Idiosyncratic Volatility, its Expected Variation, and the Cross-Section of Stock Returns

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First version: October 31, 2016
This version: January 29, 2018

Abstract

We offer a novel perspective on the negative relation between idiosyncratic volatility (IVOL) and expected returns. We show that the IVOL puzzle is largely driven by a mean-reversion behavior of the stocks' volatilities, which is not captured by a simple historic measure of IVOL. In doing so, we make use of option implied information to extract the expected mean-reversion speed of IVOL in an almost model-free fashion. Together with the current level of IVOL this method allows us to identify stocks' expected IVOL changes in a very general setting. Under the assumption of IVOL carrying a positive price of risk (Merton (1987)) we offer a explanation for the puzzle. In a horse race we show that the mean-reversion speed is superior to the most prominent competing explanations. All our findings are robust to different measures of IVOL and various stock characteristics.

Keywords: Options, Stock Returns, Idiosyncratic Volatility, Volatility-of-Volatility

JEL: G12, G13

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We thank Vikas Agarwal, Torben Andersen, René Flacke, Lorenzo Garlappi, Michael Hofmann, Namhee Matheson, Tamara Nunes, Daniel Schmidt, Michael Semenischev, Julian Thimme, Viktor Todorov, Marliese Uhrig-Homburg, Michael Ungeheuer, Alex Weissensteiner, participants at the 2018 Zurich Workshop on Asset Pricing, the *Asset Allocation under Parameter Uncertainty* Workshop in Bozen, the 5th Paris Financial Management Conference, the 6th Annual OptionMetrics Research Conference, the 24th Annual Meeting of the German Finance Association, the 2017 SoFiE Financial Econometrics Summer School at the Kellogg School of Management, the 34th International Conference of the French Finance Association, the 16th Colloquium on Financial Markets in Cologne, and seminar participants at the Finance Center Münster for helpful comments and suggestions.

1 Introduction

The higher the exposure to systematic risk, the higher the expected return of an asset. This fundamental relation between systematic risk and returns is one of the cornerstones in asset pricing. In contrast to systematic risk, the relation between idiosyncratic risk and expected returns offers a less clear picture until now. In classical asset pricing theory it has been common sense to assume that idiosyncratic risk is either positively priced (Merton (1987)) or has no pricing impact at all (Sharpe (1964), Lintner (1965)). However, in the seminal work of Ang et al. (2006) both classical assumptions are challenged by the finding of a negative relation between the realized idiosyncratic return volatility (IVOL) and subsequent returns.[1] According to their findings assets which have a high idiosyncratic risk level yield lower returns than assets with a low idiosyncratic risk level. Since this finding seems irreconcilable with classical approaches, the negative relation between idiosyncratic risk and future returns has become known as the IVOL puzzle.

We exploit information from the cross-section of stock options to offer a resolution for the IVOL puzzle under the assumption that idiosyncratic risk carries a positive price of risk. Central in finding a negative IVOL-return relation so far is the measurement of IVOL. The measure proposed by Ang et al. (2006) is based on realized returns and thus purely historic in nature (see Fu (2009)). By making use of option prices, we link the pure historic measure of IVOL to forward looking expectations embedded in these option prices. In doing so, we make use of the mean-reverting behaviour of IVOL to identify expected IVOL changes. In contrast to other studies, our method allows us not only to analyze the relation between expected IVOL and returns itself, but also to analyze why there is a negative relation between the realized IVOL and subsequent returns. To do so, we use the time series information from the cross-section of option prices to estimate the volatility

[1] In the following we use idiosyncratic risk and firm specific risk synonymously for IVOL.

of expected idiosyncratic volatility (IVOLVOL) and show theoretically as well as empirically that our measure of IVOLVOL can be used as a proxy for the expected mean-reversion speed of IVOL. In doing so, we employ an almost model-free approach to measure IVOLVOL. This approach is directly linked to the measurement of realized IVOL. Our method uses techniques from the literature on model-free moments (Demeterfi et al. (1999), CBOE (2016)). We calculate implied volatilities for the expected return distribution of every stock in our cross-section. Using a simple linear model we decompose the variation in these implied volatilities into a part driven by the variation of the implied market volatility and a part left unexplained by the market, which we use to estimate our measure of IVOLVOL.

For our empirical analysis, we use more than 20 years of daily stock and stock options data and focus on assets with the highest liquidity in stock and option trading only. Despite having a subset of the whole stock universe the negative IVOL-return relation is prevalent in this sample too. The median size and trading volume of the firms lie in the 80%-percentile of the universe taken in comparable studies. Still, a single sort on IVOL yields a significant monthly return and Fama-French 3-factor alpha (Fama and French (1993)) for a low-minus-high portfolio of 1.19% and 1.72%, respectively. This negative relation is not puzzling though, but can be explained by a lack in the pure historic measurement of IVOL. Controlling for the expected future idiosyncratic risk level which we infer by augmenting the historic measure of IVOL with our measure of IVOLVOL resolves this finding. A conditional double sort on realized IVOL and our measure of IVOLVOL, revels that the difference in monthly returns (alpha) between low IVOL and high IVOL stocks 0.23% (0.69%) vanishes for stocks with a low IVOLVOL. However, if the IVOLVOL is high, the difference in return (alpha) between low and high realized IVOL strongly increases 2.17% (2.74%) and is highly statistically significant.

The amplification of the negative relation between realized IVOL and subsequent returns for high IVOLVOL, is not puzzling though. In essence, our measure of IVOLVOL measures the time series variation in the expectation of idiosyncratic variances. It is therefore closely connected to the mean-reversion speed of the underlying IVOL dynamics. We show that this intuition holds theoretically as well as empirically. If stocks show a high IVOLVOL, the estimated mean-reversion speed is significantly larger than for low IVOLVOL stocks. This observation holds irrespectively from the realized IVOL level. Moreover, the estimated mean-reversion levels revel that low realized IVOL is expected to increase, while high realized IVOL is expected to decrease. Combining the insights of these two findings, we can infer the expected change in IVOL levels. For slow mean-reversion (low IVOLVOL), the IVOL level is expected to stay rather constant. Thus, the historical measure of idiosyncratic risk is a rather good proxy for expected IVOL. On the other hand, low (high) realized IVOL is expected to be subject to a strong increase (decrease) if the mean-reversion speed is high (high IVOLVOL). In this case realized IVOL is a rather bad proxy for expected IVOL.

Under the assumption of IVOL caring a positive price of risk (Merton (1987)), the differences in the mean-reversion speed of the IVOL dynamics explain the findings from the double sort. Remember, if IVOL has a slow mean-reversion speed and thus is expected to change little over the next month, the return of the low-minus-high IVOL portfolio is statistically not distinguishable from zero. For these stocks, the realized IVOL level is a rather good proxy for the expected IVOL level. Thus, if realized IVOL is high (low) in the current period and it is expected to mean revert only slowly, investors demand high (low) subsequent returns, expecting idiosyncratic risk to decease (increase) by rather small amounts.[2] On the other hand, if IVOL is expected to be exposed to a fast mean-reversion, e.g., in the case of high IVOLVOL, investors adjust their expected returns. Thus, if idiosyncratic risk of a stock is cur-

[2] We find the mean-reversion effect to be still existent for stocks with low mean-reversion speed (although less strong), which eventually blurs the pure positive effect.

rently high (low) investors demand lower (higher) returns, because they expect the idiosyncratic risk to decrease (increase) by a relatively large amount. This effect explains the observed pattern in the double sort analysis and eventually resolves the IVOL puzzle.

There exists a variety of different attempts to explain the negative IVOL-return relation. In a comparative analysis we show that our measure for mean-reversion in IVOL captures indeed an important facet of the IVOL puzzle which was omitted by the literature so far. In doing so, we use the method from Hou and Loh (2016) and document that our measure surpasses and dominates the explanatory power of competing explanations. A classification of stocks with respect to the expected mean-reversion speed (proxied by IVOLVOL levels) times the lagged level of realized IVOL helps to explain around 60% of the IVOL anomaly. This interaction term is very closely connected to our previous conditional analysis. In contrast, controlling for the mean-reversion speed, other explanatory variables make up no more than 18% in total. Consequently, our measure combined with others helps to explain around 65% – 75% of the total negative IVOL-return relation.[3] For the test set of alternative explanations we follow closely Hou and Loh (2016). We include different (co-)skew measures as well as the retail trading proportion (RTP) as proxies for lottery preferences. Further, we test for market frictions by including lagged returns, the liquidity measure of Pastor and Stambaugh (2003), the proportion of zero returns and the relative bid-ask spread.

Finally, a robustness analysis confirms that our results are robust to a different measurement of IVOL as well as a different IVOLVOL measure and different portfolio sorting schemes. The results cannot be explained by stock liquidity, short-sale constraints or size. In addition, we show that investors trade options in the direction of expected changes of idiosyncratic risk, by assuming corresponding changes in the

[3] Hou and Loh (2016) find values between 29 – 54% for the combined explanatory power of the most established measures.

stocks price as a compensation for idiosyncratic risk.

The remainder of the paper is structured as follows. In the subsequent Section 2 we discuss the literature which is closest to our research. Section 3 elaborates on the concept of measuring mean-reversion in idiosyncratic risk and its link to expected returns. In Section 4 we describe our data and methods to calculate IVOL and IVOLVOL. Section 5 contains our main results. There we show the existence of the IVOL puzzle in our dataset and afterwards reason it with expected IVOL changes. The robustness analysis is conducted in Section 6 and, last, Section 7 concludes.

2 Literature Review

Our paper is related to different strands of the literature, focusing on the relation between idiosyncratic risk and expected returns as well as possible resolutions for the IVOL puzzle. Ang et al. (2006) are the first to document the negative IVOL-return relation. Stocks with low realized idiosyncratic risk exhibit higher subsequent returns than stocks with high realized idiosyncratic risk. They show in a sequential paper (Ang et al. (2009)) that the IVOL anomaly is prevalent in different markets and thus robust. However, the robustness of the negative relation is questioned by Bali and Cakici (2008) who argue that the effect is mainly driven by small stocks and the portfolio weighting scheme. We add to the literature by concentrating on the biggest stocks only. We provide strong evidence for the existence of a robust and generally negative IVOL-return relation for the subset of stocks which belongs to the 60% top size quantile of the whole universe.

The literature argues that the incorporation of expected future idiosyncratic risk is crucial for understanding the IVOL-return relation. Fu (2009) and Peterson and Smedema (2011) use EGARCH models to proxy expectations about idiosyncratic risk innovations and show that the puzzle vanishes after accounting for those.

They find the expected IVOL-return relation to be positive. However, Fink et al. (2012) question these methods by showing that the former studies are prone to a look-ahead bias. After controlling for the set of information they find the IVOL anomaly still to be prevalent. In contrast to the former authors Rachwalski and Wen (2016) argue that investors only price perceived IVOL and thus incorporate idiosyncratic information only with a lag. Following them, the puzzle merely stems from mis-measurement of current IVOL, which can be proxied by perceived idiosyncratic risk, measured in terms of realizations far in the past, and current IVOL. However, none of the former studies make use of implicit information from the options market to extract expectations about IVOL but only use stock prices. Historic stock prices lack the forward-looking features options provide. Thus, option prices add a valuable facet in analyzing the risk-return relation. We add to the literature by extracting a measure for expected IVOL innovations in an almost model-free manner, using stock option prices.

Offering another possible resolution to the IVOL puzzle, some authors connect the anomaly to short-sale constraints. Shleifer and Vishny (1997) find that idiosyncratic risk dampens the willingness to short-sell. Stambaugh et al. (2015) and Boehme et al. (2009) argue that the underperformance of high IVOL stocks stems merely from short-sale constrained stocks, since they are too expensive. The results of Figlewski and Webb (1993), Phillips (2011) and Lin and Lu (2015), amongst others, show that stocks with liquid options trading are significantly less prone to short-sell constraints and their effects.[4] Our sample consists of large stocks with highly liquid options only. Consequently, we add to this discussion by providing evidence that the IVOL anomaly is strongly prevalent in our stock universe where short-sale constraints are very weak.

[4]In addition, Hu (2017) documents that active option trading improves the overall market information environment. Blau and Wade (2013) show that if investors' short-selling costs are high they turn to the option markets to set up synthetic short positions.

We are not the first to use stock options in a joint analysis with idiosyncratic risk.[5] Aliouchkin (2015) looks at the cross-section of S&P100 options and calibrates a flexible model for the dynamics of stock prices. Subsequently, he extracts moments of the expected return distributions. He finds that the absolute idiosyncratic skewness and co-skewness is negatively related to future returns. We differ from his research in several dimensions. First, we use a considerably bigger cross-section of stocks, since we have on average 383 firms per month in our sample. Second, we use almost model-free methods and do not rely on a parametrized model. Third, we explicitly focus on expected innovations of idiosyncratic volatility.

Other authors extract information of expected idiosyncratic risk using low parametrized models. Dennis et al. (2006), Diavatopoulos et al. (2008), Moll and Huffman (2016), amongst others, employ regression models to estimate betas from the stock return distribution and calculate implied idiosyncratic volatilities from option prices and aggregate implied volatility. They find that implied idiosyncratic risk is priced and that investors take into account its innovations. However, different from our approach, their measure for expected IVOL levels can get negative since their defined IVOL innovations are only loosely connected to the measurement of IVOL by factor models in the style of Ang et al. (2006). Further, these authors do not focus explicitly on big stocks with the most liquid options. Thus, we add to the literature, in at least four ways. First, our method makes use of the he full information embedded in daily implied volatility levels on the single stock level (VIX^i).[6] For

[5] Cao and Han (2013) show that delta-hedged option returns are decreasing in IVOL. Elkamhi et al. (2011) use a measure for informed option trading and show that the more uniformed option traders the lower stock returns. Bégin et al. (2016) calibrate parametric models on single stock level for 260 stocks using options. They show that only idiosyncratic jump risk matters for the equity risk premium. Diffusive idiosyncratic risk is not priced.

[6] We measure implied volatilities for single stocks (VIX^i) as suggested by the CBOE (2016) for the S&P500. This is done by the CBOE for selected stocks too (currently Amazon, Apple, Goldman Sachs, Google and IBM), as well as selected ETFs.

its calculation we convert American option prices to European prices to get rid of the early-exercise premium and thus have more exact results of the implied volatilities. Therefore our measures are more consistent and potentially less biased compared to other work. Second, we explicitly focus on stocks with the most liquidity in option trading and make use of the information embedded in the whole time-series of daily option prices, rather than concentrating on end-of-month observations. Third, we measure expected innovations with the help of the expected mean-reversion speed and not by the difference between the current and option implied IVOL level. In doing so, our method differs from the existing literature and uses information from the option markets only and not from the underlying return distribution to infer expected IVOL changes.[7] Fourth, our method relies on a model-free relation between current IVOL levels and expected innovations. Our approach is thus consistent with the measurement of IVOL relative to a factor model.

Another strand of the literature finds that higher order risks on individual firm level such as skewness, co-skewness and volatility-of-volatility are closely related to the IVOL-return relation as well. Boyer et al. (2010) show that expected idiosyncratic skewness is negatively correlated with returns. In an extensive study Conrad et al. (2013) document a strong negative impact of individual risk neutral skewness on future returns. Harvey and Siddique (2000) find a significant risk-premium for market skewness. These findings are supported by Dittmar (2002) and Schneider et al. (2017) who provide further empirical evidence as well as theoretical explanations. Baltussen et al. (2014) document that realized volatility-of-volatility is negatively related to future returns.[8] Our paper adds to this literature and shows that higher

[7]The cited literature uses the CAPM beta to measure the exposure of firm specific volatility to aggregate (market) volatility. Our results show that both are only weakly correlated. On average the correlations of the option implied beta to the one month CAPM beta, one year CAPM beta and the beta of Frazzini and Pedersen (2014) is 15.63%, 19.55% and 20.46%, respectively.

[8]Chen et al. (2014) find the same results using high-frequency data. Bali et al. (2017) relate large changes in IVOL to firm-level news.

order idiosyncratic risk helps to explain the IVOL anomaly. Further, we show that the information contained in IVOLVOL with respect to the IVOL-return relation is different from the established (co-)skewness measures.

3 Idiosyncratic Risk and its Expectation

It is well recognized that volatility is mean-reverting and that the incorporation of this feature is essential for pricing risk.[9] Idiosyncratic volatility captures the risk of a stock in excess of its systematic risk and is naturally bounded from below as well as from above. In the extreme, a stock's volatility can either fully depend on the market or can be idiosyncratic volatility only. Consequently, IVOL should mean-revert, too, and the mean-reversion effect should affect prices, as long as we follow Merton (1987) and assume IVOL to carry a positive price of risk.[10] This is the cornerstone of our method helping us to explain the documented negative realized IVOL-return relation. Depending on the mean-reversion speed, realized IVOL is either a good or bad proxy for expected IVOL. This explains why studies relying on a pure historic measure of IVOL find a negative relation.

If idiosyncratic risk is mean-reverting, it is quite reasonable for a portfolio of very high (low) IVOL stocks that these realized IVOL levels lie on average above (below) the long-run mean and deviate from it by a large amount.[11] Clearly, the expected mean-reversion speed is directly related to how fast this deviation is expected to vanish. In order to infer the expected changes in IVOL, a measure for the expected mean-reversion speed of IVOL is thus central for our analysis. In theory as well as empirically the time series volatility of expected idiosyncratic volatility, our measure of IVOLVOL, serves as a natural proxy for the mean-reversion speed.

[9] See for example Merville and Pieptea (1989) and Heston (1993).

[10] We validate the existence of mean-reversion in IVOL levels in the empirical section.

[11] In the empirical part we show this assumption to hold.

To demonstrate the theoretical relationship we assume that IVOL follows a simple Ornstein-Uhlenbeck process:

$$d\text{IVOL}_t^i = \kappa^i \left(\overline{\text{IVOL}}^i - \text{IVOL}_t^i \right) dt + \sigma_{\text{IVOL}}^i dW_t^i. \tag{1}$$

where κ^i denotes the mean-reversion speed, $\overline{\text{IVOL}}^i$ the mean-reversion level of IVOL^i, dW_t^i describes a Wiener process, scaled by σ_{IVOL}^i.[12] If τ is some time-scale, for example one month, the conditional expectation of IVOL at time t is quite standard and is given by a weighted average of the current level of IVOL and its long-run mean:

$$\mathbb{E}_t \left[\text{IVOL}_{t+\tau}^i \right] = \text{IVOL}_t^i e^{-\kappa^i \tau} + \overline{\text{IVOL}}^i \left(1 - e^{-\kappa^i \tau} \right). \tag{2}$$

The larger κ^i, the more weight will be pushed to the long-run mean and thus the larger the expected changes in IVOL towards its long-run mean. Given a time series of expected IVOLs e.g., expectations for the next month IVOL on a daily basis over this month, we define our measure of IVOLVOL as:

$$\text{IVOLVOL}_t^i = \text{std} \left[\mathbb{E}_s \left[\text{IVOL}_{s+\tau}^i \right]_{s \in \{t-\tau:t\}} \right]. \tag{3}$$

Therefore, IVOLVOL estimates the realized variation in the expectations of IVOL over a certain period. This variation will be larger, the larger κ^i.[13] The intuition is that for a high κ^i, the expected IVOL will change by a larger amount from s to $s+1$ (e.g., from day to day) towards its long-run mean and that these larger innovations

[12] For simplicity we assume the parameters to be the same under $\mathbb{P}$ and $\mathbb{Q}$. This assumption will have no qualitative impact on our results, as long as the parameters for the different measures are positively related. Further we assume the mean-reversion speed and the long run mean to be time invariant. This is no server restriction, since our later analysis concentrates on rather short holding periods of one month.

[13] The dynamics for the expectations in Equation(2) are given by $d\mathbb{E}_t \left[\text{IVOL}_{t+\tau}^i \right] = 2e^{-\kappa^i t} \kappa^i \left(\overline{\text{IVOL}^i} - \text{IVOL}_t^i \right) dt + 2e^{-\kappa^i t} \sigma_{\text{IVOL}}^i dW_t^i$. These directly depend on κ^i and the distortion from the long-run mean.

drive the IVOLVOL up. Figure 1 demonstrates that this intuition is indeed correct. It shows IVOLVOL in relation to κ^i for different levels of distortion from the long run mean.[14] The higher the mean-reversion coefficient κ^i the larger IVOLVOL. Further, the increase of IVOLVOL in κ^i is stronger the higher the distortion from the long run mean. Thus, given a distortion from the long-run mean, the volatility of expected idiosyncratic volatility is a proxy for the mean-reversion speed.

The underlying mechanism can be understood best by considering the following simple example. Assume a stock has a high realized IVOL at time t which is well above its long-run mean. In such a case the drift component in equation (1) gets highly negative and thus has a major negative impact on future IVOL innovations compared to the Wiener process. A higher κ^i strengthens this impact and therefore the realized IVOL in $t+\tau$ will decrease towards its long-run mean by lager amounts, the larger κ^i. This in turn will give rise to a higher variation in the expected next month IVOL at time t and $t+\tau$. Thus, the time series volatility of expectations of IVOL rises in κ^i as long as there is a deviation from the long-run mean.[15]

Next to being a proxy for the mean-reversion speed, IVOLVOL has the notable feature of being very closely connected to the estimation method of IVOL and thus to IVOL itself. For the sake of simplicity, assume that IVOL is measured relative to the CAPM, for now. In particular, assume that IVOL_t at day t is estimated as follows:

$$R_s^i - r_s^f = \alpha_i + \beta_i \left(R_{s,M} - r_s^f\right) + \epsilon_{s,i}, \tag{4}$$

[14]We simulate 200,000 paths of IVOL, using equation (1) over one month. In every point in time we compute the expected next month IVOL, conditional on the current realization of IVOL e.g, $\mathbb{E}_t\left[\text{IVOL}^i_{t+\tau}|\text{IVOL}^i_t\right]$. IVOLVOL is measured as the mean of the standard deviations of the expectations on IVOL.

[15]In the later analysis we show that there is no significant difference in the long-run mean for differen IVOLVOL. Thus, it follows that any differences in IVOLVOL should be mainly driven by κ^i and not by the distortion from the long-run mean.

and $\text{IVOL}_t^i = \text{std}\left[\epsilon_s^i\right]_{s\in\{t-\tau:t\}}$, with $\tau = 1$ month.[16] If we calculate the risk-neutral expectation of the quadratic variation on both hand sides and assume a deterministic interest rate, we get:[17]

$$\left(\sigma_{i,s}^{\mathbb{Q}}\right)^2 = \gamma_i + \left(\beta_i^{\mathbb{Q}}\right)^2 \left(\sigma_{M,s}^{\mathbb{Q}}\right)^2 + \mathbb{E}_s^{\mathbb{Q}}\left[\int_s^{s+\tau} \left(d\epsilon_{i,s}\right)^2 ds\right] \tag{5}$$

$$= \gamma_i + \beta_{i,\sigma_M} \left(\sigma_{M,s}^{\mathbb{Q}}\right)^2 + \eta_{i,s}^{\mathbb{Q}}, \tag{6}$$

where $\left(\sigma_{i,s}^{\mathbb{Q}}\right)^2$ describes the expected variance of the individual stock returns and $\left(\sigma_{M,s}^{\mathbb{Q}}\right)^2$ the expected variance of the market portfolio returns over the future period. The process $\eta_{i,s}^{\mathbb{Q}}$ describes the risk-neutral expectation of the variance of the residuals $\epsilon_{i,s}$ over the future period. Thus, $\eta_{i,s}^{\mathbb{Q}} = \mathbb{E}_s\left[\text{IVOL}_{s+\tau}^2\right]$ defines the expected idiosyncratic variance over the next month.[18] Consequently, it proxies for the expected next month IVOL. Therefore, IVOLVOL states the realized variation in the risk-neutral expectation of IVOL:

$$\text{IVOLVOL}_t^i = \text{std}\left[\eta_{i,s}^{\mathbb{Q}}\right]_{s\in\{t-\tau:t\}}. \tag{7}$$

As shown above, IVOLVOL is a proxy for the expected mean-reversion speed of idiosyncratic volatility. Our method isolates this expectation with the use of forward looking information from option prices. As we explain more thorough in the next section, we employ model-free methods to calculate expected variations. Consequently, unlike other studies on expected idiosyncratic risk, we rely completely on the information available upon month t and thus do not face any lookahead bias. in addition, we do not use a high degree of parametrization, but provide a consistent and almost model-free framework which can handle expected IVOL innovations and realized IVOL levels at once.

[16] In the empirical part of our paper we define IVOL relative to the 3-factor Fama-French model.

[17] The assumption of deterministic interest rate is quite common when calculating expected variations. See for example Bakshi et al. (2003) or Jiang and Tian (2005).

[18] Note, that the $\epsilon_{i,s}$ are assumed to be normal distributed and consequently the quadratic variation indeed equals the variance over the sample path.

Figure 2 illustrates the estimation of IVOL and IVOLVOL to highlight the different information these measures carry. Both measures are estimated during the very same time period from $t-\tau$ to time t. IVOL relies on daily returns during this period only. Therefore, IVOL_t is a pure historic measure of the realized idiosyncratic volatility from $t-\tau$ until t. On the other hand, IVOLVOL_t is estimated using the time series of $\eta^{\mathbb{Q}}_{t-\tau:t}$, where each $\eta^{\mathbb{Q}}_s = \mathbb{E}^{\mathbb{Q}}_s\left[\text{IVOL}^2_{s+\tau}\right]$ stands for the risk-neutral expectation of next month idiosyncratic variance. Thus, although IVOLVOL_t is estimated from $t-\tau$ until time t, it carries information about the expected mean-reversion speed of IVOL for the period from t until $t+\tau$. Together with realized IVOL, this allows us to identify expected IVOL changes during the next month. In general, if a stock is currently in a state of low (high) IVOL, its idiosyncratic risk is likely to increase (decrease) over the next period due to the mean-reversion of IVOL. However, the expected mean-reversion speeds may be quite distinct for different stocks. These differences can be recovered from the variation in expected idiosyncratic volatility e.g., from our measure of IVOLVOL. For example, if a stock has low IVOL and high IVOLVOL (low/high) is expected to strongly increase. It is currently quite low and is subject to a large expected mean-reversion effect. In comparison, for a stock with the same low level of IVOL, but low IVOLVOL (low/low), the expected changes in IVOL are small. The reason is that a lower IVOLVOL level signals a smaller κ^i. Overall, the low/high stocks have higher expected idiosyncratic risk than the low/low stocks and should yield higher returns. The same reasoning applies to stocks with currently high IVOL. If a stock has high IVOL and high IVOLVOL (high/high), IVOL is expected to decrease over the next period. In contrast, stocks with low IVOLVOL have an IVOL with a higher expected persistence. For these stocks the magnitude of changes in idiosyncratic risk is expected to be less pronounced than for high IVOLVOL stocks. Consequently, if the IVOL of a stock is currently high and IVOLVOL is low (high/low), IVOL is expected to be more sticky and therefore less likely to decrease by a great amount. As a result, high/low stocks should yield higher returns than high/high stocks.

The relation between IVOL and IVOLVOL, described above, leads to two hypotheses regarding the risk-return relation of stocks with respect to idiosyncratic risk. These hypotheses should hold if expected idiosyncratic risk is positively priced (Merton (1987)) and given that IVOLVOL is a proxy for the mean-reversion effect.

Hypothesis 1: The negative IVOL-return relation should vanish for low IVOLVOL stocks, because the idiosyncratic risk of these stocks has a low mean-reversion speed. In this case the realized IVOL level is a rather good proxy for the expected future IVOL level. Hence, low (high) realized IVOL signals low (high) future expected idiosyncratic risk and thus lower (higher) returns.

Hypothesis 2: For high IVOLVOL stocks, the difference in returns between low and high IVOL stock should increase, because the idiosyncratic risk of these stocks has a high mean-reversion speed. In this case the realized IVOL is a poor proxy for expected future IVOL levels, because currently high (low) IVOL signals lower (higher) expected future IVOL.

4 Data and Methodology

This section describes the data used in the empirical part later on and explains the construction of key measures of our analysis, IVOL and IVOLVOL.

4.1 Data

We merge three different databases for the sample period from 1996/01 to 2016/04, which spans more than 20 years of data. We use daily bid/ask prices, implied volatilities, trading volumes, and open interests of American stock-options as well of SPX options and the zero yield curve from Ivy DB US provided by OptionMetrics. From

CRSP we obtain daily and monthly stock data, such as split-adjusted returns, prices, dividend amounts, dividend frequency and trading volume. Further, to calculate the book-to-market ratio we include the book-value on an annual basis from Compustat in our analysis. Last, we obtain daily Fama-French factors from Kenneth French's data library.

We calculate the option price as the mid-point of bid/ask prices. To provide a reliable data basis for our analysis we employ several filters, which are quite similar to Goyal and Saretto (2009). First, we exclude all options with zero open interest, zero volume, no implied volatilities and which violate standard no-arbitrage bounds or where the bid price is below the ask price. Second, we follow OptionMetrics' pricing approach for American options to calculate synthetic prices of corresponding European stock options. Given an implied volatility, we re-price all quoted American options using a Cox et al. (1979) (CRR) tree with 1000 steps and incorporate discrete dividends. For dividend amounts and dates we use CRSP quotes. For precision, we discard all options with a relative pricing error of the calculated CRR American option price to the quoted mid bid/ask price larger than 1%. Next, we use the same CRR trees to calculate European option prices and thereby account explicitly for the early exercise premium of American options. This is essential for the later construction of the $(\text{VIX}^i)^2$ on the single stock level. This approach is quite accurate as shown for example in Tian (2011) and Ju and Zhong (1999) and comparable to Broadie et al. (2007), since the CRR pricing model converges to the Black-Scholes model if the step size goes to zero. After these filtering methods and requiring at least 15 observations for the IVOL and IVOLVOL estimation, our whole sample comprises 3232 firms. Each month, our cross-section consists of 388 firms on average.

4.2 Measurement of IVOL and IVOLVOL

To estimate the idiosyncratic volatility on the individual stock level we follow the established approach of measuring it relative to the Fama-French 3-factor model. We first regress contemporaneously daily excess returns over one month on the three factors, excess return of the market portfolio, high-minus-low book-to-market ratio portfolio and the high-minus-low size portfolio. Afterwards, we define IVOL as the standard deviation of the model's pricing errors. This leads to the following measurement of IVOL for month t

$$R_s^i - r_{f,s} = \alpha^i + \beta_{MKT}^i MKT_s + \beta_{HML}^i HML_s + \beta_{SMB}^i SMB_s + \epsilon_s^i, \tag{8}$$

$$\text{IVOL}_t^i \equiv \text{std}\left[\epsilon_{t-\tau:t}^i\right], \tag{9}$$

where τ equals one month, $r_{f,s}$ is the risk-free rate and R_s^i are daily stock returns.

Measuring volatility of idiosyncratic volatility follows a quite similar pattern, since we define IVOLVOL as the standard deviation of a contemporaneous regression again. However, equation (6) requires to measure the risk-neutral expectations of the variation in market and stock returns. A natural and model-free measure of expected market volatility under the risk-neutral measure is the VIX, provided by the CBOE. Thus, we build on this approach and compute a VIX^i on a single stock level. This allows for computing IVOLVOL from equation (6). More accurately, for month t we regress daily VIX^i levels on the market VIX^M

$$\left(\text{VIX}_s^i\right)^2 = \gamma^i + \beta_{VIX}^i \left(\text{VIX}_s^M\right)^2 + \eta_s^i, \tag{10}$$

$$\text{IVOLVOL}_t^i \equiv \text{std}\left[\eta_{t-\tau:t}^i\right]. \tag{11}$$

To ensure reliable results of our estimation, we consider stocks only if we have at least 15 daily returns R_s^i and at least 15 daily VIX_s^i observations within a month. We therefore concentrate on stocks which have the highest liquidity in option trading. Note, that our IVOLVOL estimate proxies the exact IVOLVOL only. The calculation of our IVOL relies on the Fama-French 3-facter model, while equation (10) we assume the CAPM. However, all our results hold if we estimate IVOL relative to the CAPM or the Fama-French 5-factor model (Fama and French (2015)), as we show in the robustness part. In addition, enhancing the IVOLVOL estimation by including measures for systematic volatility and jump risks, like the realized variance or bipower variation from Corsi et al. (2010) for the S&P500 or VIX, increases the explanatory power of the model to estimate IVOLVOL only, but has little impact on the results of the later sorting exercise. Finally, imposing the CAPM by including the VIX in equation (10) only, leads to a more noisy measure of IVOLVOL and therefore should weaken our later findings, so that results with an exact estimation of IVOLVOL should be even stronger. Therefore, we stick to the most straightforward model to estimate IVOLVOL and calculate VIX_t^i of the individual stock on day t as

$$\left(\text{VIX}_t^i\right)^2 = \frac{2e^{r_{f,t}\tau}}{\tau}\left[\int_0^{F_{t,T}^i} \frac{P_t^i(K)}{K^2}dK + \int_{F_{t,T}^i}^{\infty} \frac{C_t^i(K)}{K^2}dK\right], \tag{12}$$

where $F_{t,T}^i$ is the forward price, $P_t^i(K)$ are put prices and $C_t^i(K)$ are call prices with strike K and maturity $\tau = 1$ month.[19] We use the set of options with maturity of exactly one month as long as they are available. Otherwise we use two sets, one with maturity τ_1 below one month and a second with maturity τ_2 above one month. In each case we follow Jiang and Tian (2005). We interpolate implied volatilities across strikes using spline interpolation and extrapolate using the quoted implied volatility of the highest or lowest strike, respectively. If necessary, we interpolate the implied volatilities linear across maturity to get prices of options with one month to maturity and employ the above formula. On average we use a quoted subset of 8 options per day, which is extended to roughly 10 options per day using the put-call parity, to build the $(\text{VIX}^i)^2$.

[19]In unreported results we although test the method of Bakshi et al. (2003) to measure implied variances and find that our results remain qualitatively unchanged.

5 Results

In this section we present our results. First, we analyzing the relation between realized IVOL and subsequent returns. Second, we test our hypothesis with respect to the IVOL anomaly and the IVOLVOL.

5.1 Realized Idiosyncratic Risk and Expected Returns

Ang et al. (2006) and most follow-up studies analyze the IVOL puzzle by looking at the full stock universe quoted at NYSE, AMEX and NASDAQ. In contrast, we use stocks for which options are traded only and in addition filter for option liquidity. Thus, we analyze a subsample compared to previous studies. Table 1 highlights some key differences of our sample compared to the usual NYSE, AMEX and NASDAQ sample for the same sample periods. The table reports the mean, median and percentiles of firm size and trading volume in stocks. With regard to both aspects our sample consist of the largest and most liquid stocks compared to the total universe. The median firm size of $ 1,422 million in our sample is higher than the 80% decile of all firms in the total universe. The same holds true for the median trading volume of $ 408 million.

Due to this quite different sample we analyse the relation between realized idiosyncratic risk and subsequent returns first. Table 2 reports results of single IVOL sorts. Following Ang et al. (2006) we sort stocks into quintile portfolios each month, such that their realized one month IVOL is increasing in portfolio rank and hold these portfolios over the following month. Next to mean excess returns of equally weighted and value weighted portfolios, we report Fama-French 3-factor alpha for both. Table 2 clearly indicates that the puzzle is prevalent in our sample. The high IVOL portfolio yields significantly lower alpha than the low IVOL portfolio, for both, value and equally weighted portfolios. On a return level we find only the monthly excess return for equally weighted portfolios (1.19%) to be significant. However, for value weighted portfolios, there is a significant difference in alpha of 1.21% between the low and high IVOL portfolio, while for equally weighted portfolios the difference is 1.72% and highly significant, too. Thus, these findings confirm that the negative IVOL-return relation is prevalent in our sample if we sort on a realized measure of IVOL.

Finding the negative IVOL-return relation to be prevalent is challenging for explanations based on limits of arbitrage at least in our sample. Those explanations rely on the assumption that investors might be unable to exploit an arbitrage opportunity, since they might face short selling restrictions or short selling might be too expensive. However, our sample consists of very large stocks only, with high liquidity in option and stock trading. For these stocks, short selling is considered to be comparatively easy and less expensive.[20] Even though Bali and Cakici (2008) find the negative relation between realized IVOL and subsequent returns to be quite sensitive to the portfolio weighting scheme and only to be prevalent for value weighted portfolios, we find the exact opposite result in our sample with respect to excess returns, while the weighting scheme is irrelevant for Fama-French alpha. This suggests that the puzzle might be driven by stocks which are among the smallest in our sample. Nevertheless, these stocks are still be among the largest compared to the sample in Bali and Cakici (2008), as indicated by Table 1. Therefore, this finding is no contradiction to previous studies. Interestingly, other studies often find a non-linear but hump-shaped pattern in returns or alpha when sorting on realized IVOL. In contrast, in our sample, where stocks and options are very liquid, we find returns and alphas to be strictly decreasing in IVOL. Still, the puzzle seems to be mainly driven by the high IVOL portfolio, where the drop in return or alpha is the highest.

Having very strong evidence, we conclude that the negative relation between

[20] See for example Figlewski and Webb (1993), Phillips (2011) and Lin and Lu (2015), amongst others.

the historical measure of realized IVOL and subsequent returns is evident in our sample. This holds especially for equally weighted returns.

5.2 Variation in Expected Idiosyncratic Risk and its Mean-Reversion Speed

Our hypothesis are based on two key aspects. First, we assume IVOL to be mean-reverting as has been proven for total volatility. Second, we argue that the time series variation in expected IVOL levels, expressed by our measure of IVOLVOL, is a proxy for the mean-reversion speed. If both aspects hold true, we are able to infer the direction and the strength of expected changes in IVOL by looking at the current realization of IVOL and our measure of IVOLVOL simultaneously.

In order to test the first assumption, Table 3 reports results of an augmented Dicky-Fuller test. There we test the stationarity of the IVOL time series for every stock in our sample. The null states that the time series has a unit root, while the alternative hypothesis assumes stationarity of the time series without a drift and trend. The values in the first three rows report the percentage of rejected null hypothesis in favor of the alternative hypothesis for significance levels of 10%, 5% and 1%. The last row shows the number of timer series included. The columns report results for different required minimum lengths of each time series (12 to 120 month). These results show that a large majority of the IVOL time series is stationary. At the 5% significance level e.g., the null is rejected for 78.53% (minimum of 12 observations) up to 97.58% (minimum of 120 observations) of all IVOL time series included.[21] Therefore, we conclude that IVOL is in general stationary and thus has to show a mean-reverting behavior.

[21]Note, that our sample covers two crisis periods (dot-com and financial crisis) where volatility did tent to spike up. If shorter IVOL time series mainly cover one of those periods, the lower percentage of rejected nulls might be biased by these two crises.

Next, we turn to the mean-reversion speed of IVOL and check whether our measure of IVOLVOL proxies for it. Therefore, we estimate the Ornstein-Uhlenbeck process in equation (1) for every IVOL time series using maximum likelihood.[22] Since we are interested in the direct relation between κ and IVOLVOL, we make use of the cross-sectional information every month and infer if an IVOL shows relatively high IVOLVOL in that month. In addition, we also use the cross-sectional information to infer if a stocks shows a current high or low IVOL. This allows us to test for differences in the long-run mean $\overline{\text{IVOL}}$ conditional on a certain IVOL and thus for the expected direction of IVOL changes. Therefore we obtain four states characterised by an high or low IVOL/IVOLVOL and estimate the process for these states separately. That is, given an IVOL time series, we estimate equation (1) four times and each time use observations where the IVOL belonged to the specific state only. To obtain reliable results we include stationary IVOL time series only, which have at least one year of observation. In addition, we include a given estimate only if the IVOL time series belonged at least half an year to that specific state. Table 4 reports results of this estimation. There we report the average of all cross-sectional estimates for a given state. Panel A of Table 4 reports the mean-reversion speed κ. Overall, there is a quite large dispersion in κ. The overall lowest mean-reversion speed of 0.59 is observed for low IVOL and IVOLVOL, while the overall highest mean-reversion speed of 1.14 belongs to high IVOL and IVOLVOL. Currently low IVOL seems to show on average a lower κ than high IVOL. Depending on the IVOLVOL, the differences -0.39 and -0.35 are both significant on the 1% level.[23] More important, however, are the differences between states of low and high IVOLVOL. For both, high and low IVOL, the difference is -0.13 and -0.16, respectively and highly significant

[22]In unreported results, we have checked different estimation methods. The results stay qualitatively the same.

[23]Note, to calculate any difference, an IVOL time series is required to belong to each of the two states for at least half a year. Therefore, all reported averages for the differences might differ from simply taking the difference between the reported averages of the two states.

on the 1% level. That is, a higher cross-sectional IVOLVOL indeed proxies for the mean-reversion speed of IVOL.

Panel B of Table 4 reports the estimated mean-reversion level of IVOL. Two aspects are especially notable. First, there is no significant difference in the mean-reversion levels between low and high IVOLVOL. Thus, controlling for IVOLVOL has no impact on the expected IVOL level in the long run. In addition any differences in the IVOLVOL for a given realized IVOL level, should purely stem from the mean-reversion speed and not from the distrortion from the long-run mean. Second, the mean-reversion levels of high IVOL lie below the levels of low IVOL. We find both differences to be highly statistically significant and for high IVOLVOL (-1.56%) to be slightly higher than for low IVOLVOL (-1.50%). This finding highlights two things. First, we can conclude that high IVOL is expected to decrease and low IVOL to increase. Moreover, in the long run current high IVOL is expected to lie below current low IVOL. Since the mean-reversion speed is higher for high IVOLVOL, levels are expected to change faster for these stocks. This confirms that IVOLVOL proxies for the mean-reversion speed and if we consider the current realized level of IVOL, we are able to infer the direction of the expected changes in IVOL.

After having confirmed that our measure of IVOLVOL helps inferring the strength of expected IVOL changes and that the current level of IVOL indicates the direction of expected changes, we turn our attention to the realized mean-reversion effect on the IVOL time series. As shown above, we expect that a currently low (high) IVOL will tend to increase (decrease) on average. This increase (decrease) should be stronger the higher the IVOLVOL. Table 5 reports the mean-reversion effect for different IVOL/IVOLVOL portfolios. In the table, we follow our later analysis and run a dependent double sort first. We sort our stock universe into quintile portfolios based on their IVOL and then split each portfolio into three sub-portfolios based on the IVOLVOL. For the resulting portfolios, we look at the average change in IVOL and run the following regression:

$$\Delta \text{IVOL}_{t+1}^{PF} = \alpha^{PF} + \gamma^{PF}\text{IVOL}_t + \epsilon_{t+1}^{PF}. \tag{13}$$

Here, $\Delta\text{IVOL}_{t+1}^{PF}$ is the change of the average IVOL of a given portfolio over the next month and α^{PF} is a constant. In this regression γ^{PF} gives the direction and the strength of realized IVOL on the next month IVOL chances. Table 5 reveals that the directions of the IVOL movements are as expected from the previous analysis. Regardless of the level of IVOLVOL, there is an average increase in IVOL if the current IVOL level is very low and a decrease if it is very high. However, there are differences in the magnitude of change in the levels of IVOL. While the effect for low IVOLVOL is positive only for the lowest IVOL portfolio, for high IVOLVOL the effect changes sign for the forth and fifth portfolio only. More importantly, however, the strength of the mean-reversion effect clearly depends on our measure of IVOLVOL. Conditional on low IVOL, we find an insignificant effect of 0.03 for the low and medium IVOLVOL portfolios. This is not surprising, since our previous analysis showed the highest persistence for low IVOL and IVOLVOL. In contrast, for high IVOLVOL, where the mean-reversion speed was significantly larger, the effect gains economically and statistically much more power and equals 0.16. The same holds true for high IVOL. There the effect for all IVOLVOL portfolios is statistically highly significant. While it is -0.23 for lowest IVOLVOL, it is -0.32 for the high IVOLVOL portfolio, indicating that high IVOL portfolios are subject to a higher decrease than the increase of low IVOL portfolios, which goes along with the higher mean-reversion speed for high IVOL as shown in de previous analysis. Therefore, we conclude that IVOLVOL indeed indicates the mean-reversion speed and can be used together with the historically realized IVOL to infer the direction of the expected and also realized IVOL movements.

5.3 Expected Idiosyncratic Risk and Expected Returns

Having confirmed that IVOLVOL proxies for the mean-reversion speed, we now test if the mean-reversion speed helps explaining why we observe a negative relation between the pure historic measure of IVOL and subsequent returns (see section 5.1). Our goal is to analyze if the negative relation stems from a flaw in the realized IVOL measure since IVOL is mean reverting.

Tables 6 and 7 report results for dependent 5×3 portfolio double sorts. For these, we sort our stock universe into quintile portfolios each month based on their realized IVOL level first. This gives us the usual IVOL sort we are aiming to explain. Then each IVOL portfolio is split into three independent portfolios according to the stocks measures of IVOLVOL.[24] As a starting point, Figure 3 shows the cumulative monthly log returns for the equally weighted low - high IVOL portfolios, conditional on the level of IVOLVOL. As can be seen, accounting for the mean-reversion speed of IVOL has a strong impact on the portfolio's performance. It holds the higher the IVOLVOL the larger the return of the difference portfolio. The difference portfolio for high IVOLVOL stocks yields a cumulate log-return of more than 4 over the whole sample period. This difference portfolio performs well over the whole sample. However, for low expected variation in idiosyncratic risk levels the cumulative return of a low-minus-high IVOL portfolio is slightly negative and almost zero.

For a more detaild analysis, Panel A of Table 6 reports Fama-French alphas of equally weighted portfolios for the conditional sort. In the table the extreme IVOL/IVOLVOL portfolios are of most interest, that is the low/low, low/high, high/low and high/high portfolios and the differences in alpha between those. For

[24]Note, in order to be closer to our hypothesis, we should sort the IVOL conditional on a certain mean-reversion speed level. In unreported results we check conditional double sorts, sorting on IVOLVOL first and then on IVOL. In this case, all our results are even stronger. However, this sorting schema makes it harder to put the results into perspective to the single IVOL sort. Since we aim to explain the findings in section 5.1, we stick to sorts conditional on IVOL.

these portfolios, we expect the distortion of the IVOL from its long-run mean to be the highest on average and thus the mean-reversion speed to have the highest influence. As for the single sort in section 5.1, the alpha is positive for low IVOL portfolios, being weakly significant for low IVOLVOL and highly significant for high IVOLVOL, and changes sign for higher IVOL portfolios. The alpha decreases faster in the IVOL portfolios for stocks with higher IVOLVOL. The second last column of Panel A reports the alpha for the difference between the low minus high IVOL portfolio for every IVOLVOL bucket. All results speak strongly in favor of our hypotheses. Precisely, the alpha of the difference portfolio is 0.69% if IVOLVOL is low and only weakly statistical significant. As pointed out the mean-reversion effect is much slower for these stocks and high (low) IVOL will stay rather high (low). Thus, the historically realized IVOL is a rather good proxy for the expected IVOL levels and the observed negative relation vanishes. In contrast, the significance is economically and statistically much stronger if we look at the alpha of the difference portfolio when IVOLVOL is high, which is 2.74%. Compared to the alpha of 1.72% of the previous single sort in Table 2, the negative relation between the historically realized IVOL and subsequent alpha is boosted by quite a large amount. Again, these stocks are characterised by an IVOL that is much faster mean-reverting. Therefore, it is very likely that a currently low (high) IVOL will increase (decrease) by a greater amount and thus investors demand a higher compensation for these stocks. The last row of Panel A in Table 6 reports the difference in alpha between the lowest and highest IVOLVOL portfolio for every realized IVOL bucket. Looking at these results, there is a significant negative difference in alpha between the low/low and low/high portfolios of -0.44%. On the other hand, there is a highly significant positive difference of 1.69% between the alpha of high/low and high/high portfolios. This finding is especially interesting. Low IVOL stocks earn higher returns the higher the mean-reversion speed and high IVOL stocks earn lower returns the higher the mean-reversion speed. Both findings can be explained by the expected future IVOL levels.

For high IVOLVOL and low IVOL, IVOL levels are expected to be above the levels expected when IVOL an IVOLVOL are low. On the other side, for high IVOLVOL and IVOL, the expected IVOL levels are below the expected levels when IVOL is high and IVOLVOL low as pointed out by Table 4. Thus, given a positive price of risk we necessarily should observe this pattern in alpha.

Panel B of Table 6 reports excess returns of the very same equally weighted portfolios. Overall, the results are similar to the case of the Fama-French alpha. Returns tend to be positive for low IVOL portfolios and decrease in the IVOL rank. However, they get negative only for the highest IVOL portfolios which have either a medium or high IVOLVOL. Looking at the differences in returns between low and high realized idiosyncratic risk portfolios in the second last column reveals that the difference of 0.23% between low/low and high/low is not significantly different from zero. In contrast, the difference between low/high and high/high of 2.17% is highly significant in a statistical and economical sense. Again, compared to the 1.19% of the simple single sort, this is a significant increase in the negative relation. The last row of Panel B shows that the 1.3% and -0.64% difference between low/high and high/high and between low/low and high/low, respectively are highly significant again. Once more, this is as expected in IVOL is mean-reverting with different speed and if IVOL carries a positive price of risk. Table 7 reports alpha and excess returns for conditional sorts of value weighted portfolios. The results for alpha and reurns are similar compared to equally weighted portfolios. If the IVOLVOL is low, the alpha of the difference portfolio is not significant and 0.41%. If the IVOLVOL is high, it gets significant and equals 2.11%. However given a certain IVOL level, only the difference between high/low and high/high has a highly significant alpha of 1.58%. The results for excess returns in Penal B look the same compared to alpha. Controlling for the mean-reversion speed, ensures that results are highly significant for the difference between low/high and high/high portfolio returns (1.59%). Still, we find no significance between low/low and high/low portfolio returns (0.03%), as expected. Again, the last row revels that for high IVOL returns are significantly lower if IVOLVOL is high (1.58%). Allthoug we find no significant difference between the return of low/low and low/high (-0.30%), the results for value weighted returns are especially interesting. Remember, we did not find a significant negative relation between realized idiosyncratic risk and subsequent returns for value weighted portfolios in the first place, as we show in Table 2. However, if we control for the mean-reversion speed of IVOL, we find a significant pattern for high IVOLVOL and thus not only an economically but also statistically amplification of the negative IVOL-return relation.

By construction our measure of IVOLVOL imposes a certain degree of correlation with IVOL. Remember, the time series of expected IVOL in Equation (2) depends not only on the mean-reversion speed, but on the current realization of IVOL too. That is why IVOLVOL and IVOL are theoretically and empirically correlated and it is not possible to disentangle both without loosing valuable information on κ.[25] Table 8 reports the average level of IVOL for our portfolio sorts. There the correlation of IVOL and IVOLVOL is visible. The last row reports the differences in average IVOL for different IVOLVOL levels. While all these differences are statistically significant, only the difference for the high IVOL portfolio is economically meaningful. All other differences range from 6 to 12 basis points. For the fourth portfolio already, the difference in mean IVOL levels is only -0.11% and economically negligible. In addition, the differences in mean IVOL between low and second highest IVOL are almost exactly the same for low IVOLVOL (-1.63) and high IVOLVOL (-1.62), as can be seen in the last column of Table 8. Therefore, any difference in returns of the 1 - 4 portfolio should not be driven by a different IVOL exposure but solely stem from the differences in the mean-reversion speed. Thus, in order to make sure that our findings are not solely driven by correlation matters, the last columns

[25] In the robustness part we run a double sort with a orthogonalized version of IVOLVOL. We find the results to be robust if the measure of IVOLVOL is completely uncorrelated to IVOL.

of Table 6 and 7 report the difference between the low and the second highest IVOL portfolios (1 - 4).

Table 6 and 7 confirm our previous reasoning. Although, the differences in returns and alpha decrees in absolute terms, all 1 - 4 differences except for value weighted returns stay significant for high IVOLVOL and insignificant for low IVOLVOL. For equally weighted portfolios we find a highly significant alpha of 1.53% and a significant return of 1.12%. The alpha (0.97%) for value weighted returns is a bit smaller in absolute terms but still statistically significant. Again, the negative IVOL-return relation is weaker in our sample on an aggregated level for value weighted portfolios. However, we find still a significant alpha for high mean-reversion. Even if we do not use the most extreme portfolios and if the difference portfolios have the exact IVOL exposure. Therefore, we conclude that our previous findings are not driven by differences in the IVOL exposure and thus have to be driven by the mean-reversion speed.

Reconciling the findings, we see evidence that the negative relation between the historically realized IVOL and subsequent returns (as we find it in 5.1) vanishes if we expect a low mean-reversion speed of IVOL. On the other hand, the negative relation becomes much stronger if we expect IVOL to mean-revert quickly. Both findings speak strongly in favor of our hypotheses. However, why does previous research find a negative relation between the historic measure and subsequent returns in the first place if the effects go in opposite directions? The explanation for that can bee seen if we reconsider the results from section 5.2. Even if we expect IVOL to mean-revert only slowly, the mean-reversion effect is still prevalent, although at a much lower level, and we see a general increase (decrease) for low (high) IVOL stocks. Therefore, we see an overall negative pattern.

5.4 Mean-Reversion Speed vs. Competing Explanations

All evidence thus far strongly speaks in favor that controlling for the mean-reversion speed in IVOL offers an explanation for the IVOL puzzle. However, there exists a battery of other attempts to solve the puzzle, as pointed out in section 2. If the mean-reversion speed in IVOL is the central driver in observing a negative relation between past realized IVOL levels and subsequent returns, it should be able to beat these competing explanations in a direct horse race. Therefore, we make use of the method in Hou and Loh (2016), which allows to decompose the IVOL coefficient of a cross-sectional regression of returns on IVOL, with respect to various competing explanations. To do so, we closely follow Hou and Loh (2016) and regress in a first stage realized returns on the lagged IVOL:

$$R^i_{t+1} = \alpha_{t+1} + \gamma_{t+1}\text{IVOL}^i_t + \epsilon^i_{t+1} \tag{14}$$

In this setup, a negative average γ simply captures the negative relation between the realized IVOL and subsequent returns. Panel A of Table 9 displays the intercepts and coefficients of this cross-sectional regression, where we regress the lagged IVOL^i on returns at every point in time and then average over time. In addition, it states the average size of the cross-section. Next to raw returns, we use Fama-French 3-factor alpha and stock characteristic-adjusted returns, following Daniel et al. (1997) (DGTW), as independent variable. The coefficients are all negative and range from -0.2654 for raw returns to -0.0124 for 3-factor alpha. This fact highlights once more the existence of the IVOL puzzle in our data sample, since all coefficients are significantly negative except for the Fama-French 3-factor alpha.[26]

In essence the applied method aims to analyse the potential different explana-

[26] Note, that the IVOL coefficient is not significant for Fama-French 3-factor alpha. However, this is just a matter of smaller sample due to adding the lagged IVOLVOL. Table 10 highlights that there is a significant negative coefficient for the Fama-French 3-factor alpha if we use less restrictive controls and thereby increase the average size of our cross-section.

tions carry for explaining the γ of the first stage regression. Therefore, it is necessary to capture the orthogonal information every control carries separately, as Hou and Loh (2016) point out. Thus we run a second stage regression, where we regress IVOL instantaneously on the different controls:

$$\text{IVOL}_t^i = a_t + \delta_t \text{Controls}_t^i + \zeta_t^i \tag{15}$$

where δ_t is a $n \times 1$ vector and Controls_t^i a $1 \times n$ vector. In Panel B of Table 9 we report results of this cross-sectional regression. Since it is the same for all independent variable sets in Panel A, the coefficients are identical. To test our hypothesis we include three dummy variables indicating if a stock's lagged IVOLVOL_{t-1}^i lies in the lowest, middle or highest tertile and interact these with the lagged IVOL_{t-1}^i level. This closely follows our previous analysis. According to our arguing and the previous findings, the mean reverting behaviour of IVOL drives the negative relation between the measure of realized IVOL and subsequent returns. Thus, we expect the negative influence of IVOL on returns to be strongest for $\text{D}_{t-1}^{high} \times \text{IVOL}_{t-1}^i$. For these we expect the realized IVOL_{t-1}^i to be a bad proxy for $\mathbb{E}_{t-1}[\text{IVOL}_t^i]$ and therefore displaying the negative relation between IVOL_{t-1}^i and subsequent returns. We follow Hou and Loh (2016) and include next to our $\text{D}_{t-1}^i \times \text{IVOL}_{t-1}^i$ measures the most promising alternative explanations. Precisely, we add realized skewness, risk-neutral skewness, co-skewness, the retail trading proportion (RTP), lagged returns, the proportion of zero returns, the liquidity measure of Pastor and Stambaugh (2003) and the relative bid-ask spread. In contrast to our control for the mean-reversion we add these measures instantaneously, which should induce some hurdle for our explanation. The results in Panel B show that all coefficients of our measures and all other controls except for liquidity are highly significant, and thus directly related to IVOL. Using the δ's, we can isolate the orthogonal effect the single controls carry in explaining the IVOL.

Using the results from the second regression, the method decomposes the γ

coefficient from the first regression:

$$\gamma_{t+1} = \frac{\text{Cov}\left[R^i_{t+1}, \delta_t \text{Control}^i_t\right]}{\text{Var}\left[\text{IVOL}^i_t\right]} + \frac{\text{Cov}\left[R^i_{t+1}, (a_t + \zeta^i_t)\right]}{\text{Var}\left[\text{IVOL}^i_t\right]} \tag{16}$$

where the first term states the part of γ which stems from the orthogonal variation of the controls with IVOL and the second term the residual part which is left unexplained by the controls. Thus, the computed coefficients and the residual coefficient will add up to γ from the first stage regression by design. Panel C of Table 9 clearly indicates that the proposed relation of our measures holds. $\text{D}^{high}_{t-1} \times \text{IVOL}^i_{t-1}$ has the highest impact. Compared with the other two IVOLVOL measures it is the only one showing a negative sign. It is the only control, which shows a significant coefficient for all three return proxies. It is able to explain the IVOL coefficient by significantly 62.25% up to 87.08%, while all other controls explain much less of the γ coefficient. The findings are a strong support for the IVOL puzzle being driven by the mean-reversion. This fact is bolstered when looking at the other controls. In line with Hou and Loh (2016), we find that many potential explanations have little power in explaining the puzzle. RTP is the second best performing explanation, showing an explanatory power of $-0.84\% - 15.86\%$. However, these lie far below the explanatory power of $\text{D}^{high}_{t-1} \times \text{IVOL}^i_{t-1}$. Moreover, all competing explanations taken together are in no case able to beat the explanatory power of $\text{D}^{high}_{t-1} \times \text{IVOL}^i_{t-1}$. In addition, the three IVOLVOL controls together explain by far the largest fraction of the IVOL puzzle. While all explanations together are able to explain 76.08%, 64.67% and 76.55%, our controles for the mean-reversion speed alone are responsible for 58.56%, 67.09% and 63.78%, respectively. Thus, due to controlling for the mean-reversion speed we are able to boost the explanatory power way above the reported 29%-54% in Hou and Loh (2016). As one might argue including a lagged IVOL on the one side is an unbalanced horse race, we repeat the analysis using just the IVOLVOL dummies only and do not interact with the lagged IVOL. Although we deviate with this setup from our hypothesis a bit, this method imposes a much larger hurdle in explaining the IVOL return relation compared to the previous analysis, by conditioning on the IVOLVOL level only. Results of this exercise are presented in Table 10. Interestingly, all findings stay qualitatively the same. Still, the highest IVOLVOL dummy has the highest potential in explaining the IVOL coefficient and is not beaten by any other control. It explains from 27.56% up to 37.55% of the coefficient. Taking all three dummies together, they add the most explanatory power compared to all other controls together once more. All in all, these results fully support our hypothesis and show that even simple dummies to control for mean-reversion effects are superior to many other famous explanations for the IVOL puzzle.

6 Robustness

In this section we undertake a robustness analysis with respect to the model which is used in the estimation of idiosyncratic risk. Further we provide evidence that the underperformance cannot be explained by other stock characteristics and that our reasoning is supported by option trading data.

6.1 Controlling for Measurement of IVOL

Throughout the paper we estimate idiosyncratic risk relative to the Fama-French 3-factor model. However, our IVOLVOL measure is calculated relative to the market VIX and thus measures mean-reversion effects of idiosyncratic volatility relative to the CAPM. To check if our main findings still hold in a more consistent setting we calculate IVOL relative to the CAPM and report excess returns as well as the mean-reversion effect in Table 11 and Table 12. Table 11 shows that our main findings remain unchanged overall. When we calculate idiosyncratic volatility relative to the CAPM the IVOL-return relation still vanishes completely for low IVOLVOL stocks and is strengthened for high IVOLVOL stocks. In addition, Table 12 documents that our explanation for the existence of the negative IVOL-return relation is still valid. The mean-reversion effect is still larger for high IVOLVOL stocks with signs in the right direction. Interestingly, the absolute effect remains overall on the same level, but the significance for low/high stocks is dampened to a 10% significance level.

6.2 Controlling for Correlation Effects

Our measure of IVOLVOL and IVOL are correlated by construction, as pointed out before. To provide additional evidence, next to the 1 - 4 differences in the main analyzes, that our results are not driven by this correlation we orthogonalized our measure of IVOLVOL with respect to IVOL. Therefore we run every month a cross-sectional regression of IVOLVOL on IVOL. Thereby, we regress IVOLVOL on IVOL and define the orthogonal IVOLVOL as the regression residual. This method ensures that IVOL and orth. IVOLVOL are completely uncorrelated in each point of time. Table 13 reports excess returns for an equally weighted double sort on IVOL and orth. IVOLVOL. Overall, our main results are unaffected by the orthogonalization of IVOLVOL. Still, the 1 - 5 difference is insignificant for low orth. IVOLVOL, but highly statistically and economically significant for high orth. IVOLVOL (1.71%). Note, that we loose valuable information on κ by the orthogonalization and thus find no significance for the 1 - 4 portfolios. This fact can bee seen by looking at Table 14. Still, it holds, the higher orth. IVOLVOL the higher the mean-reversion effect in IVOL. However, for high IVOL the difference in the mean-reversion effect for low and high orth. IVOLVOL is smaller compared to the previous results. Even though this is the case, our explanation is robust.

In addition, we test a different sorting scheme. Table 15 reports equally weighted results of a simple independent 2×2 double sort. Both, for alpha and excess returns the results point in the very same direction as before. For low IVOL, we find no significant difference in return or alpha for the high minus low IVOL portfolio. In contrast, the difference portfolio shows a highly significant excess return (alpha) of 0.73% (0.95%) on average for high IVOLVOL stocks. Therefore we conclude that our results are neither driven by a biased estimator of idiosyncratic risk, nor by a specific sorting technique.

6.3 Controlling for Fama-French's Five Factors

In our main analysis we calculate risk-adjusted returns relative to the Fama-French three-factor model, to remain comparable to the literature. However, Fama and French (2015) extend their model by an investment as well as a profitability factor. These factors are different from the classical ones and may proxy risk factors which are important to explain the IVOL-return relation. Thus, we calculate risk-adjusted returns relative to the five-factor model. Table 16 shows alpha for a double-sort on IVOL and IVOLVOL. The table indicates that our findings and our explanation are robust to the 5-factor model of Fama and French (2015). The returns of the difference 1 - 5 and 1 - 4 portfolios vanish for low IVOLVOL stocks and are amplified for high IVOLVOL stocks.

6.4 Controlling for Stock Characteristics

In this section we only concentrate on portfolios of high IVOLVOL stocks since we find significant differences in returns for those only. However, these results might be driven by some special stock characteristics. We first perform the usual conditional 5×3 sort on idiosyncratic risk and IVOLVOL. Afterwards we only concentrate on the stocks in the highest IVOLVOL tertil and use the same method as in Ang et al. (2006). That is, we first sort conditionally for the robustness variable in a high/low portfolio and then for IVOL. Thereupon, we average the portfolios along the robustness variable and report excess returns as well.

Table 17 reports the resulting excess returns after controlling for various characteristics. Our analysis reveals that none of the included variables are capable to explain the excess return of the low-minus-high portfolio since both stay highly economically and statistically significant. Further, for almost all control variables the magnitudes of returns are in the same range as before. In the following we will explain the economic reasons for including the control variables.

Size – We control for size since Bali and Cakici (2008) suggest that the IVOL puzzle might be driven by small stocks mainly. Controlling for size has a small marginal effect on return level. It only leads to slightly deteriorated returns for the difference portfolios.

Book-to-Market Ratio – As Fama and French (1992) and others show, the book-to-market (B/M) ratio is a strong driver of returns. The higher the ratio the higher the future realized return. It might be possible that low IVOL stocks are mainly value stocks and that high IVOL stocks are more likely to be growth stocks. This relation might explain the large returns of the low-minus-high IVOL portfolios. However, after controlling for B/M the return for the low-minus-high portfolios are still highly significantly positive.

Liquidity – Several authors argue that liquidity positively influences returns and that high IVOL stocks might be less liquid.[27] We measure liquidity risk using the liquidity beta introduced by Pastor and Stambaugh (2003) estimated over the formation period of one month. We find, even after controlling for liquidity, the excess return of the low-minus-high portfolios remain significant.

Bid-Ask Spread of Stocks – Another measure for liquidity is the bid-ask spread, which we measure as the daily average over the formation month.[28] Similar to the

[27]See for example Amihud (2002) and Liang and Wei (2012) amongst others.

[28]Brennan and Subrahmanyam (1996) argue that the spread is a measure for liquidity, although a noisy one.

findings for the liquidity beta the significance of the excess return of low IVOL stocks remains untouched.

Bid-Ask Spread of Puts – The underperformance of the high IVOL portfolio could be driven by too high prices of stocks for which investors face short sale constrains. Lin and Lu (2015) show that, since the replicating portfolio for puts includes a short position in the underlying stock, the bid-ask spread of put options is positively related to lending fees. Those fees can be interpreted as a level of short-sale constraints, because the higher the fee the more costly to short the stock. However, after controlling for the spread we still find highly significant abnormal returns for the difference portfolios.

Return Reversal – Huang et al. (2010) show that, if the whole stock universe is considered, the IVOL puzzle can on average be explained by return reversal. After controlling for return reversal the return of the difference portfolios are still highly significant in statistical and economic terms, so we do not find evidence for this explanation in our subsample.

6.5 Evidence from Option Trading

In this section we check, if option trading is in line with the observed pattern of expected returns. Option trading captures the expectations of market participants directly and thus should give us the opportunity to confirm our results taking an invertor's perspective. In general, investors would buy more put (call) options if they expected low (high) returns. Consequently, investors should trade more (less) puts relative to calls in the case of an expected decrease (increase) of next month IVOL. In addition, the trading activity should be coupled to the magnitude of the absolute expected change of risk. If IVOL is expected to decrease (increase) by a larger amount for one stock compared to another, investors should trade more puts (calls) for this stock than for the other one.

Table 18 reports average ratios of traded put volumes to call volumes (put/call-ratio) in the portfolio formation month. In line with the literature our data shows that in general more calls are traded than puts, since the put/call-ratio is highly significant different from one for almost all portfolios.[29] For stocks of the low/low portfolio the ratio is overall the largest and investors trade the most put options. Thus, investors expect low returns for low/low stocks. This finding goes along with our reasoning, since IVOL is expected to stay low. In comparison, for high/low stocks the average put/call-ratio is the lowest. Thus, market participants expect high returns for these stocks, which is in line with currently high and persistent IVOL.

More importantly, conditional on the IVOL level we find highly significant differences in the put/call-ratios between low and high IVOLVOL buckets. If idiosyncratic risk is low, the ratio for high IVOLVOL stocks is significantly smaller than for low IVOLVOL assets. In this case, the table documents a highly significant difference of 0.09 in the put/call-ratios. Investors are more willing to trade calls on low/high stocks than on low/low stocks. Thus, they expect higher returns for low/high stocks than for low/low, which could be explained by a stronger increase in idiosyncratic risk for the low/high stocks. In a similar spirit, we find that conditional on high IVOL the put/call-ratio to be significantly larger for high IVOLVOL stocks than for low IVOLVOL assets. The difference is with -0.05 highly significant. Therefore, investors have a higher demand for puts on assets in the high/high regime. Thus, they expect lower returns for these assets. This is again in line with a large expected decline in idiosyncratic risk. In addition, the differences in put/call-ratios between low and high IVOL for given IVOLVOL buckets go along with our hypothesis too. The difference is the largest for low IVOLVOL stocks (0.23), for which investors expect the risk levels to be good distinguishable from each other. However, for high IVOLVOL we find the lowest difference of 0.09. Thus, investors expect returns to

[29] See for example, Dennis and Mayhew (2002) or Bali and Murray (2013).

converge the most. This observation also holds for the 1 - 4 difference.

All in all, we conclude that the observed patterns are as predicted. Investors trade more puts (calls) if they expect a decrease (increase) in idiosyncratic risk. This supports our hypothesis from another perspective and we find the mean-reversion in IVOL to be an overall very robust phenomenon.

7 Conclusion

Our paper analyzes the widely documented negative relation between historically realized IVOL and subsequent realized returns. Thereby, we highlight the importance of measuring expected IVOL to infer the real relation between expected idiosyncratic risk and expected returns. If IVOL obeys a mean-reversion process, a high (low) IVOL might indicate a rather large distortion from its long-run mean. In such a case it is obvious that the higher the mean-reversion speed, the larger the expected decrease (increase) in IVOL. Further, we demonstrate theoretically and empirically that the variation in expected future idiosyncratic risk will increase in the mean-reversion speed. Therefore, we propose the measure of IVOLVOL as a natural proxy for the expected mean-reversion speed in IVOL. Then the magnitude of the negative IVOL-return relation should increase in IVOLVOL.

The empirical assessment of our paper speaks strongly in favor of these hypotheses. First, we document the existence of the classical IVOL anomaly in a subsample of highly liquid stocks and options if a historic measure of IVOL is used. Stocks with high idiosyncratic volatility underperform compared to stocks with low idiosyncratic volatility. Second, we verify that the time series of IVOL is stationary. Therefore, it is reasonable to assume a mean-reversion behavior of IVOL. In addition, we show that our measure of IVOLVOL indeed proxies for the mean-reversion speed in IVOL. In essence, we can expect a higher speed of mean-reversion if the

IVOLVOL is large. Thereby, we construct our measure of IVOLVOL in an almost model-free fashion by using option implied information which we extract from the cross-section of options. Thus, our measure of IVOLVOL is entirely forward looking and can be used together with the current level of IVOL to infer the expected innovation in IVOL. Third, this interplay resolves the previously documented negative IVOL-return relation. More precisely, the negative IVOL-return relation vanishes for low IVOLVOL stocks. Here, idiosyncratic risk levels are expected to be rather constant. On the other hand, the negative IVOL-return relation is economically and statistically bolstered, looking at high IVOLVOL stocks. Again, the idiosyncratic risk of these stocks is expected to move in opposite direction and thus investors demand a higher (lower) compensation for current low (high) IVOL stocks, which are expected to suffer a larger increase (decrease) in idiosyncratic risk.

In a comparative analysis in the style of Hou and Loh (2016) we show that IVOLVOL is distinct from other explanations for the IVOL anomaly. Our proxy for mean-reversion can explain at least 65% of the total IVOL anomaly on its own. Other explanatory variables, which proxy market frictions or the lottery preference of investors, only manage to explain 18%.

Moreover, our results are not driven by stock characteristics other than expectations about idiosyncratic risk. In a robustness analysis we rule out stock liquidity and short-sale constraints. Further, we show that our results are insensitive to different measures of IVOL and IVOLVOL. In addition, our findings are backed up by options data.

Our paper gives a new perspective on the existence of the negative IVOL-return relation for the biggest, most liquid stocks if a historic measure of IVOL is used. In addition, it demonstrates that this observation can be explained by a mean-reversion in idiosyncratic volatility.

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Stock Fundamentals

		Mean	Median	Lowest 20%	Next 40%	Next 60%	Next 80%	Highest 90%
Sample	Size	6504	1422	422	988	2225	6387	14778
	Dollar Vol.	1133	408	126	283	582	1407	2531
Whole	Size	1618	168	37	104	281	947	2438
Universe	Dollar Vol.	259	19	2	10	39	169	469

Table 1: The table shows descriptives of size and Dollar Vol, the average monthly trading Dollar volume, in million $ each. Whole Universe covers all stocks from the three exchanges AMEX, NYSE and NASDAQ. Sample is the subsample of the universe, which we use throughout the paper. The sample only contains stocks for which we have at least 15 days of return and VIX^i observations for some month. We only use stock characteristics for the months the stock is included. The sample period is 1996/01–2016/04.

Single Sort on Idiosyncratic Volatility

	1 LOW	2	3	4	5 HIGH	1 - 5
	Equally Weighted					
Excess Return	0.98^{***}	0.89^{**}	0.87^{*}	0.66	-0.21	1.19^{**}
	(0.33)	(0.41)	(0.52)	(0.63)	(0.7)	(0.54)
FF3-Alpha	0.48^{***}	0.24^{*}	0.07	-0.24	-1.24^{***}	1.72^{***}
	(0.11)	(0.14)	(0.19)	(0.24)	(0.27)	(0.31)
	Value Weighted					
Excess Return	0.91^{***}	0.79^{**}	0.70	0.69	0.13	0.77
	(0.31)	(0.40)	(0.49)	(0.59)	(0.71)	(0.55)
FF3-Alpha	0.46^{***}	0.19	0.01	-0.13	-0.75^{**}	1.21^{***}
	(0.08)	(0.15)	(0.21)	(0.23)	(0.33)	(0.36)

Table 2: The table shows monthly excess returns, averaged over the sample period 1996/01–2014/12, and Fama-French 3-factor alpha for different portfolios. We sort stocks into five equally/value weighted portfolios basing on realized IVOL in formation month. Then, we calculate excess returns for the next month. We calculate IVOL relative to the Fama-French 3-factor model. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Stationarity of IVOL

	Required Observations			
	12	24	60	120
10% Level	86.32	88.14	95.24	99.21
5% Level	78.53	80.58	90.32	97.58
1% Level	62.23	64.18	75.17	88.92
Number of Stocks	3,209	3,110	2,582	1,904

Table 3: The table shows results of an augmented Dicky-Fuller test for an unit root against a stationary time series. Values report the percentage of rejected null hypothesis against the alternative of a stationary process without drift and trend for the 10, 5 and 1% significance level. To include a stock's time series at least 12, 24, 60 or 120 observations of the monthly IVOL are required. The last row reports the number of time series included to the test.

Mean-Reversion of IVOL

		Ranking on Idiosyncratic Volatility		
		1 LOW	2 HIGH	1 - 2
Panel A		Mean-Reversion Speed κ		
IVOLVOL	1 LOW	0.59^{***} (0.03)	0.84^{***} (0.04)	-0.39^{***} (0.05)
	2 HIGH	0.61^{***} (0.04)	1.14^{***} (0.05)	-0.35^{***} (0.07)
	1 - 2	-0.13^{***} (0.05)	-0.16^{***} (0.07)	
Panel B		Mean-Reversion Level $\overline{\text{IVOL}}$		
IVOLVOL	1 LOW	1.90^{***} (0.23)	1.62^{***} (0.29)	1.50^{***} (0.60)
	2 HIGH	2.70^{***} (0.30)	2.22^{***} (0.14)	1.56^{***} (0.56)
	1 - 2	-0.62 (0.44)	-0.41 (0.38)	

Table 4: The table shows the average maximum likelihood estimates for an Ornstein-Uhlenbeck process. The mean-reversion level is given in percentage. The cross-sectional information is used to infer if a stock shows high IVOL and/or IVOLVOL. The estimation is done for every time series conditional on a certain regime separately. All IVOL time series with at least 12 months of observation are used. The estimates for a certain regime are used only if the IVOL was observed in it for at least 6 Months.

Mean Reversion Effect

		Ranking on Idiosyncratic Volatility				
		1 LOW	2	3	4	5 HIGH
IVOLVOL	1 LOW	0.02	-0.13^{***}	-0.10^{***}	-0.11^{***}	-0.25^{***}
		(0.04)	(0.03)	(0.03)	(0.03)	(0.03)
	2	0.00	-0.03	-0.02	-0.09^{***}	-0.28^{***}
		(0.04)	(0.04)	(0.03)	(0.03)	(0.03)
	3 HIGH	0.12^{***}	0.08^{*}	0.05	-0.04	-0.31^{***}
		(0.05)	(0.05)	(0.04)	(0.04)	(0.04)

Table 5: The table reports the mean reversion effect in IVOL for different IVOL/IVOLVOL portfolios. For each portfolio we measure the mean reversion effect by running the regression $\Delta \text{IVOL}_{t+1}^{PF} = \alpha^{PF} + \gamma^{PF}\text{IVOL}_t^{PF} + \epsilon_{t+1}^{PF}$, where IVOL_t^{PF} is the average portfolio IVOL in month t. The portfolio formation date is month t. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Equally Weighted Double Sorts

		Ranking on Idiosyncratic Volatility						
		1 LOW	2	3	4	5 HIGH	1 - 5	1 - 4
Panel A		FF3-Alpha						
IVOLVOL	1 LOW	0.26^{*}	0.21	0.15	0.21	-0.42	0.69^{*}	0.05
		(0.15)	(0.16)	(0.26)	(0.30)	(0.32)	(0.38)	(0.33)
	2	0.46^{***}	0.23	0.03	-0.08	-1.29^{***}	1.74^{***}	0.53^{*}
		(0.13)	(0.15)	(0.25)	(0.29)	(0.35)	(0.39)	(0.30)
	3 HIGH	0.70^{***}	0.28	0.03	-0.83^{**}	-2.04^{***}	2.74^{***}	1.53^{***}
		(0.16)	(0.27)	(0.27)	(0.33)	(0.35)	(0.40)	(0.35)
	1 - 3	-0.44^{**}	-0.06	0.12	1.04^{***}	1.61^{***}		
		(0.20)	(0.25)	(0.30)	(0.38)	(0.40)		
Panel B		Excess Return						
IVOLVOL	1 LOW	0.65^{**}	0.73^{***}	0.82^{*}	0.99^{*}	0.41	0.23	-0.34
		(0.28)	(0.33)	(0.48)	(0.59)	(0.67)	(0.58)	(0.46)
	2	0.99^{***}	0.88^{**}	0.84	0.84	-0.18	1.16^{**}	0.15
		(0.35)	(0.38)	(0.55)	(0.66)	(0.75)	(0.59)	(0.46)
	3 HIGH	1.28^{***}	1.07^{*}	0.95	0.16	-0.89	2.17^{***}	1.12^{**}
		(0.40)	(0.57)	(0.62)	(0.72)	(0.79)	(0.65)	(0.53)
	1 - 3	-0.64^{***}	-0.34	-0.13	0.83^{**}	1.30^{***}		
		(0.25)	(0.32)	(0.34)	(0.40)	(0.48)		

Table 6: The table shows Fama-French 3-factor alpha and excess returns for equally-weighted portfolios from a conditional 5×3 double sort on IVOL and IVOLVOL, over the sample period 1996/01–2016/04. We sort stocks first on IVOL and subsequently on IVOLVOL. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Value Weighted Double Sorts

		Ranking on Idiosyncratic Volatility						
		1 LOW	2	3	4	5 HIGH	1 - 5	1 - 4
Panel A		FF3-Alpha						
IVOLVOL	1 LOW	0.49*** (0.14)	0.38** (0.15)	0.00 (0.22)	0.00 (0.29)	0.09 (0.36)	0.41 (0.40)	0.49 (0.31)
	2	0.32** (0.12)	0.11 (0.19)	0.17 (0.26)	0.07 (0.30)	−1.24*** (0.45)	1.56*** (0.47)	0.25 (0.31)
	3 HIGH	0.61*** (0.18)	−0.01 (0.27)	0.28 (0.35)	−0.36 (0.44)	−1.50*** (0.54)	2.11*** (0.60)	0.97** (0.47)
	1 - 3	−0.12 (0.24)	0.38 (0.26)	−0.28 (0.38)	0.36 (0.51)	1.58** (0.62)		
Panel B		Excess Return						
IVOLVOL	1 LOW	0.84** (0.27)	0.85** (0.33)	0.58* (0.42)	0.70 (0.53)	0.81 (0.67)	0.03 (0.57)	0.14 (0.40)
	2	0.80** (0.33)	0.72* (0.38)	0.89 (0.54)	0.94 (0.67)	−0.25 (0.75)	1.04* (0.60)	−0.15 (0.48)
	3 HIGH	1.14*** (0.42)	0.77* (0.58)	1.11 (0.65)	0.63 (0.81)	−0.45 (0.99)	1.59** (0.81)	0.51 (0.58)
	1 - 3	−0.30 (0.28)	0.08 (0.36)	−0.53 (0.43)	0.07 (0.54)	1.26* (0.69)		

Table 7: The table shows Fama-French 3-factor alpha and excess returns for value-weighted portfolios from a conditional 5×3 double sort on IVOL and IVOLVOL, over the sample period 1996/01–2016/04. We sort stocks first on IVOL and subsequently on IVOLVOL. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Mean IVOL Level

		Ranking on Idiosyncratic Volatility						
		1 LOW	2	3	4	5 HIGH	1 - 5	1 - 4
IVOLVOL	1 LOW	0.89 (0.14)	1.43 (0.22)	1.91 (0.29)	2.52 (0.38)	3.69 (0.56)	−2.80 (0.42)	−1.63 (0.25)
	2	0.97 (0.15)	1.47 (0.22)	1.95 (0.3)	2.58 (0.39)	4.05 (0.61)	−3.08 (0.46)	−1.60 (0.24)
	3 HIGH	1.01 (0.16)	1.50 (0.23)	1.98 (0.3)	2.63 (0.40)	4.87 (0.72)	−3.86 (0.57)	−1.62 (0.25)
	1 - 3	−0.12 (0.02)	−0.07 (0.01)	−0.06 (0.01)	−0.11 (0.02)	−1.18 (0.18)		

Table 8: The table shows mean IVOL levels over the sample period 1996/01–2016/04, from a conditional 5×3 double sort on IVOL and IVOLVOL. We calculate IVOL relative to the three-factor Fama-French model. Newey-West adjusted standard errors are stated in parentheses.

Decomposition of IVOL Puzzle

	Raw Returns		Alpha - FF3		Returns - DGTW adj.	
	Coeff.	Expl.	Coeff.	Expl.	Coeff.	Expl.
Panel A: $IVOL_t$ on $Return_{t+1}$						
Intercept	0.0118^{***}		0.0004^{***}		0.0042	
	(0.0039)		(0.0002		(0.0029)	
IVOL	-0.2654^{*}		-0.0124		-0.2598^{**}	
	(0.1426)		(0.0078)		(0.1289)	
Mean N	229.52		229.52		229.52	
Panel B: Controls on $IVOL_t$						
Intercept	0.0114^{***}		0.0114^{***}		0.0114^{***}	
	(0.0024)		(0.0024)		(0.0024)	
$D^{low}_{t-1} \times IVOL_{t-1}$	0.1432^{***}		0.1432^{***}		0.1432^{***}	
	(0.0284)		(0.0284)		(0.0284)	
$D^{mid}_{t-1} \times IVOL_{t-1}$	0.3218^{***}		0.3218^{***}		0.3218^{***}	
	(0.0501)		(0.0501)		(0.0501)	
$D^{high}_{t-1} \times IVOL_{t-1}$	0.3767^{***}		0.3767^{***}		0.3767^{***}	
	(0.0578)		(0.0578)		(0.0578)	
$Skew_{real.}$	-0.0005^{***}		-0.0005^{***}		-0.0005^{***}	
	(0.0002)		(0.0002)		(0.0002)	
$Skew_{RN}$	0.0023^{***}		0.0023^{***}		0.0023^{***}	
	(0.0005)		(0.0005)		(0.0005)	
CoSkew	-0.0053^{***}		-0.0053^{***}		-0.0053^{***}	
	(0.0023)		(0.0023)		(0.0023)	
RTP	0.0000		0.0000		0.0000	
	(0.0000)		(0.0000)		(0.0000)	
LagRet	0.0002^{***}		0.0002^{***}		0.0002^{***}	
	(0.0015)		(0.0015)		(0.0015)	
ZeroRet	0.0004^{***}		0.0004^{***}		0.0004^{***}	
	(0.0001)		(0.0001)		(0.0001)	
Liquidity	229.71		229.71		229.71	
	(379.75)		(379.75)		(379.75)	
Spread	103.68^{***}		103.68^{***}		103.68^{***}	
	(39.71)		(39.71)		(39.71)	
Panel C: Decomposition of IVOL Coefficient						
$D^{low}_{t-1} \times IVOL_{t-1}$	0.0071	-2.66%	0.0011^{*}	-9.06%	0.0052	-1.99%
	(0.0149)		(0.0006)		(0.0126)	
$D^{mid}_{t-1} \times IVOL_{t-1}$	0.0027	-1.03%	0.0014	-10.93%	-0.0068	2.62%
	(0.0300)		(0.0013)		(0.0273)	
$D^{high}_{t-1} \times IVOL_{t-1}$	-0.1669^{**}	$62.25\%^{***}$	-0.0109^{***}	$87.08\%^{**}$	-0.1650^{**}	$63.15\%^{***}$
	(0.0814)		(0.0047)		(0.0794)	
$Skew_{real.}$	-0.0016	0.59%	0.0001	-0.65%	0.0017	-0.65%
	(0.0087)		(0.0005)		(0.0087)	
$Skew_{RN}$	-0.0018	0.68%	-0.0008	6.04%	-0.0046	1.77%
	(0.0083)		(0.0005)		(0.0080)	
CoSkew	0.0018	-0.68%	0.0002	-1.72%	0.0007	-0.26%
	(0.0032)		(0.0002)		(0.0031)	
RTP	-0.0420^{**}	15.66%	0.0001	-0.84%	-0.0414^{*}	15.86%
	(0.022)		(0.0010)		(0.0213)	
LagRet	0.0062	-2.30%	0.0001	-0.93%	0.0059	-2.25%
	(0.0238)		(0.0011)		(0.0225)	
ZeroRet	-0.0131	4.90%	0.0008	-6.52%	-0.0135	5.17%
	(0.0271)		(0.0015)		(0.0263)	
Liquidity	-0.0048	1.80%	-0.0002	1.71%	-0.0030	1.15%
	(0.0084)		(0.0006)		(0.0078)	
Spread	0.0084	-3.15%	-0.0001	0.49%	0.0072	-2.76%
	(0.0073)		(0.0003)		(0.0066)	
$Controls_{total}$	-0.2039^{**}	76.08%	-0.0081^{*}	64.67%	-0.2000^{**}	76.58%
	(0.1095)		(0.0047)		(0.0965)	
Residual	-0.0641	23.92%	-0.0044	$35.33\%^{*}$	-0.0612^{**}	23.42%
	(0.0614)		(0.0037)		(0.0553)	

Table 9: The table shows a multivariate analysis for the decomposition of the idiosyncratic volatility puzzle following Hou and Loh (2016). Panel A shows the average coefficients and mean size of the sample of cross-sectional regressions of monthly (risk-adjusted) returns on IVOL. DGTW refers to risk adjusted returns from Daniel et al. (1997). Panel B shows results from a multivariate regression of IVOL on a set of explanatory variables. Panel C shows the normalized covariation of the explanatory variables as well as their overall explanatory power for the IVOL anomaly. We calculate IVOL relative to the 3-factor Fama-French model and IVOLVOL relative to the market VIX as $\left(\text{VIX}^i_s\right)^2 = \gamma^i + \beta^i_{VIX}\left(\text{VIX}^M_s\right)^2 + \eta^i_s$, $\text{IVOLVOL}^i_t \equiv \text{std}\left[\eta^i_{t-30D:t}\right]$. Liquidity is the liquidity beta of Pastor and Stambaugh (2003) and $Skew_{real.}$ is the realized skewness of raw daily returns in the formation month. $Skew_{RN}$ is the risk neutral skew from Bakshi et al. (2003) at the end of formation month. CoSkew is the coskewness measure in Chabi-Yo and Yang (2010). RTP is the retail trading proportion. LagRet is the one month lagged return, ZeroRet is the proportion of zero returns and Spread is the average relative bid-ask-spread in formation month. $Controls_{total}$ is the sum of the variables explanatory power and residual captures the unexplained part of the IVOL-return relation. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Decomposition of IVOL Puzzle

	Raw Returns		Alpha - FF3		Returns - DGTW adj.	
	Coeff.	Expl.	Coeff.	Expl.	Coeff.	Expl.
Panel A: $IVOL_t$ on $Return_{t+1}$						
Intercept	0.0120***		0.0004***		0.0044	
	(0.0037)		(0.0001)		(0.0027)	
IVOL	−0.2801***		−0.0144***		−0.2708***	
	(0.1329)		(0.0066)		(0.1221)	
Mean N	315.81		315.81		315.81	
Panel B: Controls on $IVOL_t$						
Intercept	0.0122***		0.0122***		0.0122***	
	(0.0026)		(0.0026)		(0.0026)	
D_t^{low}	−0.0028***		−0.0028***		−0.0028***	
	(0.0010)		(0.0010)		(0.0010)	
D_t^{mid}	0.0029***		0.0029***		0.0029***	
	(0.0010)		(0.0010)		(0.0010)	
D_t^{high}	0.0111***		0.0111***		0.0111***	
	(0.0019)		(0.0019)		(0.0019)	
$Skew_{real.}$	−0.0003		−0.0003		−0.0003	
	(0.0002)		(0.0002)		(0.0002)	
$Skew_{RN}$	0.0036***		0.0036***		0.0036***	
	(0.0008)		(0.0008)		(0.0008)	
CoSkew	−0.0025		−0.0025		−0.0025	
	(0.0021)		(0.0021)		(0.0021)	
RTP	0.0000		−0.0000		−0.0000**	
	(0.0001)		(0.0000)		(0.0001)	
LagRet	0.0034***		0.0034***		0.0034***	
	(0.0013)		(0.0013)		(0.0013)	
ZeroRet	0.0004***		0.0004***		0.0004***	
	(0.0001)		(0.0001)		(0.0001)	
Liquidity	202.50		202.50		202.50	
	(348.61)		(348.61)		(348.61)	
Spread	129.31***		129.31***		129.31***	
	(41.50)		(41.50)		(41.50)	
Panel C: Decomposition of IVOL Coefficient						
D_t^{low}	−0.0171	6.14%	−0.0012	9.01%	−0.0241	8.99%
	(0.0692)		(0.0023)		(0.0565)	
D_t^{mid}	−0.0085	3.06%	0.0011	-8.15%	−0.0043	1.60%
	(0.0203)		(0.0015)		(0.0225)	
D_t^{high}	−0.0889	31.97%	−0.0051	37.55%	−0.0739	27.56%
	(0.0939)		(0.0046)		(0.0834)	
$Skew_{real.}$	0.0049	-1.77%	0.0001	-1.03%	0.0080	-2.98%
	(0.0066)		(0.0004)		(0.0065)	
$Skew_{RN}$	−0.0126	4.53%	−0.0009	6.75%	−0.0144	5.35%
	(0.0096)		(0.0006)		(0.0089)	
CoSkew	0.0045	-1.64%	0.0002	-1.68%	0.0036	-1.34%
	(0.0031)		(0.0001)		(0.0030)	
RTP	−0.0336	12.10%	−0.0004	3.08%	−0.0339	12.66%
	(0.0183)		(0.0010)		(0.0184)	
LagRet	−0.0031	1.10%	0.0001	-0.51%	−0.0035	1.31%
	(0.0191)		(0.0010)		(0.0180)	
ZeroRet	−0.00230	8.27%	−0.0017	12.25%	−0.0256	9.53%
	(0.0261)		(0.0013)		(0.0262)	
Liquidity	−0.0060	2.15%	−0.0001	0.53%	−0.0041	1.53%
	(0.0068)		(0.0003)		(0.0067)	
Spread	0.0077	-2.79%	0.0002	-1.26%	0.0072	-2.68%
	(0.0058)		(0.0002)		(0.0055)	
$Controls_{total}$	−0.1755**	63.14%	−0.0077**	56.54%	−0.1650**	61.54%
	(0.1002)		(0.0039)		(0.0877)	
Residual	−0.1025**	36.86%	−0.0059**	43.46%	−0.1031**	38.46%
	(0.0594)		(0.0032)		(0.0536)	

Table 10: The table shows a multivariate analysis for the decomposition of the idiosyncratic volatility puzzle following Hou and Loh (2016). Panel A shows the average coefficients and mean size of the sample of cross-sectional regressions of monthly (risk-adjusted) returns on IVOL. DGTW refers to risk adjusted returns from Daniel et al. (1997). Panel B shows results from a multivariate regression of IVOL on a set of explanatory variables. Panel C shows the normalized covariation of the explanatory variables as well as their overall explanatory power for the IVOL anomaly. We calculate IVOL relative to the 3-factor Fama-French model and IVOLVOL relative to the market VIX as $\left(\text{VIX}_s^i\right)^2 = \gamma^i + \beta_{VIX}^i \left(\text{VIX}_s^M\right)^2 + \eta_s^i$, $\text{IVOLVOL}_t^i \equiv \text{std}\left[\eta_{t-30\text{D}:t}^i\right]$. Liquidity is the liquidity beta of Pastor and Stambaugh (2003) and $Skew_{real.}$ is the realized skewness of raw daily returns in the formation month. $Skew_{RN}$ is the risk neutral skew from Bakshi et al. (2003) at the end of formation month. CoSkew is the coskewness measure in Chabi-Yo and Yang (2010). RTP is the retail trading proportion. LagRet is the one month lagged return, ZeroRet is the proportion of zero returns and Spread is the average relative bid-ask-spread in formation month. $Controls_{total}$ is the sum of the variables explanatory power and residual captures the unexplained part of the IVOL-return relation. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Returns of Equally Weighted Portfolios - IVOL relative to CAPM

		Ranking on Idiosyncratic Volatility						
		1 LOW	2	3	4	5 HIGH	1 - 5	1 - 4
IVOLVOL	1 LOW	0.43	0.75^{***}	0.78	0.81	0.04	0.39	-0.31
		(0.29)	(0.34)	(0.48)	(0.62)	(0.71)	(0.60)	(0.46)
	2	0.80^{***}	0.71^{*}	0.73	0.80	-0.21	1.00^{*}	0.10
		(0.36)	(0.43)	(0.57)	(0.71)	(0.79)	(0.60)	(0.45)
	3 HIGH	0.92^{***}	0.74	0.56	0.07	-1.05	1.97^{***}	1.05^{**}
		(0.40)	(0.54)	(0.64)	(0.75)	(0.83)	(0.64)	(0.51)
	1 - 3	-0.48^{*}	0.01	0.22	0.74	1.10^{***}		
		(0.25)	(0.31)	(0.37)	(0.47)	(0.48)		

Table 11: The table shows a robustness analysis for the existence of the IVOL-return relation. We report monthly excess returns, averaged over the sample period 1996/01–2016/04, from a conditional 5×3 double sort on IVOL and IVOLVOL. We calculate IVOL relative to the CAPM. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Mean Reversion Effect in IVOL - IVOL relative to CAPM

		Ranking on Idiosyncratic Volatility				
		1 LOW	2	3	4	5 HIGH
IVOLVOL	1 LOW	0.02	-0.15^{***}	-0.10^{***}	-0.12^{***}	-0.25^{***}
		(0.04)	(0.03)	(0.03)	(0.03)	(0.03)
	2	0.00	-0.06	-0.04	-0.08^{***}	-0.26^{***}
		(0.04)	(0.04)	(0.04)	(0.03)	(0.03)
	3 HIGH	0.11^{***}	0.08^{*}	0.03	-0.07^{*}	-0.30^{***}
		(0.04)	(0.04)	(0.04)	(0.04)	(0.04)

Table 12: The table shows robustness analysis for the mean reversion effect. We report the mean reversion effect κ^{PF} in IVOL for different IVOL/IVOLVOL portfolios, where we calculate IVOL relative to the CAPM. For each portfolio we measure the mean reversion effect κ^{PF} by running the regression $\text{IVOL}_{t+1}^{PF} - \text{IVOL}_{t}^{PF} = \alpha^{PF} + \kappa^{PF}\text{IVOL}_{t}^{PF} + \epsilon_{t+1}^{PF}$, where IVOL_{t}^{PF} is the average portfolio IVOL in month t. The portfolio formation date is month t. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Returns of Equally Weighted Portfolios - IVOLVOL orthogonal to IVOL

		Ranking on Idiosyncratic Volatility						
		1 LOW	2	3	4	5 HIGH	1 - 5	1 - 4
orth. IVOLVOL	1 LOW	0.77^{**} (0.34)	0.72^{**} (0.35)	0.81^{*} (0.48)	0.90 (0.60)	0.34 (0.70)	0.42 (0.59)	-0.13 (0.44)
	2	1.17^{***} (0.36)	0.93^{**} (0.41)	0.79 (0.54)	0.69 (0.62)	-0.28 (0.74)	1.45^{**} (0.61)	0.48 (0.43)
	3 HIGH	1.01^{***} (0.33)	1.01^{*} (0.53)	0.98 (0.63)	0.39 (0.74)	-0.70 (0.79)	1.71^{***} (0.65)	0.63 (0.56)
	1 - 3	-0.25 (0.18)	-0.29 (0.28)	-0.17 (0.34)	0.51 (0.38)	1.04^{**} (0.52)		

Table 13: The table shows a robustness analysis for the existence of the IVOL-return relation. We report monthly excess returns, averaged over the sample period 1996/01–2016/04, from a conditional 5×3 double sort on IVOL and a orthogonal IVOLVOL. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Mean Reversion Effect in IVOL - IVOLVOL orthogonal to IVOL

		Ranking on Idiosyncratic Volatility				
		1 LOW	2	3	4	5 HIGH
orth. IVOLVOL	1 LOW	-0.13^{***} (0.04)	-0.15^{***} (0.03)	-0.10^{***} (0.03)	-0.12^{***} (0.03)	-0.29^{***} (0.03)
	2	-0.01 (0.04)	-0.01 (0.04)	-0.03 (0.04)	-0.07^{**} (0.03)	-0.27^{***} (0.03)
	3 HIGH	0.31^{***} (0.05)	0.09^{**} (0.05)	0.07^{*} (0.04)	-0.04 (0.04)	-0.30^{***} (0.04)

Table 14: The table shows robustness analysis for the mean reversion effect. We report the mean reversion effect κ^{PF} in IVOL for different IVOL/IVOLVOL portfolios. IVOLVOL is orthogonal to IVOL. For each portfolio we measure the mean reversion effect κ^{PF} by running the regression $\text{IVOL}_{t+1}^{PF} - \text{IVOL}_t^{PF} = \alpha^{PF} + \gamma^{PF}\text{IVOL}_t^{PF} + \epsilon_{t+1}^{PF}$, where IVOL_t^{PF} is the average portfolio IVOL in month t. The portfolio formation date is month t. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Independent Double-Sort - Equally Weighted

		Ranking on Idiosyncratic Volatility		
		1 LOW	2 HIGH	1 - 2
		Return		
IVOLVOL	1 LOW	0.90*** (0.34)	0.87* (0.52)	0.03 (0.27)
	2 HIGH	0.95* (0.55)	0.22 (0.66)	0.73** (0.32)
	1 - 2	−0.05 (0.29)	0.65** (0.29)	
		FF3-Alpha		
IVOLVOL	1 LOW	0.34*** (0.11)	0.15 (0.23)	0.19 (0.20)
	2 HIGH	0.16 (0.26)	−0.79*** (0.22)	0.95*** (0.23)
	1 - 2	0.18 (0.25)	0.93*** (0.21)	

Table 15: The table shows monthly excess returns, averaged over the sample period 1996/01–2016/04, and 3-factor Fama-French alphas for portfolios from unconditional 2×2 double sorts. We sort stocks into four equally weighted portfolios basing on realized IVOL/IVOLVOL in formation month. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Risk Adjusted Returns - FF5

		Ranking on Idiosyncratic Volatility						
		1 LOW	2	3	4	5 HIGH	1 - 5	1 - 4
IVOLVOL	1 LOW	0.07 (0.15)	0.14 (0.17)	0.26 (0.29)	0.58* (0.30)	0.24 (0.28)	−0.17 (0.33)	−0.52* (0.31)
	2	0.29** (0.12)	0.20 (0.16)	0.28 (0.25)	0.52* (0.31)	−0.68** (0.34)	0.97*** (0.37)	−0.23 (0.30)
	3 HIGH	0.76*** (0.17)	0.61** (0.27)	0.54** (0.26)	−0.24 (0.32)	−1.34*** (0.36)	2.10*** (0.44)	1.00*** (0.35)
	1 - 3	−0.69*** (0.19)	−0.47* (0.25)	−0.28 (0.37)	0.82** (0.40)	1.58*** (0.41)		

Table 16: The table shows Fama-French five factor alpha for conditional sorted IVOL/IVOLVOL portfolios for the sample period 1996/01–2016/04. We calculate IVOL relative to the Fama-French three factor model. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Robustness Analysis: Stock Characteristics

	Ranking on Idiosyncratic Volatility						
Controlling for	1 LOW	2	3	4	5 HIGH	1 - 5	1 - 4
Size	0.20	−0.22	−0.44	−0.73***	−1.76***	1.95***	0.92***
	(0.19)	(0.22)	(0.30)	(0.28)	(0.35)	(0.35)	(0.27)
B/M	0.27*	0.06	−0.41	−0.79**	−2.21***	2.48***	1.05***
	(0.16)	(0.27)	(0.28)	(0.32)	(0.38)	(0.40)	(0.34)
Liquidity	0.25	0.13	−0.28	−1.01***	−2.03***	2.28***	1.26***
	(0.16)	(0.22)	(0.29)	(0.34)	(0.35)	(0.37)	(0.36)
Volume	0.24	−0.03	−0.02	−1.09***	−2.08***	2.32***	1.33***
	(0.16)	(0.23)	(0.30)	(0.35)	(0.36)	(0.38)	(0.35)
Bid-Ask: Stock	−0.04	−0.06	−0.50*	−0.99***	−1.34***	1.29***	0.94***
	(0.21)	(0.23)	(0.26)	(0.28)	(0.28)	(0.32)	(0.31)
Bid-Ask: Put	0.24	−0.09	−0.11	−0.74**	−2.24***	2.48***	0.98***
	(0.15)	(0.26)	(0.26)	(0.37)	(0.34)	(0.36)	(0.38)
Return Reversal	0.08	0.18	−0.21	−0.93***	−2.06***	2.14***	1.01***
	(0.16)	(0.22)	(0.28)	(0.33)	(0.38)	(0.40)	(0.36)

Table 17: The table shows a robustness analysis for the IVOL phenomenon conditional on the high IVOLVOL regime, for the sample period 1996/01–2016/04. We report Fama-French 3-factor alpha. First, we sort stocks on stock characteristics in a high/low fashion and subsequently on IVOL basing on realizations in formation month. Then, we calculate next month returns and afterwards we average again along characteristics. Size is the market capitalization. We measure Liquidity by the liquidity beta of Pastor and Stambaugh (2003) and Volume by the average stock trading volume of one month. The bid-ask spreads for stocks and puts are calculated as the difference of bid- and ask-prices divided by the mid-prices. We measure return reversal as in Huang et al. (2010) by returns in formation month. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Put/Call-Ratio

		Ranking on Idiosyncratic Volatility						
		1 LOW	2	3	4	5 HIGH	1 - 5	1 - 4
IVOLVOL	1 LOW	0.89	0.76^{**}	0.74^{**}	0.71^{***}	0.66^{***}	0.23^{***}	0.19^{***}
		(0.13)	(0.11)	(0.11)	(0.10)	(0.10)	(0.04)	(0.03)
	2	0.81	0.77^{**}	0.75^{**}	0.71^{***}	0.68^{***}	0.14^{***}	0.10^{***}
		(0.12)	(0.11)	(0.11)	(0.11)	(0.1)	(0.02)	(0.02)
	3 HIGH	0.80^{*}	0.80^{*}	0.76^{**}	0.75^{**}	0.71^{***}	0.09^{***}	0.05^{***}
		(0.12)	(0.12)	(0.11)	(0.11)	(0.11)	(0.02)	(0.02)
	1 - 3	0.09^{***}	-0.04^{**}	-0.02^{*}	-0.05^{***}	-0.05^{***}		
		(0.03)	(0.02)	(0.01)	(0.02)	(0.02)		

Table 18: The table shows monthly put/call-ratio, averaged over the sample period 1996/01–2016/04, for a conditional 5×3 double sort on IVOL and IVOLVOL. First, we sort stocks on IVOL and subsequently on IVOLVOL. The put/call-ratio are the one-month averages of put trading volume devided by call trading volume. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. The test hypothesis for the ratios is that the ratios are equall to one, whereas the hypothesis for the differences is that they equall zero. Newey-West adjusted standard errors are stated in parentheses.

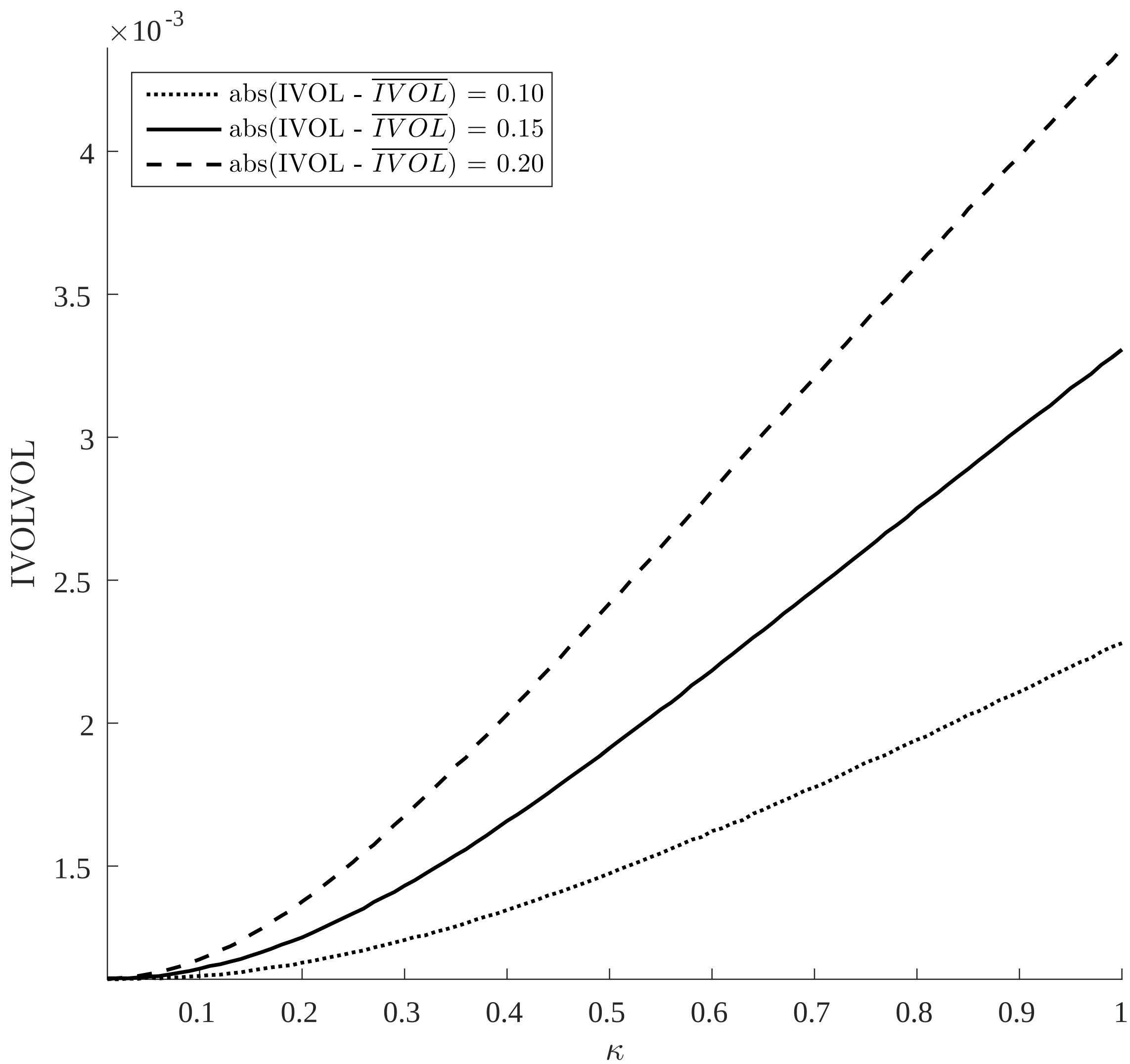

Figure 1: The figure displays the dependence of IVOLVOL on the mean-reversion coefficient κ^i in our model from section 2 for different IVOL levels. The model's coeffiecents are $\overline{\text{IVOL}}^i = 0.15$, $\sigma^i_{\text{IVOL}} = 0.01$. We simulate 200,000 paths of IVOL, using equation (1) over one month to generate the data.

Estimation of IVOL and IVOLVOL

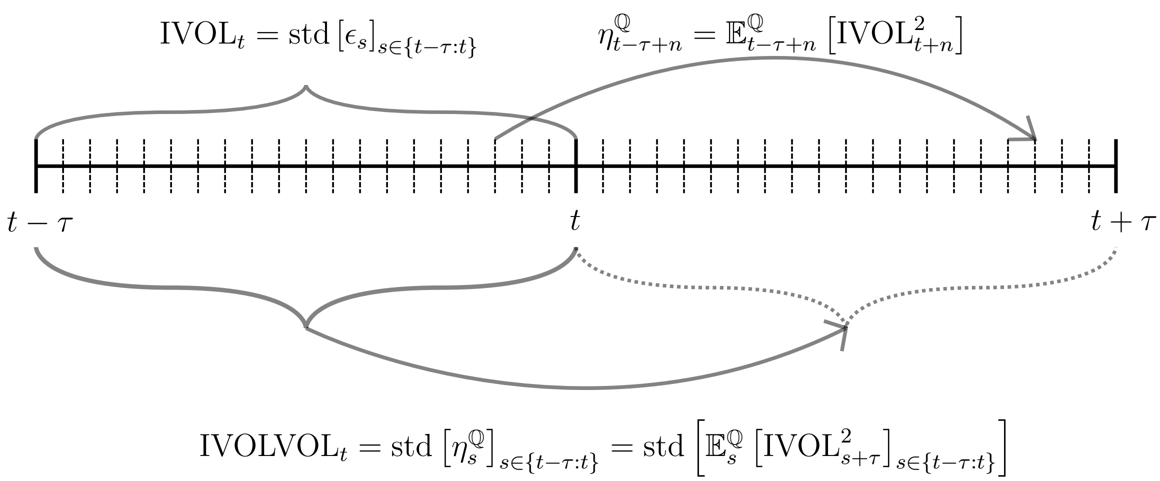

Figure 2: The figure illustrates the measurement of IVOL and our measure of IVOLVOL. The IVOL is estimated using daily realized returns from month $t-\tau$ to t. IVOLVOL is estimated using the daily risk-neutral expectations during month $t-\tau$ to t for the next month expected idiosyncratic variance.

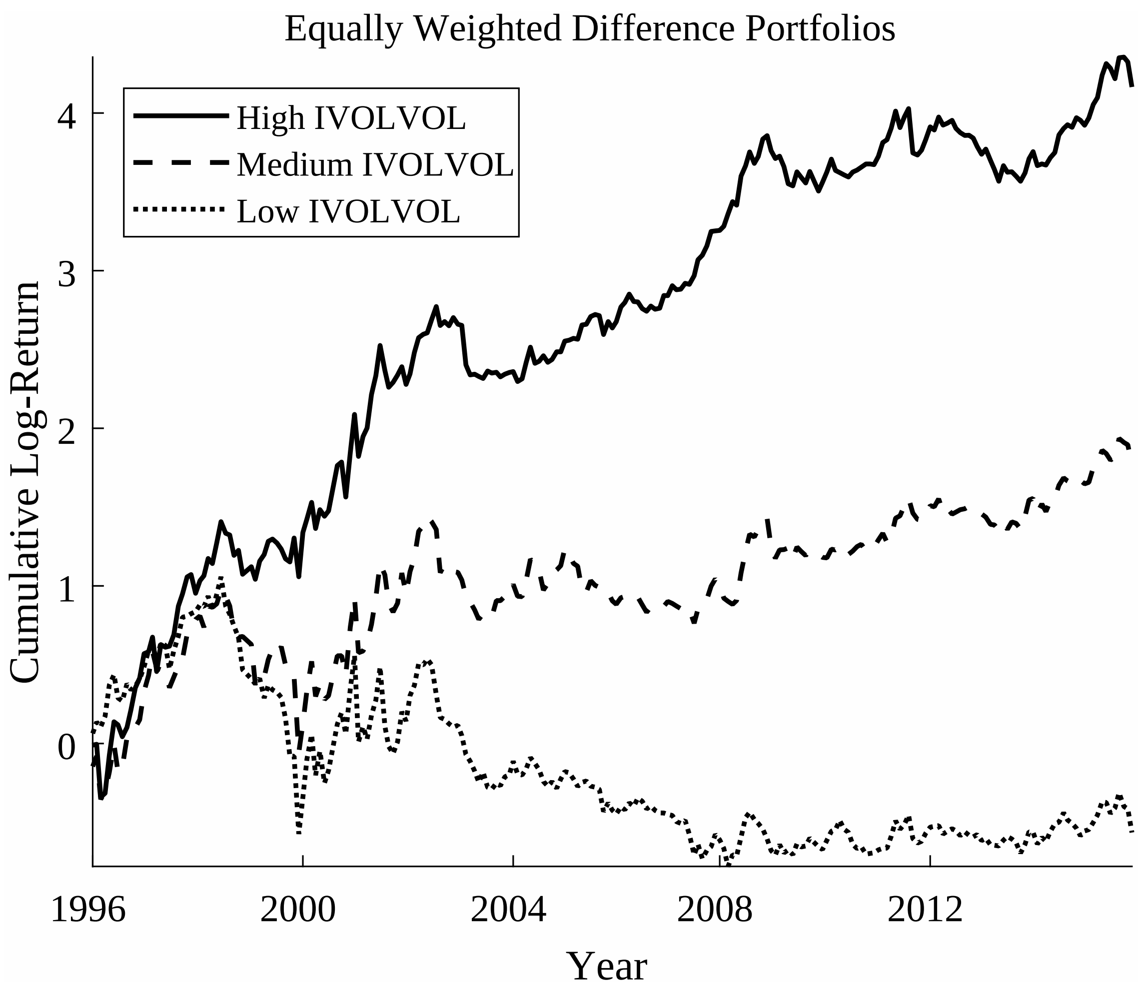

Figure 3: The figure displays cumulative log-returns of low-minus-high IVOL difference portfolios across different IVOLVOL regimes. Each month we conditionally sort stocks first into five idiosyncratic risk quintiles and subsequently into three IVOLVOL tertziles. Afterwards, for each IVOLVOL regime we calculate returns of equally weighted low-minus-high IVOL portfolios over the next month and accumulate.