System 1, System 2, and Speculative Trading

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ABSTRACT

Loss aversion and overconfidence are arguably the two most studied behavioral biases in finance, and yet often considered having contradictory effects on risk taking. Overconfident investors are generally more prone to take-on risk, whereas loss averse investors tend to be more cautious. We study their marginal impacts on trading. We propose a model in which rational investors and investors who are jointly loss averse and overconfident, disagree over public signals. The proposed theory succeeds to rationalize asymmetries in returns: It generates a positive correlation between volume and aggregate information, a high-volume return premium, positive unconditional skewness and explains cross-sectional variation in skewness at the firm level.

I. Introduction

In his international best-seller, "Thinking fast and Slow", summarizing decades of research in cognitive psychology, Kahneman (2011) introduces metaphorically System 1 and System 2 as

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fictitious characters. These two cognitive systems aim to represent how thought can emerge in two different ways. The former produces fast, intuitive, and often biased thinking, while the later produces slow, more deliberate, effortful and often rational thinking. Overconfidence and loss aversion, are often considered as being the key attributes of System 1, and yet sometimes viewed as contradictory in the way they influence investors' attitude toward risk. In connection with the contradictory nature of overconfidence and loss aversion within System 1, Daniel Kahneman states:[1]

> "We are loss averse independently of optimism… loss aversion exists independently of risk and independently of uncertainty. We are optimistic even when we do not have control… Clearly, there is an obvious evolutionary advantage to loss aversion but there is probably a biological advantage to optimism too, they are contradictory in a way, but they are both useful."

The model we propose is inspired by this systems' metaphor. We think of the market as consisting of two types of investors. "System 1 investors" who are simultaneously loss averse and overconfident or optimistic about the relevance of the information, and "System 2 investors" also called "rational speculators" who form fully rational expectations about security returns.[2] Based on this so-called dual-process cognitive theory, we propose a stylized model of speculative trading

[1] Daniel Kahneman pronounced this statement during his lecture of its book "thinking fast and slow" (Kahneman, (2011)) held at the UBS International Center of Economics in Society of the University of Zurich, on April 16, 2013. The full video lecture is available at: https://www.ubscenter.uzh.ch/en/events/opinions/kahneman.html.

[2] Supported by the dual-systems' characteristics, we will thereafter assume the first group of traders (System 1) as being presumably identified as non-professional, retail investors while the second group (System 2) as being more sophisticated and experienced traders acting also as the market makers.

able to capture the interaction between these two types of agents. It succeeds particularly to reconcile the following striking features about skewness and trading-volume at the firm level:

1. Positive (negative) correlation between volume and high-priced (low-priced) stocks
2. High-volume return premium. (short-run positive abnormal return following unusually large trading-volume shock)
3. Average positive skewness in individual-firm returns
4. Negative correlation between skewness and trading volume, past return, and firm size

To our knowledge, no previous theoretical model of investor behavior has been able to explain jointly these phenomena. We think it is important to fill this gap. Unlike the majority of models anchored in the rational expectation paradigm and traditional asset pricing theories, we develop a theory that speaks directly to the joint behavior of trading volume and asset prices, and to asymmetries in price changes. Our model draws inspiration from Hong and Stein (2007) who propose a disagreement model that gives a central role to trading volume.

Our main intuition behind this paper is motivated by the simple observation that the two cognitive biases known as *loss aversion* and *overconfidence* are systematically called upon to explain some empirical evidence for which neoclassical finance has a much difficulty in explaining, while they have a priori opposite impacts on an investor's willingness to bear risk (overconfidence is positively related to risk taking whereas loss aversion is negatively related to

it). For instance, in isolation, overconfidence provides a good explanation to the *active investing puzzle*,[5] whereas loss aversion provides a good explanation to the *equity premium puzzle*.[6]

We extend the model of Harris and Raviv (1993) on speculative trading when agents agree to disagree, and short-selling is not possible. We assume that there are two types of speculative traders. One group is risk neutral and perfectly assesses the released information, and the other group is loss averse and overconfident in the sense that its members think that the signal is more informative than it really is. We therefore model overconfidence as the belief of an agent that the information is more accurate than it is. As in Harris and Raviv (1993), we assume that traders start with common prior beliefs about the return on a particular asset. As information flows, the forecasts by agents of the two groups oscillate, since each trader updates his belief about returns using his own model of the relationship between the news and asset return. The two groups agree on whether a particular signal is favorable or unfavorable, but they disagree on the extent to which information is important. Speculators in the irrational group increase (decrease) their probability of high returns more upon receipt of favorable (unfavorable) information than those in the irrational group. However, unlike the Harris and Raviv (1993) model, trading does not occur necessarily when information switches from favorable to unfavorable. Loss aversion deters investors from trading unless the extent of their information is sufficiently large (above a certain

[5]The active investing puzzle, a term coined by Daniel and Hirshleifer (2015) refers to the excessive trading of individual investors. It is widely accepted that the total volume of trades in financial markets is excessively important to be explained by traditional asset pricing model, and that the most prominent behavioral explanation of such excessive trading is overconfidence (Barberis and Thaler, 2003).

[6] The equity premium puzzle (Mehra and Prescott, 1985) refers to the observation that given the return of stocks and bonds over past century, an abnormally high level of risk aversion would be necessary to explain why investors are willing to hold bonds at all. The myopic loss aversion concept introduces by Benartzi and Thaler (1995, 1999) represents arguably the most prominent behavioral explanation of the equity premium puzzle.

positive threshold). This threshold is partly responsible for the asymmetric predictions of the models.

We derive a closed-form solution for the equilibrium price and for the expected trading volume conditional on aggregate information. We observe first, that at any time *t* the maximum conditional volume occurs when aggregate information is positive. We then demonstrate that aggregate information is positively correlated with trading volume. This result is consistent with the well-accepted positive correlation between contemporaneous volume and return. It is also consistent, although more subtly, with the tendency for glamour high-priced stocks to have significantly higher turnover than low-priced value stocks reported, among others, by Harris and Raviv (2007), as well as the very strong lead-lag relationship between returns and turnover that are both market-wide and at the individual security level reported by Statman, Thorley, and Vorkink (2006). The second result indicates that the correlation between aggregate information and speculative volume increases with loss aversion. Based on assumptions we made on the nature of institutional traders and retail investors, this relation implies novel predictions about fixed-firm characteristics associated with the correlations between price levels, returns and trading volume. We then show that the model generates a high-volume return premium, which is defined as the propensity for individual stocks with unusually large trading volume shocks to experience large subsequent returns. The model in Gervais, Kaniel and Mingelgrin (2001) provides an alternative behavioral explanation to Merton's (1987) recognition hypothesis. Several testable predictions arise from the proposed theory, in line with empirical evidence reported in the work of Kaniel, Ozoguz and Starks (2012). Particularly, the theory supports the negative correlation between the levels of premium and institutional ownership.

Finally, we consider the model's implications for the observed level of skewness in stock returns. The model succeeds in reconciling several striking empirical facts that have been difficult to rationalize within a single theory framework. Our theory thus generates positive average skewness, a standard empirical feature of individual stock returns. Also, our theory explains the cross-sectional variation in skewness at the firm level reported by Chen, Hong and Stein (2001), and in particular the fact that skewness is negatively correlated with trading volume, past return, and firm size.

Our paper contributes more generally to a growing literature that uses loss aversion and overconfidence to explain financial phenomena.

Researchers have found indeed that loss aversion helps to explain many striking empirical evidence in finance. Benartzi and Thaler (1995, 1999) explain the equity-premium puzzle, Barberis, Huang and Santos (2001) find asset returns have high mean, are excessively volatile and are significantly predictable in the time series. Gomes (2005) and Berkelaar et al. (2004) apply loss aversion to portfolio choice and find that loss averse investors abstain from holding stocks, unless they expect the equity premium to be quite high. Barberis and Huang (2001) explain the "value effect"[7], McQueen and Vorkink, (2004) rationalize the GARCH effect in stock returns, Dittmann, Maug, and Spalt (2010) succeed to explain under some assumptions, observed compensation practices, Pasquariello (2014) explain patterns related to liquidity and price efficiency. More recently, Ouzan (2016) succeeds to explain stylised facts about market crashes within an asymmetric information framework.

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[7] The *value effect* or *value premium* refers to the phenomenon that stocks with low ratios of price to fundamentals have higher average return.

Several authors, among them, Kyle and Wang (1997), Odean (1998b), Daniel, Hirshleifer, and Subrahmanyam (1998), and Caballé and Sákovics (2003), find as it was briefly mentioned earlier, that models of overconfidence predict a high trading volume. Besides the excessive trading prediction, Odean (1998b), shows that there is a positive correlation between the presence of overconfident traders and the volatility of asset prices. Daniel, Hirshleifer, and Subrahmanyam (1998), in their model show that overreaction and self-attribution bias can reconcile short-run positive autocorrelation with long-run negative autocorrelation. They provide indeed, evidence on short-term momentum with long-term mean reversal. Daniel, Hirshleifer, and Subrahmanyam's (1998) theory provides an explanation for market underreaction[8]. Daniel and Titman (1999) find also that momentum is stronger for growth stocks. Lee and Swaminathan (2000) and Glaser and Weber (2003) demonstrate that momentum is stronger among high-turnover stocks. Scheinkman, and Xiong (2003) and Hong Scheinkman, and Xiong (2006) study the joint behavior of volume and overpricing. Finally, Burnside, Han, Hirshleifer, and Wang (2011) based upon investor overconfidence; explain why high-interest-rate currencies tend to appreciate relative to low-interest-rate currencies[9]. Recently, Daniel and Hirshleifer (2015) provided an excellent survey on models of overconfidence that can plausibly explain patterns that are puzzling from the perspective of fully rational models.

The remainder of this paper is organized as follows. Section II, presents the disagreement model of trading between rational and irrational investors. Section III derives the equilibrium. Section IV derives the main results regarding the evolution of trading volume and prices. Section

[8] Market underreaction refers here to the pattern that stock price reactions, after public event, are of the same sign as post event long-run abnormal returns.
[9] This market anomaly is known as the forward premium puzzle.

V discusses the implications of the model on the high-volume return premium. Section VI derives results on skewness in asset returns and discusses the main model predictions relative to skewness. Finally, section VII concludes and proposes avenues for further research.

II. The Model

The model described in this paper is based on differences of opinions. There are two groups of traders, namely group $\langle S_1 \rangle$ (for System 1) and group $\langle S_2 \rangle$ (for System 2). $\langle S_2 \rangle$ consists of fully rational, risk neutral speculators. We may consider $\langle S_2 \rangle$ as sophisticated and experienced traders, typically institutional investors. $\langle S_1 \rangle$ however, which may be identified as non-professional, retail investors, is loss averse and exhibits overconfidence toward the precision of the information released. Both groups of speculators trade at dates $t = 1,...,T$ shares of an asset that makes a single random payment $\boldsymbol{R}$ at date T. There are one risk-free asset and one risky asset. Shares of the risky asset are traded for a riskless asset. We neglect discounting; the risk-free asset is thus a claim to one unit of terminal-period wealth.

We follow the model of differences of opinion of Harris and Raviv (1993) and we let the final payoff $\boldsymbol{R}$ be either high (H) or low (L) with $L \leq H$. We assume that the probabilities of high and low payoff are equal.[10] At each date $t = 1,...,T$, new public information is revealed, and all investors receive the same public signal. Following the reception of the public information, speculators update their beliefs and may trade at a price P_t . We may think of public signals as

[10] For tractability, we assume that traders realize that the true prior probabilities are equal.

macroeconomic news, dividend announcement, quarterly earnings, merger announcement, acquisition announcement, political events or any release of information that influences the future prospect of the stock price. The signals are independent and identically distributed conditional on the true payoff. The rational group (System 2) knowns the true distribution of the public signal given the final payoff $\boldsymbol{R} \in \{H, L\}$ denoted by $\delta_{\langle S_2 \rangle}(s|\boldsymbol{R})$. It is given by

$$\delta_{\langle S_2 \rangle}\left(s|H\right) = \delta_{\langle S_2 \rangle}\left(-s|L\right) = \begin{cases} k_{\langle S_2 \rangle} a_{\langle S_2 \rangle}{}^{s} & \text{for} \quad s \geq 0 \\ k_{\langle S_2 \rangle} b_{\langle S_2 \rangle}{}^{-s} & \text{for} \quad s < 0 \end{cases} \tag{1}$$

This distribution is chosen in order to allow the true posterior at date t to depend on the history of the signal only through the cumulative signal. The parameters $a_{\langle S_1 \rangle}$ and $b_{\langle S_2 \rangle}$ parameters strictly between 0 and 1, and $k_{\langle S_2 \rangle}$ is a constant required to make $\delta_{\langle S_2 \rangle}\left(s|H\right)$ and $\delta_{\langle S_2 \rangle}\left(-s|L\right)$ density functions. Using Bayes' rule, after observing a history of signals $s^t = (s_1, ..., s_t)$, the true posterior probability that $\boldsymbol{R} = H$ is

$$\pi_{\langle S_2 \rangle}\left(H|s^t\right) = \frac{\prod_{\tau=1}^{t} \delta_{\langle S_2 \rangle}\left(s_\tau|H\right)}{\sum_{\kappa=\{H,L\}} \prod_{\tau=1}^{t} \delta_{\langle S_2 \rangle}\left(s_\tau|\kappa\right)} = \left[1 + \theta_{\langle S_2 \rangle}^{m}\right]^{-1} \equiv \pi_{\langle S_2 \rangle}^{H}(m) \tag{2}$$

where $m = s_1 + ... + s_t$ is the cumulative signal, and $\theta_{\langle S_2 \rangle} = b_{\langle S_2 \rangle} / a_{\langle S_2 \rangle}$. The independence assumption of the signals and the specific form of the likelihood function imply that signals are *additive* and the posterior depends on the signal history only through cumulative signal m[11]. Therefore, since

[11] For instance, at $t = 2$, to infer the likelihood of the final payoff, the stream $s_1 = 1$, and $s_2 = 3$ of public information is equivalent to $s_1 = 6$, and $s_2 = -2$.

the posterior depends on the signal history only through the cumulative signal, we substitute m for s_t and drop the subscript in the posterior. The probability that $\boldsymbol{R} = L$, given m, is $\pi^{H}(m) = 1 - \pi^{L}(m)$. Since we assign larger values of the signal as more favorable information, we assume that the posterior probability of high outcome is increasing in m $(\theta_{\langle S_2 \rangle} < 1)$. $\theta_{\langle S_2 \rangle}$ can be interpreted as an inverse measure of the quality of the signal[12]. For example, if $\theta_{\langle S_2 \rangle} = 0$, then any positive signal results in a posterior that assigns probability 1 to $\boldsymbol{R} = H$, and any negative signal results in a posterior that the probability 1 to $\boldsymbol{R} = L$. Conversely, for $\theta_{\langle S_2 \rangle} = 1$, the posterior is independent of the signal.

After having defined the true distribution of the signal and the payoffs, we now consider the beliefs of the speculators regarding the distribution of the signal and the payoff. All speculators know the correct prior that $\boldsymbol{R}$ can be either H or L with equal probabilities. Differences of opinion are generated by assuming that speculators have different model of interpreting the signals. After observing the signal, each speculator revises her belief regarding the final payoff, using Bayes' rule and his own model (likelihood function) of the relation between signal and the payoff. The rational group, as already mentioned, has a true model and updates its belief regarding the true distribution. We assume that each speculator is absolutely convinced that her model is correct. Indeed, each group believes the other group is basing its decision on an incorrect model. All information including all speculators models are assuming to be common knowledge, so speculators do not attempt to infer the prices from the behavior of other speculators. This model,

[12] We refer the true quality of the signal as a measure interpreted by the rational group, since we assume that this group of traders is rational and therefore interprets truly the quality of the signal.

in line with other models of differences of opinion (Harris and Raviv, (1993), Kandel and Pearson (1995)), suggests that even when all investors observe the same information they may induce to trade with one another[13]. In order to generate important trading-volume the model combines heterogeneous priors with the assumption that the investors do not fully update their beliefs based on each other trading decisions. Investors agree to disagree in equilibrium[14].

The model of the irrational group (System 1) shares the same functional form as the rational group (System 2). Consequently, the resulting posteriors also exhibit the same functional form as the true posteriors. In particular, speculators of $\langle S_1 \rangle$ realize that the signals are *i.i.d.* conditional on the final payoff, but they do not know the true density functions $\delta_{\langle S_2 \rangle}\left(s \middle| \boldsymbol{R}\right)$. Instead, group $\langle S_2 \rangle$ believes that the conditional density of a signal *s* given final payoff $\boldsymbol{R}$ is given by $\delta_{\langle S_1 \rangle}\left(s \middle| \boldsymbol{R}\right)$ which represents the same functional form as the true density function but with different parameters.

Since both groups are to have models resulting in different posterior beliefs, they have different values of θ. In the present model, $\theta_{\langle S_1 \rangle} < \theta_{\langle S_2 \rangle}$. Group $\langle S_1 \rangle$ typically overestimates the precision of the signal and believes the signal is of higher quality (more informative) than group $\langle S_2 \rangle$ and therefore, responds to a given signal history to a greater extent. The irrational group (System 1) amplifies the sentiment concerning the public news, resulting that the traders within this group are

[13] Notice that, traditional rational expectation models unlike difference of opinions models where trading volume takes its source in speculation, have hard time to rationalize the enormous observed volume in financial markets with motives of hedging or portfolio rebalancing per se.
[14] Glaser and Weber (2004) test the two distinct features of overconfidence namely, *miscalibration of subjective probability* and the *better-than-average effect* on trading volume and conclude that the *better-than-average effect/ difference of opinions* models better explain the enormous volume observed in financial markets.

more optimistic when the cumulative signal is positive and are more pessimistic when *m* is negative. When $m = 0$, both groups revert to their prior beliefs. Namely, that the prior is high with probability 0.5.

Figure 1 depicts the posterior beliefs of both groups (probability of high outcome in function of the cumulative signal m). Equation (2) reflects that $\pi_H(m)$ is monotone increasing in m and is concave for $m > 0$ and convex for $m < 0$. These characteristics imply that larger cumulative signals indicate greater likelihood of high final payoff and that the posterior is more sensitive to changes in cumulative signal when beliefs are more diffuse (i.e. when the absolute value of *m* is very low).

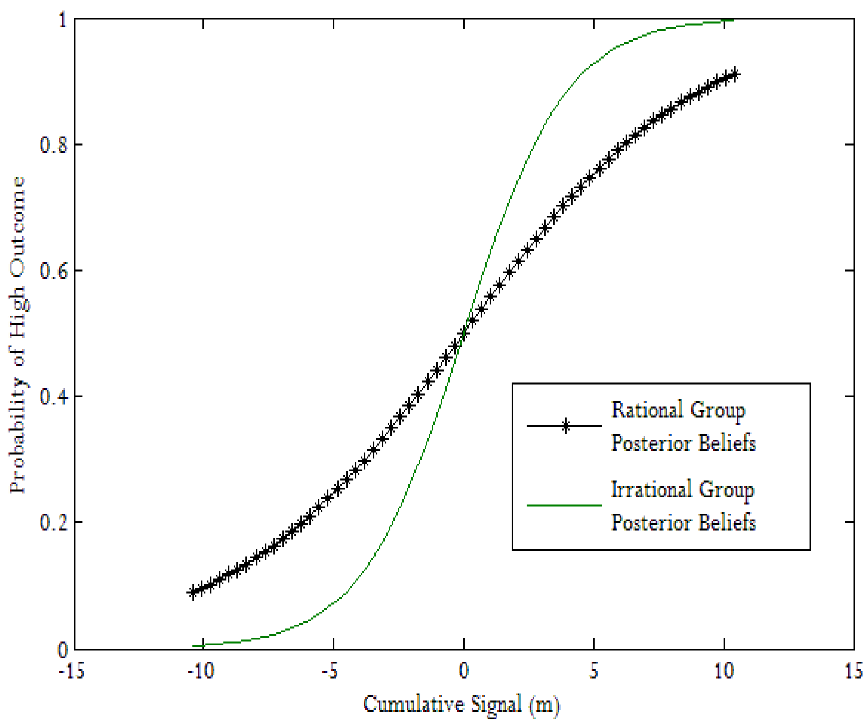

Figure 1. Probability of High Outcome The probability of high outcome is displayed as a function of the cumulative signal *m* for both groups of speculators. The irrational group is more responsive to the signal than the rational group. For a positive (negative) signal cumulative, the irrational group values the asset more highly (little).

Figure 1 depicts the probability of high outcome in function of the cumulative signal m. Larger cumulative signal reflects the fact that larger cumulative signals indicate greater likelihood of high final payoff and the posterior is more sensitive to changes in cumulative signal when beliefs are more diffuse.

The two groups do not share the same preferences and utility function. The rational group is risk neutral and the irrational group is loss averse. The utility functions of both groups are depicted in Figure 2. The linearity of utility function is convenient for tractability and is appropriate since we are interested primarily in volume generated by speculation as opposed to hedging for life cycle consideration. In this model, there is no trading except for speculative purpose. The only difference is that the utility function of the irrational group is a piecewise linear function and it is kinked at the origin. The irrational group's loss aversion is captured by the following utility function:

$$U_{\langle S_1 \rangle}(\pi, \lambda) = \begin{cases} \pi & \text{for} \quad \pi > 0 \\ \lambda\pi & \text{for} \quad \pi < 0 \end{cases} \tag{3}$$

where π in our model represents the realised gain or loss and the coefficient λ stands for the degree of loss aversion. As we can see in Figure 2, the utility function of the irrational group has the same curvature than the rational group. Linearity of the utility functions implies that demand functions are infinitely elastic. Therefore, any trader will seek to buy an infinite number of shares at any price below his reservation price. Following the model of Harris and Raviv (1993), to make the equilibrium well defined, we must assume that there is a fixed number of shares available.

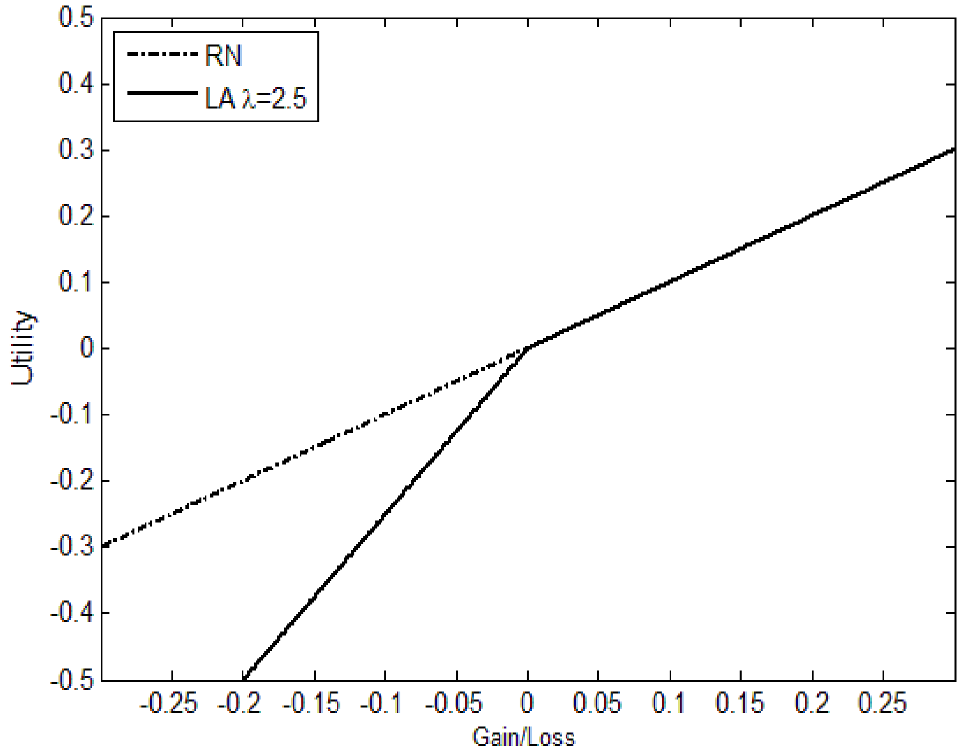

Figure 2. Utility of gains and losses. The dotted line represents the utility function of rational group $\langle S_2 \rangle$ and the solid line refers to the utility function irrational group $\langle S_1 \rangle$ with a coefficient of loss aversion of 2.5.

This implies that short-sales are not allowed. Before we introduce the equilibrium, we ought to recall the concept of *mental accounting*, a term coined by Thaler (1980). It refers to the process by which people think about and evaluate their financial transactions. A question that arises in applying our analysis to the cross section at the firm level: Over which gains, and losses is the investor loss averse? When investors are loss averse over changes in total wealth, we are speaking about *broad framing*, in contrast with *narrow framing* when investors are loss averse over changes in the value of their portfolio of stocks or over changes in the value of individual stocks that they own. If we want to consider the proposed model as an equilibrium at the individual security level, we need to assume *narrow framing.* Numerous experimental studies suggest that when doing their

mental accounting, people engage in narrow framing, that is, they often appear to pay attention to narrowly defined gains and losses.[15]

III. Equilibrium

To define in this setting the equilibrium price, we must make assumptions on how the market is organised**.** For tractability, we assume that in every period the rational group has sufficient market power to offer a price on a take-it-or-leave-it basis. This price will equal to the price taking group's reservation price. Therefore, we restrict the price taker to be one particular group, namely the irrational group. This restriction is in line with our main assumption concerned the nature of both rational and irrational groups. Under the model assumptions, small and individual investors are viewed as irrational agents while large traders, (e.g. institutional traders) as rational investors. If we follow this interpretation, market makers give preferential treatment to the large traders by filling their orders at the reservation price of the small investors[16].

The second assumption concerned the nature of preferences of the irrational group. In line with recent results of behavioral economics developed by a Barberis and Xiong (2012), a number of

[15] This may reflect a concern for non-consumption sources of utility, such as regret, which are often more naturally experienced over narrowly framed gains and losses. If one of an investor's many stocks performs poorly, the investor may experience a sense of regret over the specific decision to buy that stock. In other words, individual stock gains and losses can be carriers of utility in their own right, and the investor may take this into account when making decisions. Barberis and Huang (2001) study the equilibrium behavior of firm-level stock returns when investors are loss averse and exhibit narrow framing in their mental accounting. They find typical individual stock return has a high mean and excess volatility, and there is a large "value premium".

[16] This assumption allows us to make the equilibrium tractable. Following the explanation of Harris and Raviv (1993), if we don't impose this market structure and given that the equilibrium is competitively set in each period, then the price in any period in which there is a trade will equal the reservation price of the buyer. This occurs because the buyer has an infinitely elastic demand, but the supply is bounded. Since the buyer's group changes from period to period, each group reservation price in any period will involve that group's expectation of the of the first group's expectation, and so forth which is becoming quickly intractable.

authors have suggested that investors derive utility from realizing gains and losses on assets they own. In this work, we borrow besides loss aversion the realized utility concept and assume that the reference point from which traders derive utility is set at the end of the trading periods (at time *T*) when the payoff is realized, and consumption occurs. This assumption makes the analysis much more tractable without scarifying too much realism. Therefore, since the asset generates the payoff just at the last period and the traders know that they have the possibility to trade until the last period, the expected utility optimal demand at each period for the irrational group is

$$E_{\langle S_1\rangle}\left(U_{\langle S_1\rangle}(\pi,\lambda)\middle|m\right)=\begin{cases} q\left[(H-P_t)\pi_{\langle S_1\rangle}\left(H\middle|m\right)+\lambda(L-P_t)\left(1-\pi_{\langle S_1\rangle}\left(H\middle|m\right)\right)\right] & \text{for } q\geq 0 \\ q\left[\lambda(H-P_t)\pi_{\langle S_1\rangle}\left(H\middle|m\right)+(L-P_t)(1-\pi_{\langle S_1\rangle}\left(H\middle|m\right)\right] & \text{for } q<0 \end{cases} \tag{4}$$

where λ represents the coefficient of loss aversion, and q the demand for the risky asset. Maximizing the conditional expected utility gives an unbounded demand for a sufficient large cumulative signal in absolute value

$$|m|>\frac{1}{\ln\left(1/\theta_{\langle S_1\rangle}\right)}\left[\ln(\lambda)+\ln\left(\frac{P_t-\mathrm{L}}{H-P_t}\right)\right], \tag{5}$$

and zero elsewhere.

To derive the equilibrium price, we must define first the rational group’s current expectation of the payoff and it is equal to

$$E_{\langle S_2\rangle}\left[\boldsymbol{R}\middle|m\right]=H\pi_{\langle S_2\rangle}\left(H\middle|m\right)+L\pi_{\langle S_2\rangle}\left(L\middle|m\right)=\left(H-L\right)\pi^{H}_{\langle S_2\rangle}\left(m\right)+L. \tag{6}$$

Let assume that at time *t* the rational group holds the risky asset. In order to be willing and able to sell it, two conditions should be fulfilled. First, the cumulative signal should be positive. This condition arises from the differences of opinion of groups $\langle S_1 \rangle$ and $\langle S_2 \rangle$. If at time *t-1*, the cumulative signal was negative and at time *t*, it becomes positive, the irrational group becomes optimistic while he was previously pessimistic about the information released. It arises because the groups "switch sides" in their posterior belief, and thus in their current expectation of the final payoff. The second condition implies that the cumulative signal is sufficiently large to ensure that the condition of (5) is met for positive value of *m*. This condition is driven by loss averse preferences of irrational traders[17]. Note that if $\lambda = 1$, only the first condition holds and we retrieve the condition of Harris and Raviv (1993). Since we assume that the rational group has sufficient market power to propose a price on a take-it-or-leave-it basis, the price-taking group always engages in trades that they believe have zero net present value. We already assume, however, in order to make the equilibrium well defined, that there is a fixed number of shares available and short-sales are not allowed. Therefore, the reservation price of the price-taking group and the price of the risky asset at date t is the price P_t^* that solves[18]

$$\frac{1}{\ln\left(1/\theta_{\langle S_1 \rangle}\right)}\left[\ln(\lambda)+\ln\left(\frac{P_t - \mathrm{L}}{H - P_t}\right)\right] = m, \tag{7}$$

and it is equal to

[17] We already assume that the irrational group can be considered as individual non-sophisticated traders or retail investors.

[18] We solve equation (5) with equality for positive demand since short-sales are not allowed.

$$P_t^* = \frac{H + \lambda \theta_{\langle S_1 \rangle}^m L}{1 + \lambda \theta_{\langle S_1 \rangle}^m}. \tag{8}$$

The reservation price is smaller than the irrational group's current expectation of the final payoff unless $\lambda = 1$. Loss aversion, indeed, induces the price-taking group to accept buying the risky asset at a lower price. The rational group however will accept to sell the risky asset only if the cumulative signal *m* is above the threshold[19]

$$\psi = \ln(\lambda) / \ln(\theta_{\langle S_2 \rangle} / \theta_{\langle S_1 \rangle}). \tag{9}$$

As in the model of Harris and Raviv (1993), trades will occur only when the two groups "switch sides". The novelty here is that the threshold is not zero and it depends both on the precision of the public signal of both groups and on loss aversion unless $\lambda = 1$. If the coefficient of loss aversion equals to one, we retrieve the same result as in Harris and Raviv (1993). When the cumulative signal is below ψ , the risky asset is held by the rational group, while when it is above ψ the risky asset is entirely held by the irrational group of traders. The price changes according to the reservation price of the irrational group each time information appears. We assume that the equilibrium price is always given by equation (8) even in period in which there are no gain to trade[20]. We can state our first lemma.

[19] The rational group would accept to sell the risky asset only if its current expectation of the payoff (Equation (4)) is above the equilibrium price (Equation (8)). Therefore, the threshold refers to the cumulative signal *m* that solves $(H + \lambda \theta_{\langle S_1 \rangle}^m L) / (1 + \lambda \theta_{\langle S_1 \rangle}^m) = (H - L)\pi_{\langle S_2 \rangle}^H(m) + L.$

[20]In such period when the cumulative signal does not cross the threshold $\psi,$ any price between P_t^* and the rational group current expectation results indeed in no trade.

Lemma 1. *The equilibrium price P_t^* increases with overconfidence and decreases with loss aversion.*

Proof: Recall that $1/\theta_{\langle S_2 \rangle}$ is a measure of overconfidence $\left(\theta_{\langle S_1 \rangle} < \theta_{\langle S_2 \rangle}\right)$. And that loss version is given by λ .It is straightforward to show that $\frac{\partial P_t^*}{\partial\left(1/\theta_{\langle S_1 \rangle}\right)} > 0$, and $\frac{\partial P_t^*}{\partial \lambda} < 0$. Q.E.D.

Figure 3 illustrates the equilibrium price in function of cumulative signal and the rational group's current expectation of final payoff. This plot emphasizes the nature of the equilibrium. Trades in that model are jointly impacted by loss aversion and differences of opinion. The rational group knows that to be able to sell the risky asset to the other group of traders, they need to set the price sufficiently low compared to the cumulative signal. This so called "distance" between *m* and P_t^* highlights the very nature of loss aversion. The equilibrium price set by the market maker is always below the current expectation of the irrational group $\langle S_1 \rangle$. Therefore, on average loss aversion featured in conjunction with short sales constraints reduce overpricing and increase underpricing whereas overconfidence increases mispricing with no asymmetry. The trading process and the characteristic of the equilibrium highlight how overconfidence and loss aversion are intertwined and reveal their joint impacts on investors. As we can see from equation (8) overconfidence and loss aversion on one hand, have opposite effect; i.e., the former increases the equilibrium price and the later decreases it[21]. Nonetheless, on the other hand, these biases differ in the way they spur investors to react to information. Overconfidence in this model allows speculative trading and forces the irrational group to exploit what they consider as a mispricing[22],

[21] This effect reflects somewhat the contradicting aspect of these two major cognitive biases.

[22] Remember that both groups consider that the other group falsely interprets the signal. The more the group $\langle S_1 \rangle$ is overconfident, the more he views the security as being mispriced.

whereas although loss aversion reduces their incentive to trade, it creates as it was already emphasized in a different setting, in previous works by Pasquariello (2014) and later on by Ouzan (2016) a no trade region in the optimal demand[23]. In order to trade, the rational group has to propose a sufficiently low price compared to the aggregate information.

Disagreement is not a sufficient condition for trading anymore. Traders need to disagree on the accuracy of the signal in order to trade but they need also to consider the quality of the signal. If for instance, the aggregate quality of the signal is judged insufficient (the so-called "distance" between aggregate information and the price is too small) the market maker cannot propose a price below its current expectation of the payoff. The market maker becomes incapable to increase the price-taker perceived signal quality and therefore trades does not happen although traders' sentiment has just shifted[24]. This phenomenon is primarily responsible for the asymmetric properties on speculative volume that we further develop in the following sections.

IV. Speculative Volume

Having described a simple model based on heterogeneous beliefs, we now investigate the relation between volume and return. Recall that the equilibrium price in any period in our model is the reservation price of the price-taking group; namely $\langle S_1 \rangle$, even if at that period no trades take place. Let $v_{t+1}(m,s)$ denote the volume at $t+1$ and $s_t = s.$ $v_{t+1}(m,s)$ is equal to 0 if m and $m+s$ are both either greater or smaller than ψ, and 1 otherwise. The next lemma provides an explicit

[23] The kink at the origin of the utility function, which characterises loss aversion as displayed in Figure 2 is primarily responsible for the so-called "no-trade" region. That's differs fundamentally and economically from simply being the mirror opposite of overconfidence.
[24] Last period rational traders where more optimistic than irrational traders while it is now the opposite.

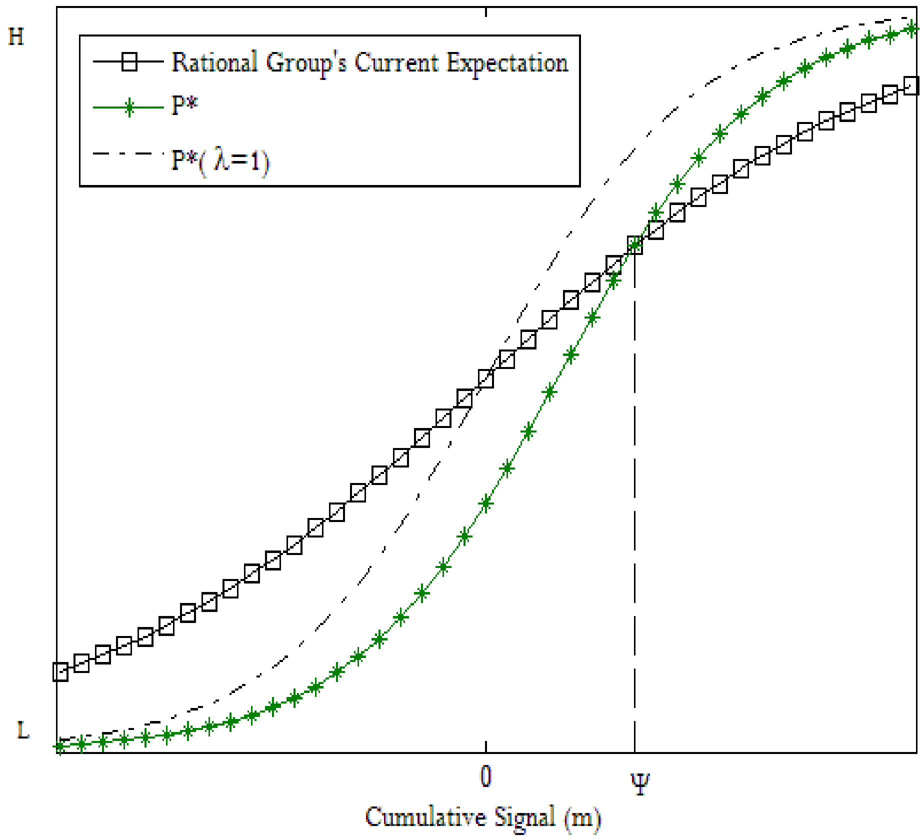

Figure 3. Equilibrium Price. The equilibrium prices for $\lambda > 1$ and $\lambda = 1$ are displayed as a function of the cumulative signal *m*. The equilibrium price intersects the risky asset group's current expectation (reservation price) at $m = \psi$.

function for the expected volume conditional on cumulative signal at any time *t* and it equals the probability of positive volume at $t+1$ given the cumulative signal $m_t = m$ at t

Lemma 2. The conditional expected volume at $t+1$ given the cumulative signal $m_t = m$ is

$$\bar{V}(m) = E^{ra}\left[v_{t+1} \middle| m_t = m\right] = \frac{\pi^H_{\langle S_2 \rangle}(m)}{\ln(a_{\langle S_2 \rangle}) + \ln(b_{\langle S_2 \rangle})} \begin{cases} b^m_{\langle S_2 \rangle}\left[b^{-\psi}_{\langle S_2 \rangle}\ln(a_{\langle S_2 \rangle}) + a^{-\psi}_{\langle S_2 \rangle}\ln(b_{\langle S_2 \rangle})\right] & m \geq \psi \\ a^{-m}_{\langle S_2 \rangle}\left[b^{\psi}_{\langle S_2 \rangle}\ln(a_{\langle S_2 \rangle}) + a^{\psi}_{\langle S_2 \rangle}\ln(b_{\langle S_2 \rangle})\right] & m < \psi \end{cases} \tag{10}$$

Proof: The derivation of the conditional volume is reported in the appendix A.

Figure 4 plots the expected next period volume given the current cumulative signal, for different levels of loss aversion. One can observe that the maximum expected volume arises when $m = \psi$. Our first result shows that the speculative volume is larger on average in period of positive aggregate information (see appendix B for formal proof).

Proposition 1. *Aggregate information and the speculative volume are positively correlated*.

Both practitioners and academics are aware of the tendency for higher volume to accompany higher price levels (both in time series and in the cross section). Proposition 1 is indeed in line with the well-accepted positive correlation between *contemporaneous* volume and return (Karpoff (1987); Stoll and Whaley (1987); Bessembinder and Seguin (1993); Bessembinder, Chan, and Seguin (1996); Chordia, Roll, and Subrahmanyam (2000); and Lo and Wang (2000), Harris and Raviv (2007)). In addition, this result also supports the empirical evidence that share turnover is positively related to lagged returns (Statman, Thorley, and Vorkink (2006)). Signals in the present model are additive,

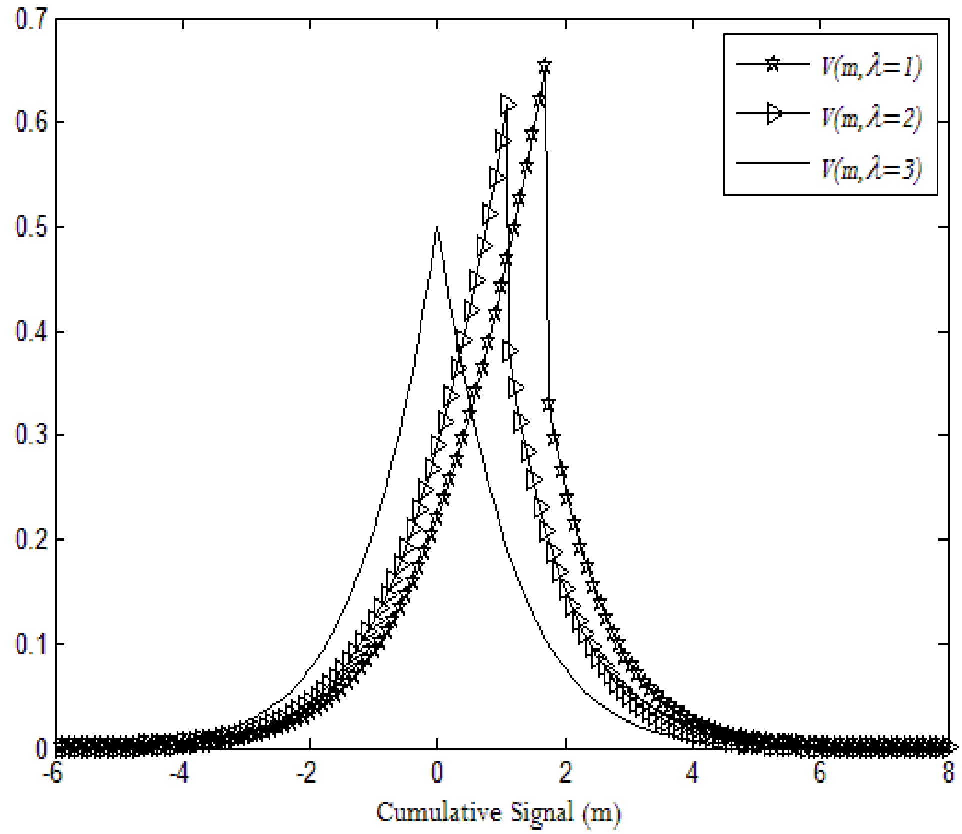

Figure 4. Expected Volume. The expected next period volumes given the current cumulative signal are displayed for different levels of loss aversion.

therefore, the correlation is valid also when the positive information has been accumulated in the past, implying therefore positive past returns. This result supports also, although more subtly, the work of Hong and Stein (2007). In their study, they extend the traditional link between "overtrading" and bubbles documented by Ofek and Richardson (2002) and Kindlenberg, and Aliber (2005). They show that the positive correlation between price level relative to their fundamental values and trading volume exists not only in bubble-like situations but also in tranquil time. Alternatively, to put it differently, they show that “glamour stocks tend to have higher volume than low priced value stocks”. Proposition 1 demonstrates that this relation is indeed verified using

aggregate information as proxy for future growth rather than using stock's ratio of price to fundamentals[25].

The plot of the conditional volume (Figure 4) exhibits a discontinuity around the threshold ψ. This discontinuity of the expected conditional volume increases with loss aversion and it arises from the symmetric distribution of the public information and the asymmetric nature of the equilibrium. It is straightforward to demonstrate the following result (proposition).

Proposition 2. *The correlation between speculative volume and aggregate information increases with loss aversion.*

Following the intuition of Barberis, Huang, and Santos (2001) on how investors frame their losses and gains[26], Proposition 2 might predict that the correlation between price level and trading volume is stronger in bull market rather than in bear market. After a big loss in the stock market, investors experience a sense of regret over the decision to invest in stocks and may be even more reluctant to realise further loss than usual. In Barberis, Huang and Santos's (2001) model, to capture the influence of prior outcomes they introduce an historical benchmark level[27]. Since we don't study in the present work an intertemporal asset pricing model but rather a disagreement model that captures the joint behavior of speculative volume and asset price, we simply make the

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[25] Stocks deemed as glamour stocks are usually considered to have strong growth potential. We define in our setting "glamour stocks" by stocks receiving positive coverage (aggregate information). Positive coverage should represent a good proxy for high ratios of market value to earnings, cash flows, or book value. The positive prospect of one share impounded into aggregate information have a direct impact on the market price while may not be yet incorporated into book value, current cashflows, or earnings.
[26] The utility function in their asset pricing model come from fluctuations in financial wealth.
[27] In their model, Barberis, Huang and Santos (2001) use historical benchmark as a fictitious secondary benchmark to determine the magnitude of the utility received from a particular gain or loss.

assumption that irrational, less sophisticated traders, during bear market are more loss averse than usual. Thus, the first testable implication of our model refers to the average abnormal positive correlation between price level and trading volume during bear market.

In the same vein, a second testable implication arising from Proposition 2, is related to number of institutional investors for a given stock. The proposed stylised model implies that all traders within a particular group exhibit the same preferences. Moreover, we attribute the irrational group to be non-institutional traders. Therefore, it is fair to assume that the ratio of institutional traders for a given stock is negatively correlated to the degree of loss aversion within the model used to price that particular stock. Therefore, our model predicts, all things being equal, that the correlation between trading volume and price level is on average greater for stocks with a higher concentration of less sophisticated traders (retails investors). Moreover, several studies indeed confirm the conventional wisdom that institutional investment increases with firm size. According to O'brien and Bushnan (1990), it is because the firm size can be used to establish prudence on investment in legal case. Falkenstein (1996) documents that U.S. mutual funds tilt their portfolios towards large firms, and Gompers and Metrick (2001) find that American institutions invest in firms that are larger, more liquid, and have had relatively low returns during her previous year. Ferreira and Matos (2008) confirm this finding internationally, and find that all institutional investors, whatever their geographic origin, share a preference for the stock of large and widely held firms. The model suggests that a firm with a less sophisticated traders or as a buy product a firm of smaller size, would exhibit a greater correlation between trading volume and price level ceteris paribus. As Feng and Seasholes (2005) pointed out, trading experience and investor sophistication dampen the reluctance to realize losses. Statman, Thorley, and Vorkink (2006) find evidence that support our

assumption both from an ownership structure and size standpoints. They demonstrate that the positive correlation between turnover and lagged returns is more pronounced in small-cap stocks and in earlier periods where individual investors hold a greater proportion of shares.

V. High-Volume Return Premium

Gervais, Kaniel and Mingelgrin (2001) report that individual stocks with unusually large trading volume over periods of a day or a week tend to experience large return over the subsequent month. This hypothesis refers to the *high-volume return premium.* The main explanation of this intriguing empirical evidence is provided by investor recognition hypothesis of Merton (1987), also known as the visibility hypothesis. Gervais, Kaniel and and Mingelgrin (2001) reinforce investors' incomplete information hypothesis. They show that the excess market-adjusted return that occurs after a stock receives substantial positive shocks is not merely a by-product effect of return autocorrelation nor of the momentum effects that Jagadeesh and Titman (1993) document. Moreover, Kaniel, Ozoguz and Starks (2012) extend the initial work on the high-volume return premium across 41 different countries in both developed and emerging markets. The authors find that the high-volume return premium represents a pervasive phenomenon and it is present in almost all countries of their sample. Using Merton's (1987) recognition hypothesis as a guide, they find that the magnitude of the premium is associated with some country and firm characteristics. However, at the firm level, several measures of visibility do not corroborate the investor recognition hypothesis. Namely, one of their most puzzling finding is the positive correlation between analyst coverage and the high-volume return premium. Kaniel, Ozoguz and Starks' (2012) result suggests that firms that are more closely followed by analysts or S&P are more likely

to experience high return following a strong volume shock. Interestingly, they show that the high-volume return premium increases in the existence of analyst coverage or S&P coverage but not in the level of analyst coverage.

The high-volume return premium can be seen in our context, before we demonstrate it formally as a gradual information flow version of Proposition 1.[28] To motivate our future development, the analyst coverage story may indeed in the contrary, support the proposed behavioral explanation of the premium. The S&P coverage or the presence of analyst coverage increases the likelihood of the presence of non-specialists[29]. While the number of analysts might mitigate this result. Bhushan (1989) reports that the aggregate demand for analyst services increases as more institutions hold shares in a firm or the percentage held by them increases[30].

We depart from the visibility hypothesis and demonstrate mathematically in the following equations that the high-volume return premium is required because of the presence of irrational traders that exhibit jointly overconfidence and loss aversion.

The unconditional volume v at time $t+1$ is equal to

$$\int_{-\infty}^{+\infty} E_{\langle S_2 \rangle}\left[v_{t+1} \middle| m_t = m \right] g_t(m) \cdot dm, \tag{11}$$

[28] We demonstrate that the maximum volume appears when there is a positive aggregate information. Due to gradual information flow, it may take a certain time however for the positive signal to be incorporated into the stock price.
[29] We assume indeed that non-sophisticated and non-specialist investors are naturally a priory aware of narrow pool of firms, so the presence of analysts should inherently increase their holding of covered stocks to a greater extent than specialists.
[30] It suggests that while analyst coverage attract non-sophisticated traders, the level of coverage increases with institutional traders and therefore mitigate the former result as reported in Kaniel, Ozoguz and Starks (2012).

where $g_t(m)$ refers to the unconditional density function of the cumulative signal *m*. Let define the expected return between *t* and *T*, as $E_t\left[r_T\right]=\frac{(H+L)}{2P_t}-1$. Our third result shows the existence of the high-volume return premium (see the Appendix B for formal proof).

Proposition 3. *The expected return increases with the average unconditional volume.*

Proposition 3 indicates that a positive volume shock generates on average a positive expected return. We demonstrate in Appendix B that the average unconditional volume increases with loss aversion, i.e. $\frac{\partial \bar{V}}{\partial \psi}>0$. The comparative static argument for the next result is quite straightforward. Let consider two models with different degree of loss aversion λ_1, and λ_2 with $\lambda_1>\lambda_2$. Therefore, after let say a public announcement, the average volume shock will be greater for the model with λ_1, ceteris paribus. And therefore, from Proposition 3, we can state that the model with λ_1 will in average generate a greater premium than the model with λ_2. We state this result formally as Proposition 4.

Proposition 4. *The high-volume return premium increases with loss aversion.*

A high concentration of loss averse traders should have therefore, on average, a positive effect on the premium. A shock in loss aversion parameter due for instance, to economic downturn should also increase the average future return on the asset. Several testable implications arise from Proposition 4. Using the behavioral explanation as a guide, we may be able to investigate firm and country characteristics associated with the premium. The first obvious direct relation with our assumption is the negative correlation that should exist between institutional trading and the

premium. Furthermore, in the time series as well we should find evidence that the high-volume return premium is more pronounced in time of market downturn.

Let confront our theory with Merton's theory (1987). Indubitably, information environment limits the investors who are aware of a firm's securities to a subset of investing population. Therefore, the reduction in cost of capital after a volume shock associated with the visibility hypothesis can hardly be refuted. Nonetheless, some of the main implications of Merton's (1987) recognition hypothesis are in direct opposition with the behavioral motives presented in the paper. For instance, according to our model, a broader investor base increases the propensity of less sophisticated traders and therefore should increase the correlation between future return and trading volume, whereas following the visibility hypothesis it should decrease it[31]. We thus need to examine the visibility hypothesis alongside with the proposed behavioral explanation. Miller (1977) and Mayshar (1983) claim that the holders of a particular stock will on average tend to be more optimistic about its prospect. As Gervais Kaniel and Mingelgrin (2001) pointed out, it is especially true if taking negative position is rendered difficult by institutional constraints on short-selling. Since in our model we impose short-sales constraints on both type of investors, the potential sellers are largely restricted to current stockholders whereas the set of potential buyers, after any shock that attracts the attention of the investors include a large fraction of the market. Our model might be set nicely within the visibility hypothesis framework. Some market implications may corroborate the visibility hypothesis with the proposed behavioral explanation. For instance, at the country level, Kaniel, Ozoguz and Starks (2012) find that the premium is higher for countries with more listed companies per urban population. According to their interpretation

[31] The larger the investor base the more visible is already the particular stock.

of Merton's (1987) argument, when a country has more listed companies, the investor base for each of the companies individually would be expected to be smaller and therefore a shock in volume should amplify the premium. Likewise, following the assumption of the number of irrational traders, the high-volume return premium should be high where there is a high concentration of listed companies per urban population. An important number of listed companies for a particular country implies in general a well-functioning financial market, where many small and heterogeneous participants are interacting[32]. Kaniel, Ozoguz and Starks (2012), report also that firms with low institutional holdings have a great high-volume return premium that do firms with large institutional holdings. This result supports strongly the proposed behavioral argument as well as the visibility hypothesis. Although the investor recognition hypothesis assumes that visibility is more important for individual investors, this finding represents the strongest support to our theory.

A crucial question regarding the high-volume return premium is whether there exist implementable economic trading strategies. The evidence that investors can take advantage of market anomalies in general and from the high-volume return premium in particular are mixed. Lesmond, Schill and and Zhou (2004) show that transaction costs prevent profitable strategy execution. They find that those stocks that generate large momentum returns are precisely those stocks with high trading costs. The effect of transactions cost has been found also to be detrimental on possible trading profits for the high-volume return-premium. However as Kaniel, Ozoguz, and

[32]In the work of Kaniel, Ozoguz and Starks (2012), although their results indicate that the magnitude of the high-volume premium is not different in emerging countries than it is in developed countries that are not in G-7, stocks in a G-7 country are more susceptible to the effect of extreme volume shock. This is consistent with both the Merton (1987) hypothesis and our hypothesis. Since these markets have more individuals per capita who participate in the stock market, there may be a relatively greater proportion of retail, non-sophisticated investors, who would be also susceptible to investor visibility shock.

Starks (2012) report that institutional investors could not profitably exploit the high volume return premium. This observation hints that it would be worthwhile to analysis economic trading strategy based on both behavioral and visibility hypothesis. After having controlled for transaction cost, stock held by an important fraction of retail investors represent according to Kaniel, Ozoguz and Starks (2012), a viable trading strategy.

VI. Skewness

Aggregate stock market returns display negative skewness, the propensity to have market downward movement on the aggregate level with greater probability rather than upward market movement. A vast literature tends to explain this styled fact about the distribution of aggregate stock returns (e.g. Fama (1965), Christie (1982), Pyndick (1984), French, Schwert, and Stambaugh (1987), Bekaert and Wu (2000), Hong and Stein (2003), and Yuan (2005), Alburquerque (2012), Chang, Christoffersen, Jacobs (2013)). However, this evidence contrasts with the fact that firm-level returns are positively skewed. Alburquerque (2012) succeeds to reconcile these two counterintuitive phenomena. His model explains the positive skewness at the firm level, and generate negative *coskewness* in the market portfolio[33].

In our model since we have only one risky asset, we thus inevitably depict an incomplete picture of the market. However, since we do not introduce heterogeneity in news arrival nor multiple assets, neither correlation between assets, it is fair to say that our model aims to speak at the firm level rather than at the aggregate level. Whether it is the correlation between trading volume and high-priced stocks, or the high-volume return premium, we focus in this study at striking empirical

[33] To generate a positive firm level skewness, Alburqueque (2012), shows that the unconditional distribution of equilibrium returns is a mixture of normal distribution. To generate a negative *coskewness*, he introduces heterogeneity in firm's announcement events.

evidence that arise to a set of particular stocks rather than to the market in general. We already conjectured that the degree of loss aversion depends on prior gains and losses and we already assumed *narrow framing*. We will see shortly that our model generates unconditional positive skewness. We are able also to speak directly about the joint behavior of return skewness and speculative volume.

Figure 3 suggests that returns are positively skewed. In our model, it is easy to show that the equilibrium price at time t is convex (concave) for $m > \ln(1/\lambda)/\ln(\theta_{\langle S_1 \rangle})$ $\left(m < \ln(1/\lambda)/\ln\left(\theta_{\langle S_1 \rangle}\right)\right)$. Therefore, since $\ln(1/\lambda)/\ln\left(\theta_{\langle S_1 \rangle}\right) > 0$ and the distribution of the aggregate information is centered and symmetric, one can figure out that upward movements occur more often than downward movements, unless $\lambda = 1$. When irrational traders do not exhibit loss aversion, the model does not generate any skewness. We state the result formally as Proposition 5 (see Appendix C for a formal proof).

Proposition 5. *Asset returns are positively skewed unless* $\lambda = 1$.

The conjunction of loss aversion displayed by irrational traders and short-sales constraint is mainly responsible for the positive skewness of asset return. Intuitively, loss averse traders dislike losses much more than they like gains. Therefore, Group $\langle S_1 \rangle$ will engage in trades only if they think they can realise gains more often than losses. Consequently, the market maker[34] proposes a price to the loss averse traders accordingly. This feature in addition to the short-sales constraint generate as a buy product positive skewness of asset return. It is easy to show that the positive

[34] Recall that the rational group under our assumption is indeed the market maker.

skewness increases with loss aversion. Proposition 6 establishes the relation between skewness and loss aversion.

Proposition 6. *Positive skewness increases with loss aversion.*

The proof of Result 6 is merely an extension of the proof of Result 5. As in Results 2 and 4, several testable implications arise from Result 6. The first implication would be the presence of more positive skewness among stocks held by a greater number of institutional traders, ceteris paribus. Furthermore, in the cross section, securities that have experienced recent drop in their prices should exhibit more positive skewness in their subsequent returns. Chen, Hong and Stein (2001) find robust evidence about conditional skewness in line with implications generated by our model. Beyond the general evidence, broadly accepted, that skewness is positive at the firm level and negative for the market as a whole, they come up with three robust findings. First, that positive skewness will be more pronounced for small firms, ceteris paribus. Second, they find also that when past returns have been high, skewness is forecasted to become more negative and reciprocally, when past returns have been low, skewness is forecasted to become more positive. Third, that negative skewness is greater in stocks that have experienced an increase in trading volume. Whereas the work of Chen, Hong and Stein (2001) aims to test the theory developed in Hong and Stein (2003), the size effect and influence of past return on skewness do not speak directly to the predictions of their model.

In line with those arguments, our model may support the relation between size and skewness found in Chen, Hong and Stein (2001). Retail investors are indeed more likely to hold small firm stocks, indicating within our context, a greater degree of loss aversion and therefore a more positive skewness in asset return. After having established, a connection between loss aversion

and skewness we are interested to expand our analysis to skewness conditional on trading volume. In the same spirit as Hong and Stein (2003), we are able to show that a positive correlation between negative skewness and trading volume or to put it differently, in average, an increase in trading volume generates a decrease in skewness. We state the result formally in Result 7.

Proposition 7. *Skewness and speculative volume are negatively correlated.*

One way to persuade ourselves that Proposition 7 holds is by observing (equation (10)) that the maximum conditional volume appears when $m = \psi$. At that point, the conditional skewness is negative[35], while we already shown that the overall unconditional skewness is positive. Therefore, we can infer that negative skewness is more pronounced in stocks experiencing a positive shock in volume, unless $\lambda = 1$. We provide a more rigorous demonstration, of the negative correlation between skewness and trading volume, using numerical (Monte Carlo simulation) techniques alongside with analytical analysis. We report it in appendix C.

Interestingly, the proposed behavioral explanation succeeds to bring arguments in favor of the three robust findings highlighted by Chen, Hong and Stein (2001), whereas Hong and Stein's (2003) theory supports mainly only the relation between volume and skewness. At the fundamental level, although, the proposed model differs from the model of Hong and Stein (2003), we both integrate disagreement in the way different groups of traders update their information and some restrictions for investors in taking short positions. In their model, differences of opinion and short-sales constraints are responsible for the correlation between skewness and volume but do not

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[35] Notice that $\frac{\ln(1/\lambda)}{\ln(\theta_{\langle S_1 \rangle})} < \psi$, therefore, the price of the risky asset when $m = \psi$, is locally concave, suggesting negative skewness of asset return between t and $t+1$.

permit to talk about fixed-firm characteristics impacting conditional skewness. Other studies have already documented the effect of past returns and size on conditional skewness. Harvey and Siddique (2000) and Cao, Coval, and Hirshleifer (2002) show that there is negative conditional skewness after periods of positive returns. Harvey and Siddique (2000) report that skewness is more negative on average for large firms by market capitalization. Boyer**,** Mitton, and Vorkink (2010) find strong negative cross-sectional relation between average returns and expected skewness. They find also that other firm characteristics are also important predictors of idiosyncratic skewness, including idiosyncratic volatility, turnover, firm size, and industry designation. It is worthwhile to mention that Boyer**,** Mitton, and Vorkink (2010) follow indeed the approach of Chen, Hong, and Stein (2001) to derive their results. The motivation of their study was primarily to test the prediction of recent theories that stock with high idiosyncratic skewness should have low expected returns[36] unlike Chen, Hong, and Stein (2001) where their primary motivation was to test the relation between trading volume and skewness.

There are indeed, several different theories that explain only partially empirical evidence on skewness of asset return. The proposed model hope to encompass a broader understanding of skewness at the cross section. Hong and Stein (2001) attribute the impact of past returns on skewness to stochastic bubbles[37]. While they evoke the discretionary-disclosure to explain the

[36] Mitton and Vorkink (2007), develop a model incorporating heterogeneous investor preference for skewness, predicting lower expected return for stocks with idiosyncratic skewness. Barberis and Huang (2008) show that when investors have cumulative prospect theory, positively skewed securities will earn lower average returns. Brunnermeier and Parker (2005) and Brunnermeier, Gollier, and Parker (2007) solve an endogenous-probabilities model that produces similar asset pricing implications for skewness.

[37] Quote "In the context of a bubble model, high past returns, or a low book-to-market value imply that the bubble has been building up for a long time, so that there is a larger drop when it pops and prices fall back to fundamentals."

effect of size. Our study represents the advantage to coming up with a model that captures the three patterns documented above in a more integrated fashion.[38]

VII. Conclusion

We develop a disagreement model of speculative trading that captures for the first time the joint effect of loss aversion and overconfidence, on equilibrium price, speculative volume, and higher moments. Although a priori contradicting, these two key cognitive biases, display distinct features and do not merely cancel one another. Their unique interaction permits to generate features in line with striking empirical evidence that any of them can generate on their own. Moreover, this paper enriches also our understanding of their complex and marginal impacts on speculation.

The model can help shed light indeed on different, a priori not related, cross-sectional phenomena observed in the stock market. Namely, the model generates first, a positive correlation between public aggregate information and speculative volume. This feature provides an explanation for the observed correlation between high priced stocks and speculative volume (Hong and Stein (2007)). The model generates also a high-volume return premium (Gervais, Kaniel, and Mingelgrin (2001)), and permits to fill the gap between cross-sectional evidence and fixed firm characteristic that fail to be rationalize using the widely accepted Merton's (1987) recognition investor hypothesis. For example, the proposed model may explain simultaneously why the high-volume return premium is decreasing with firm's size while it is increasing with the existence of analyst coverage (Kaniel Ozoguz and Starks (2012)). Our model also supports the negative correlation between the concentration of institutional traders and the magnitude of the premium.

[38] This statement should however be taken with a grain of salt, since it is based on assumptions that we made on institutional traders and their elimination of behavioral biases through experience, as well as on their propensity to hold large-cap stocks.

Since the *visibility* hypothesis is hardly refutable, and short-sales constraints are assumed in the proposed theory as well as it amplifies Merton's (1987) recognition investor hypothesis, we should not see the proposed behavioral explanation as a substitute for the Merton's hypothesis but rather as complementary theories. Using both theories as guide, this work strongly suggests that it should exist economically implementable strategies based on the high-volume return premium. The model also supports a variety of stylised fact related to the skewness of asset return. The model succeeds to reconcile cross-sectional variation in skewness at the firm level (skewness is negatively correlated with trading volume, past return, and firm size) with the fact that on average skewness in individual firm return is positive.

It is important however to put our model's compelling supports of the impact of fixed-firm characteristics on trading volume and on conditional skewness, described above, into some perspective. In order to establish the influence of firm size or past return on volume and conditional skewness, we assume: (1) *narrow framing*; (2) firm size as a proxy of institutional trading; (3) institutional traders are immune from behavioral biases (overconfidence and loss aversion in our specific case). Although, some empirical evidence attests these facts, a rigorous empirical study should be conducted in order to strengthen the validity of the proposed theory.

Finally, a straightforward avenue emerges from this work. Questions of market efficiency, price-impact and survival of irrational traders in the long run are crucial to our understanding of financial markets. Several authors have been interested by those questions and developed consumption-based asset price models where both irrational traders and rational trades are present (Hirshleifer, and Luo, (2001), Kogan, Wang, Ross, and Westerfield (2006), and Easley and Yang (2015)). Although the proposed model underlies the trading mechanism between rational and

irrational traders, the purpose of this work is not to speak directly to questions of efficiency in financial markets. Whereas, it is the principal motivation behind the interest in the survival and price impact of irrational traders. Therefore, one could borrow the concept introduced in this study and develop a consumption-based asset-pricing model where irrational traders are naturally jointly loss averse[39] and overconfident (optimistic) about the prospect of economy[40].

Appendix A

Derivation of $\bar{V}(m)$

From the definition of the true posterior given by equation (2), it follows immediately that

$$\pi^{H}_{\langle S_2\rangle}(m)=\frac{a^{m}_{\langle S_2\rangle}}{a^{m}_{\langle S_2\rangle}+b^{m}_{\langle S_2\rangle}} \quad \text{and} \quad \pi^{L}_{\langle S_2\rangle}(m)=\pi^{H}_{\langle S_2\rangle}(-m)=\frac{b^{m}_{\langle S_2\rangle}}{a^{m}_{\langle S_2\rangle}+b^{m}_{\langle S_2\rangle}}. \tag{A1}$$

We get the same relation, for the irrational group, substituting the subscript *ra* with i*rr*. Following Bayes' rule, the true density of the current signal given the cumulative signal *m* is

$$f(s/m)=\delta_{\langle S_2\rangle}\left(s\middle|H\right)\pi_{\langle S_2\rangle}(m)+\delta_{\langle S_2\rangle}\left(s\middle|L\right)\pi_{L}(m)=\frac{\delta_{\langle S_2\rangle}(s/H)\pi^{H}_{\langle S_2\rangle}(m)}{\pi^{H}_{\langle S_2\rangle}(m+s)} \tag{A2}$$

The expected volume when $m>\psi$ $(m\le\psi)$ refers to the probability given the cumulative signal at time *t* that the public signal at time *t+1* is in the range $s\in(-\infty,\psi-m)$ $\left(s\in(\psi-m,\infty)\right)$. Replacing $k_{\langle S_2\rangle}$ with its value is required to make $\delta_{\langle S_2\rangle}(s/H)$ and $\delta_{\langle S_2\rangle}(s/L)$ density functions:

$$\frac{1}{k_{\langle S_2\rangle}}=\int_0^{\infty}(a^{s}_{\langle S_2\rangle}+b^{s}_{\langle S_2\rangle})ds=-\frac{1}{\ln(a_{\langle S_2\rangle})}-\frac{1}{\ln(b_{\langle S_2\rangle})}. \tag{A3}$$

[39] Loss aversion in those studies are modeled with recursive preference representation as in Barberis and Huang, (2009) and Easley and Yang (2015).
[40]In those models, investors who are optimistic about the state of the economy typically overestimate the rate of growth of the aggregate endowment.

We thus get for $m > \psi$

$$\int_{-\infty}^{\psi-m} -\frac{b_{ra}^{-s}(a_{\langle S_2\rangle}^{m+s}+b_{\langle S_2\rangle}^{m+s})\ln(a_{\langle S_2\rangle})\ln(b_{\langle S_2\rangle})}{\left(a_{\langle S_2\rangle}^{m}+b_{\langle S_2\rangle}^{m}\right)\left[\ln(a_{\langle S_2\rangle})+\ln(b_{\langle S_2\rangle})\right]}ds = \frac{a_{\langle S_2\rangle}^{m}b_{\langle S_2\rangle}^{m}\left[b_{\langle S_2\rangle}^{-\psi}\ln\left(a_{\langle S_2\rangle}\right)+a_{\langle S_2\rangle}^{-\psi}\ln\left(b_{\langle S_2\rangle}\right)\right]}{\left(a_{\langle S_2\rangle}^{m}+b_{\langle S_2\rangle}^{m}\right)\left(\ln\left(a_{\langle S_2\rangle}\right)+\ln\left(b_{\langle S_2\rangle}\right)\right)} \quad \text{(A4)}$$

and for $m \le \psi$ we have

$$\int_{\psi-m}^{\infty} -\frac{a_{\langle S_2\rangle}^{s}(a_{\langle S_2\rangle}^{m+s}+b_{\langle S_2\rangle}^{m+s})\ln(a_{\langle S_2\rangle})\ln(b_{\langle S_2\rangle})}{\left(a_{\langle S_2\rangle}^{m}+b_{\langle S_2\rangle}^{m}\right)\left[\ln(a_{\langle S_2\rangle})+\ln(b_{\langle S_2\rangle})\right]}ds = \frac{b_{\langle S_2\rangle}^{\psi}\ln(a_{\langle S_2\rangle})+a_{\langle S_2\rangle}^{\psi}\ln(b_{\langle S_2\rangle})}{\left(a_{\langle S_2\rangle}^{m}+b_{\langle S_2\rangle}^{m}\right)\left(\ln(a_{\langle S_2\rangle})+\ln(b_{\langle S_2\rangle})\right)} \quad \text{(A5)}$$

Replacing the posterior belief of the rational group with equation (2) leads to equation (10).

■

Let define as $\varphi_t(m)$, the unconditional density of the cumulative signal at date *t*. We next show that $\varphi_t(m)$ is symmetric with respect to $m=0$. The proof is by induction on *t*. For $t=1$, $\varphi_t(m) = f(m|0) = f(-m|0)$ since $\pi_{\langle S_2\rangle}^{H}(m) = \pi_{\langle S_2\rangle}^{L}(0)$. From (A2) it follows that $f(s|-m) = f(s|m)$. Now let assume that $\varphi_t(m) = \varphi_t(-m)$ for all $t < n$, by induction

$$\varphi_t(m) = \int_{-\infty}^{\infty} \varphi_t\left(m|m_{t-1}=x\right)\, \varphi_{n-1}(x)dx$$

$$= \int_{-\infty}^{\infty} f\left(m-x|x\right)\, \varphi_{n-1}(x)\, dx$$

$$= \int_{-\infty}^{\infty} f\left(m+x|-x\right)\, \varphi_{n-1}(-x)\, dx$$

$$= \int_{-\infty}^{\infty} f\left(-m-x|x\right)\, \varphi_{n-1}(x)\, dx$$

$$= \int_{-\infty}^{\infty} f\left(-m|m_{n-1}=x\right)\, \varphi_{n-1}(x)\, dx = \varphi_t(-m)$$

■

Having proved that the unconditional distribution of the cumulative signal is symmetric, we are ready to demonstrate Proposition 1.

Proposition 1. $Cov_t\left[\bar{V}(m), m\right] > 0$.

Proof. Since the unconditional distribution is symmetric, $E\left[m\right] = 0$.

$Cov_t\left[\bar{V}(m), m\right] = E\left[\bar{V}(m) \cdot m\right] = \int_{-\infty}^{\infty} \bar{V}(m) \cdot m \cdot \varphi_t\left(m\right)\, dm$. Let decompose this integral into the sum of three distinct integrals: I_1, I_2, and I_3, according to the brackets $m_1 \in \left[-\infty, -\psi\right]$, $m_2 \in \left[-\psi, \psi\right]$, and $m_3 \in \left[\psi, \infty\right]$ respectively. It is easy to show that $I_3 + I_1 > 0$, since for all $m_1 = -m_3$, $\bar{V}(m_3) > \bar{V}(m_1)$, and $m \cdot \varphi_t\left(m\right)$ is an odd function.

It remains now to demonstrate that $I_2 > 0$. $\bar{V}(m_2)$ is an increasing and positive function within the range $m_2 \in \left[-\psi, \psi\right]$. It is easy to show that the integrand of I_2 is positive (negative) function for all $m_2 > 0$ ($m_2 < 0$). Moreover for any value of $m_2 \in \left[0, \psi\right], \bar{V}(m_2) > \bar{V}(-m_2)$, and thus the integrand $\left|\bar{V}(m_2) \cdot m_2 \cdot \varphi_t\left(m_2\right)\right| > \left|\bar{V}(-m_2) \cdot \left(-m_2\right) \cdot \varphi_t\left(-m_2\right)\right|$. Therefore $I_2 > 0$.

■

Appendix B

Proposition 3. $\dfrac{\partial E\left[r_T\right]}{\partial \bar{V}} > 0$.

Starting with the equation for the expected return on a security between period *t* and *T*, $\partial E_t\left[r_T\right] = \dfrac{(H+L)}{2P_t} - 1$, we have that $\dfrac{\partial E_t\left[r_T\right]}{\partial P_t} = -\dfrac{(H+L)}{4P_t} < 0$. Using Lemma 1 $\left(\dfrac{\partial P_t}{\partial \lambda} < 0\right)$ it follows that

$$\frac{\partial E\left[r_T\right]}{\partial \lambda}=\frac{\partial E\left[r_T\right]}{\partial P_t}\cdot\frac{\partial P_t}{\partial \lambda}=\theta_{\langle S_1\rangle}^{m}\left(\frac{H-L}{2P_t\left(1+_t\lambda\theta_{\langle S_1\rangle}^{m}\right)}\right)^2>0. \qquad \text{(B1)}$$

We now want to show that $\frac{\partial\bar{V}}{\partial\lambda}=\frac{\partial\bar{V}}{\partial\psi}\frac{\partial\psi}{\partial\lambda}>0.$ We make naturally the straightforward assumption, that equilibrium price cannot take negative value (H and L are positive). The threshold $\psi=\ln(\lambda)/\ln(\theta_{\langle S_2\rangle}/\theta_{\langle S_1\rangle})$ increases with loss aversion i.e. $\frac{\partial\psi}{\partial\lambda}>0$. All that is left to show is that $\frac{\partial\bar{V}(m)}{\partial\psi}>0.$ Recall that $\pi_{\langle S_2\rangle}^{H}(\psi)=\frac{a_{\langle S_2\rangle}^{\psi}}{a_{\langle S_2\rangle}^{\psi}+b_{\langle S_2\rangle}^{\psi}}=\frac{1}{1+\theta_{\langle S_2\rangle}^{\psi}}$. Using the fundamental theorem of algebra, $\frac{\partial\bar{V}(m)}{\partial\psi}$ is equal to

$$\frac{\partial}{\partial\psi}\int_{-\infty}^{\psi}\frac{\left[b_{\langle S_2\rangle}^{\psi}\ln(a)+a_{\langle S_2\rangle}^{\psi}\ln(b_{\langle S_2\rangle})\right]\varphi_t\left(m\right)}{(a_{\langle S_2\rangle}^{m}+b_{\langle S_2\rangle}^{m})\left[\ln(a_{\langle S_2\rangle})+\ln(b_{\langle S_2\rangle})\right]}dm+\frac{\partial}{\partial\psi}\int_{\psi}^{\infty}\frac{a_{\langle S_2\rangle}^{m}b_{\langle S_2\rangle}^{m}\left[b_{\langle S_2\rangle}^{-\psi}\ln\left(a_{\langle S_2\rangle}\right)+a_{\langle S_2\rangle}^{-\psi}\ln\left(b_{\langle S_2\rangle}\right)\right]}{\left(a_{\langle S_2\rangle}^{m}+b_{\langle S_2\rangle}^{m}\right)\left[\ln\left(a_{\langle S_2\rangle}\right)+\ln\left(b_{\langle S_2\rangle}\right)\right]}\varphi_t\left(m\right)dm$$

(B2)

$$=\frac{\left(b_{\langle S_2\rangle}^{\psi}-a_{\langle S_2\rangle}^{\psi}\right)\left[\ln(a_{\langle S_2\rangle})-\ln(b_{\langle S_2\rangle})\right]}{\left(a_{\langle S_2\rangle}^{\psi}+b_{\langle S_2\rangle}^{\psi}\right)\left[\ln(a_{\langle S_2\rangle})+\ln(b_{\langle S_2\rangle})\right]}\varphi_t\left(\psi\right)>0,$$

where the last inequality follows from the fact that $1\geq a_{\langle S_2\rangle}>b_{\langle S_2\rangle}>0$, and $\varphi_t\left(\psi\right)>0$. Then using the chain rule method $\left(\frac{\partial E[r_T]}{\partial\lambda}=\frac{\partial E[r_T]}{\partial\bar{V}}\frac{\partial\bar{V}}{\partial\lambda}\right)$ and given that $\frac{\partial\bar{V}}{\partial\lambda}>0$ and that $\frac{\partial E\left[r_T\right]}{\partial\lambda}>0$, $\frac{\partial E\left[r_T\right]}{\partial\bar{V}}>0$ follows.

■

Appendix C

Proposition 5. *The unconditional skewness is positive.*

Proof. We define skewness as being the third moments of the distribution of returns between *t* and *t*+1:

$E^{ra}\left[R_{t+1}^{3}\right]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(P_{t+1}-P_{t}\right)^{3} f\left(s|m\right)\varphi_{t}\left(m\right)dsdm$. From (A2) it follows that $f\left(s|-m\right)= f\left(-s|m\right)$ and $f\left(s|m\right)= f\left(-s|-m\right)$. We can decompose the unconditional skewness into the following sum of four integrals $II_1+II_2+II_3+II_4=$

$$+\int_{0}^{\infty}\int_{0}^{\infty}\left(P_{t+1}-P_{t}\right)^{3} f\left(s|m\right)\varphi_{t}\left(m\right)ds\ dm+\int_{-\infty}^{0}\int_{-\infty}^{0}\left(P_{t+1}-P_{t}\right)^{3} f\left(s'|m'\right)\varphi_{t}\left(m'\right)ds'\ dm'. \qquad \text{(C1)}$$

$$+\int_{-\infty}^{0}\int_{0}^{\infty}\left(P_{t+1}-P_{t}\right)^{3} f\left(s|m\right)\varphi_{t}\left(m\right)ds\ dm+\int_{0}^{\infty}\int_{-\infty}^{0}\left(P_{t+1}-P_{t}\right)^{3} f\left(s'|m'\right)\varphi_{t}\left(m'\right)ds'\ dm'$$

To demonstrate positive skewness, we show that $II_1+II_2>0$ and $II_3+II_4>0$. First recall that $\varphi_{t}\left(m\right)=\varphi_{t}\left(-m\right)$ and $f\left(s|m\right)= f\left(-s|-m\right)$. Let define $\Gamma(m,s)=P_{t+1}-P_{t}$. For any $s>0$ and $m>0$, we set $m'=-m$ and $s'=-s$. Therefore we get $\varphi_{t}\left(m\right)\cdot f\left(s|m\right)=\varphi_{t}\left(m'\right)\cdot f\left(s'|m'\right)$. Moreover $\Gamma(m,s)>0,\ \Gamma(m',s')<0$,an

$\Gamma(m,s)+\Gamma(m',s')=\dfrac{(H-L)\theta_{\langle S_2\rangle}^{m}(\theta_{\langle S_2\rangle}^{2m+s}-1)(\theta_{\langle S_2\rangle}^{s}-1)(\lambda-1)\lambda(1+\lambda)}{\left(\theta_{\langle S_2\rangle}^{m}+\lambda\right)\left(\theta_{\langle S_2\rangle}^{m+s}+\lambda\right)\left(\lambda\theta_{\langle S_2\rangle}^{m}+1\right)\left(\lambda\theta_{\langle S_2\rangle}^{m+s}+1\right)}>0$. Consequently, for any $s>0$ and $m>0$, $m'=-m$ and $s'=-s$, the integrand of II_1 is greater than the integrand of -II_2 . Therefore $II_1>-II_2$. Using the same reasoning, and the fact that $f\left(s|-m\right)= f\left(-s|m\right)$, one can

show that $II_3 > -II_4$. Notice also that if $\lambda = 1$, $\Gamma(m,s) + \Gamma(m',s') = 0$ and therefore $E^{\langle S_2 \rangle}\left[R_{t+1}^3\right] = 0$

.

■

Proposition 7. $Cov_t\left[Sk(m), V(m)\right] < 0.$

We define $Sk(m)$ as the conditional skewness at *t+1* given cumulative signal *m*. We already demonstrated in Result 1 that trading volume and aggregate information are positively correlated. Thus, to demonstrate Result 7, we can show that $Cov_t\left[Sk(m), m\right] < 0.$

where $Cov_t\left[Sk(m), m\right] = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} m\left(P_{t+1} - P_t\right)^3 f\left(s \middle| m\right) \varphi_t\left(m\right) ds\ dm$.

We divide the integral into the sum of four integrals (same intervals as in the demonstration of Proposition 5) and we proceed to a change of variables to permit the integration over a positive range of *m* and *s*. Since the integral of a sum of functions equals to the sum of integrals, therefore, we get

$$Cov_t\left[Sk(m), m\right] = \int\limits_0^{\infty}\int\limits_0^{\infty} \Omega(s,m)\Pi(s,m)\varphi_t\left(m\right) ds\ dm \tag{C2}$$

where $\Omega(m,s) = \dfrac{mk_{\langle S_2 \rangle}(H-L)^s \theta_{\langle S_2 \rangle}{}^{sm}(\theta_{\langle S_2 \rangle}{}^{s} - 1)\lambda^s a_{\langle S_2 \rangle}^s}{1+\theta_{\langle S_2 \rangle}^m}$,

and $$\Pi(s,m) = \left(\begin{array}{l} -\dfrac{\left(1+\theta_{\langle S_2 \rangle}^{m+s}\right)}{\left(\theta_{\langle S_1 \rangle}^m + \lambda\right)^3 \left(\theta_{\langle S_1 \rangle}^{m+s} + \lambda\right)^3} + \dfrac{\left(\theta_{\langle S_2 \rangle}^m + \theta_{\langle S_2 \rangle}^s\right)}{\left(1+\theta_{\langle S_1 \rangle}^m \lambda\right)^3 \left(\theta_{\langle S_1 \rangle}^s + \lambda\theta_{\langle S_1 \rangle}^m\right)^3} \\ + \dfrac{\left(\theta_{\langle S_2 \rangle}^m + \theta_{\langle S_2 \rangle}^s\right)}{\left(\theta_{\langle S_1 \rangle}^m + \lambda\right)^3 \left(\theta_{\langle S_1 \rangle}^m + \lambda\theta_{\langle S_1 \rangle}^s\right)^3} - \dfrac{\left(1+\theta_{\langle S_2 \rangle}^{m+s}\right)}{\left(1+\theta_{\langle S_1 \rangle}^m \lambda\right)^3 \left(1+\lambda\theta_{\langle S_1 \rangle}^{m+s}\right)^3} \end{array} \right).$$

We see immediately that $\Omega(m,s) < 0$ for all $s > 0$, and, $m > 0$. Thus, one way to demonstrate the negative covariance is to show that $\Pi(s,m) > 0$ for all $s > 0$, and, $m > 0$. Unfortunately, it is not possible to demonstrate this relation analytically. Using numerical techniques we can show $\Pi(s,m) > 0$ for all $s > 0$, and, $m > 0$, when $2.02 > \lambda > 1$, and $1 > \theta_{\langle S_1 \rangle} > \theta_{\langle S_2 \rangle} + 0.048 > 0$. We can easily conjecture that the integral is always positive for larger degree of loss aversion. To conceive it visually, let take for example, a higher degree of loss aversion, namely $\lambda = 3$, and find the global minimum given all the constraints. We find a global minimum at -0.015 where $\theta_{\langle S_1 \rangle} = 0.343$ $\theta_{\langle S_2 \rangle} = 0.393$, $m = 0.95$, $s = 1.4$. The figure below plots $\Pi(1.4, m)$ and $\Pi(s, 0.95)$ respectively.

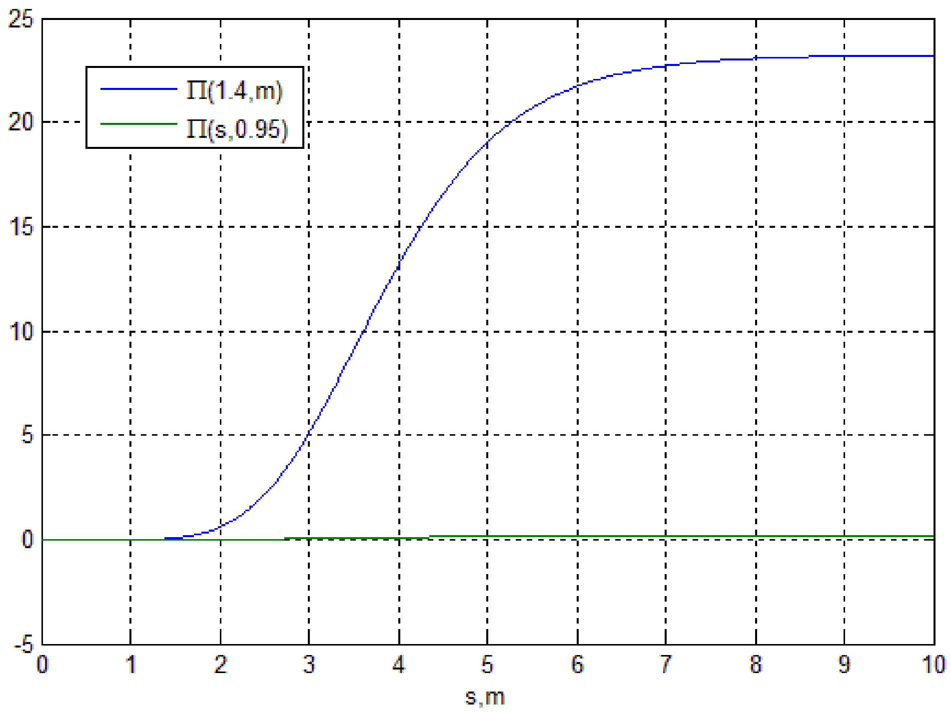

Figure C1. Plots of $\Pi(1.4, m)$ and $\Pi(s, 0.95)$, where the *x-axis* refers to *s* and *m* respectively.

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