Demand-supply imbalance risk and long-term swap spreads*

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Abstract

We develop a model in which long-term swap spreads are determined by preferred habitat investors' demand for swaps, constrained intermediaries' supply of swaps, and compensation for the risk that spreads temporarily widen due to future shocks to demand or supply. Empirically, we identify these separate demand and supply factors, and assess their respective contributions to the level of swap spreads and the returns on swap spread trades.

Keywords: limits to arbitrage, swap spreads, covered interest parity

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1 Introduction

An interest rate swap is an agreement to exchange a series of floating interest payments based on the future realizations of a short-term reference rate (the "floating leg") for a series of fixed interest payments (the "fixed leg"). With over $350 trillion of outstanding notional value as of 2020, the market for interest rate swaps is the world's largest derivatives market.[1] These swaps are used by investors and borrowers to manage their exposures to interest-rate risk.[2]

Since their advent in the late 1980s, the fixed rates on swaps had always remained above the corresponding rates on like-maturity government bonds. In other words, swap spreads—the difference between swap rates and government bond rates—had been uniformly positive. However, beginning in the 2008 Global Financial Crisis, the fixed rates on long-dated swaps (e.g. 30-year swaps) have fallen below government bond yields, resulting in negative long-dated swap spreads.[3] These negative swap spreads, which seemingly represent a pure arbitrage opportunity, have been alternatively attributed to large increases in the demand for long-dated swaps from end-users or to rising balance-sheet costs at the financial intermediaries who supply swaps.[4]

In this paper, we separately identify investors' demand for swaps and financial intermediaries' supply of swaps, and decompose the variation in swap spreads into the respective contributions of these two forces. Moreover, we highlight a additional key determinant of swap spreads that has been overlooked in the recent literature: financial intermediaries need to be compensated for the risk that spreads may temporarily widen due to future shocks to demand or supply. We show that compensation for this risk explains a significant fraction of the returns to swap spread arbitrage.

In considering demand and supply forces jointly, our work speaks to the different nature of financial market dislocations observed, for instance, during the 2008 Global Financial Crisis and

[1] https://stats.bis.org/statx/srs/table/d5.1

[2] Historically, the floating leg of most swaps was tied to the 3-month London Interbank Offer Rate (LIBOR) which is an indicative unsecured 3-month borrowing rate for global money-center banks. In recent years, swaps tied to overnight interbank unsecured borrowing rates (Overnight Index Swaps (OIS) tied to the Fed Funds Effective Rate) and swaps tied to overnight interbank secured borrowing rates (swaps tied to the Secured Overnight Financing Rate (SOFR)) have become more popular and are replacing LIBOR-based swaps.

[3] See Boyarchenko et al. (2018) for an overview of the negative swap spreads witnessed since 2008.

[4] See Klingler and Sundaresan (2019) and Jermann (2020) for a demand and a supply perspective on negative swap spreads, respectively.

at the onset of COVID-19 pandemic in early 2020. During the 2008 Global Financial Crisis, the strength of intermediary balance sheets deteriorated significantly, prompting their rapid withdrawal from arbitrage trades. In contrast, intermediary balance sheets remained strong after the onset of COVID-19, but their intermediation capacity was outstripped by large swings in investor demand, notably in the Treasuries market.

We build a model in which the demand of preferred habitat investors, such as MBS investors and pension funds, for receiving the fixed rate (and paying the floating rate) in long-term interest rate swaps is not naturally offset by opposing demands from other investors and has to be accomodated by risk-averse and leverage-constrained financial intermediaries. Intermediaries' binding leverage constraints limit the arbitrage between the swap market and the Treasury market, where intermediaries hedge the interest rate risk of their swap positions, opening the door to non-zero swap spreads which are a failure of the Law of One Price. These binding leverage constraints mean that swap spreads are driven by both shocks to end-investors' net demand for swaps and shocks to intermediaries' net worth (which can either relax or tighten their leverage constrains).

Demand and supply shocks—shocks to end-investors' net demand and shocks to intermediaries' wealth, respectively—naturally induce different co-movements between swap spreads and intermediaries' swap positions, allowing us to separately identify these shocks using sign restrictions. Specifically, positive demand shocks increase (the magnitude of) both swap spreads and intermediaries' swap positions, while suppy shocks decrease (the magnitude of) swap spreads but increase (the magnitude of) swap positions. We find that both supply and demand shocks contribute to the variation in swap spreads after 2009, approximately in equal measure. Moreover, shifting hedging demand from MBS investors appears to be the main driver of net investor demand for swaps.

In addition to the interplay between the demand and supply, our model points to *arbitrage risk* as a key determinant of long-dated swaps spreads: arbitraging long-dated swap spreads is not only balance sheet-consuming, but it also risky for financial intermediaries. Specifically, future shocks to either demand or supply may temporarily widen swap spreads, lending to capital losses for financial intermediaries. As a result, intermediaries will require a compensation for *this supply and demand risk*, increasing the magnitude of swap spreads at longer maturities. Thus, our model predict that measures of intermediaries' positions in the swap spread arbitrage trade should predict the returns to this trade, even after controlling for measures of intermediaries' balance-sheet costs.

Furthermore, we should expect that end-user demand should be a stronger predictor than supply of the returns to swap spread arbitrage. Intuitively, positive demand shocks increase both the compensation intermediaries require for committing their scarce balance-sheet to swap arbitrage as well as the compensation they required for bearing future swap spread risk. By contrast, a negative supply shock—i.e., a negative shock to intermediaries' wealth—reduces intermediaries' exposure to swap spread risk and the associated risk compensation, while increasing the compensation required for using balance sheet. Since these two effects partially offset each other, supply shocks have a smaller impact on the returns to swap arbitrage than demand shocks. We verify this predicted asymmetry in the data, providing evidence in support of the arbitrage risk channel.

Finally, the model also points to the possible determinants of the balance-sheet cost associated with intermediating swaps. To the extent that intermediaries allocate their scarce balance sheet between swap arbitrage and other risky investments, the market price of risk represents the opportunity cost of committing the balance sheet to the swap trade. Thus, in the model, there is a link between risk premia and swap spreads, which does not rely on intermediaries being the marginal investor in the market for the risky assets. Our findings are in line with this prediction.

Our model builds on Vayanos and Vila (2021) and, as a result, is related to the growing literature on demand factors in the term structure of interest rates; see Greenwood and Vayanos (2014), Hanson (2014), Malkhozov et al. (2016), Gourinchas et al. (2020), and Greenwood et al. (2020), among others. Our model is different from the aforementioned papers along two dimensions. First, we consider long-maturity swap spreads that arise because of limits to arbitrage rather than long-maturity bond yields. Second, we allow for the variation in both the demand and the supply in the swap market, and use the model predictions to disentangle these two forces empirically.

Our work is also related to De Long et al. (1990) who show that noise traders can create a risk in the price of an asset that deters rational arbitrageurs from aggressively betting against them. In their model, an equilibrium with noise trader risk can exist alongside a more standard equilibrium in which arbitrageurs eliminate all mispricings and noise trader risk does not arise.[5] In contrast, we show that noise trader risk does not have rely on the special type of equilibrium considered in De Long et al. (1990). In our model, the violations of the law of one price arise because intermediaries, who play the role of arbitrageurs, are subject to a leverage constraint. Once these violations are present, they are amplified by the risk of demand and supply shocks. Our results are also related to Spiegel (1998) as our model, which features demand shocks, attains multiple equilibria that qualitatively differ from each other. While we discuss the existence of other equilibria of our model, in the predictions and empirical tests we are focusing on the unique stable equilibrium of the model.

Swap rates and Treasury yields have been extensively studied in previous literature. A stream of literature calibrates dynamic term structure models to understand the dynamics of swap spreads (see, e.g., Duffie and Singleton (1997), Lang et al. (1998), Collin-Dufresne and Solnik (2001), Liu et al. (2006) and Feldhütter and Lando (2008), among others). The ones closest to our paper are Klingler and Sundaresan (2019) and Jermann (2020) who focus on the impact of the demand and the supply channels in isolation on negative swap spreads.

Finally, our work is also related to Cohen et al. (2007), Chen et al. (2018), and Goldberg and Nozawa (2020) who identify demand and supply shifts in shorting, index option, and corporate bond markets, respectively. In our paper, we focus on demand and supply forces in the swap market. Moreover, we show how shifts in demand and supply constitute a source of risk for the arbitrageurs in this market, who consequently require a compensation reflected in swap long-maturity spreads.

The rest of the paper is organized as follows. Section 2 provides a theoretical framework to study the impact of demand and supply risk on swap spreads and derives testable predictions. Section 4 presents the various data sources used and our main empirical findings. Section 5 concludes. An Online Appendix gathers additional results omitted from the main paper.

[5] Loewenstein and Willard (2006) argue that the equilibrium with noise trader risk has several unappealing properties.

2 Model

2.1 Demand and supply of swaps

Time is discrete and is indexed by t. In the model, Treasuries and swaps are bonds with identical payoff. Preferred-habitat investors demand swaps that are supplied to them by intermediaries, who then hedge their interest rate risk in the Treasury market. Interest rate swap spreads are the difference between the fixed rate in a swap and the yield of a Treasury security of the same maturity. The return on the (long) swap spread trade is then approximated by

$$\tau s_t^\tau - (\tau - 1)\, s_{t+1}^{\tau-1}$$

for maturity-τ zero-coupon claims, and

$$\frac{1}{1-\delta} s_t - \frac{\delta}{1-\delta} s_{t+1}$$

for a perpetuity with coupon declining at rate δ, where $s_t^\tau \equiv y_t^{\tau,swap} - y_t^{\tau,Treasury}$ and $s_t \equiv y_t^{swap} - y_t^{Treasury}$.[6] Historically, at least since the 1980's, swap rates have been higher than the corresponding Treasury yields, meaning that most swap spreads have been positive, however, after the 2008 Global Financial Crisis, they turned negative.

Our focus is on how the demand for and supply of long-term swaps affects the swap term structure. For this purpose, to simplify the analysis, we model preferred-habitat investors' demand for the swap perpetuity only. Note that the perpetuity coupon decline rate δ determines the duration of the demand. We think about preferred-habitat investors as pension funds and insurance companies who demand long-duration assets but for balance sheet considerations prefer to obtain the desired duration structure by investing in swaps. We specify the demand curve to be decreasing

[6] We think about Treasury prices as determined outside of our framework and focus only on the swap spreads, which determine the pricing of swaps relative to the Treasuries.

in the swap spread in the form

$$d_t = \bar{d} + \zeta_t^d + \gamma s_t \tag{1}$$

with $\bar{d}, \gamma \geq 0$ constants and ζ_t^d representing preferred-habitat investors' demand shocks.

A sequence of short-lived leverage-constrained mean-variance-optimizing competitive intermediaries allocate their balance sheet between the swap arbitrage and an outside risky investment opportunity with excess return r, whose first two moments are exogenously given by $\mathrm{E}_t[r_{t+1}] = \bar{r} > 0$ and $\mathrm{Var}_t[r_{t+1}] = \sigma_r^2$. If x_t denotes date-t intermediaries' position in the swap spread perpetuity and z_t the position in the alternative investment opportunity, they maximize a mean-variance objective of their next period wealth:

$$\max_{z_t, x_t} \mathrm{E}_t[w_{t+1}] - \frac{\alpha}{2}\mathrm{Var}_t[w_{t+1}], \tag{2}$$

subject to the budget constraint

$$w_{t,t+1} = w_t + \left(\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}s_{t+1}\right)x_t + r_{t+1}z_t$$

and the margin/leverage constraint

$$\kappa_x |x_t| + \kappa_z |z_t| \leq w_t, \tag{3}$$

with $\alpha > 0$ risk aversion parameter and $\kappa_x, \kappa_z > 0$ capital requirement parameters in the swap spread and the other investment, respectively. We assume the wealth that date-t intermediaries are born with is given by $w_t = \bar{w} + \zeta_t^w$, where $\bar{w} > 0$.

Finally, since swaps are in zero net supply, market clearing requires $d_t + x_t = 0$.[7] In particular, if preferred habitat investors demand a positive amount of the swap spread, $d_t > 0$, intermediaries need to step in and supply these $(-x_t) = d_t$ units; if preferred habitat investors are short in the swap

[7]To keep the analysis tractable, the market-clearing condition is specified in terms of the swap spread, as opposed to writing separate market-clearing conditions for the swap market and Treasury market. We abstract from modelling other agents' demand for either of these assets.

spread, $d_t < 0$, intermediaries $x_t > 0$.

We posit an autoregressive structure for preferred-habitat investors' demand (ζ_t^d) and intermediaries' wealth (ζ_t^w) shocks:

$$\zeta_{t+1} = \rho \zeta_t + \varepsilon_t,$$

where $\zeta_t = [\zeta_t^d, \zeta_t^w]'$ is the vector of shocks, $\rho = diag\,(\rho_d, \rho_w)$ is the diagonal matrix of the AR(1) coefficients $\rho_d, \rho_d \in [0, 1)$, $\varepsilon_t = [\varepsilon_t^d, \varepsilon_t^w]'$ is the vector of structural demand and supply (wealth) shocks, and $\text{Var}_t\left[\zeta_{t+1}^i\right]$ =$\text{Var}\left[\varepsilon_{t+1}^i\right] = \sigma_i^2$, $i = w, d$ denote the conditional variance of the shocks. For parsimony, we further assume that structural demand and supply shocks are orthogonal to each other and the risky investment return, $\text{Cov}_t\left[\varepsilon_{t+1}^w, \varepsilon_{t+1}^d\right]$ =$\text{Cov}_t\left[\varepsilon_{t+1}^w, r_{t+1}\right]$ =$\text{Cov}_t\left[\varepsilon_{t+1}^d, r_{t+1}\right] = 0$. Finally, we assume that the model parameters are such that we can consider

$$0 < d_t < \frac{1}{\kappa_x} w_t < \frac{\kappa_z}{\kappa_x} \frac{\bar{r}}{\alpha \sigma_r^2} \tag{4}$$

in approximation, as the only relevant case.

Conditions (4) imply that the swap demand from end users is always positive but smaller than the intermediaries' wealth, and that the intermediaries' leverage constraint (3) is always binding. In other words, we focus on the case in which preferred-habitat investors' demand fixed rate payments and this demand is met by constrained intermediaries who then hedge their risk in the Treasuries market: $x_t < 0$. This case is consistent with the prime dealers positioning in Treasuries in the aftermath of the Global Financial Crisis, a period in which swap spreads turned negative. The conditions rule out non-linearities due to occasionally binding constraints; we leave the extension of the model to such cases for future research.

2.2 Equilibrium

From (2) and (3), intermediaries' first-order conditions with respect to x_t and z_t are

$$\frac{1}{1-\delta} s_t - \frac{\delta}{1-\delta} \text{E}_t\left[s_{t+1}\right] - \alpha x_t \left(\frac{\delta}{1-\delta}\right)^2 \text{Var}_t\left[s_{t+1}\right] - \psi_t \kappa_x sgn\left(x_t\right) = 0, \tag{5}$$

and

$$\bar{r} - \alpha\sigma_r^2 z_t - \psi_t \kappa_z sgn(z_t) = 0, \tag{6}$$

where ψ_t is the multiplier associated with the leverage constraint (3). As $\bar{r} > 0$ and by definition $\psi_t \geq 0$, it must be that $z_t \geq 0$; moreover, assuming $d_t > 0$ and that the constraint (3) binds, (5) and (6) imply

$$\psi_t = \frac{\bar{r} - \alpha\sigma_r^2 z_t}{\kappa_z} = \frac{\bar{r} - \alpha\sigma_r^2 \frac{w_t + \kappa_x x_t}{\kappa_z}}{\kappa_z}. \tag{7}$$

Combining (5) and (5), we obtain that intermediaries' swap demand satisfies

$$\overbrace{\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right]}^{\text{Expected excess return}} = \overbrace{\alpha\Omega x_t}^{\substack{\text{Compensation for} \\ \text{supply-and-demand risk}}} + \overbrace{-\frac{\kappa_x}{\kappa_z}\left(\bar{r} - \alpha\sigma_r^2\left(\frac{w_t + \kappa_x x_t}{\kappa_z}\right)\right)}^{\substack{\text{Compensation for using scarce} \\ \text{balance-sheet space } (-\kappa_x\psi_t)}}, \tag{8}$$

where $\Omega = \left(\frac{\delta}{1-\delta}\right)^2 \mathrm{Var}_t\left[s_{t+1}\right]$ measures the magnitude of the swap spread risk. Equation (8) highlights the two key forces that shape expected excess returns on swaps: compensation for supply-and-demand risk and compensation for using scarce balance-sheet space.[8]

To solve the model, we conjecture that swap spreads s_t are linear functions of the 2×1 state vector $\xi_t = [\xi_t^d, \xi_t^w]'$. As shown in the Appendix, a rational expectations equilibrium of our model is a fixed point of an operator involving the "price-impact" coefficients which govern how these two state variables impact swap spreads s_t. Specifically, the market clearing condition $d_t = -x_t$ implicitly defines an operator which gives the expected returns—and, hence, the price-impact coefficients—that will clear markets when investors believe the risk of holding assets is determined

[8] If $d_t < 0$, (7) is replaced by

$$\psi_t = \frac{\bar{r} - \alpha\sigma_r^2 z_t}{\kappa_z} = \frac{\bar{r} - \alpha\sigma_r^2 \frac{w_t - \kappa_x x_t}{\kappa_z}}{\kappa_z}, \tag{9}$$

and intermediaries' optimal demand is

$$\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right] = \alpha\Omega x_t + \kappa_x\psi_t = \alpha\Omega x_t + \frac{\kappa_x}{\kappa_z}\left(\bar{r} - \alpha\sigma_r^2\frac{w_t - \kappa_x x_t}{\kappa_z}\right). \tag{10}$$

by some initial set of price-impact coefficients. A rational expectations equilibrium of our model is a fixed point of this operator.

Combining the conjectured linear form, the demand curve (1), the supply curve (8), and market clearing $d_t + x_t = 0$, after some algebra leads to the following equilibrium swap spreads and swap positions:

Theorem 1 *The equilibrium swap spreads and swap positions in the generic long-term swap spread are given by*

$$\begin{bmatrix} s_t \\ x_t \end{bmatrix} = \begin{bmatrix} A_0 \\ -\left(\bar{d} + \gamma A_0\right) \end{bmatrix} + \begin{bmatrix} A_d & A_w \\ -\left(1 + \gamma A_d\right) & -\gamma A_w \end{bmatrix} \begin{bmatrix} \zeta_t^d \\ \zeta_t^w \end{bmatrix}, \tag{11}$$

where

$$A_d = -\frac{\alpha \left[\left(\frac{\kappa_x}{\kappa_z} \right)^2 \sigma_r^2 + \Omega \right]}{\frac{1-\rho_d \delta}{1-\delta} + \alpha\gamma \left[\left(\frac{\kappa_x}{\kappa_z} \right)^2 \sigma_r^2 + \Omega \right]} < 0, \tag{12}$$

$$A_w = \frac{\frac{1}{\kappa_x} \alpha \left(\frac{\kappa_x}{\kappa_z} \right)^2 \sigma_r^2}{\frac{1-\rho_w \delta}{1-\delta} + \alpha\gamma \left[\left(\frac{\kappa_x}{\kappa_z} \right)^2 \sigma_r^2 + \Omega \right]} > 0, \text{ and} \tag{13}$$

$$A_0 = \frac{\frac{1}{\kappa_x} \alpha \left(\frac{\kappa_x}{\kappa_z} \right)^2 \sigma_r^2 \bar{w} - \frac{\kappa_x}{\kappa_z} \bar{r} - \alpha \left[\left(\frac{\kappa_x}{\kappa_z} \right)^2 \sigma_r^2 + \Omega \right] \bar{d}}{1 + \alpha\gamma \left[\left(\frac{\kappa_x}{\kappa_z} \right)^2 \sigma_r^2 + \Omega \right]} = \frac{\frac{1}{\kappa_x} \alpha \left(\frac{\kappa_x}{\kappa_z} \right)^2 \sigma_r^2 \bar{w} - \frac{\kappa_x}{\kappa_z} \bar{r} + \frac{1}{\gamma} \bar{d}}{1 + \alpha\gamma \left[\left(\frac{\kappa_x}{\kappa_z} \right)^2 \sigma_r^2 + \Omega \right]} - \frac{1}{\gamma} \bar{d} < 0, \tag{14}$$

with

$$\Omega = \left(\frac{\delta}{1-\delta} \right)^2 \left(A_d^2 \sigma_d^2 + A_w^2 \sigma_w^2 \right). \tag{15}$$

Equations (12), (13), and (15) define a system of higher-order polynomial equations in A_d, A_w, and Ω.

To understand equilibrium swap spreads, we start by considering a few special cases. First, in the deterministic case with $\sigma_d^2 = \sigma_w^2 = 0$, there is a straightforward, unique equilibrium. In fact, in absence of demand and supply risk we have $\Omega = 0$, and the fixed point problem in equations (12), (13), and (15) is degenerate. As long as $\kappa_x > 0$, demand and supply still impact swap spreads, but due to the lack of demand and supply risk, the swap spread is constant and negative, given by (14).[9] Introducing intermediary wealth shocks while keeping demand shocks shut down, i.e., $\sigma_w^2 > \sigma_d^2 = 0$, then leads to these shocks affecting equilibrium swap spreads, and the risk of fire sales after suffering adverse wealth shocks means intermediaries demand higher expected returns on swap arbitrage, pushing A_0 to more negative territories.

Second, consider the model that features demand shocks, but intermediaries are not subject to a leverage constraint, $\kappa_x = 0$ (thus supply shocks do not matter). In this case the model has three equilibria. When demand is stochastic, the fixed-point problem is non-degenerate: the risk of holding assets depends on how swap spreads react to future demand and supply shocks. For example, if investors think demand shocks will have a large impact on swap spreads, they will perceive swaps as being highly risky. As a result, investors will only absorb demand shocks if they are compensated by large spread changes and high future expected returns, making the initial belief self-fulfilling. This kind of logic means that (i) an equilibrium only exists when investors' risk aversion α is sufficiently low relative to the volatility of demand shocks and (ii) the model admits multiple equilibria.

In the unique stable equilibrium we have $A_0 = A_d = A_w = 0$, and hence $s_t = 0$ for all t: the Law of One Price (LoOP) holds and swap spreads are always zero. This is the only stable equilibrium in the sense that it is robust to a small perturbation in investors' beliefs regarding equilibrium price impact.[10] The model has two additional self-fulfilling equilibria, however. When $\kappa_x = 0$, (13) yields $A_w = 0$ for these equilibria, too, but the fact that intermediaries have to accommodate demand shocks leads to multiple A_d values that solve (12) and (15), in the spirit of Spiegel (1998).

[9]When $\sigma_d^2 = \sigma_w^2 = \kappa_x = 0$, $A_0 = 0$ is the only equilibrium – swap spreads are zero as the no arbitrage principle would suggest.

[10]Equilibrium non-existence and multiplicity are common in models like ours where short-lived investors absorb shocks to the supply of infinitely-lived assets. Consistent with Samuelson's (1947) "correspondence principle," the unique stable equilibrium has comparative statics that accord with standard intuition. By contrast, the comparative statics of the unstable equilibria are usually counterintuitive. For instance, at an unstable equilibrium, an increase in the volatility of short rate shocks can reduce the impact that supply shocks have on equilibrium prices. By contrast, in the stable equilibrium, an increase in the volatility of short rate shocks always increases the impact of supply shocks on equilibrium prices. For previous treatments of these issues, see, e.g., De Long et al. (1990), Spiegel (1998), Watanabe (2008), Banerjee (2011), Albagli (2015), and Greenwood et al. (2018).

In these two additional self-fulfilling equilibria, the LoOP fails a la De Long et al. (1990) because all agents believe that the LoOP will fail, whereas in the stable equilibrium the LoOP holds because agents believe it will hold. Our three self-fulfilling equilibria, as opposed to the two of Spiegel (1998), originate from the fact that preferred habitat investors' demand itself can depend on the level of the swap spread as long as $\gamma > 0$; when $\gamma \to 0$, these collapse to only two equilibria. The $s_t = 0$ equilibrium then acts as the low-volatility equilibrium and the second as the high-volatility (in fact, infinitely volatile) equilibrium.[11]

Finally, the introduction of the leverage constraint on intermediaries pushes A_w above zero, which then also impact the three possible equilibrium A_d coefficients and the corresponding A_0 constants. Thus, our model features multiplicity because of the demand shocks; intermediaries' constraints then make the stable equilibrium non-trivial, that is, to feature a non-zero swap spread. In this equilibrium, the LoOP does not simply fail a la De Long et al. (1990); no rational intermediary can expect that the LoOP will hold in the future, because their leverage constraint should be binding with positive probability. In the rest of the analysis we focus on this equilibrium.

2.3 Predictions

2.3.1 Identification of demand and supply shocks

Consider the impact of demand and supply shocks, ε_t^d and ε_t^w, based on (11)-(13). In particular, we have the following result:

Proposition 2 *We have $A_d < 0$, $A_w > 0$, $-(1+\gamma A_d) < 0$ and $-\gamma A_w < 0$. Therefore, demand shocks decrease the swap spread and equilibrium swap positions, whereas shocks to intermediaries' wealth increase the swap spread while decreasing swap positions.*

Therefore, the ε_t^w and ε_t^d shocks can be identified through the sign of their respective effect on the swap spreads and the equilibrium swap positions.

[11]Our low-volatility equilibrium features no swap spread volatility because swap spreads have zero fundamental value as opposed to the dividend paid by the stock in Spiegel (1998).

2.3.2 Term structure

The implications for the shape of the term structure of swap spreads cannot be directly inferred from (11)-(14). Indeed, by varying the coupon decline rate δ, we change both the maturity of the swap and the characteristics of the swap demand. In order to study the term structure, we can derive the swap spreads for non-traded maturity-τ zero-coupon swaps from the first order condition of the constrained problem that would also allow trading in maturity-τ zero-coupon swaps besides the long-maturity swap and the outside risky opportunity:

$$0 = \tau s_t^{(\tau)} - (\tau - 1)\,\mathrm{E}_t\left[s_{t+1}^{(\tau-1)}\right] - \alpha\frac{\delta}{1-\delta}(\tau-1)\,\mathrm{Cov}_t\left[s_{t+1}, s_{t+1}^{(\tau-1)}\right] x_t + \psi_t\kappa_x. \tag{16}$$

We conjecture and verify an equilibrium with affine swap spreads to obtain the following result:

Theorem 3 *The equilibrium maturity-n swap spread is*

$$s_t^{(\tau)} = A_0^{(\tau)} + A_d^{(\tau)}\zeta_t^d + A_w^{(\tau)}\zeta_t^w, \tag{17}$$

with the exact functional forms for $A_0^{(\tau)}$, $A_d^{(\tau)}$, and $A_w^{(\tau)}$ provided in the Appendix.

From (17), we find that the one-period swap spread is given by

$$s_t^{(1)} = -\kappa_x\psi_t = A_0^{(1)} + A_d^{(1)}\zeta_t^d + A_w^{(1)}\zeta_t^w,$$

where

$$\begin{bmatrix} A_0^{(1)} \\ A_d^{(1)} \\ A_w^{(1)} \end{bmatrix} = \begin{bmatrix} (1+\alpha\gamma\Omega)\,A_0 + \alpha\Omega\bar{d} \\ \left(\frac{1-\rho_d\delta}{1-\delta} + \alpha\gamma\Omega\right)A_d + \alpha\Omega < 0 \\ \left(\frac{1-\rho_w\delta}{1-\delta} + \alpha\gamma\Omega\right)A_w > 0 \end{bmatrix}.$$

Note that $A_0 - A_0^{(1)} = -\alpha\Omega\left(\bar{d} + \gamma A_0\right) = -\alpha\Omega\mathrm{E}[d_t] < 0$, implying that swap spreads are negative and their term structure is downward sloping (i.e., upward sloping in absolute terms) on average. The average slope is a function of intermediaries' risk aversion α, the swap spread risk Ω, and the average swap demand of preferred habitat investors, $\bar{d} + \gamma A_0$.

2.3.3 Drivers of swap spreads and swap spread returns

Next we want to understand swap spreads and returns on swap spread trades in more details. Iterating (5) forward and using $s_t^{(1)} = -\kappa_x \psi_t$, we can write

$$s_t = (1-\delta) \sum_{i=0}^{\infty} \delta^i \mathrm{E}_t \left[s_{t+i}^{(1)} sgn\left(d_{t+i}\right) \right] - \alpha \left(1-\delta\right) \Omega \sum_{i=0}^{\infty} \delta^i \mathrm{E}_t \left[d_{t+i} \right]. \tag{18}$$

The first right-hand side term in equation (18) corresponds to a version of the expectation hypothesis – it is the expected cost of committing intermediary balance sheet (which can be measured by the one-period spread) in future periods combined with the expectation of the direction of trade of the preferred habitat investors; when preferred habitat investors are expected to demand swaps and intermediaries need to supply them, the term simply depends on the expectation of future one-period swaps, $\mathrm{E}_t \left[s_{t+i}^{(1)} \right]$. The second term in (18) is a risk premium term that arises because of the risk associated with long-term swap spreads. This term depends on the riskiness of swaps, Ω, that is determined endogenously, and on the expected future demand that intermediaries need to match in the swap trade.

Consider now the special case when preferred-habitat investors' demand does not depend on the swap rate, that is, $\gamma = 0$. It is easy to confirm that under this assumption $s_t^{(1)} = -\kappa_x \psi_t = A_0^{(1)} + A_d^{(1)} \zeta_t^d + A_w^{(1)} \zeta_t^w$, does not depend on demand risk (σ_d^2) and supply risk (σ_w^2). However, for all $\tau \geq 2$, $s_t^{(\tau)}$ will depend on demand risk (σ_d^2) and supply risk (σ_w^2). In fact, in an analogy with Vayanos and Vila (2021), $s_t^{(1)} = -\kappa_x \psi_t$ plays the role of the short rate, and this short-term spread has nothing to do with demand and supply risk. At the same time, long-term swap spreads depend on the expected path of short-term spreads and the risk that shocks to demand and supply will alter swap rates. However, when $\gamma > 0$, $s_t^{(1)} = -\kappa_x \psi_t$ will depend on demand and supply risk, since the scale of intermediary balance sheets depends on risk.

Next we express expected returns on the swap spread arbitrage trade. First, we obtain the following result:

Proposition 4 *The expected return on the swap spread trade over one period is*

$$\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}E_t\left[s_{t+1}\right] = \kappa_x \psi_t sgn\left(x_t\right) + \alpha\Omega x_t = s_t^{(1)} sgn\left(d_t\right) - \alpha\Omega d_t. \tag{19}$$

This result is a simple corollary of (5). In fact, the expected return is the sum of the compensation for committing intermediary balance sheet to the swap spread trade over one period and of the compensation for the exposure to the swap spread risk. When preferred habitat investors have a positive demand for swaps, $d_t > 0$, this expression simplifies to $s_t^{(1)} - \alpha\Omega d_t < 0$ since $s_t^{(1)} < 0$. Note that (19) can also be re-written in terms of the return in excess of the one period spread s_t^1.

With the above in mind, and using (11), the expected return on swap spread trades can also be represented as a function of demand and supply shocks. In particular, we obtain two results:

Proposition 5 *The expected return on the generic long-term swap spread trade is given by*

$$\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}E_t\left[s_{t+1}\right] = B_0 + B_w\zeta_t^w + B_d\zeta_t^d,$$

where

$$B_0 = A_0,\ B_d = \frac{1-\rho_d\delta}{1-\delta}A_d < 0,\ \textit{and}\ B_w = \frac{1-\rho_w\delta}{1-\delta}A_w > 0.$$

In particular, we find that the swap spread risk Ω mitigates the magnitude of the supply factor loading B_w while it increases (in absolute value) the demand factor loading B_d. To understand this differential effect of risk, note that a negative shock to intermediaries wealth (a negative supply shock) simultaneously increases the compensation the intermediary requires for committing her balance sheet to the swap trade and reduces her position in the swap trade, thereby decreasing the compensation she requires for the exposure to the swap spread risk. Thus, we expect the predictive power of the supply factor to be mitigated by the two opposing forces, which is refected in the smaller magnitude of the loading B_w. In contrast, an increase in the demand for swaps increases both the balance sheet cost and the exposure to the swap spread risk.[12]

Expected returns also have a term structure, as the next result suggests:

Proposition 6 *The expected return on the generic long-term swap spread trade is given by*

$$\tau s_t^{(\tau)} - (\tau - 1) E_t \left[s_{t+1}^{(\tau-1)} \right] = B_0^{(\tau)} + B_d^{(\tau)} \zeta_t^d + B_w^{(\tau)} \zeta_t^w, \tag{20}$$

where $B_d^{(\tau)}$ is negative and downward sloping across maturities, while $B_w^{(\tau)}$ is positive and constant across maturities.

Importantly, we find that $B_w^{(\tau)}$ is constant across maturities. Intuitively, this constancy reflects the fact that the $B_w^{(\tau)} \zeta_t^w$ term in (20) purely reflects compensation for scarce balance sheet space and all swaps, irrespective of maturity, consume the amount of balance sheet space per unit notional. By contrast, the $B_d^{(\tau)} \zeta_t^d$ term reflects both compensation for balance sheet space and compensation for risk. Further, the fact that $B_d^{(\tau)}$ is decreasing in τ reflects the fact that intermediaries need greater compensation for the risk of holding longer-term swaps.

Finally, our framework allows to study swap spread volatility. In particular, we obtain the following result:

Proposition 7 *The conditional and unconditional volatilities of swap spreads are given by*

$$Var_{t-1} \left[s_t^{(n)} \right] = \left(A_d^{(n)} \right)^2 \sigma_d^2 + \left(A_w^{(n)} \right)^2 \sigma_w^2$$

and

$$Var \left[s_t^{(n)} \right] = \left(A_d^{(n)} \right)^2 Var \left[\zeta_t^d \right] + \left(A_w^{(n)} \right)^2 Var \left[\zeta_t^w \right].$$

Both variances are either decreasing or hump-shaped across maturities.

[12]Relatedly, note that $B_d < A_d^{(1)} < 0$ and $0 < B_w < A_w^{(1)}$.

2.3.4 Swap spreads and the opportunity cost

As intermediaries need to choose in equilibrium between allocating their scarce balance sheet to swap spread arbitrage or the outside risky investment opportunity, equilibium swap spreads naturally depend on the expected return and risk of this investment opportunity. In particular, we have the following result:

Proposition 8 *Equilibrium swap spreads and positions are affected by the expected return and risk of the risky investment opportunity, $\bar{r}$ and σ_r^2. In particular, $\frac{dA_0}{d\bar{r}} < 0$, which implies that the swap spread s_t decreases in $\bar{r}$ while the intermediary position x_t increases in $\bar{r}$.*

Note from (11) and (14) that, when intermediaries allocate their balance sheet between the swap trade and investment into risky investment opportunity, the level of swap spreads depends on the market price of risk associated with the risky investment opportunity, $\bar{r}$. The link between the market price of risk and swap spreads does not rely on intermediaries being the marginal investor in the market for the risky asset. Instead, this link arises because $\bar{r}$ represents the opportunity cost of committing the balance sheet to the swap trade. Thus, the direction of the causality is from the risky asset market to the intermediaries, and not the other way around, unlike in existing intermediary asset pricing models.

3 Data

4 Empirical evidence

4.1 The market for interest rate swaps

We test the predictions of our model in the market for long-maturity interest rate swaps. In this market, large groups of investors—insurers, pension funds, MBS investors, banks, and corporates—are "natural receivers of fixed rate," as highlighted by the Treasury Borrowing Advisory Committee.

For instance, insurers and pension funds receive fixed to manage the duration of their liabilities without committing their balance sheet which can be used for less liquid or higher expected return investments. Klingler and Sundaresan (2019) argue that the incentive to do this increased as pension funds became significantly underfunded in the aftermath of the Global Financial Crisis. MBS investors use swaps to offset the drops in their portfolios' duration which occur when the probability of mortgage pre-payments increases; see Hanson (2014) and Malkhozov et al. (2016). As such, the MBS investors' demand for receiving fixed is likely to be highly time-varying. Finally, banks and corporates also on net contribute to the demand for receiving fixed; see TBAC.

The net demand for receiving fixed rate is not offset by the "natural" supply of fixed rate payments. The resulting imbalance has to be absorbed by intermediaries who toggle between swap and Treasuries markets. An intermediary who pays fixed and takes a matching long position in the Treasuries will earn the difference between the Treasuries yield and the swap rate, the negative of the swap spread. In addition, the intermediary will also earn any difference between the floating rate she receives in the swap and the funding rate for Treasuries. For instance, for the LIBOR swap, the intermediary will earn the spread between the LIBOR and the GC rates. Importantly, prior to swap maturity which could be up to 30 years ahead, the intermediary can suffer a mark-to-market loss if Treasury yields increase relative to swap rates.[13]

In sum, the overall structure of the swap market is in line with the model in Section 2.

4.2 Swap spreads and positions

We focus on the 30-year LIBOR swap spread, shown on Figure 1, which has been consistently negative since 2009, an apparent violation of the LoOP. Indeed, an investor paying fixed rate in the swap and taking a matching long position in Treasuries will earn not only the negative of the swap spread but also the historically positive difference between the LIBOR and the GC rate for Treasuries. The 30-year OIS swap spread, which is stripped from the LIBOR-GC spread, is even

[13] See Boyarchenko et al. (2018) and Jermann (2020) on the mechanics of swap trades and the role of intermediaries as suppliers of fixed rate in the swap market.

more negative, making the apparent LoOP-violation even more striking. The 30-year OIS swap spread is also shown on Figure 1 for a shorter time period for which it is available.

Our model connects swap spreads to intermediary swap positions. However, the data on swap quantities intermediated by the dealers is not directly available. Therefore, in line with our premise that dealers' intermediation consists of paying fixed in the swap market and taking an offsetting long position in Treasuries, we consider Primary Dealers' net Treasuries positions instead. Supporting this choice, we note that Primary Dealers' interest rate book Value-at-Risk (VaR) fell considerably after the Global Financial Crisis and stayed at a stable low level ever since.[14] Moreover, the correlation between the net position and the VaR is close to zero. This suggests that dealers' net position in Treasuries does not correspond to a directional investment and instead is, at least partly, offset by a position in interest rate derivatives. As shown on Figure 1, Primary Dealers switched from a short net position to a long net position in 2009. Interestingly, the timing of the switch in net position sign coincides with the drop of the 30-year swap spread into negative territory.

4.3 Supply and demand decomposition

In order to uncover the relative contribution of dealers' supply and investors' demand to the variation in the 30-year swap spread, we consider a vector auto-regression (VAR)

$$\begin{bmatrix} \text{30y swap spread}_t \\ \text{PD net position}_t \end{bmatrix} = C_0 + \sum_{i=1}^{L} C_i \begin{bmatrix} \text{30y swap spread}_{t-i} \\ \text{PD net position}_{t-i} \end{bmatrix} + \xi_t, \tag{21}$$

where C_i is a vector of constant terms, C_i are matrices of auto-regressive coefficients, and ξ_t is the vector of reduced-form VAR residuals. We identify the structural supply and demand shocks $\varepsilon_t = \left[\varepsilon_t^d, \varepsilon_t^w\right]'$ by imposing the sign restrictions implied by Proposition 2: demand shocks widen the negative swap spread and equilibrium swap positions; in contrast, supply shocks widen the negative swap spread but shrink swap positions. Thus, structural shocks are related to reduced-form VAR

[14] We thank Chris Anderson for sharing his series of average Primary Dealers' interest rate book VaR.

residuals by the mapping

$$\xi_t = \underbrace{\begin{bmatrix} - & - \\ + & - \end{bmatrix}}_{A} \varepsilon_t,$$

where the sign-restricted rotation matrix A corresponds to the matrix in equation (11) of the model. In addition, just as in the model, we assume that supply and demand shocks are orthogonal. We estimate the model using the pure sign restrictions approach of Uhlig (2005) standard in the macroeconomic literature, but also used in, among others, Cohen et al. (2007), Chen et al. (2018), and Goldberg and Nozawa (2020). We set lag length to $L = 4$, but our results are robust to various changes in the specification.

Figure 2 illustrates the historical decomposition of demeaned spreads by plotting the contribution of demand and supply shocks separately. It is imminent that both demand and supply forces play an important role, and that their relative importance has a time-arying nature. For instance, after 2015, quite a few regulatory changes hit the intermediary sector, showing up as important shocks to the supply side that pushed swap spreads down significantly.

Table 2 provides further details on the relative contribution of these shocks to the dynamics of swap spreads. The upper panel shows our results from performing a variance decomposition. We find that the two types of shocks constribute similarly to the variation in spreads at all horizons, with supply shocks becoming more important at longer horizons. In addition, the lower panel collects pairwise correlations of the two types of shocks with known determinants of asset prices such as the Treasury term premium, the noise measure of Hu et al. (2013) that measure the average deviation of observed yields from a fitter yield curve, stock volatility measured by VIX, and the Treasury implied volatility. We find that both demand and supply forces play an important role. While both types of shocks are normalized such that positive shocks push swap spreads into more negative territory, the panel highlghts that supply and demand shocks have starkly different impact on these variables, with the opposite sign.

Recall that, according to Proposition 8, equilibrium swap spread s_t decreases in the expected return of the outside invesment opportunity, $\bar{r}$, whereas the intermediary position x_t increases in $\bar{r}$. This also means that changes in $\bar{r}$, if we allowed for time variation in the model (in the Appendix we present an extension that allows for time-varying $\bar{r}$), would be equivalent to supply shocks, because it captures the opportunity cost of supplying swaps. The results of Panel B are in line with our model predictions. Supply factors comove positively with all these measures of risk premia, especially with the noise and volatility measures, and actually negatively to demand.

On the demand side what stands out is actually the correlation with the term premium. While our model is silent about the origin of demand factors, only refers to those investors as preferred-habitat investors, we can further look into potential causes of this negative correlation. Table 3 collects the results from regressing the swap spread and the contribution of demand shocks to this spread on a couple of well-known determinants of the Treasury term premium such as (i) MBS dollar duration that proxies for the hedging demand of MBS investors (see Hanson (2014) and Malkhozov et al. (2016)), (ii) the pension underfunding measure of Klingler and Sundaresan (2019) that proxies for pension funds' demand for the fixed leg of swaps, and (iii) corporate issuance. We find that in our sample (January 2009 to June 2018) MBS dollar duration is the driving force behind the variation in both swap spreads and the demand factors. Lending support to this result, Figure 3 shows the time series of MBS dollar duration and the demand factor, and the patterns are almost mirror images of each other. In fact, we would expect that as MBS duration decreases, preferred-habitat investors become hungry for additional duration. But since their balance sheets are occupied by MBS and they have little cash on hand, they cannot simply go to the Treasury market and buy bonds, and instead end up in the derivative market to obtain synthetic duration, driving their demand for swaps up. It is interesting to note that the R^2 in the demand regressions are even higher than those from the swap spread—of course, the latter is in part due demand and in part due to supply.

4.4 Arbitrage risk

Our model has a range of predictions that we take to the data.

Figure 1 also shows the 10-year swaps spread, which reaches negative territories from time to time, but less so than the 30-year spread, therefore suggesting an interesting term structure dimension to swap spreads in the sense that longer maturity spreads are more negative. These observations are in line with our theoretical framework, as according to our model, the term structure of spreads is on average downward-sloping. This is apparent from (18) that simplifies to

$$s_t = (1-\delta)\sum_{i=0}^{\infty}\delta^i \underbrace{\mathrm{E}_t\left[s_{t+i}^{(1)}\right]}_{<0} - \alpha\,(1-\delta)\,\Omega\sum_{i=0}^{\infty}\delta^i \underbrace{\mathrm{E}_t\left[d_{t+i}\right]}_{>0} \tag{22}$$

when $d_t > 0$. Long-term swap spreads depend on the expected future costs of the balance sheet, but since they are actually risky investments, they require a higher compensation for risk-averse investors, which then lead to an even more negative level of swap spreads. The size of the risk premium term depends on Ω that represents the riskiness of swap arbitrage. In fact, plotting the time series of swap spreads with different maturities we find that longer maturities correspond to more negative spreads after 2009.

Finally, we revisit our predictions about the relationship of returns and the risk channel. First, Proposition 4 states that holding period returns on the swap spread arbitrage (paying fixed rate on the swap and buying Treasuries to match the cash flows) comprise of two terms: the one-period arbitrage spread that measures the cost of the balance sheet over one period (in the short term), e.g. the GC repo-OIS spread, and a compensation for the riskiness of the trade, which is prorportional to the quantity of swap spread arbitrage. Therefore, quantities should predict holding period returns on swap spread trades, because they measure how much intermediaries are exposed to the risk – this is in fact why we use the net primary dealer positions.

Second, the model highlights the different effect of demand versus supply-side shocks. Proposition 5 states that demand shocks have a negative coefficient ($B_d < 0$) and supply shocks have a positive coefficient ($B_w > 0$) in predicting arbitrage trade returns. Further, recall that a supply shock increases balance sheet cost but reduces exposure to arbitrage risk. In contrast, demand shock increases both balance sheet cost and arbitrage risk exposure. Therefore, the impact of demand shocks is amplified by the risk channel, visible via the additional Ω risk term, as opposed to supply shocks, hence the demand factor should be a stronger predictor of future excess return on the swap spread arbitrage than the supply factor.

Table 4 collects our results. In fact, we find support that our quantity measure (PD net position) is strongly predicting returns on the arbitrage trade, both at the 3-month and at the 12-month horizon, even after controlling for other institutional drivers of swap spreads such as the GC repo-OIS spread and the LIBOR-OIS spread. Further, when turning to predicting returns with the demand and supply factors that are extracted from the VAR decomposition, we find that supply is insignificant, whereas demand has a strong predicting power for the returns on the arbitrage trade, for both holding periods, with R^2s around 15% for the short horizon and 55% at the longer horizon. Therefore, we conclude that arbitrage risk is also an important determinant of swap spreads, beyond the usual impacts of demand and supply shocks on the levels of swap spreads.

We run several robustness checks, including one about the period after 2018. As pointed out by Fleming and Rosenberg (2008), Tresury issuances and significant changes in Fed bond holdings impact dealers' inventory, and we would expect that several events unrelated to the mechanisms discussed here, such as the TCJA-related issuance in 2018, the Fed balance sheet unwinding starting from 2018, and then additional Fed Treasury purchases in 2020 could hinder our results. In fact, when we extend the sample to end 2020, prime dealers' position is no longer significant in the previous regressions, but controlling for issuance and Fed holdings, we recover significance with the same signs as before.

5 Conclusion

We find that both investors' demand and dealers' supply play an important yet distinct role in explaining the variation in long-maturity swap spreads. In considering demand and supply forces jointly, our work speaks to the different nature of observed episodes of financial market dislocations, as argued by policymakers and academics, such as during the 2008 Global Financial Crisis and at the onset of COVID-19 pandemic in early 2020.

In addition to the interplay between the demand and the supply forces, we show that arbitrage risk is a key determinant of longer-maturity LoOP violations. Indeed, our model highlights that once LoOP violations arise because of limits to arbitrage, they are amplified by the presence of arbitrage risk. We also provide empirical support for all of these channels.

Our framework does not only speak to negative swap spreads, but can also be applied to other limited arbitrage settings—for instance, to deviations from the covered interest parity (CIP). We leave this for future research.

References

Albagli, E. 2015. Investment horizons and asset prices under asymmetric information. *Journal of Economic Theory* 158:3787–837.

Banerjee, S. 2011. Learning from Prices and the Dispersion in Beliefs. *Review of Financial Studies* 24:3023–3068.

Boyarchenko, N., P. Gupta, N. Steele, and J. Yen. 2018. Negative Swap Spreads. *Federal Reserve Bank of New York Economic Policy Review* 24:1–14.

Chen, H., S. Joslin, and S. X. Ni. 2018. Demand for Crash Insurance, Intermediary Constraints, and Risk Premia in Financial Markets. *Review of Financial Studies* 32:228–265.

Cohen, L., K. B. Diether, and C. J. Malloy. 2007. Supply and Demand Shifts in the Shorting Market. *Journal of Finance* 62:2061–2096.

Collin-Dufresne, P., and B. Solnik. 2001. On the term structure of default premia in the swap and Libor markets. *Journal of Finance* 56:1095–1115.

De Long, B. J., A. Shleifer, L. H. Summers, and R. J. Waldmann. 1990. Noise Trader Risk in Financial Markets. *Journal of Political Economy* 98:703–738.

Duffie, D., and K. J. Singleton. 1997. An econometric model of the term structure of interest-rate swap yields. *Journal of Finance* 52:1287–1321.

Feldhütter, P., and D. Lando. 2008. Decomposing swap spreads. *Journal of Financial Economics* 88:375–405.

Fleming, M. J., and J. V. Rosenberg. 2008. How Do Treasury Dealers Manage Their Positions? *Federal Reserve Bank of New York Staff Report* .

Goldberg, J., and Y. Nozawa. 2020. Liquidity Supply in the Corporate Bond Market. *Journal of Finance* forthcoming.

Gourinchas, P.-O., W. Ray, and D. Vayanos. 2020. A Preferred-Habitat Model of Term Premia and Currency Risk. Working paper.

Greenwood, R., S. G. Hanson, and G. Y. Liao. 2018. Asset Price Dynamics in Partially Segmented Markets. *Review of Financial Studies* 31:3307–3343.

Greenwood, R., S. G. Hanson, J. C. Stein, and A. Sunderam. 2020. A Quantity-Driven Theory of Term Premiums and Exchange Rates. Working paper, harvard business school.

Greenwood, R., and D. Vayanos. 2014. Bond Supply and Excess Bond Returns. *Review of Financial Studies* 27:663–713.

Hanson, S. G. 2014. Mortgage Convexity. *Journal of Financial Economics* 113:270–299.

Hu, G. X., J. Pan, and J. Wang. 2013. Noise as Information for Illiquidity. *Journal of Finance* 68:2341–2382.

Jermann, U. 2020. Negative Swap Spreads and Limited Arbitrage. *Review of Financial Studies* 33:212–238.

Klingler, S., and S. Sundaresan. 2019. An Explanation of Negative Swap Spreads: Demand for Duration from Underfunded Pension Plans. *Journal of Finance* 74:675–710.

Lang, L. H., R. H. Litzenberger, and A. L. Liu. 1998. Determinants of interest rate swap spreads. *Journal of Banking and Finance* 22:1507–1532.

Liu, J., F. A. Longstaff, and R. E. Mandell. 2006. The market price of risk in interest rate swaps: The roles of default and liquidity risks. *Journal of Business* 79:2337–2359.

Loewenstein, M., and G. A. Willard. 2006. The Limits of Investor Behavior. *Journal of Finance* 61:231–258.

Malkhozov, A., P. Mueller, A. Vedolin, and G. Venter. 2016. Mortgage Risk and the Yield Curve. *Review of Financial Studies* 29:1220–1253.

Spiegel, M. 1998. Stock Price Volatility in a Multiple Security Overlapping Generations Model. *Review of Financial Studies* 11:419–447.

Uhlig, H. 2005. What are the effects of monetary policy on output? Results from an agnostic identification procedure. *Journal of Monetary Economics* 52:381–419.

Vayanos, D., and J.-L. Vila. 2021. A Preferred-Habitat Model of the Term Structure of Interest Rates. *Econometrica* 89:77–112.

Watanabe, M. 2008. Price Volatility and Investor Behavior in an Overlapping Generations Model with Information Asymmetry. *Journal of Finance* 63:229–272.

Table 1: **Summary statistics:** This table reports the means and the standard deviations of the 30-year swap spread (Swap spread), the spread between the 3-month general collateral repo rate and the 3-month OIS rate (GC repo-OIS), the Primary Dealers' net position in coupon-bearing Treasury securities (PD net), the Primary Dealers' gross position in coupon-bearing Treasury securities (PD gross), and the spread between the 3-month USD LIBOR rate and the 3-month general collateral repo rate (LIBOR-GC repo). Data are weekly and run from July 2001 to December 2008 (01-08) and from January 2009 to December 2020 (09-20).

	Mean		St. dev.	
	01-08	09-20	01-08	09-20
Swap spread, bps	45	-25	16	15
GC repo - OIS, bps	-8	7	12	8
PD net, bln	-96	59	36	64
PD gross, bln	350	392	65	60
LIBOR - GC repo, bps	35	18	44	18

Table 2: **Supply and demand factors:** Panel A reports the forecast error variance decomposition of the 30-year swap spread from the structural VAR identified using sign restrictions as described in Section X. Variables included in the VAR are the 30-year swap spread and the Primary Dealers' net position in coupon-bearing Treasury securities. Panel B reports the correlation coefficients of the demand factor (Demand) and the supply factor (Supply) with, respectively, the Adrian, Crump, and Moench (2013) term premium (Term premium), the Hu, Pan, and Wang (2013) term structure noise (Noise), the VIX volatility index (Stock volatility), the VXTY Treasury volatility index (Treasury volatility), and the 30-year swap spread (Swap spread). The demand factor and the supply factor are defined in such a way that an increase in either factor corresponds to more negative swap spread. Data are weekly and run from January 2009 to June 2018.

Panel A: Swap spread variance decomposition

Horizon (h), weeks	Supply	Demand
1	0.50	0.50
4	0.46	0.54
12	0.49	0.51
52	0.65	0.35

Panel B: Correlations

	Suppply	Demand
Term premium	0.20	−0.62
Noise	0.30	−0.16
Stock volatility	0.31	−0.16
Treasury volatility	0.40	−0.28
Swap spread	−0.76	−0.87

Table 3: **Drivers of swap demand:** This table reports the slope and intercept coefficients from regressions of, respectively, the 30-year swap spread and the demand factor on aggregate dollar duration of mortgage-backed securities (MBS dollar duration) and the Klingler and Sundaresan (2019) pension fund underfunding factor. All variables are in 3-month changes. Data are weekly in regressions (1) and (3), are quarterly in regressions (2), and run from January 2009 to June 2018. Newey and West (1987) standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Swap spread		Demand	
	(1)	(2)	(3)	(4)
MBS dollar duration	0.787***	0.624***	−0.335***	−0.257***
	(0.175)	(0.153)	(0.064)	(0.081)
Pension underfunding		30.075		15.230
		(67.376)		(23.573)
Corporate issuance		−8.427		12.540
		34.612		(10.451)
Constant	−0.030	0.361	0.260	0.175
	(0.807)	(1.469)	(0.286)	(0.401)
Observations	482	37	482	37
Adjusted R^2	0.120	0.020	0.165	0.113

Table 4: **Swap spread and swap spread trade holding period returns:** This table reports the slope and intercept coefficients from regressions of, respectively, the 30-year swap spread (regressions 1-2), the 3-month holding period return on the 30-year swap spread trade (regressions 3-5), and the 12-month holding period return on the 30-year swap spread trade (regressions 6-8) on the spread between the 3-month general collateral repo rate and the 3-month OIS rate (GC repo-OIS), the Primary Dealers' net position in coupon-bearing Treasury securities (PD net), the Primary Dealers' outstanding balances of Treasury securities in through financing arrangements (PD sec. in) the supply (Supply) and demand (Demand) factors, and the spread between the 3-month USD LIBOR rate and the 3-month general collateral repo rate (LIBOR-GC repo). Data are weekly and run from January 2009 to June 2018. Newey and West (1987, 1994) standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Spread		3-month returns			12-month returns		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
GC repo - OIS	−1.319***	−1.300***	0.407	−2.585		8.574	−0.333	
	(0.236)	(0.253)	(3.035)	(2.961)		(6.481)	(5.136)	
PD net		−0.050		1.618***			5.414***	
		(0.037)		(0.381)			(0.821)	
PD sec. in		−0.035***		0.242*			1.230***	
		(0.006)		(0.136)			(0.196)	
Supply					−2.905			6.192
					(3.197)			(7.665)
Demand					12.920***			52.809***
					(2.876)			(5.252)
LIBOR - GC repo	−0.081	−0.107*	1.527	1.750	1.451	5.493***	6.539***	4.766***
	(0.064)	(0.057)	(1.384)	(1.243)	(1.249)	(1.828)	(1.368)	(1.366)
Constant	−15.536***	52.908***	−12.003	−506.807**	−8.583	−103.376	−2555.930***	−47.988
	(2.219)	(12.616)	(30.958)	(256.891)	(27.795)	(64.546)	(387.606)	(45.193)
Observations	495	495	495	495	495	495	495	495
Adjusted R^2	0.269	0.360	0.020	0.145	0.148	0.072	0.437	0.552

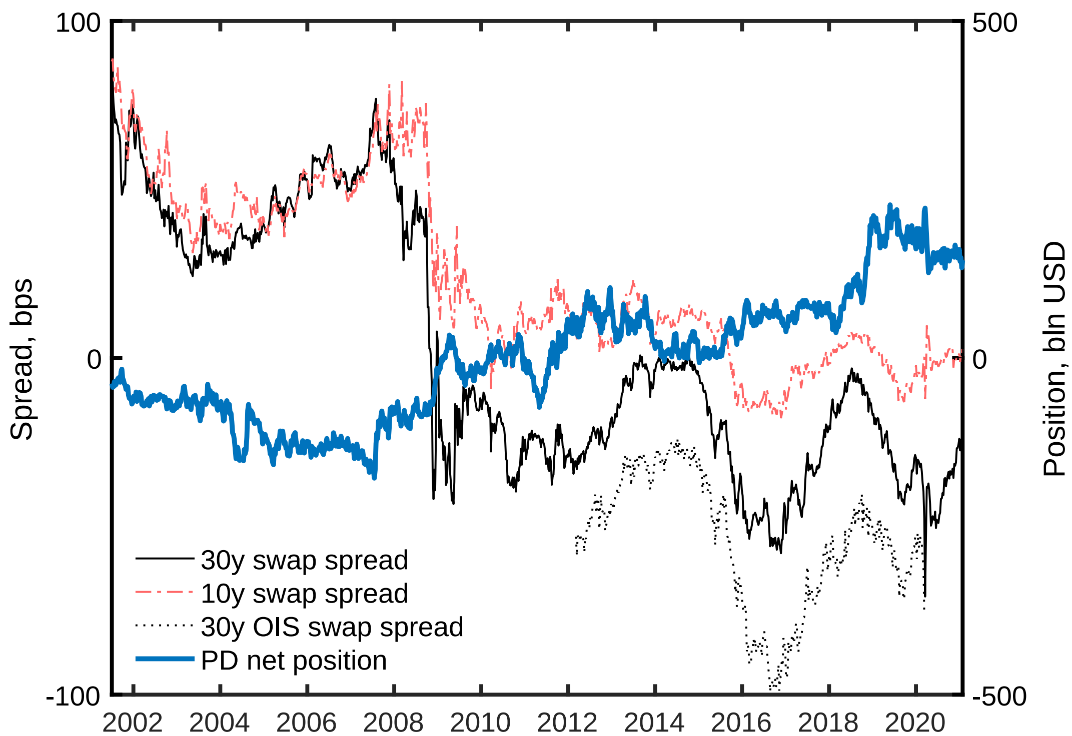

Figure 1: **Swap spread and Primary Dealers' position:** This figure shows the 30y LIBOR swap spread (30y swap spread), the 10y LIBOR swap spread (10y swap spread), the 30y OIS swap spread (30y OIS swap spread), and the Primary Dealers' net position in coupon-bearing Treasury securities (PD net position). Data are weekly and run from January 2001 to December 2020.

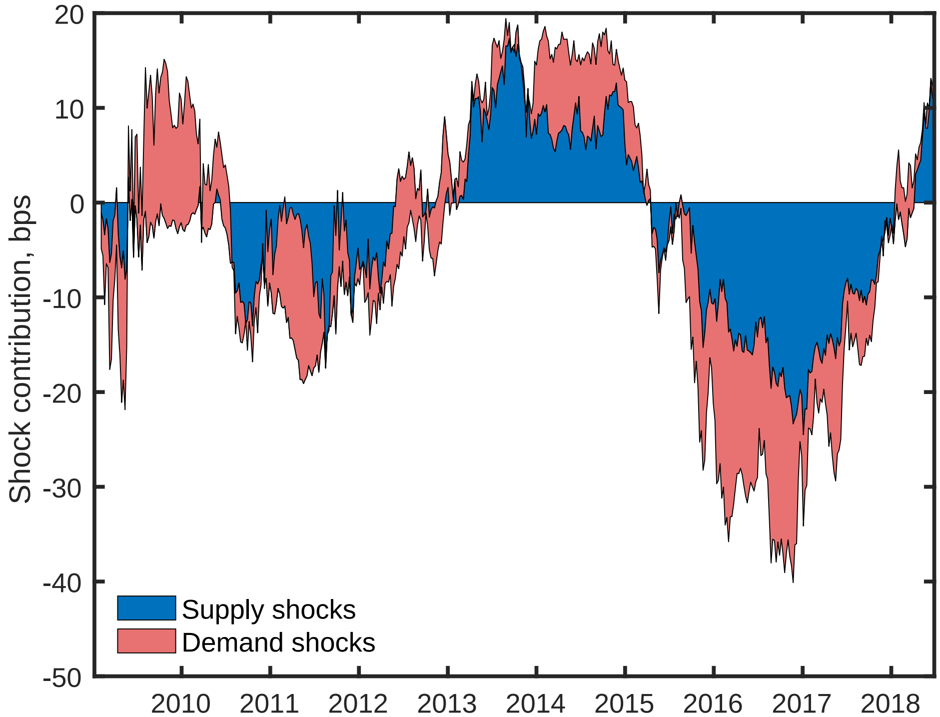

Figure 2: **Swap spread historical decomposition:** This figure shows the contribution of demand and supply shocks to the 30y swap spread. Data are weekly and run from January 2009 to June 2018.

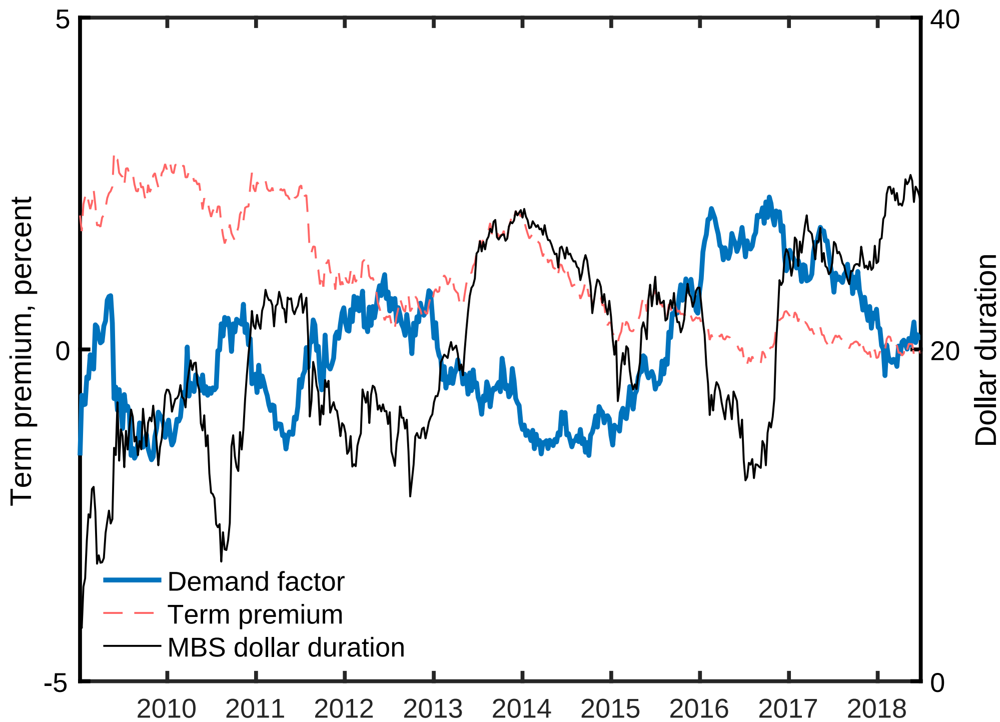

Figure 3: **Demand factor and aggregate MBS dollar duration:** This figure shows the series of the demand component of the 30y swap spread (Demand factor), the Adrian, Crump and Moench (2013) term premium (Term premium), and the aggregate dollar duration of U.S. mortgage-backed securities (MBS dollar duration). Data are weekly and run from January 2009 to June 2018.

A Proofs and derivations

Proof of Theorem 1. Using (2) and (3) to write the Lagrangian as

$$\begin{aligned}
\mathcal{L}_t &= \left(\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right]\right)x_t + \mathrm{E}_t\left[r_{t+1}\right]z_t - \frac{\alpha}{2}\left(\frac{\delta}{1-\delta}\right)^2 \mathrm{Var}_t\left[s_{t+1}\right]x_t^2 \\
&+\alpha\frac{\delta}{1-\delta}\mathrm{Cov}_t\left[r_{t+1}, s_{t+1}\right]x_t z_t - \frac{\alpha}{2}\mathrm{Var}_t\left[r_{t+1}\right]z_t^2 - \psi_t\left(\kappa_x\left|x_t\right| + \kappa_z\left|z_t\right| - w_t\right) \\
&= \left(\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right]\right)x_t + \bar{r}z_t - \frac{\alpha}{2}\left(\frac{\delta}{1-\delta}\right)^2 \mathrm{Var}_t\left[s_{t+1}\right]x_t^2 - \frac{\alpha}{2}\sigma_r^2 z_t^2 \\
&-\psi_t\left(\kappa_x\left|x_t\right| + \kappa_z\left|z_t\right| - w_t\right),
\end{aligned}$$

where in the second step we conjectured $\mathrm{Cov}_t\left[r_{t+1}, s_{t+1}\right] = 0$, since demand and wealth shock innovations, which in equilibrium determine the swap spread, are independent of r_{t+1}.

Intermediaries' first order conditions with respect to x_t and z_t are then (5) and (6), respectively. where ψ_t is the multipier associated with the leverage constraint (3). As $\bar{r} > 0$, and by definition $\psi_t \geq 0$, it must be the case that $z_t \geq 0$ and hence (6) simplifies to

$$\psi_t = \frac{1}{\kappa_z}\left(\bar{r} - \alpha\sigma_r^2 z_t\right). \tag{23}$$

Substituting it into (5), we obtain

$$\alpha\Omega x_t = \frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right] - \frac{\kappa_x}{\kappa_z}sgn\left(x_t\right)\left(\bar{r} - \alpha\sigma_r^2 z_t\right), \tag{24}$$

where $\Omega \equiv \left(\frac{\delta}{1-\delta}\right)^2 \mathrm{Var}_t\left[s_{t+1}\right]$.

Suppose now that in equilibrium $d_t > 0$, i.e., preferred-habitat investors are long in swaps. Market clearing then implies $x_t < 0$, thus, $sgn\left(x_t\right) = -1$ in (24). Combining this with a binding (3), after some algebra, yields (8) and

$$z_t = \frac{\frac{1}{\kappa_z}\alpha\Omega w_t + \left(\frac{\kappa_x}{\kappa_z}\right)^2\bar{r} + \frac{\kappa_x}{\kappa_z}\left(\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right]\right)}{\alpha\left[\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2 + \Omega\right]}. \tag{25}$$

In the $\delta = 0$ case ($\frac{1}{1-\delta} = 1$) we obtain optimal intermediary demand (26). From here, the swap level and equilibrium positions are the same as in (11).For $\delta = 0$, swap spread risk does not matter ($\Omega = 0$), and the model essentially becomes static. In this case intermediary demand is simply

$$x_t = -\frac{1}{\alpha\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2}\frac{\kappa_x}{\kappa_z}\left(\frac{1}{\kappa_z}\alpha\sigma_r^2 w_t - \bar{r}\right) + \frac{1}{\alpha\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2}s_t. \tag{26}$$

Combining with the demand curve (1) and the market clearing condition, the swap level and equilibrium positions become

$$\begin{bmatrix} s_t \\ x_t \end{bmatrix} = \begin{bmatrix} \frac{\alpha\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2\frac{1}{\kappa_x}\bar{w}-\frac{\kappa_x}{\kappa_z}\bar{r}-\alpha\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2\bar{d}}{1+\alpha\gamma\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2} \\ -\frac{\gamma\alpha\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2\frac{1}{\kappa_x}\bar{w}-\gamma\frac{\kappa_x}{\kappa_z}\bar{r}+\bar{d}}{1+\alpha\gamma\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2} \end{bmatrix} + \begin{bmatrix} -\frac{\alpha\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2}{1+\alpha\gamma\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2} & \frac{\alpha\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2}{1+\alpha\gamma\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2}\frac{1}{\kappa_x} \\ -\frac{1}{1+\alpha\gamma\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2} & -\gamma\frac{\alpha\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2}{1+\gamma\alpha\sigma_r^2\left(\frac{\kappa_x}{\kappa_z}\right)^2}\frac{1}{\kappa_x} \end{bmatrix} \begin{bmatrix} \zeta_t^d \\ \zeta_t^w \end{bmatrix}.$$

This one-period case provides us with a few intuitions: the ζ_t^w and ζ_t^d shocks can be identified through the sign of their respective effect on swap spreads and positions; the level of swap spreads depends on the market price of risk $\bar{r}$ associated with the risky investment opportunity, i.e., the opportunity cost of committing the balance sheet to the swap trade.

Suppose now we have $\delta > 0$. Combining (8) with the demand curve (1) and the market-clearing condition $d_t + x_t = 0$, we can write

$$\alpha\left[\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2 + \Omega\right]d_t - \frac{1}{\kappa_z}\frac{\kappa_x}{\kappa_z}\alpha\sigma_r^2 w_t + \frac{\kappa_x}{\kappa_z}\bar{r} + \left(\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right]\right) = 0. \tag{27}$$

Conjecturing a solution in the form of

$$s_t = A_0 + A_d\zeta_t^d + A_w\zeta_t^w,$$

we have

$$\mathrm{E}_t\left[s_{t+1}\right] = A_0 + A_d\rho_d\zeta_t^d + A_w\rho_w\zeta_t^w \text{ and } \mathrm{Var}_t\left[s_{t+1}\right] = \frac{1}{\left(\frac{\delta}{1-\delta}\right)^2}\Omega = A_d\sigma_d^2 + A_w^2\sigma_w^2,$$

whereas demand can be written as

$$d_t = \bar{d} + \zeta_t^d + \gamma s_t = \bar{d} + \gamma A_0 + \left(1+\gamma A_d\right)\zeta_t^d + \gamma A_w\zeta_t^w.$$

Substituting the conjectured form into (27), after some algebra we obtain the equilibrium coefficients provided in (13)-(14).

If instead in equilibrium $d_t < 0$, i.e., preferred-habitat investors are short in swaps. [To be added] ■

A Equilibrium existence and multiplicity

Lemma 9 *There always exists a solution to the equation system (12)-(13). Further, there might exist multiple equilibria of the model.*

Proof. Equations (12)-(13) can be rearranged to obtain

$$\left(1+\gamma A_d\right)\left[\frac{1-\rho_d\delta}{1-\delta} - \frac{1-\rho_w\delta}{1-\delta} + \frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\frac{1}{A_w}\right] = \frac{1-\rho_d\delta}{1-\delta},$$

therefore, once one of the A_is is pinned down, the other is uniquely determined; in particular, a higher A_d corresponds to a higher A_w and vice versa. Because in equilibrium we have $\Omega = \left(\frac{\delta}{1-\delta}\right)^2 \left(A_d^2\sigma_d^2 + A_w^2\sigma_w^2\right)$, (13) implies

$$\begin{aligned}\frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\frac{1}{A_w} &= \frac{1-\rho_w\delta}{1-\delta} + \alpha\gamma\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2 \\ &+\alpha\gamma\left(\frac{\delta}{1-\delta}\right)^2\left[\frac{\sigma_d^2}{\gamma^2}\left(\frac{\frac{1-\rho_d\delta}{1-\delta}}{\frac{1-\rho_d\delta}{1-\delta} - \frac{1-\rho_w\delta}{1-\delta} + \frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\frac{1}{A_w}} - 1\right)^2 + A_w^2\sigma_w^2\right].\end{aligned} \tag{28}$$

Note that when $A_w \searrow 0$ (as it is always positive when it exists), the LHS of (28) diverges to infinity whereas the RHS has a finite limit. On the other hand, when $A_w \to \infty$, the LHS converges to 0 whereas the RHS's limit is ∞. Since both sides are continuous functions of A_w, there always exists a solution; in fact we always have an odd number of solutions. From (28) this odd number cannot be above 5 as the equation is actually equivalent to a 5th order polynomial in A_w.

Note also that since $\alpha, \gamma, \Omega \geq 0$, (13) implies

$$A_w \leq \bar{A}_w \equiv \frac{1-\delta}{1-\rho_w\delta}\frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2,$$

which means we must have

$$\frac{\frac{1-\rho_d\delta}{1-\delta}}{\frac{1-\rho_d\delta}{1-\delta} - \frac{1-\rho_w\delta}{1-\delta} + \frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\frac{1}{A_w}} \leq 1.$$

The LHS of (28) is trivially decreasing in A_w, whereas, after some algebra, the RHS satisfies

$$\frac{dRHS}{dA_w} \propto \frac{\sigma_d^2}{\gamma^2}\left(\frac{\frac{1-\rho_d\delta}{1-\delta}}{\frac{1-\rho_d\delta}{1-\delta} - \frac{1-\rho_w\delta}{1-\delta} + \frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\frac{1}{A_w}} - 1\right)\frac{\frac{1-\rho_d\delta}{1-\delta}\frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\frac{1}{A_w^2}}{\left(\frac{1-\rho_d\delta}{1-\delta} - \frac{1-\rho_w\delta}{1-\delta} + \frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\frac{1}{A_w}\right)^2} + A_w\sigma_w^2.$$

In particular, at $A_w = 0$,

$$\frac{dRHS}{dA_w} \propto -\frac{\sigma_d^2}{\gamma^2}\frac{\frac{1-\rho_d\delta}{1-\delta}}{\frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2} < 0,$$

while at $A_w = \bar{A}_w$ we have

$$\frac{dRHS}{dA_w} \propto \frac{1-\delta}{1-\rho_w\delta}\sigma_w^2\frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2 > 0.$$

[To be continued.] ■

B Equilibrium with $\gamma = 0$

Assuming $\gamma = 0$, the system (12)-(14) simplifies to

$$
\begin{aligned}
A_d &= -\frac{1-\delta}{1-\rho_d\delta}\alpha\left[\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2+\left(\frac{\delta}{1-\delta}\right)^2\left(A_w^2\sigma_w^2+A_d^2\sigma_d^2\right)\right], \qquad (29)\\
A_w &= \frac{1-\delta}{1-\rho_w\delta}\frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2>0, \text{ and}\\
A_0 &= \frac{1}{\kappa_x}\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\bar{w}-\frac{\kappa_x}{\kappa_z}\bar{r}-\alpha\left[\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2+\Omega\right]\bar{d}.
\end{aligned}
$$

In particular, (29) is equivalent to

$$
0=\left(\frac{\delta}{1-\delta}\right)^2\sigma_d^2A_d^2+\frac{1}{\alpha}\frac{1-\rho_d\delta}{1-\delta}A_d+\left[\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2+\left(\frac{\delta}{1-\delta}\right)^2A_w^2\sigma_w^2\right]
$$

and thus has two real roots, given by

$$
A_d=\frac{-(1-\rho_d\delta)\pm\sqrt{(1-\rho_d\delta)^2-4\left(\frac{\kappa_x}{\kappa_z}\delta\alpha\sigma_d\sigma_r\right)^2\left[1+\left(\frac{1}{\kappa_z}\frac{\delta}{1-\rho_w\delta}\alpha\sigma_r\sigma_w\right)^2\right]}}{2\frac{\delta^2}{1-\delta}\alpha\sigma_d^2},
$$

if

$$
\frac{\kappa_x}{\kappa_z}\frac{\delta}{1-\rho_d\delta}\alpha\sigma_d\sigma_r\sqrt{1+\left(\frac{1}{\kappa_z}\frac{\delta}{1-\rho_w\delta}\alpha\sigma_r\sigma_w\right)^2}<\frac{1}{2};
$$

and, as long as these roots exists, they both satisfy the economically meaningful restriction $A_d < 0$ as required. Importantly, the two roots behave differently when $\delta \to 0$: while

$$
\lim_{\delta\to 0}A_{d,-}=-\frac{1}{\alpha\sigma_d^2}\lim_{\delta\to 0}\frac{1-\rho_d\delta}{\delta^2}=-\infty,
$$

we have

$$
\lim_{\delta\to 0}A_{d,+}=-\left(\frac{\kappa_x}{\kappa_z}\right)^2\alpha\sigma_r^2.
$$

Proof of Theorem 3. We allow intermediaries to trade previously non-traded maturity-τ zero-coupon swaps. The Langrangian is then given by

$$\begin{aligned}\mathcal{L}_t &= \left(\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right]\right)x_t + \mathrm{E}_t\left[r_{t+1}\right]z_t + \left(\tau s_t^{(\tau)} - (\tau-1)\,\mathrm{E}_t\left[s_{t+1}^{(\tau-1)}\right]\right)x_t^{(\tau)} \\ &\quad -\frac{\alpha}{2}\left(\frac{\delta}{1-\delta}\right)^2 \mathrm{Var}_t\left[s_{t+1}\right]x_t^2 - \frac{\alpha}{2}(\tau-1)^2\,\mathrm{Var}_t\left[s_{t+1}^{(\tau-1)}\right]\left(x_t^{(\tau)}\right)^2 - \frac{\alpha}{2}\mathrm{Var}_t\left[r_{t+1}\right]z_t^2 \\ &\quad +\alpha\frac{\delta}{1-\delta}\mathrm{Cov}_t\left[r_{t+1},s_{t+1}\right]x_t z_t + \frac{\alpha}{2}(\tau-1)\,\mathrm{Cov}_t\left[r_{t+1},s_{t+1}^{(\tau-1)}\right]z_t x_t^{(\tau)} \\ &\quad -\frac{\alpha}{2}\frac{\delta}{1-\delta}(\tau-1)\,\mathrm{Cov}_t\left[s_{t+1},s_{t+1}^{(\tau-1)}\right]x_t x_t^{(\tau)} - \psi_t\left(\kappa_x\left|x_t + x_t^{(\tau)}\right| + \kappa_z\left|z_t\right| - w_t\right).\end{aligned}$$

Writing down the FOCs, restricting our attention to the same equilibria as before, and setting $x_t^{\tau} = 0$, we obtain (5), (6), and (16).

Combining (5) and (16), we write

$$\begin{aligned}-\kappa_x\psi_t &= \tau s_t^{(\tau)} - (\tau-1)\,\mathrm{E}_t\left[s_{t+1}^{(\tau-1)}\right] - \alpha\frac{\delta}{1-\delta}(\tau-1)\,\mathrm{Cov}_t\left[s_{t+1},s_{t+1}^{(\tau-1)}\right]x_t \\ &= \frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right] - \alpha\Omega x_t.\end{aligned} \tag{30}$$

Assuming that swap spreads are an affine function of demand and wealth factors,

$$s_t^{(\tau)} = A_0^{(\tau)} + A_d^{(\tau)}\zeta_t^d + A_w^{(\tau)}\zeta_t^w, \tag{31}$$

we can write

$$\mathrm{E}_t\left[s_{t+1}^{(\tau-1)}\right] = A_0^{(\tau-1)} + \rho_d A_d^{(\tau-1)}\zeta_t^d + \rho_w A_w^{(\tau-1)}\zeta_t^w \text{ and } \mathrm{Cov}_t\left[s_{t+1},s_{t+1}^{(\tau-1)}\right] = A_d A_d^{(\tau-1)}\sigma_d^2 + A_w A_w^{(\tau-1)}\sigma_w^2.$$

Combining these with the equilibrium s_t and x_t from (11) and plugging these all into (30), we obtain that the functions $A_0^{(\tau)}$, $A_d^{(\tau)}$, and $A_w^{(\tau)}$ solve the linear difference equation system

$$\begin{bmatrix}\tau A_0^{(\tau)} \\ \tau A_d^{(\tau)} \\ \tau A_w^{(\tau)}\end{bmatrix} = \Phi_1\begin{bmatrix}(\tau-1)\,A_0^{(\tau-1)} \\ (\tau-1)\,A_d^{(\tau-1)} \\ (\tau-1)\,A_w^{(\tau-1)}\end{bmatrix} + \Phi_0,$$

with

$$\Phi_1 = \begin{bmatrix}1 & -\alpha\frac{\delta}{1-\delta}\sigma_d^2\left(\bar{d}+\gamma A_0\right)A_d & -\alpha\frac{\delta}{1-\delta}\sigma_w^2\left(\bar{d}+\gamma A_0\right)A_w \\ 0 & \rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2\left(1+\gamma A_d\right)A_d & -\alpha\frac{\delta}{1-\delta}\sigma_w^2\left(1+\gamma A_d\right)A_w \\ 0 & -\alpha\gamma\frac{\delta}{1-\delta}\sigma_d^2 A_d A_w & \rho_w - \alpha\gamma\frac{\delta}{1-\delta}\sigma_w^2 A_w^2\end{bmatrix},\ \Phi_0 = \begin{bmatrix}\left(1+\alpha\gamma\Omega\right)A_0 + \alpha\Omega\bar{d} \\ \left(\frac{1-\rho_d\delta}{1-\delta}+\alpha\gamma\Omega\right)A_d + \alpha\Omega \\ \left(\frac{1-\rho_w\delta}{1-\delta}+\alpha\gamma\Omega\right)A_w\end{bmatrix},$$

and the initial conditions $A_0^{(0)} = A_d^{(0)} = A_w^{(0)} = 0$, since $s_t^{(0)} = 0$. In particular, the 1-period swap rate at time t is given by

$$s_t^{(1)} = -\kappa_x\psi_t = A_0^{(1)} + A_d^{(1)}\zeta_t^d + A_w^{(1)}\zeta_t^w,$$

where

$$\begin{bmatrix} A_0^{(1)} \\ A_d^{(1)} \\ A_w^{(1)} \end{bmatrix} = \Phi_0.$$

Note that $A_0 - A_0^{(1)} = -\alpha\Omega \mathrm{E}[d_t] < 0$, implying that the term structure of swap spreads is negative and downward sloping on average.

The solution of the difference equation system is given by

$$\begin{bmatrix} \tau A_0^{(\tau)} \\ \tau A_d^{(\tau)} \\ \tau A_w^{(\tau)} \end{bmatrix} = \left(\mathbf{1} - \Phi_1\right)^{-1} \left(\mathbf{1} - \Phi_1^{\tau}\right) \Phi_0.$$

In the special case with $\gamma = 0$, the difference equations for $\tau A_d^{(\tau)}$ and $\tau A_w^{(\tau)}$ simplify to

$$\begin{aligned} \tau A_d^{(\tau)} &= \left(\rho_d - \alpha \frac{\delta}{1-\delta} \sigma_d^2 A_d\right) (\tau - 1)\, A_d^{(\tau-1)} - \alpha \frac{\delta}{1-\delta} \sigma_w^2 A_w \,(\tau - 1)\, A_w^{(\tau-1)} + \left(\frac{1-\rho_d\delta}{1-\delta} A_d + \alpha\Omega\right) \text{ and} \\ \tau A_w^{(\tau)} &= \rho_w \,(\tau - 1)\, A_w^{(\tau-1)} + \frac{1-\rho_w\delta}{1-\delta} A_w, \end{aligned}$$

which imply

$$A_w^{(\tau)} = \frac{1-\rho_w\delta}{1-\delta} A_w \frac{1}{\tau} \frac{1-\rho_w^{\tau}}{1-\rho_w}$$

and

$$\begin{aligned} A_d^{(\tau)} &= \frac{1}{\tau}\left[\kappa_1 \frac{1 - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)^{\tau}}{1 - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)} + \kappa_2 \frac{\rho_w^{\tau} - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)^{\tau}}{\rho_w - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)}\right] \\ &= -\left[\alpha \left(\frac{\kappa_x}{\kappa_z}\right)^2 \sigma_r^2 \underbrace{\frac{1}{\tau} \frac{1 - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)^{\tau}}{1 - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)}}_{>0} + \kappa_2 \underbrace{\left(\frac{1}{\tau} \frac{1 - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)^{\tau}}{1 - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)} - \frac{1}{\tau} \frac{\rho_w^{\tau} - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)^{\tau}}{\rho_w - \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2 A_d\right)}\right)}_{>0}\right] \end{aligned}$$

with

$$\kappa_1 = \left(\frac{1-\rho_d\delta}{1-\delta} A_d + \alpha\Omega\right) - \kappa_2 = -\alpha \left(\frac{\kappa_x}{\kappa_z}\right)^2 \sigma_r^2 - \kappa_2 < 0$$

and

$$\kappa_2 = \alpha \frac{\delta}{(1-\delta)^2} \sigma_w^2 \frac{1-\rho_w\delta}{1-\rho_w} A_w^2 > 0.$$

Notice that

$$\frac{dA_w^{(\tau)}}{d\tau} = \underbrace{\frac{1-\rho_w\delta}{1-\delta}\frac{A_w}{1-\rho_w}}_{>0}\frac{d}{d\tau}\frac{1-\rho_w^\tau}{\tau}$$

But since

$$\frac{d}{d\tau}\frac{1-\rho^\tau}{\tau} = \frac{\frac{d}{d\tau}\left(1-\rho^\tau\right)\tau - \left(1-\rho^\tau\right)}{\tau^2} = -\frac{\left(1-\rho^\tau\right)+\rho^\tau\tau\ln\rho}{\tau^2} < 0$$

Regarding the shape of the functions $A_w^{(\tau)}$ and $A_d^{(\tau)}$, we have the following results:

Lemma 10 *For* $0 < \rho < 1$ *and* $\tau \geq 1$ *integer, the function*

$$F\left(\rho,\tau\right) = \frac{1-\rho^\tau}{\tau}$$

is positive, decreasing, and convex in τ.

Proof. It is straightforward that F is positive. To show that it is decreasing in τ, notice that

$$F\left(\rho,\tau\right) = \frac{1-\rho^\tau}{\tau} > F\left(\rho,\tau+1\right) = \frac{1-\rho^{\tau+1}}{(\tau+1)}$$

iff

$$(\tau+1)\left(1+\rho+...+\rho^{\tau-1}\right) > \tau\left(1+\rho+...+\rho^\tau\right)$$

i.e.

$$1+\rho+...+\rho^{\tau-1} > \tau\rho^\tau,$$

but this is straightforward since $\rho^k > \rho^\tau$ for all $k < \tau$. ■

From here, in the $\gamma = 0$ case $A_w^{(\tau)}$ is positive and decreasing. Further, we can show that in the brackets on the RHS of $A_d^{(\tau)}$, both functions are positive and decreasing, therefore, $A_d^{(\tau)}$ is the difference of two positive and decreasing functions. In fact, we can show that $A_d^{(\tau)}$ is negative and U shaped. Finally, we can express $A_0^{(\tau)}$ from

$$\tau A_0^{(\tau)} = (\tau-1)A_0^{(\tau-1)} - \alpha\frac{\delta}{1-\delta}\left(\bar{d}+\gamma A_0\right)\left[\sigma_d^2 A_d(\tau-1)A_d^{(\tau-1)} + \sigma_w^2 A_w(\tau-1)A_w^{(\tau-1)}\right] + \Phi_{0,0}.$$

In the general case, we can write

$$\begin{aligned}
\tau A_d^{(\tau)} &= \left(\rho_d - \alpha\frac{\delta}{1-\delta}\sigma_d^2\left(1+\gamma A_d\right)A_d\right)(\tau-1)A_d^{(\tau-1)} - \alpha\frac{\delta}{1-\delta}\sigma_w^2\left(1+\gamma A_d\right)A_w(\tau-1)A_w^{(\tau-1)} + \Phi_{0,d} \\
\tau A_w^{(\tau)} &= -\alpha\gamma\frac{\delta}{1-\delta}\sigma_d^2 A_d A_w(\tau-1)A_d^{(\tau-1)} + \left(\rho_w - \alpha\gamma\frac{\delta}{1-\delta}\sigma_w^2 A_w^2\right)(\tau-1)A_w^{(\tau-1)} + \Phi_{0,w}
\end{aligned}$$

$$\left(X\tau A_d^{(\tau)} + \tau A_w^{(\tau)}\right) = \left(X\Phi_{1,dd} + \Phi_{1,wd}\right)(\tau-1)A_d^{(\tau-1)} + \left(X\Phi_{1,dw} + \Phi_{1,ww}\right)(\tau-1)A_w^{(\tau-1)} + \left(X\Phi_{0,d} + \Phi_{0,w}\right)$$

with

$$\left(X\tau A_d^{(\tau)} + \tau A_w^{(\tau)}\right) = \left(X\Phi_{1,dw} + \Phi_{1,ww}\right)\left[\frac{\left(X\Phi_{1,dd} + \Phi_{1,wd}\right)}{\left(X\Phi_{1,dw} + \Phi_{1,ww}\right)}(\tau - 1)\, A_d^{(\tau-1)} + (\tau - 1)\, A_w^{(\tau-1)}\right] + \left(X\Phi_{0,d} + \Phi_{0,w}\right)$$

$$X = \frac{\left(X\Phi_{1,dd} + \Phi_{1,wd}\right)}{\left(X\Phi_{1,dw} + \Phi_{1,ww}\right)}$$

$$\begin{aligned} X^2\Phi_{1,dw} + \left(\Phi_{1,ww} - \Phi_{1,dd}\right) X - \Phi_{1,wd} &= 0 \\ X &= \frac{-\left(\Phi_{1,ww} - \Phi_{1,dd}\right) \pm \sqrt{\left(\Phi_{1,ww} - \Phi_{1,dd}\right)^2 + 4\Phi_{1,dw}\Phi_{1,wd}}}{2\Phi_{1,dw}} \\ X &= \frac{-\left(\Phi_{1,ww} - \Phi_{1,dd}\right) \pm \sqrt{\left((\rho_w - \rho_d) + \alpha\frac{\delta}{1-\delta}\left[\sigma_d^2\left(1 + \gamma A_d\right) A_d - \gamma\sigma_w^2 A\right.\right.}}{2\Phi_{1,dw}} \end{aligned}$$

From here, we can show that equilibrium coefficients are qualitatively similar as in the $\gamma = 0$ case. [To be added.] ■

Proof of Proposition 5. The expected return on the swap spread trades can also be represented as a function of demand and supply shocks. First, regarding the generic swap spread trade opportunity, we can write

$$\frac{1}{1-\delta}s_t - \frac{\delta}{1-\delta}\mathrm{E}_t\left[s_{t+1}\right] = B_0 + B_d\zeta_t^d + B_w\zeta_t^w,$$

where

$$B_0 = A_0,\ B_d = \frac{1-\rho_d\delta}{1-\delta}A_d < 0, \text{ and } B_w = \frac{1-\rho_w\delta}{1-\delta}A_w > 0.$$

Notice further, that

$$0 > A_d^{(1)} = \frac{1-\rho_d\delta}{1-\delta}A_d + \alpha\Omega\underbrace{\left(1+\gamma A_d\right)}_{>0} > \frac{1-\rho_d\delta}{1-\delta}A_d = B_d$$

while

$$A_w^{(1)} = \frac{1-\rho_w\delta}{1-\delta}A_w + \alpha\gamma\Omega A_w > \frac{1-\rho_w\delta}{1-\delta}A_w = B_w > 0.$$

■

Proof of Proposition 6. We write

$$\tau s_t^{(\tau)} - (\tau - 1)\,\mathrm{E}_t\left[s_{t+1}^{(\tau-1)}\right] = B_0^{(\tau)} + B_d^{(\tau)}\zeta_t^d + B_w^{(\tau)}\zeta_t^w,$$

where

$$
\begin{aligned}
B_0^{(\tau)} &= \tau A_0^{(\tau)} - (\tau-1)\, A_0^{(\tau-1)} \\
&= \alpha\frac{\delta}{1-\delta}\sigma_d^2\left(\bar{d}+\gamma A_0\right)A_d\left[\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2-\kappa_2\frac{1-\rho_w}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\right]\frac{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^{\tau-1}}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)} \\
&\quad +\alpha\frac{\delta}{1-\delta}\left(\bar{d}+\gamma A_0\right)\left[\sigma_d^2A_d\kappa_2\frac{1-\rho_w}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}-\sigma_w^2A_w^2\frac{1-\rho_w\delta}{1-\delta}\right]\frac{1-\rho_w^{\tau-1}}{1-\rho_w} \\
&\quad +\Phi_{0,0},
\end{aligned}
$$

$$
\begin{aligned}
&= \alpha\frac{\delta}{1-\delta}\sigma_d^2\left(\bar{d}+\gamma A_0\right)A_d\left[\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2-\alpha\frac{\delta}{(1-\delta)^2}\sigma_w^2\frac{1-\rho_w\delta}{1-\rho_w}A_w^2\frac{1-\rho_w}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\right]\frac{1-(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma}{1-(\rho_d-\alpha\frac{\delta}{1-\delta}} \\
&\quad -\alpha\frac{\delta}{1-\delta}\left(\bar{d}+\gamma A_0\right)\sigma_w^2A_w^2\frac{1-\rho_w\delta}{1-\delta}\frac{\rho_w-\rho_d}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\frac{1-\rho_w^{\tau-1}}{1-\rho_w} \\
&\quad +\Phi_{0,0},
\end{aligned}
$$

$$
\begin{aligned}
&= \underbrace{\alpha\frac{\delta}{1-\delta}\sigma_d^2\left(\bar{d}+\gamma A_0\right)\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2A_d}_{<0}\frac{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^{\tau-1}}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)} \\
&\quad -\underbrace{\alpha\frac{\delta}{1-\delta}\left(\bar{d}+\gamma A_0\right)\sigma_w^2\frac{1-\rho_w\delta}{1-\rho_w}A_w^2}_{>0}\left[\begin{array}{c}\alpha\frac{\delta}{(1-\delta)^2}\sigma_d^2A_d\frac{1-\rho_w}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\frac{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^{\tau-1}}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\\+\frac{\rho_w-\rho_d}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\frac{1-\rho_w^{\tau-1}}{1-\rho_w}\end{array}\right] \\
&\quad +\Phi_{0,0},
\end{aligned}
$$

[To be finished.] and

$$
B_w^{(\tau)} = \tau A_w^{(\tau)} - (\tau-1)\,\rho_w A_w^{(\tau-1)} = \frac{1-\rho_w\delta}{1-\delta}A_w
$$

constant across maturities, while

$$
\begin{aligned}
B_d^{(\tau)} &= \kappa_1\frac{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^{\tau}}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}+\kappa_2\frac{\rho_w^{\tau}-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^{\tau}}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)} \\
&\quad -\rho_d\left[\kappa_1\frac{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^{\tau-1}}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}+\kappa_2\frac{\rho_w^{\tau-1}-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^{\tau-1}}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\right],
\end{aligned}
$$

or

$$
\begin{aligned}
B_d^{(\tau)} &= -\left[\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\frac{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^\tau}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}+\kappa_2\left(\frac{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^\tau}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}-\frac{\rho_w^\tau-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^\tau}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\right)\right] \\
&\quad +\rho_d\left[\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2\frac{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^{\tau-1}}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}+\kappa_2\left(\frac{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)^{\tau-1}}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}-\frac{\rho_w^{\tau-1}-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}{\rho_w-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\right.\right.
\end{aligned}
$$

Note that in the limit $\alpha\delta = 0$ this simplifies to

$$
\begin{aligned}
B_d^{(\tau)} &= \left[\kappa_1\frac{1-\rho_d^\tau}{1-\rho_d}+\kappa_2\frac{\rho_w^\tau-\rho_d^\tau}{\rho_w-\rho_d}\right]-\rho_d\left[\kappa_1\frac{1-\rho_d^{\tau-1}}{1-\rho_d}+\kappa_2\frac{\rho_w^{\tau-1}-\rho_d^{\tau-1}}{\rho_w-\rho_d}\right] \\
&= \kappa_1+\kappa_2\rho_w^{\tau-1} = -\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2-\kappa_2\left(1-\rho_w^{\tau-1}\right),
\end{aligned}
$$

meaning $B_d^{(\tau)}$ is negative and downward sloping across maturities. In the general case, at $\tau = 1$ it becomes

$$
B_d^{(1)} = -\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2 < 0,
$$

whereas $\tau \longrightarrow \infty$ it is

$$
B_d^{(\infty)} = -\frac{(1-\rho_d)}{1-\left(\rho_d-\alpha\frac{\delta}{1-\delta}\sigma_d^2A_d\right)}\left[\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2+\kappa_2\right] < -\alpha\left(\frac{\kappa_x}{\kappa_z}\right)^2\sigma_r^2 < 0.
$$

Hence, it is reasonable to conjecture that $B_d^{(\tau)}$ is also negative and downward sloping across maturities in the general case [To be finished]. ■

Proof of Proposition 7. From (17), conditional and unconditional swap spread volatilies are given by

$$
\mathrm{Var}_{t-1}\left[s_t^{(\tau)}\right] = \mathrm{Var}\left[A_0^{(\tau)}+A_d^{(\tau)}\zeta_t^d+A_w^{(\tau)}\zeta_t^w\right] = \left(A_d^{(\tau)}\right)^2\sigma_d^2+\left(A_w^{(\tau)}\right)^2\sigma_w^2 \tag{32}
$$

and

$$
\mathrm{Var}\left[s_t^{(\tau)}\right] = \mathrm{Var}\left[A_0^{(\tau)}+A_d^{(\tau)}\zeta_t^d+A_w^{(\tau)}\zeta_t^w\right] = \left(A_d^{(\tau)}\right)^2\mathrm{Var}\left[\zeta_t^d\right]+\left(A_w^{(\tau)}\right)^2\mathrm{Var}\left[\zeta_t^w\right]. \tag{33}
$$

Since $A_w^{(\tau)}$ is positive and monotone decreasing and $A_d^{(\tau)}$ is negative and U shaped, both variances are either downward sloping or hump shaped. ■

B Additional tables and figures

Table A1: **Swap spread and swap spread trade holding period returns, alternative positions:** This table reports the slope and intercept coefficients from regressions of, respectively, the 30-year swap spread (regressions 1-2), the 3-month holding period return on the 30-year swap spread trade (regressions 3-4), and the 12-month holding period return on the 30-year swap spread trade (regressions 5-6) on the spread between the 3-month general collateral repo rate and the 3-month OIS rate (GC repo-OIS), the Primary Dealers' gross position in coupon-bearing Treasury securities (PD gross), the Primary Dealers' long position in coupon-bearing Treasury securities plus the outstanding balances of securities in through financing arrangements (PD long + sec. in), and the spread between the 3-month USD LIBOR rate and the 3-month general collateral repo rate (LIBOR-GC repo). Data are weekly and run from January 2009 to December 2020. Newey and West (1987, 1994) standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Spread		3-month returns		12-month returns	
	(1)	(2)	(3)	(4)	(5)	(6)
GC repo - OIS	−1.303***	−1.378***	1.165	0.995	11.435*	11.276**
	(0.235)	(0.227)	(2.885)	(2.768)	(6.155)	(5.549)
PD gross	0.011		0.530**		1.999***	
	(0.014)		(0.384)		(0.392)	
PD long + in		−0.026***		0.264**		1.214***
		(0.005)		(0.122)		(0.183)
LIBOR - GC repo	−0.065	−0.118**	2.246*	1.899	8.206***	7.200***
	(0.068)	(0.059)	(1.269)	(1.326)	(1.805)	(1.559)
Constant	−20.318***	40.473***	−232.586**	−572.433**	−935.760***	−2676.237***
	(5.922)	(10.721)	(95.472)	(250.523)	(173.351)	(401.028)
Observations	495	495	495	495	495	495
Adjusted R^2	0.270	0.329	0.050	0.064	0.160	0.264

Table A2: **Swap spread and swap spread trade holding period returns, full sample:** This table reports the slope and intercept coefficients from regressions of, respectively, the 30-year swap spread (regressions 1-3), the 3-month holding period return on the 30-year swap spread trade (regressions 4-6), and the 12-month holding period return on the 30-year swap spread trade (regressions 7-9) on the spread between the 3-month general collateral repo rate and the 3-month OIS rate (GC repo-OIS), Primary Dealers' net position in coupon-bearing Treasury securities (PD net), Primary Dealers' net position in coupon-bearing Treasury securities orthogonalized with respect to a time trend, coupon-bearing Treasury securities held by the Federal Reserve and the 12-month average of coupon-bearing Treasury securities net issuance (PD net orth.), and the spread between the 3-month USD LIBOR rate and the 3-month general collateral repo rate (LIBOR-GC repo). Data are weekly and run from January 2009 to December 2020. Newey and West (1987, 1994) standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Spread			3-month returns			12-month returns		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
GC repo - OIS	−0.936***	−0.889***	−0.954***	−0.256	−0.862	−1.075	2.658	−1.320	−1.206
	(0.151)	(0.210)	(0.149)	(2.097)	(2.956)	(2.257)	(4.160)	(6.059)	(4.592)
PD net		−0.009			0.116			0.789	
		(0.024)			(0.382)			(0.800)	
PD net orth.			0.011			0.842*			3.759***
			(0.034)			(0.467)			(0.884)
LIBOR - GC repo	−0.085*	−0.087*	−0.094*	1.058	1.073	0.778	6.051***	6.231***	4.473***
	(0.049)	(0.048)	(0.050)	(1.137)	(1.135)	(1.126)	(1.657)	(1.599)	(1.609)
Constant	−16.832***	−16.575***	−16.375***	−10.069	−12.941	0.337	−126.835***	−144.273***	−69.743
	(2.025)	(2.002)	(1.996)	(27.476)	(27.377)	(27.317)	(62.933)	(61.073)	(59.951)
Observations	626	626	614	615	615	614	576	576	576
Adjusted R^2	0.229	0.229	0.233	0.008	0.007	0.023	0.062	0.069	0.140

Table A3: **Swap spread and swap spread trade holding period returns, pre-2009 sample:** This table reports the slope and intercept coefficients from regressions of, respectively, the 30-year swap spread (regressions 1-2), the 3-month holding period return on the 30-year swap spread trade (regressions 3-4), and the 12-month holding period return on the 30-year swap spread trade (regressions 5-6) on the spread between the 3-month general collateral repo rate and the 3-month OIS rate (GC repo-OIS), Primary Dealers' net position in coupon-bearing Treasury securities (PD net), and the spread between the 3-month USD LIBOR rate and the 3-month general collateral repo rate (LIBOR-GC repo). Data are weekly and run from December 2001 to December 2008. Newey and West (1987, 1994) standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Spread		3-month returns		12-month returns	
	(1)	(2)	(3)	(4)	(5)	(6)
GC repo - OIS	−0.644***	−0.530***	−2.717	−1.545	7.820*	11.092**
	(0.171)	(0.172)	(2.028)	(2.222)	(4.608)	(5.024)
PD net		−0.010**		−1.028**		−2.869***
		(0.045)		(0.464)		(1.090)
LIBOR - GC repo	−0.148***	−0.128**	−1.710***	−1.498**	−3.317**	−2.726*
	(0.049)	(0.050)	(0.633)	(0.621)	(1.574)	(1.443)
Constant	44.444***	34.727***	−12.894	−112.987***	20.741	−258.775**
	(1.945)	(5.227)	(20.876)	(46.050)	(53.603)	(111.740)
Observations	370	370	370	370	370	370
Adjusted R^2	0.333	0.368	0.137	0.158	0.216	0.263

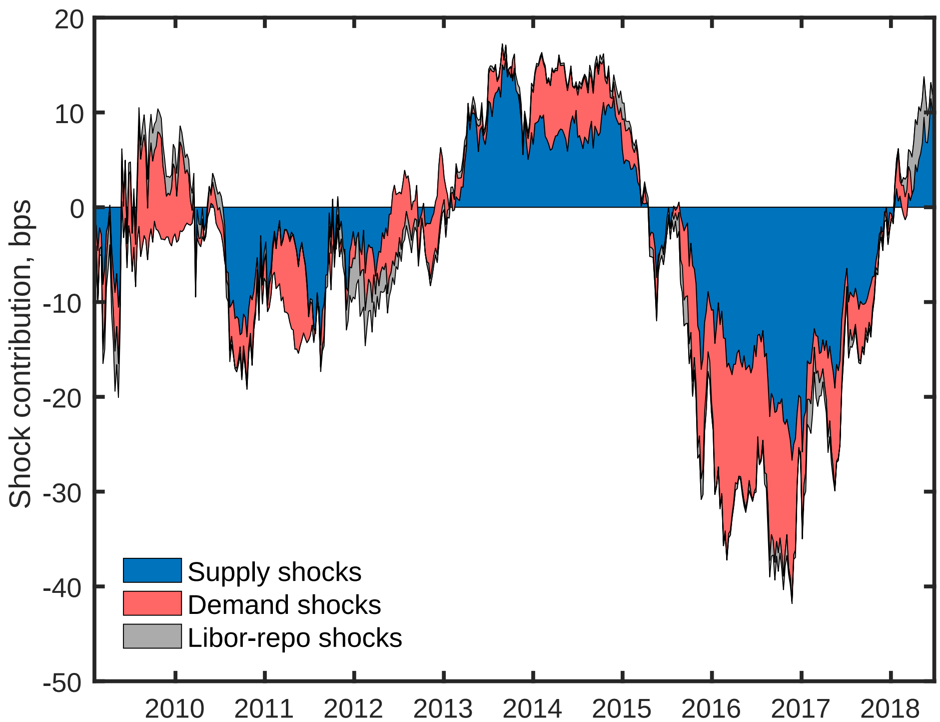

Figure A1: **Swap spread historical decomposition:** This figure shows the contribution of supply, demand, and LIBOR-Repo shocks to the 30y swap spread. Data are weekly and run from January 2009 to June 2018.