When Uncertainty and Volatility Are Disconnected: Implications for Asset Pricing and Portfolio Performance*

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Abstract

We analyze an environment where the uncertainty in the equity market return and its volatility are both stochastic, and may be potentially disconnected. We solve a representative investor's optimal asset allocation and derive the resulting conditional equity premium and risk-free rate in equilibrium. Our empirical analysis shows that the equity premium appears to be earned for facing uncertainty, especially high uncertainty that is disconnected from lower volatility, rather than for facing volatility as traditionally assumed. Incorporating the possibility of a disconnect between volatility and uncertainty significantly improves portfolio performance, over and above the performance obtained by conditioning on volatility only.

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"You would think if uncertainty was high, you'd have a bit more volatility."

William Dudley, New York Fed President, February 15, 2017.

1. Introduction

Although the notions of uncertainty and volatility are often used interchangeably, the two concepts are inherently different: volatility measures the dispersion of short-term shocks around a long-term mean, while uncertainty measures the difficulty to forecast more general aspects of the distribution, including its long-term mean. It is natural to expect uncertainty and volatility to be positively correlated, as the quote above implies. For example, Amengual and Xiu (2018) show that the resolution of monetary policy uncertainty is associated with declines in volatility. However, there have been several episodes in which either volatility was high and uncertainty was significantly lower or vice versa. For instance, the US 2016 presidential election generated some uncertainty about long-term economic and other policies, but was surprisingly characterized by very low levels of stock market volatility. Similarly, the UK's exit from the EU (Brexit) involved substantial uncertainty about trade, growth, and immigration policies for the UK and the EU, but had a barely noticeable impact on short-term volatility in their respective stock markets. These recent events are examples of situations in which a disconnect between the two variables appeared because uncertainty was substantially higher than volatility. By contrast, the stock market dynamics during the financial crisis in 2008 and the initial stock market reaction to the diffusion of the Covid-19 pandemic in the spring of 2020 are examples of situations in which a disconnect occurred due to a higher increase in volatility compared to the rise in uncertainty. Interestingly, in the months following the stock market crash in March 2020, volatility declined much faster than uncertainty, leading to a switch in the nature of disconnect, characterized instead by uncertainty being higher than volatility.

Figure 1 contains a scatter plot of volatility and uncertainty at the weekly frequency between January 1986 and December 2020, proxied respectively by realized volatility computed from high-frequency data and the economic policy uncertainty index (EPU) of Baker et al. (2016). The figure shows that the two variables, although generally positively correlated, are far from being perfect substitutes for one another. And the degree of connection between uncertainty and volatility appears to vary across periods, consistent with the anecdotal evidence that can be gleaned from the specific episodes mentioned above.

INSERT FIGURE 1 ABOUT HERE

Motivated by these empirical observations, we construct a novel asset pricing model in which uncertainty and volatility are two separate stochastic processes, whose degree of connection is stochastic. We show that, in the context of the simple regimes exhibited in Figure 1, our model delivers a different equity premium and risk-free rate and consequently a different portfolio strategy in regimes in which uncertainty and volatility are high or low at the same time (connected), compared to regimes in which one of them is significantly higher than the other (disconnected). We show that incorporating this potential disconnect leads to substantially improved forecasts of the equity premium and portfolio performance.

We model uncertainty using the framework of a robust control problem pioneered by Hansen and Sargent.[1] In this framework, the investor recognizes that he is unable to know exactly the true underlying model and is therefore subject to model uncertainty. As a result, he considers instead a reference model that represents his best estimate of the true underlying model. However, due to model uncertainty, he acknowledges that his reference model may be different from the true one, and he would like his optimal consumption and portfolio to be robust to this possibility. Accordingly, he assumes that the true underlying model lies in a set of alternative models. To make sure that his model uncertainty concerns are realistic, he only considers alternative models that are statistically difficult to distinguish from the reference model and are therefore difficult to reject empirically. Accordingly, the investor imposes an upper bound on the growth rate of the alternative models' "distance" to the reference model, which we measure using relative entropy. The upper bound on entropy determines model uncertainty. When the bound is large (respectively, small), alternatives that are statistically far from the reference model will fall below (respectively, above) the bound, and the investor will have low (respectively, high) confidence on the reference model. A key contribution of our paper is that we generalize model uncertainty (captured by the upper bound on relative entropy growth) to be driven by the product of stochastic volatility and a new "disconnect" stochastic process, that drives a wedge between volatility and uncertainty. Because the investor is interested in policies that are robust across the set of alternative models, he optimally evaluates his policies under the worst-case alternative in the set of models satisfying his entropy growth constraint. For that reason, the dynamics of volatility and disconnect, which drive the entropy bound or model uncertainty, have an important impact on asset prices and associated optimal policies.

We first solve the model in partial equilibrium, computing in closed form a representative log-utility investor's optimal policies for consumption and portfolio choice. We show that the investor's optimal equity holding consists of the standard myopic term, which is inversely proportional to stock return variance, and a new (yet still myopic as befits a logarithmic investor) uncertainty correction

[1] See, for example, Anderson et al. (2003), Hansen et al. (2006), Cogley et al. (2008), Hansen and Sargent (2008), and Hansen and Sargent (2011). For a discussion and comparison of the max-min expected utility of Gilboa and Schmeidler (1989) and robust control theory, see Hansen and Sargent (2001) and Hansen et al. (2006).

term, which is the optimal response to the potential disconnect between volatility and uncertainty. We find that the contemporaneous interaction of volatility and uncertainty plays an important role in determining optimal portfolios. In particular, the sensitivity of portfolio weights to changes in volatility depends on the level of model uncertainty. Accordingly, the trajectory of portfolio weights in a given period depends on the joint dynamics of volatility and uncertainty, including whether they are connected or disconnected.

Given the optimal asset allocation by a representative investor, we then solve for the equilibrium equity premium and risk-free rate, which are non-linear functions of both stochastic volatility and disconnect processes. Our model predicts that the uncertainty term embedded in disconnect generates a flight-to-quality-like correlation among asset returns. In high-uncertainty periods investors require a high equity premium to hold the risky asset and are willing to accept a low risk-free rate to hold the safe asset. The presence of stochastic volatility may diminish or amplify these effects depending on whether it is declining or increasing, respectively. The interaction of volatility with a possibly disconnected uncertainty means that our model can generate a high equity premium in a low volatility environment, whenever the disconnect is high. These empirical patterns were observed in the period surrounding the US 2016 election, among other disconnect episodes.

Our model's results can help explain the challenges faced by previous empirical studies trying to establish a risk-return trade-off using volatility alone as a measure of risk. While most theoretical asset pricing models imply a positive relationship between return and risk,the empirical evidence for such a trade off is mixed or inconclusive, and depends on the sample period and methodology, including whether total or idiosyncratic volatility is considered.[2] Although our model also implies a positive relationship between return and volatility, it adds a new component in the equity premium associated with the disconnect between volatility and uncertainty, which is also theoretically positive. Including this component allows our model to more accurately reproduce asset pricing patterns. For instance, our model suggests that both positive and negative risk-return trade offs are possible, according to uncertainty been high or low, respectively.

Empirically, we find that once a disconnect is possible, stock returns respond negatively to contemporaneous return volatility (consistent with the prevalence of the leverage effect in the data but inconsistent with the predictions of most theoretical asset pricing models), but respond positively, and consistently across horizons, to the uncertainty component in the model (as predicted by our model). Our results therefore show that allowing for uncertainty to be disconnected from volatility makes uncertainty a much better variable than volatility in terms of generating a trade-off with expected returns: it appears from our results that the equity premium is earned mainly for facing

[2]See, e.g., Merton (1980), French et al. (1987), Glosten et al. (1993), Goyal and Santa-Clara (2003), Ang et al. (2009) and Campbell et al. (2018).

uncertainty, especially high uncertainty that is disconnected from lower volatility, rather than for facing volatility per se.

We then go on to evaluate the portfolio performance of a reference investor who predicts future excess returns using our estimated relation between stock excess return, volatility, and uncertainty. We find that our model significantly improves portfolio performance relative to both existing unconditional and conditional asset pricing models, including those that time volatility but do not account for its potential disconnect from uncertainty,[3] and those that allow for the presence of uncertainty but not for its disconnect from volatility. These results are valid when back-testing the model in-sample but also out-of-sample.

Our work is motivated by a growing literature that analyzes the effect of uncertainty on the equity premium, both theoretically and empirically. For instance, the theoretical work of Pastor and Veronesi (2012) and Pastor and Veronesi (2013) shows that political uncertainty shocks command a risk premium, and that stocks become more correlated and volatile in periods of elevated political uncertainty. On the empirical side, Brogaard and Detzel (2015) show that uncertainty positively forecasts excess returns and that innovations in uncertainty carry a significantly negative risk premium, while Bali et al. (2017) find that the difference between returns on portfolios with the highest and lowest uncertainty beta is negative and highly significant. The empirical results in both papers are consistent with the notion that investors care not only about expected returns and risk, but also about uncertainty.

In existing models that do not account for model uncertainty, time-varying volatility drives the optimal consumption and investment policies of a risk-averse agent, but there is no mechanism to account for the degree of uncertainty attached to the assumptions of the model (see, e.g., Chacko and Viceira (2005), Liu (2007), and Drechsler and Yaron (2011)). On the other hand, in models in which the agent is uncertainty-averse but volatility is constant, the optimal portfolio strategy is to reduce the exposure to risky assets when uncertainty increases (see, e.g., Trojani and Vanini (2000), Trojani and Vanini (2004), Maenhout (2004), Maenhout (2006), and Illeditsch (2011)).[4] Since

[3]The volatility-managed portfolios in Moreira and Muir (2017), which take less risk when volatility is high, produce large alphas and Sharpe ratios. We show that incorporating disconnect as an additional variable achieves even higher portfolio performance.

[4]Liu et al. (2005) consider a setup with constant diffusive volatility but in which the stock price is subject to jump risk and the investor is averse to uncertainty with respect to jumps. They show that their model is consistent with several empirical patterns of option prices. Jahan-Parvar and Liu (2014) show that a production-based asset-pricing model with regime-switching productivity, constant volatility in each regime, and ambiguity aversion can reproduce, among other asset prices empirical patterns, predictability of excess return by investment-capital, price-dividend, and consumption-wealth ratios. In contrast to these papers, in our endowment economy model volatility and uncertainty predict excess returns, and this predictability generates high portfolio performance, especially in periods of high uncertainty that is disconnected from low volatility.

volatility and returns tend to be negatively correlated, the resulting conservative asset allocation makes the investor forgo much of the upside of asset markets in situations in which volatility fails to materialize despite levels of uncertainty above or close to average, such as the aftermaths of the US 2016 election, Brexit, and the months that followed the stock market crash of March 2020 driven by the pandemic. Including uncertainty and volatility separately allows the investor to take advantage of such situations.

We are aware of two papers that have ventured to incorporate both stochastic volatility and uncertainty, as we do. Drechsler (2013) considers a general model with stochastic model uncertainty, stochastic volatility and jumps. His model differs from ours in at least two dimensions. First, his focus is on reproducing the empirical properties of index options and the variance premium, while our focus is on stock excess return predictability and the characterization of asset prices, especially in periods of high disconnect between uncertainty and volatility. An important feature of our model, which is not present in Drechsler (2013), is that volatility affects model uncertainty, and this effect is stochastic. Second, we go beyond equilibrium asset pricing properties, and consider a regression-based portfolio approach to evaluate how the predictions from our model affect portfolio performance, and find that our model's stock excess return predictability significantly increases portfolio performance in and out of sample. Faria and da Silva (2016) study optimal asset allocation for an investor subject to stock return stochastic volatility and constant ambiguity uncertainty. Like us, they are interested in the impact of model uncertainty on optimal portfolios in a setup with stochastic volatility. However, because they assume constant model uncertainty, their setup is not suited to study the joint dynamics of volatility, uncertainty, and their disconnect on the optimal portfolio decision. Moreover, they do not study asset prices in equilibrium and their main focus is on the marginal impact of model uncertainty on investors' hedging demands, which they find to be very low.

The rest of the paper is organized as follows. Section 2 sets up the model, in which volatility and uncertainty are potentially disconnected. Section 3 solves for the optimal consumption and portfolio allocation taking prices as given. Section 4 solves for the equilibrium asset prices' dynamics, including the conditional equity premium and risk-free rate. Section 5 describes the data and our empirical analysis. Section 6 evaluates the performance of our model's implied portfolio strategy in the full sample and in specific high disconnect regimes. Section 7 shows that our results also hold out-of-sample. Section 8 concludes.

2. A framework with volatility and uncertainty disconnected

We consider an infinite horizon expected utility maximization problem where a representative investor chooses his consumption level and allocates his funds between a risk-less and a risky asset, which exhibits stochastic volatility. For this purpose, the investor employs a particular model which presumably represents his best estimate of the risky asset's dynamics under a benchmark or reference probability measure. However, the investor fears that the model he uses is potentially misspecified, i.e., he believes that the true model could lie in a larger set of alternative models that are statistically difficult to distinguish from the reference model. To mitigate the effect of potential model misspecification on his utility, we show in Section 3 that the investor chooses optimal consumption and portfolio holdings that are robust against small perturbations of the reference model.

We start by specifying the set of alternative models for asset returns which the investor considers as plausible alternatives. Following the Hansen-Sargent approach, the set of alternative models considered by the investor consists of those models whose relative entropy (or Kullback-Leibler) distance from the reference model is bounded. Models at a small distance from the reference model should be statistically difficult to distinguish from it.

The new element in our approach is that we generalize the upper bound on relative entropy growth. We first let stochastic volatility affect this upper bound rendering the set of alternative models time-varying, i.e., the bound is higher when there is higher volatility and vice versa. To avoid perfect correlation between model uncertainty and volatility, however, we introduce an additional stochastic process which drives the degree of connection between uncertainty and volatility. This setup allows us to study market scenarios in which, simultaneously, uncertainty can be high while volatility is low and vice versa: those are the "HL" and "LH" regimes characterized in Figure 1.

2.1 Asset prices dynamics under the reference and robust measures

Assume a complete, filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ satisfying the usual assumptions. $\mathbb{P}$ denotes the reference (or physical) probability measure. We consider a Lucas tree economy (see Lucas (1978)) with a single risky asset with price S_t and a risk-free asset with price B_t. The risky asset represents a perpetual claim on the stream of aggregate dividends in the economy, whose exogenous dynamics are given by

$$\frac{dD_t}{D_t} = \mu_D dt + \sigma_D v_t dW_t^D, \tag{1}$$

where μ_D and σ_D are constant parameters. Under the reference measure, the dynamics of the asset prices are

$$\frac{dB_t}{B_t} = r_{f,t}dt, \quad B_0 > 0, \tag{2}$$

$$\frac{dS_t}{S_t} = \left(\mu_{S,t} - \frac{D_t}{S_t}\right)dt + \sigma_S v_t dW_t^S, \quad S_0 > 0, \tag{3}$$

$$dv_t = \kappa_v(\theta_v - v_t)dt + \sigma_v v_t dW_t^v, \quad v_0 > 0, \tag{4}$$

where $r_{f,t}$ denotes the risk-free rate, $\mu_{S,t}$ is the expected total return of the risky asset and v_t is its stochastic volatility. These two Brownian motions are correlated with correlation coefficient $\rho_{Sv} \in [-1,1]$, so that $\mathbb{E}[dW_t^S dW_t^v] = \rho_{Sv}dt$. The stochastic volatility process in Equation (4) follows an SV-GARCH diffusion, where κ_v, θ_v, and σ_v are positive parameters denoting the speed of mean reversion, the long run level, and the volatility of volatility, respectively.[5]

To introduce the possibility of model misspecification, we specify a set of alternative or worst-case robust dynamics which are statistically close to the reference dynamics in Equation (3). Consider a set of equivalent probability measures $\mathbb{P}^\vartheta$ such that

$$\frac{dS_t}{S_t} = \left(\mu_{S,t} - \frac{D_t}{S_t} + \sigma_S v_t h_t\right)dt + \sigma_S v_t dW_t^{S,\vartheta}, \tag{5}$$

where $W_t^{S,\vartheta} = (W_t^{S,\vartheta})_{t\geq 0}$ is a Brownian motion under $\mathbb{P}^\vartheta$. Equation (5) states that the expected total return on the risky asset has changed from $\mu_{S,t}$ under $\mathbb{P}$ to $\mu_{S,t} + h_t\sigma_S v_t$ under $\mathbb{P}^\vartheta$ where $(h_t)_{t\geq 0}$ is a continuous $\mathcal{F}_t$-measurable function. In what follows, h_t is referred to as the drift perturbation function. Equation (5) reveals that the effect of model uncertainty in the drift induced by h_t is significantly affected by volatility. That is, low levels of volatility mitigate the impact of uncertainty while high levels of volatility amplify it. Moreover, as we will show later, the optimal drift perturbation h_t will be itself a function of both volatility and uncertainty, thereby inducing a non-linear stochastic stock return drift in equilibrium.

2.2 Introducing disconnect

Equations (3) and (5) imply that the Brownian motions under the reference and robust measures are related by

$$dW_t^S = dW_t^{S,\vartheta} + h_t dt, \tag{6}$$

[5] The choice of the SV-GARCH diffusion process for volatility is essentially without loss of generality for a logarithmic investor, and has no effect on any of our main theoretical or empirical results.

and from Girsanov's theorem we can determine that the change from the reference to the robust measure is given by the exponential martingale:

$$\vartheta_t = \exp\left\{\int_0^t h_s dW_s^S - \int_0^t \frac{h_s^2}{2} ds\right\}, \tag{7}$$

with dynamics

$$\frac{d\vartheta_t}{\vartheta_t} = h_t dW_t^S. \tag{8}$$

The process ϑ_t restricts the size of alternative models that are statistically difficult to distinguish from the reference model. Given two probability measures $\mathbb{P}$ and $\mathbb{P}^\vartheta$, growth in the entropy of $\mathbb{P}^\vartheta$ relative to $\mathbb{P}$ over the time interval $[t, t+\Delta t]$ is defined as

$$G(t, t+\Delta t) \equiv \mathbb{E}_t^\vartheta\left[\log\left(\frac{\vartheta_{t+\Delta t}}{\vartheta_t}\right)\right], \qquad \forall t \geq 0, \tag{9}$$

where $\mathbb{E}_t^\vartheta[\cdot]$ denotes the conditional expectation up to time t with respect to $\mathbb{P}^\vartheta$. The investor seeks to be robust against departures from $\mathbb{P}$ up to a certain point: the set of admissible model misspecification is restricted to

$$\left\{\vartheta_t : \mathcal{R}(\vartheta_t) \equiv \lim_{\Delta t \to 0} \frac{G(t, t+\Delta t)}{\Delta t} \leq \frac{1}{2}\epsilon^2 \mathcal{U}_t^2,\ \mathcal{U}_t := \eta_t v_t,\ \epsilon \geq 0,\ \eta_t \geq 0,\ v_t \geq 0,\ \forall t \geq 0\right\}, \tag{10}$$

where the stochastic process $\mathcal{U}_t$ represents the amount of uncertainty that the investor is seeking robustness against and the constant parameter ϵ is a measure of the investor's uncertainty aversion: an investor with higher ϵ seeks robustness against a larger set of alternative models.

The stochastic process η_t denotes the disconnect process we add to the robust control framework. From Equation (10) we have

$$\eta_t := \frac{\mathcal{U}_t}{v_t} \tag{11}$$

so high levels of η_t capture situations in which the economy is in the "HL" regime, characterized by high uncertainty and low volatility, while low levels of η_t are observed when the economy is in the "LH" regime, characterized by low uncertainty and high volatility. By contrast, in the two connected regimes "HH" and "LL", η_t tends to be away from extreme values, and closer to its normalized mean value set to 1. Of course, the variables in Equation (11) are continuous so the notion (and number) of discrete regimes is simply a convenient low-dimensional representation of the investment environment that plays no formal role in the analysis of the model.

To fully specify the model, we assume that η_t is a positive process that, like v_t, follows a SV-GARCH diffusion

$$d\eta_t = \kappa_\eta(\theta_\eta - \eta_t)dt + \sigma_\eta \eta_t dW_t^\eta, \tag{12}$$

where κ_η, θ_η, and σ_η are positive parameters denoting the speed of mean reversion, the long run level, and the volatility of disconnect, respectively.[6] Disconnect and volatility are correlated with correlation coefficient $\rho_{\eta v} \in [-1, 1]$, so that $\mathbb{E}[dW_t^\eta dW_t^v] = \rho_{\eta v} dt$. Since volatility is correlated with the stock price, it follows that disconnect and the stock price are also correlated. Their correlation coefficient is $\rho_{S\eta} \in [-1, 1]$, and hence $\mathbb{E}[dW_t^S dW_t^\eta] = \rho_{S\eta} dt$.[7]

3. Optimal portfolio allocation in partial equilibrium

To solve the optimization problem, we use dynamic programming under constraints to convert the entropy constraint into a penalty on perturbations from the reference model. We then derive the optimal robust solution to the investor's investment and consumption problem in closed form.

We start by formulating the investor's robust control problem. Let X_t denote the investor's wealth and ω_t be the percentage of wealth (or portfolio weight) invested in the risky asset; $1 - \omega_t$ is invested in the risk-free asset. The investor consumes at an instantaneous rate C_t and assumes the risky asset evolves according to the dynamics specified in Equation (5). Accordingly, the dynamics of investor's wealth X_t follow

$$\begin{aligned} dX_t &= \omega_t X_t \left(\frac{dS_t + D_t dt}{S_t} \right) + (1 - \omega_t) X_t \frac{dB_t}{B_t} - C_t dt \\ &= \left(X_t \left[r_{f,t} + \omega_t \left(\mu_{S,t} - r_{f,t} + \sigma_S v_t h_t \right) \right] - C_t \right) dt + \omega_t X_t v_t \sigma_S dW_t^{S,\vartheta}, \end{aligned} \tag{13}$$

starting from an initial endowment $X_0 > 0$. The investor derives logarithmic utility from consumption, has subjective discount rate $\beta > 0$, and solves the infinite horizon problem

$$\sup_{\{C_s, \omega_s\}_{t \le s < \infty}} \inf_{\{h_s\}_{t \le s < \infty}} \mathbb{E}_t^\vartheta \left[\int_t^\infty e^{-\beta s} \log(C_s) ds \right], \tag{14}$$

subject to the entropy growth constraint in Equation (10) and the wealth dynamics in Equation (13). In partial equilibrium, the investor solves this problem taking the dynamics of asset prices as given.

Define the value function

$$V(X_t, v_t, \eta_t) = \sup_{\{C_s, \omega_s\}_{t \le s < \infty}} \inf_{\{h_s\}_{t \le s < \infty}} \mathbb{E}_t^\vartheta \left[\int_t^\infty e^{-\beta s} \log(C_s) ds \right], \tag{15}$$

[6] As it was the case with volatility, the choice of the SV-GARCH diffusion process to model disconnect is essentially without loss of generality in the case of a logarithmic investor, and has no effect on any of our main theoretical or empirical results.

[7] We specify our theoretical model to be as general as possible, including correlations between volatility, disconnect and the stock market. As it will become clear in the following sections, however, our empirical analysis does not rely on these assumptions.

associated with the optimal stochastic robust control problem in Equation (14), where the endogenous drift perturbation function h_t, as we will show, will be state-dependent in volatility v_t and disconnect η_t. The perturbed Hamilton-Jacobi-Bellman (HJB) equation characterizing the optimal solution is

$$
\begin{aligned}
0 = \sup_{\{C_t,\omega_t\}} \inf_{\{h_t\}} \Bigg\{ & \log(C_t) + \frac{\partial V}{\partial X}\left(X_t\left[r_{f,t} + \omega_t\left(\mu_{S,t} - r_{f,t} + \sigma_S v_t h_t\right)\right] - C_t\right) \\
& + \frac{1}{2}\frac{\partial^2 V}{\partial X^2} v_t^2 X_t^2 \sigma_S^2 \omega_t^2 + \frac{\partial V}{\partial v}\kappa_v(\theta_v - v_t) + \frac{1}{2}\frac{\partial^2 V}{\partial v^2}\sigma_v^2 v_t^2 + \frac{\partial V}{\partial \eta}\kappa_\eta(\theta_\eta - \eta_t) \\
& + \frac{1}{2}\frac{\partial^2 V}{\partial \eta^2}\sigma_\eta^2 \eta_t^2 + \frac{\partial^2 V}{\partial X \partial v}\omega_t X_t \rho_{Sv}\sigma_S \sigma_v v_t^2 + \frac{\partial^2 V}{\partial X \partial \eta}\omega_t X_t \rho_{S\eta}\sigma_S\sigma_\eta \eta_t v_t + \frac{\partial^2 V}{\partial \eta \partial v}\rho_{\eta v}\sigma_\eta \sigma_v \eta_t v_t \Bigg\},
\end{aligned}
\tag{16}
$$

subject to:[8,9]

$$
\mathcal{R}(\vartheta_t) = \frac{1}{2}h_t^2 \leq \frac{1}{2}\epsilon^2 \mathcal{U}_t^2,\ \mathcal{U}_t := \eta_t v_t. \tag{17}
$$

The robust optimal control problem is solved in two steps. First, the investor chooses a set of worst-case drift perturbation functions $\{h_s\}_{t\leq s<\infty}$. Second, the investor selects admissible consumption and portfolio holdings $\{C_s, \omega_s\}_{t\leq s<\infty}$ that maximize his expected utility of consumption under the worst-case scenario. The solution for the optimal robust policies is characterized in the following proposition.

Proposition 1. *The optimal consumption, perturbation function, and portfolio policy are given, respectively, by*

$$
\textit{Consumption}: \quad C_t^* = \beta X_t. \tag{18}
$$

$$
\textit{Perturbation}: \quad h_t^* = -\epsilon \eta_t v_t. \tag{19}
$$

$$
\textit{Portfolio weight}: \quad \omega_t^* = \underbrace{\frac{\mu_{S,t} - r_{f,t}}{\sigma_S^2 v_t^2}}_{\textit{reference demand}} - \underbrace{\frac{\epsilon}{\sigma_S}\eta_t}_{\textit{uncertainty correction}}. \tag{20}
$$

The investor consumes a constant fraction of wealth that is equal to the subjective discount rate β, which is typical of logarithmic utility. Interestingly, the optimal drift perturbation function h_t^* is driven by the interaction of stochastic disconnect and volatility, and is further amplified by uncertainty aversion.[10] This implies that, for a given level of uncertainty aversion, the drift adjustment can be high even when volatility is low, if there is high disconnect (HL regime). The second term in the demand function for the risky asset in Equation (20) represents the adjustment made by the investor to his portfolio holdings in the presence of disconnect. It is negative and increasing in magnitude in the investor's degree of uncertainty aversion and in the level of disconnect. The correction

[8] Appendix A.1 shows that $\mathcal{R}(\vartheta_t) = h_t^2/2$.

[9] It can be shown that, under mild assumptions, the solution we provide satisfies the transversality condition: $\lim_{t\to\infty} \mathbb{E}^{\vartheta}\left[V(t, X_t, \eta_t, v_t)\right] = 0$. A formal proof of this sufficient condition is available from the authors upon request.

[10] While in principle there are two roots for the solution of $h_t^* = \pm\epsilon\eta_t v_t$, we focus on the realistic solution with $h_t^* = -\epsilon\eta_t v_t < 0$, which prevails when the expected return of the risky asset under the reference measure is positive.

term is largest when η_t is largest, i.e., in the HL regime where uncertainty is significantly higher than volatility, and lowest when η_t is lowest, i.e., in the LH regime characterized by uncertainty being significantly lower than volatility. In general equilibrium, the equity premium will adjust to induce the investor to nevertheless hold the risky asset. For this to happen, the equity premium must be relatively higher in the HL regime and lower in the LH regime, which turn out to be key predictions of the model that match the empirical evidence.

4. Equilibrium asset prices

An equilibrium is a specification of the dynamics of the risky and risk-less asset prices, including $\mu_{S,t} = \mu_S(v_t, \eta_t)$ and $r_{f,t} = r_f(v_t, \eta_t)$, combined with a set of optimal (robust) consumption and investment policies that support continuous clearing in the markets for the consumption good and the risky asset:

$$C_t = D_t, \tag{21}$$

$$\omega_t = 1. \tag{22}$$

The next proposition characterizes the equilibrium risk-free rate $r_{f,t}$ and the conditional equity premium under the reference measure, which can be expressed as

$$\frac{1}{dt}\mathbb{E}_t\left[\frac{dS_t + D_t dt}{S_t}\right] - r_{f,t} = \mu_{S,t} - r_{ft}, \tag{23}$$

given the dynamics for the stock price S_t in Equation (3).

Proposition 2. *In equilibrium, the price-dividend ratio is* $\frac{S_t}{D_t} = \frac{1}{\beta}$ *and, under the reference measure* $\mathbb{P}$*, the equilibrium equity premium and risk-free rate are given, respectively, by*

$$\textit{Equity premium}: \quad \mu_{S,t} - r_{f,t} = \sigma_D^2 v_t^2 + \epsilon\sigma_D\eta_t v_t^2. \tag{24}$$

$$\textit{Risk-free rate}: \quad r_{f,t} = \beta + \mu_D - \sigma_D^2 v_t^2 - \epsilon\sigma_D\eta_t v_t^2. \tag{25}$$

The equity premium and risk-free rate are time-varying and non-linear functions of volatility and disconnect. In particular, the uncertainty term embedded in disconnect generates a flight-to-quality-like correlation among asset returns. In periods of increasing uncertainty (increasing disconnect, for a given level of volatility) the demand for risky assets decreases and the demand for safe assets increases. Accordingly, in periods of increasing uncertainty the investor requires a higher equity premium to hold the risky asset, and is willing to accept a lower risk-free rate to hold the risk-less asset in equilibrium. The presence of stochastic volatility may diminish or amplify these effects depending on whether it is declining or increasing, respectively. In the following sections, we show that our model is consistent with the observed dynamics of asset prices during high disconnect episodes precisely because both volatility and disconnect jointly drive asset prices.

The equilibrium equity premium and risk-free rate can be also understood in the context of the optimal portfolio policy obtained in Proposition 1. The negative uncertainty correction implies that the representative investor is more conservative and prefers to hold less of the risky asset and more of the risk-free asset compared to a reference investor. In general equilibrium, he must allocate all his wealth to the risky asset. Therefore, the last term in the equity premium boosts up the reward from holding the risky asset just enough so that he optimally chooses to allocate all his wealth to the risky asset. By the same token, the risk-free rate has to decrease in general equilibrium, so the investor optimally chooses not to hold the risk-free asset at all.

5. Empirical analysis

5.1 Data

The S&P 500 (logarithmic) returns including dividends obtained from the CRSP database serve as a proxy for the risky-asset return. The risk-free rate is the three-month (constant maturity) Treasury bill rate, which we obtain from the Fred St. Louis Database (Ticker "DGS3MO"). Both series are available at the daily frequency from January 1, 1986 until December 31, 2020; we aggregate them to the weekly frequency.[11]

We construct our weekly measure of uncertainty $\mathcal{U}_t$ using the daily news-based economic policy uncertainty (EPU) index developed by Baker et al. (2016).[12] The daily EPU index is constructed by counting in the archives of well over one thousand US newspapers the number of articles that contain at least one term related to economic policy uncertainty from the following list: "uncertain," "uncertainty," "economic," "economy," "Congress," "deficit," "Federal Reserve," "legislation," "regulation," and "White House." The daily EPU time series is volatile, and we smooth it by computing its moving average over the past week (5 trading days) to construct the uncertainty measure we use at the weekly frequency.

The diffusive term of the stock price process is $\sigma_{S,t} := \sigma_S v_t$ where $\sigma_S = 1$, since we match v_t to the square root of realized variance, which is computed daily from aggregated intraday returns

[11] In our sample, there are 77 dates for which there is no three-month Treasury bill data available. These dates roughly correspond to the Martin Luther King Jr. Day, President's Day, and Columbus Day. For those dates, we proxy the three-month Treasury bill rate by its quote on the previous day.

[12] Section 5.3 shows that our results continue to hold when considering alternative uncertainty measures.

sampled at five minutes frequency (78 observations per trading day). To obtain a weekly measure of realized variance, we sum the daily realized variance estimates. Based on the definition of disconnect in Equation (11), we construct an empirical proxy for η_t at the weekly frequency by computing the ratio of uncertainty as measured by EPU to realized volatility, and we then normalize this ratio to have an average value over the full sample equal to one. This normalization is for ease of interpretation of the level of disconnect relative to its mean: when η_t is far away from one there is high disconnect between uncertainty and volatility, and when η_t is close to one disconnect is low.

INSERT TABLE 1 HERE

Table 1 summarizes key asset pricing moments (including uncertainty, volatility, and disconnect) for the full sample and for three high-disconnect sub-periods over which we will examine in more detail the performance of the model: Financial crisis (from July 2007 until March 2009), US 2016 election (from July 2016 until January 2018), and Covid-19 (from January 2020 until December 2020). Over the full sample, the average stock excess return is 7.7%, with a relatively high level of the average risk-free rate compared to recent values. Given the paths of uncertainty $\mathcal{U}_t$ and volatility v_t, our normalized measure of disconnect η_t has an average value of 1 over the full sample by construction.

Compared to the full sample results, during the financial crisis period the average stock excess return was extremely negative (-33.4%) and both volatility and uncertainty were very high. However, despite a very high level of average uncertainty (about 30% higher than in the full sample), the extremely high level of average volatility (about 60% higher than in the full sample) implies this is a high disconnect period, with average level of disconnect about 40% lower than in the full sample.

The period surrounding the US 2016 election was characterized by a higher average stock excess return (more than double its level in the full sample) and significantly lower average volatility, with slightly lower average uncertainty. Even though average uncertainty in this period was relatively close to its full-sample average (only about 15% lower than in the full sample), because average volatility was extremely low during this period (about 40% below its average level in the full sample) this is a high disconnect period, with average level of disconnect about 30% higher than in the full sample.

Finally, the Covid-19 period was characterized by high average stock excess returns, similar to those around the US 2016 election, but with considerably higher average volatility and uncertainty. Despite the extremely high level of average volatility, comparable to the one observed during the financial crisis, the even more extreme level of average uncertainty (close to 3 times its level in the full sample) implies that this is a high disconnect period, with average level of disconnect about

50% higher than in the full sample. While disconnect was on average high during the Covid-19 subperiod, it fluctuated over time, mainly driven by volatility. Initially, disconnect declined sharply due to a high spike in volatility in March 2020, while stock prices dropped. Subsequently, it recovered and reached very high levels, as volatility declined in the following months, while financial markets recovered. Meanwhile, uncertainty remained high during most of the Covid-19 period.

INSERT FIGURE 2 ABOUT HERE

In Figure 2, we plot the time series of volatility and uncertainty, as well as the resulting disconnect process. From Panel A, we observe that uncertainty is substantially more volatile than volatility: the standard deviation of the EPU index is 50.1% compared to only 7.3% for realized volatility. While less volatile, realized volatility is more persistent than uncertainty: the first order autocorrelation of volatility is 0.87, compared to 0.74 for the EPU index. In Panel B, we plot the associated normalized disconnect time series. Since it is constructed as a normalized ratio of uncertainty and volatility, it is also volatile and persistent. The shaded areas in the charts mark the three high-disconnect periods described above: the financial crisis, the US 2016 election, and Covid-19. High disconnect occurs whenever η_t is far away from 1.

Consistent with Figure 1, the correlation between volatility and uncertainty is only 0.44: while the two series comove on average, they are often disconnected. Regressing uncertainty on volatility and a constant yields an adjusted R^2 of about 25%. Therefore, about 75% of the time series variation in uncertainty cannot be explained by volatility—the gap which we attribute to disconnect in the model.

5.2 Equity premium and risk-free rate in different regimes

Proposition 2 implies that the equity premium and the risk-free rate are specific functions of v_t and η_t. While these variables are continuous in the model, and we will analyze them as such, it is useful for interpretation purposes to start by thinking in terms of discrete regimes corresponding to ranges of values: High volatility with high uncertainty (HH), high volatility with low uncertainty (HL), low volatility with high uncertainty (LH), and low volatility with low uncertainty (LL), as described in Figure 1. For the purpose of this discussion, "high" and "low" are defined based on threshold levels for v_t and η_t, given by their respective mean plus one half of their standard deviation:[13] observations above (respectively, below) the threshold level are "high" (respectively, "low").

[13] The analysis presented here is robust to alternative thresholds definitions, which we omit for brevity.

Table 2 reports the average stock market excess return and risk-free rate at the two-week horizon in each regime. It also shows the estimated unconditional probability of being in each regime.

INSERT TABLE 2 ABOUT HERE

The most frequent regime is LL (both volatility and uncertainty are low, η_t close to one), and is empirically characterized by a high average risk-free rate and a low average stock excess return. Regime HH (both volatility and uncertainty are high, η_t close to one) exhibits the lowest average risk-free rate, and considerably higher average stock excess return than in the LL regime. Moving on to the disconnected regimes, we find that the average stock excess return is highest in the HL regime (high uncertainty with low volatility, high η_t) while the average stock excess return in the LH regime (low uncertainty with high volatility, low η_t) dominates only the one in the LL regime. The empirical results are precisely what Proposition 2 implies: the stock market provides a sizable compensation for bearing uncertainty, especially in regimes in which uncertainty is high and disconnected from volatility (HL regime, high η_t), compared to the premium earned solely for bearing volatility.

The results in Table 2 correspond to a fixed investment horizon of two weeks. Figure 3 shows the results as the investment horizon ranges from $\tau = 1$ to 12 weeks.

INSERT FIGURE 3 ABOUT HERE

Panel A in Figure 3 plots the stock excess return for each regime as a function of the investment horizon τ. Consistent with our results in Table 2, high-uncertainty regimes (HH and HL) deliver high subsequent stock excess returns for all the horizons considered. The stock excess return in low-uncertainty regimes (LL and LH) are also positive for most horizons (with the only exception for $\tau = 12$ in the LH regime), but they are significantly lower in magnitude compared to those achieved in high-uncertainty regimes. To incorporate the volatility dimension in the comparison, Panel B plots the Sharpe ratios, obtained by scaling the average realized stock excess returns by their respective standard deviation. By this metric, the HL regime achieves the highest performance for most horizons (with the only exception being for $\tau = 11$ weeks), outperforming the results achieved in regime HH. It then follows that, while high-uncertainty regimes offer a high subsequent stock excess return in general (Panel A), regimes with high uncertainty and simultaneous high volatility deliver subsequent more volatile excess returns, leading to investments in regime HL outperforming those in regime HH for any risk-averse investor (Panel B). This is a consequence of volatility persistence (as discussed above, volatility is highly persistent, and in particular more persistent than uncertainty). Finally, Panel C plots the risk-free rate across regimes; the results are also consistent with the evidence contained in Table 2 and with the prediction of Proposition 2. For instance, the risk-free rate is lower during high-uncertainty regimes (HH and HL), and is highest in the most likely regime characterized by low uncertainty and low volatility (LL).

Next, we examine the predictions of the model contained in Proposition 2 by regressing both the conditional equity premium and risk-free rate on the variance (v_t^2) and uncertainty term ($v_t^2\eta_t$), using the continuum of their values and without distinguishing discrete regimes. Figure 4 shows the results of those regressions.

INSERT FIGURE 4 ABOUT HERE

We start describing the impact of volatility and uncertainty on the stock excess return, shown in the left panels of Figure 4. Panel A reports the estimated coefficient from regressing the stock excess return on realized variance (v_t^2) only, with the shaded area representing a 95% confidence interval. It shows that there is a positive albeit insignificant relationship between the stock excess return and its volatility. In Panel C we do a similar analysis, but adding the uncertainty term ($v_t^2\eta_t$) to the regression, as required by Proposition 2. We obtain that both volatility and the uncertainty term are statistically significant at the 5% level (except for short horizons up to two weeks). The estimated regression coefficient for the volatility term is negative, in line with the leverage effect. For the uncertainty term, however, the sensitivity is positive and consistent with the model's prediction. Panel E shows that our model including both volatility and uncertainty delivers a substantially higher (in-sample) adjusted R^2.

Moving on to the impact of volatility and uncertainty on the risk-free rate, we report the results in the panels on the right side of Figure 4. Panel B reports the estimated coefficient from regressing the risk-free rate on realized variance (v_t^2) only, with the shaded area representing a 95% confidence interval. It shows that there is a significantly negative relationship between the risk-free rate and the stock return volatility, which is consistent with our result in Proposition 2. In Panel D we do a similar analysis, but adding the uncertainty term ($v_t^2\eta_t$) to the regression, as required by Proposition 2. We obtain a significantly negative relationship between the risk-free rate and uncertainty, while volatility becomes insignificant. This shows that uncertainty is the main driver of the risk-free, and it has a negative sign consistent with our result in Proposition 2. Finally, Panel F shows that our model including both volatility and uncertainty delivers a substantially higher (in-sample) adjusted R^2 for the risk-free rate.

It is clear from our results that uncertainty plays a prominent role in characterizing both the equity premium and the risk-free rate. One possible interpretation is through the lens of a flight-to-quality-like effect. In periods of high uncertainty the demand for risky assets decreases and investors require a higher equity premium to hold the risky asset. Similarly, in periods of high uncertainty the demand for safe assets increases, and their required yields decline accordingly. This is consistent with uncertainty been positively related to the equity premium and negatively related to the risk-free rate as indicated by Proposition 2, a relation for which we find empirical validation in the data. By contrast, our empirical analysis shows that volatility affects stock excess returns mainly via the leverage effect, and has a negligible impact on the risk-free rate.

Our results on the impact of volatility and uncertainty on the equity premium help explain the challenges faced by previous empirical studies trying to establish a risk-return trade-off using volatility alone as a measure of risk. Comparing panels A and C, it appears that volatility as the sole state variable struggles to generate the variation in the equity premium that is present in the data; the effect of volatility on excess returns in the data is not as clear cut as standard financial theory suggests. By averaging periods when the trade-off between volatility and return works in the expected direction with periods when it does not, the regression in Panel A ends up with a still positive, but barely significant effect. Once we include both volatility and uncertainty in Panel C, volatility is no longer tasked with playing this dual role and we obtain a clean separation of the effect of volatility (negative) and uncertainty (positive) on the equity premium.

Summing up, we find fairly strong support in the data for the model's predictions as given in Proposition 2, most notably the important impact of uncertainty, with the provision that volatility has a negative effect on the equity premium and a negligible effect in the risk-free rate.

5.3 Robustness to alternative measurements of uncertainty

As explained in Section 5.1, we construct our weekly measure of uncertainty $\mathcal{U}_t$ using the daily economic policy uncertainty (EPU) index developed by Baker et al. (2016). In this section, we summarize the results obtained using alternative measures of uncertainty.

Different approaches to measure uncertainty have been employed in the literature.[14] Several papers consider the newspaper-based methodology of Baker et al. (2016), but with a different selection of words in order to target different types of uncertainty. For example, Husted et al. (2020) select words related to measure more specifically monetary policy uncertainty, while Caldara and Iacoviello (2019) consider a list of words tailored to identify geopolitical uncertainty. Moreover, besides the news-based component used in its daily version, the monthly frequency EPU index from Baker et al. (2016) contains two additional components added to gauge uncertainty regarding the federal tax code, by counting the number of federal tax code provisions set to expire in future years, and to

[14]Cascaldi-Garcia et al. (2020) extensively survey and assess the literature on uncertainty measurement.

quantify disagreement among economic forecasters.[15] The monthly EPU has been further disentangled into several sub-indexes of specific categories, with words lists associated to each of them. [16] The categories are: economic policy, monetary policy, fiscal policy, taxes, government spending, health care, national security, entitlement programs, regulation, trade policy, and sovereign debt and currency crises. Finally, Baker et al. (2021) construct Twitter economic uncertainty indicators, and they show they behave similar to the newspaper-based EPU index.

There are several alternative approaches which measure uncertainty without relying on text analysis from newspapers. Scotti (2016) uses macroeconomic news and survey forecasts to construct an ex-post realized measure of uncertainty about the state of the economy. In contrast to sentiment-based uncertainty measures that rely on analysts' forecasts, Jurado et al. (2015) construct a monthly index of macroeconomic uncertainty as an aggregate volatility of statistical forecasts for hundreds of economic series. In turn, Izhakian (2020) defines ambiguity as probability perturbations (uncertain probabilities) and aversion to ambiguity as aversion to the mean-preserving spreads in these probabilities. He then constructs an uncertainty measure based on the expected volatility of probabilities across the relevant events.

To make sure our results are not driven by the specific uncertainty measure we use, we reproduced the same analysis that led to the construction of Figure 4 for all the uncertainty measures described in this subsection. Despite these measures being constructed very differently, and being available at varying frequencies ranging from daily to monthly frequency, we find the results highlighted in Figure 4 are broadly consistent for all the measures considered, with the main caveat that some monthly measures tend to lead to less statistically significant results, as expected given the lower number of observations available.

All the alternative uncertainty measures we considered are not constructed as measures of risk premia, as otherwise the uncertainty measures used would most likely be correlated by construction with the equity premium due to the strong co-movement of all risk premia. For example, given that we use realized volatility as our volatility measure, the VIX would not be a desirable uncertainty measure. This is because the marginal contribution of VIX to explain the equity premium would be its difference with realized volatility, which is the square root of the variance risk premium.[17]

[15] In addition, the news-based component is different from the one used in the daily series. For the monthly frequency, the focus is only on ten major US newspapers, and the word search is more specific.

[16] For example, articles that fulfill the requirements to be coded as EPU and also contain the term "Federal reserve" would be included in the monetary policy uncertainty sub-index.

[17] Bollerslev et al. (2009) construct a model in which both the volatility and the volatility-of-volatility of consumption growth rate drive the equity premium. They proxy the latter using the variance risk premium and show empirically that it is able to predict the stock excess return. In contrast to their paper, our theory considers model uncertainty rather than volatility-of-volatility of consumption growth rate (which is constant in our model). Empirically, we refrain

We conclude that our results are robust to several alternative measurements of uncertainty, and hence are most likely not driven by our specific choice of the daily EPU index to construct our weekly uncertainty measure.

6. Disconnect-managed portfolios

We now examine the performance of portfolios managed using our formula for the equity premium, and compare them to the performance of portfolios managed based on an equity premium that depends only on volatility, and to the performance of unconditional, hence passive, portfolios. Recall that from Proposition 2, the model-implied equity premium is given by:[18]

$$\text{ERP}_t = \mu_{S,t} - r_{f,t} = v_t^2 + \epsilon \eta_t v_t^2. \tag{26}$$

This implies that a forecast for the equity premium can be obtained from the regression

$$r_{e,t+\tau} := r_{t,t+\tau} - r_{f,t,t+\tau} = \beta_0 + \beta_v v_t^2 + \beta_\eta \eta_t v_t^2 + \epsilon_{t+\tau}, \tag{27}$$

for any horizon τ. Estimating this equation via OLS we obtain

$$\widehat{r}_{e,t+\tau} = \widehat{\beta}_0 + \widehat{\beta}_v v_t^2 + \widehat{\beta}_\eta \eta_t v_t^2, \tag{28}$$

where $\widehat{r}_{e,t+\tau}$ represents the estimated equity premium at horizon τ. Finally, we plug this expression into the optimal policy of a standard logarithmic reference investor, which gives us for that investor an optimal estimated portfolio weight of the form

$$\widehat{\omega}_t^* = \frac{\mu_{S,t} - r_{f,t}}{v_t^2} = \frac{\widehat{\beta}_0}{v_t^2} + \widehat{\beta}_v + \widehat{\beta}_\eta \eta_t. \tag{29}$$

Accordingly, in this section we try to assess whether a reference logarithmic investor could achieve better portfolio performance by taking into account that the equity premium is a function of both volatility and uncertainty, as suggested by our model in Proposition 2.

For a standard model with stochastic volatility but no uncertainty, we re-estimate Equation (27) but regressing on the volatility term only. For the portfolio performance analysis, we multiply the estimated portfolio weights in Equation (29) by a constant $L = 0.15$, which is set to make both dynamic portfolio strategies on average about 100% invested in the index. This is for ease of comparison with a benchmark passive investor who allocates all his wealth to the risky asset.

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from using risk premia to predict the equity premium, and consider instead uncertainty measures, consistent with our modeling framework.

[18] Since we estimate the diffusive part of the stock price process via the realized volatility estimator, we set $\sigma_S = \sigma_D = 1$.

We now turn to comparing portfolio strategies over the full sample, and over the sub-periods representing different volatility and uncertainty regimes. We compare an unconditional, passive, buy-and-hold indexing strategy, $\omega_t^I = 1$, a portfolio strategy in which the stock excess return is obtained by conditioning on volatility only, and a portfolio strategy in which the stock excess return is obtained conditioning on both volatility and uncertainty.

We highlight that all the portfolio comparisons in this section correspond to a full in-sample analysis with the advantage of hindsight. In other words, we back-test all the portfolio strategies considered. In Section 7, we show that adding uncertainty as a predictor of future stock excess return improves forecast accuracy out-of-sample, over and above the forecast accuracy that can be obtained by using only volatility as a predictor.

6.1 Full Sample

We start the portfolio analysis by comparing the above mentioned three different asset allocation strategies (unconditional, conditional on volatility only, conditional on volatility and uncertainty) over the full sample January 1986 - December 2020, at horizons of two and four weeks.[19] Panels A and B in Figure 5 plot the time series of optimal portfolio weights, while panels C and D report the resulting cumulative wealth processes.

INSERT FIGURE 5 ABOUT HERE

The dotted black line represents the unconditional passive buy-and-hold indexing strategy, the dashed blue line represents the active portfolio strategy conditioning on volatility only, and the solid red line represents the active portfolio strategy conditioning on both volatility and uncertainty. Panels A and B show that both active strategies use leverage extensively, as the portfolio holdings ω_t are very often above one. The two active portfolio strategies are correlated, consistent with the idea that on average volatility and uncertainty are connected and hence comove. They diverge mainly in periods of high disconnect. For example, in the period around the financial crisis characterized by high disconnect driven by a very high level of uncertainty and an extremely high level of volatility, the active strategy conditioning on both volatility and uncertainty was more conservative than the one conditioning on volatility only, thereby avoiding the significant stock market losses in this period. By contrast, in the period around the US 2016 election which was characterized by levels of uncertainty close to average and simultaneously very low volatility (hence high disconnect), the active strategy conditioning on both volatility and uncertainty was more aggressive than the one conditioning on

[19]Different horizons of up to 12 weeks deliver similar results.

volatility only, taking advantage of the significant stock market gains during this period. Panels C and D show that the portfolio strategy based on the model (conditioning on both volatility and uncertainty) achieves the highest associated wealth compared to both a strategy conditioning on volatility only and an unconditional passive indexing strategy.

We then evaluate the risk and performance of the wealth excess return associated with each portfolio strategy. We consider the following statistics: (i) average wealth excess return, (ii) standard deviation of the wealth excess return, (iii) maximum draw-down $\left(\mathcal{MDD}^{\%}\right)$,[20] (iv) 5% Value-at-Risk (VaR),[21] (v) Sharpe ratio, and (vi) Treynor ratio. Figure 6 summarizes key return statistics for the portfolios.

INSERT FIGURE 6 ABOUT HERE

Panel A of Figure 6 shows that the active portfolio strategy conditioning on both volatility and uncertainty achieves higher excess return on average than the portfolio strategy conditioning on volatility only and the unconditional passive investment strategy for all horizons. Panel B shows that this is not due to excessive risk taking as the standard deviation of the three strategies considered are close to one another, and none of them exhibits the lowest standard deviation across all horizons. Panels C and D show that the portfolio strategy conditioning on both volatility and uncertainty preserves the investor's wealth very effectively, as it achieves the lowest maximum drawdown over most horizons and the lowest value-at-risk in all horizons considered. Finally, panels E and F show that the portfolio strategy conditioning on both volatility and uncertainty delivers the highest risk-adjusted return, as it achieves the highest Sharpe and Treynor ratios for all horizons considered.

We now evaluate the alpha and beta components of the returns for each portfolio strategy. Figure 7 summarizes the results.

INSERT FIGURE 7 ABOUT HERE

Panel A in Figure 7 shows that the active portfolio strategy conditioning on both volatility and uncertainty achieves the highest (CAPM) alpha for all horizons considered. Panel B shows that

[20] Let W_t denote the current level of wealth process and let $\overline{W}_t := \max_{s\in[0,T]} W_s$ be its running maximum. The drawdown at time t measures the drop of wealth from its running maximum, and is defined as $\mathcal{D}_t := \overline{W}_t - W_t,\ \forall t \geq 0$. The maximum draw-down at time t denoted by $\mathcal{MDD}_t$ records the worst performance that could have happened in the past. It is defined as the running maximum of the drawdown process, given by $\mathcal{MDD}_t := \max_{\tau\in(0,t)} \mathcal{D}_\tau$. Finally, to express the maximum drawdown as the highest proportional wealth loss in the past, we compute $\mathcal{MDD}^{\%} = \mathcal{MDD}_t/\overline{W}_t \in (0,1)$.

[21] The 5% Value-at-Risk is defined as the maximum possible loss (expressed in percentage terms) during the investment period, after excluding all worse outcomes whose combined probability is at most 5%.

the overall systematic risk beta is similar for both active strategies, with none of them achieving a lower beta for all horizons considered. Interestingly, all the estimated beta coefficients are lower than one, which indicates that the large wealth excess returns are not driven by excessive systematic risk taking.[22] Finally, panels C and D show that the active portfolio strategy conditioning on both volatility and uncertainty achieves the highest alpha for all horizons considered, whether we use CAPM (Panel A), Fama-French three-factor (Panel C), or five-factor (Panel D) models as benchmark.

6.2 High-Disconnect Regimes

Figure 2 clearly established that there are several prominent disconnect periods in recent financial markets history. When describing Table 1 we further characterized the main dynamics of uncertainty and volatility in those periods, which led to them been categorized as high-disconnect regimes. In addition, Table 2 and Figure 3 suggest that the impact of disconnect on stock excess return can be very different depending on the relative levels of volatility and uncertainty. In this subsection, we analyze portfolio performance from following the same three strategies implemented in Section 6.1, but focusing on specific sub-periods characterized by high disconnect: the financial crisis, the US 2016 election, and the Covid-19 crisis. Rather than conditioning on regimes as was done for illustrative purposes in Table 2 and Figure 3, here we condition on the continuum of volatility and uncertainty values, as in Section 6.1.

6.2.1 Financial Crisis

We start by analyzing the 2008 financial crisis, characterized by high uncertainty and extremely high volatility. Table 1 shows that over the two-year time period from July 2007 until March 2009 the average stock market excess return was -33.40%. Accordingly, the excess return of wealth associated with all portfolio strategies considered were negative around this period. For that reason, in this subsection we refrain from using the Sharpe and Treynor ratios, as they are not straightforward to interpret for ranking purposes when average excess returns are negative. Instead, we analyze the average and standard deviation of wealth excess returns.

INSERT FIGURE 8 ABOUT HERE

[22]The adjusted R^2 obtained from a regression of the wealth excess return for each active portfolio strategy onto the stock market return are 0.6 (conditioning on volatility only) and 0.5 (conditioning on volatility and uncertainty), respectively. This implies that about half of the wealth excess return variance of the active portfolio strategies is not driven by exposure to the aggregate stock market.

Panels A and B on Figure 8 show that the active portfolio strategies achieve a higher terminal wealth than the index during this sub-period. At the two-weeks horizon, the wealth evolution associated to conditioning on volatility only is very similar to the one associated to conditioning on both volatility and uncertainty (Panel A). However, when considering a one-month horizon, the portfolio conditioning on volatility and uncertainty generates the highest associated level of wealth (Panel B).

Panel C shows that the active portfolio strategies achieve higher excess return on average than the passive index strategy for all horizons. However, there is no clear dominant active strategy based on this metric, as none of them achieves the highest wealth excess return consistently across all horizons considered. Panel D shows that the higher excess return achieved by the active strategies is not due to excessive risk taking, as the standard deviation of their wealth excess return is lower than the one associated with the passive index strategy. The standard deviation of wealth excess return associated with the active strategies are very similar to one another for all horizons. Panels E and F show that the active portfolio strategies do equally well in wealth preservation, as they obtain similar levels of maximum drawdown and value-at-risk for most horizons. By contrast, the unconditional passive index strategy is less effective at wealth preservation, as its levels of maximum drawdown and value-at-risk are about double their corresponding levels for its active portfolio strategy counterparts at all horizons.

6.2.2 US 2016 election

We now consider the period surrounding the US 2016 election, from July 2016 until January 2018. During this period, uncertainty remained relatively close to its average value while realized volatility reached record lows. In Figure 9 we plot the wealth evolution along with risk-adjusted returns for each portfolio strategy around the US 2016 election.

INSERT FIGURE 9 ABOUT HERE

Panels A and B show that the conditional active portfolio strategies achieve a higher terminal wealth than the unconditional indexing strategy during this sub-period. At the two-weeks horizon the portfolio conditioning on volatility and uncertainty generates the highest associated level of wealth (Panel A). When considering a one-month horizon, the wealth evolution associated to conditioning on volatility only is similar to the one associated to conditioning on both volatility and uncertainty. Panels C and D show that the portfolio conditioning on both volatility and uncertainty achieves the highest risk-adjusted returns in most situations (with the only exception of the one-month horizon, in which conditioning only on volatility slightly dominates), as measured by the Sharpe and Treynor ratios. In addition, it is worth noting that while both active strategies outperform the index, this is partly due to the use of high leverage (the portfolio weights for the active strategies tend to be high, sometimes above 2). Finally, panels E and F show that the active portfolio strategies would not preserve the investor's wealth effectively over this period, as they result in higher maximum drawdowns and value-at-risk than the passive index across all horizons. Comparing the two active strategies only, conditioning on both volatility and uncertainty does a better job at wealth preservation, achieving lower maximum drawdown and value-at-risk across all horizons.

6.2.3 Covid-19

Finally, we consider the recent Covid-19 pandemic crisis, characterized by extremely high volatility (the average volatility in this period is about double the average volatility in the full sample) and comparatively even more extremely high uncertainty (the average uncertainty in this period was close to 3 times the average uncertainty in the full sample).

The Covid-19 pandemic triggered strong market reactions. At the peak of the crisis in mid March 2020, stock market volatility exceeded the highest levels previously seen during the financial crisis and caused equity markets to rapidly lose value, and then subsequently recover. This period is particularly interesting to study within our framework, because of the different timing in the dynamics of volatility and uncertainty. Uncertainty increased substantially in late February and remained at very elevated levels throughout most of the sample period. By contrast, volatility initially spiked sharply and then quickly reverted to levels closer to its full-sample mean. In terms of our measure of disconnect, this implies η_t fell substantially in late February and early March but then increased and stayed high for the remainder of the sample period. In short, around this period the market experienced a regime transition from a more connected regime characterized by extremely high uncertainty and extremely high volatility to a more disconnected regime characterized by extremely high uncertainty and moderately high (but no longer extreme) volatility. In Figure 10 we collect the wealth evolution for all portfolio strategies together with a set of portfolio performance measures. The period considered in this analysis is from January 2020 until December 2020.

INSERT FIGURE 10 ABOUT HERE

The top two panels of Figure 10 show that the portfolio conditioning on both volatility and uncertainty performs well throughout the entire Covid-19 period. Panel A shows that, at the two-week horizon, besides protecting the investor's wealth at the onset of the crisis in March and April

2020, the strategy also took advantage of the subsequent recovery of equity markets by rapidly increasing its equity portfolio share. Accordingly, at this horizon the wealth level achieved by the portfolio conditioning on both volatility and uncertainty is the highest. Panel B shows that at the monthly horizon the wealth evolution of the passive index and the portfolio conditioning on both volatility and uncertainty are very similar. By contrast, for the same horizon, the portfolio conditioning on volatility only achieved the lowest wealth level, incurring cumulative losses by the end of 2020. Notably, the portfolio conditioning on both volatility and uncertainty was almost fully invested in the equity market, compared to only half of wealth invested in equities for the portfolio conditioning on volatility only. The high uncertainty around this period led to higher subsequent stock excess return, and only the portfolio conditioning on both volatility and uncertainty took advantage of this compensation for uncertainty, which the portfolio conditioned on volatility alone could not exploit.

Panels C and D show that the portfolio strategy conditioning on both volatility and uncertainty delivers the highest risk-adjusted return, as it achieves the highest Sharpe and Treynor ratios for most horizons considered (all except for the monthly horizon, in which conditioning on volatility and uncertainty delivers a risk-adjusted return similar to the one obtained by the passive index strategy). By contrast, the portfolio strategy conditioning on volatility only obtains the worst performance in most horizons, including negative Sharpe and Treynor ratios in some of them. Finally, panels E and F show that the active portfolio strategies preserve wealth more effectively than the passive index strategy, as they obtain lower maximum drawdowns and value-at-risk for most horizons (again all except for the monthly horizon, in which the three strategies achieve very similar wealth preservation). Meanwhile, both active portfolio strategies preserve wealth equally well; neither results in a lower maximum drawdown or value-at-risk for all horizons.

7. Out-of-sample analysis

In Section 6, all the portfolio comparisons made were in-sample. It was a back-testing exercise, performed with the advantage of hindsight. In this section, we examine the same three prediction models for the equity premium, but out-of-sample. We show that adding uncertainty as a predictor of future stock excess return improves forecast accuracy out-of-sample, compared to forecasts conditioning on volatility only, or based on the unconditional average stock excess return. Because in the context of our regression-based portfolio analysis higher stock excess return forecast accuracy drives higher portfolio performance, the results we provide here support the notion that the higher portfolio performance from conditioning on both volatility and uncertainty shown in Section 6 holds also out of sample.

We start by partitioning the full sample in half. The estimation sample consists on the first half of observations (from January 1986 until June 2003), leaving the rest of the sample for implementing the forecasts out-of-sample.[23] We run similar regressions to the one in Equation (27), with the stock excess return regressed on: (i) only an intercept (unconditional model), (ii) an intercept and volatility, and (iii) an intercept, volatility, and uncertainty. We run the regressions and make forecasts over horizons ranging from 1 to 12 weeks. Starting from using the first half of observations in the full sample, we update our estimation one week at a time by using a rolling expanding window.

Using the estimated regression coefficients, we go on to predict out of sample stock excess returns over horizons $\tau = 1, ..., 12$ weeks in the remaining observations available (starting from the July 2003 - December 2020 period). To evaluate the forecast performance we compare the ex-ante forecast of stock excess return according to each of the three regression based models considered above with the corresponding ex-post realized values, by means of the root mean squared error (RMSE). Table 3 shows the results.

INSERT TABLE 3 ABOUT HERE

Each column in Table 3 reports the RMSE of a forecast of stock excess return for each regression model considered. Remarkably, the results show that the richer model (conditioning on volatility and uncertainty) is more accurate (lower RMSE) than conditioning only on volatility, or not conditioning at all, for all horizons considered except the one-week horizon. This establishes that the in-sample higher performance of the conditional model based on volatility and uncertainty was not due to overfitting in-sample.

Conditioning only on volatility does not achieve similar success. For horizons from 1 to 7 weeks, forecasting the stock excess return conditioning on volatility is more accurate than not conditioning at all. However, for longer horizons from 8 to 12 weeks, not conditioning provides a better forecast than conditioning on volatility. By contrast, for all horizons considered, conditioning on volatility and uncertainty leads to more accurate forecasts of the stock excess return than not conditioning.

Overall, our results clearly show that a forecast of stock excess return conditioning on both volatility and uncertainty is superior to both a forecast conditioning on volatility only and a forecast based on the unconditional mean. Since in our regression-based portfolio analysis any increase in portfolio performance must be driven by higher accuracy in the stock excess return forecast, our results suggest that the high portfolio performance achieved by conditioning on volatility and uncertainty shown in Section 6 holds also out of sample.

[23] We replicated the analysis using for the estimation sample other fractions of the full sample, including 40%, 60%, 70%, and 80%. In all cases, the results were qualitatively the same.

8. Conclusion

In this paper, we develop a novel and tractable framework in which volatility and model uncertainty are both stochastic, are possibly disconnected, and affect the optimal consumption and portfolio choice of an investor who is averse to both. By allowing uncertainty and volatility to be disconnected we achieve a better in-sample description and better out-of-sample forecast of the equity premium.

In particular, we show that equity returns respond negatively to contemporaneous volatility (consistent with the prevalence of the leverage effect in the data), but respond positively, and consistently across horizons, to the uncertainty component in the model. Our results therefore show that the equity premium appears to be earned for facing uncertainty, especially high uncertainty that is disconnected from lower volatility, rather than for facing volatility as traditionally assumed.

This has important implications for optimal portfolio allocation and asset prices. The model predicts that periods of high uncertainty are followed by subsequently higher excess returns, which are even higher in disconnected regimes characterized by uncertainty being significantly higher than volatility. Both are salient features of the data. Accordingly, a reference investor forecasting the stock excess return conditioning on both volatility and uncertainty achieves superior portfolio performance, over and above the one achieved by conditioning on volatility only, or by not conditioning at all.

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Appendix: Proofs

A.1 Derivation of the relative entropy growth

Given the definition of the entropy growth in Equation (9) and its bound implied by the set of admissible model misspecification in Equation (10), we can write:

$$\mathcal{R}(\vartheta_t) := \left.\frac{d}{ds}\right|_{s=0} \mathbb{E}_t^{\vartheta}\left[\log \vartheta_{t+s}\right]. \tag{A.1}$$

We then compute the relative entropy growth as follows:

$$\begin{aligned}\mathcal{R}(\vartheta_t) &= \left.\frac{d}{ds}\right|_{s=0} \mathbb{E}_t^{\vartheta}\left[\int_t^{t+s} h_u dW_u^S - \int_t^{t+s} \frac{h_u^2}{2} du\right] \\ &= \left.\frac{d}{ds}\right|_{s=0} \mathbb{E}_t^{\vartheta}\left[\int_t^{t+s} h_u dW_u^{S,\vartheta} + \int_0^{t} \frac{h_u^2}{2} du\right] \\ &= \left.\frac{d}{ds}\right|_{s=0} \int_t^{t+s} \frac{h_u^2}{2} du = \frac{h_t^2}{2},\end{aligned} \tag{A.2}$$

where the first equality follows from the expression for the change of measure ϑ_t in Equation (7), the second one uses the relation between the Brownian motions under the reference and robust measures in Equation (6), and the last one is an application of the Leibnitz rule of differentiation of integrals.[24]

A.2 Proof of Proposition 1

To find a solution of the problem in Equation (16) subject to the entropy growth constraint in Equation (17), we formulate the Lagrangian for the constrained optimization problem:

$$\begin{aligned}\sup_{\{C_t,\omega_t\}} \inf_{\{h_t,\theta\}} \Bigg\{ & \log(C_t) + \frac{\partial V}{\partial X}\left[X_t(r_{f,t} + \omega_t\left(\mu_{S,t} - r_{f,t} + v_t h_t \sigma_S\right)) - C_t\right] \\ & + \frac{1}{2}\frac{\partial^2 V}{\partial X^2} v_t^2 X_t^2 \sigma_S^2 \omega_t^2 + \frac{\partial V}{\partial v}\kappa_v(\theta_v - v_t) + \frac{1}{2}\frac{\partial^2 V}{\partial v^2}\sigma_v^2 v_t^2 + \frac{\partial V}{\partial \eta}\kappa_\eta(\theta_\eta - \eta_t) \\ & + \frac{1}{2}\frac{\partial^2 V}{\partial \eta^2}\sigma_\eta^2 \eta_t^2 + \frac{\partial^2 V}{\partial X \partial v}\omega_t X_t \rho_{Sv}\sigma_S\sigma_v v_t^2 + \frac{\partial^2 V}{\partial X \partial \eta}\omega_t X_t \rho_{S\eta}\sigma_S\sigma_\eta \eta_t v_t + \frac{\partial^2 V}{\partial \eta \partial v}\rho_{\eta v}\sigma_\eta\sigma_v\eta_t v_t \\ & + \theta\left(\frac{1}{2}h_t^2 - \frac{1}{2}\epsilon^2 \mathcal{U}_t^2\right)\Bigg\},\end{aligned} \tag{A.3}$$

subject to $C_t \geq 0$ and $\theta \geq 0$, where θ is the multiplier for the relative entropy constraint in Equation (17).

[24] Leibniz rule states that for a continuous function f, $\frac{d}{du}\left(\int_{a(u)}^{b(u)} f(x,u)\,dx\right) = \int_{a(u)}^{b(u)} \frac{\partial f}{\partial u}\,dx + f\big(b(u),u\big)\cdot b'(u) - f\big(a(u),u\big)\cdot a'(u)$, where the functions $a(u)$ and $b(u)$ are both continuous and both have continuous derivatives in u.

Solving for the Lagrangian in Equation (A.3) leads to the following optimal policies:[25]

$$C_t^* = \frac{1}{\frac{\partial V}{\partial X}}, \tag{A.4}$$

$$h_t^* = -\epsilon \mathcal{U}_t = -\epsilon \eta_t v_t, \tag{A.5}$$

$$\omega_t^* = -\frac{\frac{\partial V}{\partial X}\left(\mu_{S,t} - r_{f,t} + v_t h_t^* \sigma_S\right) + \frac{\partial^2 V}{\partial X \partial v_t} \rho_{Sv} \sigma_S \sigma_v v_t^2 + \frac{\partial^2 V}{\partial X \partial \eta} \rho_{S\eta} \sigma_S \sigma_\eta \eta_t v_t}{\frac{\partial^2 V}{\partial X^2} v_t^2 X_t \sigma_S^2}. \tag{A.6}$$

Finally, given logarithmic preferences, we conjecture that the value function is of the form:[26]

$$V(X, \eta, v) = \frac{\log(X)}{\beta} + \phi(\eta, v). \tag{A.7}$$

Plugging this conjecture of the value function into the optimal policies in equations (A.4), (A.5), and (A.6), respectively, leads to the expressions in the proposition.

A.3 Proof of Proposition 2

The equity premium in Equation (24) follows from combining the optimal portfolio policy in Equation (20) with the market clearing condition for the stock in Equation (22).

The price-dividend ratio given in the proposition follows from combining the optimal consumption policy in Equation (18) with the market clearing condition for consumption in Equation (21). In addition, one needs to take into account that because $C_t = D_t$ for all t, the price of a claim on the future stream of consumption must be equal to the price of a claim on the future stream of dividends, which implies that $X_t = S_t$.

Because the price-dividend ratio is constant, the dynamics of the stock price and dividends are related by $\frac{dS_t}{S_t} = \frac{dD_t}{D_t}$, which implies both have the same drift and diffusion (and $W_t^D = W_t^S \quad \forall t$). Identifying terms using equations (1) and (3), and using the equilibrium price-dividend ratio, we get

$$\mu_{S,t} = \beta + \mu_D, \tag{A.8}$$

$$\sigma_S = \sigma_D. \tag{A.9}$$

The risk-free rate in Equation (25) follows from combining the equity premium in Equation (24) with the stock return drift and volatility in equations (A.8) and (A.9).

[25] While in principle there are two roots for the solution of $h_t^* = \pm\epsilon\mathcal{U}_t$, we focus on the realistic solution with $h_t^* = -\epsilon\mathcal{U}_t < 0$, which prevails when the expected return of the risky asset under the reference measure is positive.

[26] It can be verified that this conjecture provides a solution to the problem, with $\phi(\eta, v)$ satisfying an ordinary differential equation, which may be solved in closed form depending on the assumed functional form of $\mu_{S,t}$ and $r_{f,t}$.

	Full Sample	Financial Crisis	US 2016 election	Covid-19
Time Period Begin	Jan. 1986	Jul. 2007	Jul. 2016	Jan. 2020
Time Period End	Dec. 2020	Mar. 2009	Jan. 2018	Dec. 2020
Average stock excess return (%)	7.70	-33.40	17.29	16.97
Median stock excess return (%)	11.73	-35.70	18.97	23.96
Average risk-free rate (%)	3.20	2.02	0.67	0.38
Median risk-free rate (%)	3.14	1.77	0.54	0.32
Average uncertainty $\mathcal{U}_t$	101.21	130.02	85.00	284.30
Median uncertainty $\mathcal{U}_t$	84.81	106.29	78.84	257.55
Average stock return volatility v_t (%)	17.70	28.16	10.51	28.94
Median stock return volatility v_t (%)	11.02	15.37	7.22	18.82
Average disconnect $\eta_t = \mathcal{U}_t/v_t$ (scaled)	1.00	0.59	1.29	1.49
Median disconnect $\eta_t = \mathcal{U}_t/v_t$ (scaled)	0.85	0.58	1.25	1.42

Table 1: Asset pricing moments

Notes: This table presents the average and median annualized stock excess return, risk-free rate, uncertainty ($\mathcal{U}_t$), annualized volatility (v_t), and disconnect (η_t). These are computed over the full sample (from January 1986 until December 2020) and during three sub-periods associated with high disconnect: Financial crisis (from July 2007 until March 2009), US 2016 election (from July 2016 until January 2018), and Covid-19 (from January 2020 until December 2020). All the data is nominal and sampled at the weekly frequency.

		Volatility Level	
		High	Low
Uncertainty Level	High	$\bar{r}_e = 23.17\%,\ \bar{r}_f = 1.29\%$ $\mathbb{P}[\text{HH}] = 9.23\%$	$\bar{r}_e = 26.33\%,\ \bar{r}_f = 2.39\%$ $\mathbb{P}[\text{HL}] = 10.82\%$
	Low	$\bar{r}_e = 5.56\%,\ \bar{r}_f = 3.1\%$ $\mathbb{P}[\text{LH}] = 10.36\%$	$\bar{r}_e = 3.07\%,\ \bar{r}_f = 3.62\%$ $\mathbb{P}[\text{LL}] = 69.59\%$

Table 2: Stock excess return and risk-free rate in different uncertainty and volatility regimes.

Notes: This table reports the estimated unconditional probability of each market regime (HH, HL, LH, and LL). It also shows the average (annualized) risk-free rate and stock excess return at a 2-weeks horizon in each regime. The threshold values for volatility and uncertainty are given by their mean plus one half of their standard deviation. We say that volatility and uncertainty are high (respectively, low), when they are above (respectively, below) their threshold values. Both volatility and uncertainty are sampled at the weekly frequency from January 1986 until December 2020.

τ	Unconditional	Volatility	Volatility and Uncertainty
1	1.558	1.552	1.557
2	1.989	1.986	1.978
3	2.277	2.274	2.258
4	2.527	2.522	2.503
5	2.754	2.750	2.725
6	2.950	2.946	2.915
7	3.130	3.126	3.088
8	3.288	3.289	3.241
9	3.438	3.444	3.389
10	3.590	3.595	3.538
11	3.743	3.762	3.703
12	3.888	3.932	3.870

Table 3: Root mean squared error ($\times 100$) for different out-of-sample stock excess return forecasts.

Notes: This table reports the root mean squared error (RMSE), multiplied by 100 for ease of illustration purposes, from out-of-sample forecasts of stock excess return using horizons 1 through 12 weeks, based on three different models: (i) $\widehat{r}_{e,t+\tau} = \widehat{\beta}_0$ (unconditional mean equal to 7.70%), (ii) $\widehat{r}_{e,t+\tau} = \widehat{\beta}_0 + \widehat{\beta}_v v_t^2$ (conditioning on volatility), and (iii) $\widehat{r}_{e,t+\tau} = \widehat{\beta}_0 + \widehat{\beta}_v v_t^2 + \widehat{\beta}_\eta \eta_t v_t^2$ (conditioning on volatility and uncertainty). A lower RMSE indicates a better forecast performance. The estimation period starts by using the first half of the full sample: from January 1986 until June 2003. We then update our estimation one week at a time by using a rolling expanding window. The out-of-sample period corresponds to the remaining observations in our sample (starting from the July 2003 - December 2020 period).

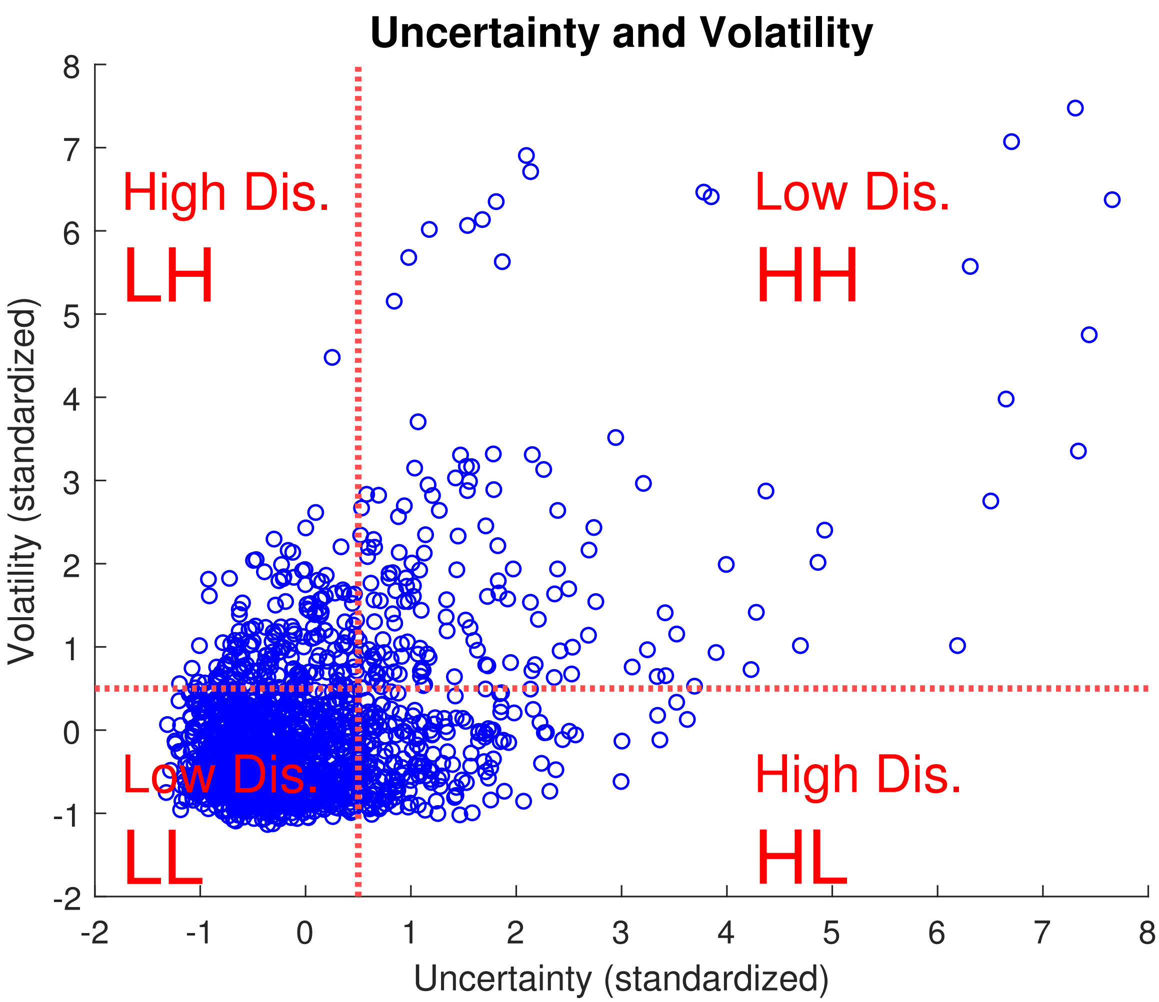

Figure 1: Uncertainty and volatility regimes

Notes: The figure shows a scatter plot of standardized uncertainty (proxied by the economic policy uncertainty index EPU_t) and volatility (proxied by realized volatility). Both are sampled at the weekly frequency from January 1986 until December 2020. The threshold values for volatility and uncertainty are given by their mean plus one half of their standard deviation. We say that volatility and uncertainty are high (respectively, low), when they are above (respectively, below) their threshold values. High disconnect occurs when either uncertainty is high while volatility is low (denoted "HL") or when uncertainty is low while volatility is high (denoted "LH"). In the other two quadrants, uncertainty and volatility are in sync and disconnect is low.

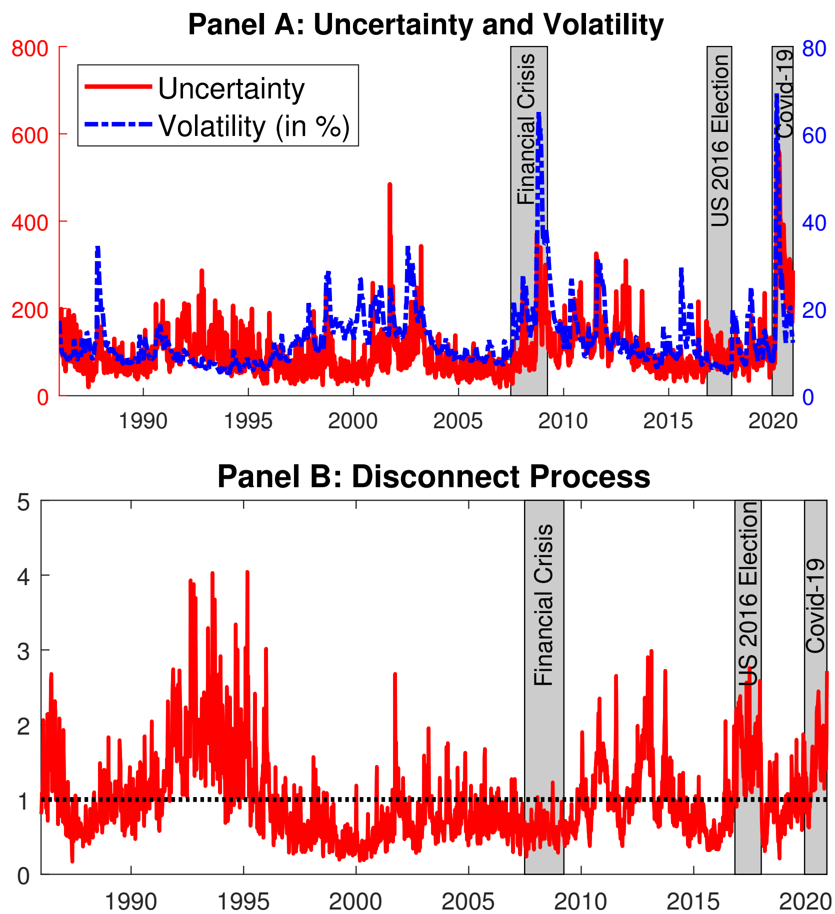

Figure 2: Time series of volatility, uncertainty and disconnect

Notes: Panel A shows uncertainty (proxied by the economic policy uncertainty index EPU_t) and volatility (proxied by realized volatility). Panel B shows the disconnect process (η_t) defined as the ratio of the EPU index divided by realized volatility, normalized to have a mean of 1. The sample period is from January 1986 until December 2020, and the data is sampled at the weekly frequency. The shaded areas correspond to three sub-periods associated with high disconnect: Financial crisis (from July 2007 until March 2009), US 2016 election (from July 2016 until January 2018), and Covid-19 (from January 2020 until December 2020).

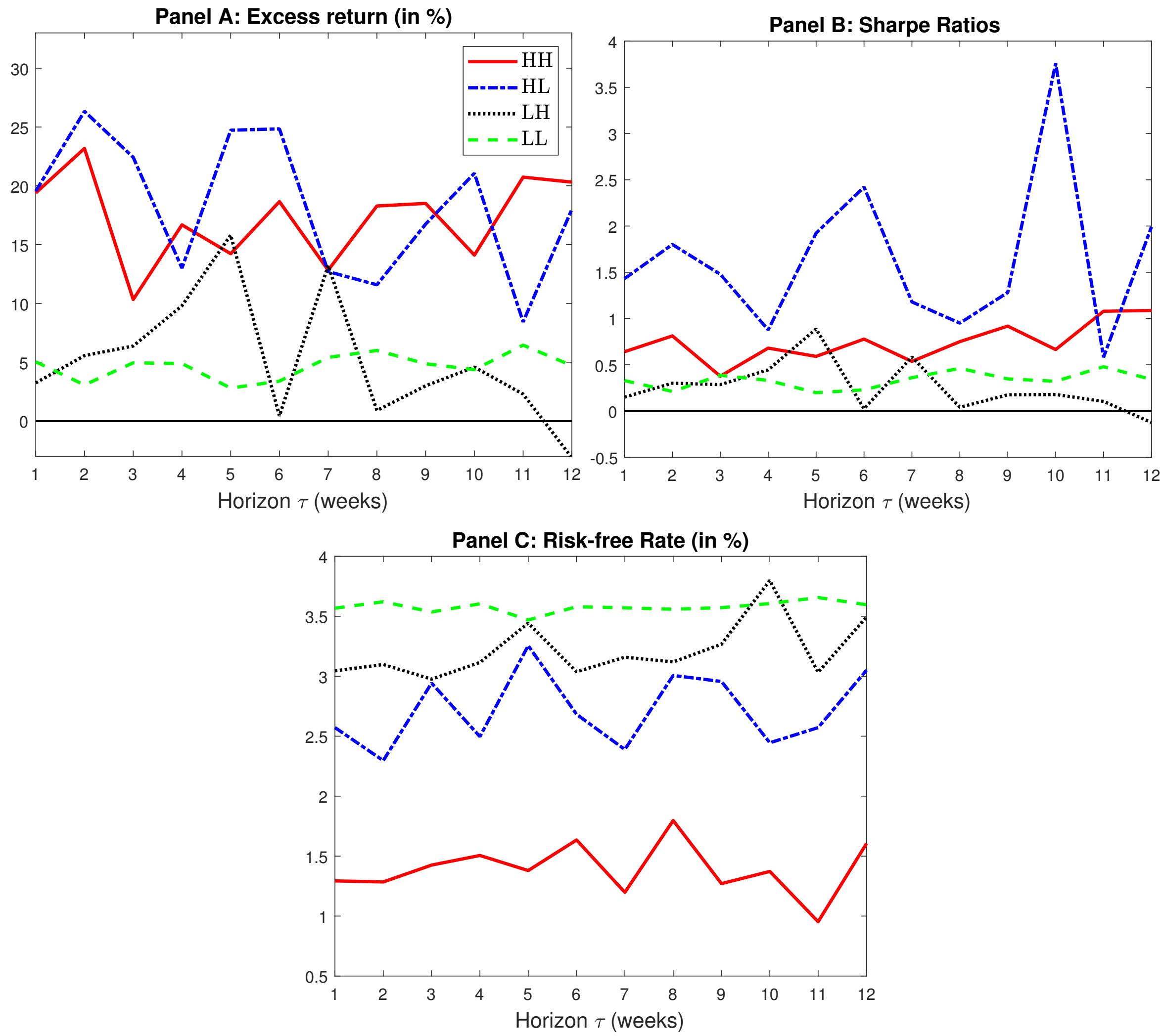

Figure 3: Stock excess return, Sharpe ratio, and risk-free rate conditional on regimes of uncertainty and volatility

Notes: This figure shows the annualized average stock excess return (in %), Sharpe ratio, and risk-free rate (in %) over a given horizon $\tau = 1, \ldots, 12$ weeks, conditional on being in one of the four market regimes: (i) both volatility and uncertainty are high (HH), (ii) high uncertainty and low volatility (HL), (iii) low uncertainty and high volatility (LH), or (iv) both low uncertainty and low volatility (LL). The threshold values for volatility and uncertainty are given by their mean plus one half of their standard deviation. We say that volatility and uncertainty are high (respectively, low), when they are above (respectively, below) their threshold values. The sample period is from January 1986 until December 2020, and the data is sampled at the weekly frequency.

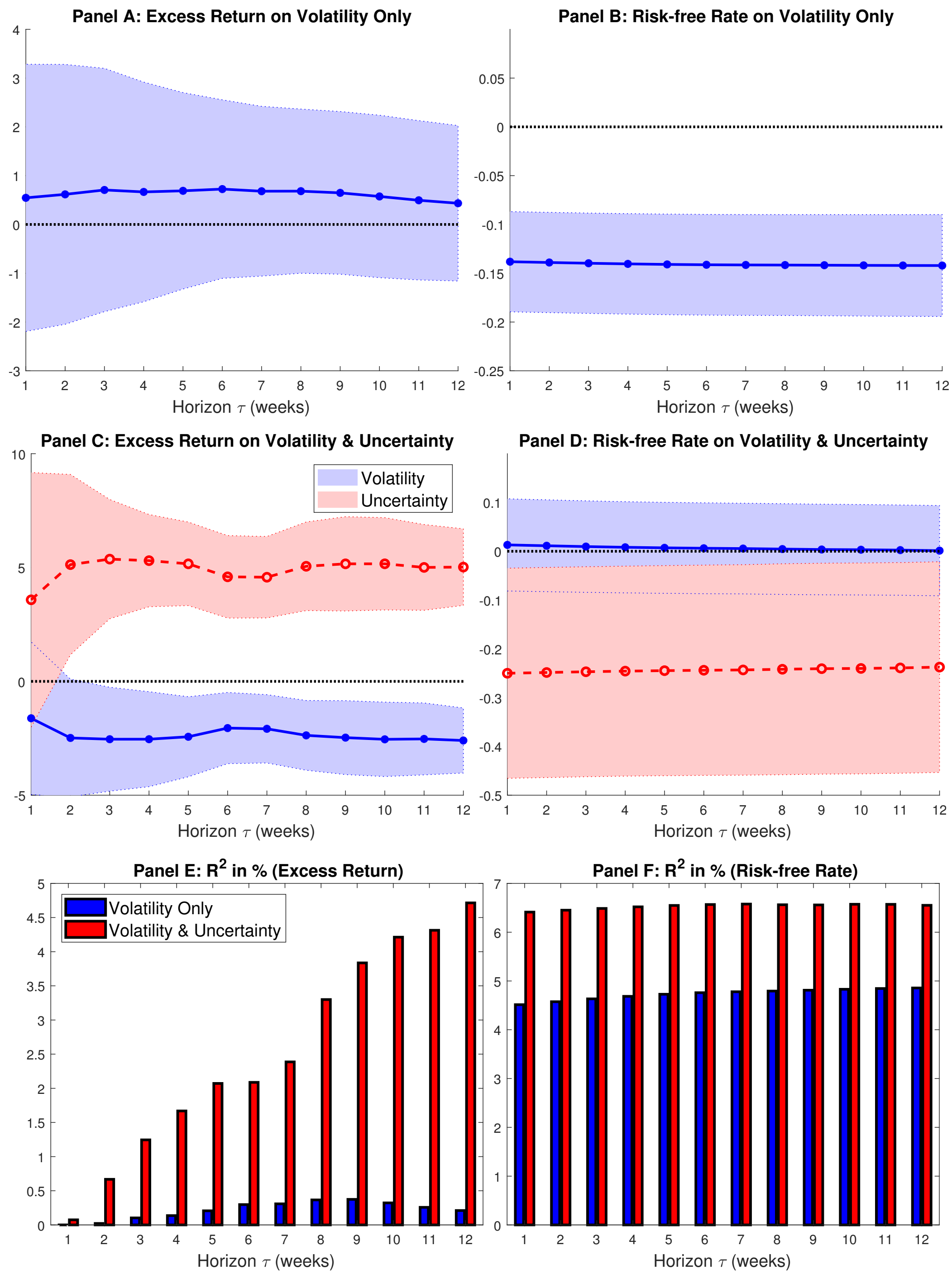

Figure 4: Regression of stock excess return and risk-free rate on volatility and uncertainty

Notes: The left charts correspond to regressions of stock excess return on volatility and uncertainty, and the charts on the right show regressions of the risk-free rate on volatility and uncertainty. All regressions are run over horizons of up to 12 weeks. Panels A and B show estimated coefficients from univariate regressions on a variance term only. Panels B and C show estimated coefficients from bivariate regressions on variance and an uncertainty term (based on Proposition 2). The shaded areas represent 95% confidence intervals constructed using auto-correlation and heteroskedasticity corrected standard errors (HAC). Panels D and E show the adjusted R^2 (in %) from the uni- and bi-variate regressions in panels A, B, C and D. The sample period is from January 1986 until December 2020, and the data is sampled at the weekly frequency.

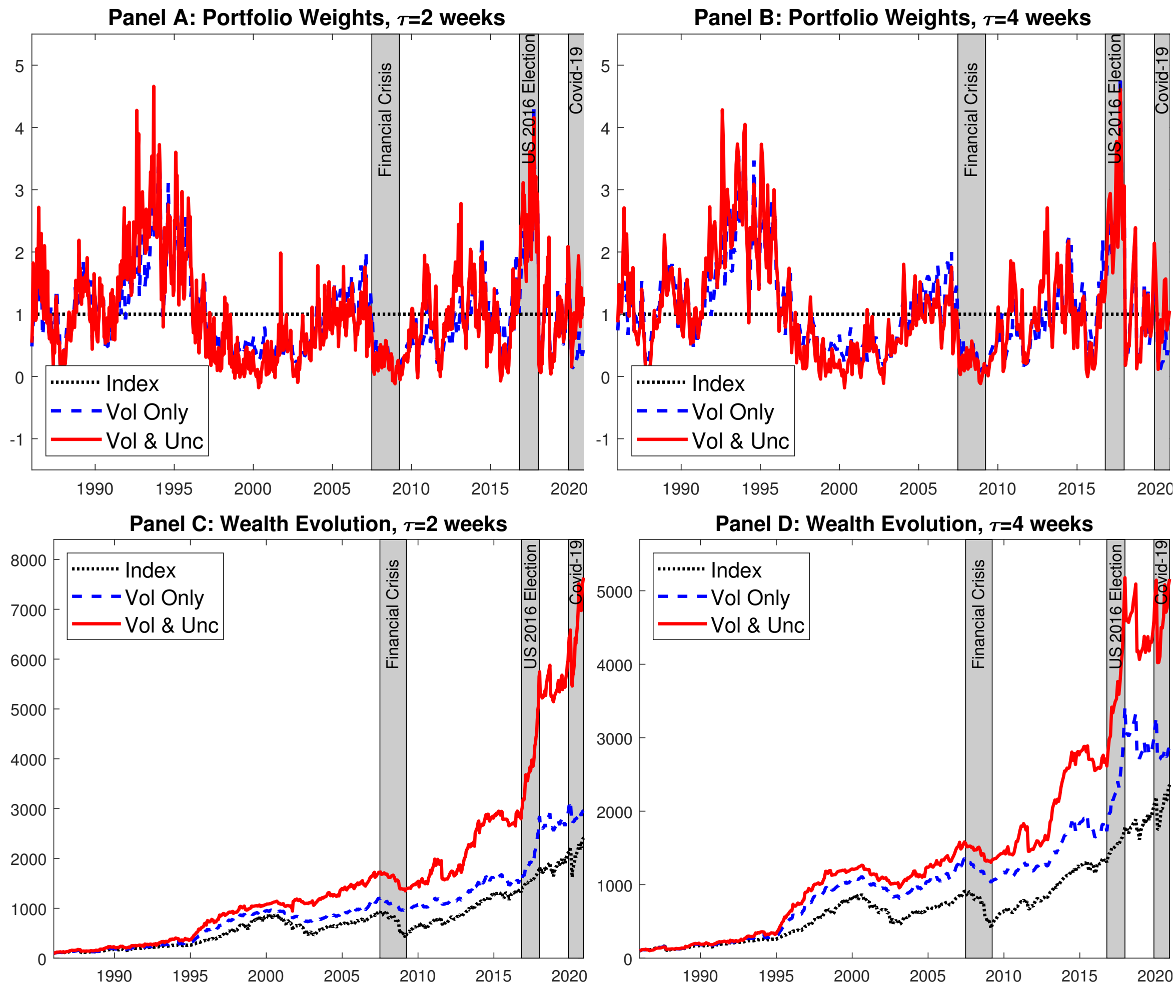

Figure 5: Portfolios and wealth evolution: full sample

Notes: Panels A and B show for two- and four-weeks horizons, respectively, the equity portfolio weights according to three strategies: "Index" consists on investing only in the stock ($\omega_t = 1$), "Vol Only" forecasts the equity premium conditioning on volatility only, and "Vol & Unc" forecasts the equity premium conditioning on both volatility and uncertainty. Panels C and D show for two- and four-weeks horizons, respectively, the cumulative wealth evolution associated with each portfolio strategy in Panels A and B. The initial wealth in panels C and D is fixed to 100. The investment and model estimation periods are both from January 1986 until December 2020. The shaded areas correspond to three sub-periods associated with high disconnect: Financial crisis (from July 2007 until March 2009), US 2016 election (from July 2016 until January 2018), and Covid-19 (from January 2020 until December 2020).

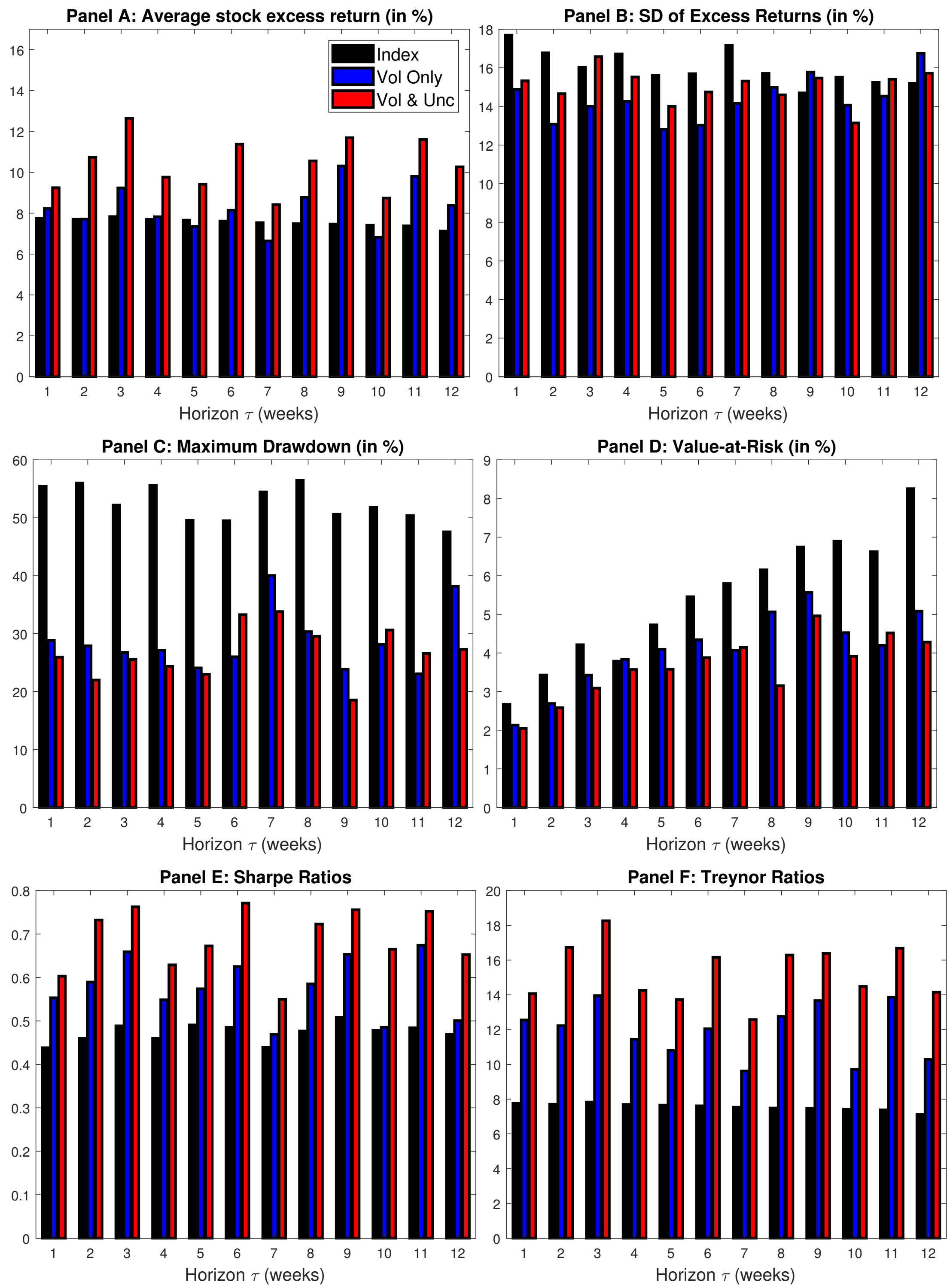

Figure 6: Portfolio performance measures: full sample

Notes: This figure reports measures of risk-adjusted wealth excess returns for: index investing, portfolio conditioning on volatility only, and portfolio conditioning on volatility and uncertainty. The statistics shown are: average and standard deviation of stock excess return (in %), maximum drawdown (in %), 5% Value-at-Risk (in %), Sharpe ratio, and Treynor ratio. The investment and model estimation periods are both from January 1986 until December 2020.

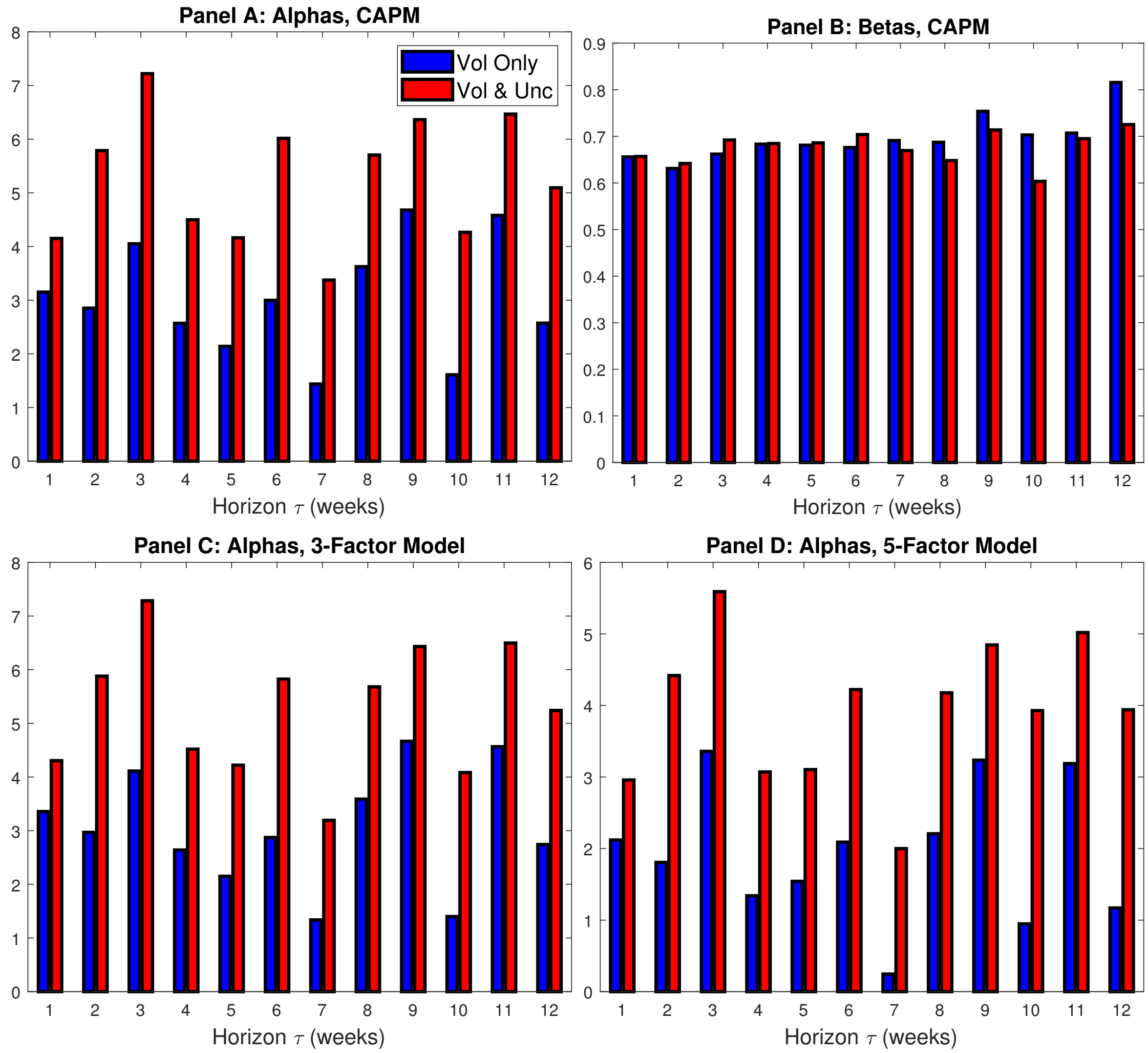

Figure 7: Alphas and betas: full sample

Notes: Panels A and B show the estimated alpha and beta from a CAPM model, respectively. These are obtained by regressing the excess return of wealth associated with each active portfolio strategy onto the excess return of the equity market. Panels C and D show estimated alphas from alternative models. The three-factor alpha is obtained from performing the regressions onto the excess return of the market, the SMB ("Small Minus Big") factor, and the HML ("High Minus Low") factor. The five factor alpha is obtained from regressing onto the excess return of the market, the SMB factor, the HML factor, the RMW ("Robust Minus Weak") factor, and the CMA ("Conservative Minus Aggressive") factor. The investment and model estimation periods are both from January 1986 until December 2020.

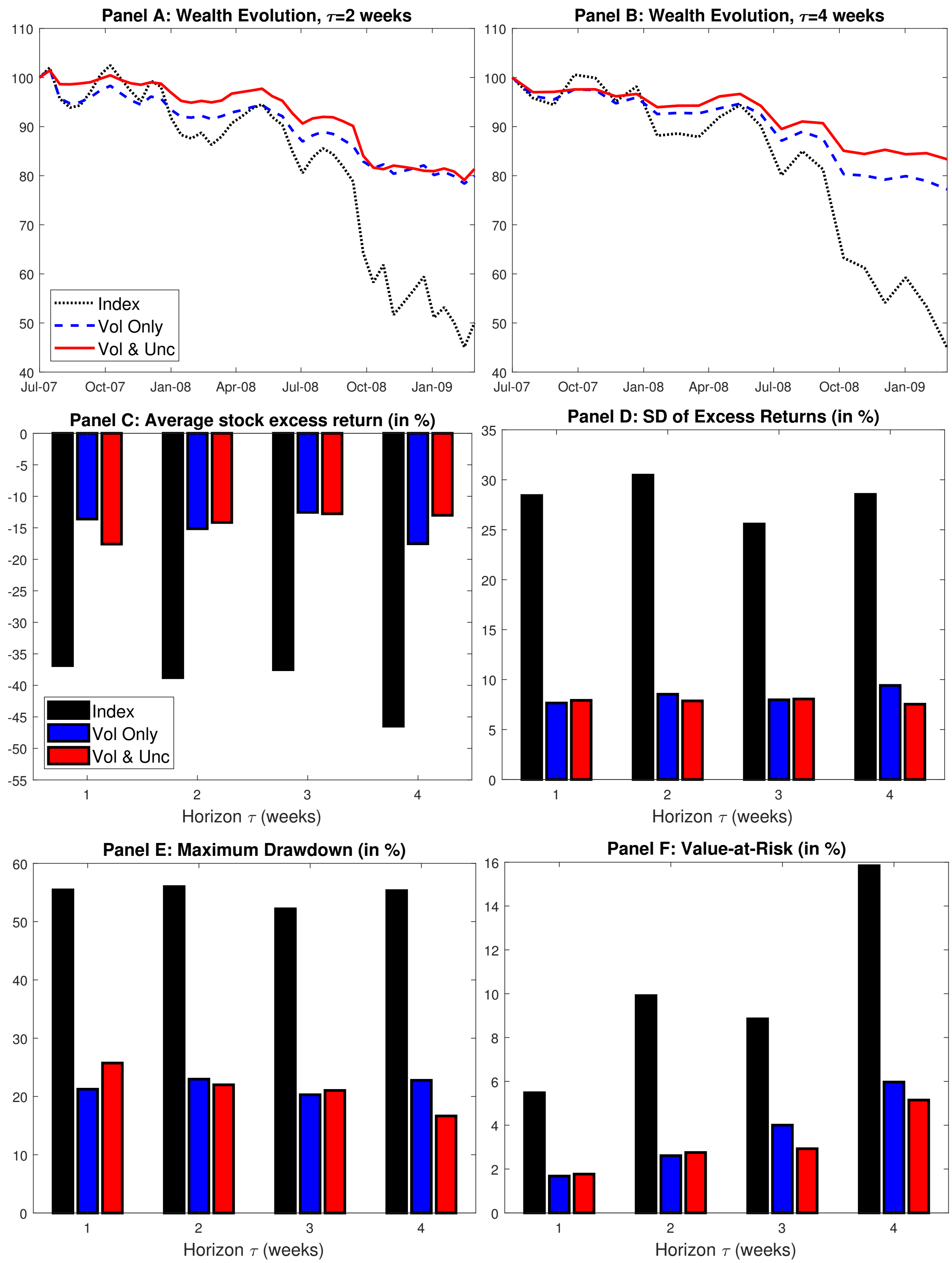

Figure 8: Wealth evolution and portfolio performance measures around the 2008 Financial Crisis

Notes: The elements in this figure match closely those from figures 5 and 6, except that in this figure the investment period is restricted to the financial crisis period (from July 2007 until March 2009). The estimation period is from January 1986 until December 2020. The initial wealth in panels A and B is fixed to 100.

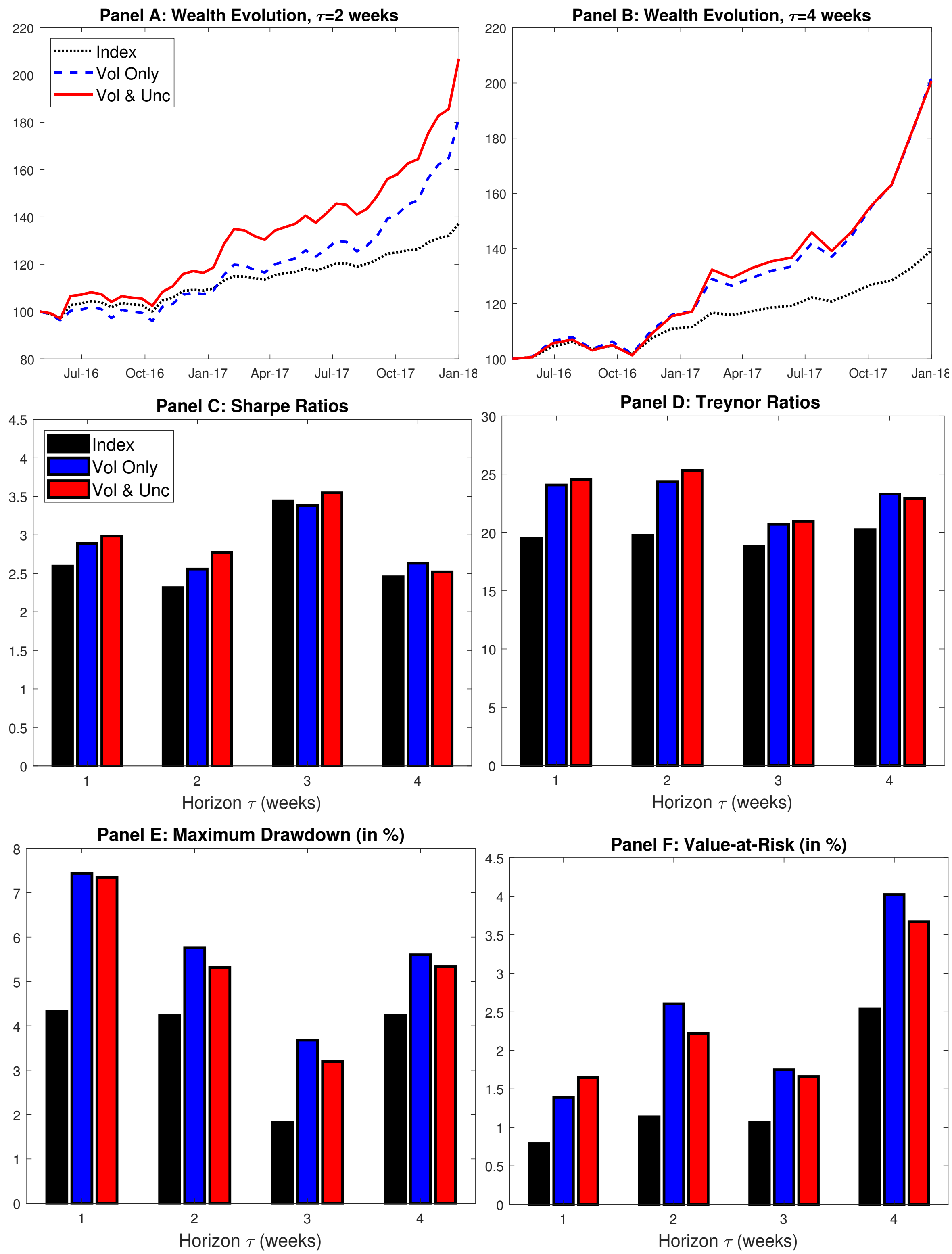

Figure 9: Wealth evolution and portfolio performance measures around the 2016 US Election

Notes: The elements in this figure match closely those from figures 5 and 6, except that in this figure the investment period is restricted to the 2016 US election period (from July 2016 until January 2018). The estimation period is from January 1986 until December 2020. The initial wealth in panels A and B is fixed to 100.

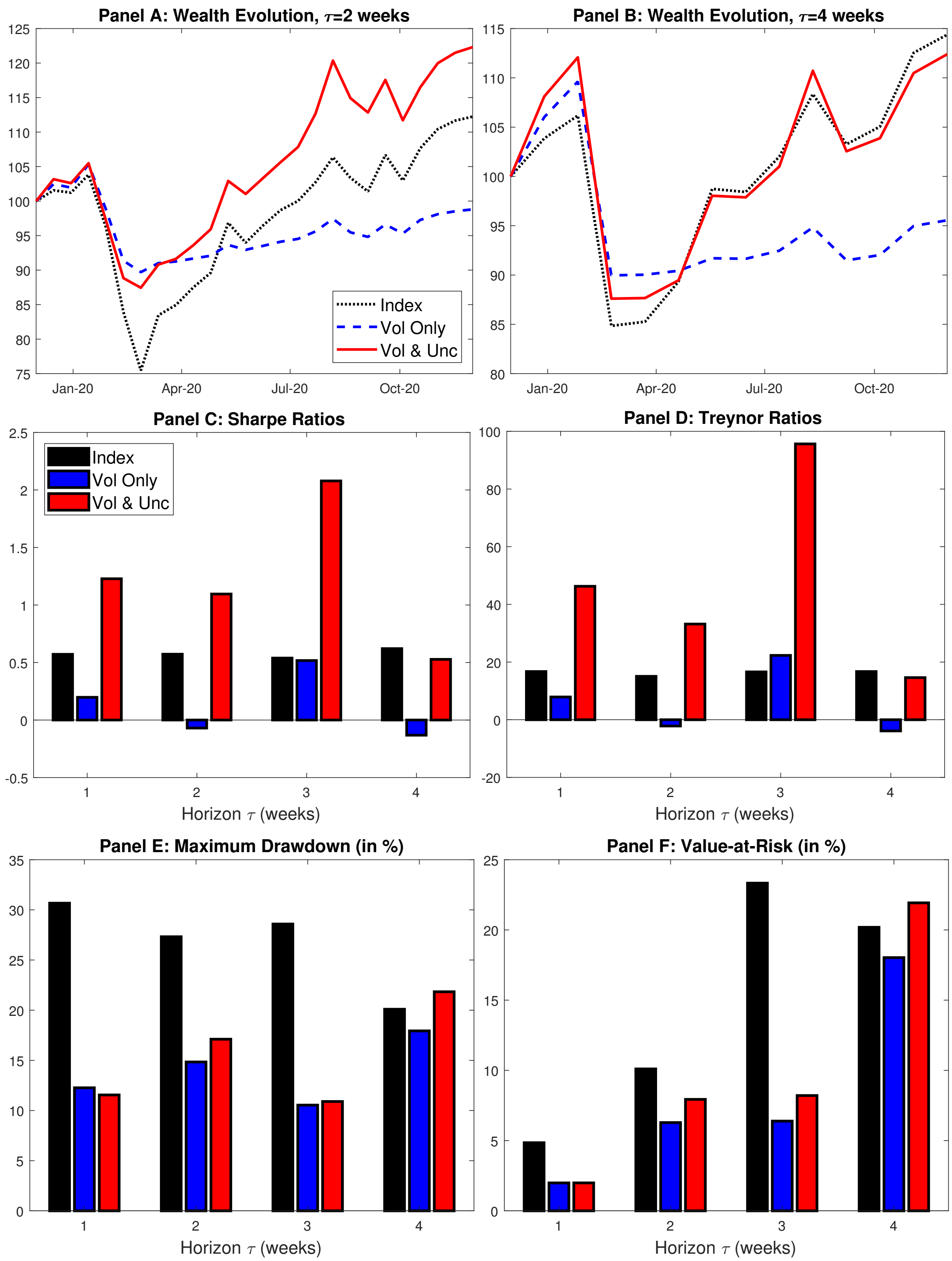

Figure 10: Wealth evolution and portfolio performance measures around the COVID-19 pandemic

Notes: The elements in this figure match closely those from figures 5 and 6, except that in this figure the investment period is restricted to the COVID-19 pandemic period (from January 2020 until December 2020). The estimation period is from January 1986 until December 2020. The initial wealth in panels A and B is fixed to 100.