Operating Leverage, Risk Taking and Coordination Failures*

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June 2018

Abstract

We study an economy with demand spillovers where firms' decisions to produce are strategic complements. Firms have access to an increasing returns to scale technology and choose their operating leverage trading off higher fixed costs for lower variable costs. The choice of operating leverage determines the firm's systematic risk, that is, determines how responsive the firm's profits are to an aggregate labor productivity shock, which is the only risk factor in this economy. We show that firms take excessive risk as they do not internalize that higher operating leverage increases the likelihood of a coordination failure where output is inefficiently depressed across the economy. More generally, our analysis suggests that individual risk-taking decisions aggregate into excessive output volatility in the presence of strategic complementarities among agents.

Keywords: Coordination failure, operating leverage, systematic risk

*We would like to thank the Montreal Institute of Structured Finance and Derivatives for financial support. E-mail addresses: matthieu.bouvard@mcgill.ca and adolfo.demotta@mcgill.ca. Preliminary version. Comments are welcome.

1 Introduction

Strategic complementaries, which often generate multiple equilibria and cause coordination failures, arise in wide variety of settings. They can be caused, for instance, by thick market externalities, technological complementaries, imperfect competition and demand spillovers, or imperfect information.[1] These complementarities have been particularly useful in macroeconomics in explaining why economies sometimes appear to overreact to small shocks.[2] Strategic complementaries have been also extensively used in the explanation of financial crises.[3] Our objective is to study risk-taking in an economy that exhibits such strategic complementarities and that may suffer from a coordination failure.

Specifically, we consider an imperfectly competitive economy with aggregate demand spillovers as in Murphy et al. (1989). The economy is populated by representative consumers who are endowed with labor and who collectively own all the firms in the economy. The labor is subject to a productivity shock, which is the only risk factor in this economy. The economy also has a continuum of sectors that produce final goods. Each sector, has a competitive fringe of firms with a constant returns to scale technology and a unique firm with access to an increasing returns to scale technology. This increasing returns to scale technology generates strategic complementarities across sectors: an increase in profits in one sector translates into a greater demand for goods of the other sectors, which allows firms in these other sectors to leverage their own increasing returns to scale technologies and increase profits. There are two dates in this economy. In the final date, after the labor productivity shock is realized, production takes place

[1] Early examples include Hart (1982), Diamond (1982), Weitzman (1982), Bryant (1983), Kiyotaki (1985), Shleifer (1986).

[2] See Cooper (1998, 1999) from an overview of the effect of strategic complementaries in macroeconomics.

[3] See Freixas and Rochet (1999) for an example of coordination failures in the banking system and Obstfeld and Rogoff (1997) for an example of currency crises.

in each sector by either the competitive fringe of firms or by the firm with the increasing returns to scale technology. In the initial date, each firm with access to the increasing returns to scale technology chooses its operating leverage trading off higher fixed costs for lower variable costs. In this economy, the choice of operating leverage is akin to the firm's choice of systematic risk as operating leverage will determine how responsive a firm's profits are to the aggregate labor productivity shock. Specifically, when productivity is high, the firm benefits from operating leverage and is able to meet its demand at a lower cost. However, when productivity is low, operating leverage reduces profits as the demand is not large enough to compensate the firm for the high fixed costs. In equilibrium, the combination of all firm's operating leverage choices endogenously determines aggregate risk, that is, how the labor productivity shock affects the economy's aggregate income.

We start by considering a benchmark case in which workers do not earn any rents when working for the increasing returns to scale firm, that is, the case in which workers are just paid their outside option. In such case, despite the presence of demand spillovers, there is a unique production equilibrium in the final date and this equilibrium is efficient. Moreover, firms choose the socially optimal operating risk in the initial stage and therefore, the amount of risk in the economy is also socially efficient.

Next we consider the case in which workers earn rents that, for instance, may arise due to a moral hazard problem. As shown in Murphy et al. (1989), when workers earn rents, the production decisions in the final date are not socially optimal as firms tend to underinvest. Specifically, when the sum of the workers' rents and the firm's profits is positive, a firm with negative profits does not invest while it would socially optimal to do so, as it would lead to a higher aggregate income and utility for the representative consumer. In addition, the economy now features multiple equilibria in the production stage. Specifically, for intermediate realizations of the labor productivity shock, an equilibrium in which all firms with the increasing returns to scale technology invest coexists with one in which none of them invest. Since our objective is to study risk taking in the initial date, the indeterminacy of the production equilibrium associated to the presence of multiple equilibria complicates the analysis. To circumvent this indeterminacy, we use global game techniques to pin down a unique equilibrium. In this equilibrium, all firms with the increasing returns to scale technology invest if the labor productivity shock is above an endogenously determined threshold. In other words, the equilibrium has the appealing economic property that the economy is more likely to suffer a coordination failure when the productivity is low, that is, when the economic fundamentals are weak. Therefore, there are two related sources of inefficiency in the production equilibrium of the final date: firms tend to underinvest because they fail to internalize the rents earned by the workers, and firms also tend to underinvest because of coordination failures.

Having characterized the unique production equilibrium in the final date, we proceed to study the risk choices in the initial date. We first show that investment inefficiencies in the final date leads firms to increase their operating leverage. Intuitively, because of the workers' rents and the coordination failures, firms now only invest in the final date when the realized labor productivity shock is high, that is, when demand is high, which makes operating leverage more profitable. Notice that when deciding their operating leverage in the initial date, firms face a real options problem as they can always choose not to invest when labor productivity is low in the final date. The underinvestment inefficiencies has the effect of increasing the investment threshold in the final date, which induces firms to increase their risk exposure by choosing a higher operating leverage in the initial date.

Second, we consider the constrained social optimum in which firms only continue in the final date when they earn a profit and show that the economy exhibits excessive risk taking in the initial date.[4] Intuitively, when a firm chooses its operating leverage, it takes the operating leverage of all other firms and, therefore, the equilibrium investment threshold, as given. Consequently, firms do not internalize the effect that their combined risk choices have on this endogenously determined investment threshold. The social planner, however, does internalize the effect and has incentives to decrease the operating leverage, which lowers the equilibrium threshold and prevents coordination failures. This excessive operating leverage in the initial date leads to excessive risk and output volatility.

Our paper is related to a stream of literature in macroeconomics that grounds low aggregate output realizations into coordination failures. In particular, Cooper and John (1988) provide a general framework to analyze economies in which agents' actions are strategic complements, and examine different ways in which the underlying externalities across agents may arise. In our paper, demand complementarities originate from a combination of imperfect competition and increasing-return-to-scale technologies as in Shleifer and Vishny (1988), Kiotaki (1988) and Murphy et al. (1989). In these papers, coordination motives can generate multiple equilibria that can be Pareto-ranked, illustrating how economies may be trapped into inferior low-output equilibria.

While we build on these early contribution to micro-found coordination failures in a production economy, we take advantage of global games techniques to sharpen the predictions of these models. As shown in seminal work by Carlsson and Van Damme (1993) introducing dispersed information in games of strategic complementarities can lead to a unique equilibrium prediction

[4] We also show that there is excessive risk taking with respect to the unconstrained social optimum.

pinned down by the realization of an underlying economic fundamental. Chamley (1999) or Bebchuk and Goldstein (2011) use this insight to study economies where agents' investment decisions exhibit a generic form of complementarity. Schaal and Taschereau-Dumouchel (2015) introduce global games in a dynamic macro-model with demand complementarities, and show how transitory shocks can trigger long-lasting periods of depressed investment and recession. More generally, global games have found several applications, notably currency attacks (Morris and Shin, 1988) and bank runs (Rochet and Vives, 2004, Goldstein and Pauzner, 2005). In this type of setting, Guimaraes and Morris (2007) show that risk-averse agents' exposures to an underlying source of risk affect the outcome of the coordination game they play, but do not model the origin of these exposures. Goldstein and Pauzner (2004) study how contagion across economies subject to coordination failures propagates through the portfolios of diversified investors. We instead focus on firms' productive decisions to calibrate their exposures to aggregate shocks by adjusting their operating leverage. Our model relates coordination failures to excessive risk-taking, which stands in contrast with Lamont (1995) where coordination failures stems from firms' inability to take on financial leverage because of moral hazard. In Farhi and Tirole (2012), strategic complementarities in banks' financial leverage are driven by the anticipation of bailouts, which is distinct from the demand externalities that operate in our framework.

2 Model

We consider a two-period economy with a representative consumer who has a utility function $\exp\left[\int_0^1 \ln x(q)\right] dq$ defined over a unit interval of goods indexed by q. There is a unit interval of consumers each endowed with L units of labor that they supply inelastically. Therefore, L defines the amount of labor in the economy, which we assume that is drawn from a uniform distribution defined over $[0, \overline{L}]$. Different realizations of L can be viewed as an economy-wide labor productivity shock, which is the only risk factor in this economy.

Each good q is produced by a sector, and each sector consists of two types of firms. A competitive fringe of firms with a constant returns to scale technology that produces one unit of output using one unit of labor and a unique firm, hereafter "the monopolist", with access to an increasing returns to scale technology. This increasing returns to scale technology requires a fixed number of units of labor, $F(s_q) = \frac{s_q}{\alpha - s_q}$, and allows to produce one unit of output at a marginal cost of $\alpha - s_q$ units of labor, where $\alpha < 1$, $s_q \geq 0$, and $F'(.) > 0$, $F''(.) > 0$, $F(0) = 0$ and $\lim_{s \to 1} F(s) = \lim_{s \to 1} F'(s) + \infty$. In this set-up, s_q is a choice variable that trades-off variable and fixed costs and determines each monopolist firm q operating leverage. Specifically, the increasing returns to scale technology has a lower marginal cost than the competitive fringe constant returns to scale technology, i.e., $\alpha < 1$, but the monopolist can choose to further decrease its marginal cost, i.e., to increase s_q, at the expense of increasing its fixed cost $F(s_q)$. Consumers own all the profits of this economy.

We assume that workers in the monopolist firms earn rents. In particular, workers receive their outside option, which is determined by the wage in competitive fringe, plus a fraction β of the firm's revenues. We assume that $\alpha + \beta < 1$ which guarantees that the monopolist firm has a lower marginal costs even after accounting for the worker's rent. As it will become clear below, the role of the workers' rent is similar to the one in Murphy et al. (89), that is, to introduce a wedge between the social and the private value of production for the monopolist, which results in multiple equilibrium and in the possibility of a coordination failure. We also assume that $\overline{L} > 2$, which rules out a corner solution in which all firm choose zero operating leverage, i.e., $s_q = 0$.

There are two relevant dates in the economy. At $t = 0$, each monopolist firm q chooses its operating leverage, i.e., s_q, which determines its exposure to the labor productivity shock, L. At $t = 1$, after L is realized, production takes places in each sector by either the monopolist or the competitive fringe, and consumers consume the output using their endogenously determined income, y. Throughout, we take the wage that workers earn in the competitive fringe as the numeraire.

3 The Benchmark Case

We start by studying the case in which workers do not earn any rents, i.e., $\beta = 0$, which serves as a benchmark to highlight the firms' risk choices, i.e., $\{s_q\}_{q\in[0,1]}$, in this economy in the absence of coordination failures.

Consider first the consumption decision at $t = 1$. The representative consumer chooses the consumption basket, $\{x(q)\}_{q\in[0,1]}$, that maximizes his utility given the prices of the different goods, $\{x(q)\}_{q\in[0,1]}$, and his available income, y

$$\max_{\{x(q)\}} \exp\left[\int_o^1 \ln x(q)\right] dq \quad \text{s.t.} \quad \int_o^1 p(q)x(q) = y, \tag{1}$$

From the above optimization problem, it immediately follows that consumers spend the same percentage of their income in each good and, consequently, when income is y, the consumer can be thought of as spending y on every good $x(q)$.

Consider next the production decision at $t = 1$. We assume that the monopolist maximizes its profit taking the demand curve as given and operates (or invests) only if it can earn a profit at the price it charges. The monopolist is sector q solves the following optimization problem:

$$\max_{p(q)} p(q)x(q) - (\alpha - s_q)x(q) - F(s_q) \quad \text{s.t.} \quad x(q) = \frac{y}{p(q)} \text{ and } p(q) \leq 1, \tag{2}$$

where the worker's wage is set equal to one, i.e., the numeraire. The first constraint, $x(q) = \frac{y}{p(q)}$, is the demand function, and the second constraint reflects that if the monopolist sets its price above one, it loses all its sales to the competitive fringe.[5] Since the demand function is unit-elastic, the monopolist sets the price highest possible, $p(q) = 1$, and produces $x(q) = y$ units of output. Therefore, if the monopolist produces, it generates profits

$$\pi_q(y) = (1 - \alpha + s_q)y - F(s_q), \tag{3}$$

and it will produce when $\pi_q(y) \geq 0$, that is, when $y \geq \frac{F(s_q)}{1-\alpha+s_q}$.

The production and consumption decisions at $t = 1$ endogenously determined the aggregate income y. Let n be the fraction of sectors in the economy in which a monopolist is active. The aggregate income at $t = 1$ is the sum of profits and labor income, L, that is:

$$y = \int_0^n (1 - \alpha + s_i)y - F(s_i)di + L. \tag{4}$$

Solving, it yields the aggregate income y as a function of the fraction of sectors in the economy in which a monopolist is active n, the production technologies of the active monopolists $\{s_i\}_{i\in[0,n]}$, and the amount of effective labor L:

$$y = \frac{L - \int_0^n F(s_i)di}{1 - \int_0^n 1 - \alpha + s_i di}. \tag{5}$$

The numerator of (5) is the amount of labor used in the economy for actual production of output, after the fixed costs. The inverse of the denominator, i.e., $\left(1 - \int_0^n (1 - \alpha + s_i)di\right)^{-1}$, is the multiplier that shows the effect on the aggregate income of an additional unit of effective labor L. Notice that a unit increase in L, raises aggregate income by more than one unit as long as $n > 0$, i.e., $\left(1 - \int_0^n (1 - \alpha + s_i)di\right)^{-1} > 1$ if $n > 0$. Intuitively, when a firm makes a profit, it

[5]Since the firms in the competitive fringe require one unit of labor to produce one unit of output, the competitive price would be equal to the worker's wage in the competitive fringe, which we take as the numeraire.

increases the demand of the other firms in the economy, which now are can make a better use of their increasing returns to scale technologies.

Plugging the aggregate income in (5) into the profits in (3), yields the following expression for the firm's profits as a function of n, $\{s_i\}_{i\in[0,n]}$ and L:

$$\pi_q = (1-\alpha+s_q)\frac{L-\int_0^n F(s_i)di}{1-\int_0^n 1-\alpha+s_i di} - F(s_q). \tag{6}$$

Consequently, monopolist q produces if

$$\frac{L-\int_0^n F(s_i)di}{1-\int_0^n 1-\alpha+s_i di} > \frac{F(s_q)}{1-\alpha+s_q}. \tag{7}$$

From equation (7), the lemma below follows:

Lemma 1 *A monopolist firm is more likely to produce at $t=1$ the lower its operating leverage s_q and the higher the effective labor L.*

Intuitively, the higher the effective labor L, the higher the demand y, and the larger the profits π_q and, the lower the operating leverage, the smaller the aggregate income needed to generate positive profits, as a higher fixed cost requires a higher level of aggregate income to justify production.[6]

Finally, at $t=0$, each monopolist q chooses its operating leverage s_q, that is, the ratio of fixed to variable costs. When choosing s_q, the monopolist firm faces the following trade-off: on the one hand, higher operating leverage decreases the probability of the monopolist being active at $t=1$ (see Lemma 1) and decreases profits for low-levels of aggregate income (i.e., $\frac{\partial\pi_q(y)}{\partial s_q}<0$ if $y<\frac{1}{(1-s_q)^2}$) but increases profits for high-levels of aggregate income (i.e., $\frac{\partial\pi_q(y)}{\partial s_q}>0$

[6]It is immediate to verify that $\partial\frac{F(s_q)}{s_q}/\partial s_q > 0$.

if $y > \frac{1}{(1-s_q)^2}$). Notice also that the choice of operating leverage determines the firm's systematic risk as s_q is the sensitivity of the firm's profits to aggregate income y, i.e., $\partial\pi_q/\partial y = s_q$, which in turn, is proportional to the sensitivity of the firm's profits to the aggregate shock L, i.e., $\partial\pi_q/\partial L = s_q(1 - \int_0^n (1-\alpha+s_i)di)^{-1}$. The next lemma summarizes this discussion.

Lemma 2 *An increase in operating leverage (i) decreases the firm's profits for low-levels of aggregate income, (ii) increases the firm's profits for high-levels of aggregate income, and (iii) increases the sensitivity of the firm's profits to the aggregate shock L.*

Hereafter, we focus on symmetric equilibria in which all monopolist firms choose non-cooperatively the same operating leverage s. Imposing symmetry in equation (7), given L, s, and n, a firm chooses to produce if

$$L \geq \frac{F(s)}{1-\alpha+s}. \tag{8}$$

Notice that this condition does not depend on n. While the number of active monopolist firms does affect the profits of the other active monopolists, it does not affect the sign of these profits. Intuitively, if a monopolists q makes a profit on its own, the other (identical) monopolists that are active are also making a profit and consequently, these other active monopolist are exerting a positive demand externality on monopolist q, a externality that further increases its originally positive profits. Alternatively, if a monopolists q were to suffer a loss on its own, other (identical) monopolists would also be suffering a loss and consequently, these other active monopolist would be imposing a negative demand externality on monopolist q, a externality that would further decrease its already negative profits. In other words, in equilibrium, the demand externalities have the effect of increasing profits or decreasing negative profits profits, but these externalities do not change the sign of these profits and consequently of the the production decisions. As we will see, this will not be the case when workers earn rents, i.e., $\beta > 0$.

Using the condition in (7) and imposing symmetry, at $t = 1$, if $L < \frac{F(s)}{1-\alpha+s}$, no monopolist are active ($n = 0$) and the aggregate income is just the amount of effective labor, $y = L$, and, if $L \geq \frac{F(s)}{1-\alpha+s}$, all monopolist are active ($n = 1$) and $y = \frac{L-F(s)}{\alpha-s}$ and therefore, the aggregate income y given L and s can be written as:[7]

$$y(s, L) \equiv \max\left\{\frac{L - F(s)}{\alpha - s}, L\right\}. \tag{9}$$

To find the equilibrium, consider a unilateral deviation where firm q chooses operating leverage s_q different from all other firms' operating leverage, s_{-q}. Firm q produces when profits are positive, i.e., $(1 - \alpha + s_q)\, y(s_{-q}, L) - F(s_q) \geq 0$, or equivalenly when

$$L \geq L(s_q, s_{-q}) \equiv \begin{cases} (\alpha - s_{-q})\dfrac{F(s_q)}{1 - \alpha + s_q} + F(s_{-q}) & \text{if } s_q \geq s_{-q}, \\ \dfrac{F(s_q)}{1 - \alpha + s_q} & \text{if } s_q < s_{-q}. \end{cases} \tag{10}$$

Let

$$\pi(s_q, s_{-q}) \equiv \int_{\min\{L(s_q,s_{-q}),\overline{L}\}}^{\overline{L}} (1 - \alpha + s_q)\, y(s_{-q}, L) - F(s_q)\; dL \tag{11}$$

be firm q's profit (to a factor $1/\overline{L}$) if it chooses s_q and all other firms choose s_{-q}, then s^* is a symmetric equilibrium if

$$s^* \in \operatorname*{argmax}_s \pi(s, s^*). \tag{12}$$

Let $L(s)$ be the minimum effective labor that induces all identical monopolist with operating leverage s to produce, i.e., $L(s) \equiv L(s, s) = \frac{F(s)}{1-\alpha+s}$. The next proposition characterizes the unique symmetric equilibrium at $t = 0$:

[7] Notice that if $L \geq \frac{F(s)}{1-\alpha+s}$ then $\frac{L-F(s)}{\alpha-s} \geq L$.

Proposition 1 *There is a unique symmetric equilibrium* $s^* \in]0, \alpha[$ *at* $t = 0$*, which is defined by the the following condition*

$$E\left[y|L \geq L(s^*)\right] - F'\left(s^*\right) = 0.$$

The unique symmetric equilibrium s^*, which is strictly greater than zero and smaller than α, increases in $\overline{L}$. Intuitively, an increase in $\overline{L}$ induces the firm to take on more risk as increasing its operating leverage allows the firm to benefit from the high realizations of the aggregate shock L and, in the case of a low realization of L, the firm can choose not to operate. It is interesting to note that there is a single symmetric equilibrium at $t = 1$ despite the fact that firms exert demand externalities on each other, that is, the strategic complementary among firms generated by the demand externality does not lead to multiple equilibria. As in Murphy et al. (89), a monopolist operates as long as it makes a profit and, while the magnitude of this profit depends on n, i.e., the number of monopolist operating at $t = 1$, whether the firm makes a profit or would suffer a loss does not.[8] Intuitively, if a monopolist makes a profit operating by itself, then the only equilibrium is one in which all monopolist operate as by operating it makes operating even more profitable. Alternatively, if a monopolist were to make a loss operating by itself, then the only equilibrium is one in which no monopolist operates as, if they all were to operate, it would increase the loss from operating.

Once characterized the equilibrium risk choice s^*, we proceed to study the socially optimum risk choice s^{FB}. Specifically, we still assume that, at $t = 1$, monopolist firms operate only when they make a profit and that consumers maximize their utility by spending all they available income and study the risk choice s at $t = 0$ that maximizes the expected utility of the representative consumer. Since in equilibrium, all goods have a price of one (i.e., the marginal cost

[8] As already explained, this is reflected in the fact that n does not enter the continuation condition in equation 8.

in the competitive fringe) and since consumers spend the same amount y in each good, the consumer's utility in equilibrium income is equal to his income, $\exp\left[\int_o^1 \ln x(q)\right] dq = y$. Therefore, at $t = 0$, the social planner would choose the s that maximizes the expected income at $t = 1$ or equivalently, since the labor income equals L and hence does not depend on s, the social planner would choose the s that maximizes expected profits. The next proposition characterizes the socially optimum risk choice s^{FB}.

Proposition 2 *The socially optimum risk choice equals the equilibrium risk choice, i.e.,* $s^{FB} = s^*$.

Proposition 2 shows that risk taking in this economy is socially optimal. That is, each firm, by choosing the operating leverage s_q that maximizes its expected profits at $t = 0$, also maximizes the expected demand externalities that through these profits have on the other firms. Intuitively, the firm only operates when it makes a profits and, conditionally operating, it chooses the operating leverage that maximizes profits and hence that has maximizes the demand externality.

4 Coordination Failures and Risk Taking

In this section we consider the general case in which workers are able to extract a rent in the monopolist firm. Specifically, workers receive a percentage β of the revenue on top of the their outside option wage of 1. We model the worker's rent as a fraction of the revenue so that this rent scales up with the firm. In addition, the rent being a fraction of the revenue also implies that the rent does not directly impact the choice of operating leverage, that is, the monopolist firm will not have an incentive to shift between variable and fixed costs in order to reduce the rent. This modelling choice is isomorphic to the one in Murphy et al. (89), which assumes that workers must pay a wage premium to workers in the monopolist sector.[9] In fact the only difference is that Murphy et al. (89) considers the wage premium as non-pecuniary costs in the worker's utility function and we assume that it is just a pure rent transfer.

The analysis of the consumption decision at $t = 1$ is identical to the one in which $\beta = 0$, that is, all goods have the same expenditure share and therefore, the representative consumer's income y can be thought of as spending on every good $x(q)$ in the unit interval. At $t = 1$, a monopolist q produces if it can earn a profit given aggregate income y and its production technology s_q. Because the monopolist only retains a fraction $(1 - \beta)$ of revenue, it now solves the following optimization problem:

$$\max_{p(q)} p(q)x(q)(1-\beta) - (\alpha - s_q)x(q) - F(s_q) \;\; \text{s.t.} \;\; x(q) = \frac{y}{p(q)} \text{ and } p(q) \leq 1, \tag{13}$$

where, once again, we take the worker's wage in the competitive fringe as the numeraire. As in the case in which $\beta = 0$, since the demand function is unit-elastic, the monopolist sets the highest possible price, i.e., $p(q) = 1$, and produces $x(q) = y$ units. Therefore, if the monopolist operates, its profits are

$$\pi_q(y) = (1 - \beta - \alpha + s_q)y - F(s_q), \tag{14}$$

and it will operate when $\pi_q(y) > 0$, that is, when the aggregate income is large enough:

$$y \geq \frac{F(s_q)}{1 - \beta - \alpha + s_q}. \tag{15}$$

Let n be the fraction of sectors in the economy in which a monopolist is active. The aggregate

[9] In our set up profits are $\pi(x_q) = p_q x_q(1-\beta) - [(\alpha - s_q)x_q - F(s_q)]$ where p_q is the price and x_q the quantity produced. If we rewrite $\pi'(x_q) = p_q x_q - \frac{1}{1-\beta}[(\alpha - s_q)x_q - F(s_q)]$, $\frac{1}{1-\beta} - 1 = \frac{\beta}{1-\beta}$ would correspond to the wage premium in Murphy et al. (89).

income y at $t = 1$ is the sum of the profit and the labor income

$$y = \int_0^n (1 - \beta - \alpha + s_i)y - F(s_i) + \beta y \, di + L. \tag{16}$$

Solving for y in equation (16), we derive y as a function of the fraction of sectors in the economy in which a monopolist is active n:

$$y = \frac{L - \int_0^n F(s_i)di}{1 - \int_0^n 1 - \alpha + s_i di}. \tag{17}$$

Plugging equation (17) into equation (14), it follows that

$$\pi_q = (1 - \beta - \alpha + s_q)\frac{L - \int_0^n F(s_i)di}{1 - \int_0^n 1 - \alpha + s_i di} - F(s_q), \tag{18}$$

and that monopolist q produces if

$$\frac{L - \int_0^n F(s_i)di}{1 - \int_0^n 1 - \alpha + s_i di} > \frac{F(s_q)}{1 - \beta - \alpha + s_q}, \tag{19}$$

from which the following lemma follows:

Lemma 3 *A monopolist firm is more likely to produce at* $t = 1$ *the lower its operating leverage* s_q *and the higher the effective labor* L*, and the lower the workers' rent* β*.*

Similarly to the case in which $\beta = 0$, a monopolist firm q is more likely to produce the higher the effective labor L and the lower its operating leverage s_q. In addition, a monopolist firm q is now also more likely to produce the lower the workers' rents β. While the production condition for $\beta = 0$ and $\beta > 0$ (i.e., equations 7 and 19) are similar, there is an important economic difference. To see this, consider the case in which all monopolist firms choose the same operating leverage at $t = 0$, i.e., $s_i = s_q = s$ for all i. Condition (19) can be then written as

$$\frac{L - nF(s)}{1 - n(1 - \alpha + s)} > \frac{F(s)}{1 - \alpha - \beta + s}. \tag{20}$$

Condition (20) implies that a firm does not continue if no other firm continues (i.e., if $n = 0$) when

$$L < \frac{F(s)}{1 - \alpha - \beta + s}, \tag{21}$$

and that a firm continues when all other firms continue (i.e., $n = 1$) when

$$L > \frac{(1 - \beta)F(s)}{1 - \alpha - \beta + s}. \tag{22}$$

Notice that as long as $\beta > 0$, there will multiple equilibria at $t = 1$ when

$$L \in \left(\frac{(1 - \beta)F(s)}{1 - \alpha - \beta + s}, \frac{F(s)}{1 - \alpha - \beta + s} \right). \tag{23}$$

The intuition is the same as the in Murphy et al. (1988). When $\beta = 0$, a firm operates only when it earns a profit that is, when by operating the firm exerts a positive demand spillover effect on the other firms. When $\beta > 0$, a firms still only continues when it earns a profit, however, now a firm that makes a loss could still have a net positive demand spillover on the other firms through the workers' rents. In other words, when $\beta = 0$, the externality of a monopolist firm being active on the aggregate demand only comes from its profits, while when $\beta > 0$, the externality on the aggregate demand comes from its profits as well as the workers' rents. Consequently, when $\beta > 0$, there can be situations in which if no other monopolist operates, a monopolist does not produce but, if all other monopolist produce, because the demand spillovers generated by the rents earn by the other firms' workers, the monopolist makes profits and produces. The following Proposition summarizes the discussion.

Proposition 3 *If all firms choose the same operating leverage at $t = 0$, then at $t = 1$: (i) if $L > \frac{F(s)}{1-\alpha-\beta+s}$, all firms operate; (ii) if $L < \frac{(1-\beta)F(s)}{1-\alpha-\beta+s}$, no firm operates; and (iii) if $L \in \left(\frac{(1-\beta)F(s)}{1-\alpha-\beta+s}, \frac{F(s)}{1-\alpha-\beta+s}\right)$, there is multiple equilibria, one equilibrium in which no firm operates and* *another equilibrium in which all firms operate, which Pareto dominates the one in which no firm operates.*

The presence of multiple equilibria captures the idea that the economy can suffer from a coordination failure. Specifically, when there are multiple equilibria, the economy can be trapped in an equilibrium in which no firm operates, while this equilibrium is Pareto dominated by an equilibrium in which all firms operate. The existence of multiple equilibria at $t = 1$ makes it difficult to predict the equilibrium operating leverage (the risk choice) at $t = 0$, as this equilibrium depends on the anticipated production equilibrium at $t = 1$.

Next, we use global games techniques as a refinement to obtain a unique production equilibrium at $t = 1$ that will allow us to study the operating leverage decision at $t = 0$. Specifically,suppose all firms have chosen s at $t = 0$ and that each monopolist firm q observes a noisy signal of L, $l_q = L + \xi_q$ where ξ is uniform on $[-\varepsilon, \varepsilon]$, at $t = 1$. Assume that firm q follows a threshold strategy, that is, the firm operates at $t = 1$ if l_q is above a threshold l^*. Then, for a given realization of L, the number of monopolist firms that operate is

$$n(L) \equiv \begin{cases} 1 & \text{if } L > l^* + \varepsilon \\ \frac{L+\varepsilon-l^*}{2\varepsilon} & \text{if } L \in [l^* - \varepsilon, l^* + \varepsilon] \\ 0 & \text{if } L < l^* - \varepsilon \end{cases} \tag{24}$$

At the threshold, a firm with signal l^* must be indifferent between operating and not operating, that is, the following condition must hold:

$$\frac{1}{2\varepsilon} \int_{l^*-\varepsilon}^{l^*+\varepsilon} (1 - \alpha - \beta + s) \frac{L - n(L)F}{1 - n(L)(1 - \alpha + s)} - F \, dL = 0. \tag{25}$$

Then, taking the limit when $\varepsilon \to 0$, the next proposition follows.

Proposition 4 *If all monopolist firms choose the same operating leverage s at $t = 0$ and ε tends to 0, then at $t = 1$, all monopolist operate if and only if $L \geq L^T(s)$ and all monopolist do not operate if and only if $L < L^T(s)$, where*

$$L^T(s) \equiv \frac{F}{1-\alpha+s} + \frac{\beta F}{(1-\alpha-\beta+s)\ln\left(\frac{1}{\alpha-s}\right)}.$$

Notice that threshold $L^T(s)$ is always higher than the first-best threshold $\frac{F}{1-\alpha+s}$ in equation 8 as long as $\beta > 0$. There are two reasons for this result. First, because the workers' rents, the minimum L required to sustain an equilibrium in which all firms operate is larger. Specifically, from equation 22, for all firms to operate, L needs to be larger than $\frac{(1-\beta)F(s)}{1-\alpha-\beta+s}$, which increases with beta β as long as $s < \alpha$.[10] Second, even when L is large enough to sustain an equilibrium in which all firms operate, there is a range of realizations of L above $\frac{(1-\beta)F(s)}{1-\alpha-\beta+s}$ such that firms fail to coordinate and all end up not operating. Indeed, the threshold $L^T(s)$ in Proposition 4 falls within the multiple equilibria region, i.e., $L^T(s) \in \left(\frac{(1-\beta)F(s)}{1-\alpha-\beta+s}, \frac{F(s)}{1-\alpha-\beta+s}\right)$. In other words, the global games refinement partitions the multiple equilibria region in two, and chooses the equilibrium in which all firms produce for large enough realizations of L, i.e., for $L \in \left(L^T(s), \frac{F(s)}{1-\alpha-\beta+s}\right)$, and chooses the equilibrium in which all firms fail to produce for low enough realizations of L, i.e., for $L \in \left(\frac{(1-\beta)F(s)}{1-\alpha-\beta+s}, L^T(s)\right)$. Therefore, the refinement links the probability of a coordination failure to the economy's fundamentals, in our specific case, the aggregate labor productivity shock L. The next lemma summarizes the discussion.

Lemma 4 *If all monopolist firms choose the same operating leverage s at $t = 0$, then at $t = 1$, the economy suffers from a coordination problem when $L \in \left(\frac{(1-\beta)F(s)}{1-\alpha-\beta+s}, L^T(s)\right)$, that is, a monopolist firm would be willing to operate if all the other firms operate but the equilibrium is one in which no firm invest.*

[10] When $s \to \alpha$ then $F(s) \to +\infty$, therefore in any equilibrium $s < \alpha$.

Once characterized the production equilibrium at $t = 1$, we proceed to analyze the choice of operating leverage (and hence, of risk) at $t = 0$. As in the case in which $\beta = 0$, we focus on symmetric equilibrium in which all monopolist firms choose non-cooperatively the same equilibrium $s_q = s$ and choose to operate at $t = 1$ when $L \geq L^T(s)$. Therefore, the aggregate income y at $t = 1$ given L and s can be written as:

$$y_\beta(s, L) \equiv \max \left\{ \begin{array}{ll} \frac{L-F(s)}{\alpha-s} & \text{if } L \geq L^T(s) \\ L & \text{if } L < L^T(s) \end{array} \right\}. \tag{26}$$

To find the equilibrium, consider a unilateral deviation where firm q chooses operating leverage s_q different from all other firms' operating leverage, s_{-q}. Firm q produces when profits are positive, i.e., $(1 - \alpha - \beta + s_q)\, y_\beta(s_{-q}, L) - F(s_q) \geq 0$, that is, when

$$L \geq L_\beta(s_q, s_{-q}) \equiv \left\{ \begin{array}{ll} \max\left\{(\alpha - s_{-q})\dfrac{F(s_q)}{1-\alpha-\beta+s_q} + F(s_{-q}), L^T(s_{-q})\right\} & \text{if } s_q \geq s_{-q}, \\ \min\left\{\dfrac{F(s_q)}{1-\alpha-\beta+s_q}, L^T(s_{-q})\right\} & \text{if } s_q < s_{-q}. \end{array} \right. . \tag{27}$$

Let

$$\pi_\beta(s_q, s_{-q}) \equiv \int_{\min\{L_\beta(s_q,s_{-q}),\overline{L}\}}^{\overline{L}} (1 - \alpha - \beta + s_q)\, y(s_{-q}, L) - F(s_q)\; dL \tag{28}$$

be firm q's profit (to a factor $1/\overline{L}$) if it chooses s_q and all other firms choose s_{-q}, then s^*_β is a symmetric equilibrium if

$$s^*_\beta \in \operatorname*{argmax}_s \pi_\beta(s, s^*_\beta). \tag{29}$$

Next we characterized the equilibrium.

Proposition 5 *If at $t = 0$, a symmetric equilibrium exists, s^*_β this equilibrium is interior, i.e., $s^*_\beta \in]0, \alpha[$, unique, and defined by the the following condition*

$$E\left[y|L \geq L^T(s^*_\beta)\right] - F'\left(s^*_\beta\right) = 0.$$

There is a $\overline{L}$ large enough such that a unique symmetric equilibrium exists.

The condition that defines the equilibrium in Proposition 5, $E\left[y|L \geq L^T(s^*_\beta)\right] - F'\left(s^*_\beta\right) = 0$, is similar to the one in Proposition 1 $E\left[y|L \geq L(s^*)\right] - F'\left(s^*\right) = 0$, except that in Proposition 5 firms operate at $t = 1$ when $L > L^T(s^*_\beta)$ while in Proposition 5 firms operate at $t = 1$ when $L > L(s^*)$. Economically, this constitutes an important difference as in the case in which $\beta > 0$, when $L = L^T(s^*_\beta)$ firms make profits while if $\beta = 0$, when $\beta = 0$, firms $L = L(s^*)$ firms just break even, which again highlights the coordination failure in this economy for $L \in \left(\frac{(1-\beta)F(s^*_\beta)}{1-\alpha-\beta+s^*_\beta}, L^T(s^*_\beta)\right)$. From the comparison between $L^T(.)$ and $L(.)$, we obtain the following corollary.

Corollary 1 *Let s^* be a symmetric equilibrium when $\beta = 0$ and s^*_β be a symmetric equilibrium when $\beta > 0$, then $s^* < s^*_\beta$.*

The corollary implies that operating leverage increases with β. Intuitively, for a given operating leverage s, the higher the beta, the higher the threshold $L^T(s)$. Consequently, each firm now benefits from having a higher operating leverage as the aggregate income conditioning on other firms operating is higher as now firms only operate for higher realizations L.

Once characterized the equilibrium risk choice s^*_β, we proceed to study its efficiency. We continue to assume that, at $t = 1$, monopolist firms operate only when they make a profit and that consumers maximize their utility by spending all they available income. As we saw in Lemma 4, now firms may choose not to continue when it is socially optimal to so. Therefore, we are considering constrained efficiency, that is, the socially optimal risk choice at $t = 0$ given that the economy can suffer from a coordination failure at $t = 1$.[11]

[11]Notice that Proposition 2 and Corollary 1 implies that $s^*_\beta > s^* = s^{FB}$, that is, the operating leverage when there are coordination failures is going to be larger the first-best operating leverage when there are no coordination failures.

Since in equilibrium all goods have a price of one and since consumers spend the same amount y in each good, the consumer's utility in equilibrium income is equal to his income, $\exp\left[\int_o^1 \ln x(q)\right] dq = y$. Therefore, at $t = 0$, the social planner would choose the s that maximizes the expected income at $t = 1$ or equivalently, since the labor income equals L and hence does not depend on s, the social planner would choose the s that maximizes the sum of expected profits and the workers' rents. The next proposition characterizes the constrained efficiency of the equilibrium risk choice.

Proposition 6 *In any symmetric equilibrium s^*_β, there is excessive risk taking.*

Proposition 6 shows that the equilibrium is constrained inefficient and specifically that in equilibrium there is excessive risk taking. The intuition for this result comes from the fact that the economy can suffer from a coordination failure and that firms when choosing its operating leverage do not internalize the effect that their risk choices have on the probability of such coordination failure. Indeed, as indicated in Proposition 4, the economy suffers from a coordination failure when L falls bellow $L^T(s^*_\beta)$, which is increasing in s^*_β. While this threshold depends of the operating leverage of all firms in the economy, each individual firms when choosing its own operating leverage takes this threshold as given and hence, ignores the negative externality imposed on the other firms through the effect on the threshold of its choice of operating leverage.

It is important to notice that when $\beta = 0$, firms also take the operating threshold of all other firms as given. However, when $\beta = 0$, firms at the operating threshold, i.e., at $L = L(s^*)$, make zero profits while, when $\beta > 0$, firms at the operating threshold, i.e., at $L = L^T(s^*_\beta)$, make positive profits. In other words, excessive risk-taking is the product of coordination failures among firms which we pin down through global games techniques. To drive this point further down, consider the equilibrium in Proposition 3 and instead of applying a global games treatment, simply assume that firms always coordinate on the Pareto-superior production equilibrium: for any common leverage s, they produce if and only if $L \geq (1-\beta)\frac{F(s)}{1-\alpha-\beta+s}$. We then obtain the following result

Corollary 2 *If at $t = 1$, firms coordinate production decisions on the Pareto-superior equilibrium in Proposition 3, then ex-ante risk-taking is constrained-efficient at $t = 0$.*

5 Discussion

Proposition 6 delivers the main message of the paper, that is, the idea that when the economy can suffer from coordination failures, firms take excessive risk as they do not internalize the effect that their risk choices have on the probability of a coordination failure. Following some of the seminal papers in the literature, e.g., Hart (1982), Kiyotaki (1988), Murphy et al. (1989), Shleifer (1986), Shleifer et al. (1988), Weitzman (1982), etc., we have illustrated this effect in the context of a model with demand externalities. However, excessive risk taking can also arise in other situations with strategic complementarities and coordination failures. For instance, strategic complementaries can also work through investment and technological externalities (Bryant, 1983) or through search externalities (Diamond, 1982).

In general, as Propositions 1 and 2 illustrate, the presence of demand externalities is not enough to generate multiple equilibria and the possibility of a coordination failure. In the model, what generates multiple equilibria is the fact that workers earn rents. As shown in Murphy et al. (1989), there are other reasons that can lead to the existence of multiple equilibria in the presence of demand externalities, including workers demanding a wage premium or firms using resources to invest at one point in time (which decreases aggregate demand today) that generate labor savings at a later point (which increases aggregate demand tomorrow). Therefore, while our modelling choice is both technically convenient as well economically relevant (there are numerous reasons that explain why workers can earn rents), it is by no means the only reason that can generate a coordination failure in the context of the modelled economy.[12] From an empirical point of view, the analysis indicates that economies in which workers are able to capture rents will tend to be more prone to coordination failures and to excessive risk taking (Corollary 1).

In a similar model, Shleifer and Vishny (1988) shows that the presence of aggregate demand spillovers can lead firms to underinvestment even in the absence of a coordination failure. Their underinvestment result is related to the fact that the multiplier, which depends on the number of active monopolist firms, is not constant across states. In our model, all monopolist firms make the same risk choice at $t = 0$ and consequently all have the same technology and make the same operating decision at $t = 1$. Specifically, either all firms operate or no firm operates depending on the realization of the aggregate shock L at $t = 1$. This implies that, in equilibrium, the demand multiplier is constant across all states L in which the firm operates, that is, when $L \geq L^T(s^*_\beta)$. Specifically, when $n = 1$, the multiplier is constant and equal to $\frac{1}{\alpha - s^*} > 1$.[13] We have shown that in the absence of coordination failures ($\beta = 0$) with homogenous firms, firms' risk choices are socially optimal (Proposition 2). We have also shown that coordination failures ($\beta > 0$) lead to excessive risk taking (Proposition 6). Consider next the case of no coordination failures ($\beta = 0$) and but with two types of monopolist firms with a different investment technology. Specifically, while all monopolist firms still have the same fixed costs $F(s_q) = \frac{s_q}{\alpha - s_q}$, now, we assume that in a fraction n_1 of the sectors, the monopolist firms have marginal cost $\alpha + \varphi_1 - s_q$

[12] As explained before, our modelling of the workers' rents is isomorphic to the modelling in Murphy et al., (89). See footnote 4.

[13] The demand multiplier refers to increase in aggregate income for each dollar of profits that a firm generates. In the model, as we explained following equation 5, the multiplier equals $\left(1 - \int_0^n (1 - \alpha + s_i) di\right)^{-1}$.

with $\alpha + \varphi_1 < 1$ while, in a fraction n_2 $(= 1 - n_1)$ of the sectors, the monopolist firms have marginal cost marginal cost of $\alpha + \varphi_2 - s_q$, with $\alpha + \varphi_2 < 1$ and $\varphi_1 \neq \varphi_2$. We also assume that $\overline{L} > 2/\alpha$ to rule out corner solutions but otherwise the model remains the same. The next proposition characterizes the efficiency of the equilibrium.

Proposition 7 *Any symmetric equilibrium in which all type-1 firms choose s_1^* and all type-2 firms choose s_2^* is constrained inefficient and features insufficient risk-taking.*

Proposition 7 shows that with heterogenous firms and no coordination failures there is insufficient risk-taking, which contrasts with the excessive risk-taking result of Proposition 6. This highlights the difference between the investment problem at $t = 1$ and the risk-taking problem at $t = 0$. Specifically, while in both set ups –the set up in Proposition 6 and the one in Proposition 7– there is underinvestment (i.e.,sometimes firms do not operate at $t = 1$ when it would be socially efficient to do so), the risk-taking inefficiency goes in opposite direction.[14] To understand the intuition behind the insufficient risk-taking result of Proposition 7, consider first the benchmark case with $\beta = 0$ and homogenous firms (as in Propositions 1 and 2). In general, while firms choose their operating leverage to maximize expected profits, the social planner's objective is to maximize the sum over each realization of L of the product of the profits and the demand multiplier. When $\beta = 0$ and firms are homogeneous, all monopolist firms are active when $L > L(s)$ and the multiplier is also constant and equal to $\frac{1}{\alpha - s^*}$ for all $L > L(s)$. The constant multiplier makes maximizing the sum of the product of the profits and the demand

[14]In the set up of Proposition 7 with heterogeneous firms, there is underinvestment because firms underweight the profits that they make in the higher productivity states (higher realizations of L) when there are a lot of firms operating and therefore, the demand multiplier is larger. (This underinvestment problem is similar to the one in Shleifer and Vishny, 1988.) In the set up of Proposition 7, there is underinvestment because firms fail to fully internalize the demand externalities from operating. (This underinvestment problem is similar to the one in Murphy et al., 1989.)

multiplier equivalent to maximizing expected profits, which implies that the firms' risk choices are efficient as Proposition 2 states. Consider now the case of Proposition 7 with $\beta = 0$ and heterogeneous firms. Assume without loss of generality that in equilibrium type-1 firms have a lower operating threshold, L_1, than type-2 firms, L_2, that is, assume that for $L \in [L_1, L_2[$ only type-1 firms operate while for $L \in [L_2, \overline{L}]$ both types operate. In that case, the demand multiplier is $(1-n_1(1-\alpha-\varphi_1+s_1))^{-1}$ for $L \in [L_1, L_2[$ while is $(1-n_1(1-\alpha-\varphi_1+s_1)-n_2(1-\alpha-\varphi_2+s_2))^{-1}$ for $L \in [L_2, \overline{L}]$. Since $(1-n_1(1-\alpha-\varphi_1+s_1))^{-1} < (1-n_1(1-\alpha-\varphi_1+s_1)-n_2(1-\alpha-\varphi_2+s_2))^{-1}$ and extra dollar of profits is more valuable from the social point of view when $L \in [L_1, L_2[$ than when $L \in [L_2, \overline{L}]$ while for the point of view of type-1firms the extra dollar is equally valuable, that is, from the social point of view type-1firms overweight profits in the $[L_1, L_2[$ region while underweight profits in the $[L_2, \overline{L}]$. Consequently, since operating leverage has the effect of shifting profits from lower to higher realizations of L, type-1 firms tend to have too little operating leverage. Finally notice as well, that the contrast between the excessive risk-taking result of Proposition 6 and the insufficient risk-taking result of Proposition 7 also highlights the fact that the excessive risk-taking is specifically associated to the existence of coordination failures.

The analysis has implications for public policy as the government may be able to decrease the probability of a coordination failure by controlling risk-taking. For instance, the presence of a public sector that internalizes demand externalities and therefore, that takes less risk can help reducing the probability of an economy-wide coordination failure. It is important to notice that the effect of the public sector on the probability of a coordination failure would work directly through its own risk-taking behaviors as well as indirectly through the risk-taking behavior of the private sector. Intuitively, as seen in Proposition 5, firms choose their operating leverage by equation at the margin their expected demand to cost of increasing the operating leverage, that is,

$$E[y|L > L^T(s^*)] = F'(s^*). \tag{30}$$

The presence of a public sector with lower risk and consequently, with lower operating, would have the effect of reducing the operating threshold of the private sector and, as a consequence, also of reducing its risk taking behavior. From the empirical point of view, the analysis suggests that economies with a large public sector will tend to be less volatile.

Finally, an important assumption in terms of the timing of the model is the fact that firms choose their technology –their operating leverage– at $t = 0$, before the realization of the aggregate labor productivity shock at $t = 1$. Our interpretation of the model is one in which firms choose a technology and operate in a business as usual situation, and the economy suffers a labor productivity shock at some point in time. Since changing the cost structure is something that takes time or may not be feasible at all, at that point, firms have to decide whether to continue operating or to foreclose given their current cost structure. In the model, for tractability, we assume that firms do not operate at $t = 0$. However, one can think of an alternative model in which firms operate in both dates. In this alternative model, the choice of technology at $t = 0$ would depend on the known productivity of labor at $t = 0$ as well as on the expected productivity of labor at $t = 1$. However, the same economic effect that leads to excessive risk-taking in the paper would also be present in that model, that is, firms would still not internalize the effect that their risk choices at $t = 0$ have of the probability of a coordination failure at $t = 1$.

6 Conclusion

The role of demand externalities and the possibility of a systemic crises in which the economy is trapped in a low activity equilibrium has been extensively studied in the literature. In essence, this literature views economies as interdependent systems in which agents need to coordinate their investment and consumptions decisions. The paper shows that this type of economies feature excessive risk-taking as agents do not internalize the effect that their risk choices have on the probability that the economy suffers a coordination failure. The analysis has implications for public policy as well as for the role of the public sector. Specifically, a public sector that internalizes the effect that its risk choices have on social welfare can help coordinating the economy and reduce the probability that the economy ends up trapped in a low activity Pareto-dominated equilibrium.

While the model emphasizes the role of demand externalities because of its economic importance, there are other sources of strategic complementaries –search frictions, technological spillovers, investment complementarities– that can generate multiple equilibria and the possibility of a coordination failure. In general, we expect the same economic forces to apply in these alternative set-ups. Indeed, as long as risk increases the probability of a coordination failure and agents do not internalize this effect, the agents will incur in excessive risk-taking.

Strategic complementarities are also particularly important in the financial system as illustrated by recurrent systemic crises. Indeed, its liquidity transformation role combined with the fact that financial institutions constantly interact in a highly interdependent system makes the financial system particularly vulnerable to a coordination failure. Our analysis then suggest that economies that feature a more concentrated financial system, for instance, the Canadian economy, may be more stable as the large financial institutions will internalize to a larger extent the effect that their risk choices have of the probability of a systemic financial crises. Alternatively, it also indicates that the economies with a more disperse financial system will require stricter regulation to curve excessive risk-taking.

The model is a one-factor model, that is, the unique source of risk is the labor productivity shock. In general however, there can be multiple sources of risk and risks can be either systematic or idiosyncratic. The existence of different types of risk and its interaction can have important implications for the probability of a systemic crisis. For instance, going back to the example of the financial system, in a more concentrated financial system, on the one hand, financial institutions will do a better job internalizing the externalities caused by their risk choices but, on the other hand, their idiosyncratic risks will not be as well diversified and can become an important source of aggregate risk. The interaction between different types of risks is an interesting avenue for future research.

References

Angeletos, George-Marios, and Ivan Werning, 2006, Crises and prices: Information aggregation, multiplicity, and volatility, *American Economic Review* 96, pp. 1720-1736.

Bryant, John, 1983, A simple rational expectations Keynes-type model, *Quarterly Journal of Economics* 98, pp. 525-528.

Carlsson, Hans, and Eric Van Damme, 1993, Global payoff uncertainty and risk dominance, *Econometrica* 61, pp. 989-1018.

Chamley, Christophe, 1999, Regime switches, *Quarterly Journal of Economics* 114, pp. 869-905.

Cooper, Russell, 1994, Equilibrium selection in imperfectly competitive economies with multiple equilibria, *Economic Journal* 104, pp. 1106-1122.

Cooper, Russell, and Andrew John, 1988, Coordinating coordination failures in keynesian models, *Quarterly Journal of Economics* 103, pp. 441-463.

Cooper, Russell, 1999, *Coordination games, complementarities and macroeconomics*, Cambridge University Press.

Diamond, Peter, 1982, Aggregate demand management in search equilibrium, *Journal of Political Economy* 90, pp. 881-894.

Fahri, Emmanuel and Jean Tirole, 2012, Collective moral hazard, maturity mismatch, and systemic bailouts, *American Economic Review* 102, pp. 60-93.

Freixas, Xavier, and Jean-Charles Rochet, 1997, *Microeconomics of Banking*, MIT Press.

Goldstein, Itay, and Ady Pauzner, 2004, Contagion of self-fulfilling financial crises due to diversification of investment portfolios, *Journal of Economic Theory* 119, pp. 151-183.

Goldstein, Itay, and Ady Pauzner, 2005, Demand deposit contracts and the probability of bank runs, *Journal of Finance* 60, pp. 1293-1328.

Guimaraes, Bernardo, and Stephen Morris, 2008, Risk and wealth in a model of self-fulfilling currency attacks, *Journal of Monetary Economics* 54, pp. 2205-2230.

Hart, Oliver, 1982, A model of imperfect competition with Keynesian features, *Quarterly Journal of Economics* 97, pp. 109-38.

Kiyotaki, Nobuhiro, 1988, Multiple expectations equilibria under monopolistic competition, *Quarterly Journal of Economics* 103, pp, 695-714.

Lamont, Owen, 1995, Corporate-debt overhang and macroeconomic expectations, *American Economic Review* 85, pp. 1106-1117.

Morris, Stephen and Hyun Shin, 1998, Unique equilibrium in a model of self-fulfilling currency attacks, *American Economic Review* 88, pp. 587-597.

Morris, Stephen and Hyun Shin, 2003, Global games: Theory and applications, *Advances in Economics and Econometrics* (Proceedings of the Eighth World Congress of the Econometric Society), edited by M. Dewatripont, L. Hansen and S. Turnovsky; Cambridge University Press.

Murphy, Kevin, Andrei Shleifer, and Robert Vishny, 1989, Industrialization and the big push, *Journal of Political Economy 97,* pp. 1003-1026.

Obstfeld, Maurice and Kenneth Rogoff, 1997, *Foundations of International Macroeconomics*, MIT Press.

Rochet, Jean-Charles and Xavier Vives, 2004, Coordination failures and the lender of last resort: Was Bagehot right after all? *Journal of the European Economic Association 2,* pp. 1116-1147

Schaal, Edouard, and Mathieu Taschereau-Dumouchel, 2015, Coordinating business cycles, working paper.

Shleifer, Andrei, 1986, Implementation cycles, *Journal of Political Economy* 94, pp. 1163-90.

Shleifer, Andrei, and Robert Vishny, 1988, The efficiency of investment in the presence of aggregate demand spillovers, *Journal of Political Economy* 96, pp. 1221-31.

Weitzman, Martin, 1982, Increasing returns and the foundations of unemployment theory, *Economic Journal* 92, pp. 787-804.

Appendix

Proof of Proposition 1

Let $L(s) \equiv L(s_q = s, s_{-q} = s)$ and assume for now that $L(s) < \overline{L}$. Assume further that firm q takes the lower bound $L(s)$ as given. Then firm q would solve the following optimization problem:

$$\max_{s_q} \frac{1}{\overline{L}} \int_{L(s)}^{\overline{L}} \pi(s_q, s) dL \tag{A.1}$$

which writes as:

$$\max_{s_q} \frac{1}{\overline{L}} \int_{L(s)}^{\overline{L}} \frac{L - F(s)}{\alpha - s}(1 - \alpha + s_q) - F(s_q)\, dL \quad \text{where } L(s) = \frac{F(s)}{1 - \alpha + s} \tag{A.2}$$

The first derivative with respect to s_q writes:

$$\int_{L(s)}^{\overline{L}} \frac{L - F(s)}{\alpha - s} - F'(s_q)\, dL \Longleftrightarrow E\left[y | L \geq L(s)\right] - F'(s_q) \tag{A.3}$$

The second derivative with respect to s_q writes:

$$-\int_{L(s_q)}^{\overline{L}} F''(s_q) < 0 \tag{A.4}$$

Therefore, given s, the unique interior solution s_q^* to A.1 would be defined by

$$E\left[y | L \geq L(s)\right] - F'\left(s_q^*\right) = 0, \tag{A.5}$$

and if we impose symmetry $s = s_q^* = s^*$ then we would have

$$E\left[y | L \geq L(s^*)\right] - F'(s^*) = 0, \tag{A.6}$$

which is the condition in Proposition 1.

Next we explore the properties of this condition, $E\left[y|L\geq L(s^{*})\right]-F'\left(s^{*}\right)=0$:

$$\begin{aligned}
\Lambda(s) &\equiv E\left[y|L\geq\frac{F(s)}{1-\alpha+s}\right]-F'\left(s\right)= && \text{(A.7)}\\
&= \frac{\frac{\overline{L}+\frac{F(s)}{1-\alpha+s}}{2}-F\left(s\right)}{\alpha-s}-F'\left(s\right)\\
&= 0.5\frac{\overline{L}-F\left(s\right)}{\alpha-s}+0.5\frac{F(s)}{1-\alpha+s}-F'\left(s\right) && \text{(A.8)}\\
&= \frac{0.5}{\alpha-s}\left(\overline{L}-\frac{s}{\alpha-s}+\frac{s}{1-\alpha+s}-\frac{2\alpha}{\alpha-s}\right)\\
&= \frac{0.5}{\left(\alpha-s\right)\left(1-\alpha+s\right)}\left[\overline{L}(1-\alpha+s)-\frac{2s^{2}+s+2\alpha\left(1-\alpha\right)}{\alpha-s}\right]
\end{aligned}$$

Therefore,

$$\text{sign}\left[\Lambda(s)\right]=\text{sign}\underbrace{\left[\overline{L}(1-\alpha+s)-\frac{2s^{2}+s+2\alpha\left(1-\alpha\right)}{\alpha-s}\right]}_{\equiv\Omega(s)} \tag{A.9}$$

$$\begin{aligned}
\frac{\partial\Omega(s)}{\partial s} &= \overline{L}-\frac{4s(\alpha-s)+2s^{2}}{(\alpha-s)^{2}}-\frac{\alpha}{(\alpha-s)^{2}}-\frac{2\alpha\left(1-\alpha\right)}{(\alpha-s)^{2}} && \text{(A.10)}\\
&= \overline{L}-\frac{-2s^{2}+4s\alpha+\alpha\left(3-2\alpha\right)}{(\alpha-s)^{2}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^{2}\Omega(s)}{\partial s^{2}} &= -\frac{\left[-4s+4s\right](\alpha-s)^{2}-2(\alpha-s)\left[-2s^{2}+4s\alpha+\alpha\left(3-2\alpha\right)\right]}{(\alpha-s)^{4}} && \text{(A.11)}\\
&= 2\frac{2(\alpha-s)^{2}+3\alpha}{(\alpha-s)^{3}}>0
\end{aligned}$$

Next, $\Omega''(.)>0$ together with $\Omega(0)=(1-\alpha)(\overline{L}-2)>0$ and $\lim_{s\rightarrow\infty}\Omega(s)=-\infty$ implies $\Omega(s)=0$ has a unique solution that belongs to $(0,\alpha)$, and therefore, (A.6) has a unique solution. Since $\Omega(0)>0$, $\frac{2s^{2}+s+2\alpha(1-\alpha)}{\alpha-s}$ cuts $\overline{L}(1-\alpha+s)$ from below at s^{*}.

Consider first a deviation to a lower s_q. Let s^{eq} be an equilibrium and consider a deviation by a firm by firm q to an s_q smaller than s^{eq}. In a deviation to $s_q<s^{eq}$ the firm would solve the following optimization problem:

$$\max_{s_q}\frac{1}{\overline{L}}\int_{\frac{F(s^{eq})}{1-\alpha+s^{eq}}}^{\overline{L}}\frac{L-F\left(s^{eq}\right)}{\alpha-s^{eq}}(1-\alpha+s_q)-F\left(s_q\right)dL+\frac{1}{\overline{L}}\int_{\frac{F(s_q)}{1-\alpha+s_q}}^{\frac{F(s^{eq})}{1-\alpha+s^{eq}}}L(1-\alpha+s_q)-F\left(s_q\right)dL \tag{A.12}$$

The first derivative w.r.t. s_q is

$$\frac{1}{\overline{L}}\int_{\frac{F(s^{eq})}{1-\alpha+s^{eq}}}^{\overline{L}}\frac{L-F\left(s^{eq}\right)}{\alpha-s^{eq}}dL+\frac{1}{\overline{L}}\int_{\frac{F(s_q)}{1-\alpha+s_q}}^{\frac{F(s^{eq})}{1-\alpha+s^{eq}}}LdL-\frac{1}{\overline{L}}\int_{\frac{F(s_q)}{1-\alpha+s_q}}^{\overline{L}}F'\left(s_q\right). \tag{A.13}$$

Imposing symmetry, $s^{eq}=s_q$, and setting first derivative in A.13 to zero, we have $\Lambda(s^{eq})=0$, which means that $s^*\in]0,\alpha[$ is the only possible equilibrium.

Notice that

$$\begin{aligned}
&\frac{1}{\overline{L}}\left[\int_{\frac{F(s^*)}{1-\alpha+s^*}}^{\overline{L}}\frac{L-F\left(s^*\right)}{\alpha-s^*}dL+\int_0^{\frac{F(s^*)}{1-\alpha+s^*}}LdL-\int_0^{\frac{F(s_q)}{1-\alpha+s_q}}LdL-\int_{\frac{F(s_q)}{1-\alpha+s_q}}^{\overline{L}}F'\left(s_q\right)dL\right] \qquad (A.14)\\
>&\frac{1}{\overline{L}}\left[\int_{\frac{F(s_q)}{1-\alpha+s_q}}^{\overline{L}}\frac{L-F\left(s_q\right)}{\alpha-s_q}dL+\int_0^{\frac{F(s_q)}{1-\alpha+s_q}}LdL-\int_0^{\frac{F(s_q)}{1-\alpha+s_q}}LdL-\int_{\frac{F(s_q)}{1-\alpha+s_q}}^{\overline{L}}F'\left(s_q\right)dL\right]\\
=&\frac{1}{\overline{L}}\left[\int_{\frac{F(s_q)}{1-\alpha+s_q}}^{\overline{L}}\frac{L-F\left(s_q\right)}{\alpha-s_q}dL-\int_{\frac{F(s_q)}{1-\alpha+s_q}}^{\overline{L}}F'\left(s_q\right)dL\right]\\
=&\Pr(L\geq\frac{F(s_q)}{1-\alpha+s_q})\left(E\left[y|L\geq\frac{F(s_q)}{1-\alpha+s_q}\right]-F'\left(s_q\right)\right)\\
=&\Pr(L\geq\frac{F(s_q)}{1-\alpha+s_q})\Lambda(s_q),
\end{aligned}$$

where the inequality follows from the fact that s^* is the unique social optimum (as we prove below in Proposition 2) and the social optimum maximizes the expected aggregate income. Notice that we showed above that, if $\alpha<1$ and $\overline{L}>2$, then sign$[\Lambda(0)]>0$ and that there is a unique s^* such that $\Lambda(s^*)=0$ where $s^*\in]0,\alpha[$. Indeed, at $s_q=s^*$, $\frac{2s_q^2+s_q+2\alpha(1-\alpha)}{\alpha-s_q}$ will cut $\overline{L}(1-\alpha+s_q)$ once from below, which implies that $\Lambda(s_q)>0$ for all $s_q<s^*$ and therefore , there is no incentive to deviate to an $s_q<s^*$.

Consider now a deviation to a higher s_q. Let s^{eq} be an equilibrium and consider a deviation by a firm by firm q to an s_q larger than s^{eq}. In a deviation to $s_q>s^{eq}$ the firm would solve the following optimization problem:

$$\max_{s_q}\frac{1}{\overline{L}}\int_{L(s_q,s^{eq})}^{\overline{L}}\frac{L-F\left(s^{eq}\right)}{\alpha-s^{eq}}(1-\alpha+s_q)-F\left(s_q\right)dL \tag{A.15}$$

where

$$L(s_q,s^{eq})=\frac{\alpha-s^{eq}}{1-\alpha+s_q}F(s_q)+F\left(s^{eq}\right). \tag{A.16}$$

The first derivative w.r.t. s_q equals

$$\frac{1}{\overline{L}}\int_{L(s_q,s^{eq})}^{\overline{L}}\frac{L-F\left(s^{eq}\right)}{\alpha-s^{eq}}-F'\left(s_q\right)dL, \tag{A.17}$$

which can be also rewritten as

$$\begin{aligned}
&\frac{\overline{L}-L(s_q,s^{eq})}{\overline{L}}\left[\frac{\frac{\overline{L}+L(s_q,s^{eq})}{2}-F\left(s^{eq}\right)}{\alpha-s^{eq}}-F'(s_q)\right] && \text{(A.18)}\\
=\;&\frac{\overline{L}-L(s_q,s^{eq})}{\overline{L}}\left[\frac{1}{2}\frac{\overline{L}-F\left(s^{eq}\right)}{\alpha-s^{eq}}+\frac{1}{2}\frac{\left(\frac{\alpha-s^{eq}}{1-\alpha+s_q}F(s_q)+F\left(s^{eq}\right)\right)-F\left(s^{eq}\right)}{\alpha-s^{eq}}-F'(s_q)\right]\\
=\;&\frac{\overline{L}-L(s_q,s^{eq})}{\overline{L}}\left[\frac{1}{2}\frac{\overline{L}-F\left(s^{eq}\right)}{\alpha-s^{eq}}+\frac{1}{2}\frac{F(s_q)}{1-\alpha+s_q}-F'(s_q)\right].\\
=\;&\frac{\overline{L}-L(s_q,s^{eq})}{\overline{L}}\left[\frac{1}{2}\frac{\overline{L}-F\left(s^{eq}\right)}{\alpha-s^{eq}}+\Psi(s_q)\right] && \text{(A.19)}
\end{aligned}$$

where $\Psi(s_q)\equiv\frac{1}{2}\frac{F(s_q)}{1-\alpha+s_q}-F'(s_q)$.

Notice that we saw before that $\Lambda(0)>0$ and that there is a unique solution $s^*\in]0,\alpha[$ such that $\Lambda(s^*)=0$. Notice also that if we impose $s_q=s^{eq}$ in the first derivative w.r.t. s_q (equation A.18) and set it equal to zero then, we have $\Lambda(s^{eq})=0$, which means that at s^* there is no local incentive to deviate to a higher s_q. Furthermore, since $s^*\in]0,\alpha[$ is the unique solution to $\Lambda(s^*)=0$, it also means that s^* is again the only possible equilibrium. Next we verify that there is no non-local incentive to deviate to an s_q higher than s^*.

We saw that at s^*, $\frac{2s^2+s+2\alpha(1-\alpha)}{\alpha-s}$ cuts $\overline{L}(1-\alpha+s)$ from below, that is, $\Lambda'(s^*)<0$, i.e.,

$$\frac{1}{2}\left|\frac{\partial\frac{\overline{L}-F(s_i)}{\alpha-s_i}}{\partial s_i}\right|_{s_i=s^*}+\frac{1}{2}\left|\frac{\partial\frac{F(s_i)}{1-\alpha+s_i}}{\partial s_i}\right|_{s_i=s^*}-F''(s^*)<0. \tag{A.20}$$

Notice that

$$\begin{aligned}
\frac{\partial\frac{\overline{L}-F(s)}{\alpha-s}}{\partial s} &= \frac{(\alpha-s)F'\left(s\right)+\overline{L}-F\left(s\right)}{(\alpha-s)^2} && \text{(A.21)}\\
&= \frac{1}{\alpha-s}\left[\frac{\overline{L}-F\left(s\right)}{\alpha-s}-F'\left(s\right)\right]
\end{aligned}$$

and that

$$\Lambda(s^*) = \frac{1}{2}\frac{\overline{L} - F(s^*)}{\alpha - s^*} + \frac{1}{2}\frac{F(s^*)}{1 - \alpha + s^*} - F'(s^*) = 0 \Rightarrow \frac{\overline{L} - F(s^*)}{\alpha - s^*} - F'(s^*) > 0. \tag{A.22}$$

Equations A.21 and A.22 imply that

$$\left|\frac{\partial \frac{\overline{L}-F(s)}{\alpha - s}}{\partial s}\right|_{s=s^*} > 0, \tag{A.23}$$

and equations A.20 and A.23 imply that

$$\Psi'(s^*) < 0. \tag{A.24}$$

Next, we calculate the derivative of $\Psi(s_q)$ w.r.t. s_q:

$$\begin{aligned}\Psi'(s_q) &= \frac{1}{2}\frac{(1-\alpha+s_q)F'(s_q) - F(s_q)}{(1-\alpha+s_q)^2} - F''(s_q) \\ &= \frac{1}{2}\left[\frac{\alpha - \alpha^2 + s_q^2}{(1-\alpha+s_q)^2(\alpha - s_q)^2} - \frac{4\alpha}{(\alpha - s_q)^3}\right] \\ &= \frac{(\alpha - \alpha^2 + s_q^2)(\alpha - s_q) - 4\alpha(1-\alpha+s_q)^2}{2(1-\alpha+s_q)^2(\alpha - s_q)^3} \\ &= \frac{-s_q^3 - 3\alpha s_q^2 - 9\alpha(1-\alpha)s_q + 9\alpha^2 - 4\alpha - 5\alpha^3}{2(1-\alpha+s_q)^2(\alpha - s_q)^3}\end{aligned} \tag{A.25}$$

Provided that $\alpha > s_q$

$$\text{sign}\{\Psi'(s_q)\} = \text{sign}\left\{-s_q^3 - 3\alpha s_q^2 - 9\alpha(1-\alpha) + 9\alpha^2 - 4\alpha - 5\alpha^3\right\}, \tag{A.26}$$

and since $\{-s_q^3 - 3\alpha s_q^2 - 9\alpha(1-\alpha) + 9\alpha^2 - 4\alpha - 5\alpha^3\}$ in s_q, then

$$\Psi'(s^*) < 0 \Rightarrow \Psi'(s) < 0 \text{ for all } s_q \in]s^*, \alpha[\tag{A.27}$$

and therefore

$$\Psi(s_q) < \Psi(s^*) \text{ for all } s_q \in]s^*, \alpha[. \tag{A.28}$$

This implies that the first derivative in A.18, which is zero at $s_q = s^{eq} = s^*$, is negative for all for all $s_q \in]s^*, \alpha[$ and $s^{eq} = s^*$ and hence, there is no incentive to deviate to an $s_q \in]s^*, \alpha[$. Finally notice that as $s_q \to \alpha$ then $F(s_q) \to +\infty$ so the agent will not deviate to an $s_q \geq \alpha$.

Therefore, $s^* \in]0, \alpha[$ such that $\Lambda(s^*) = 0$ is an equilibrium and is the unique symmetric equilibrium. *Q.E.D.*

It is useful to note that:

$$\begin{aligned} \frac{\partial \frac{F(s_i)}{1-\alpha+s_i}}{\partial s_i} &= \frac{(1-\alpha+s_i)\, F'(s_i) - F(s_i)}{(1-\alpha+s_i)^2} \\ &= \frac{\alpha}{(1-\alpha+s_i)\,(\alpha-s_i)^2} - \frac{s_i}{(\alpha-s_i)\,(1-\alpha+s_i)^2} \\ &= \frac{1}{(1-\alpha+s_i)\,(\alpha-s_i)}\left(\frac{\alpha}{\alpha-s_i} - \frac{s_i}{1-\alpha+s_i}\right) \\ &= \frac{\alpha-\alpha^2+s_i^2}{(1-\alpha+s_i)^2\,(\alpha-s_i)^2} > 0 \text{ for } \alpha \leq 1. \end{aligned} \tag{A.29}$$

Proof of Proposition 2

The social planner chooses the operating leverage s that maximizes expected profits:

$$\max_s \frac{1}{\overline{L}} \int_{L(s)}^{\overline{L}} \pi(s,s) dL \quad \text{s.t.} \quad L(s) = \frac{F(s)}{1-\alpha+s} \tag{A.30}$$

or

$$\max_s \frac{1}{\overline{L}} \int_{L(s)}^{\overline{L}} y(s,L)(1-\alpha+s) - F\,(s)\, dL \quad \text{s.t.} \quad L(s) = \frac{F(s)}{1-\alpha+s}. \tag{A.31}$$

The f.o.c. writes

$$\frac{1}{\overline{L}} \int_{L(s^{FB})}^{\overline{L}} \frac{\partial y(s^{FB},L)}{\partial s}(1-\alpha+s^{FB}) + y(s^{FB},L) - F'\left(s^{FB}\right) dL = 0 \tag{A.32}$$

Since $\frac{\partial L(s)}{\partial s}$ is increasing in s, i.e.,

$$\begin{aligned} \frac{\partial L(s)}{\partial s} &= \frac{\partial \frac{F(s)}{1-\alpha+s}}{\partial s} = \frac{(1-\alpha+s)\, F'(s) - F(s)}{(1-\alpha+s)^2} \\ &= \frac{\alpha}{(1-\alpha+s)\,(\alpha-s)^2} - \frac{s}{(\alpha-s)\,(1-\alpha+s)^2} \\ &= \frac{1}{(1-\alpha+s)\,(\alpha-s)}\left(\frac{\alpha}{\alpha-s} - \frac{s}{1-\alpha+s}\right) \\ &= \frac{\alpha-\alpha^2+s^2}{(1-\alpha+s)^2\,(\alpha-s)^2} > 0 \;\text{ for } \alpha \leq 1, \end{aligned} \tag{A.33}$$

and goes to infinity as $s \to \alpha$, and since the f.o.c evaluated as $s = 0$ (equation A.32) is positive, i.e.,

$$\begin{aligned} &\frac{1}{\overline{\overline{L}}} \int_0^{\overline{L}} \left(\frac{L}{\alpha^2} - \frac{1}{\alpha^2} \right) (1-\alpha) + \frac{L}{\alpha} - \frac{1}{\alpha} dL \qquad \text{(A.34)} \\ = \; &\frac{1}{\alpha^2 \overline{\overline{L}}} \int_0^{\overline{L}} L - 1 dL > 0 \;\; \text{for } \overline{L} > 2, \end{aligned}$$

it follows that the social planner's maximization problem has at least one solution, and any such solution is interior, i.e., $s^{FB} \in]0, \alpha[$.

The social planner equivalently also chooses the operating leverage s that maximizes the expected aggregate income, i.e.,

$$\max_s \frac{1}{\overline{\overline{L}}} \int_{L(s)}^{\overline{L}} y(s, L) dL + \frac{1}{\overline{\overline{L}}} \int_0^{L(s)} L dL \quad \text{s.t.} \quad L(s) = \frac{F(s)}{1 - \alpha + s} \tag{A.35}$$

which yields the following f.o.c. (**I Think we need to mention $y(s, L(s)) = L(s)$ to get to the foc**:

$$\frac{1}{\overline{\overline{L}}} \int_{L(s^{FB})}^{\overline{L}} \frac{\partial y(s^{FB}, L)}{\partial s} dL = 0 \tag{A.36}$$

Putting together the two f.o.c. (equations A.32 and A.36) it follows that

$$\frac{1}{\overline{\overline{L}}} \int_{L(s^{FB})}^{\overline{L}} y(s^{FB}, L) - F' \left(s^{FB} \right) dL = 0, \tag{A.37}$$

that is,

$$E \left[y | L \geq L(s^{FB}) \right] - F'(s^{FB}) = 0 \tag{A.38}$$

We have shown in the proof Proposition 1 that the unique solution to this equation is s^*, that is, $s^{FB} = s^*$. Therefore s^* the equilibrium operating leverage also maximizes s^* the social planer's objective function. *Q.E.D.*

Proof of Lemma 3

The result that a monopolist firm is more likely to produce at $t = 1$ the higher the effective labor L, and the lower the workers' rent β follows directly from condition 19. Deriving the right hand side of condition 19 w.r.t. s_q

$$\frac{\partial \frac{F(s_q)}{1-\beta-\alpha+s_q}}{\partial s_q} = \frac{F'(s_q)}{1-\beta-\alpha+s_q} - \frac{F(s_q)}{(1-\beta-\alpha+s_q)^2} \tag{A.39}$$
$$= \frac{F(s_q)}{(1-\beta-\alpha+s_q)}\left(\frac{\alpha}{(\alpha-s_q)s_q} - \frac{1}{1-\beta-\alpha+s_q}\right)$$
$$= \frac{F(s_q)}{(\alpha-s_q)s_q(1-\beta-\alpha+s_q)^2}\left(\alpha(1-\beta-\alpha+s_q)-(\alpha-s_q)s_q\right)$$
$$= \frac{F(s_q)}{(\alpha-s_q)s_q(1-\beta-\alpha+s_q)^2}\left(\alpha(1-\beta-\alpha)+s_q^2\right) > 0,$$

which implies that a monopolist firm is more likely to produce at $t = 1$ the lower its operating leverage s_q. *Q.E.D.*

Proof of Proposition 4

Suppose all firms choose s, and firm q observes a noisy version of L, i.e., a signal $l_q = L + \xi_q$ where ξ is uniform on $[-\varepsilon, \varepsilon]$. Suppose firm q produces iff $l_q > l^*$. Then, for a given realization of L, the number of firms that produce is

$$n(L) \equiv \begin{cases} 1 & \text{if } L > l^* + \varepsilon \\ \frac{L+\varepsilon-l^*}{2\varepsilon} & \text{if } L \in [l^*-\varepsilon, l^*+\varepsilon] \\ 0 & \text{if } L < l^* - \varepsilon \end{cases} \tag{A.40}$$

The firm with signal l^* must be indifferent, i.e.,

$$\frac{1}{2\varepsilon}\int_{l^*-\varepsilon}^{l^*+\varepsilon}(1-\alpha-\beta+s)\frac{L-n(L)F}{1-n(L)(1-\alpha+s)} - F\; dL = 0. \tag{A.41}$$

Consider the following change of variable: $z = \frac{L-l^*+\varepsilon}{2\varepsilon} \Leftrightarrow L = 2\varepsilon z + l^* - \varepsilon$. (A.41) becomes

$$\frac{1}{2\varepsilon}\int_0^1 \left\{(1-\alpha-\beta+s)\frac{2\varepsilon z + l^* - \varepsilon - n(2\varepsilon z + l^* - \varepsilon)F}{1-n(2\varepsilon z + l^* - \varepsilon)(1-\alpha+s)} - F\right\} 2\varepsilon dz = 0 \tag{A.42}$$
$$\Leftrightarrow \int_0^1 (1-\alpha-\beta+s)\frac{2\varepsilon z + l^* - \varepsilon - zF}{1-z(1-\alpha+s)} - F\; dz = 0 \tag{A.43}$$

and

$$n(z) \equiv \begin{cases} 1 & \text{if } z > 1 \\ z & \text{if } z \in [0,1] \\ 0 & \text{if } z < 0 \end{cases} \tag{A.44}$$

So when $\varepsilon \to 0$, then $z \to \frac{1}{2}$ and hence, $n(z) \to z$ Therefore, from equation A.42 $L(s)$ is given by,

$$\int_0^1 (1-\alpha-\beta+s)\frac{L(s)-nF}{1-n(1-\alpha+s)} - F\ dn = 0 \tag{A.45}$$

and operating:

$$\begin{aligned}
&\Leftrightarrow \int_0^1 (1-\alpha-\beta+s)\frac{L(s)-nF}{1-n(1-\alpha+s)} - F\ dn = 0 &\text{(A.46)}\\
&\Leftrightarrow \frac{1-\alpha-\beta+s}{1-\alpha+s}\int_0^1 \frac{(1-\alpha+s)L(s)-(1-\alpha+s)\,nF+F-F}{1-n(1-\alpha+s)}dn - F = 0\\
&\Leftrightarrow \frac{1-\alpha-\beta+s}{1-\alpha+s}\int_0^1 \frac{(1-\alpha+s)\,L(s)-F}{1-n\,(1-\alpha+s)} + Fdn - F = 0\\
&\Leftrightarrow \frac{1-\alpha-\beta+s}{1-\alpha+s}\int_0^1 \frac{(1-\alpha+s)\,L(s)-F}{1-n\,(1-\alpha+s)}dn - \frac{\beta}{1-\alpha+s}F = 0\\
&\Leftrightarrow (1-\alpha-\beta+s)\int_0^1 \frac{(1-\alpha+s)\,L(s)-F}{1-n\,(1-\alpha+s)}dn - \beta F = 0\\
&\Leftrightarrow (1-\alpha-\beta+s)\int_0^1 \frac{(1-\alpha+s)\,L(s)-F}{1-n\,(1-\alpha+s)}dn - \beta F = 0\\
&\Leftrightarrow -\frac{1-\alpha-\beta+s}{1-\alpha+s}\left[(1-\alpha+s)\,L(s)-F\right]\ln(\alpha-s) - \beta F = 0\\
&\Leftrightarrow L^T(s) = \underbrace{\frac{F}{1-\alpha+s}}_{\text{First Best}} + \underbrace{\frac{\beta F}{(1-\alpha-\beta+s)\ln\left(\frac{1}{\alpha-s}\right)}}_{\text{Distortion}}
\end{aligned}$$

Now, it is apparent that the RHS of (A.43) is increasing in l^*, which guarantees the uniqueness of an equilibrium in threshold strategies for any ε. Iterated deletion of strictly dominated strategies ensure global uniqueness in a setting with global strategic complementarities (e.g., Morris and Shin 2003). One potential concern in this setup is that firms' actions are not complement if L is small enough. Specifically, letting n be the fraction

$$\frac{\partial \pi}{\partial n} \geq 0 \Leftrightarrow L \geq \frac{F(s)}{1-\alpha+s}.$$

Note however, that

$$\frac{F(s)}{1-\alpha+s} < \frac{(1-\beta)F(s)}{1-\alpha-\beta+s},$$

i.e., the region where the firms' actions are not strategic complement is strictly smaller than the upper bound of the lower-dominance region. Therefore if

$$2\varepsilon < \frac{(1-\beta)F(s)}{1-\alpha-\beta+s} - \frac{F(s)}{1-\alpha+s},$$

a private signal l_q consistent with $L \geq \frac{(1-\beta)F(s)}{1-\alpha-\beta+s}$ rules out $L < \frac{F(s)}{1-\alpha+s}$, i.e., ensures that L is in the region where actions are complement. This, in turn, allows to apply the standard iterated deletion of strictly dominated strategies. It follows that the threshold equilibrium derived in (A.43) is the unique equilibrium for ε small enough, hence when $\varepsilon \to 0$.

Proof of Proposition 4

We need to show that $\frac{F}{1-\alpha+s} + \frac{\beta F}{(1-\alpha-\beta+s)\ln\left(\frac{1}{\alpha-s}\right)} > \frac{(1-\beta)F}{1-\alpha-\beta+s}$:

$$\Leftrightarrow \quad \frac{F}{1-\alpha+s} + \frac{\beta F}{(1-\alpha-\beta+s)\ln\left(\frac{1}{\alpha-s}\right)} > \frac{(1-\beta)}{1-\alpha-\beta+s} \tag{A.47}$$

$$\Leftrightarrow \quad -+\frac{\beta}{(1-\alpha-\beta+s)\ln\left(\frac{1}{\alpha-s}\right)} > \frac{1-\beta}{1-\alpha-\beta+s} - \frac{1}{1-\alpha+s}$$

$$\Leftrightarrow \quad \frac{\beta}{\ln\left(\frac{1}{\alpha-s}\right)} > \frac{(1-\alpha+s)(1-\beta)-(1-\alpha-\beta+s)}{1-\alpha+s}$$

$$\Leftrightarrow \quad \frac{1}{\alpha-s} - 1 > \ln\left(\frac{1}{\alpha-s}\right) \quad \text{which is verified for all } \alpha - s \in (0,1).$$

Next we show that $\frac{F}{1-\alpha+s} + \frac{\beta F}{(1-\alpha-\beta+s)\ln\left(\frac{1}{\alpha-s}\right)} < \frac{F}{1-\alpha-\beta+s}$:

$$\Leftrightarrow \quad \frac{F}{1-\alpha+s} + \frac{\beta F}{(1-\alpha-\beta+s)\ln\left(\frac{1}{\alpha-s}\right)} < \frac{F}{1-\alpha-\beta+s} \tag{A.48}$$

$$\Leftrightarrow \quad \frac{\beta}{(1-\alpha-\beta+s)\ln\left(\frac{1}{\alpha-s}\right)} < \frac{1}{1-\alpha-\beta+s} - \frac{1}{1-\alpha+s}$$

$$\Leftrightarrow \quad 1-(\alpha-s) < \ln\left(\frac{1}{\alpha-s}\right) \quad \text{which is verified for all } \alpha - s \in (0,1)$$

Q.E.D.

Proof of Proposition 5

I- Local Conditions: Suppose all firms choose s^*_β and firm q chooses s_q.

- If no other firm operates, firm q operates if

$$L > L_-(s_q) \equiv \frac{F(s_q)}{1-\alpha-\beta+s_q}. \tag{A.49}$$

$L'_-(s_q)$ has the sign of $\alpha(1-\alpha-\beta)+s_q^2 > 0$ (given that $\alpha+\beta<1$). Also, (A.45) implies $(1-\alpha-\beta+s^*_\beta)L^T(s^*_\beta) - F(s^*_\beta) < 0$, therefore $L_-(s^*_\beta) > L^T(s^*_\beta)$. Since $L(0) = 0 < L^T(s^*_\beta)$, there is a unique s_- such that $L_-(s_-) = L^T(s^*_\beta)$ and $s_- < s^*_\beta$. If $L < L^T(s^*_\beta)$ and $s_q < s_-$, q operates iff $L > L_-(s_q)$. If $L < L^T(s^*_\beta)$ and $s_q > s_-$, q never operates.

- If all other firms operate, firm q operates if

$$L > L_+(s_q) \equiv (\alpha - s^*_\beta)\frac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*_\beta), \tag{A.50}$$

which is strictly increasing in s_q. Also, (A.45) implies $(1-\alpha-\beta+s^*_\beta)\frac{L^T(s^*_\beta)-F(s^*_\beta)}{\alpha-s^*_\beta} - F(s^*_\beta) > 0$, therefore $L_+(s^*_\beta) < L^T(s^*_\beta)$. Since $\lim_{s\to\alpha} L_+(s) = +\infty$, there is a unique $s_+ < \alpha$ such that $L_+(s_+) = L^T(s^*_\beta)$ and $s_+ > s^*_\beta$. If $L > L^T(s^*_\beta)$ and $s_q > s_+$, i operates iff $L > L_+(s_q)$. If $L > L^T(s^*_\beta)$ and $s_q < s_+$, q always operates. This implies in particular $L'_-(s^*_\beta) = L'_+(s^*_\beta) = 0$, therefore the f.o.c. for an interior solution writes

$$\Leftrightarrow \quad E\left[y|L \geq L(s^*_\beta)\right] - F'\left(s^*_\beta\right) \tag{A.51}$$

$$\Leftrightarrow \quad \int_{L^T(s^*_\beta)}^{\overline{L}} \frac{L - F(s^*_\beta)}{\alpha - s^*_\beta} - F'(s^*_\beta)\, dL = 0$$

$$\Leftrightarrow \quad \overline{L} + L^T(s^*_\beta) = 2\frac{\alpha + s^*_\beta}{\alpha - s^*_\beta}$$

$$\Leftrightarrow \quad \overline{L} = \frac{1}{\alpha - s^*_\beta}\left[2(\alpha + s^*_\beta) - \frac{s^*_\beta}{1-\alpha+s^*_\beta} - \frac{\beta s^*_\beta}{(1-\alpha-\beta+s^*_\beta)\ln\left(\frac{1}{\alpha - s^*_\beta}\right)}\right] \tag{A.52}$$

If $s^*_\beta = 0$, the RHS of (A.52) is equal to 2. If $s^*_\beta \to \alpha$, the RHS of (A.52) tends to $+\infty$. It follows that if $\overline{L} > 2$, (A.52) has at least one solution.

Rewrite, (A.52) as

$$\alpha\overline{L} = 2\alpha + s^*_\beta \underbrace{\left(2 + \overline{L} - \frac{1}{1-\alpha+s^*_\beta} - \frac{\beta}{(1-\alpha-\beta+s^*_\beta)\ln\left(\frac{1}{\alpha-s^*_\beta}\right)}\right)}_{f(s^*_\beta)} \tag{A.53}$$

$f'(.) > 0$ and $\lim_{s\to\alpha} f(s) = 1 + \overline{L} > 0$ implies there exists $\hat{s}$ (possibly equal to 0) such that $f(s) > 0$ iff $s > \hat{s}$. $\overline{L} > 2$ implies that $s \leq \hat{s}$ cannot be solution to (A.53) since the RHS is then lower than 2α. If $s > \hat{s}$, then $f(s) > 0$ and $f'(s) > 0$ imply that the RHS of (A.53) is strictly increasing. Therefore (A.53) has a unique solution.

Hence, if there exists a symmetric equilibrium, this equilibrium is unique, interior, i.e., $s^*_\beta \in]0, \alpha[$, and defined by the f.o.c. in A.51.

The f.o.c. in A.51 gives a necessary condition for a symmetric equilibrium to exists as otherwise firm q would have incentive to deviate locally from s^*_β. (Notice that with local deviations around s^*_β the operating threshold does not change from $L^T(s^*_\beta)$. Intuitively, at $L^T(s^*_\beta)$, a firm makes a profit when all other firms operate and makes a loss when no other firm operates so a local deviation will not change that a firm makes a profit when $L > L^T(s^*_\beta)$ and a loss when $L < L^T(s^*_\beta)$.) Next we analyze the possibility of non-local (above or below) deviations from s^*_β by firm q.

II- Large Deviations:

* Consider first the case of a downward deviation: $s_q < s^*_\beta$.

Reminder: let $s_- \in (0, s^*_\beta]$ be the unique solution to $L_-(s_-) = L^T(s^*_\beta)$, where $L_-(.)$ is defined in (A.49). $s_- < s^*_\beta$ iff $\beta > 0$ and

- if $s_q < s_-$, then q invests iff $L > L_-(s_q) = \frac{F(s_q)}{1-\alpha-\beta+s_q}$($= L(s_q)$ to simplify notation),

- if $s_q \geq s_-$, then q invests iff $L > L^T(s^*_\beta)$.

The firm solves

$$\pi(s_q) = \int_{L^T(s^*_\beta)}^{\overline{L}} (1-\alpha-\beta+s_q)\frac{L - F(s^*_\beta)}{\alpha - s^*_\beta} - F(s_q)\; dL + \int_{\min\{L(s_q), L^T(s^*_\beta)\}}^{L^T(s^*_\beta)} (1-\alpha-\beta+s_q)L - F(s_q)\; dL \tag{A.54}$$

and the first derivative w.r.t. s_q is

$$\frac{\partial \pi}{\partial s_q}(s_q) = \int_{L^T(s^*_\beta)}^{\overline{L}} \frac{L - F(s^*_\beta)}{\alpha - s^*_\beta} - F'(s_q)\; dL + \int_{\min\{L(s_q), L^T(s^*_\beta)\}}^{L^T(s^*_\beta)} L - F'(s_q)\; dL. \tag{A.55}$$

Two observations:

1. If $s_q \in [s_-, s^*_\beta)$, $\pi'(s_q)$ simplifies to

$$\int_{L^T(s^*)}^{\overline{L}} \frac{L - F(s^*_\beta)}{\alpha - s^*_\beta} - F'(s_q)\; dL > 0 \tag{A.56}$$

since $F'' > 0$.

2. $\pi'(.)$ is continuous (though not differentiable at s_-), which implies in particular that when $s_q \to s_-$ from below, $\pi'(s_q) > 0$.

Suppose now $s_q < s_-$.

$$\begin{aligned} \pi''(s_q) &= -\int_{L(s_q)}^{\overline{L}} F''(s_q)\, dL - [L(s_q) - F'(s_q)]\, L'(s_q) \\ &= -[\overline{L} - L(s_q)]F''(s_q) + \frac{[F'(s_q) - L(s_q)]^2}{1-\alpha-\beta+s_q} \end{aligned} \tag{A.57}$$

where I am using (and will use repeatedly) that

$$L'(s_q) = \frac{(1-\alpha-\beta+s_q)F'(s_q) - F(s_q)}{(1-\alpha-\beta+s_q)^2} = \frac{F'(s_q) - L(s_q)}{1-\alpha-\beta+s_q} \tag{A.58}$$

Next we calculate the sign of the second derivative $\pi''(s)$ in equation A.57. Since $F''(s_q) > 0$, the sign($\pi''(s_q)$)=sign($\pi''(s_q)/F''(s_q)$)

$$\frac{\pi''(s_q)}{F''(s_q)} = -\overline{L} + \underbrace{L(s_q) + \frac{[F'(s_q) - L(s_q)]^2}{(1-\alpha-\beta+s_q)F''(s_q)}}_{\equiv G(s_q)} \tag{A.59}$$

$$\begin{aligned} G'(s_q) &= \frac{F'(s_q) - L(s_q)}{(1-\alpha-\beta+s_q)} \\ &\quad + \frac{2[F'(s_q) - L(s_q)][F''(s_q) - L'(s_q)](1-\alpha-\beta+s_q)F''(s_q)}{[F''(s_q)(1-\alpha-\beta+s_q)]^2}] \\ &\quad - \frac{[F'(s_q) - L(s_q)]^2\,[F'''(s_q)(1-\alpha-\beta+s_q) + F''(s_q)]}{[F''(s_q)(1-\alpha-\beta+s_q)]^2}. \end{aligned} \tag{A.60}$$

$G'(s_q)$ has the sign of

$$\begin{aligned}
& [F''(s_q)]^2(1-\alpha-\beta+s_q) \\
& +2[F''(s_q)-L'(s_q)](1-\alpha-\beta+s_q)F''(s_q) \\
& -[F'(s_q)-L(s_q)]\left[F'''(s_q)(1-\alpha+s_q)+F''(s_q)\right] \\
= \; & 3[F''(s_q)]^2(1-\alpha-\beta+s_q)-2[F'(s_q)-L(s_q)]F''(s_q) \\
& -[F'(s_q)-L(s_q)]\left[F'''(s_q)(1-\alpha-\beta+s_q)+F''(s_q)\right] \\
= \; & 3[F''(s_q)]^2(1-\alpha-\beta+s_q) \\
& -[F'(s_q)-L(s_q)][F'''(s_q)(1-\alpha-\beta+s_q)+3F''(s_q)] \\
= \; & \frac{12\alpha^2}{(\alpha-s_q)^6}(1-\alpha+s_q)-\frac{6\alpha^2}{(\alpha-s_q)^6}(1-\alpha+s_q) \\
& +\frac{6\alpha s_q}{(\alpha-s_q)^5}-\frac{6\alpha^2}{(\alpha-s_q)^5}+\frac{6\alpha s}{(\alpha-s)^4(1-\alpha-\beta+s_q)},
\end{aligned} \tag{A.61}$$

which, in turn has the sign of

$$g(s) \equiv \alpha(1-\alpha-\beta+s_q)^2-(1-\alpha-\beta)(\alpha-s_q)^2 \tag{A.62}$$

There two possible cases:

1. $2\alpha+\beta-1 \leq 0$

   Then $g(.)$ is concave, $g(\alpha)>0$ and $g(0)>0$, therefore, $g(.)>0$ for $s \in [0, s_-)$. It follows that $\pi''(.)$ is strictly increasing for $s \in [0, s_-)$.

2. $2\alpha+\beta-1>0$ Then $g(.)$ is convex, $g(\alpha)>0$ and $g(0)<0$. It follows that $\pi''(.)$ is first decreasing, then increasing on $[0, s_-)$.

N.B.: $\pi'(0)>0$ *and* $\pi''(s_-)<0$ *is then sufficient rule out the deviation*

Indeed, in (1), $\pi''(s_-)<0$ and π'' increasing imply $\pi''(.)<0$ on $(0, s_-)$. Then since $\pi'(s_-)>0$, $\pi'(.)>0$ on $(0, s_-)$.

In (2), $\pi''(s_-)<0$ implies either $\pi''(.)<0$ on $(0, s_-)$ (then back to previous case), or $\pi''(.)$ is first positive and then negative. Then since $\pi'(0)>0$ and $\pi'(s_-)>0$, $\pi'(.)>0$ on $(0, s_-)$.

Sufficient conditions for $\pi'(0)>0$ *and* $\pi''(s_-)<0$*:*

i) $\pi'(0) > 0$ is true if $\overline{L}$ is large enough, for instance $\overline{L} > \frac{2}{\alpha}$ is sufficient.

ii) $\pi''(s_-) < 0$:

From (A.52), if $\overline{L} \to +\infty$ then $s^* \to \alpha$, and $\overline{L}(\alpha - s^*) \to 3\alpha$. In addition, if $\overline{L} \to +\infty$ then $L^T(s^*_\beta) \to +\infty$ which implies $L(s_-) \to +\infty$, which implies $s_- \to \alpha$. Finally,

$$L(s_-)(\alpha - s^*_\beta) = \frac{s_-}{(\alpha - s_-)(1 - \alpha - \beta + s_-)}(\alpha - s^*_\beta) = L^T(s^*_\beta)(\alpha - s^*_\beta) \tag{A.63}$$

$$\Leftrightarrow \quad \frac{s_-}{1 - \alpha - \beta + s_-}\overline{L}(\alpha - s^*_\beta) = L^T(s^*_\beta)(\alpha - s^*_\beta)\overline{L}(\alpha - s_-)$$

Using, $\lim_{\overline{L}\to+\infty} L^T(s^*_\beta)(\alpha - s^*_\beta) = \alpha$, $\lim_{\overline{L}\to+\infty} \overline{L}(\alpha - s^*_\beta) = 3\alpha$ and $\lim_{\overline{L}\to+\infty} s_- = \alpha$ yields

$$\lim_{\overline{L}\to+\infty} \overline{L}(\alpha - s_-) = \frac{3\alpha}{1 - \beta} \tag{A.64}$$

Next,

$$\begin{aligned} \pi''(s_-) &= -[\overline{L} - L(s_-)]F''(s_-) + \frac{[F'(s_-) - L(s_-)]^2}{1 - \alpha - \beta + s_-} \\ &= -\left[\overline{L} - \frac{s_-}{(\alpha - s_-)(1 - \alpha - \beta + s_-)}\right]\frac{2\alpha}{(\alpha - s_-)^3} \\ &\quad + \frac{\left[\frac{\alpha}{(\alpha - s_-)^2} - \frac{s_-}{(\alpha - s_-)(1 - \alpha - \beta + s_-)}\right]^2}{1 - \alpha - \beta + s_-}, \end{aligned} \tag{A.65}$$

which has the sign of

$$-[\overline{L}(\alpha - s_-) - \frac{s_-}{(1 - \alpha - \beta + s_-)}](2\alpha) + \frac{\left[\alpha - \frac{s_-(\alpha - s_-)}{(1 - \alpha - \beta + s_-)}\right]^2}{1 - \alpha - \beta + s_-}, \tag{A.66}$$

which tends to

$$-(3\frac{\alpha}{1 - \beta} - \frac{\alpha}{1 - \beta})2\alpha + \frac{\alpha^2}{1 - \beta} = -3\frac{\alpha^2}{1 - \beta} < 0, \tag{A.67}$$

when $\overline{L} \to +\infty$. Therefore there exists $\hat{L}$ such that if $L > \hat{L}$, $\pi''(s_-) < 0$.

* Consider now the case of an upward deviation: $s_q > s^*_\beta$.

The firm continues if $L \geq \max\{(\alpha - s^*_\beta)\frac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*_\beta), L^T(s^*_\beta)\}$. The firm will not deviate to a $s_q > s^*_\beta$ such that $(\alpha - s^*_\beta)\frac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*_\beta) \leq L^T(s^*_\beta)$ as A.51 implies that

$$s^*_\beta = \arg\max_{s_q} \int_{L^T(s^*_\beta)}^{\overline{L}} \frac{L - F(s^*_\beta)}{\alpha - s^*_\beta}(1 - \alpha - \beta + s) - F(s)\; dL = 0. \tag{A.68}$$

Consider a large deviation such that $(\alpha - s^*_\beta)\frac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*_\beta) > L^T(s^*_\beta)$ The firm would solve

$$\max_{s_q} \frac{1}{\overline{L}} \int_{(\alpha-s^*_\beta)\frac{F(s_q)}{1-\alpha-\beta+s_q}+F(s^*_\beta)}^{\overline{L}} \frac{L - F\left(s^*_\beta\right)}{\alpha - s^*_\beta}(1-\alpha-\beta+s_q) - F\left(s_q\right) dL, \tag{A.69}$$

which yields the following first derivative

$$\frac{1}{\overline{L}} \int_{(\alpha-s^*_\beta)\frac{F(s_q)}{1-\alpha-\beta+s_q}+F(s^*_\beta)}^{\overline{L}} \frac{L - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} - F'\left(s_q\right) dL, \tag{A.70}$$

which also writes as

$$\frac{\overline{L} - (\alpha - s^*_\beta)\frac{F(s_q)}{1-\alpha-\beta+s_q} - F(s^*_\beta)}{\overline{L}} \left[\frac{1}{2}\frac{\overline{L} - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} + \underbrace{\frac{1}{2}\frac{F(s_q)}{1-\alpha-\beta+s_q} - F'\left(s_q\right)}_{\equiv \Psi(s_q)} \right]. \tag{A.71}$$

Therefore

$$sign(\text{first derivative}) = sign\left\{ \frac{1}{2}\frac{\overline{L} - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} + \Psi(s_q) \right\} \tag{A.72}$$

Since $L^T(s^*_\beta) > (\alpha - s^*_\beta)\frac{F(s^*_\beta)}{1-\alpha-\beta+s^*_\beta} + F(s^*_\beta)$ and

$$\frac{1}{2}\frac{\overline{L} - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} + \frac{1}{2}\frac{L^T(s^*_\beta) - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} - F'(s^*_\beta) = 0 \tag{A.73}$$

then

$$\frac{1}{2}\frac{\overline{L} - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} + \Psi(s^*_\beta) < 0. \tag{A.74}$$

Also

$$\frac{1}{2}\frac{\overline{L} - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} + \Psi(0) = \frac{1}{2}\frac{\overline{L} - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} - \frac{1}{\alpha} > \frac{1}{2\alpha}\left(\overline{L} - 2\right) > 0 \tag{A.75}$$

where the first inequality follows from

$$\frac{\overline{L} - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} > \frac{\overline{L}}{\alpha} \Rightarrow \alpha F\left(s^*_\beta\right) < s^*_\beta \overline{L} \Rightarrow \frac{\alpha}{\alpha - s^*_\beta} < \overline{L} \tag{A.76}$$

which holds since

$$0.5\overline{L} + 0.5L^T(s^*) = \frac{\alpha + s^*_\beta}{\alpha - s^*_\beta} \Rightarrow \overline{L} > \frac{\alpha + s^*_\beta}{\alpha - s^*_\beta} > \frac{\alpha}{\alpha - s^*_\beta}. \tag{A.77}$$

From A.74 and A.75, $\Psi(0) > \Psi(s^*_\beta)$, and therefore, $\Psi'(\widehat{s}) < 0$ for some $\widehat{s} < s^*_\beta$. Notice also

$$\begin{aligned}
\frac{\partial \Psi(s_q)}{\partial s_q} &= \frac{1}{2}\frac{s_q^2 + \alpha\left(1-\beta-\alpha\right)}{(1-\alpha-\beta+s_q)^2(\alpha-s_q)^2} - \frac{2\alpha}{(\alpha-s_q)^3} \\
&= \frac{\left(s_q^2 + \alpha\left(1-\beta-\alpha\right)\right)(\alpha-s_q) - 4\alpha(1-\alpha-\beta+s_q)^2}{2(1-\alpha-\beta+s_q)^2(\alpha-s_q)^2} \\
&= \frac{-s_q^3 - 3\alpha s_q^2 - 9\alpha\left(1-\beta-\alpha\right)s_q + 9\alpha^2\left(1-\beta\right) - 4\alpha\left(1-\beta\right)^2 - 5\alpha^3}{2(1-\alpha-\beta+s_q)^2(\alpha-s_q)^2}
\end{aligned} \tag{A.78}$$

which implies that if $\Psi'(s_q) < 0$ for all $s_q > \widehat{s}$ and therefore, that $\Psi(s^*_\beta) > \Psi(s_q)$ for all $s_q > s^*_\beta$. Since $\Psi(s^*_\beta) > \Psi(s_q)$ for all $s_q > s^*_\beta$, from A.74 it follows that

$$sign(\text{first derivative}) = sign\left\{\frac{1}{2}\frac{\overline{L} - F\left(s^*_\beta\right)}{\alpha - s^*_\beta} + \Psi(s_q)\right\} < 0 \text{ for all } s_q > s^*_\beta, \tag{A.79}$$

and therefore, there are no incentives to deviate to an $s_q > s^*_\beta$ such that $(\alpha - s^*_\beta)\frac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*_\beta) > L^T(s^*_\beta)$. *Q.E.D.*

Proof of Corollary 1

From Proposition 5, s^*_β is defined by $E\left[y|L \geq L^T(s^*_\beta)\right] = F'\left(s^*_\beta\right)$, Proposition 5 s^* is defined by $E\left[y|L \geq L(s^*)\right] = F'\left(s^*\right)$. Since

$$L(s) = \frac{F(s)}{1-\alpha+s} < \frac{(1-\beta)F(s)}{1-\alpha-\beta+s} < L^T(s) \text{ for any } s < \alpha \tag{A.80}$$

where the first inequality follows from $\frac{\partial \frac{1-\beta}{1-\alpha-\beta+s}}{\partial \beta} > 0$ and the second from Proposition **4**. Let s^* satisfy that $E\left[y|L \geq L(s^*)\right] = F'\left(s^*\right)$ and consider an equilibrium $s^*_\beta \leq s^*$:

$$E\left[y|L \geq L^T(s^*_\beta)\right] - F'\left(s^*_\beta\right) > E\left[y|L \geq L(s^*_\beta)\right] - F'\left(s^*_\beta\right) \tag{A.81}$$

where the inequality follows from A.80. From the proof of Proposition 1, we know that if s^* is an equilibrium that $E\left[y|L \geq L(s)\right] - F'\left(s\right)$ is zero for $s = s^*$ and is positive for all $s \in [0, s^*[$, therefore, from A.81, it follows that $E\left[y|L \geq L^T(s^*_\beta)\right] - F'\left(s^*_\beta\right) > 0$, and therefore, $s^*_\beta \leq s^*$ cannot be an equilibrium, a contradiction. *Q.E.D.*

Proof of Proposition 6

The social planner maximizes over s

$$W(s) \equiv \int_{L^T(s)}^{\overline{L}} (1-\alpha-\beta+s)\frac{L-F(s)}{\alpha-s} - F(s)\ dL \tag{A.82}$$

The first-order derivative writes

$$\int_{L^T(s)}^{\overline{L}} \frac{L-F(s)}{\alpha-s} - F'(s)\ dL + (1-\alpha-\beta+s)\int_{L^T(s)}^{\overline{L}} \frac{\partial}{\partial s}\left[\frac{L-F(s)}{\alpha-s}\right]\ dL \tag{A.83}$$
$$-\left[(1-\alpha-\beta+s)\frac{L^T(s)-F(s)}{\alpha-s} - F(s)\right]\frac{\partial L^T}{\partial s}$$

Consider the three above terms in turn

- We have shown that $\int_{L^T(s)}^{\overline{L}} \frac{L-F(s)}{\alpha-s} - F'(s)$ is 0 at s^*_β and strictly negative for $s > s^*_\beta$.

- Next,

$$\int_{L^T(s)}^{\overline{L}} \frac{\partial}{\partial s}\left[\frac{L-F(s)}{\alpha-s}\right] = \frac{1}{\alpha-s}\int_{L^T(s)}^{\overline{L}} \frac{L-F(s)}{\alpha-s} - F'(s), \tag{A.84}$$

  which, as above, is 0 at s^*_β and strictly negative for $s > s^*_\beta$.

- Finally, at the operating threshold $L^T(s)$, firms make a strictly positive profit:

$$(1-\alpha-\beta+s)\frac{L^T(s)-F(s)}{\alpha-s} - F(s) > 0 \tag{A.85}$$

  for any s. In addition, $\frac{\partial L^T}{\partial s} > 0$.

It follows, that $W'(s) < 0$ for any $s \geq s^*_\beta$, i.e., there is excessive risk-taking in equilibrium. *Q.E.D.*

Proof of Corollary ??

Suppose that for any (common) leverage s, firms coordinate on the Pareto-superior equilibrium in the production game, i.e., produce if and only if

$$L^+(s) \geq (1-\beta)\frac{F(s)}{1-\alpha-\beta+s}.$$

Consider firm q with leverage s_q when all other firms have leverage s_{-q}. Firm q produces if and only if

$$L \geq L^{+}(s_q, s_{-q}) \equiv \begin{cases} \min\left\{\dfrac{F(s_q)}{1-\alpha-\beta+s_q}, L^{+}(s_{-q})\right\} & \text{if } s_q < s_{-q} \\ (\alpha - s_{-q})\dfrac{F(s_q)}{1-\alpha-\beta+s_q} + F(s_{-q}) & \text{if } s_q \geq s_{-q} \end{cases} \tag{A.86}$$

As earlier a necessary condition for an interior equilibrium s^* is

$$\frac{\partial}{\partial s_q} \int_{L^{+}(s_q,s^*)}^{\overline{L}} \frac{L - F(s)}{\alpha - s^*}(1-\alpha-\beta+s_q) - F(s_q)\, dL \Bigg|_{s_q = s^*} = 0 \tag{A.87}$$

Note that $L^{+}(s,s) = L^{+}(s)$, and for any s,

$$\frac{L^{+}(s) - F(s)}{\alpha - s}(1-\alpha-\beta+s) - F(s) = 0. \tag{A.88}$$

Therefore, a marginal change in s_q around s^* only affects the integral in (A.87) through its integrand, and (A.87) reduces to

$$\int_{L^{+}(s^*)}^{\overline{L}} \frac{L - F(s^*)}{\alpha - s^*} - F'(s^*)\, dL = 0. \tag{A.89}$$

Consider now the social planner's problem:

$$\max_s \frac{1}{\overline{L}} \int_{L^{+}(s)}^{\overline{L}} y(s,L)(1-\alpha-\beta+s) - F(s)\, dL, \tag{A.90}$$

where, as earlier, $y(s,L) = \frac{L-F(s)}{\alpha-s}$. Using (A.88) again, (A.90) reduces to

$$\frac{1}{\overline{L}} \int_{L^{+}(s)}^{\overline{L}} \frac{\partial y(s,L)}{\partial s}(1-\alpha-\beta+s) + y(s,L) - F'(s)\, dL = 0 \tag{A.91}$$

Equivalently, the social planner maximizes net aggregate output, i.e.,

$$\max_s \frac{1}{\overline{L}} \int_{L^{+}(s)}^{\overline{L}} (1-\beta) y(s,L) dL + \frac{1}{\overline{L}} \int_{0}^{L^{+}(s)} L dL. \tag{A.92}$$

Since $y[s, L^{+}(s)] = L^{+}(s)$, the f.o.c. for the above problem reduces to

$$\int_{L^{+}(s^{FB})}^{\overline{L}} (1-\beta) y(s^{FB}, L) dL = 0.$$

Combining this with (A.89) shows that (A.91) holds at s^*, i.e., the equilibrium leverage satisfies the first-order condition of the social planner.

Proof of Proposition 7

- If both produce type of firms operate:

$$\text{Income: } y(s_1, s_2, L) \equiv \frac{L - n_1 F(s_1) - n_2 F(s_2)}{1 - n_1(1 - \alpha - \varphi_1 + s_1) - n_2(1 - \alpha - \varphi_2 + s_2)} \tag{A.93}$$

$$\text{Profits of type-i firm: } \pi_i(s_1, s_2, L) \equiv y(s_1, s_2, L)(1 - \varphi_i - \alpha + s_i) - F(s_i) \tag{A.94}$$

$$\text{Threshold for type 2 firms if type 1 firms operate, } L_2(s_1, s_2)\text{: } \pi_2(s_1, s_2, L_2(s_1, s_2)) = 0 \tag{A.95}$$

- If (w.l.o.g.) only type-1 firms operate:

$$\text{Income: } y(s_1, L) \equiv \frac{L - n_1 F(s_1)}{1 - n_1(1 - \alpha - \varphi_1 + s_1)} \tag{A.96}$$

$$\text{Profits of type i firm: } \pi_i(s_1, L) = \frac{L - n_1 F(s_1)}{1 - n_1(1 - \alpha - \varphi_1 + s_1)}(1 - \alpha - \varphi_i + s_i) - F(s_i) \tag{A.97}$$

$$\text{Threshold for type 1 firms if type 2 firms do not operate, } L_1(s_1)\text{: } \pi_1(s_1, L_1(s_1)) = 0$$

Next we prove the following claim:

Claim Let (s_1^*, s_2^*) be a symmetric equilibrium in which all type-1 firms choose s_1^* and all type-2 firms choose s_2^* and assume w.l.o.g. that $L_1(s_1^*) < L_2(s_1^*, s_2^*)$, then

$$E[y|L > \ L_1(s_1^*)] - F'(s_1^*) = 0 \tag{A.98}$$

$$E[y|L > \ L_2(s_1^*, s_2^*)] - F'(s_2^*) = 0. \tag{A.99}$$

Proof:

Consider the local incentives to deviate of a type-1 firm taking the operating leverage of all the other firms as given.

For $s_q < s_1^*$, the profits of type-1 are:

$$\begin{aligned} &\int_{L_2(s_1^*, s_2^*)}^{\overline{L}} y(s_1^*, s_2^*, L)\,(1 - \alpha - \varphi_1 + s_q) - F(s_q)dL \\ &+ \int_{L_1(s_1^*)}^{L_2} y(s_1^*, L)\,(1 - \alpha - \varphi_1 + s_q) - F(s_q)dL \\ &+ \int_{\frac{F(s_q)}{1 - \alpha - \varphi_1 + s_q}}^{L_1(s_1^*)} L\,(1 - \alpha - \varphi_1 + s_q) - F(s_q)dL \end{aligned} \tag{A.100}$$

The first derivative w.r.t. s_q writes:

$$\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*, L) - F'(s_q)dL \\ + \int_{L_1(s_1^*)}^{L_2} y(s_1^*, L) - F'(s_q)dL \\ + \int_{\frac{F(s_q)}{1-\alpha-\varphi_1+s_q}}^{L_1(s_1^*)} L - F'(s_q)dL \tag{A.101}$$

For s_q to be an equilibrium, it must hold that at $s_q = s_1^*$ the first-order derivative must be greater or equal to zero, that is, since $L_1(s_1^*) = \frac{F(s_1^*)}{1-\alpha-\varphi_1+s_1^*}$:

$$\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*, L) - F'(s_1^*)dL + \int_{L_1(s_1^*)}^{L_2} y(s_1^*, L) - F'(s_1^*)dL \\ = \frac{E[y|L \geq L_1(s_1^*)] - F'(s_1^*)}{\Pr(L \geq L_1(s_1^*))} \geq 0. \tag{A.102}$$

Consider now a local deviation such that $s_q > s_1^*$, the profits of type-1 are:

$$\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*, L)\,(1-\alpha-\varphi_1+s_q) - F(s_q)dL \\ + \int_{\frac{1-n_1(1-\alpha-\varphi_1+s_1^*)}{1-\alpha-\varphi_1+s_q}F(s_q)+n_1F(s_1^*)}^{L_2} y(s_1^*, L)\,(1-\alpha-\varphi_1+s_q) - F(s_q)dL \tag{A.103}$$

The first derivative w.r.t. s_q writes:

$$\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*, L) - F'(s_q)dL \\ + \int_{\frac{1-n_1(1-\alpha-\varphi_1+s_1^*)}{1-\alpha-\varphi_1+s_q}F(s_q)+n_1F(s_1^*)}^{L_2(s_1^*,s_2^*)} y(s_1^*, L) - F'(s_q)dL \tag{A.104}$$

For s_q to be an equilibrium, it must hold that at $s_q = s_1^*$, the f.o.c. must be smaller or equal to zero, that is, since

$$\frac{1-n_1(1-\alpha-\varphi_1+s_1^*)}{1-\alpha-\varphi_1+s_1^*}F(s_1^*) + n_1F(s_1^*) = L_1(s_1^*), \tag{A.105}$$

$$\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*, L) - F'(s_1^*)dL + \int_{L_1(s_1^*)}^{L_2(s_1^*,s_2^*)} y(s_1^*, L) - F'(s_1^*)dL \\ = \frac{E[y|L \geq L_1(s_1^*)] - F'(s_1^*)}{\Pr(L \geq L_1(s_1^*))} \leq 0 \tag{A.106}$$

Therefore, putting the two necessary conditions together:

$$E[y|L > L_1(s_1^*)] - F'(s_1^*) = 0 \tag{A.107}$$

A similar argument will get the following necessary condition for a symmetric equilibrium is:

$$E[y|L > L_2(s_1^*, s_2^*)] - F'(s_2^*) = 0 \tag{A.108}$$

(Notice that (i) $L_1(s_1^*) = L_2(s_1^* \, s_2^*)$ cannot be an equilibrium since for $s_1^* = s_2^*$, $L_1(s^*) \neq L_2(s^*)$ and (ii) $s_i^* \in]0, \alpha[$.) Q.E.D.

This completes the proof of the claim. Next we proceed to proove Proposition 7. Again, let (s_1^*, s_2^*) be a symmetric equilibrium in which all type-1 firms choose s_1^* and all type-2 firms choose s_2^* and assume w.l.o.g. that $L_1(s_1^*) < L_2(s_1^*, s_2^*)$. Assuming $L_1(s_1) < L_2(s_1, s_2)$, the social planner chooses s_1 and s_2 to maximize

$$\begin{aligned} W(s_1, s_2) = n_1 & \int_{L_2(s_1,s_2)}^{\overline{L}} y(s_1, s_2)\left(1 - \alpha - \varphi_1 + s_1\right) - F(s_1) dL \\ + n_2 & \int_{L_2(s_1,s_2)}^{\overline{L}} y(s_1, s_2)\left(1 - \alpha - \varphi_2 + s_2\right) - F(s_2) dL \\ + n_1 & \int_{L_1(s_1)}^{L_2(s_1,s_2)} y(s_1)\left(1 - \alpha - \varphi_1 + s_1\right) - F(s_1) dL. \end{aligned} \tag{A.109}$$

The first derivatives of equation A.109 write:

$$\begin{aligned} & n_1 \int_{L_2(s_1,s_2)}^{\overline{L}} y(s_1, s_2) - F'(s_1) dL \\ & + \left[n_1\left(1 - \alpha - \varphi_1 + s_1\right) + n_2\left(1 - \alpha - \varphi_2 + s_2\right)\right] \int_{L_2(s_1,s_2)}^{\overline{L}} \frac{\partial y(s_1, s_2)}{\partial s_1} dL \\ & + n_1 \int_{L_1(s_1)}^{L_2(s_1,s_2)} y(s_1) - F'(s_1) dL \\ & + n_1 \int_{L_1(s_1)}^{L_2(s_1,s_2)} \frac{\partial y(s_1)}{\partial s_1}\left(1 - \alpha - \varphi_1 + s_1\right) dL \end{aligned} \tag{A.110}$$

and

$$\begin{aligned} & n_2 \int_{L_2(s_1,s_2)}^{\overline{L}} y(s_1, s_2) - F'(s_2) dL \\ & + \left[n_1\left(1 - \alpha - \varphi_1 + s_1\right) + n_2\left(1 - \alpha - \varphi_2 + s_2\right)\right] \int_{L_2(s_1,s_2)}^{\overline{L}} \frac{\partial y(s_1, s_2)}{\partial s_2} dL \end{aligned} \tag{A.111}$$

Since

$$\frac{\partial y(s_1,s_2)}{\partial s_1} = \tag{A.112}$$

$$= \frac{-n_1 F'(s_1)}{1-n_1(1-\alpha-\varphi_1+s_1)-n_2(1-\alpha-\varphi_2+s_2)}$$

$$+ n_1 \frac{L-n_1F(s_1)-n_2F(s_2)}{(1-n_1(1-\alpha-\varphi_1+s_1)-n_2(1-\alpha-\varphi_2+s_2))^2}$$

$$= \frac{n_1[y(s_1,s_2)-F'(s_1)]}{1-n_1(1-\alpha-\varphi_1+s_1)-n_2(1-\alpha-\varphi_2+s_2)}$$

$$\frac{\partial y(s_1,s_2)}{\partial s_2} = \frac{n_2[y(s_1,s_2)-F'(s_2)]}{1-n_1(1-\alpha-\varphi_1+s_1)-n_2(1-\alpha-\varphi_2+s_2)} \tag{A.113}$$

$$\frac{\partial y(s_1)}{\partial s_1} = \frac{n_1[y(s_1)-F'(s_1)]}{1-n_1(1-\alpha-\varphi_1+s_1)} \tag{A.114}$$

then

$$\frac{n_1\int_{L_2(s_1,s_2)}^{\overline{L}} y(s_1,s_2)-F'(s_1)dL}{1-n_1(1-\alpha-\varphi_1+s_1)-n_2(1-\alpha-\varphi_2+s_2)} + \frac{n_1\int_{L_1(s_1)}^{L_2(s_1,s_2)} y(s_1)-F'(s_1)dL}{1-n_1(1-\alpha-\varphi_1+s_1)} \tag{A.115}$$

and

$$\frac{n_2\int_{L_2(s_1,s_2)}^{\overline{L}} y(s_1,s_2)-F'(s_2)dL}{1-n_1(1-\alpha-\varphi_1+s_1)-n_2(1-\alpha-\varphi_2+s_2)} \tag{A.116}$$

Notice that at $(s_1,s_2)=(s_1^*,s_2^*)$

$$\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*,s_2^*)-F'(s_1^*)dL + \int_{L_1(s_1^*)}^{L_2(s_1^*,s_2^*)} y(s_1^*)-F'(s_1^*)dL = 0 \tag{A.117}$$

$$\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*,s_2^*)-F'(s_2^*)dL = 0 \tag{A.118}$$

Therefore, at $(s_1,s_2)=(s_1^*,s_2^*)$, the first of the first derivatives it follows:

$$\frac{n_1\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*,s_2^*)-F'(s_1^*)}{1-n_1(1-\alpha-\varphi_1+s_1^*)-n_2(1-\alpha-\varphi_2+s_2^*)} + \frac{n_1\int_{L_1(s_1^*)}^{L_2(s_1^*,s_2^*)} y(s_1^*)-F'(s_1^*)dL}{1-n_1(1-\alpha-\varphi_1+s_1^*)} \tag{A.119}$$

$$= \frac{n_1\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*,s_2^*)-F'(s_1^*)dL}{1-n_1(1-\alpha-\varphi_1+s_1)-n_2(1-\alpha-\varphi_2+s_2)} - \frac{n_1\int_{L_2(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*,s_2^*)-F'(s_1^*)dL}{1-n_1(1-\alpha-\varphi_1+s_1)} > 0$$

That is

$$\frac{\partial W(s_1^*,s_2^*)}{\partial s_1} > 0 \;\text{ and }\; \frac{\partial W(s_1^*,s_2^*)}{\partial s_2} = 0 \tag{A.120}$$

Q.E.D.