ONE GLOBAL VILLAGE?
COMPETITION IN THE INTERNATIONAL ACTIVE FUND MANAGEMENT INDUSTRY

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Abstract. We introduce an international active fund management industry model in which competing managers, each having heterogeneous incentives (effort productivities, costs) for searching domestic versus foreign investment opportunities. In equilibrium, incentive heterogeneity leads to a novel prediction: increasing foreign competitiveness, which improves (worsens) domestic manager incentives, induces an increase (decrease) of both domestic performance and size. Empirically, we find that 30 global markets' performance and size, on average, decrease with U.S. concentration. This evidence is consistent with our theoretical predictions but is inconsistent with extrapolation of single-country (implying homogeneous incentives) equilibria to one "global village" [e.g., Feldman, Saxena, and Xu (2020)].

JEL Codes: G10, G15, G20, L10
Keywords: Active management, Mutual funds, Global fund markets, Global village, Effort, Performance, Market concentration, Competition, Herfindahl-Hirschman index, Industry size, Alpha

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1 Introduction

The competitive environment faced by fund managers plays a critical role in determining the performance, fees, and size in the active fund management industry (AFMI). Recent articles study, both theoretically and empirically, how scale, skill, and competitiveness influence U.S. AFMI outcomes (i.e., in a single dominant country[1]) [e.g., Pastor and Stambaugh (2012) (PS); Pastor, Stambaugh, and Taylor (2015); and Feldman, Saxena, and Xu (2020) (FSX)]. International evidence suggests that AFMI outcomes are heterogeneous across countries [e.g., Dyck, Lins, and Pomorski (2013); Khorana, Servaes, and Tufano (2005, 2009); Chan, Covrig, and Ng (2005); and Ferreira, Keswani, Miguel, and Ramos (2012, 2013); Ferreira, Matos, and Pires (2018)] and that there are significant cross-effects where foreign economies influence domestic fund manager outcomes [e.g., Goldstein and Pauzner (2004); Defond, Hu, Hung, and Li (2011); Jotikasthira, Lundblad, and Ramadorai (2012); Yu and Wahid (2014)]. This international evidence creates the need for theoretical models that provide economic insights into how foreign economies affect domestic AFMIs, especially for smaller economies that are likely to experience substantial spill-over effects from a dominant foreign country. In this paper, we study, theoretically and empirically, how foreign AFMI competitiveness influences the performance and size of a domestic AFMI.

Our two-country model has important features distinguishing it from the single-country setting (e.g., PS, FSX): fund managers in our model spend two types of effort exploring investment opportunities. One type targets investment opportunities in the domestic stock market, whereas the other type targets opportunities in the foreign stock market. We allow the productivity of efforts for finding mispriced assets, and the costs of these efforts, to be heterogeneous for the domestic and foreign countries.[2,3]

Other features of our model follow PS and FSX: fund managers, competing for investment funds, maximize expected net (of management fees) alphas by choosing management fees and costly effort levels. Infinitely many mean-variance risk-averse investors maximize their portfolios Sharpe ratios, by allocating their wealth across domestic active funds and a passive benchmark portfolio (here, the benchmark includes both domestic and foreign

[1] Dominant in the sense that it is little influenced by other countries' AFMIs.
[2] For example, the costs and productivity of finding alpha in Sydney, Australia, or Silicon Valley are likely to be heterogenous for an Australian active manager.
[3] We do not extend the model to include more than two countries as this unnecessarily increases the complexity of the model without adding to its intuition: a three-country model can be represented as a two-country model with a domestic country and an aggregation of the two foreign countries as a single foreign country.

stocks).[4,5] Alpha production is subject to decreasing returns to scale at the industry level, as in PS and FSX, and at fund levels, as in Berk and Green (2004), FSX, and Feldman and Xu (2020).

Studying how changes in the competitive environment in a foreign AFMI (FAFMI) influence outcomes in a domestic AFMI (DAFMI), we introduce a model of DAFMI/FAFMI equilibrium with endogenous performance, size, fees, and managerial effort under a continuum of DAFMI and FAFMI concentration levels.[6] Our model analyzes how and by which mechanisms FAFMI concentration affects DAFMI. By allowing real-world heterogeneities to influence fund managers' decisions on how to allocate their efforts between domestic and foreign stock markets, we obtain novel predictions. All else being equal, a change in the concentration of one AFMI induces effort productivities (for finding mispriced opportunities) and effort costs to change, incentivizing effort re-allocation across the domestic and foreign stock markets.

As is the case in the real world, DAFMI/FAFMI concentration levels (competitiveness) affect alpha production functions and cost functions. Managers' competition for investment funds and investors' portfolio choices determine equilibrium fund sizes, performance, and direct benefits.[7] Our main theoretical findings are that an increase in FAFMI concentration changes[8] both DAFMI performance and size in the same direction. In this setting, equilibrium expected net alpha and size depend on a key quantity: the *direct benefits*, for DAFMI, defined as the sum over domestic and foreign stock markets of gross alphas produced by managers' efforts, minus the sum of the costs of these efforts. Further, the direction of the change of equilibrium expected net alpha and size depends on the direction of change of direct benefits. Precisely, if and only if (holding DAFMI concentration unchanged) higher FAFMI concentration induces higher (lower) DAFMI direct benefits, then it induces higher (lower)

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[4] In the model, both foreign and domestic active funds aim to outperform the international passive benchmark consisting of foreign and domestic stocks. However, we do not allow investors to allocate directly to foreign active funds, reflecting real-world barriers, such as transaction and information costs, taxes, and biases, to invest in domestic funds. For example, we assume that a representative Australian retail investor does not convert AUD savings to USD then fulfill various regulatory and legal requirements before investing in a U.S. active fund. Instead, he or she invests in an Australian active fund that looks for investment opportunities in both U.S. and Australian markets.

[5] As we argue in Section 2, a case in which both countries' investors and active funds invest in both countries' active funds falls under the analysis of FSX.

[6] For brevity, we henceforth omit the word "levels" in the terms "concentration levels," "performance levels," "expected net alpha levels," "effort levels," "cost levels," "investment levels," etc.

[7] We prove the existence and uniqueness of this equilibrium below, see Proposition 0, its proof intuition, and its proof in the appendix. The underlying intuition is simple. Competing managers, who do not offer competitive expected net alphas to investors, who maximize their portfolios' Sharpe ratios, do not attract investments, therefore, cannot exist. Thus, in a Nash equilibrium, all mangers offer the same expected net alphas, and managers' skills determine the sizes of their wealth under management. Because of the optimization problems of managers and investors, each has a unique optimum; the Nash equilibrium is unique.

[8] Increase or decrease; both first and second order.

DAFMI fund expected net alphas and size.[9]

Intuitively, as competitiveness increases in FAFMI, there are lower incentives to exert effort to search for investment opportunities in the foreign stock market,[10] thereby increasing incentives and wealth available to DAFMI funds to search for investment opportunities in the domestic stock market. Consequently, these changes make it more difficult to find investment opportunities domestically. However, these changes could also decrease managers' salary levels and then decrease the effort costs of searching for investment opportunities. If the effect on the costs overwhelms the effect on the return from investments, then these changes increase DAFMI size and performance. Which of these effects dominates is an empirical question, depending of parameter values, a question that we answer in the empirical part of this paper.

Our analysis is related to that of FSX, though FSX does not address, theoretically or empirically, the question of how competitiveness in one market's AFMI affects other markets' AFMIs. While a single dominant market analysis is reasonable for the U.S., due to its relative magnitude, it is unlikely that a smaller market's AFMI, subject to substantial influence from foreign fund managers and investors, behaves in the same way, as if such international effects could be largely ignored. Alternatively, one could conceive a one global village[11] version of the single-country model of FSX, in which fund managers in all countries face the same effort productivities and costs for global investment opportunities.[12] In such a model, the decision of whether to spend more effort in finding investment opportunities in the domestic stock market than that effort spent in the foreign stock market is irrelevant. The relevant decision is only about how much total effort to spend, given the opportunities and costs of effort. Such a model is plausible and has distinct empirical predictions compared to our international model, in which effort heterogeneities play an important role in determining AFMIs outcomes.

Specifically, the one global village model (with no country-level heterogeneities) predicts that the performance and size of AFMIs in all countries will respond in the same direction to change in a global AFMI competitive environment. For example, if the dominant U.S. AFMI becomes more competitive, resulting in a decrease in incentives (productivities net

[9] We also provide second-order analysis. In equilibrium, concave DAFMI expected net alphas in FAFMI concentration imply concave DAFMI direct benefits in FAFMI concentration. In turn, concave DAFMI direct benefits in FAFMI concentration imply concave DAFMI size in FAFMI concentration. On the other hand, equilibrium convex DAFMI size in FAFMI concentration implies convex direct benefits in FAFMI concentration, and convex DAFMI fund expected net alphas in FAFMI concentration.

[10] Lower (higher) such incentives induce lower (higher) optimal effort allocated to find alphas.

[11] Thomas Friedman defines the global village as a world "tied together into a single globalized marketplace and village." [Poll (2012)].

[12] That is, a fund manager based in Sydney, Australia faces the same opportunities and costs in finding investment opportunities in Silicon Valley as in Sydney.

of costs) to exert effort in the U.S. stock market, it will also reduce incentives of foreign fund managers to find alphas there. In this one global village model, foreign active fund managers cannot have improved incentives to exert effort domestically when U.S. incentives decline. As a result, net alphas will decrease together globally, and investors will re-allocate wealth from all active funds to the global passive benchmark, resulting in a decrease in the global AFMI size.

In contrast, our model with international competition allows for DAFMI and FAFMI to respond in different directions when competitiveness in FAFMI increases. By modeling two types of effort for the two countries and their heterogeneous productivities and costs, our model allows for the re-allocation of optimal fund manager effort across countries. For example, consider the following scenario. As AFMI competitiveness increases in the U.S., there are higher costs and lower productivity for domestic active managers to search for investment opportunities in the U.S. stock market. However, domestic fund managers find better local opportunities to enhance productivity and reduce costs (e.g., they are able to better attract U.S. and domestic talent or to acquire U.S. knowledge and technology at more attractive prices, so they become more productive at finding investment opportunities domestically). This leads to an increase in the domestic AFMI's alpha and size in response to an increase in the U.S. AFMI competitiveness. The one global village model does not allow this scenario.

Our empirical evidence is inconsistent with a one global village interpretation of FSX, and it significantly supports a model with international competition, as does our model here. Using the Normalized-Herfindahl-Hirschman Index (NHHI) and other indices as concentration (competitive environment) measures, we study 30 active global equity AFMIs, considering each one as a domestic AFMI (DAFMI), and analyze their fund net alphas and size associations with both the domestic and the U.S. equity AFMI concentration, thus calling U.S. market the foreign AFMI (FAFMI). Pooling all the markets' data together and using multiple hypothesis tests, we find that domestic AFMI fund net alphas and sizes are, on average, both significantly negatively associated with the U.S. NHHI. Consistent with our theoretical results, DAFMI net alphas and sizes move, on average, in the same direction in response to changes in FAFMI concentration. Furthermore, we find that, on average, global (DAFMI) markets' sensitivity to U.S. (FAFMI) concentration is higher than to their own concentration. This suggests that the heterogeneous international effects we model are of first-order importance for markets outside the U.S.

We also empirically study all 30 individual pairs of the 30 DAFMIs, keeping FAFMI fixed to be that of the U.S. The effects for these pairs are expected to be heterogeneous across countries and depend on how U.S. concentration affects incentives (that is, effort productivities and costs) in various markets. We find that six (one) DAFMI markets' fund net alphas and size, on average, are both significantly negatively (positively) associated with the FAFMI NHHI, whereas nine DAFMI markets' fund net alphas and size both are insignificantly associated with the FAFMI NHHI. Only for 3 of the 30 DAFMI markets are the associations with FAFMI NHHI inconsistent with our theoretical prediction; that is, DAFMI alpha and size do not move in the same direction when FAFMI NHHI changes.[13] Furthermore, after we combine our sample's European DAFMI markets into one EU DAFMI market, we find that this EU DAFMI market's fund net alphas and size both are, on average, significantly negatively associated with the FAFMI NHHI.

Dyck, Lins, and Pomorski (2013) find that alphas are generally larger in emerging market equity, followed by non-U.S. developed markets, and are smallest in U.S. markets. These results are consistent with the cross-sectional notion that alphas are higher in less competitive markets. This result is consistent with our model but is distinct from our key empirical result: in contrast to a cross-sectional relation, we examine a spill over relation where *changes* in competitiveness of the U.S. markets influences the performance and size in *other* markets.

Our empirical results are robust to several empirical specifications. We use Pastor, Stambaugh, and Taylor's (2015) (PST) recursive demeaning estimator to address endogeneity, reverse causality, fund fixed effects, and omitted-variable-related issues when studying the FAFMI concentration–DAFMI net alpha relation.[14] We also use Zhu's (2018) estimator in studying this relation for a robustness check and find consistent results. We use vector auto-regression (VAR) techniques to account for simultaneity in determining DAFMI size and FAFMI concentration in our robustness checks and find consistent results with those that do not use the VAR techniques. We control for survival bias by using Morningstar Direct's global database, which contains both surviving and terminated funds. Our empirical results are also robust to the use of alternative estimation methods and concentration measures.

Our findings provide implications for fund managers, investors, and regulators. Our evidence suggests that it is important to understand how changes in the internal competitive environment of dominant countries influences the incentives for finding investment

[13] In Finland (Chile and Taiwan), fund net alpha is significantly negatively (positively) associated with FAFMI NHHI, but DAFMI size is significantly positively (negatively) associated with FAFMI NHHI.

[14] The recursive-demeaning estimator by Pastor, Stambaugh, and Taylor's (2015) also controls the effects that a first-difference approach can control.

opportunities in smaller economies. The findings also suggest that current low and decreasing concentration in the U.S. AFMI can benefit non-U.S. AFMIs. These are typically good times for smaller AFMIs to try to improve domestic effort productivity by acquiring talent and technology at more attractive rates. Policy changes or mergers in the U.S. AFMI that substantially increase its concentration are likely to negatively affect non-U.S. AFMIs by re-allocating incentives away from domestic markets toward finding investment opportunities in the U.S.

While we focus on AFMI concentration, our model provides a framework to analyze other international AFMI heterogeneities. For example, differences in regulation, capital gains taxes, and transaction costs can be modeled as heterogeneously influencing fund manager costs for finding investment opportunities. Differences in equity market development, noise trading levels, and sophistication of non-AFMI investors can be modeled as heterogeneously influencing fund manager productivity. These factors will have theoretical predictions for international differences in AFMI alpha and size. Our model can also be extended to allow for multiple domestic/foreign AFMIs as well as heterogeneous investors in foreign and domestic AFMIs with different levels of home bias, risk aversion, and background risks. Modeling such features can provide new testable implications for international differences in AFMI alpha and size. We leave these and other such extensions for future research.

Section 2 develops the theoretical model, Section 3 presents the empirical methods and results, and Section 4 concludes.

2 Theoretical Framework

We develop a theoretical framework for modeling the effects of DAFMI and FAFMI concentrations on DAFMI managerial efforts, fees, performance, size, and direct benefits. For simplicity, we consider a two-country international model, in which each country has an AFMI with competing fund managers who invest in stocks and infinitely many mean-variance risk-averse investors who allocate their wealth to a passive international benchmark portfolio and active funds.

Consider three possibilities of investment across the two countries. If each country's investors and AFMI managers invest in both countries' AFMIs and stocks, respectively, we can consider the two countries as one global village with one AFMI. FSX studies this case.

If each country's investors and AFMI managers can invest only in their own country, we have two separate AFMIs. Each country's AFMI is, again, modeled in FSX.

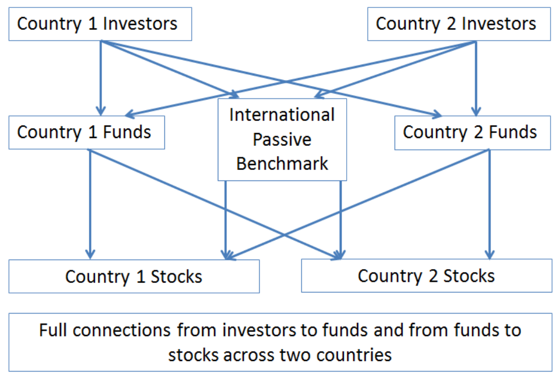
If, however, due to transaction and information costs, each country's investors invest only in DAFMI, whereas fund managers, facing lower transaction and information costs, invest in both countries' stocks, then a new DAFMI/FAFMI model is required. In such a model, fund managers compete domestically for wealth to manage, but both DAFMI and FAFMI concentrations affect managers' gross alpha production and effort costs. We introduce such a model in the following sections. We note that this model is applicable to more complex situations. For instance, a European country's AFMI might be closely related to some other European countries' AFMIs, and also be related to the U.S. AFMI. Then, we can regard these European countries' AFMIs, in aggregate, as one DAFMI and the U.S. AFMI as FAFMI. Then, we can use our model to study how the FAFMI (U.S.) concentration affects the DAFMI (these European markets in aggregate).

Figure 1 illustrates these three cross-country investment possibilities.

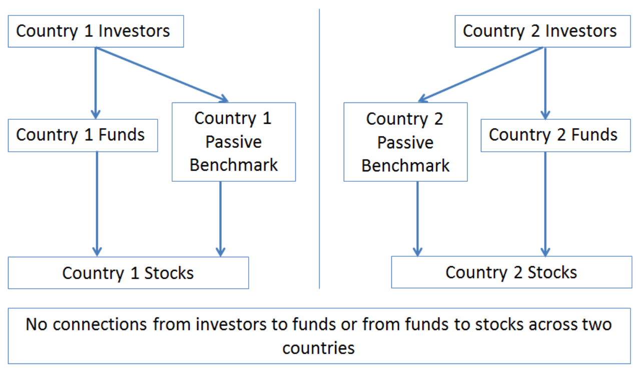
Figure 1 Three Cases of a Two-Country Model.

This figure shows the three cases of two-country models. In the first case, investors invest in both countries' AFMIs and managers invest in both countries' stocks. The two countries can be regarded as one AFMI "global village." In the second case, investors can invest only in DAFMI and fund managers can only invest in domestic stocks. The two countries' AFMIs are separate. In the third case, investors invest only in DAFMI, whereas fund managers invest in stocks of both countries. Each country's fund managers compete for domestic investments, but both DAFMI and FAFMI concentrations affect gross alpha production and effort costs in each country's AFMI.

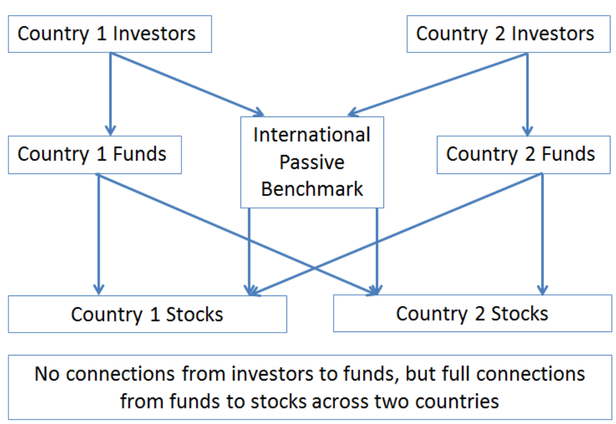
First Case



Second Case



Third Case – The Case in This Study



2.1 Setting

The economy consists of two countries, Country 1 and Country 2, and one period. We denote each Country k's, $k = 1,2$, parameters by superscripts. For simplicity and without loss of generality, we assume the countries' currency exchange rate is one.[15] Country k has two types of agents: M^k, $M^k > 1$, active fund managers, and N^k, $N^k \to \infty$, infinitely many investors. Fund managers in both countries are risk-neutral, invest in both countries' stocks, and maximize fund profits by optimally choosing proportional management fees and effort. Mean-variance risk-averse investors in both countries allocate their wealth between a passive international (including both domestic and foreign stocks) benchmark portfolio and domestic active funds (DAFMI), maximizing their portfolios' Sharpe ratios. All investors are small; thus, individual investors' do not affect fund sizes.

Due to the economy's internal symmetry with respect to Country 1 and 2, it is sufficient to focus on one country only. We denote Country 1 (2) as domestic (foreign), and its AFMI as DAFMI (FAFMI).

Fund Managers' Problem

Manager i in Country 1 observes alpha production functional forms and cost functional forms, their parameters, and domestic and foreign concentration levels, fund sizes, and fees. She maximizes her economic profits by allocating effort in each country, and fee rates, subject to nonnegative allocations and nonnegative profit rate. Mathematically,

$$\max_{e_i^{11}, e_i^{12}, f_i^1} s_i^1 [f_i^1 - C_i^1(e_i^{11}, e_i^{12};\ s_i^1, H^1, H^2)], \tag{1}$$

subject to

$$f_i^1 - C_i^1(e_i^{11}, e_i^{12}; s_i^1, H^1, H^2) \geq 0, \tag{2}$$

$$e_i^{11} \geq 0, \tag{3}$$

$$e_i^{12} \geq 0, \tag{4}$$

[15] Realistically, currency risk is likely to influence investor portfolios as their domestic and foreign consumption requires different numeraires. This risk induces a hedging demand. However, this hedging demand can be directly managed via currency derivatives and, thereby, separated from problems of choosing active versus passive equity management. Specifically, we assume that the passive benchmarks are hedged to local currencies; e.g. U.S. investors use their benchmark as MSCI World (hedged to USD) while Australian investors use MSCI World (hedged to AUD). In our setting, active equity managers do not have skills in predicting exchange rates, so they hedge implicit currency bets when investing internationally. Thus, alpha arises only from the selection or timing of equities, after hedging out currency risk. Our preferred way of modeling this fully hedged scenario is assuming that the exchange rate is one. This is a reasonable assumption that captures our model property that currency risks are orthogonal to the issues we study, keeping our model parsimonious.

$$f_i^1 \geq 0, \tag{5}$$

where s_i^1, f_i^1, e_i^{11}, e_i^{12}, and $C_i^1(e_i^{11}, e_i^{12};\ s_i^1, H^1, H^2)$ represent manager i's fund size, (nonnegative) proportional management fee, (nonnegative) effort spent in Country 1's stock market, (nonnegative) effort spent in Country 2's stock market,[16] and average (per dollar) cost function, where H^1 and H^2 represent Country 1 and Country 2 AFMI (DAFMI and FAFMI) concentrations, respectively. We define the domain of H^1 and H^2 as $[0, 1)$, where {0} represents a fully competitive market and {1} represents a monopolistic market.[17] Also, inequality (2) shows that manager i's profit rate should be nonnegative to survive.

Following FSX, we assume that the marginal diversification benefits of investing in an additional fund are trivial, such that managers compete for investments over net alphas. Manager i has to maximize her fund expected net alpha given fund size and AFMI concentrations. Thus, as in FSX, we can transform manager i's profit maximizing problem (1) to an equivalent problem of maximizing expected net alpha:

$$\max_{e_i^{11}, e_i^{12}, f_i^1} \mathrm{E}(\alpha_i^1 | D), \tag{6}$$

subject to constraints (2), (3), (4), and (5).

Proof. See the appendix.

The proof is similar to the one in FSX. Its intuition is as follows. Under competition, funds that offer higher expected net alphas draw (all) investments. The possibility (threat) that other managers increase fund profits by improving expected net alphas, and their fund sizes, forces managers to maximize expected net alphas to "survive." Thus, funds must offer similar expected net alphas, leading to a unique Nash equilibrium. We note that this aspect of the equilibrium is similar to that in PS; but in addition to their result, we show that it holds also in the case of a finite number of managers.

Manager i's average cost function has the following form.[18]

$$C_i^1(e_i^{11}, e_i^{12};\ s_i^1, H^1, H^2) = c_0^1 + c_{1,i}^1 s_i^1 + c_2^{11}(e_i^{11};\ H^1, H^2) \tag{7}$$

[16] We remind the reader that the first superscript designates the manager's country, and the second superscript designates the country where stock effort was directed.
[17] The open right boundary of the concentrations' domain implies that managers are competing.
[18] To simplify our model, we assume there is no interaction between effort and size in the average cost function because it is unlikely that fund size affects managers' per dollar efforts. We also assume that there is no interaction between concentration and size in the average cost function because it is unlikely that concentration affects managers' average cost sensitivities to fund size. Nevertheless, even if these interacting effects exist, they tend to be small in comparison to the effect of other terms in the average cost function.

$$+c_2^{12}(e_i^{12};\ H^1, H^2),$$

where c_0^1 and $c_{1,i}^1$ are constants and $c_2^{11}(e_i^{11};\ H^1, H^2)$ and $c_2^{12}(e_i^{12};\ H^1, H^2)$ are costs, conditional on countries' concentrations, due to e_i^{11} and e_i^{12} efforts directed at Country 1's stock market and Country 2's stock market, respectively. Each fund's operation cost is positive, so $c_0^1 > 0$. Also, we assume decreasing returns to scale at the fund level, so fund average cost increases with fund size, i.e., $c_{1,i}^1 > 0$. $c_2^{11}(e_i^{11};\ H^1, H^2)$ and $c_2^{12}(e_i^{12};\ H^1, H^2)$ have the following functional characteristics:

- nonnegative, i.e., $c_2^{11}(0;\ H^1, H^2) = 0$, $c_2^{12}(0;\ H^1, H^2) = 0$, $\forall\, H^1, H^2$; $c_2^{11}(e_i^{11};\ H^1, H^2) > 0$, $\forall\, e_i^{11} > 0, H^1, H^2$; and $c_2^{12}(e_i^{12};\ H^1, H^2) > 0$, $\forall\, e_i^{12} > 0, H^1, H^2$;
- increasing convex in effort, as we assume increasing marginal cost for each unit of effort, i.e., $c_{2\,e_i^{11}}^{11}(e_i^{11};\ H^1, H^2) > 0$, $c_{2\,e_i^{11},e_i^{11}}^{11}(e_i^{11};\ H^1, H^2) > 0$, $\forall\, e_i^{11}, H^1, H^2$, and $c_{2\,e_i^{12}}^{12}(e_i^{12};\ H^1, H^2) > 0$, $c_{2\,e_i^{12},e_i^{12}}^{12}(e_i^{12};\ H^1, H^2) > 0$, $\forall\, e_i^{12}, H^1, H^2$.

The different $c_{1,i}^1$'s across DAFMI funds imply differences in DAFMI fund-level decreasing returns to scale parameters, as $c_{1,i}^1$'s measure differences in the rate at which managers' costs in generating gross alpha increase with size. This setting of fund-level decreasing returns to scale is consistent with the empirical findings in the literature.[19] We now introduce two terms, an DAFMI individual manager skill and DAFMI aggregate skill.

DAFMI fund manager skill. In our model, ${c_{1,i}^1}^{-1}$ is the source of heterogeneous manager ability/skill. A more skilled manager is one who has lower total variable costs of active management for the same AUM and gross alpha.

Aggregate DAFMI skill. DAFMI aggregate skill is the sum of individual managers' skills, $\sum_{i=1}^{M^1}\left({c_{1,i}^1}^{-1}\right)$. In our model, DAFMI is more skilled when the sum of its mangers' skills is higher.

We show below that higher DAFMI aggregate skill corresponds to higher DAFMI size and that higher individual DAFMI fund managers' skill, relative to other managers, corresponds to a higher relative size of their fund. (See Proposition 1 and the discussion following Lemma 1.)

[19] Van Binsbergen, Kim, and Kim (2020, Panel A) document the heterogeneity of fund managers' decreasing returns to scale. Other empirical studies, such as Chen, Hong, Huang, and Kubik (2004) and Yan (2008), also show that different fund types experience different levels of decreasing returns to scale.

By spending efforts, manager i improves her fund net alpha. Manager i's net alpha has the following form.

$$\alpha_i^1 = a^1 - b^1 \frac{S^1}{W^1} + A^{11}(e_i^{11};\ H^1, H^2) + A^{12}(e_i^{12};\ H^1, H^2) - f_i^1, \tag{8}$$

where a^1 and b^1 are positive constants and b^1 is the industry-level decreasing returns to scale rate, with conditional mean and variance

$$\mathrm{E}\left(\begin{matrix} a^1 \\ b^1 \end{matrix}\middle| D\right) \triangleq \begin{pmatrix} \widehat{a^1} \\ \widehat{b^1} \end{pmatrix}, \quad \mathrm{Var}\left(\begin{matrix} a^1 \\ b^1 \end{matrix}\middle| D\right) \triangleq \begin{pmatrix} \sigma_{a^1}^2 & \sigma_{a^1 b^1} \\ \sigma_{a^1 b^1} & \sigma_{b^1}^2 \end{pmatrix}, \tag{9}$$

where D is in managers' and investors' information sets. Equation (8) is based on the alpha production structure in PS and FSX. The information structure in Definitions (9) follows PS. For simplicity, we assume $\sigma_{a^1 b^1} = 0$. Parameter W^1 is the country's total wealth, and S^1 is DAFMI size (controlled by investors). The gross alpha production functions $A^{11}(e_i^{11};\ H^1, H^2)$ and $A^{12}(e_i^{12};\ H^1, H^2)$ quantify the impact of e_i^{11} and e_i^{12}, respectively. $A^{11}(e_i^{11};\ H^1, H^2)$ and $A^{12}(e_i^{12};\ H^1, H^2)$ have the following functional characteristics:

- nonnegative, i.e., $A^{11}(0;\ H^1, H^2) = 0$, $A^{12}(0;\ H^1, H^2) = 0$, $\forall\, H^1, H^2$ and $A^{11}(e_i^{11};\ H^1, H^2) > 0, \forall\, e_i^{11} > 0, H^1, H^2$, $A^{12}(e_i^{12};\ H^1, H^2) > 0, \forall\, e_i^{12} > 0, H^1, H^2$;
- increasing concave in effort, as we assume marginal productivity of efforts is decreasing, i.e., $A_{e_i^{11}}^{11}(e_i^{11};\ H^1, H^2) > 0, A_{e_i^{11}, e_i^{11}}^{11}(e_i^{11};\ H^1, H^2) < 0, \forall\, e_i^{11}, H^1, H^2$ $A_{e_i^{12}}^{12}(e_i^{12};\ H^1, H^2) > 0, A_{e_i^{12}, e_i^{12}}^{12}(e_i^{12};\ H^1, H^2) < 0, \forall\, e_i^{12}, H^1, H^2$;
- $A^{11}(e_i^{11};\ H^1, H^2)$ increases with H^1 and has positive cross partial derivative with respect to H^1 and e_i^{11}, as higher H^1 implies more unexplored investment opportunities and higher efficiency in using fund industry resources in Country 1, i.e., $A_{H^1}^{11}(e_i^{11};\ H^1, H^2) > 0, A_{e_i^{11}, H^1}^{11}(e_i^{11};\ H^1, H^2) > 0, \forall\, e_i^{11} > 0, H^1, H^2$;
- $A^{11}(e_i^{11};\ H^1, H^2)$ increases with H^2 because a higher H^2 implies more unexplored opportunities in Country 2, diverting managerial efforts, leaving more unexplored opportunities in Country 1, and improving effort productivity in Country 1 as well. That is, $A_{H^2}^{11}(e_i^{11};\ H^1, H^2) > 0, A_{e_i^{11}, H^2}^{11}(e_i^{11};\ H^1, H^2) > 0, \forall\, e_i^{11}, H^1, H^2$;
- $A^{12}(e_i^{12};\ H^1, H^2)$ increases with H^2 and has positive cross partial derivative with respect to H^2 and e_i^{12}, as higher H^2 implies more unexplored investment opportunities in Country 2, i.e., $A_{H^2}^{12}(e_i^{12};\ H^1, H^2) > 0$, $A_{e_i^{12}, H^2}^{12}(e_i^{12};\ H^1, H^2) > 0$, $\forall\, e_i^{12} > 0, H^1, H^2$;

- $A^{12}(e_i^{12};\ H^1, H^2)$ increases with H^1 because a higher H^1 implies more unexplored opportunities in Country 1, diverting managerial efforts, leaving more unexplored opportunities in Country 2, and improving effort productivity in Country 2 as well, i.e., $A_{H^1}^{12}(e_i^{12};\ H^1, H^2) > 0,\ A_{e_i^{12},H^1}^{12}(e_i^{12};\ H^1, H^2) > 0, \forall\ e_i^{12}, H^1, H^2$.

From Equation (8) and Definitions (9), manager i's fund expected net alpha is[20]

$$\mathrm{E}(\alpha_i^1 | D) = \widehat{a^1} - \widehat{b^1}\frac{S^1}{W^1} + A^{11}(e_i^{11};\ H^1, H^2) + A^{12}(e_i^{12};\ H^1, H^2) - f_i^1. \tag{10}$$

We define the direct benefits of efforts exerted by DAFMI managers in the domestic stock market e_i^{11} and in the foreign stock market e_i^{12}, as follows:

$$B^{11}(e_i^{11};\ H^1, H^2) \triangleq A^{11}(e_i^{11};\ H^1, H^2) - c_2^{11}(e_i^{11};\ H^1, H^2), \tag{11}$$

$$B^{12}(e_i^{12};\ H^1, H^2) \triangleq A^{12}(e_i^{12};\ H^1, H^2) - c_2^{12}(e_i^{12};\ H^1, H^2). \tag{12}$$

These two terms are important for social planners and policy makers, as they capture the direct benefits of e_i^{11} and e_i^{12}, respectively, in terms of increase in gross alpha production minus the corresponding effort costs.

$B^{11}(e_i^{11};\ H^1, H^2)$ and $B^{12}(e_i^{12};\ H^1, H^2)$ capture the direct benefit from effort exerted in active fund management in terms of an increase in gross alpha production minus the effort cost. We interpret *benefits* generally, allowing them to be positive or negative.

Whether manager i's marginal direct benefits of initial effort in each county's stock market are positive [i.e., $B_{e_i^{11}}^{11}(0;\ H^1, H^2) > 0, B_{e_i^{12}}^{12}(0;\ H^1, H^2) > 0, \forall\ H^1, H^2$] is an important condition affecting the equilibrium. If this condition is not met, no effort is exerted, as in PS (see Proposition PS, Section 2.3 in FSX). Whether the sensitivity of manager i's direct benefits at optimal effort is positive or not [i.e., $\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1} > 0\ (\leq 0)$ and $\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2} > 0\ (\leq 0)$] is also an important condition affecting the

[20] Following FSX, investors observe the passive benchmark and the AFMI funds' returns. The difference between these returns comes from three components: net alphas, the common risk factor, and idiosyncratic risks. As the distributions of the common risk and idiosyncratic risk are common knowledge, investors know the likelihood functions of the net alphas. Given prior beliefs of net alphas, they form posteriors and update their beliefs. In our one-period model, there is no dynamic Bayesian updating, but we suggest that investors reach a fixed-point equilibrium. Further, because investors observe fees and fund sizes (thus, industry size), they can also infer the sum of $A^{11}(e_i^{11};\ H^1, H^2)$ and $A^{12}(e_i^{12};\ H^1, H^2)$. For simplicity and brevity, we depress the notation of $\hat{A}^{11}(e_i^{11};\ H^1, H^2)$ and $\hat{A}^{12}(e_i^{12};\ H^1, H^2)$ in favor of $A^{11}(e_i^{11};\ H^1, H^2)$ and $A^{12}(e_i^{12};\ H^1, H^2)$, as these two functions are deterministic.

equilibrium.[21]

Investors' Problem

Country 1's infinitely many mean-variance risk-averse investors invest in M^1 funds, earning returns, $\mathbf{r_F^1}$, a $M^1 \times 1$ vector with elements $r_{F,i}^1$, $i = 1, \dots, M^1$, in excess of the risk-free rate. The model of $\mathbf{r_F^1}$ is

$$\mathbf{r_F^1} = \boldsymbol{\alpha}^\mathbf{1} + \boldsymbol{\beta}^\mathbf{1} r_p + x^1 \boldsymbol{\iota}_{\mathbf{M^1}} + \boldsymbol{\varepsilon}^\mathbf{1}, \tag{13}$$

where $\boldsymbol{\alpha}^\mathbf{1}$ is a $M^1 \times 1$ vector of fund net alphas in Country 1, with each element as α_i^1, $i = 1, \dots, M^1$; and $\boldsymbol{\beta}^\mathbf{1}$ is the beta loading of each fund to an international benchmark portfolio. To simplify the framework, we assume each fund has beta loading equal to one, with respect to the international benchmark portfolio,[22] so that $\boldsymbol{\beta}^\mathbf{1}$ is the same as the $M^1 \times 1$ unit vector $\boldsymbol{\iota}_{\mathbf{M^1}}$. r_p is the international benchmark's return in excess of the risk-free rate, with mean μ_p, $\mu_p > 0$, and variance σ_p^2, $\sigma_p^2 > 0$. x^1 is the common risk factor of fund returns in Country 1, with mean 0 and variance $\sigma_{x^1}^2$, $\sigma_{x^1}^2 > 0$. $\boldsymbol{\varepsilon}^\mathbf{1}$ is a $M^1 \times 1$ vector of fund idiosyncratic risk factors in Country 1, and each of its elements is ε_i^1, $i = 1, \dots, M^1$, which has mean 0 and variance $\sigma_{\varepsilon^1}^2$, $\sigma_{\varepsilon^1}^2 > 0$. The parameters μ_p, σ_p^2, $\sigma_{x^1}^2$, and $\sigma_{\varepsilon^1}^2$ are constants, common knowledge to both investors and managers.

Investor *j*'s portfolio return (in excess of the risk-free rate) is

$$r_j^1 = \boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^\mathbf{T}} \mathbf{r_F^1} + (1 - \boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}}) r_p = r_p + \boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^\mathbf{T}} (\boldsymbol{\alpha}^\mathbf{1} + x^1 \boldsymbol{\iota}_{\mathbf{M^1}} + \boldsymbol{\varepsilon}^\mathbf{1}), \tag{14}$$

where $\boldsymbol{\delta}_\mathbf{j}^\mathbf{1}$ is a $M^1 \times 1$ vector of weights that investor *j* allocates to the M^1 funds, with each element as $\delta_{j,i}^1$, and superscript T is a transpose operator. Investor *j*'s problem is[23]

$$\max_{\boldsymbol{\delta}_\mathbf{j}^\mathbf{1}} \frac{\mathrm{E}\left(r_j^1 \middle| D\right)}{\sqrt{\mathrm{Var}\left(r_j^1 \middle| D\right)}}, \tag{15}$$

subject to

$$\delta_{j,i}^1 \geq 0, \qquad \forall i, \tag{16}$$

$$\boldsymbol{\delta}_\mathbf{j}^{\mathbf{1}^\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}} \leq 1. \tag{17}$$

[21] See also, Proposition 3 and the "proof intuition" to it.
[22] This is a common assumption, as active funds usually have diversified portfolios. See the discussion in Pastor and Stambaugh (2012).
[23] Recall Footnote 20.

Constraints (16) and (17) imply that investors cannot short sell funds, or short sell the international benchmark portfolio. To simplify our analysis, we assume that, in equilibrium, all investors have the same weights allocated to funds (i.e., a symmetric equilibrium), such that

$$\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^*} = \boldsymbol{\delta}_{\mathbf{k}}^{\mathbf{1}^*}, \qquad \forall j \neq k. \tag{18}$$

In this case, in equilibrium, the fund industry size in Country 1 is

$$\frac{S^{1^*}}{W^1} = \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^{*\mathbf{T}}} \boldsymbol{\iota}_{\mathbf{M}^1}, \qquad \forall j. \tag{19}$$

We also note that, as in PS and FSX (see PS, pp. 748–750, including Footnote 6, and references therein, and FSX, Footnote 4), DAFMI's and FAFMI's active search for net alphas might have indirect effects not modeled here. It might drive security prices toward their true values; it might induce firms to improve governance and performance and to reduce agency costs. It might induce transfer of wealth from less productive firms or investors to more productive ones. As discussed in PS, FSX, and elsewhere in the literature, gross alphas are zero-sum. We note that this is the case regardless of whether any manager's direct and or indirect benefits are non-zero or zero.

We are now ready to characterize, in the following propositions, lemma, and corollaries, the IAFMI equilibrium, induced by managers choosing optimal effort in each country, and optimal fees. That is, we characterize DAFMI equilibrium expected net alphas, Sharpe ratios, effort, fee rates, direct benefits of effort, DAFMI size, and DAFMI funds' market shares. In Proposition 0, we formally state the DAFMI Nash equilibrium. In Proposition 1, we describe the qualitative properties of this equilibrium; and in Lemma 1, we describe technical properties of the DAFMI equilibrium used to prove Proposition 0 and 1. We present the two propositions and lemma in a sequence, and then provide the proofs intuition.

We first define DAFMI equilibrium optimal allocations.[24] Let

- $\mathbf{e}^{\mathbf{11}^*}$ be an $M^1 \times 1$ vector with Country 1 managers' optimal effort allocations to Country 1 stocks, $e_i^{11^*}$,
- $\mathbf{e}^{\mathbf{12}^*}$ be an $M^1 \times 1$ vector with Country 1 managers' optimal effort allocations to Country 2 stocks, with components, $e_i^{12^*}$,
- $\mathbf{f}^{\mathbf{1}^*}$ be an $M^1 \times 1$ vector with Country 1 managers' optimal fee allocations, $f_i^{1^*}$, and
- $\boldsymbol{\delta}^{\mathbf{1}^*}$ be an $M^1 \times N^1$ matrix with vectors of Country 1 investors' optimal wealth weights

[24] This is sufficient for describing the IAFMI equilibrium because of the DAFMI–FAFMI symmetry.

allocations to funds, $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^*}$.

PROPOSITION 0. Unique Nash Equilibrium.

There exists a unique Nash equilibrium, $\{\mathbf{e}^{\mathbf{11}^*}, \mathbf{e}^{\mathbf{12}^*}, \mathbf{f}^{\mathbf{1}^*}, \boldsymbol{\delta}^{\mathbf{1}^*}\}$.

Proof of Proposition 0. See the appendix.

The following proposition that characterizes the equilibrium.

PROPOSITION 1. For manager i, $i = 1, \dots, M^1$, if initial effort inputs generate positive direct benefits of effort, then in the DAFMI equilibrium induced by managers choosing optimal effort-fee combinations, $(e_i^{11^*}, e_i^{12^*}, f_i^{1^*})$, DAFMI size, $\frac{S^{1^*}}{W^1}$, and DAFMI fund market shares, $\frac{S_i^{1^*}}{S^{1^*}}, \forall i$, adjust such that the following properties are satisfied.

1. Competition drives managers' economic profits to zero, so they can only charge break-even fees.
2. Higher managers' aggregate skill results in higher DAFMI size.
3. Higher managers' relative skill results in higher DAFMI fund market share (relative fund size).
4. Managers offer the same market competitive expected net alphas.
5. Managers offer the same market competitive Sharpe ratios.
6. Investors hold the same DAFMI portfolio weights (which are proportional to DAFMI fund sizes).
7. Equilibrium efforts and fees are the same across funds.
8. Equilibrium DAFMI direct benefits of effort are the same across funds.

Proof of Proposition 1. See the appendix. The proof intuition is below.

To prove Proposition 1, we use the seven results of the following Lemma 1, which characterize properties of the IAFMI equilibrium. (The following statement of Lemma 1 is mostly verbal. An analytical statement of the Lemma, with all relevant mathematical expressions, is in the Appendix.)

LEMMA 1. For every manager i, $i = 1, \dots, M^1$, if initial effort inputs generate positive direct benefits of effort, the DAFMI equilibrium induced by managers choosing optimal effort-fee combinations, $(e_i^{11^*}, e_i^{12^*}, f_i^{1^*})$, has the following properties.

1. Fees are equal to costs:
2. The impact of marginal effort, in either country, on gross alpha is set to be equal to the marginal average costs of effort in the respective country, thus manager i's marginal direct benefits of effort (in either country) under the optimal effort are zero.

3. When either country's concentration is higher, DAFMI equilibrium optimal efforts in either country are higher (lower) if and only if higher concentration induces a larger (smaller) marginal effort impact on gross alphas than on costs in the respective country.
4. Whether each country's higher concentrations induce higher equilibrium optimal fees depends on whether they induce changes in equilibrium DAFMI sizes and in equilibrium optimal efforts in each country that are aggregately positive.
5. When either country's concentrations are higher, equilibrium manager i's direct benefits of effort in the respective country are higher (lower) if and only if higher concentrations induce, in the respective country, a larger (smaller) impact on gross alphas than on costs.
6. Pairwise relative DAFMI fund sizes, $s_i^{1^*}/s_j^{1^*}$, are inversely proportional to their corresponding cost coefficients, $c_{1,j}^1/c_{1,i}^1$, (where $c_{1,i}^1$ is the intensity of fund-level decreasing returns to scale in gross alpha production).
7. DAFMI fund market shares $s_i^{1^*}/S^{1^*}$s are $s_i^{1^*}/S^{1^*} = \left[c_{1,i}^1 \sum_{j=1}^{M^1} \left(c_{1,i}^1\right)^{-1}\right]^{-1}$, $\forall i$.

Analytical statement and proof of Lemma 1. See the appendix. The proof intuition is below.

The proof of the existence and uniqueness of the Nash equilibrium is similar to the single-country one in FSX. Competing for investments, DAFMI managers maximize fund expected net alphas by choosing optimal efforts and fees in each country, earning zero economic profits (break-even fees) in equilibrium. The reason for the latter is the following. If DAFMI managers increase fees, they would lower fund expected net alphas and lose all investments. If DAFMI managers decrease fees, they would become insolvent—incurring negative cash flows (costs higher than fees). Deviating from equilibrium effort would also induce a loss of investments (if decreasing effort) or insolvency (if increasing effort). Therefore, DAFMI managers have no incentive to deviate.

Also, as there are no diversification benefits across funds, DAFMI managers who attempt to provide higher expected net alphas attract investments. Consequently, due to decreasing returns to scale in performance, on the one hand, and increasing fund costs, on the other hand, "alpha gains" are more than mitigated by a (break-even) fee increase, resulting in an overall decrease in expected net alpha. Thus, in equilibrium, the allocation of investments, or fund sizes, sets expected net alphas to be equal across funds. If DAFMI fund managers cannot produce the DAFMI highest expected net alpha, even for an infinitesimal fund size, they lose all investments and go out of the market.

In addition, as DAFMI funds have the same expected net alphas, they have the same expected returns. As the source of DAFMI fund returns' variance is the same across funds, the DAFMI fund return variance is the same across funds. Therefore, DAFMI managers offer the same competitive Sharpe ratio. Because investors cannot obtain a higher Sharpe ratio, they have no incentive to deviate.

These conditions result in a DAFMI unique Nash equilibrium in which neither DAFMI investors nor DAFMI managers have incentives to deviate from their chosen strategies.

If an increase in either country's concentrations induces a higher (lower) marginal effort impact on DAFMI gross alphas than a marginal effort impact on DAFMI costs, in the respective country, then DAFMI managers optimally choose, in each country, higher (lower) efforts in producing fund net alphas. If an increase in either country's concentrations induces higher DAFMI equilibrium optimal efforts in the respective country and DAFMI managers' costs are driven higher, then equilibrium break-even fees are higher.

Higher concentrations in each country have two effects on manager i's direct benefits of effort in the respective country. First, in the respective country, they directly affect gross alpha production functions, $A^{11}(e_i^{11};\ H^1, H^2)$ and $A^{12}(e_i^{12};\ H^1, H^2)$, and costs, $c_2^{11}(e_i^{11};\ H^1, H^2)$ and $c_2^{12}(e_i^{12};\ H^1, H^2)$, being parameters of each of these functions. Second, in the respective country, they affect DAFMI equilibrium optimal efforts, consequently changing the respective country's gross alphas and costs. In equilibrium, the latter (net) effect is zero because managers keep increasing DAFMI efforts in each country until, in each country, the marginal effort impact on gross alphas is equal to the marginal effort impact on costs. Thus, the effect of higher concentration through effort on gross alphas, in each country, is cancelled out by its effects on costs. Therefore, in DAFMI equilibrium (as the net second effect is zero), changes in either country's concentrations affect gross alphas and costs through the (direct) first effect only. Consequently, if higher, either country's concentrations induce higher direct impacts on gross alphas than on costs in the respective country. DAFMI manager i's direct benefits of effort, in this country, increase in the respective concentration.

DAFMI managers' different costs of producing gross alphas (skills) induce different fund sizes in equilibrium. There is a separation between the determination processes of DAFMI size (that is, DAFMI weight in total wealth, $\frac{S^{1}}{W^1}^*$) and DAFMI fund market shares (that is, relative fund sizes within DAFMI). The former is driven by DAFMI managers' aggregate skill (cost), and the latter by DAFMI managers' relative skills (costs). In other words, how DAFMI investors weight the funds inside DAFMI, or investors' "optimal DAFMI portfolio," could be unaffected by how DAFMI investors weight the DAFMI as a whole relative to the passive benchmark. This separation property facilitates later results.

For convenience in describing the equilibrium in the following propositions, we define the equilibrium optimal expected net alphas of an initial marginal investment in the DAFMI (i.e., where $S^1 = 0$) as $X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)$. Quantitatively,

$$X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) \triangleq \widehat{a^1} + A^{11}(e_i^{11};\ H^1, H^2) + A^{12}(e_i^{12};\ H^1, H^2) \\ -c_0^1 - c_2^{11}(e_i^{11};\ H^1, H^2) - c_2^{12}(e_i^{12};\ H^1, H^2). \tag{20}$$

For DAFMI to exist, we must have positive expected net alphas for initial infinitesimal investments into it, or[25]

$$X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) > 0,\ \forall\ H^1, H^2. \tag{21}$$

If Inequality (21) is violated, investors receive no advantage in diverting funds from the passive index to the DAFMI. Also, to offer meaningful results, we assume that initial marginal allocations of effort generate positive AFMI direct benefits of effort; that is,

$$B_{e_i^{11}}^{11}(0;\ H^1, H^2) > 0,\ B_{e_i^{12}}^{12}(0;\ H^1, H^2) > 0,\ \forall i, H^1, H^2, \tag{22}$$

such that the optimal effort, $(e_i^{11^*}, e_i^{12^*})$, is positive, finite, and attainable, i.e., $B_{e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right) = 0,\ B_{e_i^{12}}^{12}\left(e_i^{12^*};\ H^1, H^2\right) = 0,\ \ e_i^{11^*}, e_i^{12^*} < K,\ \ \forall i, H^1, H^2$, for some positive constant K. We focus on the case where the optimal effort is positive.

As in PS (see their Proposition 2) and FSX (see their Proposition RA2), the explicit analytic solutions for $\frac{S^{1^*}}{W^1}$ are solutions of a cubic equation and are cumbersome. The following proposition presents the cubic equation, and its corollary presents properties of its solution.

PROPOSITION 2. Equilibrium Optimal Allocations.

For manager i, $i = 1, \ldots, M^1$, we have

1. $\mathrm{E}(\alpha_i^1 | D)\big|_{\left\{\mathbf{e^{11^*}}, \mathbf{e^{12^*}}, \mathbf{f^{1^*}}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}} > 0$; and

2. the equilibrium optimal $\frac{S^{1^*}}{W^1}$ is either 1 or a real positive solution (smaller than 1), of the following first-order condition (a cubic equation) of the investors' problem [Equations (15)–(17)]. After substituting $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^{*\mathbf{T}}} \boldsymbol{\iota}_{\mathbf{M^1}} = \frac{S^{1^*}}{W^1}$, $\forall j$,

[25] The condition in Inequality (21) here is equivalent to the condition that $a > 0$ in PS. See PS, p. 747, for further discussion and insights.

$$-\gamma\sigma_{b^1}^2\left(\frac{S^{1^*}}{W^1}\right)^3-\left\{\gamma\sigma_{a^1}^2+\gamma\sigma_{x^1}^2+\widehat{b^1}+\left[\sum_{i=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\frac{S^{1^*}}{W^1} \tag{23}$$

$$+X\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right)=0,$$

where $\gamma \triangleq \mu_p/\sigma_p^2$.

Proof of Proposition 2. See the appendix.

The intuition of Proposition 2 is as follows. DAFMI investors allocate investments to funds based on their risk-return tradeoffs. Investing wealth in the DAFMI increases portfolios' risk, so they choose to limit these investments, leaving $\mathrm{E}(\alpha_i^1|D)|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}} > 0$. The risk-return tradeoff of potentially investing the last dollar, the dollar that would drive DAFMI fund expected net alphas to zero, is "in the variance favor." That is, the marginal cost of risk, of investing this last dollar, is higher than the marginal benefit of the gained net alpha. This prevents optimizing DAFMI risk-averse investors from allocating it to the DAFMI, leaving DAFMI fund expected net alphas to be positive. The properties of the cubic equation guarantee exactly one real positive root. If the positive root is larger than 1, then $\frac{S^{1^*}}{W^1}=1$.

We can now write the following corollary, characterizing DAFMI equilibrium relations between performance and size, and between the rate of returns to scale decrease and size.

COROLLARY TO PROPOSITION 2. For large enough W^1, such that $\frac{S^{1^*}}{W^1}<1$, we have the following.

1. Higher equilibrium optimal expected net alphas of an initial marginal investment in the DAFMI induce a larger equilibrium DAFMI size relative to total DAFMI wealth, or

$$\frac{d\frac{S^{1^*}}{W^1}}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}=\frac{1}{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^{1^*}}{W^1}\right)^2+\sigma_{a^1}^2+\sigma_{x^1}^2\right]+\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1}>0. \tag{24}$$

2. A higher rate of decrease in aggregate DAFMI returns to scale [fund level and industry-level, $\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1$] induces a smaller equilibrium DAFMI size, or

$$\frac{d\frac{S^{1^*}}{W^1}}{d\left\{\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}}=\frac{-\frac{S^{1^*}}{W^1}}{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^{1^*}}{W^1}\right)^2+\sigma_{a^1}^2+\sigma_{x^1}^2\right]+\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1}<0. \tag{25}$$

Proof of Corollary to Proposition 2. See the appendix.

The intuition of this corollary is as follows. A higher level of DAFMI equilibrium optimal expected net alpha of an initial marginal investment, $X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)$, attracts more investments to the DAFMI. Also, we can see that $\widehat{b^1}$ is the industry-level expected decreasing returns to scale rate coming from the alpha production function, based on current information, whereas $\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1$ may be regarded as the equilibrium decreasing returns to scale factor coming from DAFMI managers' costs of alpha production (calculated by aggregating all the fund average cost sensitivities to size, $c_{1,i}^1$'s). The latter decreasing returns to scale factor, $\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1$, is inversely proportional to DAFMI aggregate skill. Thus, the factor $\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1$ may be regarded as the combined decreasing returns to scale factor in DAMI.

The next proposition offers comparative statics that underlie our main empirical analysis. The following statement of Proposition 3 is mostly verbal. An analytical statement of the proposition, with all relevant mathematical expressions, is in the Appendix.)

PROPOSITION 3. Sensitivities of DAFMI Size and Expected Net Alphas to Concentration. Where $\frac{{S^1}^*}{W^1} < 1$, we have the following.[26]

1. Higher concentrations, in either country, induce larger (smaller) DAFMI equilibrium size and higher (lower) DAFMI equilibrium expected net alphas if and only if higher concentrations induce a larger (smaller) aggregate (over the two countries) impacts of induced optimal effort changes on gross alphas than on costs.
2. If concave in either country's concentration, DAFMI equilibrium direct benefits of efforts function indicates concave DAFMI equilibrium size in the respective concentration. (If convex in either country's concentration, DAFMI equilibrium size indicates convex, DAFMI equilibrium direct benefits of efforts function in the respective concentration.) The sensitivity of equilibrium DAFMI size to the cross partial derivative of DAFMI and FAFMI concentrations depend on signs and sizes of

[26] When $\frac{{S^1}^*}{W^1} = 1$, it is the case that, 1. $\frac{{S^1}^*}{W^1}$ is unrelated to DAFMI and FAFMI concentrations; 2. higher DAFMI/FAFMI concentrations induce higher (lower) DAFMI/FAFMI equilibrium expected net alphas if and only if higher concentrations induce a larger (smaller) impact on gross alphas than on costs; and 3. DAFMI/FAFMI equilibrium expected net alphas are concave (convex), in DAFMI/FAFMI concentrations, if and only if the DAFMI/FAFMI equilibrium direct benefit function is concave (convex), in concentrations.

several terms, including the sum of the sensitivities of DAFMI direct benefits due to efforts exerted in the domestic and foreign stock markets, to the cross partial derivative of DAFMI and FAFMI concentrations, and the product of the sums of DAFMI direct benefits sensitivities, due to efforts exerted in the domestic and foreign stock markets, to DAFMI and FAFMI concentrations, respectively.

3. Concave equilibrium expected net alphas, in either country's concentration, indicates concave, in concentration, equilibrium direct benefit function. (Convex, in concentration, equilibrium direct benefit function indicates convex, in concentration, equilibrium expected net alphas.)

   Similar to the case of equilibrium DAFMI size, the sensitivity of DAFMI equilibrium expected net alpha dependency on the cross partial derivative of DAFMI and AFMI concentrations depends on signs and sizes of several terms, including the sum of the sensitivities of DAFMI direct benefits due to efforts exerted in the domestic and foreign stock markets, to the cross partial derivative of DAFMI and FAFMI concentrations, and the product of the sums of DAFMI direct benefits sensitivities due to efforts exerted in the domestic and foreign stock markets, to DAFMI and FAFMI concentrations, respectively.

Analytical statement and proof of Proposition 3. See the appendix.

The intuition behind Proposition 3 is as follows. Changes of H^1 affect both DAFMI cost and productivity of efforts exerted in alpha production in both domestic and foreign stock markets. In turn, such changes affect equilibrium DAFMI expected net alpha, $\mathrm{E}(\alpha_i^1 \middle| D)\Big|_{\left\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\right\}}$, in two stages. In the first stage, if a higher H^1 induces a larger (smaller) aggregate, across the domestic and foreign stock markets, impact on gross alphas than on costs, it increases (decreases) DAFMI managers' ability to produce expected net alphas, thereby increasing (decreasing) the DAFMI expected net alphas produced. In the second stage, DAFMI investors react to the increase (decrease) in DAFMI fund expected net alphas by increasing (decreasing) investments in funds, consequently decreasing (increasing) DAFMI expected net alphas, due to decreasing returns to scale. The risk-return tradeoff of DAFMI risk-averse investors makes their reaction to changes in DAFMI fund expected net alphas less intense. That is, they subdue their additional investments to funds when inferring higher fund expected net alphas due to risk increase, and they limit their reduction in investments to funds when observing lower fund expected net alphas due to the decrease in risk.

The first stage and second stage described above, the latter as affected by risk attitudes, result in a change of DAFMI optimal efforts in both the domestic and foreign stock markets. DAFMI new optimal efforts, in turn, affect DAFMI alphas productions and the efforts costs producing it in both the domestic and foreign stock markets. The overall outcome depends on the aggregate—across the domestic and foreign stock markets—relative sensitivities to DAFMI concentration of—the domestic and foreign stock markets—alpha production functions, on the one hand, and of the efforts cost functions, on the other. Indeed, we formally show that whether a higher H^1 increases equilibrium DAFMI expected net alpha, $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$, depends on whether it has a larger impact on DAFMI gross alphas than on the costs producing it [i.e., the sign of $\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\Big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$, depends on the sign of $\frac{dB^{11}(e_i^{11^*};H^1,H^2)}{dH^1}+\frac{dB^{12}(e_i^{12^*};H^1,H^2)}{dH^1}=A_{H^1}^{11}(e_i^{11^*};\,H^1,H^2)+A_{H^1}^{12}(e_i^{12^*};\,H^1,H^2)-c_{2\,H^1}^{11}(e_i^{11^*};\,H^1,H^2)-c_{2\,H^1}^{12}(e_i^{12^*};\,H^1,H^2)$[27] (as we show in Lemma 1.5 above). Thus, a higher H^1 induces a larger equilibrium DAFMI expected net alpha if and only if it induces higher equilibrium DAFMI direct benefits, $[B^{11}(e_i^{11^*};\,H^1,H^2)+B^{12}(e_i^{12^*};\,H^1,H^2)]$. This explains the expected net alpha part of Proposition 3.1.

If a higher H^1 induces a larger (smaller) impact on gross alphas than on costs, then it attracts more (fewer) investments to the DAFMI [if investors have additional wealth to allocate to funds (i.e., $\frac{S^{1^*}}{W^1}<1$)]. This explains the size part of Proposition 3.1.

The intuition regarding H^2 in Proposition 3.1 is similar.

Examining the second-order effects of DAFMI concentration on DAFMI size, we first note that changes in H^1 that induce a larger $\frac{S^{1^*}}{W^1}$ result in a larger allocation to DAFMI funds and, in turn, in a higher investors' overall portfolio risk. Mean-variance risk-averse investors facing risk-return tradeoffs respond to an increase in marginal portfolio risks, holding other parameters constant, by optimally lowering investment in funds. Thus, how changes in H^1 affect changes in equilibrium $\frac{S^{1^*}}{W^1}$ depends on how changes in H^1 affect this risk-return tradeoff. The implications for the second-order derivative $\frac{d^2\frac{S^{1^*}}{W^1}}{d{H^1}^2}$ are in the proof of Proposition 3, which

[27] This total derivative of DAFMI direct benefits with respect to H^1 is the same as its partial derivative with respect to H^1.

expresses this tradeoff analytically by identifying $\frac{d^2\frac{S^{1^*}}{W^1}}{dH^{1^2}}$ as a sum of two addends. The first addend is negative (positive) if the sum of the direct benefits functions is concave (convex) in H^1, and the second one is always negative. This shows that a concave sum of the direct benefits functions in H^1 implies an $\frac{S^{1^*}}{W^1}$ concave in H^1; and a convex $\frac{S^{1^*}}{W^1}$ in H^1 implies a convex sum of the direct benefits functions in H^1.

The intuition regarding H^2 in Proposition 3.2 is similar to that of H^1. The intuition regarding the cross partials in Proposition 3.2 is straightforward, as second-order derivatives become cross partial derivatives and squares of first-order derivatives become products of first-order derivatives with respect to both countries' concentrations.

This explains Proposition 3.2.

Similarly, examining the second-order effects of DAFMI concentration on expected net alphas, we show that as H^1 changes, the change of marginal $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$, i.e., $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$, is positively proportional to the second-order change in the sum of the direct benefit functions plus an adjustment term that captures the effects of risk. This adjustment term ensures that, holding all other parameters constant, if investors' marginal portfolios risks of investing in funds are higher, investors optimally invest less in funds. In doing so, they exert a smaller negative impact on expected net alphas; thus, a higher H^1 induces a higher marginal $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$. We can see that if the second-order derivative of the sum of the direct benefits functions is positive, $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$ must be positive, whereas if $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$ is negative, the second-order derivative of the sum of the direct benefits functions must be negative.

The intuition regarding H^2 in Proposition 3.3 is similar to that of H^1. The intuition regarding the cross partials in Proposition 3.3, similar to that in Proposition 3.2.

This explains Proposition 3.3.

When investors have no additional wealth to allocate to funds, i.e., $\frac{S^{1^*}}{W^1} = 1$, they exert no impact on marginal $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$, making the marginal equilibrium optimal expected net alphas depend only on the effect of H^1 and H^2 on managers' ability to produce net alphas.

We are now ready for the following proposition.

PROPOSITION 4. Relation between skill, market share, and net alpha.

When $\frac{S^{1^*}}{W^1} < 1$, a decrease (increase) in DAFMI manager i's skill, ${c_{1,i}^1}^{-1}$, while DAFMI manager j's skill, ${c_{1,j}^1}^{-1}$, $\forall j \neq i$, is unchanged induces

1. a decrease (increase) in $\frac{s_i^{1^*}}{S^{1^*}}, \forall i$, and an increase (decrease) in $\frac{s_j^{1^*}}{S^{1^*}}, \forall j \neq i$, and
2. a decrease (increase) in $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$ and a decrease (increase) in $\mathrm{E}(\alpha_j^1|D)\big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}, \forall j \neq i$.

Proof of Proposition 4. See the appendix.

According to Proposition 4, a decrease in DAFMI manager i's skill leads to a decrease in i's market share, $\frac{s_i^{1^*}}{S^{1^*}}$. Some of the assets that fund i loses are invested in all other funds, thereby increasing the market share of all other funds.

Also, a higher skill (lower $c_{1,i}^1$), affects $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$ in two stages. In the first stage, it decreases DAFMI manager i's average cost and, thus, induces higher fund expected net alphas. As DAFMI manager i offers a higher fund expected net alpha, investments shift into DAFMI fund i from other DAFMI funds, making all those funds' expected net alphas higher due to decreasing returns to scale at the fund level. At the second stage, an increase in DAFMI fund expected net alphas attracts investments into DAFMI, which in turn drives down DAFMI funds' expected net alphas due to decreasing returns to scale at the industry level. Where $\frac{S^{1^*}}{W^1} < 1$, DAFMI investors' portfolio risks increase (decrease) when they invest more (less) in DAFMI. Thus, they subdue DAFMI investment increases when observing an increase in DAFMI fund expected net alphas, and they limit investment reductions when observing a decrease in DAFMI fund expected net alphas. Thus, DAFMI investors' risk aversion mitigates the countered effect at the second stage and makes the first stage's effect dominant.

Where $\frac{S^{1^*}}{W^1} = 1$, DAFMI investors have no additional wealth to allocate to funds, so their investments have no impact on DAFMI marginal equilibrium optimal expected net alphas, causing the first stage's effect to dominate.

2.2 Endogenous Market Concentrations

Our model allows for an endogenous measure of DAFMI and FAFMI concentrations. Modeling an endogenous measure of concentration facilitates the use of available and prevalent empirical measures. If we define H^1 and H^2 to be Herfindahl-Hirschman indices (HHI), which is the sum of market shares squared, then H^1 and H^2 are endogenous to our model.[28] Using funds' equilibrium market share, as identified in Lemma 1.7, we can write the equilibrium DAFMI and FAFMI concentrations, H^{1^*} and H^{2^*}, as

$$H^{1^*} \triangleq \sum_{i=1}^{M^1}\left(\frac{s_i^{1^*}}{S^{1^*}}\right)^2 = \sum_{i=1}^{M^1}\left[c_{1,i}^1 \sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-2}, \tag{26}$$

$$H^{2^*} \triangleq \sum_{i=1}^{M^2}\left(\frac{s_i^{2^*}}{S^{2^*}}\right)^2 = \sum_{i=1}^{M^2}\left[c_{1,i}^2 \sum_{j=1}^{M^2}\left(c_{1,i}^2\right)^{-1}\right]^{-2}. \tag{27}$$

We can see that H^{1^*} and H^{2^*} are determined by $c_{1,i}^1$s and $c_{1,i}^2$s. Specifically, depending on the size of $c_{1,i}^1$ relative to that $c_{1,j}^1$, $\forall j \neq i$, an increase in $c_{1,i}^1$, holding $c_{1,j}^1$, $\forall j \neq i$ constant, increases or decreases H^{1^*}.

For simplicity and brevity, we focus our discussion on DAFMI (similar results hold for FAFMI). When the DAFMI concentration is defined as the HHI, Propositions 3 and 4 imply that the relation between the $c_{1,i}^1$s, DAFMI equilibrium fund expected net alphas, and DAFMI size is complex. An increase in $c_{1,i}^1$ affects the DAFMI equilibrium fund expected net alphas in two ways: 1. its direct impact leads to lower DAFMI equilibrium fund expected net alphas (Proposition 4) and 2. depending on fund i's size relative to DAFMI rivals, it increases or decreases H^{1^*}, which consequently increases (decreases) DAFMI equilibrium fund expected net alphas if and only if $\frac{dB^{11}\left(e_i^{11^*}; H^{1^*}, H^{2^*}\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^{1^*}, H^{2^*}\right)}{dH^1} \geq 0\ (< 0)$ (Proposition 3.1). Similarly, an increase in $c_{1,i}^1$ affects the equilibrium DAFMI size in two ways: 1. its direct impact leads to an (inverse direction) DAFMI size change, and 2. it increases or decreases H^{1^*}, which consequently increases (decreases) the equilibrium DAFMI size if and only if $\frac{dB^{11}\left(e_i^{11^*}; H^{1^*}, H^{2^*}\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^{1^*}, H^{2^*}\right)}{dH^1} \geq 0\ (< 0)$ (Proposition 3.1). Thus, in the endogenous DAFMI concentration measure case, the relation between the $c_{1,i}^1$s, DAFMI equilibrium fund

[28] In an M^1-fund DAFMI, for example, the HHI could have values between the highest concentration, 1, in which one of the funds captures practically all the market share, and the lowest concentration, $1/M^1$, in which market shares are evenly divided. That is, in an M^1-funds' market HHI $\in \left[\frac{1}{M^1}, 1\right)$.

expected net alphas, and DAFMI size depend on fund i's size relative to rivals.[29]

Due to investments in the foreign stock market, DAFMI is also affected by changes in H^{2^*}. An increase in $c_{1,i}^2$ affects DAFMI equilibrium fund expected net alphas in the following way: depending on fund i's size relative to rivals', it increases or decreases H^{2^*}, which consequently increases (decreases) DAFMI equilibrium fund expected net alphas if and only if $\frac{dB^{11}\left(e_i^{11^*}; H^{1^*}, H^{2^*}\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^{1^*}, H^{2^*}\right)}{dH^2} \geq 0\ (< 0)$ (Proposition 3.1). Also, an increase in $c_{1,i}^2$ affects DAFMI size in the following way: depending on fund i's size relative to rivals', it increases or decreases H^{2^*}, which consequently increases (decreases) equilibrium DAFMI size if and only if $\frac{dB^{11}\left(e_i^{11^*}; H^{1^*}, H^{2^*}\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^{1^*}, H^{2^*}\right)}{dH^2} \geq 0\ (< 0)$ (Proposition 3.1). Notice that an increase in $c_{1,i}^2$ does not have direct impact on DAFMI fund expected net alphas and size, as that of $c_{1,i}^1$. Its impact on DAFMI is only through its impact on FAFMI concentration, H^{2^*}.

Thus, when the market concentration is endogenous, the relations between DAFMI concentration and DAFMI equilibrium fund expected net alphas and size are more complex. On the other hand, the relations between FAFMI concentration and the DAFMI equilibrium fund expected net alphas and size are similar to those under the exogenous concentrations framework.

Please see the discussion in FSX regarding the industry characteristics affecting equilibrium markets' concentration and why modeling those here would unnecessarily complicate our model. As long as real-world concentration is not exactly determined by the $c_{1,i}^1$s (or any other exogenous parameter of our model), we are back to the case that when concentration is exogenous (that is, has an exogenous component), our predictions remain unaltered regarding the relation between changes in exogenous DAFMI concentration level, the DAFMI equilibrium fund expected net alphas, and DAFMI size.

Similarly to FSX, we now proceed with an empirical analysis of the benefits and costs of changing concentrations of DAFMI and FAFMI using the version of our model with endogenous concentrations. This version of our model befits available data of empirical market

[29] We believe that our cost function, Equation (18), is a concise one that captures essential effects within our model. To assure that all our functional form restrictions of the non-specialized model (exogenous concentration), which we deem basic and simple, hold in the specialized one (endogenous measure of industry concentration); however, we need to impose additional, technical, "second-order," parameter restrictions. For brevity and simplicity, we do not impose these restrictions. We call the parameter values that make the specialized model abide by these restrictions *plausible*. We later confirm that the said technical restrictions are not empirically binding. That is, imposing these restrictions would not change our empirical results. In other words, the empirically estimated parameters fall within the plausible parameters range.

concentrations, such as the HHI. Popular empirical market concentration measures, such as HHI, are functions of rivals' relative sizes. We use empirical techniques to control potential endogeneity of market concentration measures.

Whether DAFMI fund net alphas and DAFMI size move in the same direction as DAFMI (FAFMI) concentration become empirical questions. Further, in cases where active fund management creates value, if fund net alphas and DAFMI size increase with DAFMI (FAFMI) concentration, our model predicts positive marginal direct benefits of efforts, for plausible parameter values. Both signs of benefits sensitivity to changing concentrations are plausible alternatives to a null hypothesis of no benefits of active fund managers' efforts.

In the following empirical analysis, we use alternative empirical measures of concentration to evaluate robustness to issues such as endogeneity. We also control for potential endogeneity of DAFMI size and alpha using lagged measures of concentration and the recursive demeaning estimator of PST.

3 Empirical Study

We analyze the concentration–net alpha and concentration–DAFMI size relations using international data of active equity mutual funds. We regard the U.S. AFMI as FAFMI, whose concentrations might affect another market's DAFMI net alphas and size. This is because the U.S. has the largest AFMI, which influences global DAFMIs. We analyze how DAFMI concentrations and, more importantly, how the FAFMI concentration influence global DAFMI net alphas and sizes.

3.1 Methodology

We describe our concentration measures, fund net alpha estimates, and our econometric models in this section.

Concentration Measures

Following FSX, and many other empirical papers, we use the following three indices to measure AFMI concentrations. The indices i, j, and t indicate the fund, the market, and the time, respectively.

1. Herfindahl-Hirschman Index (HHI)

$$HHI_{j,t} = \sum_{i=1}^{M_{j,t}} MS_{i,j,t}^2. \tag{28}$$

2. Normalized-Herfindahl-Hirschman Index (NHHI)

$$NHHI_{j,t} = \frac{M_{j,t} \times HHI_{j,t} - 1}{M_{j,t} - 1}. \tag{29}$$

3. 5-Fund-Index (5FI)

$$5FI_{j,t} = \sum_{i=1}^{5} MS_{i,j,t}. \tag{30}$$

Here $MS_{i,j,t}$ is the market share of a fund in its market, measured as the fund's asset under management divided by the total assets under management in its market, and the $MS_{i,j,t}$'s in the $5FI_{j,t}$ are those of the largest five funds in market j. $M_{j,t}$ is the number of funds in market j. As some markets tend to have a large number of funds and others tend to have a small number of funds in our sample period, we focus on the results of using NHHI as the market concentration measure because it adjusts the effect of the number of funds on market concentrations [Cremers, Nair, and Peyer (2008)]. For a robustness check, we redo the analyses using HHI and 5FI.

Style-Matching Model and Net Alpha Estimation

Following FSX, we develop our style-matching model to estimate funds' passive benchmarks and then calculate fund net alphas. We use the following return-generating process:

$$R_{i,j,t} = \alpha_{i,j,t} + b_{i,j,t}^{1} F_{j,t}^{1} + \cdots + b_{i,j,t}^{n_j} F_{j,t}^{n_j}, \tag{31}$$

where $R_{i,j,t}$ is the return net of management fees of an active fund, $\alpha_{i,j,t}$ is the fund net alpha, and $F_{j,t}^{1}$ through $F_{j,t}^{n_j}$ are the factors constructing the benchmark portfolio returns. We require the benchmark portfolio to be an international passive benchmark portfolio, so $F_{j,t}^{1}$ through $F_{j,t}^{n_j}$ include returns net of management fees of domestic tradable index funds of different asset classes, a U.S. large-cap equity tradable index fund, and a domestic risk-free asset.[30] We include a U.S. large-cap equity tradable index fund because it can be a potential factor in this

[30] After matching the styles of the active funds, we compare the fund net alphas of active funds from different categories. See, for example, discussions in Sharpe (1992) and Berk and Binsbergen (2015).

international passive benchmark.[31,32] Coefficients $b_{i,j,t}^{1}$ through $b_{i,j,t}^{n_j}$ represent the loadings, and n_j is the number of these factors in a particular market. In our algorithm, in each fund market, we minimize the variance of the residual when projecting $R_{i,j,t}$ on $F_{j,t}^{1}$ through $F_{j,t}^{n_j}$, and we constrain the coefficients $\hat{b}_{i,j,t}^{1}$ through $\hat{b}_{i,j,t}^{n_j}$ to be positive and sum up to one (as investors cannot short sell funds). We use a 5-year rolling window, from months $t-60$ to $t-1$, to estimate $\hat{b}_{i,j,t}^{1}$ through $\hat{b}_{i,j,t}^{n_j}$. The predicted value $\hat{b}_{i,j,t}^{1}F_{j,t}^{1}+\cdots+\hat{b}_{i,j,t}^{n_j}F_{j,t}^{n_j}$ is the international passive benchmark at time t, and we estimate $\alpha_{i,j,t}$ by subtracting $R_{i,j,t}$ from $\hat{b}_{i,j,t}^{1}F_{j,t}^{1}+\cdots+\hat{b}_{i,j,t}^{n_j}F_{j,t}^{n_j}$. We note that our empirical design of identifying passive benchmarks using matching tradable index funds fits our theoretical structure, which assumes the appropriate international passive benchmarks for each fund.

Our style-matching method is similar to the style-matching model developed by Sharpe (1992). Also, as our passive benchmark is tradable, our net alpha estimation is consistent with the Berk and Binsbergen (2015) argument that to measure the value added by a fund, its performance should be compared to the next-best investment opportunity available to investors. Moreover, our style-matching passive benchmark is similar to the characteristic-based benchmark developed by Daniel, Grinblatt, Titman, and Wermers (1997). Our model is similar to the style-matching model of FSX except that ours contains an additional U.S. large-cap equity tradable index fund besides domestic tradable index funds.

Concentration–Net Alpha Relation

Pastor, Stambaugh, and Taylor (2015) (PST) develop a recursive demeaning (RD) estimator to control endogeneity bias. We adopt their method here to analyze the concentration–net alpha relation. The model we use is

$$\overline{\alpha_{\iota,J,t}} = \beta_1\overline{NHHI_{J,t-1}^{D}} + \beta_2\overline{NHHI_{t-1}^{US}} + \overline{Controls_t} + \overline{\varepsilon_{\iota,J,t}}, \tag{32}$$

where the superscription *D* and *US* represent the domestic and the U.S. concentration measures, respectively. The bar above the variables represents the recursive forward-demeaning operator.

[31] In many of the markets that we study, during early years of the data sample, the U.S. large-cap equity index funds are the only U.S. equity index funds available to investors, due to their higher liquidity. Other index funds, such as the U.S. small-cap equity index funds and the U.S. all-equity index funds, were not available in these markets during those years. Also, we use the Vanguard 500 Index Fund as the U.S. large-cap equity tradable index fund, whose returns move closely with the returns of the U.S. stock market, and even move closely with the returns of global markets. Thus, this index fund can represent a factor in the international passive benchmark.

[32] A list of these index funds is in an online Data Appendix.

The recursive forward-demeaned value of a time-series variable X_t is

$$\overline{X_t} = X_t - \frac{1}{T-t+1}\sum_{s=t}^{T} X_{s,} \tag{33}$$

where T is the total number of observations of this time-series.

Following the literature [e.g., FSX, PST, and Spiegel and Zhang (2013)], we use the lagged values of the following variables as control variables:

- *Market Share*, which is calculated as a fund's net assets under management (AUM) divided by the sum of all funds' net AUM in the same month;
- *Fund Volatility*, which is the standard deviation of a fund's net return in the last 12 months;
- *Fund Age*, which is calculated as the number of months since the fund's inception month;
- *DAFMI Size*, which is the sum of funds' net assets under management in a DAFMI market, divided by this market's stock market capitalization.

The variable *Time Trend*, which is equal to one for the first observation and increases by one over each month, is also included to control any time-trend effect in the fund net alphas.

The recursive demeaning method that we use addresses the omitted fund fixed effect. An additional concern is that reverse causality could exist between fund market shares and fund net alphas because when fund net alphas are higher, corresponding asset values increase and funds attract investments, both leading to a higher market share. To address this endogeneity issue, we use the recursive backward-demeaned market share to instrument for the recursive forward-demeaned market share, following PST and FSX.[33] For a robustness check, we also use the lagged market share with a constant term to instrument for the recursive forward-demeaned market share, following Zhu (2018).

Our model is similar to the concentration–net alpha model in FSX, except for the following: 1. whereas FSX studies how U.S. AFMI concentration affects U.S. AFMI net alphas, our international model uses the U.S. AFMI concentration as FAFMI concentration, and studies the associations of this FAFMI concentration with fund net alphas in other global DAFMI markets; 2. our model includes more fund-level controls, such as fund volatility and fund age.

We also run the model in Equation (32) and estimate the coefficients at each DAFMI market.

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[33] The recursive backward-demeaned value of a time-series variable X_t is $\underline{X_t} = X_t - \frac{1}{t-1}\sum_{s=1}^{t-1} X_s$. PST shows that this method can address both the omitted fund fixed effect issue and the reverse causality issue.

Concentration–DAFMI Size Relation

Our panel regression model is

$$DAFMI\_Size_{j,t} = \beta_0 + \beta_1 NHHI^{D}_{j,t-1} + \beta_2 NHHI^{US}_{t-1} + Controls_t + \varepsilon_{j,t}, \qquad (34)$$

where $DAFMI\_Size_{j,t}$ is the DAFMI size of each global market. We include lagged *DAFMI Size*, *Time Trend*, and market fixed effects as control variables. We also perform the model in Equation (34) at each DAFMI market and estimate the coefficients without market fixed effects but with Newey-West estimates of standard error.

3.2 Data

We obtain our data from the Global Databases of Morningstar Direct. Our sample contains 30 active equity mutual fund markets. Due to data availability, most of these markets have observations from 1999, so we set our sample period from the beginning of 1999 to the end of 2015 and use monthly data.[34] Our online Data Appendix supplements the data description below.

The active equity mutual fund filter and the sample development method are similar to those in FSX. We use keywords in Morningstar to identify active equity mutual funds. We require the mutual funds to be open-ended and non-restricted. In each mutual fund market dataset, we exclude index funds, enhanced index funds, funds of funds, and in-house funds of funds. Also, we require funds to be classified as "Equity" in the Global Broad Category Group, and we further identify equity funds based on their Morningstar Category. Next, we use the fund identification provided by Morningstar to aggregate fund share class-level information to fund-level information. To have sufficient observations of net alphas for each fund to mitigate measurement error, we require each of our active equity mutual funds to have at least ten years' return observations, as we use a five-year rolling window to estimate fund net alphas.[35]

The index funds used in the style-matching model are also from Morningstar. We require index funds to have no missing observations in our sample period so that the style-matching model is consistent and stable. The information of the risk-free rate of each market is provided by the International Financial Statistics on the official website of International Monetary Fund (IMF).

For each market, the DAFMI size is calculated as total funds' net assets under

[34] As we use a 5-year rolling window to estimate fund net alphas, the tests contain observations from the beginning of 2004 to the end of 2015.
[35] We also omit some rare cases in which there is a gap with more than five years' return observations missing.

management divided by domestic stock market capitalization, which is a relative size measure and which is consistent with FSX and PST. Each market's fund net assets under management and stock market capitalization are also provided by the Global Databases of Morningstar Direct.

All the fund returns are net of administrative and management fees and other costs taken out of fund assets; thus, the fund alphas we estimate are net alphas (net of fees). For comparison purpose and to be consistent with our international model, we measure the fund returns, risk-free returns, fund net assets under management, and stock market capitalization in U.S. dollars. We define a fund's age as the number of months since the fund's inception month, which is also provided by the Global Databases of Morningstar Direct.[36]

Table 1 reports the summary statistics of these global active equity mutual fund markets. Panel A presents the summary statistics of market-level variables. It shows that the average *DAFMI Size* greatly varies across the global markets, from around 6.5% in Canada to 0.015% in Germany. The market concentration also greatly varies across the global markets. The average *NHHI* value ranges from around 0.36 in Austria to around 0.01 in Taiwan. Panel B shows the summary statistics of fund-level variables. The average R-squared of the style-matching model are quite high in each market (ranging from 97% in Chile to 83% in Mexico), with a low standard deviation in each market. This result indicates that our style-matching benchmarks perform well in tracking the style of the active equity mutual funds, so it is unlikely that our style-matching models omit relevant factors in developing the passive benchmarks. Also, most DAFMI markets' average *Net Return* and *Net Alphas* are slightly positive with large standard deviations. In most markets, the average fund market share is small, showing that there are no dominating funds in these markets, and fund volatilities are large compared to fund returns. Asian markets (e.g., mainland China, India, Japan, Korea, and Thailand) tend to have younger funds, whereas European markets (e.g., France, Germany, and the United Kingdom) tend to have older funds.

[36] The inception date information of funds in the Israel market is not available in the Global Databases of Morningstar Direct, so for funds in this market, we use the month of the first observation of a fund's returns as this fund's inception month.

Table 1 Summary Statistics.

Monthly data is used. Panel A reports the summary statistics for market-level data, and Panel B reports those for fund-level data. We report the number of observations, mean, and standard deviation of each variable. *DAFMI Size* is the sum of funds' net assets under management in a DAFMI market divided by this market's stock market capitalization, and it is in decimal. *NHHI*, *HHI*, and *5FI* are Normalized-Herfindahl-Hirschman Index, Herfindahl-Hirschman Index, and the 5-Fund-Index, respectively, and they are in decimals. *Net Return* and *Net Alpha* are fund net return and fund net alpha, respectively, and they are in percentages, and net of administrative and management fees and other costs taken out of fund assets. The *Style-Matching Model* R^2 is the R^2 when estimating the style-matching model, and it is in decimal. *Market Share* is calculated as a fund's net assets under management (AUM) divided by the sum of all funds' net AUM in the same month.

Panel A: Summary statistics for market-level data

		DAFMI Size			*NHHI*			*HHI*			*5FI*		
Global Market	Number of Funds	Number of Observations	Mean	Standard Deviation	Number of Observations	Mean	Standard Deviation	Number of Observations	Mean	Standard Deviation	Number of Observations	Mean	Standard Deviation
Australia	257	144	0.0374	0.0065	144	0.0180	0.0048	144	0.0212	0.0056	144	0.2325	0.0404
Austria	7	144	0.0178	0.0062	144	0.3612	0.2337	144	0.4164	0.2322	144	0.8918	0.0823
Belgium	20	144	0.0020	0.0022	144	0.1224	0.1193	144	0.1744	0.1388	144	0.6914	0.1673
Brazil	102	144	0.0017	0.0020	144	0.0237	0.0159	144	0.0283	0.0189	144	0.2785	0.0971
Canada	299	144	0.0651	0.0049	144	0.0127	0.0007	144	0.0149	0.0012	144	0.1818	0.0103
Chile	25	144	0.0090	0.0130	144	0.0248	0.0082	144	0.0530	0.0168	144	0.3888	0.0771
China (Mainland)	4	144	0.0026	0.0011	139	0.1039	0.0637	144	0.2193	0.2031	144	0.7666	0.2222
Denmark	24	144	0.0128	0.0039	144	0.0507	0.0158	144	0.0760	0.0223	144	0.5091	0.0780
Finland	27	144	0.0077	0.0051	144	0.1014	0.0504	144	0.1804	0.0898	144	0.7198	0.2124
France	180	144	0.0109	0.0033	144	0.0251	0.0100	144	0.0286	0.0107	144	0.2908	0.0488
Germany	89	144	0.0001	0.0001	144	0.0630	0.0321	144	0.0730	0.0357	144	0.5237	0.1016
Greece	14	144	0.0180	0.0061	144	0.1530	0.2439	144	0.1992	0.2373	144	0.7045	0.1416
Hong Kong	14	144	0.0008	0.0005	130	0.1330	0.1003	144	0.2812	0.2675	144	0.8170	0.1219
India	38	144	0.0034	0.0023	144	0.0837	0.1337	144	0.1713	0.2265	144	0.5272	0.3156
Israel	82	144	0.0189	0.0137	144	0.0169	0.0070	144	0.0257	0.0068	144	0.2572	0.0566
Italy	47	144	0.0063	0.0030	144	0.0268	0.0119	144	0.0397	0.0180	144	0.3318	0.0944
Japan	317	144	0.0034	0.0018	144	0.0976	0.1387	144	0.0995	0.1385	144	0.3622	0.1757
Korea	210	144	0.0580	0.0295	144	0.0162	0.0044	144	0.0186	0.0058	144	0.2115	0.0518
Mexico	29	144	0.0003	0.0001	144	0.1466	0.1146	144	0.1789	0.1293	144	0.6584	0.1391
Netherlands	15	144	0.0021	0.0041	144	0.0841	0.0763	144	0.1462	0.0979	144	0.7077	0.1512
Norway	45	144	0.0267	0.0099	144	0.0573	0.0247	144	0.0770	0.0260	144	0.5101	0.0819
Portugal	17	144	0.0022	0.0016	117	0.0942	0.0364	117	0.1483	0.0320	117	0.7513	0.0601
Singapore	13	144	0.0015	0.0009	144	0.1785	0.1053	144	0.2974	0.1723	144	0.8351	0.1044
South Africa	78	144	0.0197	0.0039	144	0.0466	0.0111	144	0.0559	0.0147	144	0.4269	0.0559
Spain	85	144	0.0027	0.0025	144	0.0237	0.0086	144	0.0318	0.0083	144	0.2977	0.0400
Sweden	90	144	0.0203	0.0097	144	0.0441	0.0484	144	0.0665	0.0819	144	0.4012	0.2286
Switzerland	118	144	0.0063	0.0030	144	0.0242	0.0190	144	0.0321	0.0244	144	0.2877	0.1122
Taiwan	98	144	0.0212	0.0191	144	0.0094	0.0011	144	0.0160	0.0006	144	0.1841	0.0086
Thailand	132	144	0.0141	0.0075	144	0.0201	0.0055	144	0.0254	0.0060	144	0.2713	0.0392
United Kingdom	379	144	0.0073	0.0054	144	0.0438	0.0605	144	0.0525	0.0786	144	0.3510	0.2222

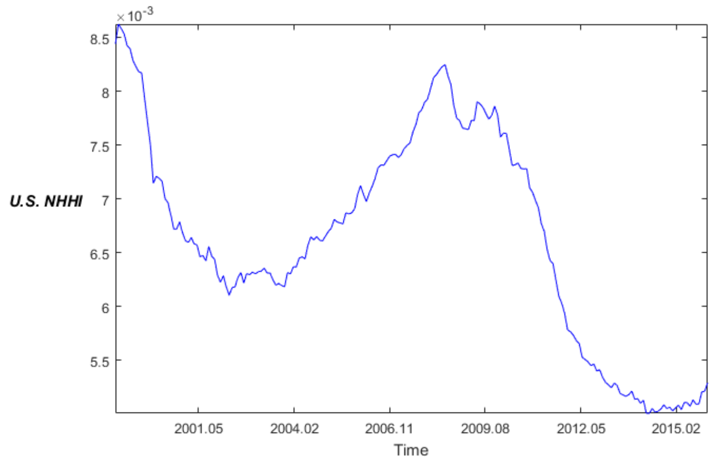
Panel B: Summary statistics for fund-level data

	Net Return			*Net Alpha*			*Stlye-Matching Model R^2*			*Market Share*			*Fund Volatility*			*Fund Age*		
Global Market	Number of Observations	Mean	Standard Deviation	Number of Observations	Mean	Standard Deviation	Number of Observations	Mean	Standard Deviation	Number of Observations	Mean	Standard Deviation	Number of Observations	Mean	Standard Deviation	Number of Observations	Mean	Standard Deviation
Australia	28,197	0.7645	7.1380	28,197	0.2154	1.9923	28,197	0.9285	0.0790	27,951	0.0044	0.0092	28,197	6.6662	2.5895	28,197	156.9449	78.0510
Austria	571	1.1370	8.5657	571	0.1817	2.2183	571	0.9018	0.0416	318	0.2302	0.3016	571	6.8843	3.6184	571	193.8091	47.3629
Belgium	2,690	0.7719	6.1115	2,690	0.2272	1.8962	2,690	0.8959	0.0799	1,949	0.0642	0.0909	2,690	5.2739	2.6627	2,690	167.2743	94.7546
Brazil	11,283	0.3186	10.0539	11,283	0.2377	5.0543	11,283	0.9021	0.0835	11,283	0.0049	0.0131	11,283	8.6979	3.5834	11,283	154.4623	75.3139
Canada	34,810	0.5163	6.1113	34,810	0.0100	1.9951	34,810	0.8877	0.1086	32,147	0.0038	0.0070	34,810	5.4672	2.4536	34,810	201.7699	132.5100
Chile	1,500	1.2362	6.0915	1,500	0.1442	1.0964	1,500	0.9724	0.0273	1,500	0.0292	0.0213	1,500	6.0348	2.1127	1,500	177.1000	86.3065
China (Mainland)	325	0.7791	7.7435	325	0.1558	2.4760	325	0.9479	0.0517	325	0.1131	0.0710	325	7.6592	2.7960	325	102.8031	24.3654
Denmark	2,773	1.2043	6.3738	2,773	0.1216	1.3192	2,773	0.9486	0.0748	2,726	0.0333	0.0389	2,773	5.7295	2.5818	2,773	191.8370	89.3553
Finland	3,356	0.8177	7.0131	3,356	0.0488	1.7025	3,356	0.9293	0.0752	1,927	0.0695	0.0918	3,356	6.3940	2.7714	3,356	163.8603	78.1918
France	22,324	0.5189	6.2557	22,324	0.1273	1.9821	22,324	0.8749	0.1212	21,627	0.0049	0.0116	22,324	5.7187	2.3752	22,324	195.1456	98.0176
Germany	10,630	0.7717	6.8077	10,630	0.1068	2.0592	10,630	0.9066	0.1017	9,348	0.0144	0.0295	10,630	6.1183	2.9736	10,630	226.7954	169.4476
Greece	1,656	-0.4618	9.2689	1,656	0.0328	2.0276	1,656	0.9410	0.0463	982	0.0920	0.1236	1,656	7.5792	3.4792	1,656	142.0888	67.2920
Hong Kong	2,134	0.8363	6.3093	2,134	0.1679	2.1306	2,134	0.8640	0.1671	1,417	0.0860	0.1046	2,134	5.6537	2.4568	2,134	145.2816	74.5456
India	11,090	1.2264	8.5167	11,090	0.3846	2.7072	11,090	0.8884	0.0931	3,186	0.0138	0.0207	11,090	8.1268	3.0045	11,090	138.6034	53.6679
Israel	8,466	0.6473	6.9214	8,466	0.1620	2.9836	8,466	0.8287	0.1345	8,466	0.0098	0.0132	8,466	6.2083	2.7682	8,466	155.5122	54.7284
Italy	5,601	0.2624	6.9582	5,601	0.1239	1.2957	5,601	0.9547	0.0569	5,236	0.0165	0.0201	5,601	6.2208	2.7494	5,601	160.6245	59.0760
Japan	35,633	0.3950	5.0012	35,633	0.0656	2.1802	35,633	0.8585	0.1433	35,574	0.0029	0.0196	35,633	4.6409	1.8697	35,633	138.5075	61.9765
Korea	16,819	0.5343	7.4214	16,819	0.0128	1.8854	16,819	0.9491	0.0413	16,769	0.0020	0.0050	16,819	6.8830	3.3583	16,819	109.7323	30.4874
Mexico	3,320	0.5156	6.6944	3,320	0.0493	2.6913	3,320	0.8277	0.1744	2,896	0.0343	0.0627	3,318	5.9836	2.6956	3,320	202.8283	71.8646
Netherlands	1,876	0.6674	6.7878	1,876	0.1517	2.4881	1,876	0.8732	0.1522	1,425	0.0757	0.0821	1,876	6.0162	2.7520	1,876	207.7729	142.1011
Norway	5,738	1.0073	8.4386	5,738	0.0864	2.0225	5,738	0.9316	0.0703	5,588	0.0223	0.0344	5,738	7.6985	3.2807	5,738	171.9840	71.9108
Portugal	1,778	0.1448	7.7343	1,778	0.0488	2.1276	1,778	0.9292	0.0242	1,739	0.0646	0.0737	1,778	7.0545	2.3312	1,778	172.8661	38.0430
Singapore	1,710	0.8435	6.4475	1,710	0.0121	1.3539	1,710	0.9375	0.0551	1,285	0.1016	0.1434	1,710	5.5903	2.9420	1,710	188.2772	72.5187
South Africa	8,995	0.8079	7.2570	8,995	0.1223	2.2113	8,995	0.9137	0.0712	8,589	0.0142	0.0265	8,995	6.8175	2.3513	8,995	159.0649	107.1109
Spain	9,962	0.5168	7.0045	9,962	0.0025	1.2959	9,962	0.9559	0.0758	9,734	0.0115	0.0170	9,962	6.2243	2.9537	9,962	157.4677	59.5060
Sweden	11,197	1.0388	7.2233	11,197	0.0449	1.5148	11,197	0.9468	0.0620	9,015	0.0132	0.0199	11,003	6.5627	2.8655	11,003	177.5179	89.7282
Switzerland	12,788	0.7914	5.2876	12,788	0.0479	1.9316	12,788	0.8940	0.1129	11,708	0.0088	0.0157	12,788	4.7589	2.0713	12,788	163.4837	104.7498
Taiwan	7,308	0.4173	5.5388	7,308	0.1297	2.4830	7,308	0.8365	0.0665	7,308	0.0085	0.0087	7,308	5.8419	2.3938	7,308	183.4614	49.5105
Thailand	14,135	1.0660	6.7499	14,135	0.1447	1.5856	14,135	0.9479	0.0414	14,127	0.0054	0.0107	14,135	6.6426	2.0872	14,135	139.0712	52.6587
United Kingdom	48,939	0.6505	5.4471	48,939	0.0118	1.5585	48,939	0.9206	0.0774	31,019	0.0039	0.0144	48,939	4.8214	2.1785	48,939	226.4449	146.5886

Figure 2 illustrates the monthly NHHI of the U.S. active equity mutual fund market from January 1999 to December 2015. It shows that the concentration of this U.S. market decreases substantially from January 1999 to the end of 2003. After that, it started to increase gradually and decreased again, reaching the lowest point at the current time.

Figure 2 NHHI of the U.S. Active Equity Mutual Fund Market.

The *U.S. NHHI* value is in decimals. Sample period is from January 1999 to December 2015.



3.3 Empirical Results

Table 2 reports the empirical results of the concentration–net alpha relation. We find that, on average, DAFMI net alphas are significantly negatively associated with *DAFMI NHHI* and *U.S. NHHI*. However, the absolute value of the coefficient of *U.S. NHHI* is much larger than that of *DAFMI NHHI*, showing that a small change in the concentration in the U.S. AFMI, say 0.01 change in *U.S. NHHI*, has a much larger impact on the DAFMI net alphas than the same magnitude change in *DAFMI NHHI* [regarding model specification (3), 136.5698/0.4526 ≈ 302 times in magnitude]. If we consider the values of the standardized coefficients, i.e., the t-statistics, we find that the absolute value of the t-statistics of *U.S. NHHI* is 13.96, which is still three times larger than that of *DAFMI NHHI*, 4.64.

Also, *DAFMI NHHI* does not have much power in explaining DAFMI net alphas, as illustrated by the small R-squared in model specification (1). After we add *U.S. NHHI* in model specifications (2) and (3), these models' explanation power improves, as the R-squareds of these two model specifications almost triple.[41] Further including *lagged Market Share* in the

[41] As shown in the literature, fund net alphas, or risk-adjusted abnormal returns, are difficult to explain, so the small R-squared values of the models in Table 2 are consistent with the findings in the literature.

controls reduces the sample size by more than 13%, and reduces the R-squared and adjusted R-squared (as we use the instrumental variable regression here to instrument for the forward-demeaned market share with its backward-demeaned value); but we still find consistent results in the coefficients of *DAFMI NHHI* and *U.S. NHHI*. If we use *HHI* and *5FI* as concentration measures, we find consistent results, though *DAFMI 5FI* is not significant.

In unreported robustness tests, we instrument for the forward-demeaned market share with the lagged market share and a constant term, following Zhu (2018). Also, we test model (32) in Table 2 by using fund fixed-effect regressions instead of the RD method. In addition, we extend the sample period to earlier years, starting from 1997, when quite a few markets do not have observations, and redo the tests in Table 2. We find consistent results in all these tests.

Our framework provides an appealing explanation for our interesting finding that *U.S. NHHI* has larger impact on DAFMIs net alphas than *DAFMI NHHI* has. Changes in U.S. AFMI concentration affect the alpha productivity and effort costs of searching investment opportunities in the U.S. stock markets; consequently, by a substitution effect, the changes affect the incentives of managers to search for opportunities in global stock markets. As the U.S. AFMI is much larger than global DAFMI markets, the impact of the U.S. AFMI concentration changes would be larger than that of DAFMIs concentration changes.

Table 3 reports the empirical results of the concentration–DAFMI size relation. We find that, on average, DAFMI size is significantly negatively associated with *U.S. NHHI* but insignificantly associated with *DAFMI NHHI*. In particular, 0.01 increase in *U.S. NHHI*, on average, decreases DAFMI size (relative to the domestic stock market capitalization) by more than 10 basis points, as shown in model specifications (2) and (3). As the global stock markets' capitalizations are huge, this represents an economically significant decrease in DAFMI size measured in dollar. Also, the R-squared and adjusted R-squared of each model specification in Table 3 are close to 90%, showing that DAFMI size is well explained by the models. When the concentrations are measured by *HHI* or *5FI*, we find consistent results.

In unreported tests using a panel VAR model, regarding DAFMI size and DAFMI concentration measures as endogenous and the U.S. concentration measures as exogenous, we find that DAFMI size has insignificant impact on DAFMI concentration measures. Thus, results in Table 3 are not affected by bias created by the reverse causality between DAFMI size and DAFMI concentration measures.

Table 2 Concentrations and Fund Net Alpha.

The dependent variable is *Net Alpha* and is in percentage. The variables are recursively forward-demeaned. *DAFMI Size* is the sum of funds' net assets under management in a DAFMI market divided by this market's stock market capitalization, and it is in decimal. DAFMI and U.S. *NHHI*, *HHI*, and *5FI* are Normalized-Herfindahl-Hirschman Index, Herfindahl-Hirschman Index, and the 5-Fund-Index of the corresponding market, respectively, and they are in decimals. *Time Trend* is equal to one for the first observation and increases by one over each month. For each fund, *Market Share* is calculated as a fund's net assets under management (AUM) divided by the sum of all funds' net AUM in the same month. *Fund Volatility* is the standard deviation of a fund's net return in the last 12 months. *Fund Age* is calculated as the number of months since the fund's inception month. Standard errors are clustered by fund and presented in parentheses. The symbols ***, **, and * represent the 1%, 5%, and 10% significant level, respectively, in a two-tail *t*-test.

	(1)	(2)	(3)	(4)	(5)	(6)
Lagged DAFMI NHHI	-0.1708*		-0.4526***	-0.7435***		
	(0.0885)		(0.0875)	(0.1604)		
Lagged U.S. NHHI		-129.5473***	-136.5698***	-137.8691***		
		(6.3192)	(6.4891)	(9.8781)		
Lagged DAFMI HHI					-0.7267***	
					(0.2179)	
Lagged U.S. HHI					-137.8569***	
					(9.5359)	
Lagged DAFMI 5FI						-0.1680
						(0.1589)
Lagged U.S. 5FI						-12.9864***
						(0.8676)
Lagged Market Share				0.3988	1.0544	1.0427
				(3.9721)	(3.9514)	(4.1441)
Lagged Fund Volatility	0.0176***	0.0344***	0.0340***	0.0325***	0.0325***	0.0335***
	(0.0025)	(0.0026)	(0.0026)	(0.0029)	(0.0029)	(0.0029)
Lagged Fund Age	0.1982***	0.1654***	0.1715***	0.1788***	0.1794***	0.1788***
	(0.0282)	(0.0290)	(0.0288)	(0.0301)	(0.0301)	(0.0304)
Lagged DAFMI Size	0.4154	-0.5494	-0.5533	-1.2676	-1.2699	-1.7595*
	(0.7707)	(0.7387)	(0.7360)	(0.9190)	(0.9305)	(0.9035)
Time Trend	-0.2009***	-0.1703***	-0.1768***	-0.1845***	-0.1851***	-0.1850***
	(0.0285)	(0.0293)	(0.0291)	(0.0304)	(0.0305)	(0.0308)
Number of Observations	325,318	325,318	325,318	282,251	282,251	282,251
R-squared	0.0009	0.0024	0.0026	0.0025	0.0024	0.0024
Adjusted R-squared	0.0009	0.0024	0.0026	0.0025	0.0024	0.0023

Table 3 Concentrations and DAFMI Size.

The dependent variable is *DAFMI Size*. *DAFMI Size* is the sum of funds' net assets under management in a DAFMI market divided by this market's stock market capitalization, and it is in decimal. DAFMI and U.S. *NHHI*, *HHI*, and *5FI* are Normalized-Herfindahl-Hirschman Index, Herfindahl-Hirschman Index, and the 5-Fund-Index of the corresponding market, respectively, and they are in decimals. *Time Trend* is equal to one for the first observation and increases by one over each month. Market fixed effects are controlled. Standard errors are clustered by market and presented in parentheses. The symbols ***, **, and * represent the 1%, 5%, and 10% significant level, respectively, in a two-tail *t*-test.

	(1)	(2)	(3)	(4)	(5)
Lagged DAFMI NHHI	-0.0017		-0.0020		
	(0.0015)		(0.0016)		
Lagged U.S. NHHI		-0.1213**	-0.1733**		
		(0.0469)	(0.0654)		
Lagged DAFMI HHI				-0.0016	
				(0.0010)	
Lagged U.S. HHI				-0.1932**	
				(0.0751)	
Lagged DAFMI 5FI					-0.0011*
					(0.0006)
Lagged U.S. 5FI					-0.0167**
					(0.0063)
Lagged DAFMI Size	0.9390***	0.9384***	0.9373***	0.9361***	0.9353***
	(0.0120)	(0.0122)	(0.0126)	(0.0131)	(0.0133)
Time Trend	-0.0000	-0.0000	-0.0000*	-0.0000*	-0.0000*
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Constant	0.0014**	0.0022***	0.0033**	0.0037**	0.0051**
	(0.0006)	(0.0008)	(0.0013)	(0.0015)	(0.0020)
Number of Observations	4,268	4,320	4,268	4,290	4,290
R-squared	0.8878	0.8876	0.8880	0.8880	0.8878
Adjusted R-squared	0.8877	0.8875	0.8879	0.8879	0.8877

To test whether DAFMI markets' fund net alphas and size are both significantly associated with the concentration measures, we conduct multiple hypothesis tests on whether β_1 in Equation (32) and β_1 in Equation (34) are both significantly different from zero, and on whether β_2 in Equation (32) and β_2 in Equation (34) are both significantly different from zero. The tests are based on the p-values from model specification (4) in Table 2 and model specification (3) in Table 3. As the p-values are estimated based on t-test statistics, multiple hypothesis methods assuming nonnegative correlation are appropriate. Thus, we choose the Sidak method, a one-step method, and the Holland-Copenhaver method, a step-down method [see the discussions in Newson and the ALSPAC Study Team (2003)]. The uncorrected critical p-value is set to be 0.1, and then we calculate the corrected critical p-values based on the Sidak method and the Holland-Copenhaver method.

Results in Table 4 show that DAFMI markets' fund net alphas and size both significantly decrease with *U.S. NHHI* because both $\beta_2 = 0$ in Equation (32) and $\beta_2 = 0$ in Equation (34) are rejected, in the Sidak method and in the Holland-Copenhaver method. In fact, if we set the uncorrected critical p-value to 0.05 instead of 0.1, both β_2 in Equation (32) and β_2 in Equation (34) still pass the multiple hypothesis tests. Thus, we are 95% confident that these two significant results are real.

On the other hand, we cannot be 90% confident that DAFMI markets' fund net alphas and size both significantly decrease with *DAFMI NHHI* because β_1 in Equation (34) does not pass the multiple hypothesis tests. We find consistent test results where concentrations are measured by *HHI* and *5FI*.

The findings that, on average, DAFMI markets' fund net alphas and size, both, decrease with the U.S. AFMI concentration, is consistent with the predictions of our theoretical model, under both the exogenous concentration framework and the endogenous concentration framework. Based on our empirical and theoretical results, we conclude that higher concentrations in the U.S. AFMI induce smaller aggregate impacts of induced optimal effort-level changes on gross alphas than on costs. In other words, higher U.S. AFMI concentration induces lower DAFMI direct benefits.

Table 4 Results of Multiple Hypothesis Tests.

This table presents the results of the tests on whether *Net Alpha* and *DAFMI Size* are both significantly associated with DAFMI concentration measures and whether they are both significantly associated with U.S. concentration measures. The tests are based on the *p*-values from model specifications (4), (5), (6) in Table 2 and (3), (4), (5) Table 3. The uncorrected critical *p*-value is set to be 0.10. The corrected *p*-value and the rejection decision of the Sidak method and the Holland-Copenhaver method are presented.

Panel A: NHHI Results			Sidak		Holland–Copenhaver	
	Coefficient	P-Value	Critical P-Value	Reject	Critical P-Value	Reject
Net Alpha--Lagged DAFMI NHHI	-0.7435	0.0000	0.0513	Yes	0.0513	Yes
DAFMI Size--Lagged DAFMI NHHI	-0.0020	0.2228	0.0513	No	0.1000	No
Net Alpha--Lagged U.S. NHHI	-137.8691	0.0000	0.0513	Yes	0.0513	Yes
DAFMI Size--Lagged U.S. NHHI	-0.1561	0.0122	0.0513	Yes	0.1000	Yes

Panel B: HHI Results			Sidak		Holland–Copenhaver	
	Coefficient	P-Value	Critical P-Value	Reject	Critical P-Value	Reject
Net Alpha--Lagged DAFMI HHI	-0.7267	0.0009	0.0513	Yes	0.0513	Yes
DAFMI Size--Lagged DAFMI HHI	-0.0015	0.1582	0.0513	No	0.1000	No
Net Alpha--Lagged U.S. HHI	-137.8569	0.0000	0.0513	Yes	0.0513	Yes
DAFMI Size--Lagged U.S. HHI	-0.1751	0.0158	0.0513	Yes	0.1000	Yes

Panel C: 5FI Results			Sidak		Holland–Copenhaver	
	Coefficient	P-Value	Critical P-Value	Reject	Critical P-Value	Reject
Net Alpha--Lagged DAFMI 5FI	-0.1680	0.2902	0.0513	No	0.1000	No
DAFMI Size--Lagged DAFMI 5FI	-0.0010	0.0797	0.0513	No	0.0513	No
Net Alpha--Lagged U.S. 5FI	-12.9864	0.0000	0.0513	Yes	0.0513	Yes
DAFMI Size--Lagged U.S. 5FI	-0.0149	0.0087	0.0513	Yes	0.1000	Yes

Table 5 reports the results of the concentrations–net alpha relations and the concentrations–DAFMI size relations in Equations (32) and (34), respectively, for individual DAFMI markets.[42] We show only the results of relevant coefficients. We also conduct similar multiple hypothesis tests with the same critical *p*-values and check, for each individual DAFMI market, whether β_1 in Equation (32) and β_1 in Equation (34) are both significantly different from zero and whether β_2 in Equation (32) and β_2 in Equation (34) are both significantly different from zero. The statistics of multiple hypothesis tests are unreported for brevity.

We find that in six (one) DAFMI markets, Austria, Germany, Israel, Japan, Spain, and the United Kingdom (Australia), on average, fund net alphas and sizes are both significantly negatively (positively) associated with *U.S. NHHI*, whereas nine DAFMI markets' fund net alphas and size are both insignificantly associated with *U.S. NHHI*. On the other hand, on average, in only one (two) DAFMI market, Finland (Chile and Taiwan), fund net alpha is significantly negatively (positively) associated with *U.S. NHHI*, but its size is significantly positively (negatively) associated with *U.S. NHHI*. These results show that, in general, DAFMI markets' fund net alphas and size are more likely to move in the same direction when *U.S. NHHI* changes, than to move in opposite directions. This finding is consistent with the prediction of our theoretical model under both the exogenous concentration framework and the endogenous concentration framework.

Regarding the association with the *DAFMI NHHI*, we find that in only three (one) DAFMI markets, Germany, Greece, and Singapore (Australia) fund net alphas and size are both significantly negatively (positively) associated with *DAFMI NHHI*. Also, five DAFMI markets' fund net alphas and size are both insignificantly associated with *DAFMI NHHI*. On the other hand, on average, in three (one) DAFMI markets, Hong Kong, Sweden, and the United Kingdom (Spain), fund net alpha is significantly positively (negatively) associated with *DAFMI NHHI*, but size is significantly negatively (positively) associated with *DAFMI NHHI*.

In unreported robustness checks, we replace *NHHI* by *HHI* and *5FI*, and redo these tests. We find consistent results.

The current low and probably decreasing concentration in the U.S. AFMI, given the tradeoff of higher U.S. AFMI concentration is not changed, would benefit (harm) other global DAFMI markets whose fund net alphas and size are, on average, negatively (positively) associated with the *U.S. NHHI*. Our results show that, on average, other global DAFMI markets

[42] When testing the concentrations–net alpha relation for each DAFMI market, we do not include the fund market share as a control because it is insignificant in the tests and including it significantly reduces the sample sizes of some small markets. We exclude the fund market share in the controls to reduce noise in the estimations.

in our sample are likely to benefit from the decreasing concentration in the U.S. AFMI.

Robustness Check: Regarding European Union Markets as One DAFMI Market

Investors in the European Union (EU) might invest in active funds of any of the EU member countries with low transaction costs due to the policies, such as the same currency used in EU and the low capital control inside the EU. Thus, we regard the EU markets in our sample as one DAFMI market, calculate its concentration measures and DAFMI size, and redo our analyses. The EU markets in our sample include Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Netherlands, Portugal, Spain, Sweden, and the United Kingdom, and all of these 13 countries joined the EU before the start of our sample period.

After this combination, we have a total of 18 DAFMI markets in our sample. In unreported tests, we redo our analyses in Table 2 and Table 3, and find consistent results in the signs, magnitudes, and significances of the coefficients.

In unreported tests, we also perform the analyses of Table 5 on the EU market, and check how its fund net alphas and DAFMI size change with concentrations. We find that in the EU market, the DAFMI net alphas and size are both significantly and negatively associated with *DAFMI NHHI* and *U.S. NHHI* (based on results of similar multiple hypothesis tests), and the effect of a small change in *U.S. NHHI* are much larger than the effects of the same magnitude change in *DAFMI NHHI*. These results also imply that we are not in one global village, consistent with what we have found before.

We do not combine other DAFMI markets. For example, we do not combine the Asian markets into one DAFMI market. This is because large transaction costs would discourage investors in one market (e.g., mainland China) from investing in active funds operating in another market (e.g., Japan), making each of these DAFMI markets an individual market in practice.

Table 5 Results of Each Market.

Panel A shows the results of the concentrations–net alpha relations for each DAFMI market, and the dependent variable is fund *Net Alpha*. Standard errors are clustered by fund and presented in parentheses. Panel B shows the results of the concentrations–DAFMI size relations for each market, and the dependent variable is *DAFMI Size*. Newey-West estimates of standard error with a maximum lag of 12 months are used and presented in parentheses. In both panels, concentrations are measured by *NHHI*, and only the coefficients of the *DAFMI NHHI* and the *U.S. NHHI* are reported. Control variables in the tests of Panel A and B are the same as those in model specification (3) of Table 2 and Table 3, respectively. Control variables' coefficients are omitted here for brevity. The symbols ***, **, and * represent the 1%, 5%, and 10% significant level, respectively, in a two-tail *t*-test.

Panel A: Results of concentrations–net alpha relations

	Lagged DAFMI NHHI		*Lagged U.S. NHHI*		Number of	R-squared
Global Market	Coefficient	Standard Error	Coefficient	Standard Error	Observations	
Australia	125.9292***	(16.5322)	309.5224***	(50.9527)	28,029	0.0438
Austria	5.1260***	(0.9187)	-2,647.3727***	(391.4927)	564	0.0556
Belgium	1.8650***	(0.6542)	-9.1151	(49.7206)	2,668	0.0073
Brazil	0.5451	(3.8677)	-96.1587*	(52.2873)	11,219	0.0065
Canada	-181.1875***	(22.0536)	-173.1231***	(26.7814)	34,675	0.0103
Chile	-66.6638**	(25.2895)	560.1942**	(223.1512)	1,475	0.0545
China (Mainland)	-16.9914	(31.0978)	-1,026.2887	(821.8890)	321	0.0383
Denmark	4.1505	(4.5418)	-61.8472	(67.0540)	2,763	0.0123
Finland	4.0763***	(0.7630)	-250.5316***	(41.9122)	3,329	0.0216
France	6.0672***	(1.7516)	113.6250***	(18.5782)	22,144	0.0106
Germany	-4.2801***	(0.7191)	-185.8201***	(34.0415)	10,541	0.0124
Greece	-1.3648***	(0.3569)	-54.5851	(78.0598)	1,639	0.0134
Hong Kong	0.5265**	(0.2475)	-31.2086	(108.8858)	2,125	0.0132
India	4.3950***	(0.8540)	55.8574	(53.6853)	10,981	0.0257
Israel	44.3736***	(5.5980)	-555.0263***	(61.6503)	8,384	0.0306
Italy	-7.7472***	(2.3477)	-167.6262***	(50.5915)	5,554	0.0037
Japan	-0.6904***	(0.2309)	-155.3339***	(17.8868)	35,407	0.0098
Korea	99.0272***	(7.2094)	-131.6193***	(39.3773)	16,609	0.0235
Mexico	-1.2081	(1.7108)	-595.0665***	(116.2500)	3,289	0.0393
Netherlands	-9.7028*	(5.0168)	-14.5984	(51.6035)	1,861	0.0164
Norway	-23.9439***	(2.8096)	-347.1046***	(38.0491)	5,724	0.0280
Portugal	29.3836***	(4.2338)	-962.4869***	(85.5555)	1,761	0.0504
Singapore	-0.8709**	(0.3721)	50.6522	(100.5254)	1,705	0.0196
South Africa	30.6163***	(7.0941)	-276.1948***	(30.7909)	8,951	0.0247
Spain	-18.9087***	(2.2604)	-127.5332***	(29.1495)	9,877	0.0184
Sweden	1.6164**	(0.6218)	34.8206	(22.7690)	10,965	0.0048
Switzerland	5.3854***	(1.7370)	26.1425	(27.9872)	12,724	0.0065
Taiwan	221.3754***	(56.7624)	434.9734***	(89.0874)	7,210	0.0083
Thailand	12.6860***	(2.4723)	-155.7064***	(36.4270)	14,045	0.0182
United Kingdom	1.8723***	(0.2298)	-32.4324*	(17.9394)	48,779	0.0048

Panel B: Results of concentrations–DAFMI size relations

Global Market	*Lagged DAFMI NHHI* Coefficient	*Lagged DAFMI NHHI* Standard Error	*Lagged U.S. NHHI* Coefficient	*Lagged U.S. NHHI* Standard Error	Number of Observations	R-squared
Australia	0.6249***	(0.2359)	0.7778*	(0.4535)	144	0.8559
Austria	0.0028	(0.0026)	-0.7475*	(0.4508)	144	0.8266
Belgium	0.0021	(0.0021)	-0.0868	(0.0853)	143	0.8852
Brazil	0.0061	(0.0070)	0.2491	(0.1517)	144	0.8845
Canada	0.1890	(0.2570)	-0.1675	(0.2102)	144	0.8559
Chile	0.1389	(0.0843)	-0.8077**	(0.4009)	144	0.9672
China (Mainland)	-0.0007	(0.0008)	0.0854	(0.0557)	138	0.8222
Denmark	0.0074	(0.0181)	0.2887	(0.1745)	144	0.8703
Finland	-0.0010	(0.0020)	0.1541**	(0.0777)	144	0.9817
France	-0.0040	(0.0248)	0.1278	(0.1567)	144	0.9281
Germany	-0.0019***	(0.0005)	-0.0478***	(0.0163)	144	0.2883
Greece	-0.0072***	(0.0021)	-1.7732***	(0.5678)	144	0.7541
Hong Kong	-0.0002**	(0.0001)	-0.0416***	(0.0133)	129	0.9885
India	-0.0006	(0.0009)	0.1122	(0.0733)	143	0.9139
Israel	0.1948	(0.1251)	-4.9103*	(2.7980)	144	0.8292
Italy	0.0086	(0.0094)	-0.1340	(0.1474)	144	0.9417
Japan	0.0006	(0.0007)	-0.0875**	(0.0439)	144	0.9049
Korea	0.3201	(0.3657)	0.1406	(0.7746)	144	0.8945
Mexico	-0.0001*	(0.0001)	-0.0055	(0.0047)	143	0.9780
Netherlands	-0.0001	(0.0028)	0.0811	(0.2892)	144	0.0019
Norway	0.0063	(0.0088)	0.0357	(0.1987)	144	0.9772
Portugal	0.0223	(0.0138)	-0.1995	(0.1405)	116	0.9015
Singapore	-0.0003*	(0.0002)	-0.0426	(0.0411)	144	0.9604
South Africa	-0.0066	(0.0222)	0.0809	(0.0669)	144	0.9682
Spain	0.0051*	(0.0028)	-0.0915***	(0.0309)	144	0.9928
Sweden	-0.0226*	(0.0131)	-0.7917	(0.5138)	144	0.9533
Switzerland	-0.0085	(0.0081)	-0.0764	(0.2098)	144	0.7317
Taiwan	0.7602	(0.6852)	-1.0715*	(0.6254)	144	0.9627
Thailand	0.0161	(0.0236)	-0.2496	(0.2070)	144	0.9701
United Kingdom	-0.0085***	(0.0025)	-0.9730***	(0.2742)	144	0.9197

4 Conclusion

We introduce a theoretical model of IAFMI equilibrium with which we investigate DAFMI performance, size, and managerial efforts under a continuum of DAFMI and FAFMI concentrations. Utilizing PS's and FSX's single-country frameworks, in which fund managers have *homogeneous* incentives (effort productivities and costs) while searching for investment opportunities, we create a two-country IAFMI framework in which in each country, due to transaction costs, information costs, and taxes, investors invest only in DAFMI funds, whereas fund managers invest in both domestic and foreign stock markets facing *heterogeneous* incentives. Gross alpha production and managerial effort costs depend on concentrations. In particular, higher FAFMI concentration implies higher incentives to explore investment opportunities in the foreign stock market because effort spent there is more productive. Moreover, this FAFMI concentration increase also induces incentives to divert managerial effort from the domestic stock market to the foreign one, affecting, in turn, incentives to spend effort to explore investment opportunities in the domestic stock market. By symmetry, higher DAFMI concentration induces similar effects.

Our model's comparative statics characterize the association between DAFMI expected net alphas and a continuum of DAFMI and FAFMI concentrations, and that between DAFMI size and a continuum of DAFMI and FAFMI concentrations. In particular, we show that, in equilibrium, if and only if higher FAFMI concentration induces higher (lower) DAFMI direct benefits does it induce higher (lower) DAFMI fund expected net alphas and size. By symmetry, a similar necessary and sufficient condition holds for higher DAFMI concentration.

In addition, the concavity of DAFMI fund expected net alphas in FAFMI concentration indicates that DAFMI direct benefits of effort are concave in FAFMI concentration. This further induces concavity of DAFMI size in FAFMI concentration. On the other hand, equilibrium convex DAFMI size in FAFMI concentration implies convex direct benefits in FAFMI concentration and, consequently, convex DAFMI fund expected net alphas in FAFMI concentration. By symmetry, similar second-order results hold for DAFMI concentration.

We specialize our model to allow for endogenous concentrations, which befits empirical market concentration measures, thus facilitating empirical studies. Although the relation between DAFMI concentration and DAFMI expected net alpha, and that between DAFMI concentration and DAFMI size, become more complex in this framework, we are still able to conclude that DAFMI fund expected net alphas and size, in equilibrium, move in the same direction as FAFMI concentration.

We use the data of 30 active equity mutual fund markets in Morningstar Direct to test our theoretical findings. We find that, on average, DAFMI fund net alphas and size decrease with the U.S. AFMI concentration. As FSX find that U.S. AFMI fund net alphas and size increase in U.S. AFMI concentration, our findings suggest that we do not live in one global village.

Our findings provide relevant implications for fund managers, investors, and regulators. If market parameters leading to the current equilibrium persist, the current low, and probably decreasing, concentration in the U.S. AFMI would benefit (harm) other global DAFMI markets whose fund net alphas and size are, on average, negatively (positively) associated with the U.S. AFMI concentration. Our empirical results suggest that, on average, other global DAFMI markets would benefit from the declining U.S. AFMI concentration.

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APPENDIX FOR ONLINE PUBLICATION

Proof of Managers' Maximization Problems Equivalence: Profits and Expected Net Alpha

We prove that when DAFMI managers maximize fund expected net alphas, they maximize profits and must do so in order to survive (that is, have wealth to manage and be solvent).[43] We also show that this maximization leads to a unique Nash equilibrium.

First, we establish that all managers offer the same level of fund expected net alpha. This is the case in PS and FSX as well, and the rationale here is the same: managers who offer expected net alpha that is lower than the highest offered by some other manager attract no investments, as diversification benefits are irrelevant, negligible to risk-averse investors, and, thus, out of the DAFMI.

Next, we show that DAFMI managers' competition drives the DAFMI (unique) expected net alphas to be the highest possible one, in which managers are still solvent; that is, managers charge break-even fees.

Suppose that managers choose profit maximizing optimal effort and fees to set DAFMI funds expected net alpha to be $\bar{\alpha}$. Without loss of generality, we assume that $\bar{\alpha}$ is between zero and the highest expected net alpha that allows solvency. We show that, in equilibrium, $\bar{\alpha}$ is the maximum fund expected net alpha that mangers can produce (while staying solvent). They do that by choosing optimal efforts $e_i^{11^*}$ and $e_i^{12^*}$ (by fulfilling the condition in Lemma 1.2) and charging a fee f_i^1 such that the fund expected net alpha is exactly $\bar{\alpha}$. Substituting $\bar{\alpha}$ into Equation (10) (our "state" equation that links efforts, alpha production functions, fees, and fund expected net alphas) yields

$$f_i^1 = \widehat{a^1} - \widehat{b^1}\frac{S^{1^*}}{W^1} + A^{11}\left(e_i^{11^*};\ H^1, H^2\right) + A^{12}\left(e_i^{12^*};\ H^1, H^2\right) - \bar{\alpha}. \tag{35}$$

Denote the profit rate of manager i, as pro_i^1, $pro_i^1 \triangleq f_i^1 - C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^1, H^1, H^2\right)$. Then, from the last definition and equation (35), we have

$$\begin{aligned}\bar{\alpha} = \widehat{a^1} - \widehat{b^1}\frac{S^{1^*}}{W^1} + A^{11}\left(e_i^{11^*};\ H^1, H^2\right) + A^{12}\left(e_i^{12^*};\ H^1, H^2\right) - pro_i^1 - \\ c_0^1 - c_{1,i}^1 s_i^1 - c_2^{11}\left(e_i^{11^*};\ H^1, H^2\right) - c_2^{12}\left(e_i^{12^*};\ H^1, H^2\right).\end{aligned} \tag{36}$$

As all managers produce the same expected net alphas, Equation (36) implies an equilibrium condition,

[43] By our model assumptions, insolvent managers are out of the DAFMI.

$$pro_i^1 + c_{1,i}^1 s_i^1 = pro_j^1 + c_{1,j}^1 s_j^1,\ \forall i,j. \quad (37)$$

Next, we consider manager i's total dollar profit function (size in dollars times the per dollar profit rate):

$$s_i^1 pro_i^1 = s_i^1[f_i^1 - c_0^1 - c_{1,i}^1 s_i^1 - c_2^{11}(e_i^{11^*};\ H^1, H^2) - c_2^{12}(e_i^{12^*};\ H^1, H^2)], \quad (38)$$

and by the first-order condition, the optimal fund size given manager i's profit is

$$s_i^{1^{opt}} = \frac{f_i^1 - c_0^1 - c_2^{11}(e_i^{11^*};\ H^1, H^2) - c_2^{12}(e_i^{12^*};\ H^1, H^2)}{2c_{1,i}^1} = \frac{pro_i^1}{2c_{1,i}^1} + \frac{s_i^1}{2}. \quad (39)$$

The latter equality is useful in presenting the optimal size relative to current size. Note that if manager i maximizes her fund's expected net alpha, the profit rate $pro_i^1 = 0$, and the condition in Equation (39) for $s_i^{1^{opt}}$ does not exist. For some manager j, $j \neq i$, it is possible that pro_j^1 is so high that $s_j^1 < s_j^{1^{opt}}$. In other words, it might be possible that some manager j, $j \neq i$, increases his (dollar) profits by increasing his fund expected net alpha, reducing profit rates and increasing (his fund) size. As manager i does not observe other managers' cost functions,[44] she must consider the above possibility [to avoid losing (all) the wealth she manages].

We now demonstrate that the possible scenario described above indeed occurs. We analyze a simple game between manager i and all other managers, denoted "$-i$". The actions of this game are to either maintain expected net alpha or improve it by an infinitesimal amount. Throughout, we assume that the diversification benefits of investing in both manager i and manager $-i$ are negligible. The payoffs are the profits of the two managers.

If manager i improves her fund expected net alpha infinitesimally and manager $-i$ does not follow, then manager i's profit change by an infinitesimal amount, say ε_i^1, and manager $-i$ receives no investments and earns no profits. If, on the other hand, manager i does not follow manager $-i$ when increasing her fund's expected net alpha infinitesimally, then manager $-i$ profits change by ε_{-i}^1, and manager i receives no investments and earns no profits. Suppose that manager i believes that manager $-i$'s strategy is to improve his or her fund expected net alpha, $\bar{\alpha}$ with (nontrivial) probability p and to maintain $\bar{\alpha}$ with probability $1-p$. Suppose that manager i's strategy is to improve her fund expected net alpha with probability θ and maintain $\bar{\alpha}$ with probability $1-\theta$.

[44] If cost functions were common knowledge, each manager could have calculated the DAFMI equilibrium independently.

The payoffs of such a game are illustrated in the following table, with the row (column) representing manager i's ($-i$'s) action, and with manager i's ($-i$'s) payoffs in the first (second) figures in the brackets.[45]

		Maintain $\bar{\alpha}$	Improve Infinitesimally
		$1-p$	p
Maintain $\bar{\alpha}$	$1-\theta$	$(pro_i^1 s_i^1, pro_{-i}^1 s_{-i}^1)$	$(0, pro_{-i}^1 s_{-i}^1 + \varepsilon_{-i}^1)$
Improve Infinitesimally	θ	$(pro_i^1 s_i^1 + \varepsilon_i^1, 0)$	$(pro_i^1 s_i^1 + \varepsilon_i^1, pro_{-i}^1 s_{-i}^1 + \varepsilon_{-i}^1)$

We show that in this game, manager i optimally chooses $\theta = 1$, until reaching the highest fund expected net alphas. (This is the break-even/zero-profits point, beyond which the manager becomes insolvent.) As manager i is a generic manager, this implies that all mangers do that. We also show that once managers reach the point of producing the highest fund expected net alphas, they are in (a Nash) equilibrium.

The expected payoff of manager i is[46]

$$\pi_i^1 = (1-p)[(1-\theta)pro_i^1 s_i^1 + \theta(pro_i^1 s_i^1 + \varepsilon_i^1)] + p\theta(pro_i^1 s_i^1 + \varepsilon_i^1). \qquad (40)$$

The first-order condition is

$$\frac{d\pi_i^1}{d\theta} = \varepsilon_i^1 + p * pro_i^1 s_i^1. \qquad (41)$$

Equation (56) shows that $\varepsilon_i^1 \to 0$ implies that $d\pi_i^1/d\theta > 0$. Thus, manager i's optimal choice to maximize π_i^1 is $\theta = 1$. That is, increasing fund expected net alphas increases profits.

As managers keep increasing fund expected net alphas, they reach a level of fund expected net alpha where $\bar{\alpha}$ is the maximum fund expected net alpha. At this point, managers' profit rates must be zero (otherwise managers could use profits to increase fund expected net alphas). Moreover, further increases of fund expected net alphas (by increasing efforts or decreasing fees) make managers insolvent. Thus, at this point, when $\bar{\alpha}$ is the optimal fund expected net alpha, ε_i^1 and ε_{-i}^1 are negative. Managers are, then, in a Nash equilibrium (Maintain $\bar{\alpha}$, Maintain $\bar{\alpha}$).

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[45] For simplicity and brevity, we do not introduce new notation to differentiate the infinitesimal profit changes when one or two players move. We use ε_i^1 and ε_{-i}^1 in both cases.

[46] Generally, ε_i^1 and ε_{-i}^1 may be positive or negative, which does not affect our results as they approach zero. If the infinitesimal profit change for manager i, when both players move, was denoted δ_i^1, Equation (56) would have been $\frac{d\pi_i}{d\theta} = \varepsilon_i^1 + p(\delta_i^1 - \varepsilon_i^1) + p \times pro_i s_i$, yielding the same result as ε_i^1 and δ_i^1 approach zero.

Therefore, each manager will improve his or her fund expected net alpha as long as it is below the maximum fund expected net alpha. Thus, managers' problems of maximizing profits is equivalent to maximizing their funds' expected net alphas.

Next, we show that that managers' optimization leads to a unique DAFMI equilibrium. Because at any fund expected net alpha below the maximizing level, managers attract no investments and have incentives to increase fund expected net alphas. Because further increasing fund expected net alpha above the maximizing level drives managers to insolvency, this Nash equilibrium is unique.

The proof for FAFMI managers is similar.

Q.E.D.

Proof of Proposition 0

$\{\mathbf{e^{11}}^*, \mathbf{e^{12}}^*, \mathbf{f^1}^*, \boldsymbol{\delta^1}^*\}$ is a Nash Equilibrium for the following reasons.

1. Given other DAFMI managers' optimal choices, a manager has incentives to not deviate from $\mathbf{e^{11}}^*$, $\mathbf{e^{12}}^*$, and $\mathbf{f^1}^*$. If a DAFMI manager deviates from $\mathbf{e^{11}}^*$, $\mathbf{e^{12}}^*$, and $\mathbf{f^1}^*$, this manager decreases the fund expected net alpha, either losing all investment or becoming insolvent. Also, DAFMI managers cannot deviate from both in offsetting ways and gain. This is because effort increases do not sufficiently improve performance to justify costs and fee increases, and effort reductions cause too great a loss of performance that cannot be returned to investors through fee reductions. The reason is that a manager's optimal effort and fee together determine his or her fund expected net alpha. If the DAFMI manager deviates from the equilibrium and produces a higher fund expected net alpha, he or she incurs a loss; and if the DAFMI manager deviates and produces a lower fund expected net alpha, he or she receives no investments. We proved these results in the previous proof of maximization problem equivalence.
2. Given DAFMI managers' and other DAFMI investors' optimal choices, a DAFMI investor has no incentive to deviate from $\boldsymbol{\delta_j^1}^*$. This is because, when there are infinitely many small mean-variance risk-averse investors, each investor's choice does not affect fund sizes and, thus, DAFMI size. Changing allocations across funds does not improve an DAFMI investor's portfolio Sharpe ratio, whereas changing allocations between the DAFMI and the passive benchmark decreases the portfolio Sharpe ratio.

$\{\mathbf{e^{11}}^*, \mathbf{e^{12}}^*, \mathbf{f^1}^*, \boldsymbol{\delta^1}^*\}$ is unique because

$\mathbf{e^{11}}^*$ is unique because, for each DAFMI fund, $e_i^{11^*}$ is the unique solution of

$B^{11}_{e_i^{11}}\left(e_i^{11^*};\ H^1, H^2\right) = 0;$

$\mathbf{e^{12}}^*$ is unique because, for each DAFMI fund, $e_i^{12^*}$ is the unique solution of $B^{12}_{e_i^{12}}\left(e_i^{12^*};\ H^1, H^2\right) = 0;$

$\mathbf{f^{1}}^*$ is unique because, for each DAFMI fund, $f_i^{1^*} - C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2\right) = 0$, where $C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2\right)$ is a deterministic function of $e_i^{11^*}$ and $e_i^{12^*}$**,** and $\mathbf{e^{11}}^*$ and $\mathbf{e^{12}}^*$ are unique;

$\boldsymbol{\delta^{1}}^*$ is unique because allocations to DAFMI funds maximize DAFMI investor portfolios' Sharpe ratios, driving fund expected net alphas to the same values. Deviating, thus, cannot help and to the extent that large deviation would affect fund sizes, they will decrease Sharpe ratios. Moreover, the uniqueness of $\mathbf{e^{11}}^*, \mathbf{e^{12}}^*$and $\mathbf{f^{1}}^*$ rules out the existence of additional equilibrium allocations. We show below (Proposition 2) that each $\boldsymbol{\delta_j^{1}}^*$ is the weights vector of DAFMI funds' "market portfolio."

The proof for FAFMI managers is similar.

Q.E.D.

Analytical Statement of Lemma 1

For every manager i, $i = 1, \dots, M^1$, if initial effort inputs generate positive direct benefits of effort [i.e., $B^{11}_{e_i^{11}}(0;\ H^1, H^2) > 0, B^{12}_{e_i^{12}}(0;\ H^1, H^2) > 0, \forall\ H^1, H^2$], the DAFMI equilibrium induced by managers choosing optimal effort-fee combinations, $(e_i^{11^*}, e_i^{12^*}, f_i^{1^*})$, has the following properties.

1. Fees are equal to costs:

$$f_i^{1^*} - C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2\right) = 0. \tag{42}$$

2. The impact of marginal effort, in either country, on gross alpha is set to be equal to the marginal average costs of effort in the respective country, thus manager i's marginal direct benefits of effort (in either country) under the optimal effort are zero:

$$\begin{aligned} A^{11}_{e_i^{11}}\left(e_i^{11^*};\ H^1, H^2\right) - c^{11}_{2\, e_i^{11}}\left(e_i^{11^*};\ H^1, H^2\right) = B^{11}_{e_i^{11}}\left(e_i^{11^*};\ H^1, H^2\right) = 0, \\ A^{12}_{e_i^{12}}\left(e_i^{12^*};\ H^1, H^2\right) - c^{12}_{2\, e_i^{12}}\left(e_i^{12^*};\ H^1, H^2\right) = B^{12}_{e_i^{12}}\left(e_i^{12^*};\ H^1, H^2\right) = 0. \end{aligned} \tag{43}$$

3. When either country's concentration is higher, DAFMI equilibrium optimal efforts in either country are higher (lower) if and only if higher concentration induces a larger (smaller) marginal effort impact on gross alphas than on costs in the respective country.

Or,

$$de_i^{11^*}/dH^1 \geq 0(<0) \text{ iff } A^{11}_{e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right) \geq 0(<0),$$

$$de_i^{12^*}/dH^1 \geq 0(<0) \text{ iff } A^{12}_{e_i^{12},H^1}\left(e_i^{12^*};\ H^1,H^2\right) - c^{12}_{2\ e_i^{12},H^1}\left(e_i^{12^*};\ H^1,H^2\right) \geq 0(<0),$$

$$de_i^{11^*}/dH^2 \geq 0(<0) \text{ iff } A^{11}_{e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right) \geq 0(<0),$$

$$de_i^{12^*}/dH^2 \geq 0(<0) \text{ iff } A^{12}_{e_i^{12},H^2}\left(e_i^{12^*};\ H^1,H^2\right) - c^{12}_{2\ e_i^{12},H^2}\left(e_i^{12^*};\ H^1,H^2\right) \geq 0(<0). \tag{44}$$

4. Whether each country's higher concentrations induce higher equilibrium optimal fees depends on whether they induce changes in equilibrium DAFMI sizes and in equilibrium optimal efforts in each country that are aggregately positive. Or,

$$\frac{df_i^{1^*}}{dH^1} = \left[\sum_{j=1}^{M^1}\left(c^1_{1,j}\right)^{-1}\right]^{-1} W^1 \frac{d\left(\frac{S^1}{W^1}\right)^*}{dH^1} + c^{11}_{2\ e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right)\frac{de_i^{11^*}}{dH^1}$$

$$+c^{12}_{2\ e_i^{12}}\left(e_i^{12^*};\ H^1,H^2\right)\frac{de_i^{12^*}}{dH^1} + c^{11}_{2\ H^1}\left(e_i^{11^*};\ H^1,H^2\right)$$

$$+c^{12}_{2\ H^1}\left(e_i^{12^*};\ H^1,H^2\right),$$

$$\frac{df_i^{1^*}}{dH^2} = \left[\sum_{j=1}^{M^1}\left(c^1_{1,j}\right)^{-1}\right]^{-1} W^1 \frac{d\left(\frac{S^1}{W^1}\right)^*}{dH^2} + c^{11}_{2\ e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right)\frac{de_i^{11^*}}{dH^2} \tag{45}$$

$$+c^{12}_{2\ e_i^{12}}\left(e_i^{12^*};\ H^1,H^2\right)\frac{de_i^{12^*}}{dH^2} + c^{11}_{2\ H^2}\left(e_i^{11^*};\ H^1,H^2\right)$$

$$+c^{12}_{2\ H^2}\left(e_i^{12^*};\ H^1,H^2\right).$$

5. When either country's concentrations are higher, equilibrium manager i's direct benefits of effort in the respective country are higher (lower) if and only if higher concentrations induce, in the respective country, a larger (smaller) impact on gross alphas than on costs. Or,

$$\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} \geq 0\ (<0) \text{ iff } A^{11}_{H^1}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\ H^1}\left(e_i^{11^*};\ H^1,H^2\right) \geq 0\ (< \tag{46}$$

0),

$$\frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1} \geq 0\ (<0) \text{ iff } A_{H^1}^{12}\left(e_i^{12^*};\ H^1, H^2\right) - c_{2\ H^1}^{12}\left(e_i^{12^*};\ H^1, H^2\right) \geq 0\ (<0),$$

$$\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} \geq 0\ (<0) \text{ iff } A_{H^2}^{11}\left(e_i^{11^*};\ H^1, H^2\right) - c_{2\ H^2}^{11}\left(e_i^{11^*};\ H^1, H^2\right) \geq 0\ (<0),$$

$$\frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2} \geq 0\ (<0) \text{ iff } A_{H^2}^{12}\left(e_i^{12^*};\ H^1, H^2\right) - c_{2\ H^2}^{12}\left(e_i^{12^*};\ H^1, H^2\right) \geq 0\ (<0).$$

6. Pairwise relative DAFMI fund sizes, $s_i^{1^*}/s_j^{1^*}$, are inversely proportional to their corresponding cost coefficients, $c_{1,j}^1/c_{1,i}^1$ (where $c_{1,i}^1$ is the intensity of fund-level decreasing returns to scale in gross alpha production).
7. DAFMI fund market shares, $s_i^{1^*}/S^{1^*}$s are $s_i^{1^*}/S^{1^*} = \left[c_{1,i}^1 \sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}$, $\forall i$.

Proof of Proposition 1 and Lemma 1.

The proof of Proposition 1.1 is in the Proof of 0.1.

To maximize $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{1^*}\}}$, manager i chooses the breakeven management fee. This is because choosing higher fee would decrease expected net alpha and choosing lower fee would induce insolvency. Moreover, changing both fees and efforts would move managers away from optimal effort. Thus,

$$f_i^{1^*} - C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2\right) = 0. \tag{47}$$

This proves Lemma 1.1.

If direct benefit of DAFMI manager i exerted to domestic stock market $B^{11}(e_i^{11};\ H^1, H^2)$ has a partial derivative with respect to effort, at zero effort, is positive, i.e., $B_{e_i^{11}}^{11}(0;\ H^1, H^2) = A_{e_i^{11}}^{11}(0;\ H^1, H^2) - c_{2\ e_i^{11}}^{11}(0;\ H^1, H^2) > 0$, then it pays to exert this effort, and the optimal level is positive, i.e., $e_i^{11^*} > 0$. The first-order condition, with respect to effort, to maximize $\mathrm{E}(\alpha_i^1|D)$ becomes

$$A_{e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right) - c_{2\ e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right) = B_{e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right) = 0. \tag{48}$$

The related second-order condition, $A_{e_i^{11}, e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right) -$

$c^{11}_{2\,e_i^{11},e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right) < 0$, is satisfied by assumptions. (This is because we assume that productivity effort decreases in scale, i.e., $A^{11}_{e_i^{11},e_i^{11}}(e_i^{11};\ H^1,H^2) < 0, \forall e_i^{11}$, and that the costs of effort increase in scale, i.e., $c^{11}_{2\,e_i^{11},e_i^{11}}(e_i^{11};\ H^1,H^2) > 0,, \forall e_i^{11}$). Thus, $e_i^{11^*}$ is a maximum. (We assume that functional forms of effort productivities and effort costs induce a finite $e_i^{11^*}$.)

By symmetries, the proof of the results regarding $e_i^{12^*}$ is similar to the one regarding $e_i^{11^*}$. This proves Lemma 1.2.

Fully differentiating (48) with respect to H^1 and H^2, we have

$$\frac{de_i^{11^*}}{dH^1} = -\frac{A^{11}_{e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\,e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right)}{A^{11}_{e_i^{11},e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\,e_i^{11},e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right)} \tag{49}$$

$$\frac{de_i^{11^*}}{dH^2} = -\frac{A^{11}_{e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\,e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right)}{A^{11}_{e_i^{11},e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\,e_i^{11},e_i^{11}}\left(e_i^{11^*};\ H^1,H^2\right)}. \tag{50}$$

Thus, the sign of $de_i^{11^*}/dH^1$ ($de_i^{11^*}/dH^2$) depends on the sign of $A^{11}_{e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\,e_i^{11},H^1}\left(e_i^{11^*};\ H^1,H^2\right)$ ($A^{11}_{e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right) - c^{11}_{2\,e_i^{11},H^2}\left(e_i^{11^*};\ H^1,H^2\right)$), because we have shown that the denominator of $de_i^{11^*}/dH^1$ ($de_i^{11^*}/dH^2$) is negative.

By symmetries, the proof of the results regarding $e_i^{12^*}$ is similar to the one regarding $e_i^{11^*}$. This proves Lemma 1.3.

The optimal DAFMI manager effort $e_i^{11^*}$ is determined only by the functions $A^{11}(e_i^{11};\ H^1,H^2)$ and $c_2^{11}(e_i^{11};\ H^1,H^2)$, which are the same across DAFMI funds. Thus, we have $e_i^{11^*} = e_j^{11^*}$ and, consequently, $B^{11}\left(e_i^{11^*};\ H^1,H^2\right) = B^{11}\left(e_j^{11^*};\ H^1,H^2\right), \forall i,j$. By symmetries, the proof of the results regarding $e_i^{12^*}$ is similar to the one regarding $e_i^{11^*}$, and we also have $e_i^{12^*} = e_j^{12^*}$ and, consequently, $B^{12}\left(e_i^{12^*};\ H^1,H^2\right) = B^{12}\left(e_j^{12^*};\ H^1,H^2\right), \forall i,j$. Because, in equilibrium, managers produce the (same) fund expected net alphas (Proposition 1.3, which we proved above in the Manager's Equivalence Problems theorem) and, as we just showed, exert the same optimal efforts (i.e., $e_i^{11^*} = e_j^{11^*}$ and $e_i^{12^*} = e_j^{12^*}, \forall i,j$), from the definition of fund net alpha in Equation (8), we have that $f_i^{1^*} = f_j^{1^*}, \forall i,j$.

These prove Proposition 1.7 and Proposition 1.8.

As $e_i^{11^*}$, $e_i^{12^*}$, and $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}}$ are the same across DAFMI funds, we further have $C_i^1(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2) = C_j^1(e_j^{11^*}, e_j^{12^*};\ s_j^{1^*}, H^1, H^2), \forall i, j$, and recall that $C_i^1(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2) = c_0^1 + c_{1,i}^1 s_i^{1^*} + c_2^{11}(e_i^{11^*};\ H^1, H^2) + c_2^{12}(e_i^{12^*};\ H^1, H^2)$. As c_0^1, $e_i^{11^*}$, $e_i^{12^*}$ and $C_i^1(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2)$ are the same across DAFMI funds, we have the following relationship between different funds' sizes and costs:

$$c_{1,i}^1 s_i^{1^*} = c_{1,j}^1 s_j^{1^*}, \forall i, j, \tag{51}$$

or $s_i^{1^*}/s_j^{1^*} = c_{1,j}^1/c_{1,i}^1, \forall i, j$.

This proves Lemma 1.6.

Summing $s_i^{1^*}/s_j^{1^*}$ with respect to i, $i = 1,2, \dots, M^1$, we have $\sum_{i=1}^{M^1} \frac{s_i^{1^*}}{s_j^{1^*}} = \frac{S^{1^*}}{s_j^{1^*}} = \sum_{i=1}^{M^1} \frac{c_{1,j}^1}{c_{1,i}^1}$. Inversing the second equality and exchanging the subscripts j and i gives

$$\frac{s_j^{1^*}}{S^{1^*}} = \left[c_{1,i}^1 \sum_{j=1}^{M^1} \left(c_{1,i}^1\right)^{-1}\right]^{-1}, \forall i. \tag{52}$$

This proves Lemma 1.7.

Using the break-even fee condition and Equation, we can write

$$\begin{aligned} f_i^{1^*} &= C_i^1(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2) \\ &= c_0^1 + c_{1,i}^1 s_i^{1^*} + c_2^{11}(e_i^{11^*};\ H^1, H^2) + c_2^{12}(e_i^{12^*};\ H^1, H^2) \\ &= c_0^1 + c_{1,i}^1 \frac{s_i^{1^*}}{S^{1^*}} \left(\frac{S^{1^*}}{W^1}\right) W^1 + c_2^{11}(e_i^{11^*};\ H^1, H^2) + c_2^{12}(e_i^{12^*};\ H^1, H^2) \\ &= c_0^1 + \left[\sum_{j=1}^{M^1} \left(c_{1,j}^1\right)^{-1}\right]^{-1} W^1 \left(\frac{S^{1^*}}{W^1}\right) + c_2^{11}(e_i^{11^*};\ H^1, H^2) \\ &\quad + c_2^{12}(e_i^{12^*};\ H^1, H^2). \end{aligned} \tag{53}$$

Fully differentiating $f_i^{1^*}$ with respect to H^1 and H^2, we have

$$\begin{aligned} \frac{df_i^{1^*}}{dH^1} = &\left[\sum_{j=1}^{M^1} \left(c_{1,j}^1\right)^{-1}\right]^{-1} W^1 \frac{d(S^1/W^1)^*}{dH^1} + c_{2\, e_i^{11}}^{11}(e_i^{11^*};\ H^1, H^2) \frac{de_i^{11^*}}{dH^1} \\ &+ c_{2\, e_i^{12}}^{12}(e_i^{12^*};\ H^1, H^2) \frac{de_i^{12^*}}{dH^1} + c_{2\, H^1}^{11}(e_i^{11^*};\ H^1, H^2) \\ &+ c_{2\, H^1}^{12}(e_i^{12^*};\ H^1, H^2), \end{aligned} \tag{54}$$

$$\frac{df_i^{1^*}}{dH^2} = \left[\sum_{j=1}^{M^1} \left(c_{1,j}^1\right)^{-1}\right]^{-1} W^1 \frac{d(S^1/W^1)^*}{dH^2} + c_{2\,e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right)\frac{de_i^{11^*}}{dH^2}$$ (55)

$$+ c_{2\,e_i^{12}}^{12}\left(e_i^{12^*};\ H^1, H^2\right)\frac{de_i^{12^*}}{dH^2} + c_{2\,H^2}^{11}\left(e_i^{11^*};\ H^1, H^2\right)$$

$$+ c_{2\,H^2}^{12}\left(e_i^{12^*};\ H^1, H^2\right).$$

Thus, whether each country's higher concentrations induce higher equilibrium optimal fees depends on whether they induce changes in equilibrium DAFMI sizes and in equilibrium optimal efforts in each country that are aggregately positive.

This proves Lemma 1.4.

Fully differentiate $B^{11}\left(e_i^{11^*};\ H^1, H^2\right)$ with respect to H^1 and H^2, and use the result $B_{e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right) = 0$, and we have

$$\frac{dB^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^1}$$

$$= B_{e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right)\frac{de_i^{11^*}}{dH^1} + A_{H^1}^{11}\left(e_i^{11^*};\ H^1, H^2\right)$$ (56)

$$- c_{2\,H^1}^{11}\left(e_i^{11^*};\ H^1, H^2\right)$$

$$= A_{H^1}^{11}\left(e_i^{11^*};\ H^1, H^2\right) - c_{2\,H^1}^{11}\left(e_i^{11^*};\ H^1, H^2\right)$$

$$\frac{dB^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^2}$$

$$= B_{e_i^{11}}^{11}\left(e_i^{11^*};\ H^1, H^2\right)\frac{de_i^{11^*}}{dH^2} + A_{H^2}^{11}\left(e_i^{11^*};\ H^1, H^2\right)$$ (57)

$$- c_{2\,H^2}^{11}\left(e_i^{11^*};\ H^1, H^2\right)$$

$$= A_{H^2}^{11}\left(e_i^{11^*};\ H^1, H^2\right) - c_{2\,H^2}^{11}\left(e_i^{11^*};\ H^1, H^2\right).$$

By symmetries, the proof of the results regarding $e_i^{12^*}$ is similar to the one regarding $e_i^{11^*}$. Thus, where either country concentrations are higher, equilibrium manager i's direct benefits of effort in the respective country are higher (lower) if and only if higher concentrations induce, in the respective country, a larger (smaller) impact on gross alphas than on costs.

This proves Lemma 1.5.

In equilibrium, all DAFMI funds' expected alphas are the same, i.e., $\mathrm{E}(\alpha_i^1 | D)\big|_{\{\mathbf{e^{11^*}}, \mathbf{e^{12^*}}, \mathbf{f^{1^*}}, \boldsymbol{\delta}^{1^*}\}}$ is the same across for all funds. Consequently, DAFMI fund expected returns $\mathrm{E}\left(r_{F,i}^1 \middle| D\right)\Big|_{\left\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\right\}} = \mathrm{E}\left(\alpha_i^1 \middle| D\right)\Big|_{\left\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\right\}} + \mu_p$ are the same in equilibrium. In addition, as DAFMI funds have the same expected alphas, they have the same expected returns. The source of DAFMI fund returns' variance is the same across funds, and $\mathrm{Var}\left(r_{F,i}^1 \middle| D\right)\Big|_{\left\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\right\}} = \sigma_p^2 + \sigma_{a^1}^2 + \left(\frac{S^{1^*}}{W^1}\right)^2 \sigma_{b^1}^2 + \sigma_x^2 + \sigma_\varepsilon^2$, $\forall i$. That is, the DAFMI fund return variance is the same across funds. Combining these results, we conclude that all managers offer the same competitive Sharpe ratio.

This proves Proposition 1.4 and 1.5.

We note that Proposition 1.3 is a direct consequence of Lemma 1.6 and 1.7.

Finally, to prove Proposition 1.2, recalling that aggregate skill is $\sum_{i=1}^{M^1}\left(c_{1,i}^1\right)^{-1}$, we differentiate $\frac{S^{1^*}}{W^1}$ by parts to get

$$\frac{d\,\frac{S^{1^*}}{W^1}}{d\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}} \frac{d\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}}{d\sum_{i=1}^{M^1}\left(c_{1,i}^1\right)^{-1}} > 0. \tag{58}$$

The inequality is correct because, from Equation (66), the first multiplicand of the LHS is negative, and because the variables in the second multiplicand of the LHS are positive, the second multiplicand is negative.

This proves Proposition 1.2.

This proves Proposition 1 except for Proposition 1.6, which is proved in the next section.

Q.E.D.

Proof of Proposition 1.6, Proposition 2, and Corollary to Proposition 2

DAFMI investor j's portfolio Sharpe ratio is

$$\frac{\mathrm{E}\left(r_j^1 \middle| D\right)}{\sqrt{\mathrm{Var}\left(r_j^1 \middle| D\right)}}$$

$$= \frac{\mu_p + \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}} \left[\widehat{a^1} - \widehat{b^1}\frac{S^{1\,*}}{W^1} + A^{11}\left(e_i^{11^*};\ H^1, H^2\right) + A^{12}\left(e_i^{12^*};\ H^1, H^2\right) - f_i^{1^*}\right]}{\sqrt{\sigma_p^2 + \left[\sigma_{a^1}^2 + \sigma_{x^1}^2 + \left(\frac{S^{1\,*}}{W^1}\right)^2 \sigma_{b^1}^2\right]\left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}}\right)^2 + \sigma_{\varepsilon^1}^2 \left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}}\right)}} \tag{59}$$

$$= \frac{\mu_p + \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}} \left\{-\frac{S^{1\,*}}{W^1}\left\{\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1 + \widehat{b^1}\right\} + X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)\right\}}{\sqrt{\sigma_p^2 + \left[\sigma_{a^1}^2 + \sigma_{x^1}^2 + \left(\frac{S^{1\,*}}{W^1}\right)^2 \sigma_{b^1}^2\right]\left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}}\right)^2 + \sigma_{\varepsilon^1}^2 \left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}}\right)}}.$$

The second equality holds because $f_i^{1^*} - C_i^1\left(e_i^{11^*}, e_i^{12^*};\ s_i^{1^*}, H^1, H^2\right) = 0$, the definition of $X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)$, $c_{1,i}^1 s_i^{1^*} = c_{1,i}^1 \frac{s_i^{1^*}}{S^{1^*}}\left(\frac{S^{1\,*}}{W^1}\right) W^1$, and Equation (52). We assume that marginal diversification benefits of investing in one more fund is trivial, so we set $\sigma_{\varepsilon^1}^2 \left(\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,\mathbf{T}} \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}}\right) \to 0$ when solving the problem. When maximizing DAFMI investor j's portfolio Sharpe ratio, we take the first-order condition with respect to $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}}$. We have

$$\frac{\mu_p}{\sigma_p^2}\left[\sigma_{a^1}^2 + \sigma_{b^1}^2 \left(\frac{S^{1\,*}}{W^1}\right)^2 + \sigma_{x^1}^2\right] \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,*\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}} - \left\{\left[\sum_{i=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1 + \widehat{b^1}\right\}\frac{S^{1\,*}}{W^1} + X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) = 0. \tag{60}$$

Notice that each small investor regards $\frac{S^{1\,*}}{W^1}$ as given since each of them cannot affect this ratio. Substitute $\gamma \triangleq \mu_p/\sigma_p^2$, $(\gamma > 0)$ and symmetric equilibrium condition $\frac{S^{1\,*}}{W^1} = \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,*\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}}$ into (60), we have

$$-\gamma\sigma_{b^1}^2\left(\frac{S^{1\,*}}{W^1}\right)^3 - \left\{\gamma\sigma_{a^1}^2 + \gamma\sigma_{x^1}^2 + \widehat{b^1} + \left[\sum_{i=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{S^{1\,*}}{W^1} + X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) = 0. \tag{61}$$

If the constraint $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}\,*\mathbf{T}} \boldsymbol{\iota}_{\mathbf{M^1}} \leq 1$ is not binding (i.e., $\frac{S^{1\,*}}{W^1} < 1$), the equilibrium optimal $\frac{S^{1\,*}}{W^1}$ is a real positive solution of this cubic equation. This is because the condition $X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right) > 0, \forall\, H^1, H^2$ (positivity of the lowest order polynomial coefficient) and the negativity of the two higher order polynomial coefficients $-\gamma\sigma_{b^1}^2$, and $-\Big\{\gamma\sigma_{a^1}^2 +$

$\gamma\sigma_{x^1}^2 + \widehat{b^1} + \left[\sum_{i=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\Big\}$, (i.e., $-\gamma\sigma_{b^1}^2 < 0$, and $-\Big\{\gamma\sigma_{a^1}^2 + \gamma\sigma_{x^1}^2 + \widehat{b^1} + \left[\sum_{i=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\Big\} < 0$) guarantee the existence of exactly one positive real solution for $\frac{S^{1^*}}{W^1}$ (and two imaginary ones). Also, as each DAFMI investor cannot affect the value of $\frac{S^{1^*}}{W^1}$, the cubic equation above shows that the solution for $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^{*\mathbf{T}}}\boldsymbol{\iota}_{\mathbf{M}^1} = \frac{S^{1^*}}{W^1}$ is unique given the parameter values and the market $\frac{S^{1^*}}{W^1}$.

If the constraint $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^{*\mathbf{T}}}\boldsymbol{\iota}_{\mathbf{M}^1} \leq 1$ is binding, (i.e., $\frac{S^{1^*}}{W^1} = 1$), there is an obviously unique solution where DAFMI investors maximize their portfolio Sharpe ratios by allocating all their wealth to the DAFMI (no international passive index holdings).

We, thus, demonstrated that $\boldsymbol{\delta}_{\mathbf{i}}^{\mathbf{1}^*} = \boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^*}, \forall i, j$, such that $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{1}^{*\mathbf{T}}}\boldsymbol{\iota}_{\mathbf{M}^1} = \frac{S^{1^*}}{W^1}$, induces a unique equilibrium.

This proves Proposition 1.6 and Proposition 2.2.

In addition, from the proofs above, we have shown that

$$\mathrm{E}\left(\alpha_i^1 \middle| D\right)\Big|_{\left\{\mathbf{e}^{\mathbf{11}^*}, \mathbf{e}^{\mathbf{12}^*}, \mathbf{f}^{\mathbf{1}^*}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}} = -\frac{S^{1^*}}{W^1}\left\{\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1 + \widehat{b^1}\right\} + X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right). \tag{62}$$

Also, by taking variance on both sides of the alpha production function, we have

$$\mathrm{Var}\left(\alpha_i^1 \middle| D\right)\Big|_{\left\{\mathbf{e}^{\mathbf{11}^*}, \mathbf{e}^{\mathbf{12}^*}, \mathbf{f}^{\mathbf{1}^*}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}} = \sigma_{a^1}^2 + \left(\frac{S^{1^*}}{W^1}\right)^2 \sigma_{b^1}^2. \tag{63}$$

By substituting (62) into (61) and rearranging, we have

$$\mathrm{E}\left(\alpha_i^1 \middle| D\right)\Big|_{\left\{\mathbf{e}^{\mathbf{11}^*}, \mathbf{e}^{\mathbf{12}^*}, \mathbf{f}^{\mathbf{1}^*}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}} = \frac{S^{1^*}}{W^1}\gamma\left[\sigma_{x^1}^2 + \mathrm{Var}\left(\alpha_i^1 \middle| D\right)\Big|_{\left\{\mathbf{e}^{\mathbf{11}^*}, \mathbf{e}^{\mathbf{12}^*}, \mathbf{f}^{\mathbf{1}^*}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}}\right]. \tag{64}$$

Because all the components on the right-hand side of the equation above are positive, we have $\mathrm{E}\left(\alpha_i^1 \middle| D\right)\Big|_{\left\{\mathbf{e}^{\mathbf{11}^*}, \mathbf{e}^{\mathbf{12}^*}, \mathbf{f}^{\mathbf{1}^*}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}} > 0$. The intuition is that a portfolio with allocations to DAFMI funds and the international passive benchmark is always riskier (i.e., higher portfolio return variance) than a portfolio with allocations only to the international passive benchmark. If $\mathrm{E}\left(\alpha_i^1 \middle| D\right)\Big|_{\left\{\mathbf{e}^{\mathbf{11}^*}, \mathbf{e}^{\mathbf{12}^*}, \mathbf{f}^{\mathbf{1}^*}, \boldsymbol{\delta}^{\mathbf{1}^*}\right\}} = 0$, because of a sufficiently large amount of investment in DAFMI funds, DAFMI investors can always improve their portfolio Sharpe ratios (in particular, reduce their portfolios risk) by shifting wealth from DAFMI to the international passive benchmark.

Thus, we should have $\mathrm{E}(\alpha_i^1|D)\big|_{\{\mathbf{e}^{11^*},\mathbf{e}^{12^*},\mathbf{f}^{1^*},\boldsymbol{\delta}^{1^*}\}} > 0$ to induce investments to DAFMI funds.

This proves Proposition 2.1.

This proves Proposition 2.

Where $\frac{S^{1^*}}{W^1} < 1$, fully differentiating (61) with respect to $X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)$ and $\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1$, respectively, we have

$$\frac{d\frac{S^{1^*}}{W^1}}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)} = \frac{1}{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^{1^*}}{W^1}\right)^2+\sigma_{a^1}^2+\sigma_{x^1}^2\right]+\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1} > 0, \tag{65}$$

$$\frac{d\frac{S^{1^*}}{W^1}}{d\left\{\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}} = \frac{-\frac{S^{1^*}}{W^1}}{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^{1^*}}{W^1}\right)^2+\sigma_{a^1}^2+\sigma_{x^1}^2\right]+\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1} < 0. \tag{66}$$

The inequalities above hold because the parameters are positive.

This proves the Corollary of Proposition 2.

Q.E.D.

Analytical Statement of Proposition 3

Sensitivities of DAFMI Size and Expected Net Alphas to Concentration.

Where $\frac{S^{1^*}}{W^1} < 1$, we have the following.[47]

1. Higher concentrations, in either country, induce larger (smaller) DAFMI equilibrium size and higher (lower) DAFMI equilibrium expected net alphas if and only if higher concentrations induce a larger (smaller) aggregate (over the two countries) impacts of induced optimal effort changes on gross alphas than on costs.

   The analytical statements of the verbal statements are as follows. Regarding DAFMI equilibrium size sensitivity to DAFMI concentration, we have

$$\frac{d\frac{S^{1^*}}{W^1}}{dH^1} = \frac{d\frac{S^{1^*}}{W^1}}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]; \tag{67}$$

   thus,

[47] When $\frac{S^{1^*}}{W^1} = 1$, it is the case that, 1. $\frac{S^{1^*}}{W^1}$ is unrelated to DAFMI and FAFMI concentrations; 2. higher DAFMI/FAFMI concentrations induce higher (lower) DAFMI/FAFMI equilibrium expected net alphas if and only if higher concentrations induces a larger (smaller) impact on gross alphas than on costs; and 3. DAFMI/FAFMI equilibrium expected net alphas are concave (convex), in DAFMI/FAFMI concentrations, if and only if the DAFMI/FAFMI equilibrium direct benefit function is concave (convex), in concentrations.

$$\frac{d\frac{S^{1}}{W^{1}}^{*}}{dH^{1}} \geq 0\ (<0) \text{ iff } \frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1} \geq 0\ (<0).$$

The analytical statements regarding DAFMI equilibrium size sensitivity to foreign concentration are

$$\frac{d\frac{S^{1}}{W^{1}}^{*}}{dH^{2}} = \frac{d\frac{S^{1}}{W^{1}}^{*}}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)}\left[\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2}\right]; \tag{68}$$

thus,

$$\frac{d\frac{S^{1}}{W^{1}}^{*}}{dH^{2}} \geq 0\ (<0) \text{ iff } \frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2} \geq 0\ (<0).$$

The analytical statements regarding DAFMI equilibrium expected net alpha sensitivity to DAFMI concentration are

$$\left.\frac{d\mathrm{E}\left(\alpha_i^1 \middle| D\right)}{dH^1}\right|_{\left\{\mathbf{e}^{11^*}, \mathbf{e}^{12^*}, \mathbf{f}^{1^*}, \boldsymbol{\delta}^{1^*}\right\}} = \left[\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1}\right]$$
$$\times \left\{1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\} \frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)}\right\}; \tag{69}$$

thus,

$$\left.\frac{d\mathrm{E}\left(\alpha_i^1 \middle| D\right)}{dH^1}\right|_{\left\{\mathbf{e}^{11^*}, \mathbf{e}^{12^*}, \mathbf{f}^{1^*}, \boldsymbol{\delta}^{1^*}\right\}} \geq 0\ (<0) \text{ iff } \frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^1} \geq 0\ (<0).$$

The analytical statements regarding DAFMI equilibrium expected net alpha sensitivity to foreign concentration are

$$\left.\frac{d\mathrm{E}\left(\alpha_i^1 \middle| D\right)}{dH^2}\right|_{\left\{\mathbf{e}^{11^*}, \mathbf{e}^{12^*}, \mathbf{f}^{1^*}, \boldsymbol{\delta}^{1^*}\right\}} = \left[\frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2}\right]$$
$$\times \left\{1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\} \frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*}; H^1, H^2\right)}\right\}; \tag{70}$$

thus,

$$\left.\frac{d\mathrm{E}\left(\alpha_i^1 \middle| D\right)}{dH^2}\right|_{\left\{\mathbf{e}^{11^*}, \mathbf{e}^{12^*}, \mathbf{f}^{1^*}, \boldsymbol{\delta}^{1^*}\right\}} \geq 0\ (<0) \text{ iff } \frac{dB^{11}\left(e_i^{11^*}; H^1, H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*}; H^1, H^2\right)}{dH^2} \geq 0\ (<0).$$

2. If concave in either country's concentration, DAFMI equilibrium direct benefits of efforts function indicates concave DAFMI equilibrium size in the respective concentration. (If convex in either country's concentration, DAFMI equilibrium size

indicates convex, DAFMI equilibrium direct benefits of efforts function in the respective concentration.) The sensitivity of equilibrium DAFMI size to the cross partial derivative of DAFMI and FAFMI concentrations depend on signs and sizes of several terms, including the sum of the sensitivities of DAFMI direct benefits due to efforts exerted in the domestic and foreign stock markets, to the cross partial derivative of DAFMI and FAFMI concentrations, and the product of the sums of DAFMI direct benefits sensitivities, due to efforts exerted in the domestic and foreign stock markets, to DAFMI and FAFMI concentrations, respectively.

The analytical statements of the above verbal statements regarding second-order sensitivity of equilibrium DAFMI size to DAFMI concentration are

$$\frac{d^2\frac{S^1}{W^1}^*}{dH^{1^2}} = \frac{d\frac{S^1}{W^1}^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}}\right] - 6\gamma^1\sigma_{b^1}^2\frac{S^1}{W^1}^*\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]^2\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3; \quad (71)$$

thus,

if $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}} \leq 0$ then $\frac{d^2\left(S^1/W^1\right)^*}{dH^{1^2}} \leq 0$, and if $\frac{d^2\left(S^1/W^1\right)^*}{dH^{1^2}} \geq 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}} \geq 0$.

The analytical statements regarding second-order sensitivity of DAFMI size to FAFMI concentration are

$$\frac{d^2\frac{S^1}{W^1}^*}{dH^{2^2}} = \frac{d\frac{S^1}{W^1}^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{2^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{2^2}}\right] - 6\gamma^1\sigma_{b^1}^2\frac{S^1}{W^1}^*\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^2}\right]^2\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3; \quad (72)$$

thus,

if $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{2^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{2^2}} \leq 0$ then $\frac{d^2\left(S^1/W^1\right)^*}{dH^{2^2}} \leq 0$, and if $\frac{d^2\left(S^1/W^1\right)^*}{dH^{2^2}} \geq 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{2^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{2^2}} \geq 0$.

The analytical statements regarding the cross partial derivative sensitivity of equilibrium DAFMI size to DAFMI and FAFMI concentrations are

$$\frac{d^2\frac{S^1}{W^1}^*}{dH^1dH^2} = \frac{d\frac{S^1}{W^1}^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1dH^2} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1dH^2}\right]$$

$$-6\gamma^1\sigma_{b^1}^2\frac{S^1}{W^1}^*\left[\frac{d\frac{S^1}{W^1}^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3 \tag{73}$$

$$\times\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^2}\right];$$

thus, the sign of the cross partial derivative of $\frac{S^1}{W^1}^*$ with respect to H^1 and H^2 depends on the signs and magnitudes of $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1dH^2} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1dH^2}$, $\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}$, and $\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^2}$.

3. Concave equilibrium expected net alphas in either country's concentration, indicates concave, in concentration, equilibrium direct benefit function. (Convex, in concentration, equilibrium direct benefit function indicates convex, in concentration, equilibrium expected net alphas.)

Similar to the case of equilibrium DAFMI size, the sensitivity of DAFMI equilibrium expected net alpha dependency on the cross partial derivative of DAFMI and AFMI concentrations depends on signs and sizes of several terms, including the sum of the sensitivities of DAFMI direct benefits due to efforts exerted in the domestic and foreign stock markets, to the cross partial derivative of DAFMI and FAFMI concentrations, and the product of the sums of DAFMI direct benefits sensitivities due to efforts exerted in the domestic and foreign stock markets, to DAFMI and FAFMI concentrations, respectively.

The analytical statements of the verbal statements regarding second-order sensitivity of equilibrium DAFMI expected net alpha to DAFMI concentration are

$$\left.\frac{d^2\mathrm{E}\left(\alpha_i^1\middle|D\right)}{d{H^1}^2}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\right\}} = \left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{d{H^1}^2} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{d{H^1}^2}\right]\Bigg\{1 -$$

$$\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\Bigg\} + 6\gamma\sigma_{b^1}^2\frac{S^1}{W^1}^*\Bigg\{\widehat{b^1} + \tag{74}$$

$$\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\Bigg\}\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]^2\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3;$$

thus,

if $\left.\frac{d^2\mathrm{E}\left(\alpha_i^1\middle|D\right)}{d{H^1}^2}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\right\}} \le 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{d{H^1}^2}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{d{H^1}^2} \le 0$ and

if $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{d{H^1}^2}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{d{H^1}^2} \ge 0$, then $\left.\frac{d^2\mathrm{E}\left(\alpha_i^1\middle|D\right)}{d{H^1}^2}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\right\}} \ge 0$.

(The fact that equilibrium expected net alpha is concave in H^1 indicates that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\, H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\, H^1,H^2\right)$ with respect to H^1 is negative, and the fact that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\, H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\, H^1,H^2\right)$ with respect to H^1 is positive indicates that equilibrium expected net alpha is convex in H^1.)

The analytical statements regarding second-order sensitivity of equilibrium DAFMI expected net alpha to FAFMI concentration are

$$\left.\frac{d^2\mathrm{E}\left(\alpha_i^1\middle|D\right)}{d{H^2}^2}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\right\}} = \left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{d{H^2}^2}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{d{H^2}^2}\right]\Bigg\{1 - \left\{\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\Bigg\} + 6\gamma\sigma_{b^1}^2\frac{{S^1}^*}{W^1}\Bigg\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\Bigg\}\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^2}+\frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^2}\right]^2\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3; \quad (75)$$

thus,

if $\left.\frac{d^2\mathrm{E}\left(\alpha_i^1\middle|D\right)}{d{H^2}^2}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\right\}} \le 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{d{H^2}^2}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{d{H^2}^2} \le 0$ and

if $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{d{H^2}^2}+\frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{d{H^2}^2} \ge 0$, then $\left.\frac{d^2\mathrm{E}\left(\alpha_i^1\middle|D\right)}{d{H^2}^2}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\right\}} \ge 0$.

(The fact that equilibrium expected net alpha is concave in H^2 indicates that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\, H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\, H^1,H^2\right)$ with respect to H^2 is negative, and the fact that the sum of the second-order derivatives of $B^{11}\left(e_i^{11^*};\, H^1,H^2\right)$ and $B^{12}\left(e_i^{12^*};\, H^1,H^2\right)$ with respect to H^2 is positive indicates that equilibrium expected net alpha is convex in H^2.)

The analytical statements regarding the cross partial derivative sensitivity of equilibrium DAFMI expected net alpha to DAFMI and FAFMI concentrations are

$$\left.\frac{d^2\mathrm{E}\left(\alpha_i^1\middle|D\right)}{dH^1dH^2}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}\mathbf{1}^*\right\}} = \left[\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1dH^2} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1dH^2}\right]\Bigg\{1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d\left(\frac{S^1}{W^1}\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\Bigg\} + 6\gamma\sigma_{b^1}^2 \frac{S^1}{W^1}^* \Bigg\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\Bigg\}\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^2}\right]\left[\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)}\right]^3, \tag{76}$$

thus, the sign of $\left.\frac{d^2\mathrm{E}\left(\alpha_i^1\middle|D\right)}{dH^1dH^2}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}\mathbf{1}^*\right\}}$ depends on the signs and magnitudes of $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1dH^2} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1dH^2}$, $\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}$, and $\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^2} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^2}$.

Proof of Proposition 3

Also, where $\frac{S^1}{W^1}^* < 1$, by the chain rule, we have

$$\frac{d\,\frac{S^1}{W^1}^*}{dH^1} = \frac{d\,\frac{S^1}{W^1}^*}{dX\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right)}\frac{dX\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right)}{dH^1}$$

$$= \frac{d\,\frac{S^1}{W^1}^*}{dX\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right)}\left[A_{H^1}^{11}\left(e_i^{11^*};\ H^1,H^2\right) + A_{H^1}^{12}\left(e_i^{12^*};\ H^1,H^2\right) - c_{2\,H^1}^{11}\left(e_i^{11^*};\ H^1,H^2\right) - c_{2\,H^1}^{12}\left(e_i^{12^*};\ H^1,H^2\right)\right] \tag{77}$$

$$= \frac{d\,\frac{S^1}{W^1}^*}{dX\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right)}\left[\frac{dB^{11}\left(e_i^{11^*};\ H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\ H^1,H^2\right)}{dH^1}\right].$$

Recall that $\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)} > 0$. Thus, we have that $\frac{d\left(S^1/W^1\right)^*}{dH^1} \geq 0\ (< 0)$ if and only if $\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1} \geq 0\ (< 0)$.

Fully differentiating $\frac{d\frac{S^{1}}{W^{1}}^{*}}{dH^{1}}$ with respect to H^{1} again, we have

$$\frac{d^{2}(S^{1}/W^{1})^{*}}{dH^{1^{2}}}=\frac{d(S^{1}/W^{1})^{*}}{dX\left(e_{i}^{11^{*}},e_{i}^{12^{*}};H^{1},H^{2}\right)}\left[\frac{d^{2}B^{11}\left(e_{i}^{11^{*}};H^{1},H^{2}\right)}{dH^{1^{2}}}+\frac{d^{2}B^{12}\left(e_{i}^{12^{*}};H^{1},H^{2}\right)}{dH^{1^{2}}}\right]+$$

$$\frac{d^{2}(S^{1}/W^{1})^{*}}{dX\left(e_{i}^{11^{*}},e_{i}^{12^{*}};H^{1},H^{2}\right)^{2}}\left[\frac{dB^{11}\left(e_{i}^{11^{*}};H^{1},H^{2}\right)}{dH^{1}}+\frac{dB^{12}\left(e_{i}^{12^{*}};H^{1},H^{2}\right)}{dH^{1}}\right]$$

(78)

$$=\frac{d(S^{1}/W^{1})^{*}}{dX\left(e_{i}^{11^{*}},e_{i}^{12^{*}};H^{1},H^{2}\right)}0\left[\frac{d^{2}B^{11}\left(e_{i}^{11^{*}};H^{1},H^{2}\right)}{dH^{1^{2}}}+\frac{d^{2}B^{12}\left(e_{i}^{12^{*}};H^{1},H^{2}\right)}{dH^{1^{2}}}\right]-$$

$$6\gamma\sigma_{b^{1}}^{2}\frac{S^{1}}{W^{1}}^{*}\left[\frac{dB^{11}\left(e_{i}^{11^{*}};H^{1},H^{2}\right)}{dH^{1}}+\frac{dB^{12}\left(e_{i}^{12^{*}};H^{1},H^{2}\right)}{dH^{1}}\right]^{2}\left[\frac{d(S^{1}/W^{1})^{*}}{dX\left(e_{i}^{11^{*}},e_{i}^{12^{*}};H^{1},H^{2}\right)}\right]^{3}.$$

The second equality holds because, by differentiating (65) with respective to $X\left(e_{i}^{11^{*}},e_{i}^{12^{*}};\ H^{1},H^{2}\right)$ again, we have

$$\frac{d^{2}(S^{1}/W^{1})^{*}}{dX\left(e_{i}^{11^{*}},e_{i}^{12^{*}};\ H^{1},H^{2}\right)^{2}}=-6\gamma\sigma_{b^{1}}^{2}\frac{S^{1}}{W^{1}}^{*}\left[\frac{d(S^{1}/W^{1})^{*}}{dX\left(e_{i}^{11^{*}},e_{i}^{12^{*}};\ H^{1},H^{2}\right)}\right]^{3} \tag{79}$$

and then substitute the result.

Notice that $6\gamma^{1}\sigma_{b^{1}}^{2}\frac{S^{1}}{W^{1}}^{*}>0$, $\left[\frac{dB^{11}\left(e_{i}^{11^{*}};H^{1},H^{2}\right)}{dH^{1}}+\frac{dB^{12}\left(e_{i}^{12^{*}};H^{1},H^{2}\right)}{dH^{1}}\right]^{2}>0$, and $\frac{d(S^{1}/W^{1})^{*}}{dX\left(e_{i}^{11^{*}},e_{i}^{12^{*}};H^{1},H^{2}\right)}>0$. Thus, if $\frac{d^{2}B^{11}\left(e_{i}^{11^{*}};H^{1},H^{2}\right)}{dH^{1^{2}}}+\frac{d^{2}B^{12}\left(e_{i}^{12^{*}};H^{1},H^{2}\right)}{dH^{1^{2}}}\leq 0$, then $\frac{d^{2}(S^{1}/W^{1})^{*}}{dH^{1^{2}}}\leq 0$, and if $\frac{d^{2}(S^{1}/W^{1})^{*}}{dH^{1^{2}}}\geq 0$, then $\frac{d^{2}B^{11}\left(e_{i}^{11^{*}};H^{1},H^{2}\right)}{dH^{1^{2}}}+\frac{d^{2}B^{12}\left(e_{i}^{12^{*}};H^{1},H^{2}\right)}{dH^{1^{2}}}\geq 0$.

Following similar mathematics, we can prove the results of $\frac{d(S^{1}/W^{1})^{*}}{dH^{2}}$, $\frac{d^{2}(S^{1}/W^{1})^{*}}{dH^{2^{2}}}$, and $\frac{d^{2}(S^{1}/W^{1})^{*}}{dH^{1}dH^{2}}$ where $\frac{S^{1}}{W^{1}}^{*}<1$.

Where $\frac{S^{1}}{W^{1}}^{*}=1$, $\frac{S^{1}}{W^{1}}^{*}$ does not depend on H^{1} or H^{2}.

Moreover, where $\frac{S^{1}}{W^{1}}^{*}<1$, fully differentiating (62) with respect to H^{1}, we have

$$\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$$

$$= -\frac{d(S^1/W^1)^*}{dH^1}\left\{\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1 + \widehat{b^1}\right\} + \left[\frac{dB^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^1}\right] \tag{80}$$

$$= \left[\frac{dB^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^1}\right]\left\{1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d(S^1/W^1)^*}{dX\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)}\right\}.$$

By (65), we have

$$1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d\left(\frac{S^1}{W^1}\right)^*}{dX\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)} = \frac{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^1}{W^1}^*\right)^2 + \sigma_{a^1}^2 + \sigma_{x^1}^2\right]}{\gamma\left[3\sigma_{b^1}^2\left(\frac{S^1}{W^1}^*\right)^2 + \sigma_{a^1}^2 + \sigma_{x^1}^2\right] + \widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1} > 0. \tag{81}$$

The last inequality holds because the values of all the parameters and variables in the equation are positive. Then, from the result of this inequality, Equation (80) implies that $\left.\frac{d\mathrm{E}\left(\alpha_i^1|D\right)}{dH^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}} \geq 0\ (< 0)$ if and only if $\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1} \geq 0\ (< 0)$.

Also, differentiate $\left.\frac{d\mathrm{E}\left(\alpha_i^1|D\right)}{dH^1}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\}}$ again with respect to H^1. Using the result of (79), we have

$$\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}}$$

$$= \left[\frac{d^2B^{11}\left(e_i^{11^*};\ H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};\ H^1,H^2\right)}{dH^{1^2}}\right]\Bigg\{1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d(S^1/W^1)^*}{dX\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right)}\Bigg\}$$

$$- \left[\frac{dB^{11}\left(e_i^{11^*};\ H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\ H^1,H^2\right)}{dH^1}\right]\left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d^2(S^1/W^1)^*}{dX\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right)^2} \tag{82}$$

$$= \left[\frac{d^2B^{11}\left(e_i^{11^*};\ H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};\ H^1,H^2\right)}{dH^{1^2}}\right]\Bigg\{1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d(S^1/W^1)^*}{dX\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right)}\Bigg\}$$

$$+ 6\gamma\sigma_{b^1}^2\frac{S^1}{W^1}^*\left[\frac{dB^{11}\left(e_i^{11^*};\ H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\ H^1,H^2\right)}{dH^1}\right]^2\left[\frac{d(S^1/W^1)^*}{dX\left(e_i^{11^*},e_i^{12^*};\ H^1,H^2\right)}\right]^3.$$

Notice that $6\gamma\sigma_{b^1}^2\frac{S^1}{W^1}^* > 0$, $\left[\frac{dB^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^1}\right]^2 > 0$, $\frac{d\left(S^1/W^1\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)} > 0$, and $1 - \left\{\widehat{b^1} + \left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1} W^1\right\}\frac{d\left(\frac{S^1}{W^1}\right)^*}{dX\left(e_i^{11^*},e_i^{12^*};H^1,H^2\right)} > 0$. Thus, if $\left.\frac{d^2\mathrm{E}\left(\alpha_i^1|D\right)}{dH^{1^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}} \leq 0$, then $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}} \leq 0$, and if $\frac{d^2B^{11}\left(e_i^{11^*};H^1,H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};H^1,H^2\right)}{dH^{1^2}} \geq 0$, then $\left.\frac{d^2\mathrm{E}\left(\alpha_i^1|D\right)}{dH^{1^2}}\right|_{\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta}^{1^*}\}} \geq 0$.

Where $\frac{S^{1^*}}{W^1} = 1$, fully differentiating (62) with respect to H^1, we have

$$\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\right|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{1^*}\}} = \frac{dX\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)}{dH^1} = = \frac{dB^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^1} \tag{83}$$

$$\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{1^*}\}} = \frac{d^2B^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^{1^2}}. \tag{84}$$

Following similar mathematics, we can prove the results of $\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^2}\right|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{1^*}\}}$, $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{2^2}}\right|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{1^*}\}}$, and $\left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^1dH^2}\right|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{1^*}\}}$.

Putting the results in this section together, where $\frac{S^{1^*}}{W^1} < 1$, we have

$$\frac{d\frac{S^{1^*}}{W^1}}{dH^1} \geq 0(<0) \Leftrightarrow = \frac{dB^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^1} + \frac{dB^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^1} \geq 0(<0) \Leftrightarrow \left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dH^1}\right|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{1^*}\}} \geq 0(<0) \tag{85}$$

$$\frac{d^2(S^1/W^1)^*}{dH^{1^2}} > 0 \Rightarrow \frac{d^2B^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^{1^2}} > 0 \Rightarrow \left.\frac{d^2\mathrm{E}(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{\mathbf{e^{11^*}},\mathbf{e^{12^*}},\mathbf{f^{1^*}},\boldsymbol{\delta}^{1^*}\}} > 0 \tag{86}$$

$$\left.\frac{d^2E(\alpha_i^1|D)}{dH^{1^2}}\right|_{\{e^{11^*},e^{12^*},f^{1^*},\delta^{1^*}\}} < 0 \Rightarrow \frac{d^2B^{11}\left(e_i^{11^*};\ H^1, H^2\right)}{dH^{1^2}} + \frac{d^2B^{12}\left(e_i^{12^*};\ H^1, H^2\right)}{dH^{1^2}} < 0 \Rightarrow \frac{d^2(S^1/W^1)^*}{dH^{1^2}} < 0. \tag{87}$$

This proves Proposition 3.

Q.E.D.

Proof of Proposition 4

Where $\frac{{S^1}^*}{W^1} < 1$, if we fully differentiate $\frac{{S^1}^*}{W^1}$ with respect to $c_{1,i}^1$, we have

$$\frac{d\frac{{S^1}^*}{W^1}}{dc_{1,i}^1} = \frac{-\frac{{S^1}^*}{W^1}W^1}{\left\{\gamma\left[3\sigma_{b^1}^2\left(\frac{{S^1}^*}{W^1}\right)^2+\sigma_{a^1}^2+\sigma_{x^1}^2\right]+\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\left(\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right)^2\left(c_{1,i}^1\right)^2} < 0. \tag{88}$$

The first equality holds because the derivative of $X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)$ with respect to $c_{1,i}^1$ is 0, and the last inequality holds because all the parameter and variable values in the equation above are positive and we have a negative sign in the numerator.

Also, fully differentiating $\mathrm{E}(\alpha_i^1|D)\big|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\right\}}$ with respect to $c_{1,i}^1$, and substituting the result of $\frac{d\frac{{S^1}^*}{W^1}}{dc_{1,i}^1}$ above, we have

$$\left.\frac{d\mathrm{E}(\alpha_i^1|D)}{dc_{1,i}^1}\right|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\right\}} = -\left\{\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\frac{d\left(\frac{S^1}{W^1}\right)^*}{dc_{1,i}^1} - \frac{-\left(\frac{S^1}{W^1}\right)^*W^1}{\left(\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right)^2\left(c_{1,i}^1\right)^2} \tag{89}$$

$$= \frac{-\gamma\left[3\sigma_{b^1}^2\left(\frac{{S^1}^*}{W^1}\right)^2+\sigma_{a^1}^2+\sigma_{x^1}^2\right]\frac{{S^1}^*}{W^1}W^1}{\left\{\gamma\left[3\sigma_{b^1}^2\left(\frac{{S^1}^*}{W^1}\right)^2+\sigma_{a^1}^2+\sigma_{x^1}^2\right]+\widehat{b^1}+\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1\right\}\left(\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right)^2\left(c_{1,i}^1\right)^2} < 0.$$

The last inequality holds because all the parameter and variable values in the equation above are positive and we have a negative sign in the numerator.

Where $\frac{{S^1}^*}{W^1} = 1$, $\frac{d\left(S^1/W^1\right)^*}{dc_{1,i}^1} = 0$, and as $\mathrm{E}(\alpha_i^1|D)\big|_{\left\{\mathbf{e^{11}}^*,\mathbf{e^{12}}^*,\mathbf{f^1}^*,\boldsymbol{\delta^1}^*\right\}} = -\left\{\left[\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right]^{-1}W^1+\widehat{b^1}\right\}+X\left(e_i^{11^*}, e_i^{12^*};\ H^1, H^2\right)$, we have

$$\left.\frac{d\mathrm{E}\left(\alpha_i^1 \middle| D\right)}{dc_{1,i}^1}\right|_{\left\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\right\}} = -\frac{-W^1}{\left(\sum_{j=1}^{M^1}\left(c_{1,i}^1\right)^{-1}\right)^2\left(c_{1,i}^1\right)^2} < 0. \tag{90}$$

The last inequality holds all the parameter and variable values are positive and we have a negative sign in the numerators. The result of $\left.\frac{d\mathrm{E}\left(\alpha_j^1 \middle| D\right)}{dc_{1,i}^1}\right|_{\left\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\right\}}, \forall j \neq i$ are similar.

This proves Proposition 4.2.

As $\frac{s_i^{1^*}}{S^{1^*}}$ decreases in $c_{1,i}^1$ whereas $\frac{s_j^{1^*}}{S^{1^*}}, \forall i \neq j$, increases in $c_{1,i}^1$, from results above, we find that $\mathrm{E}(\alpha_i^1 | D)|_{\left\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\right\}}$ and $\frac{s_i^{1^*}}{S^{1^*}}$ are increasing/decreasing in the same direction due to changes in $c_{1,i}^1$, and that $\mathrm{E}\left(\alpha_j^1 \middle| D\right)|_{\left\{\mathbf{e}^{\mathbf{11}^*},\mathbf{e}^{\mathbf{12}^*},\mathbf{f}^{\mathbf{1}^*},\boldsymbol{\delta}^{\mathbf{1}^*}\right\}}$ and $\frac{s_j^{1^*}}{S^{1^*}}, \forall i \neq j$ are increasing/decreasing inversely due to changes in $c_{1,i}^1$, whether $\frac{S^{1^*}}{W^1} < 1$ or $\frac{S^{1^*}}{W^1} = 1$.

This proves Proposition 4.1.

This proves Proposition 4.

Q.E.D.