A General Equilibrium Model of Credit Default Swap (CDS) Markets*

Ajay Subramanian
Georgia State University
asubramanian@gsu.edu

April 30, 2021

Abstract

We develop an analytically tractable general equilibrium model to analyze the welfare effects of CDS trading and CDS regulation under aggregate uncertainty. If available equity capital is below a threshold, any equilibrium of the basic economy with no CDS markets features firm default and underinvests in firms relative to the efficient allocation. For low aggregate risk levels, there is a unique equilibrium of the economy with unregulated CDS markets in which bondholders are fully insured. Investment is efficient, and the efficient allocation can be implemented via transfers alone. For intermediate aggregate risk, the unregulated CDS economy overinvests, and a margin or collateral requirement on CDS sellers that becomes more stringent as aggregate risk increases is necessary for efficiency. When aggregate risk is high, the CDS market breaks down. A collateral requirement restores equilibrium and efficiency, but it must be maximally stringent and accompanied by a capital requirement that restricts CDS supply.

Key Words: Credit Risk, Credit Default Swaps, Welfare Effects, Regulation

JEL Classification Codes: G22, G28, D52

*I thank seminar audiences at the Georgia Institute of Technology, Monash University (Melbourne, Australia), University of Queensland (Brisbane, Australia), Georgia State University, and the International Finance and Banking Society (IFABS) Conference (Medellin, Colombia) for valuable comments. The usual disclaimers apply.

1 Introduction

Credit default swaps (CDS), which facilitate the sharing of firm default risk, were at the epicenter of the 2007-2009 financial crisis. CDS markets effectively broke down as CDS sellers such as the American International Group (AIG) were unable to meet their liabilities due to massive declines in their asset values. Following the crisis, policy makers focused attention on the lack of regulation of CDS markets, which exacerbated the crisis' severity (e.g., Augustin et al. (2016)). The 2010 Dodd-Frank Act and the Basel III guidelines propose significant new regulations such as centralized clearing, margin and capital requirements on CDS sellers (Jarrow (2011)). A growing literature examines the impact of such regulations on CDS markets within partial equilibrium frameworks. As Augustin et al. (2016) emphasize, however, the analysis of CDS trading and regulation necessitates a general equilibrium framework that incorporates (i) incomplete markets and the role of CDS in facilitating credit risk sharing; (ii) the impact of CDS markets on the real economy; and (iii) the effects of aggregate risk and correlation among firm defaults on CDS markets. We contribute to the literature by developing such a framework.

Our parsimonious and analytically tractable model generates novel insights into the welfare effects of CDS trading and the design of CDS market regulation. If the aggregate equity capital in the economy is below a threshold, any equilibrium of the basic economy with no CDS markets features firm defaults and *underinvests* in productive firms relative to the efficient allocation. Equilibria of the basic economy are unaffected by aggregate risk. The efficient allocation, however, features full insurance for bondholders and a constant level of investment in firms when aggregate risk is below a threshold, but imperfect insurance for bondholders and decreasing investment in the real economy when aggregate risk increases above the threshold. We then examine the impact of unregulated CDS trading and how CDS market regulation can be designed to implement efficient allocations. For low levels of aggregate risk, there is a unique equilibrium of the unregulated CDS market economy in which bondholders are fully insured and firm investment is efficient so that the efficient allocation can be implemented via transfers alone. For intermediate aggregate risk, the CDS market economy *overinvests* in firms. To achieve efficiency, it is necessary to impose a margin or collateral requirement on CDS sellers that restricts their investment in risky firms. For high aggregate risk levels, the CDS market breaks down. A *maximally stringent* collateral requirement and a capital requirement that restricts CDS supply restore equilibrium and efficiency. Our results show how aggregate risk impacts the effectiveness of CDS markets and CDS market regulation.

We model a single-period economy with continuums of risk-averse bondholders, risk-neutral equityholders, and risk-neutral entrepreneurs. Agents of each type are identical ex ante. There is a single consumption/capital good and a risk-free asset or savings technology providing a constant return that agents cannot short-sell. Bondholders and equityholders are each endowed with capital. Entrepreneurs have no capital, but operate firms with risky concave production technologies that they finance via equity and debt. Bondholders invest their capital in the risk-free asset and bonds issued by a *single* firm. Equityholders invest in the risk-free asset and equity stakes in an arbitrary number of firms. Equity and debt markets are *competitive*, that is, all agents take equity and debt prices *as given* with the prices being determined endogenously by clearing of the markets for *each firm's* equity and debt securities. If multiple firms' bonds offer a bondholder the same maximum expected utility, then the bondholder randomly chooses a single firm to invest in with uniform probability. If multiple firms offer the same maximum expected equity return, each equityholder invests uniformly among them. Hence, bondholders are undiversified, while equityholders are fully diversified.[1]

Firm technology shocks are identically and binomially distributed, but are not independent due to *aggregate shocks.* A positive (negative) aggregate shock leads to a higher (lower) proportion of firms experiencing positive shocks. The difference between the proportions of successful firms in the two aggregate states is the *aggregate risk* in the economy. Although agents know the distributions of the firm-level and aggregate shocks, neither the realization of the aggregate shock nor the identities of the firms that experience positive or negative shocks are known *ex ante.*

Any equilibrium of the basic economy is *symmetric,* that is, all firms raise the same amounts of capital and have identical capital structures. The intuition is as follow. Each bondholder randomly chooses a single firm to invest in with uniform probability when she is indifferent between multiple firms. If equityholders are indifferent among multiple firms, they diversify their capital uniformly among them. To ensure that the markets for *each firm*'s bonds and equity clear, therefore, all firms must offer the same expected utility to bondholders and the same expected equity return in equilibrium. Hence, firms have identical capital structures, and debt and equity returns are, respec-

[1]We can extend the model to incorporate bondholder investments in multiple firms. Our results hold as long as bondholders hold imperfectly diversified bond portfolios (due to search costs, participation costs, etc) so that there are gains from trading securities such as CDS.

tively, identically distributed across firms. Moreover, in a competitive equilibrium, the expected marginal return from each firm's technology equals the expected returns from its debt and equity to ensure that any deviations from its optimal financing decisions are suboptimal.

There exist potentially multiple equilibria. If the total equity capital in the economy is below a threshold, any equilibrium features default by the representative firm when it fails as it is unable to raise sufficient equity capital to prevent insolvency. As bondholders are risk-averse, they demand a risk premium to invest in firm bonds so that the expected return on firm bonds exceeds the risk-free return. Hence, the expected equity return and the expected marginal return from the representative firm's technology also exceed the risk-free return so that equityholders invest all their capital in firms. Therefore, a default equilibrium is essentially determined by the amount of capital that bondholders invest in firms. A default equilibrium must simultaneously satisfy two relations between bondholders' investment in firms and the expected bond return. The first relation—the *firm technology curve*—is that the expected marginal return from the representative firm's technology and, therefore, the expected bond return decline with the firm's debt capital by the strict concavity of the firm's technology. The second relation—the *debt demand curve*—expresses the demand for firm bonds as a function of the expected bond return that could, in general, be non-monotonic depending on bondholders' utility function. The two curves intersect at least once, thereby determining an equilibrium, with the intersection being *unique* if the debt demand curve is increasing. If the total equity capital in the economy exceeds the threshold, the expected marginal return from firms' technologies (hence, the expected debt and equity returns) equal the risk-free return. Debt is risk-free, and equityholders and bondholders are indifferent between firms and the risk-free asset.

A default-free equilibrium is efficient, and there is no need for CDS markets. We, therefore, assume that the total equity capital is below the threshold which ensures that any equilibrium is a default equilibrium that is inefficient because markets are incomplete. Indeed, bondholders are undiversified and, therefore, exposed to firm-specific default risk, and there is no security contingent on the aggregate shock that all investors can trade. In fact, default equilibria do not depend at all on the aggregate risk level because each bondholder invests in a single firm without caring about whether it is exposed to the aggregate shock. Risk-neutral equityholders care only about the expectations of equity returns and not the correlations among them due to aggregate risk. No agent knows the realization of the aggregate shock ex ante or the firms that are exposed to the aggregate shock.

We derive the Pareto efficient allocation that maximizes the total expected utility of bondholders subject to the expected payoffs of equityholders and entrepreneurs being no less than their respective equilibrium payoffs in the basic economy.[2] The efficient allocation balances the trade-off between insuring bondholders against aggregate risk and maximizing output by investing in risky firms. If aggregate risk is below a threshold, the efficient investment level maximizes expected firm output with the remaining capital being invested in the risk-free asset to fully insure bondholders. If aggregate risk exceeds the threshold, however, there is insufficient capital in the low aggregate state to fully protect bondholders against aggregate risk because equityholders' and entrepreneurs' limited liability constraints are binding. As aggregate risk increases above the threshold, the tradeoff between maximizing expected firm output and achieving optimal risk-sharing causes the efficient investment to decrease with aggregate risk. The basic economy underinvests in firms relative to the efficient investment level because risk-averse bondholders, who are exposed to firm-specific risk, invest too little capital in firms.

Next, we introduce *competitive* CDS markets in the basic economy. Each bondholder invests her capital in bonds issued by a *single* firm, CDS contracts on the bonds, and the risk-free asset. We consider the general scenario in which each bondholder could hold fully covered or naked CDS positions. Equityholders can buy or sell CDS on an arbitrary number of firms. As CDS contracts are in zero net supply and bondholders optimally buy CDS contracts, equityholders must hold net short positions. Equityholders sell CDS contracts with the lowest expected cost to them and invest in securities—firm equity or the risk-free asset—that offer the highest expected returns. If multiple CDS contracts have the same expected cost, equityholders diversify uniformly among them. Similarly, if multiple equity securities offer the same expected return, equityholders invest uniformly among them. As in the basic economy, any equilibrium must be symmetric with the intuition being similar. To preclude profitable deviations, the expected equity, bond and CDS returns on the representative firm are all equal to the expected marginal return from its technology. In contrast with the basic economy, however, aggregate risk crucially impacts the existence and properties of the CDS market equilibrium.

[2]If there are multiple equilibria, we consider the equilibrium in which the representative firm's leverage is lowest purely for concreteness. Our analysis holds for any selected equilibrium.

(i) For low aggregate risk levels, there is a unique equilibrium where investment in firms is efficient, and bondholders are fully insured as in the efficient allocation. The intuition is that, because the expected return on a firm's bonds equals the expected CDS return in equilibrium, it is optimal for each bondholder to be fully insured by investing all his capital in bonds and CDS. Further, aggregate risk is low enough that CDS sellers are always able to meet their CDS liabilities in equilibrium.

(ii) For intermediate aggregate risk, the unique equilibrium features *overinvestment* in firms relative to the efficient allocation with bondholders again being fully insured. As the expected bond return equals the expected CDS return, it is again optimal for each bondholder to be fully insured by investing all his capital in bonds and CDS. Aggregate risk is still low enough that equityholders are able to fully meet their CDS liabilities, but they invest all their capital in firms because the expected equity return exceeds the risk-free return. At intermediate aggregate risk levels, however, bondholders must bear aggregate risk in the efficient allocation, and the economy should invest some capital in the risk-free asset. Hence, the CDS market equilibrium is inefficient as it features overinvestment in firms and bondholders are fully insured when they should, in fact, bear aggregate risk.

(iii) For high aggregate risk levels, there is no equilibrium, that is, the CDS market breaks down. Indeed, as the expected bond return equals the expected CDS return, bondholders continue to demand full insurance against default risk. However, the aggregate risk is high enough that equityholders are unable to meet their CDS liabilities following a negative aggregate shock. In other words, there are no market-clearing CDS prices because the demand for full insurance by bondholders cannot be met by the supply of CDS by CDS sellers. Our results formally justify necdotal observations that CDS markets broke down during the financial crisis because CDS prices did not appropriately reflect the effects of aggregate risk (Jarrow (2011)).

We examine whether appropriately designed regulatory policies can ensure the existence of equilibrium for all aggregate risk levels and implement the efficient allocation. We consider instruments such as margin/collateral and capital requirements as well as transfers among agents that can be implemented via fees and payouts managed by institutions such as a centralized exchange or taxes/subsidies imposed by a fiscal authority. Given our normative objectives, we refer to a single entity—a regulator—who implements the different instruments described above.

(i) For low aggregate risk levels, there is a unique equilibrium of the regulated CDS economy such that the efficient allocation can be implemented via transfers alone. Indeed, as discussed above, the unique equilibrium of the unregulated CDS economy features efficient investment for low aggregate risk, but the allocation of output among agents is inefficient. Hence, we can achieve efficiency via the redistribution of output through transfers.

(ii) For intermediate aggregate risk, the efficient allocation can be implemented via transfers and a margin requirement on CDS sellers that becomes more stringent as aggregate risk increases. When aggregate risk is above a threshold, the efficient allocation necessitates nonzero investment in the risk-free asset, which increases with aggregate risk. In the absence of a margin requirement, CDS sellers (equityholders) invest all their capital in risky firms as the expected equity return exceeds the risk-free return. It is, therefore, essential to impose a margin requirement to force CDS sellers to make the requisite investment in the risk-free asset.

(iii) For high aggregate risk, the efficient allocation can only be implemented via a *maximally stringent* margin requirement, which requires CDS sellers to invest their entire capital endowment in the risk-free asset, and a capital requirement that restricts the capital they can raise by issuing CDS. Indeed, if aggregate risk is sufficiently high, the collateral requirement on CDS sellers becomes binding at the upper limit. Hence, to ensure that the economy as a whole invests efficiently in the risk-free asset, bondholders must be induced to invest more capital in the risk-free asset to match the efficient investment level. This can be achieved by limiting CDS supply via a capital requirement, which limits the default insurance that bondholders can purchase via CDS, thereby inducing them to increase their risk-free asset investment.

Several studies examine the effects of CDS trading on the cost of debt in *partial equilibrium* frameworks. Morrison (2005) and Parlour and Winton (2013) argue that firms' ability to hedge their credit exposure reduces their monitoring incentives, thereby resulting in higher borrowing costs. Bolton & Oehmke (2011) suggest a potential increase in a firm's cost of debt because creditors become empty when they maintain their control rights in firms despite losing their economic interest through the purchase of CDS. Che and Sethi (2014) analyze the effects of credit derivatives on default risk and the cost of debt in a model with heterogeneous beliefs. They show that CDS may induce optimistic investors to sell CDS instead of buying bonds, thereby limiting the allocation of capital to productive investment. Oehmke & Zawadowski (2015) show that CDS may push optimistic investors to migrate from the bond market to the CDS market.

Hakenes & Schnabel (2010) argue that risky borrowers may receive increased credit with default insurance provided by CDS, but Biais, Heider & Hoerova (2016) argue that this could also lead to excessive risk-taking. Several empirical and theoretical studies show that CDS have negative welfare effects stemming from asymmetric information (Duffee & Zhou 2001, Thompson 2010, Chakraborty, Chava & Ganduri 2015), counterparty risk (Stephens & Thompson 2014), and contagion because of credit risk transfer (Allen & Carletti 2006). However, Duffee & Zhou (2001), Thompson (2010), Allen & Carletti (2006) and Norden, Buston & Wagner (2014) argue that the aforementioned negative effects of CDS may be offset by more efficient risk-sharing that lowers firm borrowing costs. Danis and Gamba (2018) use a calibrated model that incorporates the positive and negative aspects of CDS markets to show that firm value increases by 2.9% on average with the introduction of a CDS market.

Because the aforementioned studies analyze partial equilibrium settings, they are not designed to address questions of CDS market efficiency and regulation that represent the main focus of our study. Fostel and Geanakoplos (2016) study how credit derivatives influence production in a general equilibrium model with incomplete markets where promises must be backed by collateral. The main model for their analysis has risk-neutral agents with heterogeneous beliefs. They show that credit derivatives lead to underinvestment relative to the Arrow-Debreu equilibrium and can lead to the non-existence of equilibrium by eliminating production. Darst and Refayet (2018) examine the impact of CDS trading on the type of debt that firms issue. In their model, investors are risk-neutral, but have heterogenous beliefs, and firms are fully financed by safe or risky debt.[3]

We complement the above studies by examining the welfare effects of CDS in a general equilibrium model with heterogeneous risk preferences, but symmetric information among market participants. We also differ significantly by showing how aggregate risk influences unregulated CDS markets and the optimal design of CDS market regulation. Margin or collateral requirements are inefficient for low aggregate risk levels as they lead to *underinvestment*. For intermediate aggregate risk, unregulated CDS markets lead to *overinvestment* in risky firms. It is necessary to impose a collateral requirement on CDS sellers that restricts their investment in risky firms to achieve efficiency. When aggregate risk is high, the unregulated CDS market breaks down because bondholders

[3]A less closely related strand of literature concerns the design of settlement mechanisms of CDS contracts (e.g, see Chernov, Gorbenko and Makarov (2013), Peivandi (2015), Du and Zhu (2017)).

demand full default insurance that CDS sellers are unable to provide. As CDS prices do not appropriately reflect the impact of aggregate risk, markets fail to clear and there is no equilibrium. A collateral requirement is necessary for efficiency with a capital requirement also being required when aggregate risk is high enough that the collateral requirement is maximally stringent. Equilibrium nonexistence, therefore, arises due to reasons different from those in Fostel and Geanakoplos (2016), where it arises due to production being eliminated so that there is no underlying asset on which CDS can be written.

2 The Basic Economy

We first model the basic economy without credit default swaps (CDS). The economy has one period with two dates 0 and 1. There are three sets of agents: a continuum of measure 1 of risk-averse *bondholders;* a continuum of measure 1 of risk-neutral *equityholders*; and a continuum of measure 1 of risk-neutral *entrepreneurs* who establish and operate firms. Agents of each type are ex ante identical. The measure of each set of agents is normalized to 1 to simplify the notation; none of the results hinges on this. All agents have access to a savings technology or risk-free asset with constant returns to scale that provides a constant return per unit of capital that we normalize to 1. Agents cannot short-sell the risk-free asset.

Each bondholder is endowed with 1 unit of capital at date 0 that he can invest in a portfolio of the risk-free asset and bonds issued by a single firm. We can extend the analysis to allow for bondholders to invest in multiple firms' bonds. Our main implications are unaltered as long as bondholders hold incompletely diversified portfolios due to transaction costs (e.g., search costs, market participation costs) so that each bondholder is exposed to firm-specific default risk that can be transferred through CDS, that is, there are potential gains from CDS trading. Each equityholder is endowed with K units of capital that she invests in the risk-free asset and equity stakes in firms. Equityholders can be viewed as sophisticated investors with no a priori restrictions on the number of firms they can invest in. Entrepreneurs have no initial endowments and establish firms by raising debt and equity capital. We alternately refer to "entrepreneurs" as "firms" wherever there is no danger of confusion.

2.1 Firms

Each firm indexed by $n \in [0, 1]$ invests the capital it raises in a risky, concave production technology. Specifically, for k units of capital invested, the firm's payoff is $\Lambda(k)$ if its technology succeeds with probability $1 - q$ and $\alpha\Lambda(k)$ if it fails with probability q, where $\Lambda(\cdot)$ is strictly increasing and concave, and $\alpha \in (0, 1)$. We can also allow for firm-specific shocks to the production function $\Lambda(.)$ without altering our implications; we avoid introducing them to simplify the exposition. $\Lambda(.)$ satisfies the Inada conditions,

$$\Lambda'(0) = \infty; \ \Lambda'(\infty) = 0. \tag{1}$$

Production technologies are, therefore, identically distributed across firms, but are not independent due to the presence of aggregate, undiversifiable risk. Specifically, with probability $1 - q$, there is a positive aggregate shock in which case a proportion ω^H of firms succeed, while the remaining proportion $1 - \omega^H$ fail. With probability q, there is a negative aggregate shock in which case a proportion $\omega^L < \omega^H$ succeed, while the remaining proportion $1 - \omega^L$ fail. As firms are ex ante identical, any firm's probability of success *conditional* on a positive (negative) aggregate shock is $\omega^H(\omega^L)$. Because firms have an identical success probability of $1 - q$, it follows that the expectation (with respect to the probability distribution of the aggregate shock) of the proportion of successful firms is $1 - q$. Hence, the parameters ω^H, ω^L satisfy

$$(1 - q)\omega^H + q\omega^L = 1 - q \tag{2}$$

Let

$$\tau = \omega^H - \omega^L. \tag{3}$$

As (3) indicates, τ is the spread between the proportions of successful firms in the "high" and "low" aggregate states. Accordingly, we refer to the parameter τ as the *aggregate risk* in the economy. All agents know τ, ω^H, ω^L. However, neither the realization of the aggregate shock nor the identities of the firms that experience positive or negative shocks are known *ex ante*.

Each firm raises equity capital from equityholders and debt capital from bondholders. A debt security or bond has the general form, $(p_B; (P_B^s, P_B^f))$, where p_B is the bond price, and P_B^s, P_B^f , are the bond payoffs when the firm succeeds and fails, respectively, with $P_B^f \leq P_B^s$ and the inequality being strict iff the firm defaults. Here, the superscripts 's' and 'f' denote success or failure of the firm. Similarly, an equity security or share of stock issued by the firm has the form, $(p_E;(P^s_E, P^f_E))$ with $P^f_E = 0$ if the firm defaults. What matters ultimately are the total amounts of debt and equity capital that each firm raises, which equal the number of units of each security (debt or equity) that it issues multiplied by the security price. The definition of a security is, therefore, arbitrary. Consequently, without loss of generality, we can choose a convenient normalization where the price of a bond is 1 and its payoffs upon the firm's success and failure are $R^s_B = \frac{P^s_B}{p_B}; R^f_B = \frac{P^f_B}{p_B}$, respectively (see Section 14.1 of Ljungvist and Sargent (2018)). Hence, an individual bond can be characterized as $\widetilde{R}_B \equiv (R^s_B, R^f_B)$, where $\widetilde{R}_B$ is a random variable that denotes the *return* of the bond, which is contingent on the firm's realized state. We can similarly characterize a share of the firm's stock by its return, $\widetilde{R}_E \equiv (R^s_E, R^f_E)$. Throughout the paper, we use a 'tilde' to denote a random variable and remove the 'tilde' to denote its realizations in different states.

Firm $n \in [0,1]$ raises equity capital, E_n, from equityholders and debt capital, B_n, from bondholders via equity and debt contracts, $\widetilde{R}_{E,n}$ and $\widetilde{R}_{B,n}$, respectively, where the additional subscript indicates that the contracts could vary across firms. Capital markets are *competitive* so that firms, equityholders and bondholders *take the set of equity and debt returns,* $\left\{\widetilde{R}_{E,n}, \widetilde{R}_{B,n}; n \in [0,1]\right\}$ *,as given* when they make their capital demand and supply decisions. Importantly, therefore, each firm chooses the amounts of equity and debt capital that it raises, but the returns on its equity and debt are determined endogenously via clearing of the markets for the firm's stock and bonds. The set of equity and bond returns for all firms is feasible iff for all $n \in [0,1]$,

$$R^f_{B.n} \leq R^s_{B,n} \text{ with } R^f_{B.n} = R^s_{B,n} \text{if } R^f_{E,n} > 0. \tag{4}$$

The above captures the absolute priority of bonds upon firm bankruptcy. The return on bonds when a firm fails must be no greater than the return when the firm succeeds with the return upon failure being strictly lower iff the firm defaults and its equity return is zero. The firm n's demand for equity and debt capital, taking the equity and debt returns, $\left\{\widetilde{R}_{E,n}, \widetilde{R}_{B,n}\right\}$, as given, is

$$(E_n, B_n) = \arg\max_{(x,y)} (1-q)\left(\Lambda(x+y) - R^s_{E,n}\cdot x - R^s_{B.n}\cdot y\right) \\ + q\left(\alpha\Lambda(x+y) - R^f_{E,n}\cdot x - R^f_{B.n}\cdot y\right). \tag{5}$$

2.2 Investors: Equityholders and Bondholders

Each equityholder invests his capital endowment K in a portfolio of the risk-free asset and equity stakes in firms. Recall that agents cannot short-sell the risk-free asset. Equityholders take the risk-free asset return and firm equity returns, $\left\{\widetilde{R}_{E,n}; n \in [0,1]\right\}$, as given when they make their investment decisions. A risk-neutral equityholder invests all his capital in the risk-free asset if

$$1 > \sup_{n\in[0,1]} \mathbb{E}\left[\widetilde{R}_{E,n}\right]. \tag{6}$$

If the strict inequality above is reversed, the equityholder invests all his capital in firms. If multiple firms offer the same (maximum) expected equity return, we assume that equityholders invest their capital uniformly among them so that they are fully diversified. If condition (6) holds with equality, then equityholders are indifferent between investing in the risk-free asset and firms. In this scenario too, if an equityholder chooses to invest in firms, she diversifies her capital uniformly among them.

Each risk-averse bondholder has a strictly increasing, strictly concave and twice differentiable utility function, $U(.)$. The bondholder invests his capital endowment of 1 unit in a portfolio of the risk-free asset and bonds issued by a single firm taking the set of bond returns, $\left\{\widetilde{R}_{B,n}; n \in [0,1]\right\}$ of all firms as given. If a bondholder chooses bonds issued by a firm $n \in [0,1]$, then his investments in bonds and the risk-free asset maximize his expected utility, that is,

$$b_n = \arg \sup_{x\in[0,1]} \mathbb{E}\left[U\left((1-x) + x\widetilde{R}_{B.n}\right)\right] \tag{7}$$

By the strict concavity of the utility function, U, there is a unique solution to (7) that can be expressed as

$$b_n = g(R^s_{B,n}, R^f_{B,n}) \tag{8}$$

for some function g. As the utility function U is continuously differentiable, the function g is continuous. The bondholder chooses the firm $n \in [0,1]$ that offers him the maximum expected utility, that is,

$$n \in \arg \sup_{m\in[0,1]} \mathbb{E}\left[(1-b_m) + b_m\widetilde{R}_{B.m}\right], \tag{9}$$

where b_m solves (7). If multiple firms offer the bondholder the same maximum expected utility, then the bondholder randomly chooses a single firm to invest in with uniform probability.

2.3 Equilibrium Conditions

The following conditions must hold in equilibrium of the economy:

1. Each bondholder solves the optimization programs (7) and (9), that is, he chooses how much capital to invest in the risk-free asset and bonds issued by a single firm to maximize his expected utility taking the risk-free return and firm bond returns as given. If multiple firms offers the same maximum expected utility to a bondholder, he randomly chooses a single firm among them to invest in with uniform probability.

2. Each equityholder chooses her portfolio of investments in the risk-free asset and firm equity to maximize her expected payoff taking as given the risk-free return and firm equity returns. If multiple firms offer the same maximum expected equity return, equityholders invest their capital uniformly among them.

3. Each firm solves (5), that is, it maximizes its expected profit by choosing how much equity and debt capital to raise and investing the capital it raises in its production technology. The firm takes the returns on its equity and debt as given when it makes its financing decisions.

4. All agents and firms are protected by limited liability. Further, debt has absolute priority in insolvency. Hence, in equilibrium, the equity and debt returns of each firm $n \in [0, 1]$ must satisfy

$$\begin{aligned} R^s_{E,n} \cdot E_n + R^s_{B.n}.B_n &\leq \Lambda(E_n + B_n) \\ R^f_{E,n} \cdot E_n + R^f_{B.n}.B_n &\leq \alpha\Lambda(E_n + B_n) \\ R^f_{B.n}.B_n &= \min\left(R^s_{B.n}.B_n, \alpha\Lambda(E_n + B_n)\right) \end{aligned} \tag{10}$$

5. The markets for *each firm's* equity and debt clear in equilibrium, that is, the demand for each firm's debt equals the supply, and the demand for each firm's equity equals the supply. Note that, consistent with markets being competitive, the constraints (10) hold *in equilibrium.* That is, individual agents make their

decisions taking security prices/returns as given. The equilibrium variables that are endogenously determined by market clearing satisfy (10).

3 Equilibria of the Basic Economy

We begin by showing that any equilibrium of the basic economy is symmetric, that is, all firms have identical capital structures.

Proposition 1 (Equilibrium Symmetry). *If an equilibrium exists, it is symmetric. The expected marginal return from each firm's technology equals the expected returns from its debt and equity.*

The intuition is as follows. First, by equilibrium condition 1 in Section 2.3, each bondholder invests in a single firm that provides him with the maximum expected utility. Hence, to ensure that the market for *each* firm's bonds clears, all firms must offer the same maximum expected utility to bondholders in equilibrium. As each bondholder randomly chooses a single firm to invest in with uniform probability when all firms offer the same maximum expected utility, it follows that firms raise the same amount of debt capital in equilibrium. Second, by equilibrium condition 2 in Section 2.3, all firms must offer the same maximum expected equity return to ensure that the equity market for each firm clears. As equityholders diversify their capital uniformly among firms that offer the same maximum expected equity return, it follows that firms also raise the same amount of equity capital in equilibrium. It then follows that debt and equity returns are, respectively, identically distributed across firms. Finally, by the necessary and sufficient first order condition of each firm's optimization problem (5), the expected return of a firm's debt or equity in equilibrium must be equal to the expected marginal return from its technology.

As any equilibrium is symmetric, we can focus on a *representative* firm without loss of generality. The analysis of equilibria requires some definitions. Define X as the unique solution to the following equation:

$$\overbrace{(1-q)\Lambda^{'}\left(X\right)+q\alpha\Lambda^{'}\left(X\right)}^{\text{expected marginal return from firm technology}} \quad = \quad \overbrace{1}^{\text{risk-free return}} \quad . \tag{11}$$

There exists a unique X by the Inada conditions, (1), and the fact that $\Lambda'(.)$ is strictly decreasing since $\Lambda(.)$ is strictly concave. The threshold, X, is the level of total capital such that, if it were invested uniformly in the set of firms of mass 1 (so that each firm receives X), then the expected marginal return from each firm's technology would be equal to the risk-free return. We assume that

$$\alpha \Lambda (X) < 1. \tag{12}$$

The above condition holds if the parameter α that determines the productivity of a firm's technology in the bad state is below a threshold. The condition simplifies the analysis and exposition by avoiding the consideration of uninteresting scenarios that do not alter our main implications. We are now ready to characterize equilibria of the economy. We use the superscript '*' to denote equilibrium variables.

Theorem 1 (Equilibrium Existence and Characterization)**.** *There exist potentially multiple equilibria of the economy. Define*

$$K_1 = X - \alpha\Lambda(X). \tag{13}$$

1. *If the total available equity capital, $K < K_1$,firms default when their technologies fail in any equilibrium, and equityholders invest all their capital in firms so that each firm raises equity capital $E^* = K$. There is a unique equilibrium if bondholders' demand function g defined in (8) is increasing in its first argument, ceteris paribus.*[4]

2. *If $K \geq K_1$,then firms remain solvent in any equilibrium. There exists a continuum of equilibria in which each firm raises capital X. The expected marginal return from each firm's technology, and the expected returns on its debt and equity all equal the risk-free return. The equilibria differ only in the relative investments by bondholders and equityholders in firms as long as the total investment is* X. *In any equilibrium, bondholders invest at most $B^* = \alpha\Lambda(X)$ in firms and their remaining capital in the risk-free asset. Equityholders invest $E^* = X - B^*$ in firm equity and their remaining capital in the risk-free asset.*

[4]By the analysis in Fishburn and Porter (1976), a sufficient condition for this to hold is that

$$U'(x) + U''(x)(x-1) > 0. \tag{14}$$

As shown by Fishburn and Porter (1976), the above condition, which is trivially satisfied for $x \leq 1$, holds for power utility functions, $x^c, c \in (0,1)$, and logarithmic utility functions.

To understand the equilibria, we analytically derive the equilibrium conditions here and provide additional details in the Appendix. Suppose that the total equity capital $K < K_1$,where K_1 is defined in (13). Consider a *candidate* (symmetric) equilibrium where the equity capital is invested uniformly among firms so that $E^* = K$. In the Appendix, we show that the representative firm must default in any equilibrium. Intuitively, when the available equity capital is below a threshold, the representative firm's equity capital buffer in any equilibrium is insufficient to prevent default when it fails. Let B^* denote the total debt capital raised by all firms in the candidate equilibrium, and $\widetilde{R}^*_E, \widetilde{R}^*_B$ denote the equity and bond returns, respectively, of the representative firm. The equilibrium conditions are as follows.

$$R^{f*}_E = 0; R^{f*}_B = \frac{\alpha\Lambda\left(E^* + B^*\right)}{B^*} = \frac{\alpha\Lambda\left(K + B^*\right)}{B^*} \tag{15}$$

$$\begin{aligned} \mathbb{E}[\widetilde{R}^*_E] = (1-q)R^{s*}_E &= \mathbb{E}[\widetilde{R}^*_B] = (1-q)R^{s*}_B + qR^{f*}_B \\ &= (1-q)\Lambda'\left(K + B^*\right) + q\alpha\Lambda'\left(K + B^*\right) \end{aligned} \tag{16}$$

$$B^* = \arg\sup_{x\in[0,1]} (1-q)U\left[(1-x) + xR^{s*}_B\right] + qU\left[(1-x) + xR^{f*}_B\right] \tag{17}$$

Conditions (15) express the fact that the equity return is zero when a firm's technology fails by the absolute priority of debt. Condition (16) expresses that the expected returns of equity and debt must equal the expected marginal return of the firm's technology by Proposition 1. Condition (17) is the demand for the representative firm's debt by bondholders. The conditions imply two key relations between the expected return on the representative firm's debt, $\mathbb{E}[\widetilde{R}^*_B]$, and the amount of debt capital, B^*, that it raises, which together determine an equilibrium.

$$\mathbb{E}[\widetilde{R}^*_B] = (1-q)\Lambda'\left(K + B^*\right) + q\alpha\Lambda'\left(K + B^*\right) \tag{18}$$

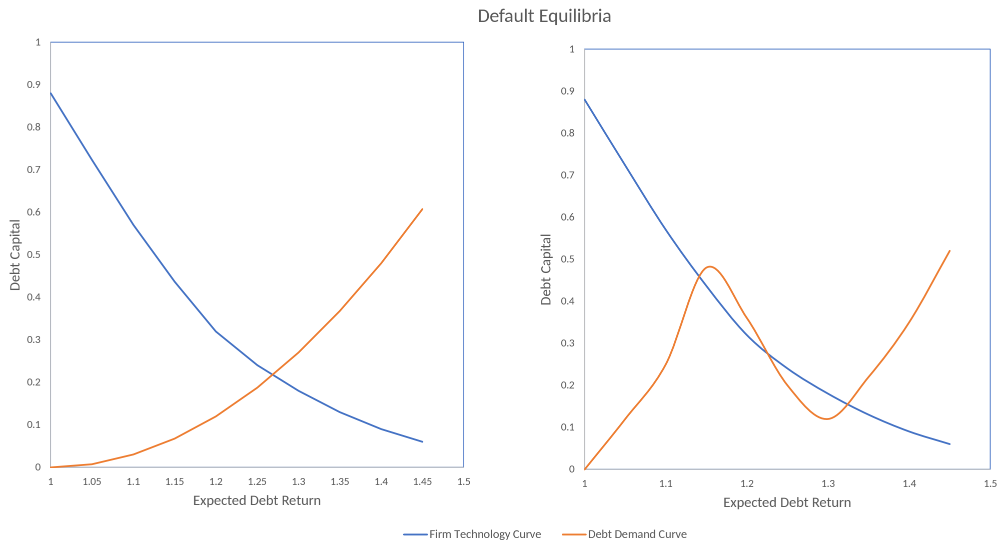
$$\begin{aligned} B^* = \arg\sup_{x\in[0,1]} (1-q)U&\left[(1-x) + \frac{x}{1-q}\left(\mathbb{E}[\widetilde{R}^*_B] - q\frac{\alpha\Lambda\left(K + B^*\right)}{B^*}\right)\right] \\ &+qU\left[(1-x) + x\frac{\alpha\Lambda\left(K + B^*\right)}{B^*}\right] \end{aligned} \tag{19}$$

The relation, (18) follows immediately from (16) and determines a strictly decreasing relation between $\mathbb{E}[\widetilde{R}^*_B]$ and B^* because the production function, $\Lambda(.)$, is strictly concave. We refer to this as the *firm technology curve.* We obtain (A6) by using the fact that $R^{f*}_B = \frac{\alpha\Lambda(K+B^*)}{B^*}$ by (15) and $R^{s*}_B = \frac{\mathbb{E}[\widetilde{R}^*_B] - qR^{f*}_B}{1-q}$ by (16). In the Appendix, we show that (19) has a unique solution that determines a second relation between $\mathbb{E}[\widetilde{R}_B^*]$ and B^*, which we refer to as the *debt demand curve*. For a general bondholder utility function, U, the debt demand curve could vary non-monotonically, but is increasing if bondholders' demand function g defined in (8) is increasing in its first argument, ceteris paribus. In the Appendix, we show that the firm technology and debt demand curves intersect at least once in the region, $\mathbb{E}[\widetilde{R}_B^*] > 1$ with the intersection being unique if the debt demand curve is increasing. An intersection point satisfies all the conditions (15)–(17) and, therefore, determines a default equilibrium. Figure 1 illustrates the default equilibria with the left panel showing the case where the debt demand curve is increasing so that the equilibrium is unique, and the right panel showing the case where the debt demand curve is non-monotonic.

The representative firm's capital structure and the returns on its debt and equity are, thereby, determined *endogenously* in competitive equilibrium of the model via clearing of the markets for the firm's debt and equity securities.. This is in sharp contrast with *partial equilibrium* models of capital structure that focus solely on the *supply* of debt and equity. In such models, firms can issue arbitrary amounts of debt and equity that are bought by investors at exogenously determined prices. In particular, in our general equilibrium setting, it is *not* an equilibrium for the representative firm (recall that any equilibrium must be symmetric by Proposition 1) to choose zero or low leverage to avoid default when $K < K_1$. First, the representative firm would invest suboptimally low capital in its technology. Second, the demand for firm bonds by bondholders would exceed the supply so that the market for each firm's bonds would not clear. The equilibrium capital structure of each firm, therefore, features default when it fails.

The default equilibria characterized in Part 1 of Theorem 1 *do not depend* at all on the aggregate risk τ. Recall that each bondholder invests in a single firm. Equityholders are risk-neutral and, therefore, care only about the expectations of equity returns and not the correlations among them. The representative firm raises equity and debt financing without caring about the correlation of its technology shock with other firms' shocks. No agent knows the realization of the aggregate shock ex ante or the firms that are exposed to the aggregate shock. Hence, the equilibrium conditions (15)-(17) do not depend on the aggregate risk, τ. As we see in the next section, the Pareto efficient allocations corresponding to the default equilibria *do* depend on aggregate risk. This implies that a default equilibrium is not Pareto efficient, which is not surprising given that markets are incomplete so that the first welfare theorem need not apply.

Figure 1: Equilibria with Default



When $K \geq K_1$, the economy features default-free equilibria as shown by Part 2 of the theorem. Intuitively, when the available equity capital exceeds a threshold, firms' equity capital buffers are high enough that debt is risk-free in any equilibrium. By (11), each firm raises total capital X in any equilibrium. Indeed, a total capitalization exceeding X would imply that the expected marginal return from its technology is below the risk-free return in which case all capital in the economy would be invested in the risk-free asset. Default-free equilibria are Pareto efficient as risk-averse bondholders face no risk so that their total expected utility cannot be increased while also ensuring that the expected payoffs of equityholders and entrepreneurs are not decreased. Hence, there is no need for additional securities such as CDS that facilitate risk-sharing between equityholders and bondholders. We, therefore, restrict attention to economies that feature default by firms, that is, we assume henceforth that $K < K_1$ so that Part 1 of Theorem 1 applies.

Assumption 1. *The total equity capital, K, in the economy is less than K_1. In the case where bondholders' demand function g does not satisfy the condition in Part 1 so that there may be multiple equilibria, we focus on the equilibrium with the lowest firm leverage.*

When there are multiple equilibria, we select the equilibrium with lowest firm leverage purely for concreteness. Our subsequent analysis and results do not depend on the particular choice of equilibrium. As will become clear in the next sections, the selected equilibrium merely serves to pin down the Pareto optimal allocation that serves as a benchmark to examine the impact of CDS markets.

4 Efficient Allocations

The equilibrium allocation of the basic economy with default by firms is inefficient because markets are incomplete. First, bondholders are undiversified and, therefore, exposed to idiosyncratic, firm-specific default risk. Second, there is no security contingent on the realization of the aggregate shock that all investors can trade. No agent knows the realization of the aggregate shock ex ante or the firms that are exposed to the aggregate shock. We now characterize Pareto efficient allocations in the economy. Specifically, we consider a *hypothetical* social planner who holds the total available capital, $1+K$, in the economy, invests the capital in the risk-free asset and firms, and allocates the payoffs among agents: bondholders, equityholders and entrepreneurs/firms. In other words, as is standard, we consider a *centralized economy* in which there are no markets for equity and debt. In subsequent sections, we examine the implementation of the efficient allocations via trading in decentralized debt, equity and CDS markets.

We derive the Pareto optimal allocation that maximizes the total expected utility of bondholders subject to the expected payoffs of equityholders and entrepreneurs/firms being no less than their respective expected payoffs in equilibrium of the basic economy. Let Δ_E and Δ_F be the expected payoffs of equityholders and entrepreneurs in the default equilibrium of the basic economy (recall Assumption 1). The Pareto optimal allocation must be contingent only on the aggregate state of the economy because firm-specific risk can be completely eliminated via diversification. Suppose the planner invests a proportion, β, of the total capital in the economy, $1 + K$, in firms with the remaining proportion $1 - \beta$ invested in the risk-free asset.

With probability q, the economy is in the low aggregate state where a proportion ω^L of firms succeed, while the remaining proportion $1 - \omega^L$ of firms fail. In the low aggregate state, the total expected payoff from firms, $M^L(\beta)$, is a function of β, where

$$M^L(\beta) = \overbrace{\omega^L \Lambda\left(\beta(1+K)\right)}^{\text{expected payoff of successful firms}} + \overbrace{(1-\omega^L)\alpha\Lambda\left(\beta\left(1+K\right)\right)}^{\text{expected payoff of failed firms}} \tag{20}$$

Similarly, with probability $1-q$, the economy is in the high aggregate state where a proportion ω^H of firms succeed, while the remaining proportion $1-\omega^H$ of firms fail. In the high aggregate state, the total expected payoff from firms, $M^H(\beta)$, as a function of β, is

$$M^H(\beta) = \overbrace{\omega^H \Lambda\left(\beta(1+K)\right)}^{\text{expected payoff of successful firms}} + \overbrace{(1-\omega^H)\alpha\Lambda\left(\beta\left(1+K\right)\right)}^{\text{expected payoff of failed firms}} \tag{21}$$

Let E^H, F^H and E^L, F^L be the respective total allocations to the equityholders and entrepreneurs in the high and low aggregate state, respectively. Let B^H and B^L be the allocation to the bondholders in the high and low state, respectively. The social planner's problem, which determines the efficient allocation, is as follows.

$$\max_{\{\beta, B^L, B^H, E^L, F^L, E^H, F^H\}} qU\left(B^L\right) + (1-q)U\left(B^H\right) \text{ subject to} \tag{22}$$

$$(1-q)E^H + qE^L \geq \Delta_E;\ (1-q)F^H + qF^L \geq \Delta_F; \tag{23}$$

$$B^L + E^L + F^L = (1+K)(1-\beta) + M^L(\beta) \tag{24}$$

$$B^H + E^H + F^H = (1+K)(1-\beta) + M^H(\beta) \tag{25}$$

$$E^L, F^L, E^H, F^H \geq 0 \tag{26}$$

Constraints (23) express the conditions that risk-neutral equityholders and entrepreneurs must obtain at least their respective expected payoffs in the equilibrium of the basic economy. Constraints (24) and (25) are the planner's budget feasibility constraints in each aggregate state L and H, respectively. Constraints (26) are the limited liability constraints for equityholders and entrepreneurs. We now characterize the efficient allocation using the subscript 'eff' to denote variables in the efficient allocation.

Theorem 2 (Efficient Allocation)**.** *There exists a threshold level of the aggregate risk, τ_1, that determines the efficient allocation as follows.*

1. ***No Risk Borne by Bondholders**: Suppose the aggregate risk $\tau \leq \tau_1$. (a) If $K+1 < X$, $\beta_{eff} = 1$ so that the social planner invests all the capital in the economy in firms, and the expected marginal return of each firm exceeds the risk free return. (b) If $K+1 \geq X$, the social planner invests capital X in firms and*

the remaining capital in the risk-free asset. The expected marginal return of each firm equals the risk free return. (c) Bondholders bear no risk and $B^H_{\mathit{eff}} = B^L_{\mathit{eff}} \geq 1$. (d) The total expected output of the economy, and the expected allocations to bondholders, equityholders and entrepreneurs do not vary with aggregate risk.

2. ***Bondholders Bear Aggregate Risk**: Suppose the aggregate risk $\tau > \tau_1$. (a) The efficient proportion β_{eff} of total capital invested in firms strictly decreases with the aggregate risk τ. It approaches the investment proportion in the default equilibrium of the basic economy as $\tau \to 1$. (b) The expected marginal return of each firm is greater than the risk-free return. (c) Bondholders bear aggregate risk so that $B^L_{\mathit{eff}} < B^H_{\mathit{eff}}$ with $(1-q)B^H_{\mathit{eff}} + qB^L_{\mathit{eff}} > 1$. (d) Equityholders and entrepreneurs get no payoffs in the low aggregate state, that is, $E^L_{\mathit{eff}} = F^L_{\mathit{eff}} = 0$. (e) The total expected output of the economy and the expected allocation to bondholders decrease with aggregate risk. The expected allocations to equityholders and entrepreneurs do not vary.*

The social planner faces a tradeoff between maximizing expected output and achieving optimal risk-sharing among agents. When the aggregate risk τ is lower than a threshold, τ_1, there is sufficient capital even in the low aggregate state for the social planner to fully insure bondholders against aggregate risk. The planner optimally invests as much capital as possible in firms as long as the expected marginal return from each firm's technology exceeds the risk-free return. Hence, if $K + 1 \leq X$, the planner invests all the capital in the economy in firms. If $K + 1 > X$, the planner invests capital X in firms since this is the trigger level of capital at which the expected marginal return of each firm is exactly the risk-free return by (11). The efficient proportion of all capital in the economy invested in firms, β_{eff}, is therefore fixed and does not vary with τ. As the proportion of all capital in the economy invested in firms does not vary with aggregate risk, the total output of the economy from investments in firms and the risk-free asset and, therefore, the respective efficient allocations to bondholders, equityholders and enterpreneurs also do not vary.

When the aggregate risk τ exceeds the threshold, τ_1, there is insufficient capital in the low aggregate state to provide full protection to bondholders against aggregate risk because equityholders' and entrepreneurs' limited liability constraints are binding. As aggregate risk increases above the threshold, the tradeoff between maximizing expected production and achieving optimal risk-sharing causes the planner to lower the investment in risky firms and increase investment in the risk-free asset. Hence, the total output of the economy decreases, and the expected marginal return from investment in the representative firm exceeds the risk-free return. Because the efficient allocation guarantees equityholders and entrepreneurs their respective expected payoffs in the basic economy, their expected payoffs do not vary with aggregate risk as the equilibrium of the basic economy does not depend on aggregate risk. Since the total output of the economy decreases with aggregate risk, however, the expected payoff to bondholders must decrease with aggregate risk .

The basic economy *underinvests* relative to the efficient investment level when $\tau < 1$. The intuition is that, as discussed in Section 3, the equilibrium of the basic economy does not depend on the aggregate risk level. Because bondholders in the basic economy are undiversified, they are exposed to firm-specific risk and, therefore, invest an inefficiently high level of capital in the risk-free asset and underinvest in firms. Consequently, the overall level of investment in firms is lower than the efficient level when aggregate risk is low. When $\tau = 1$, however, firms' technology shocks are perfectly correlated so that bondholders' lack of diversification is costless. Hence, the equilibrium of the basic economy is Pareto efficient.

5 Credit Default Swap Markets

We now introduce markets for credit default swaps (CDS) that provide insurance to risk-averse bondholders against firm-specific default risk. The economy is as described in Section 2 except that we also have traded CDS on firms that are sold by equityholders and can be purchased by bondholders.

5.1 CDS Markets

Consider a firm $n \in [0, 1]$. As we discussed in Section 2.1, we can define a basic CDS security so that its price is 1. A CDS on the firm's debt is, therefore, defined by the return, $\widetilde{R}_{CDS,n} \equiv (0, R_{CDS,n})$, where 0 is the return on the CDS contract upon firm success and $R_{CDS,n}$ is the return if the firm fails. As with debt and equity markets, CDS markets are also *competitive* with buyers and sellers *taking the returns on CDS contracts as given* when they make their buying and selling decisions, respectively. Bondholders invest their capital in a portfolio comprising bonds issued by a single firm, CDS contracts on the bond, and the risk-free asset to maximize their expected utility. If multiple firms offer the same maximum expected utility to bondholders, each bondholder randomly chooses a single firm to invest in with uniform probability. Hence, if a bondholder chooses to invest in firm $n \in [0,1]$, he solves the following optimization problem:

$$(b_n, c_n) = \arg \sup_{x,y \in [0,1]} \mathbb{E}\left[U\left((1-x-y) + x\widetilde{R}_{B.n} + y\widetilde{R}_{CDS.n}\right)\right], \tag{27}$$

$$x + y \leq 1 \tag{28}$$

The bondholder, therefore, chooses the amounts of capital to invest in bonds and CDS contracts on the bonds, (x, y), to maximize his expected utility subject to the budget constraint, (28). Note that we allow for bondholders to purchase no CDS at all (that is, $y = 0$); invest no capital in bonds (that is, $x = 0$),thereby holding *naked* CDS positions; or hold interior positions in bonds and CDS ($0 < x, y < 1$). Hence, each bondholder could, in principle, end up imperfectly insured against firm-specific default risk, fully insured, or even overinsured with a higher payoff when the firm defaults depending on his CDS holdings. We show, however, that bondholders demand full insurance in any equilibrium, that is, they trade bonds and CDS so that they are fully protected against firm default risk.

Equityholders can buy or sell CDS contracts on an arbitrary number of firms. Without loss of generality, we can focus on the *net* position of the representative equityholder in CDS contracts. In equilibrium, as CDS contracts are in zero net supply and bondholders are long CDS contracts, equityholders must hold net short positions. To simplify the exposition, therefore, we assume at the outset that the representative equityholder sells CDS contracts. Each equityholder invests her capital—the initial endowment of K plus the proceeds from selling CDS—in a portfolio of the risk-free asset and firm equity. Risk-neutral equityholders sell CDS contracts with the lowest expected cost to them and invest in securities—firm equity or the risk-free asset—that offer the highest expected returns. Each equityholder, therefore, solves the following optimization problem:

$$C = \arg\sup_{c \geq 0} \left[\overbrace{(K+c)\max\left(\sup_{n\in[0,1]} \mathbb{E}\left[\widetilde{R}_{E,n}\right], 1\right)}^{\text{expected asset payoff}} - \overbrace{c \inf_{m\in[0,1]} \mathbb{E}\left[\widetilde{R}_{CDS,m}\right]}^{\text{expected CDS costs}} \right] \tag{29}$$

The first term in the objective function on the R.H.S. above indicates that the equityholder invests her total capital—the initial endowment K plus the capital raised by selling CDS, c—in a portfolio of the risk-free asset and firm equity that provides her the maximum expected payoff. As in the basic economy, if multiple firms offer the same maximum expected return, the equityholder invests uniformly among them. The second term indicates that the equityholder sells CDS with the lowest expected cost to her. Analogous to the asset side of the CDS seller's balance sheet, if multiple CDS contracts have the same expected cost to the equityholder, the equityholder diversifies uniformly among them, that is, the equityholder sells the same number of units of each CDS.

Each firm's objective is as in the basic economy and is given by (5), that is, it raises equity and debt capital and invests the capital it raises in its production technology.

5.2 Equilibrium Conditions

The following conditions must hold in equilibrium of the CDS market economy.

1. Each bondholder chooses his portfolio of investments in the risk-free asset, bonds issued by a single firm and CDS contracts on the firm to maximize his expected utility taking as given the risk-free return, bond returns and CDS returns. If multiple firms offers the same maximum expected utility to a bondholder, he randomly chooses a single firm to invest in with uniform probability.

2. Each CDS seller/equityholder chooses her portfolio of CDS contracts to sell and her investments in the risk-free asset and firm equity to maximize her expected profit taking as given the risk-free return, equity returns and CDS returns. If multiple CDS contracts have the same minimum expected cost to equityholders, equityholders sell the same number of units of each contract. If multiple equity securities offer the same maximum expected return, equityholders invest the same amount of capital in each security.

3. Firms maximize their expected profits by choosing how much equity and debt capital to raise and investing the capital they raise in their production technologies. Firms take as given bond and equity returns when they make their decisions.

4. All agents and firms are protected by limited liability. In equilibrium, the equity and debt returns of each firm $n \in [0,1]$ must satisfy the constraints (10).

5. In addition, each equityholder/CDS seller's total CDS liability in any state cannot exceed the payoff from her assets. If equityholder $m \in [0,1]$ raises capital C_m by selling CDS, $\widetilde{R}_{A,m}$ denotes the random return on the assets of the equityholder, and $\widetilde{R}_{L,m}$ denotes the return on the portfolio of her CDS liabilities, then we must have

$$\overbrace{(K + C_m)\,\widetilde{R}_{A,m}}^{\text{equityholder/CDS seller asset payoff}} \geq \overbrace{C_m \widetilde{R}_{L,m}}^{\text{equityholder/CDS seller total liability}} \tag{30}$$

   Note that, consistent with markets being competitive, the above constraints hold *in equilibrium*. That is, individual agents make their decisions taking security prices/returns as given. The equilibrium variables that are endogenously determined by market clearing satisfy (30).

6. The markets for *each firm's* equity, debt and CDS clear in equilibrium.

5.3 Equilibrium

As in the basic economy without CDS markets, an equilibrium (if it exists) is symmetric. We use the superscript '**' to denote equilibrium variables in the economy to distinguish them from those of the basic economy in Section 3.

Proposition 2 (Equilibrium Symmetry with CDS Markets)**.** *If an equilibrium exists, it is symmetric. The expectation of the marginal return from the representative firm's technology, its equity return, bond return and CDS return are all equal, that is,*

$$\mathbb{E}\left[\widetilde{R}^{**}\right] = \mathbb{E}\left[\widetilde{R}_E^{**}\right] = \mathbb{E}\left[\widetilde{R}_B^{**}\right] = \mathbb{E}\left[\widetilde{R}_{CDS}^{**}\right], \tag{31}$$

where $\widetilde{R}^{**}$ is the marginal return from the representative firm's technology.

The intuition is quite similar to that of Proposition 1. First, the linearity of equityholders'/CDS sellers' objective (29) in the amount of capital raised via CDS issuance implies that, in equilibrium, each CDS seller must make zero expected profits from selling CDS contracts and investing the resulting capital. Consequently, the minimum expected return of CDS contracts, which coincides with the expected marginal cost of CDS issuance to equityholders, must equal the expected return on equityholders' assets. Second, as the market for bonds and CDS for *each* firm must clear, each firm offers the *same* maximum expected utility to bondholders. Hence, each firm raises the same amount of debt capital in equilibrium, and each equityholder raises the same amount of capital via CDS issuance. Third, all firms must offer the same maximum expected equity return to ensure that the equity market for each firm clears. Hence, firms also raise the same amount of equity capital in equilibrium. It then follows that debt, equity and CDS returns are, respectively, identically distributed across firms. Fourth, by the necessary and sufficient first order condition of each firm's optimization problem (5), the expected return of a firm's debt or equity in equilibrium must be equal to the expected marginal return from its technology. If the expected marginal return of each firm's technology exceeds the risk-free return, then equityholders optimally invest all their capital in firms so that the expected asset return of each equityholder is $\mathbb{E}\left[\widetilde{R}^{**}\right]$. As the expected return of each equityholder's CDS portfolio equals the expected return on her assets, (31) holds. If the expected marginal return of each firm's technology equals the risk-free return, then equityholders are indifferent between investing in firms and the risk-free asset so that (31) again holds.

We now characterize equilibria of the economy with CDS contracts.

Theorem 3 (CDS Market Equilibrium).

1. *Suppose that $K + 1 < X$. There exists a unique equilibrium of the CDS market economy iff the aggregate risk τ is below a threshold, $\tau_2 \in (0, 1)$. In the equilibrium, each firm defaults on its debt when it fails. Bondholders invest all their capital in bonds and CDS contracts, and are fully insured. Equityholders invest all their capital in firms. The expected marginal return on each firm's technology exceeds the risk-free return. If aggregate risk, τ, exceeds the threshold τ_2, there is no equilibrium.*

2. *Suppose that $K + 1 \geq X$. If the aggregate risk is below a threshold, $\tau_3 \in (0, 1)$, (this differs in general from the threshold τ_2 in Part 1), there exists a continuum of equilibria. In each equilibrium, the representative firm raises total capital X and defaults on its debt when it fails. Equilibria differ only in the relative investments by bondholders and equityholders in firms as long as each firm raises total capital X. The expected marginal return on each firm's technology as well as the expected return on bonds, equity and CDS contracts equal the risk-free return. Bondholders are fully insured and indifferent between investing in bonds, CDS contracts and the risk-free asset. Equityholders are indifferent between investing in firm equity*

and the risk-free asset. If aggregate risk exceeds the threshold τ_3, there is no equilibrium.

We discuss key elements of the proof here and provide additional details in the Appendix. As any equilibrium is symmetric by Proposition 2, we can focus on the representative bondholder. Let B^{**} and C^{**} be the investments by the bondholder in bonds and CDS, respectively, in a *candidate* equilibrium of the economy. Let $\widetilde{R}_B^{**} \equiv \left(R_B^{s**}, R_B^{f**}\right)$ be the return on the bonds held by the bondholder and R_{CDS}^{**} be the return on the CDS contract on the representative firm when it defaults. The representative bondholder's optimal investments in bonds and CDS contracts maximize his expected utility and, therefore, solve the following problem.

$$(B^{**}, C^{**}) \equiv \arg \max_{x,y \in [0,1]} \mathbb{E}\left[U\left((1-x-y) + x\widetilde{R}_B^{**} + y\widetilde{R}_{CDS}^{**}\right)\right] \text{ such that} \tag{32}$$

$$x + y \leq 1 \tag{33}$$

By (31), $\mathbb{E}\left[\widetilde{R}_B^{**}\right] = \mathbb{E}\left[\widetilde{R}_{CDS}^{**}\right]$. The first order conditions of the above optimization program then imply that it is optimal for the bondholder to invest in bonds and CDS contracts so that she is *fully insured* against firm default risk, that is, she receives the same payoff upon firm success and failure.

Suppose that $K + 1 < X$. As the total capital invested in firms is at most $K + 1$ in any equilibrium, it follows from (11) that the expected marginal return from the representative firm's technology exceeds the risk-free return in any equilibrium, that is, $\mathbb{E}\left[\widetilde{R}^{**}\right] > 1$. By (31), therefore, $\mathbb{E}\left[\widetilde{R}_E^{**}\right] = \mathbb{E}\left[\widetilde{R}^{**}\right] > 1$ so that equityholders invest all their capital in firms. Further, as bondholders must be fully insured in equilibrium as noted earlier, and $\mathbb{E}\left[\widetilde{R}_B^{**}\right] = \mathbb{E}\left[\widetilde{R}_{CDS}^{**}\right] = \mathbb{E}\left[\widetilde{R}^{**}\right] > 1$ by (31), bondholders do not invest any capital in the risk-free asset.

We now argue that, in any equilibrium, the representative firm must default on its debt when it fails. Suppose, to the contrary, that the representative firm does not default. We must then have $R_B^{s**} = R_B^{f**} = \mathbb{E}\left[\widetilde{R}_B^{**}\right] > 1$. As bonds are risk-free, and provide a return greater than one, it is optimal for the representative bondholder to invest all his capital in bonds so that $B^{**} = 1$. Hence, $B^{**}R_B^{f**} > 1$. Equityholders invest all their capital in firms as the expected equity return exceeds the risk-free return. Hence, the representative firm raises capital $K+1$ and has a payoff of $\alpha\Lambda(K+1)$ when it fails. To avoid default, therefore, we must have $B^{**}R_B^{f**} \leq \alpha\Lambda(K+1) < 1$, where the last inequality follows from (12) as $K + 1 < X$. But this is a contradiction as we just argued that $B^{**}R_B^{f**} = R_B^{f**} > 1$ if the representative firm does not default.

Therefore, the following conditions must hold in equilibrium.

$$B^{**} + C^{**} = 1 \tag{34}$$

$$B^{**}R_B^{s**} = B^{**}R_B^{f**} + C^{**}R_{CDS}^{**} \tag{35}$$

$$B^{**}R_B^{f**} = \alpha\Lambda\left(K+1\right) \tag{36}$$

$$\mathbb{E}\left[\widetilde{R}_B^{**}\right] = (1-q)R_B^{s**} + qR_B^{f**} = \mathbb{E}\left[\widetilde{R}^{**}\right] = (1-q+q\alpha)\Lambda'(K+1) \tag{37}$$

$$\mathbb{E}\left[\widetilde{R}_{CDS}^{**}\right] = qR_{CDS}^{**} = \mathbb{E}\left[\widetilde{R}^{**}\right] \tag{38}$$

Equation (34) says that bondholders optimally invest all their capital in bonds and CDS contracts. Equation (35) expresses the condition that bondholders are fully protected against firm default risk and, therefore, receive the same payoff regardless of the firm's state. Indeed, $B^{**}R_B^{s**}$, is the bondholder's total payoff if the firm succeeds and delivers the promised bond return, R_B^{s**}, while $B^{**}R_B^{f**} + C^{**}R_{CDS}^{**}$ is the payoff if the firm fails so that bonds deliver the return, R_B^{f**}, and CDS contracts on the firm's bonds provide the return, R_{CDS}^{**}. Equation (36) expresses the absolute priority of bonds upon firm bankruptcy. Equation (37) and (38) are the conditions that the expected bond and CDS returns equal the expected marginal return from the representative firm's technology by (31).

In the Appendix, we show that there is a unique solution to the system of equations (34)–(38). Note that the aggregate risk τ, *does not appear* in any of the equations so that the solution to the system of equations *does not depend* on τ. The reason is that each bondholder invests in bonds and CDS of a *single* firm. CDS sellers/equityholders care about the expected payoff of their assets and the expected cost of their CDS holdings and not the correlations among their equity or CDS positions due to aggregate risk. Each firm maximizes the expected payoff from its technology without caring about its correlation with other firms' technologies. No agent knows the realization of the aggregate shock ex ante or the firms that are exposed to the aggregate shock.

The solution to (34)–(38) determines an equilibrium of the CDS market economy iff the additional equilibrium condition (30) is also satisfied. That is, the CDS seller's payoff in every state is sufficient to deliver the contractual payoff, $C^{**}R_{CDS}^{**}$ to the representative bondholder if his bonds default. As equityholders fully diversify their capital across firms, they are exposed only to aggregate risk. It is, therefore, necessary and sufficient that the representative equityholder's payoff in the low aggregate state is greater than or equal to her total CDS liability. In the Appendix, we show that the equityholder's asset payoff in the low aggregate state is $(K+C^{**})(1-\tau)\mathbb{E}\left[\widetilde{R}^{**}\right]$,and her total CDS liability is $\frac{\tau+q(1-\tau)}{q}C^{**}\mathbb{E}\left[\widetilde{R}^{**}\right]$. It then follows that we have an equilibrium iff

$$(K+C^{**})(1-\tau) \geq \frac{\tau+q(1-\tau)}{q}C^{**}. \tag{39}$$

As the representative firm defaults in any equilibrium, each bondholder invests nonzero capital in CDS so that $C^{**} > 0$. In addition, as discussed above, C^{**} does not depend on τ. Hence, the L.H.S. of (39) decreases with the aggregate risk τ, while the R.H.S. increases with τ as $q < 1$. Further, the L.H.S. strictly exceeds the R.H.S. for $\tau = 0$, and is strictly less than the R.H.S. for $\tau = 1$. Hence, the inequality holds iff τ is below a threshold, $\tau_2 \in (0, 1)$.

Intuitively, the reason why equilibrium fails to exist when aggregate risk exceeds a threshold is because bondholders demand full protection against firm-specific default risk via their CDS trades. However, CDS sellers are only able to offer full protection to bondholders if aggregate risk is below a threshold. In other words, when aggregate risk is sufficiently high, demand for CDS never equals the supply at any CDS price so that the CDS market breaks down. We again emphasize here that bondholders and CDS sellers/equityholders *take the CDS returns as given* when they make their CDS buying and selling decisions. For the representative firm, there exists a CDS return (equivalently, price) that clears the market iff aggregate risk is below a threshold.

Suppose now that $K + 1 \geq X$ but $K < K_1 = X - \alpha\Lambda(X)$ by Assumption 1. Note that $K_1 > X - 1$ by (12). In equilibrium, the representative firm must must raise capital X so that the expected marginal return from its technology equals the risk-free return by (11). Suppose, to the contrary, that the representative firm's capitalization is less than X. The expected marginal return from its technology then exceeds the risk-free return by (11) so that the (common) expected return on bonds, equity and CDS contracts also exceeds the risk-free return. Hence, it is optimal for the total capital in the economy, $K + 1$, to be invested in firms in any equilibrium, but $K + 1 \geq X$ by assumption, which leads to a contradiction. The representative firm's capitalization in equilibrium clearly cannot exceed X as the expected marginal return from its technology would then be less than the risk-free return by (11) so that all the capital in the economy would be invested in the risk-free asset, which is again a contradiction. Hence, the representative firm's capitalization in any equilibrium is exactly X, and the (common) expected return on bonds, equity and CDS contracts equals the risk-free return.

As $K < K_1$ by Assumption 1, we can again show (see the Appendix) that any equilibrium necessarily features default by the representative firm when it fails. Indeed, as we show in the Appendix, K_1 is exactly the threshold level of equity capital above which the representative firm is able to avoid default in equilibrium. Because the expected returns on bonds and CDS contracts are equal, it is again optimal for bondholders to buy CDS contracts so that they are fully insured. As in Part 1, in equilibrium, equityholders must be able to meet their CDS liabilities, which is only possible if aggregate risk is below a threshold by arguments similar to those in Part 1. Because bondholders and equityholders are indifferent between investing in firms and the risk-free asset, there is now a continuum of equilibria that differ only in the relative allocations of capital to firm securities and the risk-free asset as long as the total capital raised by the representative firm is X. We provide the details in the Appendix.

5.4 Aggregate Risk and CDS Market Equilibrium

Suppose that $K + 1 < X$. The consideration of the case where $K + 1 \geq X$ is similar. Part 1 of Theorem 3 shows that the equilibrium of the economy with CDS markets exists iff aggregate risk is below a threshold, τ_2. Recall that τ_1 is the threshold aggregate risk level below which the efficient level of investment in firms is constant, and above which it declines with aggregate risk as described in Theorem 2. In general, the threshold, τ_1 could be greater than, equal to, or less than the threshold, τ_2. Theorems 3 and 2 then lead to the following corollary.

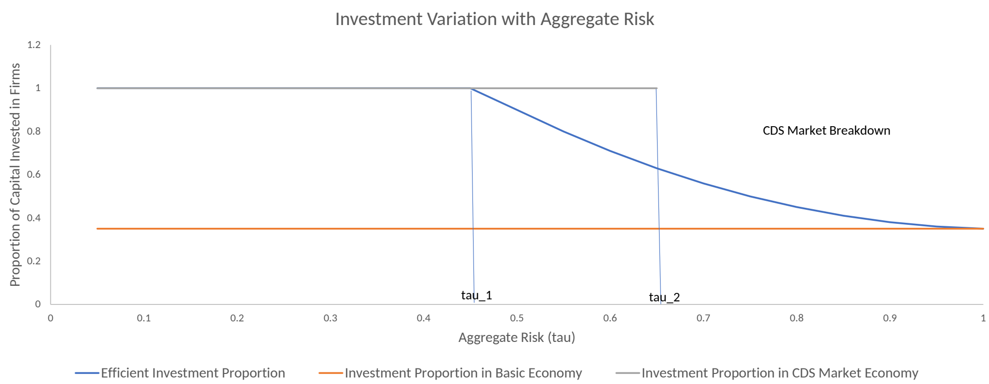
Corollary 1 (Aggregate Risk and CDS Market Equilibrium). *1. If $\tau \leq \min(\tau_1, \tau_2)$, a unique CDS market equilibrium exists in which all capital is invested in firms, and investment is efficient. CDS markets fully protect bondholders against risk.*

2. If $\min(\tau_1, \tau_2) < \tau \leq \max(\tau_1, \tau_2)$, then (a) EITHER $\tau_2 \leq \tau_1$ so that the equilibrium of the CDS market economy does not exist; (b) OR $\tau_2 > \tau_1$ so that the equilibrium of the CDS market economy exists, but, in equilibrium, the economy overinvests in firms relative to the efficient level.

3. If $\tau > \max(\tau_1, \tau_2)$, the CDS market economy has no equilibrium.

Figure 2 illustrates the results of the corollary by showing the variation of the proportion of capital invested in firms in the CDS market economy and comparing it

Figure 2: Impact of Aggregate Risk on Investment



with the corresponding investment proportions in the equilibrium of the basic economy described in Theorem 1 as well as the efficient allocation described in Theorem 2. We illustrate the case where $\tau_1 < \tau_2$ and $K + 1 < X$.

As shown by the figure, the basic economy underinvests in firms relative to the efficient allocation for all τ. The introduction of CDS markets causes the level of investment by the economy to equal the efficient level as long as $\tau \leq \tau_1$. For $\tau \in (\tau_1, \tau_2]$, the equilibrium of the CDS market economy exists, but is inefficient in two aspects. First, bondholders must bear aggregate risk in the efficient allocation as shown by Part 2 of Theorem 2, but are fully protected against risk in the CDS market economy. Second, as shown by the figure, the CDS market economy *overinvests* in firms relative to the efficient allocation. In this region, by the discussion following Theorem 2, the efficient level of investment in firms declines with aggregate risk. The investment in equilibrium of the CDS economy, however, does not depend on aggregate risk at all. Intuitively, in this region, CDS sellers/equityholders, who care only about their expected payoffs and not the correlations among firm returns created by aggregate risk, overinvest in risky firms relative to the efficient allocation. Finally, if $\tau > \tau_2$, the CDS market breaks down as there are no CDS prices at which demand for CDS is matched by the supply. Indeed, as noted earlier, if equilibrium exists, it is unaffected by aggregate risk. However, if aggregate risk is sufficiently high, the "no default" condition that

CDS sellers/equityholders msut be able to meet their liabilities in the low aggregate state is violated.

The result that the CDS market breaks down when aggregate risk exceeds the threshold, τ_2, provides a formal underpinning for anecdotal observations by commentators that CDS markets broke down during the financial crisis because CDS prices did not appropriately reflect the effects of aggregate risk (Jarrow (2011)). It is worth pointing out here that we can reinterpret the breakdown of CDS markets for $\tau > \tau_2$ as a reversion of the economy to the basic economy of Section 2 with no CDS markets. In other words, the breakdown of CDS markets implies that there is no CDS trading at all. With this interpretation, equilibrium exists for all levels of aggregate risk with the equilibrium corresponding to that of the basic economy for $\tau > \tau_2$. This alternate interpretation does not impact our results pertaining to CDS market regulation.

6 CDS Market Regulation

Unregulated CDS markets do not, in general, achieve efficiency. Indeed, Corollary 1 shows that, depending on the level of aggregate risk, an equilibrium of the CDS market may not exist at all and, if one exists, the equilibrium allocation may not be efficient. Even with the availability of CDS to share firm-specific default risk, the CDS market economy may be inefficient for reasons similar to the inefficiency of the basic economy of Section 2. Bondholders are undiversified as they invest in bonds and CDS of a single firm, and markets are incomplete as there is no security contingent on the realization of the aggregate shock that all investors can trade. We now examine whether appropriately designed regulation of CDS markets, while otherwise maintaining the constraints of the economy described above, can implement the efficient allocation characterized in Theorem 2.

6.1 Regulatory Instruments

We consider existing and proposed instruments to regulate CDS markets. To facilitate our normative analysis here, we introduce a hypothetical "regulator" who combines the roles of different institutions that influence CDS markets such as a centralized exchange and a fiscal authority.

Margin/Collateral and Capital Requirements on CDS Issuers

The regulator can require CDS issuers (equityholders) to hold a minimum proportion of their initial capital, K, in risk-free margin accounts as collateral. The margin or collateral requirement for a CDS issuer takes the following form:

$$s \geq \mu K, \tag{40}$$

for some $\mu \in [0,1]$, where s is the CDS issuer's holdings in the risk-free asset or cash.

The regulator can also impose a constraint on CDS issuance, which limits an issuer's total CDS liability relative to its capital by requiring that it cannot raise more than a proportion of its initial capital K via CDS contracts. Specifically, if c is the capital an issuer raises via CDS contracts, then

$$c \leq \nu K, \tag{41}$$

for some $\nu \geq 0$. Inequalities (40) and (41) represent constraints on the assets and liabilities of CDS issuers. If there is no margin requirement, then $\mu = 0$. If there is no capital requirement, then $\nu = \infty$.

Transfers

The regulator can implement transfers among agents. Such transfers can be implemented via fees and payouts managed by a centralized exchange or through taxes/subsidies imposed by a fiscal authority. For expositional convenience, we employ the terminology of "taxes and subsidies" when we discuss regulatory transfers.

Consider the representative firm. Let $(t_B^{j,k}, t_{CDS}^{j,k}); j \in \{H, L\}; k \in \{s, f\}$ denote the *proportional* tax on the payouts of the firm's bonds and CDS, respectively, in the aggregate state j and idiosyncratic state k, where the idiosyncratic state represents the success or failure of the representative firm in which the bondholder invests. Similarly, let $t_E^{j,k}$ denote the proportional tax on equity payments of the representative firm and $T_F^{j,k}; j \in \{H, L\}; k \in \{s, f\}$ denote the *lump sum* tax on the payoff of the firm's entrepreneur. Although we can reformulate our analysis with proportional taxes for entrepreneurs too, it is convenient to use lump sum taxes because entrepreneurs hold no capital initially. If any of these taxes take negative values, then they represent subsidies. Regulatory payments respect the limited liability of all agents, and the regulator maintains a *balanced budget* in *each* aggregate state, that is, the cash inflow to the regulator via taxes must equal the cash outflow via subsidies.

6.2 Security Returns

The presence of taxes creates wedges between *before-tax* and *after-tax* returns of securities. The before-tax return of a security is its *marginal cost* to the issuer, while the after-tax return is the *return* received by the buyer. Let $\widetilde{z}_E$ denote the marginal cost of equity financing (equity payout per unit of equity capital) for the representative firm and $\widetilde{z}_B$ denote the marginal cost of debt financing (debt payout per unit of debt capital). Analogously, let $\widetilde{z}_{CDS}$ denote the marginal cost of a CDS contract to the representative CDS seller/equityholder. If $\widetilde{R}_E, \widetilde{R}_B, \widetilde{R}_{CDS}$ denote the *net* (after taxes or subsidies) returns on the representative firm's equity, bonds and CDS contracts, respectively, then

$$\widetilde{R}_E = \widetilde{z}_E - \widetilde{t}_E; \widetilde{R}_B = \widetilde{z}_B - \widetilde{t}_B; \widetilde{R}_{CDS} = \widetilde{z}_{CDS} - \widetilde{t}_{CDS} \tag{42}$$

where $\widetilde{t}_E, \widetilde{t}_B, \widetilde{t}_{CDS}$ are random variables denoting the proportional taxes on equity, bond and CDS payouts, respectively. In contrast with the basic economy of Section 2 and the unregulated CDS market economy described of the previous section, the returns to equityholders, bondholders, and CDS buyers, $(\widetilde{R}_E, \widetilde{R}_B, \widetilde{R}_{CDS})$, also depend on the aggregate state in general as taxes/subsidies are contingent on the aggregate state.

Accordingly, the CDS return on the representative firm's debt is now defined by the nonnegative random vector, $\widetilde{R}_{CDS} \equiv \left(R^H_{CDS}, R^L_{CDS}\right)$, where R^i_{CDS} is the return received by the holder of the CDS contract when the firm defaults on its debt and the aggregate state is $i \in \{H, L\}$. Our notation here reflects the fact that, because a CDS contract pays nothing when a firm succeeds, it is fully determined by its returns upon firm default in the aggregate states, H and L. The net returns on the representative firm's bonds and equity depend on the aggregate and idiosyncratic states as the taxes/subsidies, $\widetilde{t}_B, \widetilde{t}_E$ could depend on both states. Consequently, the representative bond and equity returns are defined by the nonnegative random vectors, $\widetilde{R}_B \equiv \left(R_B^{(H,s)}, R_B^{(H,f)}, R_B^{(L,s)}, R_B^{(L,f)}\right)$ and $\widetilde{R}_E \equiv \left(R_E^{(H,s)}, R_E^{(H,f)}, R_E^{(L,s)}, R_E^{(L,f)}\right)$, respectively.

6.3 Equilibrium Conditions

A firm n raises equity and debt capital, taking the marginal costs of its equity and debt contracts, $\{\widetilde{z}_{E,n}, \widetilde{z}_{B,n}\}$, as given.

$$(E_n, B_n) = \arg\max_{(x,y)} (1-q)\left(\Lambda(x+y) - z^s_{E,n}.x - z^s_{B,n}.y\right) \tag{43}$$
$$+ q\alpha\left(\Lambda(x+y) - z^f_{E,n}.x - z^f_{B,n}.y\right).$$

If a bondholder chooses to invest in firm n, then he solves the following:

$$(b_n, c_n) \equiv \arg\sup_{x,y\in[0,1]} \mathbb{E}\left[U\left((1-x-y) + x\widetilde{R}_{B,n} + y\widetilde{R}_{CDS.n}\right)\right] \text{ such that} \tag{44}$$
$$x + y \leq 1 \tag{45}$$
$$y \leq \nu K \tag{46}$$

Constraint (45) is the budget constraint for the bondholder. Constraint (46) incorporates the capital requirement (41) on the CDS sellers, which restricts the supply of CDS contracts. The investment by the bondholder in CDS contracts cannot exceed the amount of capital that the representative CDS seller is allowed to raise via CDS contracts. Note that the expectation in (44) is over aggregate and idiosyncratic states as the bond and CDS returns depend on both states in general. As in the basic and unregulated CDS market economies, the bondholder chooses a *single* firm that offers him the maximum expected utility.

Each equityholder/CDS seller's optimization problem reflects the margin and capital requirements it faces and is given by

$$C = \arg\sup_{c\in[0,\mu K]}\left[\overbrace{\mu K + (K + c - \mu K)\max\left(\sup_{n\in[0,1]} \mathbb{E}\left[\widetilde{R}_{E,n}\right], 1\right)}^{\text{expected asset payoff}} - \overbrace{c\inf_{m\in[0,1]} \mathbb{E}\left[\widetilde{z}_{CDS,m}\right]}^{\text{expected CDS costs}}\right] \tag{47}$$

As indicated above, the amount of CDS capital issued cannot exceed μK as reflected by the capital requirement, (41). The CDS seller must invest at least capital μK in the risk-free asset. She invests her remaining capital, $K + c - \mu K$, in a portfolio of the risk-free asset and firm equity that provides her the maximum expected payoff. The second term indicates that the equityholder sells CDS with the lowest expected cost to her. The marginal cost of each CDS contract to the issuer could differ from the return to bondholders because of taxes/subsidies.

The equilibrium conditions are then directly analogous to those in Section 5.2 with the above descriptions of each agent's actions. We avoid repeating them here. As in the unregulated economy, an equilibrium (if it exists) is symmetric. We use the superscript '***' to denote equilibrium variables in the regulated economy to distinguish them from the corresponding variables of the basic economy in Section 3 and the economy with unregulated CDS markets in Section 5.

Proposition 3 (Equilibrium Symmetry with CDS Regulation)**.** *If an equilibrium exists, it is symmetric. The expected marginal return from each firm's technology, the expected marginal cost of equity, and the expected marginal cost of debt are equal, that is,*

$$\mathbb{E}\left[\widetilde{R}^{***}\right] = \mathbb{E}\left[\widetilde{z}_E^{***}\right] = \mathbb{E}\left[\widetilde{z}_B^{***}\right], \tag{48}$$

where $\widetilde{R}^{***}$ is the marginal return from the representative firm's technology. The expected equity return of each firm equals the marginal expected cost of a CDS contract.

$$\mathbb{E}\left[\widetilde{R}_E^{***}\right] = \mathbb{E}\left[\widetilde{z}_{CDS}^{***}\right] \tag{49}$$

The intuition is quite similar to that of Proposition 2. In equilibrium, the expected marginal return on each firm's technology must equal its expected marginal cost of financing. Further, as firms must be indifferent between equity and debt financing in equilibrium, the marginal costs of equity and debt financing must be equal, which leads to (48). The linearity of a CDS seller's objective function in the amount of CDS capital implies that it must make zero additional profits from raising capital via CDS issuance and investing the capital. Consequently, the expected marginal cost of a CDS contract to the representative equityholder must equal the expected equity return.

6.4 Implementation of Efficient Allocation

We now show that the efficient allocation chracterized in Theorem 2 can be implemented via an equilibrium of the economy with regulated CDS markets. Let us denote the efficient allocation by $(\widetilde{B}_{\text{eff}}, \widetilde{E}_{\text{eff}}, \widetilde{F}_{\text{eff}}, \beta_{\text{eff}})$ where$\widetilde{B}_{\text{eff}} \equiv (B_{\text{eff}}^H, B_{\text{eff}}^L)$; $\widetilde{E}_{\text{eff}} \equiv (E_{\text{eff}}^H, E_{\text{eff}}^L)$, $\widetilde{F}_{\text{eff}} \equiv (F_{\text{eff}}^H, F_{\text{eff}}^L)$ are, respectively, the payoffs to bondholders, equityholders and entrepreneurs/firms in the efficient allocation, and β_{eff} is the proportion of total capital that is invested in firms. Recall that the efficient allocation depends only on the aggregate state, H or L. Consider a *candidate* regulated equilibrium that implements the efficient allocation. Let B^{***} and C^{***} be the investments by the representative bondholder in bonds and CDS, respectively, in the equilibrium. Let $\widetilde{R}_B^{***} \equiv \left(R_B^{(H,s)***}, R_B^{(H,f)***}\right)$ be the return on the bonds held by the bondholder where $R_B^{(H,s)***}, R_B^{(H,f)***}$ are the returns upon firm success and failure, respectively. Let $\widetilde{R}_{CDS}^{***} \equiv \left(R_{CDS}^{H***}, R_{CDS}^{L***}\right)$ be the return on the representative CDS contract. We recall our standing Assumption 1 that $K < K_1$.

6.4.1 Low Aggregate Risk

Suppose first that the aggregate risk, $\tau \leq \tau_1$ so that we are in Case 1 of Theorem 2.

Theorem 4 (Efficient Implementation: Low Aggregate Risk)**.** *Suppose that $\tau \leq \tau_1$. 1. If $K+1 < X$, the efficient allocation can be implemented in a regulated equilibrium with just transfers among agents via taxes and subsidies. No margin or capital requirements on CDS issuers are required. 2. If $K+1 \geq X$, in addition to taxes and subsidies, it is necessary to impose a margin requirement on CDS issuers.*

When $\tau \leq \tau_1$, bondholders bear no risk in the efficient allocation by Case 1 of Theorem 2 with $B_{\text{eff}}^H = B_{\text{eff}}^L \geq 1$. We can ensure that bondholders, indeed, purchase full insurance in decentralized markets by designing taxes/subsidies so that the expected returns to bondholders from bonds and CDS contracts are equal.

If $K + 1 < X$, then all the capital in the economy is invested in firms in the efficient allocation. Further, by Case 1 of Theorem 2, $B_{\text{eff}}^H = B_{\text{eff}}^L > 1$, that is, bondholders' return in the efficient allocation exceeds the risk-free return. As we show in the Appendix, we can ensure that bondholders invest all their capital in bonds and CDS in the decentralized economy via taxes/subsidies which ensure that the return to bondholders matches the return in the efficient allocation. By Case 1 of Theorem 1, equityholders invest all their capital, K, in firms in equilibrium of the basic economy and their expected return exceeds the risk-free return of one. Hence, their expected payoff, Δ_E,which equals their payoff in the efficient allocation by (23) and Theorem 2, satisfies $\frac{\Delta_E}{K}>1$. In the efficient regulated equilibrium, therefore, equityholders invest all their capital in firms. Hence, there should be no margin requirement imposed on CDS sellers as it would lead to underinvestment.

If $K + 1 \geq X$, then $B_{\text{eff}}^H = B_{\text{eff}}^L = 1$, that is, bondholders' return in the efficient allocation equals the risk-free return. Bondholders are indifferent between investing in firms and the risk-free asset so that it is still weakly optimal for bondholders to invest all their capital in bonds and CDS in the decentralized economy. When $K+1 \geq X$, the proportion of capital invested in firms in the efficient allocation is strictly less than one by Theorem 2. Because $\frac{\Delta_E}{K} > 1$, however, equityholders would invest all their capital in firms without any constraint on their assets. To ensure that the economy invests the efficient proportion of capital in firms, which is strictly less than one, a collateral or margin requirement must be imposed on CDS sellers.

6.4.2 Intermediate Aggregate Risk

Now consider the case where $\tau > \tau_1$ so that we are in Case 2 of Theorem 2.

Theorem 5 (Efficient Implementation: Intermediate Aggregate Risk)**.** *For any $K < K_1$ (recall Assumption 1), there exists $\overline{\tau}_1 > \tau_1$ with $\overline{\tau}_1 \leq 1$ such that for $\tau \in (\tau_1, \overline{\tau}_1]$,the efficient allocation can be implemented with taxes/subsidies and a margin requirement that increases with aggregate risk.*

By Part 2 of Theorem 2, bondholders bear aggregate risk in the efficient allocation. Moreover, the efficient proportion of capital invested in firms, β_{eff}, declines with aggregate risk regardless of whether $K+1 < X$ or $K+1 \geq X$,which is why we don't need to consider these cases separately as in Theorem 4. To implement the efficient allocation, some capital in the economy must be invested in the risk-free asset in equilibrium of the regulated economy with the risk-free asset investment increasing with aggregate risk. However, as discussed earlier, equityholders' expected payoff, Δ_E,in the efficient allocation (see (23)) satisfies $\frac{\Delta_E}{K}>1$. Consequently, without any constraint on their assets, equityholders would invest all their capital in firms. To ensure that the economy invests the efficient proportion of capital in firms, therefore, a collateral or margin requirement must be imposed on equityholders/CDS sellers, which forces them to invest capital in the risk-free asset so that the total investment in the risk-free asset by the economy matches the efficient allocation. Further, the margin requirement increases with aggregate risk because the efficient proportion of capital that is invested in firms, β_{eff}, declines with aggregate risk by Part 2 of Theorem 2.

For general parameter values, as the margin requirement on CDS sellers increases with aggregate risk, it could be equal to one at a level of aggregate risk $\overline{\tau}_1 < 1$. If this is the case, then the margin requirement on CDS sellers becomes *maximally stringent* where they are forced to invest all their initial capital in the risk-free asset. For $\tau > \overline{\tau}_1$, therefore, compelling CDS sellers to invest their entire initial capital endowments in the risk-free asset alone is insufficient to ensure that the economy as a whole invests the efficient proportion of capital in the risk-free asset. Of course, there could be specific parameter constellations for which the margin requirement never becomes maximally stringent and the efficient allocation can be implemented via a margin requirement for all $\tau > \tau_1$. In general, however, this need not hold.

6.4.3 High Aggregate Risk

Now consider the case where $\overline{\tau}_1 < 1$ and $\tau \in (\overline{\tau}_1, 1]$.

Theorem 6 (Efficient Implementation:High Aggregate Risk)**.** *For any $K < K_1$ (recall Assumption 1) and for $\tau \in (\overline{\tau}_1, 1]$, the efficient allocation can be implemented with taxes/subsidies, a maximally stringent margin requirement $\mu = 1$, and a capital requirement ν given by (A70).*

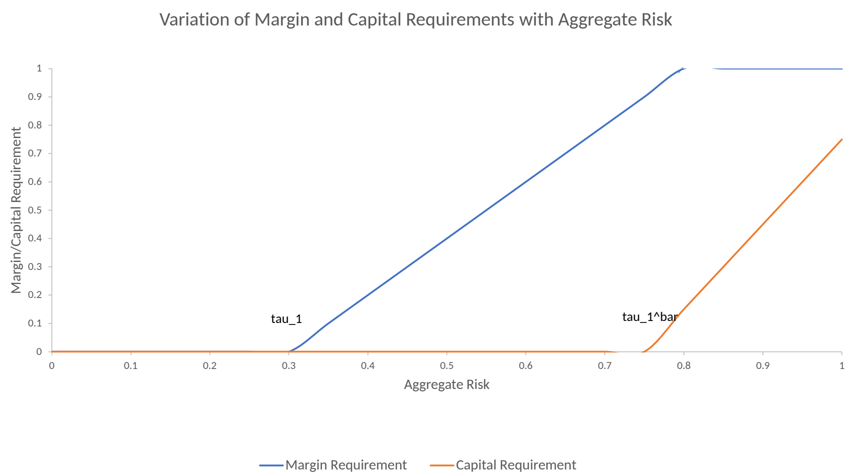
As discussed above, when aggregate risk $\tau > \overline{\tau}_1$, compelling CDS issuers/equityholders to invest their entire initial capital in the risk-free asset alone is insufficient to match the level of investment in the efficient allocation. In general, bondholders must also invest nonzero capital in the risk-free asset and equityholders face a maximally stringent margin requirement that forces them to invest their initial capital in the risk-free asset. In a scenario where the CDS market regulator cannot *directly* control bondholders' outside investments in the risk-free asset, bondholders can be *indirectly induced* to invest the requisite amount of capital in the risk-free asset to match the efficient level of investment by constraining the supply of CDS contracts via a capital requirement on CDS issuers. As the constraint on CDS supply leaves bondholders underinsured, they invest some capital in the risk-free asset. The intuition for the expected taxes/subsidies is as discussed after Theorem 4.

In summary, we see that the efficient allocation can be implemented by the introduction of CDS markets provided the markets are appropriately regulated. Figure 3 shows the variation of margin and capital requirements with aggregate risk.

7 Conclusions

We derive analytical insights into the welfare effects of CDS trading and the design of CDS market regulation in a parsimonious and tractable general equilibrium model. If the aggregate equity capital in the economy is below a threshold, any equilibrium

Figure 3: Variation of Margin and Capital Requirements with Aggregate Risk



of the basic economy with no CDS markets features firm defaults and *underinvests* in productive firms relative to the efficient allocation. Equilibria of the basic economy are unaffected by aggregate risk. The efficient allocation, however, features full insurance for bondholders and a constant level of investment in firms when aggregate risk is below a threshold, but imperfect insurance for bondholders and decreasing investment in the real economy when aggregate risk increases above the threshold. For low aggregate risk levels, the introduction of unregulated CDS trading leads to full insurance for bondholders efficient firm investment so that the efficient allocation can be implemented simply via transfers. For intermediate aggregate risk, the CDS market economy *overinvests* in firms, and a margin or collateral requirement on CDS sellers is necessary for efficiency. For high aggregate risk levels, the CDS market breaks down. A maximally stringent collateral requirement along with a capital requirement that restricts CDS supply restore equilibrium and efficiency.

In future research, it would be interesting to develop a structural model that can deliver quantitative insights into the regulation of CDS markets as well as incorporate agency frictions stemming from moral hazard and/or adverse selection.

References

[1] Allen F, Carletti E. 2006. Credit risk transfer and contagion. *Journal of Monetary Economics* 53:89–111

[2] Arping S. 2014. Credit protection and lending relationships. *Journal of Financial Stability* 10:7–19

[3] Ashcraft AB, Santos JA. 2009. Has the CDS market lowered the cost of corporate debt? *J. Monetary Economics* 56:514–23

[4] Augustin, Patrick, Marti G. Subrahmanyam, Dragon Y. Tang, and Sarah Q. Wang, 2016, Credit Default Swaps: Past, Present, and Future, *Annual Review of Financial Economics*. 2016. 8:10.1–10.22

[5] Biais B, Heider F, Hoerova M. 2016. Risk-sharing or risk-taking? Counterparty risk, incentives and margins. *Journal of Finance*. 71, 1669-1698.

[6] Chakraborty I, Chava S, Ganduri R. 2015. Credit default swaps and moral hazard in bank lending. *Working paper*, Georgia Institute of Technology.

[7] Che YK, Sethi R. 2014. Credit market speculation and the cost of capital. *American Economic Journal: Microeconomics*. 1:1–34

[8] Chernov, Mikhail, Alexander S. Gorbenko, and Igor Makarov, 2013, CDS auctions, *Review of Financial Studies* 26, 768–805.

[9] Danis A, Gamba A. 2018. The real effects of credit default swaps. *Journal of Financial Economics* 127, 51–76.

[10] Darst, M. and E. Refayet 2018. Credit Default Swaps in General Equilibrium: Endogenous Default and Credit-Spread Spillovers. *Journal of Money, Credit and Banking,* 50, 1901-1933.

[11] Du, Songzi and Haoxiang Zhu, 2017. Are CDS Auctions Biased and Inefficient? *Journal of Finance* 72, 2589–2628.

[12] Duffee GR, Zhou C. 2001. Credit derivatives in banking: useful tools for managing risk? *Journal of Monetary Economics* 48:25–54

[13] Fishburn, Peter C and R. Burr Porter, 1976. Optimal Portfolios with One Safe and One Risky Asset: Effects of Changes in Rate of Return and Risk. *Management Science* 22, 1064-1073.

[14] Fostel A, Geanakoplos J. 2016. Financial innovation, collateral and investment. *American Economic Journal:Macroeconomics* 8, 242–284.

[15] Hakenes H, Schnabel I. 2010. Credit risk transfer and bank competition. *Journal of Financial Intermediation* 19:308–32

[16] Jarrow RA. 2011. The economics of credit default swaps. *Annual Review of Financial Economics* 3:235–57

[17] Ljungvist, L. and T. Sargent, 2018. *Recursive Macroeconomic Theory.* MIT Press, Fourth Edition.

[18] Morrison AD. 2005. Credit derivatives, disintermediation, and investment decisions. *Journal of Business* 78:621–48

[19] Norden L, Buston CS, Wagner W. 2014. Financial innovation and bank behavior: evidence from credit markets. *Journal of Economic Dynamics and Control* 43:130–45

[20] Oehmke M, Zawadowski A. 2015. Synthetic or real? The equilibrium effects of credit default swaps on bond markets. *Review of Financial Studies.* 28:3303–37

[21] Oehmke M, Zawadowski A. 2017. The anatomy of the CDS market. *Review of Financial Studies* 30, 80–119.

[22] Parlour CA, Winton A. 2013. Laying off credit risk: loan sales versus credit default swaps. *Journal of Financial Economics* 107:25–45

[23] Peivandi, Ahmad, 2015, Participation and unbiased pricing in CDS settlement mechanisms, Working paper.

[24] Thompson JR. 2010. Counterparty risk in financial contracts: Should the insured worry about the insurer? *Quarterly Journal of Economics.* 125:1195–252

A General Equilibrium Model of Credit Default Swap (CDS) Markets

Online Appendix

Appendix: Proofs

Proof of Proposition 1

The proof follows along the lines of the discussion after the statement of the proposition.

We first show that each firm must offer the same maximum expected utility to bondholders and, therefore, all firms raise the same amount of debt capital from bondholders in equilibrium. Indeed, each bondholder solves the optimization problems (7) and (9). If bondholders do not obtain the same maximum expected utility from all firms, then no bondholder would invest in firms offering the lowest expected utility. Hence, the market for the bonds of such firms would not clear. As each bondholder randomly chooses a single firm to invest in with uniform probability when multiple firms offer the same maximum expected utility, it follows that all firms raise the same amount of debt capital in equilibrium.

Similarly, each firm offers the same maximum expected equity return to equityholders and all firms raise the same amount of equity capital. Indeed, by the second equilibrium condition in Section 2.3, equityholders invest their capital uniformly in firms that offer the same maximum expected equity return when the maximum expected equity return, and in the risk-free asset when the risk-free return is greater than the maximum expected equity return. When the maximum expected equity return equals the risk-free return, equityholders are indifferent between investing in firms and the risk-free asset. If they do invest in firms, they invest uniformly in all firms offering the same maximum expected equity return.

By Steps 1 and 2, all firms have identical capital structures and raise the same total capital. Hence, equity and bond returns must be identically distributed across firms.

The first order conditions of each firm's optimization problem (5) are as follows.

$$(1-q)\Lambda^{'}(E_n+B_n)+q\alpha\Lambda^{'}(E_n+B_n)=(1-q)R^s_{E,n}+qR^f_{E,n}=(1-q)R^s_{B,n}+qR^f_{B,n}. \tag{A1}$$

By Step 3, it follows that the expected marginal return from each firm's technology—the first expression in (A1)—equals the expected return on its equity and debt; the second and third expressions in (A1). Q.E.D.

Proof of Theorem 1

We split the proof into several steps for clarity.

Step 1. By Proposition 1, any equilibrium is symmetric. Suppose first that $K < K_1 = X - \alpha\Lambda(X)$. Consider a candidate symmetric equilibrium where the equity capital K is invested uniformly among firms so that $E^* = K$. Suppose that firms default when their technologies fail in the candidate equilibrium. We later show in Step 3 that firms must default when they fail in any equilibrium. Let B^* denote the total debt capital raised by all firms in the equilibrium, and $\widetilde{R}_E^*, \widetilde{R}_B^*$ denote the equity and bond returns, respectively, of the representative firm. We reproduce the conditions that determine the equilibrium here for convenience.

$$R_E^{f*} = 0; R_B^{f*} = \frac{\alpha\Lambda\left(E^* + B^*\right)}{B^*} = \frac{\alpha\Lambda\left(K + B^*\right)}{B^*} \tag{A2}$$

$$\mathbb{E}[\widetilde{R}_E^*] = (1-q)R_E^{s*} = \mathbb{E}[\widetilde{R}_B^*] = (1-q)R_B^{s*} + qR_B^{f*} = (1-q)\Lambda^{'}\left(K + B^*\right) + q\alpha\Lambda^{'}\left(K + B^*\right) \tag{A3}$$

$$B^* = \arg\max_{x\in[0,1]} (1-q)U\left[(1-x) + xR_B^{s*}\right] + qU\left[(1-x) + xR_B^{f*}\right] \tag{A4}$$

The conditions imply two relations between the expected return on the firm's debt, $\mathbb{E}[\widetilde{R}_B^*]$, and the amount of debt capital, B^*, that together determine an equilibrium.

$$\mathbb{E}[\widetilde{R}_B^*] = (1-q)\Lambda^{'}\left(K + B^*\right) + q\alpha\Lambda^{'}\left(K + B^*\right) \tag{A5}$$

$$B^* = \arg\max_{x\in[0,1]} (1-q)U\left[(1-x) + \frac{x}{1-q}\left(\mathbb{E}[\widetilde{R}_B^*] - q\frac{\alpha\Lambda\left(K + B^*\right)}{B^*}\right)\right] + qU\left[(1-x) + x\frac{\alpha\Lambda\left(K + B^*\right)}{B^*}\right] \tag{A6}$$

Relation (A5) is a strictly decreasing relation between $\mathbb{E}[\widetilde{R}_B^*]$ and B^* because $\Lambda'(.)$ is strictly decreasing as $\Lambda(.)$ is strictly concave. *Step 2.* We now show that, for a given

$\mathbb{E}[\widetilde{R}_B^*] > 1$, there is a unique solution, B^*, to the fixed point problem, (A6). Further, B^* increases monotonically with $\mathbb{E}[\widetilde{R}_B^*]$ if bondholders' demand function g defined in (8) is increasing in its first argument.

We first argue that there is a unique solution, $B_0 \in (0, \alpha\Lambda(X))$ to the equation $B = \alpha\Lambda(K + B)$. Indeed, $0 < \alpha\Lambda(K)$ and $\alpha\Lambda(X) > \alpha\Lambda(K + \alpha\Lambda(X))$ because $K < K_1 = X - \alpha\Lambda(X)$. Hence, $B < \alpha\Lambda(K + B)$ for $B = 0$, and $B > \alpha\Lambda(K + B)$ for $B = \alpha\Lambda(X)$. As Λ is strictly concave, a unique solution $B_0 \in (0, \alpha\Lambda(X))$ therefore exists to the equation $B = \alpha\Lambda(K + B)$. Further, $\alpha\Lambda(K + B) < B$ for $B \in (B_0, 1)$. In particular,

$$\alpha\Lambda(K + 1) < 1 \text{ for } K < K_1 \tag{A7}$$

Next, we will show that (A6) has a unique solution $B^* \in (B_0, 1]$. Since, as argued above, $\alpha\Lambda(K+B^*) < B^*$ for $B^* \in (B_0, 1]$, $R_B^f = \frac{\alpha\Lambda(K+B^*)}{B^*} < 1$.

Let $f(B^*)$ denote the R.H.S. of (A6) as a function of B^*. We show that $f(B^*)$ is monotonically decreasing in B^* for $B^* \in (B_0, 1]$. First, observe that

$$B^*\Lambda^{'}(K+B^*) \leq B^*\Lambda'(B^*) < \Lambda(B^*) < \Lambda(K+B^*),$$

as $\Lambda(.)$ is strictly increasing and concave. The above implies that $B^*\Lambda^{'}(K+B^*)-\Lambda(K+B^*) < 0$ so that $R_B^f = \frac{\Lambda(K+B^*)}{B^*}$ strictly decreases with B^*. Now suppose $f(B^*) \in (0,1)$ is an interior solution of the R.H.S. of (A6). The necessary and sufficient first order condition is

$$(1-q)U^{'}\left[(1-f(B^*)) + \frac{f(B^*)}{1-q}\left(\mathbb{E}[\widetilde{R}_B^*] - qR_B^f\right)\right]\left[\frac{\mathbb{E}[\widetilde{R}_B^*] - qR_B^f}{1-q} - 1\right] \tag{A8}$$
$$+qU^{'}\left[(1-f(B^*)) + f(B^*)R_B^f\right][R_B^f - 1] = 0$$

Since R_B^f decreases with B^*,to show that $f(B^*)$ *declines* with B^* it suffices to show that $f(B^*)$ *increases* with R_B^f. By the implicit function theorem, it is sufficient to show that the derivative of the expression on the L.H.S. above with respect to R_B^f is strictly positive. The derivative is

$$-qU^{'}\left[(1-f(B^*)) + \frac{f(B^*)}{1-q}\left(\mathbb{E}[\widetilde{R}_B^*] - qR_B^f\right)\right] + qU^{'}\left[(1-f(B^*)) + f(B^*)R_B^f\right]$$
$$-qU^{"}\left[(1-f(B^*)) + \frac{f(B^*)}{1-q}\left(\mathbb{E}[\widetilde{R}_B^*] - qR_B^f\right)\right] f(B^*)\left[\frac{\mathbb{E}[\widetilde{R}_B^*] - qR_B^f}{1-q} - 1\right] \tag{A9}$$
$$+qU^{"}\left[(1-f(B^*)) + f(B^*)R_B^f\right] f(B^*)[R_B^f - 1]$$

We note that, because $R_B^s = \frac{\mathbb{E}[\widetilde{R}_B^*] - qR_B^f}{1-q} > R_B^f$, and $U^{'}$ is strictly decreasing by the strict concavity of U,

$$-qU^{'}\left[(1-f(B^*)) + \frac{f(B^*)}{1-q}\left(\mathbb{E}[\widetilde{R}_B^*] - qR_B^f\right)\right] + qU^{'}\left[(1-f(B^*)) + f(B^*)R_B^f\right] > 0.$$

We showed earlier that $R_B^f < 1$ for $B^* \in (B_0, 1]$. Since $\mathbb{E}[\widetilde{R}_B^*] > 1$, we clearly must have $R_B^s > 1$. It then follows that, as $U^{"} < 0$, the third and fourth expressions in (A9) are both positive. Hence, $f(B^*)$ increases with R_B^f so that $f(B^*)$ decreases with B^*.

Suppose now that $f(B^*)$ takes boundary values. First, suppose that $f(B_0^*) = 0$ for some $B_0^* \in [0,1]$. Then the above analysis implies that $f(B^*) = 0$ for $B^* > B_0^*$ as $f(.)$ is strictly decreasing on intervals of B^* where $B'(.) \in (0,1)$. Similarly, if $f(B_1^*) = 1$ for some $B_1^* \in (0,1)$,then $f(B^*) = 1$ for $B^* < B_1^*$.

From the above, we conclude that the fixed point problem (A6) can now be expressed as $B^* = f(B^*)$, where the L.H.S. obviously increases with B^*, while the R.H.S.

decreases monotonically with B^*. We now show that the two curves intersect in the interval $(B_0, 1]$, which is the unique solution of (A6).

For $B^* = B_0$, $R_B^f = \frac{\alpha\Lambda(K+B^*)}{B^*} = 1$, since $\alpha\Lambda(K + B_0) = B_0$ by definition. This implies that bondholders are guaranteed a return of at least the risk-free return even when the representative firm fails when $B^* = B_0$. Since $\mathbb{E}[\widetilde{R}_B^*] > 1$, it follows that bondholders optimally invest all their capital in bonds so that $f(B_0) = 1$. Hence, $f(B_0) > B_0$. Clearly, $f(1) \leq 1$ as $f(B^*) \in [0, 1]$. It then follows that the fixed point problem, (A6), has a unique solution $B^* \in (B_0, 1]$.

Finally, the solution of the fixed point problem expressed as a function of $\mathbb{E}[\widetilde{R}_B^*]$ is monotonically increasing if bondholders' demand function g defined by the R.H.S. of (A6) increases with R_B^{*s}. The reason is that, keeping B^* fixed, the R.H.S. of (A4) increases with R_B^{*s}, while the L.H.S. is clearly unchanged.

Step 3. We now show that, for $K < K_1$, there can be no equilibrium in which the representative firm is solvent when its technology fails. Suppose, to the contrary, that there exists an equilibrium in which the representative firm does not default. The maximum possible investment in the representative firm in any equilibrium must be X as the expected marginal return from each firm's technology is strictly less than the risk-free return when the investment exceeds X. Hence, for debt to be risk-free, the maximum debt level must be $\alpha\Lambda(X)$, which is less than 1 by (12). Because $K < K_1$, $K + B^* < K_1 + \alpha\Lambda(X) = X$. It then follows from (11) that the expected marginal return from the representative firm's technology exceeds the risk-free return of 1. Hence, by Proposition 1, the expected debt return exceeds the risk-free return. Since debt is risk-free, it is optimal for bondholders to invest their entire capital in firms, which contradicts the conclusion above that the maximum debt level must be $\alpha\Lambda(X) < 1$.

Step 4. We now establish the existence of a default equilibrium for $K < K_1$. Relation (A5) is a decreasing relation between $\mathbb{E}[\widetilde{R}_B^*]$ and B^* that we denote as $B^* = M(\mathbb{E}[\widetilde{R}_B^*])$ where $M(.)$ is a decreasing function. By the preceding analysis, relation (A6) defines a second relation between B^* and $\mathbb{E}[\widetilde{R}_B^*]$ that we denote as $B^* = N(\mathbb{E}[\widetilde{R}_B^*])$. $N(.)$ is an increasing function if bondholders' demand function g defined by (8) increases in its first argument.

Looking at (A5), if $\mathbb{E}[\widetilde{R}_B^*] = 1$,then $(1 - q)\Lambda'(K + B^*) + q\alpha\Lambda'(K + B^*) = 1$ by (A3). It then follows from (11) that we must have $B^* = X - K > 0$ as $K < K_1 < X$. Hence, $M(1) > 0$. Looking at relation (A6), when $\mathbb{E}[\widetilde{R}_B^*] \to 1$,the debt level that solves the fixed point problem tends to zero. This is because, when the expected debt return equals the risk-free return, but debt is risky, risk-averse bondholders optimally invest no capital in firms. Hence, $N(1) = 0$. Therefore, $M(1) > N(1)$. When $B^* = 0$ in (A5), $\mathbb{E}[\widetilde{R}_B^*] = (1 - q)\Lambda'(K) + q\alpha\Lambda'(K) = \Delta > 1$ so that $M(\Delta) = 0$. However, looking at (A6), $N(\Delta) > 0$ because bondholders invest nonzero capital in firm bonds when the expected debt return exceeds one. Therefore, $M(\Delta) < N(\Delta)$. Hence, the graphs of the functions $M(.)$ and $N(.)$ intersect at least once in the interval $(1, \Delta)$. It follows that there exists $\mathbb{E}[\widetilde{R}_B^*] \in (1, \Delta)$ such that $M(\mathbb{E}[\widetilde{R}_B^*]) = N(\mathbb{E}[\widetilde{R}_B^*])$. This intersection point determines an equilibrium. Moreover, the intersection is unique if $N(.)$ is increasing, which follows if bondholders' demand function $g(.)$ is increasing in its first argument.

Step 5. Suppose now that $K \geq K_1 = X - \alpha\Lambda(X)$. We see that any $B^* \in [0, \alpha\Lambda(X)]$ determines an equilibrium in which firms do not default. Indeed, if we set $E^* = X - B^*$, the total capital raised by each firm is X so that the expected marginal return from the representative firm's technology, its expected debt and equity return all equal the risk-free return. Therefore, debtholders and equityholders are indifferent between investing in firms and the risk-free asset. Hence, any $B^* \in [0, \alpha\Lambda(X)]$,indeed, determines a default-free equilibrium. Q.E.D.

Proof of Theorem 2

By Proposition 1, the expected returns on each firm's debt and equity are equal to the expected marginal return from its technology. By Theorem 1, we therefore have

$$\Delta_E = K\left((1-q) + q\alpha\right)\Lambda'\left(K + B^*\right) \tag{A10}$$
$$\Delta_F = \left((1-q) + q\alpha\right)\left[\Lambda\left(K + B^*\right) - \left(K + B^*\right)\Lambda'\left(K + B^*\right)\right] \tag{A11}$$

To simplify the problem, set

$$\Delta_E + \Delta_F = \Delta$$
$$E^i + F^i = N^i; i \in \{H, L\}$$

Consider the following planning problem.

$$\max_{\{\beta, B^L, B^H, N^L, N^L\}} qU(B^L) + (1-q)U(B^H) \text{ subject to} \tag{A12}$$
$$B^H + N^H \leq (1+K)(1-\beta) + M^H(\beta) \tag{A13}$$
$$B^L + N^L \leq (1+K)(1-\beta) + M^L(\beta), \tag{A14}$$
$$qN^L + (1-q)N^H \geq \Delta \tag{A15}$$

Given a solution to (A12), we can construct a solution to (22) by using a transfer between entrepreneurs and equity holders. Limited liability and an increasing bondholder utility function together imply that we can replace (A12) with the following problem:

$$\max_{\{\beta,B^L,B^H,N^L,N^L\}} qU(B^L)+(1-q)U(B^H) \text{ subject to}$$

$$B^L \leq (1+K)(1-\beta)+M^L(\beta)=A_1(\beta) \quad \text{(A16)}$$

$$B^H \leq (1+K)(1-\beta)+M^H(\beta)=A_2(\beta) \quad \text{(A17)}$$

$$qB^L+(1-q)B^H \leq qA_1(\beta)+(1-q)A_2(\beta)-\Delta \quad \text{(A18)}$$

$$=(1+K)(1-\beta)+qM^L(\beta)+(1-q)M^H(\beta)-\Delta=A_0(\beta)$$

At the optimum, (A18) must clearly be binding. Hence there are two cases:

Case 1: Inequality (A18) is binding, but inequality (A16) is not binding. In this case, since bondholders are risk averse, we have

$$B^H=B^L=\max_{\beta\in[0,1]}(1+K)(1-\beta)+qM^L(\beta)+(1-q)M^H(\beta)-\Delta=\max_{\beta\in[0,1]}A_0(\beta)=A_0(\beta_{\text{eff}}) \quad \text{(A19)}$$

By (20) and (21), the efficient proportion of capital invested in firms is given by (A21). This is, indeed, the optimum iff there is enough total capital in the low aggregate state to pay bondholders A_0. Therefore,

$$A_0(\beta_{\text{eff}}) \leq A_1(\beta_{\text{eff}})$$

which is equivalent to $\tau \leq \tau_1$ where

$$\tau_1=\frac{\Delta_E+\Delta_F}{(1-q)(1-\alpha)\Lambda(\beta_{\text{eff}}(1+K))}, \quad \text{(A20)}$$

with

$$\beta_{\text{eff}}=\min\left(\frac{X}{1+K},1\right). \quad \text{(A21)}$$

If $K+1<X$, $\beta_{\text{eff}}=1$, that is, all the capital in the economy is invested in firms. It then follows from (11), that the expected marginal return of each firm's technology strictly exceeds the risk-free return. If $K+1\geq X$, then it follows from (A21) that the expected marginal return of each firm's technology equals the risk-free return.

Case 2: Inequality (A18) and (A16) are both binding. In this case we have:

$$B^H=\frac{A_0(\beta)}{1-q}-\frac{qB^L}{1-q} \quad \text{(A22)}$$

The planning problem reduces to:

$$\max_{\beta}[(1-q)U(\frac{A_0(\beta)}{1-q}-\frac{qB^L}{1-q})+qU(B^L)] \quad \text{(A23)}$$

In order to prove that $\beta_{\text{eff}}(\tau)$ is decreasing with τ, it suffices to consider the case where $\beta_{\text{eff}}(\tau) \in (0,1)$ so that the first and second order conditions for the optimization problem (A23) are necessary and sufficient. To show that $\beta_{\text{eff}}(\tau)$ decreases with τ, it is enough to show that the cross derivative of the objective function (A23) is negative by the implicit function theorem because its second derivative with respect to β evaluated at $\beta_{\text{eff}}(\tau)$ is negative by the second order condition for the optimization problem (A23). The cross derivative is:

$$\frac{\partial^2}{\partial\beta\partial\tau}[(1-q)U(B^H) + qU(B^L)] =$$
$$q(1-q)(1-\alpha)\Lambda'(\beta_{\text{eff}}(1+K))(U'(B^H) - U'(B^L)) +$$
$$q(1-q)(1-\alpha)\Lambda(\beta_{\text{eff}}(1+K))[(\gamma_H\Lambda'(\beta_{\text{eff}}(1+K)) - 1)U''(B^H) - (\gamma_L\Lambda'(\beta_{\text{eff}}(1+K)) - 1)U''(B^L)]$$

where

$$\gamma_H = \omega^H + (1-\omega^H)\alpha; \gamma_L = \omega^L + (1-\omega^L)\alpha$$

Concavity of the utility function U implies that the first term is negative since $U'(B^H) < U'(B^L)$. Hence, it is enough to show the last term is negative as well. It suffices to show the following:

$$(\gamma_H\Lambda'(\beta_{\text{eff}}(1+K)) - 1)U''(B^H) - (\gamma_L\Lambda'(\beta_{\text{eff}}(1+K)) - 1)U''(B^L) < 0$$

Since $U'' < 0$, it is enough to show that

$$A^H = \gamma_H\Lambda'(\beta_{\text{eff}}(1+K)) - 1 \geq 0$$
$$A^L = \gamma_L\Lambda'(\beta_{\text{eff}}(1+K)) - 1 \leq 0$$

The first order condition for β_{eff} in the planning problem is

$$(1-q)A^H U'(B^H) + qA^L U'(B^L) = 0.$$

Since $U'(B^H), U'(B^L) > 0$, A^H and A^L must have opposite signs. Note that:

$$A^H - A^L = (\gamma_H - \gamma_L)\Lambda'(\beta_{\text{eff}}(1+K)) = (\tau + (1-\tau)\alpha)\,\Lambda'(\beta_{\text{eff}}(1+K)) > 0,$$

where the last equality follows from (3). Hence $A^H > 0 > A^L$.

If $K+1 < X$, then $\beta_{\text{eff}}(\tau_1) = 1$ by the analysis of Case 1, and the expected marginal return of each firm when $\tau = \tau_1$ exceeds the risk-free return. Since the capital invested in firms weakly decreases with τ, it follows that the expected marginal return of each firm exceeds the risk-free return for $\tau > \tau_1$. If $K+1 \geq X$, then the expected marginal return of each firm equals the risk-free return when $\tau = \tau_1$. By the above analysis, the capital invested in firms strictly decreases with τ for $\tau > \tau_1$. Hence, the expected marginal return of each firm strictly exceeds the risk-free return for $\tau > \tau_1$.

We next show that the expected return to bondholders must exceed the risk-free return in the efficient allocation. Indeed, by Part 1 a) of Theorem 1, the expected bond return strictly exceeds the risk-free return in the equilibrium of the basic economy. Since the social planner maximizes bondholders' expected utility, while maintaining the expected payoffs of equityholders and entrepreneurs at their equilibrium levels in the basic economy, the expected bond return must exceed the risk-free return in the efficient allocation.

If $\tau = 1$, it follows from (2) and (3) that $\omega^H = 1, \omega^L = 0$ and all firm shocks are perfectly correlated. In this case, the lack of diversification of bondholders is costless so that the competitive equilibrium of the basic economy is Pareto efficient. Indeed, the equilibrium condition (15) along with the perfect correlation of firm shocks implies that bondholders receive the entire output of the economy in the low aggregate state as in the efficient allocation, that is, constraint (A16) in the optimization program that determines the efficient allocation is binding. Constraint (A18) is also satisfied with equality by the competitive equilibrium allocation. Finally, when $\tau = 1$, the competitive equilibrium allocation maximizes bondholders' expected utility subject to the above constraints and, therefore, coincides with the efficient allocation. In particular, therefore, the efficient level of investment coincides with the investment level in the competitive equilibrium. Because the efficient investment level is weakly decreasing with τ by the preceding analysis, and is strictly greater than the competitive equilibrium level for $\tau < \tau_1$, it follows that the basic economy underinvests in firms relative to the efficient allocation for $\tau < 1$ with the investment levels being equal for $\tau = 1$.

Finally, we show that the expected payoff to bondholders declines with the aggregate risk for $\tau > \tau_1$. Indeed, as (A18) is binding,

$$qB^L_{\text{eff}} + (1-q)B^H_{\text{eff}} = (1+K)\left(1-\beta_{\text{eff}}\right) + qM^L(\beta_{\text{eff}}) + (1-q)M^H(\beta_{\text{eff}}) - \Delta = A_0(\beta_{\text{eff}}) \tag{A24}$$

As β_{eff} strictly decreases with τ, the total expected output of the economy,

$$(1+K)\left(1-\beta_{\text{eff}}\right) + qM^L(\beta_{\text{eff}}) + (1-q)M^H(\beta_{\text{eff}})$$

strictly decreases so that the expected payoff to bondholders, $A_0(\beta_{\text{eff}})$, also strictly decreases. Q.E.D.

Proof of Proposition 2

The proof is very similar to that of Proposition 1 with the only additional consideration being the trading of CDS contracts. The linearity of CDS sellers' optimization problem, (29), implies that we must have

$$\max\left(\sup_{n\in[0,1]} \mathbb{E}\left[\widetilde{R}_{E,n}\right], 1\right) = \inf_{m\in[0,1]} \mathbb{E}\left[\widetilde{R}_{CDS,m}\right]$$

The above simply expresses the observation that each CDS issuer must make zero expected profits from CDS contracts to avoid a profitable deviation where the issuer sells an infinite number of CDS contracts (if the expected profit is positive) or zero contracts (if the expected profit is zero). Hence, the expected return on the assets of an equityholder/CDS seller must equal the expected marginal cost of CDS issuance so that CDS sellers are indifferent to the amount of CDS they issue in equilibrium. The amount of CDS issued, therefore, meets bondholders' demand for CDS in equilibrium. Further, the expected marginal cost of a CDS contract to CDS sellers/equityholders must equal the expected return to CDS buyers. The expected equity and debt returns of all firms must be equal by the arguments in Proposition 1. The expected CDS returns must also be equal by similar arguments to ensure that the market for each firm's CDS clears. Q.E.D.

Proof of Theorem 3

By the first order conditions of bondholders' optimization program, (32) and (33), we obtain the following Euler equations for the bond and CDS returns:

$$\mathbb{E}\left[U'\left((1-B^{**}-C^{**})+x\widetilde{R}_B^{**}+y\widetilde{R}_{CDS}^{**}\right)\widetilde{R}_B^{**}\right]=\mathbb{E}\left[U'\left((1-B^{**}-C^{**})+x\widetilde{R}_B^{**}+y\widetilde{R}_{CDS}^{**}\right)\widetilde{R}_{CDS}^{**}\right].$$

Since $E[\widetilde{R}_B^{**}]=E\left[\widetilde{R}_{CDS}^{**}\right]$ by (31), and U is strictly concave, the above implies that $(1-B^{**}-C^{**})+x\widetilde{R}_B^{**}+y\widetilde{R}_{CDS}^{**}$ is, in fact, deterministic and does not vary across states, that is, the bondholder demands full insurance in equilibrium.

1. Suppose that $K+1<X$

We reproduce the equilibrium conditions derived in the discussion after the statement of the theorem for convenience.

$$B^{**}+C^{**}=1 \tag{A25}$$

$$B^{**}R_B^{s**}=B^{**}R_B^{f**}+C^{**}R_{CDS}^{**} \tag{A26}$$

$$B^{**}R_B^{f**}=\alpha\Lambda\left(K+1\right) \tag{A27}$$

$$\mathbb{E}\left[\widetilde{R}_B^{**}\right]=(1-q)R_B^{s**}+qR_B^{f**}=\mathbb{E}\left[\widetilde{R}^{**}\right]=(1-q+q\alpha)\Lambda'(K+1) \tag{A28}$$

$$\mathbb{E}\left[\widetilde{R}_{CDS}^{**}\right]=qR_{CDS}^{**}=\mathbb{E}\left[\widetilde{R}^{**}\right] \tag{A29}$$

We show that there is a unique solution to the above system of equations. First, by (A26), (A27), and (A29),

$$R_B^{s**}=\frac{\alpha\Lambda(K+1)}{B^{**}}+\left(\frac{1}{B^{**}}-1\right)\frac{E\left[\widetilde{R}^{**}\right]}{q} \tag{A30}$$

$$R_B^{f**}=\frac{\alpha\Lambda(K+1)}{B^{**}} \tag{A31}$$

Plugging the above in (A28), we have

$$(1-q)\frac{\alpha\Lambda(K+1)}{B^{**}} + (1-q)\left(\frac{1}{B^{**}}-1\right)\frac{E\left[\widetilde{R}^{**}\right]}{q} + q\frac{\alpha\Lambda(K+1)}{B^{**}} = E\left[\widetilde{R}^{**}\right],$$

which can be rewritten as

$$\frac{\alpha\Lambda(K+1)}{B^{**}} + \frac{1-q}{q}\left(\frac{1}{B^{**}}-1\right)E\left[\widetilde{R}^{**}\right] = E\left[\widetilde{R}^{**}\right].$$

We can solve the above to obtain

$$\left[\alpha\Lambda(K+1) + \frac{1-q}{q}E\left[\widetilde{R}^{**}\right]\right]\frac{1}{B^{**}} = \frac{E\left[\widetilde{R}^{**}\right]}{q}$$

so

$$B^{**} = \frac{\left[q\alpha\Lambda(K+1) + (1-q)E\left[\widetilde{R}^{**}\right]\right]}{E\left[\widetilde{R}^{**}\right]}$$

As $\alpha\Lambda(K+1) < E\left[\widetilde{R}^{**}\right]$,we see that $0 < B^{**} < 1$. Plugging this into (A30), (A31) and (A29), we see that there is a unique solution to the system of equations (A25)-(A29). We make the important observation that the solution does not depend at all on the aggregate risk, τ. Hence, the vector of candidate equilibrium variables, $\left\{B^{**}, C^{**}, \widetilde{R}_B^{**}, \widetilde{R}_{CDS}^{**}, \widetilde{R}^{**}\right\}$ does not depend on τ.

The solution determines an equilibrium iff the representative CDS seller/equityholder is actually able to deliver the CDS return, R_{CDS}^{**}. That is, the CDS seller's payoff in every state is sufficient to deliver the contractual payoff, $C^{**}R_{CDS}^{**}$ to the representative bondholder if his bonds default. As equityholders fully diversify their capital across firms, they are exposed only to aggregate risk. It is, therefore, necessary and sufficient that the representative equityholder's payoff in the low aggregate state is greater than or equal to the contractual CDS payoff, $C^{**}R_{CDS}^{**}$.

Let us now derive the equityholder's asset payoff in the low aggregate state. As a firm's assets accrue to its bondholders when it fails, the payoff to equityholders when a firm fails is zero. Hence, an equityholder obtains a positive payoff from a firm only when it succeeds, that is, $R_E^{s**} > 0, R_E^{f**} = 0$. Therefore,

$$\mathbb{E}\left[\widetilde{R}_E^{**}\right] = (1-q)R_E^{s**} = \mathbb{E}\left[\widetilde{R}^{**}\right] = (1-q+q\alpha)\Lambda'(K+1), \tag{A32}$$

where the second equality follows from (31). In the low aggregate state, a proportion ω^L of firms succeed. The representative equityholder's total payoff in the low aggregate state is, therefore,

$$\text{Equity Payoff in Low Aggregate State}=(K+C^{**})\omega^L R_E^{s**} = (K+C^{**})\frac{\omega^L}{1-q}\mathbb{E}\left[\widetilde{R}^{**}\right]$$
$$=(K+C^{**})(1-\tau)\mathbb{E}\left[\widetilde{R}^{**}\right]$$

where the last equality above follows from (2) and (3). A proportion $1-\omega^L$ of firms fail in the low aggregate state. Hence, the total CDS liability in the low aggregate state is $(1-\omega^L)C^{**}R_{CDS}^{**}$. It follows that the solution to (A26)-(A29) determines an equilibrium iff

$$(K+C^{**})(1-\tau)\mathbb{E}\left[\widetilde{R}^{**}\right] \geq (1-\omega^L)C^{**}R_{CDS}^{**} = (1-\omega^L)C^{**}\frac{\mathbb{E}\left[\widetilde{R}^{**}\right]}{q} = \frac{\tau+q(1-\tau)}{q}C^{**}\mathbb{E}\left[\widetilde{R}^{**}\right], \tag{A33}$$

where the first equality above follows from the fact that $qR_{CDS}^{**} = \mathbb{E}\left[\widetilde{R}^{**}\right]$ by (A29) and the second equality follows from (2) and (3). From the above, we see that a solution determines an equilibrium iff

$$(K+C^{**})(1-\tau) \geq \frac{\tau(1-q)+q}{q}C^{**}$$

The L.H.S. of the above strictly decreases with the aggregate risk, τ, at all, while the R.H.S. strictly increases with τ. Further, C^{**} does not depend on τ. The L.H.S. exceeds the R.H.S. for $\tau=0$, since $K+C^{**} > C^{**}$. The R.H.S. exceeds the L.H.S. for $\tau=1$ as the L.H.S is zero, while the R.H.S. is strictly positive. Hence, the above condition holds iff the aggregate risk, τ, is below a threshold, $\tau_2 \in (0,1)$.

2. Suppose that $K+1 \geq X$
   By (11),
$$(1-q)\Lambda^{'}(K+1) + q\alpha\Lambda^{'}(K+1) \leq 1. \tag{A34}$$

As all agents can invest in the risk-free asset, the total capital invested in firms in any equilibrium must be X so that the expected marginal return from the representative firm's technology, $\mathbb{E}\left[\widetilde{R}^{**}\right]$, equals the risk-free return. By (31), the expected returns on bonds, equity and CDS contracts equal the risk-free return. Bondholders invest in bonds and CDS contracts so that they are fully protected against firm default risk, but are now indifferent between investing in the risk-free asset and firms.

We argue that any equilibrium must feature default by the representative firm when its technology fails. Suppose, to the contrary that the representative firm does not default when it fails. We must then have $R_B^{s**} = R_B^{f**} = \mathbb{E}\left[\widetilde{R}_B^{**}\right] = 1$. Because the total capital raised by the firm must be X, its payoff upon failure is $\alpha\Lambda(X)$. Hence, the maximum debt capital that the firm can raise is $\alpha\Lambda(X)$. Further, because debt is risk-free, and it is optimal for bondholders to bear no risk, they invest no capital in CDS. Hence, CDS sellers/equityholders raise no capital via CDS issuance. It then follows that the maximum total capital that the representative firm can raise is $K + \alpha\Lambda(X) < X$ as $K < K_1 = X - \alpha\Lambda(X)$ by Assumption 1. However, this contradicts the fact that each firm raises capital X in any equilibrium.

If S^{**} denotes the investment in the risk-free asset by bondholders in any equilibrium, the equilibrium conditions are as follows.

$$B^{**} + C^{**} + S^{**} = 1 \tag{A35}$$

$$B^{**} R_B^{s**} = B^{**} R_B^{f**} + C^{**} R_{CDS}^{**} \tag{A36}$$
$$B^{**} R_B^{f**} = \alpha\Lambda\left(X\right) \tag{A37}$$
$$(1-q)R_B^{s**} + qR_B^{f**} = \mathbb{E}\left[\widetilde{R}^{**}\right] = 1 \tag{A38}$$
$$qR_{CDS}^{**} = \mathbb{E}\left[\widetilde{R}^{**}\right] = 1 \tag{A39}$$

The equilibrium conditions (A35)-(A39) are analogous to (A25)-(A29) except that bondholders also invest in the risk-free asset.

Equityholders are indifferent between investing in firms and the risk-free asset. The representative equityholder holds capital $K + C^{**}$. Suppose that the equityholders invests E^{**} in firms and $K + C^{**} - E^{**}$in the risk-free asset. The total investment by bondholders and equityholders in firms must be X, that is,

$$B^{**} + E^{**} = X \tag{A40}$$

Proceeding as in Part 1, we obtain the following equations for the bond returns upon firm success and failure.

$$R_B^{s**} = \frac{\alpha\Lambda(X)}{B^{**}} + \left(\frac{1 - S^{**}}{B^{**}}\right)\frac{1}{q} \tag{A41}$$
$$R_B^{f**} = \frac{\alpha\Lambda(X)}{B^{**}} \tag{A42}$$

Plugging the above in (A38), we have

$$(1-q)\frac{\alpha\Lambda(X)}{B^{**}} + (1-q)\left(\frac{1-S^{**}}{B^{**}} - 1\right)\frac{E\left[\widetilde{R}^{**}\right]}{q} + q\frac{\alpha\Lambda(X)}{B^{**}} = E\left[\widetilde{R}^{**}\right].$$

We can solve the above to obtain

$$\left[\alpha\Lambda(X) + \frac{1-q}{q}\left(1-S^{**}\right)E\left[\widetilde{R}^{**}\right]\right]\frac{1}{B^{**}} = \frac{E\left[\widetilde{R}^{**}\right]}{q}$$

so

$$B^{**} = \frac{\left[q\alpha\Lambda(X) + (1-q)\left(1-S^{**}\right)E\left[\widetilde{R}^{**}\right]\right]}{E\left[\widetilde{R}^{**}\right]} = q\alpha\Lambda(X) + (1-q)\left(1-S^{**}\right)$$

For each $S^{**} \in [0,1]$, $0 \leq B^{**} \leq$ '1. Hence, we have a continuum of solutions to the set of equations. A solution corresponds to an equilibrium iff $B^{**} + E^{**} = X$ and the default condition (A43) is satisfied. Because the total investment in firms is X, it follows from (11) that the expected marginal return on firms equals the risk-free return.

As in Part 1, a solution of the above system of equations determines an equilibrium iff equityholders can meet their CDS liabilities in the low aggregate state. By (A32), and the fact that $\mathbb{E}\left[\widetilde{R}^{**}\right] = 1$, we have $(1-q)R_E^{s**} = 1$. Hence, the representative equityholder's total payoff in the low aggregate state is now given by

$$\begin{aligned}\text{Equity Payoff in Low Aggregate State} =& \overbrace{E^{**}\omega^L R_E^{s**}}^{\text{payoff from investment in successful firms}} \\ &+ \overbrace{K + C^{**} - E^{**}}^{\text{payoff from investment in risk-free asset}} \\ =& E^{**}\frac{\omega^L}{1-q} + K + C^{**} - E^{**} \\ =& E^{**}(1-\tau) + K + C^{**} - E^{**} \\ =& K + C^{**} - \tau E^{**}\end{aligned}$$

The total CDS liability in the low aggregate state is

$$\begin{aligned}\text{Total CDS Liability in Low Aggregate State} =& (1-\omega^L)C^{**}R_{CDS}^{**} \\ =& [\tau + q(1-\tau)]C^{**}R_{CDS}^{**} \\ =& \frac{[\tau + q(1-\tau)]}{q}C^{**},\end{aligned}$$

where the second equality above follows from (2) and (3), and the third follows from the fact that q$R_{CDS}^{**} = \mathbb{E}\left[\widetilde{R}^{**}\right] = 1$ by (A39). We, therefore, have an equilibrium iff

$$K + C^{**} - \tau E^{**} \geq \frac{[\tau + q(1-\tau)]}{q}C^{**} \qquad \text{(A43)}$$

As in Case 1, the above condition is satisfied iff the aggregate risk τ is below a threshold, $\tau_3 \in (0, 1)$. Q.E.D.

Proof of Proposition 3

The symmetry of the equilibrium follows by arguments very similar to those used in the proof of Proposition 1. In equilibrium, the expected marginal return from each firm's technology must equal its expected marginal costs of financing to prevent arbitrage. Further, to preclude profitable deviations from the equilibrium financing strategy, the expected marginal costs of equity and debt financing must be equal. Each CDS issuer must make zero expected profits from CDS contracts to avoid a profitable deviation where the issuer sells an infinite number of CDS contracts (if the expected profit is positive) or zero contracts (if the expected profit is zero). Hence, the expected marginal cost of a CDS contract must equal the expected return on the CDS issuer's assets in equilibrium. Q.E.D.

Proof of Theorem 4

Step 1: Let us first consider the representative firm. By Case 1 of Theorem 2 the total capital invested in firms is $\beta_{\text{eff}}(1+K)$ In the efficient allocation. As this capital is invested uniformly in firms (whose mass is 1), the representative firm receives an investment of $\beta_{\text{eff}}(1+K)$. Hence, the firm's payoff upon failure/default is $\alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)$. Therefore, the expected marginal return from the representative firm's technology, which equals its expected marginal cost of debt and equity by (48), is

$$\mathbb{E}\left[\widetilde{R}^{***}\right] = \mathbb{E}\left[\widetilde{z}_E^{***}\right] = \mathbb{E}\left[\widetilde{z}_B^{***}\right] = \left[(1-q)+q\alpha\right]\Lambda'\left(\beta_{\text{eff}}(1+K)\right) \tag{A44}$$

By the absolute priority of debt, the entire payoff upon firm default is paid to bondholders. This, combined with the above pins down the marginal costs of debt and equity upon firm success and failure.

$$\begin{aligned} z_B^{f***} &= \alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right); z_B^{s***} = \frac{\mathbb{E}\left[\widetilde{R}^{***}\right] - qz_B^{f***}}{1-q} \\ z_E^{f***} &= 0; z_E^{s***} = \frac{\mathbb{E}\left[\widetilde{R}^{***}\right]}{1-q} \end{aligned} \tag{A45}$$

Step 2: Next, we consider the representative bondholder. By Case 1 of Theorem 2, bondholders bear no risk in the efficient allocation and $B_{\text{eff}}^H = B_{\text{eff}}^L \geq 1$. If $K+1 < X$, $B_{\text{eff}}^H = B_{\text{eff}}^L > 1$, that is, bondholders' return in the efficient allocation exceeds the risk-free return. Hence, in an efficient regulated equilibrium, bondholders invest all their capital in bonds and CDS contracts. If $K+1 \geq X$, then $B_{\text{eff}}^H = B_{\text{eff}}^L = 1$, that is, bondholders' return in the efficient allocation equals the risk-free return. In this case, bondholders are indifferent between investing in firms and the risk-free asset so that it is still weakly optimal for bondholders to invest all their capital in bonds and CDS. As we explain further below, we can construct an efficient regulated equilibrium by imposing a margin requirement on CDS sellers/equitholders that restricts their investment in firms to match the efficient allocation. Hence, regardless of whether $K+1<X$ or $K+1 \geq X$, we have

$$B^{***}+C^{***}=1 \tag{A46}$$

To ensure that the representative bondholder's payoffs match the efficient allocation, we must have

$$\begin{aligned} B^{***}R_B^{(H,s)***} &= B_{\text{eff}}^H; B^{***}R_B^{(H,f)***}+C^{***}R_{CDS}^{H***}=B_{\text{eff}}^H; \\ B^{***}R_B^{(L,s)***} &= B_{\text{eff}}^L; B^{***}R_B^{(L,f)***}+C^{***}R_{CDS}^{L***}=B_{\text{eff}}^L. \end{aligned} \tag{A47}$$

The two sets of equations express the fact that the bondholder's payoffs in the high (low) aggregate state when the representative firm succeeds or fails must be equal as the bondholder faces no idiosyncratic risk in the efficient allocation. Further, these payoffs match the efficient payoff levels.

The representative firm's payoff upon failure/default is $\alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)$, and is distributed entirely to bondholders. We impose no marginal taxes on firm payouts upon default in the implementation. Hence,

$$B^{***}R_B^{(H,f)***}=B^{***}R_B^{(L,f)***}=\alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right) \tag{A48}$$

Because $B_{\text{eff}}^H=B_{\text{eff}}^L$, we see from (A47) and (A48) that

$$R_B^{(H,s)***}=R_B^{(L,s)***}; R_B^{(H,f)***}=R_B^{(L,f)***}; R_{CDS}^{H***}=R_{CDS}^{L***} \tag{A49}$$

Finally, to ensure that bondholders buy full insurance, we must have

$$\mathbb{E}[\widetilde{R}_B^{***}]=E[\widetilde{R}_{CDS}^{***}] \tag{A50}$$

We see that B^{***} ,$R_B^{(H,s)***}, R_B^{(H,f)***}, R_{CDS}^{H***}$ are uniquely determined by (A47), (A48) and (A50). Indeed, we have

$$R_B^{(H,s)***}=\frac{B_{\text{eff}}^H}{B^{***}}; R_B^{(H,f)***}=\frac{\alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)}{B^{***}}; R_{CDS}^{H***}=\frac{B_{\text{eff}}^H-\alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)}{1-B^{***}}$$

Plugging the above in (A50), we have

$$(1-q)\frac{B_{\text{eff}}^H}{B^{***}}+q\frac{\alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)}{B^{***}}=q\frac{B_{\text{eff}}^H-\alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)}{1-B^{***}}.$$

The L.H.S. of the above is decreasing for $B^{***}\in(0,1)$, while the R.H.S. is increasing. Further, the L.H.S. is strictly greater than the R.H.S. for $B^{***}=0$, while the R.H.S.

is strictly greater for $B^{***} = 1$. Hence, there is a unique solution, $B^{***} \in (0, 1)$ to the above equation. By (A49), the bond and CDS returns are then uniquely pinned down.

Step 3: Finally, let us consider equityholders/CDS sellers. Equityholders hold capital, K, and raise capital, C^{***}, by selling CDS to bondholders. We need to consider two cases as described in Parts 1 a) and b) of Theorem 2.

Suppose first that $K+1 < X$ so that $\beta_{\text{eff}} = 1$, that is, all the capital in the economy is invested in firms in the efficient allocation. There is no capital invested in the risk-free asset, and the expected marginal return of each firm exceeds the risk free return. Hence, equityholders invest all their capital in firms in the regulated equilibrium so that $\mathbb{E}\left[\widetilde{R}_E^{***}\right] > 1$. Hence, there should be no margin requirement imposed on equityholders. By Proposition (49), the expected marginal cost of CDS contracts equals the expected equity return, that is, $\mathbb{E}[\widetilde{z}_{CDS}^{***}] = E\left[\widetilde{R}_E^{***}\right]$. Hence, the representative equityholder makes zero expected profit from selling CDS and investing the resulting additional capital she raises so that her expected payoff is $K\mathbb{E}\left[\widetilde{R}_E^{***}\right]$. As the expected payoff of the equityholder in the efficient allocation is Δ_E (given by (A10)), we must have

$$K\mathbb{E}\left[\widetilde{R}_E^{***}\right] = \Delta_E \tag{A51}$$

As there are no payouts to equity upon firm default, the above implies pins down the equity returns upon firm success.

$$R_E^{(H,s)***} = R_E^{(L,s)***} = \frac{\Delta_E}{(1-q)K}. \tag{A52}$$

Suppose now that $K+1 \geq X$ so that $\beta_{\text{eff}} < 1$. In the efficient allocation, capital X is invested in firms and $K+1-X$ in the risk-free asset. The expected marginal return from the representative firm's technology is the risk-free return. As discussed after the statement of the theorem, equityholders do not voluntarily invest in the risk-free asset in the regulated equilibrium. They must be forced to do so via a margin requirement, μ,which is chosen so that

$$\mu K = (1-\beta_{\text{eff}})(1+K). \tag{A53}$$

The above condition ensures that the total investment in firms by bondholders and equityholders equals $\beta_{\text{eff}}(1+K)$ as required by the efficient allocation. The expected equity return must be such that equityholders obtain the expected payoff, Δ_E. Hence,

$$K\left[\mu + (1-\mu)\mathbb{E}\left[\widetilde{R}_E^{***}\right]\right] = \Delta_E \tag{A54}$$

As the equity returns upon firm failure are zero, the above pins down the equity returns upon firm success.

$$R_E^{(H,s)***} = R_E^{(L,s)***} = \frac{\frac{\Delta_E}{K} - \mu}{(1-\mu)(1-q)} \tag{A55}$$

Step 4: Finally, let us consider the entrepreneurs. Their expected payoff equals their expected payoff in the efficient allocation, which is Δ_F (see (A11)). The lump sum tax on the representative entrepreneur, $\widetilde{T}_F$, is set such that their expected payoff in the regulated equilibrium equals Δ_F, that is,

$$\Delta_F = ((1-q) + q\alpha)) \left[\Lambda\left(\beta_{\text{eff}}(1+K)\right) - \left(\beta_{\text{eff}}(1+K)\right)\Lambda'\left(\left(\beta_{\text{eff}}(1+K)\right)\right)\right] - \mathbb{E}\left[\widetilde{T}_F\right] \tag{A56}$$

We can define the taxes so that they are incurred only when the representative firm succeeds. Therefore,

$$T_F^{(H,s)} = T_F^{(L,s)} = \frac{((1-q) + q\alpha)) \left[\Lambda\left(\beta_{\text{eff}}(1+K)\right) - \left(\beta_{\text{eff}}(1+K)\right)\Lambda'\left(\left(\beta_{\text{eff}}(1+K)\right)\right)\right] - \Delta_F}{1-q} \tag{A57}$$

Q.E.D.

Proof of Theorem 5

The expected marginal return from the representative firm's technology, and the expected marginal costs of debt and equity must still satisfy (A44). The marginal costs of debt and equity capital for the representative firm are given by (A45).

Now consider the representative bondholder. The representative bond and CDS returns satisfy (A47) and (A48). However, as bondholders now bear aggregate risk with $B_{\text{eff}}^L < B_{\text{eff}}^H$, it follows from (A47) that

$$R_B^{(H,s)***} > R_B^{(L,s)***}; R_B^{(H,f)***} = R_B^{(L,f)***}; R_{CDS}^{H***} > R_{CDS}^{L***} \tag{A58}$$

We construct an efficient regulated equilibrium in which bondholders invest all their capital in bonds and CDS contracts. In this equilibrium, (A46) must hold and the representative bondholder's investment choices must satisfy the necessary and sufficient first order conditions of the bondholder's optimization program described by (44) and (45). That is,

$$\mathbb{E}\left[\left(\widetilde{R}_B^{***} - \widetilde{R}_{CDS}^{***}\right)\frac{\partial U}{\partial x}\left(x\widetilde{R}_B^{***} + (1-x)\widetilde{R}_{CDS}^{***}\right)|_{x=B^{***}}\right] = 0 \tag{A59}$$

Equations (A46), (A47), (A48) and (A59) together form a system of eight equations in eight unknowns:

$$\left(B^{***}, C^{***}, R_B^{(H,s)***}, R_B^{(H,f)***}, R_B^{(L,s)***}, R_B^{(L,f)***}, R_{CDS}^{H***}, R_{CDS}^{L***}\right)$$

Indeed, we have

$$R_B^{(H,s)***} = \frac{B_{\text{eff}}^H}{B^{***}}; R_B^{(H,f)***} = \frac{\alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)}{B^{***}}; R_{CDS}^{H***} = \frac{B_{\text{eff}}^H - \alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)}{1-B^{***}}$$
$$R_B^{(L,s)***} = \frac{B_{\text{eff}}^L}{B^{***}}; R_B^{(L,f)***} = \frac{\alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)}{B^{***}}; R_{CDS}^{L***} = \frac{B_{\text{eff}}^L - \alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right)}{1-B^{***}} \tag{A60}$$

We can rewrite (A59) as

$$(1-q)\omega^H\left(R_B^{H,s***}\right)U'(B_{\text{eff}}^H) + (1-q)\left(1-\omega^H\right)\left(R_B^{H,f***} - R_{CDS}^{H***}\right)U'(B_{\text{eff}}^H) +$$
$$q\omega^L\left(R_B^{L,s***}\right)U'(B_{\text{eff}}^L) + q\left(1-\omega^L\right)\left(R_B^{L,f***} - R_{CDS}^{L***}\right)U'(B_{\text{eff}}^L) = 0$$

Re-arranging the above. we obtain

$$(1-q)\omega^H\left(R_B^{H,s***}\right)U'(B_{\text{eff}}^H) + (1-q)\left(1-\omega^H\right)\left(R_B^{H,f***} - R_{CDS}^{H***}\right)U'(B_{\text{eff}}^H) +$$
$$q\omega^L\left(R_B^{L,s***}\right)U'(B_{\text{eff}}^L) + q\left(1-\omega^L\right)\left(R_B^{L,f***} - R_{CDS}^{L***}\right)U'(B_{\text{eff}}^L) =$$
$$(1-q)\left(1-\omega^H\right)R_{CDS}^{H***}U'(B_{\text{eff}}^H) + q\left(1-\omega^L\right)R_{CDS}^{L***}U'(B_{\text{eff}}^L)$$

By (A60), the L.H.S. above decreases, while the R.H.S. increases for $B^{***} \in (0,1)$. The L.H.S. tends to ∞ as $B^{***} \to 0$, while the R.H.S. is strictly positive by substituting (A60) in the above. Hence, the L.H.S. is strictly greater than the R.H.S. for $B^{***} = 0$. For $B^{***} \to 1$, the L.H.S. tends to $-\infty$, while the R.H.S. tends to ∞. The L.H.S. is, therefore, strictly less than the R.H.S. for $\boldsymbol{B^{***} \to 1}$. It then follows that there is a unique solution that determines the bondholder's investments in bonds and CDS contracts as well as bond and CDS returns.

Let us now turn to equityholders. Equityholders hold capital, K, and raise capital, C^{***}, by selling CDS to bondholders. The regulator must ensure that capital, $(1-\beta_{\text{eff}})(1+K)$ is invested in the risk-free asset. Hence, the margin requirement, μ, on CDS sellers (equityholders) must satisfy (A53). However, because μ cannot exceed one, this is only possible if $\frac{(1-\beta_{\text{eff}})(1+K)}{K} \leq 1$. By Theorem 2, the efficient investment proportion, β_{eff}, declines with the aggregate risk τ above the threshold, τ_1. Hence, in general, there exists a threshold $\overline{\tau}_1 \leq 1$ for which $\frac{(1-\beta_{\text{eff}})(1+K)}{K} = 1$. For $\tau \leq \overline{\tau}_1$, the efficient allocation can be implemented with taxes/subsidies and a margin requirement that satisfies (A53). The equity return must then satisfy (A54). Part 2 of Theorem 2 shows that equityholders should get no payoff in the low aggregate state in the efficient allocation. Further, equityholders get no payoff upon firm failure. Therefore,

$$R_E^{(H,f)***} = R_E^{(L,s)***} = R_E^{(L,f)***} = 0.$$

Equation (A54) then pins down the equity return upon firm success in the high aggregate state.

$$R_E^{(H,s)***} = \frac{\frac{\Delta_E}{K} - \mu}{(1-\mu)(1-q)\omega^H}. \tag{A61}$$

The expected lump sum tax on entrepreneurs is determined by (A56) as before. However, by Part 2 of Theorem 2, entrepreneurs should get no payoff in the low aggregate state in the efficient allocation. If we further specify that entrepreneurs get no payoffs when their firms fail, we obtain

$$T_F^{(H,s)***} = \frac{((1-q)+q\alpha))\left[\Lambda\left(\beta_{\text{eff}}(1+K)\right) - \left(\beta_{\text{eff}}(1+K)\right)\Lambda'\left(\left(\beta_{\text{eff}}(1+K)\right)\right)\right] - \Delta_F}{(1-q)\omega^H} \tag{A62}$$

Q.E.D.

Proof of Theorem 6

Suppose that $\overline{\tau}_1 < 1$ so that the margin requirement becomes maximal in the previous proposition. If $\tau > \overline{\tau}_1$, then the regulator can only implement the efficient level of investment in the risk-free asset by inducing bondholders to invest some of their capital in the risk-free asset. Equityholders must invest their initial capital, K, entirely in the risk-free asset so that $\mu = 1$. Hence, bondholders' investment in the risk-free asset, S^{***}, satisfies

$$(1-\beta_{\text{eff}})(1+K) = K + S^{***} \tag{A63}$$

The marginal costs of equity and debt financing for firms, the returns to equityholders and the lump sum taxes on entrepreneurs can be derived as in Theorem 5. The key difference here is the representative bondholder's problem.

The regulator cannot *directly* control bondholders' investment in the risk-free asset, but can indirectly induce this investment by constraining the supply of CDS contracts. Suppose the supply of CDS contracts is restricted so that the maximum capital that a bondholder can invest in CDS contracts is C^{***}. Then the bondholder solves the following optimization program.

$$\sup_{x,y\in[0,1]} \mathbb{E}\left[U\left((1-x-y) + x\widetilde{R}_B + y\widetilde{R}_{CDS}\right)\right] \text{ such that} \tag{A64}$$

$$x + y \leq 1 \tag{A65}$$

$$y \leq C^{***} \tag{A66}$$

As the constraint (A66) is binding, we have

$$B^{***} + S^{***} = 1 - C^{***} \tag{A67}$$

To ensure that the representative bondholder's payoffs match the efficient allocation, we must have

$$B^{***}R_B^{(H,s)***} + S^{***} = B^H_{\text{eff}};\, B^{***}R_B^{(H,f)***} + C^{***}R_{CDS}^{H***} + S^{***} = B^H_{\text{eff}};$$
$$B^{***}R_B^{(L,s)***} + S^{***} = B^L_{\text{eff}};\, B^{***}R_B^{(L,f)***} + C^{***}R_{CDS}^{L***} + S^{***} = B^L_{\text{eff}}.$$
$$B^{***}R_B^{(H,f)***} = B^{***}R_B^{(L,f)***} = \alpha\Lambda\left(\beta_{\text{eff}}(1+K)\right) \tag{A68}$$

The bondholder's bond investment satisfy the necessary and sufficient first order conditions of the program (A64).

$$B^{***} = \mathbb{E}\left[\left(\widetilde{R}_B^{***} - 1\right)\frac{\partial U}{\partial x}\left(x\widetilde{R}_B^{***} + (1 - C^{***} - x) + C^{***}\widetilde{R}_{CDS}^{***}\right)|_{x=B^{***}}\right] = 0 \tag{A69}$$

For S^{***} determined by (A63), equations (A67), (A68), and (A59) together form a system of eight equations in eight unknowns:

$$\left(B^{***}, C^{***}, R_B^{(H,s)***}, R_B^{(H,f)***}, R_B^{(L,s)***}, R_B^{(L,f)***}, R_{CDS}^{H***}, R_{CDS}^{L***}\right)$$

This system has a unique solution by arguments very similar to those used in the proof of Theorem 5. The regulator can implement the above outcome by setting the capital requirement, ν, on CDS sellers (see (41)) such that

$$C^{***} = \nu K \text{ or } \nu = \frac{C^{***}}{K}. \tag{A70}$$