

Downside Risk and the Cross-section of Corporate Bond Returns

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Abstract

We rationalize in a theoretical framework the empirically documented importance of volatility and downside risk in the cross-section of corporate bond returns. Our framework features time-varying macroeconomic uncertainty and generalized disappointment aversion. The model yields three downside risk factors versus one documented in the empirical literature. We find that our factors are able to explain the cross-sectional variation of corporate bond returns both at the individual bond- and portfolio-level. Our factors provide significant explanatory power beyond other established factors in the literature. Moreover, we find that volatility downside risk matters, while pure volatility risk is marginally significant. Last, we show that our three downside risk factors subsume the documented predictability of the [Bai, Bali, and Wen \(2019a\)](#) downside risk factor.

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1 Introduction

In workhorse asset pricing models, such as [Merton \(1974\)](#) and its extensions, different claims to a firm’s assets are priced by a common set of risk factors. There is, however, increasing awareness that returns to equity and debt often behave in a way that is inconsistent with shocks to common sources of risk. A large and still growing literature has examined the common drivers of the cross-section of stock returns ([Harvey, Liu, and Zhu, 2016](#)). More recently, there has also been a growing interest in understanding the common drivers of the cross-section of *bond* returns, partly due to increasing availability of better quality price data for corporate bonds.

[Bai, Bali, and Wen \(2019a\)](#), for example, show that downside risk is one of the most prominent predictors of future bond returns. This is intuitively consistent with the concave payoff structure of bonds, which face significant exposure to large drops in a firm’s asset values, in contrast to their limited upside potential. Moreover, [Chung, Wang, and Wu \(2019\)](#) document that corporate bond returns also reflect a premium for their exposure to volatility. These empirical insights, which are intuitive, are instrumental in advancing our comprehension of the factor structure that drives corporate bond risk premiums. However, they leave open the question as to how these risk premiums can be justified from a theoretical perspective.

In this paper, we provide a theoretical justification for downside and volatility risk as factors that drive expected excess returns in the corporate bond market. Specifically, we show how risk premiums for these sources of risk arise naturally in equilibrium when investors exhibit asymmetric preferences with aversion to disappointing outcomes ([Farago and Tédongap, 2018](#)). Our starting point is an economy that evolves randomly, with time-varying fluctuations in economic uncertainty. When investor preferences are symmetric, expected excess returns may be expressed as a linear combination of the compensation that is required for the covariance between excess bond returns and the return on the aggregate market, as well as for the covariance between excess bond returns and changes in aggregate volatility. When investors exhibit, in addition, aversion towards disappointing outcomes, they command compensation for three additional sources of risk: a binary downside state factor and its interactions with the market and volatility factors. This generalized disappointment aversion five-factor model (GDA5) generates the market factor, the empirically motivated volatility factor from [Chung, Wang, and Wu \(2019\)](#), and three downside factors, instead of one as in [Bai, Bali, and Wen \(2019a\)](#).¹

We use high-quality bond-level transaction data to investigate the ability of our model to explain the cross-sectional variation in corporate bond returns. We find that the five

¹A special case of the GDA5 model yields a one-factor model, where the factor is the marginal expected shortfall. [Bai, Bali, and Wen \(2019a\)](#) downside risk factor is estimated either using a 5% value-at-risk (VaR) or 10% expected shortfall, which is similar to a downside risk factor based on the marginal expected loss. Thus, in spirit, the downside risk factor of [Bai, Bali, and Wen](#) can be viewed as a special case of the GDA5 model.

factors in the GDA5 model are priced in the cross-section of corporate bond returns and their respective prices of risk carry theoretically consistent signs. In particular, we find that volatility downside risk is strongly significant while, on the other hand, pure volatility risk is marginally significant. This result suggests that corporate bond investors care more about volatility risk in downside states of the economy than pure volatility risk. Thus, this complements and extends the existing findings in the literature (Chung, Wang, and Wu, 2019). Our results are robust to alternative measures of the market and volatility factor, different values for the model parameters, and controlling for a wide range of bond characteristics. Interestingly, we find that our model does particularly well among investment-grade bonds which constitute the large majority of the US corporate bond market.² The model also performs especially well among large-sized, short-term, and liquid bonds. In addition, we also investigate the model’s ability to explain the cross-section of bond portfolios sorted on different bond characteristics.³ We find that the GDA5 factor model has a high average adjusted R^2 ranging from 74.9% to 80.2% with insignificant intercepts. Thus, our model can also explain the cross-sectional variation of corporate bond portfolio returns.

Next, we investigate the explanatory power of the GDA5 factor model beyond alternative models from the extant literature including the Bai, Bali, and Wen (2019a) five-factor model, a stock five-factor model, and a bond five-factor model.⁴ We find that the significance of the GDA5 factors is robust to including these alternative factors. Notably, once we include our three downside risk factors, the downside risk factor (DRF) of Bai, Bali, and Wen (2019a) becomes insignificant, while our downside risk factors remain highly significant. This suggests that our theoretically motivated downside risk factors subsume the DRF factor. Thus, our results complement and extend the findings in Bai, Bali, and Wen, as we show that corporate bond investors care not only about pure downside risk, but also about market and volatility downside risk. Consistent with our corporate bond results, we also find that GDA5 model is also able to explain the cross-sectional variation in credit default swaps (CDS) returns.

Our paper is related to a growing strand of literature on the cross-section of corporate bond returns. Bai, Bali, and Wen (2019b) construct a measure of systematic risk from individual corporate bonds and show that it is priced in the cross-section of corporate bond returns. Bali, Subrahmanyam, and Wen (2019a) find that macroeconomic uncertainty predicts future bond returns. Bali, Subrahmanyam, and Wen (2019b) document long-term reversals in corporate bond returns. Cao, Goyal, Xiao, and Zhan (2019) show that the implied volatility

²According to S&P Global Fixed Income Research, investment-grade bonds represent 72% of the US corporate bond market as of January 1, 2019. In addition, these type of bonds represent 75.3% of our corporate bond sample.

³We look at bond portfolios formed based on rating and maturity (Lin, Wang, and Wu, 2011; Chung, Wang, and Wu, 2019), and on bond size, rating and maturity (Bai, Bali, and Wen, 2019a).

⁴The Bai, Bali, and Wen (2019a) five-factor model includes bond market, downside risk, liquidity risk, credit risk and short-term reversal factors. The stock five-factor model includes the Fama-French three factors (Fama and French, 1993), a momentum factor (Carhart, 1997), and a liquidity factor (Pástor and Stambaugh, 2003). The bond five-factor model includes the bond market factor (Elton, Gruber, and Blake, 1995), default and term spread factors (Fama and French, 1993), bond liquidity factor (Lin, Wang, and Wu, 2011) and a bond momentum factor (Jostova, Nikolova, Philipov, and Stahel, 2013).

from a firm’s options predicts its bond return. These papers focus exclusively on empirically motivated factors or characteristics without a theoretical foundation. In contrast, our factor model is derived from theory, we provide a theoretical justification for the existence of downside and volatility factors, and we show that our model-implied factors have predictability beyond empirically motivated factors from the extant literature.

The rest of the paper is organized as follows. In Section 2, we provide the theoretical framework of the GDA5 factor model. Section 3 details the empirical methodology to construct the factors and the description of our corporate bond sample. We examine the performance of our model in Section 4. Section 5 concludes the paper.

2 Theoretical Framework

In this section, we introduce a theoretical framework that justifies the existence of downside risk (Bai, Bali, and Wen, 2019a) and volatility risk (Chung, Wang, and Wu, 2019) factors in the cross-section of corporate bond returns. We show that these factors are generated naturally from a consumption-based asset pricing model featuring generalized disappointment aversion preferences and time-varying macroeconomic uncertainty.

Our theoretical framework follows Farago and Tédongap (2018) who combine generalized disappointment aversion with time-varying macroeconomic uncertainty to derive a five-factor model for asset returns. In the model, the representative agent has recursive preferences (Epstein and Zin, 1989; Weil, 1989) whose certainty equivalent introduces generalized disappointment aversion according to Routledge and Zin (2010). The time-varying macroeconomic uncertainty is modeled using an aggregate consumption growth that follows a Markov regime-switching process (Bonomo, Garcia, Meddahi, and Tédongap, 2010). With this setup, Farago and Tédongap (2018) show that the expected excess return of any asset can be expressed as a linear cross-sectional factor model as following

$$\mathbb{E}[R_{i,t}^e] = p_W \sigma_{iW} + p_D \sigma_{iD} + p_{WD} \sigma_{iWD} + p_X \sigma_{iX} + p_{XD} \sigma_{iXD} \quad (1)$$

with

$$\begin{aligned} \sigma_{iW} &\equiv \text{cov}(R_{it}^e, r_{Wt}) \\ \sigma_{iD} &\equiv \text{cov}(R_{it}^e, I(\mathcal{D}_t)) \\ \sigma_{iWD} &\equiv \text{cov}(R_{it}^e, r_{Wt} I(\mathcal{D}_t)) \\ \sigma_{iX} &\equiv \text{cov}(R_{it}^e, \Delta \sigma_{Wt}^2) \\ \sigma_{iXD} &\equiv \text{cov}(R_{it}^e, \Delta \sigma_{Wt}^2 I(\mathcal{D}_t)) \end{aligned} \quad (2)$$

where r_{Wt} is the log market return, $\Delta \sigma_{Wt}^2$ are changes in the market variance, and $I(\mathcal{D}_t)$ is an indicator function of the downside state \mathcal{D}_t .

Eq. (2) may be interpreted as a five-factor (GDA5) model for excess asset returns: the market factor r_{Wt} (MKT), the downside state factor $I(\mathcal{D}_t)$ (DS), the market downside factor $r_{Wt}I(\mathcal{D}_t)$ (MKTDS), the volatility factor $\Delta\sigma_{Wt}^2$ (VOL), and the volatility downside factor $\Delta\sigma_{Wt}^2I(\mathcal{D}_t)$ (VOLDS). The model predicts that the covariance risk prices satisfy $p_W > 0$, $p_{\mathcal{D}} < 0$, $p_{W\mathcal{D}} > 0$, $p_X < 0$, and $p_{X\mathcal{D}} < 0$. The intuition is as follows. An asset whose excess return is positively correlated with the market return or market return in downside states has a higher expected return to compensate investors for the exposure to pure systematic risk and systematic risk in downside states, respectively. On the contrary, an asset whose excess return is positively correlated with market volatility, downside states, and market volatility in downside states provides a hedge against these unfavorable situations and has a lower risk premium.

The downside state (event) from [Farago and Tédongap \(2018\)](#) is given by

$$\mathcal{D}_t = \left\{ r_{Wt} - a \frac{\sigma_W}{\sigma_X} \Delta\sigma_{Wt}^2 < b \right\} \quad (3)$$

where σ_W and σ_X are the standard deviations of the market factor r_{Wt} and volatility factor $\Delta\sigma_{Wt}^2$, respectively. The parameters $a > 0$ and b are functions of parameters in the preference and aggregate consumption process. Eq. (3) implies that downside states occur when the market return is low or the change in market variance is high.

Eq. (2) may be further expressed as a beta pricing model:

$$\mathbb{E}[R_{i,t}^e] = p_F^\top \sigma_{iF} = \left(\Sigma_F^\top p_F \right)^\top \Sigma_F^{-1} \sigma_{iF} = \lambda_F^\top \beta_{iF} \quad (4)$$

where the 5×1 vector $\beta_{iF} = [\beta_{iW} \ \beta_{i\mathcal{D}} \ \beta_{iW\mathcal{D}} \ \beta_{iX} \ \beta_{iX\mathcal{D}}]^\top$ may be estimated from a time-series regression of excess returns on the GDA5 factors and the 5×1 vector $\lambda_F = [\lambda_W \ \lambda_{\mathcal{D}} \ \lambda_{W\mathcal{D}} \ \lambda_X \ \lambda_{X\mathcal{D}}]^\top$ represents the corresponding prices of risk. Note also that the signs of elements of λ_F are the same as those of p_F , i.e. $\lambda_W > 0$, $\lambda_{\mathcal{D}} < 0$, $\lambda_{W\mathcal{D}} > 0$, $\lambda_X < 0$, and $\lambda_{X\mathcal{D}} < 0$, as long as $\text{cov}(r_{Wt}, \Delta\sigma_{Wt}^2) < 0$, which is the leverage effect documented in the empirical literature ([Black, 1976](#); [Christie, 1982](#)).

The GDA5 factors include a similar volatility factor as [Chung, Wang, and Wu \(2019\)](#). The authors use innovations in the VIX index as a proxy for the volatility risk factor and find that it has a negative price of risk in the cross-section of corporate bond returns. The GDA5 factor model predicts that changes in market variance will also carry a negative price of risk. Further, the GDA5 factor model predicts that in addition to the pure volatility risk, volatility downside risk also matters for excess returns.

The GDA5 downside risk factors are related to but different from the downside risk factor (DRF) from [Bai, Bali, and Wen \(2019a\)](#). Their downside risk factor is constructed from a long-short portfolio sorted on individual bonds' 5% value-at-risk (or 10% expected short-fall) controlling for bond ratings. On the other hand, the downside state factor, market downside factor, and volatility downside factor from the GDA5 factor model are derived

from a theoretical model and are directly constructed from the market return and volatility. Nevertheless, in a special case of the GDA5 model, one can obtain a one-factor model, where the factor is the marginal expected shortfall. This factor is similar to the empirical DRF based on expected shortfall. Thus, in spirit, we can view the downside risk factor of [Bai, Bali, and Wen](#) as a special case of the GDA5 model.

3 Methodology and Data

In this section, we provide details on the construction of the GDA5 factors, the corporate bond data used in our empirical analysis, and present summary statistics of our corporate bond sample.

3.1 Constructing the GDA5 Factors

To construct the GDA5 factors from Eq. (1) and (2), we need definitions of the market return, market volatility, and parameters a and b . The market portfolio is in essence unobservable ([Roll, 1977](#)). [Farago and Tédongap \(2018\)](#) use the log return of the CRSP value-weighted portfolio as the market portfolio and realized volatility of daily market returns during the month as the market volatility.⁵ We choose to use the corporate bond market return as a proxy for the market return for our baseline results for the following reasons. First, we are trying to explain the cross-section of corporate bond returns, so it is natural to use the aggregate corporate bond portfolio as the market portfolio. Second, previous literature (see e.g. [Elton, Gruber, and Blake, 1995](#); [Bai, Bali, and Wen, 2019a](#)) use the bond market return as a proxy for the market factor, so our results are more comparable if we use the same measure. Third, if corporate bond market participants are mostly large institutional investors whose investment opportunity sets include mainly corporate bonds, then the aggregate corporate bond portfolio is a better proxy for their wealth portfolio, compared with the aggregate equity portfolio. Finally, the downside states of the equity market concentrate around the financial crisis, while the downside states of the corporate bond market are more evenly distributed over the sample period, which means that we have a larger sample period when using the corporate bond market returns. Nevertheless, in Section 4.1.1 and the Internet Appendix, we find that our main results are robust to using the excess log return of the CRSP value-weighted portfolio as the market portfolio. Following [Chung, Wang, and Wu \(2019\)](#), we use the VIX index as a proxy for the market volatility. In the Internet Appendix, we find that our main results are robust to using conditional variance estimated from an EGARCH model as our proxy for the market volatility.

The values of a and b are related to parameters in the preferences and aggregate consumption process which are difficult to measure. To choose their values, we proceed as follows. First,

⁵In robustness checks, [Farago and Tédongap \(2018\)](#) proxy the market volatility by the VIX or volatility estimated from an EGARCH model.

we set $a = 1$ in our baseline results so that the two terms on the left-hand-side of Eq. (3) are balanced with each other. Second, instead of setting b exogenously, we specify a more intuitive measure, the probability (frequency) of downside states p during the sample period. Given the values of a and p , together with the time-series of the market return and volatility, we can impute the parameter b . Finally, we can use b to obtain time-series of the DS factor using Eq. (3). Using this DS factor, we can construct the MKTDS and VOLDS factors. For our baseline results, we choose $p = 25\%$ and the corresponding value for b is -0.00485 or -0.5% . In the Internet Appendix, we find that our main results are robust to a wide range of alternative values of p and a .

3.2 Data Description

Our sample consists of transactions of U.S. corporate bonds obtained from Trade Reporting and Compliance Engine (TRACE) Standard and Enhanced data available at Wharton Research Data Services (WRDS). The TRACE Enhanced data provides more information compared with the TRACE Standard data. For example, it includes transactions that were missing in the TRACE Standard data in the early stage of the initiative. It also reports uncapped trading volumes and information that facilitates the filtering procedure. However, one caveat of the TRACE Enhanced data is that its sample period ends more than 18 months earlier than the Standard version. Therefore, we use the TRACE Enhanced data from July 2002 to June 2017, and supplement it with the TRACE Standard data from July 2017 to December 2018. We also retrieve bond information from the Mergent Fixed Income Securities Database (FISD), such as the offering date, maturity date, principal amount, coupon rate, coupon payment frequency, and credit rating.

To filter the corporate bond data, we follow [Bai, Bali, and Wen \(2019a\)](#) and apply the following criteria: (1) Remove bonds issued by governments, agencies, or supranationals; (2) Remove floating-coupon bonds and bonds with less than 1 year to maturity; (3) Remove bonds that are not denominated in USD or whose issuer is not in the jurisdiction of the U.S.; (4) Remove private-placed (including those through Rule-144A), asset-backed, agency-backed, equity-linked, and convertible bonds; (5) Remove transactions that are when-issued, locked-in, with special sales conditions, with more than two days to settle, traded under \$5 or above \$1,000, or with trading volume less than \$10,000; (6) Remove canceled records and adjust corrected or reversed records following [Dick-Nielsen \(2009\)](#) and [Dick-Nielsen \(2014\)](#). Then, we follow [Bai, Bali, and Wen \(2019a\)](#) to construct monthly corporate bond returns (see Appendix A for details). The excess return is defined as the difference between the bond return and the 1-month Treasury rate, retrieved from Kenneth French’s website.⁶

⁶https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

3.3 Summary Statistics

Our final sample consists of 1,044,097 monthly bond returns from 34,844 bonds issued by 4,588 firms. Table 1 shows the time-series averages of cross-sectional summary statistics of the bond sample. The average excess return of the bonds is 0.51% per month, corresponding to an annual return of 6.3%, which is approximately half of the average excess return of U.S. stocks in the CRSP sample over the same period (13.20%). However, the average monthly standard deviation of the bonds is 2.54%, much smaller than that of U.S. stocks during this period (5.47%). We use Moody’s bond credit ratings, coded as integers from 1 to 21 with Aaa=1 and C=21. Hence, a higher value indicates lower rating and higher credit risk. The average (median) rating of the corporate bonds in our sample is 8.72 (8.07), which corresponds roughly to a Baa2 (Baa1) Moody’s credit rating. Bonds in our sample have a wide maturity range from 1 year up to greater than 30 years, with an average (median) of 9.48 (6.49) years. There is also a wide dispersion in bond sizes as the amount outstanding (par value) ranges from about \$2 million to over \$2,000 million. The average bond in the same has \$508.04 million outstanding. The illiquidity is calculated following [Bao, Pan, and Wang \(2011\)](#) and is defined as the negative value of the first-order auto-covariance of daily changes in log bond prices within each month.⁷ Most of the bonds in the sample have positive illiquidity, implying that corporate bonds are illiquid in general. Our average bond illiquidity is 0.90 which is similar to the average in [Bao, Pan, and Wang \(2011\)](#). Bonds in our sample have an average (median) coupon rate of 5.85% (5.77%), ranging from less than 1% to over 10%. Bond age is the number of years since a bond’s offering date. It ranges from less than 1 year to more than 10 years. We calculate a bond’s 5% VaR following the same method as [Bai, Bali, and Wen \(2019a\)](#) using past 36-month data, requiring at least 24 observations. The average (median) VaR5 is 3.97% (3.46%) ranging from less than 1% to over 9%. In general, the summary statistics of our sample are similar in magnitude to [Bai, Bali, and Wen \(2019a\)](#) and [Cao, Goyal, Xiao, and Zhan \(2019\)](#).

4 Empirical Results

In this section we first examine the ability of the GDA5 model in explaining the cross-sectional variation in individual corporate bond returns. We also investigate if our main results are robust to different specifications for the market and volatility factor, and different parameters for the model. We then investigate the significance of our factors among different subsamples of bonds based on credit rating, size, maturity, and illiquidity. Next, we examine the GDA5 model’s ability in explaining the cross-section of bond portfolio returns. Importantly, we also investigate if the GDA5 model provides explanatory power beyond alternative factor models. Finally, we examine the ability of the model to explain the cross-sectional variation in corporate CDS returns.

⁷To recognize a daily bond return, we require that the period between consecutive dates with non-missing bond prices be less than or equal to 3 days. This requirement is similar but stricter than the one used by [Bao, Pan, and Wang \(2011\)](#).

4.1 GDA5 Factors and the Cross-section of Corporate Bond Returns

4.1.1 Individual Bond Level Analysis

We first examine whether the GDA5 factors can explain the cross-section of individual corporate bond returns. For this purpose, we use the [Fama and MacBeth \(1973\)](#) two-step regression method. In the first step, for each bond i and each month t , we estimate GDA5 factor betas using the following time-series regression from month $t - 59$ to month t :

$$R_{it}^e = \beta_0 + \beta_{iW}r_{Wt} + \beta_{iD}I(\mathcal{D}_t) + \beta_{iWD}r_{Wt}I(\mathcal{D}_t) + \beta_{iX}\Delta\sigma_{Wt}^2 + \beta_{iXD}\Delta\sigma_{Wt}I(\mathcal{D}_t) + \varepsilon_{it} \quad (5)$$

where R_{it}^e is the excess return of bond i in month t .⁸ This yields a time-series of factor exposures (betas) for each bond in our sample. In the second step, we estimate cross-sectional regressions to test the following GDA5 factor model:

$$\mathbb{E}[R_{i,t+1}^e] = \lambda_W\beta_{iW} + \lambda_D\beta_{iD} + \lambda_{WD}\beta_{iWD} + \lambda_X\beta_{iX} + \lambda_{XD}\beta_{iXD} \quad (6)$$

For the bond-level analysis we follow the literature (see e.g. [Bao, Pan, and Wang, 2011](#)) and require each individual bond to exist in at least 75% of its respective trading period, and have a minimum of 12 months observed continuous returns. Nonetheless, in [Section 4.1.2](#), we show that our main results are robust to including individual bonds that exist in at least 50% of its respective trading period with 12 months observed continuous returns, or including all individual bonds without any of these requirements.

As discussed in [Section 3.1](#), for our main specification throughout the paper we construct the GDA5 factors using the value-weighted portfolio of all bonds in the sample as market factor (MKT), VIX as the market volatility factor, and $p = 25\%$ and $a = 1$ for the downside state parameters. In the Internet Appendix, we show that our main results are robust to different proxies for the market factor, market volatility factor or downside state parameters.

In [Table 2](#), we report cross-sectional regressions of individual bond excess returns on the GDA5 factors. In the first column, we have our main specification, where MKT is the value-weighted portfolio of all bonds in the sample. The signs of the coefficients on the factors are all consistent with the model's prediction: positive for the market (MKT) and market downside (MKTDS) factors and negative for the downside (DS), volatility (VOL), and volatility downside (VOLDS) factors. The intuition is as follows. A corporate bond whose excess return is positively correlated with the market return or market return in downside states has a higher expected return to compensate investors for the exposure to pure systematic risk and systematic risk in downside states, respectively. On the contrary, a corporate bond whose excess return is positively correlated with market volatility, downside

⁸The existing literature (see e.g. [Chung, Wang, and Wu, 2019](#)) uses, in general, a 60-month rolling window when estimating factor betas using monthly returns. In [Section 4.1.2](#), we show that our main results are robust to different choices in the rolling window.

states, and market volatility in downside states provides a hedge against these unfavorable conditions and has a lower risk premium. The coefficients of the MKT, DS, MKTDS, and the VOLDS factors are all significant at a 1% or higher significance level. Notably the prices of risk of the VOL factor has a t -statistic of -1.57 , which means that it is not statistically significant at conventional significant levels. In the second and third column, we construct the GDA5 factors with two other proxies for the corporate bond market factor: the Bank of America Merrill Lynch Corporate Master Index (from Thomson Reuters Eikon) or the Bloomberg Barclays US Corporate Bond Index (from Bloomberg), respectively. The results are in general similar to our main specification in the first column, expect that the VOL factor becomes significant at the 5% level. The adjusted R^2 is around 13% for all three GDA5 models with different bond market factor proxies. In the last column, following [Farago and Tédongap \(2018\)](#), we use the excess return of the CRSP value-weighted portfolio as a proxy for the market factor. We find that all of the factors are significant at the 5% level, with three of them significant at the 1% level. However, the average adjusted R^2 is smaller than those from the other columns using the bond market as the market factor. As discussed in [Section 3.1](#), it is more natural to proxy the market factor with one constructed from corporate bonds since we are investigating the cross-section of corporate bond returns. Overall, these results suggest that corporate bond investors care not only about pure downside risk, but also about market and volatility downside risk. In addition, our results hold regardless if which proxy we use for the market factor. In particular, we find that corporate bond investors care more about volatility downside risk than pure volatility risk, as we find that the latter is not always significantly priced.

In summary, we find that the GDA5 model performs well in explaining the cross-sectional variation of corporate bond returns. Further, the signs of estimated prices of risk are consistent with the theoretical cross-sectional predictions of the model. Last, our results suggest that volatility downside risk plays a larger role in the cross-section of corporate bond returns than pure volatility risk. This last results complements and extends the findings of [Chung, Wang, and Wu \(2019\)](#). In the next section, we will show that the above results are robust to alternative measures of the market and volatility factors, different parameter values of the GDA5 model, different rolling windows to estimate factor betas, different filters on the bond sample, and controlling for a wide range of bond characteristics.

4.1.2 Additional Checks

In this section, we conduct a range of robustness checks on our main results including using a different proxy for market volatility, a range of different model parameters, different rolling windows to estimate factor betas, and different individual bond filters. Last, we examine if our factors provide explanatory power beyond common bond characteristics including size, maturity, credit rating, coupon rate, 5% value-at-risk, age, illiquidity, and short-term reversal. All of the results are available in the Internet Appendix.

Market Volatility In our main specification, we use the VIX index as a proxy for the volatility factor. Since the VIX index is constructed from S&P 500 index options, we use an alternative measure that is calculated from bond market factors. Table A1 in the Internet Appendix shows the cross-sectional regressions with different proxies of market factors, where the volatility factor is proxied by the conditional variance estimated from an EGARCH model on the corresponding market factors. We find that our main results are robust to an alternative proxy for market volatility. Specifically, all prices of risk have theoretically consistent signs and are statistically significant, except for the VOL factor.

Downside State Parameters In our baseline results, we choose $p = 25\%$ and $a = 1$ to construct the GDA5 factors. In Table A2 in the Internet Appendix, we repeat the analysis using different values of $p \in \{20\%, 30\%, 35\%\}$ and $a \in \{0, 0.5, 2\}$ to construct the GDA5 factors. The corresponding values of b are also shown at the bottom of the table. We find that our main results are robust to alternative downside state parameters.

Rolling Window Following the existing literature (see e.g. [Fama and French, 1992](#); [Chung, Wang, and Wu, 2019](#)), we choose a 60-month rolling window for the estimation of our factor betas in Eq. (5). In Table A3 of the Internet Appendix, we choose three other rolling windows to estimate our factor betas: 24-, 36- or 48-months of monthly data, respectively. We find that our main results are robust to the choice of rolling window.

Individual Bonds Filter In our main results using individual bonds, we follow the literature (see e.g. [Bao, Pan, and Wang, 2011](#)) and keep bonds that exist in at least 75% of their respective trading period and have at least 12 months of observed continuous returns. In Table A4 of the Internet Appendix, we report results on two different samples: one that includes all bonds, and one where we keep bonds that exist in at least 50% of their respective trading months and have at least 12 months of observed continuous returns. We find that our main results hold in these two alternative samples.

Bond Characteristics Previous literature (see e.g. [Bai, Bali, and Wen, 2019a](#)) has shown that bond characteristics are important in the cross-section of corporate bond returns. Thus, we investigate if the explanatory power of the GDA5 factors is subsumed by a set of bond characteristics including: bond size, maturity, rating, coupon rate, 5% value-at-risk (VaR5), age, illiquidity, and past 1 month returns (short-term reversal). In Table A5 of the Internet Appendix, we present the cross-sectional regressions of corporate bond excess returns on the GDA5 factors, controlling for bond size, maturity, rating, coupon rate, VaR5, age, illiquidity, and short-term reversal. We find that our main results are robust to controlling for all bond characteristics.

4.1.3 Bond Characteristics Subsample Analysis

In the previous sections, we established that the GDA5 factor model is able to explain the cross-sectional variation of individual bond returns and this result is robust to a range of specifications. In this section, we are interested in examining the GDA5 model among different types of bonds. For this purpose, we investigate the model's performance in the cross-section of individual bonds for different subsamples divided by credit rating, size, maturity, or illiquidity.

In Table 3, we present the results of the cross-sectional regressions of individual bond excess returns on the GDA5 factors for different levels of bond credit rating, size, maturity, and illiquidity. The first two columns show the results for investment-grade and high-yield bonds, respectively. Similar to the full-sample result, among both investment-grade and high-yield bonds, all GDA5 factors are priced, except for the VOL factor. Notably, the adjusted R^2 in the investment-grade bond sample is 16.3% while it is 5.5% among high-yield bonds. Thus, the GDA5 factor model performs very well among investment-grade bonds, which represent the large majority of the corporate bond market and in particular 75.3% of our sample. Meanwhile, the prices of risk of the DS factor, MKTDS factor, and VOLDS factor are larger in magnitude for high-yield bonds, compared with investment-grade bonds. This implies that high-yield bond investors worry more about the downside, market downside, and volatility downside risk of their portfolios, which is a very intuitive result as these type of bonds carry significantly more downside risk.

In the third to fifth columns of Table 3, we show the results for small, medium, and large bonds. The breakpoints are \$300 million and \$600 million par value to make the three subsamples contain similar numbers of observations. In terms of adjusted R^2 the GDA5 factor model performs better as the bond size increases. We see that from the small bond sample to the large bond sample, the average adjusted R^2 increases substantially from 8.2% to 19.0%. In addition, the VOL factor is significantly priced among small-size bonds, but just marginally priced among medium-size and insignificant among large-size bonds. On the other hand, volatility downside risk is significant for all subsamples. This implies that investors focusing on small-size corporate bonds care about both pure volatility risk and volatility downside risk.

In the sixth to eighth columns of Table 3, we report the results for short-term (1–3 years to maturity), medium-term (3–7 years to maturity), and long-term (7+ years to maturity) bonds, respectively. All of the GDA5 factor prices of risk are highly significant for short-term bonds. Their magnitudes and significance decrease as the time to maturity increases. Volatility risk is not significant among medium- or long-term bonds. This means that investors with a medium- or long-term investment horizon do not care about pure volatility risk, but care about downside, market downside, and volatility downside risk. The average adjusted R^2 are similar among bonds with different maturities.

In the last three columns, we present the results for bonds with low, medium, and high levels of illiquidity. The breakpoints are 0.03 and 2 to have a balanced number of observations in

each subsample. The average adjusted R^2 increases as bond liquidity increases. Thus, our model performs best among the most liquid bonds. Similar to the full-sample result, all of the GDA5 factors except the VOL factor are priced across all illiquidity subsamples, with the exception of volatility downside risk which is insignificant among medium-illiquidity bonds.

In summary, the GDA5 factor model, in general, performs well across different subsamples based on bond characteristics. In particular, we find that the GDA5 factors are especially important for investment-grade bonds, which represent a large majority of the US corporate bond market. The factors are also relatively more important among large-size, short-term, and liquid bonds.

4.1.4 Portfolio-level Analysis

In our previous analysis, we examined the ability of the GDA5 factor model at explaining the cross-sectional variation in individual bond excess returns. In this section, we are interested in investigating our factor model in the cross-section of portfolio bond excess returns. Following [Lewellen, Nagel, and Shanken \(2010\)](#), we investigate the performance of the GDA5 factor model on portfolios formed on bond characteristics that are unrelated to our factors. Following the literature ([Lin, Wang, and Wu, 2011](#); [Chung, Wang, and Wu, 2019](#); [Bai, Bali, and Wen, 2019a](#)), we construct two set of portfolios. The first set are 25 rating/maturity portfolios formed by independently sorting bonds into 5 credit rating quintiles and 5 maturity quintiles. We then take their intersection to form $5 \times 5 = 25$ portfolios and calculate value-weighted returns of each portfolio. The second set of portfolios are 27 size/rating/maturity value-weighted portfolios formed through an independent trivariate sort on bond size, credit rating, and maturity into three terciles, respectively. We then take their intersection to form $3 \times 3 \times 3 = 27$ portfolios and calculate value-weighted returns of each portfolio.

In [Table 4](#), we present results for the cross-sectional regressions of portfolio bond excess returns on the GDA5 factors. We find similar results as for our individual bond-level regressions in [Table 2](#). The prices of risk on the GDA5 factors have theoretically consistent signs and are significant across the two portfolios, with the exception the VOL factor which is statistically insignificant. The average adjusted R^2 is high at 74.9% for the 27 size/rating/maturity portfolios and up to 80.2% for the 25 rating/maturity portfolios. In addition, we find that both the intercepts are statistically insignificant. Taken together, these results suggest that the GDA5 factor model is also able to explain the cross-section of corporate bond portfolio returns.

In [Figure 1](#), we plot the observed versus predicted excess returns for our two sets of portfolios. To calculate predicted expected excess returns, we use the prices of risk reported in [Table 4](#). Each dot in the picture represents one bond portfolio. If the GDA5 model can explain the excess returns of these two set of bond portfolios, we would expect to see

the dots aligning on the 45-degree line. In Panel A, we plot the observed versus predicted excess returns for the 25 rating/maturity portfolios, while in Panel B, we plot this for the 27 size/rating/maturity portfolios. Consistent with our results in Table 4, we see that in both panels the dots align across the 45-degree line, which indicates that the GDA5 model performs well in explaining the cross-section of returns for these bond portfolios.

In summary, following the literature, we build bond portfolios based on bond characteristics that are unrelated to our GDA5 factors. Using these portfolios as test assets instead of individual bonds, we still find that the GDA5 factor model is able to explain the cross-sectional variation in corporate bond returns. More specifically, the model yields high average adjusted R^2 of up to 80.2% and insignificant intercepts.

4.2 Alternative Factor Models

In the previous sections, we provided strong evidence that the GDA5 factor model is able to explain the cross-sectional variation of both individual and portfolio corporate bond returns. In this section, we investigate the explanatory power of the theoretically motivated GDA5 factor model beyond other alternative factors for the cross-section of corporate bond returns from the extant literature.

Following [Bai, Bali, and Wen \(2019a\)](#), we consider three alternative models: the stock five-factor model (STK5), the bond five-factor model (BND5), and the [Bai, Bali, and Wen \(2019a\)](#) five-factor model (BBW5). The STK5 factor model includes the [Fama and French \(1993\)](#) three factors (MKT, SMB, and HML), the [Carhart \(1997\)](#) momentum factor (MOM), and the [Pástor and Stambaugh \(2003\)](#) liquidity factor (LIQ). The BND5 factor model includes the bond market factor (bMKT) from [Elton, Gruber, and Blake \(1995\)](#), the default (DEF) and term (TERM) spread factors from [Fama and French \(1993\)](#), the bond momentum factor (bMOM) from [Jostova, Nikolova, Philipov, and Stahel \(2013\)](#), and the bond liquidity factor (bLIQ) from [Lin, Wang, and Wu \(2011\)](#). The BBW5 factors include the bond market (bMKT) factor, downside risk factor (DRF), liquidity risk factor (LRF), reversal factor (REV), and credit risk factor (CRF). The details about the data source and construction of all these factors can be found in Appendix B. We estimate the betas of each individual bond to these factors similarly as in Section 4.1.1 for the GDA5 factors.

We starting by controlling for the BBW5 factor model. In Table 5, we present the cross-sectional regression results of individual corporate bond excess returns on the GDA5 and the BBW5 factor models. Column (1) is the same as the first column of Table 2. In Column (2), we show the prices of risk for the BBW5 factors. Consistent with the findings in [Bai, Bali, and Wen \(2019a\)](#), we find that all the BBW5 factors have significant prices of risk at least at a 10% significance level except for the REV factor. Among these factors, bMKT, DRF, and LRF are highly significant at the 1% level. Thus, we show that the BBW5 factor model performs well even in our extended sample.⁹ In Column (3), we include the GDA5

⁹The sample used by [Bai, Bali, and Wen \(2019a\)](#) in their cross-sectional regressions is from July 2004 to December 2016.

factors and the BBW5 factors. Since both models have the same market factor we can only include it once. We find that all of the GDA5 factors have significant prices of risk at the 1% significance level with theoretically consistent signs. On the other hand, once we include our three downside risk factors, DRF becomes insignificant and its price of risk decreases from 0.371 to 0.0576. This suggests that our three downside factors subsume the downside risk factor from [Bai, Bali, and Wen \(2019a\)](#). Further, this suggests that it is not one but rather three downside risk factors that matter for the cross-section of corporate bond returns. LRF remains highly significant while CRF becomes insignificant. Since our theoretical framework does not capture the effects of liquidity, it is not surprising that LRF is still significant after including the GDA5 factors. In Column (4), we control for the BBW5 factors but exclude the REV factor as the authors show that this factor is not important for the cross-section of corporate bond returns. The results are similar to Column (3). Taken together, our results complement and extend the findings in [Bai, Bali, and Wen \(2019a\)](#), as we show that corporate bond investors not only care about pure downside risk, but also about market and volatility downside risk.

In Table 6, we investigate if our factors provide explanatory power beyond the STK5 and/or BND5 factor models. In Column (1), we again repeat the first column from Table 2. In Column (2), we present the results for the STK5 model. We find that while the stock market, momentum, and liquidity factors have significant prices of risk, their explanatory power for the cross-sectional variation in individual corporate bond returns is smaller than the GDA5 factor model, since the average adjusted R^2 of the STK5 factor model is smaller at 10.1% vs. 13.1%, respectively. In Column (3), we include both factor models. All the GDA5 factors have significant prices of risk with theoretically consistent signs, except for the VOL factor which is still insignificant. On the other hand, the stock market and momentum factors become insignificant. The stock liquidity factor remains highly significant, which is consistent with the findings in [Lin, Wang, and Wu \(2011\)](#).

In Column (4), we present the results for the BND5 factor model. Only the bond market, default spread, and bond momentum factors are significant. However, after including both models together, we find in Column (5) that none of the BND5 factors remain significant apart from the bond market factor which is the same across models. On the other hand, all of the GDA5 factors have significant prices of risk at the 1% significance level, except for the VOL factor which is significant at the 5% level. In Column (6), we include the GDA5, STK5, and BND5 factor models together. All of the GDA5 factors have significant prices of risk, excepted for the VOL factor. Among the STK5 and BND5 factors, the stock liquidity is still highly significant. Thus, it is clear that the GDA5 factors provide explanatory power beyond the factors of the STK5 and BND5 models.

In summary, we find that the factors of the GDA5 model remain significant after including alternative factors from the existing literature. Thus, our factors provide explanatory power on the cross-section of corporate bond returns beyond a wide range of alternative factors previously studied in the literature. In particular, once we include our three downside risk factors, we find that the DRF factor ([Bai, Bali, and Wen, 2019a](#)) becomes insignificant,

while our downside risk factors remain highly significant. This suggests that the DRF factor is subsumed by our three downside risk factors. Our results thus complement and extend the findings of [Bai, Bali, and Wen](#), as we show that corporate bond investors not only care about pure downside risk, but also about market and volatility downside risk.

4.3 The GDA5 Model and the Cross-section of Credit Default Swaps

In the previous sections, we found strong evidence that the GDA5 model is able to explain the cross-sectional variation of corporate bond returns beyond other established factors and bond characteristics from the extant literature. Since credit default swaps (CDS) and corporate bonds are both assets that measure credit risk, these two assets are closely related. Thus, we expect that our model would also be able to explain the cross-section of CDS returns. To investigate this, we obtain daily data on CDS spreads directly from IHS Markit for the period of January 2001 to July 2018. We follow [Augustin and Izhakian \(forthcoming\)](#) and apply the following filters on the CDS data: (1) Retain only CDS written on a U.S. parent company excluding all CDS written on subsidiaries and private firms; (2) Retain only USD denominated contracts written on senior debt with the modified restructuring credit event clause; (3) Use only the 5-year maturity CDS spread since this is the most liquid contract. Further, to obtain monthly CDS spreads, we take the monthly average of daily CDS spreads. We compute CDS returns as changes in log CDS spreads.¹⁰ We merge the CDS data with CRSP using the IHS Markit RED-CUSIP entity mapping database.¹¹ In the end, we are left with 1,090 unique CDS firms during the period of January 2001 to July 2018, with an average monthly cross-section of 484 CDS firms. We construct a CDS market factor (cMKT) as the average return of all CDS firms in our sample weighted by their stock market capitalization. Using the cMKT we construct the GDA5 factors in a similar vein as our bond GDA5 factors.

To investigate the ability of the GDA5 factor model to explain the cross-section of CDS returns, we rely on similar cross-sectional regressions as specified in Section 4.1. In Table 7, we report the results from the cross-sectional regressions for different specifications of the GDA5 factors. In Column (1) we use the same bond GDA5 factors and probability of downside events ($p = 25\%$) as for our main results on the cross-section of corporate bond returns. We find that all GDA5 factors are statistically significant at a 5% or higher significance level and the signs of estimated prices of risk are theoretically consistent. In Column (2), we use bond GDA5 factors constructed using a $p = 15\%$ probability of downside events. We find similar results as in Column (1) with the exception that now the MKT factor is not significant. In Column (3) and (4), we focus instead on the GDA5 factors which are constructed using cMKT as the market factor, and probability for a downside event of $p = 25\%$ or $p = 15\%$, respectively. We find that the GDA5 factors are all significant

¹⁰Since we compute CDS returns as the change in log CDS spreads, the theoretically predicted signs on the prices of risk for each GDA5 factor will be the exact opposite as for corporate bond returns.

¹¹In addition, we manually verify this matching by looking at all possible pairs of company name and ticker.

at a 1% significance level and the signs of estimated prices of risk are still in line with the theoretical predictions, with the exception of volatility downside risk which is insignificant when the probability of downside events is 25%.

In summary, consistent with our results for the cross-section of corporate bond returns, we find that the GDA5 factor model is able to explain the cross-sectional variation in CDS returns. We find that all the GDA5 factors are, in general, significantly priced in the cross-section of CDS returns and carry theoretically consistent signs.

5 Conclusion

We rationalize in a theoretical framework the empirically documented importance of volatility and downside risk in the cross-section of corporate bond returns. Specifically, we show how risk premiums for these sources of risk arise naturally in equilibrium when investors exhibit asymmetric preferences with aversion to disappointing outcomes (Farago and Tédongap, 2018). The model generates a five-factor (GDA5) model with a market factor, volatility factor, and three downside risk factors. Thus, our theoretical framework yields the existence of three downside factors versus one downside risk factor as empirically documented in Bai, Bali, and Wen (2019a).

We investigate the performance of the GDA5 model using high-quality individual bond-level data, and find that our model is able to explain the cross-sectional variation of corporate bond returns. We find that all of the factors carry theoretically consistent signs and are significantly priced, except for the volatility factor which is marginally significant. These results complement and extend the findings in Chung, Wang, and Wu (2019) as we show that volatility downside risk plays a larger role than pure volatility risk in the cross-section of corporate bond returns. Our main results hold for a range of specifications for the market return and volatility, the downside state parameters, and controlling for bond characteristics. In particular, our model performs especially well among investment-grade, large-size, short-term, and liquid bonds. We also examine the ability of the GDA5 model in explaining the cross-section of corporate bond portfolios. We find high adjusted R^2 s, insignificant intercepts, and significant prices of risk on our factors, except for the volatility factor. These results suggest that our model is also able to explain the cross-sectional variation in corporate bond portfolio returns.

Next, we investigate the explanatory power of the GDA5 model beyond alternative factor models from the extant literature. We find that the GDA5 factors are still significantly priced even after controlling for a wide range of established factors from the extant literature. In particular, our results suggest that the downside risk factor of Bai, Bali, and Wen (2019a) is subsumed by our three downside factors. Thus, our results complement and extend the results in Bai, Bali, and Wen, as we show that not only pure downside risk matters, but also market and volatility downside risk. Finally, we show that our factor model is also able to explain the cross-sectional variation in corporate CDS returns.

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Table 1: Time-series averages of cross-sectional summary statistics of the corporate bond sample

	N	mean	sd	p1	p5	p25	p50	p75	p95	p99
Excess return (%)	1,044,097	0.51	2.54	-6.00	-3.34	-0.72	0.38	1.64	4.76	7.80
Rating	1,000,097	8.72	4.04	1.43	3.03	5.97	8.07	10.59	16.59	19.24
Maturity	1,044,097	9.48	8.69	1.11	1.54	3.63	6.49	11.84	27.15	30.02
Bond size (million \$)	1,044,097	508.04	581.73	2.98	10.48	160.00	356.30	632.92	1577.88	2814.13
Illiquidity	569,120	0.90	4.53	-1.54	-0.17	0.02	0.14	0.59	3.94	13.40
Coupon (%)	1,044,097	5.85	1.81	0.86	3.13	4.76	5.77	6.85	8.94	10.69
Bond age	1,044,097	4.45	4.24	0.07	0.33	1.57	3.29	5.99	13.47	18.90
5% VaR (%)	464,067	3.97	2.42	0.64	1.03	2.12	3.46	5.48	8.88	9.90

Notes: This table shows time-series averages of cross-sectional summary statistics of the corporate bond sample. The excess return is the monthly bond return minus 1-month Treasury bill rate. The credit rating is Moody's credit rating for corporate bonds, coded from Aaa=1 to C=21. The maturity is the number of years until the bond's maturity date. Bond size is the amount outstanding in million USD. Illiquidity is calculated following [Bao, Pan, and Wang \(2011\)](#). The coupon rate is in percentage points. The bond age is the number of years from its offering date. The 5% VaR is calculated following [Bai, Bali, and Wen \(2019a\)](#).

Table 2: Bond-level cross-sectional regression on the GDA5 factors using different proxies for the market factor

	Sample bond market	Merrill Lynch Corporate	Barclays US Corporate	CRSP value-weighted
MKT	0.274*** (3.26)	0.250*** (2.76)	0.247*** (2.64)	0.741*** (2.88)
DS	-10.01*** (-2.96)	-8.016*** (-2.70)	-8.031*** (-2.73)	-5.092** (-2.13)
MKTDS	0.0954*** (2.96)	0.0747** (2.13)	0.0683* (1.90)	0.282*** (2.91)
VOL	-0.0149 (-1.57)	-0.0242** (-2.13)	-0.0247** (-2.17)	-0.0221*** (-3.30)
VOLDS	-0.0141*** (-2.64)	-0.0137** (-2.39)	-0.0151*** (-2.65)	-0.0105** (-2.29)
Constant	0.176*** (4.22)	0.219*** (4.71)	0.228*** (4.68)	0.354*** (4.18)
Adj. R^2	0.131	0.129	0.129	0.087

Notes: This table shows the results of the cross-sectional regressions of individual bond excess returns on the GDA5 factors. The GDA5 factors are constructed using downside state parameters of $p = 25\%$ and $a = 1$. The market factor proxy used to construct the factors is listed at the top of each column. The volatility factor is proxied by the VIX index. All prices of risk are in percentage points. The average adjusted R^2 are shown at the bottom. Standard errors are corrected according to [Newey and West \(1987\)](#). ***, **, and * denote p -values less than 0.01, 0.05, and 0.1, respectively.

Table 3: Cross-sectional regression on GDA5 factors over bonds with different characteristics

	Credit rating		Bond size			Maturity			Illiquidity		
	IG	HY	Small	Medium	Large	1-3yrs	3-7yrs	7yrs+	Low	Medium	High
MKT	0.242*** (2.70)	0.187*** (3.23)	0.305*** (4.23)	0.291*** (3.36)	0.267*** (2.87)	0.293*** (4.16)	0.254*** (3.36)	0.146** (2.09)	0.304*** (3.43)	0.231*** (3.00)	0.247*** (3.11)
DS	-6.238** (-2.13)	-10.70*** (-3.36)	-12.51*** (-3.77)	-9.177** (-2.52)	-8.919** (-2.53)	-12.15*** (-3.68)	-9.345** (-2.28)	-6.571** (-2.17)	-11.85*** (-2.96)	-6.539** (-2.40)	-9.487** (-2.59)
MKTDS	0.0703** (2.42)	0.0771*** (3.05)	0.0851** (2.55)	0.0931*** (2.88)	0.108*** (2.84)	0.108*** (3.09)	0.0878*** (2.72)	0.0572** (2.11)	0.110*** (3.23)	0.0756*** (2.82)	0.0893** (2.56)
VOL	-0.00775 (-0.88)	-0.00965 (-1.08)	-0.0222** (-2.42)	-0.0171* (-1.72)	-0.0130 (-1.05)	-0.0327*** (-3.07)	-0.0161 (-1.45)	-0.00607 (-0.76)	-0.0123 (-1.01)	-0.0112 (-1.08)	-0.0189 (-1.65)
VOLDS	-0.00821* (-1.88)	-0.0124** (-1.99)	-0.0160*** (-3.07)	-0.0147** (-2.42)	-0.0131** (-2.02)	-0.0276*** (-3.98)	-0.0137* (-1.82)	-0.00808* (-1.90)	-0.0171** (-2.51)	-0.00783 (-1.34)	-0.0140** (-2.14)
Constant	0.151*** (3.90)	0.474*** (4.45)	0.213*** (4.09)	0.163*** (3.68)	0.146*** (3.62)	0.0784*** (2.93)	0.199*** (2.84)	0.411*** (3.77)	0.106*** (2.92)	0.157*** (3.98)	0.273*** (3.37)
Adj. R^2	0.163	0.055	0.082	0.132	0.190	0.095	0.110	0.100	0.182	0.153	0.099

Notes: This table shows the results of the cross-sectional regressions of individual bond excess returns on the GDA5 factors over different bond subsamples. The first and second columns divide the sample bonds into investment-grade (Baa3 and above) and high-yield bonds (Ba1 and below). The third to fifth columns group the sample bonds based on their sizes. The breakpoints are \$300 million and \$600 million. The sixth to eighth columns show the results for short-term (1–3 years to maturity), medium-term (3–7 years to maturity), and long-term (more than 7 years to maturity) bonds. The remaining columns divide bonds according to their illiquidity (Bao, Pan, and Wang, 2011). The breakpoints are 0.03 and 2 to make the three groups have similar numbers of observations. All prices of risk are in percentage points. The average adjusted R^2 are shown at the bottom. Standard errors are corrected according to Newey and West (1987). ***, **, and * denote p -values less than 0.01, 0.05, and 0.1, respectively.

Table 4: Cross-sectional regression on the GDA5 factors using bond portfolios

	25 Rating/Maturity	27 Size/Rating/Maturity
MKT	0.550*** (4.25)	0.426*** (3.48)
DS	-21.83*** (-2.65)	-23.39*** (-2.92)
MKTDS	0.388** (1.99)	0.378** (2.32)
VOL	-0.0408 (-1.16)	-0.0114 (-0.38)
VOLDS	-0.0748*** (-3.74)	-0.0745*** (-2.68)
Constant	-0.0547 (-1.06)	0.0868 (1.54)
Adj. R^2	0.802	0.749

Notes: This table shows the results of the cross-sectional regressions of bond portfolio excess returns on the GDA5 factors. In the first column, the test portfolios are 5×5 value-weighted portfolios independently sorted on rating and maturity. In the second column, the test portfolios are $3 \times 3 \times 3$ value-weighted portfolios independently sorted on bond size, rating, and maturity. All prices of risk are in percentage points. The average adjusted R^2 are shown at the bottom. Standard errors are corrected according to [Newey and West \(1987\)](#). ***, **, and * denote p -values less than 0.01, 0.05, and 0.1, respectively.

Table 5: Cross-sectional regression on the GDA5 factors and BBW5 factors

	(1)	(2)	(3)	(4)
MKT (GDA5)	0.274*** (3.26)		0.238*** (3.03)	0.235*** (2.98)
DS (GDA5)	-10.01*** (-2.96)		-8.810*** (-4.15)	-8.529*** (-3.92)
MKTDS (GDA5)	0.0954*** (2.96)		0.0878*** (3.33)	0.0806*** (3.28)
VOL (GDA5)	-0.0149 (-1.57)		-0.0177*** (-2.71)	-0.0183** (-2.61)
VOLDS (GDA5)	-0.0141*** (-2.64)		-0.0124*** (-3.45)	-0.0132*** (-3.41)
bMKT (BBW5)		0.244*** (3.07)		
DRF (BBW5)		0.371*** (3.10)	0.0576 (1.45)	0.0635 (1.54)
LRF (BBW5)		0.160*** (3.18)	0.0617*** (2.73)	0.0642*** (2.71)
REV (BBW5)		0.00682 (0.14)	0.00624 (0.14)	
CRF (BBW5)		0.230* (1.88)	0.122 (0.99)	0.128 (1.03)
Constant	0.176*** (4.22)	0.213*** (5.28)	0.213*** (5.09)	0.218*** (5.24)
Adj. R^2	0.131	0.170	0.183	0.180

Notes: This table shows the results of the cross-sectional regressions of individual bond excess return on the GDA5 factors and the BBW5 factors. Column (1) repeats the result in first column of Table 2. Column (2) shows the prices of risk of the BBW5 factors following [Bai, Bali, and Wen \(2019a\)](#). Column (3) combines the two factor models. Column (4) removes the REV factor from the BBW5 factors. All prices of risk are in percentage points. The average adjusted R^2 are shown at the bottom. Standard errors are corrected according to [Newey and West \(1987\)](#). ***, **, and * denote p -values less than 0.01, 0.05, and 0.1, respectively.

Table 6: Cross-sectional regression on the GDA5 factors, STK5 factors, and BND5 factors

	(1)	(2)	(3)	(4)	(5)	(6)
MKT (GDA5)	0.274*** (3.26)		0.185** (2.42)		0.265*** (3.44)	0.213*** (3.06)
DS (GDA5)	-10.01*** (-2.96)		-8.269*** (-3.71)		-9.768*** (-3.90)	-8.579*** (-3.72)
MKTDS (GDA5)	0.0954*** (2.96)		0.0732*** (3.12)		0.0939*** (3.45)	0.0780*** (3.21)
VOL (GDA5)	-0.0149 (-1.57)		-0.00239 (-0.41)		-0.0184** (-2.59)	-0.00748 (-1.36)
VOLDS (GDA5)	-0.0141*** (-2.64)		-0.00691* (-1.79)		-0.0131*** (-3.20)	-0.00848** (-2.39)
MKT (STK5)		0.751*** (2.76)	0.352 (1.41)			0.256* (1.83)
SMB (STK5)		0.102 (0.92)	0.160* (1.88)			0.0653 (0.76)
HML (STK5)		0.0355 (0.28)	-0.0373 (-0.37)			-0.0523 (-0.71)
MOM (STK5)		-0.529** (-2.40)	-0.158 (-0.78)			-0.0172 (-0.14)
LIQ (STK5)		0.352*** (2.82)	0.352*** (2.68)			0.320*** (2.97)
bMKT (BND5)				0.287*** (3.41)		
DEF (BND5)				0.297** (2.49)	-0.00382 (-0.04)	-0.0339 (-0.38)
TERM (BND5)				-0.0261 (-0.12)	-0.151 (-0.79)	-0.0452 (-0.24)
bMOM (BND5)				-0.192*** (-3.16)	-0.0783 (-1.49)	-0.0863* (-1.75)
bLIQ (BND5)				0.0319 (0.47)	0.0428 (0.77)	0.0628 (1.11)
Constant	0.176*** (4.22)	0.288*** (4.03)	0.182*** (4.73)	0.165*** (4.43)	0.186*** (4.76)	0.173*** (4.58)
Adj. R^2	0.131	0.101	0.167	0.166	0.180	0.195

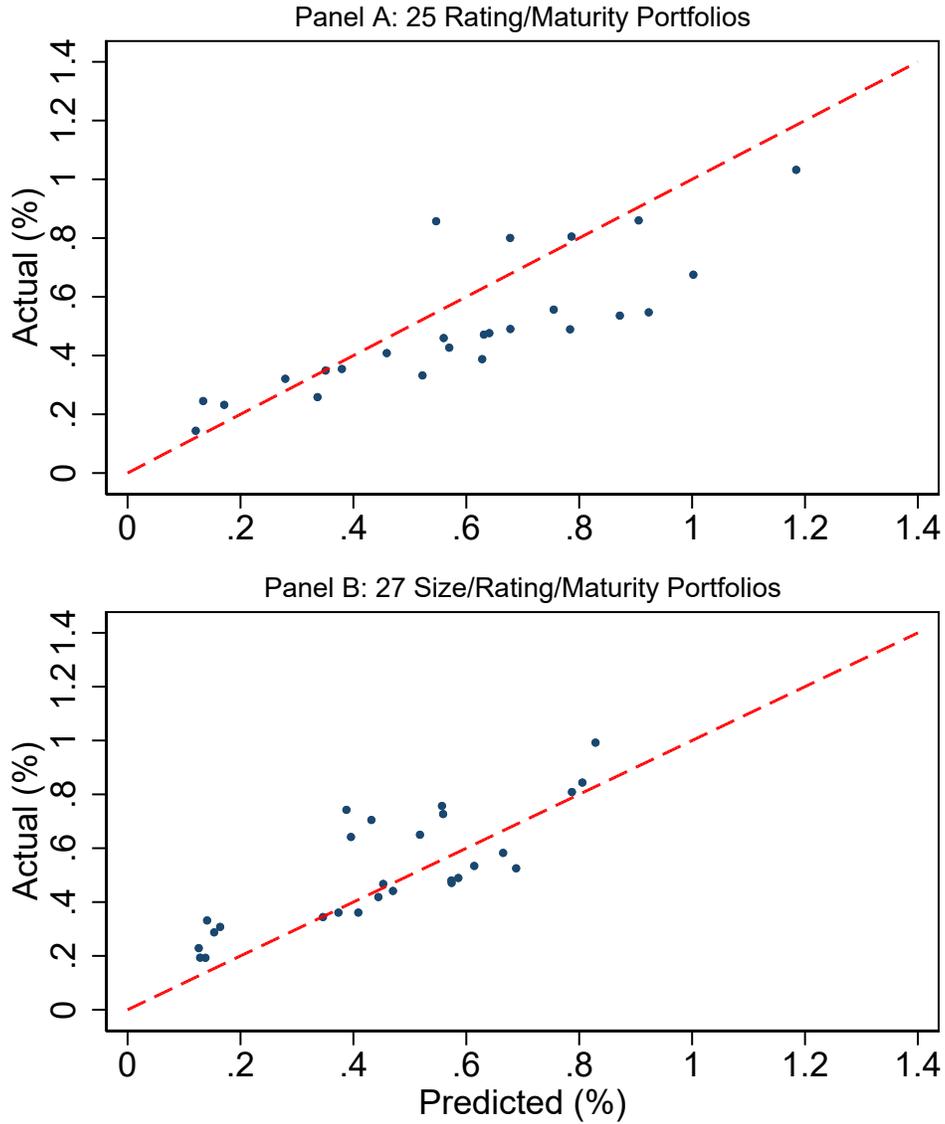
Notes: This table shows the results of the cross-sectional regressions of individual bond excess returns on the GDA5 factors, together with the STK5 factors and BND5 factors. Column (1) repeats the result in first column of Table 2. Column (2) shows the prices of risk of the STK5 factors (Fama and French, 1993; Carhart, 1997; Pástor and Stambaugh, 2003). Column (3) shows the results of the GDA5 factors controlling for the STK5 betas. Column (4) shows the prices of risk of the BND5 factors (Fama and French, 1993; Elton, Gruber, and Blake, 1995; Bao, Pan, and Wang, 2011; Jostova, Nikolova, Philipov, and Stahel, 2013). Column (5) shows the results of the GDA5 factors controlling for the BND5 betas. In Column (6), we control for both the STK5 betas and the BND5 betas. All prices of risk are in percentage points. The average adjusted R^2 are shown at the bottom. Standard errors are corrected according to Newey and West (1987). ***, **, and * denote p -values less than 0.01, 0.05, and 0.1, respectively.

Table 7: Cross-sectional regression of CDS returns on GDA5 factors

	Bond MKT		CDS MKT	
	$p = 25\%$ (1)	$p = 15\%$ (2)	$p = 25\%$ (3)	$p = 15\%$ (4)
MKT	-0.000947*** (-2.51)	-0.000405 (-1.36)	-0.00505*** (-2.52)	-0.000167*** (-4.02)
DS	0.0968*** (6.41)	0.0794*** (4.75)	0.0650*** (5.89)	0.0380*** (4.04)
MKTDS	-0.00113*** (-4.17)	-0.000917*** (-4.04)	-0.00798*** (-5.23)	-0.000103*** (-2.90)
VOL	0.000124*** (2.44)	0.0000859*** (2.83)	0.000231*** (4.13)	0.000212*** (4.35)
VOLDS	0.0000822** (2.04)	0.000109*** (3.48)	0.0000320 (1.11)	0.0000596*** (2.58)
Constant	0.0169*** (16.15)	0.0175*** (21.43)	0.0203*** (13.00)	0.0234*** (12.36)
Adj. R^2	0.047	0.054	0.038	0.037

Notes: This table shows the results of the cross-sectional regressions of CDS returns on the GDA5 factors with different proxies of market factor and different probabilities of downside events. Column (1) and (2) use the same bond market factor as in the first column of Table 2. Column (3) and (4) use a CDS market factor proxied by the average return of all CDS firms in our sample weighted by their stock market capitalization. We report the results with $p = 25\%$ in Column (1) and (3), and with $p = 15\%$ in Column (2) and (4). Standard errors are corrected according to Newey and West (1987). ***, **, and * denote p -values less than 0.01, 0.05, and 0.1, respectively.

Figure 1: Observed vs. Predicted Returns



Notes: This figure plots the observed mean excess returns versus predicted expected excess returns ($\mathbb{E}[\lambda_F^\top \beta_{iF}]$) from the GDA5 factor model using different portfolios. The prices of risk (λ_F) are estimated as in Table 4. The red dashed lines indicate the 45° line.

Appendix

A Constructing monthly corporate bond returns

We start with the transaction-level corporate bond data after applying the filters and merging the TRACE Enhanced and Standard data as described in Section 3.2. Each observation in the data set is a transaction record including bond CUSIP, trade execution date, trade execution time, transaction price, trading volume, etc. Since there are occurrences of multiple records for the same bond at the same timestamp, we first construct timestamp-level bond data. Specifically, for each bond, trade execution date, and execution time, we calculate the trading volume weighted average price, the sum of trading volumes, and the standard deviation of prices scaled by the weighted average price. We remove the observation if the scaled standard deviation is greater than 10% since such a large deviation of prices at the same time is likely due to errors. Then, we apply the filter by Rossi (2014) and obtain the filtered timestamp-level bond price data. Next, we construct daily bond price series. For each bond and each trade execution date, we calculate the trading volume weighted average price over all timestamp-level data on the day. Finally, we also apply the filter by Rossi (2014) on the daily corporate bond prices.

We follow Bai, Bali, and Wen (2019a) to calculate the monthly bond returns. For each bond and each month t , we first check if there are observations in the last 5 trading days of the month. If there is a unique one, we choose its price P_t as the bond price at the end of month t ; if there are multiple observations, we choose the one that is closest to the last day of the month; if there is no observation, the bond return in month t is set as missing. Second, we follow the same method to determine the bond price at the end of month $t - 1$, i.e. P_{t-1} . If there is no observation in the last 5 trading days of month $t - 1$, we check if there are observations in the first 5 trading days of month t . If there is a unique observation, its price is chosen to be P_{t-1} ; if there are multiple observations, the one closest to the first day of month t is chosen; otherwise, the bond return in month t is set as missing. Finally, the bond return in month t is defined as

$$R_t = \frac{(P_t + AI_t) + C_t - (P_{t-1} + AI_{t-1})}{P_{t-1} + AI_{t-1}} \quad (\text{A.1})$$

where AI_t is the accrued interest from the last coupon payment day, i.e. $P_t + AI_t$ is the full (or dirty) price of the bond, and C_t is the coupon payment in month t . Due to outliers, we winsorize the monthly bond returns at the 1% and 99% level over the whole sample.

B Constructing alternative factors

B.1 The Stock 5 factors (STK5)

We retrieve monthly series of the MKT, SMB, HML, and MOM factors from Kenneth French's website.¹ The stock liquidity factor is downloaded from Lubos Pastor's website.²

B.2 The bond 5 factors (BND5)

The bond market factor (bMKT) is proxied by the average return of all bonds in our sample weighted by their amounts outstanding. The default spread (DEF) is defined as the difference between long-term

¹http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

²<https://faculty.chicagobooth.edu/lubos.pastor/research/>

corporate bond returns and long-term government bond returns. The term spread (TERM) is defined as the difference between long-term government bond returns and 1-month Treasury rate. The data source is Amit Goyal’s website.³

The bond momentum factor (bMOM) is constructed as follows (Jostova, Nikolova, Philipov, and Stahel, 2013; Bai, Bali, and Wen, 2019a): for each month t , we group bonds into 5×5 groups sorted independently on their credit ratings and cumulative returns from month $t - 7$ to $t - 2$. Then, we calculate the equal-weighted average return of each quintile portfolio. For each credit rating quintile, we obtain return of a long-short portfolio that takes a long position in the highest past return quintile and a short position in the lowest past return quintile. Finally, we compute the average of the long-short portfolio return across all credit rating quintiles as the bMOM factor.

The bond liquidity factor (bLIQ) is calculated according to Lin, Wang, and Wu (2011). We first follow their method to calculate the bond market (non-traded) liquidity factor. The method is similar to that of Pástor and Stambaugh (2003). Then, for each bond and each month, we estimate the bond liquidity beta from the following specification using excess returns over a 5-year rolling window:

$$R_{i,t}^e = \alpha_i + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB + \beta_{i,HML}HML + \beta_{i,DEF}DEF_t + \beta_{i,TERM}TERM_t + \beta_{i,L}L_t + \varepsilon_{i,t} \quad (\text{B.1})$$

where L_t is the liquidity factor and the other variables are defined above. Finally, for each month, we sort bonds into 10 deciles based on their liquidity beta and compute the average return difference between the highest liquidity beta decile and the lowest liquidity beta decile as the bond market liquidity factor.

B.3 The Bai, Bali, and Wen (2019a) factors (BBW5)

Bai, Bali, and Wen (2019a) propose a new five-factor model for the cross-section of corporate bond returns: the bond market factor (bMKT), the downside risk factor (DRF), the liquidity risk factor (LRF), the reversal factor (REV), and the credit risk factor (CRF). We strictly follow their paper to construct these factors. Similar to the BND5 factor model, the bMKT factor is also proxied by the average return of all bonds in our sample weighted by their amounts outstanding.

Before constructing the other factors, we first calculate the downside risk and illiquidity measure. For each bond and each month, we estimate the historical 5% VaR from monthly returns in the past 36 months and use its absolute value as the downside risk measure (VaR5). The illiquidity measure (ILLIQ) is defined as $-\text{cov}_t(\Delta p_{i,t,d}, \Delta p_{i,t,d+1})$ where $\Delta p_{i,t,d} = \log(P_{i,t,d}) - \log(P_{i,t,d-1})$ is the log return of bond i on day d of month t and the covariance is calculated over all daily returns in month t (Bao, Pan, and Wang, 2011). To recognize a daily bond return, we require that the number of days between the lagged price and the current price be less than or equal to 3.

To construct the DRF, for each month we independently sort bonds into 5×5 portfolios according to their ratings and VaR5. Then, for each rating quintile, we calculate the weighted average return difference between the highest VaR5 quintile and the lowest VaR5 quintile. Finally, we compute the average long-short portfolio return across all rating quintiles as the DRF. Meanwhile, we also calculate the CRF_{VaR5} factor as the weighted average return difference between the lowest rating quintile and highest rating quintile across all VaR portfolios.

LRF and REV are generated using the same procedure. Instead of sorted on rating and VaR5, for the LRF, we sort on rating and ILLIQ; for the REV, we sort on rating and lagged monthly return. In addition, we can also calculate CRF_{LRF} and CRF_{REV} similarly. The CRF is then defined as the average of CRF_{VaR5} , CRF_{LRF} , and CRF_{REV} .

³<http://www.hec.unil.ch/agoyal/>