Correcting Alpha Misattribution in Portfolio Sorts*

Daniel Hoechle[a,b,#], Markus Schmid[c], Heinz Zimmermann[a]

[a] *University of Basel, Department of Finance, CH-4002 Basel, Switzerland*
[b] *Institute for Finance, FHNW School of Business, CH-4002 Basel, Switzerland*
[c] *Swiss Institute of Banking and Finance, University of St. Gallen, CH-9000 St. Gallen, Switzerland*

This version: June 2018

Abstract

Portfolio sorts, as commonly employed in empirical asset pricing applications, are at risk of accidentally misattributing parts of the risk-adjusted return (or "alpha") to the firm characteristic underlying the sort. Such misattribution occurs if the firm characteristic is correlated with an unobservable yet time-persistent factor. We propose a novel, regression-based methodology for analyzing asset returns. Besides handling multiple and continuous firm characteristics, our technique can also reproduce the alpha and factor exposure estimates from all variants of sorting assets into (e.g., decile) portfolios as a special case. In our empirical analysis, we find that several well-known characteristics-based factors indeed lose their predictive power when we account for firm-specific (fixed) effects.

Keywords: Portfolio sorts, Cross-section of expected returns, Tests of asset pricing models, Random effects assumption
JEL classification: C21, G14, D1

---

* We are grateful to thank Yakov Amihud, Brad Barber, Trond Doskeland, John Driscoll, Alexander Kempf, Tim Kroencke, Beni Lauterbach, Nora Laurinaityte (discussant), Juhani Linnainmaa, Ernst Maug, Hannes Mohrschladt (discussant), Stefan Ruenzi, Paul Söderlind, Martin Weber, and conference and seminar participants at the 17th Colloquium on Financial Markets in Cologne, 21st Annual Conference of the Swiss Society for Financial Market Research (SGF) in Zurich, University of Basel, University of St. Gallen, WHU – Otto Beisheim School of Management, and University of Mannheim for helpful discussions and valuable comments. This research includes a theoretical result from an unpublished, discontinued paper entitled "A Generalization of the Calendar Time Portfolio Approach and the Performance of Private Investors". A Stata program implementing our methodology, a sample dataset, and a tutorial illustrating our approach is available from the authors upon request.
# Corresponding author: Tel.: +41-61-279-1773; Email address: daniel.hoechle@fhnw.ch.
Address: Institute for Finance, FHNW School of Business, Peter Merian-Strasse 86, CH-4002 Basel, Switzerland.
E-mail addresses: markus.schmid@unisg.ch (M. Schmid), heinz.zimmermann@unibas.ch (H. Zimmermann).

1. Introduction

A standard research methodology in empirical asset pricing analyzes the pricing effects of sorting individual assets into, say, decile portfolios based on a certain firm characteristic, and then compares the (risk-adjusted) performance of the top-decile portfolio with that of the bottom-decile portfolio. Statistically significant "alphas" are considered evidence that the characteristic used for the portfolio sort offers abnormal returns. This type of an analysis, to which we henceforth refer to as the "portfolio sorts approach", relies on the fact that a well-diversified portfolio only bears systematic risk, which is non-diversifiable and possibly priced, but no unsystematic (or "firm-specific") risk, which is diversifiable and thus not priced. As a consequence, the portfolio sorts approach assumes that firm-specific effects (if present) do not have any meaningful impact on asset pricing tests.[1] In this paper, we challenge this assumption. Our formal as well as empirical analysis shows that portfolio sorts are at risk of accidentally misattributing parts of the risk-adjusted return (or "alpha") to the firm characteristic underlying the portfolio sort. Such misattribution occurs if the firm characteristic is correlated with an unobservable yet time-persistent factor. Contributing to the sparse literature analyzing the econometric properties of the portfolio sorts approach, our findings show that an apparent anomaly may disappear if the correlation between the firm characteristic underlying the portfolio sort and the time-persistent factor changes over time.[2]

We start with the casual observation that, say, Google's or Apple's outstanding performance is unlikely to be fully attributable to a single firm characteristic X, but rather might be the result of a successful combination of both observable as well as *unobservable* firm characteristics. This reasoning however stands at odds with the portfolio sorts approach due to at least two reasons. First, the portfolio sorts approach (implicitly) assumes that the full outperformance (or underperformance) of a portfolio is attributable to the *observable* firm characteristic(s) underlying the sorted portfolios. Second, by comparing the "alpha" across portfolios, the portfolio sorts approach effectively analyzes whether there are "portfolio effects", i.e., whether there are statistically significant differences across the portfolios' (risk-adjusted) long-term returns. However, conventional portfolio sorts do not allow for the inclusion of

[1] The portfolio sorts approach was introduced by Jaffe (1974) and Mandelker (1974). Recent examples of empirical asset pricing studies using the portfolio sorts approach are Baker, Bradley, and Wurgler (2011), Novy-Marx (2013), Frazzini and Pedersen (2014), Ball, Gerakos, Linnainmaa, and Nikolaev (2015), Fama and French (2015), Gerakos and Linnainmaa (2017), and Bali, Brown, and Tang (2017). The portfolio sorts approach is also widely used in other areas of empirical finance, such as for example in household finance research (e.g., Barber and Odean, 2000, 2001; Ivkovic, Sialm, and Weisbenner, 2008; Seasholes and Zhu, 2010; Korniotis and Kumar, 2013), in research on insider trading (e.g., Jeng, Metrick, and Zeckhauser, 2003), and in studies analyzing the performance of mutual funds and hedge funds (e.g., Kacperczyk, Sialm, and Zheng, 2008; Fung, Hsieh, Naik, and Ramadorai, 2008).

[2] Research covering methodological and econometric aspects of the portfolio sorts approach includes Fama (1998), Lyon, Barber, and Tsai (1999), Loughran and Ritter (2000), Kothari and Warner (2008), and Cochrane (2011).

firm-specific effects. This is problematic since alternative firm characteristics are often highly correlated. Hence, it is well possible that two apparently unrelated firm characteristics produce "top" and "bottom" portfolios with remarkably similar portfolio constituents. Moreover, a certain firm characteristic X, which has no return predictability, may accidentally "pack" a few stocks with particularly good (poor) performance into the top (bottom) portfolio. In this case, the top portfolio may outperform the bottom portfolio by a large margin, but the performance differential between the two portfolios effectively is due to a few "firm-specific effects". As a result, by only controlling for portfolio effects but *not* for firm-specific effects, the portfolio sorts approach is at risk of having low power in detecting true return anomalies.

To investigate to what extent portfolio sorts misattribute risk-adjusted returns to the firm characteristic underlying the portfolio sort, we propose a novel, regression-based methodology for analyzing asset returns. Our approach relies on estimating a firm-level panel regression with Driscoll and Kraay (1998) standard errors that are robust to general forms of cross-sectional as well as temporal dependence.[3] The model specification is such that the individual firms' monthly excess returns (over the risk-free rate) are regressed on a set of market factors (e.g., the three Fama-French factors), a series of firm characteristics (e.g., gross profitability, volatility, etc.), and all interaction terms between the market factors and the firm characteristics. By relying on standard econometrics, our technique easily handles multiple dimensions and continuous firm, fund, or investor characteristics. Furthermore, it nests all variants of the portfolio sorts approach as a special case. In fact, we prove theoretically and confirm empirically that the proposed regression approach can be specified such that it *exactly* reproduces the alpha and factor exposure estimates from all variants of sorting assets into portfolios. Hence, the proposed method shares all the statistical properties and the straightforward economic interpretation of the portfolio sorts approach. Put differently, our method constitutes a linear regression-based extension to conventional portfolio sorts. We therefore refer to our methodology as the "Generalized Portfolio Sorts" approach, or, in short, the "GPS-model".

Our formal econometric analysis shows that the portfolio sorts approach relies on the random effects assumption to hold. This directly follows from the fact that in order to replicate the results from portfolio sorts, our GPS-model needs to be estimated with pooled OLS. Since the pooled OLS estimator depends on the random effects assumption, statistical inference on portfolio sorts is therefore only valid, if firm-specific (fixed) effects are uncorrelated with the characteristic(s) underlying the sorted portfolios.

[3] For ease of exposition, but without loss of generality, the paper simply refers to "firms". However, note that our regression-based method applies to any type of subject such as individual firms, portfolios of common stocks, mutual or hedge funds, private or institutional investors, countries or districts, etc. Furthermore, our technique can also be used for evaluating the long-term performance of, say, firms, funds, and investors.

Utilizing our GPS-model, we develop a Hausman (1978) type "portfolio sorts specification test" to analyze whether the results from portfolio sorts are likely to be affected by unobservable heterogeneity across firms. Our empirical evidence shows that the random effects assumption (implicitly) underlying the portfolio sorts approach indeed is often unlikely to hold. As a result, asset pricing tests that ignore firm fixed effects are at risk of suffering from an omitted variable bias inherent in the analysis. In this context, we show formally that firm characteristics which successfully predict the cross-section of stock returns, while being unable to predict the time-series of returns, are particularly prone to suffering from omitted variable bias. Thus, a firm characteristic that predicts the cross-section of stock returns well should not be considered a good predictor for asset returns unless it is also successful in predicting the time-series of asset returns. This finding supports Cochrane's (2011, p. 1062) claim that "*time-series forecasting regressions, cross-sectional regressions, and portfolio mean returns are really the same thing*".

Providing a means to distinguish between valid factors (which are robust to violations of the random effects assumption) and invalid factors (which are vulnerable to an omitted variable bias), our "portfolio sorts specification test" contributes to the literature addressing the factor-zoo-issue raised by Cochrane (2011). Specifically, the portfolio sorts specification test proposed in this paper is complementary to the result of Harvey, Liu, and Zhu (2016). They argue that a new factor needs to clear a significant hurdle (such as having a t-statistic greater than 3.0) in order to be considered a significant determinant for the cross-section of stock returns. However, since portfolio sorts are at risk of suffering from an omitted variable bias that may result in inadvertently misattributing parts of the alpha to the firm characteristic underlying the portfolio sort, our results imply that requiring a larger t-statistic alone is an insufficient criterion for identifying relevant factors. To make a good predictor for asset returns, a factor should also be robust to violations of the random effects assumption.[4]

Our method also addresses another major shortcoming of the portfolio sorts approach: portfolio sorts are generally limited to the analysis of a small number of firm characteristics. Cochrane (2011, p. 1061), for instance, argues that while it is customary to "*sort assets into portfolios based on a characteristic […] we cannot do this with 27 variables*". Exacerbating this multidimensional challenge, the portfolio sorts approach also renders it difficult to assess the functional relationship across multiple portfolios. Researchers applying the portfolio sorts approach therefore often focus on a comparison of the top and bottom group portfolios for simplicity (Patton and Timmermann, 2010). This, however,

[4] Other recent research that addresses the factor-zoo-issue includes Bryzgalova (2016), Harvey and Liu (2017), Kozak, Nagel, and Shrihari (2017), and Pukthuanthong, Roll, and Subrahmanyam (2017). Related to this, Harvey (2017) points out that empirical finance research puts too much reliance on p-values, and that researchers are incentivized to engage in "p-hacking" in order to get their research published. Hou, Xue, and Zhang (2017) and Chordia, Goyal, and Saretto (2017) analyze "p-hacking" empirically. Their results imply that p-hacking is a serious issue, and that financial markets are more efficient than commonly assumed.

involves a loss of potentially valuable information on the relationship between the sorting characteristic and the outcome variable, usually risk-adjusted performance. Being capable to handle multivariate and continuous firm characteristics, the GPS-model easily overcomes this shortcoming of the portfolio sorts approach. Moreover, since the GPS-model allows for reproducing the results from multiple portfolio sorts by estimating a single firm-level regression, a standard Wald test can be applied for testing whether the "alphas" of a series of sorted portfolios are jointly equal to zero. Such a Wald test constitutes an easy-to-implement, yet econometrically robust, alternative to the popular "GRS-test" of Gibbons, Ross, and Shanken (1989).

In the empirical part of the paper, we illustrate the importance of accounting for firm fixed effects in asset pricing tests. To this end, we study the return predictability of four randomly chosen firm characteristics that are widely used in recent asset pricing studies. The first two characteristics we consider are related to a firm's profitability. Novy-Marx (2013) shows that "gross profitability", defined as gross profit scaled by the book value of total assets, is a better predictor for the cross-section of average stock returns than alternative measures that are based on bottom line net income, cash flows, or dividends. Challenging the findings of Novy-Marx (2013), Ball, Gerakos, Linnainmaa, and Nikolaev (2015) propose an alternative profitability measure, operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding R&D expenditures) deflated by the book value of total assets. They find operating profitability to predict the cross-section of average stock returns even better than gross profitability. Hence, gross and operating profitability are the first two characteristics considered in our empirical analysis. The other two factors we consider are the stocks' 52-week rolling beta and 52-week rolling volatility. Ang, Hodrick, Xing, Zhang (2006, 2009) and Baker, Bradley, and Wurgler (2011), among others, show that stocks with high volatility deliver low risk-adjusted returns. Baker, Bradley, and Wurgler (2011) and Frazzini and Pedersen (2014) show that high-beta stocks deliver low risk-adjusted returns.

We start our empirical analysis by evaluating the performance difference between the top and bottom quintile portfolios of stocks sorted on operating profitability. Consistent with Ball, Gerakos, Linnainmaa, and Nikolaev (2015), we find the portfolio of stocks with high operating profitability to outperform the low profitability portfolio by about 0.5% per month, statistically significant at the 1% level. We then show that our GPS-model estimated with weighted pooled OLS exactly replicates both the coefficient estimates and t-statistics from the standard portfolio sorts approach. However, when reestimating our GPS-regression model with firm fixed effects, we find operating profitability to lose its predictive power. This finding suggests that operating profitability is correlated with an unobservable yet time-persistent factor such that a significant fraction of operating profitability's "alpha" from conventional portfolio sorts is likely due to the characteristic's correlation with that unobservable return predictor. As a result, it is not operating profitability per se that has predictive power, but its time-invariant component that is absorbed by the firm fixed effects. As a consequence, operating profitability will have out-of-sample predictive power if and only if its correlation with the unobservable, time-invariant factor persists beyond our sample period.

When estimating our GPS-model with (weighted) pooled OLS, which replicates the results from standard portfolio sorts, we again find very similar results as those reported in prior research. Specifically, we find gross profitability to be significantly positively related to the Fama-French three-factor model alpha and, consistent with Baker, Bradley, and Wurgler (2011), Frazzini and Pedersen (2014), and others, we find alpha to decrease with both volatility and beta. When taking advantage of the GPS-model's capability to analyze which firm characteristics stand a multivariate test, we find that only operating profitability and the stock beta statistically significantly predict stock returns, while the coefficient estimates on gross profitability and volatility turn statistically insignificant. The finding that operating profitability statistically dominates gross profitability is consistent with Ball, Gerakos, Linnainmaa, and Nikolaev (2015). However, these results are valid only if the random effects assumption holds. To empirically test whether this assumption is justified, we employ our GPS-model to perform a Hausman (1978) type specification test. The null hypothesis of the test assumes that the random effects assumption holds. With the single exception of gross profitability, such a "GPS-model specification test" rejects the null hypothesis of the random effects assumption at the 5% level or better for each individual characteristics-based factor as well as for combinations of factors. This implies that three of the four GPS-models estimated with pooled OLS likely misattribute parts of the alpha to the respective firm characteristics. Since the GPS-model estimated with (weighted) pooled OLS is able to *exactly* reproduce the results from all variants of portfolio sorts, this is bad news for the portfolio sorts approach: alpha (and factor exposure) estimates from conventional portfolio sorts are susceptible to misattribution.

When estimating our GPS-models with firm fixed effects (i.e., with the fixed effects estimator), we obtain entirely different results. While operating profitability loses its return predictability, gross profitability remains a significant predictor for stock returns. As a result, and in stark contrast to estimating our GPS-model with pooled OLS, gross profitability turns out to be a more robust predictor for stock returns than operating profitability when firm fixed effects are accounted for. This result contradicts the findings in Ball, Gerakos, Linnainmaa, and Nikolaev (2015), but supports the argument of Novy-Marx (2013) that gross profit is a good predictor of stock returns since it is a cleaner measure of a firm's economic profitability than, say, bottom line net income. A similar result emerges when comparing the return predictability of volatility with that of the stock beta. While the stock beta turns out to be a remarkably robust predictor for (risk-adjusted) stock returns, volatility loses its predictive power when firm fixed effects are included in the analysis. As a result, only the low-beta part of the low-risk anomaly withstands tests that include firm fixed effects.

In summary, our empirical analysis demonstrates that firm-specific (fixed) effects may – even in case of apparently well-diversified portfolios – exert significant impact on the results of empirical asset pricing tests. The GPS-model proposed in this research addresses this issue. Relying on a simple, firm-level regression framework, our methodology allows to test and account for firm fixed effects in the analysis. When the GPS-model is estimated with the fixed effects estimator, it ensures valid statistical inference in empirical asset pricing tests, even if the random effects assumption is violated. The GPS-model therefore offers an important advantage compared to the portfolio sorts approach, which to date represents a major workhorse methodology in empirical asset pricing research.[5]

The remainder of the paper is organized as follows. Section 2 describes the general model setup, economic interpretation, and statistical properties of our regression-based approach to analyzing asset returns. It also introduces a framework for analyzing the cross-sectional versus time-series predictability of asset returns and for testing the validity of the random effects assumption. Section 3 empirically validates the theoretical results from Section 2 and demonstrates the importance of accounting for firm fixed effects in empirical asset pricing tests. Section 4 concludes.

2. A regression-based approach to analyzing asset returns

In this section, we start by describing the general model setup, economic interpretation, and the statistical properties of our regression-based approach to analyzing asset returns. We then demonstrate the methodology's flexibility in handling multiple dimensions and continuous firm, fund, or investor characteristics. Next, we develop a framework for analyzing the cross-sectional versus time-series predictability of asset returns and show that such an analysis is closely related to Hausman's (1978) specification test. Specifically, we show that a firm characteristic that predicts the cross-section of stock returns well only qualifies as a good predictor for asset returns if it also has predictive power for the time-series of returns.

2.1 Model setup

We propose to analyze the cross-section of stock returns using the following firm-level regression model, and to draw statistical inference based on Driscoll and Kraay (1998) standard errors that are

[5] The Fama and MacBeth (1973) procedure represents another major workhorse for analyzing stock returns in empirical asset pricing studies. Its econometric properties are studied in Petersen (2009), Vogelsang (2012), and Kamstra (2017), among others.

robust to heteroskedasticity and general forms of cross-sectional and temporal dependence (Driscoll and Kraay, 1998; Hoechle, 2007):

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + c_i + v_{it} \qquad (i = 1, \dots, N, \quad t = 1, \dots, T) \tag{1}$$

The dependent variable r_{it} is the period t (excess) return of firm i. Vector $\boldsymbol{z}_{it} = [1 \quad z_{2,it} \quad \dots \quad z_{M,it}]$ comprises a constant and a set of firm characteristics $z_{m,it}$ $(m = 2, \dots, M)$ which may vary across both firms and time. Vector $\boldsymbol{x}_t = [1 \quad x_{1t} \quad \dots \quad x_{Kt}]$ consists of a constant and a series of market-level factor variables $x_{k,t}$ $(k = 1, \dots, K)$ which only vary over time but not across firms. Popular examples of variables in vector $\boldsymbol{x}_t$ are the market excess return, the Fama and French (1993) size and value factors, or the Carhart (1997) momentum factor. With $\otimes$ denoting the Kronecker product, regression (1) comprises three types of explanatory variables: individual firm characteristics, market-level factor variables, and all interactions between firm characteristics and factor variables. Fully interacted regression model (1) thus consists of $M \times (K + 1)$ explanatory variables whose regression coefficients are stored in coefficient vector $\boldsymbol{\theta}$. Finally, c_i is an unobserved firm-specific effect with $E(c_i) = 0$, and v_{it} is the regression disturbance that is assumed to be strictly exogenous (i.e. $E[v_{it} \mid c_i, (\boldsymbol{z}_{i1} \otimes \boldsymbol{x}_1), \dots, (\boldsymbol{z}_{iT} \otimes \boldsymbol{x}_T)] = 0$ for all i and t).

If there is no unobserved heterogeneity across firms (i.e., if $c_i = 0$ for all firms) or if the firm-specific effects c_i are uncorrelated with the regressors (i.e., $E[c_i | (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)] = 0$), then regression model (1) can be estimated consistently with (weighted) pooled OLS estimation.[6] As we detail in section 2.2 below, regression (1) – with firm-specific effects c_i excluded – can be specified such that it *exactly* reproduces the results from all sorts of conventional portfolio sorts. We therefore refer to regression (1) as the "Generalized Portfolio Sorts" approach or, in short, the *GPS*-model.

Unfortunately, the random effects assumption $E[c_i | (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)] = 0$ cannot be generally justified. If unobserved heterogeneity across firms is present, it is often more convincing to allow the firm-specific effects to be correlated with the explanatory variables. In the case of $E[c_i | (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)] \neq 0$, however, the pooled OLS estimator suffers from an omitted variable bias and pooled OLS estimation of regression (1) produces inconsistent results.

The fixed effects (FE) model relaxes the random effects (RE) assumption underlying the pooled OLS estimator. Here, the firm-specific effects (c_i) are treated as unobservable random variables which may or may not be correlated with the regressors $(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)$. Therefore, the fixed effects (or within) estimator provides a means for consistently estimating GPS-model (1) even if the "firm-fixed effects"

[6] Note that the pooled OLS estimator is consistent under both the constant coefficients model as well as under the random effects (RE) model. However, under the RE model pooled OLS is inefficient compared to the FGLS random effects estimator. For details, see Cameron and Trivedi (2005, chapter 21).

(c_i) are correlated with the explanatory variables. Hence, if non-zero correlation between the firm-specific effects and the explanatory variables cannot be ruled out, GPS-model (1) should be estimated with the fixed effects estimator rather than with pooled OLS. In section 2.3 below, we discuss a Hausman (1978) type specification test for analyzing whether the RE assumption is likely to hold for a given specification of GPS-model (1).

2.2 GPS-model vs. portfolio sorts

Popularized by Fama and French's (1993, 1996) influential research, the portfolio sorts methodology became a major workhorse in empirical finance. The method offers an intuitive economic interpretation and ensures robust statistical inference even in the presence of cross-sectional and temporal dependence (Lyon, Barber, and Tsai, 1999). In this section, we demonstrate that GPS-model (1) is able to reproduce the results of all variants of conventional portfolio sorts. The GPS-model proposed in this research therefore has a sound theoretical and statistical foundation and the coefficient estimates of regression (1) offer a straightforward economic interpretation.

2.2.1 Formal exposition of the portfolio sorts approach

The portfolio sorts methodology involves two steps. In the first step, for each period t the portfolio (excess) return r_{pt} is computed for a group of firms i as follows:

$$r_{pt} = \sum_{i=1}^{N_t} w_{it} r_{it} \tag{2}$$

w_{it} denotes the beginning of period t portfolio weight of firm i ($i = 1, \dots, N_t$), and r_{it} refers to the firm's stock (excess) return in period t. The second step of the portfolio sorts approach then evaluates the (risk-adjusted) performance of portfolio p by aid of a linear K-factor time-series regression with r_{pt} from (2) as the dependent variable:

$$r_{pt} = \beta_0 + \beta_1 x_{1t} + \dots + \beta_K x_{Kt} + \varepsilon_t \tag{3}$$

In most applications, variables x_{kt} ($k = 1, \dots, K$) are specified such that (3) represents a Jensen (1968), Fama and French (1993, 2015), or Carhart (1997) type factor model. To evaluate whether the risk-adjusted performance of portfolio p is abnormally high or low, the coefficient estimate for the intercept term ($\hat{\beta}_0$), which is often referred to as the "alpha", and its statistical significance are considered.

2.2.2 Evaluating the performance of a single portfolio with the GPS-model

By estimating firm-level GPS-model (1) with pooled OLS, we can reproduce the results of time-series regression (3). To this end, we specify $\boldsymbol{z}_{it} = [\,1\,]$ and $\boldsymbol{x}_t = [\,1 \;\; x_{1t} \;\; \dots \;\; x_{Kt}]$ to obtain the following regression model:[7]

$$\begin{aligned} r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\, \boldsymbol{\theta} + v_{it} = ([\,1\,] \otimes [\,1 \;\; x_{1t} \;\; \dots \;\; x_{Kt}\,])\boldsymbol{\theta} + v_{it} \\ &= \theta_0 + \theta_1 x_{1t} + \dots + \theta_K x_{Kt} + v_{it} \end{aligned} \tag{4}$$

For ease of mathematical tractability, but without loss of generality, we limit our formal analysis to the case of a balanced panel with N firms, T time periods, and portfolio weights $w_{it} = 1/N$ (i.e., all firms are equally weighted).[8] Under these assumptions, the following result holds true:

Proposition 1 (Single Portfolio)

- ***Part A – Coefficient estimates.*** *Estimating linear regression (4) with pooled OLS yields identical coefficient estimates as estimating time-series regression (3) with OLS, i.e.,* $\hat{\theta}_k \equiv \hat{\beta}_k \quad (\forall\, k = 0, 1, \dots, K)$.
- ***Part B – Standard errors.*** *For a given lag length H, Driscoll and Kraay (1998) standard errors for coefficient estimates* $\hat{\theta}_k$ *in pooled OLS regression (4) are identical to Newey and West (1987) standard errors for coefficient estimates* $\hat{\beta}_k$ *in time-series regression (3), i.e.,* $\mathrm{SE}(\hat{\theta}_k) \equiv \mathrm{SE}(\hat{\beta}_k) \quad (\forall\, k = 0, 1, \dots, K)$.

Proof: See Appendix A.1.

Part A of Proposition 1 is an application of a well-known property from portfolio theory which says that the portfolio beta is equal to the weighted sum of individual asset betas. Part B of Proposition 1 is intuitive since the Driscoll and Kraay (1998, p. 552) "covariance matrix estimator is precisely the standard Newey and West (1987) heteroskedasticity and serial correlation consistent covariance matrix estimator, applied to the sequence of cross-sectional averages" of the moment conditions.

2.2.3 Using the GPS-model to obtain portfolio sorts

When applying the portfolio sorts approach, the analysis usually is not restricted to a single group of firms as discussed in Section 2.2.2 above. Rather, firms are sorted into a series of five, ten, or more

[7] For consistency with the portfolio sorts approach, throughout Section 2.2, we assume $c_i = 0$ (for all i) and omit the firm-specific effects c_i from the analysis. This allows us to estimate GPS-model (1) with (weighted) pooled OLS.

[8] In our empirical analysis, we also consider unbalanced panels (i.e., time-varying cross-sections) and value weighted portfolios. In Section 3, we demonstrate that our theoretical results also hold in this more general setup.

portfolios with predefined properties. In many cases, the portfolios are constructed on the basis of a single firm characteristic such as, for example, the book-to-market ratio. Occasionally, researchers also form double (or higher dimensional) sorts where portfolios are constructed based on multiple firm characteristics such as the book-to-market ratio *and* firm size. For each portfolio, the researcher then independently evaluates the risk-adjusted performance by aid of a Jensen (1968), Fama and French (1993, 2015), or Carhart (1997) type time-series regression as described in Section 2.2.1 above.

Formally, in this more general setup, the first step of the portfolio sorts approach groups the firms into characteristics-based portfolios p for which the average month t excess return, r_{pt}, is equal to

$$r_{pt} = \sum_{i=1}^{N_t} w_{it}^{(p)} z_{it}^{(p)} r_{it} \tag{5}$$

As before, r_{it} denotes the period t excess return of firm i, and $w_{it}^{(p)}$ is the firm's beginning-of-period t weight in portfolio p (with $p = 1, \dots, P$). $z_{it}^{(p)}$ is a dummy variable which is equal to one if firm i belongs to portfolio p, and zero otherwise. For each portfolio p, the weights sum up to $\sum_{i=1}^{N_t} w_{it}^{(p)} z_{it}^{(p)} = 1$, and the period t cross-section comprises $N_t = \sum_{p=1}^{P} \sum_{i=1}^{N_t} z_{it}^{(p)}$ firms.

The second step of the portfolio sorts approach then evaluates the (risk-adjusted) performance $\beta_{p,0}$ of portfolio p by aid of a linear K-factor time-series regression with r_{pt} from (5) as the dependent variable:

$$r_{pt} = \beta_{p,0} + \beta_{p,1} x_{1t} + \dots + \beta_{p,K} x_{Kt} + \varepsilon_{pt} \tag{6}$$

If the coefficient estimate for $\beta_{p,0}$ is positive (negative) and statistically significantly different from zero, then portfolio p has a positive (negative) "alpha" and generates, on average, an abnormally good (poor) return.

With GPS-model (1) it is possible to reproduce the results from time-series regression (6) for each and every portfolio p (with $p = 1, \dots, P$) by estimating a single firm-level regression with pooled OLS. As before, we specify $\boldsymbol{x}_t = [\,1 \quad x_{1t} \ \dots \ x_{Kt}\,]$. When forming vector $\boldsymbol{z}_{it}$, we recognize that the analysis needs to include a full set of P portfolio dummies $z_{it}^{(p)}$. To avoid the dummy variables trap, we omit the constant from vector $\boldsymbol{z}_{it}$ and, hence, set $\boldsymbol{z}_{it} = [\,z_{it}^{(1)} \quad z_{it}^{(2)} \ \dots \ z_{it}^{(P)}]$ to obtain

$$
\begin{aligned}
r_{it} = &\left(\left[z_{it}^{(1)} \;\; z_{it}^{(2)} \;\; \dots \;\; z_{it}^{(P)} \right] \otimes [\, 1 \;\; x_{1t} \;\; \dots \;\; x_{Kt} \,]\right) \boldsymbol{\theta} + \nu_{it} \\
= &\quad \theta_{1,0}\, z_{it}^{(1)} + \theta_{1,1}\, x_{1t} z_{it}^{(1)} + \dots + \theta_{1,K}\, x_{Kt} z_{it}^{(1)} \\
+ &\quad \theta_{2,0}\, z_{it}^{(2)} + \theta_{2,1}\, x_{1t} z_{it}^{(2)} + \dots + \theta_{2,K}\, x_{Kt} z_{it}^{(2)} \\
+ &\quad \dots \\
+ &\quad \theta_{P,0}\, z_{it}^{(P)} + \theta_{P,1}\, x_{1t} z_{it}^{(P)} + \dots + \theta_{P,K}\, x_{Kt} z_{it}^{(P)} + \nu_{it}
\end{aligned}
\qquad (7)
$$

Under the assumptions of Proposition 1, and provided that portfolios $p = 1, \dots, P$ are constant over time (i.e., $z_{it}^{(p)} = z_i^{(p)}$ for all t), the following result holds true:[9]

Proposition 2 (Portfolio sorts)

- ***Part A – Coefficient estimates.*** *For each portfolio p, pooled OLS coefficient estimates for $\theta_{p,k}$ in GPS-model (7) coincide with OLS coefficient estimates for $\beta_{p,k}$ from time-series regression (6), i.e., $\hat{\theta}_{p,k} \equiv \hat{\beta}_{p,k}$ ($\forall\, k = 0, 1, \dots, K$ and $p = 1, \dots, P$).*
- ***Part B – Standard errors.*** *For a given lag length H, Driscoll and Kraay (1998) standard errors for coefficient estimates $\hat{\theta}_{p,k}$ in GPS-model (7) coincide with Newey and West (1987) standard errors of portfolio p's coefficient estimates $\hat{\beta}_{p,k}$ from time-series regression (6), i.e.,* $\mathrm{SE}(\hat{\theta}_{p,k}) \equiv \mathrm{SE}(\hat{\beta}_{p,k})$ *($\forall\, k = 0, 1, \dots, K$ and $p = 1, \dots, P$).*

Proof: See Appendix A.2.

According to Proposition 2, the coefficient estimates of GPS-model (7) have a straightforward economic interpretation: coefficient estimate $\hat{\theta}_{p,0}$ ($p = 1, \dots, P$) measures the risk-adjusted performance (or "alpha") of portfolio p. Coefficient estimate $\hat{\theta}_{p,k}$ (with $k = 1, \dots, K$) represents portfolio p's exposure versus factor k. Note that GPS-model (7) reproduces the results of a set of P independent time-series regressions (6) by aid of a *single* linear regression on the firm level. As a result, a standard Wald test can be applied to test whether the risk-adjusted performance of the P portfolios is *jointly* equal to zero:

$$
\mathrm{H}_0\!: \theta_{1,0} = \theta_{2,0} = \ldots = \theta_{P,0} = 0 \qquad \text{vs.} \qquad \mathrm{H}_1\!:\; \theta_{p,0} \neq 0 \;\text{ for at least one } p \text{ in } 1, \dots, P \qquad (8)
$$

The multiple hypothesis test in (8) offers an alternative to the widely applied Gibbons, Ross, and Shanken (1989) or "GRS" test, a finite-sample F-test commonly used to test the joint significance of the "alphas" across a set of (e.g., decile) portfolios. Estimating GPS-model (7) with Driscoll and Kraay

[9] For Proposition 2 to hold in the general case of an unbalanced panel with time-varying portfolios, GPS-model (7) needs to be estimated with weighted pooled OLS. Details on the weighting scheme reproducing the results of portfolio sorts with value-weighted portfolios are provided in Section 3.1.

(1998) standard errors ensures that the Wald test in (8) allows for valid statistical inference if the error terms (ν_{it}) of the regression are heteroskedastic, autocorrelated, and cross-sectionally dependent.

2.2.4 Performance differences between two portfolios

The portfolio sorts approach is widely used to test whether the risk-adjusted performance (or “alpha”) of the top group portfolio differs statistically significantly from that of the bottom group portfolio. When investigating whether, say, firms with top-quintile book-to-market ratios (so-called “value stocks”) outperform firms with bottom-quintile book-to-market ratios (so-called “growth stocks”), the first step of the analysis involves computing average month t excess returns for both the top and bottom group portfolios as follows:

$$r_{p,t} = \sum_{i=1}^{N_t} w_{it}^{(p)} z_{it}^{(p)} r_{it} \tag{9}$$

As before, r_{it} is firm i’s period t excess return and $z_{it}^{(p)}$ is a dummy variable with value one if firm i belongs to group p (with $p = high,\ low$), and zero otherwise.[10] For both portfolios the beginning-of-period t portfolio weights $w_{it}^{(p)}$ sum up to $\sum_{i=1}^{N_t} w_{it}^{(p)} z_{it}^{(p)} = 1$, and the cross-section considered in the analysis comprises a total of $N_t^* = \sum_{i=1}^{N_t} z_{it}^{(low)} + \sum_{i=1}^{N_t} z_{it}^{(high)}$ firms. The period t return difference between the two portfolios is thus equal to

$$\Delta r_{p,t} = r_{high,t} - r_{low,t} \tag{10}$$

The second step of the portfolio sorts approach then evaluates the risk-adjusted performance of zero investment portfolio (10) based on a K-factor time-series regression as follows:

$$\Delta r_{p,t} = \beta_{\Delta 0} + \beta_{\Delta 1} x_{1t} + \ldots + \beta_{\Delta K} x_{Kt} + \varepsilon_{\Delta t} \tag{11}$$

If the coefficient estimate for $\beta_{\Delta 0}$ is positive (negative) and significantly different from zero, then portfolio “$high$” is considered to outperform (underperform) portfolio “low”.

With GPS-model (1), again estimated with pooled OLS, it is possible to reproduce the results of time-series regression (11). For this purpose, we set $\boldsymbol{z}_{it} = [\,1 \quad z_{it}^{(high)}]$ and $\boldsymbol{x}_t = [\,1 \quad x_{1t} \quad \ldots \quad x_{Kt}\,]$ to obtain the following firm-level regression model:

[10] For simplicity but without loss of generality, we label the portfolios as “high” and “low” here. However, subscript p could also refer to “IPO firms” and “mature firms”, “firms with a female CEO” and “firms with a male CEO”, or any other set of two portfolios that are meant to be compared, respectively.

$$
\begin{aligned}
r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \nu_{it} \\
&= \theta_{low,0} \quad + \theta_{low,1}\, x_{1t} \quad + \quad \dots \quad + \theta_{low,K}\, x_{Kt} \\
&\quad + \theta_{\Delta 0}\, z_{it}^{(high)} + \theta_{\Delta 1}\, x_{1t} z_{it}^{(high)} + \quad \dots \quad + \theta_{\Delta K}\, x_{Kt} z_{it}^{(high)} + \nu_{it}
\end{aligned} \tag{12}
$$

Under the assumptions of Proposition 1, and provided that portfolios "$high$" and "low" are constant over time, the following result holds true:[11]

Proposition 3 (Performance difference between two groups)

Part A – Coefficient estimates.

- *Pooled OLS coefficient estimates for $\theta_{\Delta k}$ in GPS-model (12) are identical to OLS coefficient estimates for $\beta_{\Delta k}$ in time-series regression (11), i.e., $\hat{\theta}_{\Delta k} \equiv \hat{\beta}_{\Delta k}$ ($\forall\, k = 0, 1, \dots, K$).*
- *Pooled OLS coefficient estimates for $\theta_{low,k}$ in GPS-model (12) are identical to OLS coefficient estimates for $\beta_{low,k}$ in time-series regression (6) for portfolio p="low", i.e., $\hat{\theta}_{low,k} \equiv \hat{\beta}_{low,k}$ ($\forall\, k = 0, 1, \dots, K$).*

Part B – Standard errors.

- *For a given lag length H, Driscoll and Kraay (1998) standard errors for coefficient estimates $\hat{\theta}_{\Delta k}$ in GPS-model (12) are identical to Newey and West (1987) standard errors for coefficient estimates $\hat{\beta}_{\Delta k}$ of time-series regression (11), i.e., $SE(\hat{\theta}_{\Delta k}) \equiv \mathrm{SE}(\hat{\beta}_{\Delta k})$ ($\forall\, k = 0, 1, \dots, K$).*
- *For a given lag length H, Driscoll and Kraay (1998) standard errors for coefficient estimates $\hat{\theta}_{low,k}$ in GPS-model (12) are identical to Newey and West (1987) standard errors for coefficient estimates $\hat{\beta}_{low,k}$ of time-series regression (6) for portfolio p="low", i.e., $\mathrm{SE}(\hat{\theta}_{low,k}) \equiv \mathrm{SE}(\hat{\beta}_{low,k})$ ($\forall\, k = 0, 1, \dots, K$).*

Proof: See Appendix A.3.

Proposition 3 shows how to specify the GPS-model when analyzing the relative performance of two portfolios. This result can be further generalized to the comparison of a certain base portfolio's (e.g., portfolio $p = 1$) performance with the performance of each other (e.g., quintile or decile) portfolio. Assuming that portfolio $p = 1$ is the base portfolio, we set $\boldsymbol{z}_{it} = [1 \;\; z_{it}^{(2)} \;\dots\; z_{it}^{(P)}]$ and $\boldsymbol{x}_t = [1 \;\; x_{1t} \;\dots\; x_{Kt}]$ to obtain the following firm-level regression:

[11] For Proposition 3 to hold in the general case of an unbalanced panel with time-varying portfolios, regression (12) has to be estimated with weighted pooled OLS. See Section 3.3 for details.

$$
\begin{aligned}
r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\, \boldsymbol{\theta} + \nu_{it} \\
&= \theta_{1,0} + \theta_{1,1}\, x_{1t} + \dots + \theta_{1,K}\, x_{Kt} \\
&+ \theta_{\Delta(2\to1),0}\, z_{it}^{(high)} + \theta_{\Delta(2\to1),1}\, x_{1t} z_{it}^{(high)} + \dots + \theta_{\Delta(2\to1),\mathrm{K}}\, x_{Kt} z_{it}^{(high)} \\
&+ \dots \\
&+ \theta_{\Delta(P\to1),0}\, z_{it}^{(P)} + \theta_{\Delta(P\to1),1}\, x_{1t} z_{it}^{(P)} + \dots + \theta_{\Delta(P\to1),K}\, x_{Kt} z_{it}^{(P)} + \nu_{it}
\end{aligned}
\tag{13}
$$

A direct consequence of Propositions 2 and 3 is as follows:

Corollary 1 (Relative performance versus a base portfolio)

- *The pooled OLS coefficient estimate for $\theta_{\Delta(p\to1),k}$ in GPS-model (13) coincides with the OLS coefficient estimate for $\beta_{\Delta k}$ in time-series regression (11) where portfolio p (with $p = 2, \dots, P$) is compared with portfolio 1, i.e., $\hat{\theta}_{\Delta(p\to1),k} \equiv \hat{\beta}_{\Delta k}$ ($\forall\ k = 0, 1, \dots, K$).*
- *For a given lag length H, the Driscoll and Kraay (1998) standard error for coefficient estimate $\hat{\theta}_{\Delta(p\to1),k}$ in GPS-model (13) coincides with the Newey and West (1987) standard error for coefficient estimate $\hat{\beta}_{\Delta k}$ of time-series regression (11) where portfolio p (with $p = 2, \dots, P$) is compared with portfolio 1, i.e., $SE(\hat{\theta}_{\Delta(p\to1),k}) \equiv \mathrm{SE}(\hat{\beta}_{\Delta k})$ ($\forall\ k = 0, 1, \dots, K$).*

■

2.3 Applications of the GPS-model beyond the scope of traditional portfolio sorts

Conventional portfolio sorts have a series of drawbacks. First, they are generally limited to the analysis of a small number of firm characteristics (Cochrane, 2011). With the portfolio sorts approach it is therefore challenging to perform robustness checks or to test for competing hypotheses. Second, it is difficult to assess the functional relationship across multiple portfolios. Researchers applying the portfolio sorts approach therefore often focus on a comparison of the top and bottom group portfolios for simplicity (Patton and Timmermann, 2010). Third, the results in Section 2.2 show that the portfolio sorts approach crucially depends on the random effects (RE) assumption to hold. This directly follows from the fact that GPS-model (1) needs to be estimated with pooled OLS (which is only consistent under the RE assumption) to replicate the results from portfolio sorts. Therefore, statistical results from portfolio sorts are biased when firm-specific effects are present and correlated with the explanatory variables.

The GPS-model proposed in this research has no such limitations and, hence, facilitates the analysis of research questions that are beyond the scope of conventional portfolio sorts. We now consider a series of such applications. Our analysis starts by discussing how to interpret the results from GPS-models that include multivariate and continuous firm characteristics. Next, we apply GPS-model (1) to study the cross-section versus time-series predictability of stock returns, thereby demonstrating that such an analysis is closely related to performing a Hausman (1978) type specification test. Finally, we discuss consistent estimation of GPS-models in the presence of firm-specific effects, and we derive a statistical test for analyzing whether the results from portfolio sorts are valid.

2.3.1 Multivariate and continuous firm characteristics

GPS-model (1) can be specified such that vector $\boldsymbol{z}_{it}$ contains multivariate binary or continuous firm characteristics. As a result, the GPS-model offers a natural solution to the "multidimensional challenge" of conventional portfolio sorts (Cochrane, 2011), and it provides a simple framework for analyzing formal tests of competing hypothesis as well as for implementing robustness checks.

Notwithstanding this flexibility, the GPS-model retains a clear-cut economic interpretation even if multiple firm characteristics are included in the analysis. To demonstrate this, we consider the case of a CAPM factor structure and two firm characteristics.[12] Denoting the period t market return in excess of the risk-free return by r_{mt}, we specify $\boldsymbol{x}_t = [1 \quad r_{mt}]$ and $\boldsymbol{z}_{it} = [1 \quad z_{1,it} \quad z_{2,it}]$. Absent firm-specific effects, we thus obtain the following GPS-model:

$$\begin{aligned} r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + v_{it} = \left([1 \quad z_{1,it} \quad z_{2,it}] \otimes [1 \quad r_{mt}]\right)\boldsymbol{\theta} + v_{it} \\ &= \left(\theta_{\alpha,0} + \theta_{\alpha,1} z_{1,it} + \theta_{\alpha,2} z_{2,it}\right) + \left(\theta_{\beta,0} + \theta_{\beta,1} z_{1,it} + \theta_{\beta,2} z_{2,it}\right) \times r_{mt} + v_{it} \\ &= \alpha_{it} + \beta_{it} \times r_{mt} + v_{it} \end{aligned} \tag{14}$$

where $\alpha_{it} = \theta_{\alpha,0} + \theta_{\alpha,1} z_{1,it} + \theta_{\alpha,2} z_{2,it}$ and $\beta_{it} = \theta_{\beta,0} + \theta_{\beta,1} z_{1,it} + \theta_{\beta,2} z_{2,it}$. The last two rows in (14) show that the GPS-model linearly decomposes the risk-adjusted performance (α_{it}) and the factor exposure (β_{it}) with respect to the firm characteristics in $\boldsymbol{z}_{it}$. The Jensen alpha (α_{it}) and beta (β_{it}) in GPS-model (14) therefore represent *conditional* measures. A simple example can illustrate this. Assume that an estimation of regression (14) yields the following result:

$$\begin{aligned} r_{it} &= \hat{\alpha}_{it} + \hat{\beta}_{it} \times r_{mt} + v_{it} \\ &= (0.2 + 0.5 z_1 - 0.8 z_2) + (0.8 - 0.3 z_1 + 0.1 z_2) \times r_{mt} + v_{it} \end{aligned}$$

In this particular case, the Jensen alpha loads positively on firm characteristic z_1 and negatively on z_2 such that the higher the value of z_1 and the lower the value of z_2 the higher is $\hat{\alpha}_{it}$. The (conditional) Jensen alpha for company A with $z_1 = 1$ and $z_2 = 0.5$ is equal to $\hat{\alpha}_A = 0.2 + 0.5 \times 1 - 0.8 \times 0.5 =$

[12] Note that the specification of vectors $\boldsymbol{x}_t$ and $\boldsymbol{z}_{it}$ can easily be extended to comprise multiple factor variables or firm characteristics without changing the logic of how to interpret the results from estimating a GPS-model.

0.3. The alpha for firm B, with $z_1 = -0.2$ and $z_2 = 0.5$, is $\hat{\alpha}_B = 0.2 + 0.5 \times (-0.2) - 0.8 \times 0.5 = -0.3$. The (conditional) beta for firm i is derived analogously.

When firm-specific effects are present and correlated with the firm characteristics in vector $\boldsymbol{z}_{it}$, (weighted) pooled OLS estimation produces biased, invalid statistical results. In this case, GPS-model (14) needs to be estimated with firm fixed effects (i.e. with the fixed effects estimator) to allow for valid statistical inference. This, however, does *not* affect the interpretation of the GPS-model results. To see this, note that the analysis in GPS-model (14) focuses on slope coefficients $\theta_{\alpha,1}$ and $\theta_{\alpha,2}$ which measure by how much the Jensen alpha changes if firm characteristics $z_{1,it}$ and $z_{2,it}$ change by one unit. By contrast, the coefficient estimate for $\theta_{\alpha,0}$ depends on the sample means of $z_{1,it}$ and $z_{2,it}$ and, hence, is of minor interest. When regression (14) is estimated with firm fixed effects, the intercept term cannot be identified and "drops out" of the regression as part of the within-transformation. However, the slope coefficients, which matter for the analysis, are consistently estimated. Therefore, interpretation of GPS-model coefficients remains the same, irrespective of whether the model is estimated with or without firm fixed effects.

2.3.2 Time-series versus cross-section predictability

In empirical asset pricing, an important question concerns the time-series versus cross-sectional predictability of asset returns (Cochrane, 2011). We now apply GPS-model (1) to formally test how well firm characteristic z_{it} predicts the time-series of asset returns as compared to the cross-section of returns. For this purpose, we start by decomposing firm characteristic z_{it} as

$$z_{it} = \bar{z}_i + \tilde{z}_{it} \tag{15}$$

where $\bar{z}_i = T_i^{-1} \sum_{t=1}^{T_i} z_{it}$ refers to firm i's time-series average of characteristic z_{it}, and $\tilde{z}_{it} = z_{it} - \bar{z}_i$ quantifies by how much the firm's period t value of z_{it} deviates from $\bar{z}_i$. Econometrically speaking, $\tilde{z}_{it}$ represents the within-transformed (or time-series demeaned) version of z_{it}. Based on (15), we therefore set vector $\boldsymbol{z}_{it}$ to $\boldsymbol{z}_{it} = [1 \quad \bar{z}_i \quad \tilde{z}_{it}]$. When specifying vector $\boldsymbol{x}_t$, we account for the fact that in the application at hand we focus on analyzing asset *returns* (rather than risk-adjusted performance) and, hence, set $\boldsymbol{x}_t = [1]$. Assuming $c_i = 0$ for all i, we obtain the following GPS-model:

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\, \boldsymbol{\theta} + v_{it} = ([1 \quad \bar{z}_i \quad \tilde{z}_{it}] \otimes [1])\, \boldsymbol{\theta} + v_{it} = \theta_0 + \theta_B \bar{z}_i + \theta_W \tilde{z}_{it} + v_{it} \tag{16}$$

Note that in regression (16), θ_B measures pure cross-sectional return predictability of firm characteristic z_{it} whereas θ_W quantifies the characteristic's time-series return predictability. In fact, GPS-model (16) is structured as Neuhaus and Kalbfleisch's (1998) variant of Mundlak's (1978) approach. Pooled OLS estimation of regression (16) thus yields the between estimate for coefficient θ_B and the within (or fixed effects) estimate for coefficient $\hat{\theta}_W$.[13] GPS-model (16) thus constitutes a "hybrid" model that combines the between (BE) estimator with the fixed effects (FE) estimator in a single regression (Allison, 2009). For our purposes, this setup is useful as it allows us to formally test whether the time-series predictability ($\hat{\theta}_W$) of firm characteristic z_{it} equals the cross-sectional predictability ($\hat{\theta}_B$).

Testing for $\hat{\theta}_B = \hat{\theta}_W$ is important for at least two reasons. First, referring to the managed-portfolio theorem, Cochrane (2011, p. 1062) argues that "*time-series forecasting regressions, cross-sectional regressions, and portfolio mean returns are really the same thing. [...] An instrument z_t in a time-series test* $0 = E[(m_{t+1}R^e_{t+1})z_t]$ *corresponds to a managed-portfolio return $R^e_{t+1}z_t$ in an unconditional test* $0 = E[m_{t+1}(R^e_{t+1}z_t)]$." As a result, testing for $\hat{\theta}_B = \hat{\theta}_W$ is of economic relevance. Second, the test is important from an econometric point of view. If the hypothesis of $\hat{\theta}_B = \hat{\theta}_W$ is rejected, then the difference between the within estimate ($\hat{\theta}_W$) and the between estimate ($\hat{\theta}_B$) is statistically significant such that the random effects (RE) assumption *cannot* be assumed to hold. GPS-model (16) therefore provides an alternative to Hausman's (1978) specification test. With $\hat{\theta}_B \neq \hat{\theta}_W$, variable z_{it} is likely to be correlated with other firm characteristics not included in the regression. As a result, rejection of hypothesis $\hat{\theta}_B = \hat{\theta}_W$ implies that pooled OLS estimation of GPS-model $r_{it} = ([1 \quad z_{it}] \otimes [1])\,\boldsymbol{\theta} + v_{it} = \theta_0 + \theta_1 z_{it} + v_{it}$ suffers from an omitted variable bias and, hence, produces biased coefficient estimates for θ_0 and θ_1. Analyzing return predictability of firm characteristic z_{it} in this case needs to account for firm fixed effects in order to ensure valid statistical inference. Put differently, a firm characteristic that predicts the cross-section of returns well should only be considered a good predictor for expected returns if it also successfully predicts the time-series of asset returns. We therefore conclude that asset pricing tests should focus on time-series return predictability, which can consistently be estimated with the within-estimator, rather than on cross-sectional return predictability, which is at risk of suffering from an omitted variable bias.

[13] Note that in the case of a balanced panel, pooled OLS estimation of regression (16) yields identical results as estimation with the (efficient) FGLS random effects estimator. If the panel is unbalanced, however, then estimation results from pooled OLS for $\hat{\theta}_B$ differ slightly from those of the FGLS random effects estimator. The reason is that the between estimator forming part of the FGLS random effects estimator weights each firm i equally, independently of the number of observations in the sample. By contrast, in pooled OLS the weight of firm i depends on the length of its time-series.

2.3.3 GPS-model specification test for the presence of fixed effects

Building on our analysis from Section 2.3.2, we now derive a Hausman (1978) type specification test, which allows us to investigate whether the random effects (RE) assumption for any given specification of a GPS-model holds. Our testing procedure starts with GPS-model (1):

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + c_i + v_{it} = \left([1 \quad z_{2,it} \quad \dots \quad z_{M,it}] \otimes [1 \quad x_{1t} \quad \dots \quad x_{Kt}]\right)\boldsymbol{\theta} + c_i + v_{it}$$

Relying on Mundlak's (1978) correlated RE assumption we model firm-specific effect c_i as

$$c_i = \left(\sum_{m=2}^{M} \xi_{m,0}\, \bar{z}_{m,it} \;+\; \sum_{m=2}^{M} \sum_{k=1}^{K} \xi_{m,k}\, \bar{q}_i^{(m,k)} \right) + u_i \equiv \overline{\boldsymbol{q}}_{\boldsymbol{i}} \boldsymbol{\xi} + u_i \tag{17}$$

where u_i is a mean zero firm-specific effect that is assumed to be uncorrelated with explanatory variables $(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)$. $\bar{z}_{m,it}$ refers to firm i's time-series average of characteristic $z_{m,it}$, and $\bar{q}_i^{(m,k)} = T_i^{-1} \sum_{t=1}^{T_i} (z_{m,it} x_{kt})$ represents the firm's time-series average of the interaction term between firm characteristic $z_{m,it}$ and factor variable x_{kt}. For ease of notation, we collect all the time-series averages in row vector $\overline{\boldsymbol{q}}_{\boldsymbol{i}}$ and store the $\xi_{m,k}$ coefficients in column vector $\boldsymbol{\xi}$.[14]

Replacing firm-specific effect c_i in GPS-model (1) by the expression in (17) we obtain the following regression model:

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \overline{\boldsymbol{q}}_{\boldsymbol{i}} \boldsymbol{\xi} + u_i + v_{it} \tag{18}$$

GPS-model (18) is structured as a correlated RE model (Mundlak, 1978). It is therefore well-known that estimating (18) with pooled OLS will yield the fixed effects (or "within") coefficient estimates for vector $\boldsymbol{\theta}$. Likewise, pooled OLS coefficient estimates for vector $\boldsymbol{\xi}$ quantify by how much the between estimator differs from the fixed effects estimator. Following Wooldridge (2010, p. 332), we can therefore test for

$$\text{H}_0\text{:} \;\; \boldsymbol{\xi} = 0 \qquad \text{vs.} \qquad \text{H}_1\text{:} \;\; \boldsymbol{\xi} \neq 0 \tag{19}$$

to obtain a regression-based variant of Hausman's (1978) specification test for GPS-model (1). To ensure robust statistical inference on our GPS-model specification test, we estimate regression (18) with Driscoll and Kraay (1998) standard errors that are robust to cross-sectional dependence, autocorrelation, and heteroskedasticity. If the null hypothesis of $\boldsymbol{\xi} = 0$ cannot be rejected, the random effects (RE) assumption is considered to hold. In this case, estimating GPS-model (1) with (weighted) pooled OLS

[14] Note that we assume that all characteristics $z_{m,it}$ vary both across firms as well as over time. If $z_{m,it}$ is time-constant, then GPS-model (1) can only be estimated with (weighted) pooled OLS and, hence, needs to rely on the RE assumption. This is a direct consequence of the fact, that the within estimator is unable to estimate the regression coefficients of time-invariant variables since time-invariant variables are absorbed by the firm-specific effect. For details, see Cameron and Trivedi (2005, Chapter 21).

allows for valid statistical inference. However, if the Wald test in (19) rejects the null hypothesis of $\boldsymbol{\xi} = 0$, then estimating the GPS-model with (weighted) pooled OLS is likely to produce inconsistent results. In the case of $\hat{\boldsymbol{\xi}} \neq 0$, GPS-model (1) should therefore be estimated with the fixed effects estimator to ensure valid statistical inference.

2.3.4 Testing the validity of conventional portfolio sorts

The results from Section 2.2 demonstrate that GPS-model (1) estimated with pooled OLS nests all variants of conventional portfolio sorts as a special case. With the pooled OLS estimator only being consistent under the random effects (RE) assumption, this therefore implies that the portfolio sorts approach also depends on the RE assumption to hold. Put differently, if firm-specific effects are present and correlated with the characteristic underlying the portfolio sort, the results from conventional portfolio sorts may inadvertently misattribute a fraction of the alpha to the firm characteristic underlying the portfolio sort.

We now utilize our results from Section 2.3.3 to form a Hausman (1978) type test for analyzing if the results from conventional portfolio sorts are statistically valid. For this purpose, we rely on GPS-model (13) where the performance of a certain base portfolio is compared with the performance of each other portfolio in the sort. Correspondingly, we specify $\boldsymbol{x}_t = [\,1 \;\; x_{1t} \;\dots\; x_{Kt}\,]$ and $\boldsymbol{z}_{it} = [\,1 \;\; z_{it}^{(2)} \;\dots\; z_{it}^{(P)}]$. Modelling the firm-specific effects as in (17) and storing the variables containing the firm-specific time-series averages in vector $\overline{\boldsymbol{q}}_i$, we estimate regression (18) with (weighted) pooled OLS and perform the Wald test in (19). If the null hypothesis of $\boldsymbol{\xi} = 0$ cannot be rejected, the random effects (RE) assumption underlying the sorted portfolios is likely to hold and, hence, the statistical results from the portfolio sorts approach can be considered valid. However, if null hypothesis $\boldsymbol{\xi} = 0$ is rejected, the portfolio sorts approach is likely to suffer from an omitted variable bias and, hence, is at risk of producing biased statistical results. In this case, estimation of a GPS-model with firm fixed effects should be preferred to performing a portfolio sorts analysis.

Note that the "portfolio sorts specification test" proposed here provides a means to address the factor-zoo-issue raised by Cochrane (2011). As such, our specification test allows to distinguish between valid factors (where the RE assumption underlying the sorted portfolios is likely to hold) and invalid factors (which are vulnerable to an omitted variable bias). Extrapolating from our empirical results in Section 3, we consider it likely that many results from prior research applying the portfolio sorts approach rest on a weak statistical foundation.

3. Empirical Analysis using the GPS-model

In this section, we illustrate the importance of accounting for firm fixed effects in empirical asset pricing tests. To this end, we study the return predictability of four randomly chosen firm characteristics that are widely used in recent asset pricing studies: gross profitability (Novy-Marx, 2013), operating profitability (Ball, Gerakos, Linnainmaa, and Nikolaev, 2015), the stocks' 52-week rolling volatility (Ang, Hodrick, Xing, Zhang, 2006, 2009; Baker, Bradley, and Wurgler, 2011), and 52-week rolling beta (Baker, Bradley, and Wurgler, 2011; Frazzini and Pedersen, 2014).

Novy-Marx (2013) shows that gross profit scaled by the book value of total assets, henceforth referred to as "gross profitability", is a better predictor for the cross-section of average stock returns than alternative measures that are based on bottom line net income, cash flows, or dividends. He argues that the good performance of gross profitability in predicting the cross-section of average stock returns is mainly due to its numerator, gross profit, being a cleaner measure of economic profitability than, say, net income. Ball, Gerakos, Linnainmaa, and Nikolaev (2015), henceforth abbreviated as BGLN, challenge the findings of Novy-Marx (2013). Their critique centers on the observation that Novy-Marx (2013) deflates net income by the book value of equity while deflating gross profit by the book value of total assets. BGLN demonstrate that the predictive power of net income and gross profit is comparable if the same deflator is used. Furthermore, they suggest an alternative profitability measure, operating profitability, which more closely relates current expenses to current revenues. Defining operating profitability as gross profit minus selling, general, and administrative expenses (excluding R&D expenditures) deflated by the book value of total assets, BGLN find this profitability measure to predict the cross-section of average stock returns even better than gross profitability.

Which characteristic is a better predictor for asset returns, gross profitability or operating profitability? – We contribute to this debate by utilizing several variants of GPS-model (1). After an empirical validation of Propositions 1 to 3, we show that the inclusion of firm fixed effects in the analysis may significantly impact on the result. We also perform Hausman (1978) type specification tests for analyzing whether the results from the portfolio sorts approach, which assumes the random effects assumption to hold, allow for valid statistical inference. Next, we demonstrate the flexibility of GPS-model (1) in handling multivariate and continuous firm characteristics. At this point, we introduce the other two characteristics-based factors, the stocks' 52-week rolling volatility (Ang, Hodrick, Xing, Zhang, 2006, 2009; Baker, Bradley, and Wurgler, 2011) and 52-week rolling beta (Baker, Bradley, and Wurgler, 2011; Frazzini and Pedersen, 2014). Finally, we conduct a horse race to examine which firm characteristics withstand a multivariate test and are robust to the inclusion of firm fixed effects. Our empirical analysis relies on the CRSP-Compustat merged database and spans the sample period from July 1963 through December 2016. We prepare the sample data as described in BGLN.

3.1 The performance of a single portfolio

To validate Proposition 1 empirically, we start with the portfolio sorts approach. Each year at the end of June, we sort the stocks into quintiles based on NYSE breakpoints and hold the portfolios for the subsequent year. For each portfolio p ($p = 1, \ldots, 5$), we then compute monthly value-weighted portfolio excess returns (r_{pt}) as

$$r_{pt} = \sum_{i=1}^{N_t} w_{it}^{(p)}\, OA_{it}^{(p)}\, r_{it} \tag{20}$$

where $w_{it}^{(p)}$ refers to the beginning-of-month t portfolio weight of stock i in quintile portfolio p, r_{it} denotes stock i's month t excess return, $OA_{it}^{(p)}$ is a dummy variable with value one if stock i in month t belongs to operating profitability portfolio p, and N_t refers to the overall month t number of stocks in the sample.

Using r_{pt} from (20) as the dependent variable, we then estimate the Fama and French (1993) three-factor model as follows:

$$r_{pt} = \alpha + \beta_{RMRF}\, RMRF_t \; + \; \beta_{SMB}\, SMB_t \; + \; \beta_{HML}\, HML_t \; + \varepsilon_t \tag{21}$$

where $RMRF_t$ is the market excess return, SMB_t denotes the return of a zero-investment size portfolio, and HML_t refers to the return of a zero-investment book-to-market portfolio.

According to Proposition 1, the results from estimating time-series regression (21) for portfolio p can be reproduced by aid of GPS-model (1) being specified as follows:

$$r_{it} = \theta_\alpha \; + \; \theta_{RMRF}\, RMRF_t \; + \; \theta_{SMB}\, SMB_t \; + \; \theta_{HML}\, HML_t \; + \; \nu_{it} \tag{22}$$

Due to the focus on a single portfolio p, GPS-model (22) only considers the subset of observations for which $OA_{it}^{(p)} = 1$ (i.e., observations on stocks that are included in operating profitability portfolio p). To reproduce the results of time-series regression (21) with GPS-model (22), we need to account for two important aspects. First, the portfolio sorts approach typically considers value-weighted rather than equal weighted portfolios. Second, the CRSP-Compustat database constitutes an unbalanced panel with time-varying cross-sections. Therefore, GPS-model (22) needs to be estimated with *weighted* pooled OLS, where observation weights are fixed such that they match the (implicit) weighting scheme underlying portfolio p's value-weighted return from expression (20). Consequently, we set the weight of observation it equal to the beginning-of-month t value weight of stock i in operating profitability portfolio p:

$$w_{it} = \frac{OA_{it}^{(p)} ME_{it}}{\sum_{i=1}^{N_t} \left(OA_{it}^{(p)} ME_{it} \right)} \tag{23}$$

where ME_{it} refers to stock i's beginning-of-month t market value of equity.

The results for the top-quintile portfolio sorted on operating profitability are reported in Table 1. The first row ("Portfolio sorts approach") reports the results from estimating portfolio-level time-series regression (21) with OLS. Statistical inference is based on Newey and West (1987) standard errors with a lag-length of three months. In line with BGLN, the top quintile portfolio has a statistically significantly positive Fama and French (1993) alpha of +0.225% per month, or +2.7% per year. In the second row, labelled as "GPS-model (weighted pooled OLS)", we report the results from estimating GPS-model (22) with weighted pooled OLS, where observation weights are set equal to the stocks' beginning-of-month t value-weights. Statistical inference relies on Driscoll and Kraay (1998) standard errors with a lag-length of three months. The results from estimating GPS-model (22) are identical with those from estimating time-series regression (21) reported in the first row.[15] This empirically validates our Proposition 1 from Section 2.2.2.

The third row, labelled as "GPS-model (standard pooled OLS)", estimates GPS-model (22) with standard pooled OLS, where all stocks are equally weighted such that microcaps receive the same weight as large caps. As a consequence, the SMB factor loading in this case is large (+0.79) and statistically significant. Moreover, the change in weights also affects the risk-adjusted performance (or alpha) which increases to +0.313% per month (or +3.75% per year). This confirms Fama and French's (2008) concern that the abundance of small- and microcap stocks can be influential for the results when observations on micro- and megacap stocks are equally-weighted.

3.2 Analyzing portfolio sorts with the GPS-model

According to Proposition 2, GPS-model (1) can be specified such that it reproduces the results of multiple sorted portfolios with a single regression on the firm-level. To validate Proposition 2 empirically, we apply the portfolio sorts procedure from Section 3.1 to each of the five quintile portfolios sorted on operating profitability. The results are reported in Panel A of Table 2. In line with BGLN, the alpha of the sorted portfolios monotonically increases with operating profitability. While the low profitability

[15] Note that the number of observations included in the estimation differs across models. The time-series regression of the portfolio sorts approach comprises 642 monthly observations whereas the weighted pooled OLS estimation of the firm-level GPS-model includes 412,443 firm-month observations.

($p = 1$) portfolio has a significantly negative alpha of -0.318% per month, the top-quintile ($p = 5$) portfolio shows a significantly positive alpha of +0.225% per month.

To reproduce the results of the portfolio sorts analysis with the GPS-model, we estimate regression (7) with weighted pooled OLS, where the weight of observation it is set equal to stock i's period t value-weight in the operating profitability portfolio it belongs to. Panel B of Table 2 reports the results in a two-dimensional matrix. The portfolio dummy variables in vector $\boldsymbol{z}_{it} = \left[OA_{it}^{(1)} \quad OA_{it}^{(2)} \quad \cdots \quad OA_{it}^{(5)} \right]$ define the columns whereas the factor variables in vector $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t]$ identify the rows. All elements in the results matrix thus represent the coefficient estimates (and t-statistics) of the interaction term between firm-characteristic $OA_{it}^{(p)}$ and factor variable (or constant) x_{kt}. A comparison of the results in Panels A and B of Table 2 shows that both the coefficient estimates and t-statistics (based on Driscoll-Kraay standard errors) of the GPS-model coincide with those from applying the portfolio sorts approach (independently) to the five quintile portfolios sorted on operating profitability. This empirically confirms the theoretical result stated in Proposition 2 from Section 2.2.3.

The GPS-model can also be specified such that it reproduces the results from "two-way portfolio sorts" by aid of a single regression on the firm-level. To illustrate this, we partially reproduce the analysis of Table 8 in BGLN. Relying on NYSE breakpoints, we sort the stocks into quintiles based on operating profitability and market capitalization.[16] As before, we form the portfolios by the end of each June and then hold them for the subsequent year. Panel A of Table 3 reports the results for the two-way sorted portfolios. For brevity, we only display the coefficient estimates (and t-values) of the alpha from estimating time-series regression (21) for each of the 25 two-way sorted portfolios. Despite a slightly different sample period, the coefficient estimates and t-values match closely with those in BGLN.

Next, we turn to GPS-model (1) and show how to replicate the results of all the two-way sorted portfolios by estimating a single regression on the firm-level. To this end, we specify $\boldsymbol{z}_{it}$ as follows:

$$\boldsymbol{z}_{it} = \left[\left[OA_{it}^{(1)} \ \ldots \ OA_{it}^{(5)} \right] \otimes \left[ME_{it}^{(1)} \ \ldots \ ME_{it}^{(5)} \right] \right]$$

Here, $OA_{it}^{(p)}$ ($p = 1, \ldots, 5$) refers to a dummy variable with value one if stock i in month t belongs to operating profitability portfolio p, and $ME_{it}^{(q)}$ ($q = 1, \ldots, 5$) is a dummy variable with value one if stock i in month t belongs to market capitalization quintile q. As before, vector $\boldsymbol{x}_t$ includes a constant and the three Fama-French factors and, hence, is specified as $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t]$. To reproduce

[16] Following BGLN, the sorts are carried out independently of each other. BGLN use the two-way portfolio sorts to investigate whether the predictive power of operating profitability for the cross-section of average returns is a market-wide phenomenon or if it is confined to certain size groups.

the results of the portfolio sorts approach, we estimate the GPS-model with weighted pooled OLS, where observations weights are fixed such that they match the value-weights of the stocks in the two-way sorted portfolios.

Panel B of Table 3 reports the results for the regression coefficients decomposing the alpha, i.e., for the coefficient estimates (and t-statistics) of the variables in vector $\boldsymbol{z}_{it}$.[17] The results are reported in a two-dimensional matrix with the operating profitability quintile dummies, $OA_{it}^{(p)}$, identifying the rows and the market capitalization quintile dummies, $ME_{it}^{(q)}$, defining the columns. Element (p, q) in the result matrix thus represents the coefficient estimate (and t-statistic) of the interaction term between operating profitability quintile dummy p and market cap quintile dummy q. When comparing the results from estimating the GPS-model (Panel B) with the results from the two-way sorted portfolios (Panel A), it is evident that they perfectly match. This provides further empirical evidence for Proposition 2.

3.3 Top versus bottom portfolio performance

To validate Proposition 3 empirically, we start by applying the portfolio sorts approach for evaluating the performance difference between the top ($p = 5$) and bottom ($p = 1$) quintile portfolios of stocks sorted on operating profitability. To this end, we first compute the month t excess returns (r_{pt}) for the top and bottom quintile portfolios as outlined in expression (20) above. We then evaluate the performance difference between the two portfolios by estimating portfolio-level time-series regression (21) with $\Delta r_{pt} = r_{5,t} - r_{1,t}$ as the dependent variable. Panel A of Table 4 reports the results. On a risk-adjusted basis, the portfolio of stocks with high operating profitability outperforms the low profitability portfolio by a significant +0.54% per month (or +6.48% per year). Despite a slightly different sample period, the coefficient estimates and t-values are very similar to those reported in Table 6 of BGLN.

To replicate the results from the portfolio sorts approach with a regression on the individual firm-level, we estimate GPS-model (12) with weighted pooled OLS. In this analysis, we only consider observations on stocks with top ($p = 5$) or bottom ($p = 1$) quintile operating profitability and set observation weights equal to the stocks' value-weights in their operating profitability portfolio. Panel B of Table 4 displays the results in a two-dimensional matrix. The elements of firm characteristics' vector $\boldsymbol{z}_{it} = \left[1 \quad OA_{it}^{(5)}\right]$ define the columns while the factor variables in $\boldsymbol{x}_t = [1 \quad RMRF_t \quad SMB_t \quad HML_t]$ identify the rows. All elements in the results matrix thus represent the coefficient estimates (and t-statistics based on Driscoll-Kraay standard errors) for the interactions between the elements in vector $\boldsymbol{z}_{it}$

[17] Note that the characteristics in vector $\boldsymbol{z}_{it}$ in fact represent the interaction terms of the variables in $\boldsymbol{z}_{it}$ with the constant in $\boldsymbol{x}_t$.

and those in vector $\boldsymbol{x}_t$. The results from estimating GPS-model (12) with weighted pooled OLS (Panel B) coincide with the results of the portfolio sorts approach in Panel A. This empirically confirms Proposition 3.

3.4 Do firm fixed effects matter for the prediction of stock returns?

The analysis in Panel A of Table 4, and by extension in Panel B of Table 4, depends on the assumption that firm-specific effects (if present) are uncorrelated with the explanatory variables in $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$, i.e., that the random effects (RE) assumption holds. In Panel C of Table 4, we reproduce the analysis from Panel B, but estimate the GPS-models with the fixed effects (or "within") estimator that is robust to violations of the RE assumption.

Most important, we find that operating profitability no longer statistically significantly predicts (risk-adjusted) stock returns when firm fixed effects are accounted for. Hence, the results in Panel C suggest that operating profitability is correlated with the firm fixed effects resulting in a significant fraction of the alpha reported in Panels A and B being misattributed to operating profitability. In other words, not operating profitability has predictive power, but a time-invariant component of it that is absorbed by the firm fixed effects. As a consequence, operating profitability is expected to have out-of-sample predictive power only if its correlation with the unobservable yet time-persistent factor persists beyond our sample period, such that operating profitability remains correlated with the fixed effects that drive the results in Panels A and B. In summary, the results in Table 4 suggest that firm fixed effects can have a major impact on the results from empirical asset pricing tests.

3.5 Using the GPS-model to test the validity of conventional portfolio sorts

As shown above, GPS-model (1), estimated with (weighted) pooled OLS, is capable to reproduce the results from conventional portfolio sorts. This, however, implies that the results from conventional portfolio sorts are valid if and only if the random effects assumption holds. We now investigate whether firm-specific effects (if present) indeed are uncorrelated with the characteristic(s) underlying the portfolio formation. To examine whether the results from conventional portfolio sorts are valid, we rely on the portfolio sorts specification test developed in Section 2.3.4.

We begin by analyzing whether portfolio sorts on operating profitability are in accordance with the random effects assumption. To this end, we specify GPS-model (1) with $\boldsymbol{z}_{it} = [\,1 \quad OA_{it}^{(2)} \;\; \ldots \;\; OA_{it}^{(5)}\,]$ and $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t]$. We then add to the regression the firm-specific time-series averages (which we store in vector $\overline{\boldsymbol{q}}_i$) of all variables in $(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)$ which vary across both firms and time.

The resulting regression model $r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \overline{\boldsymbol{q}}_i\boldsymbol{\xi} + \varepsilon_{it}$ constitutes our first specification of regression (18), which forms the basis of our portfolio sorts specification test. Panel A of Table 5 reports the results from estimating the respective regression with pooled OLS, where observation weights are set equal to the beginning-of-month t value-weights of the stocks in the quintile portfolios sorted on operating profitability. The results are displayed in matrix form. The first block of results, labelled as "Coefficient estimates on explanatory variables $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$", contains the estimation results for vector $\boldsymbol{\theta}$. With the portfolio dummies from $\boldsymbol{z}_{it}$ defining the columns and the factor variables in $\boldsymbol{x}_t$ defining the rows, element (p, k) in the results matrix represents the coefficient estimates (and *t*-statistics) for the interaction term between firm characteristic (or constant) $z_{p,it}$ and factor variable (or constant) x_{kt}. Hence, the results in the first column are for the bottom-quintile profitability portfolio while the remaining columns show by how much the results of profitability quintile portfolio p $(p = 2, 3, 4, 5)$ differ from those of the bottom-quintile portfolio. As explained in Section 2.3, the coefficient estimates for $\boldsymbol{\theta}$ represent fixed effects (FE) estimates that allow for valid statistical inference even if firm-specific effects are correlated with the variables in $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$. The results in row "1 (Intercept)" show that when controlling for firm fixed effects, the Fama-French three-factor model alpha of the top-quintile portfolio sorted on operating profitability is no longer statistically significantly different from that of the bottom-quintile profitability portfolio. The difference between the two portfolios' alpha now amounts to only +0.018% per month, which is consistent with results in Panel C of Table 4, but stands in stark contrast to the highly significant +0.542% per month alpha-difference obtained in a conventional portfolio sorts analysis which ignores firm fixed effects (see Panels A and B of Table 4).

The second block of Panel A in Table 5, labelled as "Coefficient estimates on time-series averages", contains the estimation results for vector $\boldsymbol{\xi}$. The structure of the estimation results in this block is similar to that for $\boldsymbol{\theta}$ discussed before. The coefficient estimates for the elements in $\boldsymbol{\xi}$ quantify by how much the between (BE) estimator differs from the fixed effects (FE) estimator. The results show that the difference between BE and FE coefficient estimates is particularly pronounced for the coefficient estimates quantifying the difference of quintile portfolio p's alpha $(p = 2,3,4,5)$ versus the bottom-quintile portfolio. For instance, the BE estimate for the alpha-difference between the top- and bottom-quintile portfolios deviates by a significant +1.099% per month from the respective FE estimate. As a consequence, the judgment about whether operating profitability is a good predictor of asset returns critically hinges on the random effects (RE) assumption to hold. To test for the validity of the RE assumption, we perform the Wald test in (19). This "portfolio sorts specification test" has an F-statistic of 4.612 and rejects the null hypothesis of $\boldsymbol{\xi} = 0$ at all conventional levels of statistical significance. For portfolio sorts on operating profitability the random effects assumption can therefore not be considered to hold, invalidating the results from conventional portfolio sorts.

Panel B of Table 5 reproduces the analysis from Panel A for portfolio sorts on gross profitability, defined as gross profit divided by total assets (Novy-Marx, 2013). We specify $\boldsymbol{z}_{it} = [\,1 \quad GA_{it}^{(2)} \;\dots\; GA_{it}^{(5)}\,]$, where $GA_{it}^{(p)}$ represents a dummy variable with value one if stock i in month t belongs to quintile portfolio p sorted on gross profitability. The results are quite different from those on operating profitability. Specifically, the coefficient estimate measuring by how much the alpha of the top-quintile portfolio differs from that of the bottom-quintile portfolio of +0.42% per month is positive and statistically significant. Furthermore, the differences between BE and FE coefficient estimates (i.e., the coefficient estimates for $\boldsymbol{\xi}$) are less pronounced than in Panel A when sorting on operating profitability. Correspondingly, the "portfolio sorts specification test" no longer rejects the null hypothesis of $\boldsymbol{\xi} = 0$. The respective Wald test has an F-statistic of 1.194 which corresponds to a p-value of 0.267. We therefore conclude that the statistical results from portfolio sorts on gross profitability are reliable. All else equal, stocks with high gross profitability outperform stocks with low gross profitability.

3.6 Firm fixed effects and continuous and multivariate firm characteristics

We now demonstrate that the GPS-model provides a versatile framework for investigating the predictability of stock returns as well as for analyzing the role of firm fixed effects. To determine which profitability and volatility measures offer successful return predictability, we estimate (subsets of) the following GPS-model:

$$\begin{aligned} r_{it} &= (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + e_{it} \\ &= ([1 \quad OA_{it} \quad GA_{it} \quad Vola_{it} \quad Beta_{it}] \otimes [\,1 \quad RMRF_t \quad SMB_t \quad HML_t])\,\boldsymbol{\theta} + e_{it} \end{aligned} \tag{24}$$

where OA_{it} is operating profitability, GA_{it} measures gross profitability, $Vola_{it}$ is the standard deviation of 52 weekly stock returns ending on the last Friday prior to the end of month t, and $Beta_{it}$ is the CAPM-beta of weekly stock returns estimated over the 52 weeks ending on the last Friday prior to the end of month t. The variables in vector $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t]$ represent a constant and the three Fama and French (1993) factors.

Panel A of Table 6 presents the results from estimating GPS-model (24) with weighted pooled OLS, where observation weights are set equal to the firms' month t value weights. For brevity, only the estimates for the regression coefficients decomposing the Fama-French three-factor model alpha (as a measure of the risk-adjusted performance) are reported.[18] In column (1), we estimate regression (24) with $\boldsymbol{z}_{it} = [1 \quad OA_{it}]$. The results show that alpha increases with operating profitability. The coefficient

[18] Note that conditional on firm i's characteristics in period t, the Fama-French three-factor model alpha is obtained as $\hat{\alpha}_{it} = \hat{\theta}_0 + \hat{\theta}_1 \times OPAT_{it} + \hat{\theta}_2 \times GPAT_{it} + \hat{\theta}_3 \times Vola_{it} + \hat{\theta}_4 \times Beta_{it}$. For details, see Section 2.3.1.

estimate is positive (+1.743) and highly statistically significant. A qualitatively similar result is obtained when estimating GPS-model (24) with $\boldsymbol{z}_{it} = [1 \;\; GA_{it}]$ (column 2). By contrast, the alpha decreases with volatility (column 3) and the stock beta (column 4). In sum, these univariate results are perfectly in line with the findings from previous research. Columns (5) and (6) take advantage of the GPS-model's capability to handle multivariate firm characteristics. When estimating GPS-model (24) with $\boldsymbol{z}_{it} = [1 \quad OA_{it} \quad GA_{it} \quad Vola_{it} \quad Beta_{it}]$, we find that only operating profitability (coefficient estimate: +1.105) and the stock beta (coefficient estimate: -0.303) statistically significantly predict stock returns while the coefficient estimates on gross profitability and volatility are insignificant.

The analysis in Panel A of Table 6 depends on the assumption that firm-specific effects (if present) are uncorrelated with the explanatory variables in $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$, i.e., that the random effects (RE) assumption holds. We now employ our GPS-model specification test developed in Section 2.3.3 to investigate for which regression specifications in Panel A the random effects assumption may be considered to hold. For this purpose, we extend regression (24) with the firm-level time-series averages of all variables that vary cross-sectionally and over time. We then employ a Wald test to examine whether the coefficient estimates for the time-series average variables are jointly equal to zero. If this null hypothesis cannot be rejected, the RE assumption can be assumed to hold and GPS-model (24) can be estimated with (weighted) pooled OLS. The results from the GPS-model specification test are reported in Panel B of Table 6. With the exception of the GPS-model specification in column (2), where vector $\boldsymbol{z}_{it}$ is specified as $\boldsymbol{z}_{it} = [1 \;\; GA_{it}]$, the GPS-model specification test rejects the RE assumption for all specifications in Panel A on the 5% confidence level or better. As a result, the statistical results from estimating GPS-model (24) with pooled OLS are likely to be invalid.

In Panel C of Table 6, we reproduce the analysis from Panel A, but this time we estimate the GPS-models with the fixed effects (or "within") estimator allowing firm-specific effects to be correlated with the explanatory variables in $(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)$. For some of the specifications, we observe considerable differences between the results in Panel C and those in Panel A. Specifically, in column (1), we find that operating profitability no longer statistically significantly predicts (risk-adjusted) stock returns when firm fixed effects are accounted for. Furthermore, in the multivariate analysis of column (6) which specifies vector $\boldsymbol{z}_{it}$ as $\boldsymbol{z}_{it} = [1 \quad OA_{it} \quad GA_{it} \quad Vola_{it} \quad Beta_{it}]$, the coefficient estimate of operating profitability changes sign and assumes a significantly negative value of -1.583. This result on operating profitability stands in contrast to the findings for gross profitability, whose coefficient estimate remains positive and statistically significant even when estimating regression (24) with firm fixed effects. These findings suggest that gross profitability is a more robust predictor for stock returns than operating profitability. When comparing the return predictability of volatility with that of the stock beta, we find the stock beta to be a remarkably robust predictor for (risk-adjusted) stock returns. Irrespective of the regression specification in Panel C of Table 6, the coefficient estimate for the stock beta is around -0.40, and statistically significant at the 1% level. In contrast, when accounting for firm fixed effects, volatility no longer has predictive power for the Fama-French three-factor model alpha. We therefore conclude that the 52-week rolling stock beta is a more robust predictor for stock returns than 52-week rolling volatility. More generally, we conclude that firm fixed effects can have a major impact on the results from empirical asset pricing tests.

4. Conclusion

In this paper, we propose a novel, regression-based methodology for analyzing asset returns. Our "GPS-model" relies on estimating a linear panel regression on the individual firm level, and to draw statistical inference based on Driscoll and Kraay (1998) standard errors that are robust to heteroskedasticity as well as cross-sectional and temporal dependence. Our technique easily handles continuous and multivariate firm characteristics, and it allows for the inclusion of firm fixed effects. Using formal econometric analysis, we show that our approach nests all variants of the widely-applied portfolio sorts approach, and we prove that statistical results from the portfolio sorts approach are valid if and only if the random effects (RE) assumption holds. This is a direct consequence of the fact, that to exactly reproduce the results from portfolio sorts, our GPS-model needs to be estimated with pooled OLS (which is known to depend on the RE assumption). Using our methodology, we also develop a Hausman (1978) type specification test that allows us to analyze whether the results from portfolio sorts are likely to be affected by unobserved heterogeneity across firms.

In the empirical part of the paper, we examine the relevance of accounting for firm fixed effects in empirical asset pricing tests. We do so by considering four randomly chosen characteristics that are well-known to predict asset returns: operating profitability, gross profitability, volatility, and beta. Our empirical results reveal that two out of the four tested characteristics-based factors do not withstand tests accounting for firm fixed effects, operating profitability and low volatility. We therefore conclude that the random effects assumption (implicitly) underlying the portfolio sorts approach indeed is often unlikely to hold. Specifically, our results suggest that operating profitability and low volatility are correlated with the firm fixed effects resulting in a significant fraction of the predicted alpha being misattributed to these two characteristics-based factors. In other words, not operating profitability and low volatility have predictive power, but a time-invariant component of these two variables that is absorbed by firm fixed effects. As a consequence, operating profitability and low volatility are expected to have out-of-sample predictive power only if their correlation pattern with an unobservable yet persistent return predictor remains unchanged beyond our sample period. In summary, our results suggest that firm-specific (fixed) effects can have a major impact on the results from empirical asset pricing tests. The GPS-model proposed in this paper addresses and resolves this issue. Relying on a simple, yet econometrically robust framework, the GPS-model can easily be estimated with the fixed effects (or "within") estimator, thereby ensuring valid statistical inference even in case of the random effects assumption being violated.

References

Allison, P. D. 2009. *Fixed Effects Regression Models*, Sage.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The cross-section of volatility and expected returns, *Journal of Finance* 61, 259-299.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2009, High idiosyncratic volatility and low returns: International and further US evidence, *Journal of Financial Economics* 91, 1-23.

Baker, Malcolm, Brendan Bradley, and Jeffrey Wurgler, 2011, Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly, *Financial Analysts Journal* 67, 40-54.

Bali, Turan G., Stephen J. Brown, and Yi Tang, 2017, Is economic uncertainty priced in the cross-section of stock returns?, *Journal of Financial Economics* 126, 471-489.

Ball, Ray, Joseph Gerakos, Juhani T. Linnainmaa, and Valeri V. Nikolaev, 2015, Deflating profitability, *Journal of Financial Economics* 117, 225-248.

Barber, Brad M., and Terrance Odean, 2000, Trading is hazardous to your wealth: The common stock investment performance of individual investors, *Journal of Finance* 55, 773-806.

Barber, Brad M., and Terrance Odean, 2001, Boys will be boys: Gender, overconfidence, and common stock investment, *Quarterly Journal of Economics* 116, 261-292.

Blyth, Colin R., 1972, On Simpson's Paradox and the Sure-Thing Principle, *Journal of the American Statistical Association* 67, 364-366.

Bryzgalova, Svetlana, 2016, Spurious factors in linear asset pricing models, Working Paper, Stanford University.

Cameron, A. Colin, and Pravin K. Trivedi, 2005, *Microeconometrics: methods and applications*, Cambridge University Press.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57-82.

Chordia, Tarun, Amit Goyal, and Alessio Saretto, 2017, p-hacking: Evidence from two million trading strategies, Working paper, Emory University.

Cochrane, John H., 2011, Presidential address: Discount rates, *Journal of Finance* 66, 1047-1108.

Driscoll, John C., and Aart C. Kraay, 1998, Consistent covariance matrix estimation with spatially dependent panel data, *Review of Economics and Statistics* 80, 549-560.

Fama, Eugene F., 1998, Market efficiency, long-term returns, and behavioral finance, *Journal of Financial Economics* 49, 283-306.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns of stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Fama, Eugene F., and Kenneth R. French, 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55-84.

Fama, Eugene F., and Kenneth R. French, 2008, Dissecting anomalies, *Journal of Finance* 63, 1653-1678.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1-22.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607-636.

Frazzini, Andrea, and Lasse H. Pedersen, 2014, Betting against beta, *Journal of Financial Economics* 111, 1-25.

Fung, William, Hsieh, David A., Naik, Narayan Y., and Tarun Ramadorai, 2008, Hedge funds: Performance, risk, and capital formation, *Journal of Finance* 63, 1777-1803.

Gerakos, Joseph, and Juhani Linnainmaa, 2017, Decomposing value, *Review of Financial Studies*, forthcoming.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121-1152.

Harvey, Campbell R., 2017, Presidential Address: The Scientific Outlook in Financial Economics, *Journal of Finance* 72, 1399-1440.

Harvey, Campbell R., and Yan Liu, 2017, Lucky factors, Working paper, Duke University.

Harvey, Campbell R., Yan Liu, and Heqing Zhu, 2016, …and the cross-section of expected returns. *Review of Financial Studies* 29, 5-68.

Hausman, Jerry A., 1978, Specification tests in econometrics. *Econometrica* 46, 1251-1271.

Hoechle, Daniel, 2007, Robust standard errors for panel regressions with cross-sectional dependence. *Stata Journal* 7, 281-312.

Hou, Kewei, Chen Xue, and Lu Zhang, 2017, Replicating anomalies, Working Paper, Ohio State University.

Ivkovic, Zoran, Clemens Sialm, and Scott Weisbenner, 2008, Portfolio concentration and the performance of individual investors, *Journal of Financial and Quantitative Analysis* 43, 613-656.

Jaffe, Jeffrey F., 1974, Special information and insider trading, *Journal of Business* 47, 410-428.

Jeng, Leslie A., Andrew Metrick, and Richard Zeckhauser, 2003, Estimating the returns to insider trading: A performance-evaluation perspective, *Review of Economics and Statistics* 85, 453-471.

Jensen, Michael C., 1968, The performance of mutual funds in the period 1945-1964, *Journal of Finance* 23, 389-416.

Kacperczyk, Marcin, Sialm, Clemens, and Lu Zheng, 2008, Unobserved actions of mutual funds, *Review of Financial Studies* 21, 2379-2416.

Kamstra, Mark J., 2017, Momentum, Reversals, and other puzzles in Fama-MacBeth cross-sectional regressions, Working Paper, York University.

Korniotis, George M., and Alok Kumar, 2013, Do portfolio distortions reflect superior information or psychological biases?, *Journal of Financial and Quantitative Analysis* 48, 1-45.

Kothari, S.P., and Jerold B. Warner, 2008, Econometrics of event studies, in: Eckbo, B. Espen (ed.), *Handbook of Corporate Finance: Empirical Corporate Finance, Vol. 1*, Elsevier/North-Holland, 3-36.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2017, Shrinking the Cross Section, Working Paper, University of Michigan.

Lo, Andrew W., and A. Craig MacKinlay, 1990, Data-snooping biases in tests of financial asset pricing models, *Review of Financial Studies* 3, 431-467.

Loughran, Tim, and Jay R. Ritter, 2000, Uniformly least powerful tests of market efficiency, *Journal of Financial Economics* 55, 361-389.

Lyon, John D., Brad M. Barber, and Chih-Ling Tsai, 1999, Improved methods for tests of long-run abnormal stock returns, *Journal of Finance* 54, 165-201.

Mandelker, Gershon, 1974, Risk and return: The case of merging firms, *Journal of Financial Economics* 1, 303-335.

Mundlak, Yair, 1978, On the pooling of time series and cross section data. *Econometrica* 46, 69-85.

Neuhaus, J. M., and J. D. Kalbfleisch. 1998. Between- and within-cluster covariate effects in the analysis of clustered data. *Biometrics* 54, 638-645.

Newey, Whitney K., and Kenneth D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703-708.

Novy-Marx, Robert, 2013, The other side of value: the gross profitability premium, *Journal of Financial Economics* 108, 1-28.

Patton, Andrew J., and Allan Timmermann, 2010, Monotonicity in asset returns: New tests with applications to the term structure, the CAPM, and portfolio sorts, *Journal of Financial Economics* 98, 605-625.

Petersen, Mitchell A., 2009, Estimating standard errors in financial panel data sets: Comparing approaches, *Review of Financial Studies* 22, 435-480.

Pukthuanthong, Kuntara, Richard Roll, and Avanidhar Subrahmanyam, 2017, A protocol for factor identification, Working Paper, UCLA.

Seasholes, Mark S., and Ning Zhu, 2010, Individual investors and local bias, *Journal of Finance* 65, 1987-2011.

Simpson, Edward H., 1951, The interpretation of interaction in contingency tables, *Journal of the Royal Statistical Society, Series B* 13, 238-241.

Sydsaeter, Knut, Arne Strom, and Peter Berck, 2000, *Economists' Mathematical Manual* (Springer: Berlin and Heidelberg) 3rd edn.

Vogelsang, Timothy J., 2012, Heteroskedasticity, autocorrelation, and spatial correlation robust inference in linear panel models with fixed-effects, *Journal of Econometrics* 166, 303-319.

Table 1: Single portfolio analysis

	Operating profitability: Top-quintile portfolio							
	Average Return	Three-factor model				Three-factor model statistics		
		a	b_{RMRF}	b_{SMB}	b_{HML}	*R-squared*	*N Obs.*	*N Stocks.*
(1) Portfolio sorts approach	0.574***	0.225***	0.950***	-0.075***	-0.316***	0.949	642	
	(3.10)	(4.92)	(73.78)	(-4.44)	(-18.60)			
(2) GPS-model (weighted pooled OLS)	0.574***	0.225***	0.950***	-0.075***	-0.316***	0.261	412,443	6,831
	(3.10)	(4.92)	(73.78)	(-4.44)	(-18.60)			
(3) GPS-model (standard pooled OLS)	1.070***	0.313***	1.037***	0.790***	-0.028	0.150	412,443	6,831
	(3.92)	(4.73)	(50.53)	(13.08)	(-0.63)			

This table reports the average return as well as the 3-factor model alpha along with RMRF (market), SMB (small minus big), and HML (high minus low) factor loadings for portfolios sorted by operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. The portfolio sort is based on NYSE breakpoints at the end of each June and the portfolio is held for the subsequent year. The sample period goes from July 1963 through December 2016. All results are for the quintile 5 (high) portfolio comprising the stocks with top quintile operating profitability. Row (1) reports the results from a conventional portfolio sort where the portfolio's excess return is regressed on the three Fama and French (1993) factors. Rows (2) and (3) present the results from estimating GPS-model (22). In Row (2), the regression is estimated with weighted pooled OLS, where observation weights are set equal to the beginning-of-time *t* value-weights of the stocks. In Row (3), the regression is estimated with standard pooled OLS, where all observations are equally weighted. *t*-statistics from testing for significance against a value of zero are presented in parentheses. Statistical inference for the portfolio sorts approach in Row (1) is based on Newey and West (1987) standard errors with a lag-length of three. The GPS-models in Rows (2) and (3) are estimated with Driscoll and Kraay (1998) standard errors with a lag-length of three. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 2: Analysis of quintile portfolios sorted on operating profitability

Panel A: Conventional portfolio sorts

	Quintile Portfolio				
	1 (low)	2	3	4	5 (high)
a	-0.318***	-0.116*	0.038	0.046	0.225***
	(-4.26)	(-1.86)	(0.73)	(0.88)	(4.92)
b_{RMRF}	1.092***	0.957***	0.943***	1.009***	0.950***
	(48.10)	(48.78)	(58.71)	(63.60)	(73.78)
b_{SMB}	0.212***	0.061	-0.054*	-0.048*	-0.075***
	(7.65)	(1.37)	(-1.73)	(-1.66)	(-4.44)
b_{HML}	0.175***	0.256***	0.135***	0.047	-0.316***
	(4.24)	(5.80)	(5.27)	(1.61)	(-18.60)
R-squared	0.897	0.891	0.916	0.933	0.949
N Obs.	642	642	642	642	642

Panel B: GPS-model

Vector $\boldsymbol{z}_t$ → Vector $\boldsymbol{x}_t$ ↓	$OA_{it}^{(1)}$	$OA_{it}^{(2)}$	$OA_{it}^{(3)}$	$OA_{it}^{(4)}$	$OA_{it}^{(5)}$
1 (Intercept)	-0.318***	-0.116*	0.036	0.046	0.225***
	(-4.26)	(-1.86)	(0.73)	(0.88)	(4.92)
$RMRF_t$	1.092***	0.957***	0.943***	1.009***	0.950***
	(48.10)	(48.78)	(58.71)	(63.60)	(73.78)
SMB_t	0.212***	0.061	-0.054*	-0.048*	-0.075***
	(7.65)	(1.37)	(-1.73)	(-1.66)	(-4.44)
HML_t	0.175***	0.256***	0.135***	0.047	-0.316***
	(4.24)	(5.80)	(5.27)	(1.61)	(-18.60)
R-squared	0.230				
N Obs.	1,969,221				
N Stocks	16,244				

This table reports value-weighted 3-factor model alphas along with RMRF (market excess return), SMB (small minus big), and HML (high minus low) factor loadings for portfolios sorted by operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Panel A reports the results from conventional portfolio sorts where a portfolio's excess return is regressed on the three Fama and French (1993) factors. Panel B presents the results from estimating a single GPS-model with weighted pooled OLS, where observation weights are set equal to the beginning-of-time t value-weights of the stocks. Coefficient estimates and t-statistics (in parentheses) in Panel B (GPS-model) are for the interaction of the (market-level) variables and a constant contained in vector $\boldsymbol{x}_t$ (displayed on the vertical axis) and the quintile dummy variables $OA_{it}^{(q)}$ $(q = 1, \dots, 5)$ for operating profitability in vector $\boldsymbol{z}_{it}$ (displayed on the horizontal axis). The quintile portfolios (Panel A) and dummy variables (Panel B) are formed based on NYSE breakpoints at the end of each June and then remain unchanged throughout the subsequent year. The sample period goes from July 1963 through December 2016. Statistical inference on the portfolio sorts (Panel A) is based on Newey and West (1987) standard errors with a lag-length of three. The GPS-model in Panel B is estimated with Driscoll and Kraay (1998) standard errors with a lag-length of three. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 3: Two-way portfolio sorts on operating profitability and market capitalization

Panel A: Conventional portfolio sorts

Operating profitability	Market capitalization				
	Q1	Q2	Q3	Q4	Q5
Q1	-0.40***	-0.38***	-0.30***	-0.26**	-0.25**
	(-3.76)	(-4.10)	(-3.26)	(-2.57)	(-2.43)
Q2	-0.04	-0.05	-0.06	-0.05	-0.14*
	(-0.60)	(-0.69)	(-0.66)	(-0.67)	(-1.84)
Q3	0.08	0.08	-0.01	-0.02	0.06
	(1.20)	(1.09)	(-0.15)	(-0.21)	(1.08)
Q4	0.10*	-0.01	0.10	0.09	0.04
	(1.70)	(-0.04)	(1.39)	(1.21)	(0.70)
Q5	0.31***	0.25***	0.18***	0.28***	0.23***
	(3.72)	(3.50)	(2.59)	(3.43)	(4.28)

Panel B: GPS-model

Operating profitability	Market capitalization				
	$ME_{it}^{(1)}$	$ME_{it}^{(2)}$	$ME_{it}^{(3)}$	$ME_{it}^{(4)}$	$ME_{it}^{(5)}$
$OA_{it}^{(1)}$	-0.40***	-0.38***	-0.30***	-0.26**	-0.25**
	(-3.76)	(-4.10)	(-3.26)	(-2.57)	(-2.43)
$OA_{it}^{(2)}$	-0.04	-0.05	-0.06	-0.05	-0.14*
	(-0.60)	(-0.69)	(-0.66)	(-0.67)	(-1.84)
$OA_{it}^{(3)}$	0.08	0.08	-0.01	-0.02	0.06
	(1.20)	(1.09)	(-0.15)	(-0.21)	(1.08)
$OA_{it}^{(4)}$	0.10*	-0.01	0.10	0.09	0.04
	(1.70)	(-0.04)	(1.39)	(1.21)	(0.70)
$OA_{it}^{(5)}$	0.31***	0.25***	0.18***	0.28***	0.23***
	(3.72)	(3.50)	(2.59)	(3.43)	(4.28)
R-squared	0.207				
N Obs.	1,969,221				
N Stocks	16,244				

This table reports value-weighted three-factor model alphas and t-statistics (in parentheses) for portfolios sorted by market capitalization and operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Panel A reports the results from conventional portfolio sorts where a portfolio's excess return is regressed on the Fama and French (1993) market (RMRF), size (SMB), and value (HML) factors. Panel B presents the results from estimating a single GPS-model with weighted pooled OLS, where observation weights are set equal to the beginning-of-time t value-weights of the stocks. Coefficient estimates and t-statistics (in parentheses) for the GPS-model are for the product of the market capitalization quintile dummy variables and quintile dummy variables for operating profitability. The dummy variables (Panel B) and two-way sorted portfolios (Panel A) are formed based on NYSE breakpoints at the end of each June and remain unchanged throughout the subsequent year. The market capitalization and operating profitability sorts are independent of each other. The sample period is from July 1963 through December 2016. Statistical inference for the portfolio sorts approach (Panel A) is based on Newey and West (1987) standard errors with a lag-length of three. The GPS-model in Panel B is estimated with Driscoll and Kraay (1998) standard errors with a lag-length of three. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 4: Comparison of high vs. low quintile portfolios sorted on operating profitability

Panel A: Conventional portfolio sorts

	Q1 (low)	Q5 - Q1
a	-0.318***	0.542***
	(-4.26)	(5.48)
b_{RMRF}	1.092***	-0.143***
	(48.10)	(-4.99)
b_{SMB}	0.212***	-0.287***
	(7.65)	(-8.19)
b_{HML}	0.175***	-0.491***
	(4.24)	(-10.35)
R-squared	0.897	0.335
N Obs.	642	642

Panel B: GPS-model (Pooled WLS estimation)

↓ Vector $\boldsymbol{x}_t$ / Vector $\boldsymbol{z}_{it}$ →	1	$OA_{it}^{(5)}$
1 (Intercept)	-0.318***	0.542***
	(-4.26)	(5.48)
$RMRF_t$	1.092***	-0.143***
	(48.10)	(-4.99)
SMB_t	0.212***	-0.287***
	(7.65)	(-8.19)
HML_t	0.175***	-0.491***
	(4.24)	(-10.35)
R-squared	0.224	
N Obs.	1,025,809	
N Stocks	14,705	

Panel C: GPS-model (weighted FE estimation)

↓ Vector $\boldsymbol{x}_t$ / Vector $\boldsymbol{z}_{it}$ →	1	$OA_{it}^{(5)}$
1 (Intercept)	-0.064	0.031
	(-0.66)	(0.19)
$RMRF_t$	1.091***	-0.142***
	(48.90)	(-5.03)
SMB_t	0.209***	-0.283***
	(7.69)	(-8.24)
HML_t	0.187***	-0.503***
	(4.86)	(-11.25)
within R-squared	0.225	
N Obs.	1,025,809	
N Stocks	14,705	

This table reports value-weighted 3-factor model alphas along with RMRF (market excess return), SMB (small minus big), and HML (high minus low) factor loadings for portfolios sorted on operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Panel A reports the results from conventional portfolio sorts where the bottom quintile portfolio's excess return (first column) or the return difference between the top quintile and the bottom quintile profitability portfolio (second column) is regressed on the three Fama and French (1993) factors. Statistical inference for the conventional portfolio sorts approach is based on Newey and West (1987) standard errors with a lag-length of three. Panel B reproduces the results from the conventional portfolio sorts by aid of GPS-model (12). The GPS-model is estimated with weighted pooled OLS, where observation weights are set equal to the beginning-of-time *t* value-weights of the stocks in the bottom- and the top-quintile profitability portfolio, respectively. Panel C presents the results from estimating GPS-model (12) with firm fixed effects (i.e., with the fixed effects estimator). Observation weights are set equal to those in Panel B. Coefficient estimates and *t*-statistics (in parentheses) for the GPS-models in Panels B and C are for the product of the market-level factor variables (plus a constant) contained in vector $\boldsymbol{x}_t$ (displayed on the vertical axis) and firm-characteristics vector $\boldsymbol{z}_{it}$. Vector $\boldsymbol{z}_{it}$ comprises a constant and dummy variable $OA_{it}^{(5)}$, which is one for stocks with top-quintile profitability (based on NYSE breakpoints at the end of each June) and zero otherwise. The GPS-models in Panels B and C only include firms, which belong to the top- or bottom-quintile groups of firms sorted on operating profitability. Statistical inference on the GPS-models is based on Driscoll and Kraay (1998) standard errors with a lag-length of three. The sample period is from July 1963 through December 2016. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 5: Portfolio sorts specification test

Panel A: Operating profitability

	Coefficient estimates on explanatory variables $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$ (coefficient vector $\boldsymbol{\theta}$)					Coefficient estimates on time-series averages (coefficient vector $\boldsymbol{\xi}$)			
	1	$OA_{it}^{(2)}$	$OA_{it}^{(3)}$	$OA_{it}^{(4)}$	$OA_{it}^{(5)}$	$OA_{it}^{(2)}$	$OA_{it}^{(3)}$	$OA_{it}^{(4)}$	$OA_{it}^{(5)}$
1 (Intercept)	-0.715***	0.034	0.055	-0.073	0.018	0.983***	0.712**	1.041***	1.099***
	(-4.79)	(0.33)	(0.53)	(-0.59)	(0.13)	(2.98)	(2.41)	(3.68)	(3.42)
RMRF	1.092***	-0.135***	-0.149***	-0.083***	-0.143***	-0.210	-0.138	-0.183	-0.122
	(47.89)	(-4.31)	(-5.12)	(-2.71)	(-4.97)	(-1.16)	(-0.83)	(-0.78)	(-0.43)
SMB	0.213***	-0.153***	-0.269***	-0.262***	-0.288***	0.407**	0.511**	0.393	0.285
	(7.66)	(-2.90)	(-6.35)	(-6.20)	(-8.18)	(2.09)	(2.00)	(1.46)	(0.90)
HML	0.175***	0.083	-0.037	-0.128**	-0.491***	-0.693***	-0.581**	-0.016	-0.244
	(4.23)	(1.21)	(-0.69)	(-2.08)	(-10.39)	(-2.80)	(-2.04)	(-0.05)	(-0.72)
R-squared	0.228					Portfolio sorts specification test:		*F(16, 641)*	4.612
N Obs.	1,969,221							*p-value*	0.000

Panel B: Gross profitability

	Coefficient estimates on explanatory variables $\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t$ (coefficient vector $\boldsymbol{\theta}$)					Coefficient estimates on time-series averages (coefficient vector $\boldsymbol{\xi}$)			
	1	$GA_{it}^{(2)}$	$GA_{it}^{(3)}$	$GA_{it}^{(4)}$	$GA_{it}^{(5)}$	$GA_{it}^{(2)}$	$GA_{it}^{(3)}$	$GA_{it}^{(4)}$	$GA_{it}^{(5)}$
1 (Intercept)	-0.265***	-0.074	0.078	0.130	0.420**	0.325	0.075	0.332	0.277
	(-2.96)	(-0.69)	(0.61)	(0.82)	(2.56)	(1.37)	(0.34)	(0.97)	(0.82)
RMRF	0.940***	0.077***	0.062**	0.074***	-0.029	0.092	0.064	-0.140	-0.070
	(51.13)	(2.85)	(2.28)	(2.76)	(-0.91)	(0.46)	(0.36)	(-0.56)	(-0.24)
SMB	0.007	-0.069	0.029	-0.013	-0.050	0.517**	0.772***	0.534**	0.282
	(0.26)	(-1.37)	(0.69)	(-0.29)	(-1.13)	(2.49)	(3.05)	(2.29)	(0.95)
HML	0.197***	-0.020	-0.148***	-0.431***	-0.487***	-0.556**	-0.141	-0.295	-0.449
	(5.87)	(-0.35)	(-3.37)	(-9.94)	(-8.71)	(-2.02)	(-0.46)	(-0.80)	(-1.17)
R-squared	0.236					Portfolio sorts specification test:		*F(16, 641)*	1.194
N Obs.	1,969,221							*p-value*	0.267

Table 5 – continued

This table reports the results from estimating regression $r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \overline{\boldsymbol{q}}_{\boldsymbol{i}}\boldsymbol{\xi} + \varepsilon_{it}$. Panel A contains the results for portfolio sorts on operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Vector $\boldsymbol{z}_{it}$ is specified as $\boldsymbol{z}_{it} = [\,1 \quad OA_{it}^{(2)} \;\ldots\; OA_{it}^{(5)}\,]$, where $OA_{it}^{(p)}$ $(p = 2, \ldots, 5)$ is a dummy variable with value one if firm i belongs to operating profitability quintile portfolio p. Panel B presents the results for portfolio sorts on gross profitability, defined as gross profit deflated by the book value of total assets. Here, vector $\boldsymbol{z}_{it}$ is specified as $\boldsymbol{z}_{it} = [\,1 \quad GA_{it}^{(2)} \;\ldots\; GA_{it}^{(5)}\,]$ where $GA_{it}^{(p)}$ $(p = 2, \ldots, 5)$ is a dummy variable with value one if firm i belongs to gross profitability quintile portfolio p. In both panels, vector $\boldsymbol{x}_t$ is set to $\boldsymbol{x}_t = [\,1 \quad RMRF_t \quad SMB_t \quad HML_t\,]$ with RMRF (market excess return), SMB (small minus big), and HML (high minus low) representing monthly Fama and French (1993) factor returns. Vector $\overline{\boldsymbol{q}}_{\boldsymbol{i}}$ comprises the firm-specific time-series averages from all variables in $(\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)$ that vary across both firms and time. Estimation results for coefficient vectors $\boldsymbol{\theta}$ and $\boldsymbol{\xi}$ are reported in matrix form. Each coefficient estimate and *t*-statistic (in parentheses) is for the product of the market-level factor variable in vector $\boldsymbol{x}_t$ (displayed on the vertical axis) and the portfolio dummy variable (or constant) in vector $\boldsymbol{z}_{it}$ (left panel), or for the time-series average of the interaction between the factor variable in $\boldsymbol{x}_t$ and the portfolio dummy variable in $\boldsymbol{z}_{it}$ (right panel). The regressions are estimated with weighted pooled OLS, where observation weights are set equal to the beginning-of-time *t* value-weights of the stocks in the quintile portfolios sorted on operating profitability (Panel A) and gross profitability (Panel B), respectively. The portfolio sorts specification test displayed in the lower right part of each Panel is a Wald-test on $H_0\colon \boldsymbol{\xi} = 0$. Statistical inference is based on Driscoll and Kraay (1998) standard errors with a lag-length of three months. The sample period is from July 1963 through December 2016. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Table 6: Continuous and multivariate firm characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: GPS-models estimated with weighted pooled OLS						
Constant	-0.317***	-0.248***	0.275**	0.343***	0.0960	-0.00826
	(-4.22)	(-3.46)	(2.54)	(3.02)	(0.80)	(-0.06)
Operating Profitability	1.743***					1.105***
	(4.97)					(2.61)
Gross Profitability		0.778***			0.719***	0.338
		(4.47)			(4.14)	(1.58)
Volatility			-5.423*			-0.142
			(-1.94)			(-0.05)
Beta				-0.290**	-0.315***	-0.303**
				(-2.57)	(-2.75)	(-2.52)
R-squared	0.236	0.235	0.264	0.281	0.286	0.295
N Obs.	2,115,518	2,115,518	2,289,867	2,275,370	2,073,983	2,059,734
N Stocks	17,008	17,008	19,109	19,124	16,949	16,908
Panel B: GPS-model specification test						
F(4, 641)	4.429***	2.110*	3.486***	2.400**	2.957***	5.466***
p-value of specification test	0.002	0.078	0.008	0.049	0.003	0.000
Panel C: GPS-models including firm fixed effects (weighted FE estimation)						
Constant	-0.0826	-0.335***	0.0440	0.422***	0.0927	-0.0626
	(-0.82)	(-3.32)	(0.33)	(3.05)	(0.58)	(-0.37)
Operating profitability	0.614					-1.583***
	(1.31)					(-2.63)
Gross profitability		1.003***			0.913***	1.744***
		(3.99)			(3.75)	(5.40)
52w rolling Vola			0.269			4.672
			(0.08)			(1.60)
52w rolling Beta				-0.370***	-0.385***	-0.420***
				(-2.67)	(-2.72)	(-2.95)
within R-squared	0.237	0.236	0.265	0.282	0.287	0.296
N Obs.	2,115,518	2,115,518	2,289,867	2,275,370	2,073,983	2,059,734
N Stocks	17,008	17,008	19,109	19,124	16,949	16,908

Table 6 – continued

This table reports the coefficient estimates and t-statistics (in parentheses) from GPS-models with multivariate and continuous firm characteristics in vector $\boldsymbol{z}_{it}$. The GPS-model is specified as

$$r_{it} = (\boldsymbol{z}_{it} \otimes \boldsymbol{x}_t)\,\boldsymbol{\theta} + \nu_{it} = ([1 \quad OA_{it} \quad GA_{it} \quad Vola_{it} \quad Beta_{it}] \otimes [\,1 \quad RMRF_t \quad SMB_t \quad HML_t])\,\boldsymbol{\theta} + \nu_{it}$$

OA_{it} is operating profitability (defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets). GA_{it} is gross profit deflated by the book value of total assets. OA_{it} and GA_{it} are formed at the end of each June and then remain unchanged throughout the subsequent year. $Vola_{it}$ refers to the standard deviation of weekly returns measured over rolling 52 weeks ending on the last Friday (or, in case of a bank holiday, the subsequent trading day) prior to the end of month t. Likewise, $Beta_{it}$ is the CAPM-beta of weekly returns measured over rolling 52 weeks ending on the last Friday (or, in case of a bank holiday, the subsequent trading day) prior to the end of month t. The table only displays the results for coefficient estimates (of subsets) of the variables in vector $\boldsymbol{z}_{it}$. The Fama-French three-factor model alpha (as a measure of the risk-adjusted performance) conditional on firm i's characteristics in period t is obtained as $\hat{\alpha}_{it} = \hat{\theta}_0 + \hat{\theta}_1 \times OPAT_{it} + \hat{\theta}_2 \times GPAT_{it} + \hat{\theta}_3 \times Vola_{it} + \hat{\theta}_4 \times Beta_{it}$. The results for the interactions of vector $\boldsymbol{z}_{it}$ firm characteristics with the market ($RMRF_t$), SMB_t (small minus big), and HML_t (high minus low) factors are not shown in the table. The sample period is from July 1963 through December 2016. The GPS-models in Panel A are estimated with weighted pooled OLS, where observation weights are set equal to the beginning-of-time t value-weights of the stocks. Panel B displays the results from the GPS-model specification test discussed in Section 2.3.3. To this end, the GPS-models from Panel A are extended with a series of control variables. These control variables are the firm-level time averages for all variables that vary over both the cross-section and time. The GPS-model specification test then tests by aid of a standard Wald test whether the coefficient estimates for the control variables are jointly equal to zero. Panel C reports the results from estimating the GPS-models from Panel A with the fixed effects (FE) estimator, where observation weights are set equal to the beginning-of-time t value-weights of the stocks. t-statistics test for significance against a value of zero. Statistical inference is based on Driscoll and Kraay (1998) standard errors with a lag-length of three. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).

Appendix A: Proof of Propositions 1 to 3

In this appendix, we first set the ground by rewriting both the GPS-model and the portfolio sorts approach in matrix notation. We then perform a series of basic transformations that apply to all three propositions stated in Section 2. Finally, we proof Propositions 1 to 3 mathematically. For ease of mathematical tractability and as outlined in Section 2, we thereby restrict our formal analysis to the case of a balanced panel (N firms with T regularly spaced observations), time-constant firm characteristics (i.e., $\boldsymbol{z}_{it} \equiv \boldsymbol{z}_i$), and equally weighted portfolios (i.e., $w_{it} = 1/N$). Under these simplifying assumptions, the GPS-model can reproduce the results of the portfolio sorts approach by aid of standard pooled OLS where all observations are equally weighted.

A.1 Matrix notation and proof of Proposition 1

A.1.1 Coefficient estimates and standard errors for the GPS-model

Applying matrix notation, we can write the GPS-model in (1) as[19]

$$\begin{bmatrix} r_{11} \\ r_{12} \\ \vdots \\ r_{1T} \\ r_{21} \\ \vdots \\ r_{NT} \end{bmatrix} = \left(\begin{bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \\ \vdots \\ \boldsymbol{z}_N \end{bmatrix} \otimes \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix} \right) \boldsymbol{\theta} + \begin{bmatrix} v_{11} \\ v_{12} \\ \vdots \\ v_{1T} \\ v_{21} \\ \vdots \\ v_{NT} \end{bmatrix} \tag{A1}$$

or, more briefly:

$$vec(\boldsymbol{R}) = \begin{bmatrix} \boldsymbol{\gamma}_1 \\ \boldsymbol{\gamma}_2 \\ \vdots \\ \boldsymbol{\gamma}_N \end{bmatrix} = (\boldsymbol{Z} \otimes \boldsymbol{X})\, \boldsymbol{\theta} + \boldsymbol{v} \tag{A2}$$

where $vec(\boldsymbol{R})$ represents a $(NT \times 1)$-vector of the firms' period t (excess) returns, $\boldsymbol{z}_i$ is a $(1 \times M)$-vector of firm characteristics (which are assumed to remain constant over time), $\boldsymbol{x}_t = [1 \;\; x_{1t} \;\; \cdots \;\; x_{Kt}]$ refers to a $(1 \times (K+1))$-vector of market-level variables (which apart from the constant change over time but do not vary in the cross-section), and $(\boldsymbol{Z} \otimes \boldsymbol{X})$ denotes the Kronecker product of $(N \times M)$-dimensional matrix $\boldsymbol{Z} = [\boldsymbol{z}_1' \;\; \ldots \;\; \boldsymbol{z}_N']'$ with $(T \times (K+1))$-dimensional matrix $\boldsymbol{X} = [\boldsymbol{x}_1' \;\; \ldots \;\; \boldsymbol{x}_T']'$. Estimating regression model (A2) with pooled OLS, and applying the calculus rules for the Kronecker product, yields the following coefficient estimates for $\boldsymbol{\theta}$:

$$\begin{aligned} \widehat{\boldsymbol{\theta}} &= \left((\boldsymbol{Z} \otimes \boldsymbol{X})'(\boldsymbol{Z} \otimes \boldsymbol{X})\right)^{-1}(\boldsymbol{Z} \otimes \boldsymbol{X})' vec(\boldsymbol{R}) \\ &= (\boldsymbol{Z}'\boldsymbol{Z} \otimes \boldsymbol{X}'\boldsymbol{X})^{-1}(\boldsymbol{Z}' \otimes \boldsymbol{X}') vec(\boldsymbol{R}) \\ &= ((\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{Z}' \otimes (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}') vec(\boldsymbol{R}) \end{aligned} \tag{A3}$$

[19] Throughout Appendix A, we assume $c_i = 0$ (for all i) and omit the firm-specific effects c_i from the analysis.

Next, we use the following Lemma from linear algebra (e.g., see Sydsaeter, Strom, and Berck, 2000, p. 146):

Lemma 1. *For any three matrices* $\boldsymbol{A} \in \mathbb{R}^{r,r}$, $\boldsymbol{B} \in \mathbb{R}^{r,s}$, *and* $\boldsymbol{C} \in \mathbb{R}^{s,s}$ *it holds true that* $vec(\boldsymbol{ABC}) = (\boldsymbol{C}'\otimes\boldsymbol{A})vec(\boldsymbol{B})$.

Applying Lemma 1 to expression (A3) above yields

$$\widetilde{\boldsymbol{\theta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1} \tag{A4}$$

Here, $\widetilde{\boldsymbol{\theta}}$ refers to the $((K+1)\times M)$-dimensional matrix of coefficient estimates $\hat{\theta}_{k,m}$ for the interaction of firm characteristic m (with $m = 1, \dots, M$) from vector $\boldsymbol{z}_i$ and market-level factor variable k (with $k = 0, \dots, K$) in vector $\boldsymbol{x}_t$.

We now turn to the Driscoll and Kraay (1998) covariance matrix estimator for the pooled OLS regression model in (A1). For H lags, it has the following structure:

$$\begin{aligned} \widetilde{\boldsymbol{V}}(\widehat{\boldsymbol{\theta}}) &= \left((\boldsymbol{Z}\otimes\boldsymbol{X})'(\boldsymbol{Z}\otimes\boldsymbol{X})\right)^{-1}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}\left((\boldsymbol{Z}\otimes\boldsymbol{X})'(\boldsymbol{Z}\otimes\boldsymbol{X})\right)^{-1} \\ &= ((\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1})\,\tilde{\boldsymbol{S}}_{\boldsymbol{T}}\,((\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1}) \end{aligned} \tag{A5}$$

with $\quad \tilde{\boldsymbol{S}}_{\boldsymbol{T}} = \widehat{\boldsymbol{\Omega}}_0 + \sum_{j=1}^{H}\omega_{j,H}\left(\widehat{\boldsymbol{\Omega}}_j + \widehat{\boldsymbol{\Omega}}_j'\right), \quad \widehat{\boldsymbol{\Omega}}_j = \sum_{\tau=j+1}^{T}\boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}})\,\boldsymbol{h}_{\tau-j}'(\widehat{\boldsymbol{\theta}})$,

and $\quad \boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}}) = (\boldsymbol{Z}\otimes\boldsymbol{x}_\tau)'\,\hat{\boldsymbol{v}}$

The modified Bartlett weights $\omega_{j,H} = 1 - j/(H+1)$ ensure positive semi-definiteness of $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ and smooth the sample autocovariance function such that higher order lags receive less weight.

A.1.2 Coefficient estimates and standard errors for the portfolio sorts approach

The portfolio sorts approach comprises two steps. The first step involves computing the month t average return for portfolio p as outlined in Equation (5). In our case of a balanced panel, time-constant firm characteristics, and equally weighted portfolios (i.e., $w_{it} = N_p^{-1} = \left(\sum_{i=1}^{N} z_i^{(p)}\right)^{-1}$), we can rewrite Equation (5) as follows:

$$r_{pt} = \frac{1}{N_p}\sum_{i=1}^{N} z_i^{(p)} r_{it} = \boldsymbol{r}_t'\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \tag{A6}$$

with $\boldsymbol{r}_t' = [r_{1t} \quad r_{2t} \quad \dots \quad r_{Nt}]$ and $\boldsymbol{d}_p' = \left[z_1^{(p)} \quad z_2^{(p)} \quad \dots \quad z_N^{(p)}\right]$.

Here, $z_i^{(p)}$ is a dummy variable with value one if firm i belongs to portfolio p, and zero otherwise. In the second step of the procedure, r_{pt} from (A6) is regressed on a constant and the K factor variables as outlined in Equation (6). This yields OLS coefficient estimates as follows:

$$\widehat{\boldsymbol{\beta}}_p = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\begin{bmatrix} r_{p1} \\ \vdots \\ r_{pT} \end{bmatrix} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\begin{bmatrix} \boldsymbol{r}_1'\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \\ \vdots \\ \boldsymbol{r}_T'\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \end{bmatrix}$$

$$= (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\begin{bmatrix} \boldsymbol{r}_1' \\ \vdots \\ \boldsymbol{r}_T' \end{bmatrix}\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'[\boldsymbol{\gamma}_1 \quad \boldsymbol{\gamma}_2 \quad \ldots \quad \boldsymbol{\gamma}_N]\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1}$$

$$= (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \tag{A7}$$

The formula for computing the Newey and West (1987) covariance matrix with lag length H for the regression model in Equation (6) has the following structure:

$$\widehat{\boldsymbol{V}}\left(\widehat{\boldsymbol{\beta}}_p\right) = (\boldsymbol{X}'\boldsymbol{X})^{-1}\widehat{\boldsymbol{S}}_{\boldsymbol{T}}(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

with $$\widehat{\boldsymbol{S}}_{\boldsymbol{T}} = \sum_{t=1}^{T}\hat{\varepsilon}_{pt}^2\,\boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(\hat{\varepsilon}_{p\tau}\hat{\varepsilon}_{p\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right) \tag{A8}$$

Restricting the sample to two portfolios (or groups of firms), arbitrarily denoted as $p =$ "$high$" and $p =$ "low", we can use matrix notation to rewrite Equation (10) computing the month t return difference as follows:

$$\Delta r_{p,t} = r_{high,t} - r_{low,t} = \boldsymbol{r}_t'\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_2 = \left[r_{low,t} \quad \Delta r_{p,t}\right]\begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{A9}$$

where matrix $\boldsymbol{Z}$ is specified as $\boldsymbol{Z} = [\iota \quad \boldsymbol{d}_{high}]$ and ι is a $(N \times 1)$-dimensional vector of ones. When regressing $\Delta r_{p,t}$ from (A9) on a constant and the K factor variables according to Equation (11), one obtains the following OLS coefficient estimates:

$$\widehat{\boldsymbol{\beta}}_\Delta = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\begin{bmatrix} \Delta r_{p,1} \\ \vdots \\ \Delta r_{p,T} \end{bmatrix} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_2 \tag{A10}$$

The Newey and West (1987) covariance matrix estimator for the coefficient estimates in (A10) has the same structure as the one displayed in Equation (A8), with $\hat{\varepsilon}_{\Delta t}$ replacing $\hat{\varepsilon}_t$ in the formula.

A.1.3 Proof of Proposition 1

Proposition 1 states that the GPS-model can reproduce the results of the portfolio sorts approach for the case of a single portfolio if vector $\boldsymbol{z}_i \equiv \boldsymbol{z}_{it}$ is specified as $\boldsymbol{z}_i = [\,1\,]$. In this case, matrix $\boldsymbol{Z}$ in Equation (A2) is given as $\boldsymbol{Z} = \iota$. As a result, the coefficient estimates of the GPS-model in this case are

$$\widetilde{\boldsymbol{\theta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\iota(\iota'\iota)^{-1} \tag{A11}$$

When there is only a single subject group, then $z_i^{(p)}$ for all firms i is equal to 1, i.e., $\boldsymbol{d}_p = \iota$. Consequently, the coefficient estimates for the portfolio sorts approach in Equation (A7) are equal to

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\iota(\iota'\iota)^{-1} \tag{A12}$$

As stated in Part A of Proposition 1, we thus have $\widetilde{\boldsymbol{\theta}} \equiv \widehat{\boldsymbol{\beta}}$. This completes the first part of the proof. ■

We next turn to the standard errors for the coefficient estimates. With $r_{pt} = N^{-1}\sum_{i=1}^{N} r_{it}$ and $\widetilde{\boldsymbol{\theta}} = \widehat{\boldsymbol{\beta}}$, the (estimated) residual $\hat{\varepsilon}_t$ in Equation (3) is equal to

$$\hat{\varepsilon}_t = N^{-1}\sum_{i=1}^{N} \hat{v}_{it} \equiv N^{-1}V_t \tag{A13}$$

where $\hat{v}_{it}$ is the (estimated) residual from pooled OLS regression (4). Replacing $\hat{\varepsilon}_t$ in (A8) by the corresponding term from (A13) yields

$$N^2\,\widehat{\boldsymbol{S}}_{\boldsymbol{T}} = \sum_{t=1}^{T} V_t^2\, \boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H} \omega_{j,H}\, \sum_{\tau=j+1}^{T} \Big(V_\tau V_{\tau-j}\big(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\big)\Big) \tag{A14}$$

In case of the GPS-model, we plug in $\boldsymbol{Z} = \iota$ in Equation (A5). This gives

$$\widetilde{\boldsymbol{V}}(\widehat{\boldsymbol{\theta}}) = ((\iota'\iota)^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1})\,\widetilde{\boldsymbol{S}}_{\boldsymbol{T}}\,((\iota'\iota)^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1}) = (\boldsymbol{X}'\boldsymbol{X})^{-1}\,\frac{\widetilde{\boldsymbol{S}}_{\boldsymbol{T}}}{N^2}\,(\boldsymbol{X}'\boldsymbol{X})^{-1} \tag{A15}$$

Comparing $\widetilde{\boldsymbol{V}}(\widehat{\boldsymbol{\theta}})$ in (A15) with $\widehat{\boldsymbol{V}}(\widehat{\boldsymbol{\beta}})$ from (A8) in case of a single subject group, we hence have to show that $N^{-2}\widetilde{\boldsymbol{S}}_{\boldsymbol{T}} = \widehat{\boldsymbol{S}}_{\boldsymbol{T}}$. Rewriting $\boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}})$ in (A5), we obtain:

$$\boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}}) = (\iota\otimes\boldsymbol{x}_\tau)'\,\hat{\boldsymbol{v}} = [\boldsymbol{x}_\tau' \quad \cdots \quad \boldsymbol{x}_\tau']\hat{\boldsymbol{v}} = \begin{bmatrix} \sum_{i=1}^{N} \hat{v}_{i\tau} \\ x_{1\tau}\sum_{i=1}^{N} \hat{v}_{i\tau} \\ \vdots \\ x_{K\tau}\sum_{i=1}^{N} \hat{v}_{i\tau} \end{bmatrix} = \boldsymbol{x}_\tau' V_\tau \tag{A16}$$

From (A16) it follows for $\widehat{\boldsymbol{\Omega}}_j$ in (A5) that $\widehat{\boldsymbol{\Omega}}_j = \sum_{\tau=j+1}^{T} \boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}})\,\boldsymbol{h}_{\tau-j}'(\widehat{\boldsymbol{\theta}}) = \sum_{\tau=j+1}^{T} V_\tau V_{\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j}$, and consequently

$$\widetilde{\boldsymbol{S}}_{\boldsymbol{T}} = \sum_{t=1}^{T} V_t^2\boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H} \omega_{j,H}\, \sum_{\tau=j+1}^{T} \Big(V_\tau V_{\tau-j}\big(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\big)\Big) \equiv N^2\,\widehat{\boldsymbol{S}}_{\boldsymbol{T}} \tag{A17}$$

This completes the proof. ■

A.2 Proof of Proposition 2

Proposition 2 states that the GPS-model can reproduce the results of the portfolio sorts approach for multiple sorted portfolios by estimating a single pooled OLS regression on the individual firm level. To this end, we specify vector $\boldsymbol{z}_i$ as $\boldsymbol{z}_i = [\, z_i^{(1)} \;\; z_i^{(2)} \;\ldots\; z_i^{(P)}]$. Using the definition of $\boldsymbol{d}_p$ in (A6), $(N \times P)$-matrix $\boldsymbol{Z}$ in equation (A2) is given as follows:

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{z}_1 \\ \vdots \\ \boldsymbol{z}_N \end{bmatrix} = \begin{bmatrix} z_1^{(1)} & \cdots & z_1^{(P)} \\ \vdots & & \vdots \\ z_N^{(1)} & \cdots & z_N^{(P)} \end{bmatrix} = [\boldsymbol{d}_1 \quad \cdots \quad \boldsymbol{d}_P] \tag{A18}$$

The coefficient estimates for the GPS-model are derived in (A4). Specifying matrix $\boldsymbol{Z}$ according to (A18) thus results in a $((K+1) \times P)$-dimensional matrix of coefficient estimates $\widetilde{\boldsymbol{\theta}}$. The p-th column of results matrix $\widetilde{\boldsymbol{\theta}}$ can be obtained as follows:

$$\widetilde{\boldsymbol{\theta}}_p = \widetilde{\boldsymbol{\theta}}\boldsymbol{e}_p = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_p \tag{A19}$$

where $\boldsymbol{e}_p$ is a P-dimensional vector of zeroes with a one on position p. Proposition 2 claims that $\widetilde{\boldsymbol{\theta}}_p \equiv \widehat{\boldsymbol{\beta}}_p$ where $\widehat{\boldsymbol{\beta}}_p$ refers to the portfolio p coefficient estimates from estimating the second-step time-series regression of the portfolio sorts approach. The respective coefficient estimates have been derived in Equation (A7). We thus have to proof that $\widetilde{\boldsymbol{\theta}}_p \equiv \widehat{\boldsymbol{\beta}}_p = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{R}\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1}$ or, equivalently, that

$$\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_p \equiv \boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} \tag{A20}$$

To show that (A20) indeed is an identity, we first note that $\boldsymbol{d}_p = \boldsymbol{Z}\boldsymbol{e}_{\boldsymbol{p}}$. Multiplying both sides in (A20) with $\boldsymbol{d}_{\boldsymbol{p}}'$ from the left yields

$$\begin{aligned} \boldsymbol{d}_p'\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_p &= \boldsymbol{e}_p'\boldsymbol{Z}'\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{e}_p = 1 \\ &\equiv \boldsymbol{d}_p'\boldsymbol{d}_p\left(\boldsymbol{d}_p'\boldsymbol{d}_p\right)^{-1} = 1 \end{aligned} \tag{A21}$$

This shows that $\widetilde{\boldsymbol{\theta}}_p \equiv \widehat{\boldsymbol{\beta}}_p$ and, hence, completes the first part of the proof of Proposition 2. ■

Next, we turn to the standard errors for the coefficient estimates. With $r_{pt} = N_p^{-1}\sum_{i=1}^{N} z_i^p r_{it}$ and $\widetilde{\boldsymbol{\theta}}_p = \widehat{\boldsymbol{\beta}}_p$, the (estimated) residual $\hat{\varepsilon}_{pt}$ for the portfolio sorts approach in equation (6) is equal to

$$\hat{\varepsilon}_{pt} = N_p^{-1}\sum_{i=1}^{N} z_i^{(p)}\hat{v}_{it} \equiv N_p^{-1}V_{pt} \tag{A22}$$

where $\hat{v}_{it}$ is the (estimated) residual from pooled OLS regression (7). Replacing $\hat{\varepsilon}_{pt}$ in (A8) by the corresponding term from (A22) yields

$$N_p^2\,\widehat{\boldsymbol{S}}_{\boldsymbol{T}} = \sum_{t=1}^{T} V_{pt}^2\,\boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H}\omega_{j,H}\,\sum_{\tau=j+1}^{T}\left(V_{p\tau}V_{p,\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right) \equiv \widehat{\boldsymbol{S}}_{\boldsymbol{T}}^{(p)} \tag{A23}$$

As a consequence, we finally obtain the Newey and West (1987) standard errors in case of the portfolio sorts approach as follows:

$$\widehat{\boldsymbol{V}}\left(\widehat{\boldsymbol{\beta}}_p\right) = N_p^{-2}(\boldsymbol{X}'\boldsymbol{X})^{-1}\widehat{\boldsymbol{S}}_{\boldsymbol{T}}^{(p)}(\boldsymbol{X}'\boldsymbol{X})^{-1} \tag{A24}$$

We now consider the GPS-model with matrix $\boldsymbol{Z}$ being defined according to Equation (A18). The $\left(P \times (K+1)\right)$-dimensional column vector $\boldsymbol{h}_\tau\left(\widehat{\boldsymbol{\theta}}\right)$ from (A5) in this case is equal to

$$\boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}}) = (\boldsymbol{Z}'\otimes\boldsymbol{x}_\tau')\,\widehat{\boldsymbol{v}} = \begin{bmatrix} \sum_{i=1}^{N} z_i^{(1)}\hat{v}_{i\tau} \\ x_{1\tau}\sum_{i=1}^{N} z_i^{(1)}\hat{v}_{i\tau} \\ \vdots \\ x_{K\tau}\sum_{i=1}^{N} z_i^{(1)}\hat{v}_{i\tau} \\ \sum_{i=1}^{N} z_i^{(2)}\hat{v}_{i\tau} \\ x_{1\tau}\sum_{i=1}^{N} z_i^{(2)}\hat{v}_{i\tau} \\ \vdots \\ x_{K\tau}\sum_{i=1}^{N} z_i^{(P)}\hat{v}_{i\tau} \end{bmatrix} = \begin{bmatrix} V_{1\tau} \\ x_{1\tau}V_{1\tau} \\ \vdots \\ x_{K\tau}V_{1\tau} \\ V_{2\tau} \\ x_{1\tau}V_{2\tau} \\ \vdots \\ x_{K\tau}V_{P\tau} \end{bmatrix} = (\boldsymbol{I}_P\otimes\boldsymbol{x}_\tau')\begin{bmatrix} V_{1\tau} \\ \vdots \\ V_{P\tau} \end{bmatrix} \equiv (\boldsymbol{I}_P\otimes\boldsymbol{x}_\tau')\boldsymbol{V}_\tau \qquad \text{(A25)}$$

where $\boldsymbol{I}_P$ is the P-dimensional identity matrix. With (A25) it follows for $\widehat{\boldsymbol{\Omega}}_j$ in Equation (A5) that

$$\widehat{\boldsymbol{\Omega}}_j = \sum_{\tau=j+1}^{T}(\boldsymbol{I}_P\otimes\boldsymbol{x}_\tau')\boldsymbol{V}_\tau\boldsymbol{V}_{\tau-j}'(\boldsymbol{I}_P\otimes\boldsymbol{x}_{\tau-j}) = \sum_{\tau=j+1}^{T}(\boldsymbol{V}_\tau\boldsymbol{V}_{\tau-j}')\otimes(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j}) \qquad \text{(A26)}$$

As a result, matrix $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ in (A5) can be written as follows:

$$\begin{aligned} \tilde{\boldsymbol{S}}_{\boldsymbol{T}} = & \sum_{t=1}^{T}(\boldsymbol{V}_t\boldsymbol{V}_t')\otimes(\boldsymbol{x}_t'\boldsymbol{x}_t) \\ & + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\Big((\boldsymbol{V}_\tau\boldsymbol{V}_{\tau-j}')\otimes(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j}) + (\boldsymbol{V}_{\tau-j}\boldsymbol{V}_\tau')\otimes(\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau)\Big) \end{aligned} \qquad \text{(A27)}$$

We next define $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(p,q)}$ as follows

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(p,q)} \equiv \sum_{t=1}^{T} V_{pt}V_{qt}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\Big(V_{p\tau}V_{q,\tau-j}(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j}) + V_{p,\tau-j}V_{q\tau}(\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau)\Big) \qquad \text{(A28)}$$

where V_{pt} is a scalar as in (A22) above. Consequently, matrix $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ is equal to

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}} = \begin{bmatrix} \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,1)} & \cdots & \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,P)} \\ \vdots & \ddots & \vdots \\ \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(P,1)} & \cdots & \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(P,P)} \end{bmatrix} \qquad \text{(A29)}$$

Moreover, with matrix $\boldsymbol{Z}$ being defined according to (A18), $\boldsymbol{Z}'\boldsymbol{Z}$ now is a $(P \times P)$-dimensional diagonal matrix with element (p,p) equal to N_p and all off-diagonal elements equal to zero. With $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ structured according to Expression (A29), we can thus rewrite $\widetilde{\boldsymbol{V}}(\widehat{\boldsymbol{\theta}})$ in (A5) as follows:

$$\widetilde{\boldsymbol{V}}(\widehat{\boldsymbol{\theta}}) = \begin{bmatrix} N_1^{-2}(\boldsymbol{X}'\boldsymbol{X})^{-1}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,1)}(\boldsymbol{X}'\boldsymbol{X})^{-1} & \cdots & N_1^{-1}N_P^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,P)}(\boldsymbol{X}'\boldsymbol{X})^{-1} \\ \vdots & \ddots & \vdots \\ N_1^{-1}N_P^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(P,1)}(\boldsymbol{X}'\boldsymbol{X})^{-1} & \cdots & N_P^{-2}(\boldsymbol{X}'\boldsymbol{X})^{-1}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(P,P)}(\boldsymbol{X}'\boldsymbol{X})^{-1} \end{bmatrix} \qquad \text{(A30)}$$

The second part of Proposition 2 claims that $\text{SE}(\hat{\theta}_{p,k}) = \text{SE}(\hat{\beta}_{p,k})$ for $k = 0,1,\dots,K$ and $p = 1,\dots,P$. To proof that this holds true, it is sufficient to show that $N_p^{-2}(\boldsymbol{X}'\boldsymbol{X})^{-1}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(p,p)}(\boldsymbol{X}'\boldsymbol{X})^{-1}$ in Expression (A30) is identical to $\widehat{\boldsymbol{V}}(\widehat{\boldsymbol{\beta}}_p)$ in Equation (A24) for every $p = 1,\dots,P$. This in turn is equivalent to demonstrating that $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}^{(p)}$ from (A23) coincides with $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(p,p)}$ in (A28). By comparing the two expressions we see that this holds true, which completes the proof. ■

A.3 Proof of Proposition 3

Proposition 3 states that the GPS-model can reproduce the results of the portfolio sorts approach for the case of performance differences between two portfolios. In order to compare the performance of firms in group “high” with that of firms belonging to group “low”, we have to specify vector $\boldsymbol{z}_i$ as $\boldsymbol{z}_i = [1 \;\; z_i^{(high)}]$ such that the $(N \times 2)$-dimensional matrix $\boldsymbol{Z}$ comprising the characteristics of all N firms is equal to $\boldsymbol{Z} = [\iota \quad \boldsymbol{d}_{high}]$. This matches the definition of matrix $\boldsymbol{Z}$ in Expression (A9) which is used for deriving the results of performance differences in case of the portfolio sorts approach. Estimating the GPS-model in (A2) with pooled OLS yields the coefficient estimates, structured as a $((K+1) \times 2)$-dimensional matrix, in Equation (A4).

The second column of $\widetilde{\boldsymbol{\theta}}$ in (A4) contains the coefficient estimates for the interaction terms between dummy variable $z_i^{(high)}$ and the factor variables in vector $\boldsymbol{x}_t$. In Equation (12), those coefficient estimates are named $\hat{\theta}_{\Delta k}$. To extract the coefficient estimates for $\hat{\theta}_{\Delta k}$ from $\widetilde{\boldsymbol{\theta}}$, we post-multiply Expression (A4) with $\boldsymbol{e}_2$. The resulting term coincides with the one in Equation (A10) for the portfolio sorts approach. This shows that $\hat{\beta}_{\Delta k} \equiv \hat{\theta}_{\Delta k}$ ($\forall\, k = 0, 1, \ldots, K$).

The first column of $\widetilde{\boldsymbol{\theta}}$ in (A4) comprises the coefficient estimates for subject group “low”. In Equation (12), the respective coefficient estimates are labeled as $\hat{\theta}_{low,k}$ (with $k = 0, 1, \ldots, K$). The coefficient estimates for $\theta_{low,k}$ are retrieved by post-multiplying (A4) with $\boldsymbol{e}_1$. In case of the portfolio sorts approach, we obtain the coefficient estimates for the “low” portfolio ($\hat{\beta}_{low,k}$) by repeating the analysis of (A9) and (A10) with $\boldsymbol{e}_1$ replacing $\boldsymbol{e}_2$. The resulting expressions for the portfolio sorts approach and the GPS-model again coincide. This demonstrates $\hat{\beta}_{low,k} \equiv \hat{\theta}_{low,k}$ ($\forall\, k = 0, 1, \ldots, K$) and, hence, completes the first part of the proof. ■

For the second part of the proof, we note that due to $r_{high,t} = r_{low,t} + \Delta r_{p,t}$ the following corollary holds true:

Corollary 2. $$\hat{\beta}_{high,k} = \hat{\beta}_{low,k} + \hat{\beta}_{\Delta k} = \hat{\theta}_{low,k} + \hat{\theta}_{\Delta k} \quad \text{for all } k = 0, 1, \ldots, K. \tag{A31}$$

Based on Corollary 2 and Proposition 2, and because of $r_{pt} = N_p^{-1} \sum_{i=1}^{N} z_i^{(p)} r_{it}$ (for p = “low”, “high”), residual $\hat{\varepsilon}_{\Delta t}$ in the second-step time-series regression (11) of the portfolio sorts approach is equal to

$$\hat{\varepsilon}_{\Delta t} = N_{high}^{-1} \sum_{i=1}^{N} z_i^{(high)} \hat{v}_{it} - N_{low}^{-1} \sum_{i=1}^{N} z_i^{(low)} \hat{v}_{it} \equiv N_{high}^{-1} V_{high,t} - N_{low}^{-1} V_{low,t} \tag{A32}$$

where $\hat{v}_{it}$ is the (estimated) residual from pooled OLS regression (7). Replacing $\hat{\varepsilon}_{\Delta t}$ in the Newey and West (1987) covariance matrix estimator (A8) for the coefficient estimates in (A10) by the respective expression in (A32) yields

$$\widehat{\boldsymbol{S}}_{\boldsymbol{T}} = \sum_{t=1}^{T} \left(\frac{V_{high,t}}{N_{high}} - \frac{V_{low,t}}{N_{low}}\right)^2 \boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H} \omega_{j,H} \sum_{\tau=j+1}^{T} \left(\left(\frac{V_{high,\tau}}{N_{high}} - \frac{V_{low,\tau}}{N_{low}}\right)\left(\frac{V_{high,\tau-j}}{N_{high}} - \frac{V_{low,\tau-j}}{N_{low}}\right)\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right) \tag{A33}$$

We now turn to the GPS-model with matrix $\boldsymbol{Z}$ being defined as $\boldsymbol{Z} = [\iota \quad \boldsymbol{d}_{high}]$. The $(2 \times (K+1))$-dimensional column vector $\boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}})$ from (A5) in this case is equal to

$$\boldsymbol{h}_\tau(\widehat{\boldsymbol{\theta}}) = (\boldsymbol{Z}'\otimes\boldsymbol{x}_\tau')\,\widehat{\boldsymbol{v}} = \begin{bmatrix} \sum_{i=1}^{N} \widehat{v}_{i\tau} \\ x_{1\tau}\sum_{i=1}^{N} \widehat{v}_{i\tau} \\ \vdots \\ x_{K\tau}\sum_{i=1}^{N} \widehat{v}_{i\tau} \\ \sum_{i=1}^{N} z_i^{(high)}\widehat{v}_{i\tau} \\ x_{1\tau}\sum_{i=1}^{N} z_i^{(high)}\widehat{v}_{i\tau} \\ \vdots \\ x_{K\tau}\sum_{i=1}^{N} z_i^{(high)}\widehat{v}_{i\tau} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_\tau' & \boldsymbol{x}_\tau' \\ \boldsymbol{x}_\tau' & \boldsymbol{0}' \end{bmatrix}\begin{bmatrix} V_{high,\tau} \\ V_{low,\tau} \end{bmatrix} \equiv \begin{bmatrix} \boldsymbol{x}_\tau' & \boldsymbol{x}_\tau' \\ \boldsymbol{x}_\tau' & \boldsymbol{0}' \end{bmatrix}\boldsymbol{V}_\tau \tag{A34}$$

From (A34) and $V_{high,\tau} + V_{low,\tau} \equiv V_\tau$ it follows for $\widehat{\boldsymbol{\Omega}}_j$ in Equation (A5) that

$$\widehat{\boldsymbol{\Omega}}_j = \sum_{\tau=j+1}^{T} \begin{bmatrix} V_\tau V_{\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} & V_\tau V_{high,\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} \\ V_{high,\tau}V_{\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} & V_{high,\tau}V_{high,\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} \end{bmatrix} \tag{A35}$$

As a result, matrix $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}$ in (A5) can be written in block form as follows:

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}} = \begin{bmatrix} \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,1)} & \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,2)} \\ \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(2,1)} & \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(2,2)} \end{bmatrix} \tag{A36}$$

where

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,1)} = \sum_{t=1}^{T} V_t^2(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_\tau V_{\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right)$$

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(1,2)} = \sum_{t=1}^{T} V_t V_{high,t}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_\tau V_{high,\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + V_{high,\tau}V_{\tau-j}\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)$$

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(2,1)} = \sum_{t=1}^{T} V_t V_{high,t}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_{high,\tau}V_{\tau-j}\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + V_\tau V_{high,\tau-j}\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)$$

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(2,2)} = \sum_{t=1}^{T} V_{high,t}^2(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_{high,\tau}V_{high,\tau-j}\left(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)\right)$$

Next, we rewrite matrix $(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1}$ in (A5) as

$$(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1} = \begin{bmatrix} N_{low}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1} & -N_{low}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1} \\ -N_{low}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1} & \left(N_{low}^{-1} + N_{high}^{-1}\right)(\boldsymbol{X}'\boldsymbol{X})^{-1} \end{bmatrix} \tag{A37}$$

and insert (A37) into the Driscoll and Kraay (1998) covariance matrix estimator of (A5) to obtain

$$\tilde{\boldsymbol{V}}(\widehat{\boldsymbol{\theta}}) = ((\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1})\,\tilde{\boldsymbol{S}}_{\boldsymbol{T}}\,((\boldsymbol{Z}'\boldsymbol{Z})^{-1}\otimes(\boldsymbol{X}'\boldsymbol{X})^{-1}) = \begin{bmatrix} \tilde{\boldsymbol{V}}^{(1,1)} & \tilde{\boldsymbol{V}}^{(1,2)} \\ \tilde{\boldsymbol{V}}^{(2,1)} & \tilde{\boldsymbol{V}}^{(2,2)} \end{bmatrix} \tag{A38}$$

where

$$\widetilde{\boldsymbol{V}}^{(1,1)} = N_{low}^{-2}(\boldsymbol{X}'\boldsymbol{X})^{-1}\left(\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} + \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}\right)(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

$$\widetilde{\boldsymbol{V}}^{(1,2)} = N_{low}^{-1}N_{high}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1}\left(\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}\right)(\boldsymbol{X}'\boldsymbol{X})^{-1} - \widetilde{\boldsymbol{V}}^{(1,1)}$$

$$\widetilde{\boldsymbol{V}}^{(2,1)} = N_{low}^{-1}N_{high}^{-1}(\boldsymbol{X}'\boldsymbol{X})^{-1}\left(\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}\right)(\boldsymbol{X}'\boldsymbol{X})^{-1} - \widetilde{\boldsymbol{V}}^{(1,1)}$$

$$\widetilde{\boldsymbol{V}}^{(2,2)} = N_{high}^{-2}(\boldsymbol{X}'\boldsymbol{X})^{-1}\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}(\boldsymbol{X}'\boldsymbol{X})^{-1} - \widetilde{\boldsymbol{V}}^{(1,1)} - \widetilde{\boldsymbol{V}}^{(1,2)} - \widetilde{\boldsymbol{V}}^{(2,1)}$$

According to the second part of Proposition 3, we have to show that $\widetilde{\boldsymbol{V}}^{(2,2)}$ in (A38) coincides with the Newey-West covariance matrix estimator in (A10) with $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}$ specified according to (A33). Therefore, we simplify the "sandwich" expressions in (A38) as follows

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} + \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})} = \sum_{t=1}^{T} V_{low,t}^{2}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(V_{low,\tau}V_{low,\tau-j}(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau)\right)$$

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})} = \sum_{t=1}^{T} V_{low,t}V_{high,t}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left((V_{low,\tau}V_{high,\tau-j})\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + (V_{low,\tau-j}V_{high,\tau})\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)$$

$$\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})} = \sum_{t=1}^{T} V_{high,t}V_{low,t}(\boldsymbol{x}_t'\boldsymbol{x}_t) + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left((V_{high,\tau}V_{low,\tau-j})\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + (V_{high,\tau-j}V_{low,\tau})\boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau\right)$$

and insert the resulting expressions into $\widetilde{\boldsymbol{V}}^{(2,2)}$ from (A38). This finally yields

$$\widetilde{\boldsymbol{V}}^{(2,2)} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\,\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{\Delta}\,(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

with

$$\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{\Delta} = \sum_{t=1}^{T}\left(\frac{V_{high,t}}{N_{high}} - \frac{V_{low,t}}{N_{low}}\right)^2 \boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(\left(\frac{V_{high,\tau}}{N_{high}} - \frac{V_{low,\tau}}{N_{low}}\right)\left(\frac{V_{high,\tau-j}}{N_{high}} - \frac{V_{low,\tau-j}}{N_{low}}\right)(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau)\right) \qquad \text{(A39)}$$

Since $\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{\Delta}$ in (A39) and $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}$ in (A33) coincide, this shows that $\mathrm{SE}(\hat{\theta}_{\Delta k}) = \mathrm{SE}(\hat{\beta}_{\Delta k})$ for all $k = 0, 1, \dots, K$.

The last part of Proposition (3) claims that $\widetilde{\boldsymbol{V}}^{(1,1)}$ in (A38) coincides with the Newey-West covariance matrix estimator for the second-step time-series regression of the portfolio sorts approach applied to portfolio "low". The respective Newey-West covariance estimator has been derived in Expression (A24) above with p = "low". By replacing $\tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{1})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{1},\mathbf{2})} - \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{1})} + \tilde{\boldsymbol{S}}_{\boldsymbol{T}}^{(\mathbf{2},\mathbf{2})}$ with the corresponding term derived above, we finally obtain the following expression for $\widetilde{\boldsymbol{V}}^{(1,1)}$:

$$\widetilde{\boldsymbol{V}}^{(1,1)} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\,\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{high}\,(\boldsymbol{X}'\boldsymbol{X})^{-1} \qquad \text{(A40)}$$

with

$$\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{high} = \sum_{t=1}^{T} N_{low}^{-2}V_{low,t}^{2}\,\boldsymbol{x}_t'\boldsymbol{x}_t + \sum_{j=1}^{H}\omega_{j,H}\sum_{\tau=j+1}^{T}\left(N_{low}^{-2}V_{low,\tau}V_{low,\tau-j}(\boldsymbol{x}_\tau'\boldsymbol{x}_{\tau-j} + \boldsymbol{x}_{\tau-j}'\boldsymbol{x}_\tau)\right)$$

Since $\widetilde{\boldsymbol{Q}}_{\boldsymbol{T}}^{high}$ in (A40) and $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}$ for portfolio p = "low" in (A23) are identical, this demonstrates that $\mathrm{SE}(\hat{\theta}_{\mathrm{low},k}) = \mathrm{SE}(\hat{\beta}_{\mathrm{low},k})$ for all $k = 0, 1, \dots, K$. This completes the proof of Proposition 3. ■