# Arbitrage with Financial Constraints and Market Power*

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**Abstract**

I study how financial constraints affect liquidity provision and welfare under different structures of the arbitrage industry. In competitive markets, financial constraints may impair arbitrageurs' ability to provide liquidity, thereby reducing other investors' welfare. Instead, in imperfectly competitive markets, I characterize situations in which imposing constraints on arbitrageurs leads to a Pareto-improvement relative to a no-constraint case. Further, unlike the competitive case, a drop in arbitrage capital does not always lead to a reduction in market liquidity. A subtle interaction between financial constraints and arbitrageurs' market power generates these Pareto-improvment and novel comparative statics.

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# 1 Introduction

While the LTCM and 2007-2009 crises higlighted the interactions between funding and market liquidity, our understanding of these interactions remains limited to competitive settings.[1] The reality of financial markets, however, is often closer to imperfect competition. For instance, LTCM, which initially motivated the literature, was nicknamed the "central bank of volatility", due to its dominant position in derivatives markets. LTCM is not an isolated case: many other hedge funds or banks (e.g. Amaranth, the "London whale", etc.) have been under the spotlights for becoming the dominant traders in some markets; more generally, there has been a noted increase in market power in financial markets.[2] Consistent with this evidence, transaction-level data shows that large intermediaries recognize their price impact and use optimal execution techniques to rebalance portfolios.[3] Further, financial constraints due to regulations, internal risk management, or margins imposed by financiers (e.g. brokers, repo market participants, etc.) are likely to limit the funding liquidity of these large traders, and have probably become tighter after 2007-2009.[4]

In this paper, I study the effects of imposing financial constraints on imperfectly competitive arbitrageurs. I show that financial constraints affect market liquidity and social welfare in different and sometimes opposite ways when arbitrageurs have market power. In competitive markets, binding financial constraints (i.e. a decrease in funding liquidity) impair arbitrageurs' ability to exploit profitable trading opportunities, thereby reducing market liquidity and hurting the investors who are on the other side of their trades. Instead, imposing financial constraints on imperfectly competitive arbitrageurs may in some cases *improve* both market liquidity and social welfare.

Arbitrageurs with market power may benefit from the constraints because they face a commit-

[1]For models of financially constrained arbitrage in competitive markets, see, among others, Shleifer and Vishny (1997), Gromb and Vayanos (2002, 2010, 2018), Kondor (2008), Brunnermeier and Pedersen (2009). Attari and Mello (2006) study numerically how a monopolistic arbitrageur subject to financial constraints trades, but their setting does not lend itself to welfare analysis.

[2]See for instance, De Loecker, Eeckhout, and Unger (2019), for evidence about market power. Many financial markets are dominated by a few large players. For instance, five banks represent 90% of the notional amount of derivative contracts (OCC, 2018).

[3]See, for instance, Gabaix et al., (2005), Ben-David et al. (2017), Chan and Lakonishok, 1995, and van Kervel and Menkveld (2018).

[4]Basel III has tightened capital requirements for banks, and introduced a leverage ratio and several liquidity ratios, which generate constraints that are similar in spirit to the financial constraint considered in this paper. Further, these new or tighter constraints on banks have been passed through to hedge funds and other market players through a reduction in funding (Boyarchenko et al, 2018).

ment problem, as durable good producers: once they have provided some liquidity, they cannot refrain from providing more, and reduce the arbitrage opportunity further. However, a binding constraint in the future limits their ability to provide further liquidity, mitigating their commitment problem. Surprisingly, the investors on the other side of their trades may also benefit, for two reasons. First, the only way for the arbitrageur to make the constraint binding in the future is to pledge more capital to the arbitrage early on, which speeds up risk-sharing. Second, the constraint is an imperfect commitment device: it does not prevent the arbitrageur entirely from retrading, as early capital gains generate additional collateral for later trading rounds.

**Model.** I introduce imperfect (Cournot) competition among arbitrageurs in an otherwise standard model of financially constrained arbitrage. The model has two types of investors: *hedgers* and *arbitrageurs*, which trade for two rounds and then consume. There is a risky asset traded in two segmented markets ($A$ and $B$) and a risk-free asset. In each segmented market, the risk-averse, competitive hedgers receive endowment shocks correlated with the asset payoff. For simplicity, these shocks are symmetric: hedgers in market A are overexposed to the risky asset and would like to hedge by selling, and vice versa in market B. Market segmentation prevents welfare-improving trades between the two groups. As a result, the risky asset trades at a discount in A and at a premium in B.

While hedgers are restricted to trade in their respective market, arbitrageurs can trade across markets. As the risky asset gives claims to the same cash-flows in both markets, prices converge in the final period. Thus, the *spread* between prices in markets A and B creates a textbook arbitrage opportunity for arbitrageurs, who face a relative value trade with a fixed convergence date. Arbitrageurs, however, must separately collateralize each leg of the arbitrage. Arbitrageurs' wealth serves as collateral, for both long and short positions, and must remain sufficiently large over time to absorb adverse price movements.[5] This financial constraint resembles realistic VaR constraints imposed by regulators or used by prime brokers or financiers in repo markets.[6]

[5] I use wealth and capital as synonyms. To avoid dealing with default in equilibrium, I assume that the worst change in fundamental is bounded above and below, and that arbitrageurs must fully collateralize all potential losses. This is akin to a 100% VaR constraint. Gromb and Vayanos (2002) follow a similar strategy, while Brunnermeier and Pedersen consider an $\alpha$% VaR, but do not study welfare.

[6] Brunnermeier and Pedersen, 2009, Appendix A, describe in detail the mapping of this kind of constraint to

Suppose first that arbitrageurs face financial constraints but are competitive. In equilibrium, arbitrageurs eliminate the arbitrage opportunity when capital is abundant. When capital is scarce, the financial constraint binds and prevents them from building large enough positions to eliminate the spread. As time passes, arbitrageurs earn capital gains, which increase their wealth and relax the financial constraint, so that the spread decreases. In this competitive setting, imposing financial constraints has either no effect on the equilibrium (when arbitrageurs capital is large), or prevents arbitrageurs from intermediating trades between markets, which reduces hedgers' welfare.

This result stands in sharp contrast to the effects of imposing financial constraints on arbitrageurs with market power. I show that when capital is intermediate and the risk to benefit ratio of the trade is sufficiently high, imposing constraints on arbitrageurs leads to a Pareto-improvement. In all other situations, imposing constraints has either no effect, because they never bind, or limits the liquidity provision by arbitrageurs, which reduces the welfare of at least one type of investors.[7]

The Pareto improvement follows from a subtle interaction between market power and financial constraints. Consider first a monopoly without constraints. The arbitrageur is akin to a durable good producer; he faces a commitment problem: having provided some market liquidity, the arbitrageur faces a residual demand for liquidity and cannot refrain from providing further liquidity, thereby reducing the spread further. Hedgers anticipate this behaviour, which erodes the arbitrageur's market power ex-ante.[8]

Because of these Coasian dynamics, the arbitrageur would be better off if he could commit to a trading strategy, i.e. decide ex-ante how much liquidity to provide over time and stick to it. In this case, he would trade only once at the beginning. Doing so, he would earn static monopoly profits, as is well-known in IO; this hurts hedgers relative to the no-commitment case.

With a financial constraint, the arbitrageur chooses trades sequentially. However, if the con-

real-world constraints.

[7]In the model, the numerator of the risk benefit ratio is the worst case scenario for the fundamental. In practice, it would correspond to a quantile of the return distribution, e.g. the 99% quantile.

[8]As hedgers have reduced their positions at time 0, they are less exposed to the endowment shock at time 1. To this extent, receiving liquidity / sharing risk/ buying insurance (all synonyms in this model) by trading the risky, imperfectly liquid asset against cash (the liquid asset) is akin to buying a durable good. However, in contrast to the literature on the Coase conjecture, the horizon is finite. In IO, it is typically assumed that the good is infinitely durable. Classic papers on the Coase conjecture include Stockey (1981), Bulow, (1982), Gul, Sonnenschein, and Wilson (1986).

straint binds in the future, the choice is purely mechanical (max out the constraint). To this extent, a binding constraint in the future works as a commitment device. The tightness of the financial constraints depends on the arbitrageur's trading strategy and prices. For this reason, financial constraints may bind at some date and not at other, i.e., the arbitrageur's ability to provide liquidity is *endogenously time-varying.* Further, financial constraints may limit retrading, but do not eliminate it: they merely prevent the arbitrageur from reaching his preferred position. The reason is simple: when providing liquidity, a monopoly *always* earns capital gains, which generate additional collateral and allows the arbitrageur to retrade next period.[9] It is ex-post *optimal* for the arbitrageur to keep pledging capital to the arbitrage, for lack of a better investment opportunity. The only alternative investment is the risk-free asset, which offers lower returns than the arbitrage.[10]

A welfare improvement occurs only in cases where the constraint is slack at time 0 and binding at time 1. The binding constraint at time 1 mitigates the arbitrageur's problem ex-ante; the slack constraint at time 0 allows the arbitrageur to reap the benefit of it. Indeed, the prospect of a binding constraint induces hedgers to shift some of their liquidity demand earlier. The slack constraint allows the arbitrageur to exploit this extra demand. The arbitrageur's welfare increases relative to the no commitment, no constraint case, but does not reach the perfect commitment level, because the retrading maintains some Coasian dynamics: the arbitrageur is not able to charge static monopoly prices.

Hedgers are also better off when the constraint binds at time 1 but not at time 0 than without constraint. In this case, the constraint induces faster risk-sharing but does not eliminate retrading. Instead, perfect commitment speeds up risk-sharing but eliminates retrading.[11] Receiving liquidity early matters to hedgers: the asset is conditionally riskier at time 0, because dividends news accrue every trading round.[12]

[9] By contrast, competitive arbitrageurs earn capital gains only when their constraints bind. With financial constraints, it is essential to generate more collateral for later rounds that the rents the monopolistic arbitrageur extracts in the first round cannot be diverted or repledged elsewhere.

[10] In Dow, Han, and Sangiorgi (2019), a financially constrained arbitrageur also prefers to stick to his existing position, not for a lack of better opportunities, but because it is costly to exit.

[11] Under perfect commitment, hedgers anticipate that the arbitrageur will not retrade and shift their demand to the initial trading round. The arbitrageur provides more liquidity initially to exploit this extra demand.

[12] Uncertainty about prices at time 0 gives hedgers a preference to trade early and plays the same role as a discount factor. It also creates a role for arbitrageurs in both periods. Without it, hedgers' demand would not remain downward-sloping at time 0, so that there would be no demand for liquidity. Hedgers would face a temporary

Risk-sharing speeds up endogenously: to make the financial constraint binding next round, the arbitrageur *must* trade more aggressively early on than without constraints. By taking larger positions, the arbitrageur pledges more capital to the trade now and makes it more likely to have a binding constraint next period. But doing so, he provides more liquidity and reduces the spread. The financial constraint does not eliminate retrading: in fact, the larger capital gains earned ex-ante increase the maximum position the arbitrageur can afford given the constraint. As a result, the arbitrageur accumulates larger positions than in the no commitment, no constraint case. The combination of faster risk-sharing (more liquidity early) and higher total amount of liquidity is necessary for an improvement in hedgers' welfare with a monopoly, but not with an oligopoly. In the latter, more early liquidity and enough retrading suffice.[13]

The conditions for the Pareto improvement are that capital is intermediate and the risk to benefit ratio of the trade is sufficiently high. Intuitively, if capital is very low, the arbitrageur cannot take larger positions early on without violating the constraint. But if capital is very large, he will never be financially constrained ex-post. A high risk to benefit ratio implies that positions are sufficiently capital-intensive relative to potential profits. Hence, even taking into account the capital gains from providing early liquidity, the arbitrageur's constraint does bind next period.

The risk benefit ratio also determines the comparative statics of spreads with respect to initial capital. In markets with a low risk benefit ratio, comparative statics are qualitatively similar to the competitive case: less capital may lead to higher spreads. In markets with a high risk to benefit ratio, however, a drop in capital first *reduces* the spread and then increases it. As capital drops, hedgers anticipate that the arbitrageur will trade more aggressively early on and face a binding constraint later, and eventually provide more liquidity, which reduces spreads. As capital drops further, however, the arbitrageur will no longer be able to trade aggressively early on, and spreads will increase, as in the competitive case. To the best of my knowledge, the empirical literature has not used the risk benefit ratio as a conditioning variable, and has not tested this unique prediction

risk-free asset and would flatten out demand, becoming arbitrageurs themselves.

[13]Competition among arbitrageurs erodes capital gains, so arbitrageurs' financial constraints relax less, and arbitrageurs may not provide more liquidity in total. Still, hedgers may be better off. Hence, what is essential is that the arbitrageur provides more liquidity early on and retrades, not that he provides more liquidity in total.

of the model.[14]

My results should be of interest to policy-makers regulating markets with large traders. The debate around Basel III has focused on the possibly negative effects on liquidity provision of the tightening in capital requirements. I show, however, that in some situations there are welfare *gains* from imposing financial constraints on arbitrageurs with market power.[15]

My mechanism provides a rationale for (in specific cases) imposing financial constraints on imperfectly competitive arbitrageurs, not in favor of market power itself. A standard argument in favor of market power is that pecuniary externalities arise from the fact that agents do not internalize their price effects. Thus, *given constraints*, it may be beneficial to give traders market power to curb externalities.[16] Here the competitive equilibrium is constrained efficient, so this mechanism does not arise. The mechanism of this paper is different, and to the best of my knowledge novel: *given market power*, it is in some markets socially desirable to impose financial constraints, because of the way they interact with arbitrageurs' market power.

Two additional policy implications arise: first, the model may explain why large financial institutions fund themselves more cheaply. It is often argued that this funding cost advantage results from an implicit government put.[17] In my model, large arbitrageurs are always less severely constrained than competitive arbitrageurs, because their profits lead to a larger "pledgeable income".[18] Second, the model highlights an unintended consequence of capital requirements regulation. They may increase the market power of traders by limiting their ability to participate in the market in some states.[19]

[14] The welfare result also holds in the oligopolistic case, but spreads may not be lower at all dates.

[15] One aspect that I abstract from in this paper is that binding constraints may lead to firesales and even default in equilibrium. Thus, the benefits of being constrained should be weighted against the costs generated by firesales and default. However, to the best of my knowledge, such benefits have not been highlighted before.

[16] Eisenbach and Whelan (2019) show that this argument may not always go through when imperfectly competitive traders differ in their trading needs. On a different note, Glosten (1989) shows that arbitrageur's market power can have benefits in a model of asymmetric information. When arbitrageurs (in Glosten's context, market-makers) are competitive, the market may break down when the adverse selection problem becomes extreme. A monopolistic market-maker (e.g. a specialist) can average profits over time, which reduces the likelihood of a market break-down.

[17] See e.g. Acharya, Cooley, Richardson and Walter (2010).

[18] Related ideas have been developed in the banking literature. For instance, Keeley (1990) shows that banks with market power are more likely to act prudently with regard to risk-taking, because they risk losing valuable bank charters.

[19] In the absence of constraints, the mere continuous presence on the market of a trader or his tendency to break up trades to execute block orders are factors that erode her market power. Indeed, rational investors can anticipate better prices for liquidity in the future and shift their demand.

**Related literature.** To the best of my knowledge, this paper is the first to solve the dynamic problem of imperfectly competitive traders under realistic financial constraints when all investors are rational. As a result, the paper contributes to three strands of the literature.

My first contribution is to extend the literature on the limits of arbitrage, which is cast into the competitive framework, to imperfect competition. I build on Gromb and Vayanos (2002, 2010) and Brunnermeier and Pedersen (2009). Akin to the former, the focus of this paper is on welfare.[20]

Second, I contribute to the literature on imperfect competition in financial markets by analyzing the interaction between market power and financial constraints. Several papers in this active literature model all investors as rational and emphasize the parallel with the durable goods problem studied by Coase (1972), but not study the effects of financial constraints.[21] Instead, Attari and Mello (2006) do study the effects of financial constraints on a monopolistic arbitrageur, but do not model all agents as rational, assuming that the arbitrageur faces exogenous demand curves, i.e. that hedgers do not optimize. This assumption rules out a welfare analysis and eliminates the Coasian dynamics, which are central to my results.

Finally, the paper contributes to the literature on durable good monopolists. Perhaps the most closely related paper is McAfee and Wiseman (2008), where the monopolist chooses ex-ante a capacity constraint by paying a small capacity cost. I consider similar constraints in my setting and show that, while the arbitrageur is as well off as in the no commitment no constraint case (notwithstanding the cost), hedgers are worse off than in all other cases. The reason is that capacity constraints do not eliminate retrading, but delay risk-sharing. To avoid unused capacity at time 1, the arbitrageur chooses a smaller capacity than in the no commitment case and provides the same amount of liquidity every period. Thus, an essential difference between arbitrageurs' financial constraints and the capacity constraints of goods producers is that the latter do not evolve over time. If we allow the arbitrageur to increase over time the capacity, he might do so, but hedgers will still suffer from the low initial capacity. Instead, the tightness of financial constraints depends endogenously on the trading process, so that the financial constraints help achieve both faster

[20]Other recent papers on financially constrained arbitrage include Kondor and Vayanos (2018) and Dávila and Korinek (2017), among others.

[21]See, e.g., Basak (1997), Vayanos (1999), Kihlstrom (2000), Pritsker (2009), DeMarzo and Urosevic (2007), Marinovic and Varas (2019).

risk-sharing and some retrading. This endogeneity overturns their welfare effects as commitment devices.[22] Further, if the arbitrageur could choose a time-dependent capacity ex-ante, he would never increase capacity over time. He would choose constraints inducing the perfect commitment case.

# 2 Model

I consider a standard model of financially constrained arbitrage, where arbitrageurs exploit price differences between two identical assets over time, while facing realistic capital constraints.

**Assets and timeline.** The model has three periods, indexed by $t = 0, 1, 2$. Financial markets are open at time 0 and time 1, and consumption takes place at time 2. There are two identical risky assets, A and B, and a risk-free asset with return $r_f$ normalized to 0. The risky assets trade in segmented markets at price $p_t^k$, $k = A, B$. They are both in zero net supply and pay the same dividend $D_2$ at time 2, with $D_2 = D + \epsilon_1 + \epsilon_2$. The dividend news $\epsilon_t$ are *iid* random variables with a symmetric bounded support $[-\bar{e}, \bar{e}]$, a mean of 0 and variance $\sigma^2$. The news $\epsilon_t$ is revealed to all investors at time $t$ before trading. There are two types of investors: hedgers and arbitrageurs.

**Hedgers.** In each market, there is a continuum mass one of risk-averse competitive hedgers with mean-variance preferences: $U\left(w_2^k\right) = \mathbb{E}\left(w_2^k\right) - \frac{a}{2}\mathbb{V}\left(w_2^k\right)$.[23] Every period, hedgers receive endowment shocks $s^k\epsilon_t$ that are correlated with the dividend of the risky asset, and will therefore affect their demand for the risky asset. At time $t$, hedgers' weath at is

$$w_t^k = w_{t-1}^k + s^k\epsilon_t + Y_{t-1}^k(p_t^k - p_{t-1}^k, \tag{1}$$

[22] Because of this endogeneity, there is a feedback loop between financial constraints and trading strategies, which may lead to multiple equilibria. Equilibria may coexist, because arbitrageurs choose quantities, but not hedgers' expectations. However, hedgers' expectations about whether constraints bind or not in the future determine market depth today, and therefore the arbitrageur's incentives to trade in a way that makes the constraint binding or not.

[23] Mean-variance preferences are also used for tractability reasons in, e.g., Banerjee and Green (2015). With mean-variance preferences, I consider time-constistent trading strategies for hedgers.

i.e. hedgers' wealth changes because of capital gains on the risky asset (third term) and the endowment shocks. For simplicity, the magnitude of the shock, $s^k$, is deterministic, constant over time, and symmetric across markets.[24] That is, at time $t = 1, 2$, hedgers in market A receive a shock $s^A \epsilon_t = s\epsilon_t$, while hedgers in market B receive opposite shocks, $s^B \epsilon_t = -s\epsilon_t$. As a result, A-investors have a low valuation for the risky asset, and B-investors a high valuation. Market segmentation prevents hedgers from sharing risk across markets, although they could perfectly insure each other. Therefore, assets A and B may trade at different prices in their respective market, even though their cash-flows are identical. Since the endowment shock will shift hedgers' demand up or down by $|s|$, it is convenient to think of $s$ as the net supply (in absolute value) in each market (see Section 3.2 for additional details).

**Arbitrageur(s).** There are $n \geq 1$ identical arbitrageurs, who can participate in all markets, but face financial constraints (described below). Arbitrageurs have no initial holdings of the risky asset but own initial wealth (capital) $W_0^i = \frac{W_0}{n}$, where $W_0$ is the total capital in the arbitrage industry.I focus my analysis on the comparison between the monopolistic ($n = 1$) and competitive cases (Sections 4 and 5), and consider the more general oligopolistic case in the Supplementary Appendix.

Arbitrageurs also have mean-variance preferences: $u(W_2) = \mathbb{E}(W_2) - \frac{b}{2}\mathbb{V}(W_2)$, with risk-aversion $b$. Because they have access to all markets, arbitrageur's final wealth is $W_2^i = \sum_{k=A,B} X_1^{i,k} D_2 + B_1^i$, where $X_t^{i,k} = X_{t-1}^{i,k} + x_t^{i,k}$ denotes the end-of-period position at time $t$ in asset $k$ of arbitrageur $i$, $x_t^{i,k}$ the corresponding trade, and $B_t^i = B_{t-1}^i - \sum_{k=A,B} x_t^{i,k} p_t^k$, the arbitrageur's risk-free asset holdings at the end of period $t$.

When $n$ is finite, arbitrageurs internalize their own price impact in both A and B markets. Specifically, I assume that arbitrageurs choose trades $x_t^{i,k}$ (Cournot competition), knowing hedgers'

[24]The assumption of constant shock magnitude $s$ can be relaxed at the cost of increased complexity but does not seem essential for the main results, in particular if $s$ remains of the same sign. Relaxing the assumption of deterministic shocks, however, would require a separate analysis: as shown in Gromb and Vayanos (2002), stochastic shocks lead to pecuniary externalities, so that the competitive equilibrium is not constrained efficient anymore. Market power would lead arbitrageurs to internalize some of the pecuniary externalities, opening an interaction that is not present with deterministic shocks, where the competitive equilibrium is constrained efficient. Eisenbach and Phelan (2019) start from a constrained inefficient equilibrium and study the effect of giving liquidity suppliers market power, albeit in a static setting.

demand in each market, and imposing market-clearing. The hedgers' inverted demand and market clearing define a price schedule, derived below, that links the arbitrageur's trade to the equilibrium price.

**Financial constraints.** Arbitrageurs need capital to trade the risky assets. I model the financial constraint in the same fashion as Gromb and Vayanos (2002, 2010) and Brunnnermeier and Pedersen (2009). Arbitrageurs have a margin account $V_t^k$ in each market, and their positions must be fully collateralized. That is, the arbitrageur's wealth in this account must cover the maximum possible loss on the position over the next period:

$$V_t^k \geq \max_{p_{t+1}^k} X_t^k \left( p_t^k - p_{t+1}^k \right)$$

Hence, in total, the arbitrageur's wealth must cover the total maximum loss on each account:

$$W_t \geq \sum_{k=A,B} \max_{p_{t+1}^k} X_t^k \left( p_t^k - p_{t+1}^k \right) \tag{2}$$

The presence of the financial constraint implies that arbitrageurs may not be able to fully eliminate the price differences between A and B assets. The modeling of the constraint also implies that asset A cannot be used as collateral for asset B (and vice-versa). In other words, cross-collateralization is not allowed, which can be viewed as a consequence of the assumption of market segmentation. In practice, cross-collateralization is often limited by financiers who are concerned about imperfect correlation between assets (although this would not be an issue here). Sometimes traders also voluntarily avoid cross-collateralization in order to avoid revealing their trading strategies.[25]

Given the symmetry assumptions, and in line with the literature (e.g. Gromb and Vayanos, 2002), it is natural to focus on equilibria in which the arbitrageur holds opposite positions in both assets, i.e. $X_t^{i,A} = -X_t^{i,B} = X_t^i$. Given that arbitrageurs start with no endowment in the risky

[25] For instance, Pérold (1999) reports: "LTCM inernalized most of the back-office functions associated with contractual arrangements, due to the complexity and and advanced nature of many of the firm's trades. This also helped maintain the confidentiality of its positions. LTCM chose Bear Stearns as a clearing agent partly because Bear Stearns was committed to customer business rather than being focused on proprietary trading, and thus there were fewer conflicts of interest."

assets, this implies that $x_t^{i,A} = -x_t^{i,B} = x_t^i$, for $t = 0,1$. Thus, we can rewrite the arbitrageur's budget constraint as follows:

$$W_2^i = W_0^i + \sum_{t=0,1} x_t^i \Delta_t, \text{ with } \Delta_t = p_t^B - p_t^A$$

The equation shows that by setting up opposite position in each leg of the arbitrage, arbitrageurs eliminate all fundamental risk and derive all their profits from exploiting the spread $\Delta$ between the prices of the two assets. This assumption also simplifies the financial constraint, because it implies that the risk premia on asset A and B are opposite. That is, $D_t - p_t^A = \frac{\Delta_t}{2}$, where $D_t$ is the conditional expected value of the asset at time $t$: $D_t = \mathbb{E}_t(D_2) = D_{t-1} + \epsilon_t$. This implies that $p_t^k - p_{t+1}^k = \frac{\Delta_{t+1} - \Delta_t}{2} - \epsilon_{t+1}$. As a result, we can rewrite the financial constraint (2) as follows:

$$\begin{aligned}
W_t^i &\geq \sum_{k=A,B} \max_{p_{t+1}^k} X_t^{i,k} \left( p_t^k - p_{t+1}^k \right) \\
&\geq \max_{\epsilon_{t+1}} X_t^i \left( \frac{\Delta_{t+1} - \Delta_t}{2} - \epsilon_{t+1} \right) + \max_{\epsilon_{t+1}} -X_t^i \left( -\frac{\Delta_{t+1} - \Delta_t}{2} - \epsilon_{t+1} \right) \\
&\geq 2X_t^i \left( \frac{\Delta_{t+1} - \Delta_t}{2} \right) + \max_{\epsilon_{t+1}} X_t^i \left( -\epsilon_{t+1} \right) + \max_{\epsilon_{t+1}} -X_t^i \left( -\epsilon_{t+1} \right) \\
&\geq 2|X_t^i| \bar{e} - X_t^i \left( \Delta_t - \Delta_{t+1} \right) \qquad (3)
\end{aligned}$$

The last step follows from the symmetric support of the distribution. Note that the constraint may bind upward or downward. An upward-binding constraint generates an upper bound on how much the arbitrageur can hold, e.g. for a long position, $X_t^i \leq \frac{W_t^i}{2\bar{e} - (\Delta_t - \Delta_{t+1})}$. Instead, a downward-binding constraint generates a lower bound on the arbitrageur's position, e.g. for short positions, the arbitrageur needs to hold at least $X_t^i \geq \frac{W_t^i}{-(2\bar{e} + \Delta_t - \Delta_{t+1})}$.

**VaR vs margins.** The financial constraint corresponds to a one-period VaR constraint at the 100 percent level (as implied by the assumption of full collateralization). The 100 percent level is for simplicity only, as it rules out default in equilibrium and thus makes welfare comparisons simpler[26], but the constraint is motivated by real-world regulations and margin setting practices of

[26]In particular, there is no need to compute the welfare of financiers on the other side of the constraint.

financiers.[27] An important feature of the constraint is that it is forward-looking, in the sense that it is based on both current and future prices.

The VaR constraint can also be seen as a margin constraint. Suppose that the arbitrageur holds a long position, $X_t^i \geq 0$. We can rewrite the right-hand side of inequality (3) as $2m_t^+ X_t^i$, where the $m_t^+ = \bar{e} - \frac{1}{2}(\Delta_t - \Delta_{t+1})$ denotes the margin required on the position. The properties of the margin are key for the dynamics of the model. Here, margins increase with fundamental risk $\bar{e}$ (and consequently, volatility). A more volatile asset leads to a larger potential loss on the position, which induces financiers to ask for more collateral. Margins also depend on the *change* in the spread, $\Delta_t - \Delta_{t+1}$. If financiers expect the spread to decrease, i.e. $\Delta_{t+1} \leq \Delta_t$, they reduce current margins. Hence, margins play a stabilizing role for asset prices. [28]

**Terminology.** In the literature, market liquidity refers to the price spread $\Delta_t \equiv p_t^B - p_t^A$, which resembles a bid-ask spread. However, market liquidity is a multifaceted concept. One measure of liquidity in the model is market depth, given by the slope of hedgers' inverted demand. Thus, to avoid ambiguity, I use spread instead of market liquidity for $\Delta_t$. Further, I use the expression "provide liquidity" as synonym to "provide insurance/risk-sharing". Funding liquidity relates to the tightness of the arbitrageur's financial constraint.

# 3 Benchmarks

To highlight the novel interaction betwen market power and financial constraints of the model, I first review the effect of each ingredient separately.

[27]For instance, Brunnermeier and Pedersen (2009), Appendix A, provide additional institutional details to motivate the analysis of this type of constraint.

[28]Brunnermeier and Pedersen (2009) obtain a similar constraint in their benchmark case with informed financiers. They also consider a situation in wihch financiers are assumed to be uninformed. In this case, uncertainty about whether the mispricing will decrease or not in the future can lead to procyclical, destabilizing margins. Brunnermeier and Pedersen show that a margin spiral, in which low liquidity leads to higher margins, which further limits the ability of arbitrageurs to provide liquidity, can result from the presence of uninformed financiers. This margin spiral complements and amplifies the loss spiral created by the financial constraint ("a decrease in arbitrageurs' capital impairs their ability to provide liquidity and eliminate the mispricing, which in turn reduces their capital"). Under the assumptions of this paper, there can be a loss spiral, but no margin spiral.

### 3.1 Financial constraints without market power

In a competitive equilibrium, hedgers maximize expected utility given prices, while a representative competitive arbitrageur maximize expected utility given prices and financial constraints. Note that a $^*$ denotes the competitive outcome in the paper.

**Proposition 1 (Gromb and Vayanos, 2002)** *There exists a unique competitive equilibrium:*

- *If $W_0 \geq \omega^* \equiv 2s\bar{e}$, the financial constraint never binds, the arbitrageurs absorb the supply $s$, i.e. $X_t = s$ at $t = 0, 1$, and the spread between assets A and B is always 0: $\Delta_0 = \Delta_1 = \Delta_2 = 0$*

- *If $0 \leq W_0 < \omega^*$, the financial constraint binds at $t = 0$ and $t = 1$ and the spread between assets A and B narrows over time and is closed only at $t = 2$, i.e. $\Delta_0 > \Delta_1 > \Delta_2 = 0$. The arbitrageurs' positions in asset A, $\bar{x}_0$ and $\bar{X}_1$, are the largest (long) positions allowed by the financial constraints, and satisfy:*

$$\bar{x}_0 - \bar{x}_0 \frac{a\sigma^2 (s - \bar{x}_0)}{\bar{e}} = \frac{W_0}{2\bar{e}} \tag{4}$$

$$\bar{X}_1 - \bar{X}_1 \frac{a\sigma^2 s - \bar{X}_1}{\bar{e}} = \bar{x}_0 \tag{5}$$

The equilibrium links the spread $\Delta$ to arbitrageurs' initial capital $W_0$, and takes a simple form: if arbitrageurs' capital is large enough, then arbitrageurs eliminate the arbitrage opportunity; if instead arbitrageurs start with lower capital, then the financial constraints are binding, and assets A and B trade at a positive spread, which decreases over time. An increase in the supply $s$ or in the fundamental risk (increase in $\bar{e}$) tightens proportionately the financial constraint. This is because the worst possible loss increases, so arbitrageurs need to post more collateral.

A drop in arbitrage capital has the following consequences:

**Corollary 1 (Comparative Statics in the Competitive Benchmark)** *Suppose that competitive arbitrageurs are constrained, i.e. $0 \leq W_0 < \omega^*$. A decrease in capital increases the spread, even more so if capital was initially low:*

$$\frac{\partial \Delta_t^*}{\partial W_0} < 0, \quad \frac{\partial^2 \Delta_t^*}{\partial W_0^2} < 0$$

*Instead, if the constraint is slack, equilibrium spreads and positions are independent of capital.*

**Proof.** The comparative statics follow from Proposition 1 and Corollary 3 (Appendix A.1). ■

### 3.2 Market power (monopoly) without financial constraints

A monopolistic *equilibrium* is a collection of arbitrageur's trades $(x_t)_{t=0,1}$ in each market[29] such that hedgers' demand is optimal given the anticipated price path, and such that the arbitrageur's trades maximize expected utility subject to the equilibrium price schedule.[30] In Lemma 2 in the appendix, I show that at $t = 0, 1$, hedgers' demand in market A is $Y_t = \frac{\mathbb{E}(p_{t+1})-p_t}{a\sigma^2} - s$. Inverting the demand and imposing market-clearing gives the price schedule faced by the arbitrageur:

**Lemma 1 (Price Schedules)** *Suppose that $n = 1$. At $t = 0, 1$, the price schedule faced the monopoly in market A is*

$$p_t(X_t) = \mathbb{E}(p_{t+1}) - a\sigma^2(s - X_t) \tag{6}$$

**Proof.** See Appendix A.2. ■

This equation shows that the equilibrium price today depends on the anticipated price next period. Hence setting a low price tomorrow reduces hedgers' willingness to pay today. A similar dynamic relationship between prices arise in textbook presentations of Coasian dynamics, see Tirole (1988), p. 81. While the arbitrageur does not face competition from other traders, he competes with himself over time: hedgers understand at time 0 that the arbitrageur will retrade and provide additional liquidity at time 1.

The liquidity received at time 0 is durable from hedgers' point of view, because they remain exposed to the same source of risk over time. For instance, hedgers in market A (who have some willingness to sell) will suffer less from the endowment shock at time 1 if they have already reduced their positions at time 0.[31] Risk-sharing is thus akin to a durable "insurance", and is subject to Coasian dynamics. As is well-known from IO, a monopoly can evade Coasian dynamics when he

[29]Recall that $x_t = x_t^A = -x_t^B$.

[30]As usual in the IO and finance literature, I assume that deviations by a zero mass of hedgers do not affect the course of the game.

[31]Note that this effect is maximal under our assumptions, because the exposure does not change sign.

has commitment power. I derive the equilibrium under no commitment, perfect, and imperfect commitment.

**Proposition 2 (No Commitment Equilibrium)** *The equilibrium has the following properties:*

1. *The arbitrageur buys less than the supply in each market and increases his total position over time:* $x_0^{u_0,u_1} = \frac{2}{5}s$, $X_1^{u_0,u_1} = \frac{7}{10}s$.

2. *The spread decreases over time:* $\Delta_0^{u_0,u_1} = \frac{9}{5}a\sigma^2 s$, $\Delta_1^{u_0,u_1} = \frac{3}{5}a\sigma^2 s$.

3. *The arbitrageur earns strictly positive trading profits:* $\Omega_0^{u_0,u_1} = W_0 + \frac{9}{10}a\sigma^2 s^2$.

**Proof.** This is a special case of Proposition 5. ■

As the arbitrageur has market power, he does not fully integrate markets, even though there are no financial constraints. Further, the arbitrageur splits trades to control his price impact. Because the asset is conditionnally riskier at time 0, the arbitrageur trades more aggressively in the first trading round. Indeed, hedgers are more desperate to share risk at time 0 than at time 1.[32] Since markets are not fully integrated, prices do not converge and the arbitrageur realizes trading profits by earning the spread.

**Proposition 3 (Perfect Commitment Equilibrium)** *Suppose that the arbitrageur can commit to a trading strategy. Then:*

1. *The arbitrageur trades more than in the no-commitment case at time 0 and does not trade at time 1:* $x_0^{pc} = X_1^{pc} = \frac{s}{2}$.

2. *Equilibrium spreads are larger at time 0 and time 1:* $\Delta_0^{pc} = 2a\sigma^2 s$ *and* $\Delta_1^{pc} = a\sigma^2 s$.

3. *The arbitrageur is better off, and hedgers are worse off:* $\Omega_0^{pc} = W_0 + a\sigma^2 s^2 > \Omega_0^{u_0,u_1} = W_0 + \frac{9}{10}a\sigma^2 s^2$ *and* $U_0^{pc} = -\frac{3}{4}a\sigma^2 s^2 < U_0^{u_0,u_1} = -\frac{27}{40}a\sigma^2 s^2$.

**Proof.** See Appendix A.3. ■

[32] The uncertainty about the fundamental between 0 and 1 makes the capital gain uncertain, which ensures that hedgers' demand is downward-sloping at time 0. Hedgers have no discount factor in the model, but the higher risk at time 0 plays a similar role as a discount factor.

If the arbitrageur can commit to a trading strategy, he will trade only at time 0, eliminating competition with himself over time. Note that if the arbitrageur were to trade only at time 1, he would forego the benefit of the extra demand for risk-sharing at time 0. With perfect commitment ability, not surprisingly the arbitrageur limits further the amount of risk-sharing, hurting hedgers' welfare, increasing the spreads and his own trading profits relative to the no-commitment case.

Assume now that the arbitrageur's commitment device takes the form of capacity constraints, in the spirit of McAfee and Wiseman (2008). Suppose that at time 0, before the first trading round, the arbitrageur chooses the maximum number of shares $k$ he may trade per period. A capacity $k$ costs $c(k)$, e.g. $c(k) = ck$. I look at vanishly small costs, $c \to 0$.

**Proposition 4 (Capacity Constraints)** *Suppose that the per unit capacity cost $c$ is small, but strictly positive. The arbitrageur chooses optimal capacity $k = \frac{3}{10}s = x_1^{u_0,u_1}$.*

1. *The arbitrageur trades less than in the other cases at time 0: $x_0^{cc} < x_0^{u_0,u_1} < x_0^{pc}$. Yet, at time 1, he holds a larger position than in the perfect commitment and a smaller one than in the no commitment: and $X_1^{pc} < X_1^{cc} < X_1^{u_0,u_1}$.*

2. *Equilibrium spreads at time 0 are larger than in the other cases: $\Delta_0^{u_0,u_1} < \Delta_0^{pc} < \Delta_0^{cc} = \frac{11}{5}a\sigma^2 s$. At time 1, spreads increase relative to the no commitment case and decrease relative to perfect commitment: and $\Delta_1^{u_0,u_1} < \Delta_1^{cc} = \frac{4}{5}a\sigma^2 s < \Delta_1^{pc}$.*

3. *The arbitrageur is worse off than if he could fully commit and almost as well-off as without commitment: $\Omega_0^{u_0,u_1} \approx \Omega_0^{cc} < \Omega_0^{pc}$*

4. *Hedgers ar worse off than in the other cases: $U_0^{cc} = -\frac{31}{40}a\sigma^2 s^2 < U_0^{pc} < U_0^{u_0,u_1}$.*

**Proof.** See Appendix A.4. ■

The proof shows that the arbitrageur is indifferent between two capacities: $k = x_1^{u_0,u_1}$ and $k = x_0^{u_0,u_1}$. However, with a small but positive cost, the latter is more expensive. Because of the small capacity, the arbitrageur restricts liquidity at time 0 more than in the other cases, which explains why the time-0 spread is the largest. The final position, however, is intermediate, and thus so is the time-1 spread. While the arbitrageur provide less liquidityin total than in the no

commitment case, he benefits from the larger spreads, and so achieves almost the same welfare (up to the cost). Hedgers, however, suffer from the lack of liquidity at time 0; they would be better off with higher liquidity at time 0 and a lower final position, as in the perfect commitment case. The comparison of the three cases reveals that both the total amount of liquidity (extensive margin) and the split between time 0 and time 1 matters (intensive margin) matters for hedgers' welfare. This distinction matters to understand the welfare effects of financial constraints in Section 5.

# 4 Financially constrained monopoly

The definition of equilibrium remains the same as in Section 3.2, except that in each period the monopoly's positions must satisfy the financial constraints.

**Equilibrium drivers.** Inspecting the arbitrageur's financial constraints (3) and the price schedule (6) shows that the equilibrium will be determined by two key variables: the arbitrageur's capital $W_0$ and the risk benefit ratio $\rho$, defined as follows.

**Notation 1 (Risk Benefit Ratio)** *Let* $\rho \equiv \frac{\bar{e}}{a\sigma^2 s}$ *denote the risk benefit ratio.*

The risk benefit ratio is a cost benefit ratio of the trade from the arbitrageur's point of view. Note that risk appears both in the nominator and the denominator, in different forms. In the numerator, $\bar{e}$ measures by how much the fundamental can go up or down relative to the conditional mean, thus it measures the largest potential gain or loss due to the fundamental. In the denominator, the product $a\sigma^2 s$ measures the (maximum) profitability of the arbitrageur's trade in a given market. It is determined by the amount of hedging needs from hedgers. Hedgers are more desperate to share risk if the asset is riskier (larger $\sigma^2$), if their endowment shock is larger (larger $s$), or if they are more risk averse (larger $a$).[33] Because $\bar{e}$ represents the "tail" risk of the fundamental and $\sigma^2$ is its variance, $\rho$ is large when the distribution of the fundamental has less mass in the tails.

**Equilibrium multiplicity.** Lemma 1 shows that the price schedule at time 0 depends on the expected price at time 1, which itself depends on whether the arbitrageur's constraint is binding

[33] Alternatively, one can think of the inverse of the risk benefit ratio as the maximum profit per unit of maximum risk, for each leg of the arbitrage.

at time 1. But the tighteness of the constraint itself depends on prices. Therefore, in the presence of financial constraints, equilibria can be self-fulfilling and multiple equilibria coexist. Indeed, the arbitrageur chooses a position, but does not control hedgers' expectations. The anticipation of a binding constraint next period affects hedgers' demand today, and the price at which they are ready to trade. But this price matters for the tightness of the financial constraints. Note that with a capacity constraint, the tightness of the constraint is set ex-ante and is independent of prices, so equilibrium is unique.

### 4.1 Equilibria with a slack constraint at time 1

Suppose that at time 0 hedgers anticipate that the arbitrageur is unconstrained at time 1. I conjecture that the arbitrageur chooses a position $x_0$ such that his time-1 constraint is slack, and verify under which conditions this holds. That is, given hedgers' anticipations $u_1$ (for *u*nconstrained at time 1), I determine under which conditions the arbitrageur chooses a position $x_0$ leading to state $l = \{u_1, \bar{c}_1, \underline{c}_1\}$ at time 1, where $\bar{c}_1$ denotes an upward-binding constraint and $\underline{c}_1$ a downward-binding constraint. Denoting $\Omega_0^{u_1,l}$ the value function associated with hedgers' expectations $u_1$ and state $l$, the arbitrageur chooses $x_0$ such that his time 0 expected utility $\Omega_0^u$ is $\Omega_0^u = \max_{x_0}(\Omega_0^{u_1,u_1}, \Omega_0^{u_1,\bar{c}_1}, \Omega_0^{u_1,\underline{c}_1})$.

Given that the arbitrageur takes offsetting positions across markets, his wealth is risk-free, and only the trading profits in each period enter his value functions. Each value function $\Omega_0^{u_1,l}$ is thus defined as follows:

$$
\begin{aligned}
\Omega_0^{u_1,l} = \max_{x_0 \in \mathcal{F}_0^0} \quad & W_0 + x_0 \Delta_0^{u_1}(x_0) + \Omega_1^l(x_0) \\
\text{s.t.} \quad & \Delta_0^{u_1}(x_0) = 2a\sigma^2(s - x_0) + \Delta_1^{u_1}(x_0) \\
& \text{additional consistency conditions}
\end{aligned}
$$

where $\mathcal{F}_0^0$ is the set of time-0 positions satisfying the financial constraint (3) at time 0, $\Delta_0^{u_1}(\cdot) \equiv p_t^B(\cdot) - p_t^A(\cdot)$ is the time-0 spread schedule, i.e. the difference between the price schedule in each market when hedgers assume that the arbitrageur's constraint is slack at time 1, and $\Omega_1^l$ is the

continuation value at time 1, given state $l = \{u_1, \bar{c}_1, \underline{c}_1\}$. Note that the spread schedule is fixed, in the sense that I keep hedgers' anticipation $u_1$ fixed, while the arbitrageur, given this schedule, internalizes that his trade leads to a binding or slack constraint and the associated continuation value at time 1 (i.e. $\Omega_1^l$ depends on $l$, not $u_1$). The associated consistency conditions, provided in the appendix, ensure that the arbitrageur's actions are time-consistent, e.g. if the arbitrageur sticks to the conjectured strategy, his constraint at time 1 must indeed be slack in equilibrium.

**Proposition 5 (Equilibria with slack time-1 constraint)** *There exist thresholds $\omega_0^u$, $\omega_1^u$, and $\omega^f$, that define four regions in terms of initial arbitrage capital:*

1. *In the first region, $W_0 \geq \max(\omega_0^u, \omega_1^u)$, arbitrage capital is abundant, both constraints are slack in equilibrium, and the arbitrageur holds his preferred positions $x_0^{u_0,u_1}$ and $X_1^{u_0,u_1}$ given in Proposition 2 ($\mathbf{u_0}, \mathbf{u_1}$ equilibrium).*

2. *In second region ($\max(\omega^f, \min(\omega_0^u, \omega_1^u)) \leq W_0 < \max(\omega_0^u, \omega_1^u)$) and third regions ($\max(0, \omega^f) \leq W_0 < \min(\omega_0^u, \omega_1^u)$), either there is no equilibrium with a slack constraint at time 1 (**no** $\mathbf{u_1}$), or there exists an equilibrium in which the constraint binds at time 0 but not at time 1 ($\mathbf{c_0}, \mathbf{u_1}$ equilibrium).*

   *In this equilibrium, the arbitrageur holds a smaller position at time 0, allowing him to hold his preferred position $X_1^u(x_0)$ at time 1 without violating the constraint:*

$$x_0^{c_0,u_1} < x_0^{u_0,u_1}, \quad X_1^{c_0,u_1} = X_1^u(x_0^{c_0,u_1}) = \frac{s + x_0^{c_0,u_1}}{2}$$

   *This equilibrium arises in particular in the second region when the risk benefit ratio is sufficiently low ($\rho < \frac{7}{10}$).*

3. *In the fourth region, $0 \leq W_0 < \max(0, \omega^f)$, there is little arbitrage capital, and thus there is no equilibrium with a slack constraint at time 1 (**no** $\mathbf{u_1}$).*

**Proof.** See appendix C. ■

The equilibrium takes a simple, intuitive form. When arbitrage capital is sufficiently abundant, constraints never bind, and the arbitrageur holds his preferred positions. In the opposite case,

where capital is particularly scarce, the arbitrageur cannot trade in such a way that the constraint remains slack at time 1. In between these two regions, either we are in the former case, or in an intermediate case, where the arbitrageur reduces positions at time 0 to ensure that he can hold his desired position at time 1. In other words, in such equilibria, the arbitrageur decides to save capital at time 0 to ensure that, given the time 0 position, he can trade an optimal amount at time 1. The different cases are represented in Figure 1. Note that to vary the risk benefit ratio, I hold hedgers' risk aversion $a$, the fundamental $\bar{e}$ and the variance $\sigma^2$ fixed, and vary the supply $s$.

In the second and third region, while the equilibrium can be determined analytically, it is not very tractable.[34] Thus, in Figure 2, I solve numerically for the equilibrium. The parameters chosen here show a typical case. Panel a shows in red the equilibrium that prevails in the regions where two cases may arise. Panel b eliminates the redundant information of the picture. The resulting equilibrium representation is strikingly simple. For $\rho \geq \frac{7}{10}$ (left-hand side of the picture), either capital is large enough such that no constraint binds and the arbitrageur holds his preferred position, or the arbitrageur has not enough capital to keep the constraint slack at time 1. When the risk benefit ratio is lower, an intermediate case arises, where the arbitrageur's capital is relatively low, but sufficient to allow him to keep his constraint slack at time 1. Doing so, however, requires to trade less at time 0. Intuitively, the arbitrage is profitable enough relative to the risk of the position to relax the constraint at time 1. This is because wealth at time 1 increases sufficiently thanks to the trading profits made by the arbitrageur at time 0.

There may be no equilibrium with a slack constraint at time 1 for two reasons. Either there is not enough capital, so that it is impossible for the arbitrageur to keep the constraint slack and hold his preferred position at time 1 (this is so if $W_0 < \omega^f$, where the superscript $f$ stands for floor). Or it is possible but not optimal for the arbitrageur to do so. When capital is not very abundant, and the risk benefit ratio is high enough, keeping enough dry powder at time 0 to trade his preferred position at time 1 is costly for the arbitrageur. It requires to reduce the time 0 trade away from

[34] The reason is that given hegders' anticipations, and thus the price schedule, one must check that it is indeed optimal for the arbitrageur to follow the conjectured strategy. However, deviations involve making the constraint binding at time 1. Such binding constraint implies that the effect of the position $x_0$ on the time 1 profit is no longer quadratic, so that first-order conditions become highly non-linear. Although the problem can be solved analytically, it is not very tractable. Proposition 13 in the appendix provides slightly more detailed equilibrium conditions.

his preferred level. In that case, the arbitrageur may thus deviate from the conjectured strategy and make the constraint binding at time 1. The loss from being constrained at time 1 is more than offset by the benefit of trading his preferred position at time 0 (not necessarily the same amount as if the constraint is slack at time 1). The last reason yields an endogenous threshold (represented by a dotted line in panels a and b of Figure 2) under which there is no equilibrium with a slack time 1 constraint.

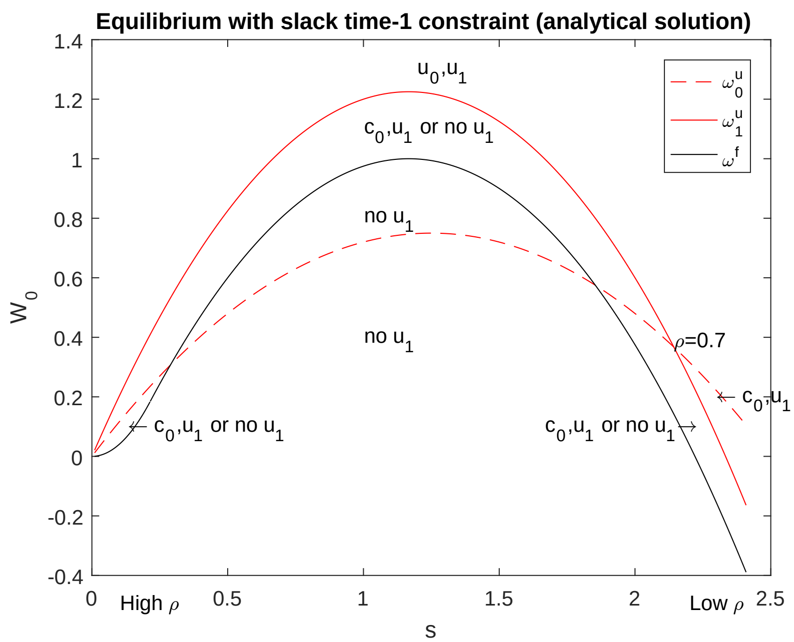


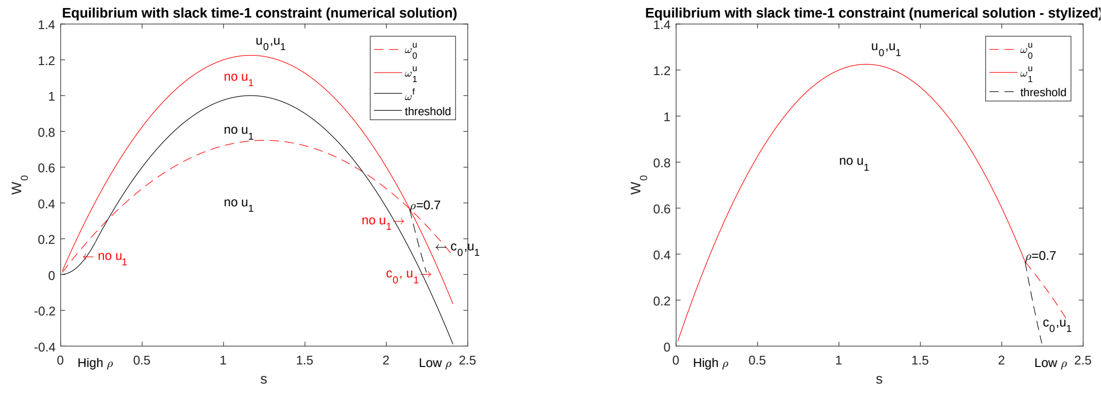

Figure 1: Equilibria with slack time-1 constraint. The parameters are $a = \sigma = 1$, $\bar{e} = 1.5$.

When the arbitrageur holds his preferred position $x_0^{u_0,u_1}$, his constraints are slack if $W_0 \geq \max(\omega_0^u, \omega_1^u)$, where $\omega_t^u$ is associated with the time-$t$ constraint. Recall that in the competitive case, a single condition ensured that *both* constraints were slack: $W_0 \geq \omega^*$. These thresholds take the following generic form.

**Proposition 6 (Trade-off between position size and profit adjustment)**

1. *The thresholds $\omega_t^u$ can be written as the sum of two terms:*

$$\omega_t^u = \underbrace{\Lambda_t^u \, s\bar{e}}_{\textit{maximum position loss}} - \underbrace{\Gamma_t^u \, a\sigma^2 s^2}_{\textit{profit adjustment}}$$




(a) (b)

Figure 2: Equilibria with slack time-1 constraint. The parameters are $a = \sigma = 1$, $\bar{e} = 1.5$.

*Similary,* $\omega^* = \Lambda^* s\bar{e} - \Gamma^* a\sigma^2 s^2$.

2. *We have:* $\Lambda_0^u < \Lambda_1^u < \Lambda^* = 2$, *and* $\Gamma^* = 0 < \Gamma_0^u < \Gamma_1^u$ *(no profit adjustment in the competitive case).*

3. *Thus, the monopoly's constraint is always slacker than the competitive arbitrageurs' constraints:* $\max(\omega_0^u, \omega_1^u) < \omega^*$.

**Proof.** Follows from the definitions of $\omega_t^u$ and $\omega^*$. ■

The form of the $\omega_t^u$ thresholds is intuitive. The first term, $\Lambda_t^u s\bar{e}$, represents the maximum potential loss caused by fundamentals. It is the product of the worst possible change in fundamental $\bar{e}$ and the arbitrageur's total exposure at time $t$, $\Lambda_t^u s$. In a competitive market, arbitrageurs fully integrate market A and B, and their total position is $s - (-s) = 2s$, as each leg of the arbitrage is of size $|s|$.[35] In a monopolistic market, the arbitrageur acquires a smaller position than competitive arbitrageurs and split orders to limit his price impact, so $\Lambda_0^u < \Lambda_1^u$ and $\Lambda_t < 2$. The second term in $\omega_t^u$, $-\Gamma_t^u a\sigma^2 s^2$, is an adjustment measuring how much (accumulated) profits due to market power relax the capital requirement. This term is zero in a competitive market, since profits are competed away. Intuitively, in the monopolistic case, financiers understand that the market being imperfectly liquid, the arbitrageur will earn trading profits, making his positions safer.

[35] That is, $\Lambda_t^u s = 2X_t$.

The profit adjustment term implies that the threshold is no longer linear but quadratic in the supply $s$ when the arbitrageur has market power. An increase in $s$ pushes prices further away from each other. Thus the arbitrageur takes a larger position in each market, making each leg more risky. However, the increase in supply also makes the arbitrage more profitable, so that the monopolist earns larger capital gains. As a result, an increase in $s$ first tightens and then loosens the constraint. By constrast, in a competitive market, an increase in $s$ always tightens the constraint.

The trade-off between the risk of the position and the profit adjustment also means that i) the threshold may be negative and thus non-binding as long as the arbitrageur starts with positive wealth (i.e. $\omega_t^u$ may be negative); this occurs if $\rho$ is small enough; and ii) that the arbitrageur may be constrained at time 0, but not at time 1. Intuitively, at time 0, the position is smaller, but so is the profit. At time 1, both the position and the profit increase (i.e. both $\Lambda_t^u$ and $\Gamma_t^u$ increase with time). If the profit increases faster than the position, the arbitrageur's constraint relaxes at time 1, even if the constraint was binding at time 0. In a competitive market, being intially unconstrained implies that the price gap between the two assets is closed. Thus wealth does not increase over time. Further, at time 1, when hedgers receive a new shock, the previous shock has been fully hedged. This implies that if arbitrageurs had enough wealth to close the price gap at time 0, they have enough wealth to do so at time 1 as well. Hence the condition boils down to a single threshold.

Conversely, in a competitive market, if the constraint binds at time 0, it also binds at time 1. It is not necessarily the case, however, when the arbitrageur has market power. The reason is simply that competitive arbitrageurs do not internalize their price impact, while the monopoly does. Because he takes into account his price impact, the monopoly takes a smaller position at time 1. The profits from time 0 may be large enough to finance this smaller position, but are never enough to finance a position that eliminates the spread.

To illustrate this point, suppose the constraint is binding at time 0 and assume that both the competitive and monopolistic arbitrageurs start with the same positive initial wealth $W_0$. In both cases, it is optimal for the arbitrageur to hold $\bar{x}_0 < s$, defined as the largest long position allowed by the financial constraint. However, the binding constraint at time 0 implies that $W_1 = 2\bar{x}_0\bar{e}$.[36]

[36] This is because $W_1 = W_0 + 2a\sigma^2 x_0(\Delta_0 - \Delta_1) = W_0 + 2a\sigma^2 x_0(s - x_0)$ and the definition of $\bar{x}_0$.

Consider now the time 1 constraint and suppose it is not binding, i.e. the arbitrageur holds $X_1$ such that $W_1 > 2X_1\bar{e} - 2a\sigma^2 X_1(\Delta_1 - \Delta_2)$. However, if the arbitrageur is competitive, he exploits the arbitrage until the marginal profit is nil, reducing the spread to 0. But $\Delta_1 - \Delta_2 = \Delta_1 = 2a\sigma^2(s - X_1) = 0$ implies that the arbitrageur wishes to hold $X_1 = s$. It is not possible, since $W_1 = 2\bar{x}_0\bar{e} < \omega^*$. Instead, the monopoly maximizes the capital gain on the arbitrage by choosing a position $X_1 = \frac{s+\bar{x}_0}{2} < s$. By reducing his position at time 1 to mitigate price impact and using the capital earned at time 0, the arbitrageur relaxes the constraint at time 1.

This insight implies that in a monopolistic market, a binding constraint at time 0 does not necessarily lead to a binding constraint at time 1. As a result, new equilibria emerge, in which the arbitrageur's constraint binds only *occasionally.* In this section, I described equilibria in which the constraint binds at time 0, but not time 1. In the next section, I consider the opposite situation.

## 4.2 Equilibria with binding time-1 constraint

There exist equilibria, in which the constraint binds upwards at time 1. However, I show in the appendix that there is no equilibrium in which the constraint binds downwards. It is intuitive: since arbitrageurs naturally want to go long the spread, the main issue arising from limited capital is that they cannot go long as much as they wish, i.e. that the constraint binds upwards. I proceed as in the previous section: I conjecture an equilibrium strategy and determine under which conditions it holds in equilibrium.

**Proposition 7 (Equilibria with binding time-1 constraint)** *Let $\omega_0^p$ and $\omega_1^p$ denote two thresholds. There are equilibria in which the arbitrageur's constraint binds upwards at time 1, as follows.*

1. *If $0 \leq \rho < \frac{3}{4}$, then $\omega_1^p < \omega_0^p$, and there are three regions in terms of arbitrage capital:*

   (a) *In the first region, with $0 \leq W_0 < \max(0, \omega^f)$, the arbitrageur's constraint binds upwards at time 0 and time 1 in equilibrium ($\mathbf{c_0}, \mathbf{c_1}$ equilibrium). This equilibrium is the same as in the constrained competitive case, for a given level of capital. The arbitrageur holds the largest (long) positions satisfying the financial constraints at each date:*

$$x_0^{c_0,c_1} = \bar{x}_0, \quad X_1^{c_0,c_1} = \bar{X}_1(\bar{x}_0)$$

(b) *In the second region, with* $\max(0, \omega^f) \leq W_0 < \max(0, \omega_1^p)$*, there are two cases, there is either no equilibrium in which the arbitrageur's constraint binds upwards at time 1 (**no** $\mathbf{c_1}$), or an equilibrium where both constraints bind as in (a).*

(c) *In the third region, with* $\max(0, \omega_1^p) \leq W_0 < \omega_0^p$ *or* $\omega_0^p \leq W_0$*, there is no equilibrium in which the arbitrageur's constraint binds upwards at time 1 (**no** $\mathbf{c_1}$).*

2. *If* $\rho \geq \frac{3}{4}$*, then* $\omega_1^p > \omega_0^p$*, and there are four regions in terms of arbitrage capital:*

   (a) *In the first region, with* $0 \leq W_0 < \omega^f$*, the equilibrium is* $\mathbf{c_0}, \mathbf{c_1}$*, as in case 1a.*

   (b) *In the second region, with* $\omega^f \leq W_0 < \omega_0^p$*, the equilibrium is the same as in 1b.*

   (c) *In the third region, with* $\omega_0^p \leq W_0 < \omega_1^p$*, there may exist an equilibrium in which the abritrageur's constraint binds upwards at time 1 and is slack at time 0 (*$\mathbf{u_0}, \mathbf{c_1}$ *equilibrium). In this equilibrium, the arbitrageur holds the same amount as in the full commitment case at time 0,* $x_0^{u_0,c_1} = x_0^{fc}$*, and the largest position satisfying the constraint at time 1:* $X_1^{u_0,c_1} = \bar{X}_1(x_0^{u_0,c_1})$.

   (d) *In the fourth region, with* $\omega_1^p \leq W_0$*, is no equilibrium in which the arbitrageur's constraint binds upwards at time 1 (**no** $\mathbf{c_1}$), as in 1c.*

As in the case of the $u_1$ equilibrium, some cases have an analytical albeit rather intractable solution. So I proceed as before and illustrate Proposition 7 using Figure 3. Then for some parameters, I determine the numerical solution in Figure 4. The graph is a typical case, and does not critically depend on the choice of parameters. Panel b of Figure 4 shows the equilibrium regions. The intuition for the form of the equilibrium is simple. With abundant capital, there is no equilibrium in which the arbitrageur's constraint binds at time 1 (no $c_1$). With little capital, constraints are likely to bind not only at time 1, but also at time 0 ($c_0, c_1$). The most interesting equilibrium arises when the risk benefit ratio is large enough ($\rho \geq \frac{3}{4}$) and arbitrage capital is intermediate ($\omega_0^p \leq W_0 < \omega_1^p$). In this case, the arbitrageur's constraint binds at time 1, but not at time 0. The conditions on arbitrage capital for this equilibrium are intuitive. The tresholds $\omega_t^p$ for this *p*artly constrained equilibrium are associated with the time $t$ constraints.[37] There must be

[37] As before, the thresholds are of the form $\omega_t^p = \Lambda_t^p s\bar{e} - \Gamma_t^p a\sigma^2 s^2$.

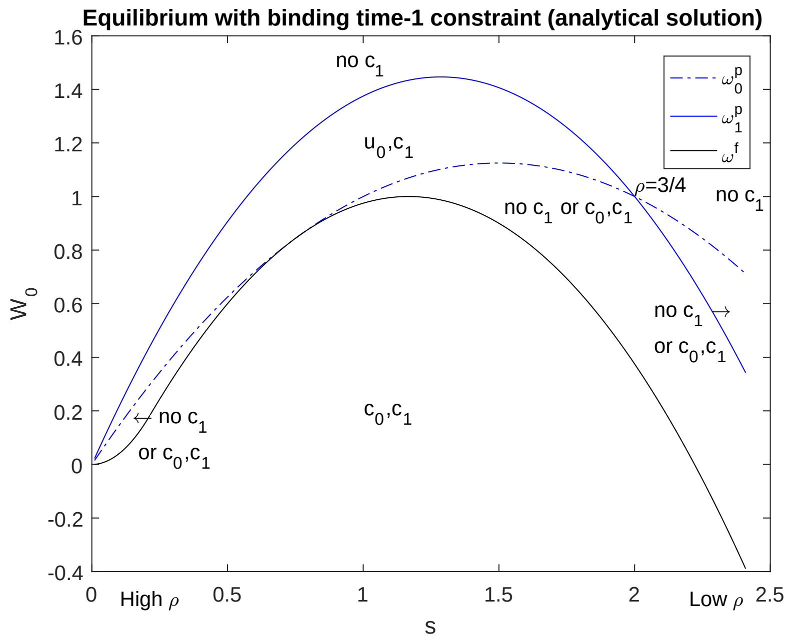
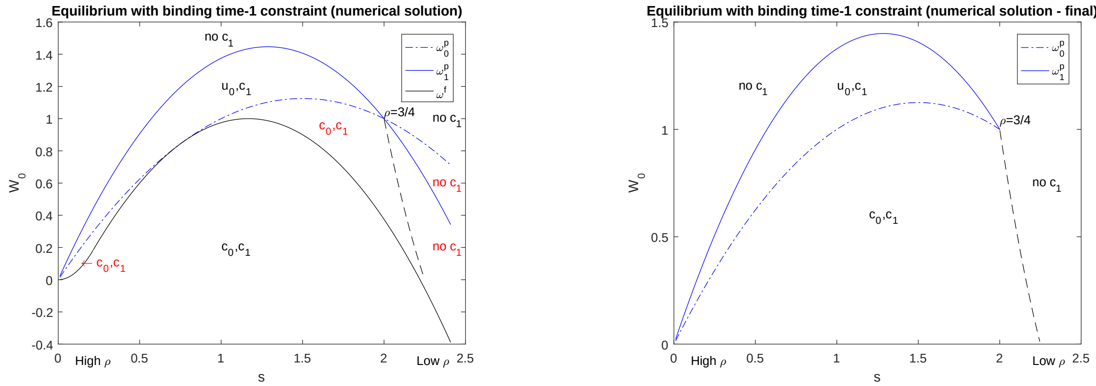



Figure 3: Equilibria with binding time-1 constraint. The parameters are $a = \sigma = 1$, $\bar{e} = 1.5$.

enough capital for the arbitrageur to be able to hold $x_0^{u_0,c_1}$ at time 0 ($W_0 \geq \omega_0^p$). However, capital cannot be large enough ($W_0 < \omega_1^p$), for otherwise, the constraint would not bind at time 1. That is, even holding a larger position that in the unconstrained equilibrium $(u_0, u_1)$, capital cannot be too large. In fact, holding a larger position at time 0 is a necessary condition for the constraint to bind at time 1. Doing so, the arbitrageur pledges more capital at time 0, increasing the chance to be constrained at time 1 (of course, not only the position but also the depth of the market, and therefore the arbitrageurs' trading profits, are different across the two equilibria). This equilibrium arises only if the risk benefit ratio is large enough. It makes sense: for the constraint to bind at time 1, it must be that the position is sufficiently risky relative to profits. Otherwise, trading is not very capital intensive, and the arbitrageur will be free to re-optimize when time 1 comes.

Having a sufficiently low level of capital does not only ensure time consistency. A low $W_0$ also ensures that deviating from being constrained at time 1 is not attractive. With sufficiently low capital, the arbitrageur must take a small position at time 0 to ensure that his time-1 constraint is slack. This reduces his time-0 profit, and thus prevents the arbitrageur from fully benefiting from the deviation.




(a) (b)

Figure 4: Equilibria with binding time-1 constraint. The parameters are $a = \sigma = 1$, $\bar{e} = 1.5$.

### 4.3 Coexistence

**Proposition 8 (Equilibria with Slack and Binding Time-1 Constraints May Coexist)**

- *There is a unique equilibrium when arbitrage capital is either sufficiently low or sufficiently high:*
  - *If* $0 \leq W_0 < \max(0, \omega^f)$*, the unique equilibrium is* $\mathbf{c_0}, \mathbf{c_1}$*.*
  - *If* $W_0 \geq \max(\omega_0^u, \omega_1^u, \omega_1^p)$*, the unique equilibrium is* $\mathbf{u_0}, \mathbf{u_1}$*.*
- *When capital is intermediate, i.e. if* $\max(0, \omega^f) \leq W_0 < \max(\omega_0^u, \omega_1^u, \omega_1^p)$*, multiple equilibria may coexist depending on the level of* $\rho$*, as detailed in Proposition 8.*

This result is illustrated by Figure 5. The result of the numerical solution is shown on Figure 6. The resulting picture is simple. There are only two regions where equilibria coexist. In these regions, capital is intermediate and the risk benefit ratio is large enough. Otherwise, the equilibrium is unique. With abundant capital, the unconstrained equilibrium $u_0, u_1$ is unique, while with scarce capital, the fully constrained equilibrium $c_0, c_1$ prevails. Further, when the risk benefit ratio is particularly low, the prediction of the model is $c_0, u_1$, i.e. the arbitrageur reduces his time-0 position to remain unconstrained. When the risk benefit ratio is larger, the fully constrained and unconstrained equilibria may overlap. If we increase further the risk benefit ratio, the unconstrained equilibrium may coexist with the $u_0, c_1$ equilibrium.

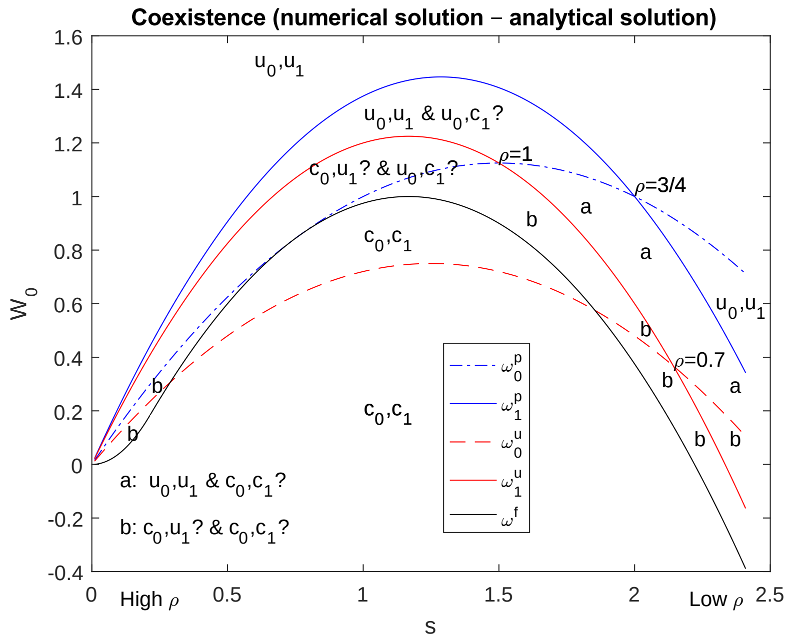



Figure 5: Coexistence. The parameters are $a = \sigma = 1$, $\bar{e} = 1.5$.

Multiple equilibria arise even though the abitrageur has market power and chooses how much to trade. Intuitively, the arbitrageur does choose quantities, but cannot pick hedgers' expectations. Since hedgers' expectations affect market depth (through the price schedule), they affect the arbitrageur's incentives to trade in one way or another, leading to self-fulfilling equilibria.

## 5 Price and welfare implications

To illustrate the results of this section, I consider a simple numerical example and compare prices and welfare within structure and across structures. Suppose that $\bar{e} = a = \sigma^2 = 1$, $s = 1.1$, and $W_0 = 0.496$. These parameters imply that $\rho = 0.909 > \frac{3}{4}$, $\omega_0^u = 0.2992$, $\omega_1^u = 0.451$, $\omega_0^p = 0.495$, $\omega_1^p = 0.59125$, and $\omega^* = 2.2$. Thus, for these parameters, $\max(\omega_0^u, \omega_1^u) < \omega_0^p \leq W_0 < \omega_1^p$. Hence the unconstrained equilibrium $(u_0, u_1)$ and the partly constrained equilibrium $(u_0, c_1)$ coexist. Further, for these parameters the competitive arbitrageurs' constraint binds at both $t = 0$ and $t = 1$. Table 1 summarizes the main quantities of interest. The example illustrates the three main points of this section:

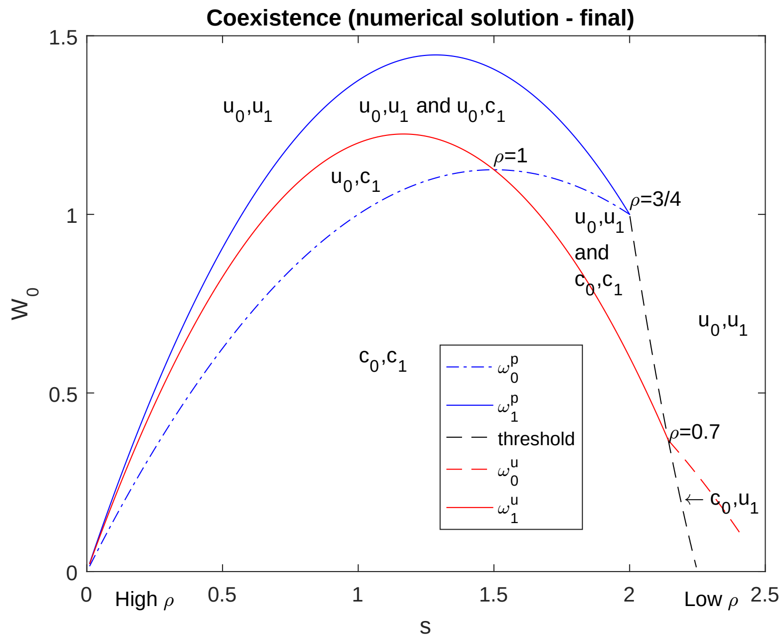



Figure 6: Coexistence. The parameters are $a = \sigma = 1$, $\bar{e} = 1.5$.

1. With a monopolistic arbitrageur, the partly constrained equilibrium Pareto-dominates the unconstrained equilibrium.

2. Imposing constraints on a competitive arbitrageur reduces welfare. However, imposing the same constraint on a monopoly with the same amount of initial capital may raise total and individual welfare.

3. Merging constrained competitive arbitrageurs into a single arbitrageur may reduce spreads at time 1.

I now consider these results in a more formal way.

## 5.1 Welfare effects

**Proposition 9** **($u_0, c_1$ Pareto-dominates $u_0, u_1$)** *Suppose that a partly constrained equilibrium $(u_0, c_1)$ exists and that it coexists with the unconstrained equilibrium $(u_0, u_1)$. Then in the partly constrained equilibrium:*

Table 1: Prices and welfare across and within market structure

| Variable | Competitive Unconstrained | Competitive Constrained | Monopoly Unconstrained | Monopoly Partly Constrained |
|---|---|---|---|---|
| $x_0$ | 1.1 | 0.55049975 | 0.44 | 0.55 |
| $X_1$ | 2.2 | 0.79363953 | 0.77 | 0.7936397 |
| $1/2\Delta_0$ | 0 | 0.85586072 | 0.99 | 0.8563603 |
| $1/2\Delta_1$ | 0 | 0.30636047 | 0.33 | 0.3063603 |
| Hedgers' welfare | 0 | -0.74354316 | -0.81675 | -0.74381802 |
| Arb. profit | 0.496 | 1.58727906 | 1.585 | 1.5872794 |
| Total welfare | 0.496 | 0.8437359 | 0.76825 | 0.84346138 |

1. *Spreads are smaller:* $\Delta_t^{u_0,c_1} < \Delta_t^{u_0,u_1}$, $t = 0, 1$;

2. *The arbitrageur holds larger positions:* $X_t^{u_0,c_1} > X_t^{u_0,u_1}$, $t = 0, 1$;

3. *Hedgers are better off;*

4. *The arbitrageur is better off if and only if* $W_0 \in [\max(\omega_1^u, \omega_0^p), w^a)$, *where* $w^a < \omega_1^p$. *This interval is non-empty if* $\rho \geq \frac{2\sqrt{5}}{5} > \frac{3}{4}$, *i.e. there exists a non-empty set of parameters such that the arbitrageur is better off.*

**Proof.** See Appendix F. ■

A key consequence of this result is that imposing financial constraints on a monopolistic arbitrageur improves social welfare in certain markets:

**Corollary 2 (Constraints on a Monopoly may be Welfare-improving)**

1. *In markets with a large amount of capital, imposing financial constraints on a monopolistic arbitrageur has no effect, because the constraint never binds.*

2. *In markets with intermediate capital* ($\omega_0^p \leq W_0 < w^a$) *and sufficiently high risk-benefit ratios* ($\rho > \frac{2\sqrt{5}}{5}$)*, imposing financial constraints on a monopolistic arbitrageur can decrease spreads and improve welfare. If the risk-benefit ratio is lower* ($\frac{3}{4} \leq \rho < \frac{2\sqrt{5}}{5}$) *and / or capital is larger* ($w^a < W_0 \leq \omega_1^p$)*, the arbitrageur is worse off.*

3. *In markets with sufficiently small capital, imposing constraints leads to a reduction in liquidity in at least one date, and at least one type of investors is worse off.*

This results provides conditions under which imposing constraints on a large arbitrageur may increase or decrease social welfare. Thus, this result may cast light on the debates about the effects of the tightening of capital and liquidity requirements that have occurred in the aftermath of the 2007-2009 crisis. These debates have mostly focused on the negative implications of the tightening of existing financial constraints or the introduction of new constraints (see e.g. Boyarchenko et al., 2018). Here instead, I show that there are benefits to regulating an arbitrageur with market power through VaR-like financial constraints in markets with high risk benefit ratios and where the arbitrageur is neither too well nor too poorly capitalized. In other situations, either the constraint has no effect, or hurts at least one type of market participant. This result is at odds with the competitive case, where financial constraints are either irrelevant or reduce hedgers' welfare.

In general, adding a friction on top of another one may bring the economy either closer or further away from the first-best. Here financial constraints interact with the arbitrageur's commitment problem. We know from the third benchmark that the arbitrageur faces a Coasian problem of competition with one-self over time, and benefits from being able to commit to trade only once. Without commitment, the arbitrageur cannot help retrading. However, the financial constraint does not rule out retrading. It serves as an endogenous and imperfect commitment device to trade *less* at time 1. Indeed, Table 1 illustrates that $x_0^{u_0,u_1} < x_0^{u_0,c_1}$ and $x_1^{u_0,u_1} > x_1^{u_0,c_1}$ (recall that $x_1 = X_1 - x_0$). However, the arbitrageur holds larger positions *at all dates* in the partly constrained equilibrium, i.e. $x_0^{u_0,u_1} > x_0^{u_0,u_1}$ and $X_1^{u_0,c_1} = x_0^{u_0,c_1} + 1_0^{u_0,c_1} > x_0^{u_0,u_1} + 1_0^{u_0,u_1} = X_1^{u_0,u_1}$.

A larger position at time 0 is inherent to the $u_0, c_1$ equilibrium: by pledging more capital early on, the arbitrageur ensures that the constraint is indeed binding at time 1. However, doing so, the arbitrageur also earns larger capital gains, which mechanically increase the position he can afford at time 1. The larger positions lead to smaller spreads, although the price impact is different across equilibria: in the $u_0, c_1$ equilibrium, hedgers demand more liquidity at time 0 in anticipation of the binding constraint at time 1, potentially increasing the spread.

*Both* the arbitrageur and hedgers are better off in the $u_0, c_1$ equilibrium, but note that the conditions are stricter for the arbitrageur to be better off. The intuition for both effects relates to the fact that the financial constraint is only an imperfect commitment device and allows some

retrading. Arbitrageurs would be better off if they were able to divert capital gains entirely between 0 and 1 and keep the same level of capital after trading at time 0. This would allow them to earn full commitment profits. For hedgers, the situation is opposite: they benefit from the fact that the arbitrageur provides more liquidity at time 0, and that he evenutally also holds a larger position at time 1, despite the binding constraint at time 1. In fact, both the increase in early liquidity and the increase in the final position are necessary to obtain hedgers' welfare gain. To see this, I compute hedgers' welfare under two counterfactual allocations.

**Proposition 10 (Why hedgers are better off)** *Suppose the $u_0, u_1$ and $u_0, c_1$ equilibria coexist.*

- *Counterfactual 1: If the time-1 position increases to $X_1^{u_0,c_1}$ without an increase in the time 0 position ($x_0^{cf1} = x_0^{u_0,u_1}$), hedgers are better off in the $u_0, c_1$ equilibrium than in the counterfactual.*
- *Counterfactual 2: If the time-1 position remains the same $X_1^{cf2} = X_1^{u_0,u_1}$ but the time 0 position increases ($x_0^{cf1} = x_0^{u_0,c_1}$), hedgers are better off in the $u_0, u_1$ equilibrium than in the counterfactual.*

*Thus, hedgers are better off because both the time-1 and time-0 positions increase in the $u_0, c_1$ equilibrium.*

**Proof.** See Appendix F. ■

This result shows that varying both the extensive margin (the total position, counterfactual 1) and the intensive margin (the amount of liquidity at time 0, counterfactual 2) are necessary to improve hedgers' welfare. It is intuitive that the increase in the final position matters: hedgers have shared more risk in the market, getting closer to the first-best.

The reason why the intensive margin – given a total amount of liquidity, how much liquidity is provided at time 0 – matters is that the asset is conditionally riskier at time 0 than at time 1, making it valuable for hedgers to share risk early. Note that the uncertainty about the fundamental between 0 and 1 is a key ingredient of the model: it implies that hedger's demand remains downward-sloping at time 0. Without it, there is no demand for liquidity at time 0. Indeed, the risky asset would be

temporarily risk-free, so that hedgers would flatten out the demand – in other words, hedgers would become arbitrageurs themselves. The price would simply equal the expected price next period and arbitrageurs would have no incentive to trade (for any trade would push the price away from the expected time 1 price, and hedgers would step in to correct this distortion).

**The oligopolistic case.** With a monopolistic arbitrageur, imposing constraints may be socially desirable in certain markets. Is it a special case? The short answer is no. In the supplementary appendix, I derive the analogs of the $u_0, u_1$ and $u_0, c_1$ equilibria with $n$ equally capitalized arbitrageurs. These equilibria may coexist, as in the monopolistic case. When they do coexist, the $u_0, c_1$ equilibrium no longer Pareto-dominates. However, if we start from a no-constraint oligopolistic economy and impose constraints, then we still obtain a Pareto-improvement under similar conditions as the monopolistic case: intermediate capital and sufficiently high risk benefit ratio. The fact that $u_0, c_1$ no longer Pareto-dominates relates to the equilibrium tightness of the constraint: capital must be sufficiently large to ensure that the $u_0, u_1$ exists, and as $n$ increases, this condition first tightens (unless $\rho$ is very large). When we compare the expected utilities between $u_0, u_1$ and $u_0, c_1$, instead, we obtain an upper bound on capital. This is because expected utility is concave in $W_0$ in the $u_0, c_1$ equilibrium, while it is linear in the $u_0, u_1$ case. I show that for $n > 1$, these two conditions cannot be satisfied simultaneously. However, if we compare a no-constraint case to a case with constraint, there is no need to take into account the lower bound for the $u_0, u_1$ equilibrium: with constraints, it is always the prevailing equilibrium.

In the oligopolistic case, I provide examples in which hedgers are better off even without an increase in the time-1 position (relative to the $u_0, u_1$ equilibrium). It suffices that the time-0 position increases. That the final position does not increase is a consequence of competition: more competition leads to lower capital gains, so that arbitrageurs' constraint at time 1 is tighter than under a monopoly. This fact implies that spreads may be reduced only at time 0 relative to a no-constraint case.

## 5.2 Price effects

**Price effects of a drop in capital.** Figures 7a to 10b show the competitive and imperfectly competitive spreads as a function of arbitrage capital at time 0 and 1.[38] For $\rho \leq \frac{3}{4}$, there is no qualitative difference between the competitive and monopolistic cases: a drop in capital leads to an increase in spread when the economy enters the region where constraints bind. For $\rho > \frac{3}{4}$, the comparative statics are qualitatively different at time 0: a drop in capital first *reduces* the spread. A further drop leads to an increase in spreads. The reduction is due to the fact that as capital drops, hedgers rationally anticipate that constraints will bind at time 1 ($u_0, c_1$ becomes the unique equilibrium as capital drops), leading to a reduction in spreads relative to the previous situation, in which $u_0, u_1$ is the equilibrium (possibly coexisting with $u_0, c_1$). Markets characterized by a high risk benefit ratio are those in which risk is large relative to profitability. Empirically, the risk could be proxied by a certain quantile of the return distribution (e.g. 99% quantile), and profitability by the P&L of trading desks involved in a given arbitrage.

**Price effects of a merger.** In the absence of constraints, the spread is always smaller with competitive arbitrageurs than with a monopolistic one. When financial constraints are present, this is no longer always the case. The example in Table 1 illustrates this point. When capital is scarce: $\Delta_0^* < \Delta_0^{u_0,c_1} < \Delta_0^{u_0,u_1}$, but $\Delta_1^{u_0,c_1} < \Delta_1^* < \Delta_1^{u_0,u_1}$. The spread is thus smaller at time 1 when the arbitrageur is partly constrained (see also Figure 7, although the difference between the competitive and monopolistic spreads is very small). More generally, merging arbitrageurs may reduce spreads at time 1.

**Proposition 11 (Merging Arbitrageurs May Reduce Time-1 Spreads)** *Suppose that competitive arbitrageurs are constrained, i.e. $W_0 < \omega^*$. Merging all arbitrageurs into a monopoly, holding capital constant, leads to a decrease in the time 1 spread*

- *If $W_0 \in [\omega_0^p, \omega_1^p[$ when the equilibrium is $u_0, c_1$,*
- *If $W_0 \in [\omega_1^u, \omega^m[$ and $\rho \geq \frac{21}{10}$ when the equilibrium is $u_0, u_1$ ($\omega_1^u < \omega^m$).*

[38] For each type of equilibrium, it is possible to write the comparative statics of the spreads with respect to capital in analytical form (see Corollaries 6 and 8 for the expression of the spreads in the different cases). However, it is difficult to do so across equilibria, hence the largely numerical approach.

From this result, we see that merging arbitrageurs always reduces the time-1 spread if the monopoly is partly constrained in equilibrium, and may reduce it if the monopoly is unconstrained. At time 0, it is possible to write conditions under which there is no such decrease in the spread. It is difficult to rule out such improvement analytically. However, in all numerical examples I considered, time-0 spreads were larger under a monopoly than constrained competitive arbitrageurs. In any event, the reduction in spread at time 1 does not lead to a welfare improvement, as the competitive equilibrium is constrained efficient (Gromb and Vayanos, 2002). Thus, in this set up, merging arbitrageurs may lead to smaller spreads, but not to a Pareto improvement.

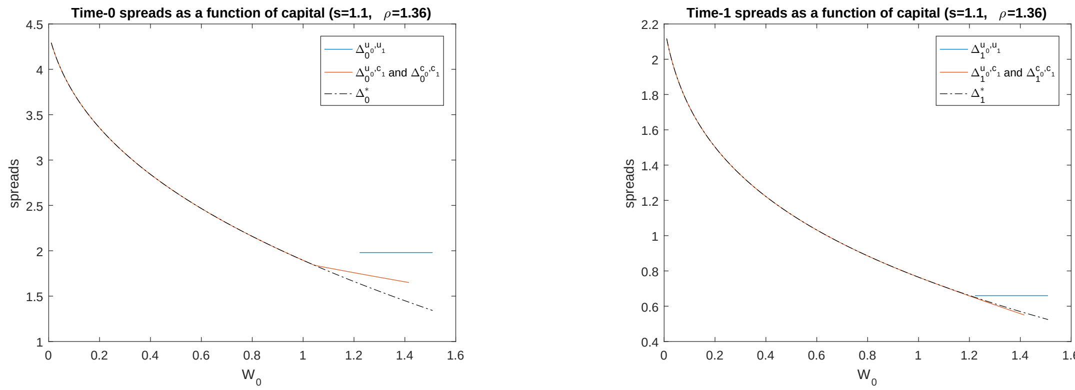
# 6 Conclusion

In this paper, I consider the effects of imperfect competition among arbitrageurs subject to realistic financial constraints. My main result is to characterize markets in which imposing these constraints may benefit both arbitrageurs and their trading counterparties (hedgers). This analysis reveals novel and subtle mechanisms through which constraints affects both types of investors in the presence of arbitrageurs' market power. On the one hand, a binding constraint can mitigate the commitment problem of an imperfectly competitive arbitrageur. On the other hand, the constraint is endogenous to the arbitrageur's trading strategy: to make a constraint binding in the future, an arbitrageur must trade more aggressively today, which speeds up the arbitrage and risk-sharing. This strategy yields capital gains, leading to some amount of retrading. The increase in early liquidity combined with sufficient retrading makes hedgers better off. These mechanisms are specific to imperfect competition: in a competitive economy, arbitrageurs take prices as given and do not recognize the commitment problem. Hence, imposing financial constraints on arbitrageurs may have diametrically different effects in different structures for the arbitrage industry.

The model may be extended to consider internal allocation of capital across trading desks or systemic risk. In my framework, an arbitrageur with market power would benefit from being able to commit to decrease her capital level in the future. This may be achieved by pledging capital gains to new trades, for instance by reallocating capital across different trading desks over time. Such effect would not arise with competitive arbitrageurs. The degree of competition among arbitrageurs

should thus result in different internal capital allocations.

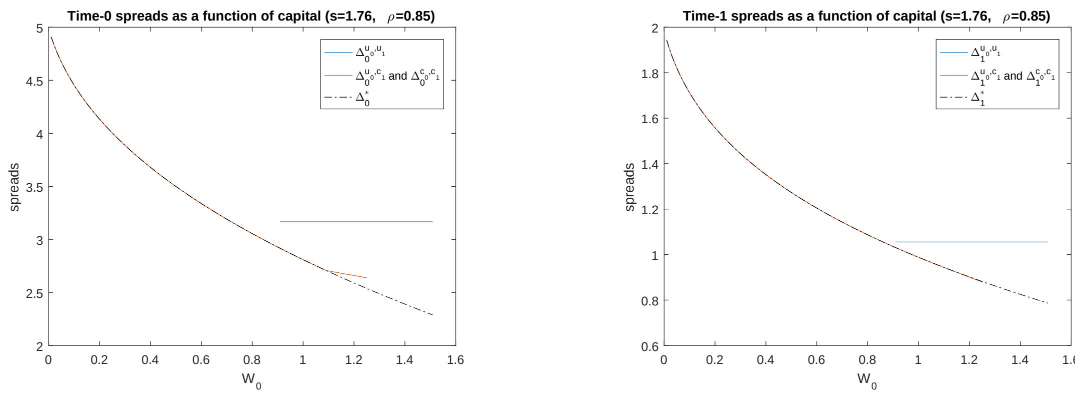
The modeling of the financial constraints precludes defaults in equilibrium. An interesting extension would consiste in allowing for the possibility of default. The introduction of a third party, e.g. a government, would allow for a discussion of the effects of systemic risk and bailouts under different market structures. An intermediate step may consist in extending the model to risky arbitrage. I study a textbook situation in which the arbitrage is risk-free. In practice, arbitrage strategies such as relative-value and convergence trading entail risk. Gromb and Vayanos (2002) show that in this case competitive arbitrageurs may not take the efficient level of risk, as they fail to internalize the effects of their strategies on others' financial constraints. With imperfect competition, one can expect that arbitrageurs would to some extent internalize the impact of their decisions, even though this would also decrease efficiency. These extensions are left for future research.




(a) Time 0

(b) Time 1

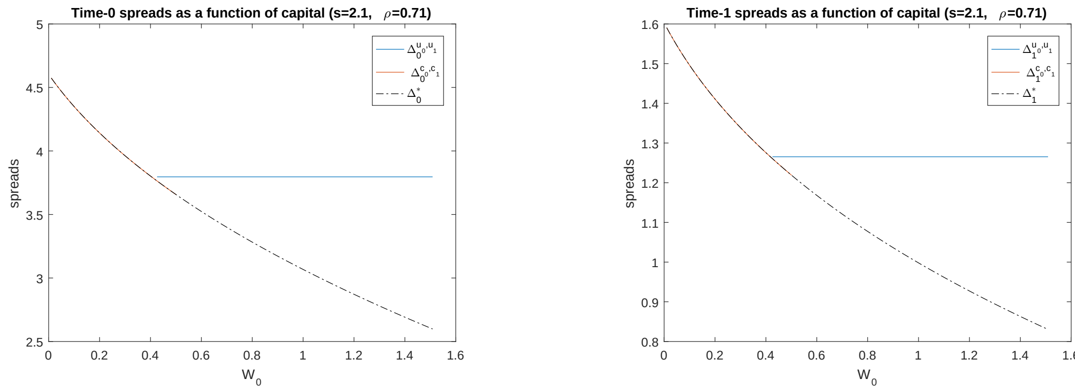
Figure 7: Spreads as a function of arbitrage capital for $\rho > 1$. The parameters are $a = \bar{e} = \sigma = 1$ and $s = 1.1$.




(a) Time 0

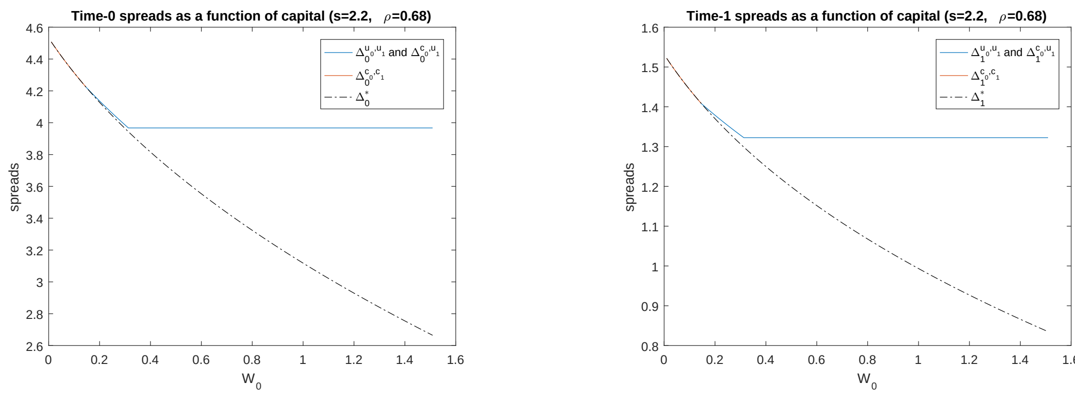
(b) Time 1

Figure 8: Spreads as a function of arbitrage capital for $\frac{3}{4} \leq \rho < 1$. The parameters are $a = \bar{e} = \sigma = 1$ and $s = 1.76$.




(a) Time 0 (b) Time 1

Figure 9: Spreads as a function of arbitrage capital for $\frac{7}{10} \leq \rho < \frac{3}{4}$. The parameters are $a = \bar{e} = \sigma = 1$ and $s = 2.1$.




(a) Time 0 (b) Time 1

Figure 10: Spreads as a function of arbitrage capital for $\rho \leq \frac{7}{10}$. The parameters are $a = \bar{e} = \sigma = 1$ and $s = 2.2$.

# Appendix

## A Benchmarks

### A.1 Competitive benchmark

**Lemma 2 (Hedgers' Demand and Certainty Equivalent)** *At time $t = 0, 1$, in market $A$*

- *Hedgers' demand is* $Y_t = \frac{\mathbb{E}(p_{t+1})-p_t}{a\sigma^2} - s$.
- *Their certainty equivalent is*

$$U_t = w_t + \frac{\sum_{\tau=t}^{T} (\mathbb{E}_t(p_{\tau+1}) - p_\tau)^2}{2a\sigma^2} - s\left(\sum_{\tau=t}^{T} (\mathbb{E}_t(p_{\tau+1}) - p_\tau)\right), \tag{7}$$

**Proof.** It is sufficient to consider hedgers in market A. Market B is similar, except that the shock is $-s$ instead of $s$. Thus, to ease notation, I drop superscript $A$.

At $t = 2$, hedgers' final wealth is $w_2 = w_1 + Y_1(p_2 - p_1) + s\epsilon_2 = w_1 + Y_1(D_1 - p_1) + (Y_1 + s)\epsilon_2$, where $D_1 = \mathbb{E}_1(D_2)$. So at time 1, hedgers solve

$$U_1 = \max_{Y_1} \ \mathbb{E}_1(w_2) - \frac{a}{2}\mathbb{V}_1(w_2) = \max_{Y_1} w_1 + Y_1(D_1 - p_1) - \frac{a\sigma^2}{2}(Y_1 + s)^2,$$

From the FOC, the optimal demand is $Y_1 = \frac{D_1 - p_1}{a\sigma^2} - s$. Thus, substituting the demand into $U_1$, we obtain the certainty equivalent at time 1,

$$U_1 = w_1 + \frac{(D_1 - p_1)^2}{2a\sigma^2} - s(D_1 - p_1)$$

At time 0, hedgers solve the following problem:

$$\begin{aligned}U_0 &= \max_{Y_0}\ \mathbb{E}_0(w_2) - \frac{a}{2}\mathbb{V}_0(w_2)\\ &= \max_{Y_0}\ \mathbb{E}_0(w_2) - \frac{a}{2}\left[\mathbb{V}_0(\mathbb{E}_1(w_2)) + \mathbb{E}_0(\mathbb{V}_1(w_2))\right]\\ &= \max_{Y_0}\ \mathbb{E}_0\left[\mathbb{E}_1(w_2) - \frac{a}{2}\mathbb{V}_1(w_2)\right] - \frac{a}{2}\mathbb{V}_0(\mathbb{E}_1(w_2))\\ &= \max_{Y_0}\ \mathbb{E}_0\left[U_1\right] - \frac{a}{2}\mathbb{V}_0(\mathbb{E}_1(w_2))\end{aligned}$$

However, as will be later verified, $p_1$ is of the form $D + \epsilon_1 +$ risk premium, where the risk premium is determinstic. Hence, $D_1 - p_1$ is independent of $\epsilon_1$. Thus, if hedgers follow the optimal strategy at time 1,

$$\begin{aligned}\mathbb{E}_0(U_1) &= \mathbb{E}_0\left(w_0 + Y_0(p_1 - p_0) + s\epsilon_1 + \frac{(D_1 - p_1)^2}{2a\sigma^2} - s(D_1 - p_1)\right)\\ &= w_0 + Y_0(\mathbb{E}_0(p_1) - p_0) + \frac{(D_1 - p_1)^2}{2a\sigma^2} - s(D_1 - p_1),\end{aligned}$$

where I used hedgers' dynamic budget constraint (1). Similarly, we compute $\mathbb{E}_1(w_2) = w_1 + Y_1(D_1 - p_1) = w_0 + Y_0(p_1 - p_0) + s\epsilon_1 + \frac{(D_1-p_1)^2}{a\sigma^2} - s(D_1 - p_1)$. Then, by the same argument as before, $\mathbb{V}_0(\mathbb{E}_1(w_2)) = \mathbb{V}_0((Y_0 + s)\epsilon_1) = (Y_0 + s)^2\sigma^2$. Thus, at time 0, hedgers solve

$$\max_{Y_0}\ w_0 + Y_0(\mathbb{E}_0(p_1) - p_0) + \frac{(D_1 - p_1)^2}{2a\sigma^2} - s(D_1 - p_1) - \frac{a\sigma^2}{2}(Y_0 + s)^2$$

The optimal demand is $Y_0 = \frac{\mathbb{E}_0(p_1)-p_0}{a\sigma^2} - s$. Substituting this demand into the maximand, we get the expresion for $U_0$ given by equation (7) with $t = 0$. ■

**Corollary 3 (Competitive Spreads and Positions in the Constrained Region)** *If* $0 \le W_0 < \omega^*$*:*

- *The arbitrageurs' positions in asset A are:* $\bar{x}_0 = \frac{a\sigma^2 s - \bar{e} + \sqrt{d_0^*}}{2a\sigma^2}$, $\bar{X}_1 = \frac{a\sigma^2 s - \bar{e} + \sqrt{d_1^*}}{2a\sigma^2}$
- *The equilibrium spreads are:*

$$\Delta_0^* = 2\left(a\sigma^2 s + \bar{e}\right) - \sqrt{d_0^*} - \sqrt{d_1^*};\quad \Delta_1^* = a\sigma^2 s + \bar{e} - \sqrt{d_1^*}, \tag{8}$$

*with* $d_0^* = \left(\bar{e} - a\sigma^2 s\right)^2 + 2a\sigma^2 W_0$ *and* $d_1^* = \left(\bar{e} - a\sigma^2 s\right)^2 + 4a\sigma^2 x_0 \bar{e}$.

**Proof.** The positions $\bar{x}_0$ and $\bar{X}_1$ are solutions to the system of equations (4)-(5). Subtitute for hedgers' demand $Y_t^k$ and plug these quantities into the market clearing equation

$$Y_t^k + \sum_{i=1}^{n} X_t^{i,k} = 0 \tag{9}$$

to obtain equilibrium prices and spreads. ■

## A.2 Monopoly without Financial Constraints

### Price schedules (Lemma 1)

**Proof.** The result follows from inverting hedgers' demand given in Lemma 2 and imposing market clearing (9). ■

Lemma 1 implies that the spread schedules $\Delta_t(\cdot) \equiv p_t^B(\cdot) - p_t^A(\cdot)$ are given by

$$\Delta_1(X_1) = 2a\sigma^2(s - X_1) = 2a\sigma^2(s - x_0 - x_1) \tag{10}$$

$$\Delta_0(x_0, x_1) = 2a\sigma^2(s - x_0) + \Delta_1(x_0, x_1) \tag{11}$$

## A.3 Monopoly with Perfect Commitment (Proposition 3)

**Proof.** If the arbitrageur commits to a trading strategy, then he chooses both $x_0$ and $x_1$ at time 0. Given the spread schedules (11)-(10), his maximization problem is as follows:

$$\max_{x_0, x_1} W_0 + x_0 \Delta_0(x_0, x_1) + x_1 \Delta_1(x_0, x_1)$$

$$\max_{x_0, x_1} W_0 + x_0 \left[4a\sigma^2(s - x_0) - 2a\sigma^2 x_1\right) + x_1 2a\sigma^2(s - x_0 - x_1)$$

From the first-order conditions, we obtain the following system:

$$\begin{cases} 4a\sigma^2(s - x_0 - x_1) = 0 \\ -4a\sigma^2 x_0 + 2a\sigma^2(s - 2x_1) = 0 \end{cases}$$

Solving we get, $x_0^{pc} = \frac{s}{2}$ and $x_1^{pc} = 0$. Thus, $X_1^{pc} = x_0$. Substituting the trades into the spread schedules gives $\Delta_0^{pc} = 2a\sigma^2 s$ and $\Delta_1^{pc} = a\sigma^2 s$. Subsituting them into the maximand, we get $\Omega_0^{pc}$. Subsitutting the spreads into (7) gives hedgers' welfare. ■

### A.4 Capacity Constraints (Proposition 4)

**Proof.** Given the capacity constraint $k$, we now have $x_0 \leq k$ and $x_1 \leq k$. Ignore the costs for now. Let's solve the arbitrageur's problem backwards. At time 1, given the spread schedule (10),

$$\max_{x_1 \leq k} 2a\sigma^2 x_1(s - x_0 - x_1)$$

The solution is $x_1 = \frac{s-x_0}{2}$ if $s - x_0 \leq 2k$ and $k$ otherwise. Hence the spread in the subgame is either $a\sigma^2(s - x_0)$ or $2a\sigma^2(s - x_0 - k)$. Suppose that the constraint is slack, then at time 0, using schedule (11), the arbitrageur solves:

$$\max_{x_0 \leq k} W_0 + 3a\sigma^2 x_0(s - x_0) + \frac{a\sigma^2}{2}(s - x_0)^2$$

Thus $x_0$ is equal to $x_0^{u_0,u_1} = \frac{2}{5}s$ or to $k$. But solving for $k$, we get $k = \frac{2}{5}s$. Since $x_1 = x_1^{u_0,u_1} = \frac{3}{10}s < k$, the constraint is indeed slack at time 1.

Suppose next that the constraint binds at time 1. Then at time 0, the arbitrageur solves

$$\max_{x_0 \leq k} W_0 + 2a\sigma^2 x_0(2s - 2x_0 - k) + 2a\sigma^2 k(s - x_0 - k)$$

The solution is $x_0 = \frac{s-x_0}{2}$ if $k \geq \frac{s}{3}$ and $k$ otherwise. Let's now show that if the constraint binds at time 1, it also binds at time 0. Suppose it is not the case, and determine the optimal capacity $k$. Afer plugging $x_0 = \frac{s-x_0}{2}$ in the previous maximand, we can rewrite the arbitrageur's optimization problem as

$$\max_{k} W_0 + a\sigma^2(s^2 - k^2)$$

The optimal is then $k = 0$. Thus, the constraint binds at time 0. Hence if the constraint binds

at time 1, it binds also at time 0. Using $x_1 = x_0 = k$ and equations (11)-(10), the arbitrageur's capacity choice can be written as:

$$\max_k W_0 + 2a\sigma^2 k(2s - 3k) + 2a\sigma^2 k(s - 2k)$$

The solution is $k = x_1^{u_0,u_1} = \frac{3}{10}s < \frac{s}{3}$, so the constraint does bind at time 0. Further, given $x_0 = k$, the constraint also does bind at time 1, because the unconstrained trade at time 1 is $\frac{7}{20}s > k$. With such capacity, the expected utility of the arbitrageur is $W_0 + \frac{9}{10}a\sigma^2 s^2$. Thus, absent costs, the arbitrageur is indifferent between capacities $k = x_0^{u_0,u_1}$ and $k = x_1^{u_0,u_1}$. However, as soon as the cost is strictly positive, the arbitrageur prefers the smaller capacity. To complete the proof, we can then calculate spreads using Lemma 1: $\Delta_1^{cc} = 2a\sigma^2(s - 2k) = \frac{4}{5}s$, and $\Delta_0^{cc} = 2a\sigma^2(2s - 3k) = \frac{11}{5}s$. Thus, $\Delta_0^{cc} - \Delta_1^{cc} = \frac{4}{5}s$. Plugging these spreads into (25) gives $U_0^{cc} = -\frac{31}{40}a\sigma^2 s^2$. Comparisons to the no commitment and perfect commitment cases follow. ■

# B Static equilibrium with market power and financial constraints

At time 1, the arbitrageur solves the following problem:

$$\begin{aligned} &\max_{x_1} \; B_0 + x_1 \Delta_1(X_1) \\ &\text{s.t. } f_1^+(X_1)\; \mathbb{1}_{X_1 \geq 0} + f_1^-(X_1)\; \mathbb{1}_{X_1 < 0} \geq 0 \end{aligned}$$

where $B_0$ is the position in the risk-free asset and $\Delta_1(X_1)$ is given by (10). The financial constraints define a set $\mathcal{F}_1 = \{X_1 \geq 0 \mid f_1^+(X_1) \geq 0\} \cup \{X_1 < 0 \mid f_1^-(X_1) \geq 0\}$.

**Notation 2 (Boundaries of $\mathcal{F}_1$)**

- *Let* $\underline{X}_1^+ \equiv \frac{a\sigma^2 s - \bar{e} - \sqrt{d_1^+}}{2a\sigma^2}$ *and* $\bar{X}_1 \equiv \frac{a\sigma^2 s - \bar{e} + \sqrt{d_1^+}}{2a\sigma^2}$ *denote the smallest and largest roots, if they exist, of* $f_1^+$*, with* $d_1^+ \equiv 2a\sigma^2 W_1 + (a\sigma^2 s - \bar{e})^2$.
- *Let* $\underline{X}_1 \equiv \frac{a\sigma^2 s + \bar{e} - \sqrt{d_1^-}}{2a\sigma^2}$ *denote the smallest root of* $f_1^-$*, with* $d_1^- \equiv 2a\sigma^2 W_1 + (a\sigma^2 s + \bar{e})^2$.

**Proposition 12 (Static Equilibrium)** *Suppose that the arbitargeur starts with wealth $W_1$ and position $x_0$. There exists a wealth threshold $\bar{W}_1^+ < 0$ such that*

- *If $W_1 < \bar{W}_1^+$, or if $\bar{W}_1^+ \leq W_1 < 0$ and $\rho > 1$, the arbitrageur has not enough capital to hold any position at time 1.*
- *If $\bar{W}_1^+ \leq W_1 < 0$ and $\rho \leq 1$, then the optimum depends on the initial position in the asset $x_0$:*
  - *If $x_0 < -s$, the arbitrageur has not enough capital to hold any position at time 1.*
  - *If $-s \leq x_0 < -s\rho$, the arbitrageur can hold only long positions in $\mathcal{F}_1 = \left[\underline{X}_1^+, \bar{X}_1\right]$, but the constraint binds downwards ($0 < X_1^u < \underline{X}_1^+ < \bar{X}_1$). It is optimal for the arbitrageur to hold $\underline{X}_1^+$.*
  - *If $x_0 \geq -s\rho$, then the arbitrageur can hold only long positions in $\mathcal{F}_1 = \left[\underline{X}_1^+, \bar{X}_1\right]$, and the constraint binds upwards ($0 < \underline{X}_1^+ < \bar{X}_1 < X_1^u$). It is optimal for the arbitrageur to hold $\bar{X}_1$.*
- *If $W_1 \geq 0$, then the arbitrageur can choose long and short positions in the segment $\mathcal{F}_1 = \left[\underline{X}_1, \bar{X}_1\right]$, with $\underline{X}_1 < 0$ and $\bar{X}_1 > 0$. The optimum depends on the initial wealth and the initial position in the asset $x_0$:*
  - *If $x_0 < -s$, then $X_1^u < 0$. If $W_1 \geq \frac{1}{2}a\sigma^2(x_0^2 - s^2) - \bar{e}(x_0 + s)$, the constraint is slack and the arbitrageur holds $X_1^u$. Otherwise, the arbitrageur's constraint binds downwards, and the arbitrageur chooses $\underline{X}_1$.*
  - *If $x_0 \geq -s$, then*
    - *If $\rho \geq 1$, then if $W_1 \geq \frac{1}{2}a\sigma^2(x_0^2 - s^2) + \bar{e}(x_0 + s) > 0$, the constraint is slack and the arbitrageur holds $X_1^u$. If $0 \leq W_1 < \frac{1}{2}a\sigma^2(x_0^2 - s^2) + \bar{e}(x_0 + s)$, the constraint binds upwards and the arbitrageur holds $\bar{X}_1$.*
    - *If $\rho < 1$, then*
      - *if $-s \leq x_0 < -s\rho$, the constraint is slack, the arbitrageur holds $X_1^u$.*
      - *if $x_0 \geq -s\rho$, the arbitrageur holds $X_1^u$ if $W_1 \geq \frac{1}{2}a\sigma^2(x_0^2 - s^2) + \bar{e}(x_0 + s) > 0$, and $\bar{X}_1$ otherwise.*

**Proof.** The discriminants of $f_1^-$ and $f_1^+$ are $4d_1^-$ and $4d_1^+$. $d_1^+ > 0$ iff $W_1 > \bar{W}_1^+ \equiv -\frac{(a\sigma^2 s-\bar{e})^2}{2a\sigma^2}$. Similarly, $d_1^- > 0$ iff $W_1 > \bar{W}_1^- \equiv -\frac{(a\sigma^2 s+\bar{e})^2}{2a\sigma^2}$. Clearly, $\bar{W}_1^- < \bar{W}_1^+ < 0$. Thus there are four cases:

*Case 1.* If $W_1 < \bar{W}_1^-$, then both $f_1^+$ and $f_1^-$ are always negative, so $\mathcal{F}_1 = \emptyset$, no constraint can be satisfied.

*Case 2.* If $\bar{W}_1^- \leq W_1 < \bar{W}_1^+$, then $f_1^+$ is always negative, so long positions are not feasible. Further, since $W_1 < 0$ and $\frac{a\sigma^2 s+\bar{e}}{a\sigma^2} > 0$, $f_1^-$ is a hump-shaped parabola with a negative intercept and two *positive* roots. Thus, if $X_1 < 0$, $f_1^-(X_1) < 0$, i.e. there is no short position satisfying the financial constraint. Thus no positions are feasible ($\mathcal{F}_1 = \emptyset$).

*Case 3.* If $\bar{W}_1^+ \leq W_1 < 0$, then short positions are still not feasible, for the same reason as in case 2. For long positions there are two subcases:

i) if $\rho > 1$, then $f_1^+$ is hump-shaped with two negative roots and a negative intercept, so there are no long positions satisfying the constraint.

ii) if $\rho \geq 1$, $f_1^+$ is hump-shaped with two positive roots. The largest root is $\bar{X}_1$ and the smallest root is $\underline{X}_1^+$, both given in Definition 2. Any position in $\mathcal{F}_1 = \left[\underline{X}_1^+, \bar{X}_1\right]$ is feasible, so $X_1^u$ must be compared to $\underline{X}_1^+$ and $\bar{X}_1$. First note that $X_1^u \geq \underline{X}_1^+$ is equivalent to $\sqrt{d_1^+} \geq -a\sigma^2 x_0 - \bar{e}$. Suppose that $-s \leq x_0 < -\frac{\bar{e}}{a\sigma^2}$. Then the right-hand side is positive, so $X_1^u \geq \underline{X}_1^+$ implies $d_1^+ \geq (a\sigma^2 x_0 + \bar{e})^2$, which is equivalent to $2a\sigma^2 W_1 \geq a^2\sigma^4(x_0^2 - s^2) + 2a\sigma^2\bar{e}(x_0 + s)$. The right-hand side is positive. But $W_1 < 0$, so this condition cannot hold, and thus $X_1^u < \underline{X}_1^+$. Given that the objective function is increasing for $X_1 \leq X_1^u$, it is optimal for the arbitrageur to hold $\underline{X}_1^+$. Suppose next that $x_0 \geq -\frac{\bar{e}}{a\sigma^2}$, then the condition $\sqrt{d_1^+} \geq -a\sigma^2 x_0 - \bar{e}$ is trivially satisfied, so $X_1^u \geq \underline{X}_1^+$. Consider then the position of $X_1^u$ relative to $\bar{X}_1$: $X_1^u \leq \bar{X}_1$ is equivalent to $\sqrt{d_1^+} \geq a\sigma^2 x_0 + \bar{e} \geq 0$. Raising both sides to the square thus implies that $2a\sigma^2 W_1 \geq a^2\sigma^4(x_0^2 - s^2) + 2a\sigma^2\bar{e}(x_0 + s)$. Again, the right-hand side is positive, while the left-hand side is negative. As a result, $X_1^u > \bar{X}_1$. Since the objective function is increasing for $X_1 \leq X_1^u$, it is optimal for the arbitrageur to hold $\bar{X}_1$.

*Case 4.* If $W_1 \geq 0$, then both $f_1^+$ has a positive and a negative root, so long positions between 0 and $\bar{X}_1$ are feasible. Further, $f_1^-$ also has both a positive and a negative root, so any position between $\underline{X}_1$ and 0 (excluded) is feasible. Thus, the constraints define a segment of feasible positions

$\mathcal{F}_1 = \left[\underline{X}_1, \bar{X}_1\right]$. Then there are two cases depending on the sign of $X_1^u$.

i) If $x_0 \geq -s$, $X_1^u \geq 0$, so the position of $X_1^u$ relative to $\bar{X}_1$ must be determined. Following the same steps as before, $\bar{X}_1 \geq X_1^u$ iff $\sqrt{d_1^+} \geq a\sigma^2 x_0 + \bar{e}$. However, $-\frac{\bar{e}}{a\sigma^2} \leq -s$ iff $\rho \geq 1$. Thus there are two cases:

a) if $\rho \geq 1$, then $X_1^u \leq \bar{X}_1$ holds if $2a\sigma^2 W_1 \geq a^2\sigma^2(x_0^2 - s^2) + 2a\sigma^2\bar{e}(s + x_0) > 0$.

b) if $\rho < 1$, then if $-s \leq x_0 < -\frac{\bar{e}}{a\sigma^2}$, the condition $\sqrt{d_1^+} \geq a\sigma^2 x_0 + \bar{e}$ is trivially satisfied, so $X_1^u \leq \bar{X}_1$, and the arbitrageur holds $X_1^u$. If instead $x_0 \geq -\frac{\bar{e}}{a\sigma^2}$, then the condition $\sqrt{d_1^+} \geq a\sigma^2 x_0 + \bar{e}$ implies $2a\sigma^2 W_1 \geq a^2\sigma^2(x_0^2 - s^2) + 2a\sigma^2\bar{e}(s + x_0) > 0$. Thus the arbitrageur holds $X_1^u$ if $W_1$ is large enough, and $\bar{X}_1$ otherwise, as the constraint binds upwards.

ii) Consider now to the case $x_0 < -s$. Then $X_1^u < 0$ and must be compared to $\underline{X}_1$. The inequality $X_1^u \geq \underline{X}_1$ is equivalent to $\sqrt{d_1^-} \geq \bar{e} - a\sigma^2 x_0$. Since $x_0 < -s$, $\bar{e} - a\sigma^2 x_0 > 0$, so raising both the side to the squares leads to $2a\sigma^2 W_1 \geq a^2\sigma^4(x_0^2 - s^2) - 2a\sigma^2\bar{e}(x_0 + s) > 0$. So for $W_1$ large enough, the arbitrageur is unconstrained and chooses $X_1^u$. Otherwise, the constraint binds downwards and the arbitrageur chooses $\underline{X}_1$, since the objective function is decreasing for $X_1 \geq X_1^u$. ■

# C Dynamic equilibrium with slack constraint at time 1

In this section, I conjecture that the arbitrageur holds an unconstrained position at time 1 and verify under which conditions it is optimal to do so. Here is the full version of Proposition 5:

**Proposition 13 (Equilibria with slack time-1 constraint)** *There exists three thresholds $\omega_0^u$, $\omega_1^u$, and $\omega^f$, with $\omega^f \equiv \omega^c \ \mathbb{1}_{\rho<7} + \hat{\omega} \ \mathbb{1}_{\rho\geq 7}$, that define four regions in terms of initial arbitrage capital:*

1. *In the first region, $W_0 \geq \max(\omega_0^u, \omega_1^u)$, arbitrage capital is abundant, both constraints are slack in equilibrium, and the arbitrageur holds his desired position at time 0 and time 1 $x_0^{u_0,u_1}$ and $X_1^{u_0,u_1}$ given in Proposition 2 ($\mathbf{u_0}, \mathbf{u_1}$)*

2. *In second region, where $\max(\omega^f, \min(\omega_0^u, \omega_1^u)) \leq W_0 < \max(\omega_0^u, \omega_1^u)$, there are two cases:*

    - *If $\rho \geq \frac{7}{10}$, an equilibrium may exist, in which the arbitrageur's constraint binds at time 0 and is slack at time 1 ($\mathbf{c_0}, \mathbf{u_1}$).*

*Namely, if* $\tilde{\Omega}_0^{u_1,u_1}(x_0^{c_0,u_1}) \geq \tilde{\Omega}_0^{u_1,\bar{c}_1}(x_0^{d*})$*, where* $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^{\bar{c}_1}} \tilde{\Omega}_0^{u_1,\bar{c}_1}(x_0)$*, the arbitrageur holds less than his desired position* $x_0^{u_0,u_1}$ *at time 0 to keep the constraint slack at time 1:*

$$x_0^{c_0,u_1} = \min(\hat{x}_0, \bar{x}_0) < x_0^{u_0,u_1}, \quad X_1^{c_0,u_1} = X_1^u(x_0^{c_0,u_1}) = \frac{s + x_0^{c_0,u_1}}{2}$$

*Otherwise, there is no equilibrium with a slack constraint at time 1.*

- *If* $\rho < \frac{7}{10}$*, an equilibrium exists, where the constraint binds at time 0 but not at time 1* ($\mathbf{c_0}, \mathbf{u_1}$)*, with*

$$x_0^{c_0,u_1} = \bar{x}_0 < x_0^{u_0,u_1}, \quad X_1^{c_0,u_1} = X_1^u(x_0^{c_0,u_1}) = \frac{s + \bar{x}_0}{2}$$

3. *In the third region, where* $\max(0, \omega^f) \leq W_0 < \min(\omega_0^u, \omega_1^u)$*, the situation is the same as in the second region with* $\rho \geq \frac{7}{10}$*. The interval* $\left[\omega^f, \min(\omega_0^u, \omega_1^u)\right)$ *is non-empty iff* $\rho \in \left[0, 3 - \frac{2}{5}\sqrt{30}\right) \cup \left(3 - \frac{2}{5}\sqrt{30}, \infty\right)$.

4. *In the fourth region,* $0 \leq W_0 < \max(0, \omega^f)$*, there is little arbitrage capital, and thus there is no equilibrium with a slack constraint at time 1 (**no*** $\mathbf{u_1}$*).*

**Proof.** The proof relies on three main steps: i) I first write the arbitrageur's objective function and payoffs from deviating, assuming that hedgers anticipate a slack constraint at time 1. The arbitrageur's maximization involves choosing a position satisfying a set of constraints. ii) I derive the sets of feasible positions and possible deviations. These sets depend on the initial level of arbitrage capital $W_0$ and the risk benefit ratio $\rho$. iii) I determine the candidate equilibrium strategy and verify conditions under which it is possible/ optimal for the arbitrageur to follow it.

## C.1 Step 1: arbitrageur's problem

**Objective functions.** Using the notations introduced in the main text, given hedgers' anticipations, the arbitrageur's problem is to choose $x_0$ (or equivalently an action $l = \{u_1, \bar{c}_1, \underline{c}_1\}$) to

maximize expected utility:

$$(\mathcal{P}^{u_1}): \quad \Omega_0^{u_1} = \max_{l=\{u_1,\bar{c}_1,\underline{c}_1\}} \Omega_0^{u_1,l}$$

The value functions $\Omega_0^{u_1,l}$ associated to each action are defined as follows:

$$\begin{aligned}
\Omega_0^{u_1,u_1} = \max_{x_0} \quad & \tilde{\Omega}_0^{u_1,u_1}(x_0) = W_0 + x_0 \Delta_0^{u_1}(x_0) + \Omega_1^u(x_0) \\
\text{s.t.} \quad & f_0^+(x_0) \geq 0 \ \mathbb{1}_{x_0 \geq 0} + f_0^-(x_0) \ \mathbb{1}_{x_0 < 0} \geq 0 \\
& f_1^+\left(\frac{s+x_0}{2}\right) \ \mathbb{1}_{x_0 \geq -s} + f_1^-\left(\frac{s+x_0}{2}\right) \ \mathbb{1}_{x_0 < -s} \geq 0 \\
& W_1(x_0) = W_0 + 2a\sigma^2 x_0(s - x_0) \geq 0
\end{aligned}$$

The objective function $\tilde{\Omega}_0^{u_1,u_1}$ relies on two premises: i) the continuation value $\Omega_1^u(x_0) \equiv \frac{1}{2}a\sigma^2(s - x_0)^2$ assumes that the arbitrageur chooses the unconstrained position at time 1, and ii) the spread schedule, based on the price schedule in each market, requires that hedgers correctly anticipate that the arbitrageur's time 1 constraint is slack in equilibrium. Given equation 11, the spread schedule is $\Delta_0^{u_1}(x_0) = \mathbb{E}_0\left[\Delta_1^u(x_0)\right] + 2a\sigma^2(s - x_0) = 3a\sigma^2(s - x_0)$, since $\Delta_1^u(x_0) = 2a\sigma^2(s - X_1^u) = a\sigma^2(s - x_0)$.

The first two constraints ensure that the arbitrageur has enough capital to hold a position $x_0$ at time 0. The next two constraints ensure that given the position established at time 0, $x_0$, the arbitrageur can indeed hold his preferred position $X_1^u(x_0)$ at time 1, be it a long or a short position. This requirement ensures that the arbitrageur's strategy is time-consistent. The arbitrageur's ability to satisfy the time-1 constraints requires positive wealth at time 1, which yields the last constraint. Otherwise, Proposition 12 shows that the arbitrageur's constraint is necessarily binding at time 1. Next, I consider the payoff from deviating towards an upward-binding constraint

at time 1.

$$
\begin{aligned}
\Omega_0^{u_1,\bar{c}_1} = \max_{x_0} \quad & \tilde{\Omega}_0^{u_1,\bar{c}_1}(x_0) = W_0 + x_0\Delta_0(x_0)^{u_1} + \Omega_1^{\bar{c}}(x_0) \\
\text{s.t.} \quad & f_0^+(x_0) \geq 0 \ \mathbb{1}_{x_0 \geq 0} + f_0^-(x_0) \ \mathbb{1}_{x_0<0} \geq 0 \\
& f_1^+\left(\frac{s+x_0}{2}\right) \ \mathbb{1}_{x_0 \geq -s} + f_1^-\left(\frac{s+x_0}{2}\right) \ \mathbb{1}_{x_0<-s} < 0 \\
& W_1(x_0) = W_0 + 2a\sigma^2 x_0(s-x_0) \geq 0
\end{aligned}
$$

The objective function includes a different continuation value at time 1. The first two constraints ensure that the position is feasible at time 0. The next two constraints ensure that, given $x_0$, the arbitrageur can indeed *not* choose his preferred position at time 1 (time consistency). The last constraint requires that wealth be positive at time 1, as Proposition 12 requires. Finally, I define the payoff from deviating towards a downward-binding constraint at time 1:

$$
\begin{aligned}
\Omega_0^{u_1,\underline{c}_1} = \max_{x_0 \in [-s,-s\rho[\cup]-\infty,-s]} \quad & \tilde{\Omega}_0^{u_1,\underline{c}_1}(x_0) = W_0 + x_0\Delta_0^{u_1}(x_0) + \Omega_1^{\underline{c}}(x_0) \\
\text{s.t.} \quad & f_0^+(x_0) \geq 0 \ \mathbb{1}_{x_0 \geq 0} + f_0^-(x_0) \ \mathbb{1}_{x_0<0} \geq 0 \\
& f_1^+\left(\frac{s+x_0}{2}\right) \ \mathbb{1}_{x_0 \geq -s} + f_1^-\left(\frac{s+x_0}{2}\right) \ \mathbb{1}_{x_0<-s} < 0 \\
& W_1(x_0) < 0 \quad \text{if } x_0 \in [-s,-s\rho[\text{, or,} \\
& 0 \leq W_1(x_0) < \frac{1}{2}a\sigma^2(x_0^2 - s^2) - \bar{e}(x_0+s) \quad \text{if } x_0 < -s
\end{aligned}
$$

The payoff is built as in the previous case. However, the last constraint requires negative wealth and $x_0$ must be chosen in the interval $[-s, -s\rho[$, as this is necessary for the constraint to bind downwards at time 1, by Proposition 12.

**Feasible positions.** Suppose first that the arbitrageur chooses $x_0$ leading $u_1$. This position must satisfy the following set of constraints. First, the position must satisfy the constraint at time 0, so $x_0 \in \mathcal{F}_0^0$, where $\mathcal{F}_0^0 = \{x_0 < 0 \mid f_0^-(x_0) \geq 0\} \cup \{x_0 \geq 0 \mid f_0^+(x_0) \geq 0\}$. I denote $\mathcal{F}_0^1$ the interval determined by the constraints at time 1. It is convenient to write $\mathcal{F}_0^1$ as the union of two intervals, one for long and one for short unconstrained positions at time 1, i.e. $\mathcal{F}_0^1 = \mathcal{F}_0^{1-} \cup \mathcal{F}_0^{1+}$, where

$\mathcal{F}_0^{1-} = \{x_0 < -s \mid f_1^-\left(\frac{s+x_0}{2}\right) \geq 0\}$ and $\mathcal{F}_0^{1+} = \{x_0 \geq -s \mid f_1^+\left(\frac{s+x_0}{2}\right) \geq 0\}$. Finally, the positive wealth constraint defines a set $\mathcal{F}_0^{pw} = \{x_0 \mid W_1(x_0) = W_0 + 2a\sigma^2 x_0(s - x_0) \geq 0\}$. The intersection of these sets thus defines a set of feasible positions

$$\mathcal{F}_0^u = \mathcal{F}_0^0 \cap \mathcal{F}_0^{u_1} \cap \mathcal{F}_0^{pw}$$

Therefore we can rewrite $\Omega_0^{u_1,u_1}$ simply as

$$\Omega_0^{u_1,u_1} = \max_{x_0 \in \mathcal{F}_0} \quad W_0 + x_0\Delta_0(x_0) + \Omega_1^{u_1}(x_0)$$

Similarly, we can define the sets $\mathcal{D}_0^{\bar{c}_1}$ and $\mathcal{D}_0^{\underline{c}_1}$ of positions leading to upward -or -downward-binding constraints at time 1. I construct these sets in detail in Section C.2.5 below. Using these notations, we can also rewrite $\Omega_0^{u_1,\bar{c}_1}$ and $\Omega_0^{u_1,\underline{c}_1}$ as follows:

$$\Omega_0^{u_1,\bar{c}_1} = \max_{x_0 \in \mathcal{D}_0^{\bar{c}_1}} \quad \tilde{\Omega}_0^{u_1,\bar{c}_1}(x_0) = W_0 + x_0\Delta_0(x_0) + \Omega_1^{\bar{c}}(x_0)$$

$$\Omega_0^{u_1,\underline{c}_1} = \max_{x_0 \in \mathcal{D}_0^{\underline{c}_1}} \quad \tilde{\Omega}_0^{u_1,\underline{c}_1}(x_0) = W_0 + x_0\Delta_0(x_0) + \Omega_1^{\underline{c}}(x_0)$$

## C.2 Step 2: Sets of feasible positions and possible deviations

The second step of the proof has two parts: First, I determine the set $\mathcal{F}_0^u$ of feasible positions at time 0, which allow the arbitrageur to trade his desired quantity at time 1 (i.e. to have a slack constraint at time 1). The set $\mathcal{F}_0^u$ is the intersection of three sets. I derive each set independently, and then study their intersection. Second, I determine the set of positions allowing the arbitrageur to deviate from the conjectured strategy. I first introduce notations for the boundaries on time-0 positions.

**Definition 1 (Time-0 Boundary Positions)**

- *Let $\underline{x}_0$ denote the smallest root of $f_0^-(x_0) = 0$ and $\bar{x}_0$ the largest root of $f_0^+(x_0) = 0$, if they exist.*
- *Let $\hat{x}_0$ and $\hat{x}_0^-$ denote the largest and smallest roots, if they exist, of $f_1^+\left(\frac{s+x_0}{2}\right) = 0$, for all*

$x_0 \geq -s$.

- *Let* $\underline{\underline{x_0}}$ *denote the smallest root of* $f_1^-\left(\frac{s+x_0}{2}\right) = 0$, *for* $x_0 < -s$.

- *Let* $\underline{z}_0$ *and* $\bar{z}_0$ *denote the smallest and largest roots of* $W_1 = W_0 + 2a\sigma^2 x_0(s - x_0) = 0$.

### C.2.1 Set $\mathcal{F}_0^0$

The set $\mathcal{F}_0^0$ is defined as $\mathcal{F}_0^0 = \{x_0 < 0 \mid f_0^-(x_0) \geq 0\} \cup \{x_0 \geq 0 \mid f_0^+(x_0) \geq 0\}$.

**Lemma 3 (Set $\mathcal{F}_0^0$)** *If* $W_0 \geq 0$, $\underline{x}_0 < 0$ *and* $\bar{x}_0 > 0$ *always exist, so* $\mathcal{F}_0^0 = [\underline{x}_0, \bar{x}_0]$, *with*

$$\underline{x}_0 = \frac{a\sigma^2 s + \bar{e} - \sqrt{d_0^-}}{2a\sigma^2}, \quad \text{where } d_0^- = 2a\sigma^2 W_0 + (a\sigma^2 s + \bar{e})^2 \tag{12}$$

$$\bar{x}_0 = \frac{a\sigma^2 s - \bar{e} + \sqrt{d_0^+}}{2a\sigma^2}, \quad \text{where } d_0^+ = 2a\sigma^2 W_0 + (a\sigma^2 s - \bar{e})^2 \tag{13}$$

**Proof.** The point follows from the fact that $W_0 \geq 0$, so that $f_0^+$ and $f_0^-$ are hump-shaped with a positive intercept, and thus always have a positive and a negative root. ■

### C.2.2 Set $\mathcal{F}_0^{u_1}$

The set $\mathcal{F}_0^{u_1}$ is defined as $\mathcal{F}_0^{u_1} = \{x_0 < -s \mid f_1^-(X_1^u) \geq 0\} \cup \{x_0 \geq -s \mid f_1^+(X_1^u) \geq 0\} \equiv \mathcal{F}_0^{u_1-} \cap \mathcal{F}_0^{u_1+}$.

**Lemma 4 (Set $\mathcal{F}_0^{u_1}$)** *Let* $\hat{\omega} \equiv 4a\sigma^2 s^2$ *and* $\omega^c \equiv \frac{7}{5}s\bar{e} - \frac{9}{10}a\sigma^2 s^2 - \frac{1}{10}\frac{\bar{e}}{a\sigma^2}$ *denote two wealth thresholds. The set* $\mathcal{F}_0^{u_1}$ *is given by*

- *If* $0 \leq \rho < 1$, $$\mathcal{F}_0^{u_1} = \begin{cases} [-s, -s\rho] & \text{if } W_0 < \omega^c \\ \left[\hat{x}_0^-, \hat{x}_0\right] & \text{if } \omega^c \leq W_0 < \hat{\omega} \\ \left[\underline{\underline{x_0}}, \hat{x}_0\right] & \text{if } W_0 \geq \hat{\omega} \end{cases},$$

- *If* $1 \leq \rho < 7$, $$\mathcal{F}_0^{u_1} = \begin{cases} \emptyset & \text{if } W_0 < \omega^c \\ \left[\hat{x}_0^-, \hat{x}_0\right] & \text{if } \omega^c \leq W_0 < \hat{\omega} \\ \left[\underline{\underline{x_0}}, \hat{x}_0\right] & \text{if } W_0 \geq \hat{\omega} \end{cases},$$

- *If* $\rho \geq 7$, $\quad \mathcal{F}_0^{u_1} = \begin{cases} \emptyset & \textit{if } W_0 < \omega^c \\ \emptyset & \textit{if } \omega^c \leq W_0 < \hat{\omega} \\ \left[\underline{\underline{x_0}}, \hat{x}_0\right] & \textit{if } W_0 \geq \hat{\omega} \end{cases}$,

$$\textit{where} \quad \underline{\underline{x_0}} \equiv \frac{2a\sigma^2 s + \bar{e} - \sqrt{\delta_0^-}}{5a\sigma^2} \quad \textit{with } \delta_0^- = 10a\sigma^2 W_0 + \bar{e}^2 + 14a\sigma^2 s\bar{e} + 9a^2\sigma^4 s^2 \tag{14}$$

$$\hat{x}_0^- \equiv \frac{2a\sigma^2 s - \bar{e} - \sqrt{\delta_0^+}}{5a\sigma^2} \tag{15}$$

$$\hat{x}_0 \equiv \frac{2a\sigma^2 s - \bar{e} + \sqrt{\delta_0^+}}{5a\sigma^2} \quad \textit{with } \delta_0^+ = 10a\sigma^2 W_0 + \bar{e}^2 - 14a\sigma^2 s\bar{e} + 9a^2\sigma^4 s^2 \tag{16}$$

**Proof.** *Short positions.* If $x_0 < -s$, the condition $W_1 \geq \frac{1}{2}a\sigma^2(x_0^2 - s^2) - \bar{e}(x_0 + s)$ must be satisfied to hold a short position at time 1 without violating the constraint. Since $W_1 = W_0 + x_0(\Delta_0 - \Delta_1) = W_0 + 2a\sigma^2 x_0(s - x_0)$, the condition is equivalent to

$$f_1^-\left(\frac{s + x_0}{2}\right) = W_0 + (s\bar{e} + \frac{1}{2}a\sigma^2 s^2) + x_0(2a\sigma^2 s + \bar{e}) - \frac{5}{2}a\sigma^2 x_0^2 \geq 0$$

Given that the intercept is positive and the parabola hump-shaped, the polynomial on the left-hand side has always a positive and a negative root, with the negative root given by (14). Then $\underline{\underline{x_0}} < -s$ is equivalent $0 < 7a\sigma^2 s + \bar{e} < \sqrt{\delta_0^-}$. Then raising both sides to the square and rearranging terms yields $W_0 > \hat{\omega}$. Hence the set of time-0 positions leading to unconstrained short positions at time 1 is

$$\mathcal{F}_0^{u_1 -} = \begin{cases} \emptyset & \text{if } W_0 < \hat{\omega} \\ \left[\underline{\underline{x_0}}, -s\right) & \text{otherwise} \end{cases}$$

*Long positions.* First, note that from proposition 12, unless $\rho < 1$ and $x_0\,[-s, -s\rho]$, the arbitrageur can hold an unconstrained position at time 1 iff $W_1 \geq \frac{1}{2}a\sigma^2(x_0^2 - s^2) - \bar{e}(x_0 + s)$. Substituting for $W_1$, the condition is equivalent to

$$f_1^+\left(\frac{s + x_0}{2}\right) = W_0 - (s\bar{e} - \frac{1}{2}a\sigma^2 s^2) + x_0(2a\sigma^2 s - \bar{e}) - \frac{5}{2}a\sigma^2 x_0^2 \geq 0 \tag{17}$$

If the polynomial in $x_0$ has no roots, which occurs if $4\delta_0^+ < 0$, i.e. if $W_0 < \omega^c$, there is no $x_0$ such that the arbitrageur holds a long position without violating the time 1 constraint.

Second, assuming that $W_0 \geq \omega^c$, we need to determine the position of $\hat{x}_0$ and $\hat{x}_0^-$ relative to $-s$. The inequality $\hat{x}_0 \geq -s$ is equivalent to $-\sqrt{\delta_0^+} \leq 7a\sigma^2 s - \bar{e}$. Hence, if $\rho < 7$, the right-hand side is positive, so the condition is satisfied, thus $\hat{x}_0 > -s$. If $\rho \geq 7$, then after raising both sides to the square and rearranging, the condition implies $W_0 \geq \hat{\omega}$ (defined in the proposition). Next, $\hat{x}_0^- \geq -s$ is equivalent to $\sqrt{\delta_0^+} \geq 7a\sigma^2 s - \bar{e}$. So if $\rho > 7$, the condition cannot be satisfied, because the right-hand side is negative. Instead, if $\rho \leq 7$, the condition implies $W_0 \geq \hat{\omega}$. To sum up,

$$\text{If } \rho < 7, \text{ then} \quad \hat{x}_0 \geq -s \quad \text{and} \quad \hat{x}_0^- \geq -s \quad \text{iff } W_0 \leq \hat{\omega}$$
$$\text{If } \rho \geq 7, \text{ then} \quad \hat{x}_0^- < -s \quad \text{and} \quad \hat{x}_0 \geq -s \quad \text{iff } W_0 \geq \hat{\omega}$$

Given that for all parameters, $\hat{\omega} > \omega^c$, we get:

- If $0 \leq \rho < 1$, $\mathcal{F}_0^{u_1+} = \begin{cases} [-s, -s\rho] & \text{if } W_0 < \omega^c \\ \left[\hat{x}_0^-, \hat{x}_0\right] & \text{if } \omega^c \leq W_0 < \hat{\omega} \\ [-s, \hat{x}_0] & \text{if } W_0 \geq \hat{\omega} \end{cases}$,
- If $1 \leq \rho < 7$, $\mathcal{F}_0^{u_1+} = \begin{cases} \emptyset & \text{if } W_0 < \omega^c \\ \left[\hat{x}_0^-, \hat{x}_0\right] & \text{if } \omega^c \leq W_0 < \hat{\omega} \\ [-s, \hat{x}_0] & \text{if } W_0 \geq \hat{\omega} \end{cases}$,
- If $\rho \geq 7$, $\mathcal{F}_0^{u_1+} = \begin{cases} \emptyset & \text{if } W_0 < \omega^c \\ \emptyset & \text{if } \omega^c \leq W_0 < \hat{\omega} \\ [-s, \hat{x}_0] & \text{if } W_0 \geq \hat{\omega} \end{cases}$

By union of $\mathcal{F}_0^{u_1-}$ and $\mathcal{F}_0^{u_1+}$, we get $\mathcal{F}_0^{u_1}$. $\blacksquare$

### C.2.3 Set $\mathcal{F}_0^{pw}$

**Lemma 5 (Set $\mathcal{F}_0^{pw}$)** *$\mathcal{F}_0^{pw} = [\underline{z}_0, \bar{z}_0]$, with*

$$\underline{z}_0 = \frac{a\sigma^2 s - \sqrt{2a\sigma^2 W_0 + a^2\sigma^4 s^2}}{2a\sigma^2} < 0, \quad \bar{z}_0 = \frac{a\sigma^2 s + \sqrt{2a\sigma^2 W_0 + a^2\sigma^4 s^2}}{2a\sigma^2} > 0 \tag{18}$$

**Proof.** Since $W_1 = W_0 + 2a\sigma^2 x_0(s - x_0)$ and $W_0 \geq 0$, $W_1 \geq 0$ requires that $x_0$ lies between $\underline{z}_0 < 0$ and $\bar{z}_0 > 0$. ■

### C.2.4 Intersection (set $\mathcal{F}_0^u$)

The set $\mathcal{F}_0^u$ is given by $\mathcal{F}_0^u = \mathcal{F}_0^0 \cap \mathcal{F}_0^{u_1} \cap \mathcal{F}_0^{pw}$. First note that:

**Lemma 6 ($\mathcal{F}_0^0$ is a subset of $\mathcal{F}_0^{pw}$)** *For any $W_0 \geq 0$, $\mathcal{F}_0^0 \subseteq \mathcal{F}_0^{pw}$.*

**Proof.** Using the definitions of $\underline{x}_0$ and $\underline{z}_0$, $\underline{x}_0 > \underline{z}_0$ is equivalent to $\bar{e} > \sqrt{d_0^-} - \sqrt{2a\sigma^2 W_0 + a^2\sigma^4 s^2}$. Given the definition of $d_0^-$, it is easy to see that the right-hand side is strictly positive. Raising both sides to the square thus preserves the order and gives after some simplificatoin

$$\sqrt{d_0^-(2a\sigma^2 W_0 + a^2\sigma^4 s^2)} > 2a\sigma^2 W_0 + a^2\sigma^4 s^2 + a\sigma^2 s\bar{e}$$

Raising both sides to the square again and simplifying, the condition becomes $3a^2\sigma^4 W_0^2 + 2a\sigma^2 W_0(a^2\sigma^4 s^2 + \bar{e}^2 + a\sigma^2 s\bar{e}) > 0$, which holds true, so $\underline{x}_0 > \underline{z}_0$.
Next, $\bar{x}_0 < \bar{z}_0$ is equivalent to $-\bar{e} + \sqrt{d_0^+} < \sqrt{2a\sigma^2 W_0 + a^2\sigma^4 s^2}$. Thei left-hand side is negative if $W_0 < \omega_0^p \equiv s\bar{e} - \frac{1}{2}a\sigma^2 s^2$, and the condition is thus satisfied. If $W_0 \geq \omega_0^p$, the condition implies after simplifying that $\bar{e} - a\sigma^2 s\sqrt{d_0^+}$. If $\rho < 1$, the left-hand side is negative, so the condition is satisfied. Otherwise, the condition boils down to $W_0 \geq 0$, which holds by assumption. ■

Lemma 6 implies that $\mathcal{F}_0^u = \mathcal{F}_0^0 \cap \mathcal{F}_0^{u_1}$. To determine the lower bound of $\mathcal{F}_0^u$, it is thus useful to place $\mathcal{F}_0^0$ relative to $-s$ and $-s\rho$.

**Corollary 4 (Position of $\mathcal{F}_0^0$ vs $-s$ and $-s\rho$)**

- *If $W_0 < \omega^* + \hat{\omega}$, then $\underline{x}_0 > -s$, i.e. $\mathcal{F}_0^0$ is to the right of $-s$.*

- *If $W_0 < \omega^* + \frac{4\bar{e}^2}{a\sigma^2}$, then $\underline{x}_0 > -s\rho$.*

**Proof.** From the definition (13), $\underline{x}_0 > -s$ is equivalent to $3a\sigma^2 + \bar{e} > \sqrt{d_0^-}$. Raising both sides to the square and rearranging the terms, the condition becomes $W_0 < 2s\bar{e} + 4a\sigma^2 s^2 = \omega^* + \hat{\omega}$. Similarly, $\underline{x}_0 > -s\rho$ is equivalent to $a\sigma^2 s + 3\bar{e} > \sqrt{d_0^-}$. Raising both sides to the square and rearranging terms gives the condition in the corollary. ■

**Corollary 5** *For any $W_0 \geq \hat{\omega}$, $\underline{x}_0 > \underline{\underline{x_0}}$, i.e. in this case the lower bound of $\mathcal{F}_0^u$ is $\underline{x}_0$.*

**Proof.** From the definitions of the thresholds, $\underline{x}_0 > \underline{\underline{x_0}}$ is equivalent to $5\sqrt{d_0^-} - 2\sqrt{\delta_0^-} < a\sigma^2 s + 3\bar{e}$. The left-hand side is negative only if $25d_0^- < 4\delta_0^-$. Substituting, the condition becomes $W_0 < \frac{3}{5}s\bar{e} + \frac{11}{10}a\sigma^2 s^2 - \frac{21}{10}\frac{\bar{e}}{a\sigma^2}$. However, for all parameters, $\hat{\omega} > \frac{3}{5}s\bar{e} + \frac{11}{10}a\sigma^2 s^2 - \frac{21}{10}\frac{\bar{e}}{a\sigma^2}$, thus $W_0 \geq \hat{\omega}$ implies $W_0 \geq \frac{3}{5}s\bar{e} + \frac{11}{10}a\sigma^2 s^2 - \frac{21}{10}\frac{\bar{e}}{a\sigma^2}$, which implies that $5\sqrt{d_0^-} - 2\sqrt{\delta_0^-} \geq 0$. Therefore, $\underline{x}_0 > \underline{\underline{x_0}}$ implies

$$\frac{1}{2}a\sigma^2 W_0 - \frac{3}{5}(a\sigma^2 s\bar{e} + a^2\sigma^4 s^2 - \bar{e}^2) < \sqrt{d_0^- \delta_0^-}$$

Then there are two cases. i) If $W_0 < \frac{6}{5}(a\sigma^2 s\bar{e} + a^2\sigma^4 s^2 - \bar{e}^2)$, the condition holds true, and $\underline{x}_0 > \underline{\underline{x_0}}$. ii) Or if $W_0 \geq \frac{6}{5}(a\sigma^2 s\bar{e} + a^2\sigma^4 s^2 - \bar{e}^2)$, $\underline{x}_0 > \underline{\underline{x_0}}$ requires

$$\begin{aligned}\frac{79}{4}W_0^2 + a\sigma^2 W_0 \left(10(a\sigma^2 s + \bar{e})^2 + 2\bar{e}^2 + 28a\sigma^2 s\bar{e} + 18a^2\sigma^4 s^2 + \frac{3}{5}(a\sigma^2 s\bar{e} + a^2\sigma^4 s^2 - \bar{e}^2)\right) \\ + (a\sigma^2 s + \bar{e})^2\left[\bar{e}^2 + 14a\sigma^2 s\bar{e} + 9a^2\sigma^4 s^2\right] - \frac{9}{25}(a\sigma^2 s\bar{e} + a^2\sigma^4 s^2 - \bar{e}^2)^2 > 0\end{aligned}$$

Inspection of the second term shows that it is positive for all parameters. Similarly, the last two terms can be written as

$$(a\sigma^2 s + \bar{e})^2\frac{216}{25}a^2\sigma^4 s^2 + (a\sigma^2 s + \bar{e})^2(\bar{e}^2 + 14a\sigma^2 s\bar{e}) - \frac{9}{25}\bar{e}^4 + \frac{18}{25}\bar{e}^2 a\sigma^2 s(a\sigma^2 s + \bar{e})$$

which is also positive. Hence, in this case as well, we get $\underline{x}_0 > \underline{\underline{x_0}}$. ■

Combining all the results, we obtain the set $\mathcal{F}_0^u$.

**Note:** I use the convention that if $a > b$, then $[a, b] = \emptyset$.

**Proposition 14 (Interval $\mathcal{F}_0^u$)**

- *If* $0 \leq \rho < 1$ *or if* $1 \leq \rho < 7$, *then* $\mathcal{F}_0^u = \begin{cases} \emptyset & \text{if } W_0 < \omega^c \\ \left[\max(\hat{x}_0^-, \underline{x}_0), \min(\hat{x}_0, \bar{x}_0)\right] & \text{if } \omega^c \leq W_0 < \hat{\omega} \\ \left[\underline{x}_0, \min(\hat{x}_0, \bar{x}_0)\right] & \text{if } \hat{\omega} \leq W_0 < \hat{\omega} + \omega^* \\ \left[\max(\hat{x}_0^-, \underline{x}_0), \min(\hat{x}_0, \bar{x}_0)\right] & \text{if } W_0 \geq \hat{\omega} + \omega^* \end{cases}$

- *If* $\rho \geq 7$, *then* $\mathcal{F}_0^u = \begin{cases} \emptyset & \text{if } W_0 < \omega^c \\ \emptyset & \text{if } \omega^c \leq W_0 < \hat{\omega} \\ \left[\underline{x}_0, \min(\hat{x}_0, \bar{x}_0)\right] & \text{if } \hat{\omega} \leq W_0 < \hat{\omega} + \omega^* \\ \left[\max(\hat{x}_0^-, \underline{x}_0), \min(\hat{x}_0, \bar{x}_0)\right] & \text{if } W_0 \geq \hat{\omega} + \omega^* \end{cases}$

**Proof.** The result follows from combining Lemma 3, Lemma 4, Lemma 5, Corollary 4, and Corollary 5. ■

It is possible to refine the characterization of the intersection as a function of primitives, but at a cost of increased analytical complexity. Therefore, I now turn to the sets of possible deviations.

### C.2.5 Possible deviations

**Lemma 7** *Suppose hedgers anticipate a slack constraint at time 1. Deviations from the arbitrageur leading to a downward-binding constraint at time 1 are either not feasible or dominated.*

**Proof.** Proposition 12 shows that there are two cases where the constraint binds downwards:

a. If $\rho \leq 1$, $-s < x_0 < -s\rho$, and $W_1 < 0$: this case cannot arise once we take into account the time-0 constraint of the arbitrageur, because of Lemma 6.

b. If $x_0 < -s$ and $0 \leq W_1 < \frac{1}{2}a\sigma^2(x_0^2 - s^2) - \bar{e}(x_0 + s)$. However, $x_0 < -s$ requires $W_0 \geq \omega^* + \hat{\omega}$ (Corollary 4). But as we will see below, for this wealth level, the unconstrained optimum $X_1^u$ is feasible, and thus the deviation leading to a downward-binding constraint is trivially

dominated. Indeed, when hedgers anticipate a slack constraint at time 1, deviations from the conjectured strategy are worth checking only if the unconstrained optimum is not feasible.

■

**Lemma 8** *Suppose hedgers anticipate a slack constraint at time 1. Deviations from the arbitrageur leading to an upward-binding constraint at time 1 must belong to the set $\mathcal{D}_0^{\bar{c}_1}$, given by*

- *If* $\rho < 1$, $\mathcal{D}_0^{\bar{c}_1} = \begin{cases} [\underline{x}_0, \bar{x}_0] & \text{if } W_0 < \omega^c \\ [\underline{x}_0, \max(\hat{x}_0, \bar{x}_0)) \cup (\min(\hat{x}_0, \bar{x}_0), \bar{x}_0] & \text{if } \omega^c \leq W_0 < \hat{\omega} \\ (\min(\hat{x}_0, \bar{x}_0), \bar{x}_0] & \text{if } \hat{\omega} \leq W_0 < \hat{\omega} + \frac{4\bar{e}^2}{a\sigma^2} \\ \left[\max(\underline{x}_0, -s\rho), \max(\underline{x}_0, -s\rho, \hat{x}_0^-)\right) \cup (\min(\hat{x}_0, \bar{x}_0), \bar{x}_0] & \text{if } W_0 \geq \hat{\omega} + \frac{4\bar{e}^2}{a\sigma^2} \end{cases}$

- *If* $1 \leq \rho < 7$, $\mathcal{D}_0^{\bar{c}_1} = \begin{cases} [\underline{x}_0, \bar{x}_0] & \text{if } W_0 < \omega^c \\ [\underline{x}_0, \max(\hat{x}_0, \bar{x}_0)) \cup (\min(\hat{x}_0, \bar{x}_0), \bar{x}_0] & \text{if } \omega^c \leq W_0 < \hat{\omega} \\ (\min(\hat{x}_0, \bar{x}_0), \bar{x}_0] & \text{if } \hat{\omega} \leq W_0 < \hat{\omega} + \omega^* \\ \left[-s, \max(-s, \hat{x}_0^-)\right) \cup (\min(\hat{x}_0, \bar{x}_0), \bar{x}_0] & \text{if } W_0 \geq \hat{\omega} + \omega^* \end{cases}$

- *If* $\rho \geq 7$, $\mathcal{D}_0^{\bar{c}_1} = \begin{cases} [\underline{x}_0, \bar{x}_0] & \text{if } W_0 < \omega^c \text{ or } \omega^c \leq W_0 < \hat{\omega} \\ (\min(\hat{x}_0, \bar{x}_0), \bar{x}_0] & \text{if } \hat{\omega} \leq W_0 < \hat{\omega} + \omega^* \\ \left[-s, \max(-s, \hat{x}_0^-)\right) \cup (\min(\hat{x}_0, \bar{x}_0), \bar{x}_0] & \text{if } W_0 \geq \hat{\omega} + \omega^* \end{cases}$

**Proof.** Proposition 12 shows that the constraint binds upwards in three cases:

- Either $W_1 < 0$ and $x_0 > -s\rho$: This case is not possible from the point of view of time 0 because $W_1 < 0 \Rightarrow x_0 \notin \mathcal{F}_0^0$ (Lemma 6).

- Or if $\rho < 1$, when $W_1 \geq 0$, $x_0 > -s\rho$ and $W_1 < \frac{1}{2}a\sigma^2(x_0^2 - s^2) + \bar{e}(x_0 + s)$. So,

$$\begin{aligned} \text{For } \rho < 1, \quad \mathcal{D}_0^{\bar{c}_1} &= \mathcal{F}_0^{pw} \cap [-s\rho, +\infty) \cap \mathcal{F}_0^0 \cap \overline{\mathcal{F}_0^{u_1}}, \quad \text{where } \overline{\mathcal{F}_0^{u_1}} \text{ is the complement of } \mathcal{F}_0^{u_1} \\ &= [-s\rho, +\infty) \cap \mathcal{F}_0^0 \cap \overline{\mathcal{F}_0^{u_1}} \end{aligned}$$

where the second equality follows from Lemma 6. Then using Lemma 4 and Corollary 4, we obtain the set given in the lemma.

- Or if $\rho \geq 1$, when $W_1 \geq 0$, $x_0 > -s$ and $W_1 < \frac{1}{2}a\sigma^2(x_0^2 - s^2) + \bar{e}(x_0 + s)$. So,

$$\text{For } \rho \geq 1, \quad \mathcal{D}_0^{\bar{c}_1} = \mathcal{F}_0^{pw} \cap [-s, +\infty) \cap \mathcal{F}_0^0 \cap \overline{\mathcal{F}_0^{u_1}} = [-s, +\infty) \cap \mathcal{F}_0^0 \cap \overline{\mathcal{F}_0^{u_1}}$$

As in the previous case, we use Lemma 4 and Corollary 4. Lemma 4 implies that we must consider separately the cases $1 \leq \rho < 7$ and $\rho \geq 7$.

■

### C.3 Step 3: Equilibrium determination

The last step of the proof consists of three parts. I first formally derive the candidate equilibrium strategy. Second, I collect and determine the order of the different relevant thresholds in terms of capital and risk benefit ratio. Third, for each case, I determine whether and /or under which conditions the arbitrageur does choose the conjectured equilibrium strategy.

**Value functions and candidate equilibrium strategy.** Given the results in Proposition 12, we can define the objective functions as follows:

$$\begin{aligned}
\tilde{\Omega}_0^{u_1,u_1}(x_0) =& W_0 + x_0\Delta_0^{u_1}(x_0) + \Omega_1^u(x_0) = W_0 + 3a\sigma^2 x_0(s - x_0) + \frac{a\sigma^2}{2}(s - x_0)^2 \\
\tilde{\Omega}_0^{u_1,\bar{c}_1}(x_0) =& W_0 + x_0\Delta_0^{u_1}(x_0) + \Omega_1^{\bar{c}}(x_0) = W_0 + x_0\Delta_0^{u_1}(x_0) + \bar{x}_1(x_0)\Delta_1^{\bar{c}}(x_0) \\
\tilde{\Omega}_0^{u_1,\underline{c}_1}(x_0) =& W_0 + x_0\Delta_0^{u_1}(x_0) + \Omega_1^{\underline{c}}(x_0) = W_0 + x_0\Delta_0^{u_1}(x_0) + \underline{x}_1(x_0)\Delta_1^{\underline{c}}(x_0)
\end{aligned}$$

**Proof.** The indirect utility at time 1 is $\Omega_1 = \max_{x_1} x_1\Delta_1$, subject to the time-1 financial constraint. When the constraint is slack,

$$\Omega_1^u = x_1^u\Delta_1^u = 2a\sigma^2 x_1^u(s - X_1^u)$$

Substituting $x_1^u = \frac{s-x_0}{2}$, we get $\Omega_1^u = \frac{a\sigma^2}{2}(s - x_0)^2$.

When the constraint binds upwards, $\Omega_1^{\bar{c}} = 2a\sigma^2\bar{x}_1(s - \bar{X}_1) = 2a\sigma^2(\bar{X}_1 - x_0)(s - \bar{X}_1)$. Similarly when the constraint binds downwards, $\Omega_1^{\underline{c}} = 2a\sigma^2(\underline{X}_1 - x_0)(s - \underline{X}_1)$. At time 0, working backwards,

and using $B_0 = B_{-1} + x_0\Delta_0 = W_0 + x_0\Delta_0$, we get $\Omega_0^{u_1,l} = W_0 + x_0\Delta_0^u + \Omega_1^l(x_0)$. ■

We can now determine under which conditions the arbitrageur's desired position satisfies the constraints at time 0 and time 1.

**Proposition 15 (Candidate equilibrium strategy with slack time-0 and -1 constraints $(\mathbf{u_0}, \mathbf{u_1})$)** *Let $\omega_0^u$ and $\omega_1^u$ denote two wealth thresholds, with $\omega_0^u = \frac{4}{5}s\bar{e} - \frac{12}{25}a\sigma^2 s^2$ and $\omega_1^u = \frac{7}{5}s\bar{e} - \frac{9}{10}a\sigma^2 s^2$, and let $\omega^u = \max(\omega_0^u, \omega_1^u)$.*

1. *If $W_0 \geq \omega^u$, then the arbitrageur can hold his preferred positions at time 0 and time 1 $x_0^{u_0,u_1} = \frac{2}{5}s$ and $X_1^{u_0,u_1} = \frac{7}{10}s$*
2. *The arbitrageur's expected utility is denoted $\Omega_0^{u_0,u_1} = \tilde{\Omega}_0^{u_1,u_1}(x_0^{u_0,u_1})$.*
3. *For any $x_0$ such that $\tilde{\Omega}_1^{\bar{c}}(x_0)$ exists, $\tilde{\Omega}_0^{u_1,\bar{c}_1}(x_0) \leq \tilde{\Omega}_0^{u_1,u_1}(x_0)$.*

**Proof.** The arbitrageur's objective function $\tilde{\Omega}_0^{u_1,u_1}$ admits a global maximum at $x_0^{u_0,u_1}$ given in the Proposition. Substituting $x_0^{u_0,u_1}$ into $X_1^u$ (Definition 2) gives $X_1^{u_0,u_1}$. Since $x_0^{u_0,u_1} > 0$, the relevant constraints are $f_0^+$ and $f_1^+$. Thus, to determine the thresholds $\omega_0^u$, $\omega_1^u$, it suffices to substitute $x_0^{u_0,u_1}$ into $f_0^+(x_0) \geq 0$ and $f_1^+\left(\frac{s+x_0}{2}\right) \geq 0$ and rearrange the terms. The fourth point follows from the fact that for any $x_0$ such that $\Omega_1^{\bar{c}}(x_0)$ exists, $\Omega_1^{\bar{c}}(x_0) \leq \Omega_1^u(x_0)$, and from the definition of $\tilde{\Omega}_0^{u_1,\bar{c}_1}$ and $\tilde{\Omega}_0^{u_1,u_1}$. ■

**Relevant wealth thresholds.** Given our analysis so far, we must order the following wealth thresholds: $\omega_0^u$, $\omega_1^u$, $\omega^c$, $\hat{\omega}$, $\hat{\omega} + \omega^*$ and $\hat{\omega} + \frac{4\bar{e}^2}{a\sigma^2}$. It is easy to see that $\hat{\omega} + \omega^*$ is larger than $\omega_0^u$, $\omega_1^u$, $\omega^c$, and $\hat{\omega}$. Similarly, when $\rho < 1$, $\hat{\omega} + \frac{4\bar{e}^2}{a\sigma^2}$ is larger than $\omega_0^u$, $\omega_1^u$, $\omega^c$, and $\hat{\omega}$.

**Lemma 9 (Wealth Threshold Ordering in Equilibrium with Slack Time-1 Constraint)** *The order of thresholds is given in Table 2.*

**Proof.** By direct calculation using threshold definition. ■

Table 2: Wealth Threshold Order for Equilibrium with Slack Time-1 Constraint

| Treshold | Greater than | Condition | Value |
|---|---|---|---|
| $\omega_0^u \equiv \frac{4}{5}s\bar{e} - \frac{12}{25}a\sigma^2 s^2$ | $\omega_1^u$ | $\rho < \frac{7}{10}$ | |
| | $\omega^c$ | $\rho < 3 - \frac{2}{5}\sqrt{30}$ or $\rho > 3 + \frac{2}{5}\sqrt{30}$ | 0.809 — 5.19 |
| | 0 | $\rho \geq \frac{3}{5}$ | |
| | $\hat{\omega}$ | $\rho > \frac{28}{5}$ | |
| $\omega_1^u \equiv \frac{7}{5}s\bar{e} - \frac{9}{10}a\sigma^2 s^2$ | $\omega^c$ | for all $\rho > 0$ | |
| | 0 | $\rho > \frac{9}{14}$ | 0.64 |
| | $\hat{\omega}$ | $\rho > \frac{7}{2}$ | |
| $\omega^c \equiv \omega_1^u - \frac{\bar{e}^2}{10a\sigma^2}$ | 0 | $7 - 2\sqrt{10} \leq \rho \leq 7 + 2\sqrt{10}$ | 0.675—13.32 |
| | $\hat{\omega}$ | never, equality for $\rho = 7$ | |
| $\hat{\omega} \equiv 4a\sigma^2 s^2$ | 0 | for any $\rho > 0$ | |

**Releveant $\rho$ thresholds.** The relevant thresholds from Table 2 and previous results are $\frac{3}{5}$, $\frac{9}{14}$, $7 - 2\sqrt{10}$, $\frac{7}{10}$,3- $\frac{2}{5}\sqrt{30}$, 1, $\frac{7}{2}$, $3 + \frac{2}{5}\sqrt{30}$, $\frac{28}{5}$, 7, $7 + 2\sqrt{10}$. If $\rho \geq 7 - 2\sqrt{10}$, wealth thresholds not necessarily positive but are always in the same order. Thus, for simplicity, I treat all the cases with $\rho < 7 - 2\sqrt{10}$ as one case. Similarly, I ignore the case $\rho > 7 + 2\sqrt{10}$, which determines the positivity of $\omega^c$, but does not affect the order of the thresholds. However, I add 1 and 7, which do not affect the order of tresholds, but affect the set of feasible positions or deviations.

Table 3: $\rho$ and Wealth Intervals for Equilibrium with Slack Time-1 Constraint

| Case | $\rho$ interval | Wealth ordering |
|---|---|---|
| 1 | $0 \leq \rho < \frac{7}{10}$ | $0 < (\omega^c, 0)^+ < (\omega_1^u, 0)^+ < (\omega_0^u, 0)^+ < \hat{\omega}$ |
| 2 | $\frac{7}{10} \leq \rho < 3 - \frac{2}{5}\sqrt{30}$ | $0 < \omega^c < \omega_0^u < \omega_1^u < \hat{\omega}$ |
| 3 | $3 - \frac{2}{5}\sqrt{30} \leq \rho < 1$ | $0 < \omega_0^u < \omega^c < \omega_1^u < \hat{\omega}$ |
| 4 | $1 \leq \rho < \frac{7}{2}$ | $0 < \omega_0^u < \omega^c < \omega_1^u < \hat{\omega}$ |
| 5 | $\frac{7}{2} \leq \rho < 3 + \frac{2}{5}\sqrt{30}$ | $\bar{0} < \omega_0^u < \omega^c < \hat{\omega} < \omega_1^u$ |
| 6 | $3 + \frac{2}{5}\sqrt{30} \leq \rho < \frac{28}{5}$ | $0 < \omega^c < \omega_0^u < \hat{\omega} < \omega_1^u$ |
| 7 | $\frac{28}{5} \leq \rho < 7$ | $0 < \omega^c < \hat{\omega} < \omega_0^u < \omega_1^u$ |
| 8 | $7 \leq \rho$ | $0 < \omega^c < \hat{\omega} < \omega_0^u < \omega_1^u$ |

**Equilibrium with slack time-1 constraint (Proposition 13).** [39]

1. If $\rho < \frac{7}{10}$, then $0 < (\omega^c, 0)^+ < (\omega_1^u, 0)^+ < (\omega_0^u, 0)^+ < \hat{\omega}$

   (a) For $0 \leq W_0 < (\omega^c, 0)^+$: There is no equilibrium with a slack constraint at time 1

[39] For brevity, I omit the ˜ on objective functions $\tilde{\Omega}_0^{u_1,l}$ in the proof.

(abbreviated **no $\mathbf{u_1}$**).

**Proof.** This is because for $W_0 < \omega^c$, $\mathcal{F}_0^u = \emptyset$ (Proposition 14). ■

(b) For $(\omega^c, 0)^+ \leq W_0 < (\omega_1^u, 0)^+$:

- If $\Omega_0^{u_1,u_1}(x_0^{c_0,u_1}) > \Omega_0^{u_1,\bar{c}_1}(x_0^{d*})$, then in equilibrium (abbreviated $\mathbf{c_0}, \mathbf{u_1}$), the constraint binds at time 0 but not at time 1, and the arbitrageur holds $x_0^{c_0,u_1} = \min(\hat{x}_0, \bar{x}_0)$, and $X_1^{c_0,u_1} = X_1^u(x_0^{c_0,u_1})$.
- Otherwise, no $u_1$.

**Proof.** If the arbitrageur trades the optimum $x_0^{u_0,u_1}$, then constraints bind upwards at time 0 and time 1: $0 < x_0^{u_0,u_1} < \min(\hat{x}_0, \bar{x}_0)$. The value function $\Omega_0^{u_1,u_1}$ is increasing for $x_0 < x_0^{u_0,u_1}$, so if the arbitrageur is willing to remain unconstrained at time 1, it is optimal to buy the largest amount such that the constraints are satisfied at time 0 and time 1, i.e. it is optimal to buy $x_0^{c_0,u_1} = \min(\hat{x}_0, \bar{x}_0)$. The alternative for the arbitrageur is between sticking to the conjectured strategy ($u_1$), but buy the constrained optimum $x_0^{c_0,u_1}$ and deviating from the conjectured strategy. The optimal deviation is $x_0^{d*}$, defined as $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^{\bar{c}_1}} \Omega_0^{u_1,\bar{c}_1}(x_0)$. If $\Omega_0^{u_1,u_1}(x_0^{c_0,u_1}) > \Omega_0^{u_1,\bar{c}_1}(x_0^{d*})$, then the arbitrageur sticks to the conjectured strategy. Otherwise, he has incentives to deviate, and thus there is no equilibrium in which the constraint is slack at time 1 in this parameter region. Note that analytical expressions exist for $x_0^{d*}$ and $\Omega_0^{u_1,\bar{c}_1}(x_0^{d*})$, but these expressions are not easily tractable. This is because $x_0^{d*}$ is a solution to a quartic equation. ■

(c) If $(0, \omega_1^u)^+ \leq W_0 < (\omega_0^u, 0)^+$, then the equilibrium is $\mathbf{c_0}, \mathbf{u_1}$, with $x_0^{c_0,u_1} = \bar{x}_0$ and $X_1^{c_0,u_1} = X_1^u(\bar{x}_0)$.

**Proof.** In this parameter region, only the time-0 constraint binds (upwards), i.e. $\bar{x}_0 < x_0^{u_0,u_1} < \hat{x}_0$. Since the arbitrageur's desired position is not feasible, we need to check for deviations. From Lemma 8, the deviation set is $[\underline{x}_0, \max(\hat{x}_0, \bar{x}_0)) \cup (\min(\hat{x}_0, \bar{x}_0), \bar{x}_0]$. First, note that any deviation belonging to $[\underline{x}_0, \max(\hat{x}_0, \bar{x}_0))$ is dominated, since by construction for any feasible $x_0$ $\Omega_0^{u_1,u_1}(x_0) \geq \Omega_0^{u_1,\bar{c}_1}(x_0)$ and given that $\Omega_0^{u_1,u_1}$ is increasing for $x_0 < x_0^{u_0,u_1}$, we have $\Omega_0^{u_1,u_1}(\bar{x}_0) \geq \Omega_0^{u_1,u_1}(x_0) \geq \Omega_0^{u_1,\bar{c}_1}(x_0)$ for any $x_0 < \bar{x}_0$. Second, deviations belonging to $(\min(\hat{x}_0, \bar{x}_0), \bar{x}_0]$ are not feasible, since this interval is empty

$(\bar{x}_0 < \hat{x}_0)$. Hence, it is optimal for the arbitrageur to hold $\bar{x}_0$ at time 0, which leads to a slack constraint at time 1. ∎

(d) If $(0, \omega_0^u)^+ \leq W_0 < \hat{\omega}$, or $W_0 \geq \hat{\omega}$, then the equilibrium is the unconstrained equilibrium of Proposition 15 (abbreviated $\mathbf{u_0}, \mathbf{u_1}$).

**Proof.** In this case, both constraints are slack, and any deviation for $x_0^{u_0,u_1}$ will decrease the time 1 payoff without changing the liquidity at time 0, i.e. without increasing trading profits at time 0. That is, for all $x_0 \in \bar{\mathcal{F}}_0^{u_1}$, by construction we have $\Omega_0^{u_0,u_1} > \Omega_0^{u_1,u_1}(x_0) \geq \Omega_0^{u_1,\bar{c}_1}(x_0)$. ∎

2. If $\frac{7}{10} \leq \rho < 3 - \frac{2}{5}\sqrt{30}$, then $0 < \omega^c < \omega_0^u < \omega_1^u < \hat{\omega}$.

   (a) If $0 \leq W_0 < \omega^c$: **no** $\mathbf{u_1}$ (same as 1a). (b) If $\omega^c \leq W_0 < \omega_0^u$: same as 1b. (c) If $\omega_0^u \leq W_0 < \omega_1^u$, then the equilibrium is $\mathbf{c_0}, \mathbf{u_1}$, with $x_0^{c_0,u_1} = \hat{x}_0$ and $X_1^{c_0,u_1} = X_1^u(\hat{x}_0)$ iff $\Omega_0^{u_1,u_1}(\hat{x}_0) \geq \Omega_0^{u_1,\bar{c}_1}(x_0^{d*})$, where $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^{\bar{c}_1}} \Omega_0^{u_1,\bar{c}_1}(x_0)$.

   **Proof.** In this parameter region, unlike in 1c, $\hat{x}_0 < x_0^{u_0,u_1} < \bar{x}_0$. The deviation set is the same as in 1c. Deviations consisting in buying less are still dominated, as in 1c. However, it is now possible to deviate by buying more than $\hat{x}_0$. Let $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^{\bar{c}_1}} \Omega_0^{u_1,\bar{c}_1}(x_0)$ denote the deviation yielding the highest utility. If $\Omega_0^{u_1,u_1}(\hat{x}_0) \geq \Omega_0^{u_1,\bar{c}_1}(x_0^{d*})$, then in equilibrium, the arbitrageur holds $\hat{x}_0$ and remain unconstrained at time 1. ∎

   (d) $\omega_1^u \leq W_0 < \hat{\omega}$ or $W_0 \geq \hat{\omega}$: $\mathbf{u_0}, \mathbf{u_1}$ (same as 1d).

3. If $3 - \frac{2}{5}\sqrt{30} \leq \rho < 1$, then $0 < \omega_0^u < \omega^c < \omega_1^u < \hat{\omega}$

   (a) If $0 \leq W_0 < \omega_0^u$ or $\omega_0^u \leq W_0 < \omega^c$: **no** $\mathbf{u_1}$ (same as 1a). (b) If $\omega^c \leq W_0 < \omega_1^u$: same as 2c. (c) If $\omega_1^u \leq W_0 < \hat{\omega}$ or $W_0 \geq \hat{\omega}$: $\mathbf{u_0}, \mathbf{u_1}$ (same as 1d).

4. If $1 \leq \rho < \frac{7}{2}$, then $0 < \omega_0^u < \omega^c < \omega_1^u < \hat{\omega}$: this is the same as case 3.

   **Proof.** The order of wealth thresholds is the same. The only difference is in the deviation set. However, what changes is the set for deviations consisting in buying, which are dominated anyway. Thus, the equilibrium is the same as in 3. ∎

5. If $\frac{7}{2} \leq \rho < 3 + \frac{2}{5}\sqrt{30}$, then $0 < \omega_0^u < \omega^c < \hat{\omega} < \omega_1^u$

   (a) If $0 \leq W_0 < \omega_0^u$ or $\omega_0^u \leq W_0 < \omega^c$: **no** $\mathbf{u_1}$ (same as 1a). (b) If $\omega^c \leq W_0 < \hat{\omega}$ or

$\hat{\omega} \le W_0 < \omega_1^u$: same as 2c. (c) If $\omega_1^u \le W_0$: $\mathbf{u_0}, \mathbf{u_1}$ (same as 1d).

6. If $3 + \frac{2}{5}\sqrt{30} \le \rho < \frac{28}{5}$, then $0 < \omega^c < \omega_0^u < \hat{\omega} < \omega_1^u$

   (a) If $0 \le W_0 < \omega^c$: **no** $\mathbf{u_1}$ (same as 1a). (b) If $\omega^c \le W_0 < \omega_0^u$: same as 1b. (c) If $\omega_0^u \le W_0 < \hat{\omega}$ or $\hat{\omega} \le W_0 < \omega_1^u$: same as 2c. (d) If $\omega_1^u \le W_0$: $\mathbf{u_0}, \mathbf{u_1}$ (same as 1d).

7. If $\frac{28}{5} \le \rho < 7$, then $0 < \omega^c < \hat{\omega} < \omega_0^u < \omega_1^u$

   (a) If $0 \le W_0 < \omega^c$: **no** $\mathbf{u_1}$ (same as 1a). (b) If $\omega^c \le W_0 < \hat{\omega}$ or $\hat{\omega} \le W_0 \omega_0^u$: same as 1b. (c) If $\omega_0^u \le W_0 < \omega_1^u$: same as 2c. (d) If $\omega_1^u \le W_0$: $\mathbf{u_0}, \mathbf{u_1}$ (same as 1d).

8. If $7 \le \rho$, then $0 < \omega^c < \hat{\omega} < \omega_0^u < \omega_1^u$

   (a) If $0 \le W_0 < \omega^c$ or $\omega^c \le W_0 < \hat{\omega}$: **no** $\mathbf{u_1}$ (same as 1a).

   **Proof.** The second case, $\omega^c \le W_0 < \hat{\omega}$, is new. The result follows from the fact that in this region, $\mathcal{F}_0^u = \emptyset$ (Proposition 14). ■

   (b) If $\hat{\omega} \le W_0 \omega_0^u$: same as 1b. (c) If $\omega_0^u \le W_0 < \omega_1^u$: same as 2c. (d) If $\omega_1^u \le W_0$: $\mathbf{u_0}, \mathbf{u_1}$ (same as 1d).

■

### C.4 Equilibrium spreads

**Corollary 6 (Equilibrium spreads in the $u_1$ equilibria)**

*In the unconstrained equilibrium (first region), equilibrium spreads are*

$$\Delta_0^{u_0,u_1} = \frac{9}{5} a\sigma^2 s, \quad \textit{and} \quad \Delta_1^{u_0,u_1} = \frac{3}{5} a\sigma^2 s \tag{19}$$

*In the partly constrained $c_0, u_1$ equilibrium (second and third regions), equilibrium spreads are*

$$\Delta_0^{c_0,u_1} = 3a\sigma^2(s - x_0^{c_0,u_1}), \quad \textit{and} \quad \Delta_1^{c_0,u_1} = 2a\sigma^2(s - X_1^u(x_0^{c_0,u_1})) = a\sigma^2(s - x_0^{c_0,u_1}) \tag{20}$$

**Proof.** Equilibrium spreads follow from substituting the equilibrium quantity (either $x_0^{u_0,u_1}$ or $x_0^{c_0,u_1}$) into the spreads schedule (11)-(10). ■

# D Equilibrium with binding time-1 constraint

I now consider equilibria with a binding constraint at time 1. The constraints may be upward-binding or downward-binding. However, I first show that it is possible to rule out equilibria with downward-bing constraint at time 1. Then, I derive the set of positions at time 0 leading to an upward-binding constraint at time 1, and the set of deviations the arbitrageur may consider. The exact form of these sets depends on the amount of initial capital $W_0$ and the ratio $\rho$. Then for each case, I determine the equilibrium or the conditions for the equilibrium.

Here is the full result:

**Proposition 16 (Equilibria with binding time-1 constraint)**

- *There are no equilibria in which the arbitrageur's constraint binds downwards at time 1.*
- *There are equilibria in which the arbitrageur's constraint binds upwards at time 1, as follows. Let $\omega_0^p \equiv s\bar{e} - \frac{1}{2}a\sigma^2 s^2$ and $\omega_1^p \equiv \frac{3}{2}s\bar{e} - \frac{7}{8}a\sigma^2 s^2$ denote two thresholds.*

  1. *If $0 \leq \rho < \frac{3}{4}$, then $\omega_1^p < \omega_0^p$, and there are three regions in terms of arbitrage capital:*
     (a) *In the first region, with $0 \leq W_0 < \max(0, \omega^f)$, the arbitrageur's constraint binds upwards at time 0 and time 1 in equilibrium ($\mathbf{c_0}, \mathbf{c_1}$ equilibrium). This equilibrium is the same as in the constrained competitive case, for a given level of capital. The arbitrageur holds $x_0^{c_0,c_1} = \bar{x}_0$, and $X_1^{c_0,c_1} = \bar{X}_1(\bar{x}_0)$.*
     (b) *In the second region, with $\max(0, \omega^f) \leq W_0 < \max(0, \omega_1^p)$, there are two cases*
        i. *If $\max(\hat{x}_0, \bar{x}_0) = \hat{x}_0$, there is no equilibrium in which the arbitrageur's constraint binds upwards at time 1 (**no** $\mathbf{c_1}$).*
        ii. *Otherwise, both constraints bind in equilibrium as in (a) iff $\Omega_0^{c_0,c_1} \geq \Omega_0^{\bar{c}_1,u_1}(x_0^{d*})$, where $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^u} \Omega_0^{\bar{c}_1,u_1}(x_0)$.*
     (c) *In the third region, with $\max(0, \omega_1^p) \leq W_0 < \omega_0^p$ or $\omega_0^p \leq W_0$, there is no equilibrium in which the arbitrageur's constraint binds upwards at time 1 (**no** $\mathbf{c_1}$).*
  2. *If $\rho \geq \frac{3}{4}$, then $\omega_1^p > \omega_0^p$, and there are four regions in terms of arbitrage capital:*
     (a) *In the first region, with $0 \leq W_0 < \omega^f$, the equilibrium is $\mathbf{c_0}, \mathbf{c_1}$, as in case 1a.*

(b) *In the second region, with* $\omega^f \le W_0 < \omega_0^p$*, the equilibrium is the same as in 1b.*

(c) *In the third region, with* $\omega_0^p \le W_0 < \omega_1^p$*, there is an equilibrium in which the abritrageur's constraint binds upwards at time 1 and is slack at time 0 (*$\mathbf{u_0}, \mathbf{c_1}$ *equilibrium) iff* $\Omega_0^{u_0,c_1} \ge \Omega_0^{\bar{c}_1,u_1}(x_0^{d*})$*, where* $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^u} \Omega_0^{\bar{c}_1,u_1}(x_0)$*.*

(d) *In the fourth region, with* $\omega_1^p \le W_0$*, is no equilibrium in which the arbitrageur's constraint binds upwards at time 1 (**no*** $\mathbf{c_1}$*), as in 1c.*

## D.1 Arbitrageur's problem

Suppose hedgers anticipate an upward-binding constraint at time 1. Let $\Omega_0^{\bar{c}_1,l}$ denote the arbitrageur's expected utility when hedgers anticipate an upward-binding constraint, and the arbitrageur chooses a trade $x_0$ leading to state $l \in \{\bar{c}_1, u_1, \underline{c}_1\}$ at time 1, i.e. an upward-binding, slack, or downward-binding constraint at time 1. The maximization problem of the arbitrageur is thus as follows.

$$(\mathcal{P}^{\bar{c}_1}): \quad \Omega_0^{\bar{c}_1} = \max_{l \in \{\bar{c}_1, u_1, \underline{c}_1\}} \Omega_0^{\bar{c}_1,l} \tag{21}$$

The expected utilities associated with state $l$ are defined as follows:

$$\begin{aligned}
\Omega_0^{\bar{c}_1,\bar{c}_1} &= \max_{x_0} \tilde{\Omega}_0^{\bar{c}_1,\bar{c}_1} = W_0 + x_0 \Delta_0^{\bar{c}_1}(x_0) + \Omega_1^{\bar{c}}(x_0) \\
&\text{s.t.} \quad f_0^+(x_0)\ \mathbb{1}_{x_0 \ge 0} + f_0^-(x_0)\ \mathbb{1}_{x_0 < 0} \ge 0 \\
&\qquad\ f_1^+\left(\frac{s + x_0}{2}\right)\ \mathbb{1}_{x_0 \ge -s} + f_1^-\left(\frac{s + x_0}{2}\right)\ \mathbb{1}_{x_0 < -s} < 0 \\
&\qquad\ W_1 = W_0 + 2a\sigma^2 x_0 (s - x_0) \ge 0
\end{aligned}$$

where $\Omega_1^{\bar{c}}(x_0) = 2a\sigma^2 \bar{x}_1(x_0)(s - \bar{X}_1(x_0))$. The third and fourth constraints ensure that in equilibrium, the arbitrageur cannot hold his desired position because his constraint binds upwards at time 1. The last constraint ensures that equilibrium wealth is positive at time 1, which is required by Proposition 12. Next, I consider the expected utlity from deviations leading to a slack constraint

at time 1.

$$\Omega_0^{\bar{c}_1,u_1} = \max_{x_0} \tilde{\Omega}_0^{\bar{c}_1,u_1} = W_0 + x_0 \Delta_0^{\bar{c}_1}(x_0) + \Omega_1^u(x_0)$$

$$\text{s.t.} \quad f_0^+(x_0)\ \mathbb{1}_{x_0 \geq 0} + f_0^-(x_0)\ \mathbb{1}_{x_0<0} \geq 0$$

$$f_1^+\left(\frac{s+x_0}{2}\right)\ \mathbb{1}_{x_0 \geq -s} + f_1^-\left(\frac{s+x_0}{2}\right)\ \mathbb{1}_{x_0<-s} \geq 0$$

$$W_1 = W_0 + 2a\sigma^2 x_0(s - x_0) \geq 0$$

The difference with the previous problem is that the constraints at time 1 are slack, leading to continuation value $\Omega_1^u$ at time 1. Finally, here is the expected utlity from deviations leading to a downward-binding constraint at time 1.

$$\Omega_0^{\bar{c}_1,u_1} = \max_{x_0 \in [-s,-s\rho[\cup]-\infty,-s]} \tilde{\Omega}_0^{\bar{c}_1,\underline{c}_1} = W_0 + x_0 \Delta_0^{\bar{c}_1}(x_0) + \Omega_1^{\underline{c}}(x_0)$$

$$\text{s.t.} \quad f_0^+(x_0)\ \mathbb{1}_{x_0 \geq 0} + f_0^-(x_0)\ \mathbb{1}_{x_0<0} \geq 0$$

$$f_1^+\left(\frac{s+x_0}{2}\right)\ \mathbb{1}_{x_0 \geq -s} + f_1^-\left(\frac{s+x_0}{2}\right)\ \mathbb{1}_{x_0<-s} < 0$$

$$W_1(x_0) < 0 \quad \text{if } x_0 \in [-s, -s\rho[, \text{ or,}$$

$$0 \leq W_1(x_0) < \frac{1}{2}a\sigma^2(x_0^2 - s^2) - \bar{e}(x_0 + s) \quad \text{if } x_0 < -s$$

**Ruling out equilibria with a downward-binding constraint**

**Lemma 10** *There is no equilibrium with a downward-binding constraint at time 1.*

**Proof.** From Proposition 12, we know that the constraint binds downwards at time 1 in two cases:

a) If $-s < x_0 < -s\rho$ and $W_1 < 0$.

b) If $x_0 < -s$ and $0 \leq W_1 < \frac{1}{2}a\sigma^2(x_0^2 - s^2) - \bar{e}(x_0 + s)$.

Case a cannot arise in equilibrium, because $W_1 < 0$ and $x_0 \in \mathcal{F}_0^0$ cannot occur simultaneously (Lemma 6). Case b requires $x_0 < -s$. However, if we assume that hedgers anticipate a downward-binding constraint, we can show that the arbitrageur always has an incentive to deviate.

When the constraint binds downwards, the arbitrageur's position is $\underline{X}_1 = \frac{a\sigma^2 s + \bar{e} - \sqrt{d_1^-}}{2a\sigma^2}$, with $d_1^- =$

$2a\sigma^2 W_1 + (a\sigma^2 s + \bar{e})^2$ (Definition 2). As a result, using equation (10), the equilibrium spread in the time-1 subgame is $\Delta_1^{\underline{c}} = 2a\sigma^2(s - \underline{X}_1) = a\sigma^2 s - \bar{e} + \sqrt{d_1^-}$. Thus, using equation (11), the time-0 spread schedule is

$$\Delta_0^{\underline{c}_1}(x_0) = 2a\sigma^2(s - x_0) + a\sigma^2 s - \bar{e} + \sqrt{d_1^-(x_0)}$$

If hedgers anticipate a downward-binding constraint at time 1, the arbitrageur's objective function at time 0 is (I drop the dependence of $d_1^-$ to $x_0$ for ease of notation)

$$\begin{aligned}
\Omega_0^{\underline{c}_1,\underline{c}_1}(x_0) &= W_0 + x_0\Delta_0^{\underline{c}_1}(x_0) + \underline{x}_1\Delta_1^{\underline{c}} \\
&= W_0 + 2a\sigma^2 x_0(s - x_0) + x_0\Delta_1^{\underline{c}} + (\underline{X}_1 - x_0)\Delta_1^{\underline{c}} \\
&= \underbrace{W_0 + 2a\sigma^2 x_0(s - x_0)}_{W_1} + \frac{\left(a\sigma^2 s + \bar{e} - \sqrt{d_1^-}\right)\left(a\sigma^2 s - \bar{e} + \sqrt{d_1^-}\right)}{2a\sigma^2}
\end{aligned}$$

Then, developing the numerator in the fraction and substituting for $d_1^-$ yields:

$$\Omega_0^{\underline{c}_1,\underline{c}_1}(x_0) = \frac{\bar{e}}{a\sigma^2}\left[\sqrt{d_1^-(x_0)} - a\sigma^2 s - \bar{e}\right]$$

Thus maximizing $\Omega_0^{\underline{c}_1,\underline{c}_1}$ with respect to $x_0$ boils down to maximizing $d_1^-$ with respect to $x_0$, which itself amounts to maximizing $W_1$. Thus,

$$x_0^{u_0,\underline{c}_1} = \arg\max W_1(x_0) = \frac{s}{2}$$

Hence, the arbitrageur's preferred position does not satisfy the requirement that $x_0 < -s$. Since $\Omega_0^{\underline{c}_1,\underline{c}_1}$ is increasing for $x_0 \leq \frac{s}{2}$, the constrained solution to the arbitrageur's problem is $x_0^{c_0,\underline{c}_1} = -s - \eta$, where $\eta$ is arbitrarily small and positive. However, at this constrained solution, it is always the case that deviating by buying more to make the constraint slack is optimal. The reason is that for any $x_0 \in \mathcal{F}_0^0$, $\Omega_0^{\underline{c}_1,\underline{c}_1}(x_0) \leq \Omega_0^{\underline{c}_1,u_1}(x_0)$, since deviating to keep the constraint slack at time 1 increases the time 1 payoff, while keeping the same price schedule at time 0. Therefore the candidate equilibrium strategy $x_0^{c_0,\underline{c}_1}$ is always dominated and cannot arise in equilibrium. ■

## D.2 Set of feasible positions and possible deviations

**Lemma 11** *Let $\mathcal{F}_0^{\bar{c}_1}$ denote the set of feasible positions with upward-binding constraint at time 1, and $\mathcal{D}_0^{u_1}$ the set of deviations leading to a slack constraint at time 1. We have:*

- $\mathcal{F}_0^{\bar{c}_1} = \mathcal{D}_0^{\bar{c}_1}$
- $\mathcal{D}_0^{u_1} = \mathcal{F}_0^{u}$

**Proof.** From Proposition 12, there are three cases in which the constraint binds upwards at time 1.

a) If $\bar{W}_1^+ \leq W_1 < 0$ and $x_0 > -s\rho$.

b) When $\rho \geq 1$, if $0 \leq W_1 < \frac{1}{2}a\sigma^2(x_0^2 - s^2) - \bar{e}(x_0 + s)$, and $x_0 \geq -s$.

c) When $\rho < 1$, if $0 \leq W_1 < \frac{1}{2}a\sigma^2(x_0^2 - s^2) - \bar{e}(x_0 + s)$ and $x_0 \geq -s\rho$.

For any set $\mathcal{A}$, let $\bar{\mathcal{A}}$ denote its complement. Case a does not materialize in equilibrium, because $W_1 < 0$ implies $x_0 \notin \mathcal{F}_0^0$ (Lemma 6). Given cases b and c, we have

$$\mathcal{F}_0^{\bar{c}_1} = \begin{cases} \mathcal{F}_0^{pw} \cup \overline{\mathcal{F}_0^{u_1}} \cup \mathcal{F}_0^0 \cup [-s, +\infty) & \text{if } \rho \geq 1 \\ \mathcal{F}_0^{pw} \cup \overline{\mathcal{F}_0^{u_1}} \cup \mathcal{F}_0^0 \cup [-s\rho, +\infty) & \text{if } \rho < 1 \end{cases}$$

This coincides with the definition of $\mathcal{D}_0^{\bar{c}_1}$ given in the proof of Lemma 8.

The set of positions leading to slack constraint at time 1 is defined as $\mathcal{D}_0^{u_1} = \mathcal{F}_0^0 \cup \mathcal{F}_0^{pw} \mathcal{F}_0^{u_1}$, i.e. to deviate from the conjectured strategy, the arbitrageur takes a position that satisfies wealth positivity, the time-0 constraint, and makes the time-1 constraint slack. This coincides with the definition of $\mathcal{F}_0^u$. ■

## D.3 Equilibrium determination

**Value functions.** I assume that hedgers anticipate an upward-binding constraint at time 1. We have ruled out equilibria with downward-binding constraints. The value functions in the remaining

cases $(l = \{\bar{c}_1, u_1, \})$ are re

$$\Omega_0^{\bar{c}_1,\bar{c}_1}(x_0) = W_0 + x_0\Delta_0^{\bar{c}_1}(x_0) + \Omega_1^{\bar{c}}(x_0) = W_0 + x_0\Delta_0^{\bar{c}_1}(x_0) + \bar{x}_1(x_0)\Delta_1^{\bar{c}}(x_0)$$

$$\Omega_0^{\bar{c}_1,u_1}(x_0) = W_0 + x_0\Delta_0^{\bar{c}_1}(x_0) + \Omega_1^{u}(x_0) = W_0 + x_0\Delta_0^{\bar{c}_1}(x_0) + x_1^{u}(x_0)\Delta_1^{u}(x_0)$$

**Proposition 17 (Candidate equilibrium strategy - slack time-0 and binding time-1 constraints)** *Let $\omega_0^p \equiv s\bar{e} - \frac{1}{2}a\sigma^2 s^2$ and $\omega_1^p \equiv \frac{3}{2}s\bar{e} - \frac{7}{8}a\sigma^2 s^2$ denote two wealth thresholds.*

1. *The function $\tilde{\Omega}_0^{\bar{c}_1,\bar{c}_1}$ admits a maximum $x_0^{u_0,c_1}$ iff $W_0 \in [\omega_0^p, \omega_1^p)$, where*

$$x_0^{u_0,c_1} = \frac{s}{2}, \quad X_1^{u_0,c_1} = \bar{X}_1(x_0^{u_0,c_1})$$

2. *The interval $[\omega_0^p, \omega_1^p)$ is non-empty iff $\rho > \frac{3}{4}$.*

3. *At this candidate equilibrium strategy, the arbitrageur's utility is*

$$\tilde{\Omega}_0^{\bar{c}_1,\bar{c}_1}(x_0^{u_0,c_1}) \equiv \Omega_0^{u_0,c_1} = \frac{\bar{e}}{a\sigma^2}\left[a\sigma^2 s - \bar{e} + \sqrt{d_1^+(x_0^{u_0,c_1})}\right],$$

$$\text{where } d_1^+(x_0^{u_0,c_1}) = 2a\sigma^2 W_0 + 2a\sigma^2 s^2 + \bar{e}^2 - 2a\sigma^2 s\bar{e}$$

4. *For any $x_0$ such that $\tilde{\Omega}_1^{\bar{c}}(x_0)$ exists, $\tilde{\Omega}_0^{\bar{c}_1,\bar{c}_1}(x_0) \le \tilde{\Omega}_0^{\bar{c}_1,u_1}(x_0)$.*

**Proof.** Let's first rewrite the objective function $\tilde{\Omega}_0^{\bar{c}_1,\bar{c}_1}$ by substituting for $\Delta_0^{\bar{c}_1}$ and $\Omega_1^{\bar{c}}$.

$$\begin{aligned}
W_0 + x_0\Delta_0^{\bar{c}_1}(x_0) + \Omega_1^{\bar{c}}(x_0) &= W_0 + 2a\sigma^2 x_0(s - x_0) + x_0\Delta_1^{\bar{c}}(x_0) + (\bar{X}_1(x_0) - x_0)\Delta_1^{\bar{c}}(x_0) \\
&= W_0 + 2a\sigma^2 x_0(s - x_0) + \bar{X}_1(x_0)\Delta_1^{\bar{c}}(x_0) \\
&= \underbrace{W_0 + 2a\sigma^2 x_0(s - x_0)}_{W_1(x_0)} + \frac{\left(a\sigma^2 s + \bar{e} - \sqrt{d_1^+(x_0)}\right)\left(a\sigma^2 s - \bar{e} + \sqrt{d_1^+(x_0)}\right)}{2a\sigma^2}
\end{aligned}$$

The last line follows from substituting for $\bar{X}_1$ and $\Delta_1^{\bar{c}}$. Then developing the numerator in the last term, substituting for $d_1^+$ and simplifying, we get:

$$W_0 + x_0\Delta_0^{\bar{c}_1}(x_0) + \Omega_1^{\bar{c}}(x_0) = \frac{\bar{e}}{a\sigma^2}\left[a\sigma^2 s - \bar{e} + \sqrt{d_1^+(x_0)}\right]$$

Therefore maximizing $\tilde{\Omega}_0^{\bar{c}_1,\bar{c}_1}$ is equivalent to maximizing $d_1^+$ subject to the constraint, which boils down to maximizing $W_1(x_0) = W_0 + 2a\sigma^2 x_0(s - x_0)$, subject to constraints. The solution is $x_0^{u_0,c_1} = \frac{s}{2}$ if $f_0^+(\frac{s}{2}) \geq 0$ and $f_1^+(\frac{3s}{4}) < 0$. The first condition requires $W_0 \geq \omega_0^p$, and the second $W_0 < \omega_1^p$.

These conditions define a non-empty interval iff $\omega_0^p < \omega_1^p$, which is equivalent to $\rho > \frac{3}{4}$.

Substituting $x_0^{u_0,c_1}$ into $W_1$ yields the equilibrium utility $\Omega_0^{u_0,c_1}$.

Finally, since for any $x_0$ such that $\Omega_1^{\bar{c}}(x_0)$ exists, $\tilde{\Omega}_1^{\bar{c}}(x_0) \leq \tilde{\Omega}_1^{u}(x_0)$, we also have, by definition of the value functions $\Omega_0^{\bar{c}_1,\bar{c}_1}$ and $\Omega_0^{\bar{c}_1,u_1}$, $\tilde{\Omega}_0^{\bar{c}_1,\bar{c}_1}(x_0) \leq \tilde{\Omega}_0^{\bar{c}_1,u_1}(x_0)$. ■

**Corollary 7** *If $\rho > \frac{3}{4}$ and $W_0 < \omega_0^p$, or if $\rho \leq \frac{3}{4}$ and $W_0 < \min(\omega_0^p, \omega_1^p)$, the candidate equilibrium strategy is*

$$x_0^{c_0,c_1} = \bar{x}_0, \quad X_1^{c_0,c_1} = \bar{X}_1(\bar{x}_0)$$

*At this strategy, the arbitrageur's expected utility is $\Omega_0^{c_0,c_1} \equiv \tilde{\Omega}_0^{\bar{c}_1,\bar{c}_1}(\bar{x}_0)$.*

**Proof.** Follows immediately from Proposition 17. ■

Note that this strategy is the same as the constrained equilibrium of the competitive case (for a given $W_0$), but with different wealth thresholds.

**Relevant wealth thresholds.** Given our analysis so far, the relevant wealth thresholds are $\omega^c$, $\omega_0^p$, $\omega_1^p$, $\hat{\omega}$, $\hat{\omega} + \omega^*$, and $\hat{\omega} + \frac{4\bar{e}^2}{a\sigma^2}$.

**Lemma 12 (Wealth Tresholds Ordering in Equilibrium with Binding Time-1 Constraint)** *The order of the wealth thresholds is given in Table 4.*

Table 4: Wealth Threshold Order for Equilibrium with Binding Time-1 Constraint

| Treshold | Greater than | Condition | Numerical value |
|---|---|---|---|
| $\omega_0^p \equiv s\bar{e} - \frac{1}{2}a\sigma^2 s^2$ | $\bar{\omega}_1^p$ | $\rho < \frac{3}{4}$ | |
| | $\omega^c$ | for any $\rho > 0$ (equality if $\rho = 2$) | |
| | 0 | $\rho \geq \frac{1}{2}$ | |
| | $\hat{\omega}$ | $\rho > \frac{9}{2}$ | |
| $\bar{\omega}_1^p \equiv \frac{3}{2}s\bar{e} - \frac{7}{8}a\sigma^2 s^2$ | $\omega^c$ | for all $\rho > 0$ | |
| | 0 | $\rho > \frac{7}{12}$ | 0.583 |
| | $\hat{\omega}$ | $\rho > \frac{13}{4}$ | |
| $\omega^c \equiv \omega_1^u - \frac{\bar{e}^2}{10a\sigma^2}$ | 0 | $7 - 2\sqrt{10} \leq \rho \leq 7 + 2\sqrt{10}$ | 0.675 - 13.32 |
| | $\hat{\omega}$ | never, equality for $\rho = 7$ | |
| $\hat{\omega} \equiv 4a\sigma^2 s^2$ | 0 | for any $\rho > 0$ | |

**Relevant risk benefit ratio thresholds.** The relevant thresholds for $\rho$ are thus, in ascending order, $\frac{1}{2}$, $\frac{7}{12}$, $7-2\sqrt{10}$, $\frac{3}{4}$, 1, $\frac{13}{4}$, $\frac{9}{2}$, 7 and $7+2\sqrt{10}$. The thresholds 1 and 7 correspond to a change in $\mathcal{F}_0^{\bar{c}_1}$. The ordering of wealth thresholds per $\rho$-interval is given in Table 5. Since the positivity of the wealth thresholds does not affect the equilibrium outcome, I group all the cases where $\rho < \frac{3}{4}$ together. Similarly, I do not distinguish the case with $\rho \geq 7 + 2\sqrt{10}$.

Table 5: $\rho$ and Wealth Intervals for Equilibrium with Binding Time-1 Constraint

| Case | $\rho$ interval | Wealth ordering |
|---|---|---|
| 1 | $\rho < \frac{3}{4}$ | $(\omega^c, 0)^+ < (\omega_1^p, 0)^+ < (\omega_0^p, 0)^+ < \hat{\omega}$ |
| 2 | $\frac{3}{4} \leq \rho < 1$ | $0 < \omega^c < \omega_0^p < \omega_1^p < \hat{\omega}$ |
| 3 | $1 \leq \rho < \frac{13}{4}$ | $0 < \omega^c < \omega_0^p < \omega_1^p < \hat{\omega}$ |
| 4 | $\frac{13}{4} \leq \rho < \frac{9}{2}$ | $0 < \omega^c < \omega_0^p < \hat{\omega} < \omega_1^p$ |
| 5 | $\frac{9}{2} \leq \rho < 7$ | $0 < \omega^c < \omega_0^p < \hat{\omega} < \omega_1^p$ |
| 6 | $7 \leq \rho$ | $(\omega^c, 0)^+ < \hat{\omega} < \omega_0^p < \omega_1^p$ |

**Equilibrium with binding time-1 constraint (Proposition 16)**

1. If $\rho < \frac{3}{4}$, $(\omega^c, 0)^+ < (\omega_1^p, 0)^+ < (\omega_0^p, 0)^+ < \hat{\omega}$

   (a) If $W_0 \leq (\omega^c, 0)^+$, in equilibrium, the arbitrageur's constraint binds upwards at time 0 and time 1 and the arbitrageur holds $x_0^{c_0,c_1} = \bar{x}_0$ and $X_1^{c_0,c_1} = \bar{X}_1(\bar{x}_0)$. The equilibrium is abbreviated $\mathbf{c_0}, \mathbf{c_1}$.

   **Proof.** In this case, it is not possible to deviate to make the constraint slack at time 1. Deviations to make the constraint downward-binding at time 1 are also not feasible.

The two cases given in the proof of Lemma 10 require either $W_1 < 0$ (case a), which contradicts $x_0 \in \mathcal{F}_0^0$ by Lemma 6, or $x_0 < -s$ (case b), which does not satisfy the time-0 constraint by Corollary 4. Further, the time-0 constraint binds upwards at the candidate equilibrium strategy, i.e. $x_0^{u_0,c_1} > \bar{x}_0$. Since $\Omega_0^{\bar{c}_1,\bar{c}_1}$ is increasing for $x_0 < x_0^{u_0,c_1}$, it is optimal for the arbitrageur to hold $\bar{x}_0$. ■

(b) If $(\omega^c, 0)^+ \leq W_0 < (\omega_1^p, 0)^+$, then

i. If $\max(\bar{x}_0, \hat{x}_0) = \hat{x}_0$, then there is no equilibrium, in which the constraint binds upwards at time 1 (abbreviated **no $\mathbf{c_1}$**).

ii. Otherwise, the equilibrium is $\mathbf{c_0}, \mathbf{c_1}$ iff $\Omega_0^{c_0,c_1} \geq \Omega_0^{\bar{c}_1,u_1}(x_0^{d*})$, where $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^u} \Omega_0^{\bar{c}_1,u_1}(x_0)$.

**Proof.** In the first case, $\bar{x}_0 < \hat{x}_0 < x_0^{u_0,c_1}$, so any position that satisfies the time-0 constraint also keeps the time-1 constraint slack. Hence, it is not possible to have an equilibrium, in which the constraint binds upwards at time 1.

In the second case, $\max(\bar{x}_0, \hat{x}_0) = \bar{x}_0$, so $\hat{x}_0 < \bar{x}_0 < x_0^{u_0,u_1}$. On the one hand, $\Omega_0^{c_1,\bar{c}_1}$ is increasing for $x_0 \leq x_0^{u_0,c_1}$, so the arbitrageur wishes to buy as much as the time-0 constraint allows. On the other hand, by buying less, in particular, by buying less than $\hat{x}_0$, the arbitrageur makes the constraint slack at time 1, increasing the time-1 expected utility but potentially decreasing the time-0 trading gains. There is no need to check for deviations leading to a downward-binding constraint at time 1 for the same reason as in case a. Thus, in equilibrium, the arbitrageur sticks to the conjectured strategy $(c_0, c_1)$ iff $\Omega_0^{c_0,c_1} \geq \Omega_0^{\bar{c}_1,u_1}(x_0^{d*})$, where $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^u} \Omega_0^{\bar{c}_1,u_1}(x_0)$.

■

(c) If $(\omega_1^p, 0)^+ \leq W_0 < (\omega_0^p, 0)^+$, then there is no equilibrium, in which the constraint binds upwards at time 1 (abbreviated **no $\mathbf{c_1}$**).

**Proof.** We now have $\bar{x}_0 < x_0^{u_0,c_1} < \hat{x}_0$. Thus, as in (b)i., there is a time-consistency problem: any position satisfying the time-0 constraint implies that the time-1 constraint is slack. Further, if there exists $x_0 \in \mathcal{F}_0^{\bar{c}_1}$ that does not also belong to $\mathcal{F}_0^{u_1}$, it leads to a lower expected utility than the corresponding deviation, because it must be smaller than $\bar{x}_0$, $\Omega_0^{\bar{c}_1,\bar{c}_1}$ is increasing for $x_0 \leq x_0^{u_0,c_1}$, and for any $x_0$ such that these functions

exist, $\Omega_0^{\bar{c}_1,\bar{c}_1}(x_0) \leq \Omega_0^{\bar{c}_1,u_1}(x_0)$ (Proposition 17). ■

(d) If $(\omega_0^p, 0)^+ \leq W_0 < \hat{\omega}$ or $W_0 \geq \hat{\omega}$, then **no** $\mathbf{c_1}$.

**Proof.** In this case, $x_0^{u_0,c_1} \leq \min(\hat{x}_0, \bar{x}_0)$, so both constraints are slack. Hence, there is again a consistency issue. The solution $x_0^{u_0,u_1}$ is based on the premise that the time-1 constraint binds, which is not confirmed for this level of arbitrage capital. Choosing a larger or smaller $x_0$ to keep the time-1 constraint binding is possible, but is trivially dominated by deviations making the time-1 constraint slack due to point 4 of Proposition 17. ■

2. If $\frac{3}{4} \leq \rho < 1$, $0 < \omega^c < \omega_0^p < \omega_1^p < \hat{\omega}$

   (a) If $0 \leq W_0 \leq \omega^c$, then $\mathbf{c_0}, \mathbf{c_1}$ (same as 1a). (b) If $\omega^c \leq W_0 < \omega_0^p$, same as 1b.

   (c) If $\omega_0^p \leq W_0 < \omega_1^p$, then the arbitrageur's constraint binds upwards at time 1 but not at time 0 in equilibrium iff $\Omega_0^{u_0,c_1} \geq \Omega_0^{\bar{c}_1,u_1}(x_0^{d*})$, where $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^u} \Omega_0^{\bar{c}_1,u_1}(x_0)$.

   **Proof.** Given that $\omega_0^p \leq W_0 < \omega_1^p$, the necessary conditions for the candidate strategy $u_0, c_1$ are satisfied. However, we need to check that it is indeed optimal for the arbitrageur to hold $x_0^{u_0,c_1}$. Deviations leading to a downward-bing constraint at time 1 are not feasible, as in case 1a. Among deviations leading to a slack constraint at time 1, the best position is $x_0^{d*} = \arg\max_{x_0 \in \mathcal{D}_0^u} \Omega_0^{\bar{c}_1,u_1}(x_0)$.[40] Thus, in equilibrium the arbitrageur chooses $x_0^{u_0,c_1}$ iff $\Omega_0^{u_0,c_1} \geq \Omega_0^{\bar{c}_1,u_1}(x_0^{d*})$. ■

   (d) If $\omega_1^p \leq W_0 < \hat{\omega}$ or $W_0 \geq \hat{\omega}$, **no** $\mathbf{c_1}$ (same as 1d).

3. If $1 \leq \rho < \frac{13}{4}$, then $0 < \omega^c < \omega_0^p < \omega_1^p < \hat{\omega}$. This is the same as case 2.

   **Proof.** The only change is in the definition of $\mathcal{F}_0^{\bar{c}_1}$, but this is had no effect on the equilibrium. ■

4. If $\frac{13}{4} \leq \rho < \frac{9}{2}$, then $0 < \omega^c < \omega_0^p < \hat{\omega} < \omega_1^p$. There is only a change in the relative position in $\hat{\omega}$, without essential difference for the form of the equilibrium. Only subcases c and d must be adjusted.

[40] While it is possible to determine this position analytically by solving the maximization problem, its form is not easily tractable: the first-order condition of the maximization problem involves solving for the root of a quartic equation on the interval $\mathcal{D}_0^u$. Therefore, for the sake of tractability and compactness of the expressions, I keep the implicit definition of the best deviation and the expected utility it generates.

(a) If $0 \leq W_0 \leq \omega^c$, then $\mathbf{c_0}, \mathbf{c_1}$ (same as 1a). (b) If $\omega^c \leq W_0 < \omega_0^p$, same as 1b. (c) If $\omega_0^p \leq W_0 < \hat{\omega}$ or $\hat{\omega} \leq W_0 < \omega_1^p$, same as 2c. (d) If $W_0 \geq \omega_1^p$, **no** $\mathbf{c_1}$ (same as 1d).

5. If $\frac{9}{2} \leq \rho < 7$, then $0 < \omega^c < \hat{\omega} < \omega_0^p < \omega_1^p$. Again, only the relative position of $\hat{\omega}$ changes, without essential difference for the equilibrium.

   (a) If $0 \leq W_0 \leq \omega^c$, then $\mathbf{c_0}, \mathbf{c_1}$ (same as 1a). (b) If $\omega^c \leq W_0 < \hat{\omega}$ or $\hat{\omega} \leq W_0 < \omega_0^p$, same as 1b. (c) If $\omega_0^p \leq W_0 < \omega_1^p$, same as 2c. (d) If $W_0 \geq \omega_1^p$, **no** $\mathbf{c_1}$ (same as 1d).

6. If $7 \leq \rho$, then $(\omega^c, 0)^+ < \hat{\omega} < \omega_0^p < \omega_1^p$. The position of $\hat{\omega}$ changes again, and extends the area in which both constraints bind upwards in equilibrium, due to a change in $\mathcal{F}_0^{\bar{c}_1}$.

   (a) If $0 \leq W_0 \leq \omega^c$ or $\omega^c \leq W_0 < \hat{\omega}$, then $\mathbf{c_0}, \mathbf{c_1}$ (same as 1a).

   **Proof.** A new case arises when $\omega^c \leq W_0 < \hat{\omega}$, as for $\rho \geq 7$, it is not possible to make the constraint slack anymore for this level of capital. ■

   (b) If $\hat{\omega} \leq W_0 < \omega_0^p$, same as 1b. (c) If $\omega_0^p \leq W_0 < \omega_1^p$, same as 2c. (d) If $W_0 \geq \omega_1^p$, **no** $\mathbf{c_1}$ (same as 1d).

### D.4 Equilibrium spreads

**Corollary 8 (Equilibrium Spreads in the $\bar{c}_1$ equilibria)** *In the $u_0, c_1$ and $c_0, c_1$ equilibria, spreads are*

$$\Delta_0^{u_0,c_1} = 2a\sigma^2 s + \bar{e} - \sqrt{d_1^+(x_0^{u_0,c_1})}, \quad \Delta_1^{u_0,c_1} = a\sigma^2 s + \bar{e} - \sqrt{d_1^+(x_0^{u_0,c_1})}$$
$$\Delta_0^{c_0,c_1} = 2(a\sigma^2 s + \bar{e}) - \sqrt{d_0^+(x_0^{c_0,c_1})} - \sqrt{d_1^+(x_0^{c_0,c_1})}, \quad \Delta_1^{c_0,c_1} = a\sigma^2 s + \bar{e} - \sqrt{d_1^+(x_0^{c_0,c_1})}$$

**Proof.** Follows from substituting equilibrium positions in the spread schedules (11)-(10). ■

## E Coexistence

Here is the full result:

**Proposition 18 (Equilibria with Slack and Binding Time-1 Constraints May Coexist)**

- *There is a unique equilibrium when arbitrage capital is either sufficiently low or sufficiently high:*
  - *If* $0 \leq W_0 < \max(0, \omega^f)$*, the unique equilibrium is* $\mathbf{c_0}, \mathbf{c_1}$.
  - *If* $W_0 \geq \max(\omega_0^u, \omega_1^u, \omega_1^p)$*, the unique equilibrium is* $\mathbf{u_0}, \mathbf{u_1}$.
- *When capital is intermediate, i.e. if* $\max(0, \omega^f) \leq W_0 < \max(\omega_0^u, \omega_1^u, \omega_1^p)$*, multiple equilibria may coexist depending on the level of* $\rho$*:*
  - *For* $0 \leq \rho < \frac{7}{10}$*, two equilibria may coexist:*
    - *If* $\omega^f \leq W_0 < \max(\omega_0^u, \omega_1^u)$*,* $\mathbf{c_0}, \mathbf{u_1}$ *may coexist with* $\mathbf{c_0}, \mathbf{c_1}$.
    - *If* $\max(\omega_0^u, \omega_1^u) \leq W_0 < \omega_1^p$*,* $\mathbf{u_0}, \mathbf{u_1}$ *may coexist with* $\mathbf{c_0}, \mathbf{c_1}$.

    *In the special case where* $0 \leq \rho < \frac{79}{140}$ *and* $\omega_1^p \leq W_0 < \omega_0^u$*,* $\mathbf{c_0}, \mathbf{u_1}$ *is the unique equilibrium, with* $x_0^{c_0,u_1} = \bar{x}_0$.
  - *For* $\frac{7}{10} \leq \rho < \frac{3}{4}$*, two equilibria may coexist:* $\mathbf{u_0}, \mathbf{u_1}$ *with* $\mathbf{c_0}, \mathbf{c_1}$*, or* $\mathbf{c_0}, \mathbf{u_1}$ *with* $\mathbf{c_0}, \mathbf{c_1}$.
  - *For* $\rho \geq \frac{3}{4}$*,* $\omega_0^u < \omega_1^u < \omega_1^p$*, and*
    - *If* $\omega^f \leq W_0 < \min(\omega_1^u, \omega_0^p)$*,* $\mathbf{c_0}, \mathbf{u_1}$ *may coexist with* $\mathbf{c_0}, \mathbf{c_1}$.
    - *If* $\omega_0^p \leq W_0 < \omega_1^u$*,* $\mathbf{c_0}, \mathbf{u_1}$ *may coexist with* $\mathbf{u_0}, \mathbf{c_1}$.
    - *If* $\omega_1^u \leq W_0 < \max(\omega_1^u, \omega_0^p)$*,* $\mathbf{u_0}, \mathbf{u_1}$ *may coexist with* $\mathbf{c_0}, \mathbf{c_1}$.
    - *If* $\max(\omega_0^p, \omega_1^u) \leq W_0 < \omega_1^p$*,* $\mathbf{u_0}, \mathbf{u_1}$ *may coexist with* $\mathbf{u_0}, \mathbf{c_1}$.

**Proof.**

**Lemma 13 (Threshold order for coexistence analysis)** *The order of wealth thresholds as a function of* $\rho$ *is given in Table 6.*

Table 6: Wealth Threshold Order To Determine Coexistence

| Treshold | Greater than | Condition | Numerical value |
|---|---|---|---|
| $\omega_0^p \equiv s\bar{e} - \frac{1}{2}a\sigma^2 s^2$ | $\omega_0^u$<br>$\omega_1^u$ | $\rho > \frac{1}{10}$<br>for any $\rho < 1$ | |
| $\bar{\omega}_1^p \equiv \frac{3}{2}s\bar{e} - \frac{7}{8}a\sigma^2 s^2$ | $\omega_0^u$<br>$\omega_1^u$ | $\rho > \frac{79}{140}$<br>$\rho > 0$ | 0.564 |

Further for $\rho \geq 7$, $\hat{\omega}$ is smaller than $\omega_0^u$, $\omega_1^u$, $\omega_0^p$, and $\omega_1^p$.

Combining tables 2, 4 and 6, the relevant thresholds for $\rho$ are thus, in ascending order, $\frac{1}{10}$, $\frac{79}{140}$, $\frac{7}{10}$, $\frac{3}{4}$, $3 - \frac{2}{5}\sqrt{30}$, 1, $3 + \frac{2}{5}\sqrt{30}$, and 7.

**Lemma 14 (Threshold order)** *The order of wealth thresholds as a function of $\rho$ is given in Table 7.*

Table 7: $\rho$ and Wealth Intervals for Coexistence Region

| Case | $\rho$ interval | Wealth ordering |
|---|---|---|
| 1 | $\rho < \frac{1}{10}$ | $\omega^c < \omega_1^u < \omega_1^p < \omega_0^p < \omega_0^u$ |
| 2 | $\frac{1}{10} \leq \rho < \frac{79}{140}$ | $\omega^c < \omega_1^u < \omega_1^p < \omega_0^u < \omega_0^p$ |
| 3 | $\frac{79}{140} \leq \rho < \frac{7}{10}$ | $\omega^c < \omega_1^u < \omega_0^u < \omega_1^p < \omega_0^p$ |
| 4 | $\frac{7}{10} \leq \rho < \frac{3}{4}$ | $\omega^c < \omega_0^u < \omega_1^u < \omega_1^p < \omega_0^p$ |
| 5 | $\frac{3}{4} \leq \rho < 3 - \frac{2}{5}\sqrt{30}$ | $\omega^c < \omega_0^u < \omega_1^u < \omega_0^p < \omega_1^p$ |
| 6 | $3 - \frac{2}{5}\sqrt{30} \leq \rho < 1$ | $\omega_0^u < \omega^c < \omega_1^u < \omega_0^p < \omega_1^p$ |
| 7 | $1 \leq \rho < 3 + \frac{2}{5}\sqrt{30}$ | $\omega_0^u < \omega^c < \omega_0^p < \omega_1^u < \omega_1^p$ |
| 8 | $3 + \frac{2}{5}\sqrt{30} \leq \rho < 7$ | $\omega^c < \omega_0^u < \omega_0^p < \omega_1^u < \omega_1^p$ |
| 9 | $7 \leq \rho$ | $\hat{\omega} < \omega_0^u < \omega_0^p < \omega_1^u < \omega_1^p$ |

**Coexistence of equilibria (Proposition 18)** **Proof.**

1. If $\rho < \frac{1}{10}$, then $\omega^c < \omega_1^u < \omega_1^p < \omega_0^p < \omega_0^u$

   (a) If $0 \leq W_0 < \omega^c$, there is a unique equilibrium: the arbitrageur's constraint binds at time 0 and time 1 ($\mathbf{c_0}, \mathbf{c_1}$). There is no equilibrium with a slack constraint at time 1 (**no** $\mathbf{u_1}$).

   (b) If $\omega^c \leq W_0 < \omega_1^u$, there are potentially multiple equilibria: $\mathbf{c_0}, \mathbf{u_1}$, if it exists, with $x_0^{c_0,u_1} = \min(\hat{x}_0, \bar{x}_0)$, and either **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{c_1}$.

   (c) If $\omega_1^u \leq W_0 < \omega_1^p$, there are potentially multiple equilibria: $\mathbf{c_0}, \mathbf{u_1}$, with $x_0^{c_0,u_1} = \bar{x}_0$, and either **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{c_1}$.

   (d) If $\omega_1^p \leq W_0 < \omega_0^p$, there is a unique equilibrium: $\mathbf{c_0}, \mathbf{u_1}$, with $x_0^{c_0,u_1} = \bar{x}_0$, and **no** $\mathbf{c_1}$.

   (e) If $\omega_0^p \leq W_0 < \omega_0^u$, there is a unique equilibrium: $\mathbf{c_0}, \mathbf{u_1}$, with $x_0^{c_0,u_1} = \bar{x}_0$, and **no** $\mathbf{c_1}$.

   (f) If $\omega_0^u \leq W_0$, there is a unique equilibrium: $\mathbf{u_0}, \mathbf{u_1}$, and **no** $\mathbf{c_1}$.

2. If $\frac{1}{10} \leq \rho < \frac{79}{140}$, then $\omega^c < \omega_1^u < \omega_1^p < \omega_0^u < \omega_0^p$

   (a) If $0 \leq W_0 < \omega^c$: $\mathbf{c_0}, \mathbf{c_1}$, same as 1a. (b) If $\omega^c \leq W_0 < \omega_1^u$: $\mathbf{c_0}, \mathbf{u_1}$, with **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (c) If $\omega_1^u \leq W_0 < \omega_1^p$: $\mathbf{c_0}, \mathbf{u_1}$, with $x_0^{c_0,u_1} = \bar{x}_0$, and either **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{c_1}$, same as 1c. (d) If $\omega_1^p \leq W_0 < \omega_0^u$: $\mathbf{c_0}, \mathbf{u_1}$, with $x_0^{c_0,u_1} = \bar{x}_0$, same as 1d. (e) If $\omega_0^u \leq W_0 < \omega_0^p$ or $\omega_0^p \leq W_0$: $\mathbf{u_0}, \mathbf{u_1}$, same as 1f.

3. If $\frac{79}{140} \leq \rho < \frac{7}{10}$, then $\omega^c < \omega_1^u < \omega_0^u < \omega_1^p < \omega_0^p$

   (a) If $0 \leq W_0 < \omega^c$: $\mathbf{c_0}, \mathbf{c_1}$, same as 1a. (b) If $\omega^c \leq W_0 < \omega_1^u$: $\mathbf{c_0}, \mathbf{u_1}$, with **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (c) If $\omega_1^u \leq W_0 < \omega_0^u$: $\mathbf{c_0}, \mathbf{u_1}$, with $x_0^{c_0,u_1} = \bar{x}_0$, and either **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{c_1}$, same as 1c. (d) If $\omega_0^u \leq W_0 < \omega_1^p$: $\mathbf{u_0}, \mathbf{u_1}$, and either **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{c_1}$. (e) If $\omega_1^p \leq W_0 < \omega_0^p$ or $\omega_0^p \leq W_0$: $\mathbf{u_0}, \mathbf{u_1}$, same as 1f.

4. If $\frac{7}{10} \leq \rho < \frac{3}{4}$, then $\omega^c < \omega_0^u < \omega_1^u < \omega_1^p < \omega_0^p$

   (a) If $0 \leq W_0 < \omega^c$: $\mathbf{c_0}, \mathbf{c_1}$, same as 1a. (b) If $\omega^c \leq W_0 < \omega_1^u$: $\mathbf{c_0}, \mathbf{u_1}$, with **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (c) If $\omega_0^u \leq W_0 < \omega_1^u$: $\mathbf{c_0}, \mathbf{u_1}$, with **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (d) If $\omega_1^u \leq W_0 < \omega_1^p$: $\mathbf{u_0}, \mathbf{u_1}$, and either **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{c_1}$, same as 3d. (e) If $\omega_1^p \leq W_0 < \omega_0^p$ or $\omega_0^p \leq W_0$: $\mathbf{u_0}, \mathbf{u_1}$, same as 1f.

5. If $\frac{3}{4} \leq \rho < 3 - \frac{2}{5}\sqrt{30}$, then $\omega^c < \omega_0^u < \omega_1^u < \omega_0^p < \omega_1^p$

   (a) If $0 \leq W_0 < \omega^c$: $\mathbf{c_0}, \mathbf{c_1}$, same as 1a. (b) If $\omega^c \leq W_0 < \omega_0^u$: $\mathbf{c_0}, \mathbf{u_1}$, with **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (c) If $\omega_0^u \leq W_0 < \omega_1^u$: $\mathbf{c_0}, \mathbf{u_1}$, with **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (d) If $\omega_1^u \leq W_0 < \omega_0^p$: $\mathbf{u_0}, \mathbf{u_1}$, and either **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{c_1}$, same as 3d. (e) If $\omega_0^p \leq W_0 < \omega_1^p$: $\mathbf{u_0}, \mathbf{u_1}$ and $\mathbf{u_0}, \mathbf{c_1}$, if it exists. (f) If $\omega_1^p \leq W_0$: $\mathbf{u_0}, \mathbf{u_1}$, same as 1f.

6. If $3 - \frac{2}{5}\sqrt{30} \leq \rho < 1$, then $\omega_0^u < \omega^c < \omega_1^u < \omega_0^p < \omega_1^p$

   (a) If $0 \leq W_0 < \omega_0^u$ or $\omega_0^u \leq W_0 < \omega^c$: $\mathbf{c_0}, \mathbf{c_1}$, same as 1a. (b) If $\omega^c \leq W_0 < \omega_1^u$: $\mathbf{c_0}, \mathbf{u_1}$, with **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (c) If $\omega_1^u \leq W_0 < \omega_0^p$: $\mathbf{u_0}, \mathbf{u_1}$, and either **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{c_1}$, same as 3d. (d) If $\omega_0^p \leq W_0 < \omega_1^p$: $\mathbf{u_0}, \mathbf{u_1}$ and $\mathbf{u_0}, \mathbf{c_1}$, if it exists, same as 5e. (e) If $\omega_1^p \leq W_0$: $\mathbf{u_0}, \mathbf{u_1}$, same as 1f.

7. If $1 \leq \rho < 3 + \frac{2}{5}\sqrt{30}$, then $\omega_0^u < \omega^c < \omega_0^p < \omega_1^u < \omega_1^p$

   (a) If $0 \leq W_0 < \omega_0^u$ or $\omega_0^u \leq W_0 < \omega^c$: $\mathbf{c_0}, \mathbf{c_1}$, same as 1a. (b) If $\omega^c \leq W_0 < \omega_0^p$: $\mathbf{c_0}, \mathbf{u_1}$, with

**no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (c) If $\omega_0^p \le W_0 < \omega_1^u$: $\mathbf{c_0}, \mathbf{u_1}$, if it exists, and $\mathbf{u_0}, \mathbf{c_1}$, if it exists. (d) If $\omega_1^u \le W_0 < \omega_1^p$: $\mathbf{u_0}, \mathbf{u_1}$ and $\mathbf{u_0}, \mathbf{c_1}$, if it exists, same as 5e. (e) If $\omega_1^p \le W_0$: $\mathbf{u_0}, \mathbf{u_1}$, same as 1f.

8. If $3 + \frac{2}{5}\sqrt{30} \le \rho < 7$, then $\omega^c < \omega_0^u < \omega_0^p < \omega_1^u < \omega_1^p$

   (a) If $0 \le W_0 < \omega^c$: $\mathbf{c_0}, \mathbf{c_1}$, same as 1a. (b) If $\omega^c \le W_0 < \omega_0^u$: $\mathbf{c_0}, \mathbf{u_1}$, with **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (c) If $\omega_0^u \le W_0 < \omega_0^p$: $\mathbf{c_0}, \mathbf{u_1}$, with **no** $\mathbf{c_1}$ or $\mathbf{c_0}, \mathbf{u_1}$, same as 1b. (d) If $\omega_0^p \le W_0 < \omega_1^u$: $\mathbf{c_0}, \mathbf{u_1}$, if it exists, and $\mathbf{u_0}, \mathbf{c_1}$, if it exists, same as 7c. (e) If $\omega_1^u \le W_0 < \omega_1^p$: $\mathbf{u_0}, \mathbf{u_1}$ and $\mathbf{u_0}, \mathbf{c_1}$, if it exists, same as 5e. (f) If $\omega_1^p \le W_0$: $\mathbf{u_0}, \mathbf{u_1}$, same as 1f.

9. If $7 \le \rho$, then $\hat{\omega} < \omega_0^u < \omega_0^p < \omega_1^u < \omega_1^p$: same as previous case, replacing $\omega^c$ by $\hat{\omega}$.

To sum up, apart from simple cases 1a and 1f, six cases arise: 1b, 1c, 1d, which are variants of each other, and 3d, 5e, 7c. When $\rho \ge \frac{3}{4}$, we can observe that 1b occurs for $\omega^f \le W_0 < \min(\omega_1^u, \omega_0^p)$, 3d for $\omega_1^u \le W_0 < \max(\omega_1^u, \omega_0^p)$, 5e for $\max(\omega_0^p, \omega_1^u) \le W_0 < \omega_1^p$, and 7c for $\omega_0^p \le W_0 < \omega_1^u$. ■ ■

# F Welfare

## F.1 Proposition 9

**Proof.** *Price effects.* Given the definition of $d_1^+(x_0)$ and the spread schedule $\Delta_0^{\bar{c}_1}(x_0)$ given in the text, we have

$$d_1^+\left(\frac{s}{2}\right) = 2a\sigma^2 W_0 + a^2\sigma^4 s^2 + (a\sigma^2 s - \bar{e})^2 \tag{22}$$

$$\Delta_1^{u_0,c_1} = 2a\sigma^2(s - \bar{X}_1(x_0^{u_0,c_1}) = a\sigma^2 s + \bar{e} - \sqrt{d_1^+\left(\frac{s}{2}\right)} \tag{23}$$

$$\Delta_1^{u_0,c_1} = \mathbb{E}_0(\Delta_1^{u_0,c_1}) + 2a\sigma^2(s - x_0^{u_0,c_1}) = 2a\sigma^2 s + \bar{e} - \sqrt{d_1^+\left(\frac{s}{2}\right)} \tag{24}$$

Thus comparing these spreads to the $u_0, u_1$ equilibrium spreads of Proposition **??**, we get:

$$\Delta_1^{u_0,c_1} < \Delta_1^{u_0,u_1} \quad \Leftrightarrow \quad \frac{2}{5}a\sigma^2 s + \bar{e} < \sqrt{d_1^+\left(\frac{s}{2}\right)}$$

Raising both sides to the square and rearranging terms gives after simplication

$$\Delta_1^{u_0,c_1} < \Delta_1^{u_0,u_1} \quad \Leftrightarrow \quad W_0 > r_1 \equiv \frac{7}{5}s\bar{e} - \frac{71}{50}a\sigma^2 s^2$$

Proceeding in the same fashion for time-0 spreads gives:

$$\Delta_0^{u_0,c_1} < \Delta_0^{u_0,u_1} \quad \Leftrightarrow \quad W_0 > r_0 \equiv \frac{6}{5}s\bar{e} - \frac{37}{25}a\sigma^2 s^2$$

Clearly, $r_0 < r_1$, so $\Delta_1^{u_0,c_1} < \Delta_1^{u_0,u_1} \Rightarrow \Delta_0^{u_0,c_1} < \Delta_0^{u_0,u_1}$. I now determine the position of the thresholds $r_0$ and $r_1$ relative to 0, $\omega_0^u$, $\omega_1^u$, $\omega_0^p$, and $\bar{\omega}_0^u$. The results are given in Table 8.

Table 8: Thresholds for price effects in Proposition 9

| Threshold | Lower than | Interval |
|---|---|---|
| $r_1$ | $< 0$ | $\rho < \frac{71}{70}$ |
| | $< \omega_1^u$ | always |
| | $< \bar{\omega}_1^p$ | always |
| | $< \omega_0^p$ | $\rho < \frac{23}{10}$ |
| | $< \omega_0^u$ | $\rho < \frac{47}{30}$ |
| $r_0$ | $< 0$ | $\rho < \frac{37}{30}$ |
| | $< \omega_0^u$ | $\rho < \frac{5}{2}$ |
| | $< \omega_1^u$ | always |
| | $< \omega_0^p$ | $\rho < \frac{49}{10}$ |
| | $< \bar{\omega}_1^p$ | always |

Further, from Lemma **??**, $\omega_0^p < \bar{\omega}_1^p$ iff $\rho > \frac{3}{4}$, so we only need to consider this region. The potential coexistence region is determined by the position of $\rho$ relative to 1 (Proposition **??**). Thus, the relevant thresholds for $\rho$ are $\frac{3}{4}$, 1, $\frac{47}{30}$, $\frac{23}{10}$, $\frac{5}{2}$, and $\frac{49}{10}$ (ignoring the positivity constraints for $r_0$ and $r_1$). Therefore, there are six cases:

1. If $\frac{3}{4} \leq \rho < 1$, then $r_0 < r_1 < \omega_0^u < \omega_1^u < \omega_0^p < \bar{\omega}_1^p$. The potential coexistence region for this interval is $[\omega_0^p, \bar{\omega}_1^p[$. Thus, if the $u_0, u_1$ and $u_0, c_1$ equilibria coexist, then $W_0 \geq \max(r_0, r_1)$, so $\Delta_t^{u_0,c_1} < \Delta_t^{u_0,u_1}$.

2. If $1 \leq \rho < \frac{47}{30}$, then $r_0 < r_1 < \omega_0^u < \omega_0^p < \omega_1^u < \bar{\omega}_1^p$. The potential coexistence region is now $[\omega_1^u, \bar{\omega}_1^p[$. If equilibria coexist, then $W_0 \geq \max(r_0, r_1)$, so $\Delta_t^{u_0,c_1} < \Delta_t^{u_0,u_1}$.

3. If $\frac{47}{30} \leq \rho < \frac{23}{10}$, then $r_0 < \omega_0^u < r_1 < \omega_0^p < \omega_1^u < \bar{\omega}_1^p$. Same as case 2.

4. If $\frac{23}{10} \leq \rho < \frac{5}{2}$, then $r_0 < \omega_0^u < \omega_0^p < r_1 < \omega_1^u < \bar{\omega}_1^p$. Same as case 2.

5. If $\frac{5}{2} \leq \rho < \frac{49}{10}$, then $\omega_0^u < r_0 < \omega_0^p < r_1 < \omega_1^u < \bar{\omega}_1^p$. Same as case 2.

6. If $\frac{49}{10} \leq \rho$, then $\omega_0^u < \omega_0^p < r_0 < r_1 < \omega_1^u < \bar{\omega}_1^p$. Same as case 2.

Thus, if the two equilibria coexist, spreads are always smaller in the $u_0, c_1$ equilibrium.

*Hedgers' welfare.* Since $\mathbb{E}_0(p_1) - p_0 = \frac{1}{2}(\Delta_0 - \Delta_1)$ and $\mathbb{E}_0(p_2 - p_1) = \frac{1}{2}\Delta_1$, we can rewrite equation (7) in Lemma 2 as

$$U_0 = \frac{(\Delta_0 - \Delta_1)^2 + \Delta_1^2}{8a\sigma^2} - \frac{s}{2}\Delta_0 \tag{25}$$

The first term represents hedgers' capital gains on their time 0 and time 1 positions. The second term represents the cost of sharing risk at a discount relative to the fundamental value (the expected value). Equation (25) gives hedgers' welfare in market A. Market B is symmetric. From (25), we get:

$$\begin{aligned} & U_0^{u_0,c_1} > U_0^{u_0,u_1} \qquad (26) \\ \Leftrightarrow \; & (\Delta_0^{u_0,c_1} - \Delta_1^{u_0,c_1})^2 - (\Delta_0^{u_0,u_1} - \Delta_1^{u_0,u_1})^2 + (\Delta_1^{u_0,c_1})^2 - (\Delta_1^{u_0,u_1})^2 > 4a\sigma^2 s(\Delta_0^{u_0,c_1} - \Delta_0^{u_0,u_1}) \end{aligned}$$

Using (23) and (24) and Proposition **??**, we get $\Delta_0^{u_0,c_1} - \Delta_1^{u_0,c_1} = a\sigma^2 s$, $\Delta_0^{u_0,u_1} - \Delta_1^{u_0,u_1} = \frac{3}{5}a\sigma^2 s$, thus condition (26) becomes

$$\frac{11}{5}a^2\sigma^4 s^2 + 2\bar{e}^2 + 2a\sigma^2 W_0 - 2(a\sigma^2 s + \bar{e})\sqrt{d_1^+\left(\frac{s}{2}\right)} > 4a\sigma^2 s\left[\frac{1}{5}a\sigma^2 s + \bar{e} - \sqrt{d_1^+\left(\frac{s}{2}\right)}\right]$$

Rearranging the terms, we can rewrite condition (26) as

$$a\sigma^2(W_0 - \omega^h) > (\bar{e} - a\sigma^2 s)\sqrt{d_1^+\left(\frac{s}{2}\right)}, \quad \text{with } \omega^h \equiv 2s\bar{e} - \tfrac{7}{10}a\sigma^2 s^2 - \tfrac{\bar{e}^2}{a\sigma^2} \tag{27}$$

We then need to place $\omega^h$ relative to 0, $\omega_0^u$, $\omega_1^u$, $\omega_0^p$, and $\bar{\omega}_1^p$, as in Table 9. Since our interval of

Table 9: Thresholds for hedgers' welfare in Proposition 9

| Threshold | Greater than | Interval |
|---|---|---|
| $\omega^h$ | $> 0$ | iff $\rho \in \left]1 - \frac{\sqrt{1.2}}{2}, 1 + \frac{\sqrt{1.2}}{2}\right[$ |
| | $> \omega_0^u$ | iff $\rho \in \left]\frac{3}{5} - \frac{\sqrt{14}}{10}, \frac{3}{5} + \frac{\sqrt{14}}{10}\right[$ |
| | $> \omega_1^u$ | iff $\rho \in \left]0, \frac{3+\sqrt{29}}{10}\right[$ |
| | $> \omega_0^p$ | iff $\rho \in \left]\frac{1}{2} - \frac{\sqrt{5}}{10}, \frac{1}{2} - \frac{\sqrt{5}}{10}\right[$ |
| | $> \bar{\omega}_1^p$ | iff $\rho \in \left]0, \frac{1}{4} + \frac{\sqrt{95}}{20}\right[$ |

interest is $\rho \geq \frac{3}{4}$, the only relevant thresholds are $\frac{3+\sqrt{29}}{10}$ ($\approx 0.88$), $\frac{3}{5} + \frac{\sqrt{14}}{10}$ ($\approx 0.97$), 1 and $1 + \frac{\sqrt{1.2}}{2}$. I add the threshold 1 as it determines the region of potential coexistence. We have thus five cases:

1. If $\frac{3}{4} \leq \rho < \frac{3+\sqrt{29}}{10}$, then $0 < \omega_0^u < \omega_1^u < \omega^h < \omega_0^p < \bar{\omega}_1^p$
2. If $\frac{3+\sqrt{29}}{10} \leq \rho < \frac{3}{5} + \frac{\sqrt{14}}{10}$, then $0 < \omega_0^u < \omega^h < \omega_1^u < \omega_0^p < \bar{\omega}_1^p$
3. If $\frac{3}{5} + \frac{\sqrt{14}}{10} \leq \rho < 1$, then $0 < \omega^h < \omega_0^u < \omega_1^u < \omega_0^p < \bar{\omega}_1^p$
4. If $1 \leq \rho < 1 + \frac{\sqrt{1.2}}{2}$, then $0 < \omega^h < \omega_0^u < \omega_0^p < \omega_1^u < \bar{\omega}_1^p$
5. If $1 + \frac{\sqrt{1.2}}{2} \leq \rho$, then $\omega^h < 0 < \omega_0^u < \omega_0^p < \omega_1^u < \bar{\omega}_1^p$

It is clear that when equilibria potentially coexist under the conditions of Proposition **??**, then $W_0 \geq \omega^h$. Therefore for any $\rho \geq \frac{3}{4}$, the left-hand side of condition (27) is positive. Instead, the right-hand side is negative for $\rho < 1$ and positive otherwise. Thus, if $\frac{3}{4} \leq \rho < 1$, condition (27) holds and $U_0^{u_0,c_1} > U_0^{u_0,u_1}$. If $\rho \geq 1$, we can raise both sides of (27) to the square to determine the trade-off. Substituting equation (22) for the expression for $d_1^+\left(\frac{s}{2}\right)$, we can rewrite (27) as:

$$a^2\sigma^4 W_0^2 - 2a\sigma^2\left[a\sigma^2\omega^h + (\bar{e} - a\sigma^2 s)^2\right]W_0 + (a\sigma^2\omega^h)^2 - (\bar{e} - a\sigma^2 s)^2\left[a^2\sigma^4 s^2 + (a\sigma^2 s - \bar{e})^2\right] > 0$$

Viewing the left-hand side as a polynomial in $W_0$, we can calculate its discriminant. After a few lines of algebra, we obtiain $4a^2\sigma^4(\bar{e} - a\sigma^2 s)^2\left[2(\bar{e} - a\sigma^2 s)^2 + 2a\sigma^2\omega^h + a^2\sigma^4 s^2\right] > 0$. After calculating

the term in parenthesis, we can write the roots as

$$W_{(1)} = \frac{a\sigma^2\omega^h + (\bar{e} - a\sigma^2 s)^2 - (\bar{e} - a\sigma^2 s)\sqrt{\frac{8}{5}a^2\sigma^4 s^2}}{a\sigma^2} = -\sqrt{\frac{8}{5}}s\bar{e} + \left(\frac{3}{10} + \sqrt{\frac{8}{5}}\right)a\sigma^2 s^2$$
$$W_{(2)} = \sqrt{\frac{8}{5}}s\bar{e} + \left(\frac{3}{10} - \sqrt{\frac{8}{5}}\right)a\sigma^2 s^2$$

It then remains to determine the position of the roots relative to $\omega_1^u$ and $\bar{\omega}_1^p$, which determine the relevant region for coexistence when $\rho \geq 1$. We have: for any $\rho$, $W_{(2)} < \bar{\omega}_1^p$ and $W_{(2)} < \omega_1^u$. Thus $W_0 \in [\omega_1^u, \bar{\omega}_1^p[$ implies $W_0 > W_{(2)}$, which implies that $U_0^{u_0,c_1} > U_0^{u_0,u_1}$. Further, note that Examples **??** and **??** show that the partly constrained equilibrium does exist, both for $\rho < 1$ and $\rho \geq 1$. Hence, if a partly constrained equilibrium exists, and coexists with the unconstrained equilibrium, hedgers are better off in the former.

*Arbitrageurs' welfare.* Starting from the arbitrageur's objective function given in the text (Lemma **??**), and substituting the equilibrium value of $d_1^+$ given by equation (22), we get the arbitrageur's utility in the partly constrained equilibrium:

$$\Omega_0^{u_0,c_1} = \frac{\bar{e}}{a\sigma^2}\left[a\sigma^2 s - \bar{e} + \sqrt{d_1^+\left(\frac{s}{2}\right)}\right] \tag{28}$$

From Proposition **??**, it is simple to calculate the arbitrageur's utility in the unconstrained equilibrium:

$$\Omega_0^{u_0,u_1} = W_0 + \frac{9}{10}a\sigma^2 s^2 \tag{29}$$

Then, $\Omega_0^{u_0,c_1} > \Omega_0^{u_0,u_1}$ is equivalent to

$$\bar{e}\sqrt{d_1^+\left(\frac{s}{2}\right)} > a\sigma^2(W_0 - \omega^a), \quad \text{with } \omega^a \equiv s\bar{e} - \tfrac{9}{10}a\sigma^2 s^2 - \tfrac{\bar{e}^2}{a\sigma^2}$$

Since $W_0 \geq \omega_0^p$, the right-hand side is positive. So raising both sides to the square preserves the

order. After some simple algebra, the condition becomes

$$-a^2\sigma^4 W_0^2 + 2a\sigma^2(\bar{e}^2 + a\sigma^2\omega^a)W_0 + \bar{e}^2\left[a^2\sigma^4 s^2 + (a\sigma^2 s - \bar{e})^2\right] - (a\sigma^2\omega^a)^2 > 0$$

Viewing the left-hand side as a polynom in $W_0$, and using the definition of $\omega^a$, we compute the discriminant. After some simplification, we obtain $\frac{4}{5}a^4\sigma^8 s^2\bar{e}^2$. Thus, we can write the roots as

$$W^a = \left(1 + \frac{1}{\sqrt{5}}\right)s\bar{e} - \frac{9}{10}a\sigma^2 s^2 > W^{a'} = \left(1 - \frac{1}{\sqrt{5}}\right)s\bar{e} - \frac{9}{10}a\sigma^2 s^2$$

Since $1 + \frac{1}{\sqrt{5}} < \frac{3}{2}$, $\bar{\omega}_1^p > W^a$. Since $1 + \frac{1}{\sqrt{5}} > \frac{7}{5}$, $W^a > \omega_1^u$. Further, $W^a \omega_0^p$ iff $\rho < \frac{2\sqrt{5}}{5}$. Thus, arbitrageurs are better off in the partly constrained equilibrium iff $W_0 \in [\max(\omega_1^u, \omega_0^p), W^a[$. ■

### F.2 Corollary 2

**Proof.** The first point is obvious. The condition under which the constraint does not bind is $W_0 \geq \max(\omega_0^u, \omega_1^u, \omega_1^p)\mathbb{1}_{\rho<0.7\cap\rho>0.75} + h\mathbb{1}_{0.7\leq\rho\leq 0.75}$, where $h$ is the threshold represented by a dotted line on Figure 9.

The second point follows from Proposition 9 and the fact that, in the absence of constraint, the unique equilibrium is $u_0, u_1$.

The third point follows from comparing hedgers' welfare in the $u_0, u_1$ vs $c_0, c_1$ or $c_0, u_1$ equilibria. From Proposition **??** and Corollary **??** [prop u1 and coro spread u1], recall that $x_0^{c_0,u_1} = \bar{x}_0 < x_0^{u_0,u_1}$ and $X_1^{c_0,u_1} = \frac{s+\bar{x}_0}{2}$. This implies that $\Delta_1^{c_0,u_1} = a\sigma^2(s - \bar{x}_0) > \Delta_1^{u_0,u_1}$ and that $\Delta_0^{c_0,u_1} - \Delta_1^{c_0,u_1} = 2a\sigma^2(s - \bar{x}_0)$. Then using (25), $U_0^{c_0,u_1} < U_0^{u_0,u_1}$ iff

$$\frac{5}{8}a\sigma^2(s - \bar{x}_0)^2 - \frac{3}{2}a\sigma^2(s - \bar{x}_0) < \frac{5}{8}a\sigma^2(s - x_0^{u_0,u_1})^2 - \frac{3}{2}a\sigma^2(s - x_0^{u_0,u_1})$$

After a few lines of simple algebra, this condition boils down to

$$\frac{5}{2}(\bar{x}_0^2 - (x_0^{u_0,u_1})^2) < a\sigma^2 s(x_0^{u_0,u_1} - \bar{x}_0)$$

Thus, $x_0^{c_0,u_1} = \bar{x}_0 < x_0^{u_0,u_1}$ implies that this condition is satisfied, so hedgers' welfare decreases when imposing constraints leads to $c_0, u_1$.

Similarly, we get

$$U_0^{c_0,c_1} = \frac{4a^2\sigma^2(s-\bar{x}_0)^2 + 4a^2\sigma^2(s-\bar{X}_1)^2}{8a\sigma^2} - \frac{s}{2}\left[2a\sigma(s-\bar{x}_0) + 2a\sigma^2(s-\bar{X}_1)\right]$$

Thus the condition $U_0^{c_0,c_1} < U_0^{u_0,u_1}$ can be simplified to

$$\frac{1}{2}a\sigma^2\left[(s-\bar{x}_0)^2 - (s-x_0^{u_0,u_1})^2 + (s-\bar{X}_1)^2 - (s-X_1^{u_0,u_1})^2\right] < a\sigma^2 s\left[x_0^{u_0,u_1} - \bar{x}_0 + X_1^{u_0,u_1} - \bar{X}_1\right],$$

which can be further reduced to

$$\frac{1}{2}a\sigma^2\left[\bar{x}_0^2 - (x_0^{u_0,u_1})^2 + \bar{X}_1^2 - (X_1^{u_0,u_1})^2\right] < 0$$

This condition holds true since when the constraint binds, $\bar{x}_0 < x_0^{u_0,u_1}$ and $\bar{X}_1 < X_1^{u_0,u_1}$ (the latter also follows from the analysis in the proof of Proposition 11). Hence hedgers' welfare also decreases when imposing constraints leads to $c_0, c_1$. ■

## F.3 Proposition 10

**Proof.** *Counterfactual 1.* I compute hedgers' welfare using equation (25), under the assumptions that $x_0^{cf1} = x_0^{u_0,u_1} = \frac{2}{5}s$ and $X_1^{cf1} = X_1^{u_0,c_1} = \frac{a\sigma^2 s - \bar{e} + \sqrt{d_1^+(x_0^{u_0,c_1})}}{2a\sigma^2}$. These quantities imply the following spreads:

$$\Delta_1^{cf1} = \Delta_1^{u_0,c_1}, \quad \Delta_0^{cf1} = 2a\sigma^2(s - x_0^{u_0,u_1} + \Delta_1^{u_0,c_1} = \frac{6}{5}s + \Delta_1^{u_0,c_1} > \Delta_0 u_0, c_1$$

Substituting into (25), we obtain

$$U_0^{cf1} = \frac{\left(\frac{6}{5}s\right)^2 + (\Delta_1^{u_0,c_1})^2}{8a\sigma^2} - \frac{s}{2}\left(\frac{6}{5}s + \Delta_1^{u_0,c_1}\right)$$

Therefore, $U_0^{cf1} < U_0^{u_0,c_1}$ can be simplified into $\frac{27}{25} > \frac{3}{4}$, which holds true.

*Counterfactual 2.* The quantities are $x_0^{u_0,c_1} = \frac{s}{2}$ and $X_1^{cf2} = X_1^{u_0,u_1} = \frac{7}{10}s$, implying that $\Delta_1^{cf2} = \Delta_1^{u_0,u_1} = \frac{3}{5}a\sigma^2 s$, $\Delta_0^{cf2} - \Delta_1^{cf2} = a\sigma^2(s - x_0^{u_0,c_1}) = a\sigma^2 s$, and $\Delta_0^{cf2} = a\sigma^2 s + \Delta_1^{cf2} = \frac{8}{5}a\sigma^2 s$. Substituting these spreads into equation (25), we get $U_0^{cf2} = -\frac{63}{100}a\sigma^2 s^2$. Compariing this welfare level to $U_0^{u_0,u_1}$, we get $U_0^{cf2} < U_0^{u_0,u_1} < U_0^{u_0,c_1}$. ■

### F.4 Proposition 11

**Proof.** *Constrained competitive vs partly constrained monopoly.* Starting from equations of the spreads (8) and (23), we see that $\Delta_1^{u_0,c_1} < \Delta_1^*$ is equivalent to $d_1^* < d_1^+\left(\frac{s}{2}\right)$. Substituting for these two expressions using (22) and Corollary 3, this condition becomes

$$2\bar{e}\sqrt{d_0^*} < 2a\sigma^2 W_0 + a^2\sigma^4 s^2 - 2a\sigma^2 s\bar{e} + 2\bar{e}^2$$

Since $W_0 \geq \omega_0^p$, the left-hand side is positive. Then raising both sides to the square and rearranging terms, the condition boils down to

$$\left[a\sigma^2 W_0 + \frac{1}{2}\left((a\sigma^2 s - \bar{e})^2 - \bar{e}^2\right)\right]^2 > 0$$

Hence the condition is always satisfied: if a partly constrained equilibrium exists, the time-1 spread is strictly smaller with a monopolistic arbitrageur than a continuum of constrained competitive arbitrageurs if $W_0 \in ]\omega_0^p, \bar{\omega}_1^p[$, and equal if $W_0 = \omega_0^p$.

*Constrained competitive vs unconstrained monopoly.* Using the spreads given in Corollary **??** and Proposition **??**, we get $\Delta_1^{u_0,u_1} < \Delta_1^*$ iff $\sqrt{d_1^*} < \frac{2}{5}a\sigma^2 s + \bar{e}$. This implies that $d_1^* < \frac{4}{25}a^2\sigma^4 s^2 + \bar{e}^2 + \frac{4}{5}a\sigma^2 s\bar{e}$. Substituting for $d_1^*$, the condition becomes:

$$\bar{e}\sqrt{d_0^*} < \bar{e}^2 - \frac{21}{50}a^2\sigma^4 s^2 + \frac{2}{5}a\sigma^2 s\bar{e} \tag{30}$$

The right-hand side is positive if $\rho^2 + \frac{2}{5}\rho - \frac{21}{50} > 0$, which is the case iff $\rho > \frac{\sqrt{46}}{10} - \frac{1}{5}$. So if $\rho < \frac{\sqrt{46}}{10} - \frac{1}{5}$, $\Delta_1^{u_0,u_1} > \Delta_1^*$ for any $W_0 < \omega^*$.

Suppose now that $\rho \geq \frac{\sqrt{46}}{10} - \frac{1}{5}$. Then raising both sides to the square, we can rearrange condition

30 as

$$W_0 < \omega^m \equiv \frac{7}{5}s\bar{e} - \frac{21}{25}a\sigma^2 s^2 + \frac{21^2}{5000}\frac{a^3\sigma^6 s^4}{\bar{e}^2} - \frac{21}{125}\frac{a^2\sigma^4 s^3}{\bar{e}}$$

I now study the position of $\omega^m$ relative to $\omega_0^u$, $\omega_1^u$, and $\omega^*$.

- $\omega^m > 0$ is equivalent to $\rho^3 - \frac{3}{5}\rho^2 - \frac{3}{25}\rho + \frac{63}{1000} > 0$. This cubic polynom has three real roots, one negative, and two positive, with $\rho_1 \approx 0.33$ and $\rho_2 \approx 0.64$. Given the sign of the first derivative, $\omega^m > 0$ iff $\rho \in [0, \rho_1[ \cup ]\rho_2, +\infty[$.

- $\omega_1^u > \omega^m$ is equivalent to $-\frac{1}{2}\rho^2 + \frac{7}{5}\rho - \frac{147}{.}200 > 0$. There are two roots, $\frac{7}{10}$ and $\frac{21}{10}$, such that $\omega_1^u > \omega^m \Leftrightarrow \rho \in \left]\frac{7}{10}, \frac{21}{10}\right[$

- $\omega_0^u > \omega^m$ is equivalent to $3\left(-\rho^3 + \frac{3}{5}\rho^2 + \frac{7}{25}\rho - \frac{147}{1000}\right) > 0$, which is equivalent to $-3(\rho - \frac{7}{10})(\rho - r_1)(\rho - r_2)$, with $r_1 = -\left(\frac{1}{20} + \frac{1}{12}\sqrt{\frac{153}{5}}\right)$ and $r_= -\frac{1}{20} + \frac{1}{12}\sqrt{\frac{153}{5}}$. As a result, $\omega_0^u > \omega^m$ iff $\rho \in \left]r_2, \frac{7}{10}\right[$.

- $\omega^* > \omega^m$ is equivalent to $\rho^3 + \frac{7}{5}\rho^2 + \frac{7}{25}\rho - \frac{147}{1000} > 0$. This polynom has two complex roots and one real root equal to approximately 0.22. Given the sign of the derivatives, $\omega^* > \omega^m$ iff $\rho > r_3 \approx 0.22$.

Given these thresholds, we obtain the following cases:

1. For $\frac{\sqrt{46}}{10} \leq \rho < \rho_2 \approx 0.64$, $\omega_1^u < \omega^m < 0 < \omega_0^u < \omega^*$, so $W_0 \geq 0$ implies $W_0 \geq \omega^m$, so for any $W_0 \in [\omega^u, \omega^*[$, $\Delta_1^{u_0,u_1} > \Delta_1^*$.

2. For $\rho_2 \leq \rho < \frac{7}{10}$, $\omega_1^u < \omega^m$ and $0 < \omega^m < \omega_0^u < \omega^*$, so $\max(\omega_0^u, \omega_1^u) \geq \omega^m$, hence $W_0 \geq \omega^u$ implies $W_0 \geq \omega^m$, so for any $W_0 \in [\omega^u, \omega^*[$, $\Delta_1^{u_0,u_1} > \Delta_1^*$.

3. For $\frac{7}{10} \leq \rho < \frac{21}{10}$, $0 < \omega_0^u < \omega^m < \omega_1^u < \omega^*$, so $\max(\omega_0^u, \omega_1^u) \geq \omega^m$, hence $W_0 \geq \omega^u$ implies $W_0 \geq \omega^m$, so for any $W_0 \in [\omega^u, \omega^*[$, $\Delta_1^{u_0,u_1} > \Delta_1^*$ (same as case 2).

4. For $\frac{21}{10} \leq \rho$, $0 < \omega_0^u < \omega_1^u < \omega^m < \omega^*$, so there are two cases:

   (a) For $W_0 \in [\omega^u, \omega^m[$, $\Delta_1^{u_0,u_1} < \Delta_1^*$.

(b) For $W_0 \in [\omega^m, \omega^*[$, $\Delta_1^{u_0,u_1} > \Delta_1^*$.

■

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