# Hedge Fund Performance under Misspecified Models

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**Abstract**

We develop a new approach for evaluating hedge fund performance. Our approach accounts for model misspecification which is expected given the multiple factors that drive hedge fund returns. It is also simple, informative about the entire alpha distribution, and designed for formal comparison tests between models. The empirical results show that the standard hedge fund models perform exactly like the CAPM and produce large and positive alphas. In contrast, performance drops significantly with alternative models that include new factors examined in the recent literature. Overall, the results suggest that hedge funds commonly follow mechanical strategies such as carry trade, time-series momentum, liquidity and correlation trading.

*Keywords:* Hedge funds, performance, model misspecification, large panel

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## 1. Introduction

Over the past three decades, the growth of the hedge-fund industry has been outstanding. The total size of the industry has increased from approximately $40 billion 1990 to close to $3 trillion at the end of 2016 (see Getmansky et al., 2015).

This strong demand from investors such as high-net-worth individuals and pension funds reflects the widely held view that hedge funds deliver positive risk-adjusted returns (*i.e.*, the alpha component). This view is supported by the arguments that hedge fund managers are more sophisticated, less constrained, and more incentivized to perform than mutual fund managers. Consistent with these arguments, the previous literature documents a strong and positive hedge fund performance—for instance, Kosowski et al. (2007) find an cross-sectional average alpha of 5.0% per year using the popular model of Fung and Hsieh (2001).[1]

A valid concern is that this strong performance is too good to be true. Hedge funds follow a wide range of mechanical trading strategies (*e.g.*, Pedersen, 2015) – in particular, they may load on "hidden" factors to generate higher average returns. This complexity strongly suggests that hedge fund models are misspecified, i.e., they do not include all the factors required to properly benchmark performance (Bollen, 2013). As a result, the estimated alphas are likely to be inflated because they are contaminated by the return premia of the omitted factors.

Misspecification calls for an extensive comparison of multiple models. This analysis provides a description of how performance varies across models. Therefore, it allows us to measure the impact of using more complex models than the CAPM – a choice commonly made in the hedge fund literature. Comparing models also provides a sharper measurement of performance. Because existing models include different sets of factors, some are likely to produce alphas that are less

[1]An non-exhaustive list of papers that also document positive hedge fund alphas includes Ackermann et al. (1999), Capocci and Hübner (2004), Buraschi et al. (2014), Rios and Garcia (2010), Getmansky et al. (2015), Liang (1999). More recently, Chen et al. (2017) estimate the entire alpha distribution and find that around 50% of the funds deliver positive alphas – a number that is substantially larger than the one documented for mutual funds (see Barras et al., 2010; Harvey and Liu, 2018).

affected by the omitted factors. Finally, this analysis measures the contribution of each factor and is useful to construct future models that are less prone to misspecification.

In this paper, we develop a novel approach to compare misspecified hedge fund models. Our approach provides an estimation of the entire alpha distribution obtained with any misspecified model. It is simple, informative, and allows for formal comparison tests. It is simple because it uses as only inputs the estimated fund alphas to compute the (i) moments, (ii) cumulative distribution function (cdf), and (iii) quantiles of the alpha distribution. It is informative because it captures the large heterogeneity in performance across funds – an analysis that cannot be performed by simply looking at the average alpha. Finally, it allows for formal comparison tests derived from a full-fledged asymptotic theory as the number of funds $n$ and return observations $T$ grow large.

Misspecification has a strong impact on the asymptotic properties of the estimated characteristics of the distribution (moments, cdf, quantile). Compared to the correctly specified case (Barras et al., 2019), they are less precisely estimated because their convergence rate depends on $T$, and not on $n$ – a result that seems surprising because these estimators are computed as cross-sectional averages (*i.e.*, we sum across funds, not over time). The reason is that the omitted factors produce a correlated structure among the fund returns. Therefore, the estimated characteristics depend on the mean of these omitted factors whose precision depends on $T$. Another important difference is that the estimated characteristics are unbiased even though they are computed using the estimated alphas (instead of the true values). This implies that the common practice of computing distribution characteristics (*e.g.*, box plots) from estimated coefficients is theoretically valid in the misspecified case.

We apply our methodology to the monthly returns of more than 13,000 hedge funds collected from five different providers (BarclayHedge, Eurekahedge, HFR, Morningstar, and TASS) over the period 1994–2016 (276 observations). Following the classification of Joenväärä et al. (2019), we sort all funds into three broad hedge fund categories: (i) equity funds (long/short and market neutral), (ii) macro funds (global macro, CTA funds), and (iii) arbitrage funds (relative value, event-driven). Using this extensive database, we then evaluate hedge fund performance using standard hedge fund models, as well alternative models based on factors recently proposed in the asset pricing literature.

Our performance analysis based on the standard hedge fund models reveals several insights. First, we observe a striking similarity between the standard models and the CAPM. The average alpha equals 2.62% per year with the CAPM, and is not statistically different from the averages obtained with the Carhart model (2.16%) and the Fung-Hsieh model (2.82%). In addition, we observe the same results for all the other distribution characteristics, including the proportion of funds with positive alphas which is above 65% for all models.

Second, our analysis indicates that the standard models are largely misspecified. Whereas the overall performance of the hedge fund industry seems economically large, the low explanatory power of the standard models provides a clear signal of misspecification. The average $R^2$ ranges between 20% and 30% across the models, leaving plenty of room for omitted factors. Therefore, these results call for an analysis of alternative models – a point forcefully made by the recent paper by Joenväärä et al. (2019) which highlights "the need for an updated benchmark model that reflects the post–2004 literature".

Using alternative models has a strong impact on hedge fund performance. Starting our analysis with equity funds, we consider an updated set of six factors. These factors are easily interpretable as they capture the excess return of mechanical strategies followed by equity funds. Adding each of these factors to the CAPM lowers hedge fund performance. For instance, adding the correlation and variance factors lowers the average alpha by 0.96% per year. Combining all factors leads to a statistically significant difference equal to 1.65% per year.

Next, we turn to the analysis of macro funds. These funds take directional positions in multiple asset classes. To capture their average returns, we, therefore, consider a set of eight global carry and time-series momentum strategies. Whereas the CAPM produces an alpha of 3.35% per year, including these eight factors yields the opposite conclusion that macro funds deliver negative performance (-0.50%).

Finally, we examine arbitrage funds which implement convergence trades by identifying similar securities that trade at different prices. Similar to the other categories, we show that the updated factors reduce the alpha from 3.04% to 1.82% per year.

The remainder of the paper is as follows. Section 2 presents our framework for evaluating hedge fund performance. Section 3 describes our novel estimation approach. Section 4 presents the hedge fund dataset and factors. Section 5 contains the empirical analysis, and Section **??** concludes. The appendix provides additional information regarding the methodology, the data, and the empirical results.

## 2. Hedge Fund Performance and Model Misspecification

### *2.1. Hedge Fund Performance*

#### *2.1.1. The Benefits of Performance Evaluation*

The objective of this paper is to evaluate the performance of individual hedge funds. In other words, we examine whether the hedge fund industry provides investors with superior returns net of fees and trading costs. Our framework exclusively focuses on performance and thus explicitly distinguishes it from skill. Whereas the two notions are commonly used interchangeably, they differ in important ways – a point forcefully made by Berk and van Binsbergen (2015). Skill is defined from the viewpoint of funds, *i.e.*, it measures whether hedge funds have unique skills that allow them to create value. In contrast, performance is defined from the viewpoint of investors, *i.e.*, it measures whether the value created by the funds, if any, is passed on to them.

Performance evaluation provides a separation between the alpha and beta components of hedge fund returns. Both components are important to guide the overall asset allocation. The alpha captures the benefits of active management – for one, a positive alpha allows the hedge fund investor to improve the risk-return trade-off of her portfolio (Treynor and Fisher, 1973). The betas capture the fund's exposures to common hedge fund factors and represent key inputs for managing the overall risk of the investor's portfolio.

Measuring performance is particularly important for hedge funds for several reasons. First, there is a commonly held view that hedge funds achieve positive returns because they load on "hidden" sources of risk orthogonal to traditional equity factors.A proper performance analysis can determine how many funds produce negative alphas and charge excessive fees to their investors. Second, the performance analysis allows for an investor-specific interpretation of the beta component of hedge fund returns. As noted by Cochrane (2013) and Pedersen (2015), some hedge fund trading strategies are not based on superior information but require technical knowledge. For instance, implementing a momentum strategy requires trading skills to mitigate the impact of transaction costs. The hedge fund investor can isolate the trading strategies that she finds harder to replicate, and attribute a positive alpha to hedge funds if they are able to implement them at a lower cost.

### *2.1.2. Measuring Performance*

The basic idea for measuring performance is straightforward. For each fund $i$ in a population of $n$ funds ($i = 1, ..., n$), we measure its performance using the net alpha $\alpha_i^*$:

$$\alpha_i^* = E[r_{i,t}] - E[r_{i,t}^B] = E[r_{i,t}] - \beta_i^{*\prime} E[f_t] = E[r_{i,t}] - \beta_i^{*\prime} \lambda \,, \tag{1}$$

where $r_{i,t}$ is the fund excess return net of trading costs and fees, and $r_{i,t}^B$ is the excess return of the benchmark portfolio assigned to fund $i$. The benchmark is defined as a linear combination of a set of tradeable hedge fund strategies $f_t$ whose risk premium vector is defined as $\lambda = E[f_t]$. The mechanical trading strategies potentially earn positive risk premia for several reasons. They may capture the impact of systematic risk, market frictions (*e.g.*, leverage constraints), or behavioral biases. In this paper, we remain agnostic on this issue – our objective is simply to avoid giving credit to the fund for following mechanical strategies that can be directly implemented by investors.

If we know the correct model, we can estimate the alpha of each fund from the following time-series regression:

$$r_{i,t} = \alpha_i^* + \beta_i^{*\prime} f_t + \varepsilon_{i,t}^* \,, \tag{2}$$

where $\varepsilon_{i,t}^*$ denotes the fund residual term. Equation (2) is interpreted as a random coefficient model (*e.g.*, Hsiao, 2003) in which the fund alpha $\alpha_i^*$ is not a fixed parameter, but a random realization from a continuum of funds. Under this sampling scheme, we can invoke cross-sectional limits to infer the entire cross-sectional alpha distribution $\phi(\alpha^*)$, which is formally referred to as the probability distribution function (pdf) (see Gagliardini et al., 2016; Barras et al., 2019).

### 2.2. *Model Misspecification*

#### 2.2.1. *The Impact of Model Misspecification*

In practice, measuring hedge fund performance is challenging because of model misspecification. This issue arises because hedge funds follow a wide range of strategies. They invest in many asset classes (e.g., equities, bonds, and derivatives), and trade in both developed and emerging markets (*e.g.* Lhabitant, 2007; Pedersen, 2015). They also engage in dynamic trading strategies which capture the time variation in betas that results from changes in economic and leverage conditions (see Ang et al., 2011; Patton and Ramadorai, 2013).[2] As a result, any model used for measuring hedge fund performance is likely to be misspecified as it only captures a limited number of strategies.

To elaborate, suppose that instead of using the correct model in Equation (2), we work with a misspecified model that only includes the factors $f_{I,t}$, but omits the factors $f_{O,t}$ (with $f_t = (f_{I,t}', f_{O,t}')'$):

$$r_{i,t} = \alpha_i + \beta_i' f_{I,t} + \varepsilon_{i,t} \,. \tag{3}$$

[2] For instance, suppose that a given fund changes its allocation to the equity market linearly based on current economic conditions measured by the demeaned variable $z_{t-1}$. In this case, the market beta equals $\beta_{m,i,t-1} = \beta_{m,i,0}^* + \beta_{m,i,1}^* z_{t-1}$, and the correct benchmark is given by $r_{i,t}^B = \beta_i^{*\prime} f_t = [\beta_{m,i,0}^*, \beta_{m,i,1}^*][r_{m,t}, z_{t-1} r_{m,t}]'$, where $r_{m,t}$ is the excess market return, and $z_{t-1} r_{m,t}$ is the excess return of the dynamic strategy.

The fund alpha $\alpha_i$ obtained with this misspecified model typically differs from the true alpha $\alpha_i^*$. To see this point, we can write the omitted factors as:

$$f_{O,t} = \alpha_O + \Psi_{O,I} f_{I,t} + u_{O,t}\,, \tag{4}$$

where $\Psi_{O,I}$ is the matrix of slope coefficients, $u_{O,t}$ is the vector of factor residuals, and $\alpha_O$ is the vector of factor alphas, *i.e.*, the risk premia of the omitted factors left unexplained by the included factors: $\alpha_O = \lambda_O - \Psi_{O,I}\lambda_I$. Writing the fund average return as $E[r_{i,t}] = \alpha_i^* + \beta_{i,I}^{*\prime}\lambda_I + \beta_{i,O}^{*\prime}\lambda_O = \alpha_i + \beta_{i,I}^{\prime}\lambda_I$, and noting that $\beta_{i,I} = \beta_{i,I}^* + \Psi_{O,I}^{\prime}\beta_{i,O}^*$, we obtain:

$$\alpha_i = \alpha_i^* + \beta_{i,O}^{*\prime}[\lambda_O - \Psi_{O,I}\lambda_I] = \alpha_i^* + \beta_{i,O}^{*\prime}\alpha_O\,, \tag{5}$$

where $\alpha_i$ is a function of the true alpha $\alpha_i^*$ and the omitted factor component $\beta_{i,O}^{*\prime}\alpha_O$.[3] Equation (5) implies that the cross-sectional alpha distribution $\phi(\alpha)$ is informative about the true alpha distribution $\phi(\alpha^*)$. However, this information is noisy because $\phi(\alpha)$ is also impacted by the distributions of the hedge fund betas on the omitted factors $f_{O,t}$.

*2.2.2. Comparison of Misspecified Models*

To mitigate the impact of misspecification, we develop a new approach for measuring performance across multiple models. This comparison analysis is important for several reasons. First, it sharpens the estimation of the true alpha distribution $\phi(\alpha^*)$. Because existing models include different sets of factors, some are likely to do a better job at reducing the omitted factor component in Equation (5). Second, this analysis determines how the estimated fund alphas vary across existing models. In particular, it measures the impact of choosing models that are significantly more com-

[3]If the factors are uncorrelated ($\Psi_{O,I}^k = 0$), Equation (5) becomes: $\alpha_i = \alpha_i^* + \beta_{i,O}^{*\prime}\lambda_O$, where the impact of each omitted factor is captured by its risk premium – a quantity that does not depend on the specific factors $f_{I,t}$ included in the misspecified model (contrary to $\alpha_O$). This assumption is largely consistent with the data because hedge funds factors tend to be weakly correlated (see Table (III)).

plex than the CAPM – a choice commonly made in the hedge fund literature.[4] Third, this analysis sheds light on the economic importance of the different factors. Focusing on the contribution of each factor is useful to construct future models that are less prone to misspecification.

To formalize this analysis, we consider a set of $K$ misspecified models. Each model $k$ ($k = 1, ..., K$) includes the factors $f^k_{I,t}$, but omits the factors $f^k_{O,t}$ (with $f_t = (f^{k\prime}_{I,t}, f^{k\prime}_{O,t})'$). Building on Equation (5), we can write the difference between the mean of the alpha distribution $\phi(\alpha^k)$ under each model $k$ and that of the true distribution $\phi(\alpha^*)$ as:

$$E[\alpha^k_i] - E[\alpha^*_i] = E[\beta^{k*}_{i,O}]'\alpha^k_O\,, \tag{6}$$

where $E[\beta^{k*}_{i,O}]$ is positive to the extent that hedge funds earn positive returns by loading on the multiple factors included in the vector $f_{I,t}$. In this case, the average alpha obtained with one specific model may significantly overestimate the true value if the number of omitted factors is large (*i.e.*, $E[\beta^{k*}_{i,O}]'\alpha^k_O$ is large). Comparing models allows us to identify values of $E[\alpha^k_i]$ that seem implausibly large.

Using Equation (5), we can also examine how the distributions $\phi(\alpha^k)$ and $\phi(\alpha^*)$ differ in terms of their variance:

$$var[\alpha^k_i] - var[\alpha^*_i] = \alpha^{k\prime}_O \Sigma^k_\beta \alpha^k_O + \alpha^{k\prime}_O cov[\beta^k_{i,O}, \alpha^*_i]\,, \tag{7}$$

where $\Sigma^k_\beta$ denotes the covariance matrix of $\beta^k_{i,O}$ across funds, and $cov[\beta^k_{i,O}, \alpha^*_i]$ denotes the vector of covariances between $\beta^k_{i,O}$ and $\alpha^*_i$ across funds. Similar to Equation (6), a comparison analysis can improve the inference on the dispersion in alpha. Under a specific model, a subset of funds may take unusually large exposures to the omitted factors to boost their returns. As these funds artificially deliver a stellar performance, the measured dispersion in alpha is biased upward (*i.e.*, $\alpha^{k\prime}_O \Sigma^k_\beta \alpha^k_O$ is large).

[4] See Getmansky et al. (2015) and Agarwal et al. (2015) for a review of hedge fund models that typically include a large number of factors across multiple asset classes.

## 3. Methodology

### *3.1. Overview of the Estimation Approach*

We now describe our novel approach for evaluating performance across models. This approach brings two main benefits. First, it explicitly allows each model to be misspecified. As discussed above, this flexibility is important in the context of hedge fund performance because all models are likely to be misspecified. Our approach reveals that accounting for model misspecification dramatically changes the statistical inference on performance evaluation.

Second, our approach applies to a large set of characteristics that determine the shape of the alpha distribution $\phi(\alpha)$. Therefore, our performance analysis departs from that of previous studies which commonly compare hedge fund models using the average alpha only. Whereas this measure captures the overall performance of the hedge fund industry, it does not account for the potentially large heterogeneity across individual funds.

We consider the following characteristics of the alpha distribution: (i) the centered moments (*e.g.*, mean, volatility, skewness), (ii) the cumulative distribution function (cdf) which measures the probability that the fund alpha is below a given threshold $a$ (*e.g.*, $a = 0$), and (iii) the quantile at a given percentile level $u$ (*e.g.*, $u = 10\%$). For each estimated quantity, we derive its asymptotic distribution as the numbers of funds and return observations grow large (simultaneous double asymptotics with $n$ and $T \to \infty$). We can, therefore, conduct a proper statistical inference to formally evaluate and compare the performance obtained with multiple models.

### *3.2. Estimation of the Fund Alpha*

Our approach uses as only inputs the estimated alphas of all funds in the population. For each fund $i$ ($i = 1, ..., n$), we run the time-series regression in Equation (3). The OLS vector of coefficients is given by

$$\hat{\gamma}_i = \hat{Q}_{x,i}^{-1} \frac{1}{T_i} \sum_t I_{i,t} x_t r_{i,t} \,, \quad (8)$$

where $I_{i,t}$ is an indicator variable equal to one if $r_{i,t}$ is observable (and zero otherwise), $T$ is the total number of periods, $T_i = \sum_t I_{i,t}$, $x_t = (1, f'_{I,t})'$, and $\hat{Q}_{x,i} = \frac{1}{T_i}\sum_t I_{i,t}x_t x'_t$ (to lighten notation, we do not superscript $\hat{\gamma}_i$, $f_{I,t}$ and $x_t$ by $k$). Because the panels of hedge fund returns is unbalanced, the random sample size $T_i$ for some funds can be very small. In this case, the inversion of matrix $\hat{Q}_{x,i}$ is numerically unstable, which yields unreliable estimates of $\hat{\gamma}_i^k$. To address this issue, we follow Barras et al. (2019) and introduce a formal fund selection rule $\mathbf{1}_i^{\chi}$ equal to one if the following two conditions are met (and zero otherwise): $\mathbf{1}_i^{\chi} = \mathbf{1}\left\{CN_i \leq \chi_{1,T}, \tau_{i,T} \leq \chi_{2,T}\right\}$, where $CN_i = \sqrt{eig_{\max}\left(\hat{Q}_{x,i}\right)/eig_{\min}\left(\hat{Q}_{x,i}\right)}$ denotes the condition number of matrix $\hat{Q}_{x,i}$, $\tau_{i,T} = T/T_i$, and $\chi_{1,T}$, $\chi_{2,T}$ denote the two threshold values.

The first condition $CN_i \leq \chi_{1,T}$ excludes funds for which the time series regression is poorly conditioned, *i.e.*, a large value of $CN_i$ indicates multicollinearity problems and ill-conditioning (*e.g.*, Belsley et al., 2004). The second condition $\tau_{i,T} \leq \chi_{2,T}$ excludes funds for which the sample size is too small. Both thresholds $\chi_{1,T}$ and $\chi_{2,T}$ increase with the sample size $T$ – with more return observations, the fund coefficients are estimated with greater accuracy which allows for a less stringent selection rule. We denote the total number of funds that satisfy this selection rule by $n_\chi = \sum_n \mathbf{1}_i^{\chi}$.

### *3.3. Statistical Inference under each Model*

#### *3.3.1. The Moments*

We now examine the characteristics of the alpha distribution $\phi(\alpha)$ under a given model $k$. To this end, we briefly present the main theoretical results for the centered moments, and the cdf and quantile of $\phi(\alpha)$. Next, we interpret these results and explain how they are impacted by model misspecification.

We denote by $M_j$ each moment of the distribution $\phi(\alpha)$, and by $\bar{M}_j$ its corresponding estimator. For instance, we have for the mean and the volatility: $M_1 = E[\alpha_i]$, $M_2 = (E[\alpha_i^2] - E[\alpha_i]^2)^{1/2}$, and $\bar{M}_1 = \frac{1}{n_\chi}\sum_i \hat{\alpha}_i \mathbf{1}_i^{\chi}$, $\bar{M}_2 = \left(\frac{1}{n_\chi}\sum_i \left(\hat{\alpha}_i^2 \mathbf{1}_i^{\chi} - \bar{M}_1^2\right)\right)^{1/2}$. The following proposition derives the asympotic distribution of each estimated moment $\bar{M}_j$ in the misspecified case.

**Proposition 1.** *As $n, T \to \infty$, such that $T/n = o(1)$,*

$$\sqrt{T}\left(\bar{M}_j - M_j\right) \Rightarrow N\left(0, V_{M_j}\right),$$

*where:*

$$V_{M_j} = \left(\eta'_{M_j} \otimes E'_1 Q_x^{-1}\right) \Omega_{ux} \left(\eta_{M_j} \otimes Q_x^{-1} E_1\right), \tag{9}$$

*and* $\eta_{M_j} = E\left[\left(\frac{\partial M_j}{\partial E[g]}\right)' \frac{\partial g}{\partial \alpha_i}\right] \beta^*_{O,i}$, *$E[g]$ is the vector of uncentered moments with* $g = [\alpha_i, ..., \alpha_i^j]'$, $E_1 = (1, 0')'$, $Q_x = E[x_t x_t']$ *and* $\Omega_{ux} = \lim_{T\to\infty} V\left[\frac{1}{\sqrt{T}} \sum_t u_{O,t} \otimes x_t\right]$.

***Proof.*** *See the Appendix.*

*3.3.2. The Cumulative Distribution Function and the Quantile*

Next, we turn to the analysis of the cumulative distribution function (cdf) and the quantile. The cdf at a given point $a$ measures the probability that $\alpha_i$ is below $a$ and is denoted by $P(a) = P[\alpha_i \leq a]$. The quantile at a given percentile $u$ is given by the inverse function: $Q(u) = P^{-1}(u)$. Their estimators denoted by $\hat{P}(a)$ and $\hat{Q}(u)$ are given by $\hat{P}(a) = \frac{1}{n_\chi} \sum_i \mathbf{1}\{\hat{\alpha}_i \leq a\} \mathbf{1}_i^\chi$ and $\hat{Q}(u) = \hat{P}^{-1}(u)$. The following proposition derives the asympotic distributions of the estimated cdf $\hat{P}(a)$ and quantile $\hat{Q}(u)$ in the misspecified case.

**Proposition 2.** *As $n, T \to \infty$, such that $T/n = o(1)$,*

$$\sqrt{T}\left(\hat{P}(a) - P(a)\right) \Rightarrow N\left(0, V_P(a)\right),$$
$$\sqrt{T}\left(\hat{Q}(u) - Q(u)\right) \Rightarrow N\left(0, V_Q(u)\right),$$

*where* $V_P(a) = \left(\eta_P(a)' \otimes E'_1 Q_x^{-1}\right) \Omega_{ux} \left(\eta_P(a) \otimes Q_x^{-1} E_1\right)$, $\eta_P(a) = E[\beta^*_{O,i} | \alpha_i = a] \phi(a)$, *$\phi(a)$ is the alpha distribution (i.e., the pdf evaluated at $a$), and* $V_Q(u) = \frac{V_P(Q(u))}{\phi(Q(u))^2}$.

***Proof.*** *See the Appendix.*

*3.3.3. Interpretation of the Results*

Propositions 1 and 2 show that the estimated characteristics of the alpha distribution (moments, cdf, quantile) all share similar properties. First, they are asymptotically normally distributed, which facilitates the construction of confidence intervals. Second, they are consistent, *i.e.*, they converge towards the true values as $n$ and $T$ grow large. Put differently, we can infer the alpha distribution $\phi(\alpha)$ even though we do not observe the fund alphas themselves, but only their estimated values ($\hat{\alpha}_i$ instead of $\alpha_i$). Third, all estimated characteristics converge at a rate equal to $\sqrt{T}$. This last result is a priori surprising because the estimated characteristics are all computed as cross-sectional averages (*i.e.*, we sum across $n$, not across $T$).

These properties depart significantly from those derived by Barras et al. (2019) for a correctly specified model. In this case, the estimated characteristics have smaller variance terms. Specifically, their convergence rate is equal to $\sqrt{n}$ – a rate that is much faster than $\sqrt{T}$ with a total population of several thousand funds. Another notable difference is that the estimated characteristics suffer from an Error-in-Variable (EIV) bias. This bias arises because the estimated fund alphas are used as as inputs ($\hat{\alpha}_i$ instead of $\alpha_i$), and must be corrected for to conduct inference on $\phi(\alpha)$. Strikingly, the EIV bias adjustment is unnecessary if the model is misspecified. Therefore, we show that the common practice of computing summary statistics based on estimated quantities is theoretically valid in the misspecified case.

To understand these differences, we need to exmamine the properties of the residual term. The residuals $\varepsilon^*_{i,t}$ ($i = 1, ..., n$) in the correctly specified case (Equation (2)) are weakly correlated across funds. In contrast, the residuals $\varepsilon_{i,t}$ ($i = 1, ..., n$) in the misspecified case (Equation (3)) are strongly correlated across funds because they include the omitted factors (via $u_{O,t}$), *i.e.*, we have $\varepsilon_{i,t} = \varepsilon^*_{i,t} + \beta^{*\prime}_{O,i} u_{O,t}$. A key implication of this strong correlation is that the estimated characteristics of the alpha distribution depend heavily on the realized value of the omitted factors.[5] The variance term is therefore larger because it inherits the slower convergence rate of the mean of the omitted factors - this rate is equal to $\sqrt{T}$ following standard convergence results in time-series analysis. In addition, the variance term dwarfs the EIV bias in magnitude, which makes the bias

[5] To illustrate, suppose that we estimate the mean $M_1$. If the mispecified model only omits one factor independent of the others (*i.e.*, $E[u_{O,t}u'_{O,t}|f_{I,t}] = V[f_{O,t}]$), and the fund betas are all equal to one (*i.e.*, $\beta_{O,i} = 1$), the variance of $\bar{M}_1$ in Equation (9) simply becomes: $V_{M_1} = c \cdot V[f_{O,t}]$, where the scalar $c$ is equal to $\left(1 + E[f_{I,t}]' V[f_{I,t}]^{-1} E[f_{I,t}]\right)^{-1}$. Therefore, the asymptotic value of $\bar{M}_1$ is entirely driven by the value taken by the omitted factor $f_{O,t}$.

adjustement procedure unncessary.

*3.4. Statistical Inference across Models*

*3.4.1. Comparison of the Moments*

We can extend our previous analysis to formally compare the moments of the alpha distribution under two different models. To this end, we denote by $\Delta M_j$ the difference between the jth moment of the alpha distribution obtained with models $k$ and $l$ and by $\Delta \bar{M}_j$ the corresponding estimator, where $\Delta M_j = M_j^k - M_j^l$ and $\Delta \bar{M}_j = \bar{M}_j^k - \bar{M}_j^l$.[6] The following proposition derives the asymptotic distribution of the estimated moment difference $\Delta \bar{M}_j$ in the misspecified case.

**Proposition 3.** *As $n, T \to \infty$, such that $T/n = o(1)$,*

$$\sqrt{T}\left(\Delta \bar{M}_j - \Delta M_j\right) \Rightarrow N\left(0, V_{\Delta M_j}\right),$$

*where $V_{\Delta M_j} = V_{M_j^k} + V_{M_j^l} - 2Cov[\bar{M}_j^k, \bar{M}_j^l]$, $V_{M_j^k}$, $V_{M_j^l}$ are given in Proposition 1, $Cov[\bar{M}_j^k, \bar{M}_j^l] = \left(\eta_{m_j^{k\prime}} \otimes E_1' Q_{x^k}^{-1}\right) \Omega_{ux}^{kl} \left(\eta_{m_j^l} \otimes Q_{x^l}^{-1} E_1\right)$, and $\Omega_{ux}^{kl} = \lim_{T\to\infty} Cov\left[\frac{1}{\sqrt{T}}\sum_t u_{O,t}^k \otimes x_t^k, \frac{1}{\sqrt{T}}\sum_t u_{O,t}^l \otimes x_t^l\right]$.*
***Proof**. See the Appendix.*

*3.4.2. Comparison of the Cumulative Distribution Function and Quantiles*

Repeating the above procedure for models $k$ and $l$, we denote by $\Delta P(a)$ the difference in cdf at the threshold $a$, by $\Delta Q(u)$ the difference in quantile at the percentile $u$, and by $\Delta \bar{P}(a)$, $\Delta \bar{Q}(u)$ their corresponding estimators, where $\Delta P(a) = P^k(a) - P^l(a)$, $\Delta Q(u) = Q^k(u) - Q^l(u)$, $\Delta \bar{P}(a) = \bar{P}^k(a) - \bar{P}^l(a)$, and $\Delta \bar{Q}(u) = \bar{Q}^k(u) - \bar{Q}^l(u)$.

The following proposition derives the asympotic distribution of the estimated cdf and quantile differences $\Delta \bar{P}(a)$ and $\Delta \bar{Q}(u)$ in the misspecified case.

[6] We assume in Proposition 3 that the two models are misspecified. If one model is correctly specified, the convergence rate of the estimated moment equals $\sqrt{n}$, which is much faster than the rate of $\sqrt{T}$ under the misspecified model. In this case, the asymptotic distribution of $\Delta \bar{M}_j$ is solely driven by the estimated moment under the misspecified model (*i.e.*, we can treat the estimated moment under the correctly specified model as known).

**Proposition 4.** *As $n, T \to \infty$, such that $T/n = o(1)$,*

$$\sqrt{T}\left(\Delta\hat{P}(a) - P(a)\right) \Rightarrow N\left(0, V_{\Delta P(a)}\right) \ ,$$
$$\sqrt{T}\left(\Delta\hat{Q}(u) - Q(u)\right) \Rightarrow N\left(0, V_{\Delta Q(u)}\right) \ ,$$

*where* $V_{\Delta P(a)} = V_{P^k(a)} + V_{P^l(a)} - 2Cov[\bar{P}^k(a), \bar{P}^l(a)]$, $V_{\Delta Q(u)} = V_{Q^k(u)} + V_{Q^l(u)} - 2Cov[\bar{Q}^k(u), \bar{Q}^l(u)]$, $V_{P^k(a)}$, $V_{P^l(a)}$, $V_{Q^k(u)}$, $V_{Q^l(u)}$ *are given in Proposition 2, and:*

$$Cov[\bar{P}^k(a), \bar{P}^l(a)] = \left(\eta_P^{k\prime}(a) \otimes E_1' Q_{x^k}^{-1}\right) \Omega_{ux}^{kl} \left(\eta_P^l(a) \otimes Q_{x^l}^{-1} E_1\right) \ . \tag{10}$$

***Proof.*** *See the Appendix.*

*3.4.3. Hypothesis Tests*

From Proposition 3, we can formally test the the null hypothesis that the jth moments $M_j^k$ and $M_j^l$ obtained with models $k$ and $j$ are equal:

$$H_0 : \Delta M_j = 0 \, . \tag{11}$$

Similarly, we can use Proposition 4 to test the null hypothesis that the cdf or the quantile remains unchanged as we move from model $k$ to model $l$:

$$H_0 : \Delta P(a) = 0 \, , \tag{12}$$

$$H_0 : \Delta Q(u) = 0 \, . \tag{13}$$

Our approach yields a straightforward testing procedure. The estimated differences in characteristics are all normally distributed, which allows for a simple computation of the rejection thresholds. Moreover, this procedure applies to the comparison of both (i) nested models (*i.e.*, one model is a restricted version of the other), and (ii) non-nested models (*i.e.*, no model is a special case of the other). This appealing property holds because of our specific framework in which both $n$ and $T$

grow large.[7]

*3.5. Estimation of the Asymptotic Variance*

To conduct statistical inference both within and across models, we need to estimate the asymptotic variance of the different estimators. The main difficulty is that each variance term in Propositions 1–4 depend on the omitted factors $u_{O,t}$ and their loadings $\beta_{O,t}$, which are not directly observable. To address this issue, we derive a consistent estimator of $V$ based on the observed residuals of each model, defined as $\hat{\varepsilon}_{i,t} = r_{i,t} - x_t'\hat{\gamma}_i$ $(i = 1, ..., n)$.

To simplify the exposition, we consider the case where $\varepsilon_{i,t}$ is independent over time.[8] We consider the following generic expressions for (i) the variance $\hat{V}$ of each estimated characteristic (*i.e.*, $\bar{M}_j, \bar{P}(a), \bar{Q}(u)$) , and (ii) the variance $\hat{V}_\Delta$ of each estimated difference in characteristics between models $k$ and $l$ (*i.e.*, $\Delta\bar{M}_j, \Delta\bar{P}(a), \Delta\bar{Q}(u)$):

$$\hat{V} = \frac{1}{n^2 T} \sum_i \sum_j \sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t} \mathbf{1}_j^\chi \tau_{j,T} I_{j,t} \hat{a}_{i,t} \hat{a}_{j,t}' \,, \tag{14}$$

$$\hat{V}_\Delta = \frac{1}{n^2 T} \sum_i \sum_j \sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t} \mathbf{1}_j^\chi \tau_{j,T} I_{j,t} \hat{a}_{i,t}^k \hat{a}_{j,t}^{l\prime} \,, \tag{15}$$

where the terms $\hat{a}_{i,t}$, $\hat{a}_{i,t}^k$, $\hat{a}_{i,t}^l$ depend on the specific characteristic. For instance, we have the following expressions for the mean $M_1$ and its difference $\Delta M_1$: $\hat{a}_{i,t} = E_1'\hat{Q}_x^{-1}\hat{\varepsilon}_{i,t}x_t$, $\hat{a}_{i,t}^k = E_1'\hat{Q}_{x^k}^{-1}\hat{\varepsilon}_{i,t}^k x_t^k$, and $\hat{a}_{i,t}^l = E_1'\hat{Q}_{x^l}^{-1}\hat{\varepsilon}_{i,t}^l x_t^l$. The appendix reports the expressions of $\hat{a}_{i,t}$, $\hat{a}_{i,t}^k$, and $\hat{a}_{i,t}^l$ for the other characteristics, and shows that $\hat{V}$ and $\hat{V}_\Delta$ are consistent estimators of $V$ and $V_\Delta$ as $n$ and $T$ grow large.

[7] In contrast, Kan and Robotti (2011) and Kan et al. (2013) show that in a setting where $n$ is fixed, the formal comparison of asset pricing models is complicated because it depends on whether they are nested or not.

[8] This assumption holds if the residual $\varepsilon_{i,t}^*$ and the omitted factors $f_{O,t}$ are independent over time. When this is not the case, we simply need to modify the variance estimator by including weighted cross-terms at different time dates (Newey and West, 1987).

## 4. Data Description

### *4.1. Hedge Fund Database*

We evaluate hedge fund performance using the monthly net-of-fee USD returns of individual funds over the period 1994-2016 (276 observations). We take several steps to mitigate the various sources of bias that arise because hedge funds are free to report their performance or not. First, we create an exhaustive universe of funds by aggregating five different data providers (BarclayHedge, Eurekahedge, HFR, Morningstar, and TASS). Because under-performing funds typically report to only one provider, combining databases offers a better representation of these funds and thus reduces the upward selection bias in individual databases (Joenväärä et al., 2019). Second, we address survivorship bias by including the dead funds from the graveyard hedge fund databases available from January 1994 onward. Third, we follow previous studies and delete the first 12 months of data to mitigate the backfill bias when funds start reporting to databases.[9] The appendix provides more detail on the construction of the hedge fund dataset.

We evaluate the performance of several hedge fund strategies across three broad hedge fund categories: equity, macro, and arbitrage. This classification facilitates the identification of relevant factors within each category and thus initially excludes multi-strategy funds and funds of funds. The first category (equity) includes long/short and market neutral funds. The second category (macro) includes global macro funds and Commodity Trading Advisors (CTAs). Finally, the third category (arbitrage) includes relative value and event-driven funds. The mapping of strategies across the different databases builds on the recent paper by Joenväärä et al. (2019) and is described in the appendix.

Table I reports summary statistics for our hedge fund dataset. For each category/strategy, we construct an equally-weighted portfolio of all existing funds at the start of each month. We then

[9] A more stringent approach is to eliminate all the return observations before the fund listing date to the database. As noted by Fung and Hsieh (2009), this approach potentially discards important information about the fund performance by eliminating all the early years of returns (in some cases, more than 5 years) – a period during which performance is typically quite strong (Aggarwal and Jorion, 2010). In addition, the listing date is not provided by all databases. For these reasons, we only report the results obtained with this alternative approach in the appendix.

report the average number of funds per month, the mean and volatility of the portfolio monthly excess return (annualized), as well as the skewness and kurtosis, and the 10th and 90th percentiles. Overall, the results are in line with those documented by Getmansky et al. (2015) over a similar period (1996–2014). For instance, they find that the mean-volatility pair equals 4.7%-3.3% for market neutral funds and 6.8%-5.9% for event-driven funds, versus 3.9%-3.2% and 6.9%-6.2% in Table I. Consistent with intuition, we find that long-short and CTA funds exhibit higher levels of volatility as they typically take more directional positions.

To conduct our fund-level performance analysis, we need to apply the two selection rules described in Section 3.2. First, we fix the minimum number of return observations equal to 36 ($\tau_{i,T} \leq 276/36$) to keep funds with short return histories. Second, we follow Barras et al. (2019) and impose that the condition number of the matrix of regressors $\hat{Q}_{x,i}$ is below 15 ($CN_i \leq 15$). Combining these two rules, we obtain a total number of 13,305 funds over our sample period.[10]

[Insert Table I about here.]

### *4.2. Hedge Fund Factors*

Building on previous work, we collect the return time-series of 28 economically-motivated factors. These factors are easily interpretable as they capture the excess return of mechanical strategies that hedge funds potentially follow. As such, they depart from purely statistical factors extracted from a PCA analysis. Using this large set of factors, we then evaluate hedge fund performance and conduct an extensive comparison of (i) standard hedge fund models and (ii) alternative models based on new combinations of factors.

Table II provides the list of factors which are presented in more detail in the appendix. Group 1 includes the set of US equity factors, which are the CRSP market index and the traditional size, value, momentum, investment, and profitability factors. It also includes the Pastor-Stambaugh (PS)

[10]The second filtering criterion is redondant with respect to the first one.

liquidity and Betting-Against-Beta (BAB) factors. Group 2 contains strategies across multiple asset classes. It includes the term and default factors for US bonds, the Goldman Sachs commodity and US dollar indices, and the two factors value and momentum "everywhere" for international equities, bonds, commodities, and currencies. Group 3 includes a set of option-based strategies, which are the correlation and variance factors inferred from options on the S&P 500 and its constituents, and three look-back option straddles on bonds, commodities, and currencies.[11] Group 4 includes the carry trade strategies for international equities, bonds (level, slope), commodities, and currencies. Group 5 includes time-series momentum (TSM) strategies constructed for international equities, bonds, commodities, and currencies.

Table II also reports summary statistics for the excess return of each factor. Consistent with previous studies, we find that the mean is positive for all factors. In other words, the mechanical strategies associated with the 28 factors all earn positive premia (*i.e.*, $\lambda > 0$). Therefore, a given fund that loads on these strategies will tend to exhibit higher average returns. We also examine the correlation across factors. To this end, we compute the correlations (in absolute value) for all factor pairs, and take an average both within and across the five groups. Table III shows that the factors are weakly correlated, even within specific groups (*e.g.*, the average correlation equals 0.19 across time-series momentum factors). Unreported results further show that only four of the 378 pairwise correlations are above 0.6 (in absolute value). Overall, these results imply that the factors capture distinct hedge fund trading strategies.

[Insert Tables II and III about here.]

[11] Contrary to the other factors, the option-based strategies are expected to deliver negative risk premia because they perform well in bad times when realized volatility/correlation is high. To maintain consistency with the other factors, we therefore multiply their returns by -1 such that the investor takes a short position in these option-based strategies.

## 5. Empirical Results

### *5.1. Performance Analysis with Standard Models*

We begin our analysis by examining the standard models for evaluating fund performance. Some of these models are very popular in the hedge fund literature because they aim at explaining the returns of multiple hedge fund strategies. In other words, they are designed as "omnibus" models that include multiple factors across several asset classes. In addition to the CAPM which serves as a natural reference point, we examine:

1. FF3: The three-factor model of Fama and French (1993) which includes the market, size, and value factors ($J = 3$);
2. AMP: The model of Asness et al. (2013) which adds the two factors value and momentum everywhere to the market index ($J = 4$);
3. CA: The four-factor model of Carhart (1997) which adds the momentum factor to FF3 ($J = 4$).
4. The five-factor model of Fama and French (2015, FF5) which adds the investment and profitability factors to FF3 ($J = 5$);
5. HL: The model of Hasanhodzica and Lo (2007) which includes the market and size factors, the term and default bond factors, and the commodity and US dollar indices ($J = 6$)
6. FH: The model of Fung and Hsieh (2001) which includes the market, size, term, and default factors, and the lookback option straddles on bonds, commodities, and currencies ($J = 7$).

For each model, we apply the approach outlined in Section 3 to estimate several characteristics of the cross-sectional alpha distribution $\phi(\alpha)$. Specifically, we report the annualized mean and volatility ($\hat{M}_1$ and $\hat{M}_2$), the annualized quantiles at 10% and 90% ($\hat{Q}(0.1)$ and $\hat{Q}(0.9)$), and the probabilities that the fund alpha is negative and positive ($\hat{P}(0)$ and $1 - \hat{P}(0)$).

A first look at Table IV, Panel A, shows that all the models produce the same average performance. The average alpha equals 2.62% per year with the CAPM and barely changes as we include more factors. For instance, it is equal to 2.44% with FF5 and 2.77% per year with the FH model. This strong similarity resonates with the analysis of Getmansky et al. (2015) which shows that a simplified version of FH with only four factors leaves the average performance of the hedge fund industry largely unchanged.

More importantly, our fund-level analysis reveals that the similarity in performance extends beyond the average. We find that all the remaining characteristics (volatility, quantiles, proportions) remain largely unchanged across models. In other words, the alpha distributions share identical dispersion and tail properties. Moreover, each model provides the same assessment of the performance of each fund. For instance, unreported results show that the fund-level correlation between the CAPM alpha and the FH alpha is equal to 0.94.

In Table IV, Panels B–D, we repeat this analysis for the three hedge fund categories (equity, macro, arbitrage). For equity funds, the largest difference in the mean and volatility across models is a mere 0.57% and 0.31% per year. These numbers remain small for the other categories. The difference in average alpha between each model and the CAPM remains below 0.94% (in absolute value) in all but three cases (AMP, CA, and FF5 for macro funds). Overall, these findings are largely consistent with those obtained for the entire population.

Taken at face value, the empirical evidence implies that the performance of the hedge fund industry is economically large. Table IV reveals that more than 60% of the funds deliver a positive alpha – a proportion that is very close to the one documented by Chen et al. (2017). In addition, a minority of funds produce a stellar performance. Across all models, we find that the top decile of funds delivers annual alphas above to 11% per year.

Alternatively, these results are due to misspecification – an interpretation that highlights the challenges faced by any model in explaining hedge fund returns. To examine this issue, we compute the adjusted $R^2$ for each fund obtained from the time-series regression in Equation (3). Using the FH model, Bollen (2013) notes that many hedge funds have a $R^2$ close to zero, yet their residual volatility cannot be diversified away. Therefore, a low $R^2$ signals misspecification, *i.e.*, it reveals the presence of omitted factors on which hedge funds load. Similarly, we find that the explanatory power of each model is relatively low. Table IV shows that the average $R^2$ ranges between 20% and 30% across the six models, leaving plenty of room for potentially omitted factors.

[Insert Table IV about here.]

In the next section, we examine whether alternative models can capture these omitted factors and mitigate the impact of misspecification. To guide the choice of these models, we study separately the three hedge fund categories (equity, macro, and arbitrage). This approach brings two benefits. First, it allows us to identify a set of plausible economic factors that are specific to each category. Second, it mitigates data-mining concerns that arise if we were to evaluate a very large number of randomly formed models.

### *5.2. Performance Analysis with Alternative Models*

#### *5.2.1. Equity Funds*

We begin our analysis with the equity category (long/short and market neutral funds). Equity funds rely on discretionary or quantitative analysis to buy undervalued stocks and sell short overvalued stocks. As a result, the returns of equity funds are potentially related to several mechanical trading strategies. First, equity funds can be exposed to correlation and variance risks. An unexpected rise in correlation/variance limits the funds' ability to balance risk between their long and short positions. Furthermore, it typically occurs in crisis times when prime brokers tighten their funding conditions (Buraschi et al., 2014). Second, equity funds take liquidity risk when they buy small-cap stocks or accommodate selling pressure in the market. Third, they have more leverage capacity than traditional investors and can therefore benefit from the undervaluation of low-risk stocks – the so-called BAB factor (Frazzini and Pedersen, 2014). Finally, equity funds invest across several international equity markets. To determine their overall allocation, they may follow carry and time-series momentum strategies which favor markets with higher dividend yields and higher past returns.

We capture these trading strategies using the following simple extensions of the CAPM:

1. COR: A correlation model which includes the correlation and variance factors of Buraschi et al. (2014) ($J = 3$);
2. LIQ: A liquidity model which includes the Pastor and Stambaugh (2003) traded liquidity factor ($J = 2$);
3. LEV: A leverage model which includes the BAB factor of Frazzini and Pedersen (2014) ($J = 2$);
4. CAR-E: A carry model which includes the equity carry factor of Koijen et al. (2018) ($J = 2$);
5. TSM-E: A time-series momentum model which includes the equity time-series momentum factor of Moskowitz et al. (2012) ($J = 2$).
6. COMB: A combined model which includes all factors above ($J = 7$).

Table V confirms that the above trading strategies help explain the returns of equity funds. We find that all six models produce a more conservative evaluation of hedge fund performance. For instance, adding the correlation and variance factors yields an average alpha equal to 0.89% per year (versus 1.85% with the CAPM). The change in performance is particularly striking with the combined model – the average alpha is equal to 0.20% per year and the proportion of funds with a positive alpha drops to 53% (versus 66% for the CAPM).

At the same time, adding more trading strategies to the CAPM does not reduce the dispersion of the alpha distribution. For one, Table V shows that the quantile at 90% remains largely constant across models. Intuitively, we expect the dispersion to shrink because the models purge the alpha of each fund from its exposures to the added factors Contrary to this view, the volatility of the combined model is actually higher than that of the CAPM (X and Y per year). It must, therefore, be the case that equity funds load on the different factors in a correlated manner.

[Insert Tables V and VI about here.]

*5.2.2. Macro Funds*

We now turn to the analysis of the macro category (global macro and CTA funds). Contrary to equity funds, macro funds invest in multiple asset classes, including equities, bonds, commodities, and currencies. In general, they take directional bets using broad economic and financial indicators (*e.g.*, GDP growth, inflation). Within each asset class, they favor assets that have a positive carry in order to earn a positive income even if prices do not move (Pedersen, 2015). They also exploit short-term trends in asset prices caused by behavioral biases, frictions, or slow moving capital. This description suggests that global carry and time-series momentum strategies potentially are important drivers of the returns of macro funds.

We incorporate these trading strategies into five extensions of the CAPM:

1. CAR-EB: A carry model which includes the equity and bond (level and slope) carry factors of Koijen et al. (2018) ($J = 3$);
2. CAR-CC: A carry model which includes the commodity and currency carry factors of Koijen et al. (2018) ($J = 3$);
3. TSM-EB: A time-series momentum model which includes the equity and bond time-series momentum factors of Moskowitz et al. (2012) ($J = 3$);
4. TSM-CC: A time-series momentum model which includes the commodity and currency time-series momentum factors of Moskowitz et al. (2012) ($J = 3$).
5. COMB: A combined model which includes all factors above ($J = 9$).

[Insert Tables VII and VIII about here.]

*5.2.3. Arbitrage Funds*

We finally examine the arbitrage category (relative value and event-driven). Arbitrage funds implement convergence trades by identifying similar securities that trade at different prices. Whereas relative value funds exploit price discrepancies in the debt market (*e.g.*, fixed income, convertible bond arbitrage), event-driven funds focus on corporate events (*e.g.*, merger arbitrage). This arbitrage strategies are potentially related to several mechanical trading strategies. First, arbitrage funds may be exposed to the correlation factor because hedging strategies become less effective under unpredictable changes in correlation. Second, they are sensitive to variance risk because they are engaged in option trading (*e.g.*, mortgage, and volatility arbitrage). Third, these funds take liquidity risk when they trade in the convertible bond market or absorb the selling pressure after merger announcements. Finally, fixed income funds may invest in the different bond markets by leveraging their positions in low-risk securities or following carry trade strategies.

We capture these trading strategies using the following extensions of the CAPM:

1. COR: The correlation model which includes the correlation and variance factors (similar to equity funds).
2. LIQ: The liquidity model which includes the liquidity factor (similar to equity funds).
3. LEV: The leverage model which includes the BAB factor (similar to equity funds).
4. CAR-EB: The equity-bond carry trade model which includes the equity and bond carry factors (similar to macro funds).

[Insert Tables IX and X about here.]

## References


Ackermann, C., McEnally, R., Ravenscraft, D., 1999. The performance of hedge funds: Risk, return, and incentives. Journal of Finance 54, 833–874. doi:10.1111/0022-1082.00129.

Agarwal, V., Mullally, K.A., Naik, N.Y., 2015. Hedge funds: A survey of the academic literature. Foundations and Trends in Finance 10, 1–111. doi:10.1561/0500000047.

Aggarwal, R.K., Jorion, P., 2010. The performance of emerging hedge funds and managers. Journal of Financial Economics 96, 238–256. doi:10.1016/j.jfineco.2009.12.010.

Ang, A., Gorovyy, S., van Inwegen, G.B., 2011. Hedge fund leverage. Journal of Financial Economics 102, 102–126. doi:10.1016/j.jfineco.2011.02.020.

Asness, C.S., Moskowitz, T.J., Pedersen, L.H., 2013. Value and momentum everywhere. Journal of Finance 68, 929–985. doi:10.1111/jofi.12021.

Barras, L., Gagliardini, P., Scaillet, O., 2019. Skill and value creation in the mutual fund industry. doi:10.2139/ssrn.3269995. Working paper.

Barras, L., Scaillet, O., Wermers, B., 2010. False discoveries in mutual fund performance: Measuring luck in estimated alphas. Journal of Finance 65, 179–216. doi:10.1111/j.1540-6261.2009.01527.x.

Belsley, D.A., Kuh, E., Welsch, R.E., 2004. Regression Diagnostics: Identifying Influential Data and Sources of Collinearity. Wiley. doi:10.1002/0471725153.

Berk, J.B., van Binsbergen, J.H., 2015. Measuring skill in the mutual fund industry. Journal of Financial Economics 118, 1–20. doi:10.1016/j.jfineco.2015.05.002.

Bollen, N.P.B., 2013. Zero-R2 hedge funds and market neutrality. Journal of Financial and Quantitative Analysis 48, 519–547. doi:10.1017/s0022109013000112.

Buraschi, A., Kosowski, R., Trojani, F., 2014. When there is no place to hide: Correlation risk and the cross-section of hedge fund returns. Review of Financial Studies 27, 581–616. doi:10.1093/rfs/hht070.

Capocci, D., Hübner, G., 2004. Analysis of hedge fund performance. Journal of Empirical Finance 11, 55–89. doi:10.1016/j.jempfin.2002.12.002.

Carhart, M., 1997. On persistence in mutual fund performance. Journal of Finance 52, 57–82. doi:10.1111/j.1540-6261.1997.tb03808.x.

Chen, Y., Cliff, M., Zhao, H., 2017. Hedge funds: The good, the bad, and the lucky. Journal of Financial and Quantitative Analysis 52, 1081–1109. doi:10.1017/s0022109017000217.

Cochrane, J.H., 2013. Finance: Function matters, not size. Journal of Economic Perspectives 27, 29–50. doi:10.1257/jep.27.2.29.

Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3–56. doi:10.1016/0304-405X(93)90023-5.

Fama, E.F., French, K.R., 2015. A five-factor asset pricing model. Journal of Financial Economics 116, 1–22. doi:10.1016/j.jfineco.2014.10.010.

Frazzini, A., Pedersen, L.H., 2014. Betting against beta. Journal of Financial Economics 111, 1–25. doi:10.1016/j.jfineco.2013.10.005.

Fung, W., Hsieh, D.A., 2001. The risk in hedge fund strategies: Theory and evidence from trend followers. Review of Financial Studies 14, 313–341. doi:10.1093/rfs/14.2.313.

Fung, W., Hsieh, D.A., 2009. Measurement biases in hedge fund performance data: An update. Financial Analysts Journal 65, 36–38. doi:10.2469/faj.v65.n3.6.

Gagliardini, P., Ossola, E., Scaillet, O., 2016. Time-varying risk premium in large cross-sectional equity data sets. Econometrica 84, 985–1046. doi:10.3982/ecta11069.

Getmansky, M., Lee, P.A., Lo, A.W., 2015. Hedge funds: A dynamic industry in transition. Annual Review of Financial Economics 7, 483–577. doi:10.1146/annurev-financial-110311-101741.

Harvey, C.R., Liu, Y., 2018. Detecting repeatable performance. Review of Financial Studies 31, 2499–2552. doi:10.1093/rfs/hhy014.

Hasanhodzica, J., Lo, A.W., 2007. Can hedge-fund returns be replicated?: The linear case. Journal of Investment Management 5, 5–45. doi:10.2139/ssrn.924565.

Hsiao, C., 2003. Analysis of panel data. Cambridge University Press. doi:10.1017/CBO9781139839327.

Joenväärä, J., Kauppila, M., Kosowski, R., Tolonen, P., 2019. Hedge fund performance: Are stylized facts sensitive to which database one uses? doi:10.2139/ssrn.1989410. Forthcoming in Critical Finance Review.

Kan, R., Robotti, C., 2011. On the estimation of asset pricing models using univariate betas. Economics Letters 110, 117–121. doi:10.1016/j.econlet.2010.11.004.

Kan, R., Robotti, C., Shanken, J., 2013. Pricing model performance and the two-pass cross-sectional regression methodology. Journal of Finance 68, 2617–2649. doi:10.1111/jofi.12035.

Koijen, R.S.J., Moskowitz, T.J., Pedersen, L.H., Vrugt, E.B., 2018. Carry. Journal of Financial Economics 127, 197–225. doi:10.1016/j.jfineco.2017.11.002.

Kosowski, R., Naik, N., Teo, M., 2007. Do hedge funds deliver alpha? A Bayesian and bootstrap analysis. Journal of Financial Economics 84, 229–264. doi:10.1016/j.jfineco.2005.12.009.

Lhabitant, F.S., 2007. Handbook of hedge funds. Wiley. doi:10.1002/9781119202028.

Liang, B., 1999. On the performance of hedge funds. Financial Analysts Journal 55, 72–85.

Moskowitz, T.J., Ooi, Y.H., Pedersen, L.H., 2012. Time series momentum. Journal of Financial Economics 104, 228–250. doi:10.1016/j.jfineco.2011.11.003.

Newey, W.K., West, K.D., 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703. doi:10.2307/1913610.

Pastor, L., Stambaugh, F., 2003. Liquidity risk and expected stock returns. Journal of Political Economy 111, 642–685. doi:10.1086/374184.

Patton, A., Ramadorai, T., 2013. On the high frequency dynamics of hedge fund risk exposures. Journal of Finance 68, 597–635. doi:10.1111/jofi.12008.

Pedersen, L.H., 2015. Efficiently inefficient. Princeton University. doi:10.2307/j.ctt1287knh.

Rios, A.D.D.L., Garcia, R., 2010. Assessing and valuing the nonlinear structure of hedge fund returns. Journal of Applied Econometrics 26, 193–212. doi:10.1002/jae.1147.

Treynor, J.L., Fisher, B., 1973. How to use security analysis to improve portfolio selection. Journal of Business 46, 66–86.

**TABLE I. Hedge Funds Descriptive Statistics**
This table reports, for each investment category, the average number of funds per month, the average and volatility excess return (annualized), the skewness and kurtosis, and the 10th and 90th percentiles.

| | Number | Mean(Ann.) | Std(Ann.) | Skewness | Kurtosis | 10% | 90% |
|---|---|---|---|---|---|---|---|
| All Funds | 2,947 | 5.95 | 5.91 | -0.28 | 4.04 | -1.58 | 2.54 |
| Equity | 1,279 | 6.97 | 8.78 | -0.48 | 4.91 | -2.48 | 3.36 |
| Long/Short | 1,132 | 7.35 | 9.59 | -0.46 | 4.89 | -2.74 | 3.69 |
| Market Neutral | 161 | 3.89 | 3.21 | -0.39 | 5.21 | -0.68 | 1.42 |
| Macro | 844 | 4.93 | 6.36 | 0.47 | 3.45 | -1.70 | 2.69 |
| CTA/Managed Futures | 449 | 4.80 | 7.70 | 0.57 | 3.63 | -2.09 | 3.22 |
| Macro | 380 | 5.23 | 5.35 | 0.29 | 3.21 | -1.45 | 2.40 |
| Arbitrage | 824 | 6.01 | 5.11 | -1.92 | 13.34 | -1.07 | 1.94 |
| Event Driven | 284 | 6.92 | 6.18 | -1.41 | 8.58 | -1.41 | 2.49 |
| Relative Value | 539 | 5.41 | 4.62 | -2.32 | 18.32 | -0.87 | 1.65 |

**TABLE II. Factors Descriptive Statistics**
This table reports the average and volatility excess return (annualized), the skewness and kurtosis, and the 10th and 90th percentiles, of the various factors used in our study.

| | Mean(Ann.) | Std(Ann.) | Skewness | Kurtosis | 10%(Ann.) | 90%(Ann.) |
|---|---|---|---|---|---|---|
| **Panel A: US Equity** | | | | | | |
| Market Index (CRSP) | 7.56 | 15.13 | -0.71 | 4.17 | -5.11 | 6.00 |
| Size | 2.19 | 10.97 | 0.46 | 7.85 | -3.56 | 3.65 |
| Value | 2.91 | 10.70 | 0.14 | 5.57 | -2.95 | 3.62 |
| Momentum | 3.35 | 7.37 | 0.64 | 5.45 | -1.85 | 2.98 |
| Investment | 4.15 | 9.70 | -0.43 | 12.10 | -2.04 | 3.21 |
| Profitability | 4.99 | 17.63 | -1.49 | 13.26 | -5.14 | 5.39 |
| Liquidity | 6.31 | 12.43 | -0.13 | 4.11 | -3.84 | 4.98 |
| Betting Against Beta | 8.58 | 13.52 | -0.52 | 5.59 | -3.56 | 4.73 |
| **Panel B: Additional Asset Classes** | Mean(Ann.) | Std(Ann.) | Skewness | Kurtosis | 10%(Ann.) | 90%(Ann.) |
| US Bond Term | -0.14 | 0.78 | -0.18 | 4.68 | -0.27 | 0.27 |
| US Bond Default | 0.02 | 0.64 | 1.22 | 19.18 | -0.15 | 0.17 |
| Commodity Index (GS) | 0.59 | 21.81 | -0.34 | 4.19 | -7.60 | 7.51 |
| US Dollar Index | 0.60 | 5.76 | -0.42 | 4.80 | -1.96 | 2.07 |
| Value Everywhere | 2.33 | 6.22 | -0.70 | 13.40 | -1.54 | 1.79 |
| Momentum Everywhere | 3.81 | 7.82 | -0.32 | 5.33 | -2.31 | 2.74 |
| **Panel C: Options** | Mean(Ann.) | Std(Ann.) | Skewness | Kurtosis | 10%(Ann.) | 90%(Ann.) |
| US Equity Correlation | 82.14 | 50.73 | -2.40 | 17.70 | -5.29 | 19.47 |
| US Equity Variance | 1.19 | 1.31 | -7.25 | 82.05 | -0.05 | 0.34 |
| Bond Straddles | 20.09 | 52.77 | -1.31 | 5.24 | -18.99 | 17.73 |
| Commodity Straddles | 6.56 | 49.48 | -1.06 | 4.53 | -20.02 | 15.98 |
| Currency Straddles | 10.24 | 67.47 | -1.36 | 5.51 | -23.31 | 20.30 |
| **Panel D: Carry Trades** | Mean(Ann.) | Std(Ann.) | Skewness | Kurtosis | 10%(Ann.) | 90%(Ann.) |
| Equity Carry Trade | 8.21 | 9.41 | 0.50 | 6.03 | -2.27 | 4.01 |
| Bond Carry Trade - Level | 3.48 | 4.44 | -0.30 | 4.61 | -1.20 | 1.85 |
| Bond Carry Trade - Slope | 0.42 | 0.45 | 1.15 | 9.43 | -0.11 | 0.18 |
| Commodity Carry Trade | 9.88 | 17.19 | 0.18 | 3.49 | -5.43 | 7.18 |
| Currency Carry Trade | 4.75 | 7.36 | -0.67 | 4.76 | -2.49 | 2.72 |
| **Panel E: Momentum** | Mean(Ann.) | Std(Ann.) | Skewness | Kurtosis | 10%(Ann.) | 90%(Ann.) |
| Equity TS momentum | 18.99 | 26.88 | 0.12 | 3.17 | -8.37 | 11.80 |
| Bond TS momentum | 17.72 | 28.32 | 0.10 | 4.24 | -8.00 | 10.72 |
| Commodity TS momentum | 10.36 | 15.32 | -0.26 | 4.66 | -4.29 | 6.22 |
| Currency TS momentum | 11.63 | 17.94 | 0.38 | 5.23 | -4.80 | 6.84 |

**TABLE III. Factors Correlations**
This table reports the average, the median, and the 10th and 90th percentiles of the pairwise correlations (in absolute value) in the universe of factors.

| | Mean | Median | 10% | 90% |
|---|---|---|---|---|
| All Factors | 0.14 | 0.09 | 0.02 | 0.32 |
| US Equity | 0.23 | 0.19 | 0.04 | 0.49 |
| Additional asset classes | 0.20 | 0.10 | 0.03 | 0.49 |
| Options | 0.25 | 0.20 | 0.12 | 0.38 |
| Carry trades | 0.09 | 0.07 | 0.05 | 0.17 |
| Momentum | 0.19 | 0.18 | 0.12 | 0.27 |

**TABLE IV. Standard models**

This table reports summary statistics for the cross-sectional distribution of the hedge funds alphas obtained with the standard models. Summary statistics are the average and the standard deviation of alphas, the proportions of negative and positive alphas, and the 10th and 90th percentiles of the alphas. We also report the average $R^2$. See Section 5.1 for a description of the standard models.

| | Mean(Ann.) | Std(Ann.) | Neg | Pos | 10%(Ann.) | 90%(Ann.) | $\bar{R}^2$ |
|---|---|---|---|---|---|---|---|
| **Panel A: All Funds** | | | | | | | |
| CAPM | 2.62 | 8.99 | 31.31 | 68.69 | -6.38 | 11.02 | 18.75 |
| FF3 | 2.23 | 8.92 | 33.30 | 66.70 | -6.93 | 10.59 | 21.64 |
| AMP | 2.24 | 9.04 | 34.05 | 65.95 | -6.81 | 10.73 | 22.65 |
| CA | 2.16 | 8.76 | 33.80 | 66.20 | -6.67 | 10.39 | 23.30 |
| FF5 | 2.44 | 9.19 | 32.98 | 67.02 | -6.72 | 11.03 | 22.55 |
| HL | 2.88 | 9.50 | 30.81 | 69.19 | -5.71 | 11.34 | 29.77 |
| FH | 2.77 | 9.12 | 30.66 | 69.34 | -6.47 | 11.19 | 26.22 |
| **Panel B: Equity Funds** | | | | | | | |
| | Mean(Ann.) | Std(Ann.) | Neg | Pos | 10%(Ann.) | 90%(Ann.) | $\bar{R}^2$ |
| CAPM | 1.85 | 8.08 | 34.02 | 65.98 | -6.76 | 9.95 | 26.32 |
| FF3 | 1.47 | 8.02 | 36.59 | 63.41 | -7.21 | 9.35 | 30.87 |
| AMP | 1.71 | 8.25 | 37.07 | 62.93 | -7.24 | 9.80 | 31.17 |
| CA | 1.42 | 7.77 | 37.04 | 62.96 | -6.97 | 9.04 | 33.29 |
| FF5 | 1.97 | 8.28 | 35.36 | 64.64 | -6.45 | 10.18 | 32.13 |
| HL | 2.42 | 8.52 | 32.83 | 67.17 | -5.98 | 10.77 | 34.96 |
| FH | 2.17 | 8.10 | 33.06 | 66.94 | -6.77 | 10.33 | 31.22 |
| **Panel C: Macro Funds** | | | | | | | |
| | Mean(Ann.) | Std(Ann.) | Neg | Pos | 10%(Ann.) | 90%(Ann.) | $\bar{R}^2$ |
| CAPM | 3.35 | 11.13 | 31.32 | 68.68 | -7.21 | 13.75 | 8.64 |
| FF3 | 2.89 | 10.96 | 33.56 | 66.44 | -8.06 | 13.30 | 9.85 |
| AMP | 2.41 | 11.21 | 35.97 | 64.03 | -8.11 | 13.01 | 12.40 |
| CA | 2.56 | 10.79 | 35.48 | 64.52 | -7.88 | 12.91 | 10.72 |
| FF5 | 2.52 | 11.32 | 35.70 | 64.30 | -8.62 | 12.80 | 10.45 |
| HL | 2.82 | 12.23 | 35.08 | 64.92 | -7.39 | 13.21 | 18.62 |
| FH | 3.27 | 11.55 | 31.55 | 68.45 | -7.74 | 13.44 | 17.67 |
| **Panel D: Arbitrage Funds** | | | | | | | |
| | Mean(Ann.) | Std(Ann.) | Neg | Pos | 10%(Ann.) | 90%(Ann.) | $\bar{R}^2$ |
| CAPM | 3.04 | 7.70 | 27.22 | 72.78 | -4.64 | 10.25 | 17.51 |
| FF3 | 2.72 | 7.78 | 28.12 | 71.88 | -5.36 | 10.16 | 19.60 |
| AMP | 2.85 | 7.59 | 27.58 | 72.42 | -4.78 | 10.13 | 20.15 |
| CA | 2.88 | 7.74 | 27.26 | 72.74 | -4.98 | 10.05 | 20.91 |
| FF5 | 3.06 | 7.97 | 26.68 | 73.32 | -4.84 | 10.64 | 20.32 |
| HL | 3.63 | 7.53 | 23.50 | 76.50 | -3.42 | 10.68 | 33.18 |
| FH | 3.16 | 7.64 | 26.14 | 73.86 | -4.66 | 10.31 | 27.27 |

**TABLE V. Equity models**
This table reports, for the equity hedge funds, the cross-sectional average and standard deviation, the proportions, and the 10th and 90th percentiles, of the annualized alphas for the equity models. We also report the average $\bar{R}^2$. See Section 5.2.1 for a description of the equity models.

| | Mean(Ann.) | Std(Ann.) | Neg | Pos | 10%(Ann.) | 90%(Ann.) | $\bar{R}^2$ |
|---|---|---|---|---|---|---|---|
| CAPM | 1.85 | 8.08 | 34.02 | 65.98 | -6.76 | 9.95 | 26.32 |
| COR | 0.89 | 9.23 | 41.25 | 58.75 | -8.62 | 9.54 | 28.73 |
| LIQ | 1.49 | 8.05 | 36.77 | 63.23 | -7.43 | 9.68 | 27.63 |
| LEV | 1.61 | 8.70 | 38.23 | 61.77 | -6.92 | 9.85 | 29.29 |
| CARE | 1.44 | 8.20 | 37.25 | 62.75 | -7.53 | 9.98 | 26.88 |
| TSME | 1.37 | 8.05 | 36.83 | 63.17 | -7.42 | 9.38 | 27.99 |
| COMB | 0.20 | 9.91 | 47.34 | 52.66 | -9.26 | 9.40 | 33.55 |

**TABLE VI. Equity models – model comparison**

This table reports, for the equity hedge funds, the cross-sectional differential of average and standard deviation, the proportions, and the 10th and 90th percentiles, of the annualized alphas for the equity models. See Section 5.2.1 for a description of the equity models.

| | CAPM | COR | LIQ | LEV | CAR-E | TSM-E | COMB |
|---|---|---|---|---|---|---|---|
| Panel A: Average | | | | | | | |
| CAPM | | 0.96 | 0.36 | 0.23 | 0.41 | 0.47 | 1.65 |
| COR | -0.96 | | -0.60 | -0.73 | -0.55 | -0.48 | 0.69 |
| LIQ | -0.36 | 0.60 | | -0.12 | 0.05 | 0.12 | 1.29 |
| LEV | -0.23 | 0.73 | 0.12 | | 0.18 | 0.24 | 1.42 |
| CAR-E | -0.41 | 0.55 | -0.05 | -0.18 | | 0.06 | 1.24 |
| TSM-E | -0.47 | 0.48 | -0.12 | -0.24 | -0.06 | | 1.18 |
| COMB | -1.65 | -0.69 | -1.29 | -1.42 | -1.24 | -1.18 | |
| Panel B: Standard Deviation | | | | | | | |
| | CAPM | COR | LIQ | LEV | CAR-E | TSM-E | COMB |
| CAPM | | -1.15 | 0.04 | -0.62 | -0.12 | 0.03 | -1.83 |
| COR | 1.15 | | 1.18 | 0.53 | 1.03 | 1.18 | -0.68 |
| LIQ | -0.04 | -1.18 | | -0.66 | -0.15 | -0.01 | -1.86 |
| LEV | 0.62 | -0.53 | 0.66 | | 0.51 | 0.65 | -1.21 |
| CAR-E | 0.12 | -1.03 | 0.15 | -0.51 | | 0.14 | -1.71 |
| TSM-E | -0.03 | -1.18 | 0.01 | -0.65 | -0.14 | | -1.86 |
| COMB | 1.83 | 0.68 | 1.86 | 1.21 | 1.71 | 1.86 | |
| Panel C: Proportion of Negative Alphas | | | | | | | |
| | CAPM | COR | LIQ | LEV | CAR-E | TSM-E | COMB |
| CAPM | | -7.23 | -2.75 | -4.21 | -3.23 | -2.81 | -13.32 |
| COR | 7.23 | | 4.48 | 3.02 | 4.00 | 4.42 | -6.09 |
| LIQ | 2.75 | -4.48 | | -1.46 | -0.48 | -0.06 | -10.57 |
| LEV | 4.21 | -3.02 | 1.46 | | 0.99 | 1.40 | -9.11 |
| CAR-E | 3.23 | -4.00 | 0.48 | -0.99 | | 0.42 | -10.10 |
| TSM-E | 2.81 | -4.42 | 0.06 | -1.40 | -0.42 | | -10.51 |
| COMB | 13.32 | 6.09 | 10.57 | 9.11 | 10.10 | 10.51 | |
| Panel D: Proportion of Positive Alphas | | | | | | | |
| | CAPM | COR | LIQ | LEV | CAR-E | TSM-E | COMB |
| CAPM | | 7.23 | 2.75 | 4.21 | 3.23 | 2.81 | 13.32 |
| COR | -7.23 | | -4.48 | -3.02 | -4.00 | -4.42 | 6.09 |
| LIQ | -2.75 | 4.48 | | 1.46 | 0.48 | 0.06 | 10.57 |
| LEV | -4.21 | 3.02 | -1.46 | | -0.99 | -1.40 | 9.11 |
| CAR-E | -3.23 | 4.00 | -0.48 | 0.99 | | -0.42 | 10.10 |
| TSM-E | -2.81 | 4.42 | -0.06 | 1.40 | 0.42 | | 10.51 |
| COMB | -13.32 | -6.09 | -10.57 | -9.11 | -10.10 | -10.51 | |
| Panel E: 10th Percentile | | | | | | | |
| | CAPM | COR | LIQ | LEV | CAR-E | TSM-E | COMB |
| CAPM | | 1.86 | 0.67 | 0.16 | 0.76 | 0.66 | 2.49 |
| COR | -1.86 | | -1.19 | -1.70 | -1.10 | -1.20 | 0.63 |
| LIQ | -0.67 | 1.19 | | -0.51 | 0.10 | -0.01 | 1.83 |
| LEV | -0.16 | 1.70 | 0.51 | | 0.61 | 0.50 | 2.34 |
| CAR-E | -0.76 | 1.10 | -0.10 | -0.61 | | -0.11 | 1.73 |
| TSM-E | -0.66 | 1.20 | 0.01 | -0.50 | 0.11 | | 1.84 |
| COMB | -2.49 | -0.63 | -1.83 | -2.34 | -1.73 | -1.84 | |
| Panel F: 90th Percentile | | | | | | | |
| | CAPM | COR | LIQ | LEV | CAR-E | TSM-E | COMB |
| CAPM | | 0.40 | 0.27 | 0.09 | -0.04 | 0.56 | 0.55 |
| COR | -0.40 | | -0.13 | -0.31 | -0.44 | 0.16 | 0.15 |
| LIQ | -0.27 | 0.13 | | -0.18 | -0.31 | 0.29 | 0.28 |
| LEV | -0.09 | 0.31 | 0.18 | | -0.13 | 0.47 | 0.46 |
| CAR-E | 0.04 | 0.44 | 0.31 | 0.13 | | 0.60 | 0.59 |
| TSM-E | -0.56 | -0.16 | -0.29 | -0.47 | -0.60 | | -0.01 |
| COMB | -0.55 | -0.15 | -0.28 | -0.46 | -0.59 | 0.01 | |

**TABLE VII. Macro models**

This table reports, for the macro hedge funds, the cross-sectional average and standard deviation, the proportions, and the 10th and 90th percentiles, of the annualized alphas for the macro models. We also report the average $\bar{R}^2$. See Section 5.2.2 for a description of the macro models.

| | Mean(Ann.) | Std(Ann.) | Neg | Pos | 10%(Ann.) | 90%(Ann.) | $\bar{R}^2$ |
|---|---|---|---|---|---|---|---|
| CAPM | 3.35 | 11.13 | 31.32 | 68.68 | -7.21 | 13.75 | 8.64 |
| COR | 2.19 | 13.00 | 37.27 | 62.73 | -9.50 | 12.85 | 11.21 |
| LIQ | 3.38 | 11.42 | 32.66 | 67.34 | -7.28 | 13.82 | 9.11 |
| LEV | 2.61 | 11.38 | 36.51 | 63.49 | -8.13 | 13.41 | 9.90 |
| CARE | 3.01 | 11.28 | 33.47 | 66.53 | -7.99 | 13.35 | 8.96 |
| TSME | 2.50 | 11.16 | 35.43 | 64.57 | -7.74 | 12.39 | 11.35 |
| COMB | 1.25 | 13.36 | 44.33 | 55.67 | -10.86 | 12.80 | 14.52 |

**TABLE VIII. Macro models – model comparison**
See Section 5.2.2 for a description of the macro models.

| | CAPM | CAR-EB | CAR-CC | TSM-EB | TSM-CC | COMB |
|---|---|---|---|---|---|---|
| Panel A: Average | | | | | | |
| CAPM | | 1.69 | 0.41 | 2.34 | 1.73 | 3.87 |
| CAR-EB | -1.69 | | -1.27 | 0.65 | 0.04 | 2.18 |
| CAR-CC | -0.41 | 1.27 | | 1.93 | 1.31 | 3.45 |
| TSM-EB | -2.34 | -0.65 | -1.93 | | -0.61 | 1.52 |
| TSM-CC | -1.73 | -0.04 | -1.31 | 0.61 | | 2.14 |
| COMB | -3.87 | -2.18 | -3.45 | -1.52 | -2.14 | |
| Panel B: Standard Deviation | | | | | | |
| | CAPM | CAR-EB | CAR-CC | TSM-EB | TSM-CC | COMB |
| CAPM | | -0.72 | -0.49 | -0.28 | 0.11 | -1.79 |
| CAR-EB | 0.72 | | 0.23 | 0.44 | 0.82 | -1.07 |
| CAR-CC | 0.49 | -0.23 | | 0.21 | 0.59 | -1.30 |
| TSM-EB | 0.28 | -0.44 | -0.21 | | 0.39 | -1.50 |
| TSM-CC | -0.11 | -0.82 | -0.59 | -0.39 | | -1.89 |
| COMB | 1.79 | 1.07 | 1.30 | 1.50 | 1.89 | |
| Panel C: Proportion of Negative Alphas | | | | | | |
| | CAPM | CAR-EB | CAR-CC | TSM-EB | TSM-CC | COMB |
| CAPM | | -8.67 | -2.82 | -14.03 | -9.52 | -22.83 |
| CAR-EB | 8.67 | | 5.85 | -5.36 | -0.85 | -14.16 |
| CAR-CC | 2.82 | -5.85 | | -11.22 | -6.70 | -20.02 |
| TSM-EB | 14.03 | 5.36 | 11.22 | | 4.51 | -8.80 |
| TSM-CC | 9.52 | 0.85 | 6.70 | -4.51 | | -13.32 |
| COMB | 22.83 | 14.16 | 20.02 | 8.80 | 13.32 | |
| Panel D: Proportion of Positive Alphas | | | | | | |
| | CAPM | CAR-EB | CAR-CC | TSM-EB | TSM-CC | COMB |
| CAPM | | 8.67 | 2.82 | 14.03 | 9.52 | 22.83 |
| CAR-EB | -8.67 | | -5.85 | 5.36 | 0.85 | 14.16 |
| CAR-CC | -2.82 | 5.85 | | 11.22 | 6.70 | 20.02 |
| TSM-EB | -14.03 | -5.36 | -11.22 | | -4.51 | 8.80 |
| TSM-CC | -9.52 | -0.85 | -6.70 | 4.51 | | 13.32 |
| COMB | -22.83 | -14.16 | -20.02 | -8.80 | -13.32 | |
| Panel E: 10th Percentile | | | | | | |
| | CAPM | CAR-EB | CAR-CC | TSM-EB | TSM-CC | COMB |
| CAPM | | 2.80 | 0.37 | 1.46 | 1.19 | 4.88 |
| CAR-EB | -2.80 | | -2.43 | -1.34 | -1.61 | 2.08 |
| CAR-CC | -0.37 | 2.43 | | 1.09 | 0.82 | 4.52 |
| TSM-EB | -1.46 | 1.34 | -1.09 | | -0.27 | 3.42 |
| TSM-CC | -1.19 | 1.61 | -0.82 | 0.27 | | 3.69 |
| COMB | -4.88 | -2.08 | -4.52 | -3.42 | -3.69 | |
| Panel F: 90th Percentile | | | | | | |
| | CAPM | CAR-EB | CAR-CC | TSM-EB | TSM-CC | COMB |
| CAPM | | 1.36 | 0.23 | 3.17 | 2.66 | 3.62 |
| CAR-EB | -1.36 | | -1.14 | 1.80 | 1.29 | 2.26 |
| CAR-CC | -0.23 | 1.14 | | 2.94 | 2.43 | 3.39 |
| TSM-EB | -3.17 | -1.80 | -2.94 | | -0.51 | 0.45 |
| TSM-CC | -2.66 | -1.29 | -2.43 | 0.51 | | 0.97 |
| COMB | -3.62 | -2.26 | -3.39 | -0.45 | -0.97 | |

**TABLE IX. Arbitrage models**

This table reports, for the arbitrage hedge funds, the cross-sectional average and standard deviation, the proportions, and the 10th and 90th percentiles, of the annualized alphas for the arbitrage models. We also report the average $R^2$. See Section 5.2.3 for a description of the arbitrage models.

| | Mean(Ann.) | Std(Ann.) | Neg | Pos | 10%(Ann.) | 90%(Ann.) | $\bar{R}^2$ |
|---|---|---|---|---|---|---|---|
| CAPM | 3.04 | 7.70 | 27.22 | 72.78 | -4.64 | 10.25 | 17.51 |
| COR | 2.24 | 8.27 | 32.20 | 67.80 | -6.31 | 9.83 | 22.50 |
| LIQ | 2.76 | 7.98 | 28.61 | 71.39 | -5.32 | 10.26 | 18.69 |
| LEV | 2.72 | 7.60 | 27.80 | 72.20 | -4.92 | 9.89 | 20.65 |
| CARE | 2.65 | 8.14 | 30.31 | 69.69 | -5.93 | 10.35 | 18.30 |
| TSME | 3.09 | 7.68 | 27.31 | 72.69 | -4.75 | 10.43 | 18.56 |
| COMB | 1.89 | 8.55 | 34.75 | 65.25 | -6.67 | 9.58 | 25.91 |

**TABLE X. Arbitrage models – model comparison**
See Section 5.2.3 for a description of the arbitrage models.

| | CAPM | COR | LIQ | LEV | CAR-EB | COMB |
|---|---|---|---|---|---|---|
| **Panel A: Average** | | | | | | |
| CAPM | 0.00 | 0.80 | 0.29 | 0.32 | 1.22 | 1.18 |
| COR | -0.80 | 0.00 | -0.51 | -0.48 | 0.42 | 0.38 |
| LIQ | -0.29 | 0.51 | 0.00 | 0.04 | 0.93 | 0.89 |
| LEV | -0.32 | 0.48 | -0.04 | 0.00 | 0.89 | 0.85 |
| CAR-EB | -1.22 | -0.42 | -0.93 | -0.89 | 0.00 | -0.04 |
| COMB | -1.18 | -0.38 | -0.89 | -0.85 | 0.04 | 0.00 |
| **Panel B: Standard Deviation** | CAPM | COR | LIQ | LEV | CAR-EB | COMB |
| CAPM | 0.00 | -0.57 | -0.28 | 0.10 | -1.10 | -0.94 |
| COR | 0.57 | 0.00 | 0.30 | 0.67 | -0.53 | -0.37 |
| LIQ | 0.28 | -0.30 | 0.00 | 0.38 | -0.82 | -0.66 |
| LEV | -0.10 | -0.67 | -0.38 | 0.00 | -1.20 | -1.04 |
| CAR-EB | 1.10 | 0.53 | 0.82 | 1.20 | 0.00 | 0.16 |
| COMB | 0.94 | 0.37 | 0.66 | 1.04 | -0.16 | 0.00 |
| **Panel C: Proportion of Negative Alphas** | CAPM | COR | LIQ | LEV | CAR-EB | COMB |
| CAPM | 0.00 | -4.98 | -1.39 | -0.58 | -8.07 | -8.30 |
| COR | 4.98 | 0.00 | 3.59 | 4.39 | -3.09 | -3.32 |
| LIQ | 1.39 | -3.59 | 0.00 | 0.81 | -6.68 | -6.91 |
| LEV | 0.58 | -4.39 | -0.81 | 0.00 | -7.49 | -7.71 |
| CAR-EB | 8.07 | 3.09 | 6.68 | 7.49 | 0.00 | -0.22 |
| COMB | 8.30 | 3.32 | 6.91 | 7.71 | 0.22 | 0.00 |
| **Panel D: Proportion of Positive Alphas** | CAPM | COR | LIQ | LEV | CAR-EB | COMB |
| CAPM | 0.00 | 4.98 | 1.39 | 0.58 | 8.07 | 8.30 |
| COR | -4.98 | 0.00 | -3.59 | -4.39 | 3.09 | 3.32 |
| LIQ | -1.39 | 3.59 | 0.00 | -0.81 | 6.68 | 6.91 |
| LEV | -0.58 | 4.39 | 0.81 | 0.00 | 7.49 | 7.71 |
| CAR-EB | -8.07 | -3.09 | -6.68 | -7.49 | 0.00 | 0.22 |
| COMB | -8.30 | -3.32 | -6.91 | -7.71 | -0.22 | 0.00 |
| **Panel E: 10th Percentile** | CAPM | COR | LIQ | LEV | CAR-EB | COMB |
| CAPM | 0.00 | 1.67 | 0.68 | 0.28 | 3.16 | 2.21 |
| COR | -1.67 | 0.00 | -0.99 | -1.40 | 1.49 | 0.53 |
| LIQ | -0.68 | 0.99 | 0.00 | -0.40 | 2.48 | 1.53 |
| LEV | -0.28 | 1.40 | 0.40 | 0.00 | 2.89 | 1.93 |
| CAR-EB | -3.16 | -1.49 | -2.48 | -2.89 | 0.00 | -0.96 |
| COMB | -2.21 | -0.53 | -1.53 | -1.93 | 0.96 | 0.00 |
| **Panel F: 90th Percentile** | CAPM | COR | LIQ | LEV | CAR-EB | COMB |
| CAPM | 0.00 | 0.42 | -0.00 | 0.36 | 0.07 | 0.57 |
| COR | -0.42 | 0.00 | -0.42 | -0.06 | -0.35 | 0.15 |
| LIQ | 0.00 | 0.42 | 0.00 | 0.36 | 0.07 | 0.57 |
| LEV | -0.36 | 0.06 | -0.36 | 0.00 | -0.29 | 0.21 |
| CAR-EB | -0.07 | 0.35 | -0.07 | 0.29 | 0.00 | 0.50 |
| COMB | -0.57 | -0.15 | -0.57 | -0.21 | -0.50 | 0.00 |