.36	1.44	1.44	1.44	1.44	1.43
Market-to-book median	1.38	1.39	1.36	1.37	1.38	1.38	1.38	1.27	1.38	1.38	1.38	1.39	1.38
Market-to-book std	0.32	0.29	0.31	0.32	0.40	0.37	0.35	0.30	0.32	0.32	0.32	0.32	0.34
Matching error mean	0.10	0.04	0.11	0.11	0.09	0.09	0.10	0.16	0.10	0.10	0.10	0.10	0.11
Matching error std	0.17	0.07	0.17	0.17	0.14	0.15	0.16	0.22	0.17	0.17	0.17	0.17	0.18

Table 6
Credit spreads

This table shows credit spread levels and credit spread sensitivities to firm parameters. The empirical levels and sensitivities are from Davydenko and Strebulaev (2007). Model-implied sensitivities for the baseline, benchmark, and debt priority models are derived with the median firm parameters in Davydenko and Strebulaev (2007) for fixed leverage and market-to-book ratios. We keep leverage and market-to-book ratios fixed by varying the investment scale factor s and the coupon c.

	Empirical	Baseline	Benchmark	Debt priority
Panel A: Credit spread levels				
Median credit spread	109	113	249	31
Panel B: Credit spread sensitivities to strategic variables				
Liquidation cost	+	+	-	+
Bargaining power	+	+	+	+
Bargaining friction	-	-	-	-
Panel C: Credit spread sensitivities to non-strategic variables				
MtB	+	+	+	-
Leverage	+	+	+	+
Volatility	+	+	+	+

Table 7
Control premia

This table summarizes statistics on control premia before distressed reorganization and non-distressed renegotiation. Quarter before event is the number of quarters before distressed reorganization or non-distressed renegotiation. Average control premium is the mean of the percentage differences between the baseline model's debt value and a no-surplus debt claim's value in a quarter. The no-surplus debt claim is derived in Appendix G. Standard deviation is the average of the intra-simulation standard deviations of control premia in a quarter.

Quarter before event	-16	-12	-8	-4	-3	-2	-1
Panel A: Distressed reorganization							
Average control premium	7.86	7.85	8.65	10.66	11.59	12.85	14.64
Standard deviation	3.45	3.36	3.58	3.88	3.93	3.91	3.70
Panel B: Non-distressed renegotiation							
Average control premium	9.46	9.82	10.25	11.21	11.66	12.32	13.41
Standard deviation	3.75	3.38	3.27	3.27	3.27	3.24	3.19