Life-cycle Risk-Taking with Personal Disaster Risk*

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Abstract

Inspired by a growing body of empirical work, this paper models a non-linear labour income process allowing for a personal disaster, such as long-term unemployment or disability, during working years. This entails an uncertain but potentially large permanent shock to earnings. Personal disaster risk allows to match the flat investment profile in age, which is observed in the US, when the calibration of both the disaster probability and the expected permanent loss in the disaster state is conservative.

Keywords: disaster risk, beta distribution, life-cycle portfolio choice, non-linear income process, unemployment risk, disability risk

JEL classification: D15, E21, G11

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1 Introduction

What will happen if my future ability to work permanently falls? It is well known that insurance against permanent shocks, such as disability and long-term unemployment, is incomplete (Guvenen and Smith, 2014; Low et al., 2010 and 2015). Young households will thus make provisions to cushion against a personal disaster, even if its occurrence is quite rare. Against this background, this paper relates the pattern of self-insurance in financial markets over the life-cycle to the possibility of a rare personal disaster during working years. It shows that personal disaster risk is able to alter lifetime *ex-ante* investment choices for the representative worker, even if *ex-post* most workers will not incur into a disaster. Additionally, it uncovers that uncertainty about the size of uninsured human capital losses, that characterizes these rare disasters, enhance the precautionary behavior of young workers. This will result in lower risk-taking at the beginning of working life, with respect to a comparable deterministic human capital loss. It is such uncertainty that allows to closely match the observed age profiles from 1992 to 2016, updated with the methods in Ameriks and Zeldes (2004), when the calibrations are conservative.

We contribute to the household finance literature by linking risk-taking in financial markets with the *ex-ante* uncertain but potentially extreme permanent impact of income shocks. Differently from the household finance literature, though, we go beyond the positive probability of zero labor income implied by the linear income process proposed by Cocco, Gomes and Maenhout (2005) and later adopted by this strand of literature. Inspired by a growing body of empirical work showing that earnings dynamics displays non-linearities (Arellano, Blundell and Bonhomme, 2017; Guvenen, Karahan, Ozkan and Song, 2016; De Nardi, Fella and Paz-Pardo, 2020) we model the income growth as a Markov process. The occurrence of the disaster state brings about a permanent income reduction of uncertain proportion. Specifically, the fraction of human capital lost follows a Beta distribution.[1] The flexibility of the Beta distribution allows to concentrate a large probability mass on the likeliest values of proportional human capital reduction, leaving the possibility of extremely unlikely but devastating realizations open. Importantly, when careers are calibrated to broadly match observed US labour market features, optimal investment in the risky asset remains

[1] This distribution may also characterize the damage caused by natural disaster (see Bhattacharjee, 2004, and Lallemant et al., 2015).

flat over the whole working life, in line with early evidence on US portfolios (Ameriks and Zeldes, 2004) which we update to 2016. This occurs even when we account for the large insurance coverage of permanent income shocks in Guvenen et al. (2016) and Guvenen and Smith (2015), which may ultimately reduce the expected human capital losses due to long-term unemployment. Without disaster risk, the implied optimal stock holding still counter-factually decreases with age before retirement, unless the long-term unemployment rate is as high as the one observed during the Great Recession (as in Bagliano, Fugazza and Nicodano, 2019).

Our results highlight the role of the non-linear income process in flattening the age profile of risk taking. With a linear income process, prior models resort to additional features to explain reduced risk taking in financial markets (see Cocco, 2004; Munk and Sorensen, 2010; Kraft and Munk, 2011; Bagliano, Fugazza and Nicodano, 2014; Hubener, Maurer and Mitchell, 2016; and Chang, Hong and Karabarbounis, 2017). In the latter study, for instance, uncertainty resolves over time thanks to agents learning about their income volatility, otherwise the linearity of labour income shocks would lead to the usual high risk taking when young. In our paper uncertainty resolves due to time passing without the occurrence of disasters. This is the reason why the young bear more labour income risk, which is the intuition pioneered in Viceira (2001) and Benzoni et al. (2007). More precisely, we model working life careers as a three-state Markov chain driving the transitions between employment, short-term unemployment and a personal disaster state. Uncertain permanent earning losses, occurring in the disaster state, represent productivity loss due to either long-term unemployment (Arulampalam (2001); Schmieder, von Wachter and Bender (2016)), or disability or both (Low and Pistaferri (2015)).

This model nests the traditional life-cycle framework in household finance (Cocco, Gomes and Menhout, 2005). Indeed, when the disaster probability is zero and/or human capital erosion is compensated by full insurance, the agents optimally reduce exposure to risky assets as they approach retirement. This pattern obtains since human capital provides a hedge against shocks to stock returns, which makes financial risk bearing generally acceptable. Investment in stocks should therefore be relatively high at the beginning of working careers, when human capital is large relative to accumulated financial wealth. Risky investment then gradually declines until retirement, as human capital decreases relative to financial wealth. When personal disaster risk is instead only partially insured [2], the above effect is moderated by the resolution of uncertainty concerning labor and pension income as the worker safely approaches retirement age. Since the risk of a disaster falls as retirement approaches, the resolution of uncertainty compensates for the hedge effect and the optimal investment in stocks is relatively flat over the life cycle.

Our model delivers additional implications concerning life-cycle choices with incomplete insurance against personal disaster risk. First, the distribution of optimal consumption growth becomes negatively skewed, due to disasters, in line with evidence on durable consumption growth (Yang, 2011). Second, personal disaster risk changes the age profile of savings thereby shrinking the heterogeneity of optimal portfolio choices across agents characterized by different career histories. Young workers increase early precautionary savings to buffer against possible, albeit rare, future disasters. Optimal early consumption consequently falls, increasing during both late working years and retirement years. Third, the average implied savings to income ratio increases, as in other life-cycle models stressing the role of earnings shocks for solving life-cycle portfolio choice puzzles. While it may appear counter-factually high, our model does not incorporate the effects of means-tested welfare programs that lead to zero optimal precautionary saving for poor households (Hubbard, Skinner and Zeldes, 1995).

This model does not address non-participation in the stock market. It should otherwise allow for correlation between stock returns and labour income shocks (see Bagliano et al. (2014) for additional conditions and Bonaparte Korniotis Kumar (2014) for empirical results) or correlation between stock returns and the skewness of labor income shocks (see Catherine, Sodini and Zhang, 2020). Likewise, prominent papers study consumption and labor market choices with permanent income shocks (Low et al., 2010 and 2015), focusing on the design of social insurance against employment and productivity risk without allowing for investments in risky assets. Our paper belongs to the household finance tradition that allows for risky investments but overlooks both moral hazard stemming from social insurance programs and the associated difference between productivity and employment risk. Finally, personal disaster risk differs from both the individual stock market disas-

[2] We do not model the option to change labor supply to buffer income shocks, as in Bodie, Merton and Samuelson (1992) and Gomes, Kotlikoff and Viceira (2008). This option is open to those who find a new job, while what drives our results is the *ex ante* possibility of a large loss in the disaster state.

ter in Fagereng, Gottlieb and Guiso (2017) and the aggregate economic collapse in the macro-finance literature (e.g. Barro (2006) - although disasters might be correlated. As Arellano et al. (2017) points out, macroeconomic disasters are statistically elusive events while disasters at the micro level happen all the time to some individuals.

The rest of the paper is organized as follows. In Section 2 we provide evidence on life-cycle portfolio holdings and institutional details on LTU, disability and social insurance for the US. Section 3 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. We detail the model calibration in Section 4 and discuss our main results in Section 5. Section 5.4 examines the ability of the model to match the observed stock-holdings in real data. Section 6 concludes the paper.

2 Households Portfolios and Personal Disasters

This section introduces to the main stylized facts concerning financial risk taking and personal disaster risk in the US. The first subsection builds on the method of Ameriks and Zeldes (2002) to examine the empirical relationship between age and conditional risky shares, i.e. the fraction of financial wealth held in risky assets conditional on participation to the stock market. These life-cycle investment profiles in US data will later be matched with the model-implied profiles.

Since such profiles will be calibrated to disaster risk, the second subsection summarizes some relevant features of disability and long-term unemployment.

2.1 Life-Cycle Profiles of Household Portfolios

We pool data from the independent cross-sectional surveys in the Survey of Consumer Finances (SCF), covering the years from 1992 to 2016. The SCF is nationally representative of households of all ages in the United States and collects detailed information on their characteristics and their investment decisions. We classify the household' financial assets into two categories: *safe* and *risky*, following Chang, Hong and Karabarbounis (2018). Safe assets include checking accounts, savings accounts, money market accounts, certificates of deposit, the cash value of life insurance, US government and state bonds, mutual funds invested in tax-free bonds and government-backed bonds, and trusts and annuities invested in bonds and money market accounts. Risky assets include stocks, stock brokerage accounts, mortgage-backed bonds, foreign and corporate bonds, mutual funds invested in stock funds, trusts and annuities invested in stocks or real estate, and pension plans that are a thrift, profit-sharing, or stock purchase plan. In Table 1, we report the summary statistics concerning both the households' financial assets composition and households main characteristics. We restrict the sample to households with positive financial assets and with head aged between 21 and 70.

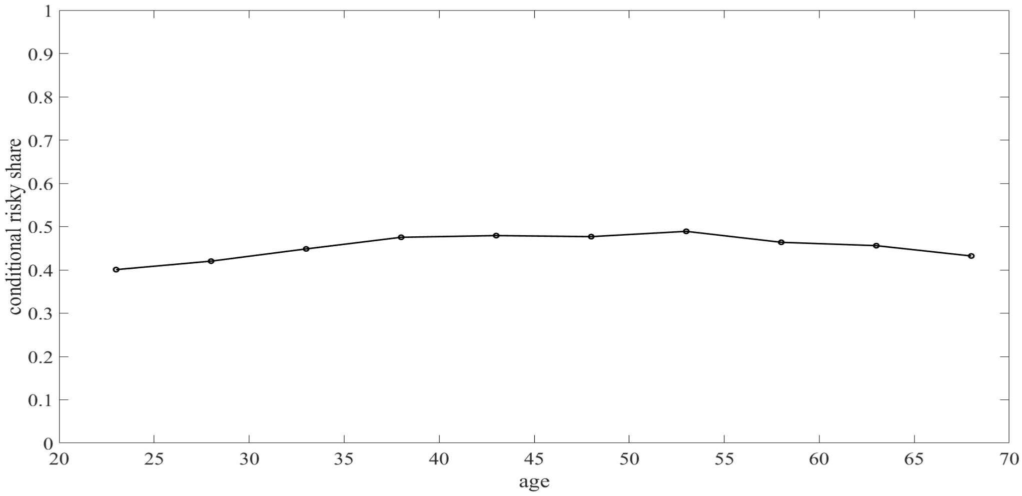
Table 1: Descriptive statistics

Wave	1992	1995	1998	2001	2004	2007	2010	2013	2016
Financial assets									
Amount ($)									
Safe	126,323	135,264	138,320	148,852	139,953	137,447	143,926	126,739	141,793
Risky	70,842	91,448	167,039	202,997	161,592	162,939	129,381	137,308	159,180
Total (Safe+Risky)	197,166	226,712	305,359	351,849	301,544	300,386	273,307	264,046	300,973
Conditional Share									
Safe	64.1%	59.7%	45.3%	42.3%	46.4%	45.8%	52.7%	48.0%	47.1%
Risky	35.9%	40.3%	54.7%	57.7%	53.6%	54.2%	47.3%	52.0%	52.9%
Men	78.6%	76.8%	76.9%	77.0%	75.7%	76.5%	76.6%	75.1%	74.2%
Age	45.6	46.2	46.5	46.5	47.5	48.2	47.6	48.2	48.9
No high school	12.5%	11.5%	10.6%	10.5%	9.4%	9.1%	8.8%	7.6%	11.3%
High school	30.1%	32.6%	31.6%	31.3%	29.9%	31.4%	30.7%	29.2%	24.9%
Some college	24.2%	27.0%	27.2%	26.0%	26.3%	26.3%	26.7%	27.2%	28.4%
College	33.2%	28.8%	30.6%	32.3%	34.4%	33.2%	33.9%	36.1%	35.4%
N (households)	3906	4302	4326	4475	4526	4423	6555	6026	6261

The table reports the average composition of households financial assets and demographic characteristics across various SCF waves ($1992-2016$). The sample is restricted to households with heads aged between 21 and 70 and with a positive amount of financial assets. Nominal variables are expressed in 2015 U.S. dollars.

In Figure 1 we report the life cycle age profile of the average conditional portfolio share invested in risky assets. The dots represents the five-year average (from age group $21-25$ to age group $66-70$). The conditional risky share is flat over the life cycle, ranging from 40% to 49%.

Figure 1: Conditional Risky Share - SCF data



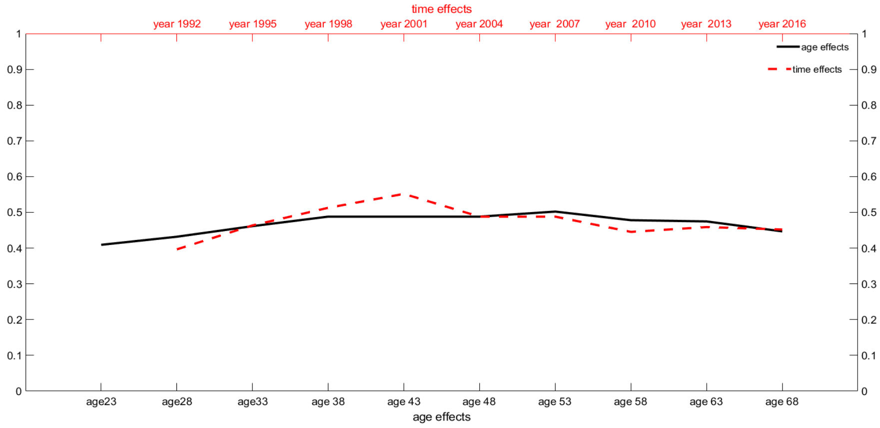
This figure displays the life cycle profile of conditional risky share of financial assets held by U.S. households grouped by five-years age classes ($21-25$,...,$66-70$). The dots represent the five-year average.

Ideally, we should distinguish the impact of age on household risk taking from that of both the calendar year and birth cohorts. However, the three effects cannot be separately identified. We therefore estimate three regression models where one effect at a time is held constant against the other two, following Ameriks and Zeldes (2002). The age dummies are constructed on the basis of five-year age groups, from 21 to 70, and the reference age group is aged between 46 and 50. Similarly, the birth year cohort dummies refer to five birth-year groups (from $1924-1928$ to $1989-1993$) and we take the cohort $1953-1958$ as the reference group. Finally, the time effects refer to the years in which the surveys are collected and we take year 2004 as the reference group. In Figure 2 panel a), we report the regression estimates of time and age effects based on OLS estimates with cohort effects excluded (dashed and solid line respectively); panel b) reports the time and cohort effects based on OLS estimates with age effects excluded (dashed and solid line respectively); finally, panel c) reports the age and cohorts effects based on OLS estimates with time effects excluded (dashed and solid line respectively).[3] The conditional risky share is remarkably flat across ages and cohorts in all the specifications. The time effects show an increase during the 1990s and a relative slowdown after 2000. Such time evolution of households' behavior may reflect both the "Internet bubble" in the '90s and higher households' background risk during the 2000s. These patterns are robust across education levels (unreported here).

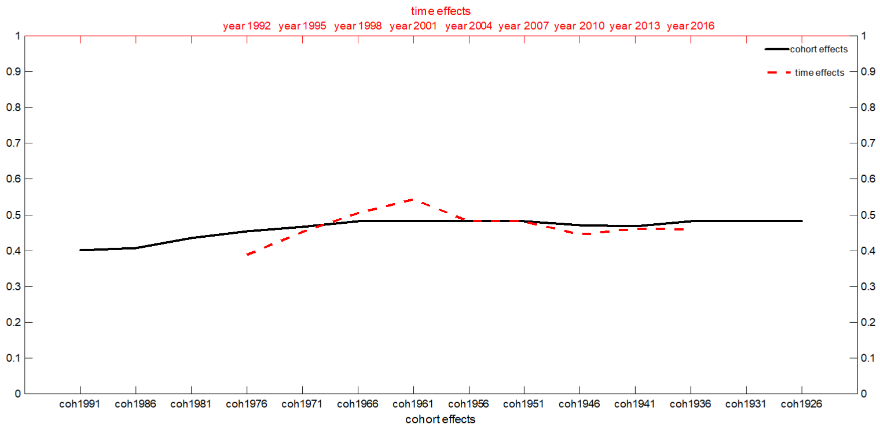
[3]We set to zero all the coefficients that are not statistically significant from zero at 5% level.

Figure 2: Age, Time and Cohort effects on Conditional Risky Share

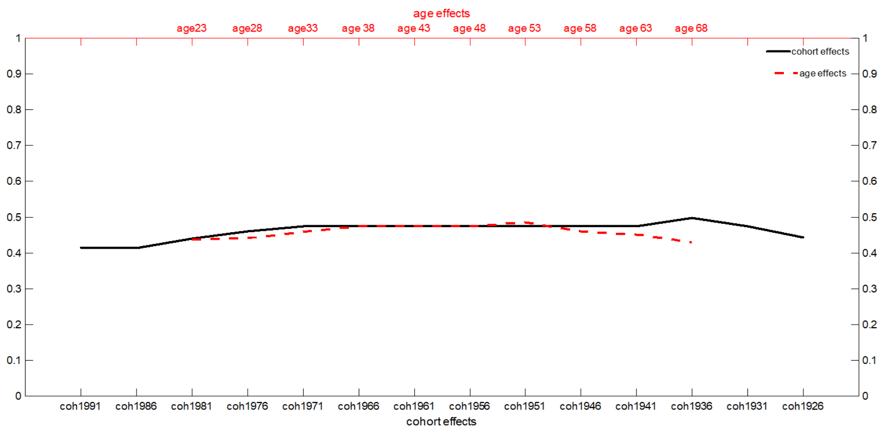
Panel (a)



Panel (b)



Panel (c)



This figure displays the estimated age, cohort and time effects on conditional risky share under different model specifications. In panel (a), the cohort effect is assumed to be constant across ages and periods; in panel (b), the age effect is assumed to be constant across cohorts and periods; in panel (c), the time effect is assumed to be the same across ages and cohorts. SCF data from 1992 to 2016 on households with head aged between 21 and 70. Coefficients not statistical significant at 5% are set to zero.

These results reveal that the conditional risky share is almost flat in age and across cohorts through 2016, confirming the patterns originally unveiled by Ameriks and Zeldes (2004).

2.2 Uninsured Personal Disasters

This section provides an assessment of human capital losses deriving from personal disasters, such as layoffs or disability. It also sheds light on estimates of the fraction of permanent income shocks that remains uninsured. In so doing, it touches upon heterogeneity across households and over the sample years. This section will thus provide the background against which we calibrate the labor income process and the insurance parameters of our model.

Unemployment may lead to persistent earnings losses, that increase in its duration because of skill deterioration and lower chances of finding a new occupation. In the United States the share of unemployed workers who were jobless for more than one year, while historically low, doubled during the Great Recession episode, reaching 24% of total unemployment in 2014 and hitting all education groups (e.g., see Kroft, Lange, Notowidigdo and Katz, 2016).[4] The chances of finding a job decrease in unemployment duration, together with unemployment benefits.[5] Early estimates of persistent earnings losses due to long-term unemployment (see Jacobson Lalonde and Sullivan, 1993a) are on the order of 25 percent of the average earnings six years after separation −relative to workers with similar characteristics that stayed with the same employer during the same episode. Guvenen et al. (2017) measure the effects of full-year nonemployment across workers with heterogeneous histories in a more recent sample (1978-2010). Earnings losses are in the order of $35-40\%$ after 10 years. Such losses mostly derive from lower chances of future employment, rather than lower earnings conditional on working. Indeed workers who are employed ten years after the shock bear smaller earnings losses −on the order of 8-10 percent. Given our focus on equity investment, it is important to stress that earnings losses are large not only for workers with low−earnings but also for those in the top 5 percent of the past earnings

[4]For instance, in 2013, the share of US unemployed workers with a high school (college) education who had been looking for work for two or more years was 12.8% (13.5%) (see Mayer, 2014).

[5]Krueger, Cramer and Cho (2014) and Kroft, Lange, Notowidigdo and Katz (2016) show that the re-employability of the long-term unemployed progressively declines over time, to the extent that they are more likely to exit the labor force than to become re-employed. The presence of more job openings does not lead to increased employment among individuals who are jobless for more than six months, and this pattern holds across all ages, industries and education levels (Ghayad and Dickens 2012).

distribution. [6]

Large negative shocks associated with health are another form of personal disaster. Mental health problems have an especially large impact on labor market outcomes, possibly because they affect prime age workers. The onset of mental illness reduces earnings initially by as much as 24%, and negative effects can last several years. Moreover, disorders reduce the probability of employment by about 14% (see Currie and Madrian (1999)).

Whether coming from layoffs or individual productivity declines, personal disaster risk is subject to incomplete insurance. Layoffs are usually partially insured by the US unemployment insurance system, but long-term unemployment is not. Personal productivity shocks are rarely insured by social welfare programs, except from major observable health problems, because of moral hazard. When awarded, disability benefits are more generous than unemployment benefits, offering a replacement rate of about 42 percent to the average worker (Gruber, 2000). The replacement rates are higher for low income people and for those who do not have employer-provided health insurance (Low and Pistaferri, 2015). And of course there may be informal insurance mechanisms, including family support. Guvenen and Smith (2014) infer the extent of overall partial insurance from a dynamic model of consumption and linear labor income shocks where agents learn about their income growth rates. The partial insurance parameter is estimated to be 0.451, implying that almost one-half of both permanent and transitory income shocks are smoothed away through mechanisms different from savings. Blundell, Pistaferri, and Preston (2008) also estimate the extent of partial insurance. While it varies across cohorts, their estimate on their overall sample is that about 36% of permanent shocks are insured (and almost 95% of transitory shocks).

Last but not least, the extent of the coverage is *ex-ante* uncertain, adding to the uncertainty of the losses experienced in a personal disaster state. For instance, the structure of disability insurance has an initial claiming stage and an appeals process, with fluctuations over time in the award rates. Importantly, such screening may be subject to error. According to Low and Meghir (2015), the probability of being rejected while having a severe work limitation exceeds 0.5.

[6] Jung and Kuhn (2019) find that a shock at the top of the earnings distribution, such as the loss of a particularly good job, is a relevant source of persistent earnings losses.

Against this background, the model will allow for residual personal losses in the personal disaster state that can be on average small but feature uncertainty as to their actual size. The calibrations will refer to the case of an expected human capital erosion as low as 20% of the permanent labor income component.

3 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest as well. The investor starts working at age t_0 and retires with certainty at age $t_0 + K$. The effective length of her life, which lasts at most T periods, is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t, the conditional survival probability at t (with $p_{t_0-1} = 1$). Investor's i preferences at date t are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-2} p_{t_0+k}\right) \left(p_{t_0+j-1}\frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1-p_{t_0+j-1})\, b\frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma}\right)\right] \quad (1)$$

where C_{it} is the level of consumption at time t, X_{it} is the amount of wealth the investor leaves as a bequest to her heirs after her death, $b \geq 0$ is a parameter capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and γ is the constant relative risk aversion parameter.

3.1 Labor and retirement income

During working life individuals receive exogenous stochastic earnings as compensation for labor supplied inelastically. Working life careers are modelled as a three-state Markov chain considering employment (e), short-term unemployment (u_1) and a disaster state characterized by long-term (u_2) unemployment. Individual labor market dynamics are driven by the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} \end{pmatrix} = \begin{pmatrix} \pi_{ee} & 1-\pi_{ee} & 0 \\ \pi_{u_1e} & 0 & 1-\pi_{u_1e} \\ \pi_{u_2e} & 0 & 1-\pi_{u_2e} \end{pmatrix} \quad (2)$$

where $\pi_{nm} = \text{Prob}\left(s_{t+1} = n | s_t = m\right)$ with $n, m = e, u_1, u_2$. If the worker is employed at t ($s_t = e$), she continues the employment spell at $t+1$ ($s_{t+1} = e$) with probability π_{ee}, otherwise she enters short-term unemployment ($s_{t+1} = u_1$) with probability $\pi_{eu_1} = 1 - \pi_{ee}$. For simplicity, we set the probability of directly entering the disaster state of long-term unemployment at zero, $\pi_{eu_2} = 0$. After being unemployed for one period, at t ($s_t = u_1$), she exits unemployment ($s_{t+1} = e$) with probability $\pi_{u_1 e}$ or becomes long-term unemployed ($s_{t+1} = u_2$) with probability $\pi_{u_1 u_2} = 1 - \pi_{u_1 e}$; that is, we set $\pi_{u_1 u_1} = 0$. Finally, if she is long-term unemployed at t ($s_t = u_2$), she either returns to employment in the following period ($s_{t+1} = e$) with probability $\pi_{u_2 e}$ or remains in the disaster state with probability $\pi_{u_2 u_2} = 1 - \pi_{u_2 e}$.

Stochastic labor income is driven by permanent and transitory shocks. In each working period, labor income Y_{it} is generated by the following process:

$$Y_{it} = H_{it} U_{it} \qquad\qquad t_0 \leq t \leq t_0 + K \tag{3}$$

where $H_{it} = F\left(t, \mathbf{Z}_{it}\right) P_{it}$ represents the permanent income component. In particular, $F\left(t, \mathbf{Z}_{it}\right) \equiv F_{it}$ denotes the deterministic trend component that depends on age (t) and a vector of individual characteristics ($\mathbf{Z}_{it}$) such as gender, marital status, household composition and education. As in Cocco, Gomes and Menhout (2005), the logarithm of the stochastic permanent component is assumed to follow a random walk process:

$$N_{it} = \log P_{it} = \log P_{it-1} + \omega_{it} \tag{4}$$

where ω_{it} is distributed as $N(0, \sigma_\omega^2)$. U_{it} denotes the transitory stochastic component and $\varepsilon_{it} = \log(U_{it})$ is distributed as $N(0, \sigma_\varepsilon^2)$ and uncorrelated with ω_{it}.

In our set-up, labor income received by the employed individual at time t depends on her past working history because we allow unemployment and its duration to affect the permanent component of labor income, H_{it}. Thus, after 1-year unemployment the permanent component H_{it} is equal to H_{it-1} eroded by a fraction Ψ_1, and after a 2-year unemployment spell the permanent component, H_{it-1}, is eroded by a fraction Ψ_2, with $\Psi_2 > \Psi_1$. This introduces non-linearity into the expected permanent labor income.[7] In compact form, the

[7] The longer the unemployment spell the larger is the worker's human capital depreciation. See Guvenen

permanent component of labor income H_{it} evolves according to

$$H_{it} = \begin{cases} F\left(t, \mathbf{Z}_{it}\right) P_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1-\Psi_1) H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1-\Psi_2) H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{5}$$

We now model the human capital erosion parameters, Ψ_1 and Ψ_2 as random variables that follow standard *Beta* distributions with shape parameters (a_j, b_j): thus, $\Psi_j \sim Beta(a_j, b_j)$.[8] This distribution allows to represent outcomes, like proportions, being defined on the continuum between 0 and 1. The standard *Beta* distribution gives the probability density of the value of Ψ_j, with $j = 1, 2$, on the interval (0,1):

$$f(\Psi_j : a_j, b_j) = \frac{\Psi_j^{a_j - 1}(1-\Psi_j)^{b_j - 1}}{B(a_j, b_j)} \tag{6}$$

where B is the beta function, thus B(a,b) plays the role of normalization constant to ensure that the total probability is 1:

$$B(a_j, b_j) = \int_0^1 t^{a_j - 1}(1-t)^{b_j - 1} dt \tag{7}$$

The expected value of Ψ_j, with $j = 1, 2$, is equal to:

$$E(\Psi_j) = \frac{a_j}{a_j + b_j} \tag{8}$$

and the variance is

$$Variance(\Psi_j) = \frac{a_j b_j}{(a_j + b_j)^2 (a_j + b_j + 1)} \tag{9}$$

In the short-term unemployment state ($s_t = u_1$) individuals receive an unemployment benefit as a fixed proportion ξ_1 of the last working year permanent income $H_{it-1} = F_{it-1} P_{it-1}$, whereas in the long-term unemployment state ($s_t = u_2$) benefits are available in proportion

et al. (2017); and Schmieder, von Wachter and Bender (2016).

[8] This modelling compactly represents the uncertainties surrounding possible future negative earnings shocks. These include the award process of disability insurance; or the differential personal impact of crisis times, such the Great Recession or the Covid one, and of ordinary business cycle contractions.

ξ_2. Thus, the income received during unemployment is

$$Y_{it} = \begin{cases} \xi_1 H_{it-1} & \text{if } \mathrm{s}_t = u_1 \\ \\ \xi_2 H_{it-2} & \text{if } \mathrm{s}_t = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{10}$$

Finally, during retirement, income is certain and equal to a fixed proportion λ of the permanent component of labor income in the last working year:

$$Y_{it} = \lambda F\left(t, \mathbf{Z}_{it_{0+l}}\right) P_{it_{0+l}} \qquad t_0 + K < t \leq T \tag{11}$$

where retirement age is $t_0 + K$, $t_0 + l$ is the last working period and λ is level of the replacement rate.

3.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding a constant gross real return R^f, and one risky asset, characterized as "stocks" yielding stochastic gross real returns R_t^s, for each period. The excess returns of stocks over the riskless asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{12}$$

where μ^s is the expected stock premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2017).

At the beginning of each period, financial resources available to the individual for consumption and saving are given by the sum of accumulated financial wealth W_{it} and current labor income Y_{it}, which we call cash on hand $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by

$$X_{it+1} = (X_{it} - C_{it})R_{it}^P + Y_{it+1} \tag{13}$$

where R^P_{it} is the investor's portfolio return:

$$R^P_{it} = \alpha^s_{it} R^s_t + (1 - \alpha^s_{it}) R^f \tag{14}$$

with α^s_{it} and $(1 - \alpha^s_{it})$ denoting the shares of the investor's portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and we assume that the investor is liquidity constrained. Consequently, the amounts invested in stocks and in the riskless asset are non negative in all periods. All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement.

3.3 Solving the life-cycle problem

In this intertemporal optimization framework, the investor maximizes the expected discounted utility over life span, by choosing the consumption and the portfolio rules given uncertain labor income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^T, \left\{\alpha^s_{it}\right\}_{t_0}^T} \left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j-1} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + \right.\right.\right.$$
$$\left.\left.\left. + \left(1 - p_{t_0+j-1}\right) b \frac{\left(X_{it_0+j}/b\right)^{1-\gamma}}{1-\gamma} \right) \right] \right) \tag{15}$$

$$s.t. \quad X_{it+1} = (X_{it} - C_{it}) \left(\alpha^s_{it} R^s_t + (1 - \alpha^s_{it}) R^f \right) + Y_{it+1} \tag{16}$$

with the labor income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it,}P_{it},s_{it}\right)=\max_{\{C_{it}\}_{t_0}^{T},\{\alpha_{it}^{s}\}_{t_0}^{T}}\left(\frac{C_{it}^{1-\gamma}}{1-\gamma}+\beta E_t\left[p_t V_{it+1}\left(X_{it+1,}P_{it+1},s_{it+1}\right)\right.\right.$$
$$\left.\left.+\left(1-p_t\right)b\frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma}\right]\right) \tag{17}$$

At each time t the value function V_{it} describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time t (X_{it}), the stochastic permanent component of income at beginning of t (P_{it}), and the labor market state $s_{it}(=e,\ u_1,\ u_2)$. The Bellman equation can be written by making the expectation over the employment state at $t+1$ explicit:

$$V_{it}\left(X_{it,}P_{it},s_{it}\right)=\max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^{s}\}_{t_0}^{T-1}}\left(\frac{C_{it}^{1-\gamma}}{1-\gamma}\right.$$
$$+\beta\left[p_t\sum_{s_{it+1}=e,u_1,u_2}\pi\left(s_{it+1}|s_{it}\right)\widetilde{E_tV}_{it+1}\left(X_{it+1,}P_{it+1},s_{it+1}\right)\right.$$
$$\left.\left.+\left(1-p_t\right)b\sum_{s_{it+1}=e,u_1,u_2}\pi\left(s_{it+1}|s_{it}\right)\frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma}\right]\right) \tag{18}$$

where $\widetilde{E_tV}_{it+1}$ denotes the expectation operator taken with respect to the stochastic variables ω_{it+1}, ε_{it+1}, and ν_{it+1}^{s}. The history dependence that we introduce in our set-up by making unemployment affect subsequent labor income prospects prevents having to rely on the standard normalization of the problem with respect to the level of P_t. To highlight how the evolution of the permanent component of labor income depends on previous individual labor market dynamics we write the value function at t in each possible state as (dropping the term involving the bequest motive):

$$V_{it}(X_{it},P_{it},e)\;=u(C_{it})\;+\beta p_t\begin{cases}\begin{cases}V_{it+1}(X_{it+1},P_{it+1},e) & \text{with prob. } \pi_{e,e}\\ \text{with }\; P_{it+1}=P_{it}e^{\omega_{it+1}} & \text{and}\\ X_{it+1}=(X_{it}-C_{it})R_{it}^{p}+F_{it+1}P_{it+1}e^{\varepsilon_{it+1}}\end{cases}\\ \\ \begin{cases}V_{it+1}(X_{it+1},P_{it+1},u_1) & \text{with prob. } 1-\pi_{e,e}\\ \text{with }\; P_{it+1}=(1-\Psi_1)P_{it} & \text{and}\\ X_{it+1}=(X_{it}-C_{it})R_{it}^{p}+\xi_1F_{it}P_{it}\end{cases}\end{cases}$$

$$V_{it}(X_{it},P_{it},u_1) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1},P_{it+1},e) \quad \text{with prob. } \pi_{u_1,e} \\ \text{with } \; P_{it+1} = (1-\Psi_1)P_{it-1}\, e^{\omega_{it+1}} = P_{it}\; e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + F_{it-1}P_{it+1}e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1},P_{it+1},u_2) \quad \text{with prob. } 1-\pi_{u_1,e} \\ \text{with } \; P_{it+1} = (1-\Psi_2)(1-\Psi_1)P_{it-1} = (1-\Psi_2)P_{it} \text{ and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} \end{cases} \end{cases}$$

$$V_{it}(X_{it},P_{it},u_2) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1},P_{it+1},e) \quad \text{with prob. } \pi_{u_2,e} \\ \text{with } \; P_{it+1} = P_{it}e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} + F_{it-2}P_{it+1}e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1},P_{it+1},u_2) \quad \text{with prob. } 1-\pi_{u_2,e} \\ \text{with } \; P_{it+1} = (1-\Psi_2)P_{it} \text{ and} \\ X_{it+1} = (X_{it}-C_{it})R^p_{it} \end{cases} \end{cases} \tag{19}$$

This problem has no closed form solution; therefore, we obtain the optimal values for consumption and portfolio shares, depending on the values of each state variable at each point in time, by means of numerical techniques. To this aim, we apply a backward induction procedure starting from the last possible period of life T and computing optimal consumption and portfolio share policy rules for each possible value of the continuous state variables (X_{it} and P_{it}) by means of the standard grid search method.[9] Going backwards, for every period $t = T-1, T-2, ..., t_0$, we use the Bellman equation (18) to obtain optimal rules for consumption and portfolio shares.

4 Calibration

Parameter calibration concerns investor's preferences, the features of the labor income process during working life and retirement, and the moments of the risky asset returns. For reference, we initially solve the model by abstracting from the unemployment risk as in Cocco, Gomes and Maenhout (2005). Then, we introduce unemployment risk and consider two scenarios: (i) unemployment spells cause permanent deterministic income losses, as in Bagliano et al. (2019), and (ii) unemployment has permanent but uncertain consequences on the worker's earnings ability.

Across all scenarios, the agent begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take

[9] The problem is solved over a grid of values covering the space of both the state variables and the controls in order to ensure that the obtained solution is a global optimum.

the conditional probability of being alive in the next period p_t from the life expectancy tables of the US National Center for Health Statistics. With regards to preferences, we set the utility discount factor $\beta = 0.96$, and the parameter capturing the strength of the bequest motive $b = 2.5$ (which bears the interpretation of the number of years of her descendants' consumption that the investor intends to save for). Finally, the benchmark value for the coefficient of relative risk aversion is $\gamma = 5$. The latter choice is relatively standard in the literature (Gomes and Michaelides 2005; Gomes, Kotlikoff and Viceira 2008) and captures an intermediate degree of risk aversion. However, Cocco, Gomes and Maenhout (2005) choose a value as high as 10 in their benchmark setting. The riskless (constant) interest rate is set at 0.02, with an expected equity premium μ^s fixed at 0.04. The standard deviation of the return innovations is set at $\sigma_s = 0.157$. Finally, we impose a zero correlation between stock return innovations and aggregate permanent labor income disturbances ($\rho_{sY} = 0$). Table 1 summarizes the benchmark values of relevant parameters.

Table 2: Calibration parameters

Description	Parameter	Value
Working life (max)	T	20 -65
Retirement (max)	$t_0 + K$	65 -100
Discount factor	β	0.96
Risk aversion	γ	5
Replacement ratio	λ	0.68
Variance of permanent shocks to labor income	σ^2_ω	0.0106
Variance of transitory shocks to labor income	σ^2_ϵ	0.0738
Riskless rate	r	0.02
Excess returns on stocks	μ^s	0.04
Variance of stock returns innovations	σ_s	0.157
Stock ret./permanent lab. income shock correlation	ρ_{sY}	0

	Unemployment no disaster	**Unemployment with disaster risk**
Unemployment benefits		
Short-term unemployed (1)	0.3	0.3
Long-term unemployed (2)	0.1	0.1
Human capital erosion		
Short-term unemployed (Ψ_1)	0	0
Long-term unemployed (Ψ_2)	0.20	(expected) 0.20
—*Beta* distribution a_2	-	*0.01*
—*Beta* distribution b_2	-	*0.04*

This table reports benchmark values of relevant parameters.

4.1 Labor income and unemployment risk

The labor income process is calibrated using the estimated parameters for US households with high school education (but not a college degree) in Cocco, Gomes and Maenhout (2005). For the high school group, the variances of the permanent and transitory shocks (ω_{it} and ε_{it} respectively) are equal to $\sigma_\omega^2 = 0.0106$ and $\sigma_\varepsilon^2 = 0.0738$. After retirement, income is a constant proportion λ of the final (permanent) labor income, with $\lambda = 0.68$. The parameter values assumed above are maintained across all scenarios.

We use data from the Current Population Survey (CPS) to calibrate the transition probabilities from employment to unemployment to reflect the risk of entering unemployment along with the observed average unemployment rates at different durations. According to the evidence based on CPS reported in Kroft, Lange, Notowidigdo and Katz (2016), the annual transition probability from employment to unemployment is 4%. Given the duration dependence and the steady decline in the annual outflow rate from unemployment to employment during the first year of unemployment (Kroft, Lange, Notowidigdo and Katz, 2016), we set the probability of leaving unemployment after the first year at 85%.

The annual transition probabilities between labor market states are chosen to match the average annual unemployment rate in the United States:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 0.85 & 0 & 0.15 \\ 0.85 & 0 & 0.15 \end{pmatrix} \tag{20}$$

Our calibration appears quite conservative, since the chance of being employed 15 months later for those who had been unemployed 27 weeks or more is only 36% (see, by comparison, the evidence on CPS data in Krueger, Cramer and Cho, 2014). Indeed, the assumed transition matrix (20) yields unconditional probabilities of being short-run (3.8%) and long-run unemployed (0.7%) in line with historical unemployment rates in US.

Well-established empirical evidence on job displacement shows that job losses affect earnings far beyond the unemployment spell, though the range of the estimated effects varies considerably. For example, the estimates for immediate losses following displacement may range from 30% (Couch and Placzek, 2010) to 40% of earnings (Jacobson, Lalond and Sullivan, 1993b). Earnings losses are shown to be persistent in a range from 15% (Couch and Placzek, 2010) to about 25% (Jacobson, LaLonde and Sullivan, 1993a) of their pre-displacement levels. These estimates abstract from the effect of unemployment duration, while Cooper (2013) finds that earnings losses are larger the longer unemployment lasts. Also, based on administrative data, Jacobson, LaLonde and Sullivan (2005) estimate that average earnings losses for displaced workers amount to 43-66% of their predisplacement wage. This body of evidence, combined with a probability of finding a job after being unemployed for 24 months as low as 40% (Kroft, Lange, Notowidigdo and Katz, 2016), leads us to calibrate a substantial expected drop in human capital following a long term unemployment spell. Thus, while Ψ_1 is kept equal to 0,[10] Ψ_2 follows a *Beta* distribution with expected value of 20% standard deviation of 33%. The calibrated distribution for Ψ_2 implies a median value for the proportional human capital erosion lower than 1%. Several studies documented the effects of unemployment on future labor income and separation rates, for example, Guevenen, Karahan, Ozkan and Song (2017) find that income losses after long term unemployment are substantial and heterogeneous. In our calibration, the long term consequences of not working for a long time are modest for the majority but possibly very large in extremely rare situations.

Unemployment benefits are calibrated according to the US unemployment insurance system. In particular, considering that the replacement rate with respect to last labor income is on average low and state benefits are paid for a maximum of 26 weeks, we set $\xi_1 = 0.3$ in case of short-term unemployment spells and set a value of $\xi_2 = 0.1$ for the long-term unemployed. No additional weeks of federal benefits are available in any state: the temporary Emergency Unemployment Compensation (EUC) program expired at the end of 2013, and no state currently qualifies to offer more weeks under the permanent Extended Benefits (EB) program.[11]

For comparison, we consider a calibration of the model without unemployment risk. This "*no unemployment risk*" scenario corresponds to the standard life-cycle set up with $\pi_{ee} = 1$ and all other entries equal to zero in the transition probability matrix (2). In addition, to highlight the effects of permanent consequences of unemployment on future earnings prospects, we consider a third calibration by adding the unemployment risk embedded in the transition probability matrix (20) with deterministic human capital erosion. In this "*unemployment with no disaster*" scenario, long-term unemployment has deterministic permanent consequences on future earnings implying a human capital loss of 20% (i.e. $\Psi_1 = 0$ and $\Psi_2 = 0.20$). This case closely corresponds to the set-up studied by Bagliano et al. (2019).

[10] In particular, for $a_1/b_1 \to 0$, the *Beta* distribution has a spike at the left end, $\Psi_1 = 0$ with probability 1, and zero probability everywhere else., i.e. there is 100% probability (absolute certainty) concentrated at the left end.

[11] Low, Meghir and Pistaferri (2010) acknowledge that layoffs are partially insured by the unemployment insurance system, while individual productivity shocks, other than major observable health shocks, are rarely insured in any formal way. As for other welfare programs, we do not model basic consumption needs and therefore overlook basic consumption insurance.

5 Results

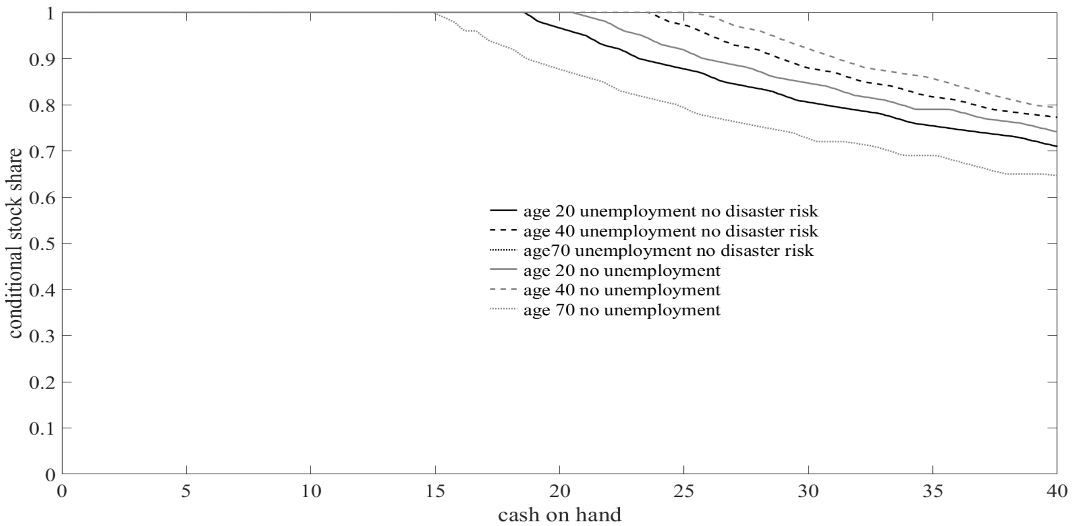
5.1 Optimal policies

Figure 3 compares investors' optimal stock shares in the case of *no unemployment* and in the case of unemployment with deterministic human capital erosion of 20%, (i.e. unemployment without disaster risk) , (panel (a)) and in our preferred scenario with "*unemployment with expected human capital erosion of* 20%*, i.e. unemployment with disaster risk*" (panel (b)). In particular, the figure plots the optimal stock share as a function of cash on hand for an average level of the permanent labor income component of investors at three different ages (20, 40, and 70). In the case with no unemployment or with unemployment but no disaster risk, the standard life-cycle results are obtained. Labor income acts as an implicit risk-free asset and affects the optimal portfolio composition depending on an investor's age and wealth. For example, at age 20 the sizable implicit holding of the risk-free asset (through human capital) makes it optimal for less-wealthy investors to tilt their portfolio towards the risky financial asset. Indeed, for a wide range of wealth levels, agents optimally choose to be fully invested in stocks. The optimal stock holding decreases with financial wealth because of the relatively lower implicit investment in (risk-free) human capital.

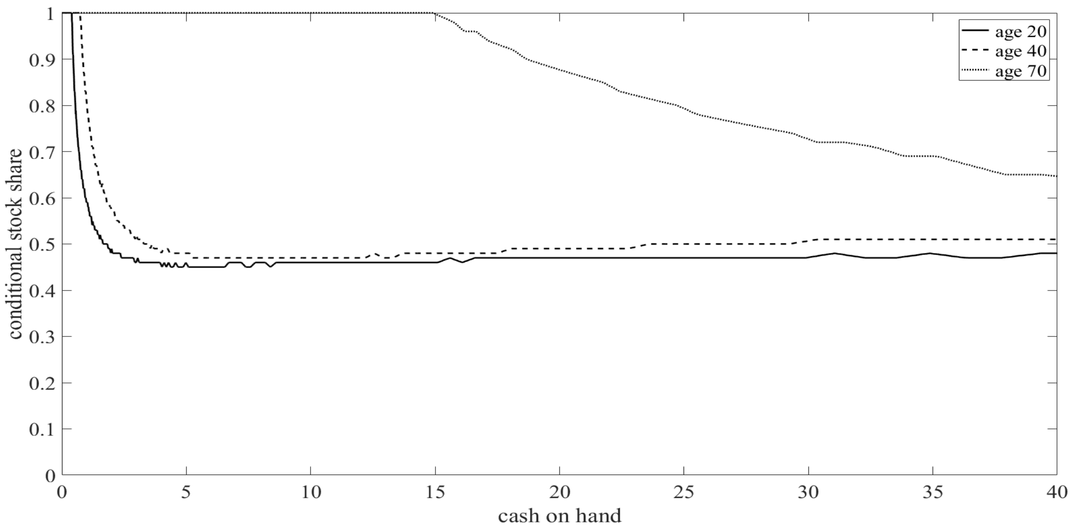
When the model is extended to allow for uncertain permanent effects of unemployment spells on labor income prospects at re-employment ("*unemployment with disaster risk*"), with the parameters governing the proportional erosion of permanent labor income set at $\Psi_1 = 0$ after one year of unemployment and at an expected $\Psi_2 = 0.2$ after 2 years, the resulting policy functions are shifted abruptly leftward. The optimal stock share still declines with financial wealth but a 100% share of investment in stocks is optimal only at very low levels of wealth. In this case, long-term unemployment implies an uncertain loss of future labor income which severely reduces the level of human capital and increases its risk at any age. Thus, for almost all levels of financial wealth, stock investment is considerably lower than in the case of no disaster risk.

Figure 3: Policy functions

(a) No unemployment - Unemployment without disaster risk



(b) Unemployment with disaster risk



This figure shows the portfolio rules for stocks as a function of cash on hand for an average level of the stochastic permanent labor income component. The policies refer to selected ages: 20, 40, and 70. Panel (a) and (b) refer respectively to the cases of no unemployment -unemployment without disaster risk and of unemployment with disaster risk. In the case of unemployment without disaster risk, the human capital loss is 20%. In the case of unemployment with disaster risk, the parameters governing the disaster risk are $\Psi_1 = 0$ and expected $\Psi_2 = 0.20$, i.e. human capital loss is uncertain with expected value equal to 20%. Cash on hand is expressed in ten thousands of US dollars.

5.2 Life-Cycle Profiles

On the basis of the optimal policy functions, we simulate the whole life-cycle consumption and investment decisions for 10,000 agents. Figure 4, panel (a), shows the average optimal equity portfolio shares plotted against age with and without disaster risk. In the case of no unemployment risk (dotted line), the well-known downward sloping pattern emerges. Over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through human capital declines with age. Consequently, at early stages of the life cycle, optimal stock investment is about 100% and decreases with age to reach around 80% at retirement. When we consider unemployment risk with deterministic human capital erosion of 20% (dashed line), the optimal portfolio share of stocks still declines with age, though being slightly lower at all ages, with a 100% optimal stock share only for very young investors.

However, when we account for the disaster risk with an expected human capital erosion of 20% (solid line), the optimal stock investment is reduced at any age and almost flat, at around 45-55%. The risk of potentially losing a substantial portion of future labor income prospects reduces the level of human capital and increases its riskiness. Because this effect is more relevant for younger workers, it induces a lower optimal stock investment conditional on financial wealth especially when young. Consequently, the age profile remains flat over the whole working life.[12] These results highlight that it is the remote possibility of encountering a potentially large negative shocks to human capital, albeit in expectation small, that dampens the incentive to invest in stocks.

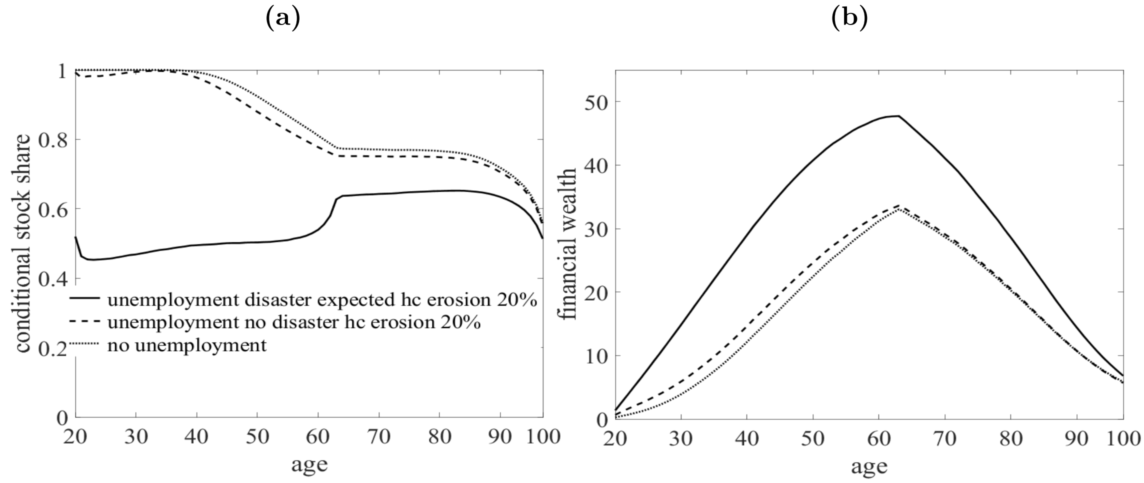
This result portrays the effects on risk-taking of the ”unusual” negative shocks that explain the consumption dynamics of US households in by Blundell, Arellano and Bonhomme (2017) throughout the earnings distribution.

The reduction in the optimal portfolio share allocated to stocks is due to higher wealth accumulation, in turn induced by larger precautionary savings.

Panel (b) of Figure 4 displays the average financial wealth accumulated over the life cycle for the three scenarios considered. In the face of possible, albeit rare, human capital depreciation, individuals accumulate substantially more financial wealth during working life to buffer possible disastrous labor market outcomes. Optimal consumption when young consequently falls, but it is much higher during both late working years and retirement years.

[12] The relatively low investment in stocks during retirement is due to the presence of a positive bequest motive.

Figure 4: Life-cycle average profiles



This figure displays the mean simulated stock investment and financial wealth accumulation life-cycle profiles. Age ranges from 20 to 100. The three cases correspond to no unemployment risk (dotted line); unemployment without disaster risk and a human capital erosion equal to 20% (dashed line); unemployment disaster risk (solid line). In the case of unemployment without disaster risk, the human capital loss is 20%. In the case of unemployment with disaster risk, the parameters governing the disaster risk are $\Psi_1 = 0$ and expected $\Psi_2 = 0.20$, i.e. human capital loss is uncertain with expected value equal to 20%. Financial wealth is expressed in ten thousands of US dollars.

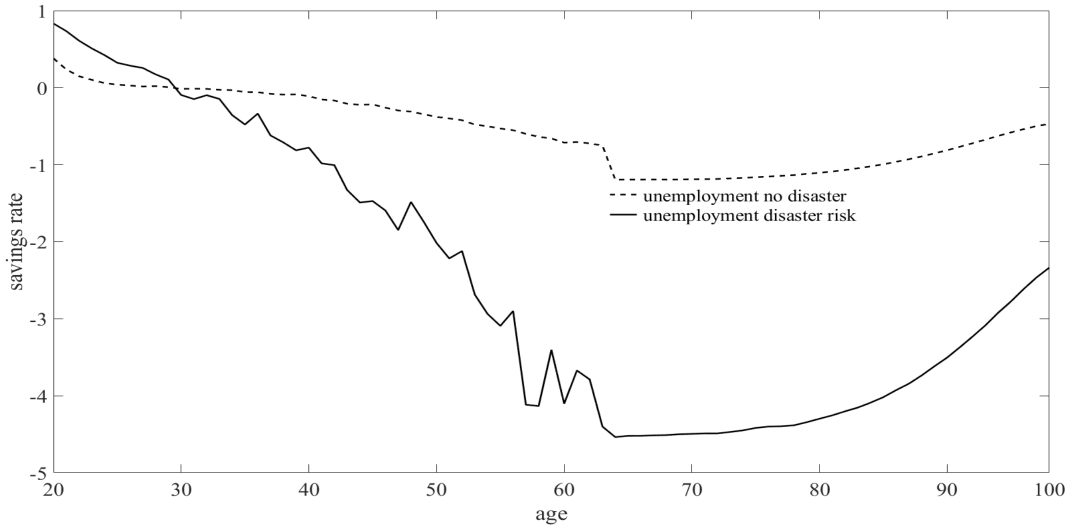
Figure 5 displays the life-cycle profile of the ratio between savings and total (financial plus labor) income[13]. When the worker is 20 years old, the average propensity to save is especially high in the disaster risk case, reaching 0.8 compared with less than 0.2 when disaster risk are absent. Such propensity monotonically decreases in age, converging to the known pattern when the worker is in her forties. The figure clearly depicts the impact on savings of the resolution of uncertainty as individuals age.[14]

These results suggest that the expectation of a higher benefit, cushioning the disastrous outcomes, mitigate the adverse impact of long term unemployment on human capital, reducing the need for cautious investing and saving during early working life. The variation of institutions across countries may thus generate different life-cycle patterns in equity investing. In this light, the decreasing stock holdings in Norwegian data (appearing in Fagereng, Gottlieb and Guiso, 2017) may be a consequence of higher long-term unemployment benefits with respect to the US.

[13] In the case of unemployment without disaster risk, the optimal decisions are very similar to the case of no unemployment, thus we report the savings ratio for the first case only

[14] Data on Norwegian households show that they engage in additional saving, shifting portfolio composition towards safe assets, in the years prior to unemployment. There is depletion of savings after the job loss (see Basten, Fagereng and Telle, 2016).

Figure 5: Life-cycle profiles of savings rate



This figure displays the savings dynamics for individuals of age 20 to 100, relative to total income (i.e. labor income plus financial income). The two cases correspond to unemployment without disaster risk (dotted line) and unemployment with disaster risk (solid line). In the case of unemployment without disaster risk, the human capital loss is 20%. In the case of unemployment with disaster risk, the parameters governing the disaster risk are $\Psi_1 = 0$ and expected $\Psi_2 = 0.20$, i.e. human capital loss is uncertain with expected value equal to 20%.

Higher savings when young obviously implies lower consumption when young. What is perhaps less obvious is whether higher wealth shields consumption from the skewness of labor income shocks. Table 2 and 3, respectively, report the mean and the standard deviation of the skewness of labor income shocks and of consumption growth.

Table 3: Skewness of labor income shocks

The table reports the mean and the standard deviation of the skewness of labor income shocks (permanent plus transitory) faced by the simulated $10,000$ investors between age t and age $t-1$.

	mean	stdev
disaster risk	-2.817	0.814
no disaster risk	-0.008	0.026

Without disaster risk, the average skewness of consumption growth is slightly negative over the working life (-0.007). With disaster risk, average skewness is negative both over the working life (-0.32).

Table 4: Skewness of consumption growth

The table reports the mean and the standard deviation of the skewness of consumption growth rates, between age t and age $t-1$, for the simulated $10,000$ investors.

	mean	stdev	mean	stdev
disaster risk	-0.32	0.49	0.018	0.27
no disaster risk	0.03	0.14	-0.07	0.17

5.2.1 Heterogeneity

The above results imply that the optimal stock investment is flat in age, even for a moderately risk averse worker. In the face of a very rare but large human capital depreciation, workers on average invest about 50% of their financial wealth in stocks. This average pattern may hide considerable differences across agents. The present section investigates the distribution across agents of both conditional optimal stock share and accumulated wealth.

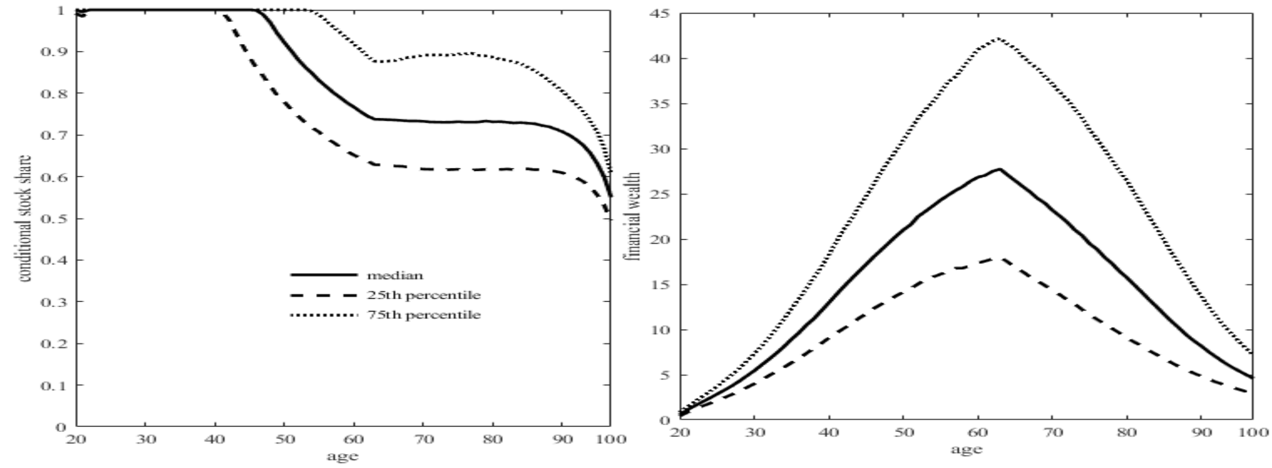
The case of unemployment without disaster risk is displayed in panels (a) and (b) of Figure 6, which show the 25^{th}, 50^{th} and 75^{th} percentiles of the distributions[15]. Both the optimal stock share and the stock of accumulated financial wealth are highly heterogeneous across workers as well as retirees. The exception is young workers as they tilt their entire portfolio towards stocks given the relatively riskless nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions displayed in Figure 3, relating optimal stock shares to the amount of available cash on hand, and on the level of cash on hand itself. Relatively steep policy functions imply that even small differences in the level of accumulated wealth result in remarkably different asset allocation choices. At the early stage of the life cycle, when accumulated financial wealth is modest, it is optimal for everybody to be fully invested in stocks. As investors grow older, different realizations of background risk induce large differences in savings and wealth accumulation. This situation pushes investors on the steeper portion of their policy functions and determines a gradual increase in the heterogeneity of optimal risky portfolio shares during their working life. After retirement, investors decumulate their financial wealth relatively slowly, due to the bequest motive, and still move along the steeper portion of their relevant policy functions; as a consequence, the dispersion of optimal shares tends to persist.

[15]In the case of unemployment without disaster risk, the optimal decisions are very similar to the case of no unemployment, thus we report the life cycle profiles for the first case only

Figure 6: Life-cycle percentile profiles

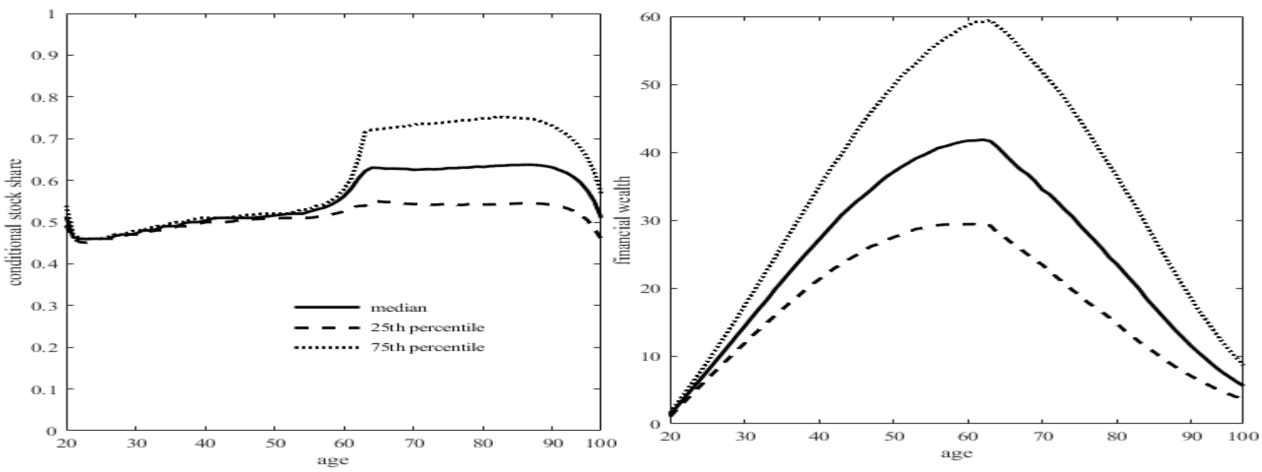
Unemployment without disaster risk

(a) **(b)**



Unemployment with disaster risk

(c) **(d)**



This figure displays the distribution of simulated equity share investment and financial wealth accumulation for individuals of age 20 to 100 in the case of unemployment without disaster risk (panels (a) and (b)) and unemployment with disaster risk (panels (c) and (d)). In the case of unemployment without disaster risk, the human capital loss is 20%. In the case of unemployment with disaster risk, the parameters governing the disaster risk are $\Psi_1 = 0$ and expected $\Psi_2 = 0.20$, i.e. human capital loss is uncertain with expected value equal to 20%. Financial wealth is expressed in ten thousands of US dollars.

Panels (c) and (d) of Figure 6 display the life-cycle distribution of stock share and financial wealth for the case with uninsured long-term unemployment. Compared with the case of no unemployment risk, the distribution of optimal stock shares is much less heterogeneous. In particular, heterogeneity shrinks during working life even for young workers, given the high human capital risk they bear at the beginning of their careers. Indeed, policy functions are relatively flat when long-term unemployment is uninsured (see panel (b) of Figure 3) implying that even large differences in the level of accumulated wealth result in homogeneous asset allocation choices.

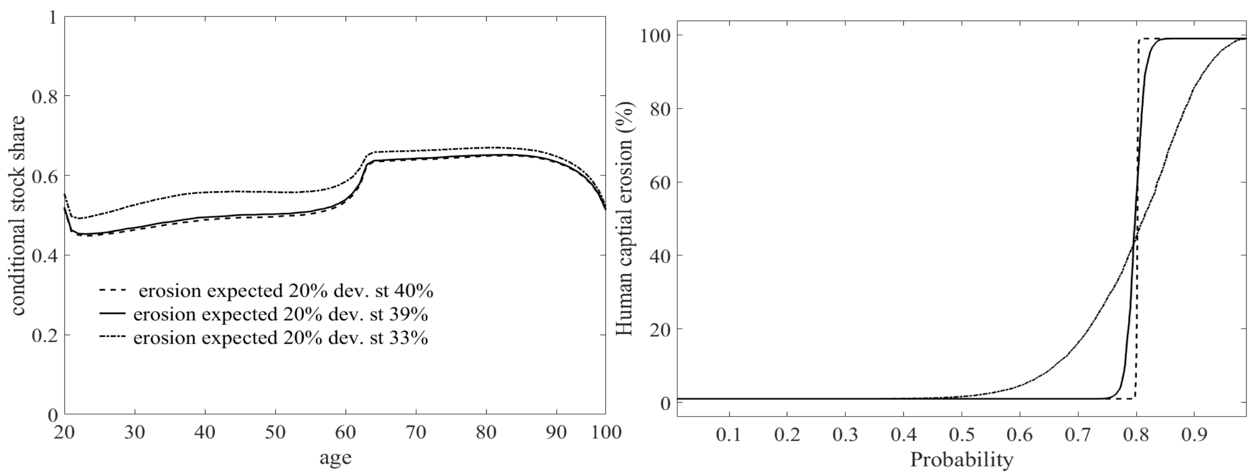
5.3 Uncertainty around the disaster risk

In this section we check the sensitivity of life-cycle profiles with respect to the uncertainty around the human capital erosion that occurs in case of long-term unemployment, captured by the two shape parameters of the Beta distribution, α_2 and β_2. In particular, we vary the two parameters to keep the expected human capital erosion equal to 20% and to allow for different degree of uncertainty. Figure 7 panel (a) shows the results of an experiment with the expected human capital erosion equal 20% and standard deviation in the range 33 – 40%. It turns out that under all distributional assumptions, life cycle profiles are very similar to the case of the benchmark case.

This outcome provides a final indication that extremely rare but potentially disastrous labor income shocks may be relevant to understand cautiousness by young workers and their limited risk taking in the stock market.

Figure 7: Uncertainty around the disaster risk

(a) **(b)**

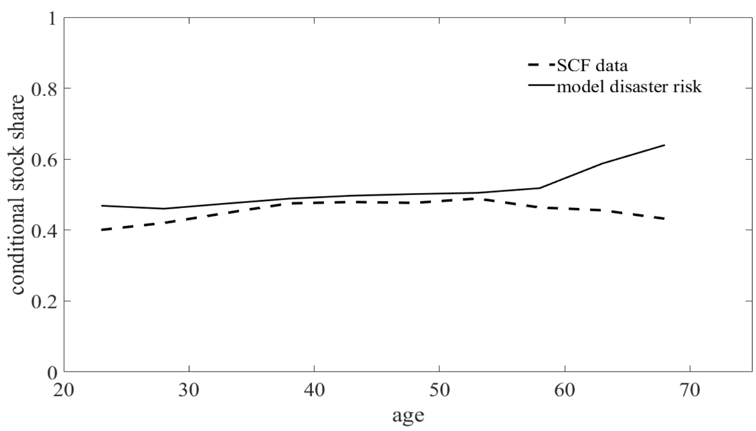


This figure displays the life-cycle profiles of optimal conditional stock holdings (panel a) for different parametrization of the distribution that governs human capital erosion. The expected erosion is 20%, the standard deviation is respectively 40% (dashed line), 39% as in the benchmark case (solid line) and 33% (dashed-dot line). Panel b reports the inverse cumulative distribution function for each parametrization.

5.4 Household portfolios with personal disaster risk: matching the empirical regularities

The key implication of our model is that optimal investment profiles are almost flat over the life cycle. In this section, we compare our results with conditional stockholdings for US male investors observed in the Survey of Consumer Finances data (waves from 1992 to 2016).

Figure 8: Life-cycle conditional stockshare profiles



This figure displays the life-cycle profiles of conditional stock holdings of age 20 to 100 observed in SCF data and obtained from the benchmark model with unemployment plus disaster risk. The parameters governing disaster risk are respectively $\Psi_1 = 0$ and expected $\Psi_2 = 0.20$.

Figure 8 compares the stock portfolio shares for stock market participants for different age classes obtained from our model with the corresponding US SCF data. The model is able to closely match the observed life-cycle pattern of equity portfolio shares conditional on participation, yielding an average value over the whole working life of 49%, to be compared with 45.6% in the data.[16]

6 Conclusions

This paper shows that even a small probability of experiencing human capital erosion of uncertain, but potentially extreme, size generates optimal conditional stock shares in line with those observed in US data, along with a skewed consumption growth distribution.

[16] We focus on average values over the whole life cycle. As highlighted by Gomes and Michaelides (2005), participation and conditional stock holding age profiles obtained from the data are not robust to the cohort and time effects assumptions.

Non-linear income shocks, that have recently become essential in consumption studies, appear to play a first order role in risk taking choices. Because of the remote possibility of a future personal disaster, younger workers face higher uncertainty concerning future income than older workers and optimally invest a higher portfolio share in the risk-free asset. These results owe to a methodological innovation in the way we model human capital erosion conditional on the occurrence of a rare disaster.

Our analysis bears implications for the design of pension plans in the US, that currently tilt the composition of optimal portfolios towards equity investments. Given the scant heterogeneity in optimal life-cycle investments induced by limited protection against a personal future disaster, the flatter design should fit workers with different career histories. More generally, the pattern of risk taking at different ages in Target Date Funds should be related to the share of uninsured disability and long-term unemployment risk.

This implication suggests that observed variation in the age pattern of stock investing by the young depends on the features of insurance coverage both across countries and across cohorts. We leave the systematic investigation of this insight for future research.

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